Light clusters and the nuclear equation of state

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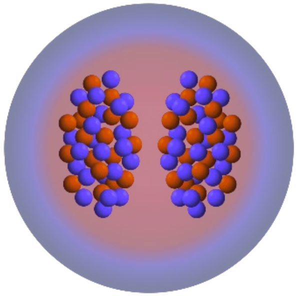
PHQMD

EMMI Workshop: Collective phenomena and the equation of state of dense baryonic matter
Darmstadt November 10-13



Outline

The hadronic equation of state (EOS) is not an observable
➔ it can only be explored by a collaboration of theory and experiment

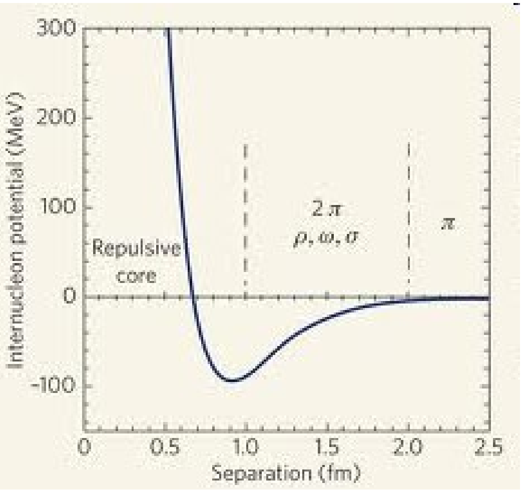
- **Which tools we have on the theoretical side**
  - many-body theory
  - transport approaches (BUU and QMD)

- **Which information do we have from HIC?**
  - kaon production (not discussed today)
  - directed and elliptic flow of protons and clusters

- **What is our present knowledge of the EOS?**

- **How can we improve?**

How to obtain theoretically the EoS from NN potential?



The Hamiltonian (in the Schrödinger equation) contains the V_{NN} potential which has a hard core :

- Made already TDHF calculations impossible
- makes also Vlasov transport calculations impossible (Bodmer 75)

Note: hard core → hard scattering has in reality not been observed in low energy collisions

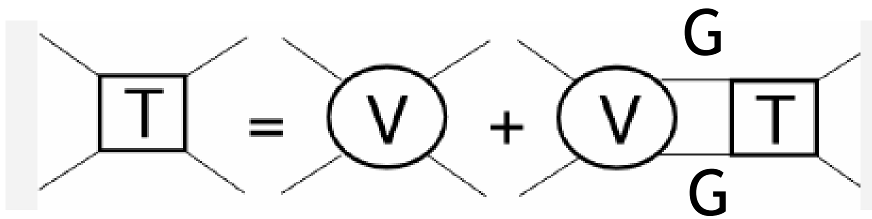
In a nuclear environment we have an effective potential which can be determined by many body techniques.
This potential enters the (time dependent) Schrödinger equation.

Colloquially we say: « Collisions are Pauli blocked », in reality complicated many body correlations are created in a nucleus .

Difficult to handle in transport approaches, several approximations possible

How to obtain theoretically the EoS: : Brueckner theory

Solution (taken over from TDHF):
In a many body system the NN potential V_{NN} has to be replaced by solution of the T(G)-matrix approach (Brueckner)



Contains info about environment

is complex

$$T_\alpha(E;q,q') = V_\alpha(q,q') + \int k^2 dk\, V_\alpha(q,k)\, G^0_{Q\overline{Q}}(E,k)\, T_\alpha(E;k,q')$$

Consequences:

V_{NN} is real → T is complex = ReT + i Im T

Replaces V_{NN} in Hamiltonian is smooth (Skyrme)

σ_{elast} collisions done identically in BUU and QMD

BUU (testp.) and QMD (part)

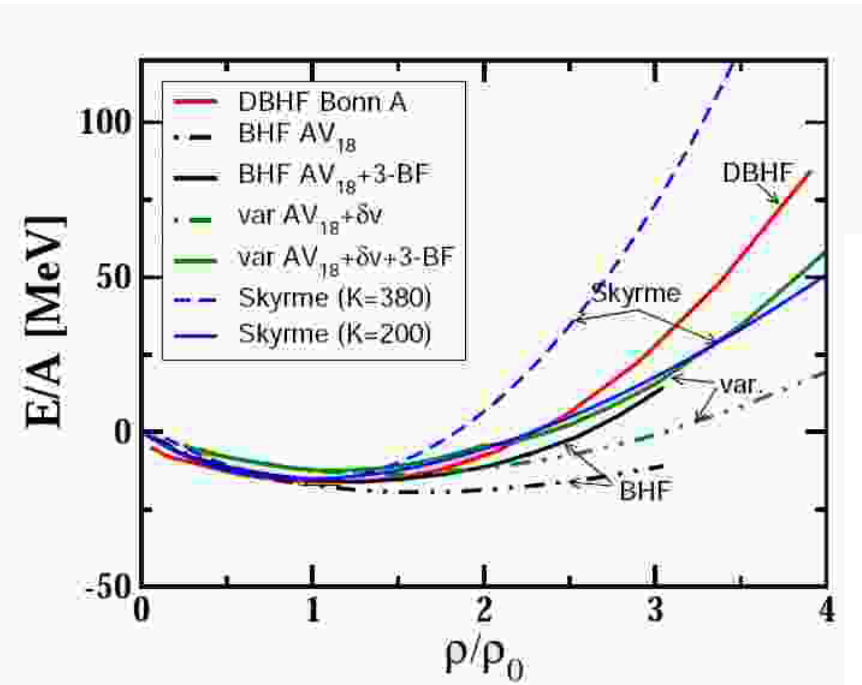
To this one adds inelastic collisions (BUU and QMD same way) !

How to obtain theoretically the EoS

Problem 1:

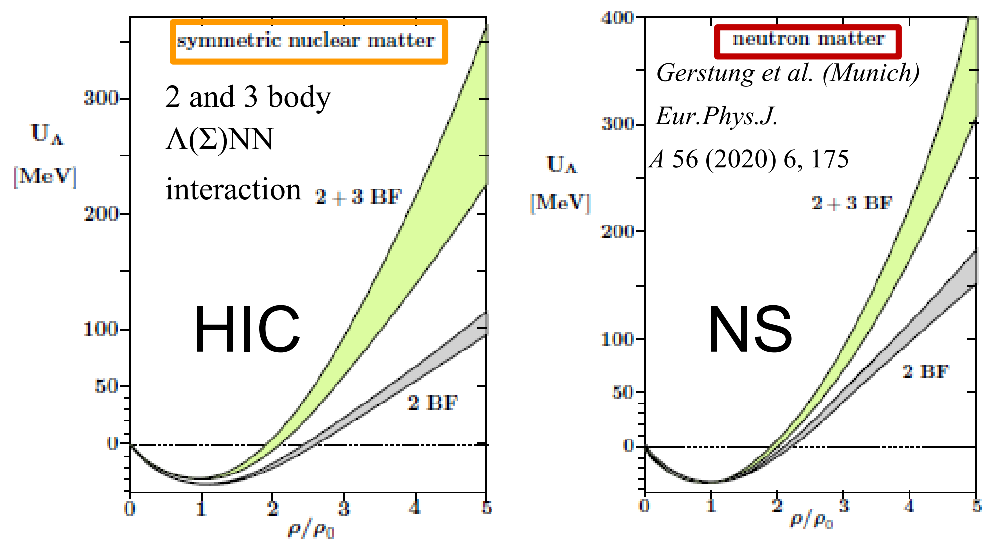
Brueckner T(G)– Matrix is an expansion of the of the many body amplitudes in $\frac{ap_F}{\hbar}$; a=range of NN pot, p_F =Fermi momentum

If $\frac{ap_F}{\hbar}$ >> 1 other many body diagrams (like 3-body) important



Different flavors of Brückner and Skyrme parametrization at T=0

Additional challenges:
3-body ΛNN potential in symmetric and asymmetric matter



EoS for $\rho >> \rho_0$ can presently not be obtained from theory

Solution: Transport Approaches

Idea: simulate the HI collision for different parametrizations of the EoS

Caveat: We want to study the EoS at high density but in heavy-ion collisions high densities phase (> ρ_0) existes only for a short time

→ We have to study how signals from the high density reach detector

→ We need a full microscopic description of the expanding system
(Hydrodynamics: not applicable at beam energies sensitive to the EOS)

How we can formulate the transport approaches*?

