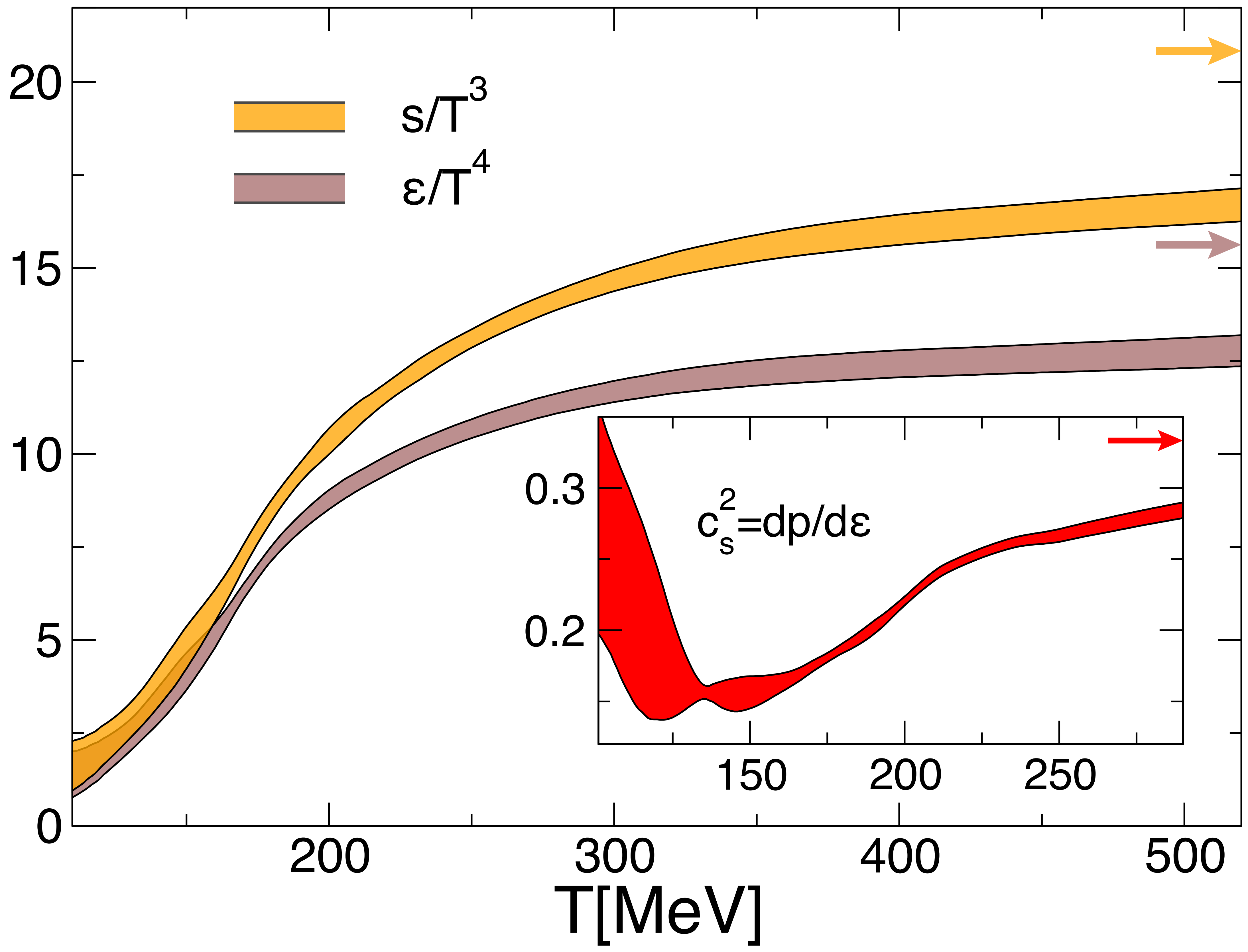
The Equation of state of dense baryonless matter at the LHC

EMMI Workshop: Collective phenomena and
the Equation-of-State of dense baryonic matter
GSI Darmstadt, Nov. 10, 2025

IPhT SACLAY

cnrs dépasser les frontières

Motivation

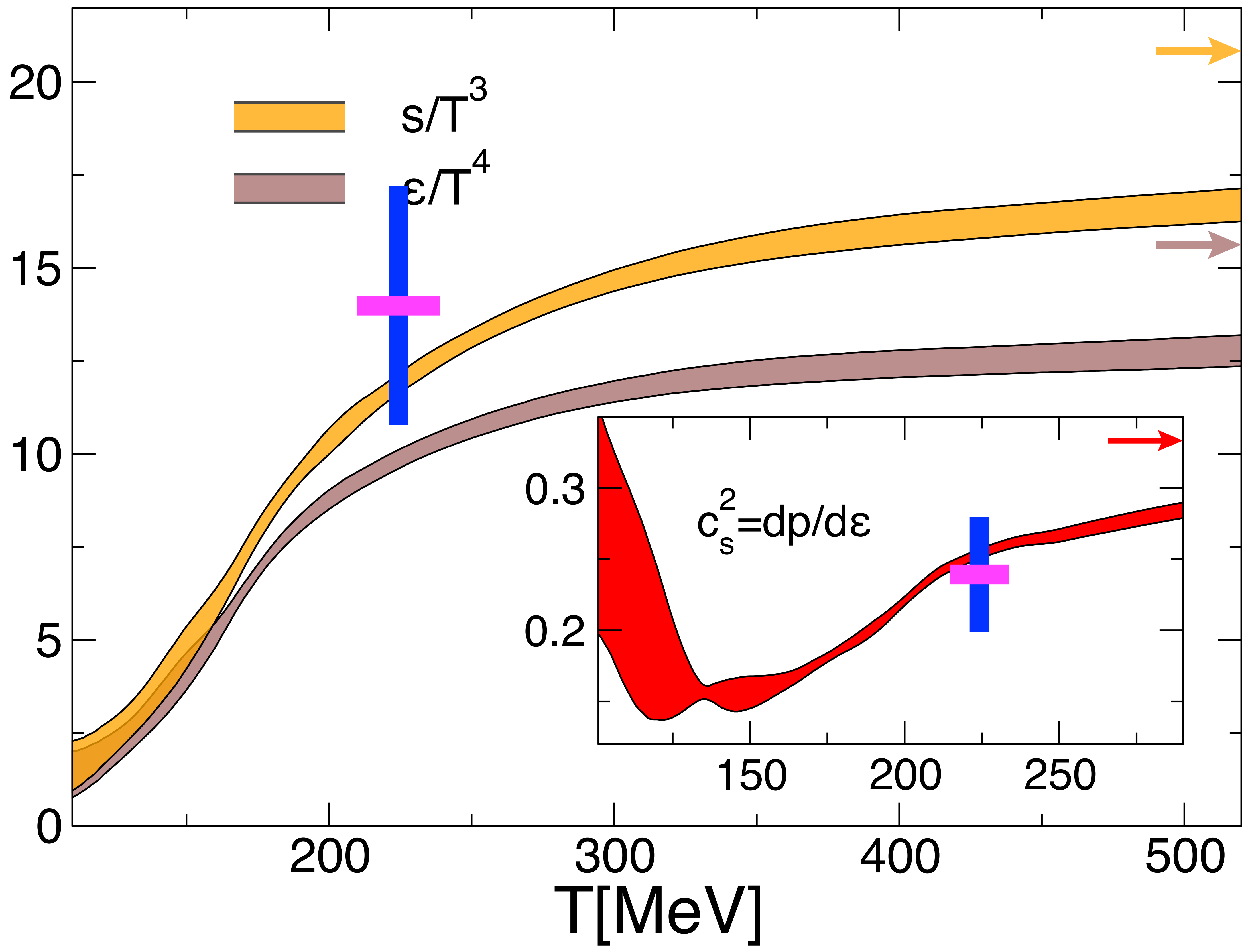


Borsanyi et al, 1309.5258

The equation of state of baryonless matter is accurately calculated in lattice QCD.

A few years back, we showed that one can measure one point on this diagram using LHC data.

Motivation

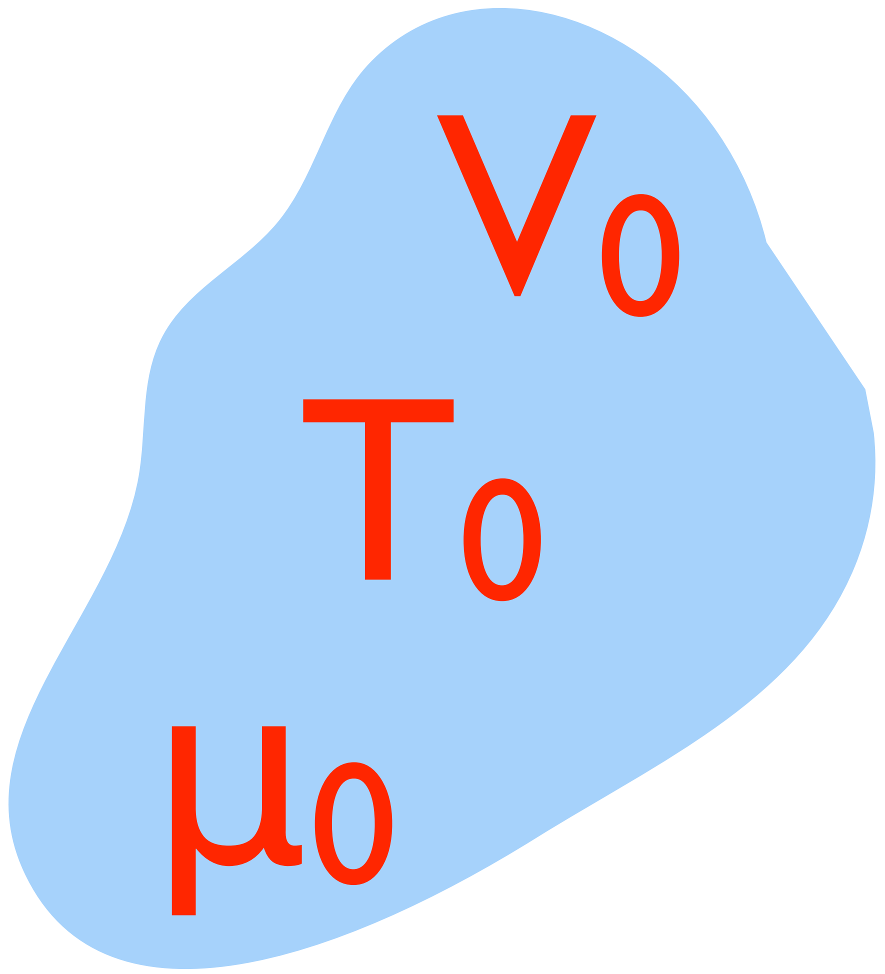


Gardim Giacalone Luzum JYO 1908.09728

My goal in this talk is to explain how we did and where the error bars (vertical and horizontal) come from.

And more generally, discuss which observables are most sensitive to EOS, and some of the difficulties in relating EOS with data.

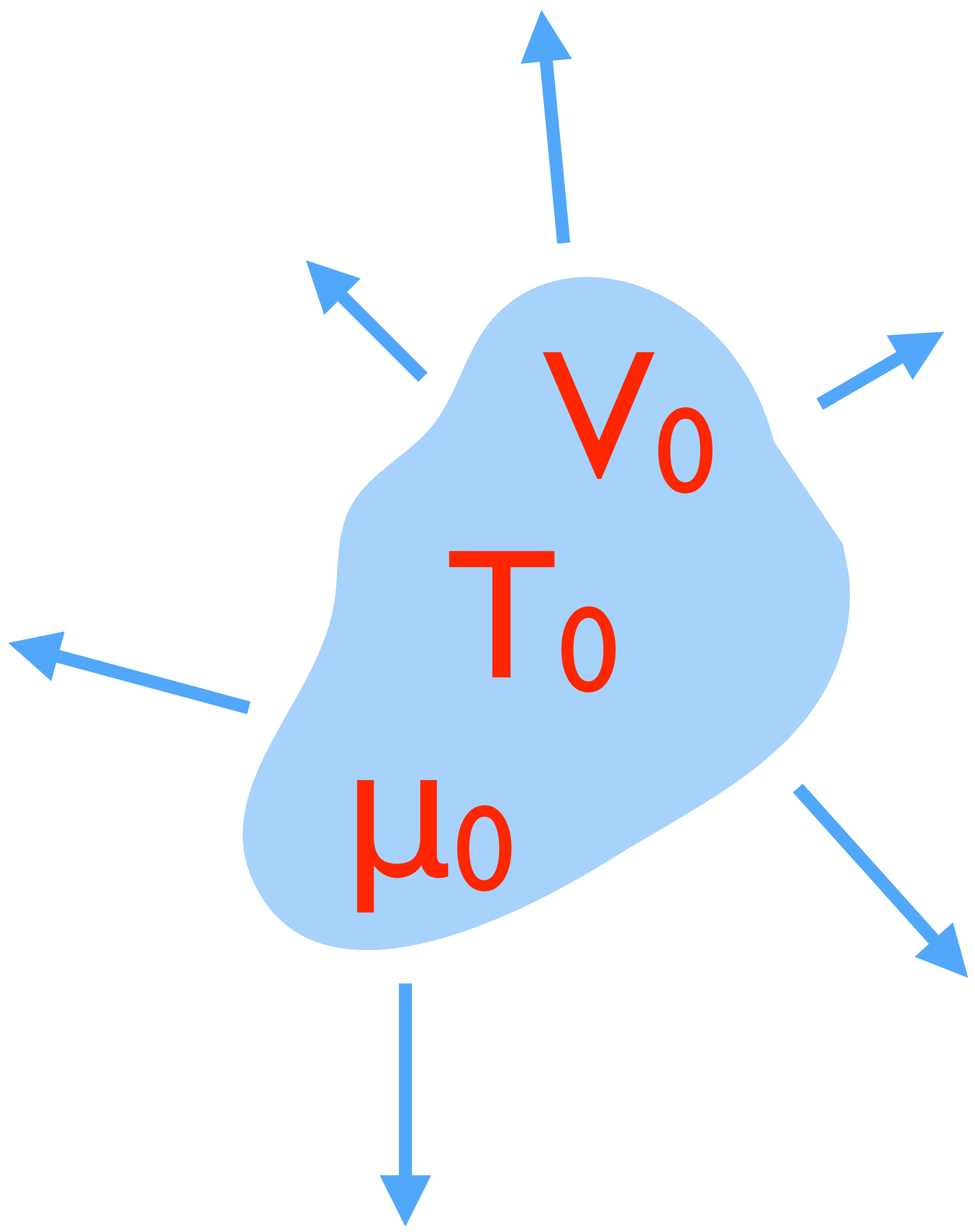
A simple thought experiment



Consider a gas at rest in thermal *equilibrium* in a closed container, with an arbitrary shape, isolated in the vacuum.

Equilibrium implies : Temperature T_0 and chemical potential μ_0 are uniform within the volume V_0.

A simple thought experiment

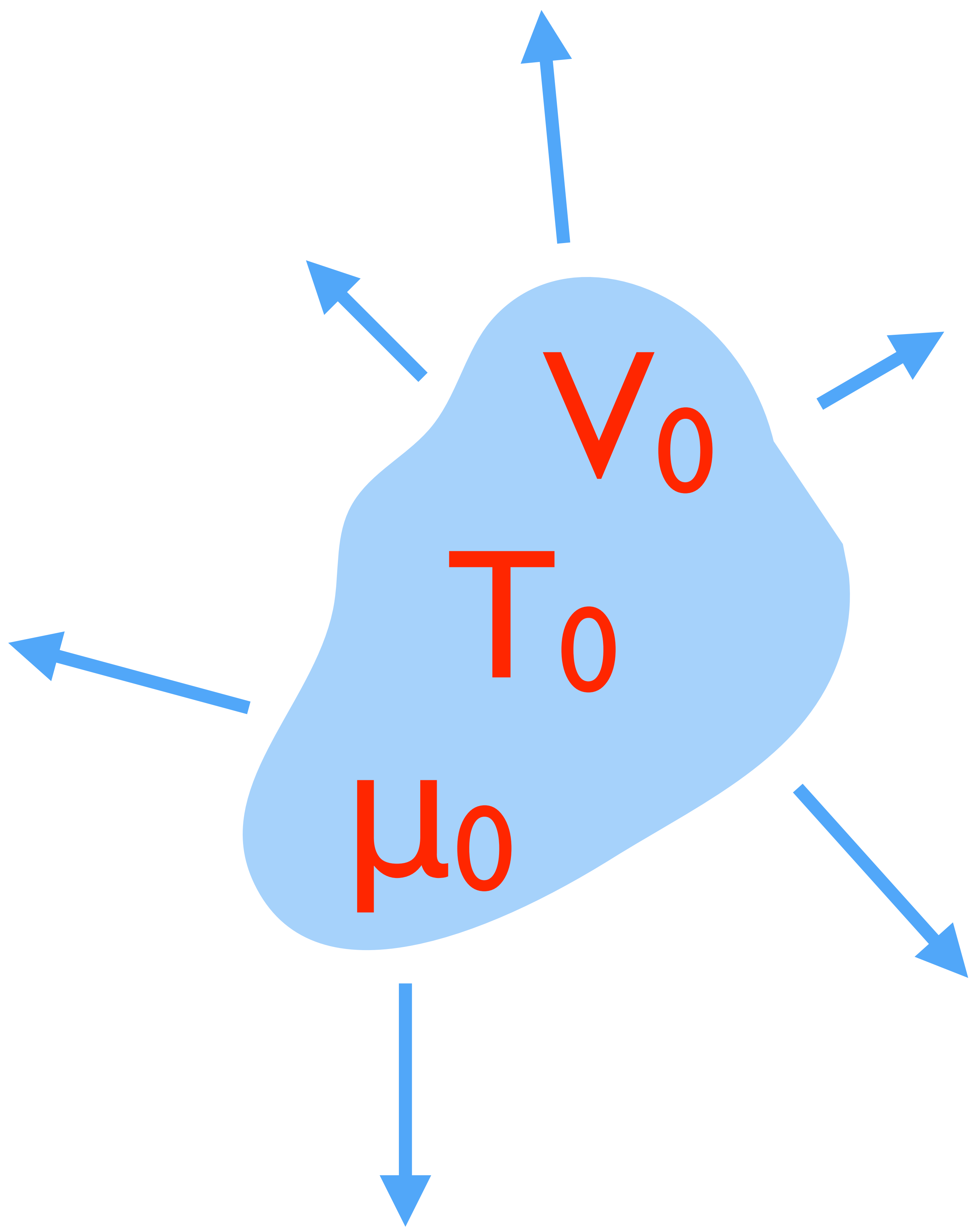


At time t=0 the container suddenly disappears.

The gas expands in all directions and cools down in a non-uniform way. The flow pattern is in general complex, anisotropic, and depends on the shape of the container.

Can we reconstruct some thermodynamic properties at t=0 by detecting outgoing particles?

Conservation laws

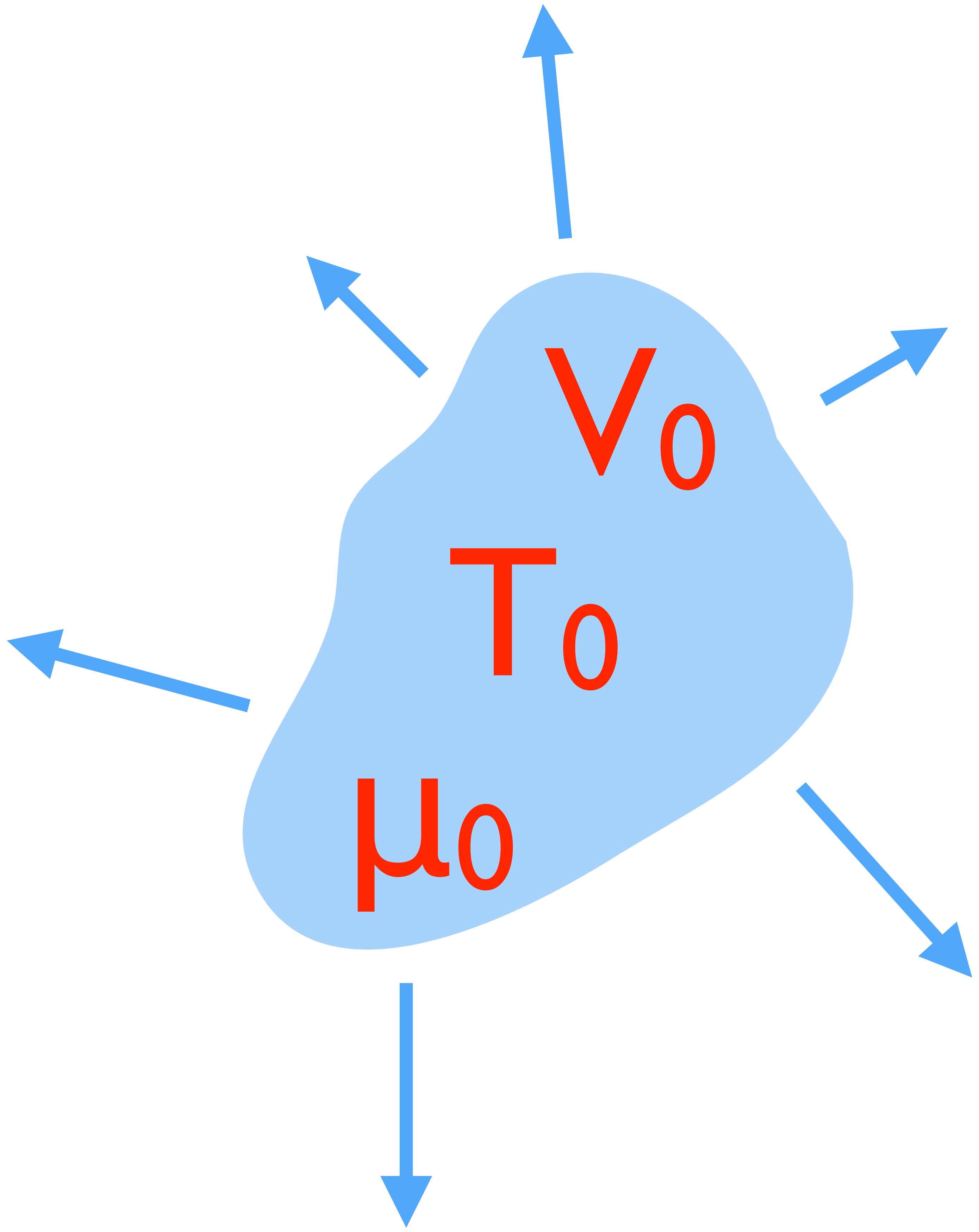


If the volume is large enough, the expansion is driven by ideal fluid dynamics. It conserves

- S=Entropy
- E=Energy
- N=Baryon number

The time history and detailed pattern of the hydrodynamic evolution are irrelevant in order to reconstruct in the initial temperature. Only conserved quantities matter.

The equation of state from conservation laws



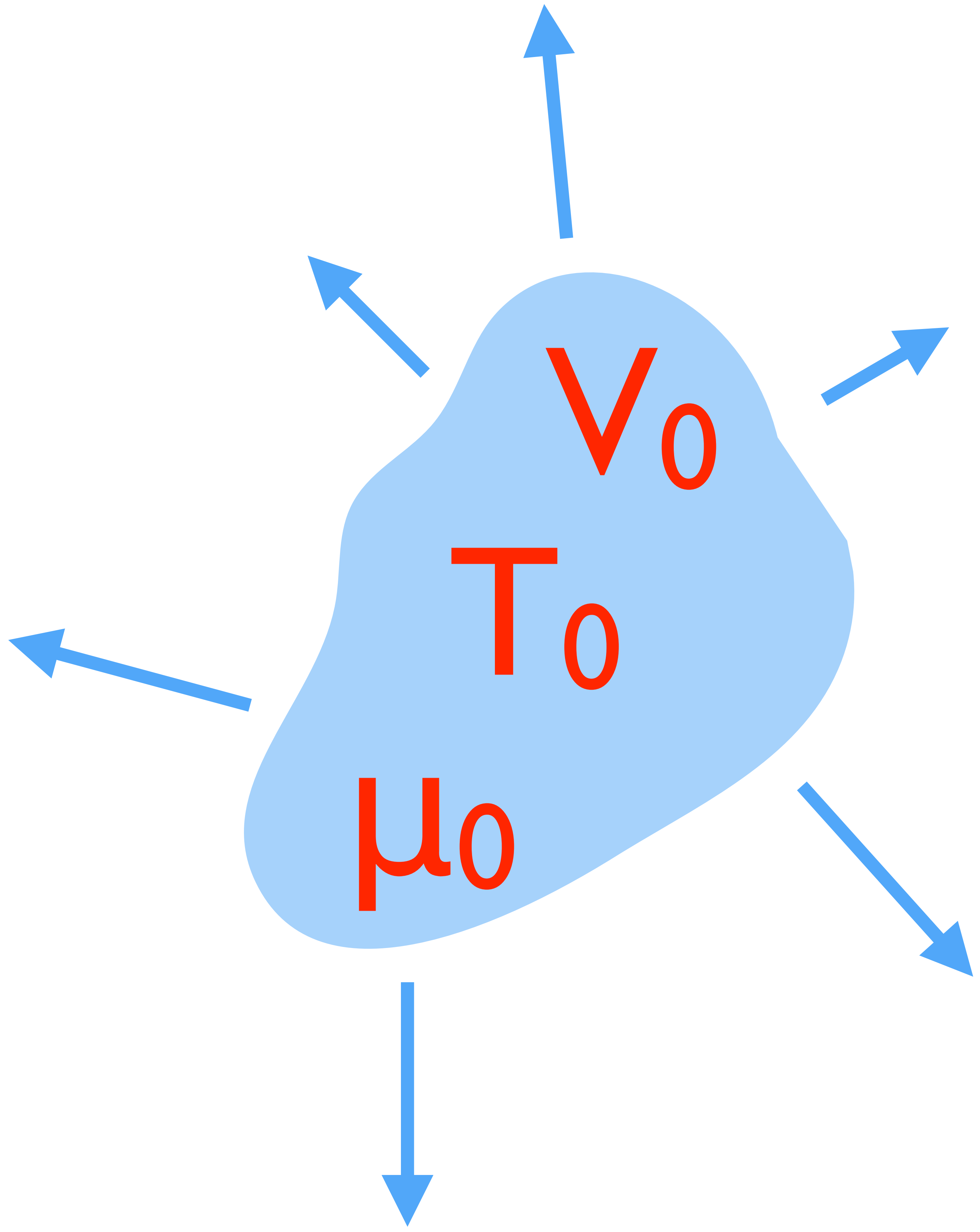
If we can reconstruct S, E and N using the detectors

and if we know the volume V_0 of the container then we have access to 3 intensive thermodynamic properties

- $s(T_0,\mu_0)=S/V_0$ entropy density
- $\varepsilon(T_0,\mu_0)=E/V_0$ energy density
- $n_B(T_0,\mu_0)=N/V_0$ baryon density

Since there are 2 parameters T and μ, we obtain 3-2=1 non-trivial constraint on the EOS.

