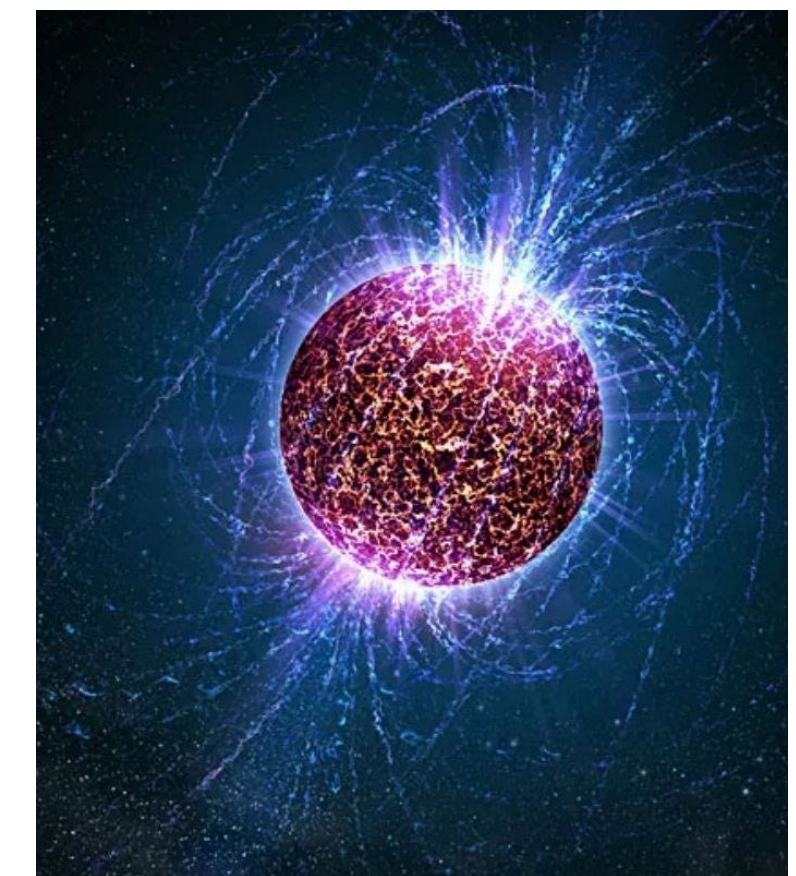
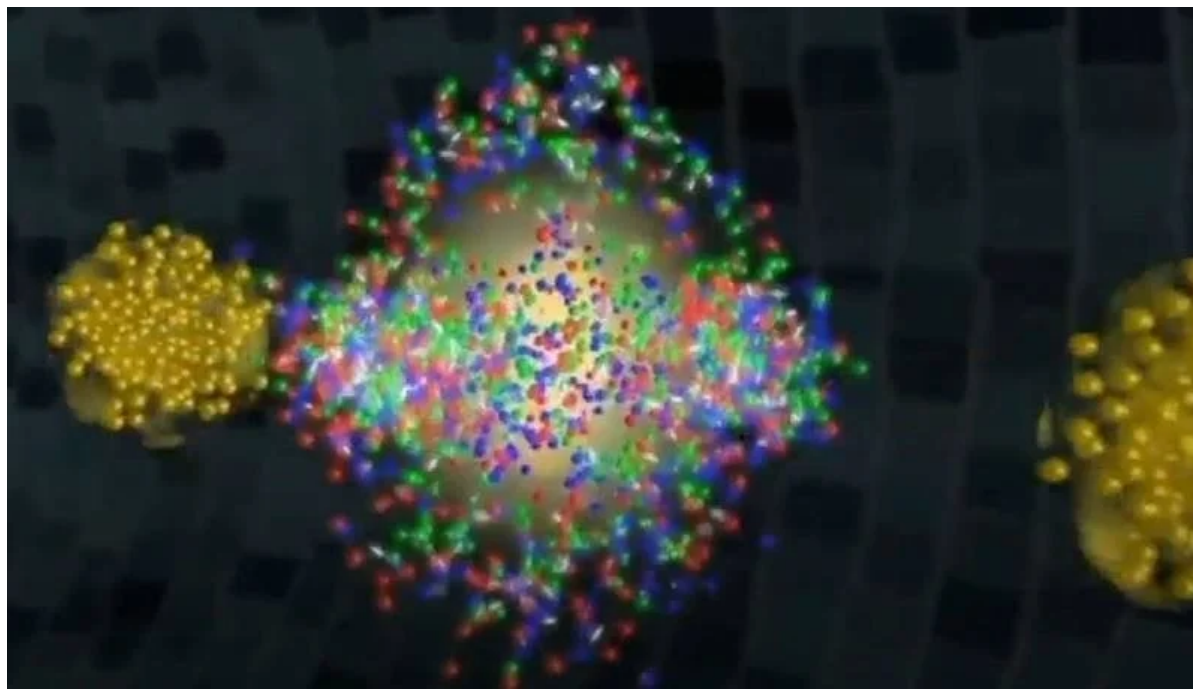




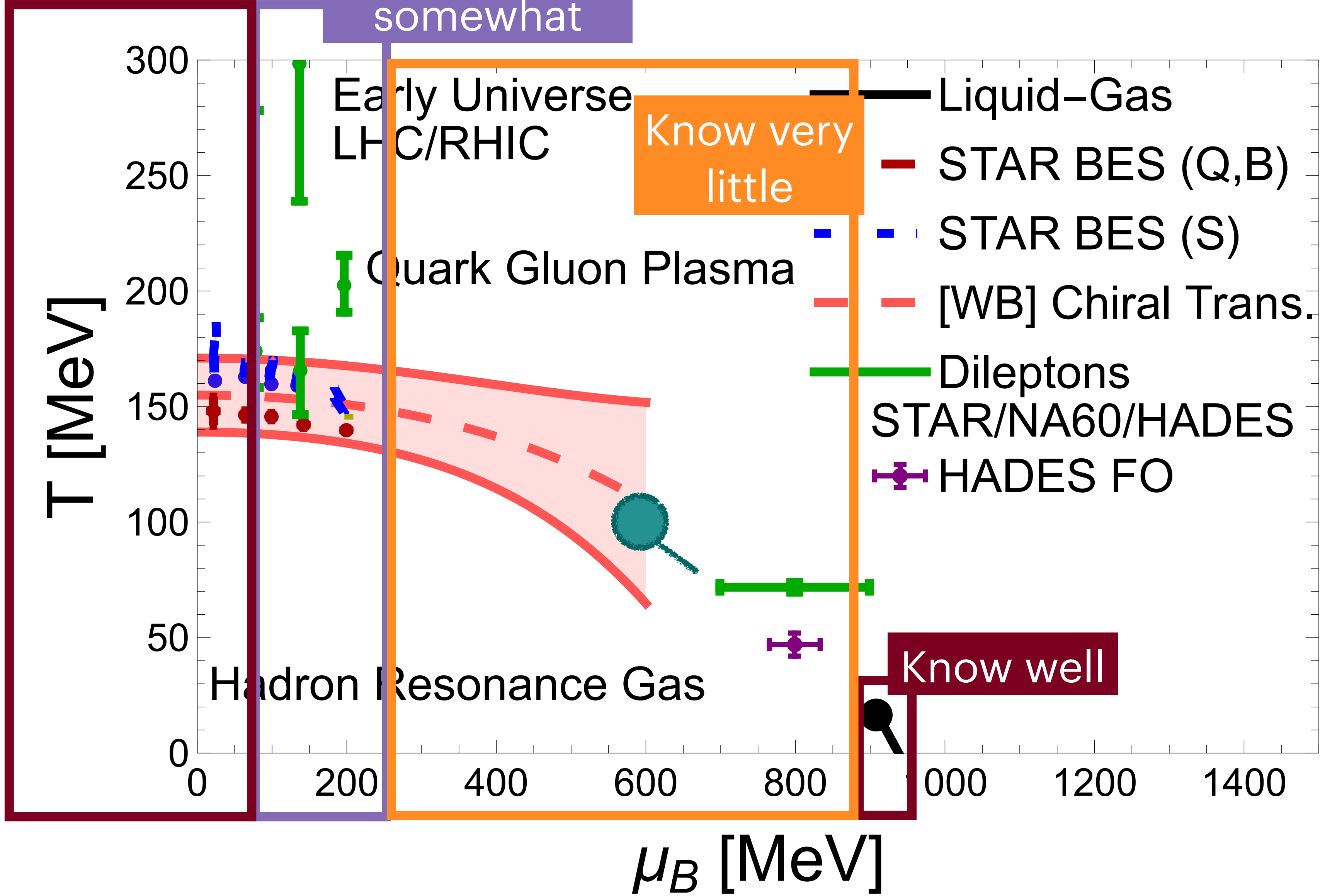
Insights from neutron stars applied to heavy-ion collisions

Jacquelyn Noronha-Hostler

Emmi Workshop GSI, Nov 2025



What we know, what we don't



Hydro simulations from $\sqrt{s} = [3, 7.7, 27] \text{ GeV}$
 Shen&Schenke, Phys. Rev. C 105, 064905 (2022)

(T, μ_B) extracted from STAR net-(p, π ,K), net-p, net-K
 fluctuations
 Alba, et al, Phys. Rev. C 101, 054905 (2020)

Chiral transition from lattice QCD
 [WB] Phys. Rev. Lett. 125, 052001 (2020)

Dilepton measurements from
 [STAR] 2402.01998 [nucl-ex]
 [HADES] Nature Phys. 15, 1040 (2019)
 [NA60] Eur.Phys.J.C59:607-623,2009

Statistical Hadronization Model
 [HADES] Phys. Rev. C 102, 054903 (2020)

Liquid-gas phase transition location
 Elliott, et al, Phys. Rev. C 87, 054622 (2013)
 μ_B estimate Vovchenko, et al, Phys. Rev. Lett. 118, 182301
 (2017)

Hints of a QCD critical point
 beginning to appear...

Isospin asymmetry

n_Q

Isospin-
Asymmetric
Matter (IAM)
 $Y_Q = \infty$

Heavy-Ion Collisions (HIC)

$Y_Q = [0.38, 0.5]$

$$Y_Q \lesssim 0.2$$
$$n_B$$

3

Optimistic Overview

New things features to look for in dense matter

Constraints from astrophysics

How do we connect the HIC and NS EOS? Symmetry energy discussions

Optimistic Overview

New things features to look for in dense matter

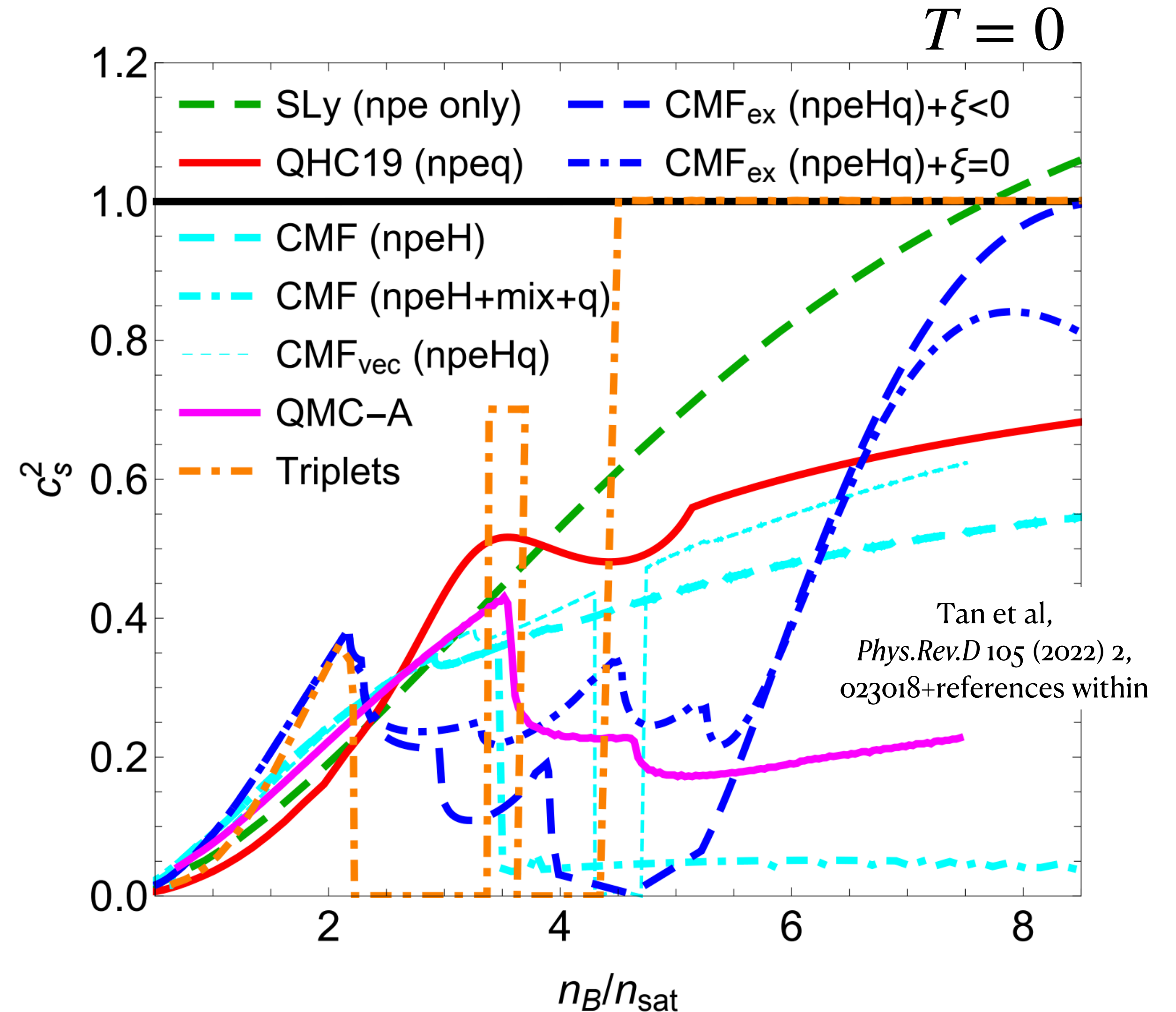
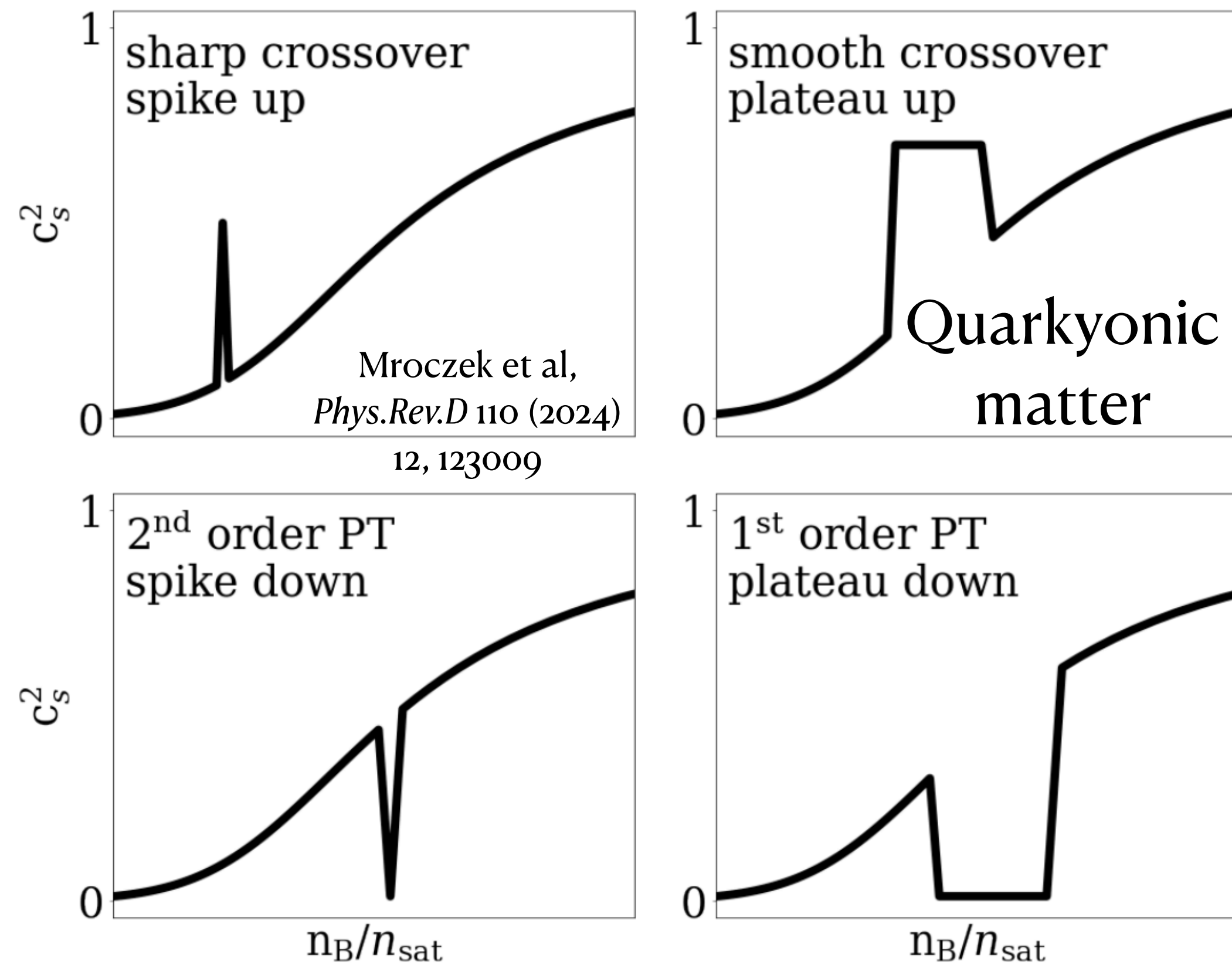
Constraints from astrophysics

How do we connect the HIC and NS EOS? Symmetry energy discussions

Speed of sound from neutron stars

Features to look for in heavy-ion collisions?

Features of interest:

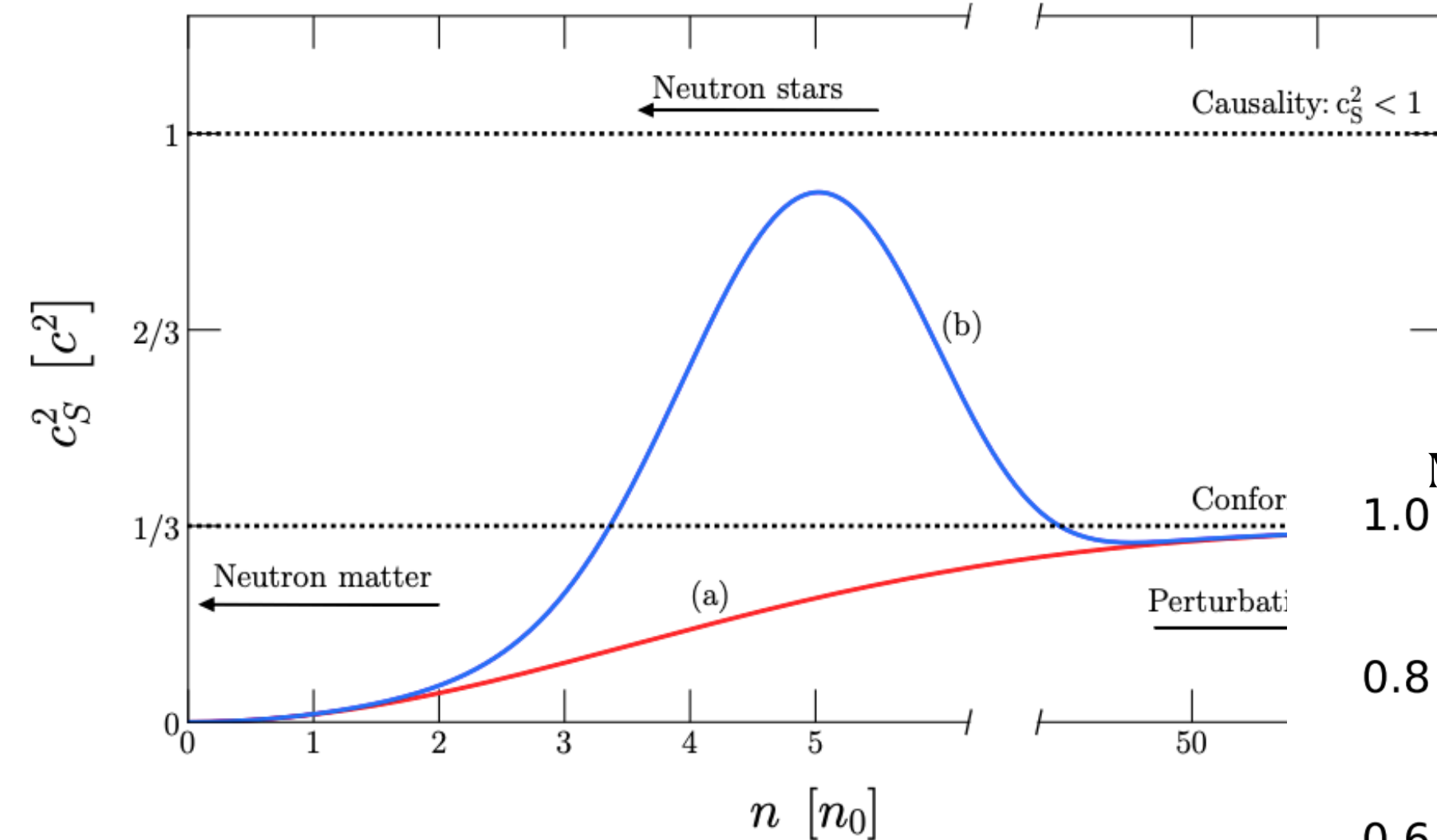


More references on bump in c_s^2

Measurements: breaking binary love relation

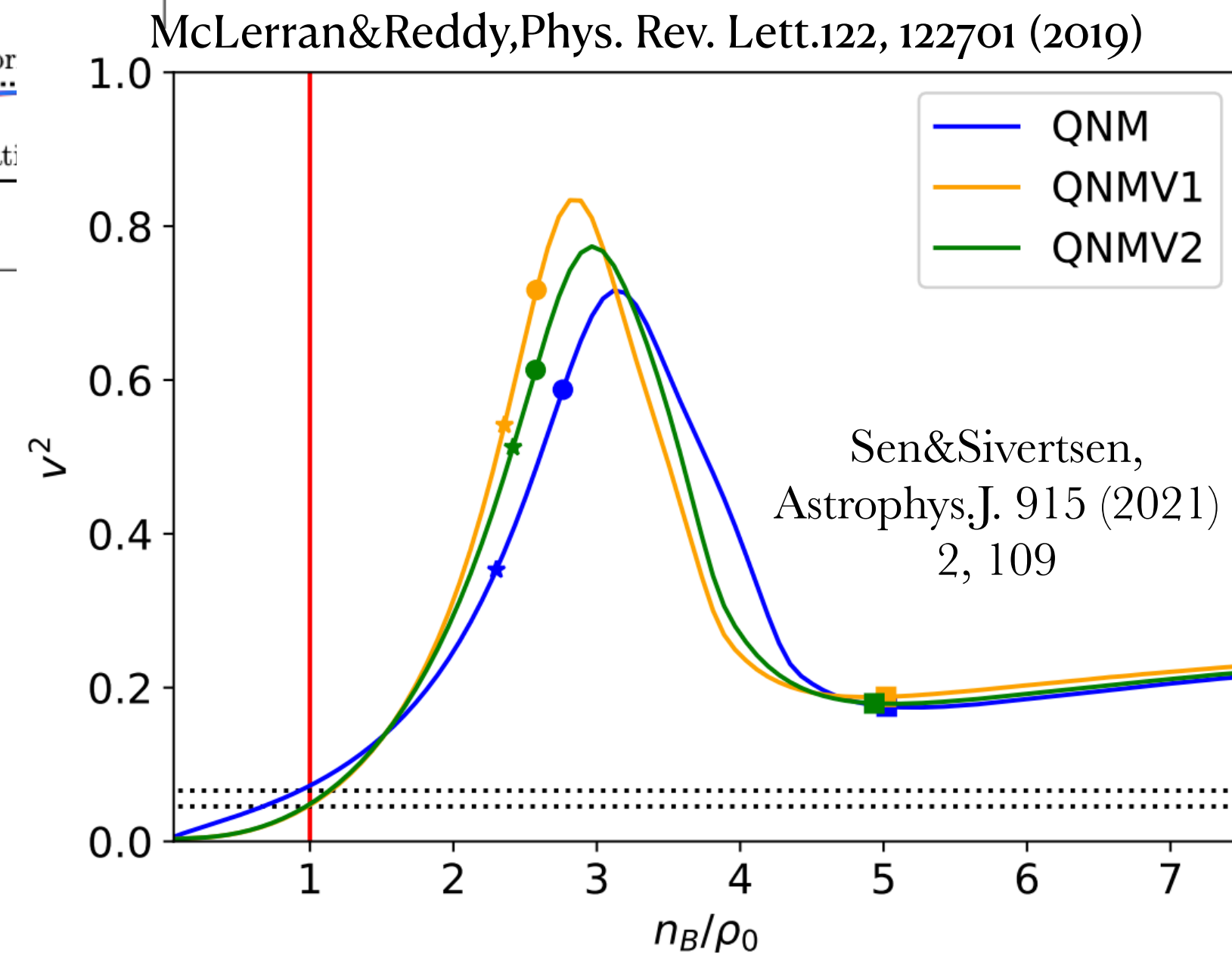
From various microscopic theories and Bayesian analyses

Steep rise in c_s^2 to reach $2M_\odot$



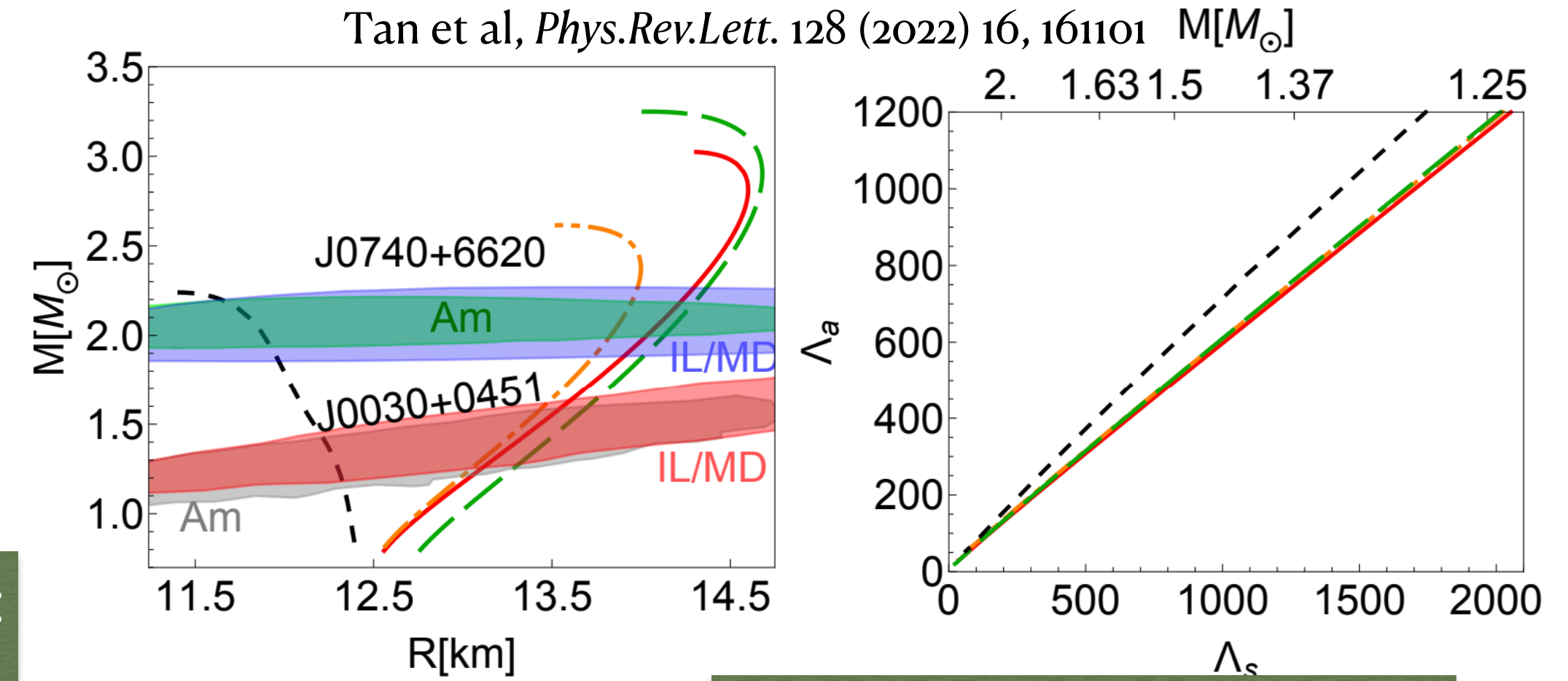
Bedaque & Steiner, Phys.Rev.Lett. 114 (2015) 3, 031103; Alford et al, Phys. Rev. D92, 083002 (2015), Ranea-Sandoval, et al, Phys. Rev. C93, 045812 I. Tews, et al, Phys. Rev. C98, 045804 (2018)