I. BUU/VUU type approaches (solve the time evolution of the one particle phase space density f(r,p,t))
- models: BUU, HSD,GiBUU, AMPT, SMASH, …
Kadanoff-Baym - PHSD

II. QMD type approaches (solve the time evolution of test wave fcts)
- models: IQMD, UrQMD, AMD, PHQMD, JAM, …

* In this talk I limit myself to the most simple nonrelativisitc versions of them

Basis of the BUU/VUU

Starting point for a quantal particle is the Schrödinger equation for a particle in a time dependent potential

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \hat{H}|\psi(t)\rangle$$

We can construct a density matrix

$$\rho(\mathbf{r},\mathbf{s},t) = \langle \mathbf{r}+\mathbf{s}/2|\psi(t)\rangle\langle\psi(t)|\mathbf{r}-\mathbf{s}/2\rangle$$

and the density matrix can be transformed in a Wigner density:

$$W(\mathbf{r},\mathbf{p},t) = \int_{-\infty}^{\infty} d\mathbf{s}\; e^{-i\mathbf{s}\mathbf{p}/\hbar}\langle \mathbf{r}+\mathbf{s}/2|\psi(t)\rangle\langle\psi(t)|\mathbf{r}-\mathbf{s}/2\rangle$$

It contains the same information but is a function of p and r,
hence of the classical phase space variables.
Very useful for semiclassial approaches

Basis of the BUU/VUU: EoM

Time evolution equation of W:

$$\frac{\partial}{\partial t}\rho = \frac{-i}{\hbar}[H,\rho] \Longrightarrow \frac{\partial}{\partial t}W(\mathbf{r},\mathbf{p},t) + \frac{\mathbf{p}}{m}\frac{\partial}{\partial \mathbf{r}}W(\mathbf{r},\mathbf{p},t) + \underbrace{\sum_{m=0}(-\hbar^2)^m \frac{1}{(2m+1)!}\left(\frac{1}{2}\right)^{2m}\left[\frac{\partial^{2m+1}}{\partial \mathbf{r}^{2m+1}}V(\mathbf{r})\right]\left(\frac{\partial}{\partial \mathbf{p}}\right)^{2m+1}W(\mathbf{r},\mathbf{p},t)}_{\hbar\to 0\ \Longrightarrow \frac{\partial V(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{p}}W(\mathbf{r},\mathbf{p},t)}$$

In the semiclassical limit

$$\hbar \to 0 \qquad\qquad \hbar \cdot \frac{\partial V(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{p}}W(\mathbf{r},\mathbf{p},t) << 1$$

We obtain a Vlasov eq.

$$\frac{\partial}{\partial t}W(\mathbf{r},\mathbf{p},t) + \frac{\mathbf{p}}{m}\frac{\partial}{\partial \mathbf{r}}W(\mathbf{r},\mathbf{p},t) + \frac{\partial V(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{p}}W(\mathbf{r},\mathbf{p},t) = 0$$

$W(\mathbf{r},\mathbf{p},t)$ **is the one nucleon distribution function**

$V(\mathbf{r})$ **is the real part of the Brueckner T(G)-matrix usually parametrized as**

$V(\mathbf{r}) = V(\rho(\mathbf{r}))$ **in diff. parametrizations for diff EoS**

Im(T) is treated as a collision term → Boltzmann equation

Solution by test particle ansatz: $W(\mathbf{r},\mathbf{p},t) = \sum^{N_N * N_T} \delta(\mathbf{r}-\mathbf{r}_i(t))\delta(\mathbf{p}_i-\mathbf{p}_i(t))$

Gives the right solution for $N_T \to \infty$

Fluctuations due to finite N_T have no physical significance

QMD time evolution

Dirac-Frenkel-McLachlan approach
A. Raab, Chem. Phys. Lett. 319, 674
J. Broeckhove et al., Chem. Phys. Lett. 149, 547

- **Dirac-Frenkel-McLachlan variational principle**: $\delta \int_{t_1}^{t_2} dt < \psi(t)| i \frac{d}{dt} - H |\psi(t) >= 0.$

Many-body wave function:

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ **for N particles (neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

[Aichelin, Phys. Rept. 202 (1991)]

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m} t\right)^2} \cdot \mathrm{e}^{i \mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i \frac{\mathbf{p}_{i0}^2(t)}{2m} t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) in coordinate and momentum space:**

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

Many-body Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$

2-body potential: $Re\ T_{i,j} = V_{i,j} = V(\mathbf{r}_i, \mathbf{r}_j, (\mathbf{p}_i, \mathbf{p}_j), t)$

Antisymmetrization is neglected since impossible to formulate collision term consistently

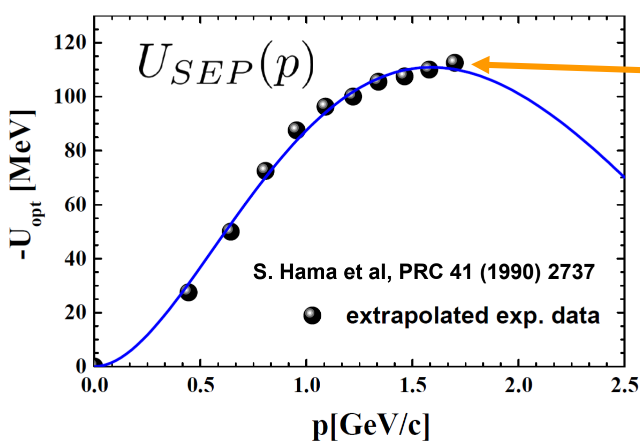
Static potentials are not sufficent

Results of pA elastic scattering show: the T(G)-matrix potential is momentum dependent

Analysis of the elastic pA data done with Dirac equation containing vector U_μ and scalar potentials U_s

Nonrelativistic reduction leads to Schrödinger equivalent potential

$$U_{SEQ}(p) = U_s + U_0 + \frac{2}{m_N}(U_s^2 - U_0^2) + \frac{U_0}{m}(\sqrt{p^2+m^2} - m)$$



Data: U_μ and U_s depend on p!!

No data for p > 1.7 GeV Ekin > 1 AGeV
→limits strongly the predictive power for FAIR

BUU: difficult to implement it for heavy-ion collisions because averaging over nuclear momenta is needed
Different schemas applied: Gale, SMASH,

Straight forward in QMD

Momentum dependent potential → EoS in PHQMD

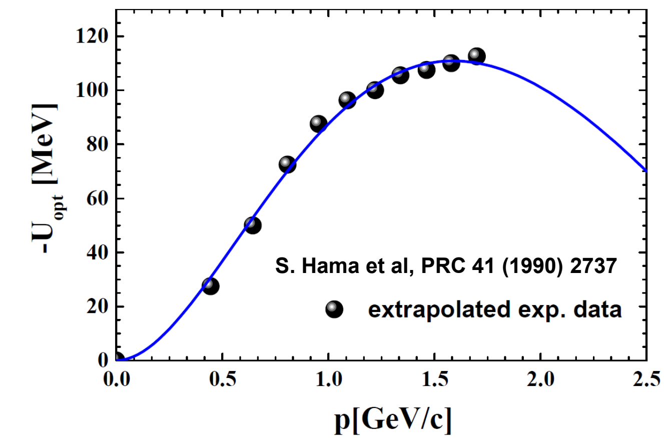
Momentum dependent potential :

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters a, b, c are fitted to the "optical" potential (Schrödinger equivalent potential U_{SEP}) extracted from elastic scattering data in pA:

$$U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p} - \mathbf{p}_1) dp_1^3}{\frac{4}{3}\pi p_F^3}$$



- **In infinite matter a potential corresponds to the EoS:**

$$E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{mom} = (a\Delta p + b\Delta p^2)\ exp(-c\sqrt{\Delta p})\ \frac{\rho}{\rho_0}$$

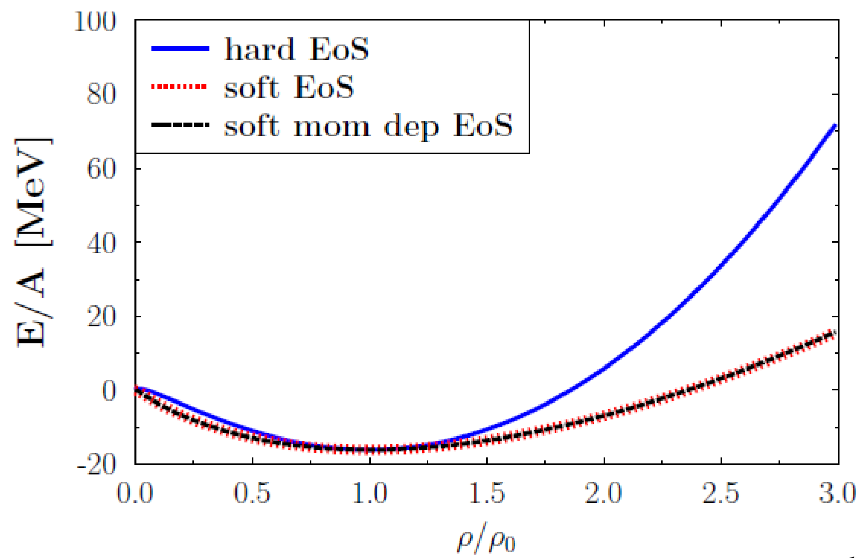
$$V_{Skyrme} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200
	a $[MeV^{-1}]$	b$[MeV^{-2}]$	c$[MeV^{-1}]$	
	236.326	-20.73	0.901	