Which observables & theory input do we need?



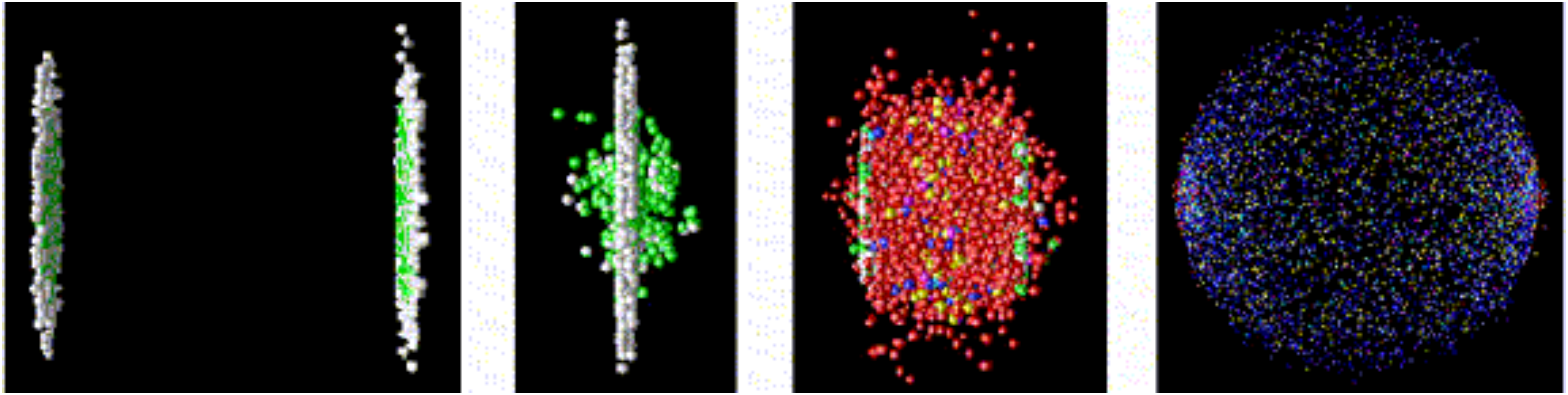
S and N inferred from charged multiplicity and net proton number.

Hanus Mazeliauskas Reygers 1908.02792

For the energy E, we need to know momenta and masses of outgoing particles.

The volume V_0 is *estimated using some theory* input rather than measured.

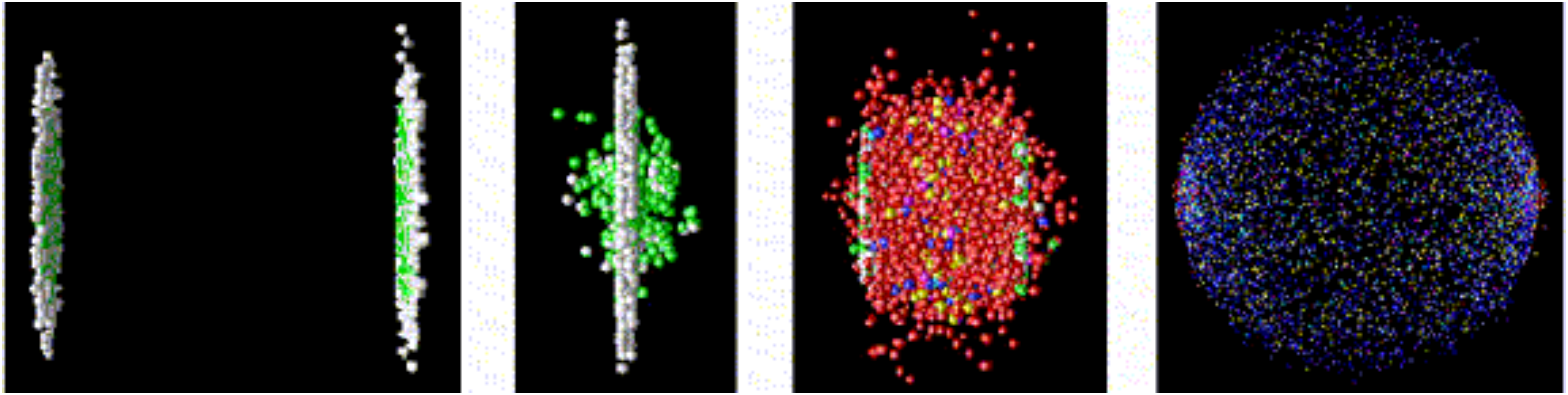
How this can be adapted to Pb+Pb @ LHC



The fluid is produced at rest in the transverse plane, but with fast longitudinal expansion.

Not uniform in the transverse plane → some averaging involved. Not a problem if the equation of state is smooth.

How this can be adapted to Pb+Pb @ LHC



We only observe a *slice of fluid* near mid-rapidity.
Energy not conserved but decreases: Work of the longitudinal pressure dE=-PdV as the system expands.
[Entropy not exactly conserved because of viscous dissipation.]

Taking into account longitudinal cooling

Our breakthrough idea in 2019 was to realize that because of longitudinal cooling, the relevant energy is no longer the initial energy of the fluid, but its final energy.

We calculate the final energy and entropy in hydrodynamics

$$E_f = \int_{freeze-out} T^{0\mu} d\sigma_\mu$$

$$S_f = \int_{freeze-out} s u^\mu d\sigma_\mu$$

Note that E_f includes the kinetic energy of the fluid.

Taking into account longitudinal cooling

If energy and entropy were conserved, one would have

$$E_f = \epsilon(T_0)V_0$$

$$S_f = s(T_0)V_0 \, .$$

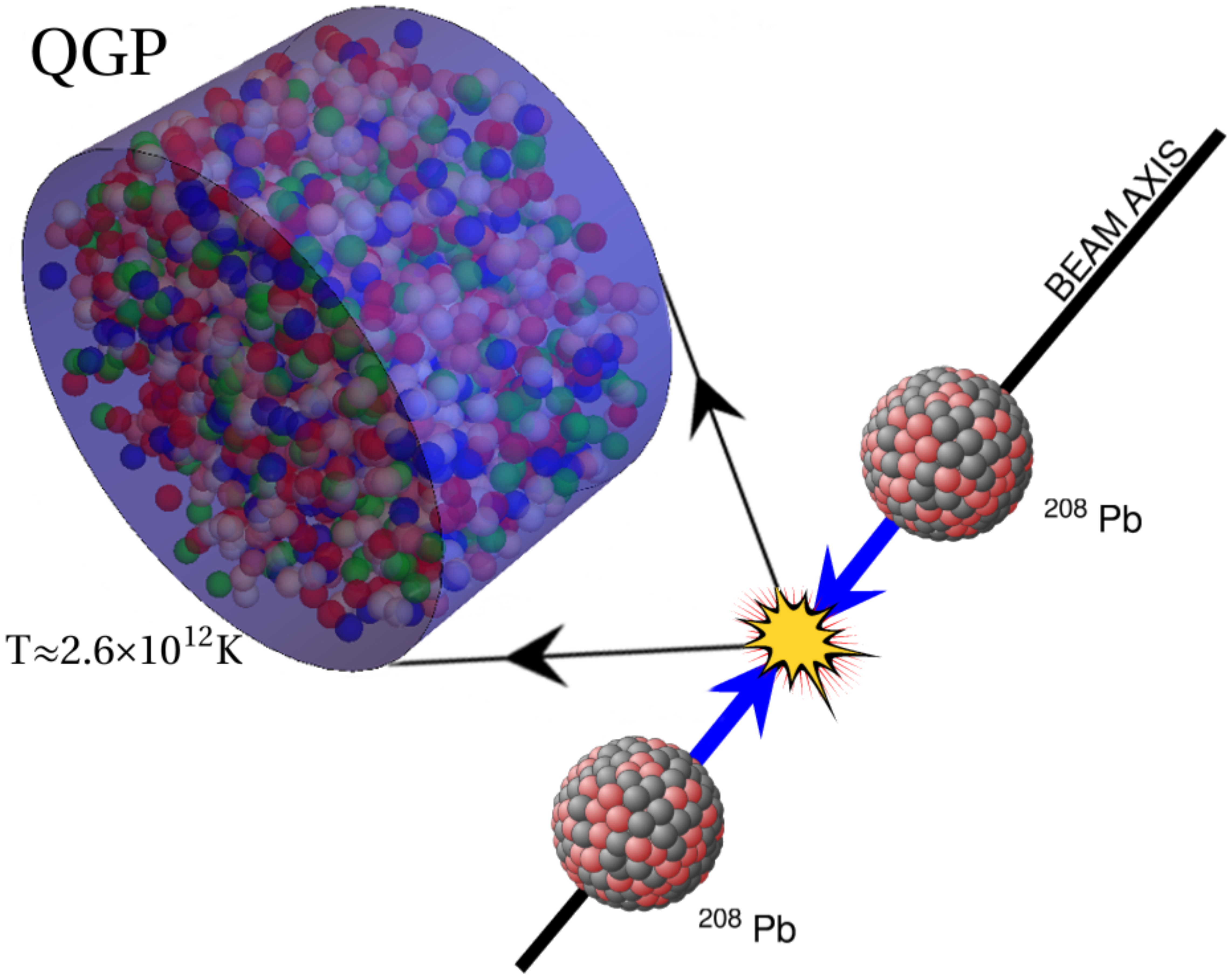
But they are not conserved. We define T_{eff} and V_{eff} by

$$E_f = \epsilon(T_{eff})V_{eff}$$

$$S_f = s(T_{eff})V_{eff} \, .$$

Knowing E_f and S_f, we can solve these eqs. for T_{eff} and V_{eff} using the EOS of the hydrodynamic calculation.

Taking into account longitudinal cooling



The final energy and entropy give us access to some intermediate, *effective* state, which I understand physically as *after longitudinal cooling and before transverse expansion.*

Note that $T_{eff} > T_{freeze-out}$ because E_f includes the kinetic energy of the fluid.

How does T_{eff} relate to observables ?

Since entropy is proportional to number of particles, and energy of a particle at midrapidity is $m_t = \sqrt{p_T^2 + m^2}$, I was expecting

$$\langle m_t \rangle \propto \epsilon(T_{eff})/s(T_{eff})$$

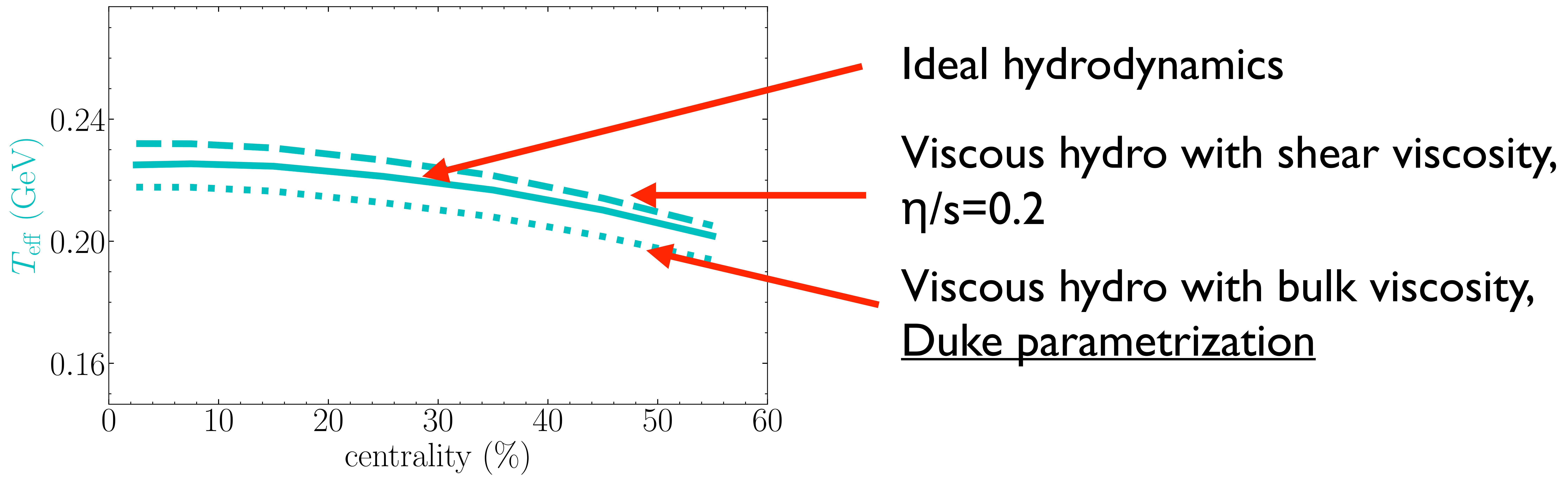
Akihiko Monnai & JYO 1707.08466

Analyzing the results of hydrodynamic calculations, Fernando Gardim found a simpler correspondence, namely,

$$\langle p_t \rangle \approx 3T_{eff}$$

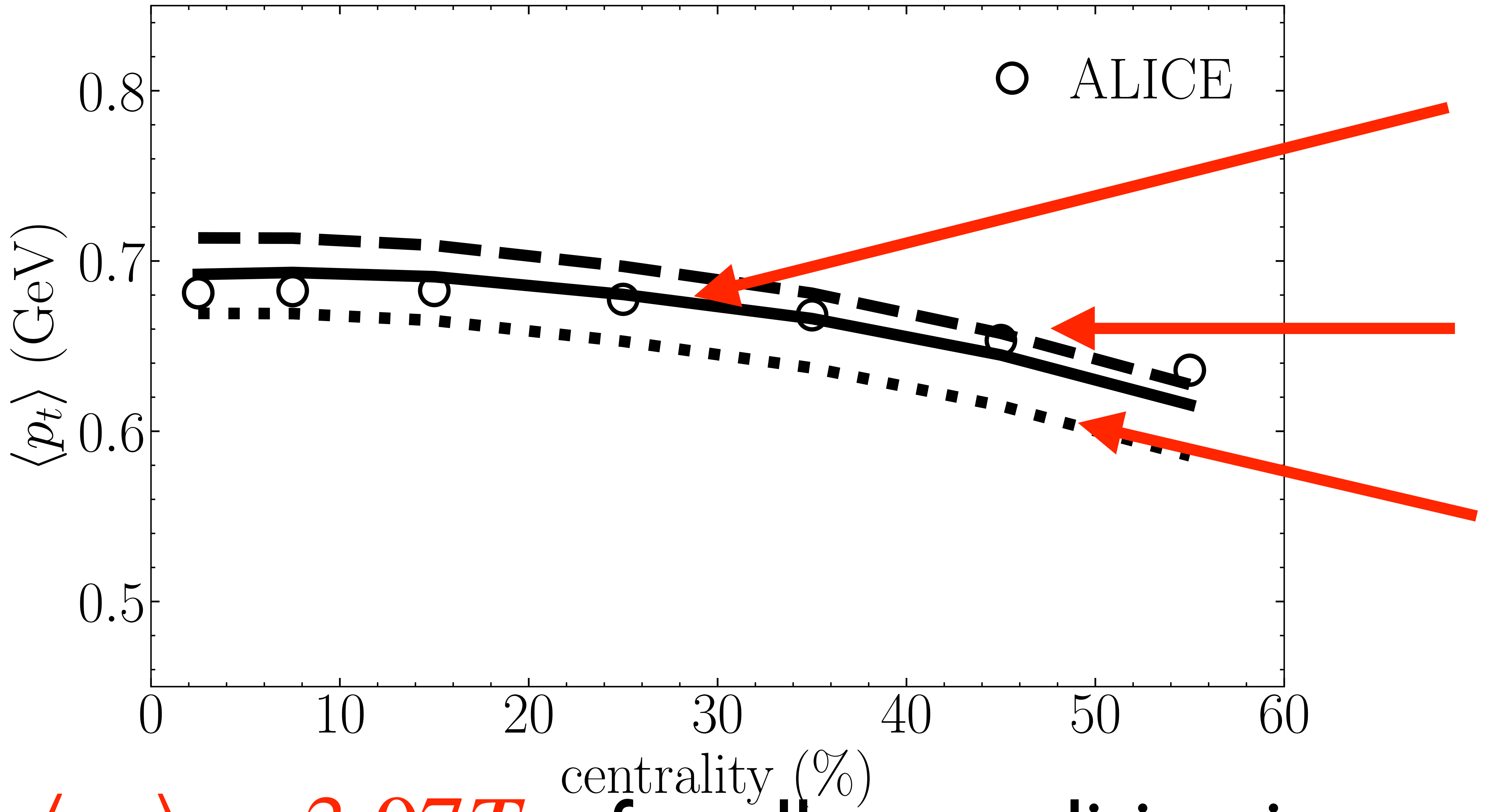
Value of T_{eff} in hydro simulations of Pb+Pb @ 5.02 TeV

We use the MUSIC code, where the initial temperature is tuned to reproduce the charged multiplicity measured by ALICE for each centrality.



Value of $\langle p_T \rangle$ in hydro simulations of Pb+Pb @ 5.02 TeV

(after resonance decays)



Ideal hydrodynamics

Viscous hydro with shear viscosity, η/s=0.2

Viscous hydro with bulk viscosity, Duke parametrization

$\langle p_T \rangle = 3.07 T_{eff}$ for all centralities, irrespective of bulk and shear viscosity!

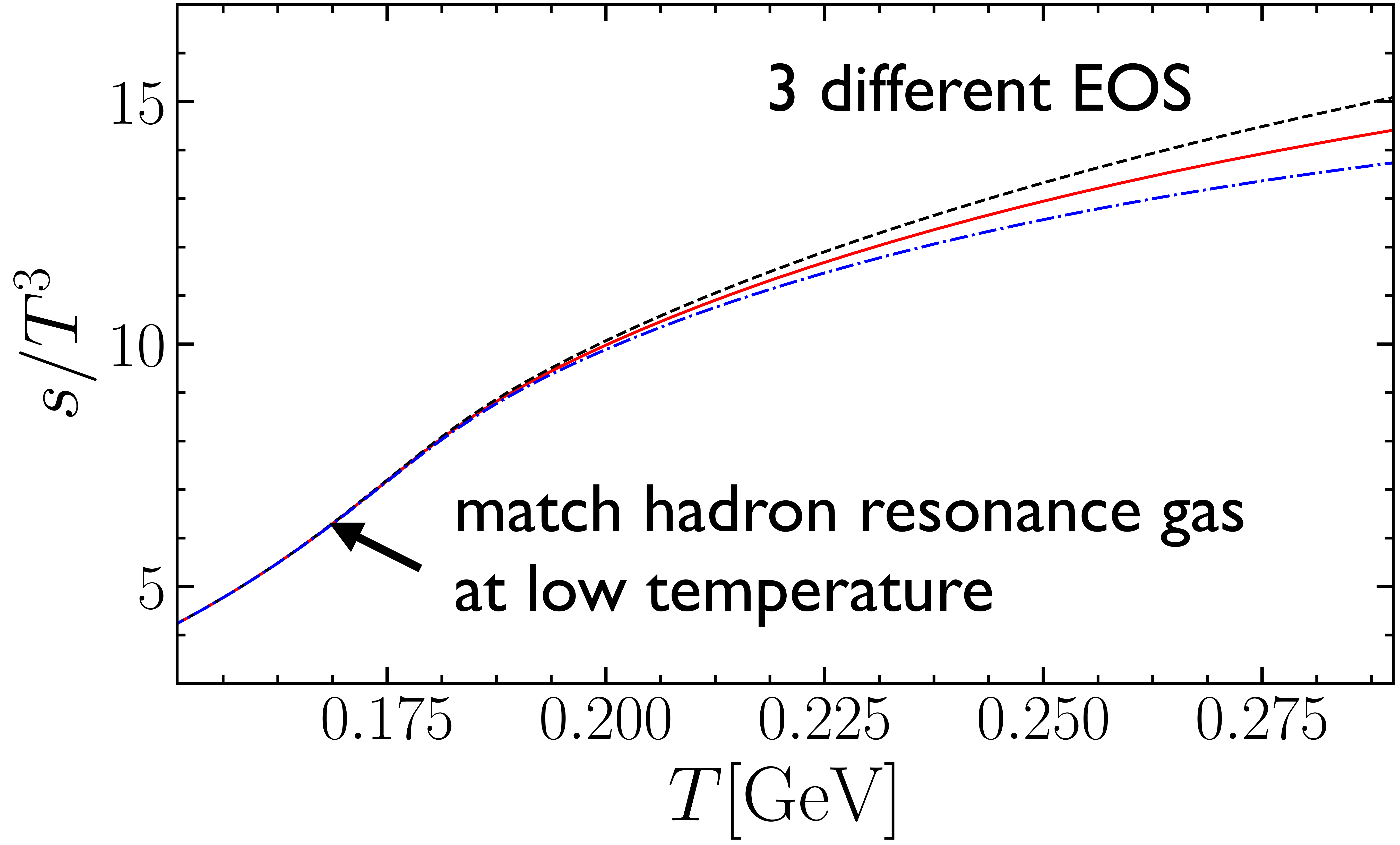
Value of T_{eff} from experiment

Extraction from data is straightforward.
ALICE measures $\langle p_t \rangle$ = 681 MeV in Pb+Pb @ 5.02 TeV in 0-5% centrality bin.

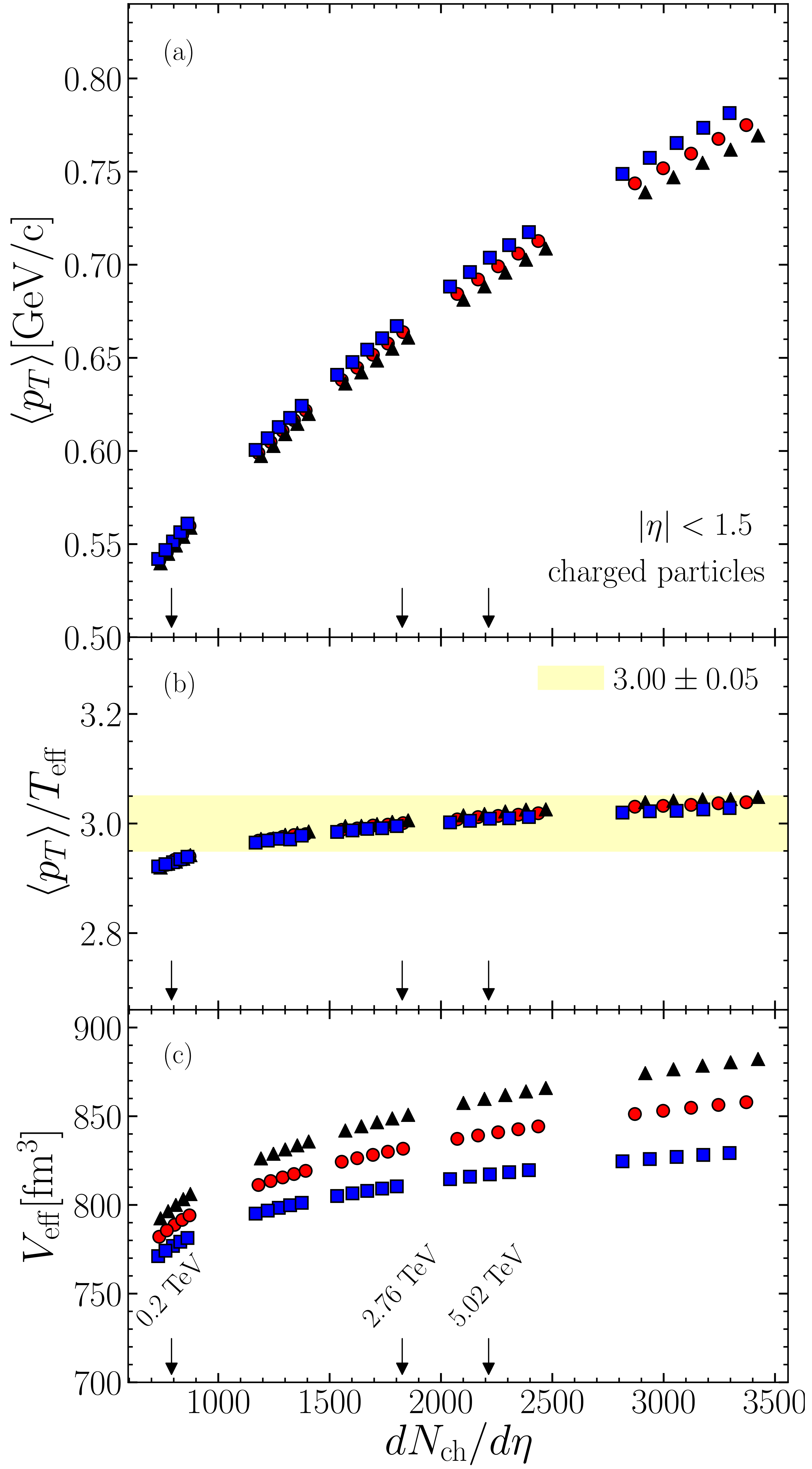
This implies $T_{eff} = \langle p_t \rangle / 3.07$ = 222 ± 9 MeV,

error on T_{eff} is from the final stages of the collision. Here we estimate it by varying the freeze-out temperature.

