Triple-Differential Yields in Heavy-Ion Collisions

Pawel Danielewicz[1] and SπRIT Collaboration

[1]Facility for Rare Isotope Beams, Michigan State University, USA

EMMI Workshop: Collective Phenomena
and Equation-of-State of Dense Baryonic Matter

GSI, Darmstadt, November 10-13, 2025

Goal: Triple-Differential Yields from Data

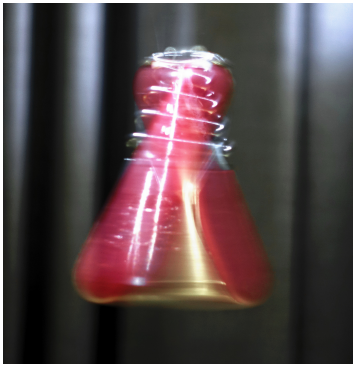
Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment

no control over plane

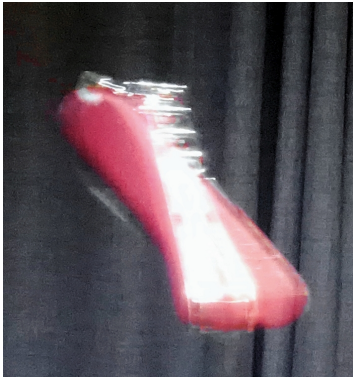
What is it?!

Goal: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment



no control over plane



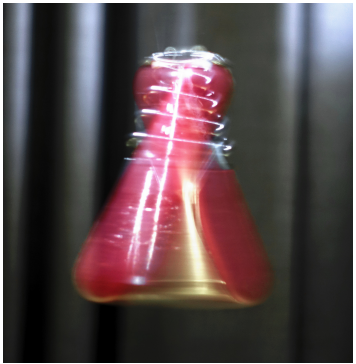
some control, v_n

Still not clear what the system is...

FRIB

Goal: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane* from Theory and Experiment



no control over plane

some control, v_n

full control, $\frac{d^3N}{dp^3}$

Claim: You can get to right panel through optical deblurring!

FRIB

Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

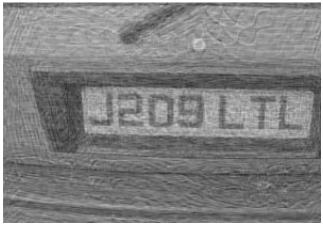
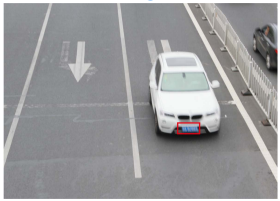


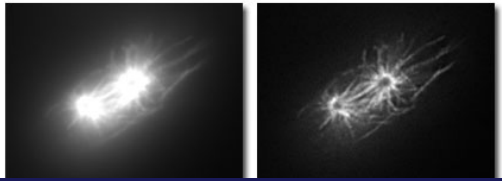
Photo of Parked Car

Fast Moving



Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon} n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\text{prod}} \simeq E_{\text{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ. Optical terminology: P - blurring or transfer function.

Bayesian Deblurring: Richardson-Lucy

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi) \, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - unknown complementary probability that ptcle has characteristic ξ while measured at ζ

Bayesian relation: number of times ptcle has characteristic in $\mathrm{d}\xi$ while measured in $\mathrm{d}\zeta$ is

$$P(\zeta|\xi) \, N(\xi) \, \mathrm{d}\xi \, \mathrm{d}\zeta = Q(\xi|\zeta) \, n(\zeta) \, \mathrm{d}\xi \, \mathrm{d}\zeta$$