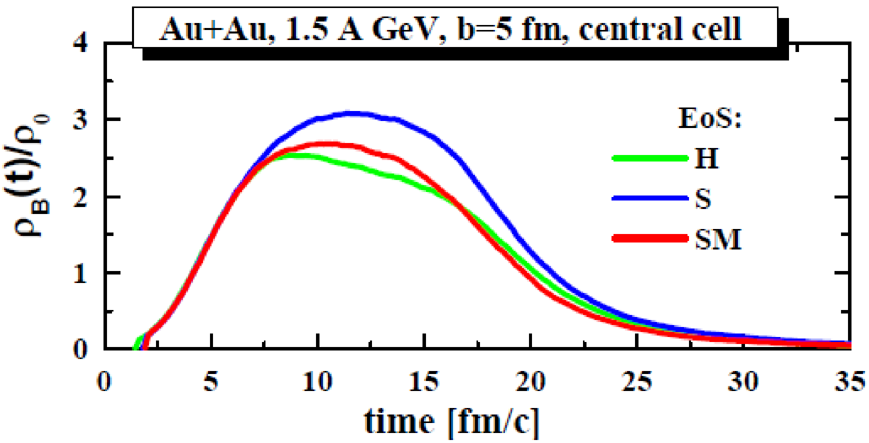
EoS for infinite cold nuclear matter at rest

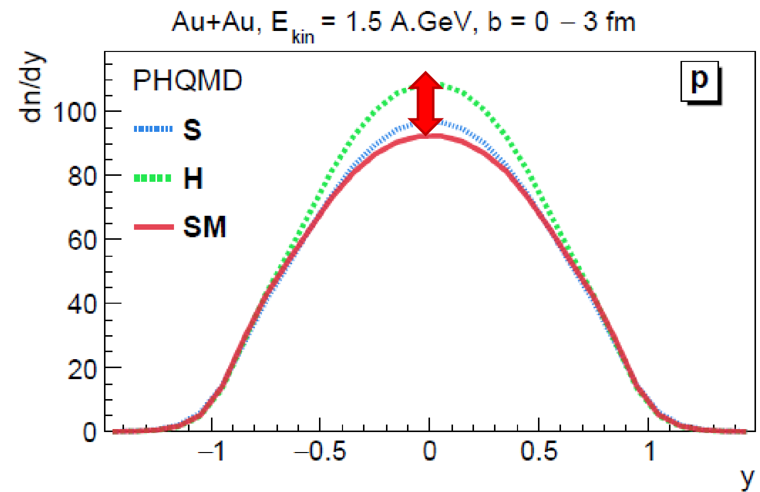


Consequences of EoS on heavy-ion dynamics

EoS: S= soft EoS, H=hard EoS, SM = soft momentum dependent EoS

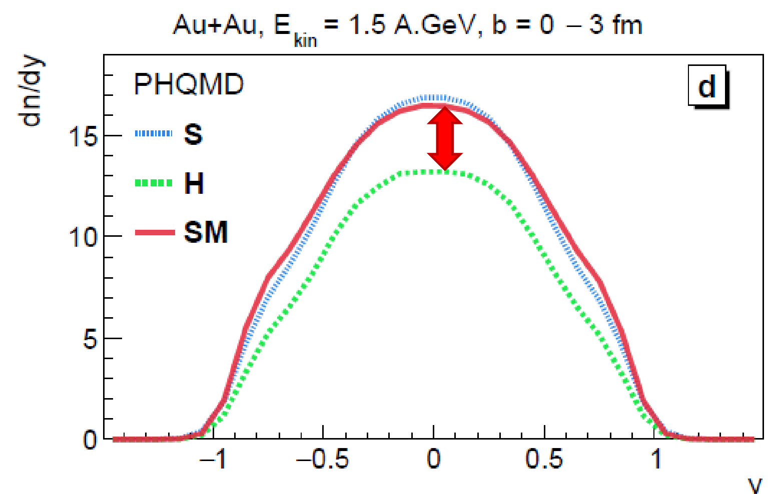
- **Nucleon dynamics is sensitive to EoS:**
  baryon density depends on the EoS





➔ **H,S,SM potentials (forces) act differently on different observables:**

1) **yield dN/dy at midrapidity:**
   protons: SM ≈ S < H
   deuterons: SM ≈ S > H



Cf. earlier studies (80^{th}, 90^{th}, …) by J. Aichelin, W. Cassing, P. Danielewicz, C. Fuchs, C.-M. Ko, U. Mosel, H. Stoeker, ….

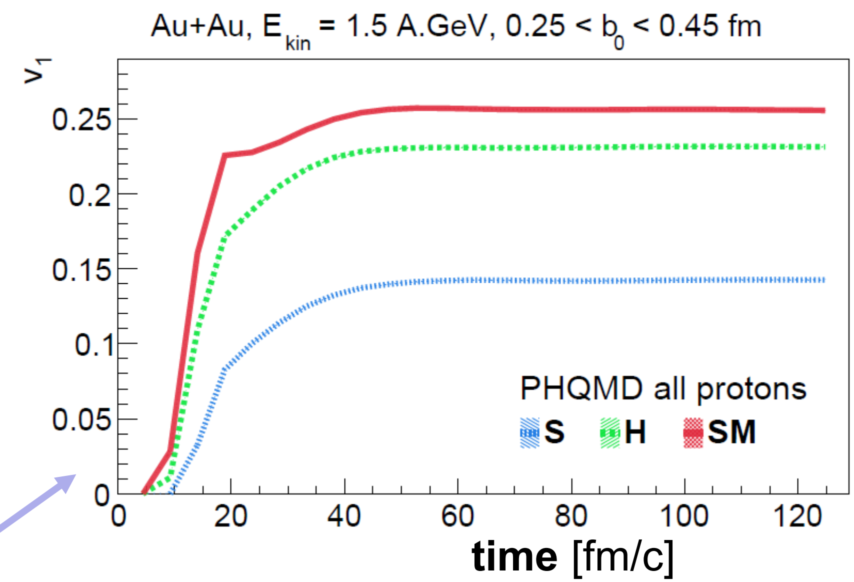
Consequences of EoS on heavy-ion dynamics

EoS: S= soft EoS, H=hard EoS, MS = soft momentum dependent EoS

➔ **H,S,SM potentials (forces) act differently on different observables:**

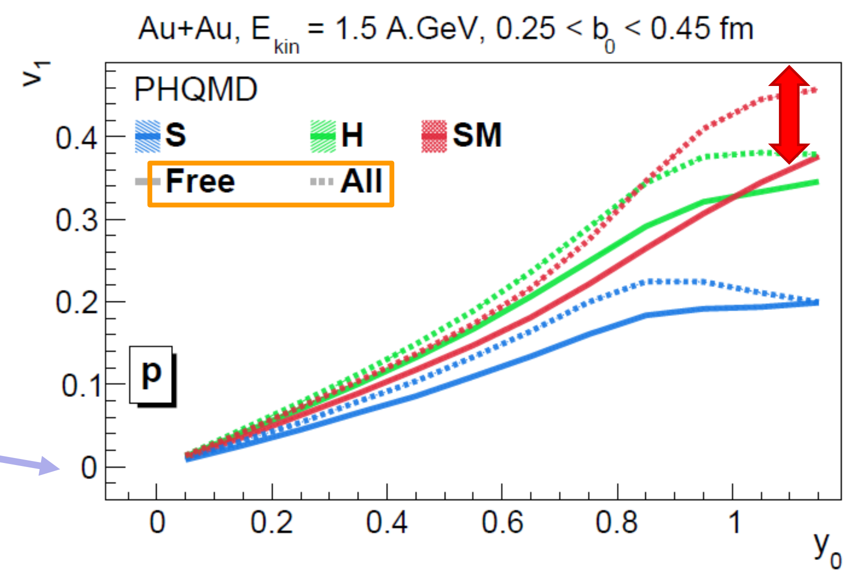
1) yield dN/dy at midrapidity:
protons: SM ≈ S < H

2) directed flow v_1:
protons: SM ≈ H > S



- **Flow v_1 with SM develops earlier than for H EoS and much earlier than for S EoS**