Recent update with more collision energies, more EOS, and UrQMD afterburner



Gardim Giannini JYO 2403.06052



Next step: entropy density at T_{eff}

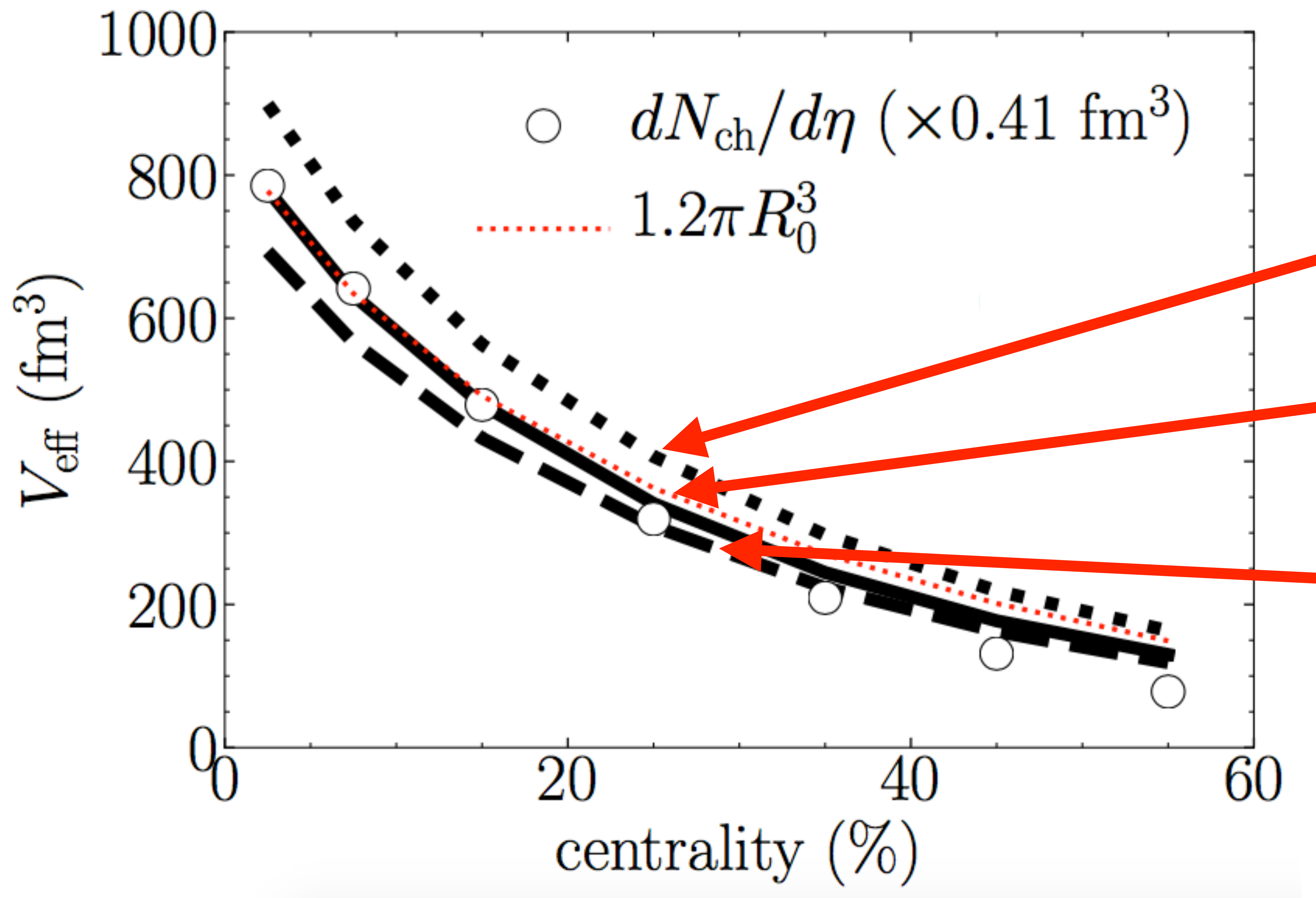
Entropy density = S/V_{eff}

S =entropy *at freeze-out*, by definition of V_{eff} and T_{eff}
S/N_{ch} = 6.7 ± 0.8 after resonance decays,
N_{ch} is measured, therefore, S is known

Hanus Mazeliauskas Reygers, 1908.02792

Effective volume V_{eff} cannot be extracted from data.
Comes from a hydrodynamic calculation.

Value of effective volume V_{eff} in hydro



Viscous hydro with bulk viscosity, Duke parametrization

Ideal hydrodynamics

Viscous hydro with shear viscosity, η/s=0.2

R_0 = initial transverse size, depends on initial-state model

V_{eff} = initial transverse area $\pi R_0^2 \times$ duration of longitudinal cooling ~$1.2R_0$, depends on transport coefficients (bulk >, shear <)

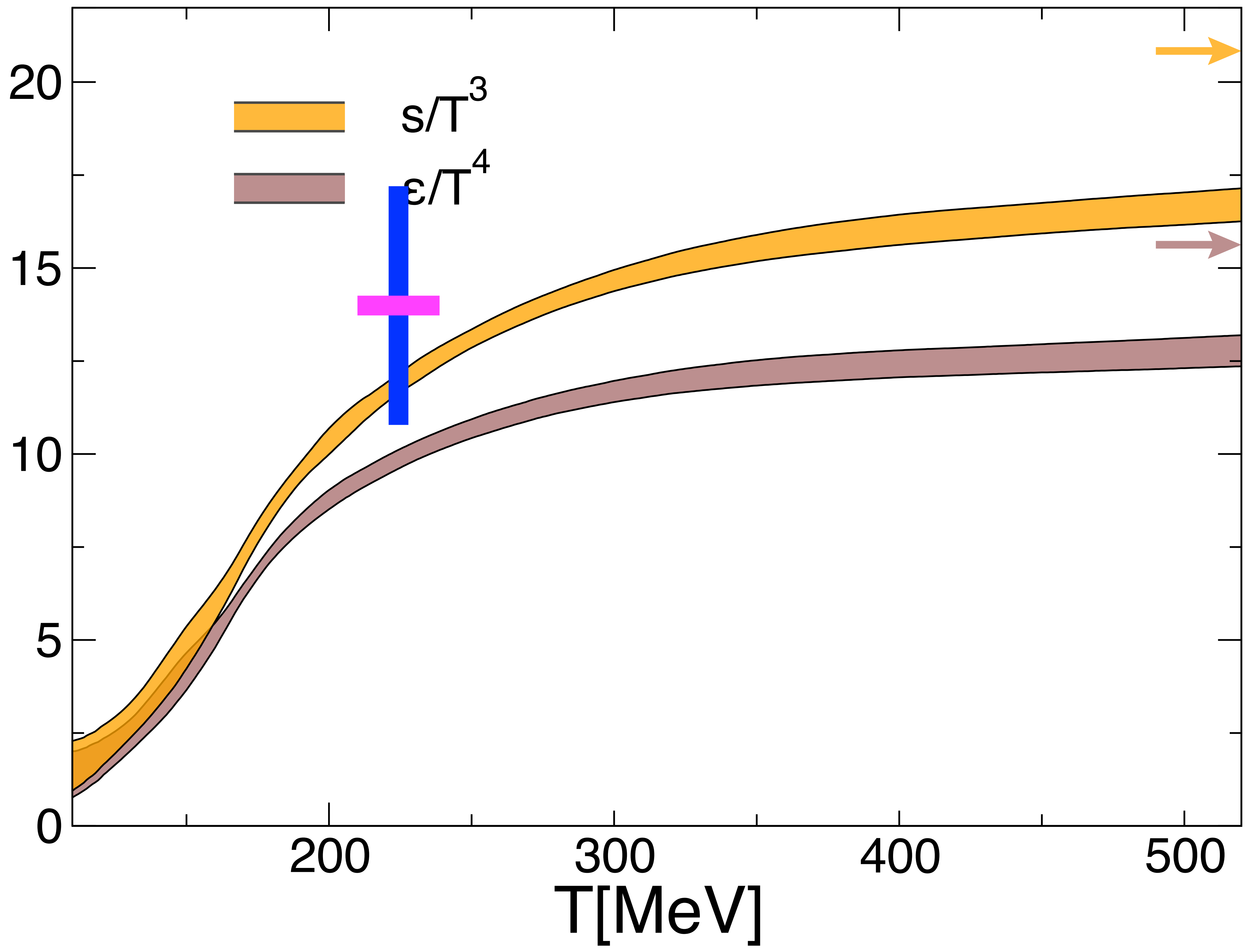
Entropy density at T_{eff}

We obtain $S/V_{eff} = s(T_{eff}) = 20 \pm 5\ \mathrm{fm}^{-3}$.

error :

40% from initial size R_0,
60% from transport coefficients

Comparison with lattice QCD



T_{eff}=222 ± 9 MeV

$s(T_{eff})/T_{eff}^3$ = 14 ± 3.5

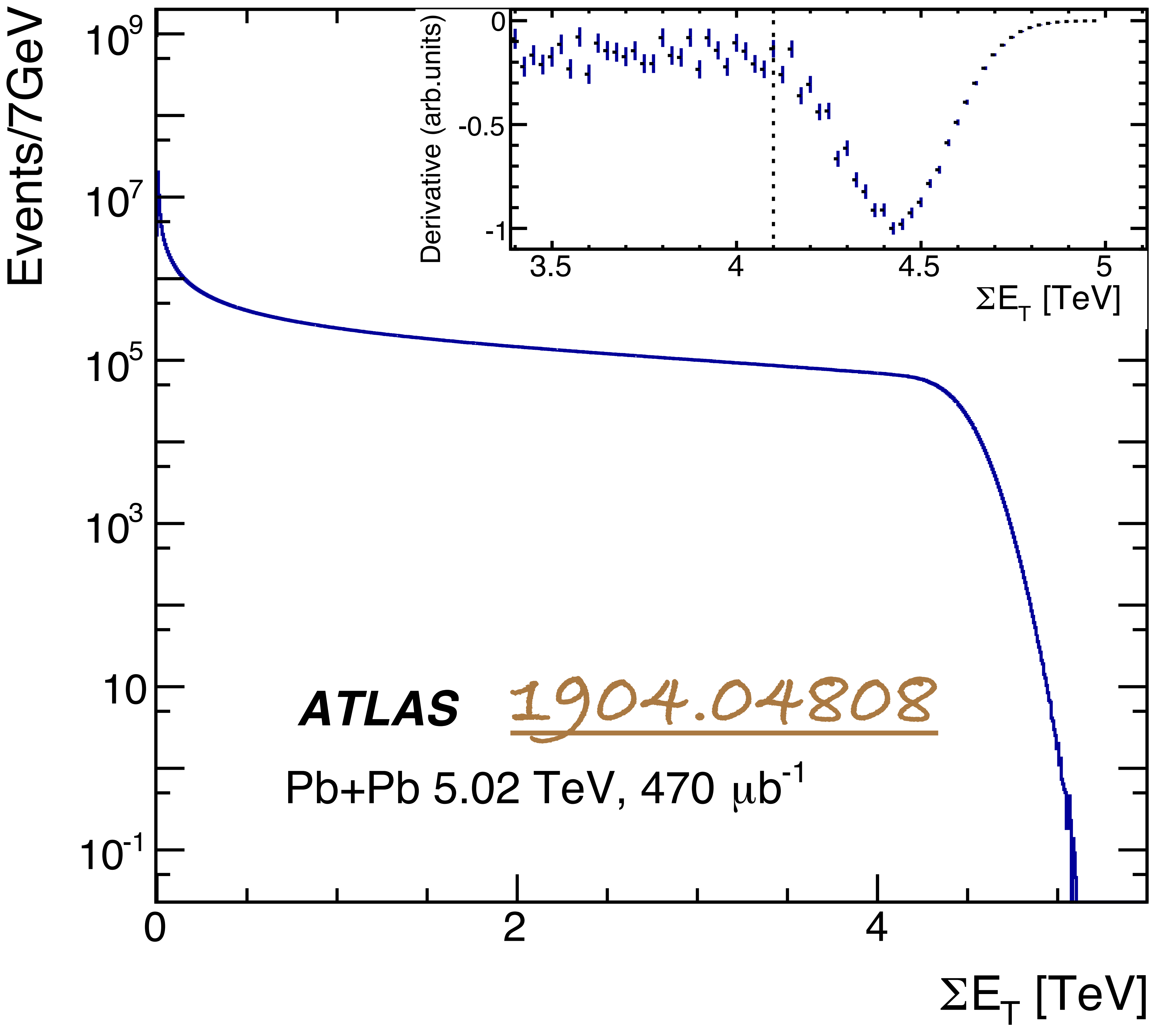
compatible with lattice.

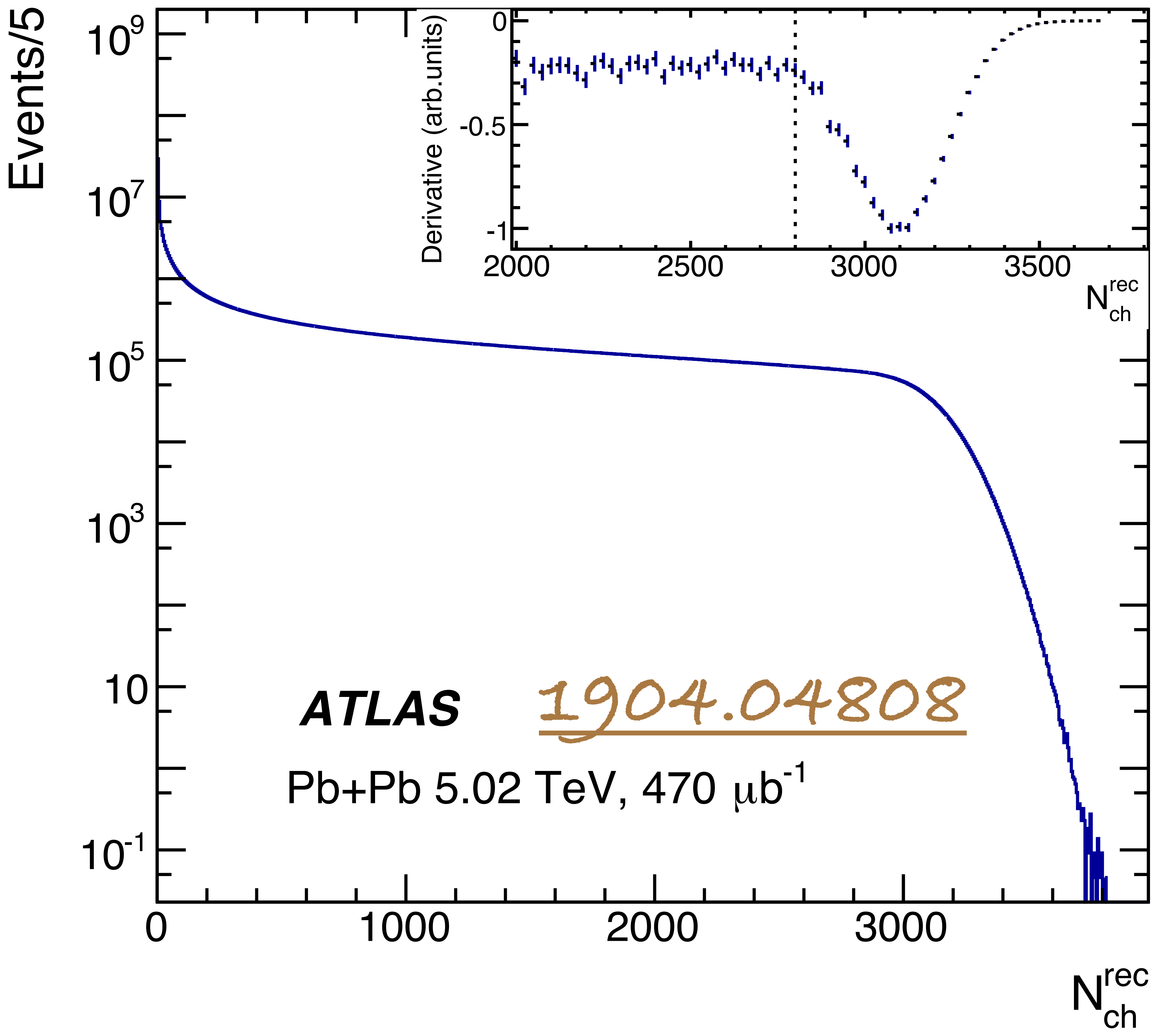
Confirms large number of degrees of freedom, implying that color is liberated: ***deconfinement observed!***

Recent developments: Volume and centrality

- It is important to know the size (& volume) of the fireball.
- Determined in particular by the impact parameter b, i.e., the *true centrality* $c = \pi b^2 / \sigma_{PbPb}$ of the collision, which the only classical parameter (negligible quantum uncertainty).
- Two collisions with the same c differ only by quantum fluctuations (e.g., from positions of nucleons within nuclei)
- Quantum fluctuations are Gaussian, i.e., simple: fluctuations of multiplicity, $\langle p_T \rangle$, anisotropic flow at fixed c are ~ Gaussian.
- First question is: What is the probability distribution of c in a given sample of events ?

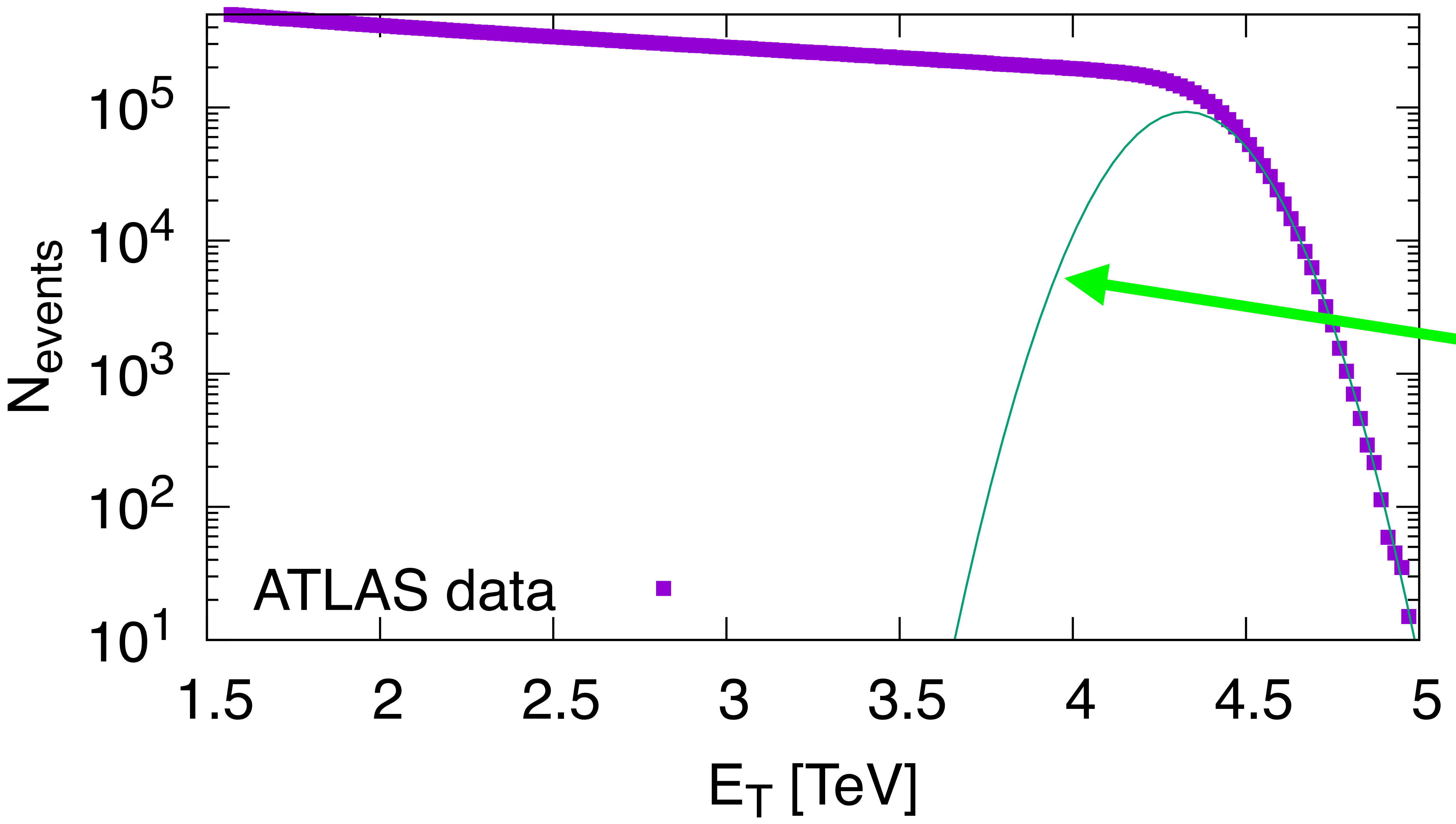
Input: distribution of the centrality classifier





We need experiments to provide the histogram of the centrality classifier.
Not all collaborations agree to share these data !

Basic assumption: Gaussian fluctuations



Distribution of E_T for $c = 0$ (head-on collisions)

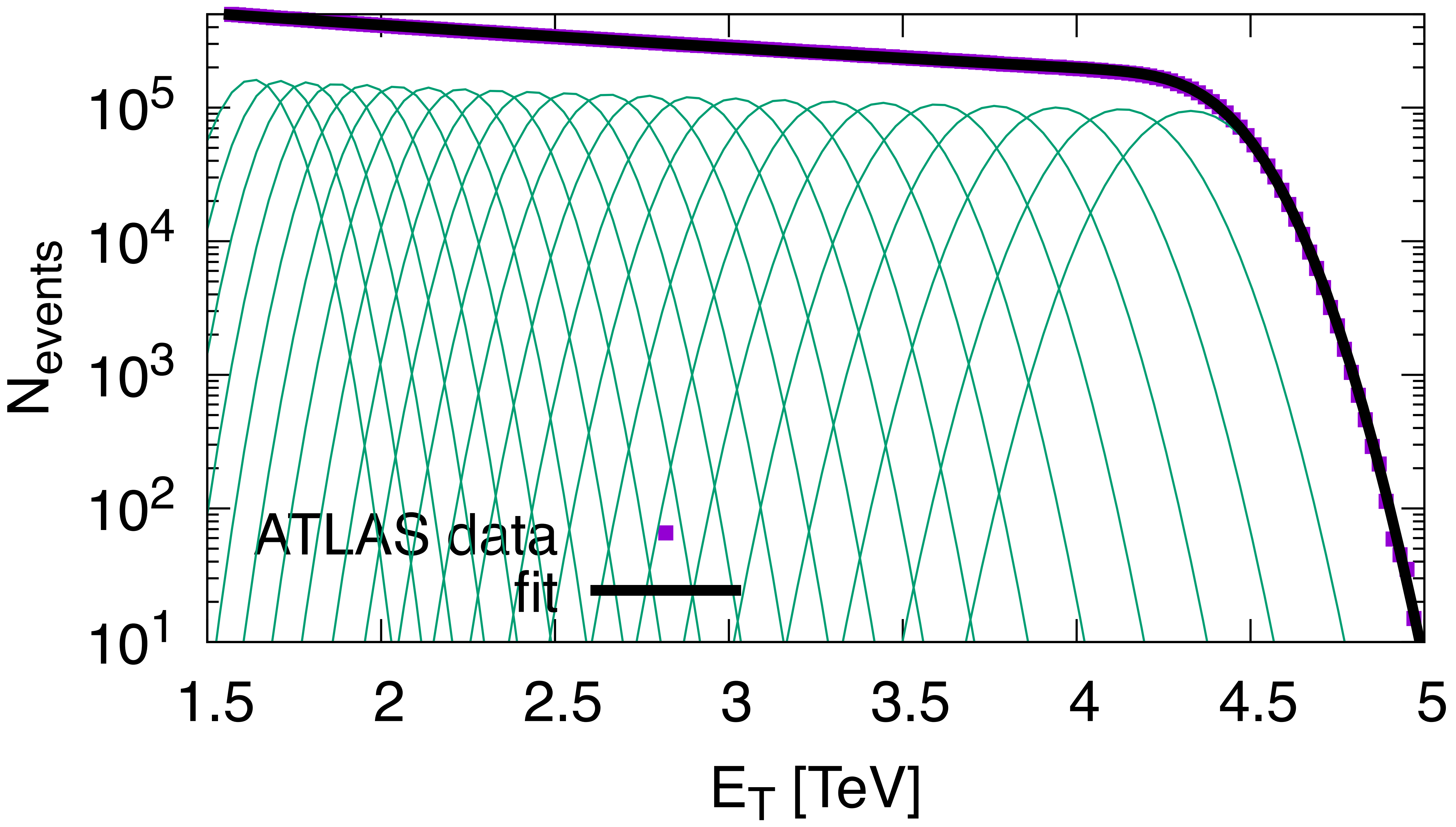
Fluctuations of E_T at fixed c are Gaussian:

$P(E_T|c)$ is a Gaussian distribution, mean $\overline{E_T}$ and width σ_{E_T} are smooth functions of c.

