EXPLORING PROTO-NEUTRON STAR MINIMUM MASS LIMITS THROUGH CHIRALLY CONSTRAINED NUCLEAR EOS

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EMMI Workshop GSI Darmstadt 2025

CRC–TR 211
Strong-interaction matter
under extreme conditions

Talk based on *Phys. Rev. C 111, 035807 (2025)*

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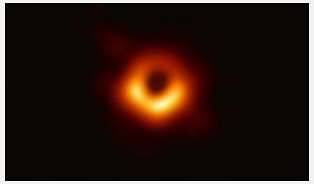
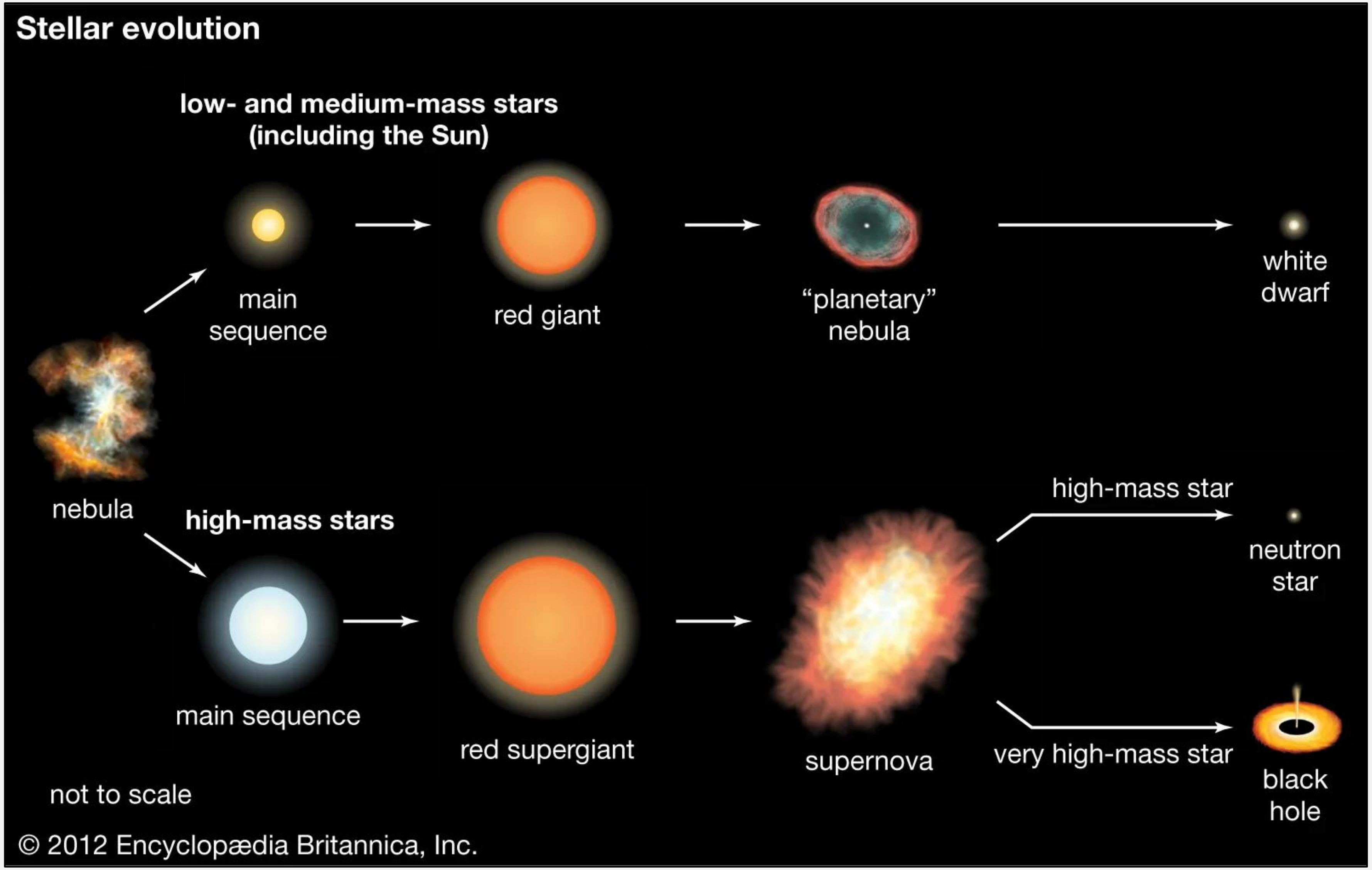
EVOLUTION OF MASSIVE STARS