One physical mechanism: Quarkyonic Matter



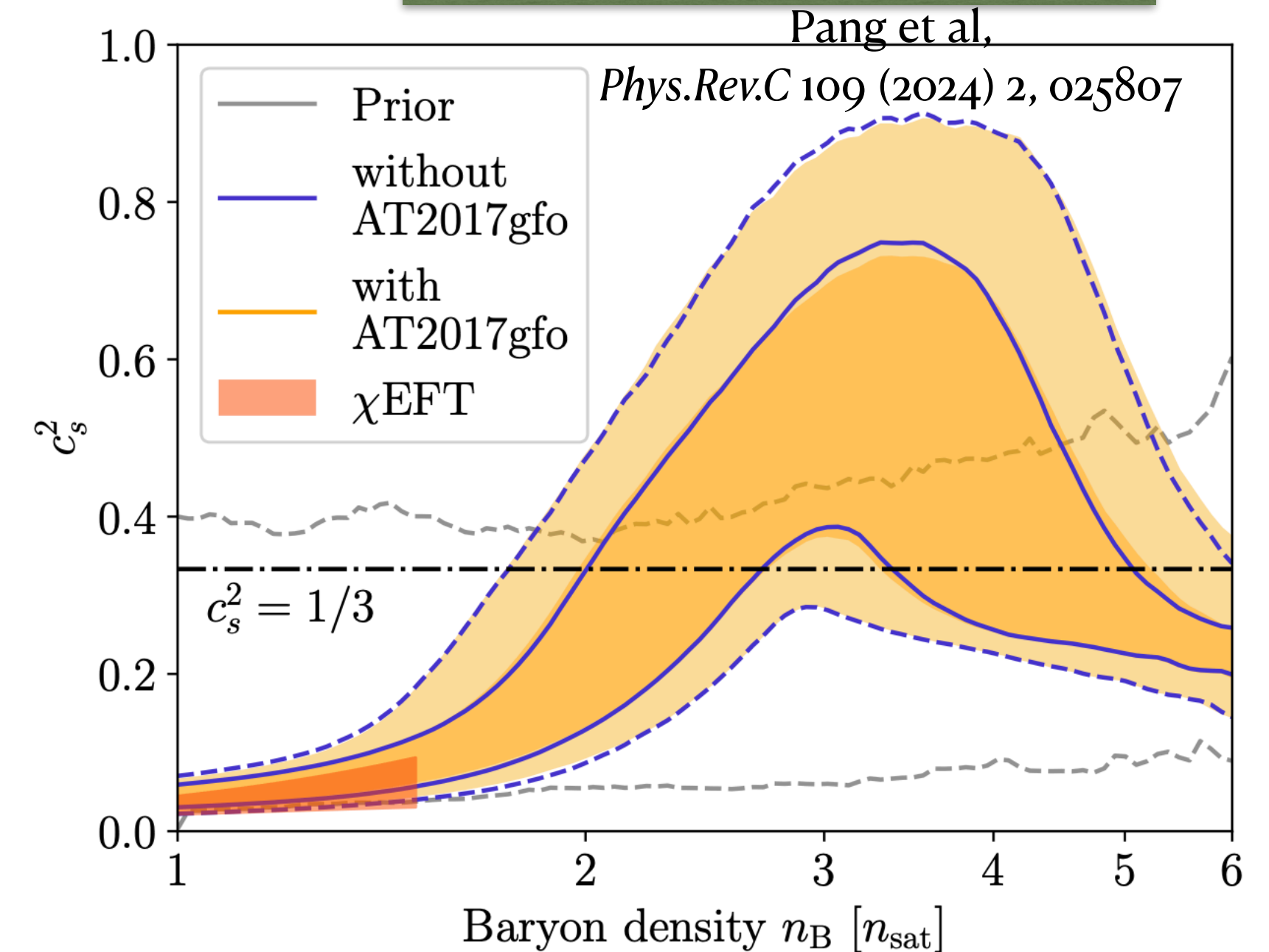
McLerran&Reddy, Phys. Rev. Lett.122, 122701 (2019)

Sen&Sivertsen, Astrophys.J. 915 (2021) 2, 109



Tan et al, Phys.Rev.Lett. 128 (2022) 16, 161101

Bayesian analysis

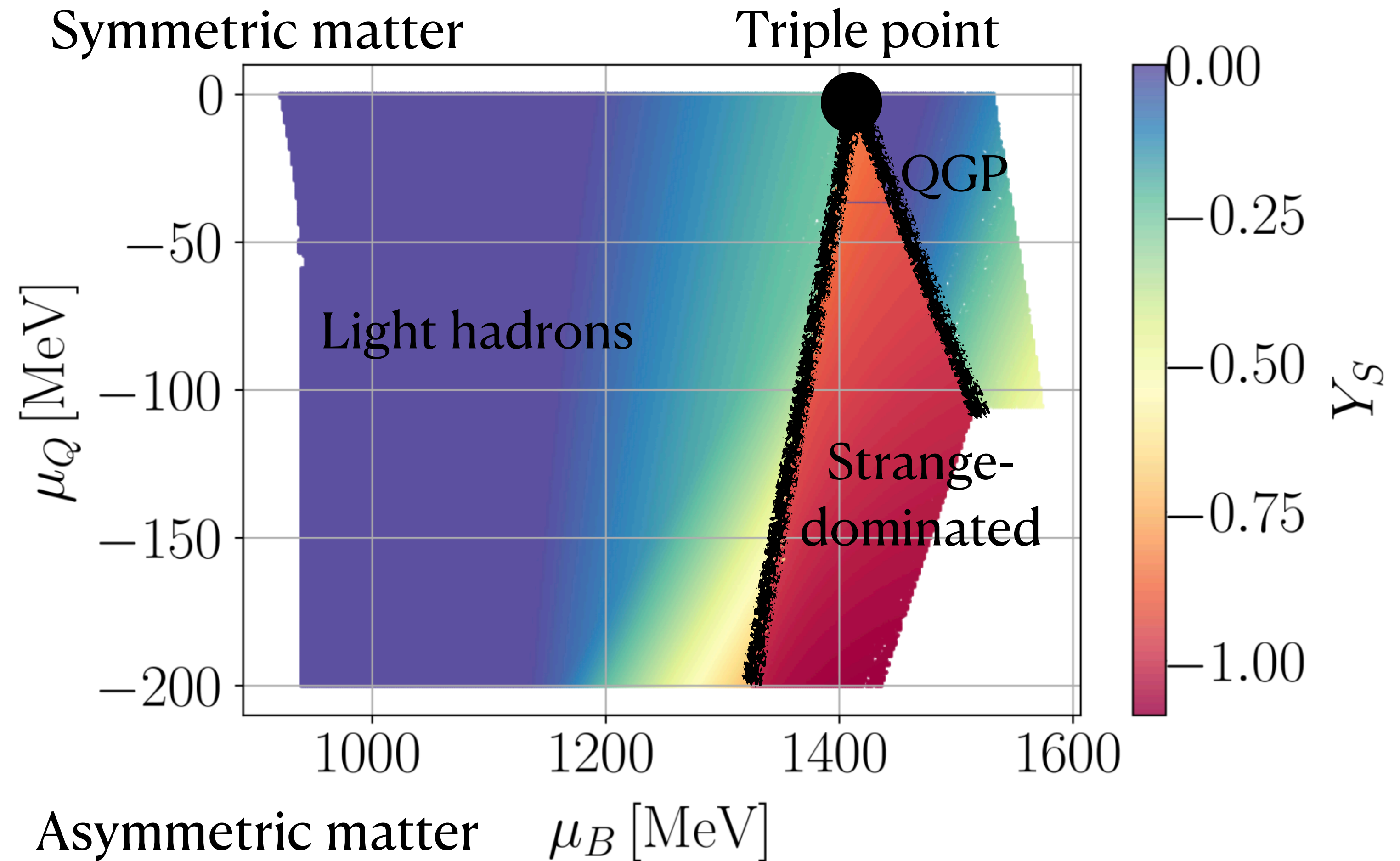
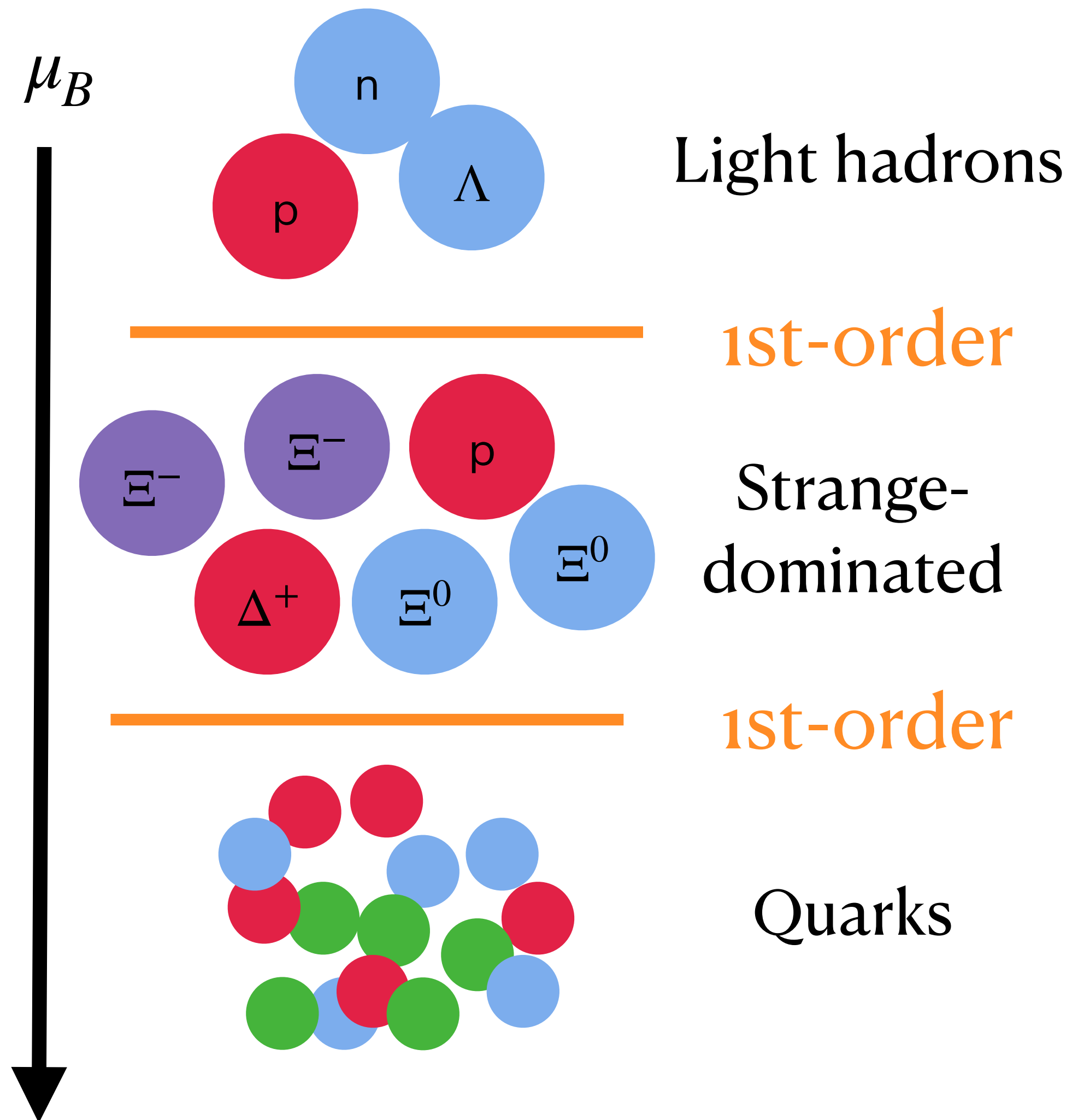


Pang et al, Phys.Rev.C 109 (2024) 2, 025807

Strangeness dominated matter



Example $T = 0$ phase diagram



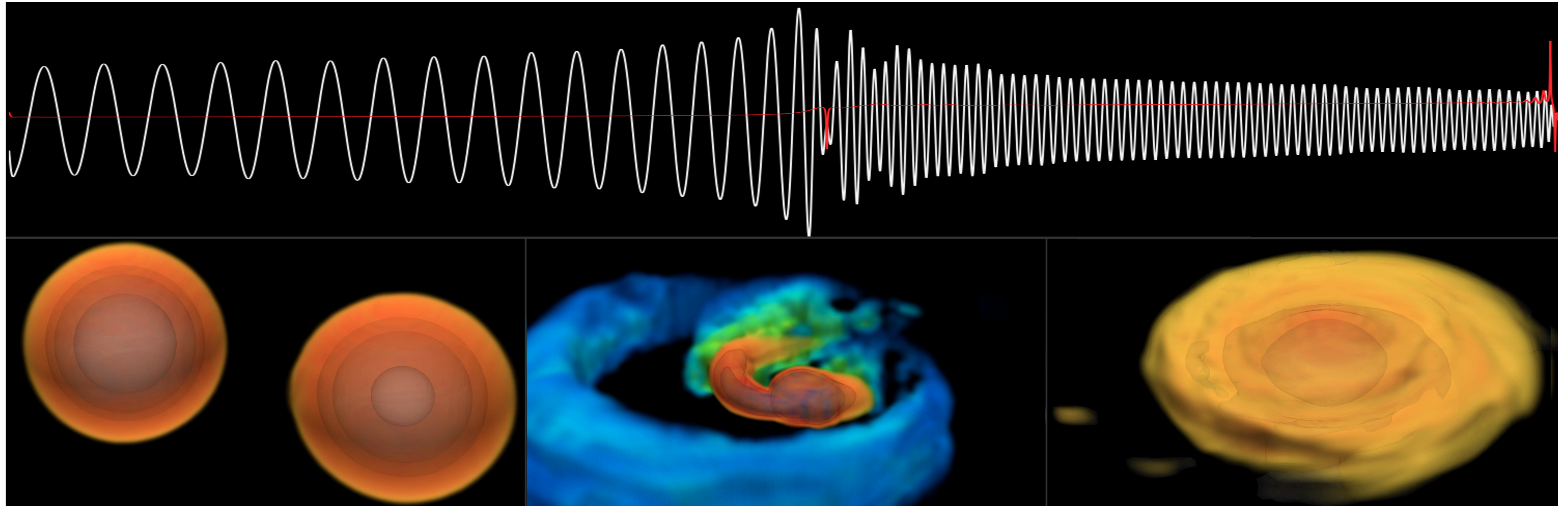
Optimistic Overview

New things features to look for in dense matter

Constraints from astrophysics

How do we connect the HIC and NS EOS? Symmetry energy discussions

Stages of a binary neutron star (NS) merger



Cold (inspiral)

$$T \sim 10^{-5} \text{ MeV}$$

Hot (merger)

$$T \sim 100 \text{ MeV}$$

Remnant

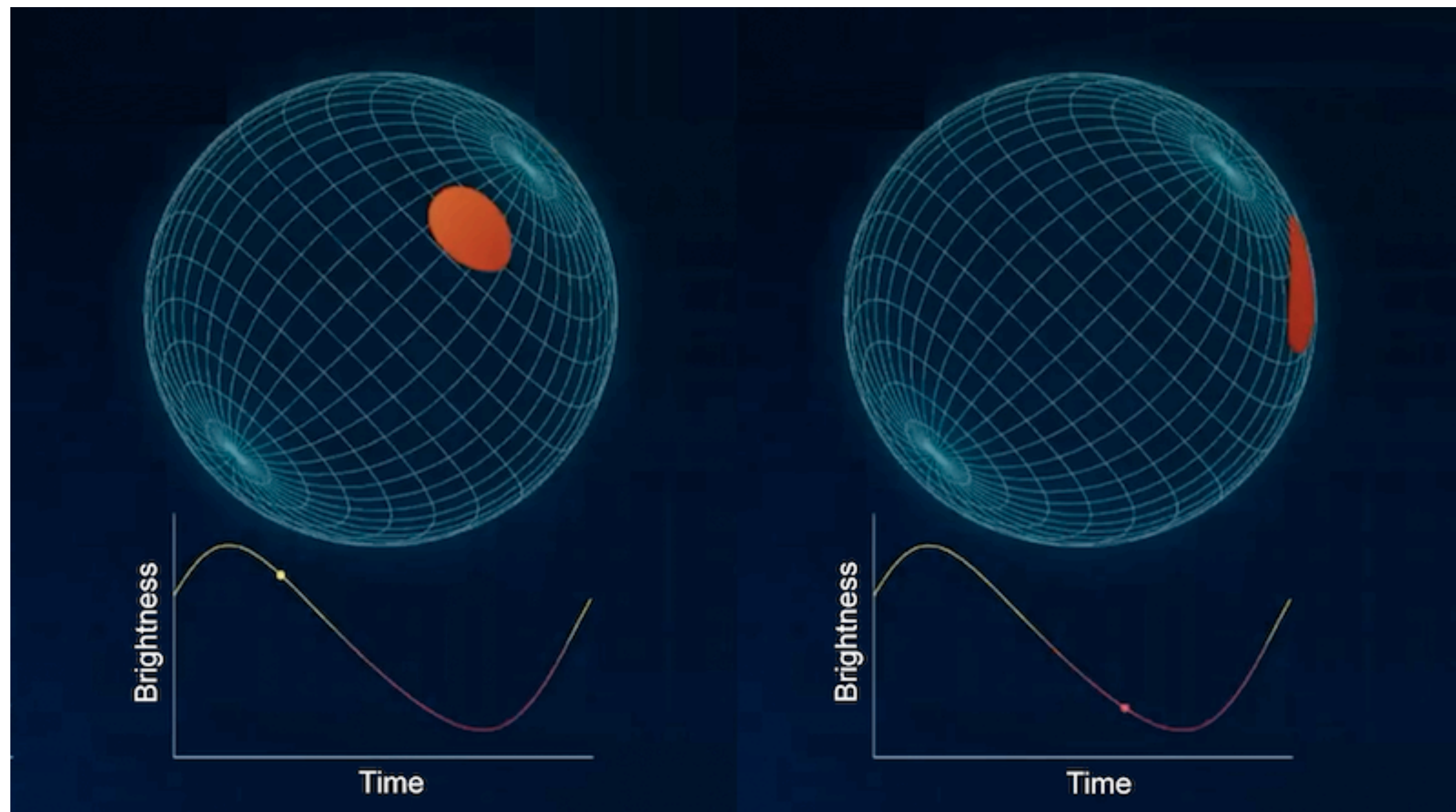
Sensitive to M_{max}

Observations here

Not yet measured

Mass-Radius of Isolated Neutron Stars

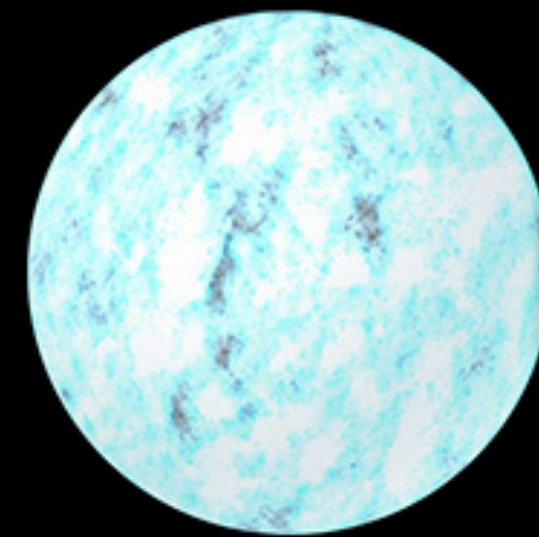
NICER collaboration



Heavier stars probe denser region of the EOS

Neutron stars

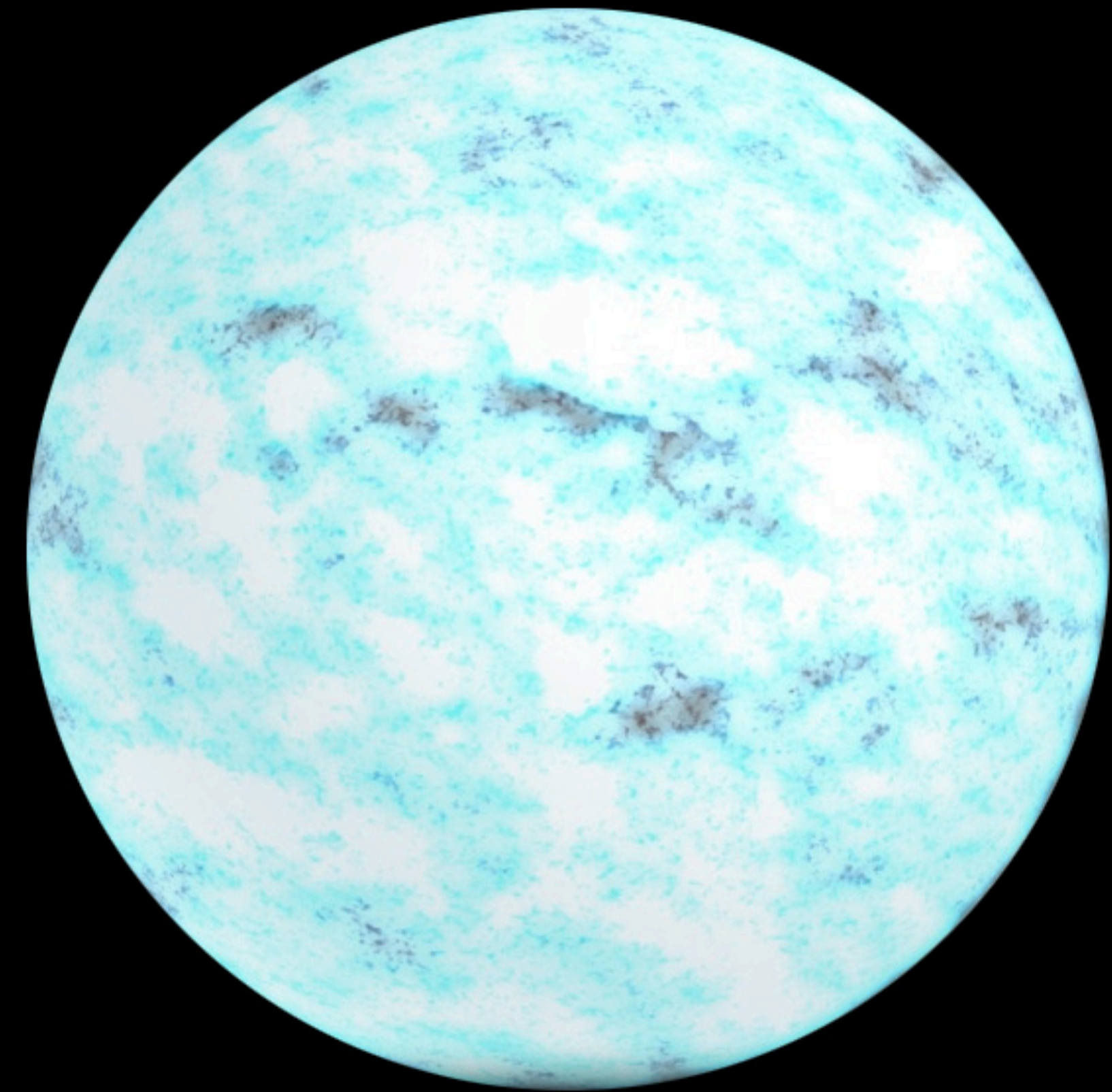
NICER:
Observations of
isolated Neutron
Star



Typical neutron star

10 km (6.2 miles)

1.4 solar masses



J0740+6620

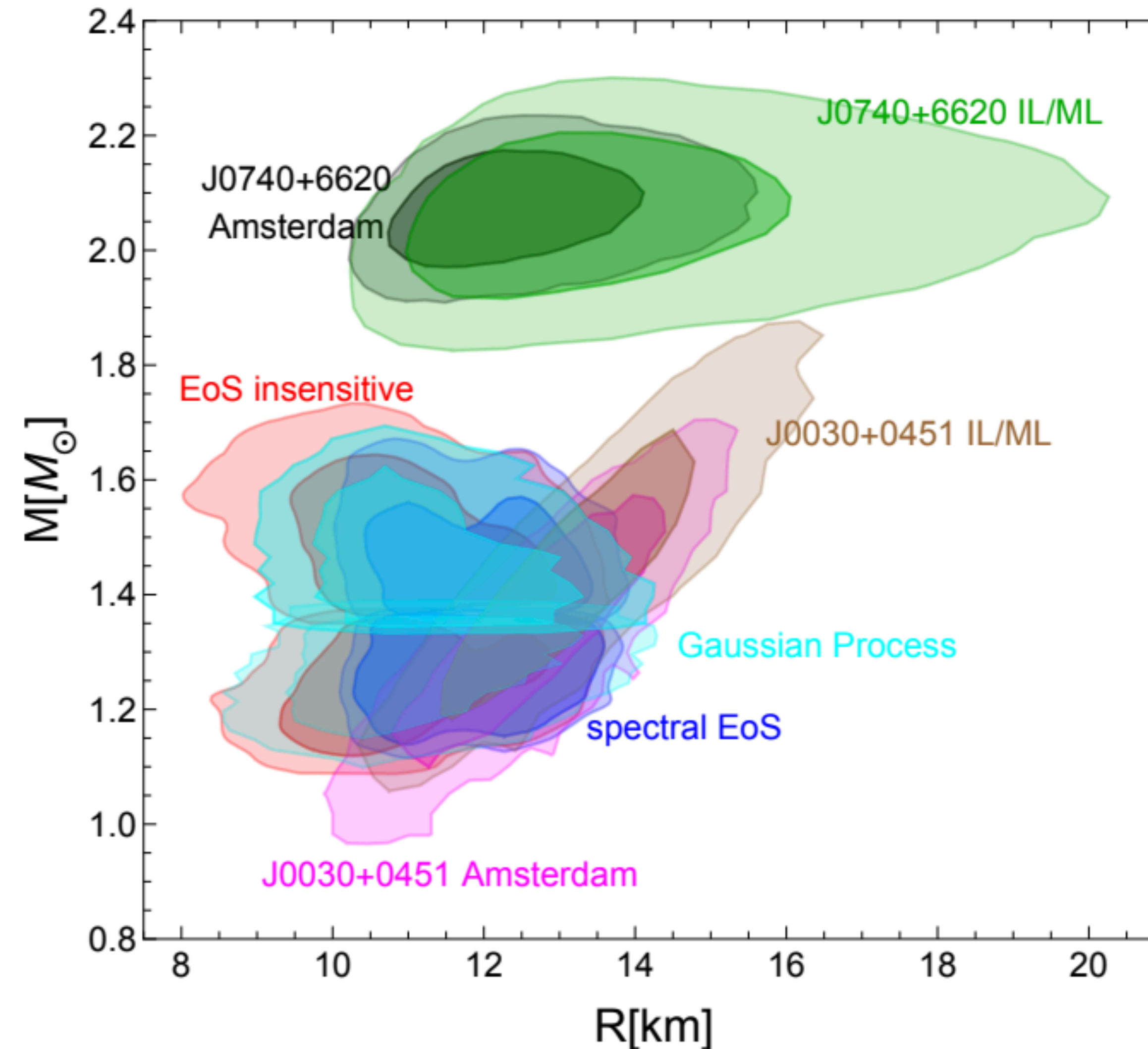
30 km (18.6 miles)