The width $\sigma_{E_T}(c = 0)$ can be read off from the tail of the distribution.

 Das Giacalone Monard JYO 1708.00081

Fitting the distribution of the centrality classifier



different Gaussians ≡ successive increments of c by 1%

We fit the distribution of E_T as an integral of Gaussians over the centrality c.
Note that experiment cannot tell us how the width varies with centrality.
We initially assumed that the ratio variance/mean was constant (cf. Poisson).

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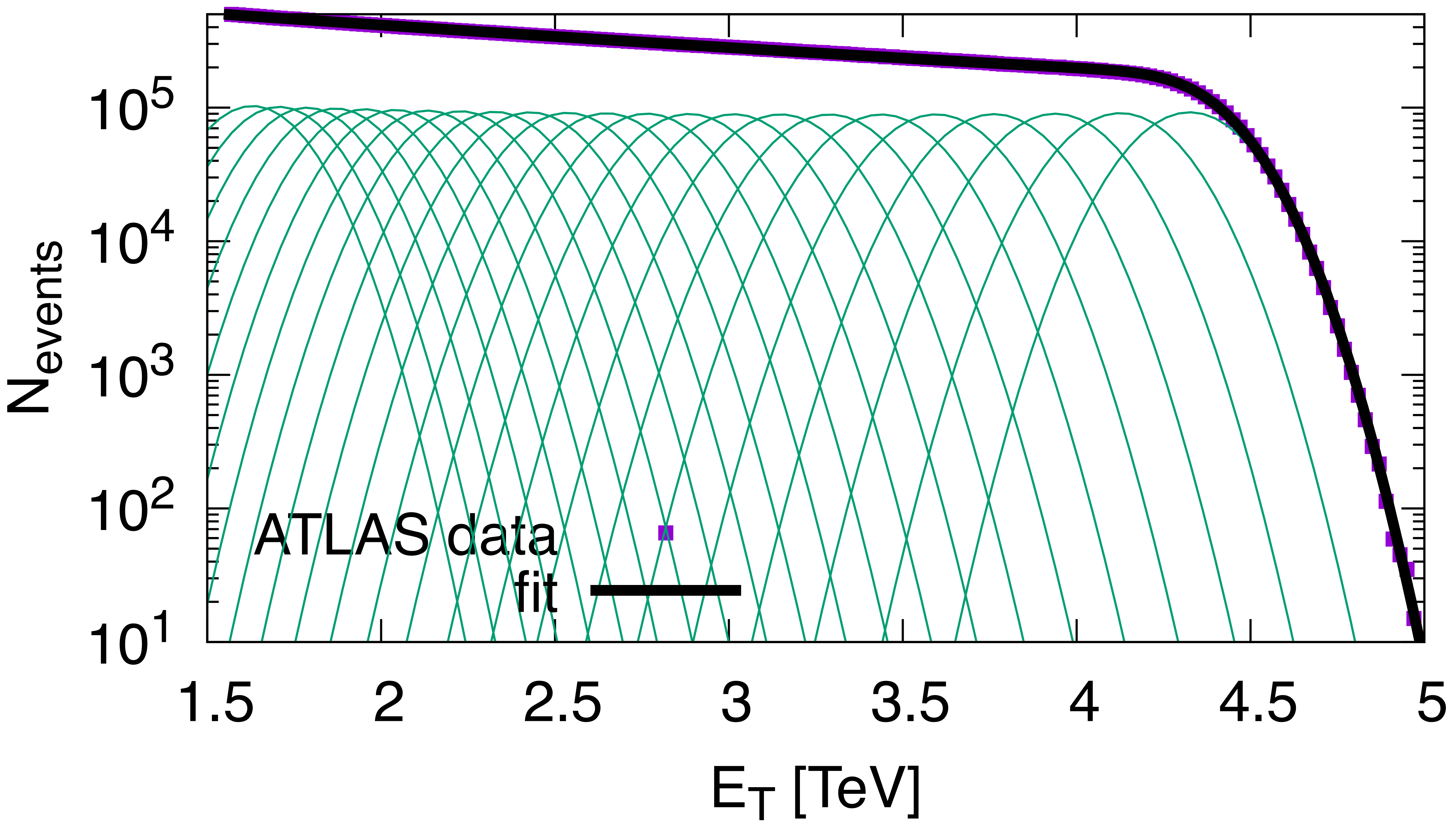
Fitting the distribution of the centrality classifier

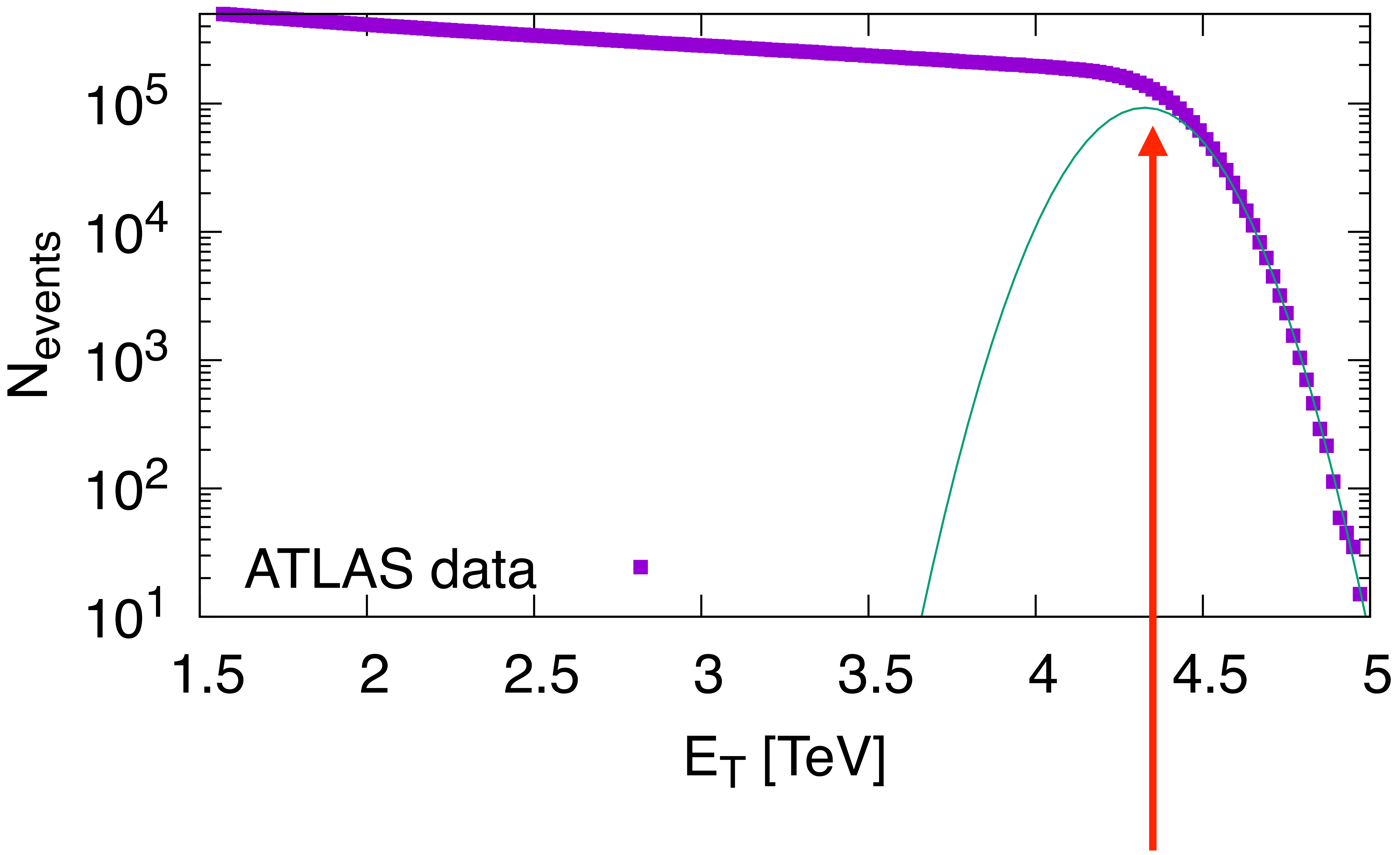


different Gaussians ≡ successive increments of c by 1%

In initial-state models, the ratio variance/mean decreases linearly as a function of the mean (binomial suppression of fluctuations), so that centrality fluctuations are broader than we initially thought for peripheral collisions.

Roubertie Verdan Kirchner JYO 2503.17035

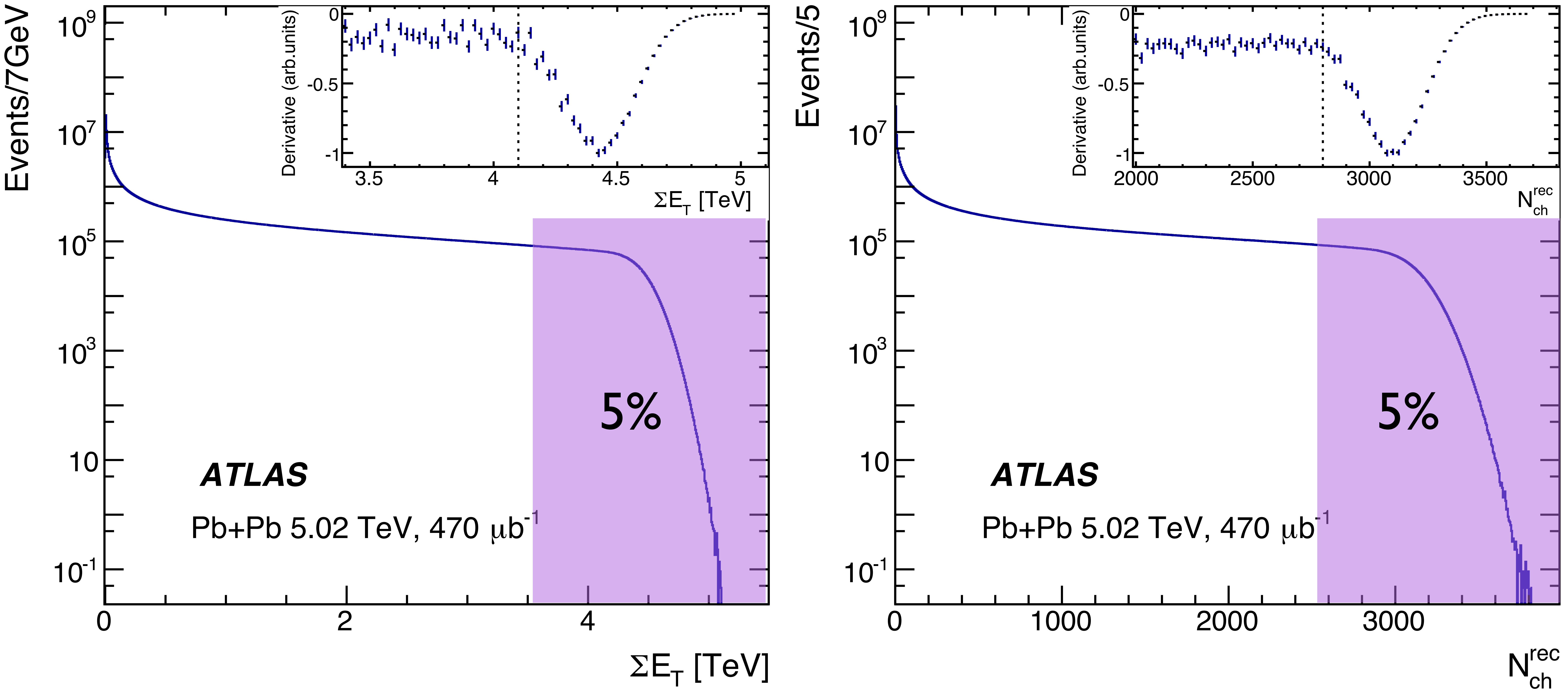
The knee



We define the knee of the distribution of E_T as the mean value of E_T for $c = 0$.
This is an output of the fit, and it is determined very precisely.
We call *ultracentral* the events above the knee.

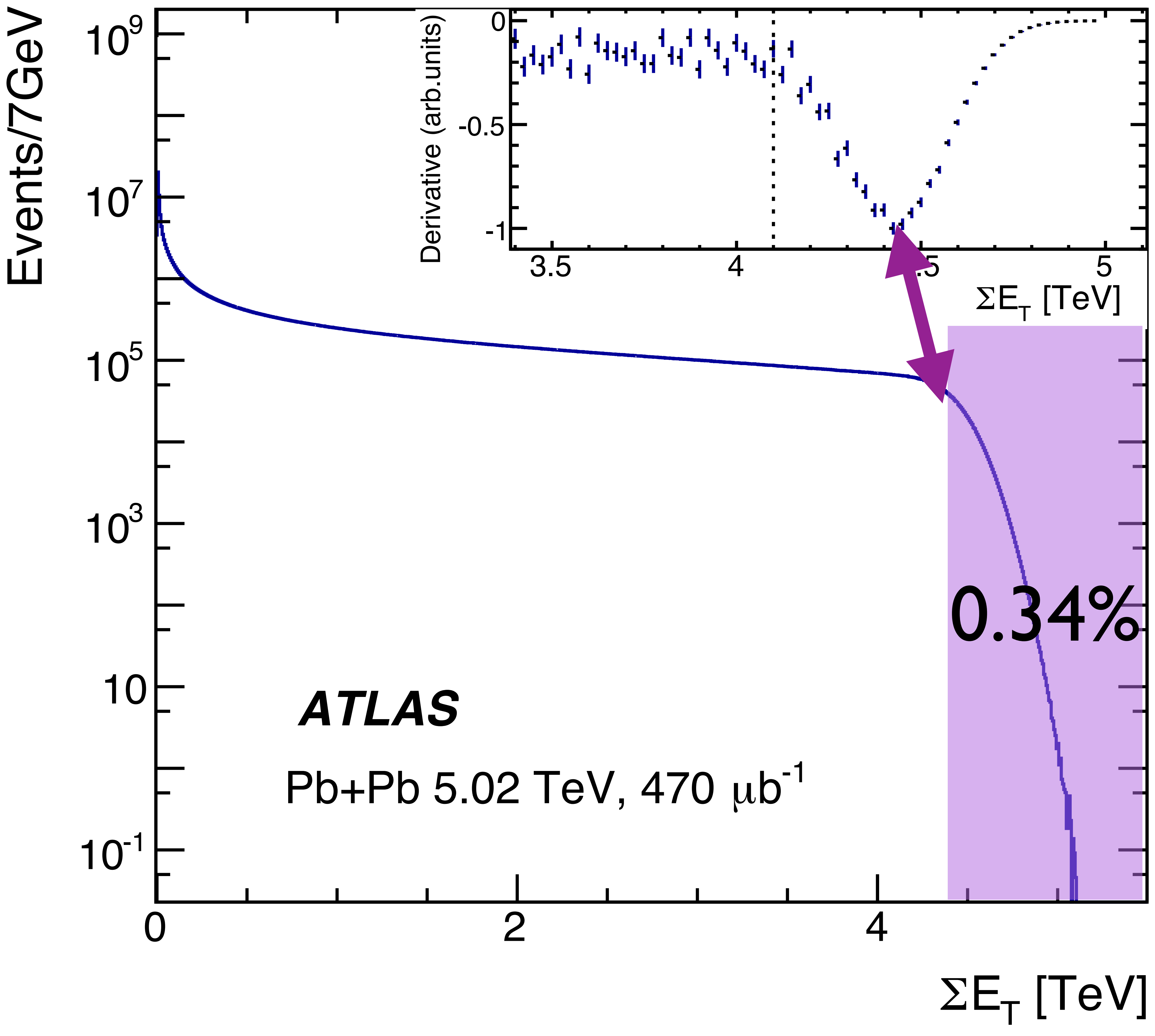
 Das Giacalone Monard JYO 1708.00081

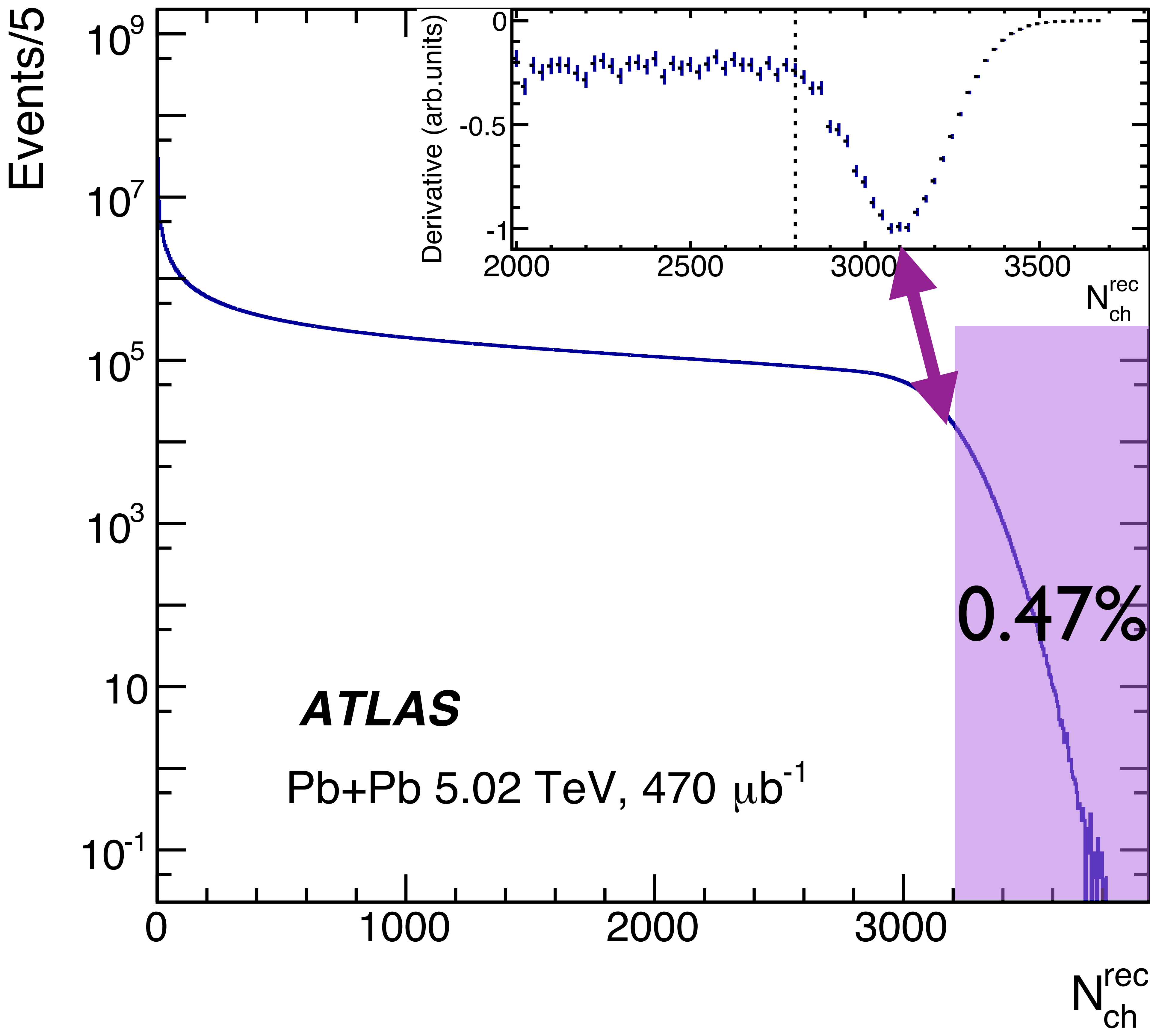
Central versus ultracentral collisions



Many analyses use 0-5% as the most central bin.