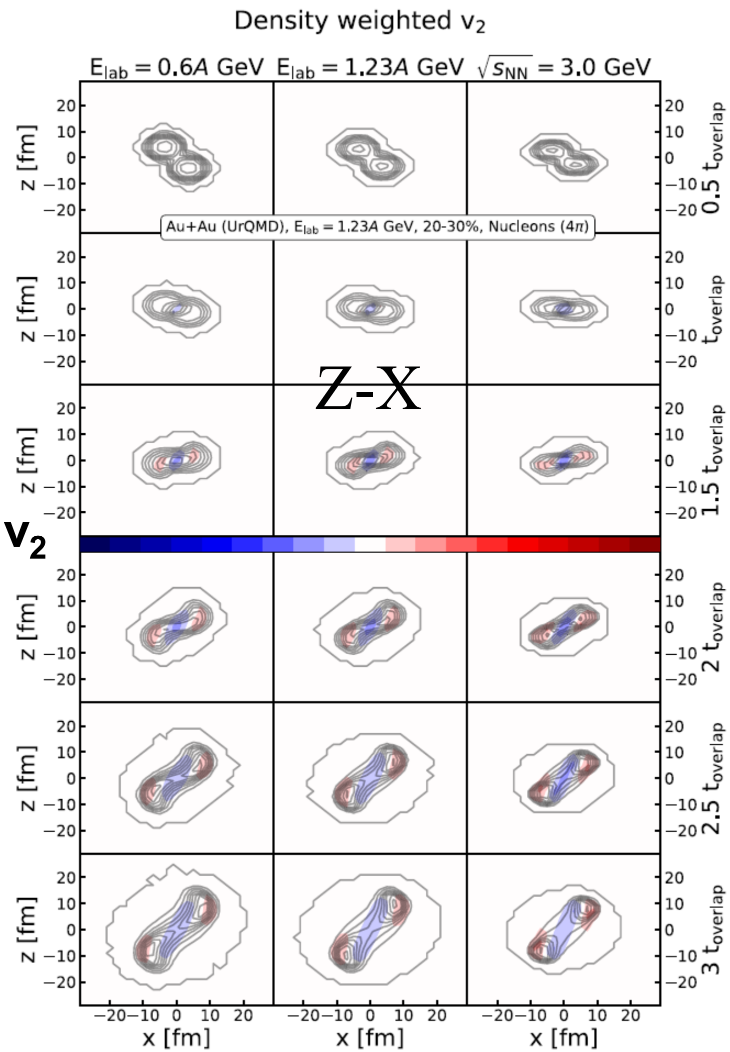
- **$v_1(y)$ of p in clusters are larger than of free (unbound) p**



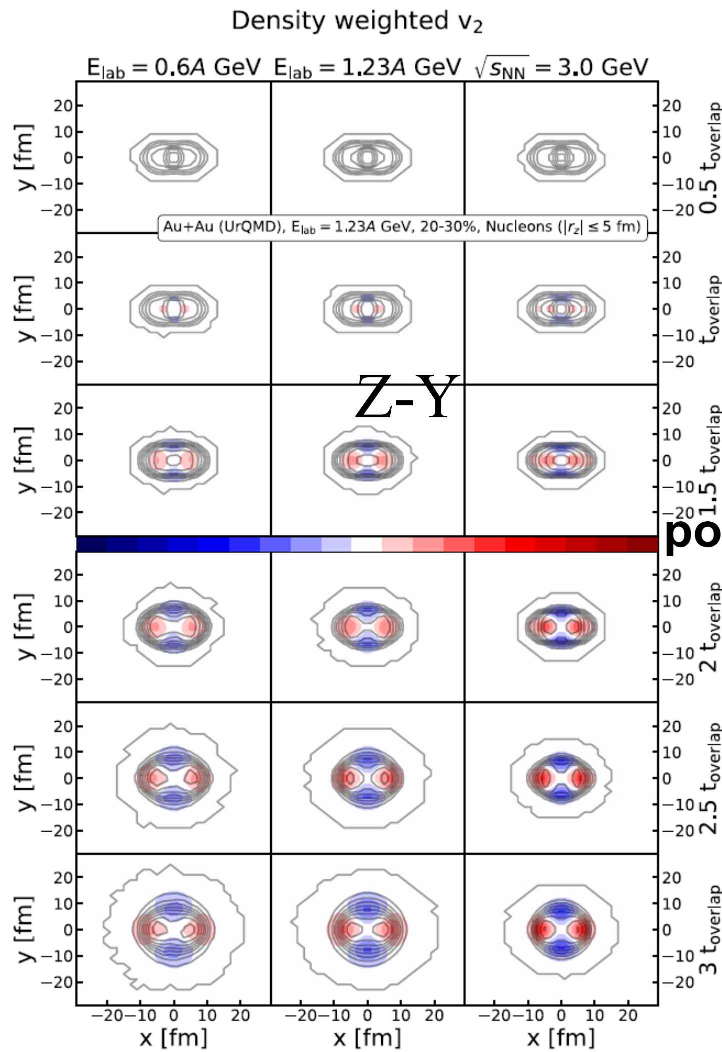
Theory is complicated but nature as well

T. Reichert PRC111, 054916

v_1 and v_2 are very complex variables



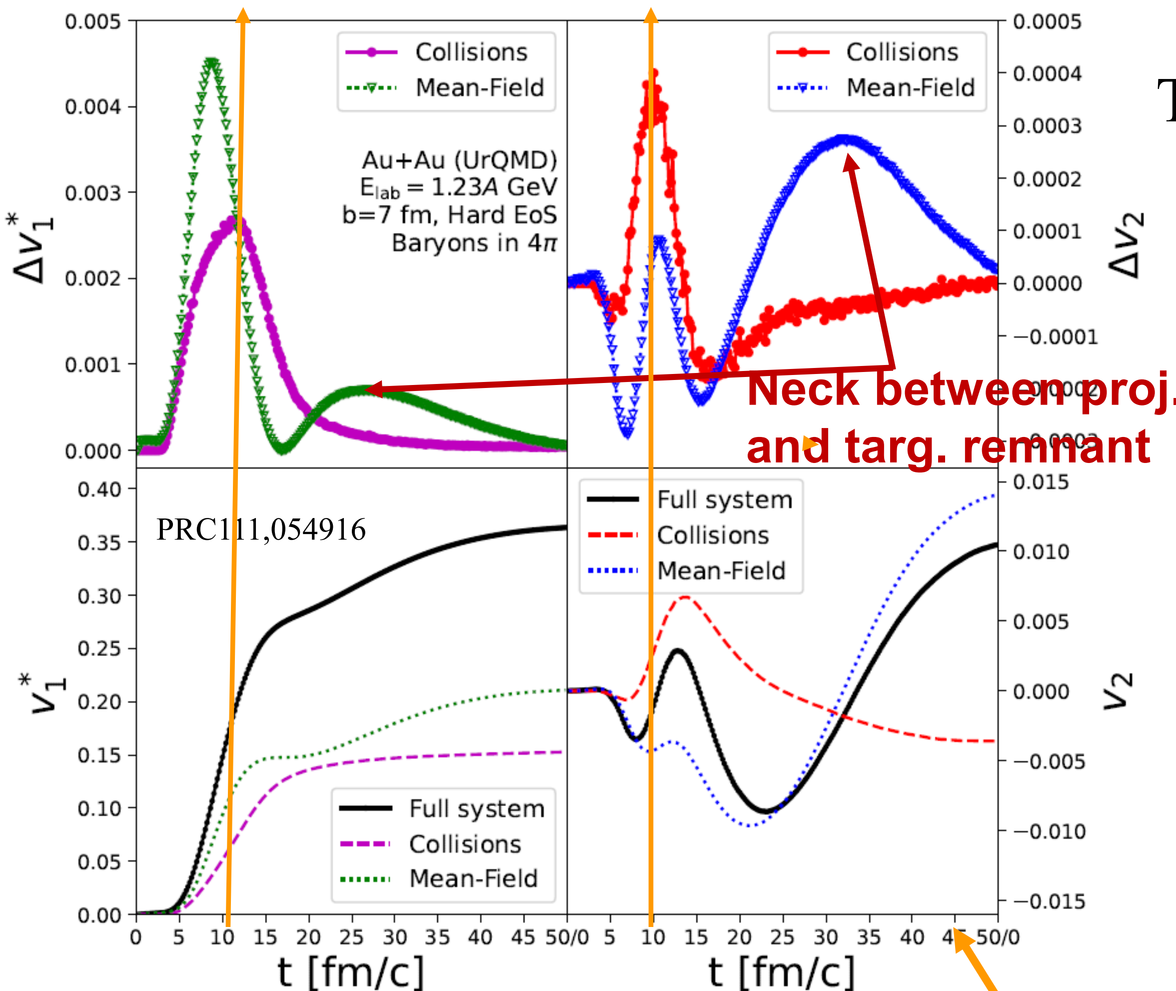
neg v_2



pos v_2

v_1 , v_2 : complex interplay between collisions and potential

4π, midrapidity similar



$T_{overlap} = 9$ fm/c

Collisions counteract potential

v_2 develops very late

$5x\ T_{overlap}$

Final v_2: more final state effect than shadowing

A lot remains still to be done before this complexity is understood

Cluster formation in PHQMD (→ E.Bratkovskaya Monday)

It is notoriously complicated to obtain from a semiclassical transport approach the probability that a genuin quantum object like a cluster is formed