2.17 solar masses

Miller, arXiv:2105.06979 [astro-ph.HE]; Astrophys.J.Lett. 887 (2019) 1, L24;
Raaijmakers, arXiv:2105.06981 [astro-ph.HE]; Astrophys.J.Lett. 887 (2019) 1, L22

Existing Mass-radius constraints

From NICER+gravitational waves



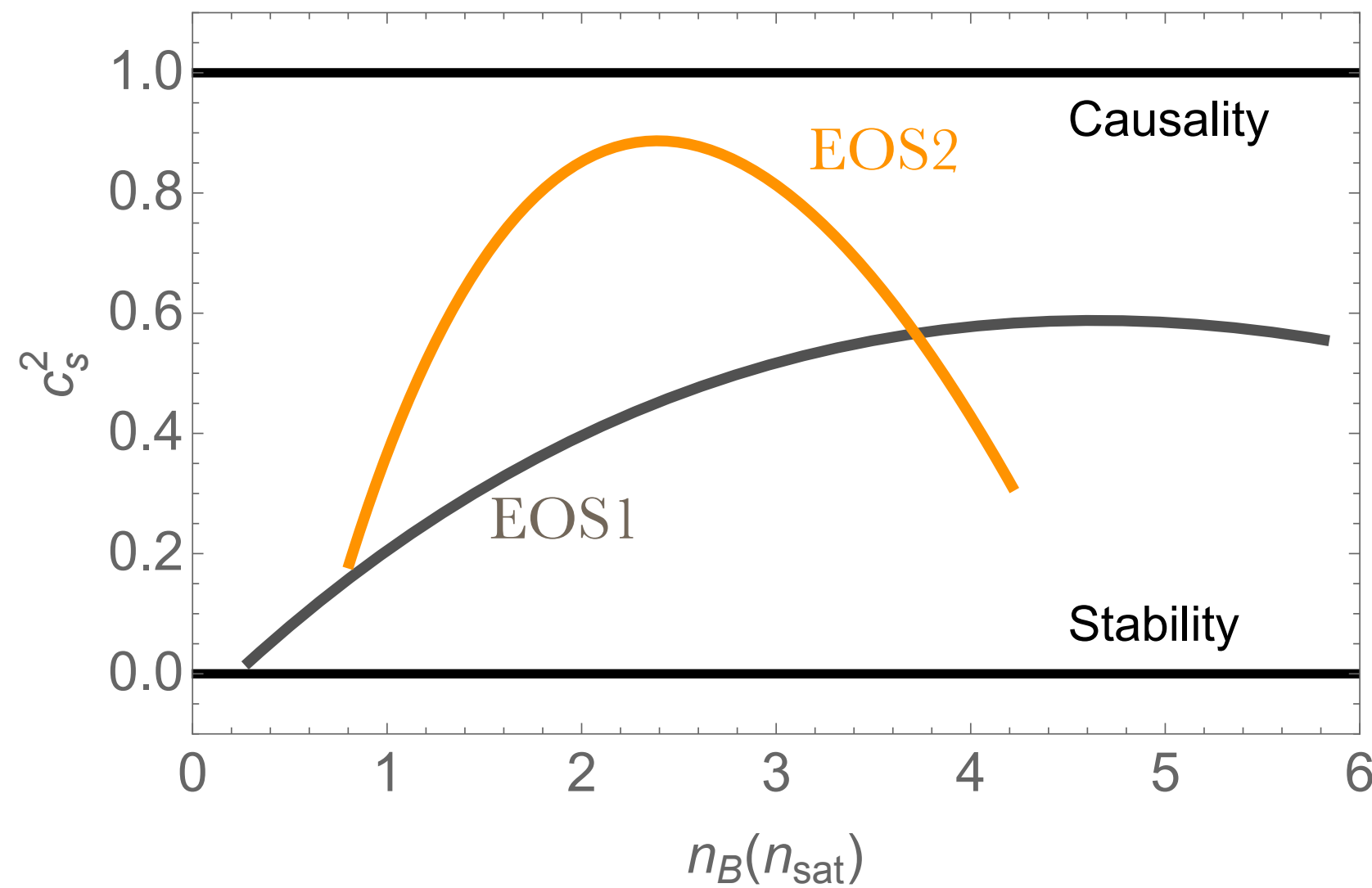
68% and 95%
confidence
regime shown

The posteriors are probability distributions, not strict error bars

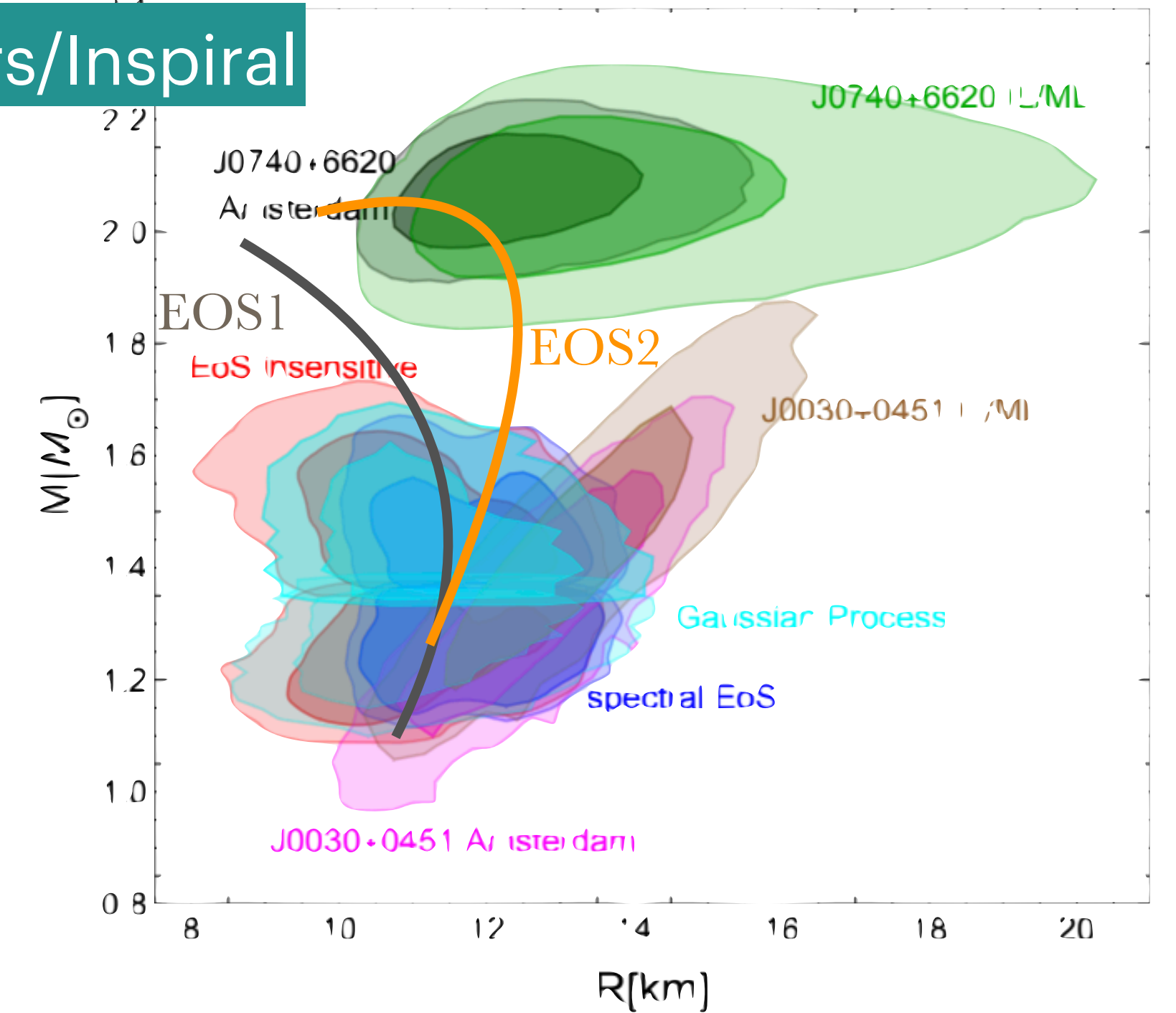
Equation of State from astrophysics observation?

Build in desired features: particles, phase transitions, interactions etc

Isolated, cold neutrons stars/Inspiral



Astrophysical constraints



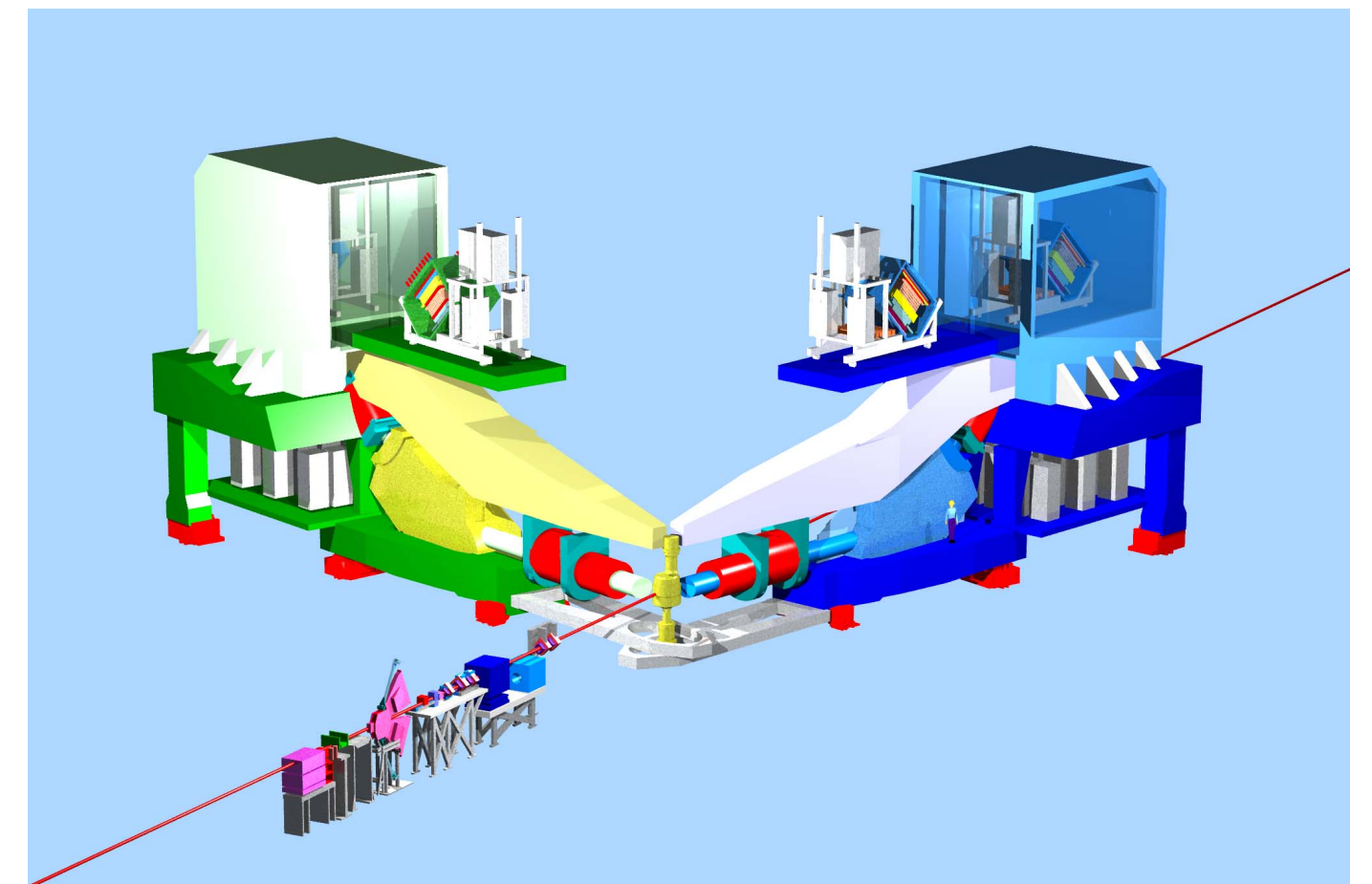
Nuclear experiments/saturation properties

Assign a likelihood to each EOS
 $Posterior \propto Likelihood \times Prior$

Independent measurements:

$$\mathcal{L} = \prod_{i=M,R,\dots} \left[\prod_{j=1}^{j(i)} \mathcal{L}(i,j) \right]$$

Theory constraints



Modified Gaussian Processes

Building in structure in c_s^2

Original Gaussian Process approach
R. Essick *Phys.Rev.D* 101 (2020) 6, 063007

- Model-agnostic approaches are common
- Gaussian processes (GPs):

EoS modeled via: $\phi(x) = \log(1/c_s^2 - 1)$, **stable and causal**

$$\phi \sim \mathcal{N}(\mu_i, \Sigma_{ij})$$

Collection of functions, behavior specified by a **mean** and **covariance kernel**

Squared-exponential is a common choice:

$$K_{\text{se}}(x_i, x_j) = \sigma^2 \exp \left[- (x_i - x_j)^2 / 2\ell^2 \right]$$

ℓ : correlation length

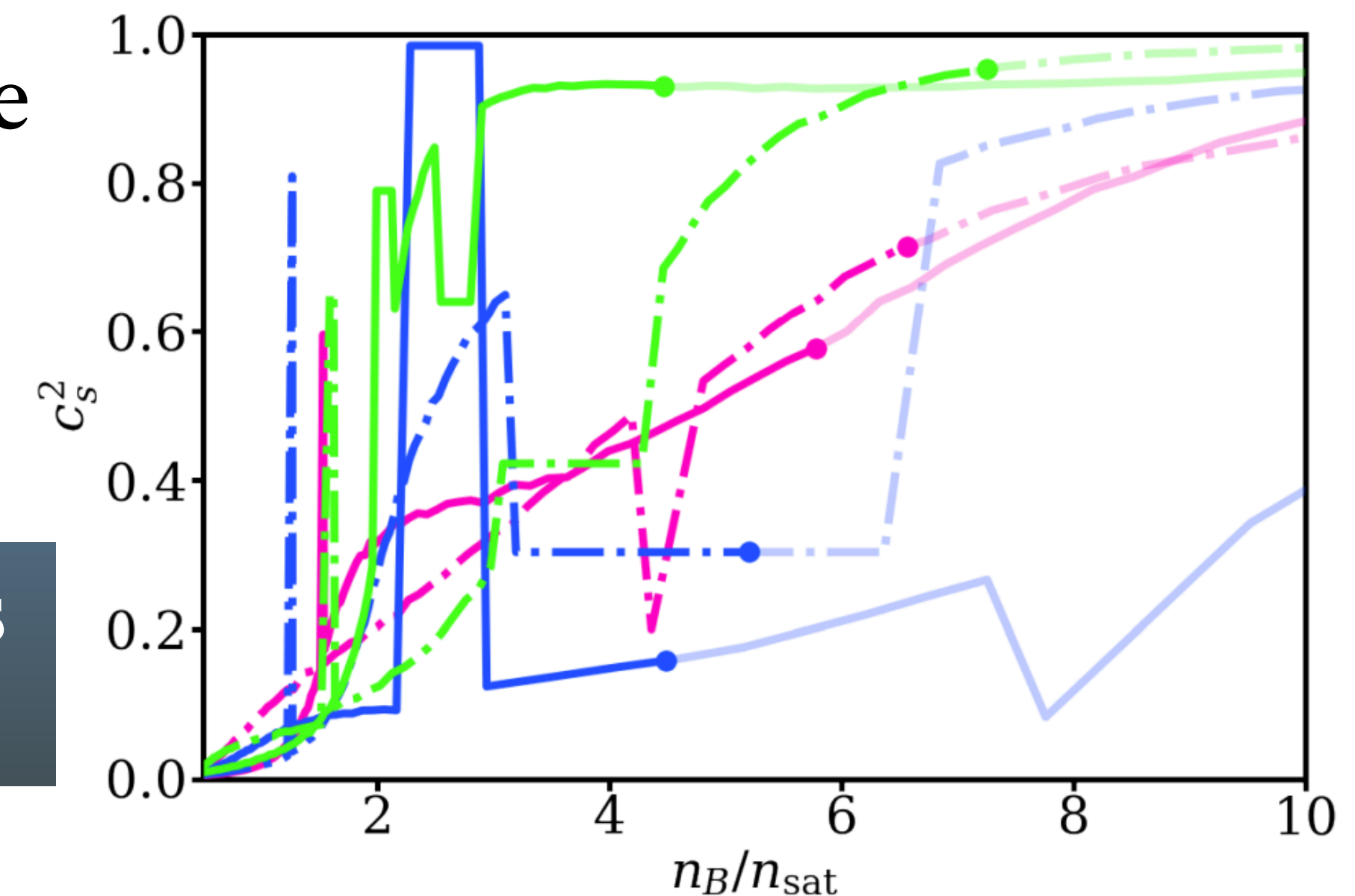
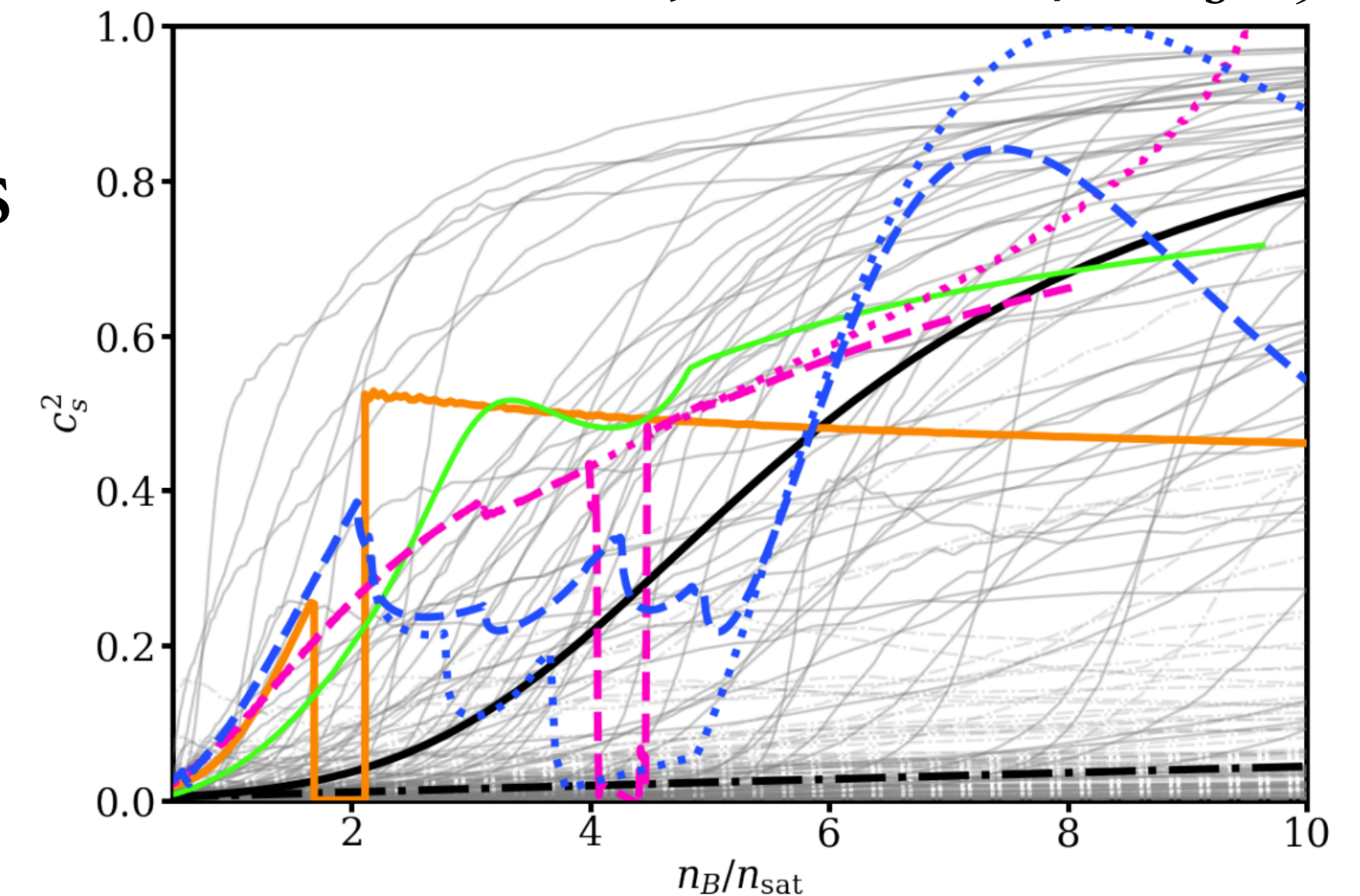
σ : correlation strength

Goal: Compare long range correlation (benchmark) versus multi-scale features

Gaussian Process
(benchmark)

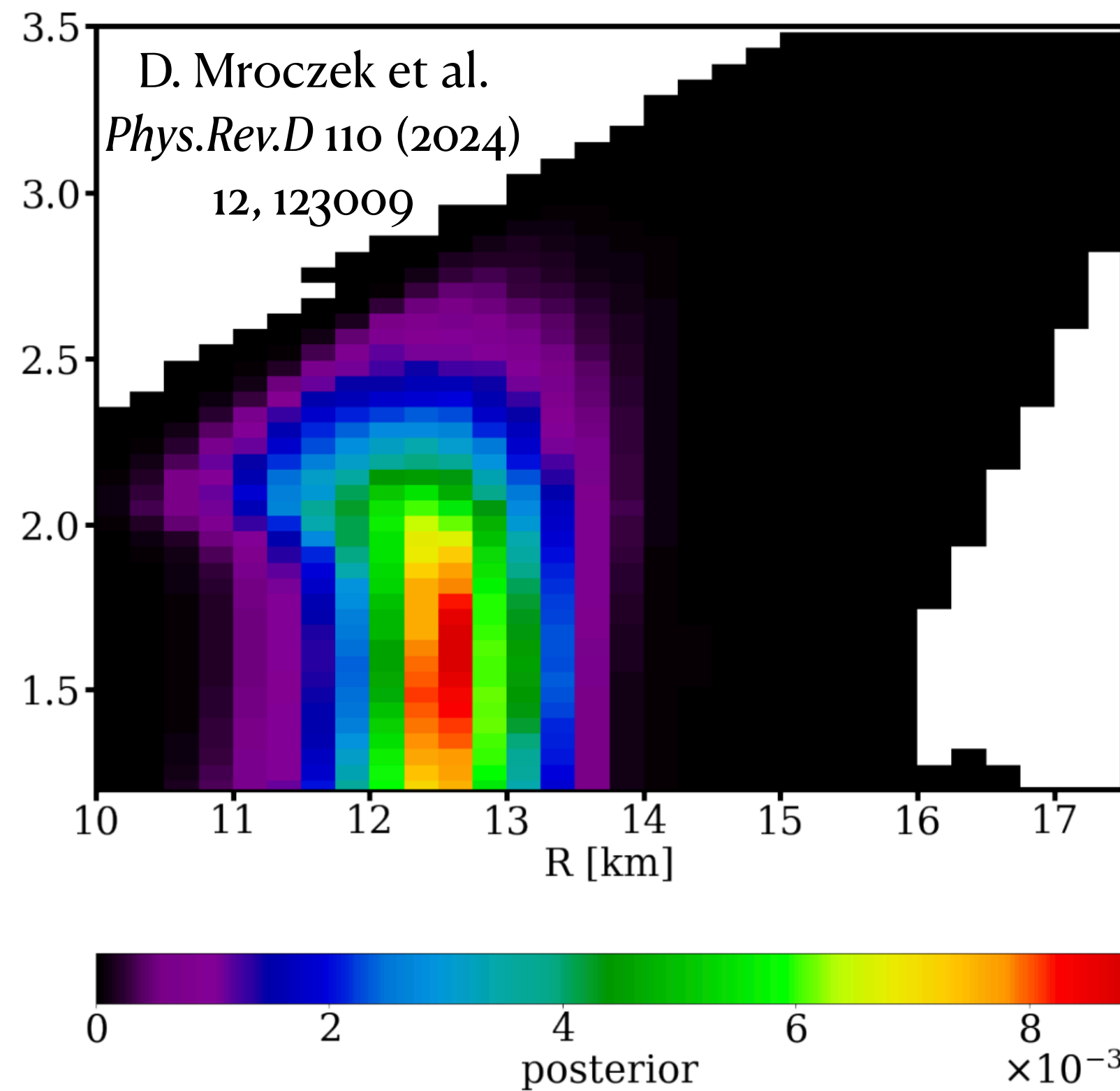
+ Structure

D. Mroczek et al. *Phys.Rev.D* 110 (2024) 12, 123009

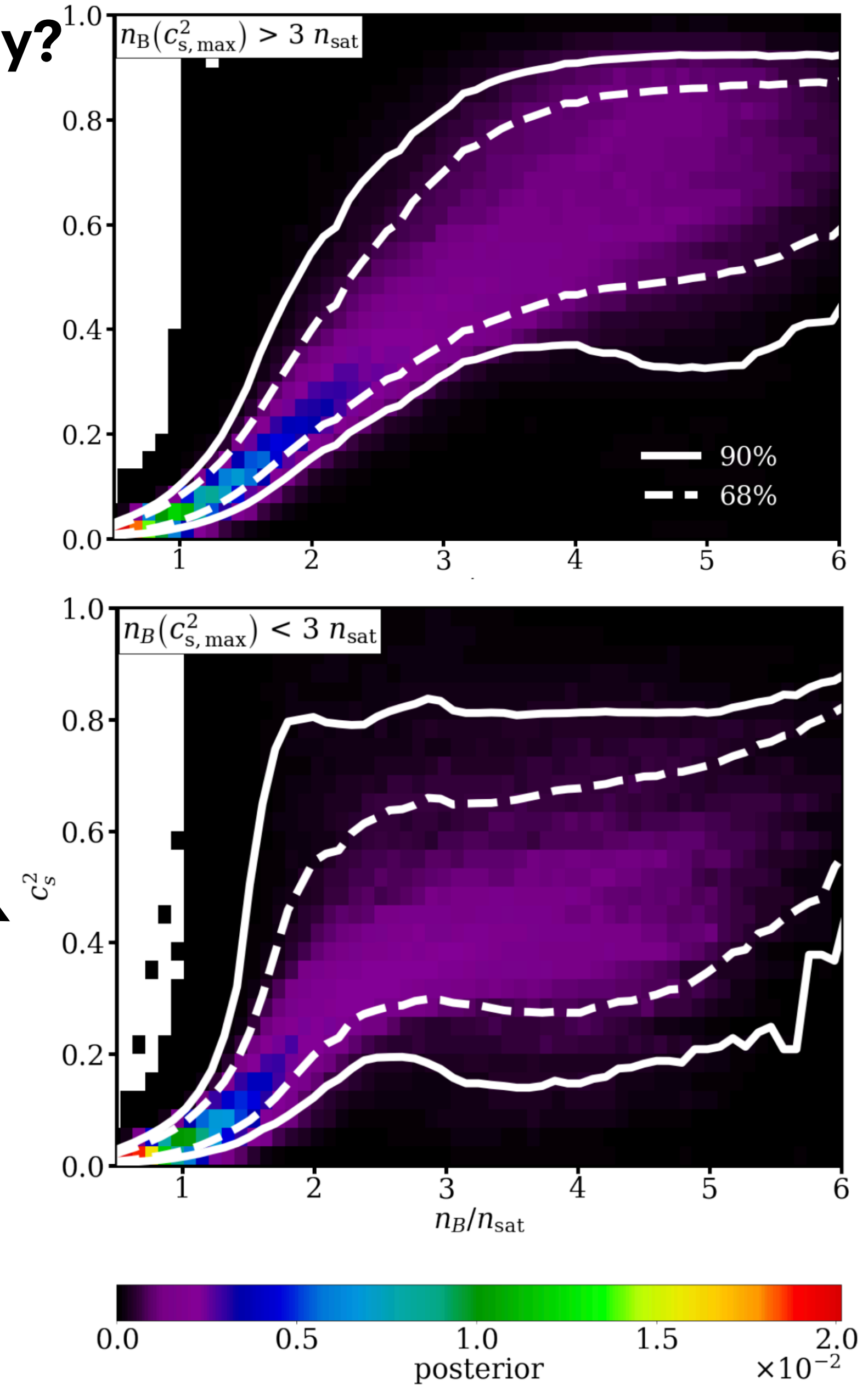


Posteriors of Mass, Radius, $c_s^2(n_B)$

What does current data tell us today?