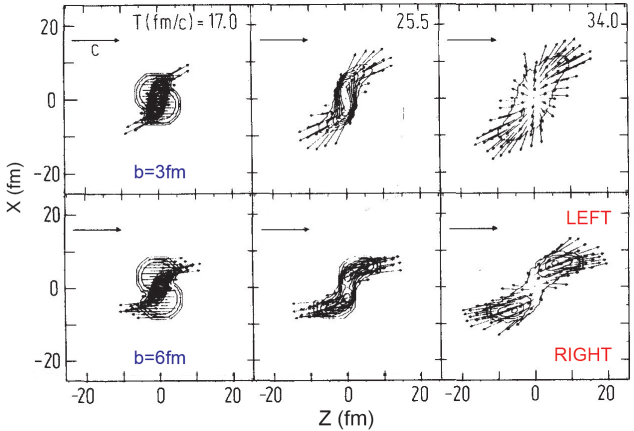
$$\text{Hence} \quad N(\xi) = \frac{\int \mathrm{d}\zeta \, Q(\xi|\zeta) \, n(\zeta)}{\int \mathrm{d}\zeta' \, P(\zeta'|\xi)}, \quad Q(\xi|\zeta) = \frac{P(\zeta|\xi) \, N(\xi)}{\int \mathrm{d}\xi' \, P(\zeta|\xi') \, N(\xi')}$$

Richardson-Lucy method solves eqs iteratively till stabilization

Reaction-Plane Deflection in Hydrodynamic Calculations

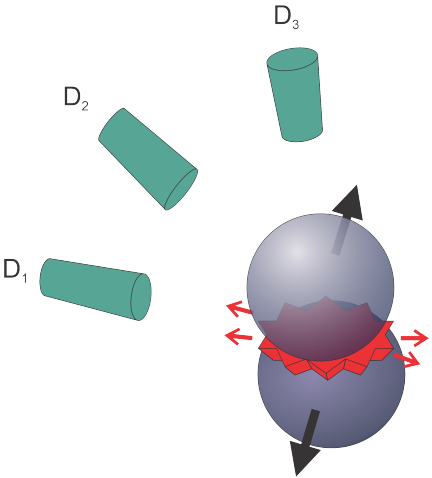
Matter dispersed in the final stage, but most likely direction of motion away from the beam, e.g., in the calculations by Buchwald for Nb + Nb at 400 MeV/nucl

Stöcker&Greiner Phys.Rep. 137(86)277



Can this be seen experimentally??

Estimating Reaction-Plane Direction w/o Self-Correlation



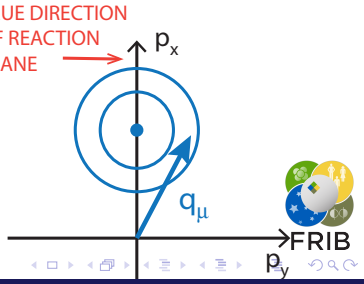
Plane direction f/particle μ estimated with

$$\mathbf{q}_\mu = \frac{1}{N}\sum_{\nu\neq\mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction, blurring features



Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

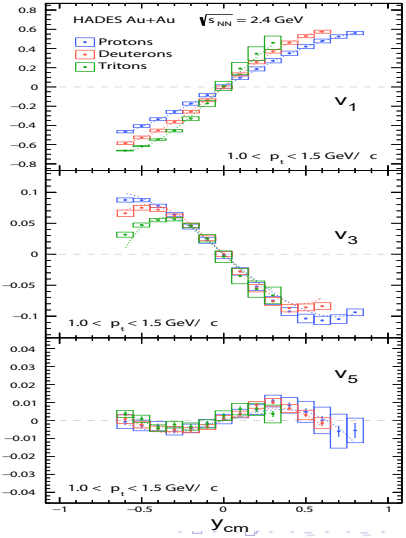
$$\langle \mathbf{p}_\mu^\perp \cdot \mathbf{q}_\mu \rangle \simeq p^\perp \langle q^x \rangle \langle \cos\phi \rangle$$

for different $p^\perp$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301

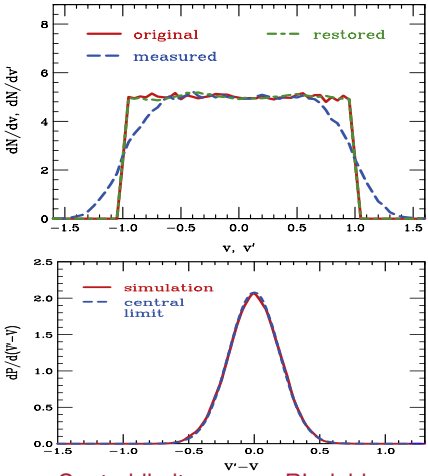


How Hard Deblurring? 1D Model First

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~Yes~~?