Up to now only phenomenoligical approaches

a) Wigner density (Danielewisz, Remler)
(in medium modification of cluster Wf has to be known)

b) Coalescence: cluster produced at a given time t if

$$\sqrt{(\mathbf{r}_i(t)-\mathbf{r}_j(t))^2} < R_{max}; \sqrt{(\mathbf{p}_i(t)-\mathbf{p}_j(t))^2} < P_{max}$$

c) Cluster produced by potential interaction (IQMD)

$$\sqrt{(\mathbf{r}_i-\mathbf{r}_j)^2} < 4fm; E_{bind} < 0$$

4 fm = range of NN potential

identifies clusters at any time

d) deuteron produced by collisions NN→ dX by cross sections

In standard PHQMD a combination of c) and d) is applied

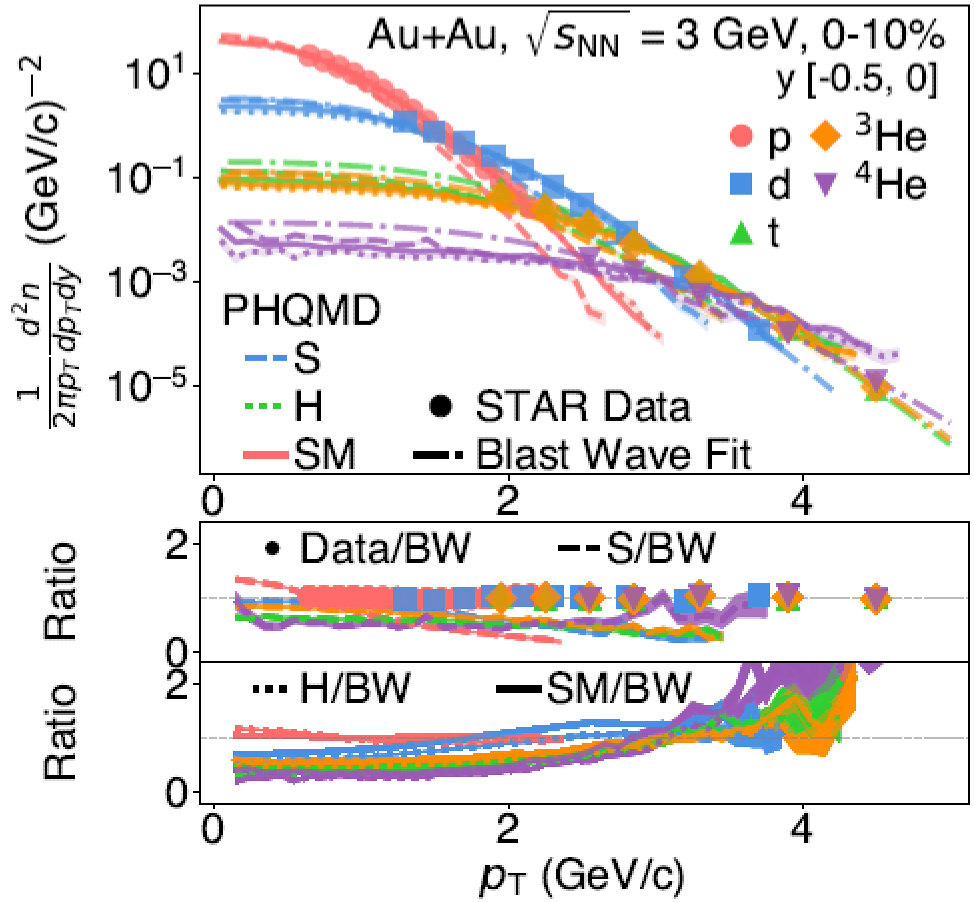
Does PHQMD describe the data ?

Before discussing v_2 we have to be sure that PHQMD describes The p_T spectra of protons and clusters

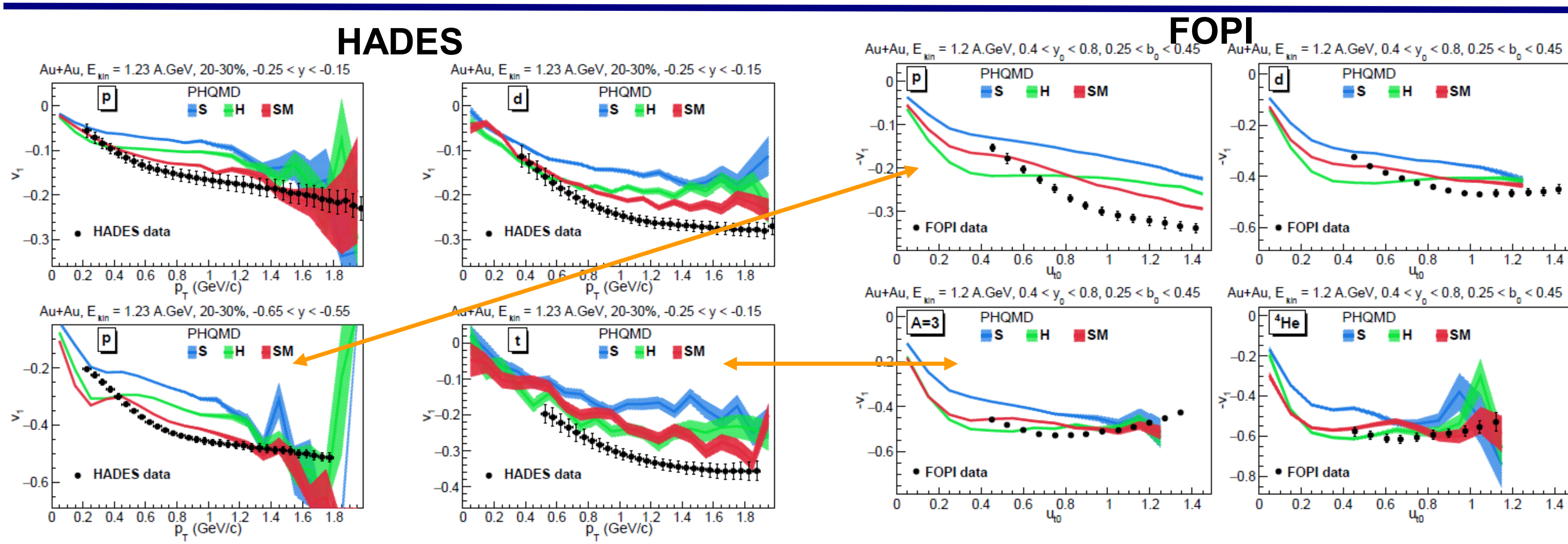
We find deviations of max 50% over 6 orders of magnitude!

Results by the STAR collaboration

2507.14255

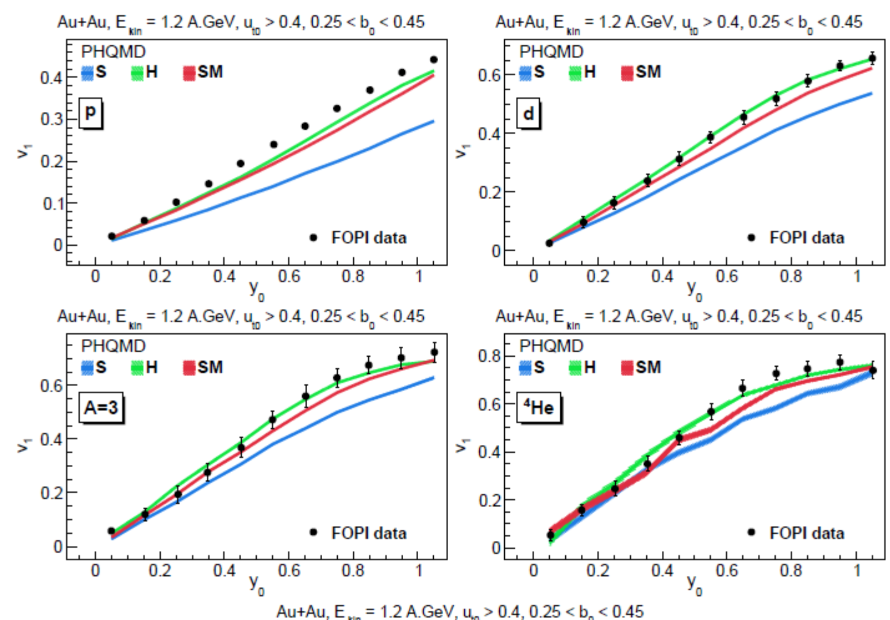


Different data sets of v_1 at 1.2 AGeV are not free of tensions



Rapidity distribution well reproduced

2411.04969



which may come from
the different kinematical range
or from tensions among the data