Both a monotonic c_s^2 and one
with a bump at $n_B = [2,3] n_{sat}$



Current data is inconclusive, need more observations! Or new constraints!

Optimistic Overview

New things features to look for in dense matter

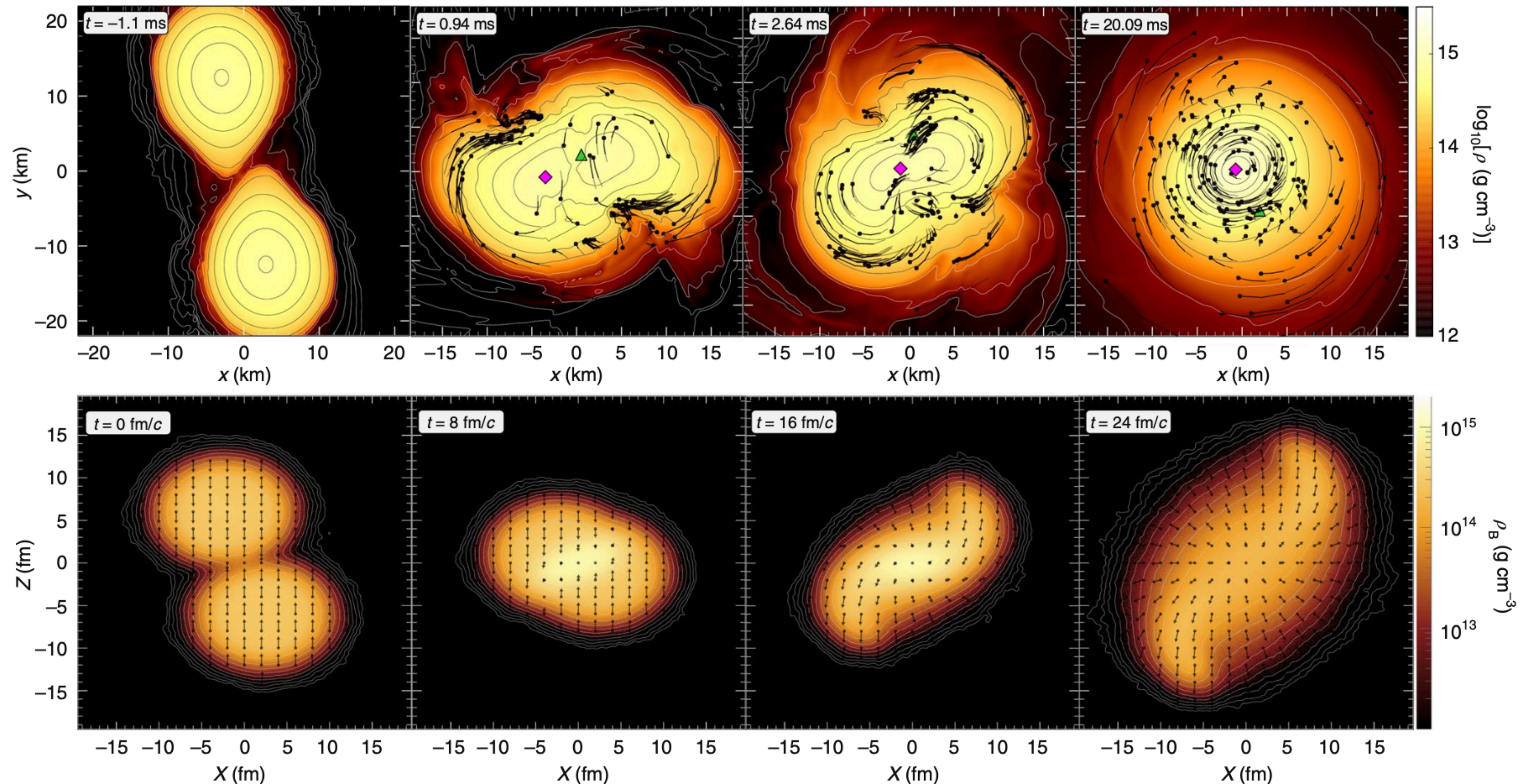
Constraints from astrophysics

How do we connect the HIC and NS EOS? Symmetry energy discussions

Heavy-ion collisions (HIC) vs Binary neutron star (NS) mergers

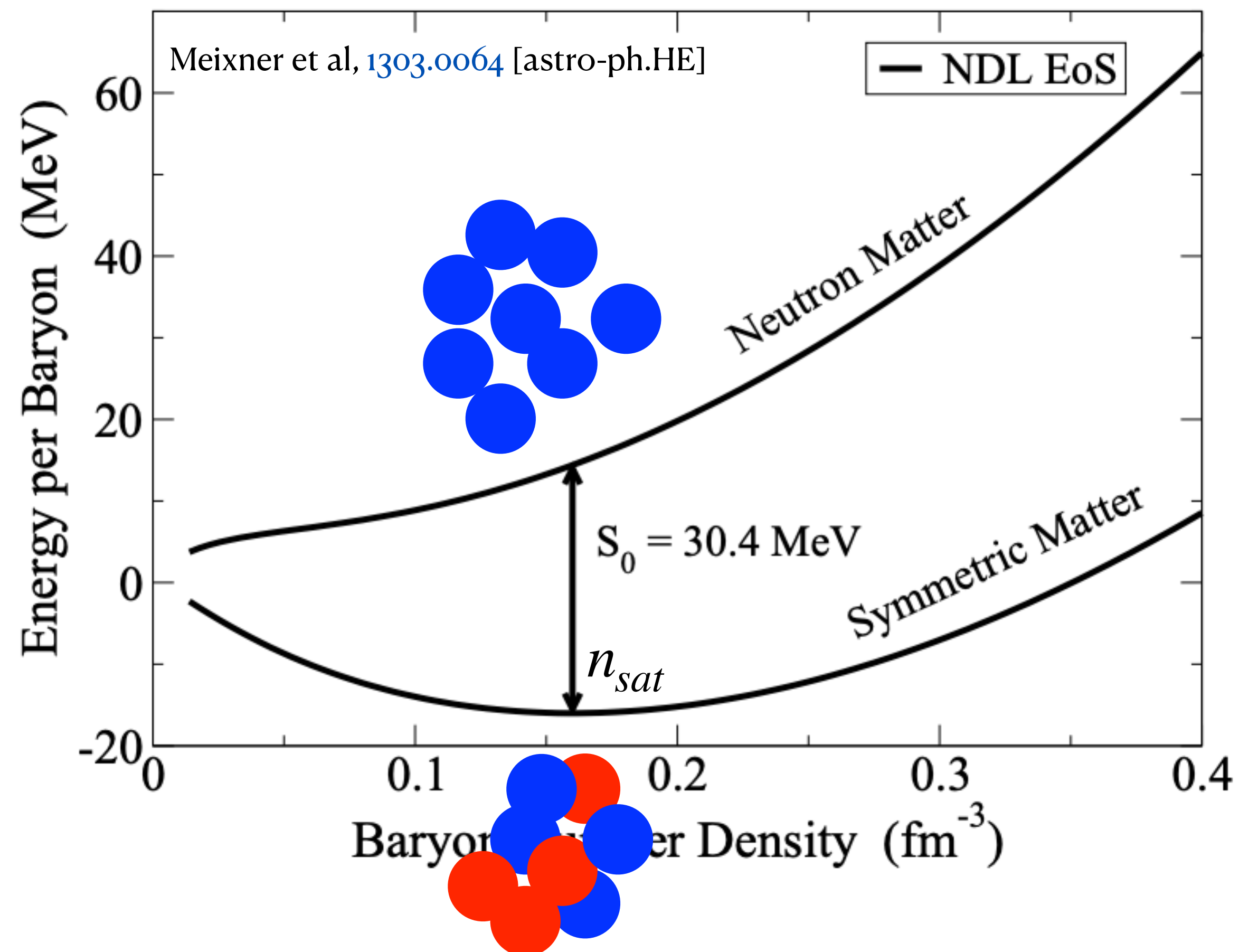
Neutron Star Mergers (Numerical Relativity)

[HADES] Nature Phys. 15 (2019) 10, 1040-1045



Symmetric nuclear matter vs Pure neutron Matter

“Symmetry Energy”



Symmetric nuclear matter

$$\mu_Q = 0$$

$$\text{Isospin asymmetry } \delta = 1 - 2Y_Q = 0$$

$$Y_Q = 0.5$$

ONLY true for neutrons and protons!

Symmetry energy

$$S(n_B) = \frac{\epsilon_{PNM}}{n_B} - \frac{\epsilon_{SNM}}{n_B}$$



What about hyperons?

Symmetry energy expansion with strangeness

Yang et al, 2504.18764 [nucl-th]

Original symmetry energy expansion
considered only protons, neutrons

isospin
asymmetry: $\delta_Q \equiv 1 - 2 Y_Q$

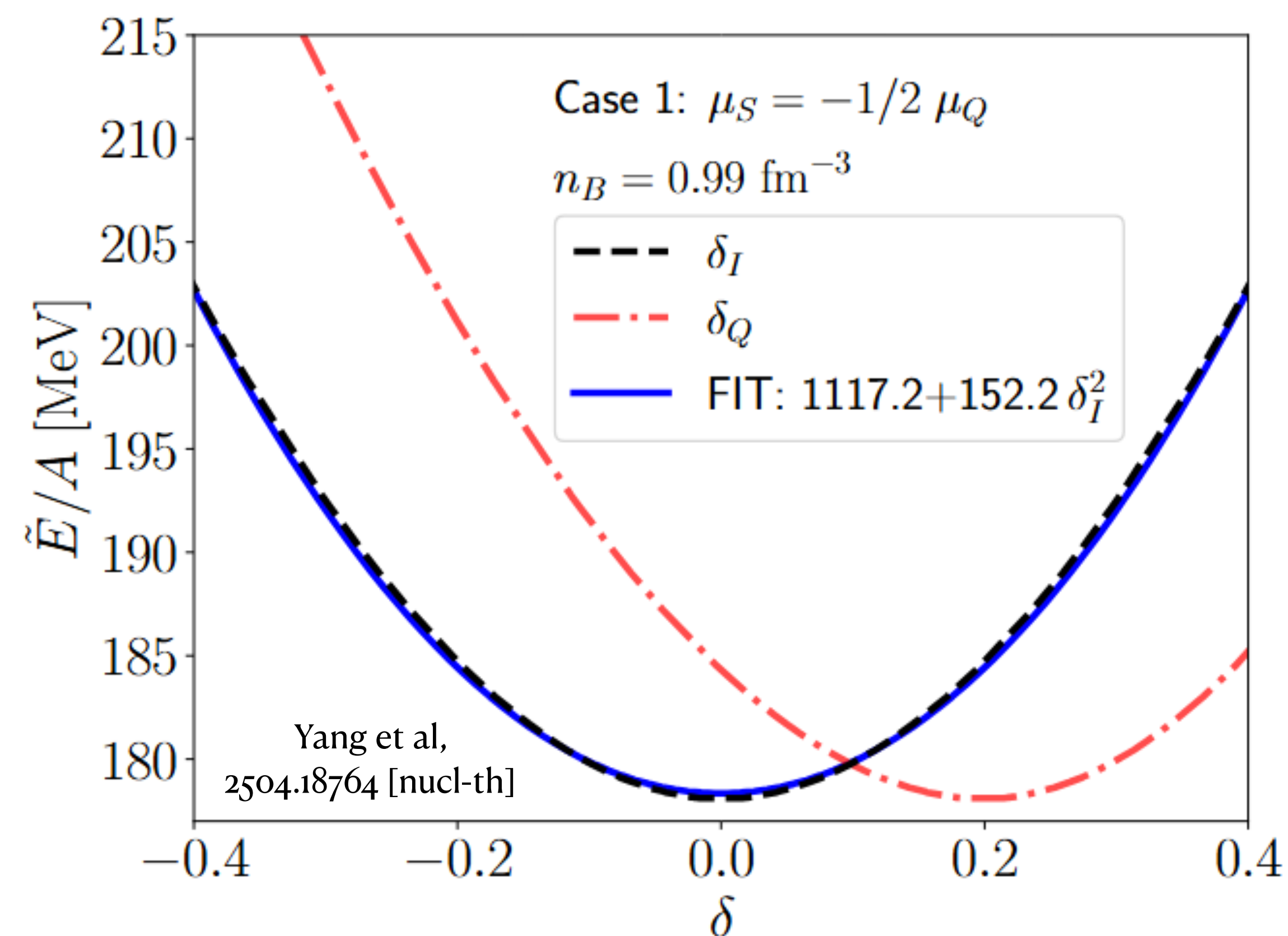
Gell-Mann-Nishijima formula

$$Q = I_z + \frac{1}{2} (B + \mathcal{S})$$

Corrected isospin asymmetry

$$\delta_I \equiv 1 + Y_S - 2 Y_Q$$

Original symmetry energy expansion
misses the ground state of nuclear
matter when hyperons are present



Chemical potentials from HIC to NS

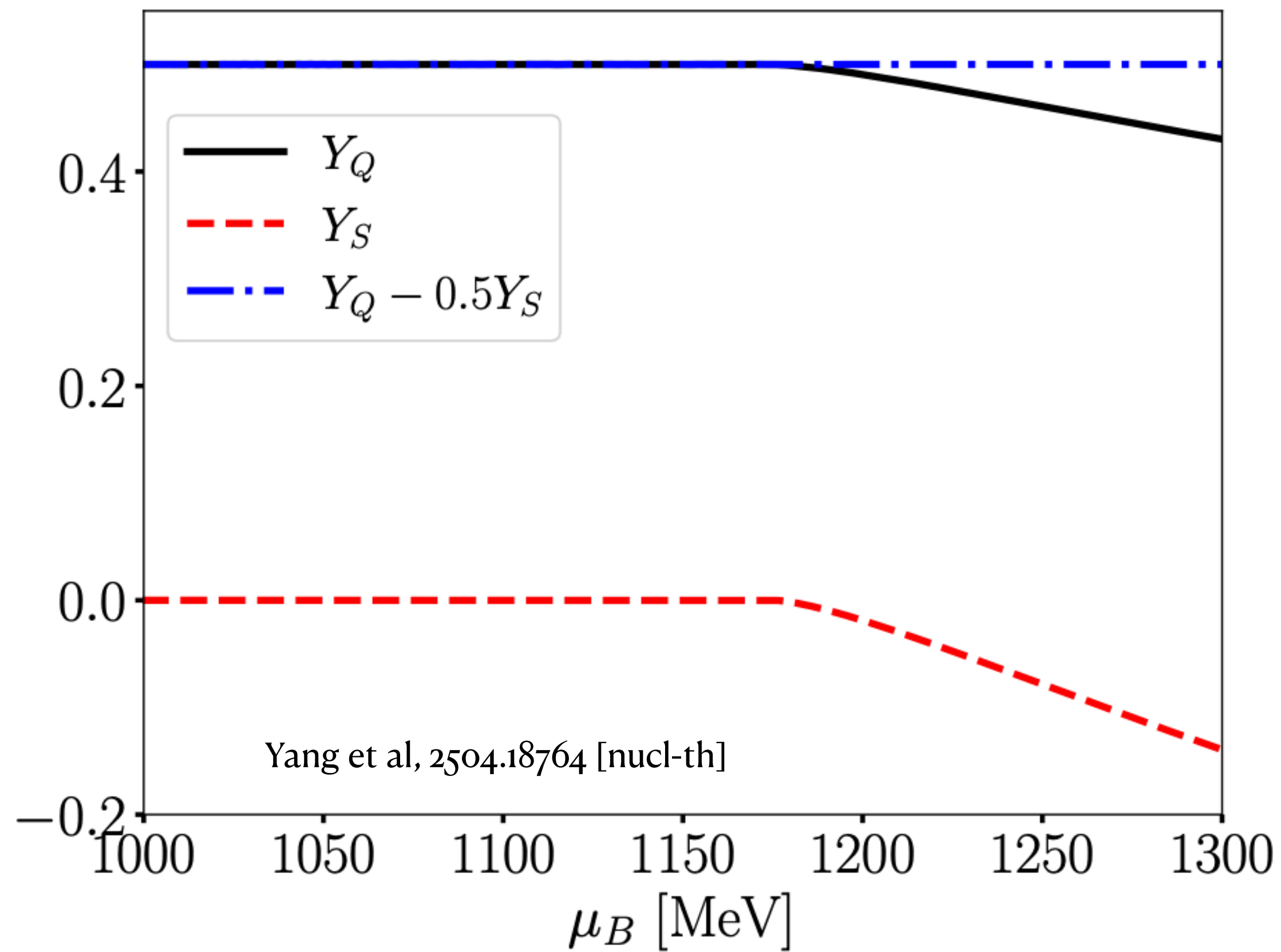
Weak β equilibrium

- Neutron stars live long enough that weak-interactions are in equilibrium. For neutrons/protons:
 $n \rightarrow p + e^- + \bar{\nu}_e$ and $p + e^- \rightarrow n + \nu_e$
that provides constraints on the chemical potentials: $\mu_n = \mu_p + \mu_e$ (free-streaming neutrinos)
- What about hyperons? For Λ 's they should also be in equilibrium i.e. $\Lambda \rightarrow p + e^- + \bar{\nu}_e$ so the chemical potentials imply $\mu_\Lambda = \mu_p + \mu_e$. So $\mu_\Lambda = \mu_n$ that implies $\mu_S = 0$!
- HOWEVER, isospin symmetry occurs for in the basis of $\mu_S = -1/2\mu_Q$ (see back-up slide for derivation). See Aryal et al, *Phys.Rev.D* 102 (2020) 7, 076016 and Yang et al, [2504.18764](#) [nucl-th]
- $\mu_Q \sim [50, 200] \text{ MeV}$, very far from 0!
- **Weak β -equilibrium is NOT isospin symmetric.** Thus, symmetry energy expansion to connect to neutron stars MUST include odd terms (skewness) terms when $Y_S \neq 0$.

Strangeness contribution to SNM

You really **CANNOT** ignore it

For SNM, $Y_Q \neq 0.5$



Wrong isospin
asymmetry

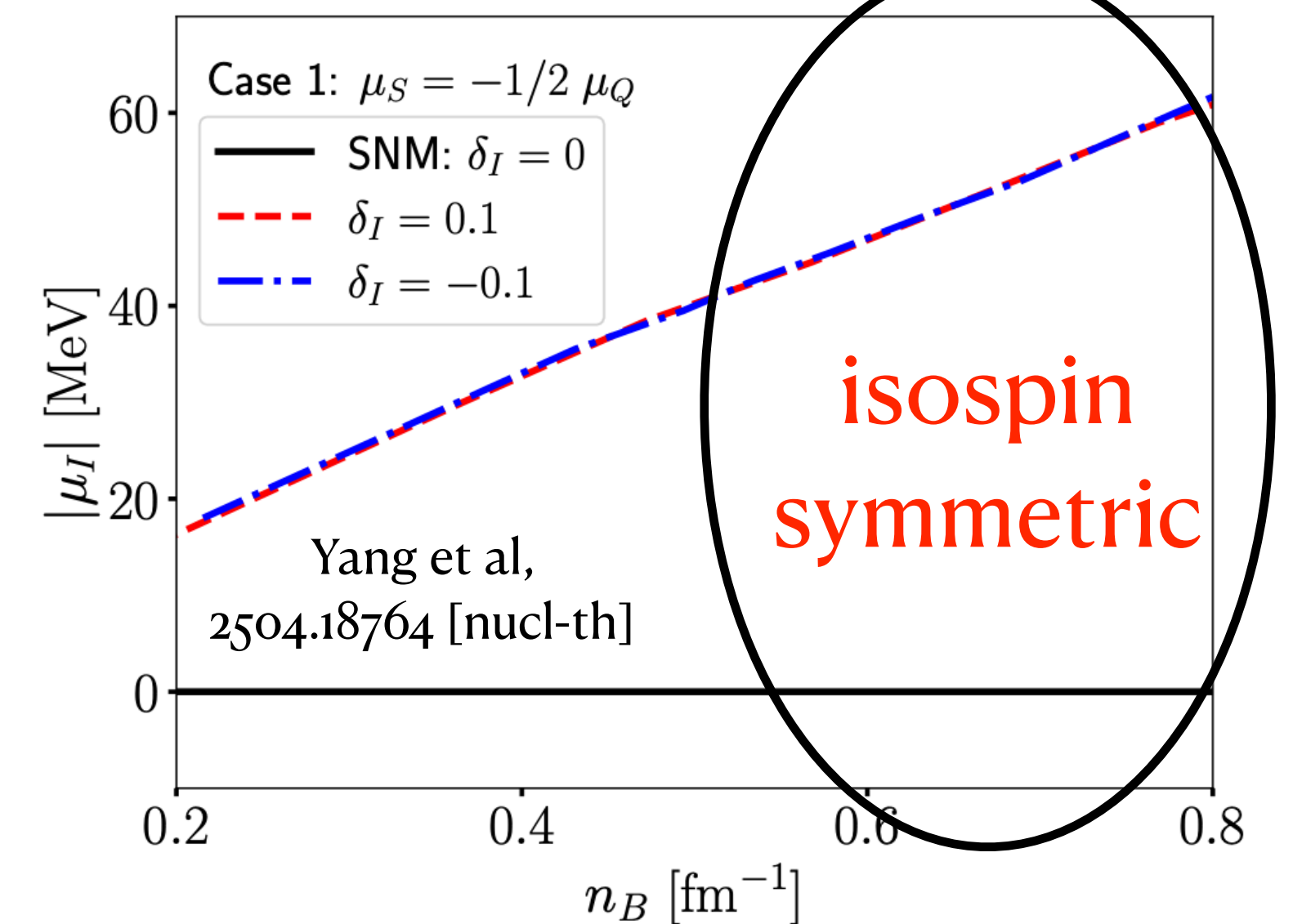
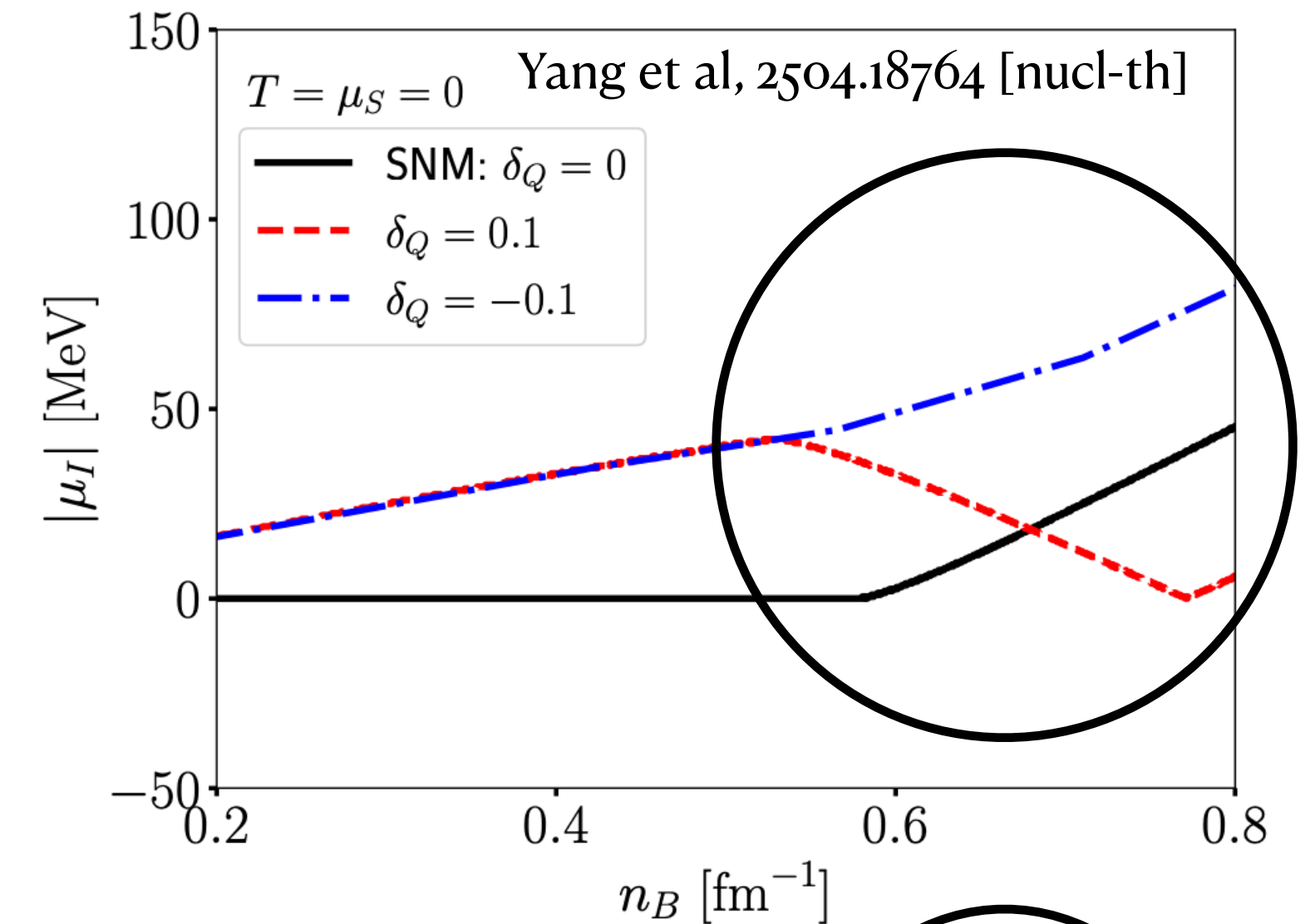
$$\delta_Q = 1 - 2Y_Q$$