Central versus ultracentral collisions

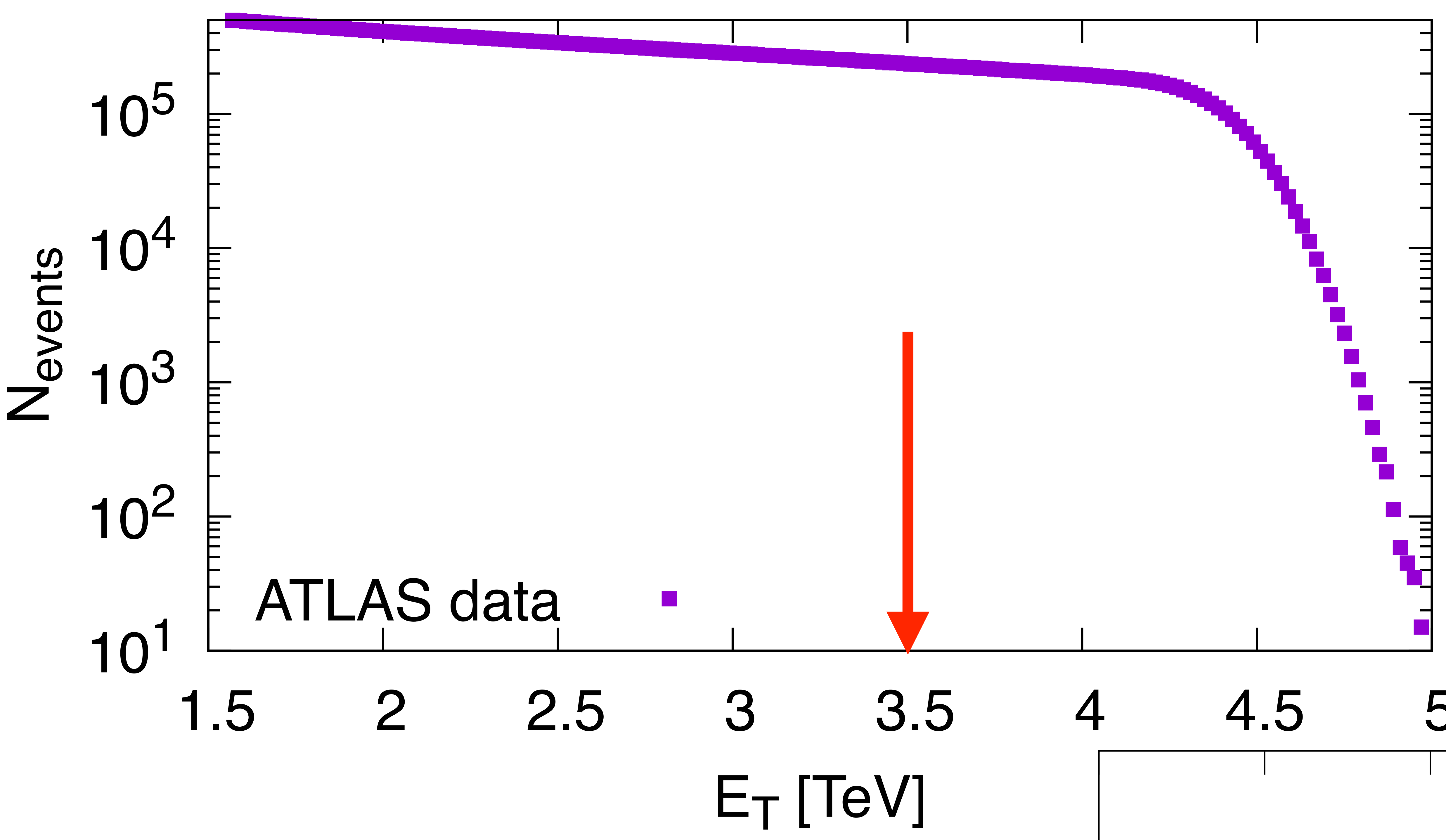




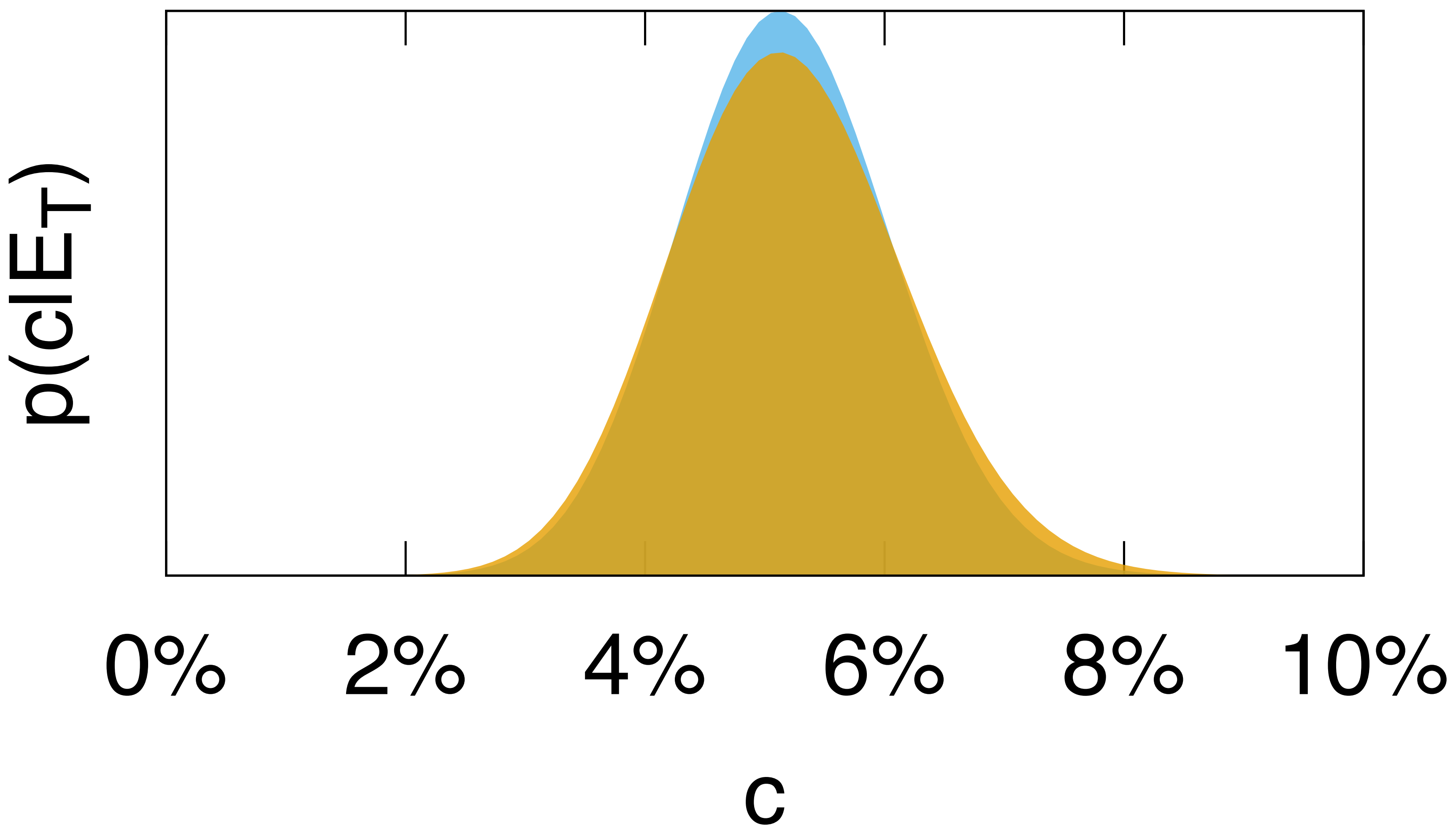
Ultracentral collisions are a much smaller fraction

(note: knee corresponds approximately to the max. slope of histogram on linear scale)

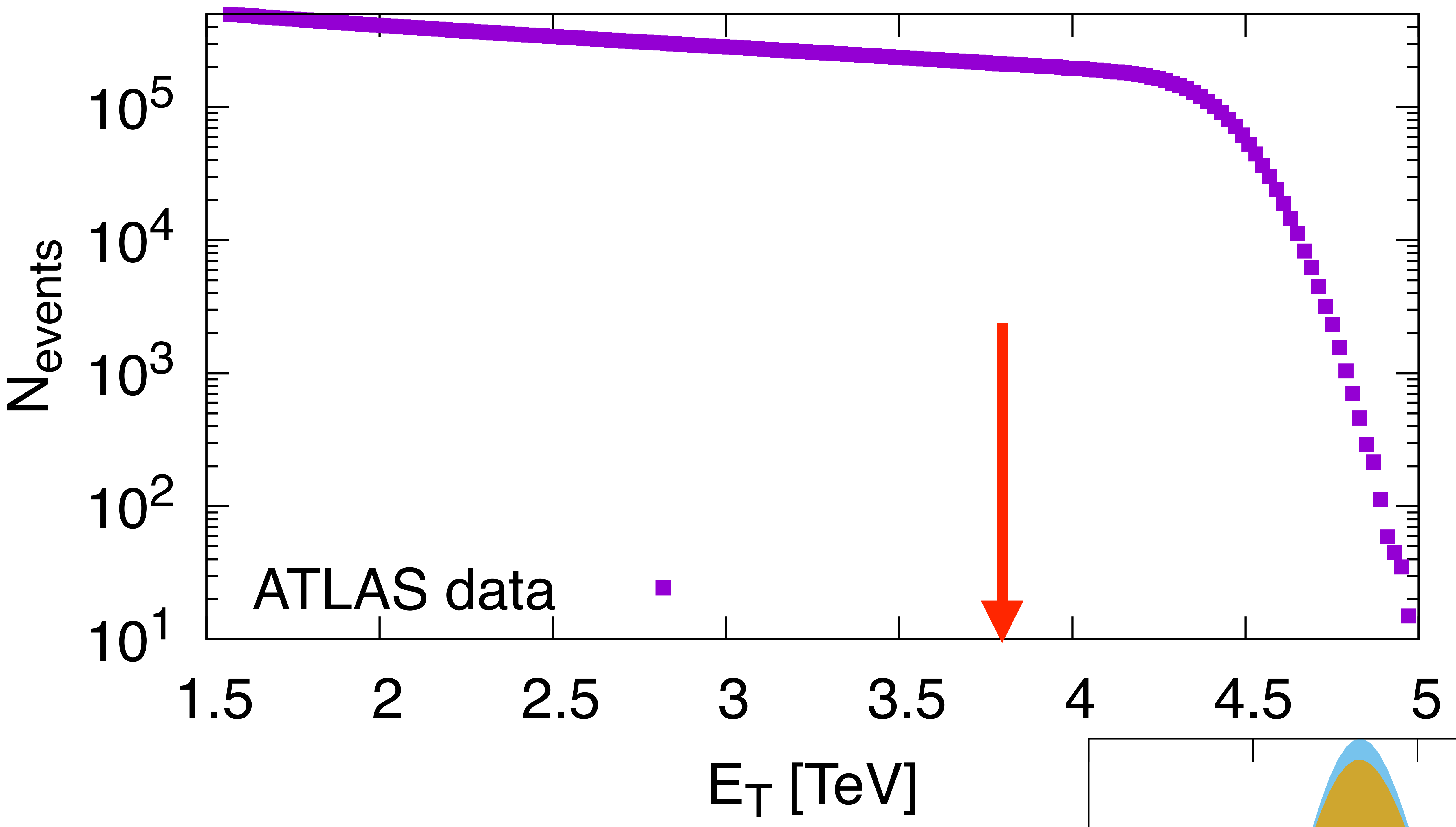
Distribution of centrality from Bayes' theorem



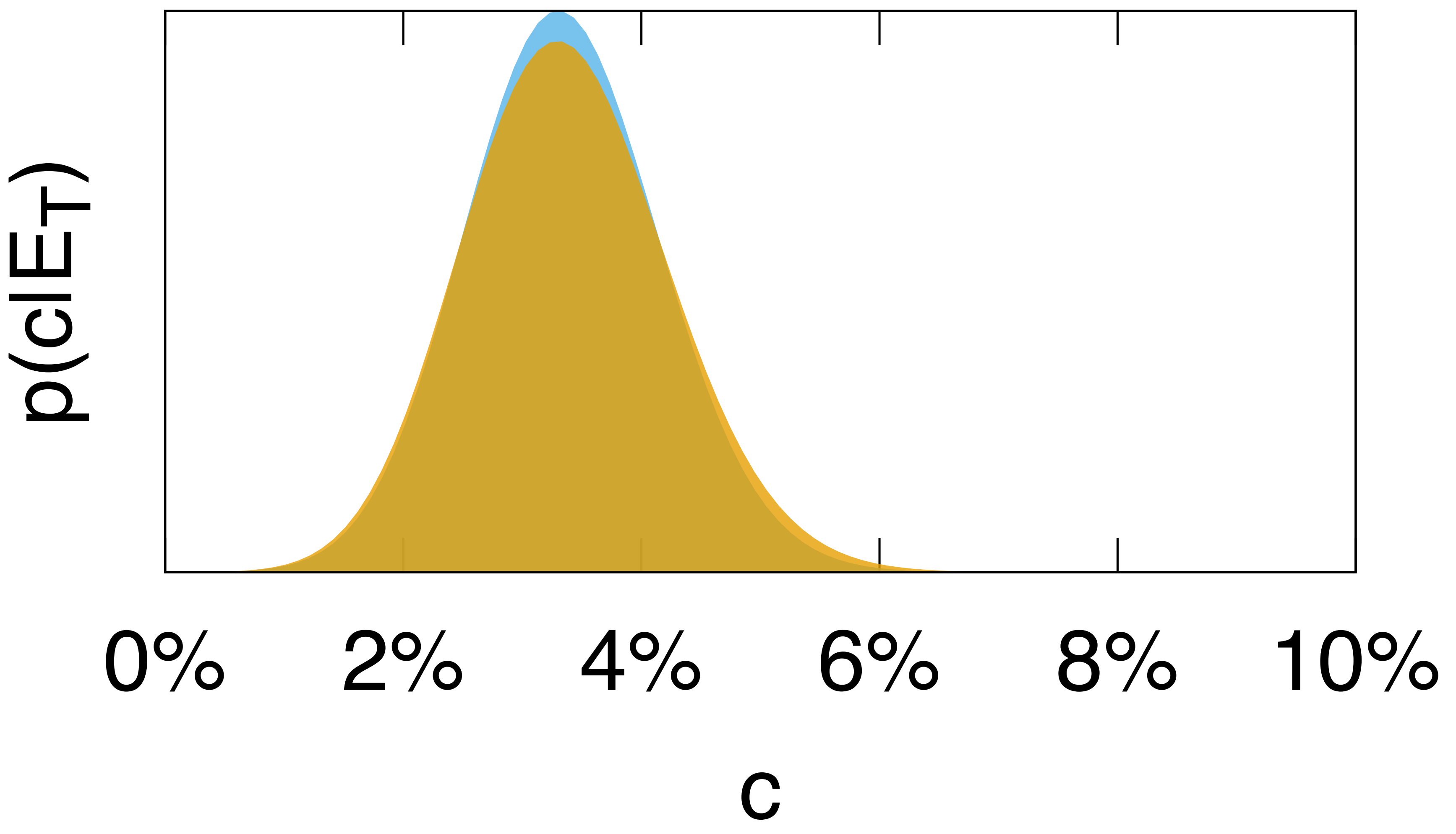
$p(c\,|\,E_T) = p(E_T\,|\,c)/p(E_T)$ is also Gaussian to a good approximation, with a std. dev. $\sigma_c \simeq 0.85\,\%$.



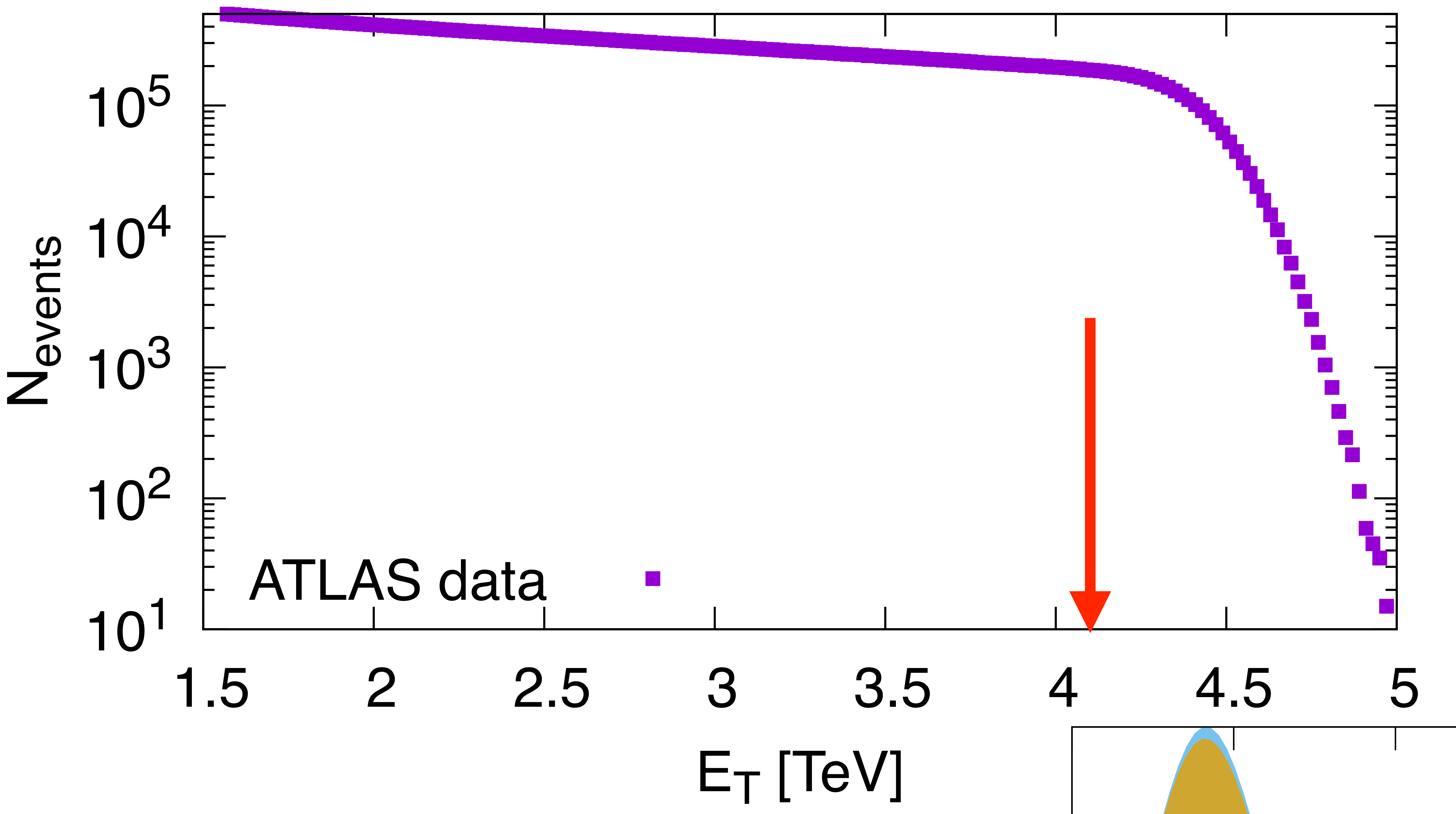
Distribution of centrality from Bayes' theorem



As E_T increases, the distribution of centrality gets shifted towards smaller values.



Distribution of centrality from Bayes' theorem



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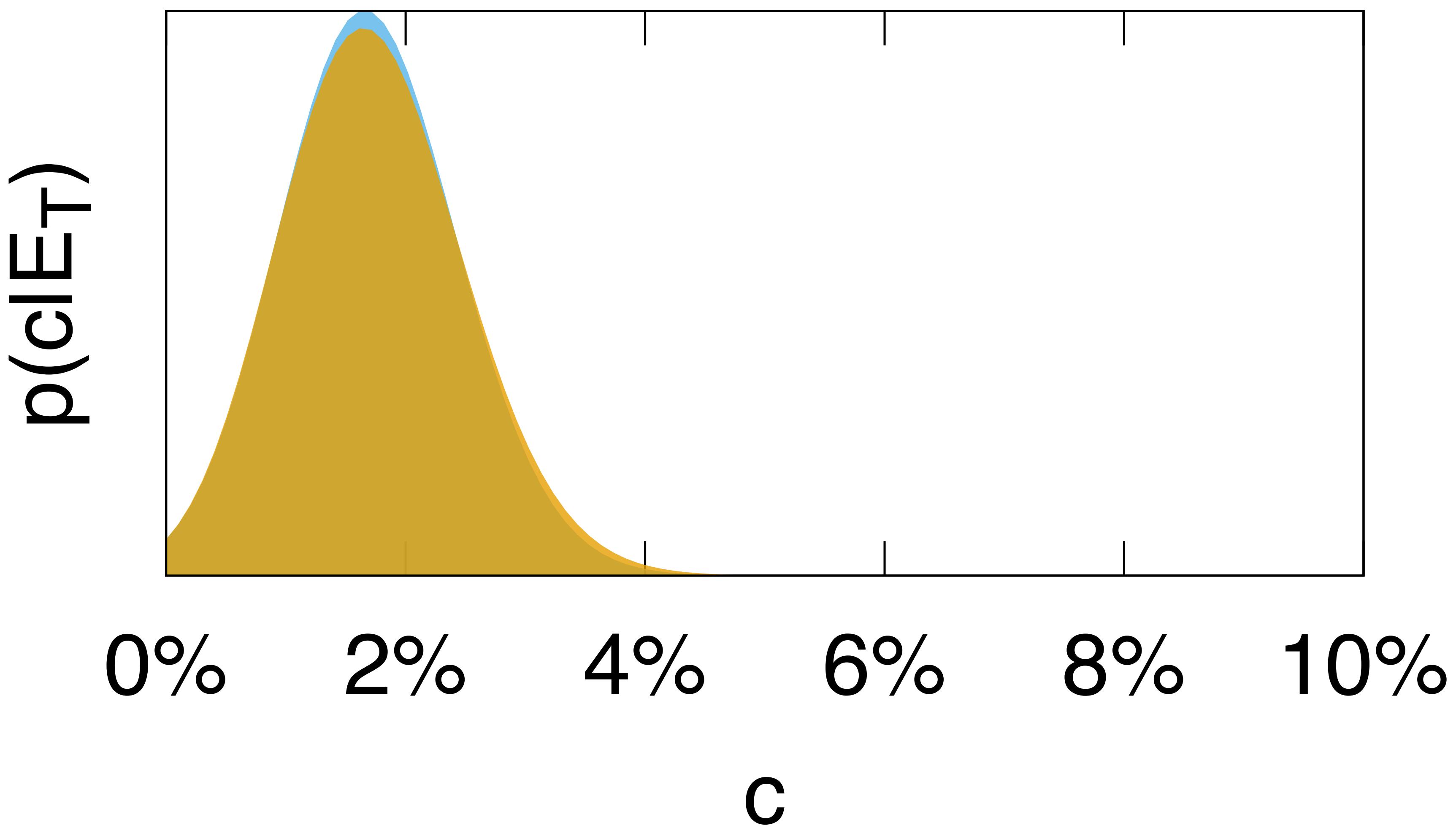
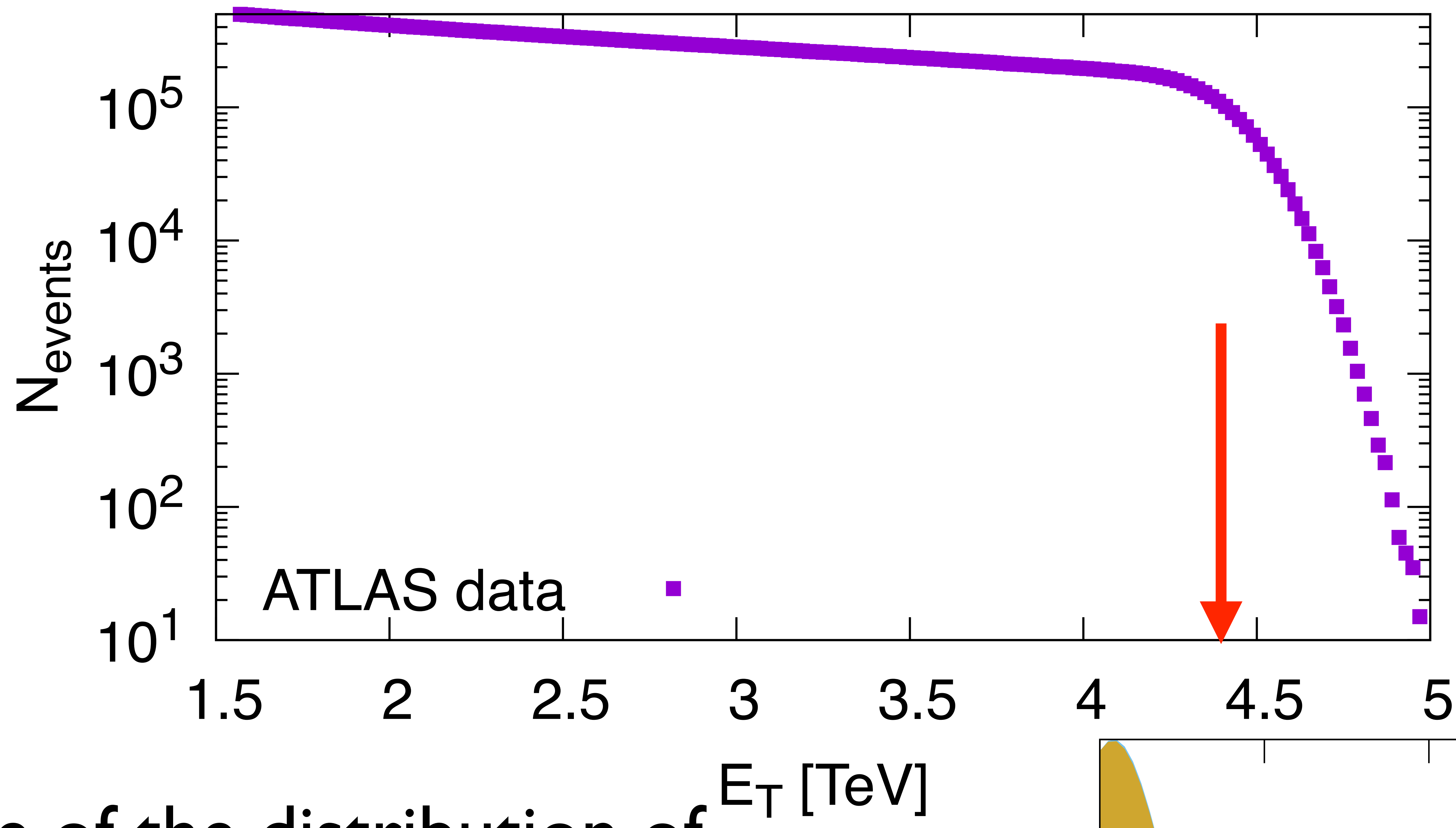
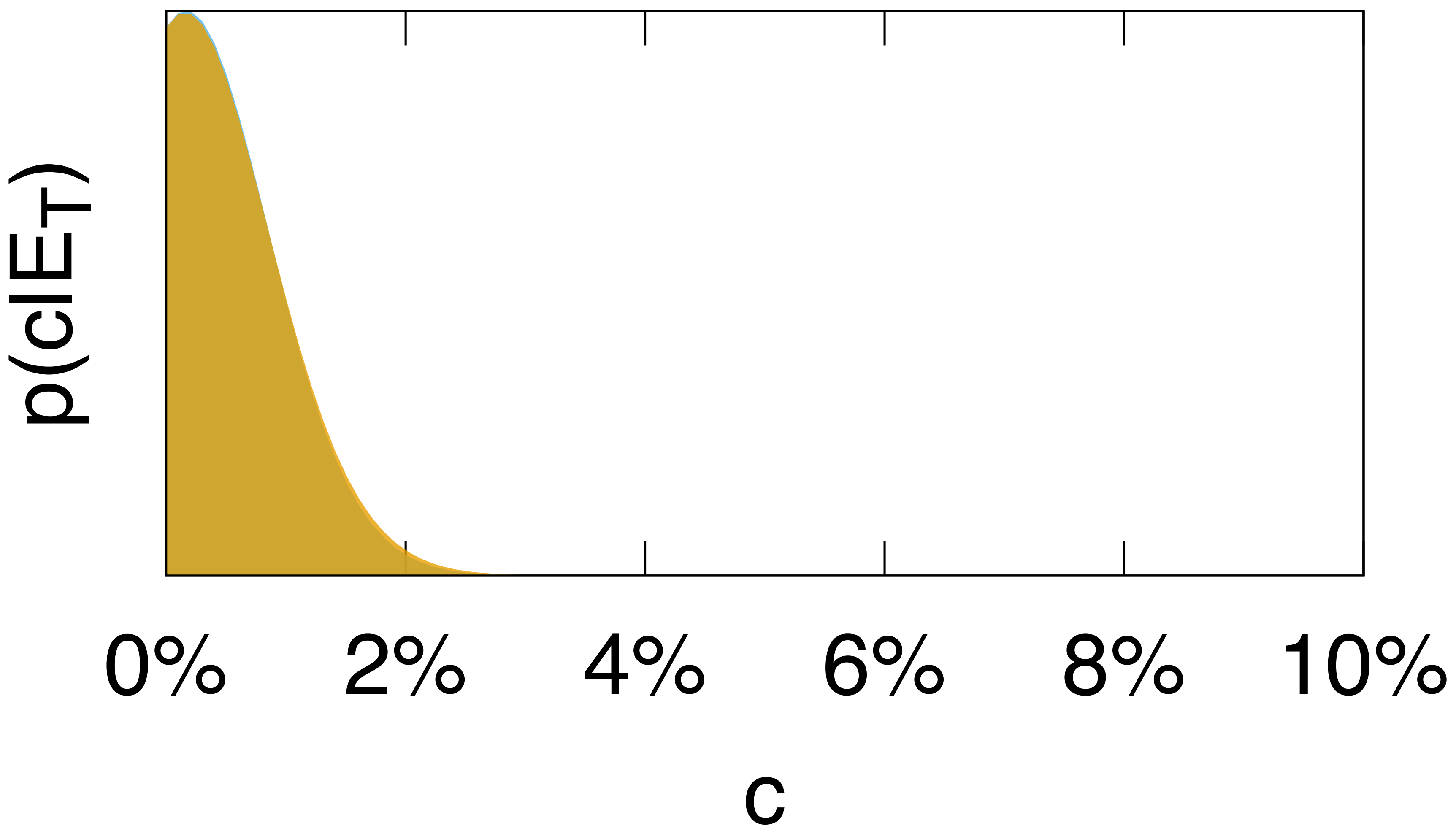