First 1D deblurring test. Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like $\mathrm{d}N/\mathrm{d}v$ in projectile cm. Task: Measuring ptcles in lab, determine $\mathrm{d}N/\mathrm{d}v$. Cm velocity V' estimated from remaining ptcles, so V' & $\mathrm{d}N/\mathrm{d}v'$ smeared:

$$\frac{\mathrm{d}N}{\mathrm{d}v'} = \int \mathrm{d}V' \, \frac{\mathrm{d}P}{\mathrm{d}V'} \, \frac{\mathrm{d}N}{\mathrm{d}v}$$



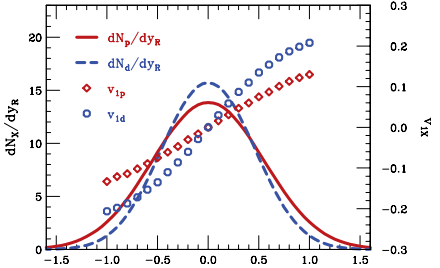
PD&Kurata-Nishimura PRC105(2022)034608

→Central-limit smear + RL deblur

3D Model for Collisions

Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$

Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl



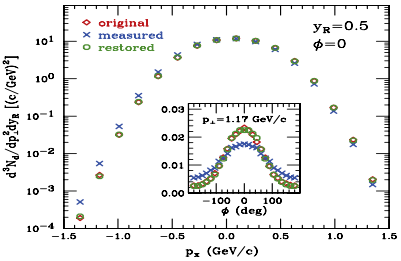
$$\frac{dN}{d\phi'} = \int_{y_R} d\Phi' \frac{dP}{d\Phi'} \frac{dN}{d\phi}$$

$$\phi' + \Phi' = \phi + \Phi$$

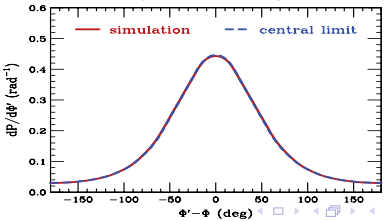
RL deblur + central-limit

Strong anisotropies restored!

Triple differential spectrum in reaction plane:



Uncertainty in reaction plane:



SπRIT 270MeV/u Measurements

The first SπRIT Time Projection Chamber (TPC) experiment was carried out in RIKEN Rare Isotope Beam Facility in 2016

Projectile-target combinations:

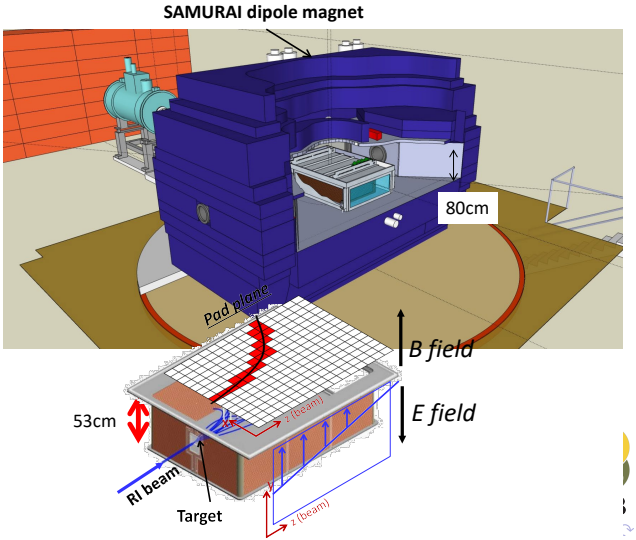
^{132}Sn + ^{124}Sn

^{124,112}Sn + ^{112,124}Sn

$\Rightarrow$ ^{108}Sn + ^{112}Sn

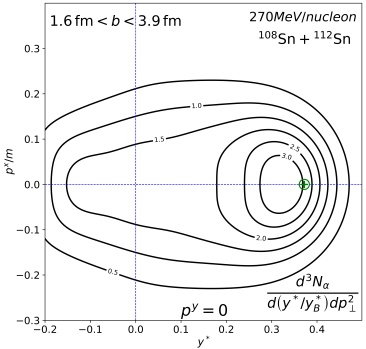
From SπRIT: Jeonghyeok Park & Mizuki Kurata-Nishimura

$\sim 450,000$ semicentral events



Triple-Differential Yields: SπRIT 270MeV/u ^{108}Sn+^{112}Sn

Distributions for *Fixed Direction of Reaction Plane* from Experiment and Theory?