Correct isospin
asymmetry

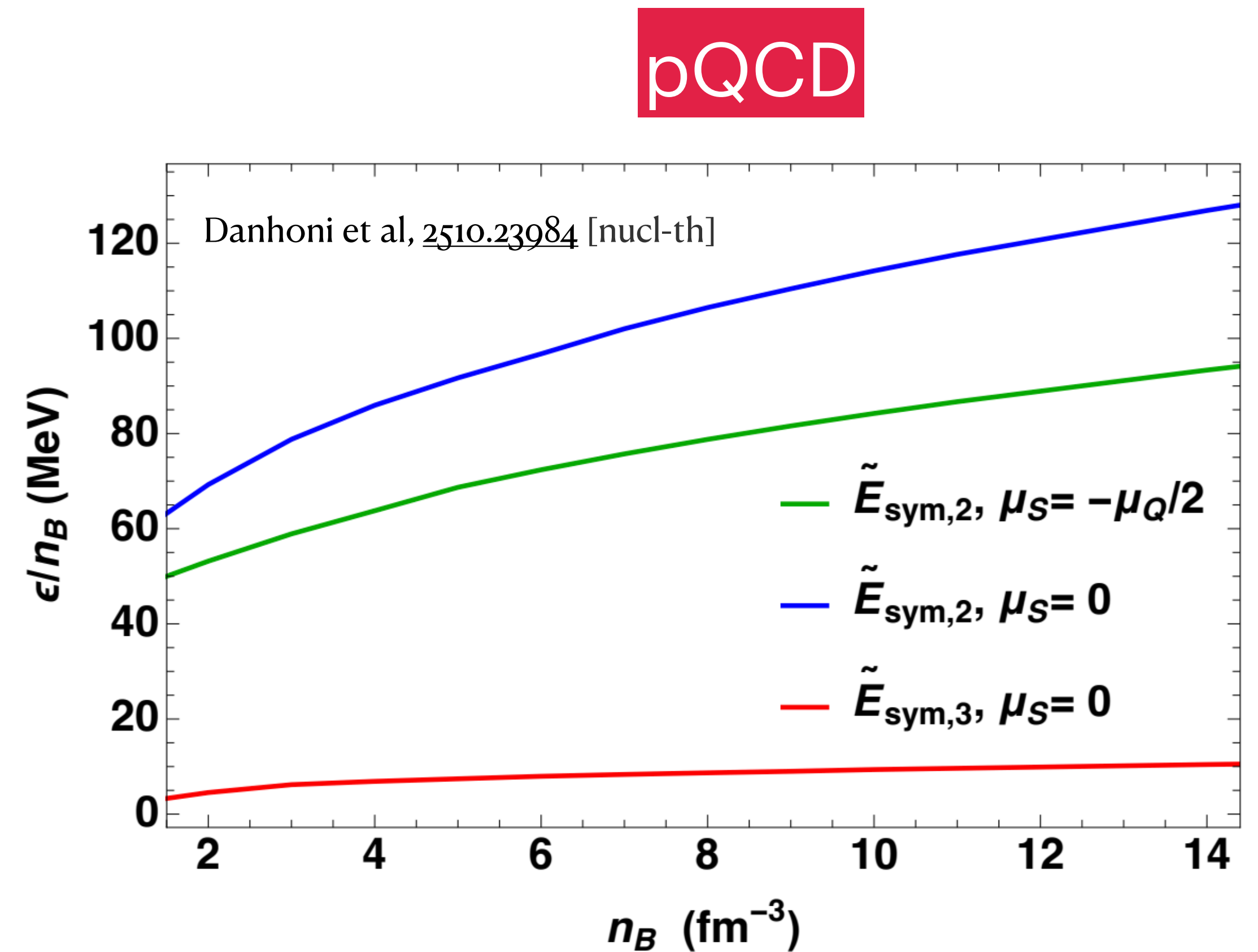
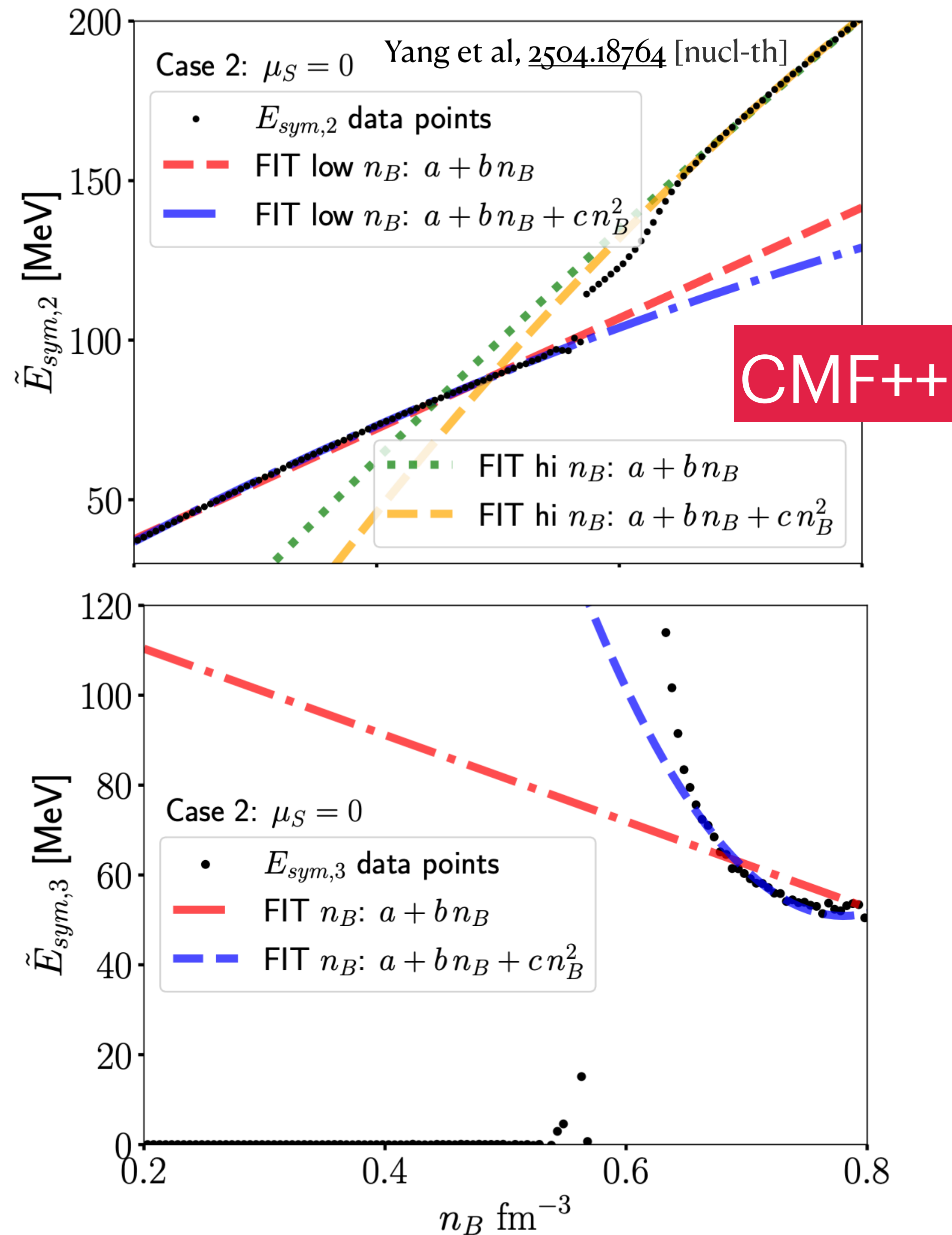
$$\delta_I = 1 - 2Y_Q + Y_S$$

PNM is meaningless for $\delta_I = 1$, then $Y_S = 2Y_Q$

NOT isospin symmetric

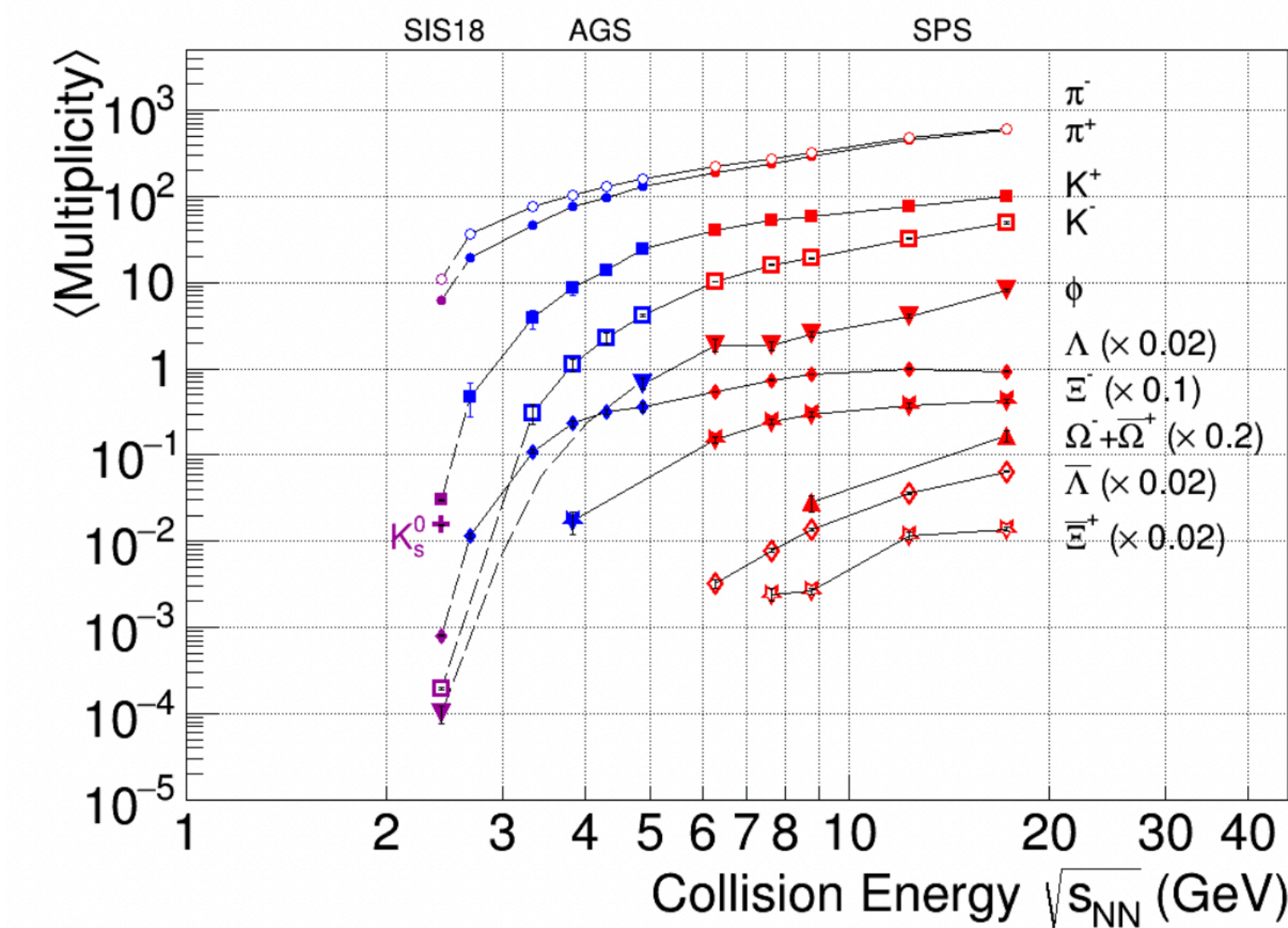
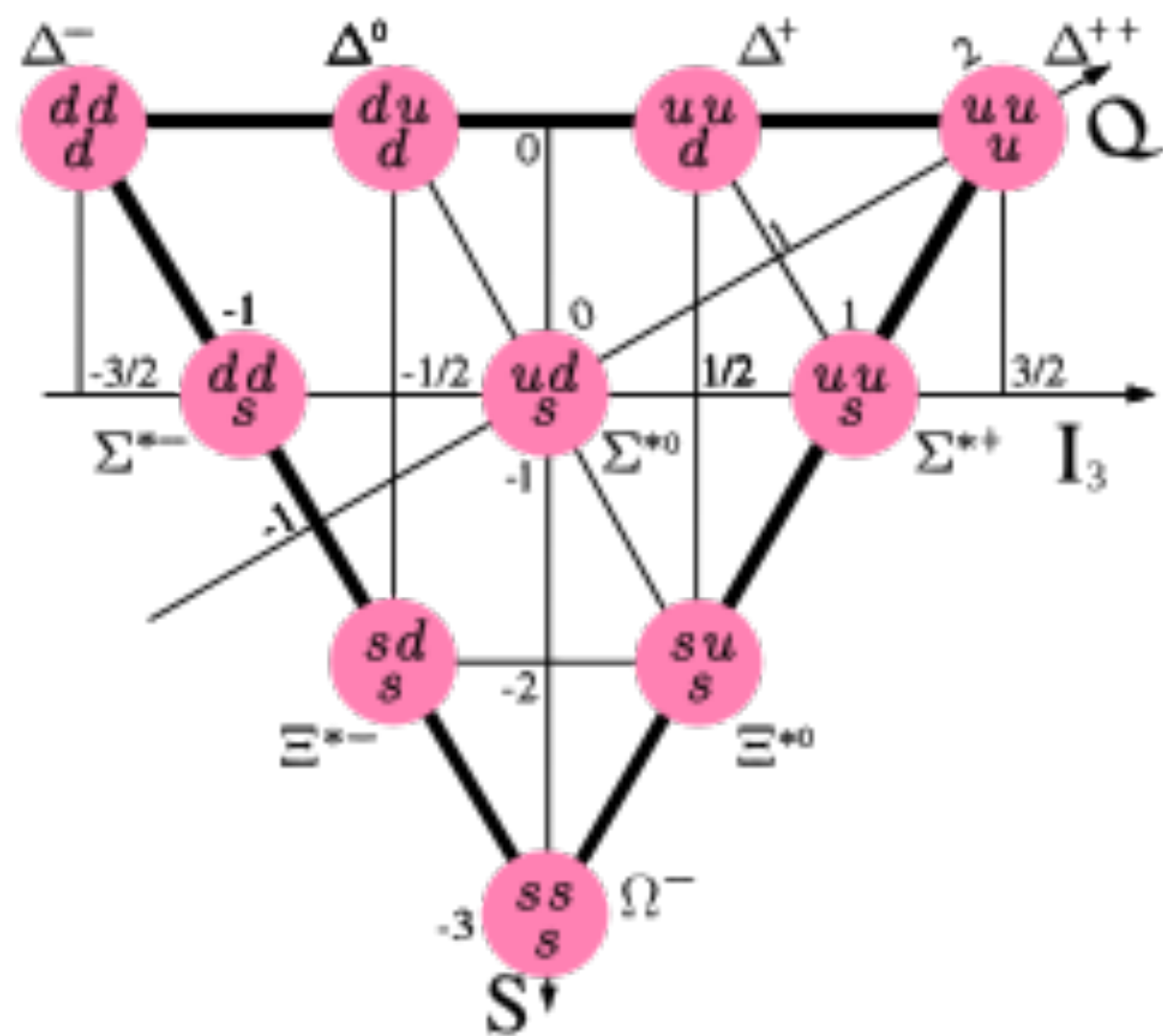


Strange symmetry energy: Chiral mean field and pQCD



Skewness term a generic feature. At β equilibrium $\mu_S = 0$, which breaks isospin symmetry when $Y_S \neq 0$

Ignoring strangeness



Blume, Markert, PPNP 66 (2011)
see also yield ratios for Si+Au Braun-Munzinger, Stachel, Wessels, Xu, PLB 344 (1995)

IGNORE Strange: Symmetry Energy Expansion

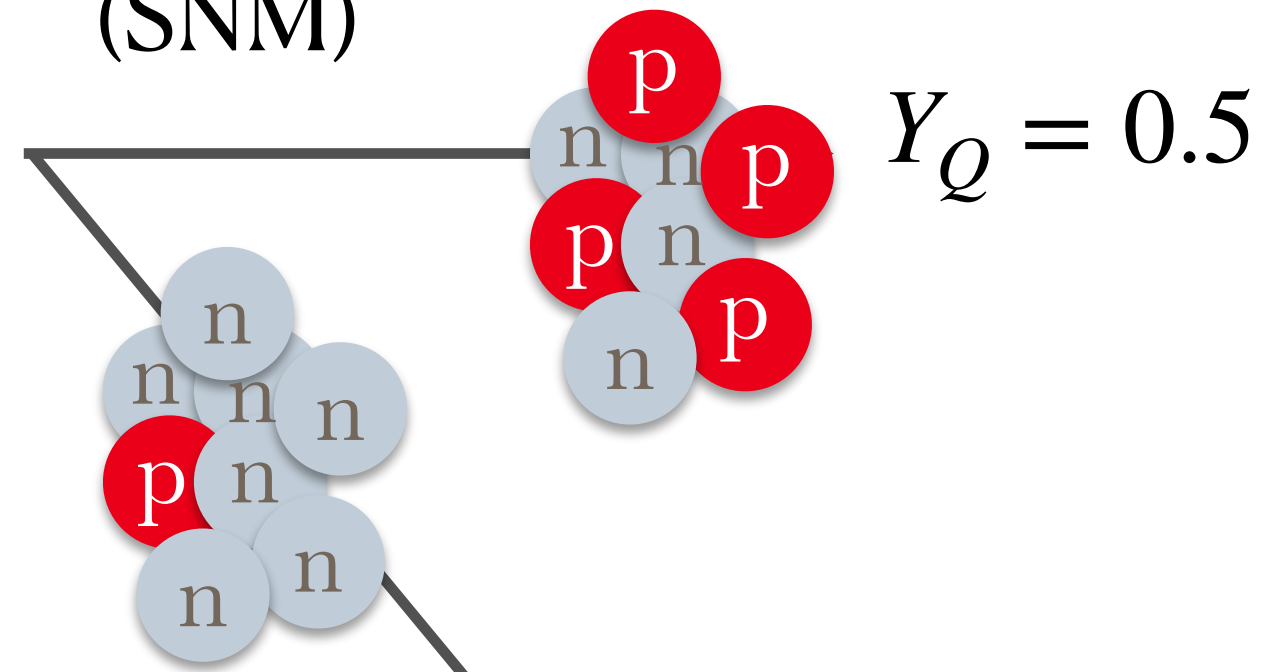
Connecting NS to HIC across Y_Q

Original symmetry energy expansion from binding energies

Bombaci & Lombardo *Phys.Rev.C* 44 (1991) 1892-1900

Symmetric matter

(SNM)



Isospin asymmetry $\delta = 1 - 2Y_Q$

where $\delta = 0$ for SNM and $\delta = 1$ for PNM

$$\frac{E_{ANM}}{N_B} = \frac{E_{SNM}}{N_B} + E_{sym,2}\delta^2 + \mathcal{O}(\delta^4)$$

Asymmetric
matter (ANM)

$Y_Q \lesssim 0.1$

Expand in δ where odd terms drop due to isospin symmetry

Convert EOS from NS to HIC, expand around n_{sat}

$$\epsilon_{HIC} = \epsilon_{NS} - 4n_B \left[E_{sym,sat} + \frac{L}{3} \left(\frac{n_B}{n_{sat}} - 1 \right) + \frac{K}{18} \left(\frac{n_B}{n_{sat}} - 1 \right)^2 + \frac{J}{162} \left(\frac{n_B}{n_{sat}} - 1 \right)^3 \right] \left[\left(Y_Q^{HIC} - Y_{Q,NS} \right) + \left(Y_{Q,NS}^2 - \left(Y_Q^{HIC} \right)^2 \right) \right]$$

$E_{sym,2}(n_B)$ 4 unknowns

To varying Y_Q^{HIC}

Yao et al, *Phys.Rev.C* 109 (2024) 6, 065803

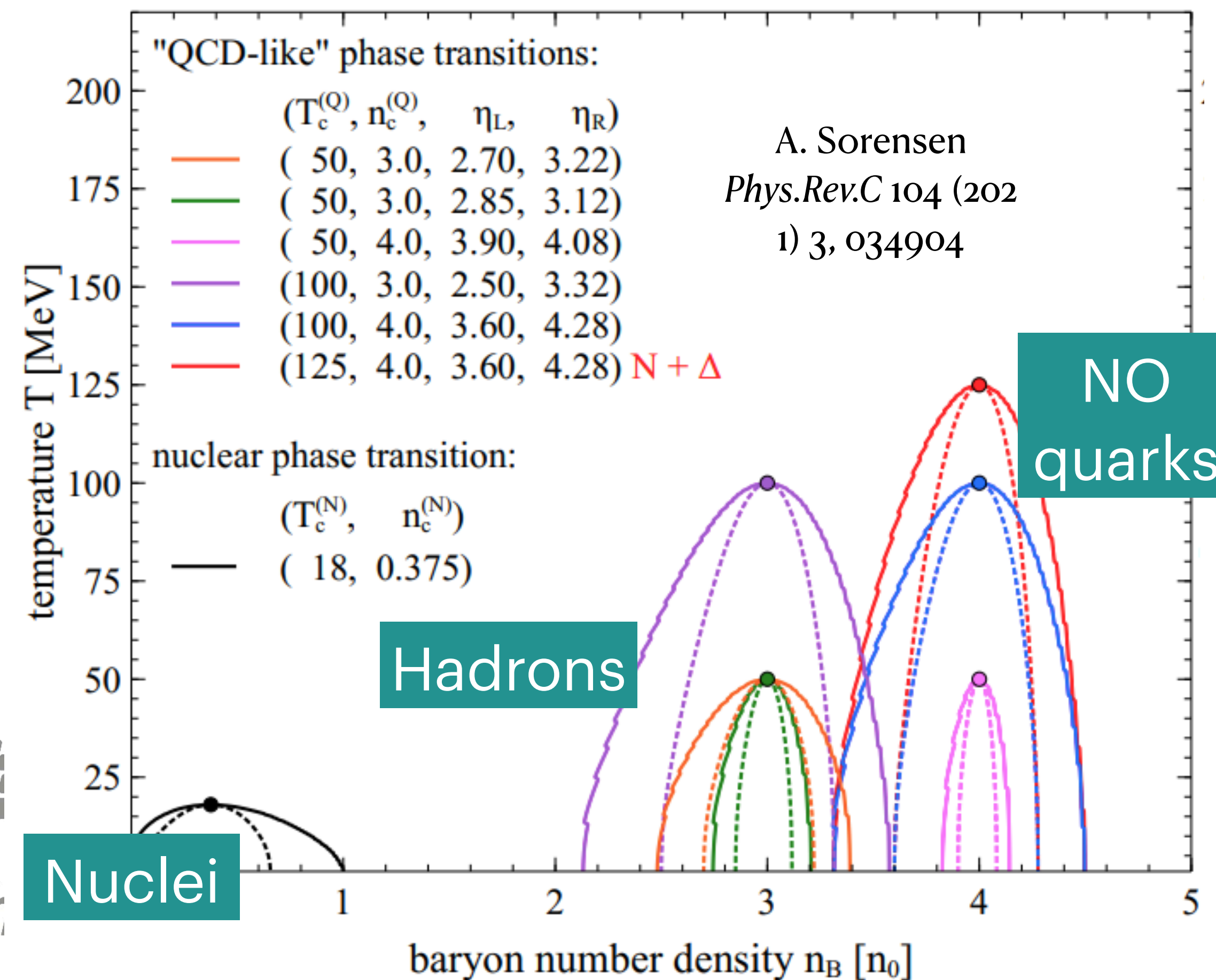
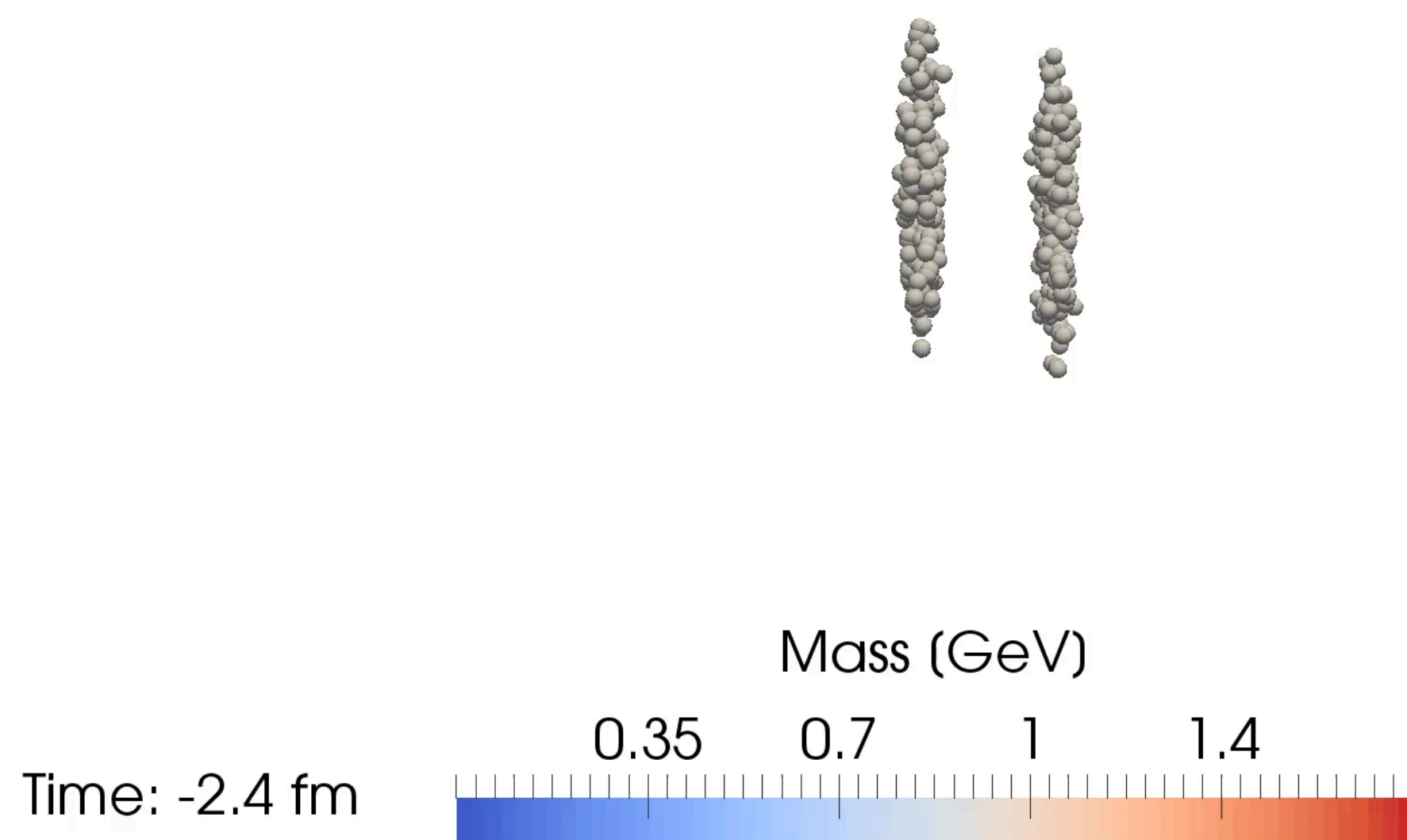
2020's Low-energy $\sqrt{s} \leq 7.7 \text{ GeV}$ HICs