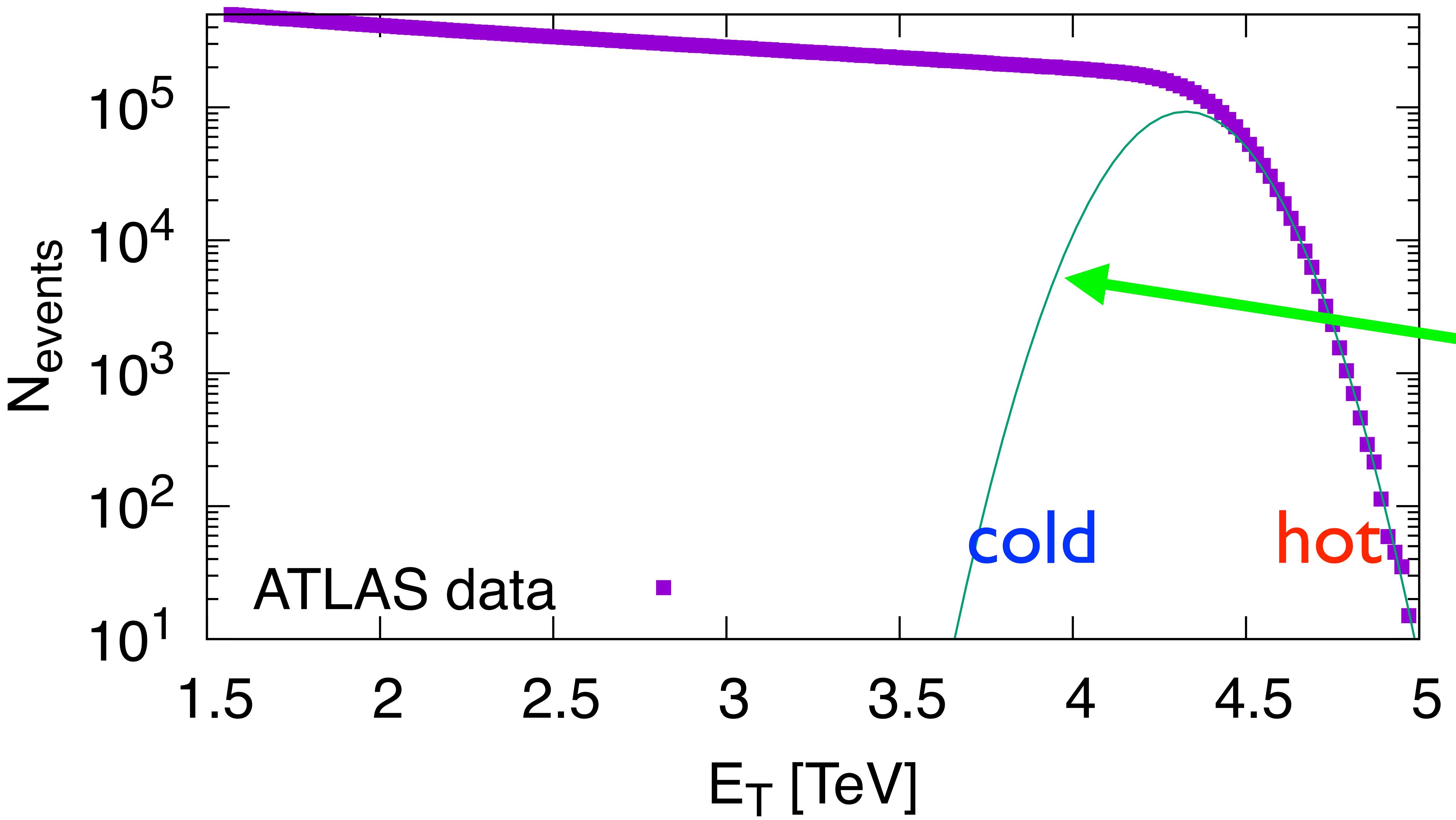
Distribution of centrality from Bayes' theorem



Above the knee of the distribution of E_T, the distribution hits the boundary at $c = 0$. No longer a Gaussian, but a *truncated* Gaussian.



The physics of ultracentral collisions



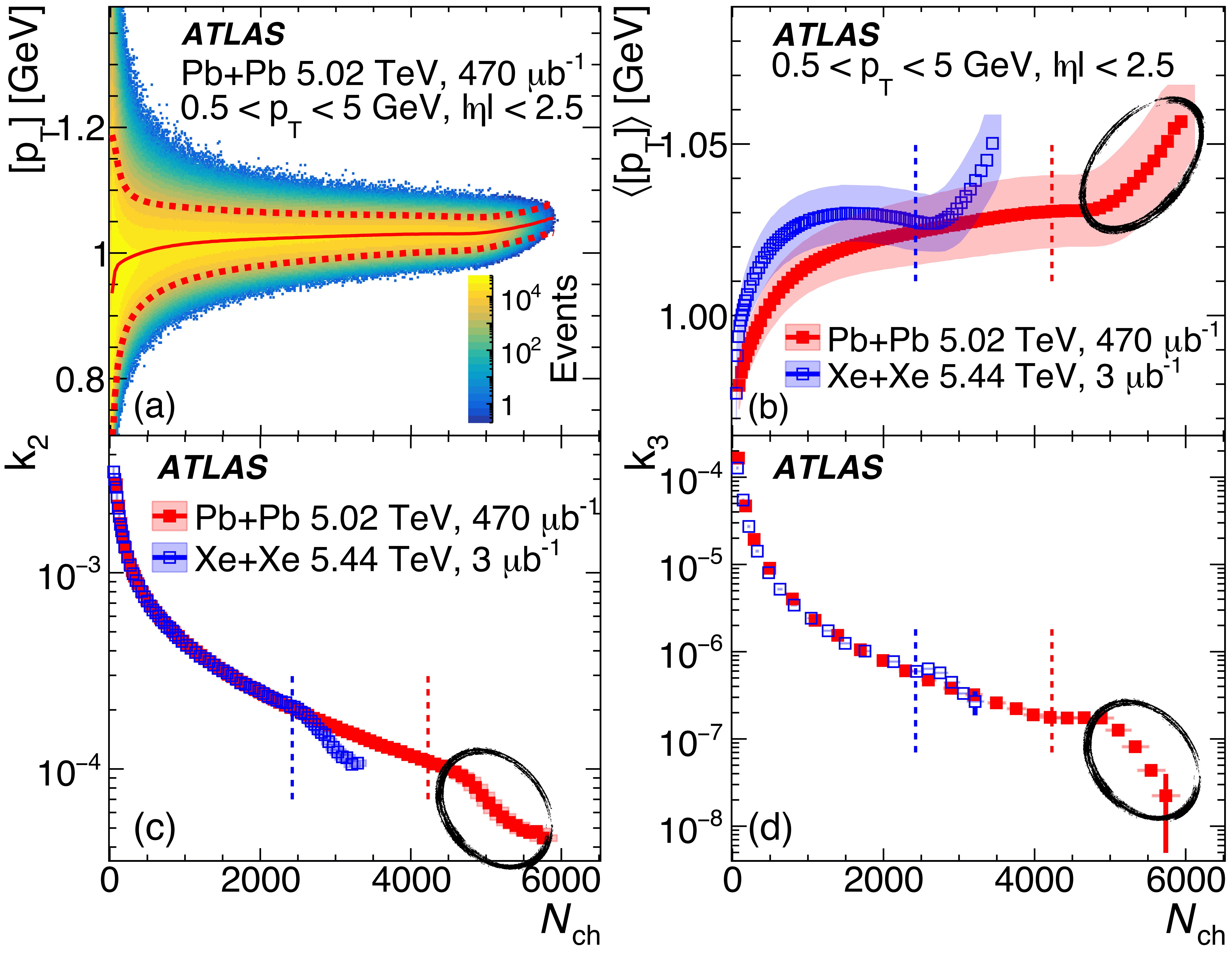
Distribution of E_T for $c = 0$ (head-on collisions)

For a Pb+Pb collision with $b = 0$, the standard deviation of the multiplicity around the knee is 4.5% (for E_T at forward rapidity, it is 3.4%). Mostly *dynamical* fluctuations at LHC.

Yousefnia Kotibhaskar Bhalerao JYO 2108.03471

If the volume is the same, collisions with larger multiplicity are hotter for fixed b.

Application to event-by-event fluctuations of $[p_T]$



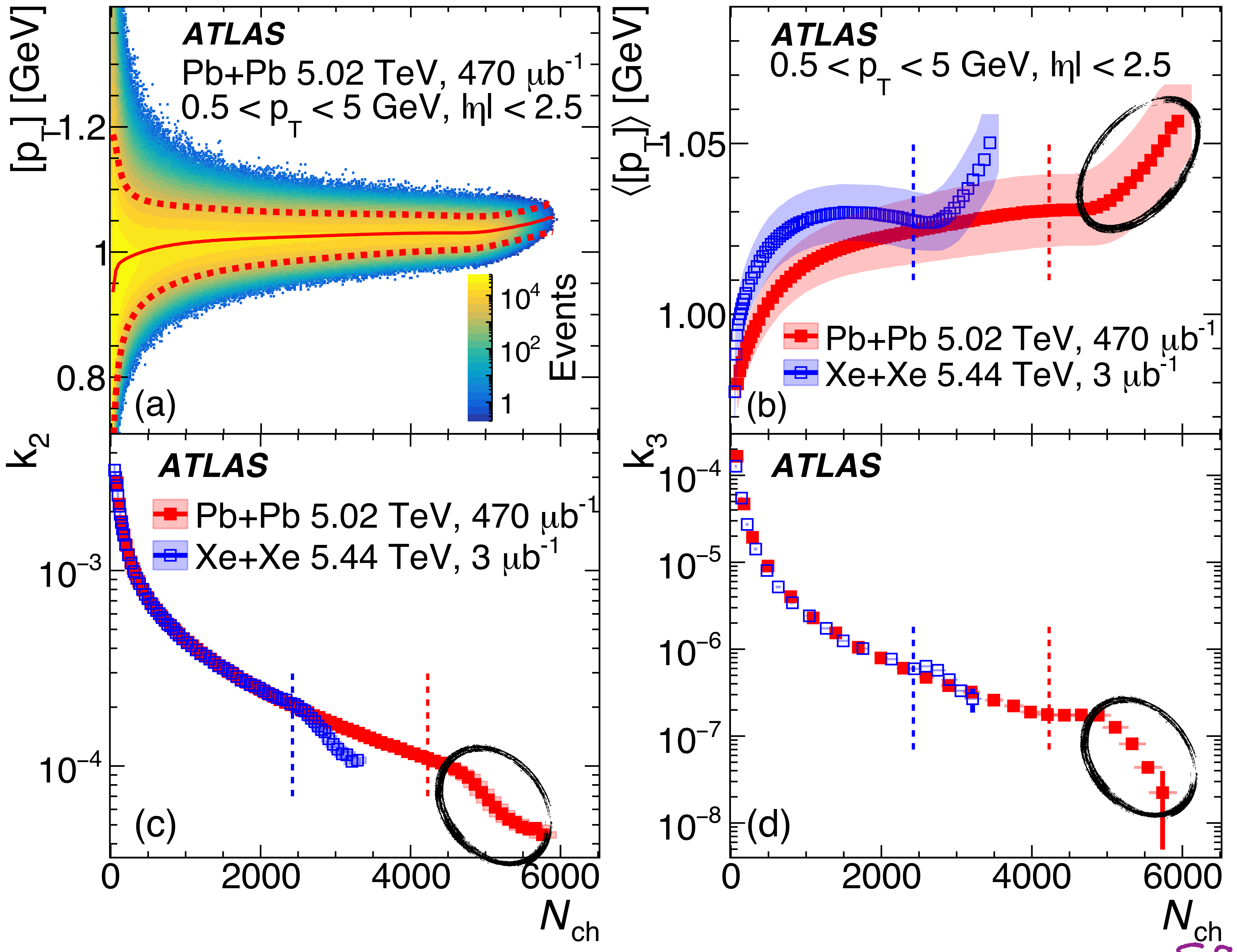
$[p_T] \equiv$ transverse momentum per particle in an event

For very large N_{ch} (ultracentral collisions):

- The mean value increases
- The relative variance k_2 decreases
- The relative skewness k_3 decreases

ATLAS 2407.06413

Application to event-by-event fluctuations of $[p_T]$

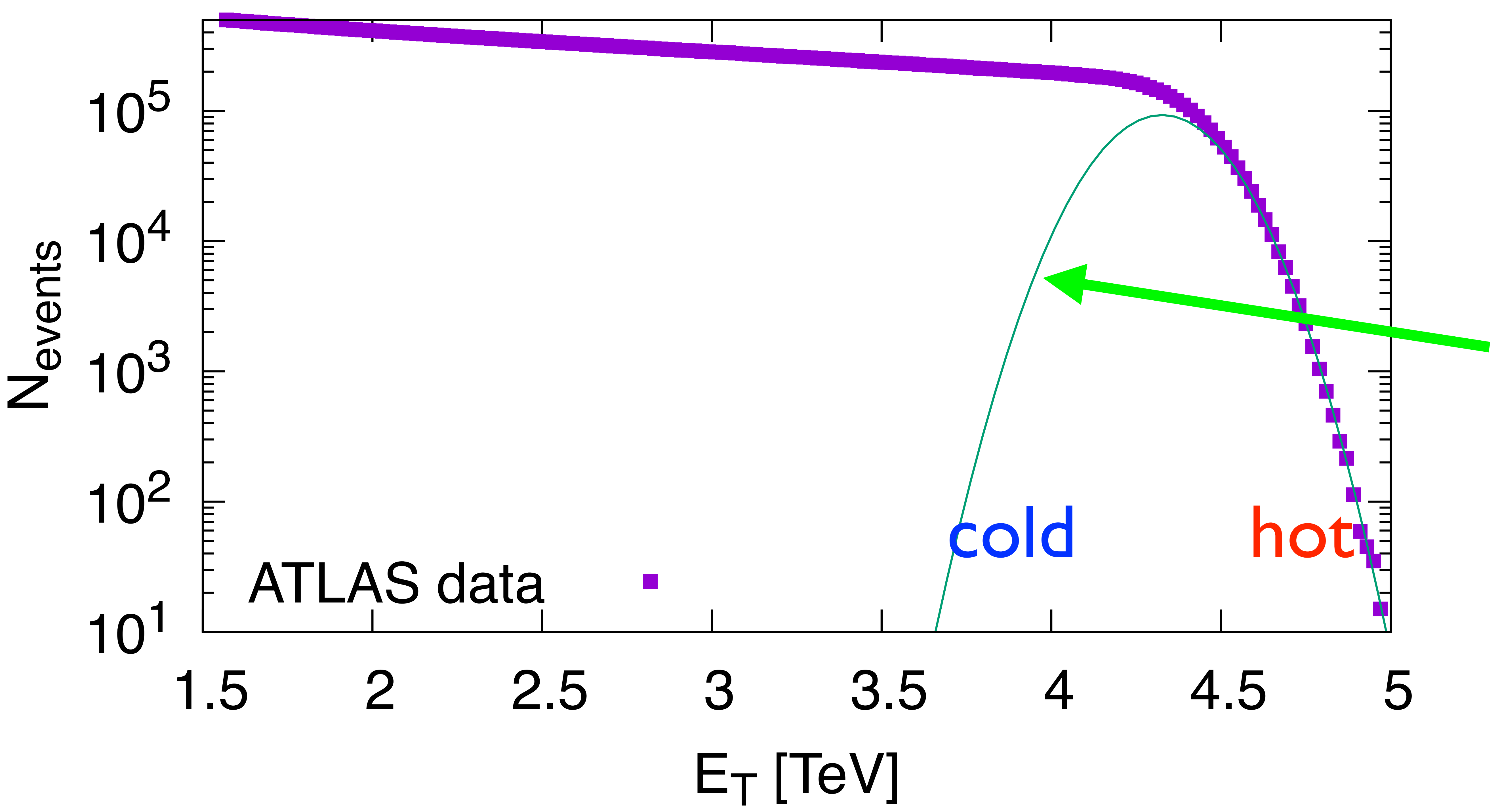


ATLAS 2407.06413

All these features are *quantitatively* understood in terms of centrality fluctuations, which gradually disappear in ultracentral collisions, and thermalization (which implies that the volume matters):

- increase of the mean
  Gardim Giacalone JYO 1909.11609
- decrease of the variance
  Samanta Bhattajia JYO 2303.15323
- non-Gaussianity
  Samanta Picchetti Luzum JYO 2306.09294

Is the volume constant in ultracentral collisions?

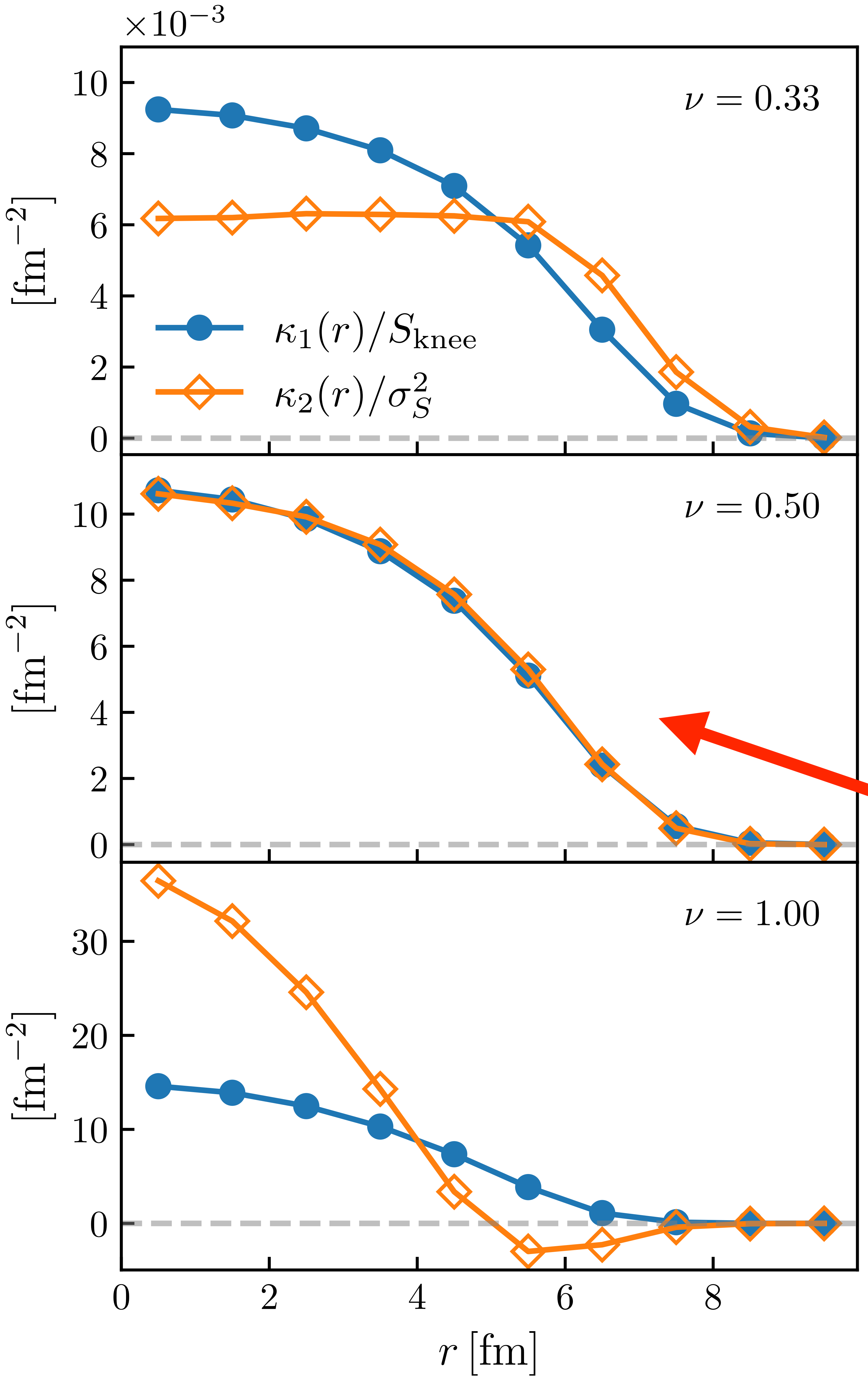


Distribution of E_T for $c = 0$ (head-on collisions)

It could be that the volume depends on the multiplicity (or E_T) at $b = 0$
This depends on the initial-state model, and how the excess multiplicity (entropy) is distributed in the transverse plane.

Nijs van der Schee 2312.04623

Is the volume constant in ultracentral collisions?



We have investigated this in a model where the initial density is proportional to $(T_A T_B)^\nu$ and studied the dependence on the exponent ν.

For each ν, we evaluate the radial distributions of the mean density and excess density due to fluctuations in collisions at b=0.

For $\nu = 0.5$, the default model where the multiplicity is proportional to number of participants, these distributions are exactly identical, which implies that the volume *is* constant.