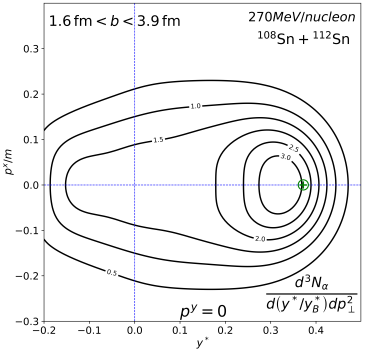


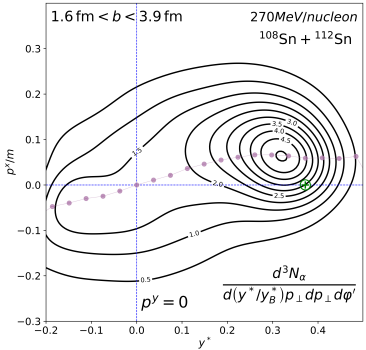
no control over plane

Boring?

Triple-Differential Yields: SπRIT 270MeV/u ^{108}Sn+^{112}Sn

Distributions for *Fixed Direction of Reaction Plane* from Experiment and Theory?





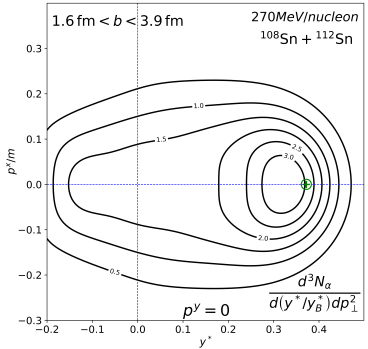
no control over plane

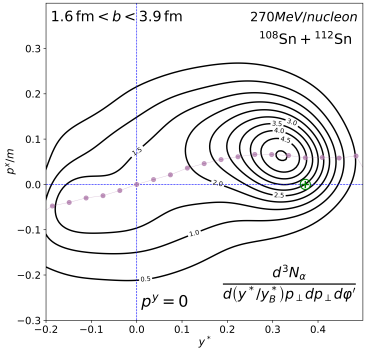
some control, $v_n' \to v_n$

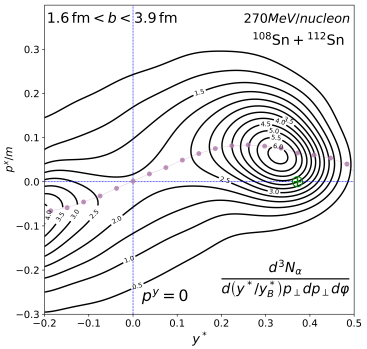
Something showing up...

Triple-Differential Yields: SπRIT 270MeV/u ^{108}Sn+^{112}Sn

Distributions for *Fixed Direction of Reaction Plane* from Experiment and Theory?







no control over plane

some control, $v_n' \rightarrow v_n$

full control, $\frac{d^3N}{dy\,dp_\perp^2}$

Something showing up...

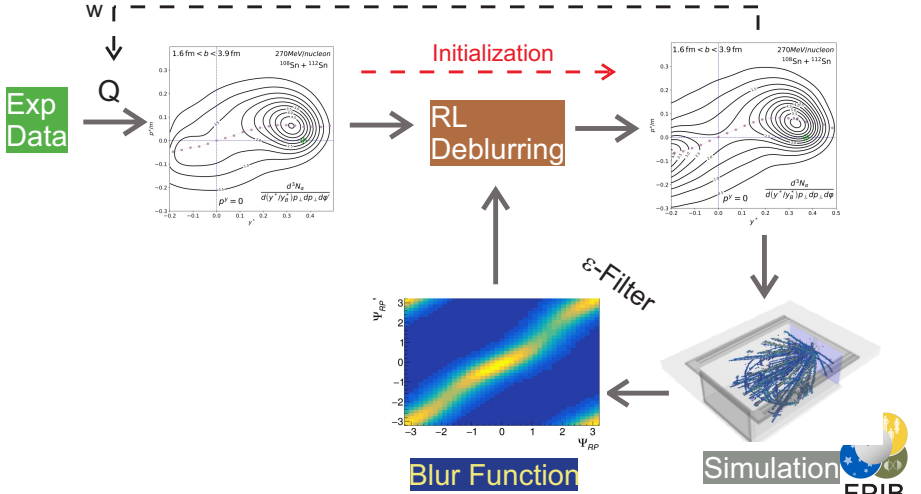
??!