$<8M_\odot$ → White Dwarf

$8M_\odot - 25M_\odot$ → Neutron Star

$> 25M_\odot$ → Black Hole





https://www.britannica.com/science/star-astronomy/Star-formation-and-evolution

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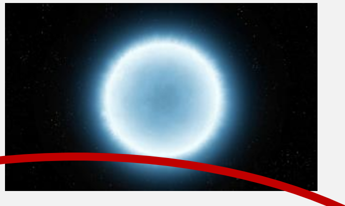

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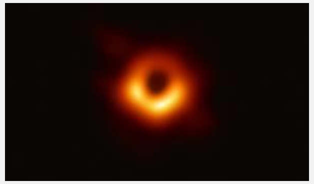

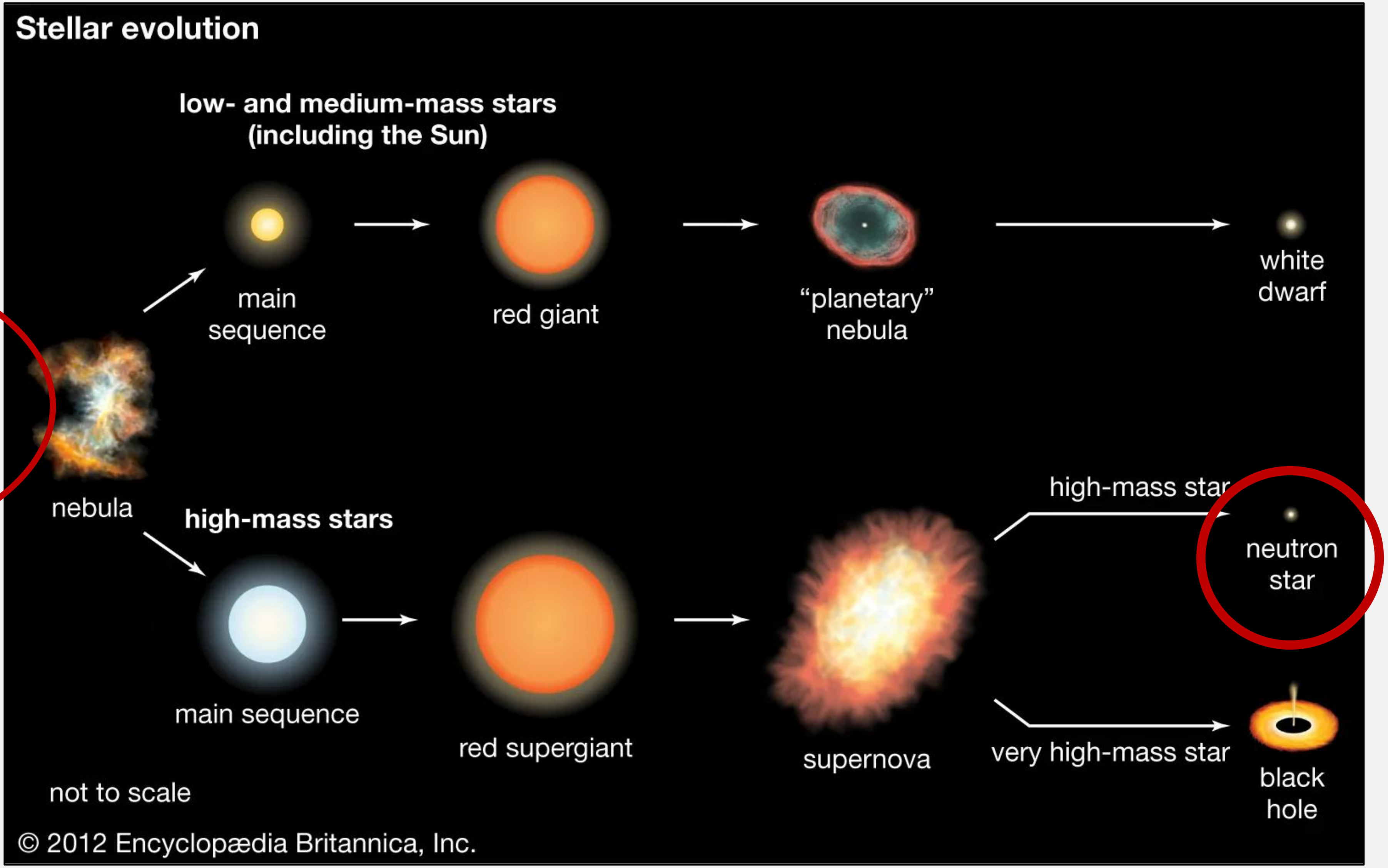

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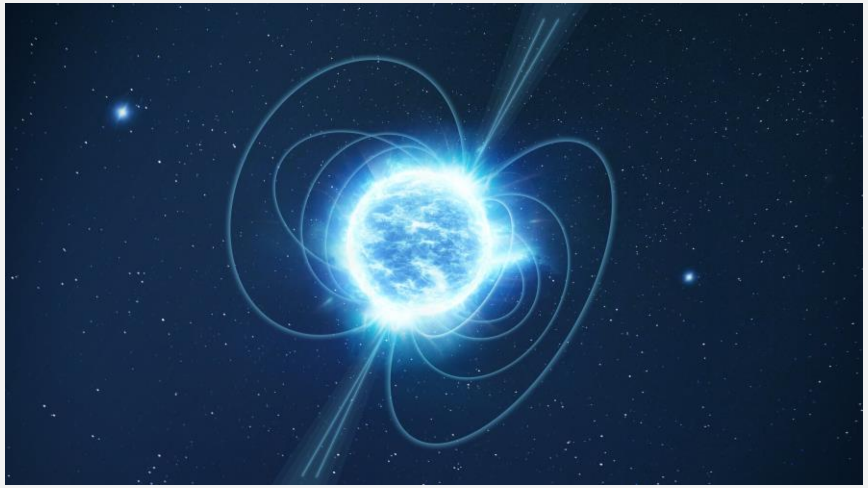