New STAR data, new interpretations

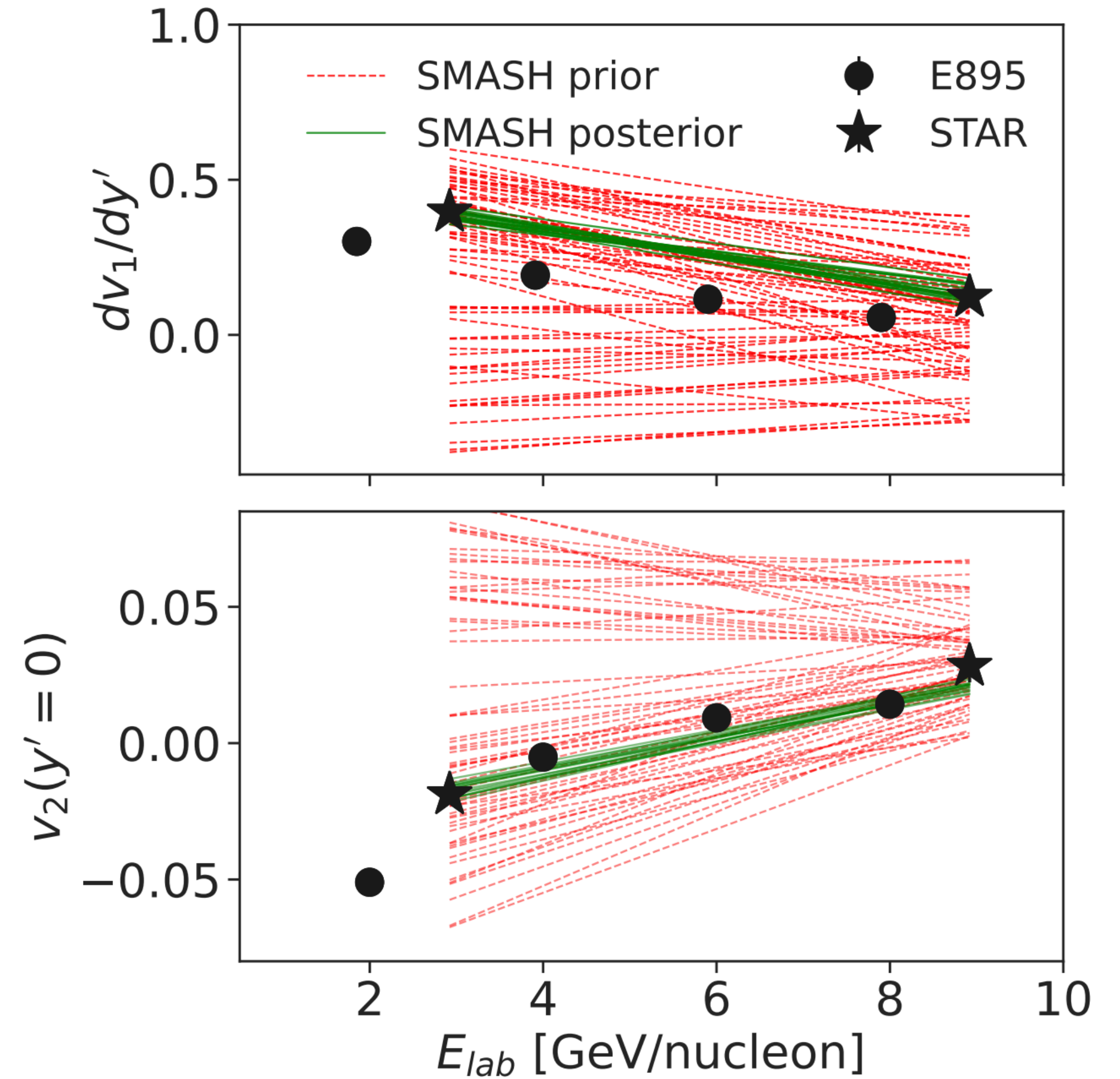
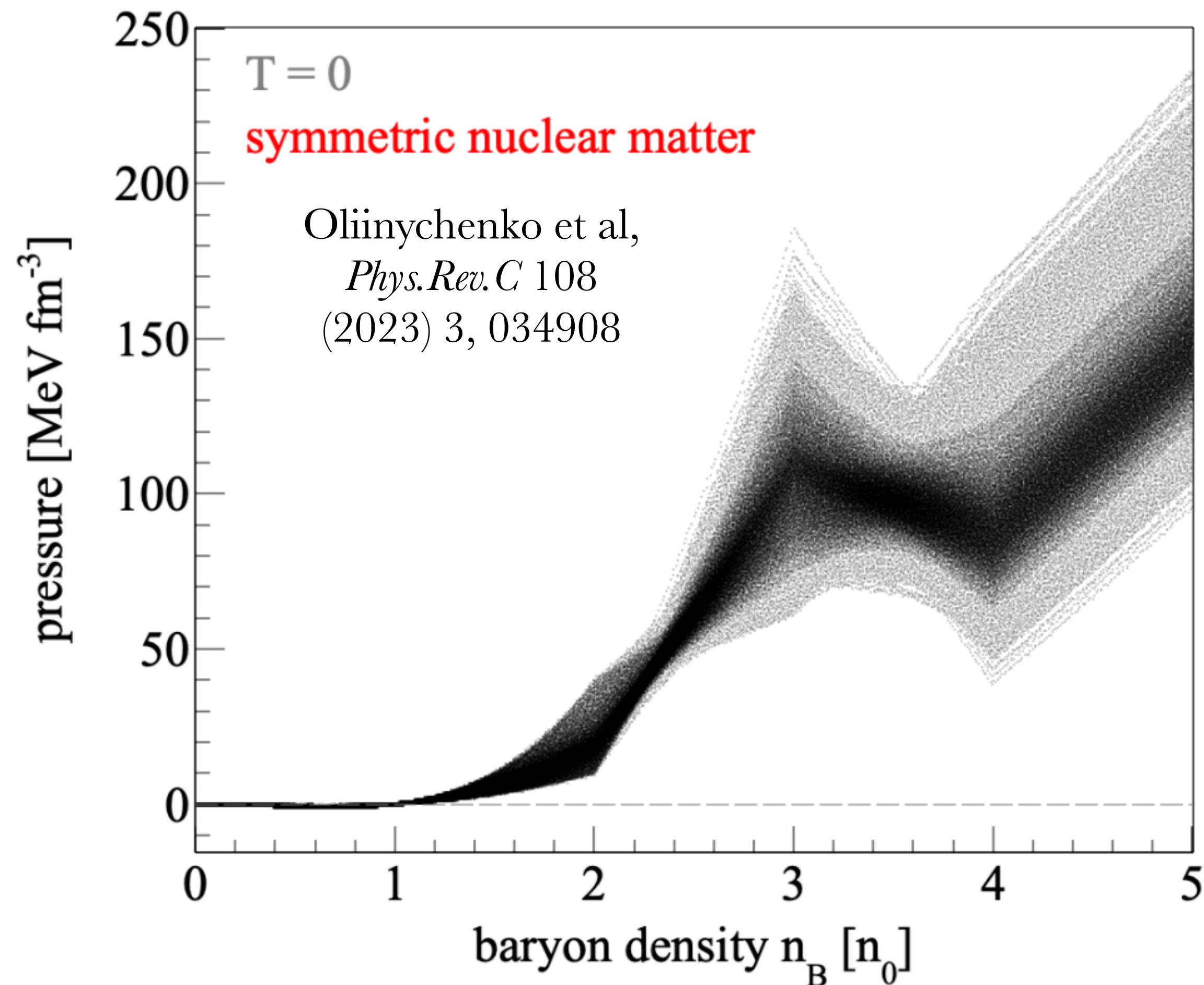
Do quark-gluons d.o.f. matter?

Build phase transitions into hadron transport

Flexible potentials $\rightarrow$ phase transitions



Peak in c_s^2 at $n_B^{peak} = [2,3] n_{sat}$ for HIC



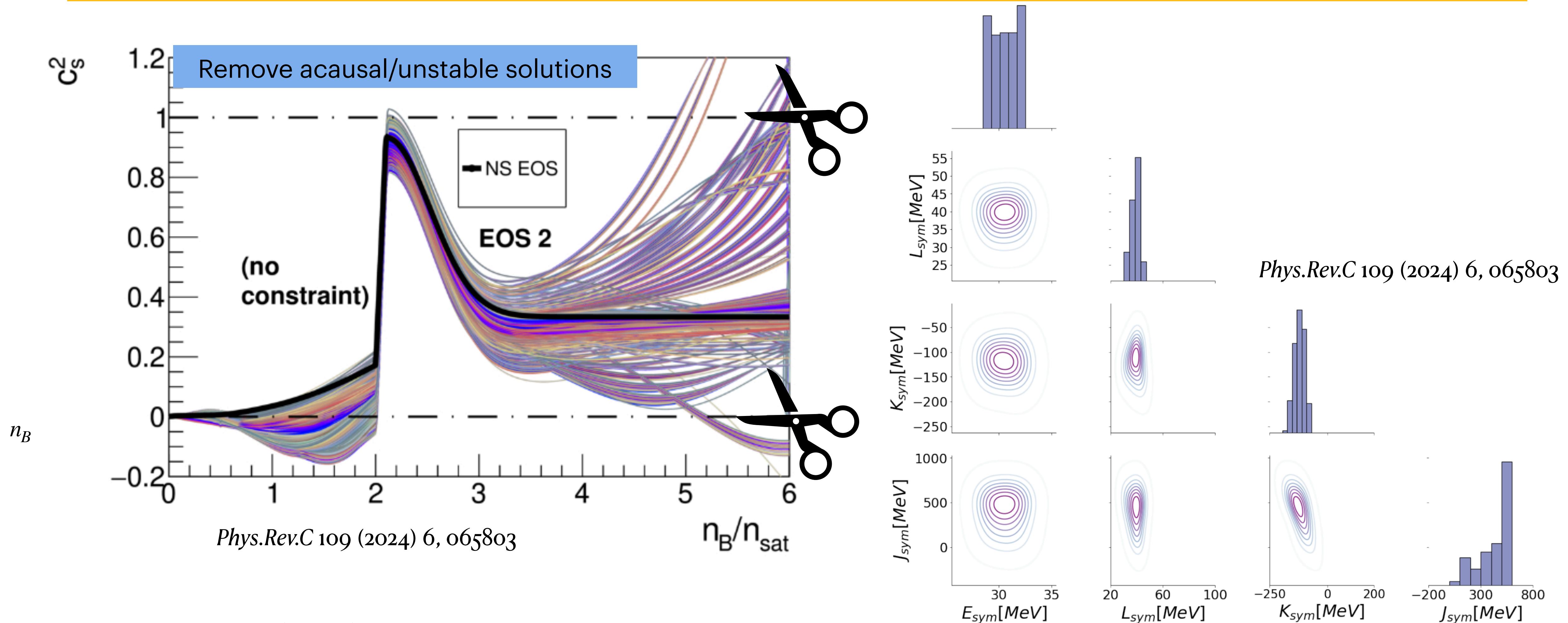
See also *Phys.Rev.Lett.* 131 (2023) 20, 202303



How do low-energy heavy-ion collisions at $T = 0$ connect to neutron stars?

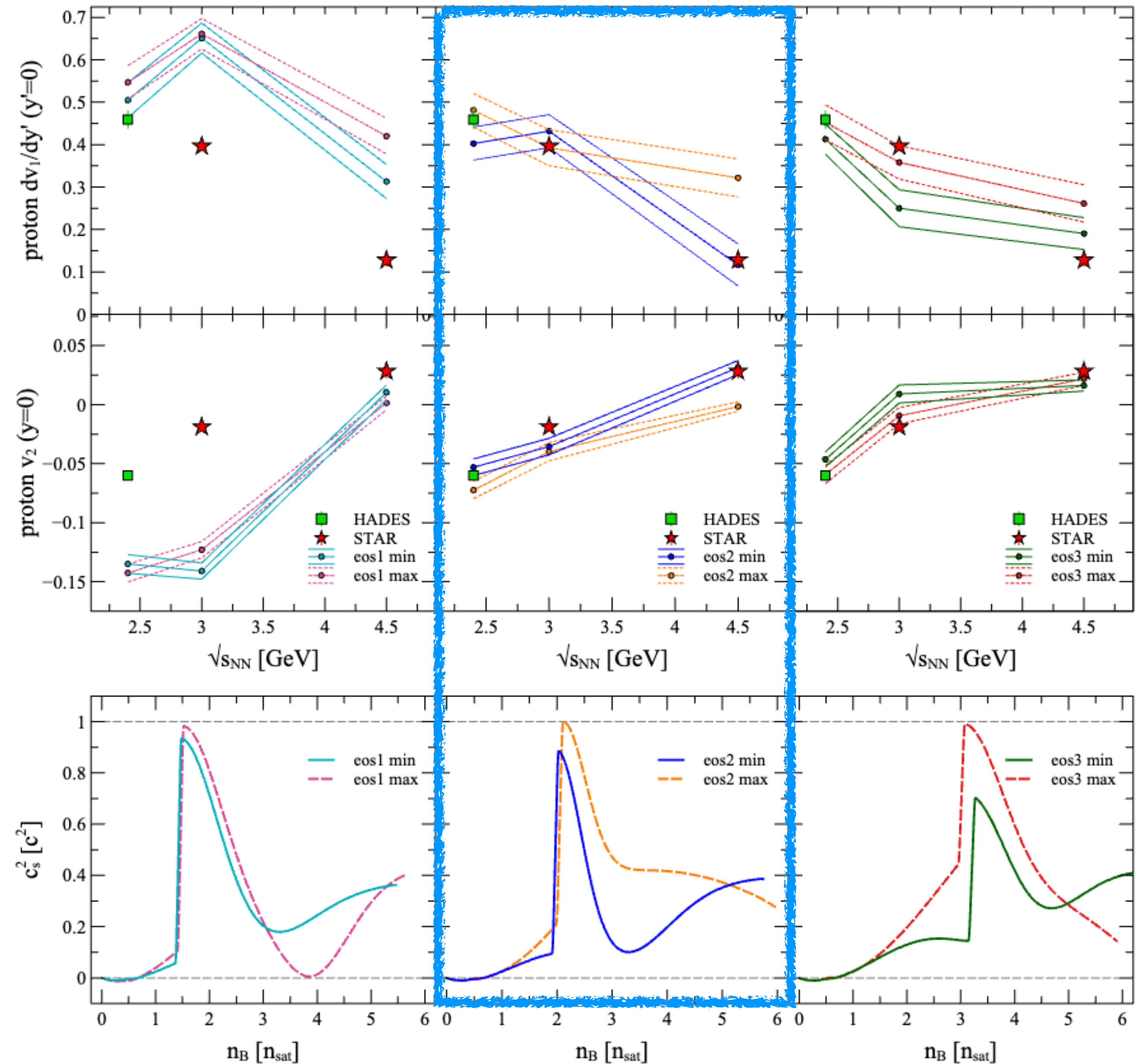
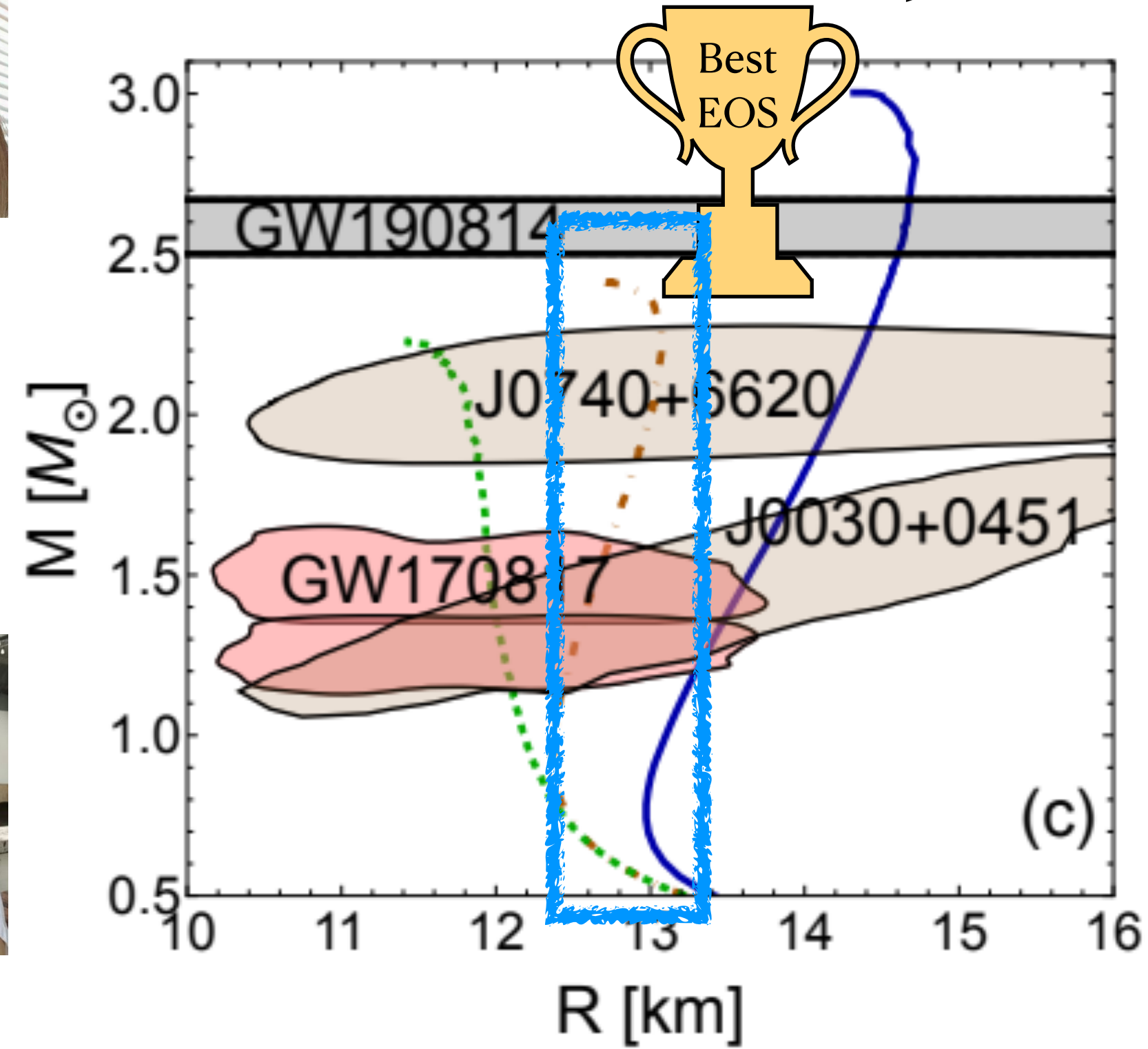
Symmetry energy expansion

Given neutron star equation of state $\rightarrow$ convert to HIC and can constrain by $0 \leq c_s^2 \leq 1$ and saturation properties.



Ultra heavy neutron star EOS works HIC

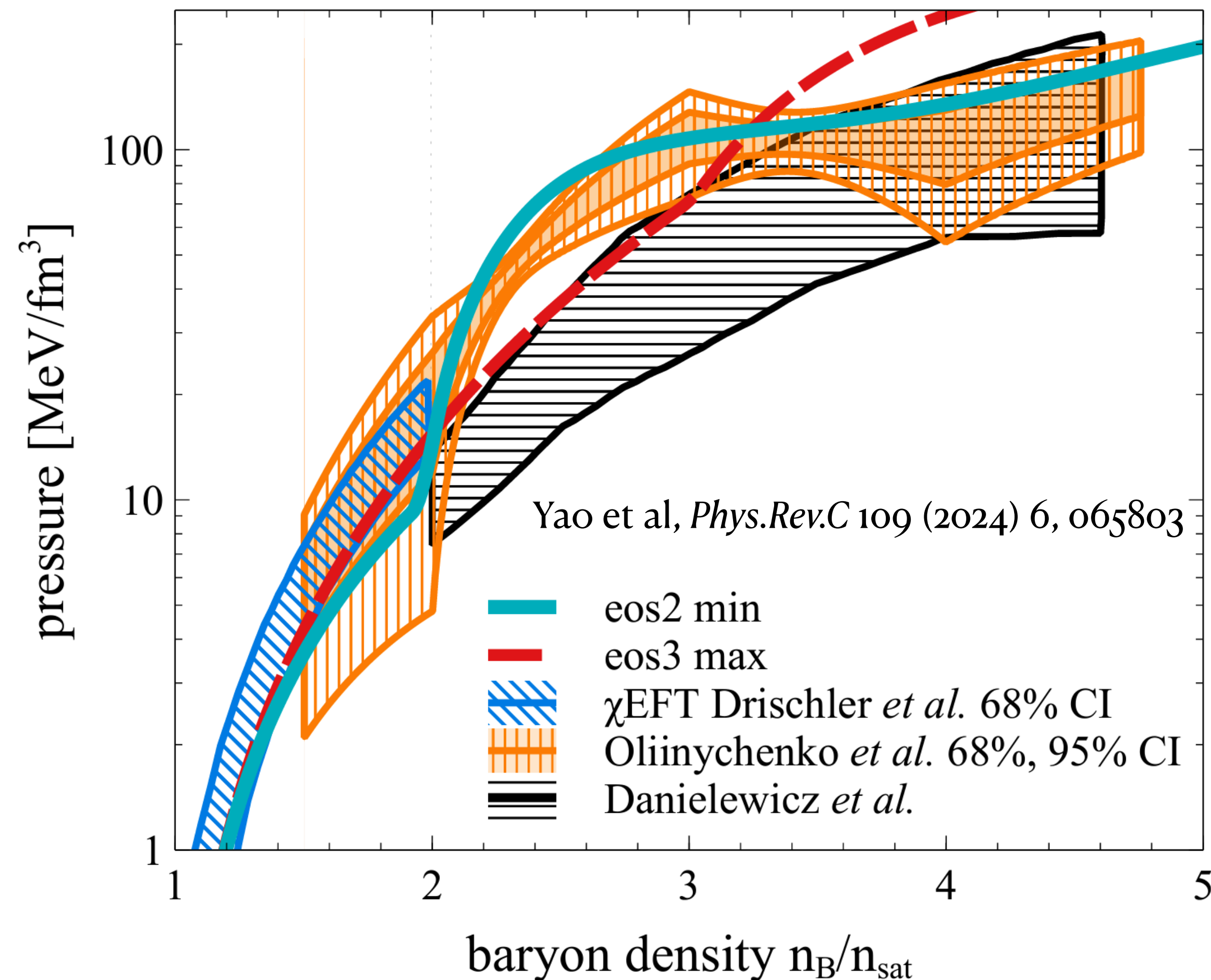
Phys.Rev.C 109 (2024) 6, 065803



Take extremes of the EOS band derived for heavy-ion collisions, run in hadron transport, and compare to heavy-ion collision flow data

Ultra heavy neutron star EOS works HIC

But, this does NOT match hadron only transport models



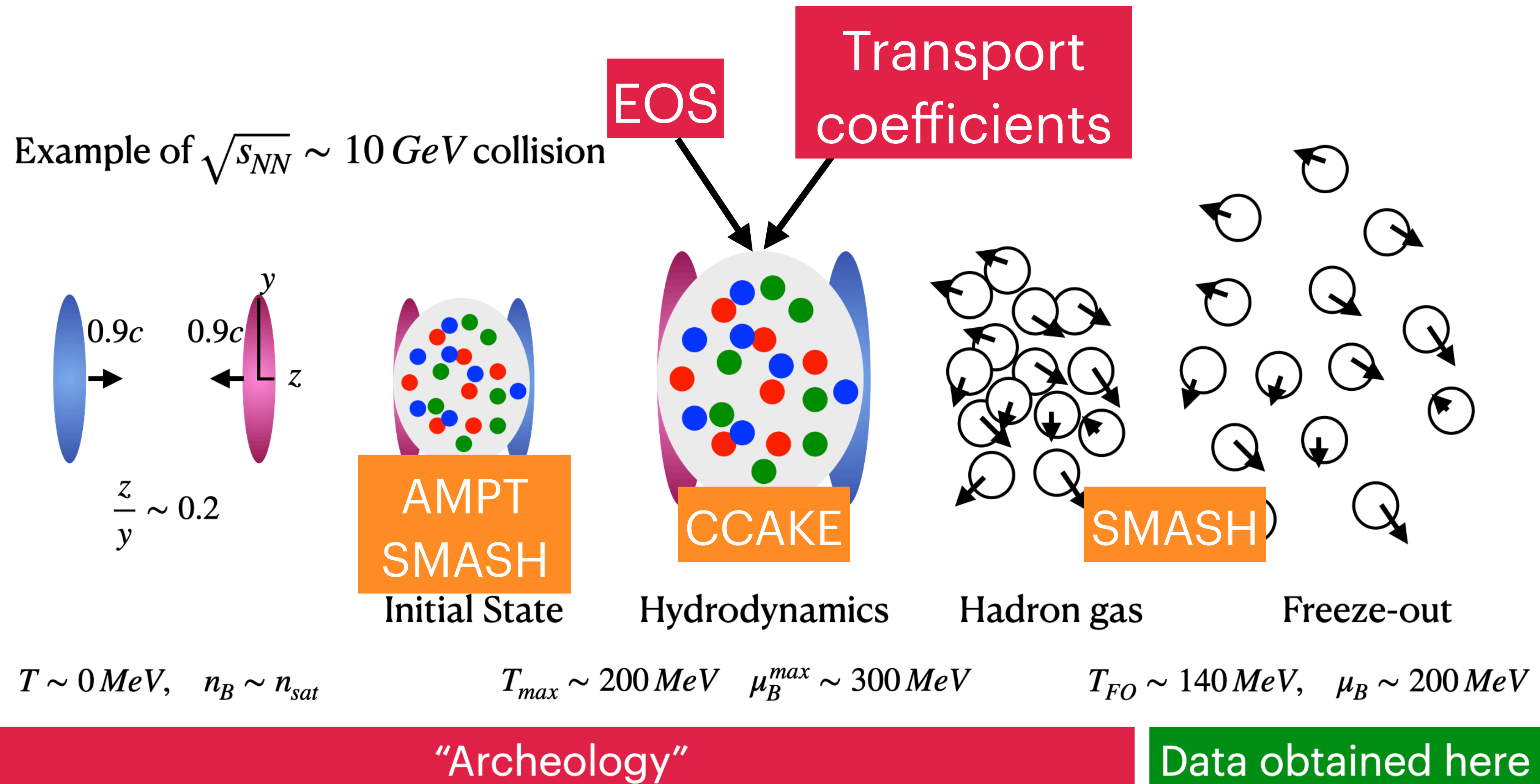
But, this is NOT the end of the story..

- Strangeness symmetry energy expansion
- Redo with hydrodynamics
- Properly include quarks
- Bayesian analysis BOTH with NS and HIC + symmetry energy coefficients
- Momentum dependence potentials
- A lot of unused HIC data still...