FRIB

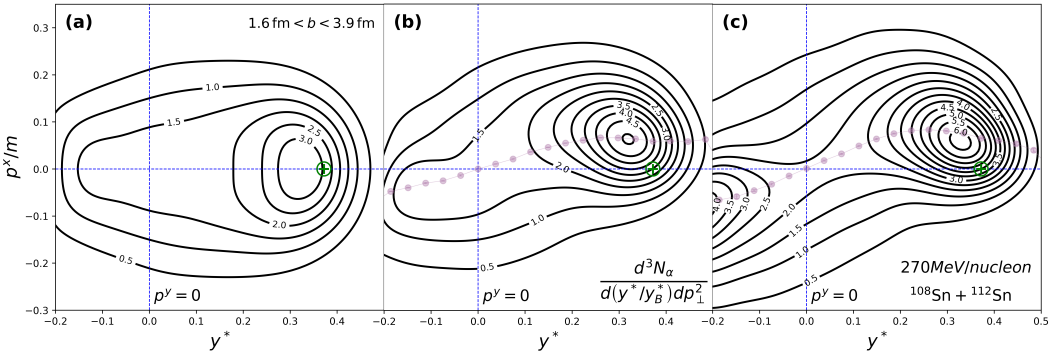
Deblurring SπRIT Data: Flow Diagram

Challenge: nonlinear problem ⇒ Self-consistency required

$$\vec{Q} = \sum_{\mu} w_{\mu} \vec{p}_{\mu}^{\perp}$$

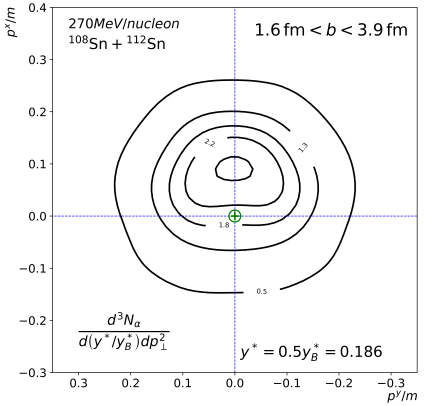


Outcome: Triple-Differential Yields Corrected for Reaction-Plane Resolution & Efficiency

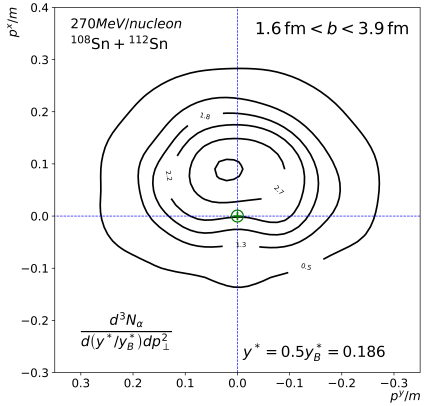


Here, 2D cut through 3D distribution of α particles

2D Cut ⊥ Beam Axis: Alpha Particles @ $y^* = y_B^*/2$



before deblurring



after deblurring: cut loaf of bread

Transverse Spectra Out- and In-Plane: p & α

Spectra generally always wider out-of-reaction plane than in-plane!