Difference between different EoS
small but S is above all data

Agreement for clusters even better
than for free protons

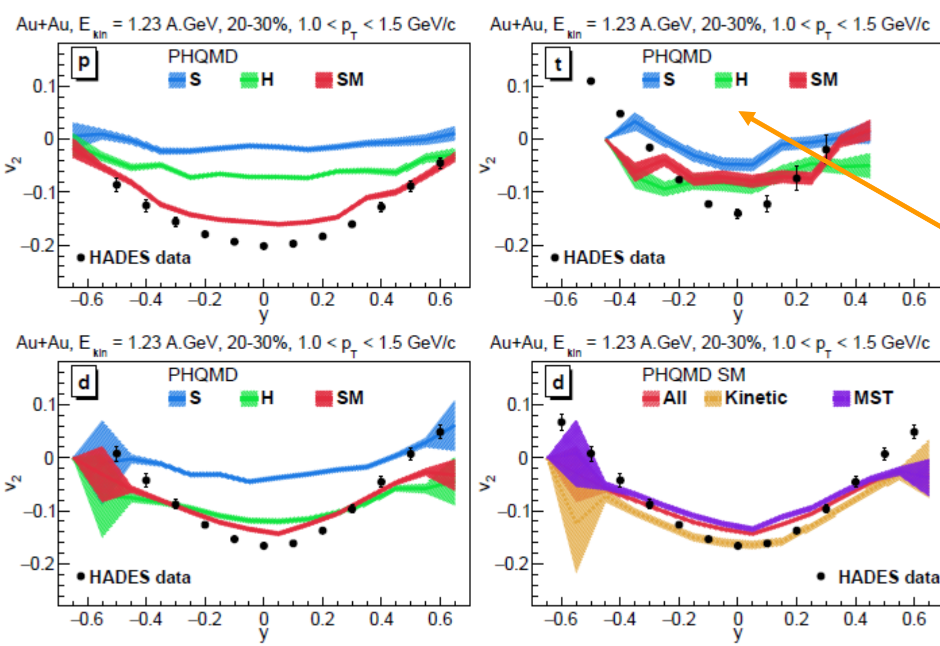
Different data set of v_2 at 1.2 AGeV are compatible

in rapidity the v_2 data set are compatible and agree with PHQMD

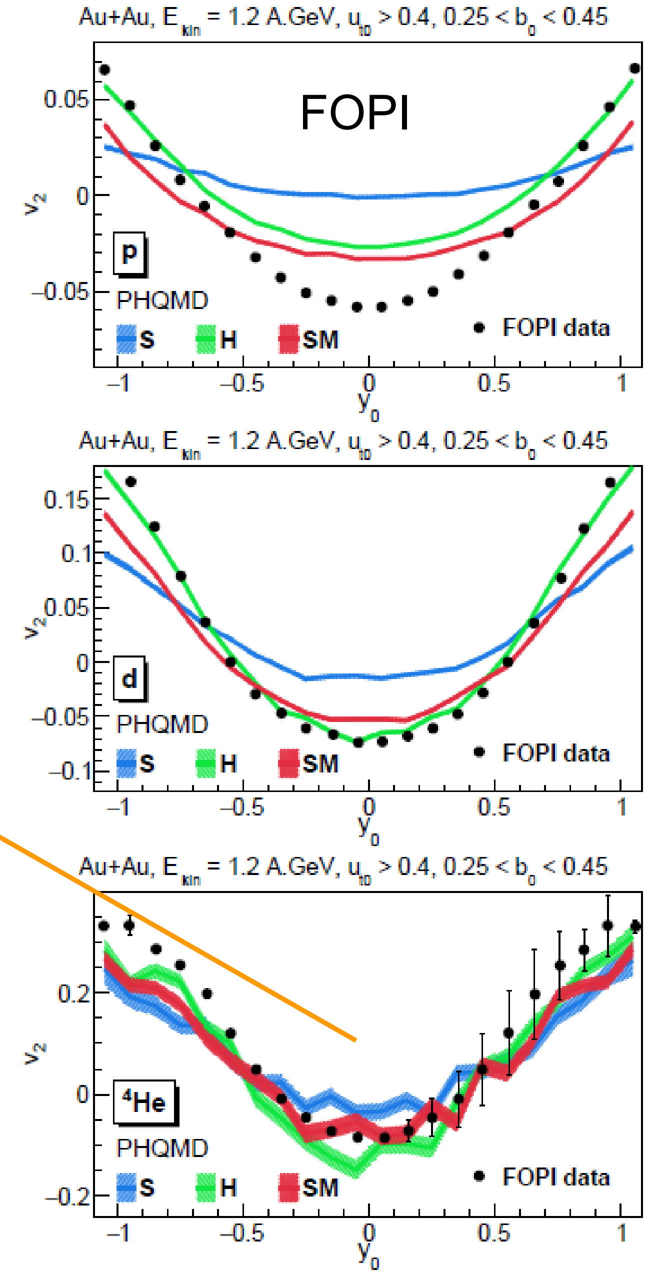
Difference for p larger than for clusters

Why Hades A=3 data differ from FOPI
A=4 data not understood

HADES



FOPI



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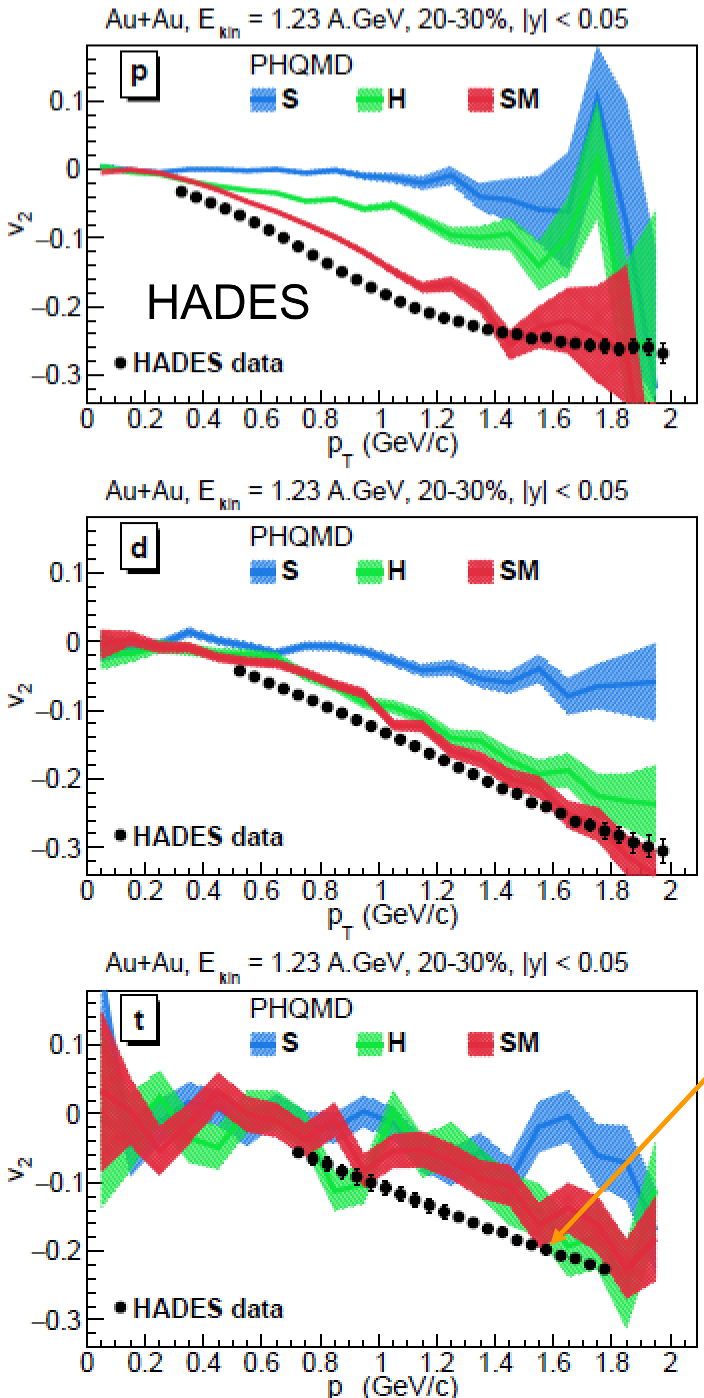