Future: hydrodynamics studies

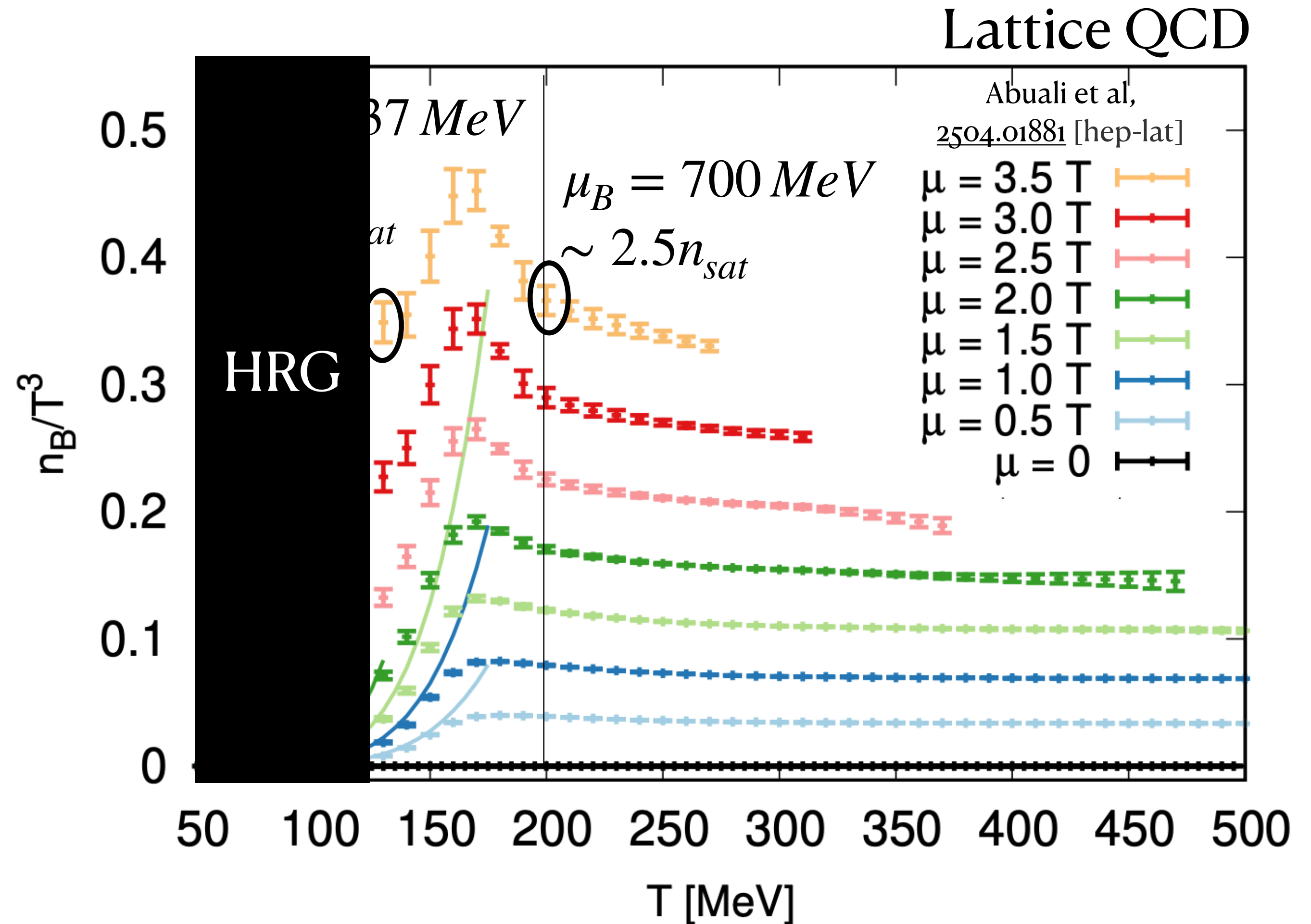
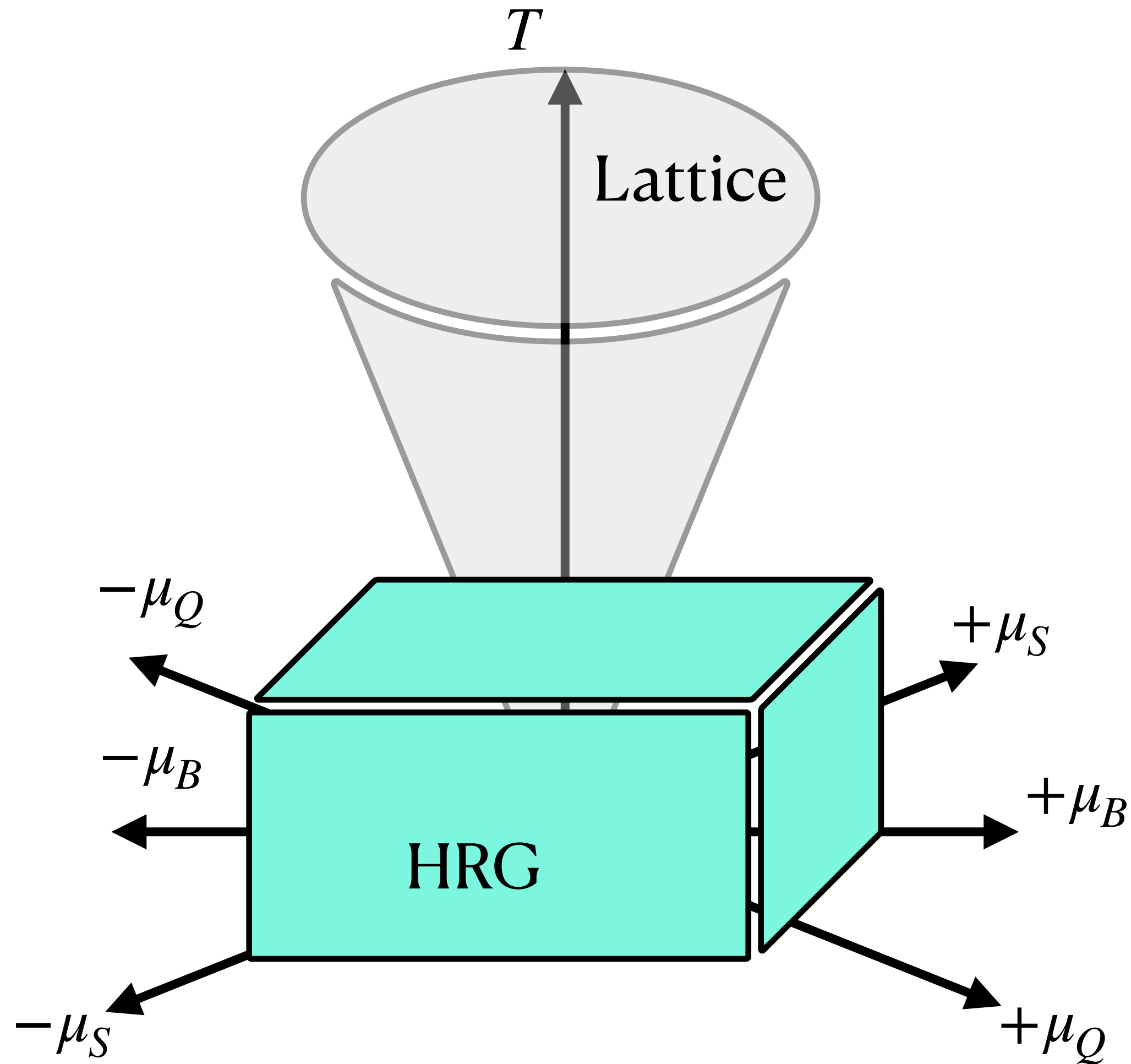
Heavy-ion Collision Framework

From initial to final state



Trento:
CCAKE: Pala et al,
SMASH:

4d EOS of heavy-ions T, n_B, n_S, n_Q



Reasonable estimates: $\mu_S/\mu_B \sim 1/3$
 $\mu_Q/\mu_B \sim -0.1$

But these are fields,
 with large fluctuations!

Lines are ideal hadron
 resonance gas (HRG)

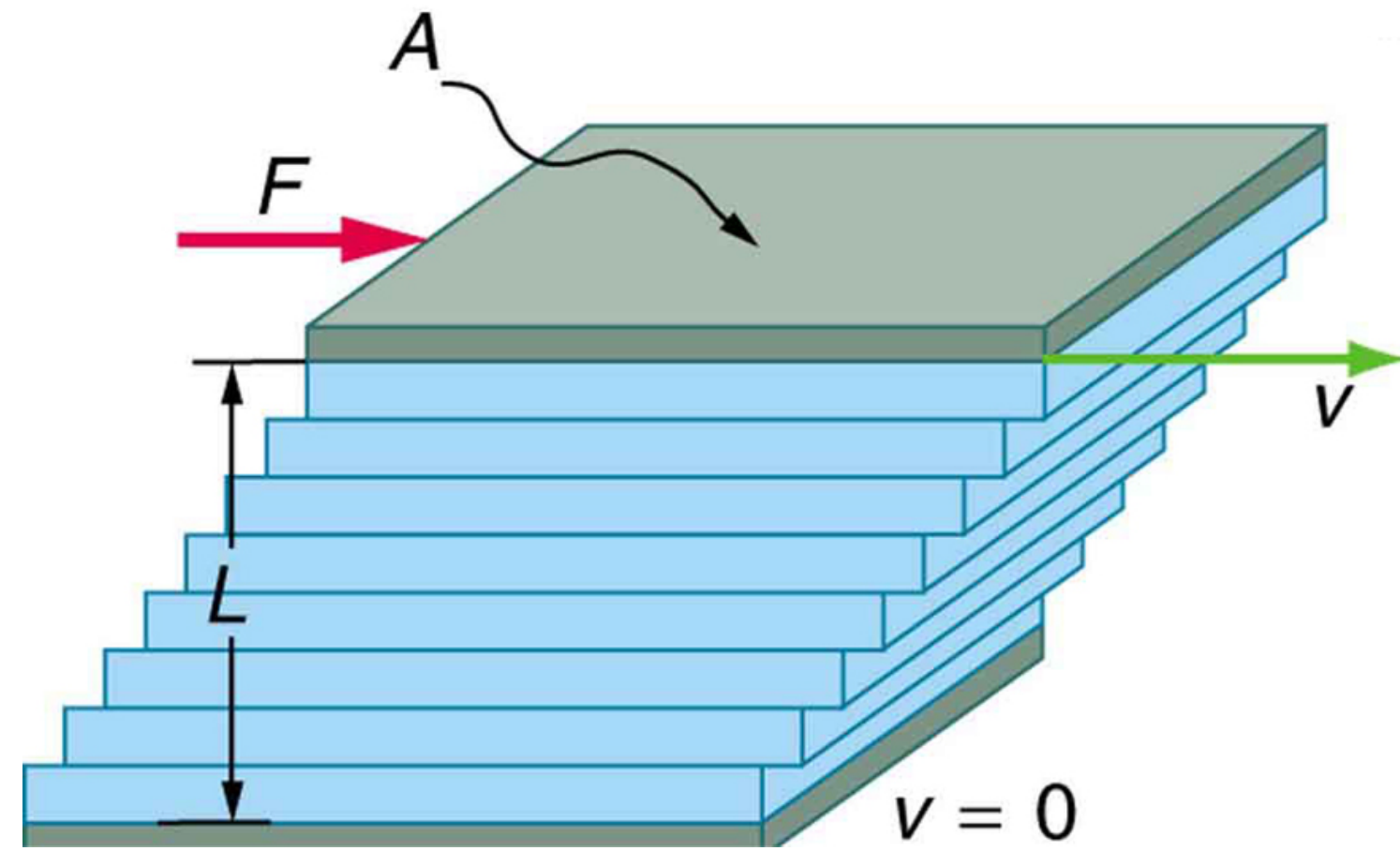
Data bars come from lattice QCD

Equations of motion for out-of-equilibrium pieces

Almaalol, Dore, JNH *Phys.Rev.D* 111 (2025) 1, 014020

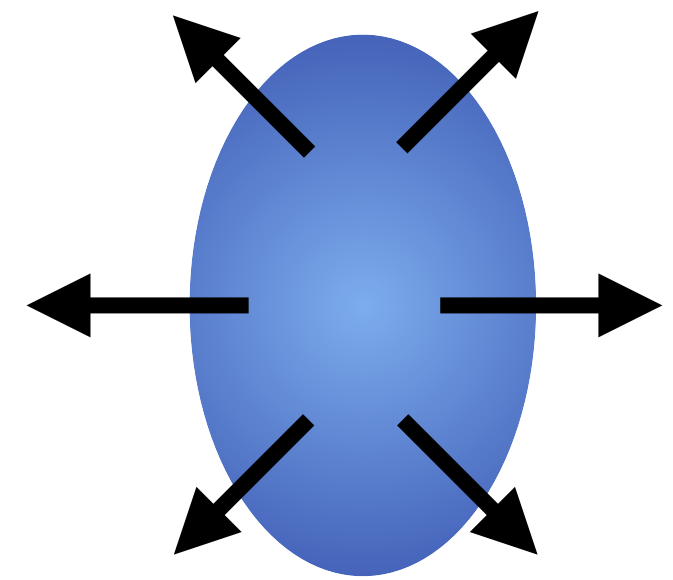


Equations NOT unique: Derived from enforcing only positive entropy production



$$\tau_{\pi} \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_{\pi}\pi^{\mu\nu}}{2}\theta - \frac{\tau_{\pi}}{2\beta_{\pi}}\dot{\beta}_{\pi}\pi^{\mu\nu} - \frac{2\eta}{\beta} \left(\gamma_1^q \nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2} n_q^{\langle\mu} \nabla^{\nu\rangle} \gamma_1^q \right)$$

$$\tau_{\Pi} \dot{\Pi} + \Pi = -(\zeta + \frac{\tau_{\Pi}}{2}\Pi)\theta - \frac{\tau_{\Pi}}{2\beta_{\Pi}}\dot{\beta}_{\Pi}\Pi - \frac{\zeta}{\beta} \left(\gamma_0^q D_{\mu} n_q^{\mu} + \frac{1}{2} n_q^{\mu} \nabla_{\mu} \gamma_0^q \right)$$

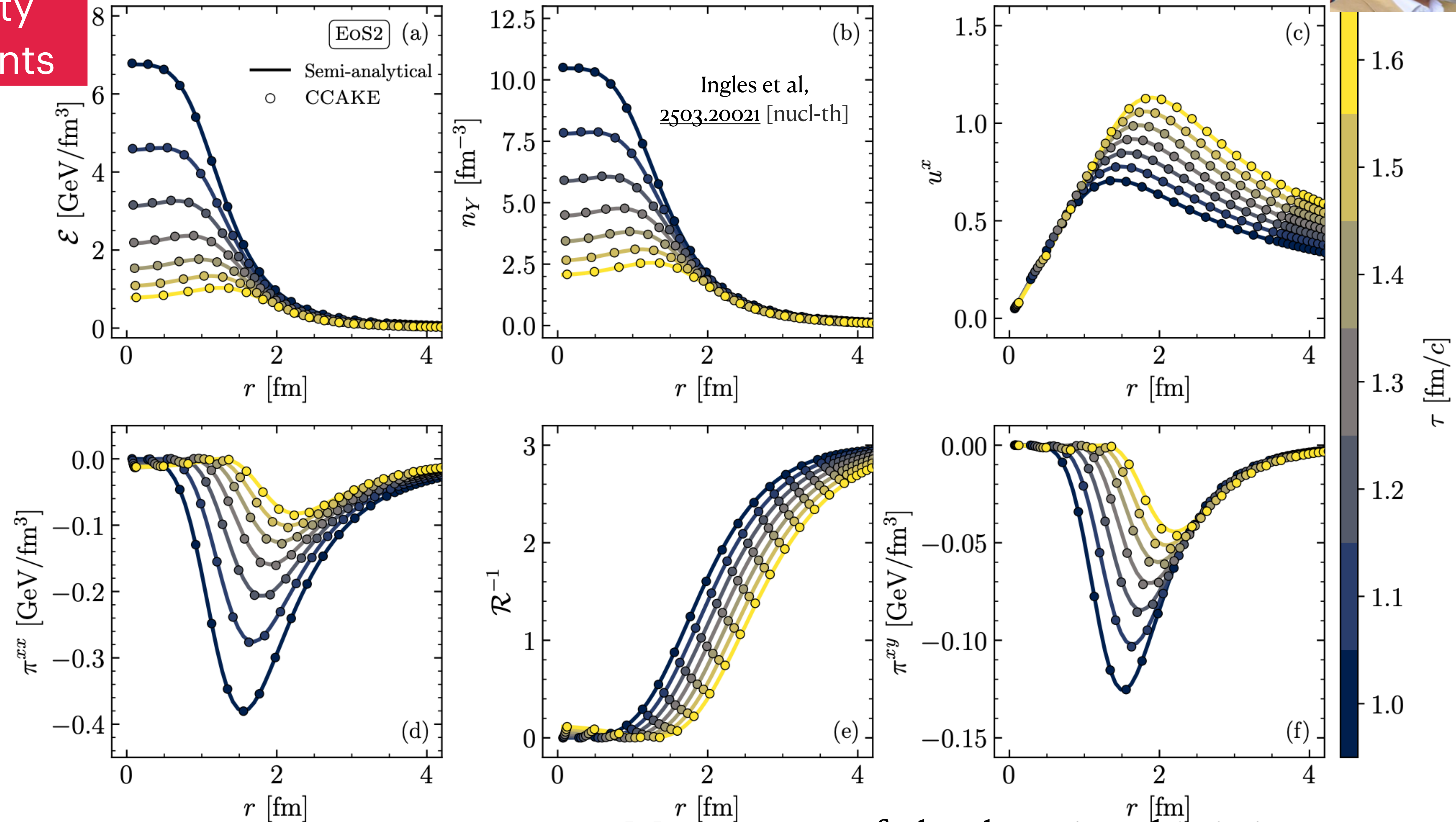


$$\tau_{qq'} \dot{n}_{q'}^{\mu} + n_q^{\mu} = -\kappa_{qq'} \nabla^{\mu} \alpha_{q'} + \frac{\tau_{qq'} n_{q'}^{\mu}}{2} \theta - \frac{\tau_{qq'}}{2\beta_{qq'}} \dot{\beta}_{qq'} n_{qq'}^{\mu} - \frac{\kappa_{qq'}}{\beta} \left(\gamma_0^{qq'} \nabla^{\mu} \Pi - \frac{\Pi}{2} \nabla^{\mu} \gamma_0^{qq'} \right) - \frac{\kappa_{qq'}}{\beta} \left(\gamma_1^{qq'} \nabla_{\nu} \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2} \nabla_{\nu} \gamma_1^{qq'} \right)$$

Ask about
Causality
+stability
constraints

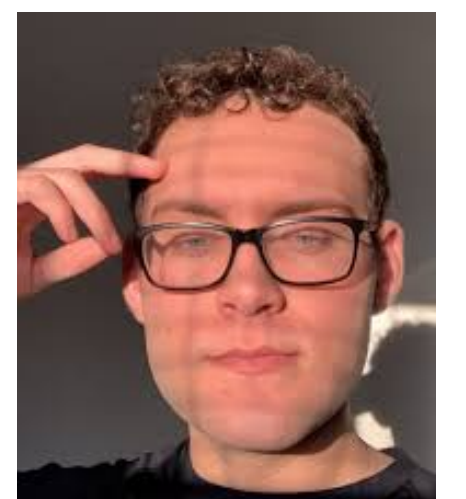
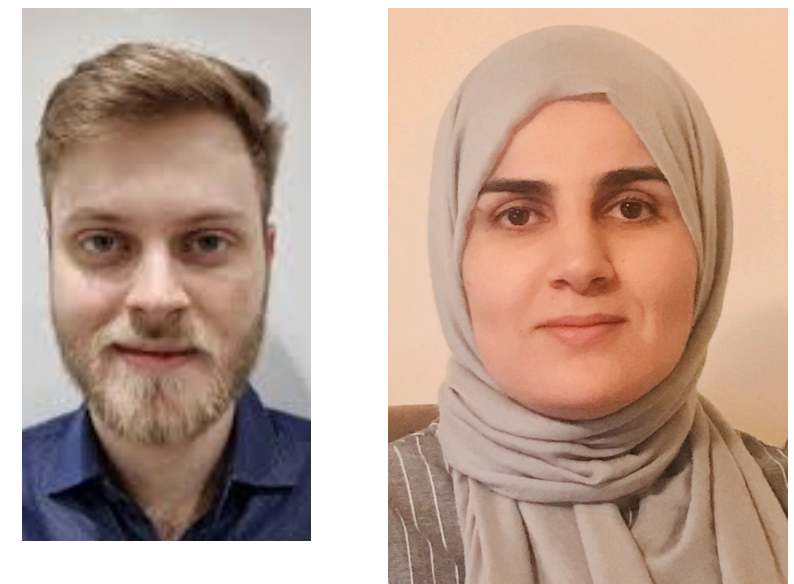
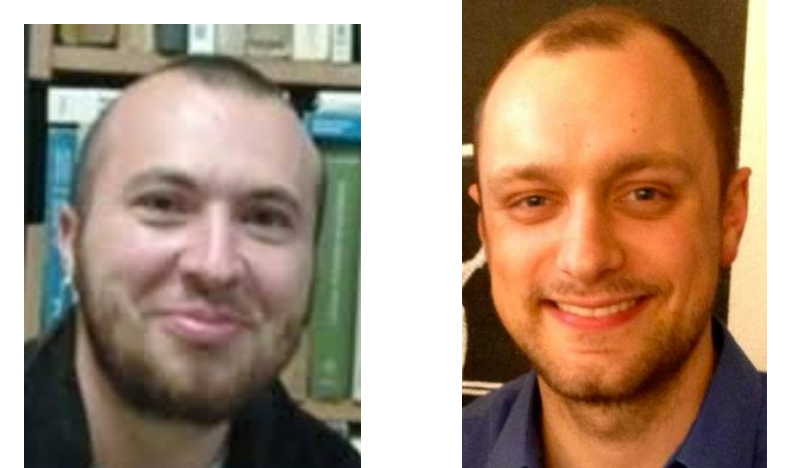
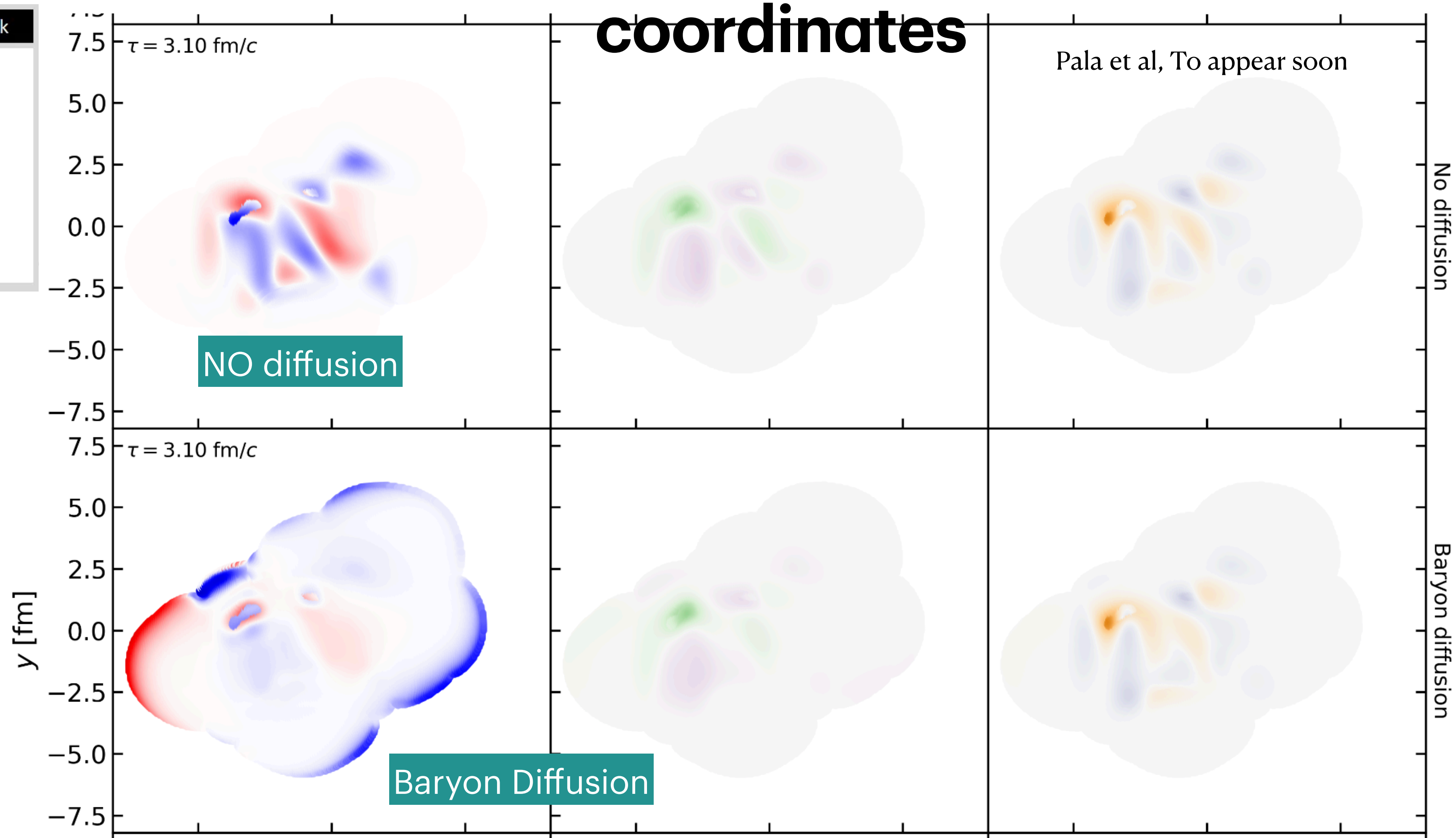
Does this actually work? Analytical checks

With viscosity, at finite densities



Coming soon: CCAKE 2.0

3+1D simulations with BSQ diffusion, on CPUs/GPUs, generalized



Pala, Kaur Virk, Almaalol, Danhoni, Yao, Long, Serenone, Salinas San Martin, Yared,
Plumberg, Oshin, Hoyer, WUC
NuclearConfectionery: Multi-stage simulation framework for modeling relativistic heavy-ion collisions," To appear soon

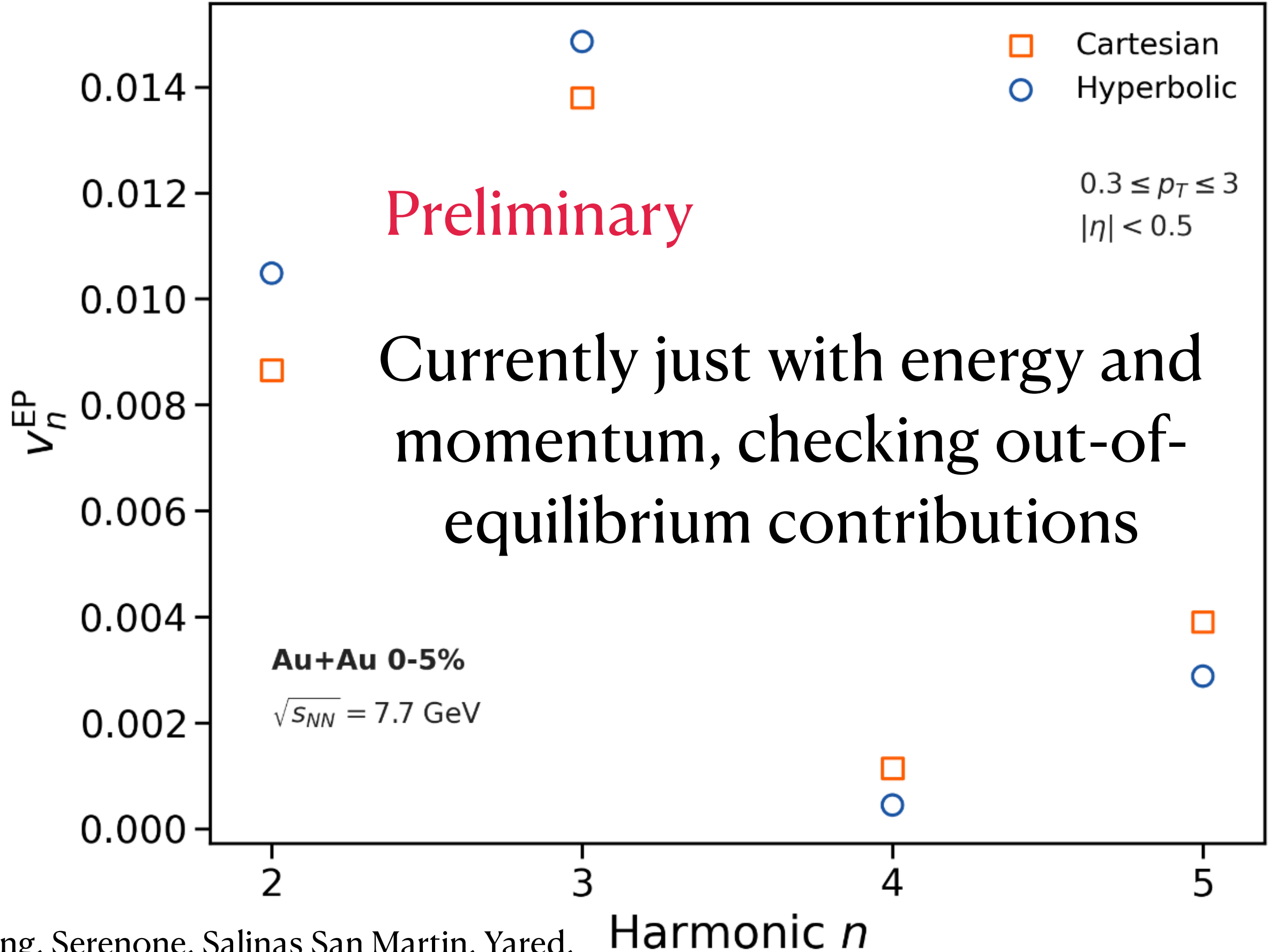
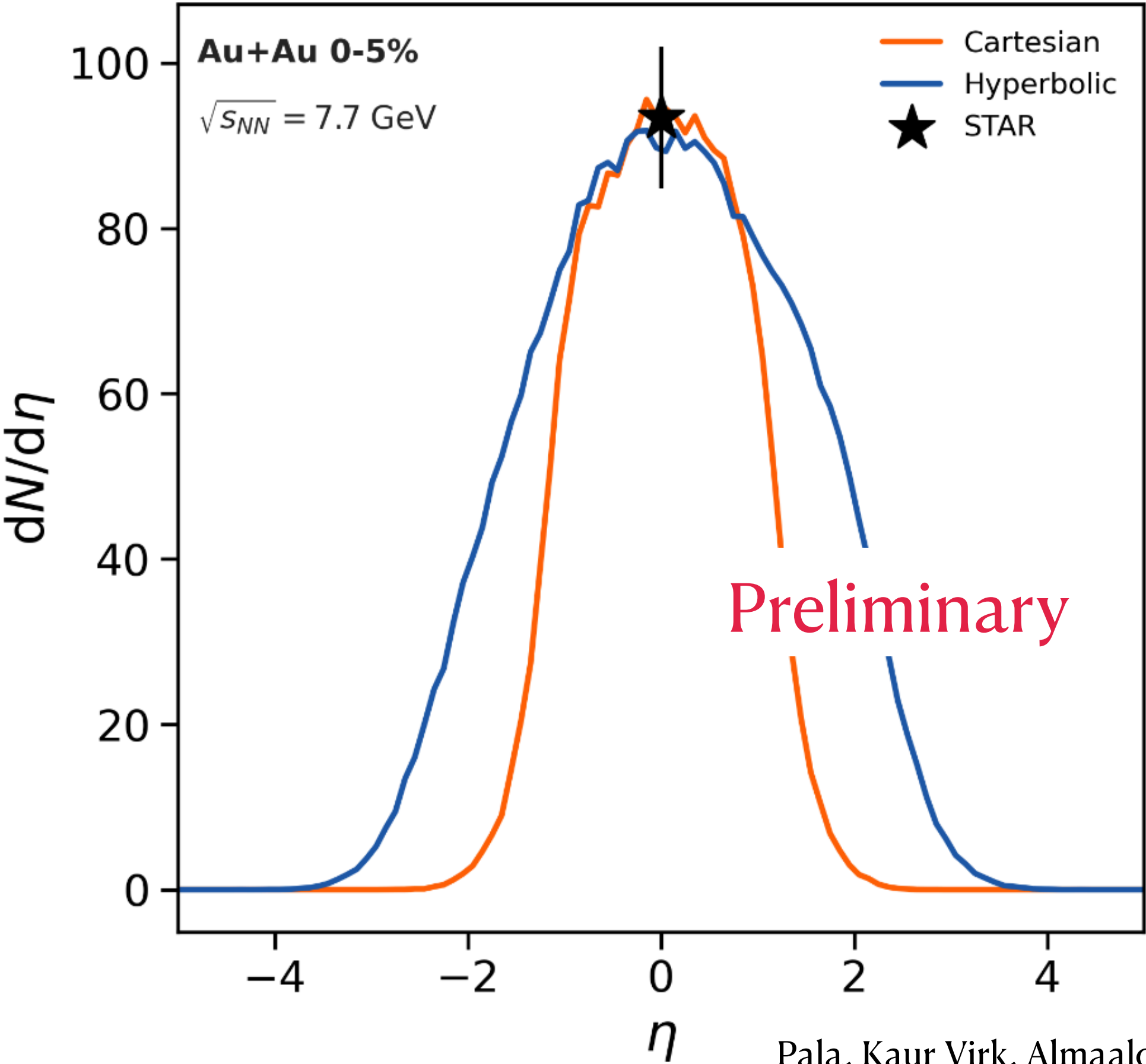
CCAKE 2.0: Cartesian vs Hyperbolic coordinates?