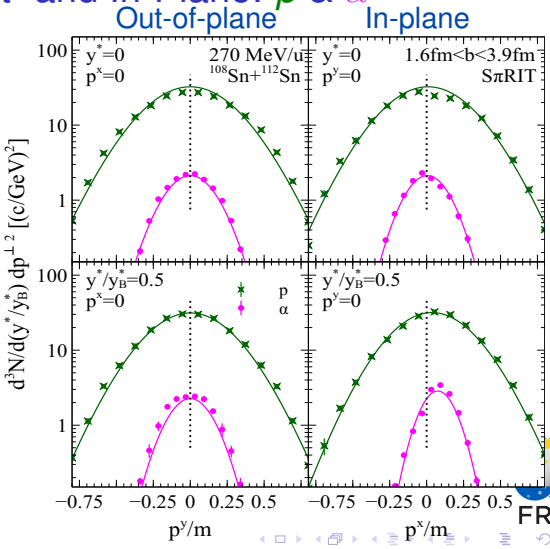
Boost up or down in reaction plane (v_s^x) away from $y^* \sim 0$.

Parametrization accurate up to 15-20%:

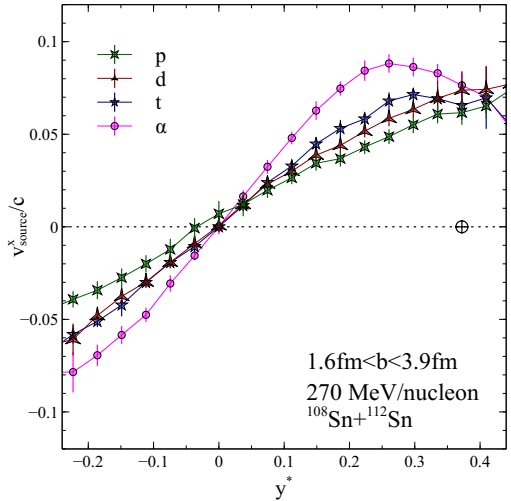
$$\frac{d^3N}{dy^* dp_\perp^2} \propto \frac{E_\perp^*}{E_\perp} \exp\left(-\frac{E_\perp^* - m}{T(\phi^*)}\right)$$

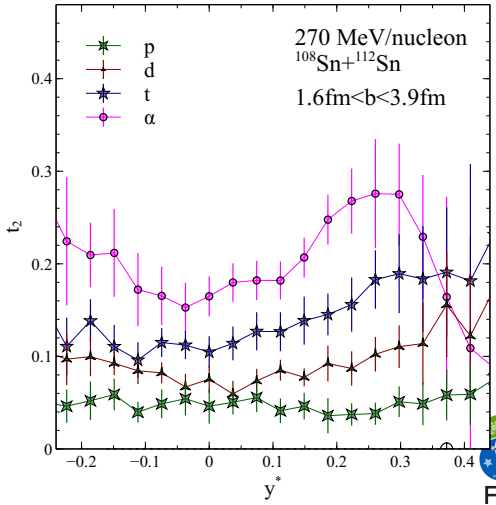
$$E_\perp^* = \gamma_s (E_\perp^* - v_s^x p^x)$$

$$\frac{1}{T(\phi^*)} = \frac{1 + t_2 \cos 2\phi^*}{T}$$

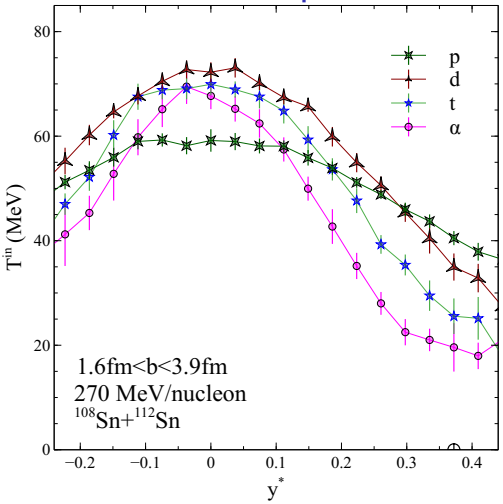


In-Plane Boost v_s^x and Quadrupole Modulation t_2 of Spectral T





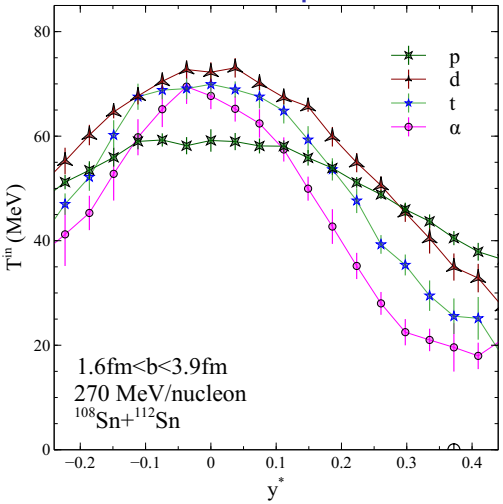
In-Plane Temperature Variation: Dependence on Mass