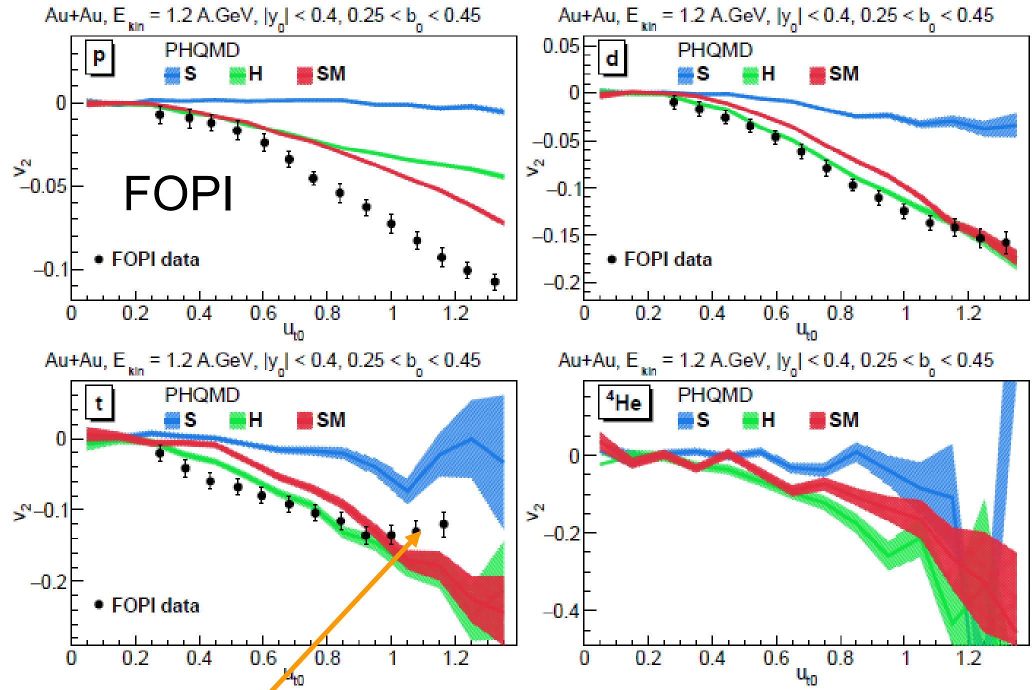

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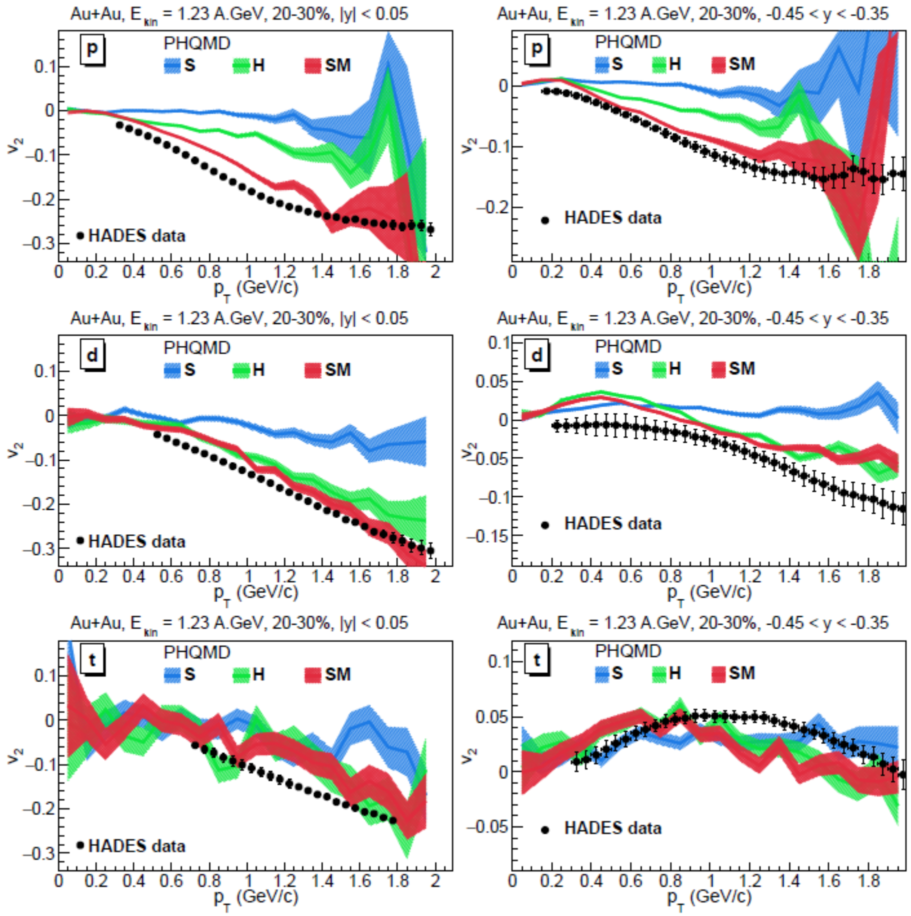
Most of p_T spectra similar for HADES and FOPI

For protons and at large p_T there are tensions

PHQMD: p and cluster reasonably reproduced

As a function of rapidity the form of the $v_2(p_T)$ changes rapidly (but in exp more than in theory)



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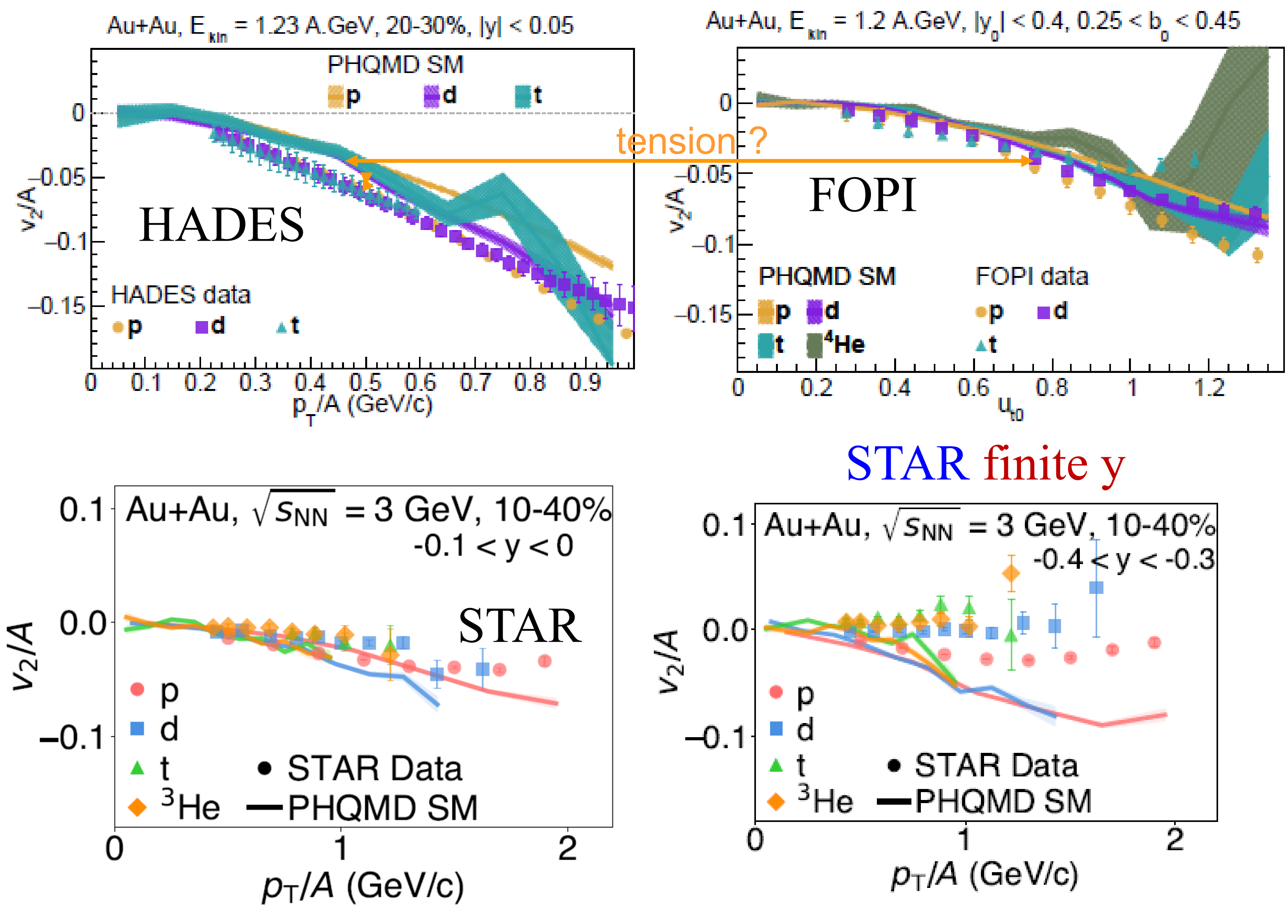
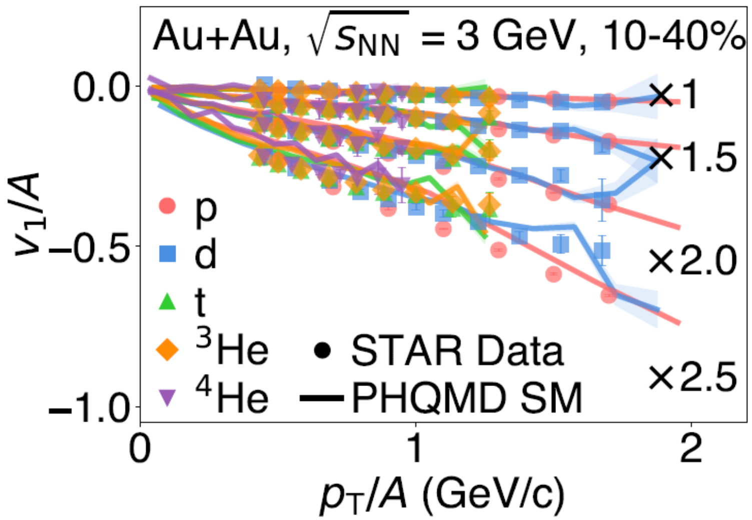
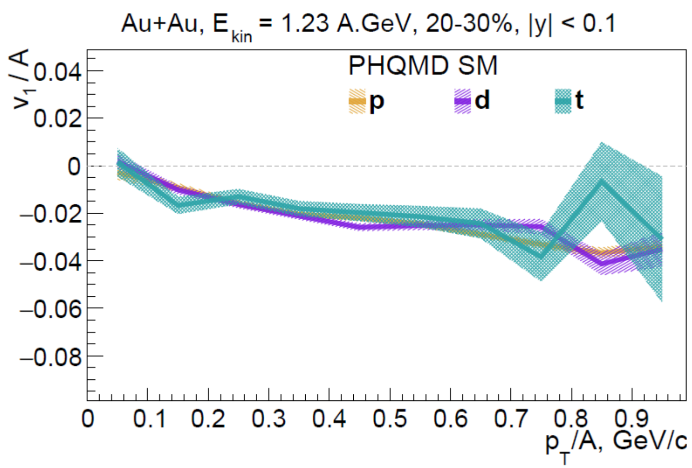
v_2 scales at midrapdity but not at finite y