AMPT+CCAKE 2.0: 3+1D simulations

Cartesian: $t_0 = 1\text{ fm}/c, N_{norm} = 0.42$

Hyperbolic: $\tau_0 = 1\text{ fm}/c, N_{norm} = 0.62$

Later contributions added as sources



Pala, Kaur Virk, Almaalol, Danhoni, Yao, Long, Serenone, Salinas San Martin, Yared, Plumberg, Gardim, JNH, "NuclearConfectionery: Multi-stage simulation framework for modeling relativistic heavy-ion collisions," To appear soon

Summary and Outlook

- Strangeness:
 - MUST include strangeness in the symmetry energy expansion $\delta_I = 1 - 2Y_Q + Y_S$
- Bumps in c_s^2 :
 - Astrophysics providing hints of large bump, must be considered in HIC, otherwise we'll get misleading results!
- Need to understand HIC (assumptions, limitations, system differences) before applying to astrophysical data
- Questions still remain on hydrodynamics vs transport at these energies
- CMB at FAIR in Germany is being built to study the regime between heavy-ions and neutron stars $\rightarrow$ scans of Z/A

Isospin symmetric chemical potentials

See Aryal et al, *Phys.Rev.D* 102 (2020) 7, 076016 and Yang et al, [2504.18764](#) [nucl-th]

In terms of B , Q and S , the first law of thermodynamics is given by

$$dE = \mu_B dB + \mu_Q dQ + \mu_S dS + \dots, \quad (17)$$

where E is the internal energy and we define the chemical potentials

$$\mu_B \equiv \left(\frac{\partial E}{\partial B} \right)_{Q,S}, \quad \mu_Q \equiv \left(\frac{\partial E}{\partial Q} \right)_{B,S}, \quad \mu_S \equiv \left(\frac{\partial E}{\partial S} \right)_{B,Q}, \quad (18)$$

We can also write the first law in terms of B , I_z and S , by using the Gell-Mann–Nishijima formula $Q = I_z + (B + S)/2$:

$$\begin{aligned} dE &= \mu_B dB + \mu_Q \left(dI_z + \frac{1}{2}dB + \frac{1}{2}dS \right) + \mu_S dS + \dots \\ &= \mu_B^{(IS)} dB + \mu_I^{(BS)} dI_z + \mu_S^{(BI)} dS + \dots, \end{aligned} \quad (19)$$

where we now define

$$\mu_B^{(IS)} \equiv \left(\frac{\partial E}{\partial B} \right)_{I_z,S}, \quad \mu_I^{(BS)} \equiv \left(\frac{\partial E}{\partial I_z} \right)_{B,S}, \quad \mu_S^{(BI)} \equiv \left(\frac{\partial E}{\partial S} \right)_{B,I_z}. \quad (20)$$

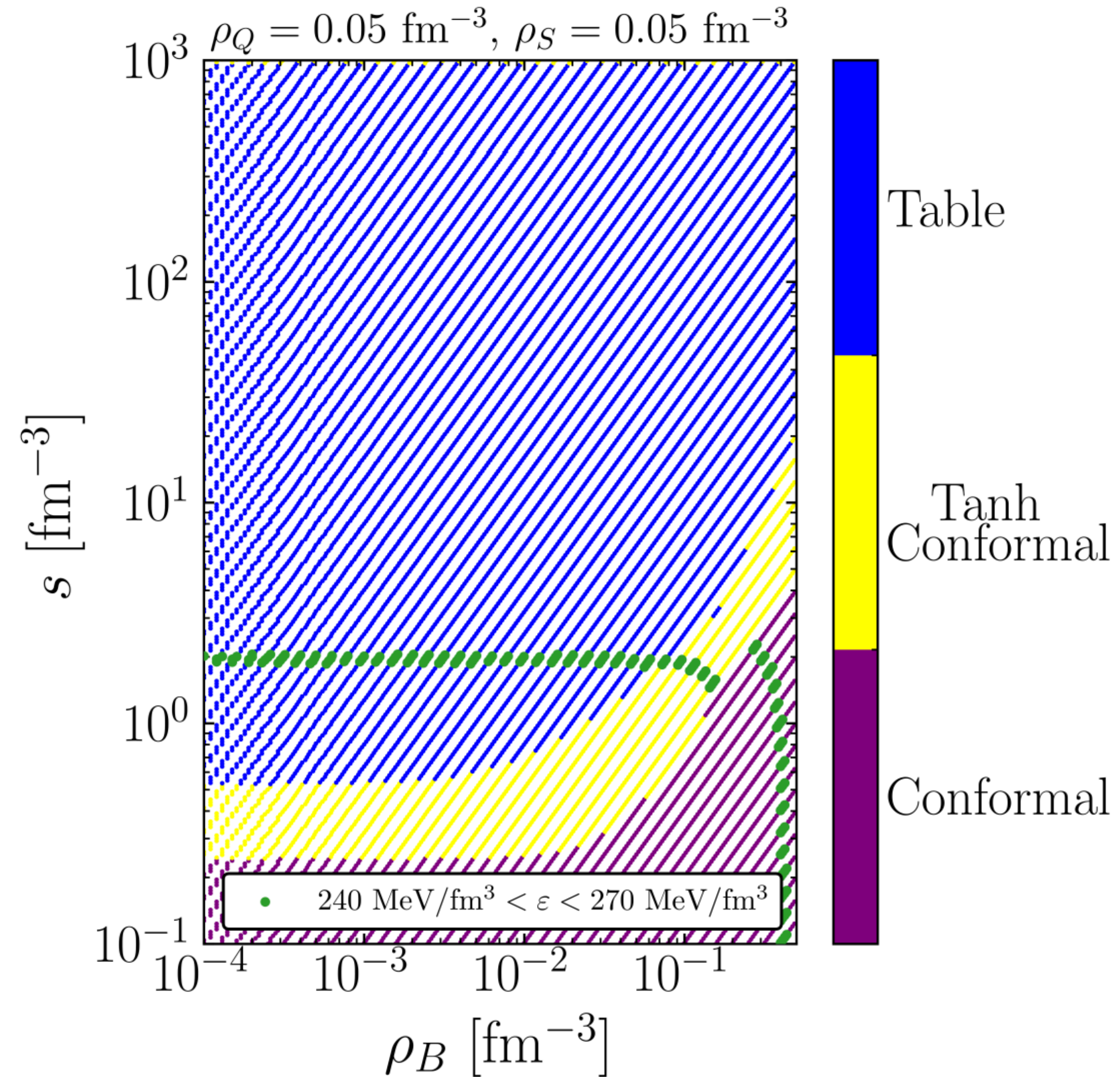
The chemical potentials in the (B, I_z, S) basis can be related to μ_B , μ_S and μ_Q by inspecting Eq. (19):

$$\mu_B^{(IS)} = \mu_B + \frac{1}{2}\mu_Q, \quad \mu_I^{(BS)} = \mu_Q, \quad \mu_S^{(BI)} = \mu_S + \frac{1}{2}\mu_Q \quad (21)$$

Breakdown of lattice QCD (Taylor series)

Back-up EOS

- Table=lattice QCD
- Low T (below freeze-out) can be replaced with HRG (see Joaquin Greta's talk from MUSES)
- Will reproduce with alternative expansion scheme, this will certainly help!



Plumberg et al, *Phys.Rev.C* 111 (2025) 4, 044905

Next Steps: extending to finite temperatures

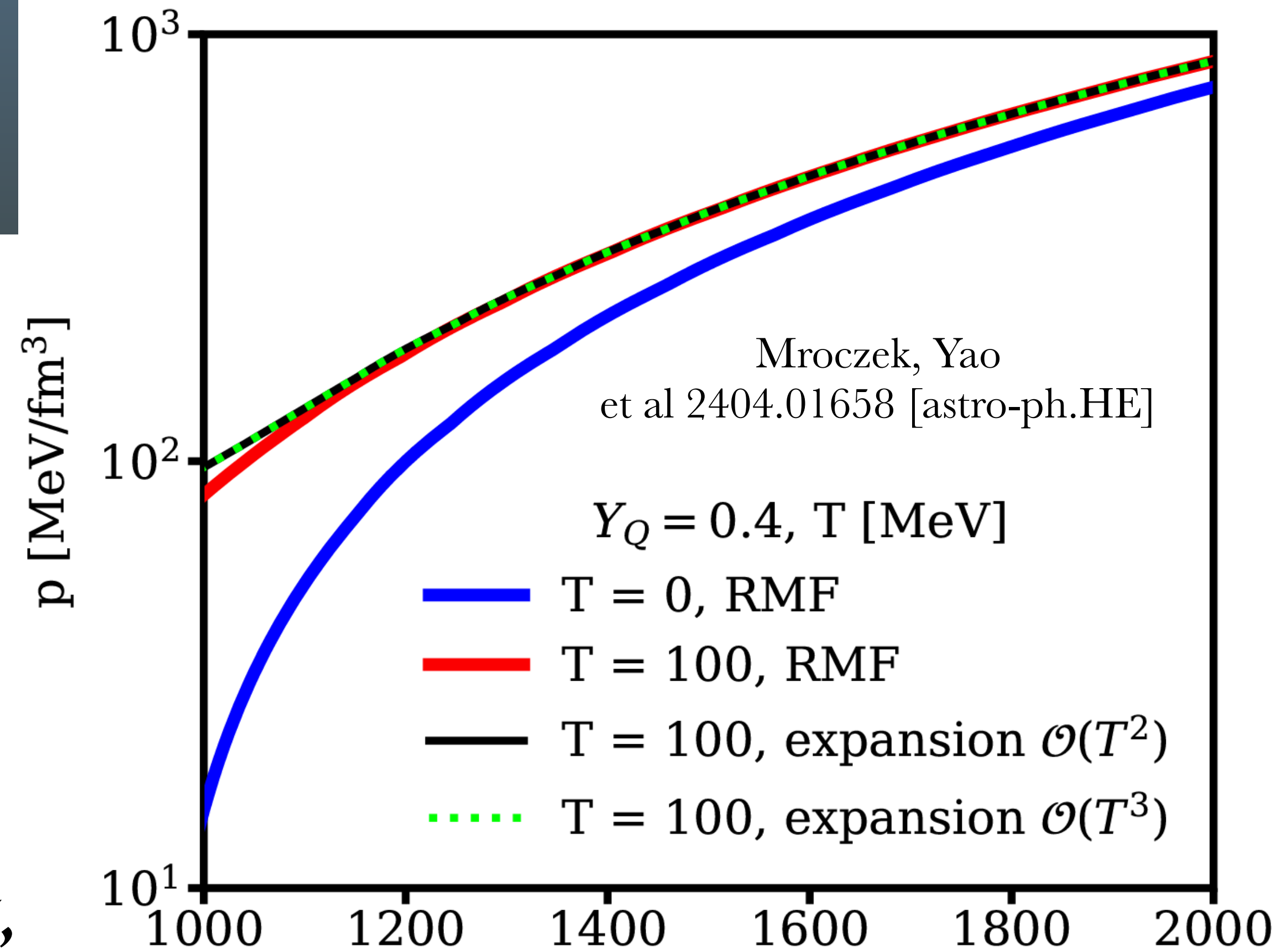
Finite T expansion in pressure

New finite T expansion can reproduce the equation of state up to $T = 100$ [MeV] at the percent level.

$$p(T, \vec{\mu}) = p_{T=0} + \frac{1}{2} \frac{\partial s}{\partial T} \bigg|_{T=0, \vec{\mu}} T^2 + \frac{1}{6} \frac{\partial^2 s}{\partial T^2} \bigg|_{T=0, \vec{\mu}} T^3 + \mathcal{O}(T^4)$$

Recall entropy is $s = \frac{\partial p}{\partial T} \bigg|_{\vec{\mu}}$

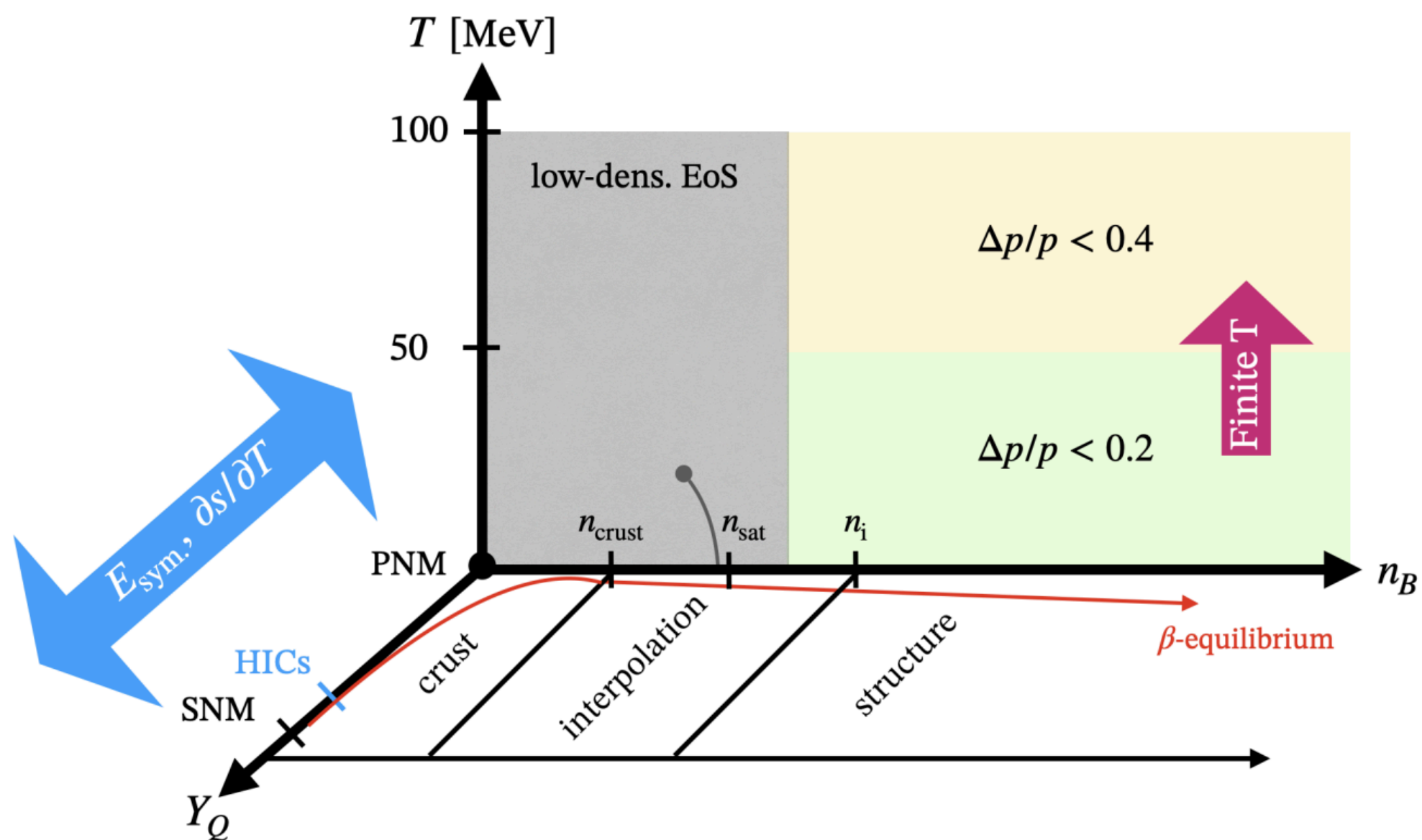
Comparisons to various models underway, extraction of these entropy derivatives as well!



Yields across $\frac{Z}{A}$ and the neutron star equation of state

Mroczek, Yao et al, 2404.01658 [astro-ph.HE]

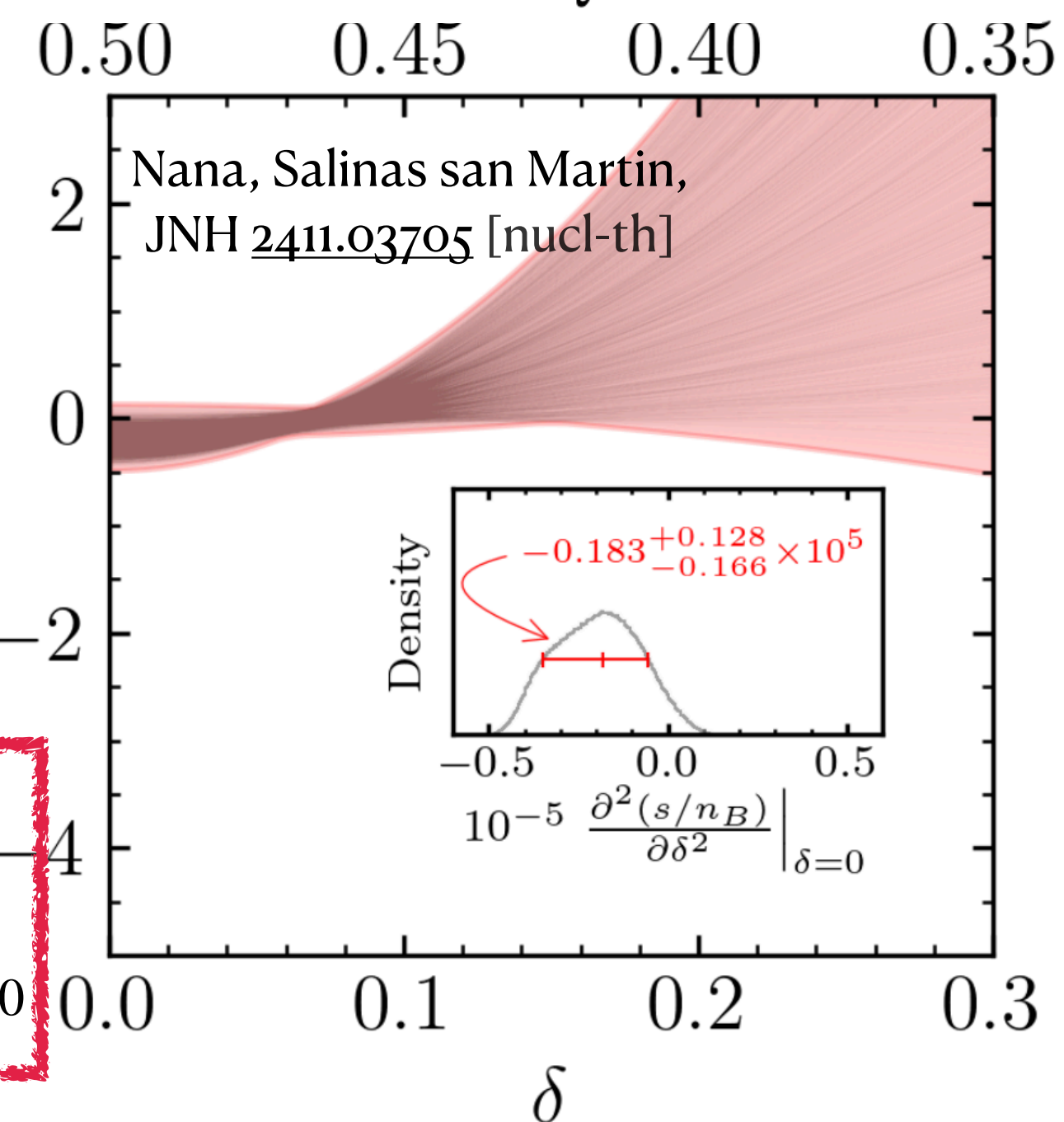
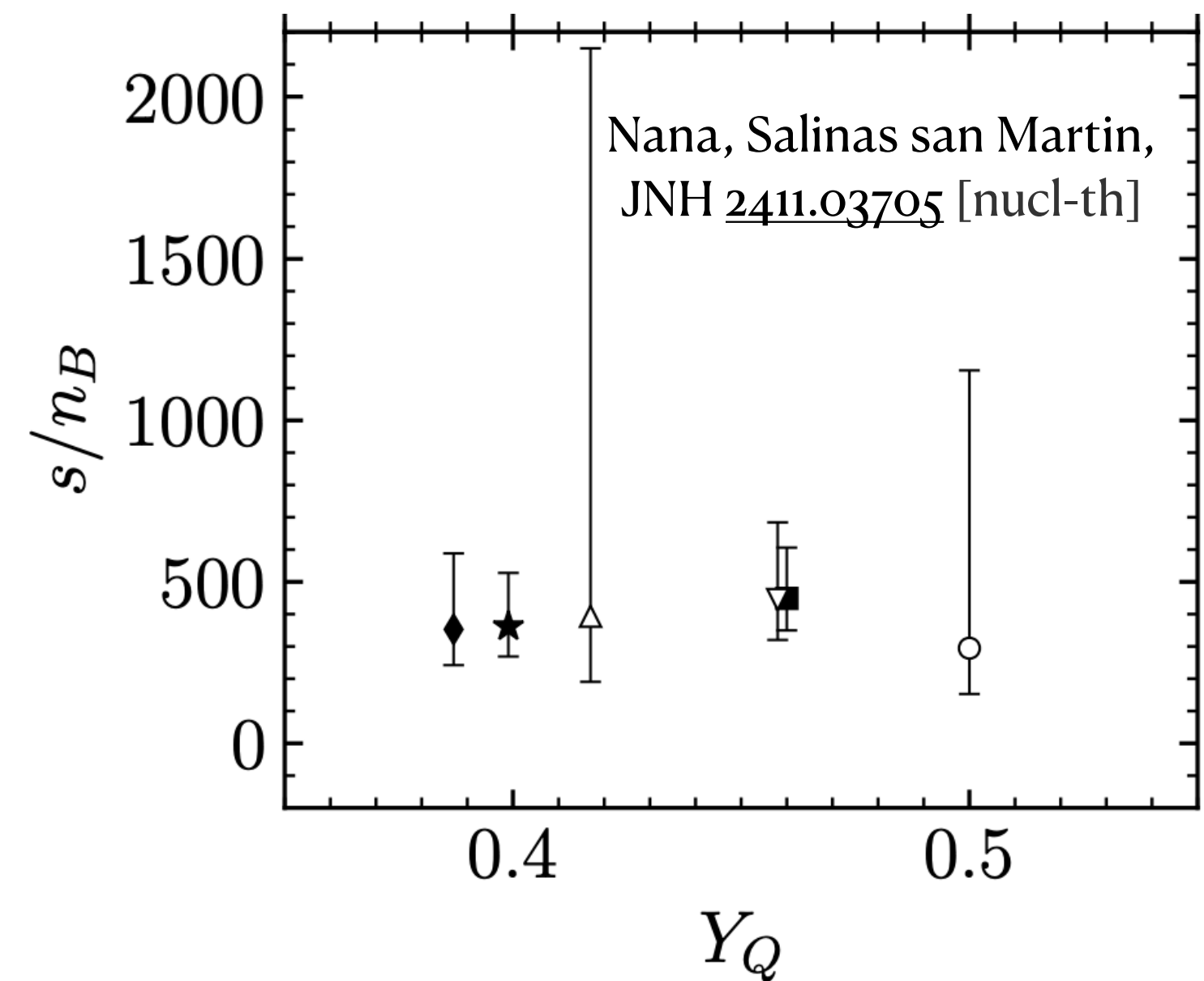
Repurposing isobars



How do we get this information?

Thermal models from heavy-ion collisions and varying Y_Q !
Proof-of-principle from RHIC

$$\sqrt{s} = 200 \text{ GeV}$$



Need information about how $\partial s/\partial T$ varies with Y_Q

$$\left. \frac{\partial s/n_B(T, n_B, Y_Q)}{\partial T} \right|_{T=0} = \frac{1}{n_B} \left. \frac{\partial s_{\text{HIC}}(T, n_B, Y_Q)}{\partial T} \right|_{T=\delta_{\text{HIC}}=0} + \frac{1}{2} \left(1 - \frac{Y_Q}{Y_Q^{\text{HIC}}} \right)^2 \left. \frac{\partial^3(s/n_B)_{\text{HIC},2}(T, n_B, \delta_{\text{HIC}})}{\partial T \partial \delta_{\text{HIC}}^2} \right|_{T=\delta_{\text{HIC}}=0}$$