Weak variation for *p*

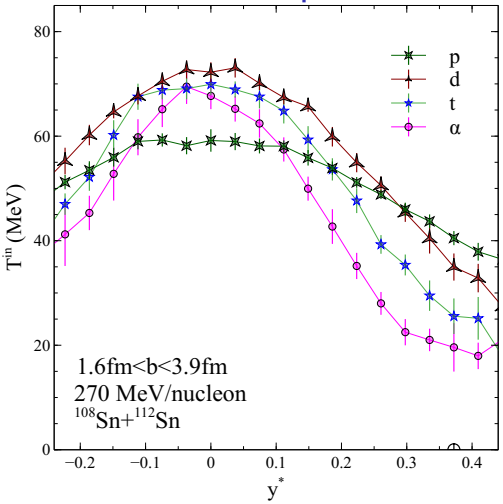
Dramatic variation for α

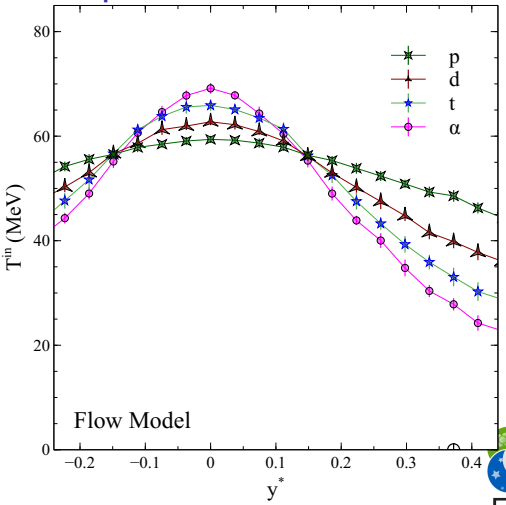
In-Plane Temperature Variation: Dependence on Mass



Flow Model: Single velocity field folded w/single spectral temperature field. Local yield ratio fixed at global

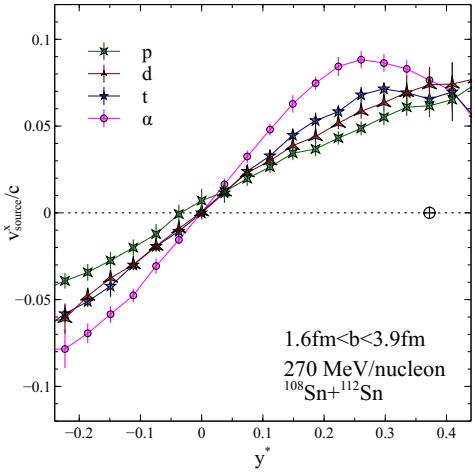
In-Plane Temperature Variation: Dependence on Mass