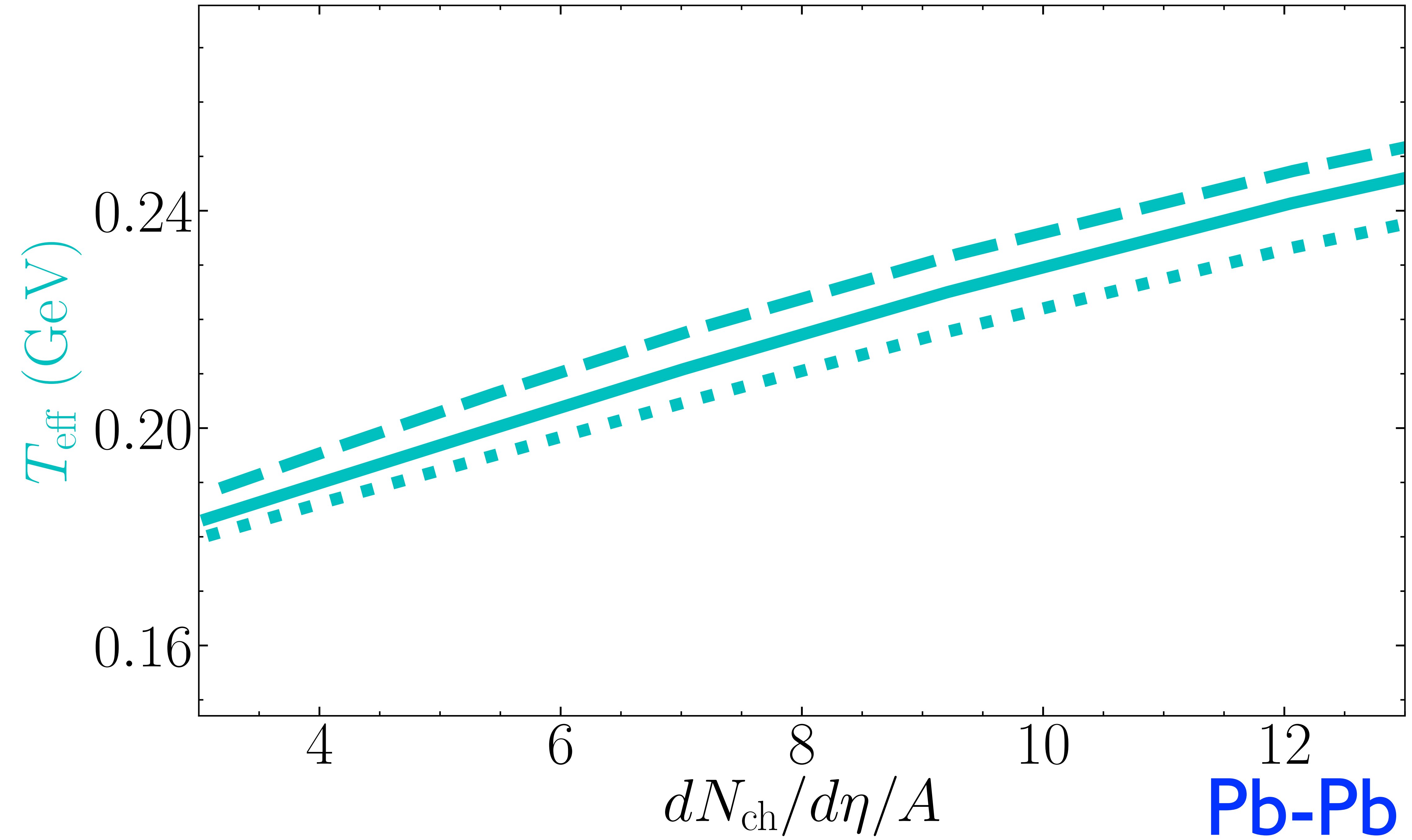
Fabian Zhou, Giacalone, JYO 2511.04605

Summary

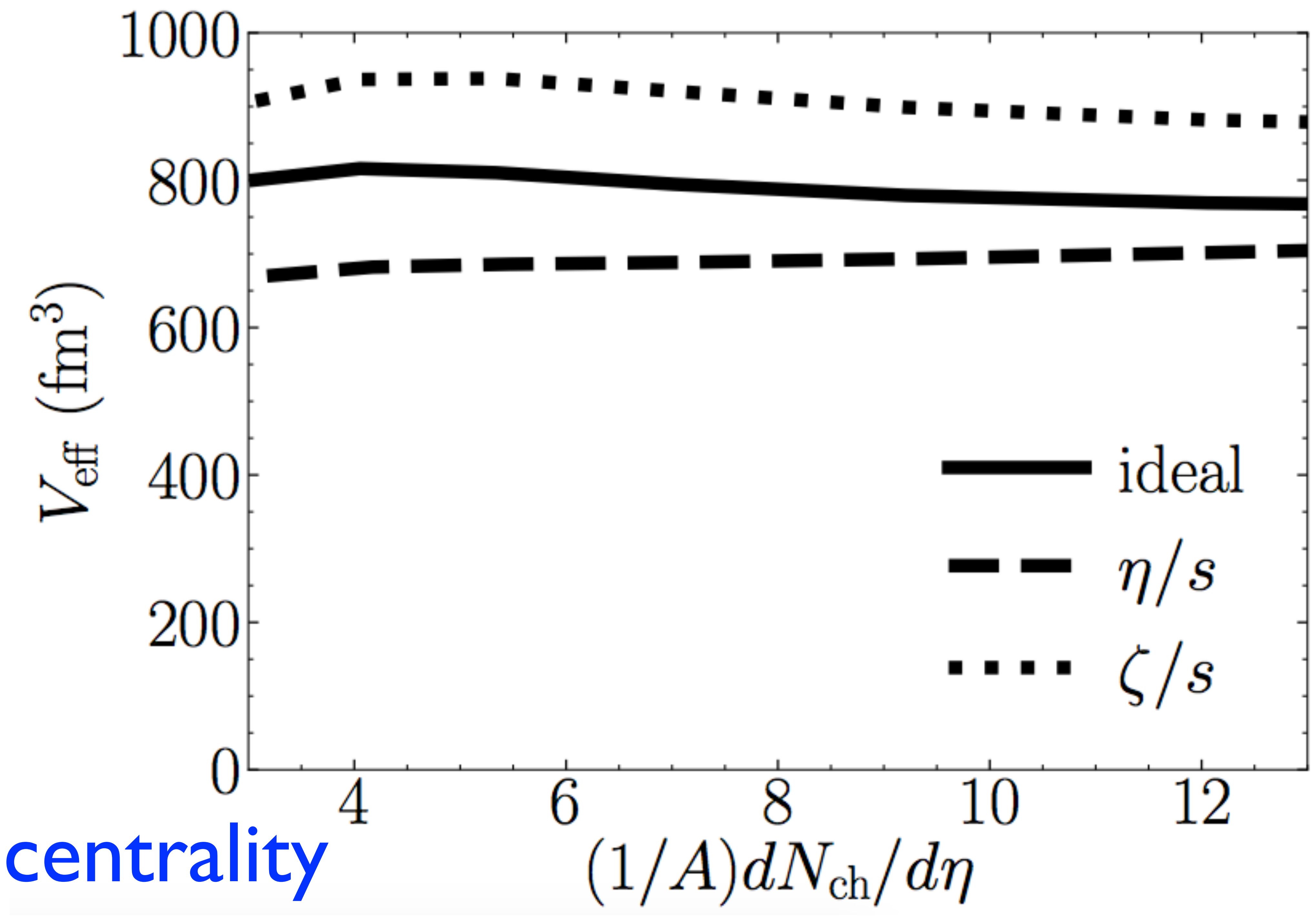
- We have a fairly robust method of reconstructing the EOS at the LHC. One collision energy = One temperature. Results agree with lattice QCD, including the evolution with collision energy (not shown in this talk).
- Uncertainty on the temperature is small and comes mostly from the modeling of the late stages of the collision. T=225 MeV is the temperature that we probe with spectra at the LHC.
- Uncertainty on the entropy density is significantly larger. It is due to uncertainties on the volume, coming in part from transport coefficients, and in part from initial state model.
- More information about volume fluctuations can be extracted from $[p_T]$ fluctuations (work in progress).

Supplementary material

Varying the collision energy

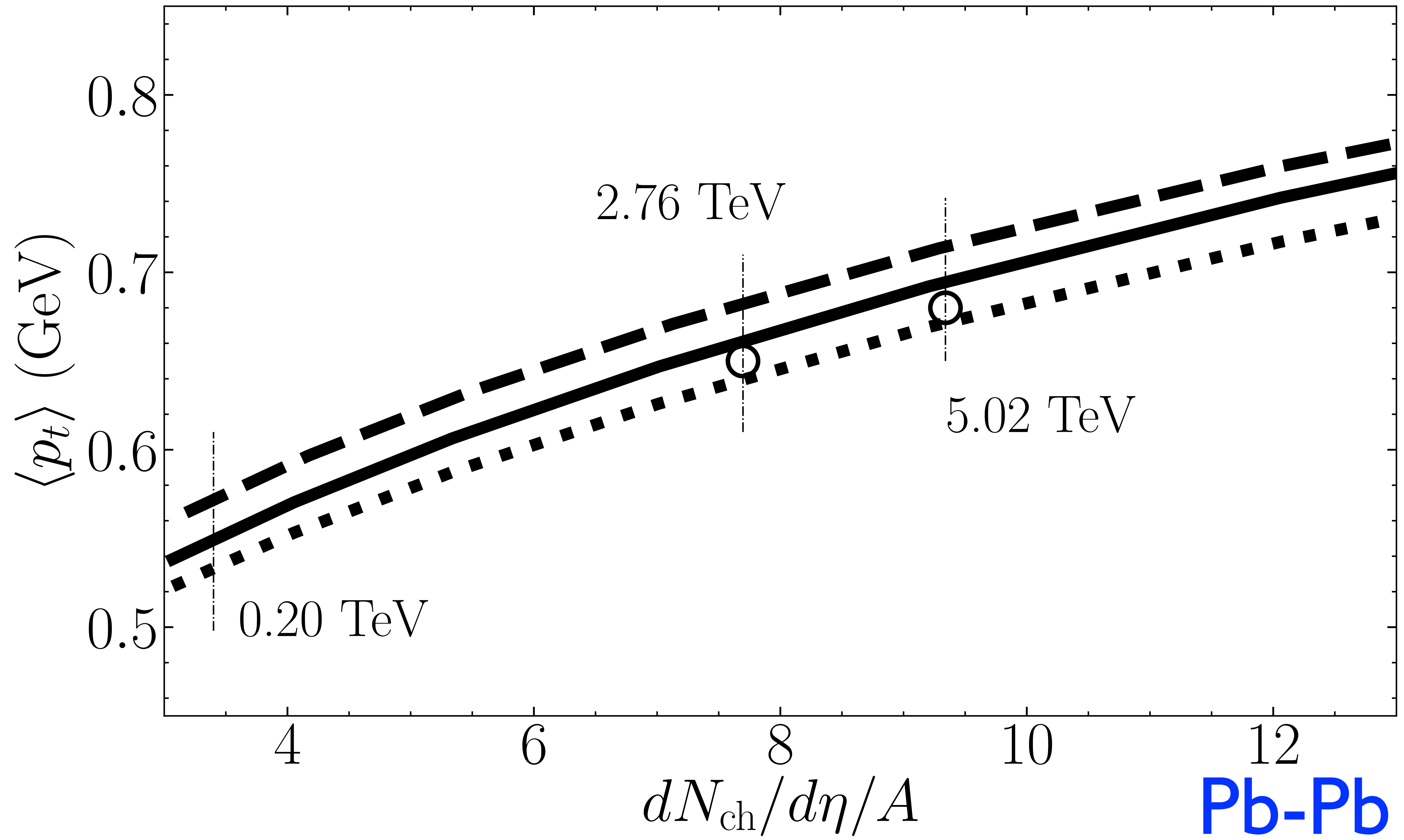


Pb-Pb 0-5% centrality



As sqrt(s) increases, T_{eff} increases, V_{eff} remains constant.
Increasing energy amounts to heating the system at constant volume.

Varying the collision energy



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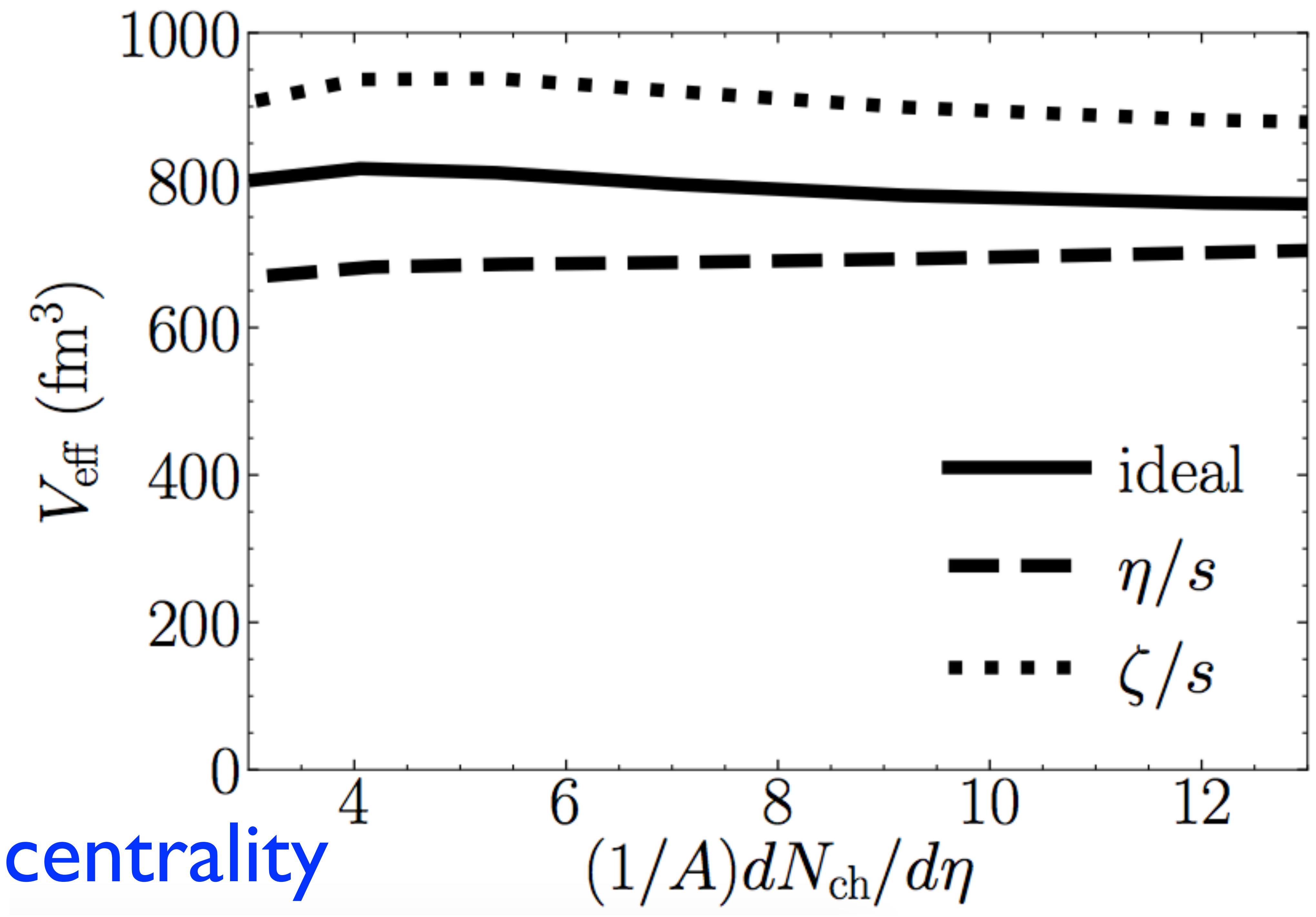
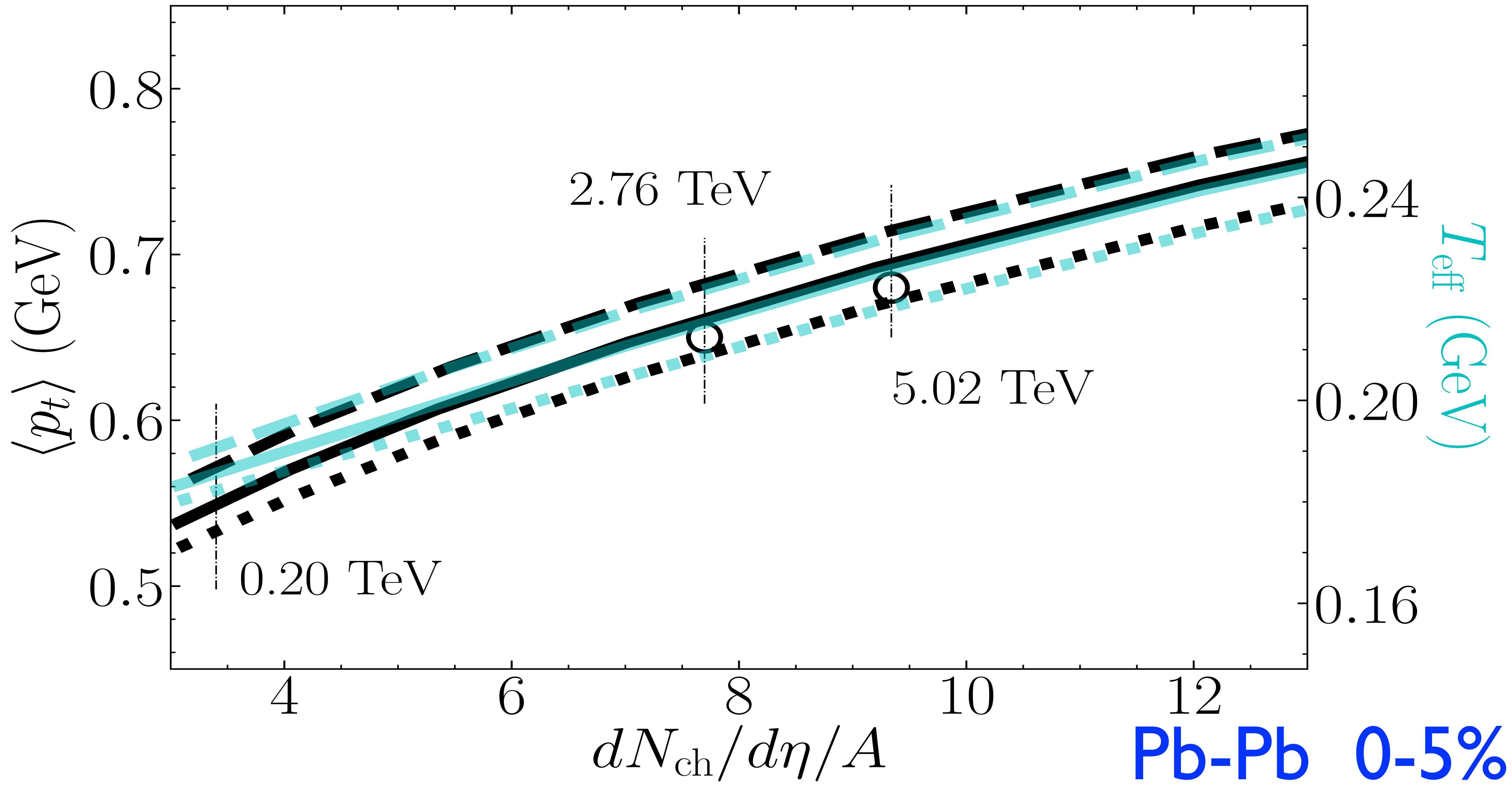


The variation of <p_t> still closely follows that of T_{eff}

Varying the collision energy



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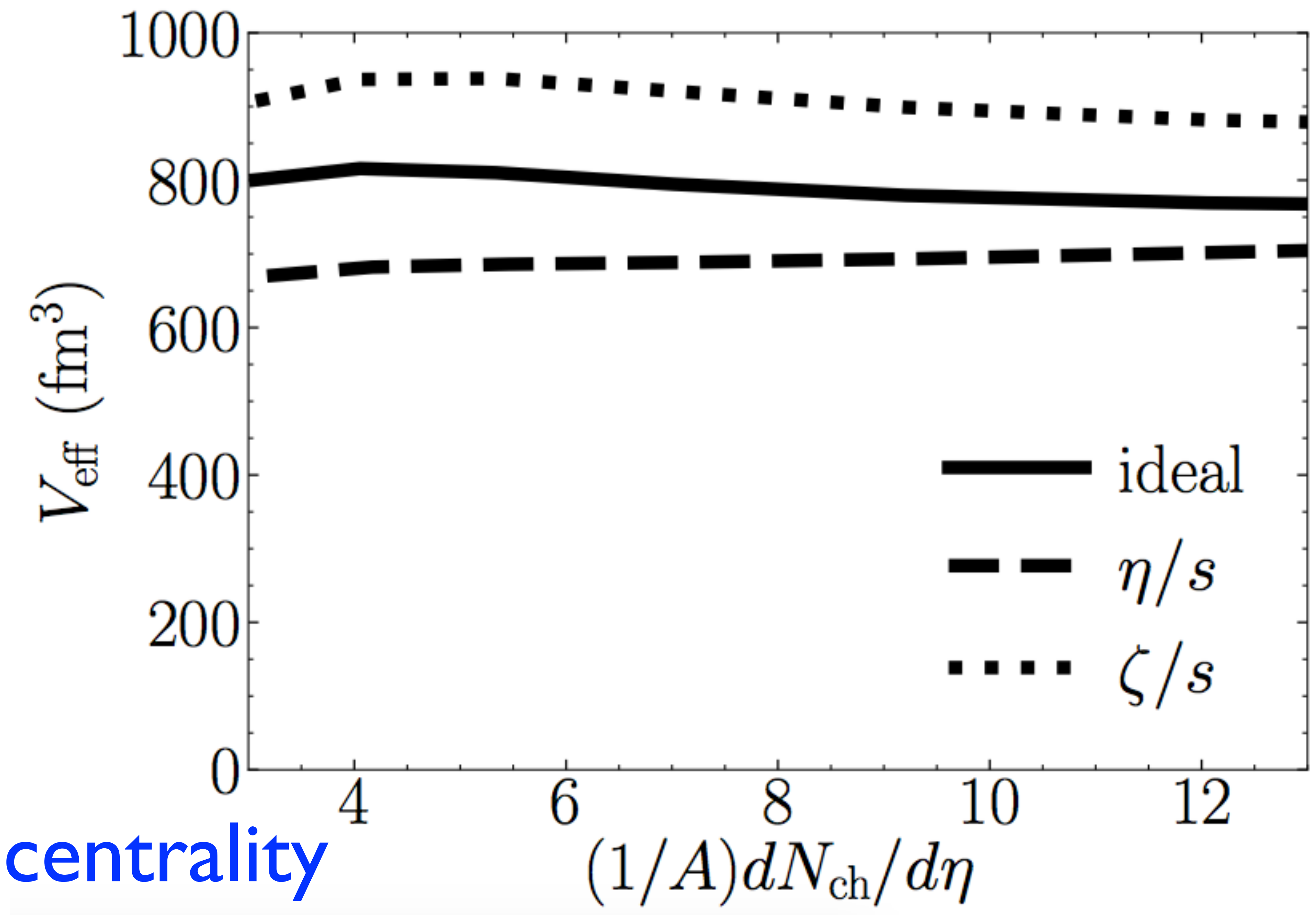
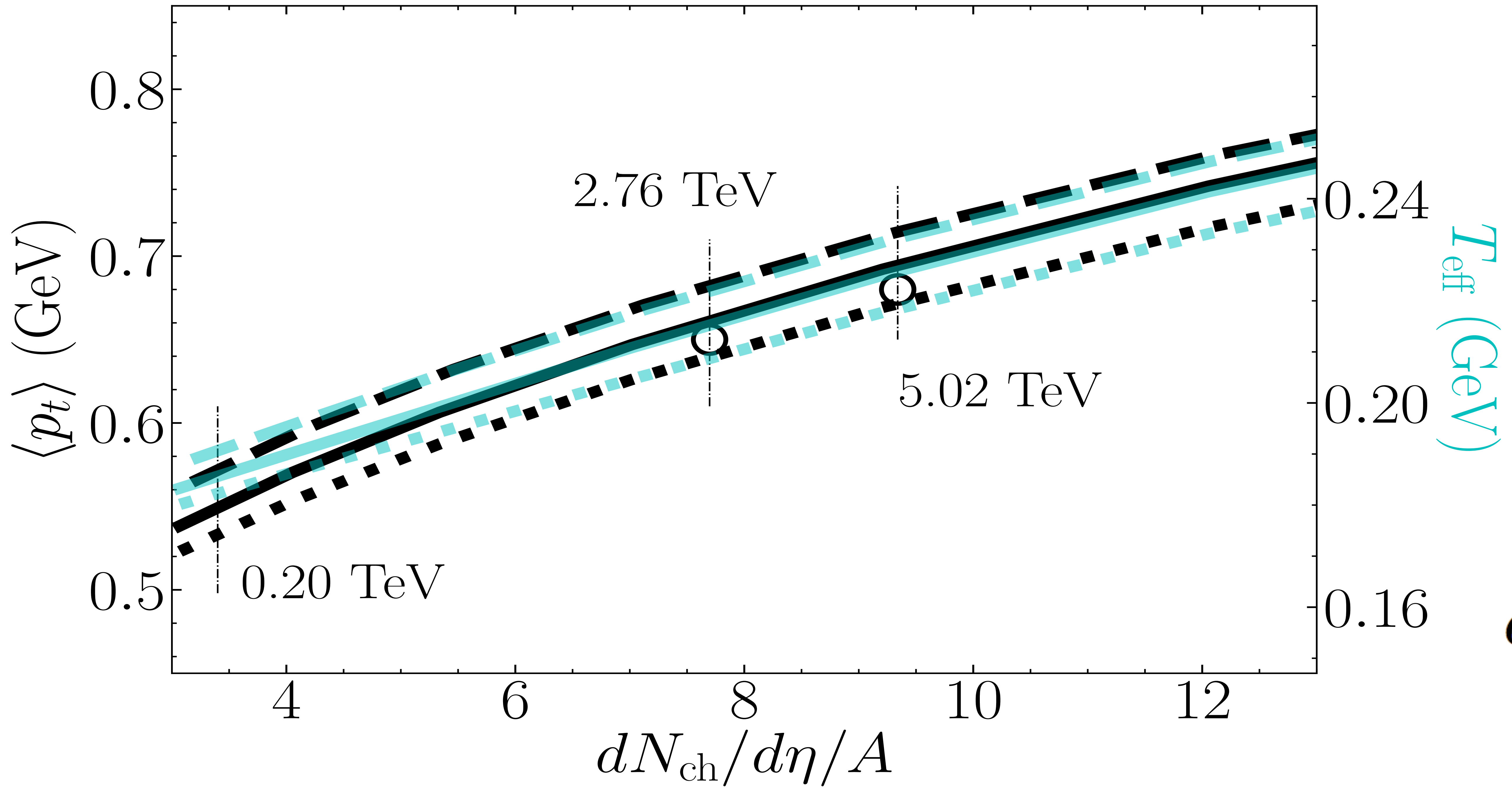