FOPI/HADES :2411.04969 STAR: 2507.14255

v_1 scales at midrapdity as well

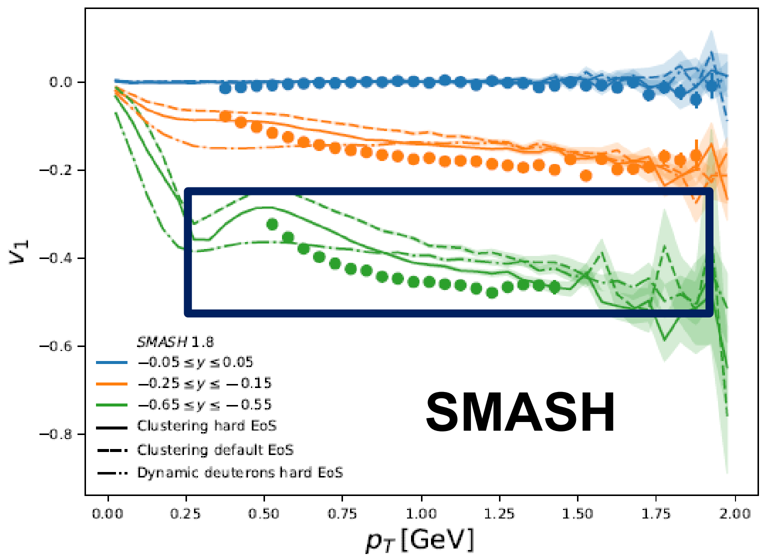




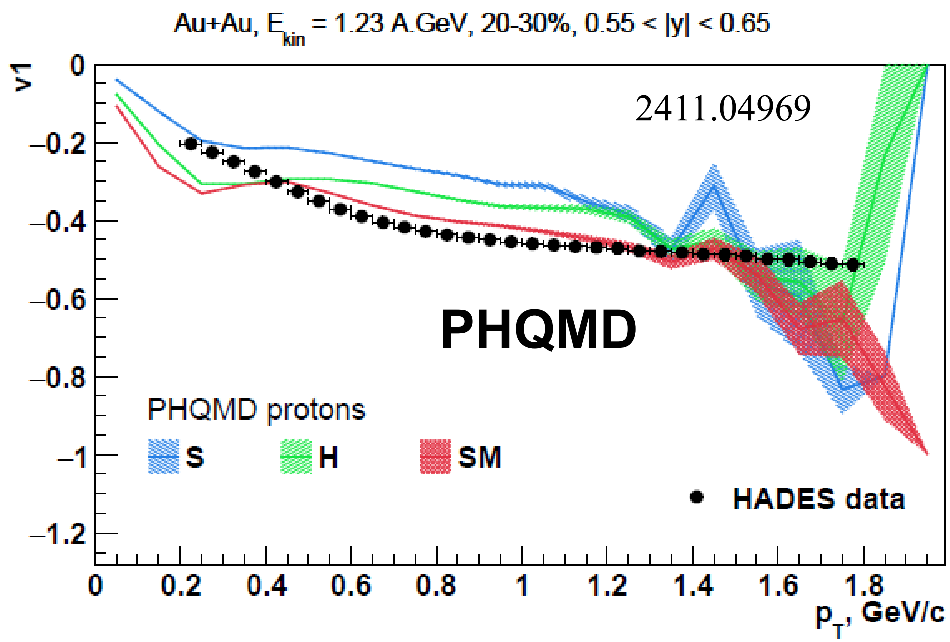
FOPI/HADES :2411.04969 STAR: 2507.14255

.. and what other models obtain for protons

Comparison of the state of the art transport models with the most precise data



Mohs, PRC 105 (2022) 3



- **Differences between the models for protons are tiny but as large as the difference between different EOS**
- **One has first to understand where the differences come from before flow can be used to nail down the EOS**
- **Systemtic experimental studies (A,E,y,p_T) would help a lot**

Summary

To extract the EoS from HI collisions is a complicated business but there is no choice
many body physics fails at high densities and temperatures
transport theories have different ingredients but should give the same EoS
the observables sensitive to the EoS reflect complicated physical processes

It can be considered as big success of the transport approaches that they can reproduce the flow data qualitativly.

For a quantitative prediction it is necessary to understand in detail what influences the results (→ Wolter) (Pauli blocking, cross section parametrization etc)

Clusters in PHQMD open a new lever arm for the study of the EoS

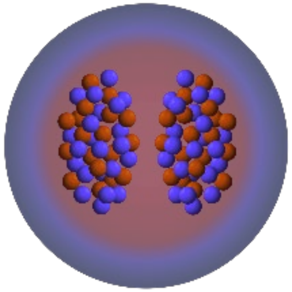
- flow of p and clusters reasonable well reproduced despite of its complex nature
- v_1 and v_2 of A ≠ 1 better produced than that of A=1
- at midrapidity we observe scaling of v_1 and v_2 as observed experimentally
- gives quantitative reasonable agreement with all cluster data
- results agree best with data for a Soft EoS with momentum dependence

→ compressibility does not change from GDR to HIC

Challenges:

exp: without precise systematic p and clusters data (v_1,v_2 (A, $\sqrt{s}$,y, p_T)) progress is difficult to imagine

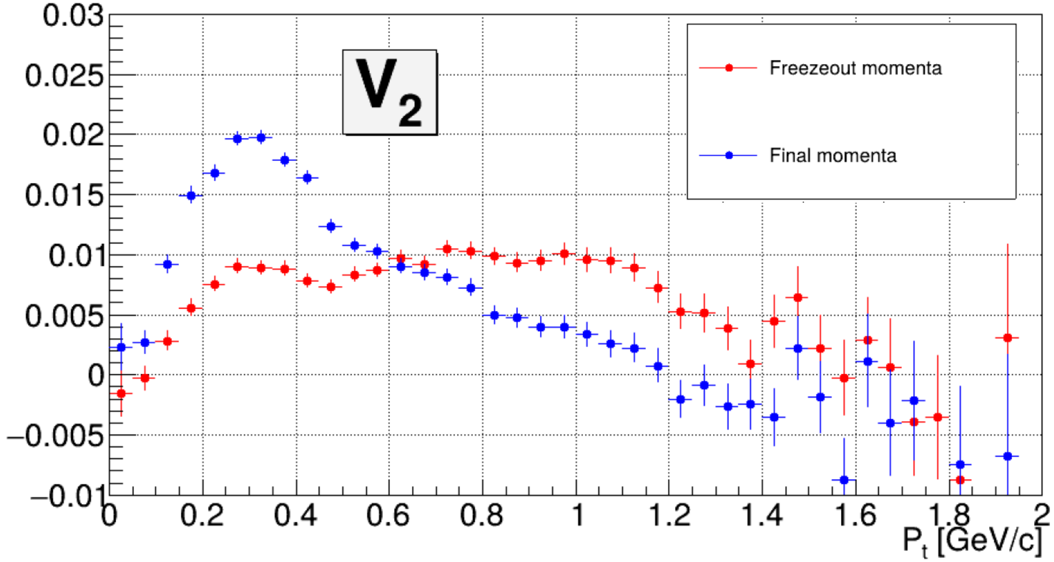
theo $U_{SEQ}(p) = U_s + U_0 + \frac{2}{m_N}(U_s^2 - U_0^2) + \frac{U_0}{m}(\sqrt{p^2+m^2} - m)$ is already dubious at E_{kin} =1.5 AGeV
fully convariant approach necessary for FAIR



Thank you for your attention !

Thanks to the Organizers !

Au+Au 3 AGeV semicentral



Tanks to D. Wielanek