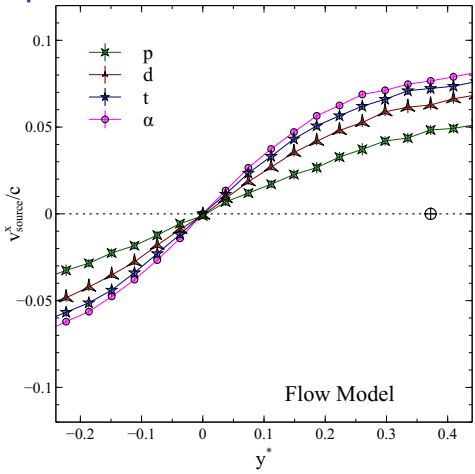




Alphas track central collision region; nucleons can come from anywhere

In-Plane Source Boost: Dependence on Mass



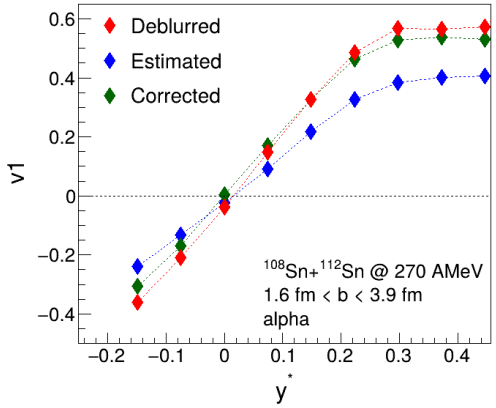


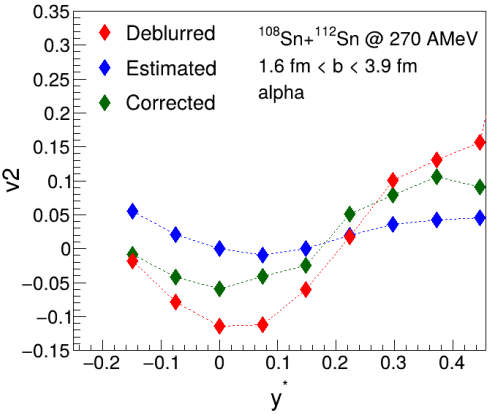
Alphas track birthplace; nucleons can come from anywhere

Back to Rami *et al.* NPA646(1999)367

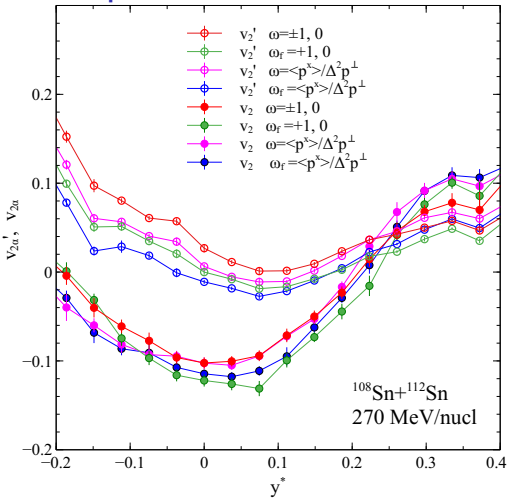
Fourier Coefficients v_n: Old vs New

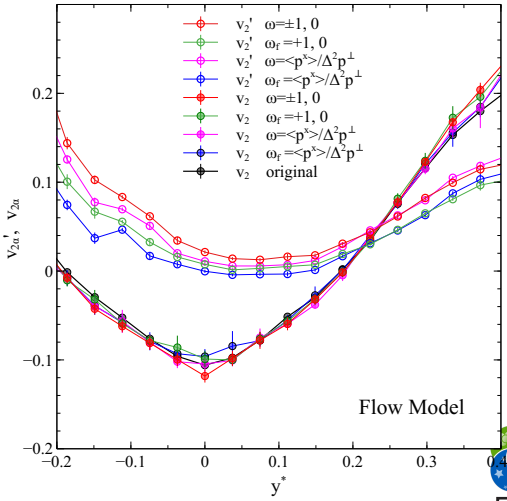
'Old'/Corrected: Kurata-Nishimura *et al.* PLB871(2025)139970. Estimated - v_n'





Dependence of Restoration on Reaction-Plane Definition





About the same results no matter how the reaction plane is defined!

Conclusions

- Reaction-plane deblurring is reality now! Detector corrections included
- Dominance of reaction plane in azimuthal correlations needed. High multiplicities + high event statistics desirable
- Much action near beam axis!
- Wealth of info & and you can retreat to v_n if you wish
- Clusters look well more promising than nucleons - structures more pronounced

few years of development; upcoming Ph.D. Thesis of Jeonghyeok Park (U Korea) & more

PD&Kurata-Nishimura PRC105(22)034608; PD *et al.* PRC108(23)L051603
Adamczewski-Musch *et al.* PRL125(20)262301 - v_n reconstruction f/azimuthal distribution

Supported by US Department of Energy under Grant US DE-SC0019209