Deviations from $<p_t>=3.07\ T_{eff}$ are negligible at LHC energy and beyond

Speed of sound c_s in the QGP



The math:

T_{eff} proportional to $<p_t>$

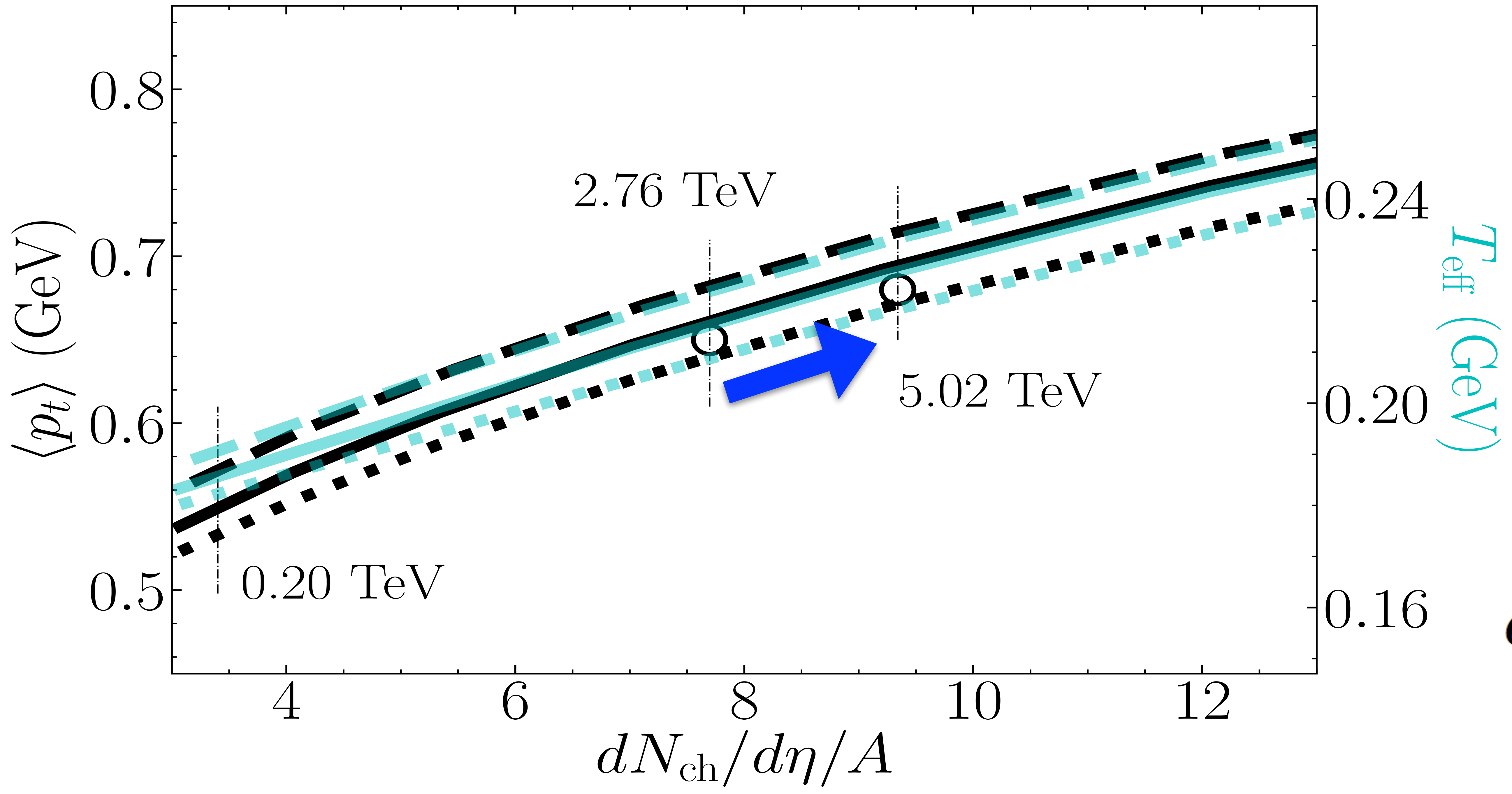
$s(T_{eff})$ proportional to $dN_{ch}/d\eta$

$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \left.\frac{sdT}{Tds}\right|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}$$

The physics:

Increasing the collision energy amounts to putting more energy into a fixed volume. Gives direct access to the compressibility=speed of sound.

Speed of sound c_s in the QGP



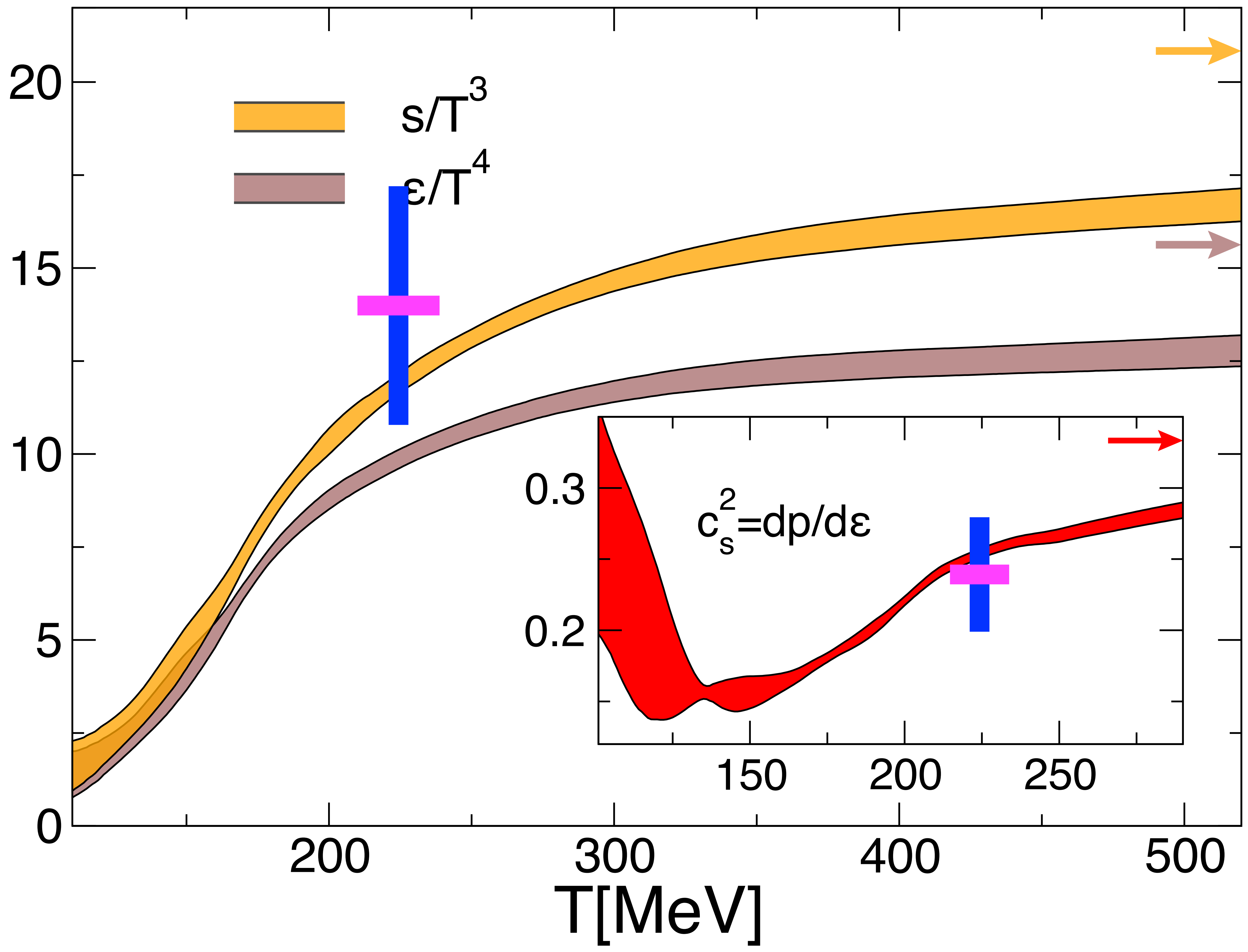
$T_{\rm eff}$ proportional to $\langle p_t \rangle$

$s(T_{\rm eff})$ proportional to $dN_{ch}/d\eta$

$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \frac{sdT}{Tds}\bigg|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}$$

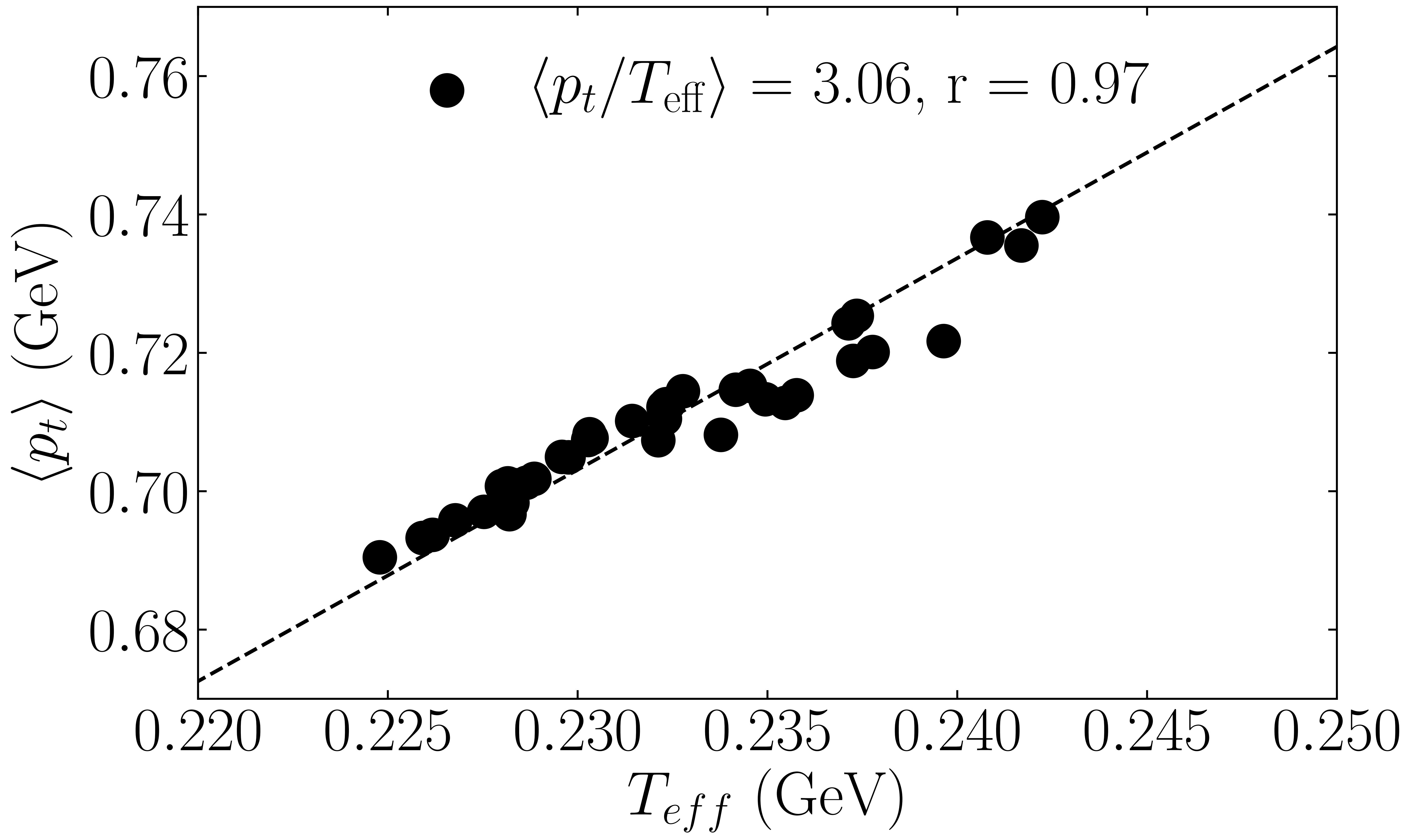
we obtain $c_s^2(T_{\rm eff}) = 0.24 \pm 0.04$ (error from variation of $V_{\rm eff}$)

Comparison with lattice QCD



compatible with lattice

Event-to-event fluctuations



T_{eff} varies event to event, but the ratio $\langle p_t \rangle / T_{eff}$ is essentially constant.

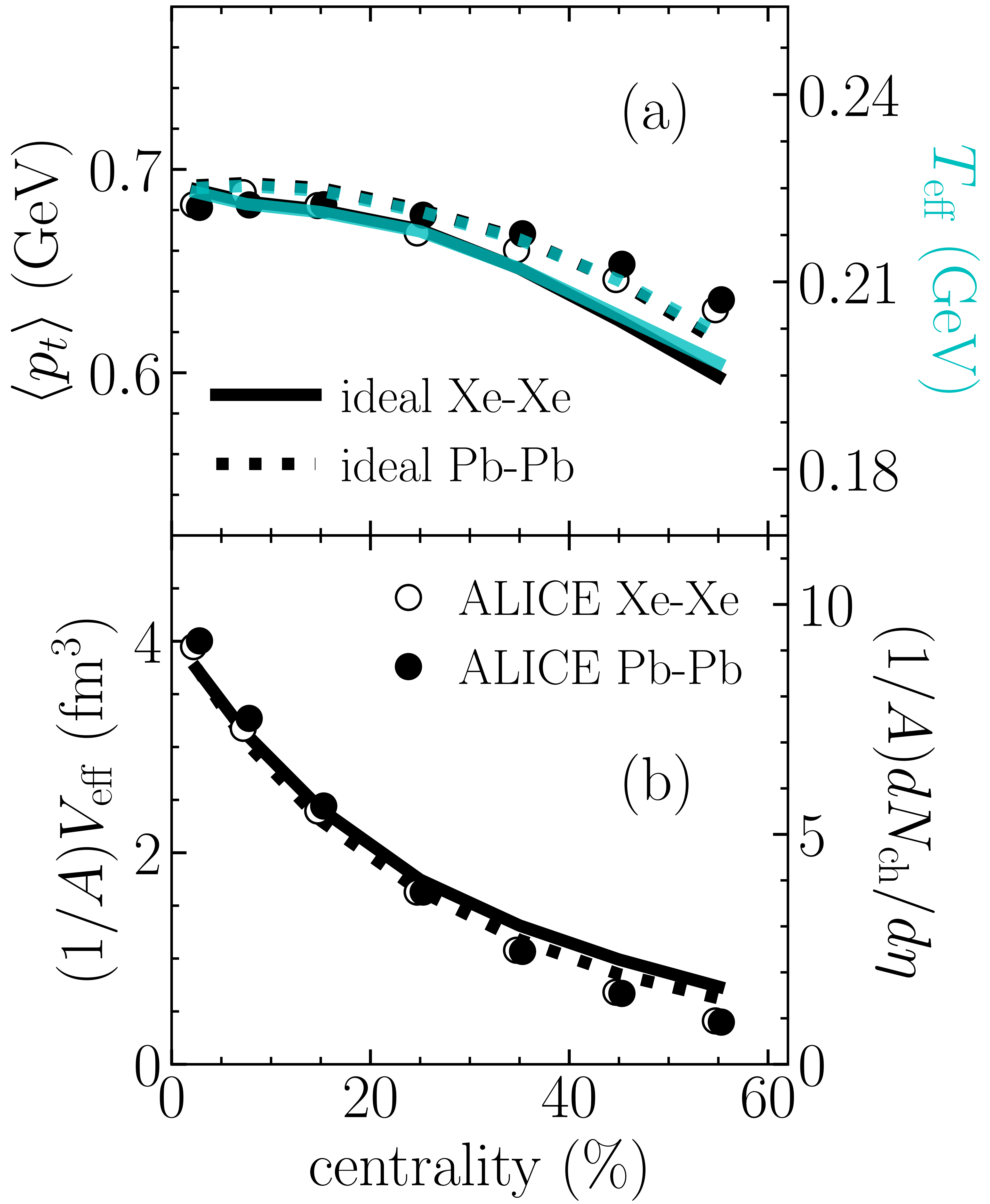
Hence, event-to-event fluctuations do not change the determination of T_{eff} from data

Definition of initial radius R_0

$$(R_0)^2 \equiv \frac{2\int_{\mathbf{r}} |\mathbf{r}|^2 s(\tau_0, \mathbf{r})}{\int_{\mathbf{r}} s(\tau_0, \mathbf{r})}$$

where $s(\tau_0, r)$ is the entropy density profile at the beginning of the hydro evolution, and integration is over the transverse plane.
The factor 2 ensures that one recovers the correct result for a uniform uniform density in a circle of radius R_0.

System-size (in)dependence

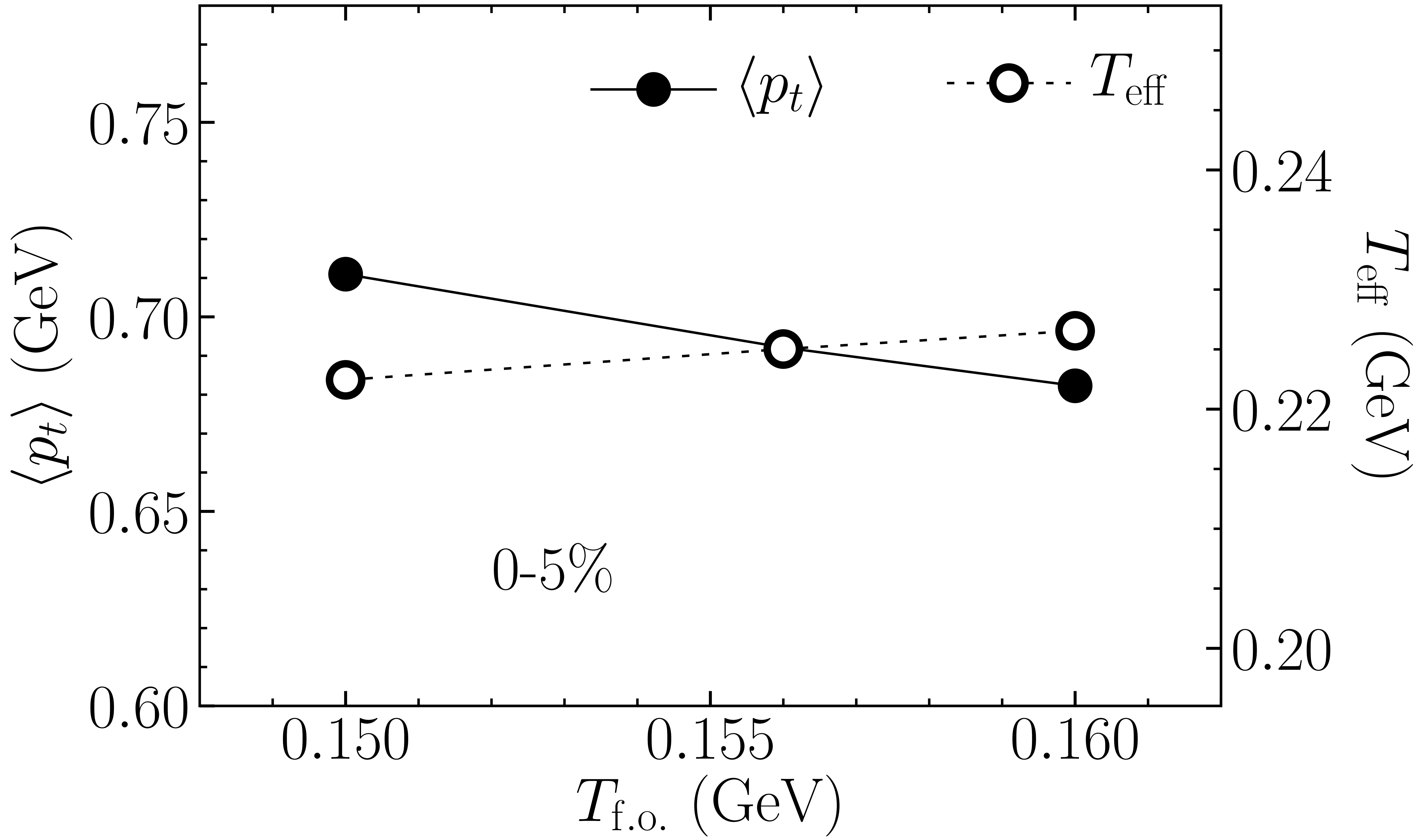


In hydro, $\langle p_t \rangle / T_{eff}$ is identical in Pb+Pb and Xe+Xe collisions.

In experiment, $\langle p_t \rangle$ is essentially the same in both systems, therefore T_{eff} is also the same.

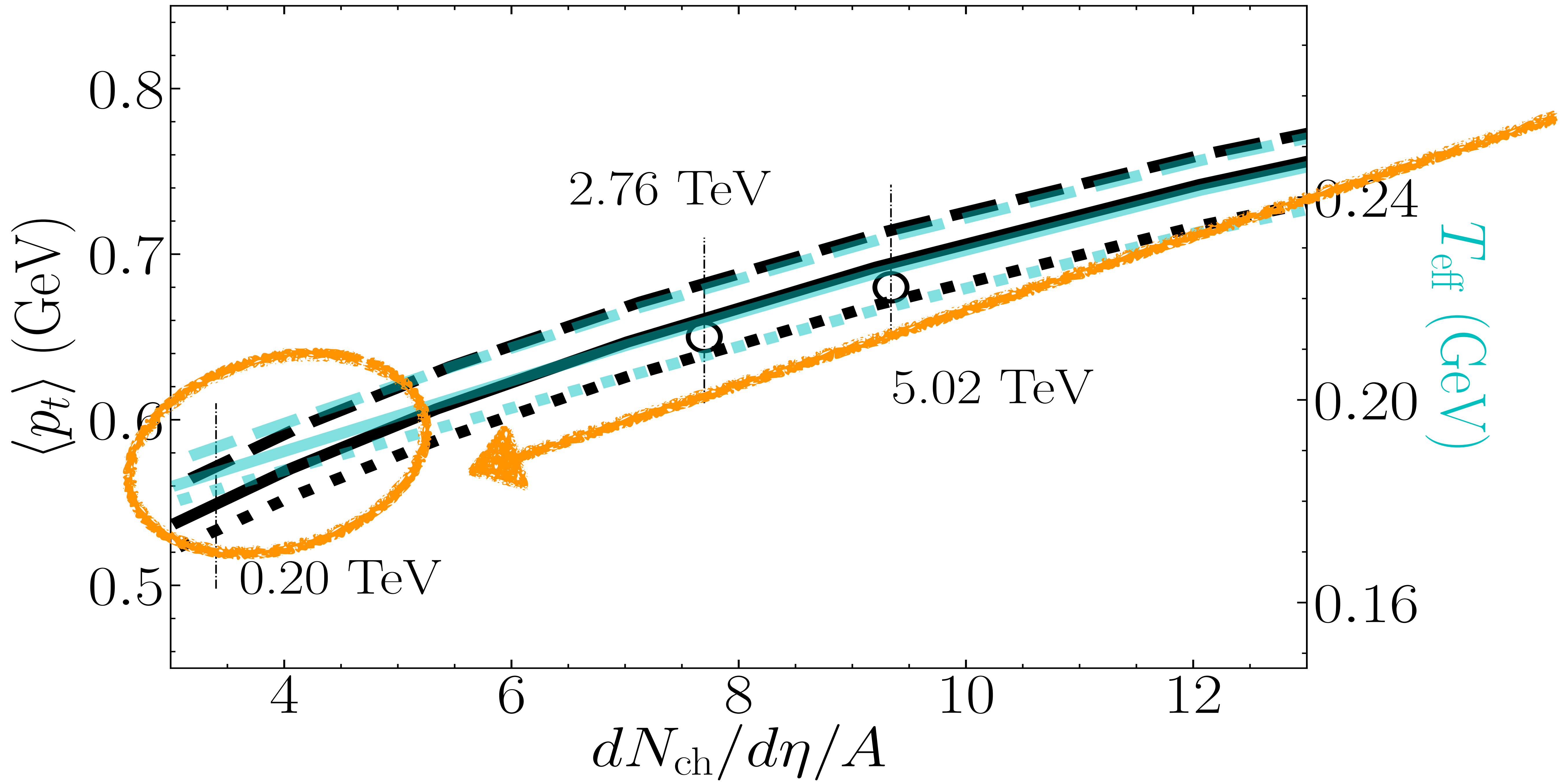
V_{eff} and the multiplicity are both proportional to A at a given centrality percentile.

Varying the freeze-out temperature



$T_{\rm eff}$ is remarkably independent of the freeze-out temperature, which confirms that the longitudinal cooling is no longer active when T<160 MeV.

Thermodynamic details



At RHIC energies, $<p_t>$ is slightly steeper than T.

Back in 1987, I showed that around the transition region, $<p_t>$ follows the energy over entropy ratio ε/s, rather than T.

In a baryonless plasma, ¾ T < ε/s < T so ε/s and T are almost proportional.
Hadron to QGP transition: T is almost constant, but ε/s keeps increasing.
This is probably what we see here.