NEUTRON STARS

	Neutron Star	**Earth**
Mass	$1.4M_{\odot} \approx 10^{30}kg$	$10^{24}kg$
Radius	$12km$	$6371km$
Density	$10^{17}kg/m^3$	$10^5kg/m^3$
Surface gravity	$10^{12}m/s^2$	$9.81m/s^2$
Rotation period	$1.4ms - 30s$	$24h$
Magnetic field	$10^4T - 10^{11}T$	$25\mu T - 65\mu T$
Central temperatures	10^9K	10^3K



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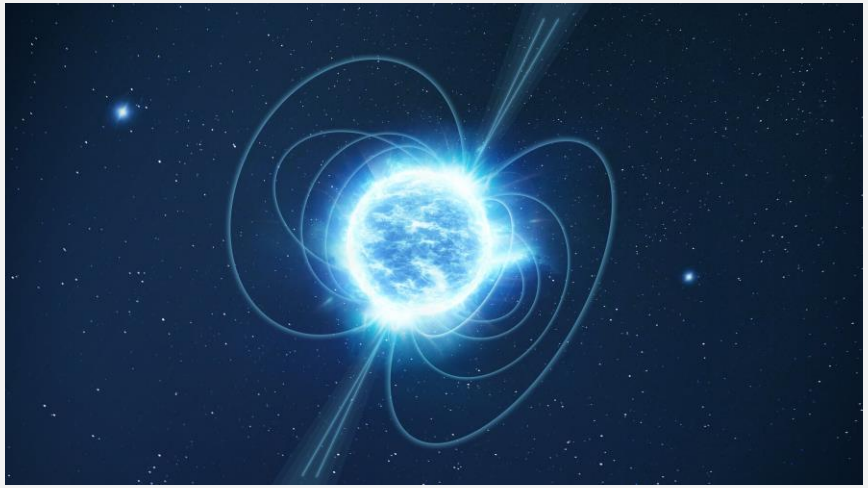
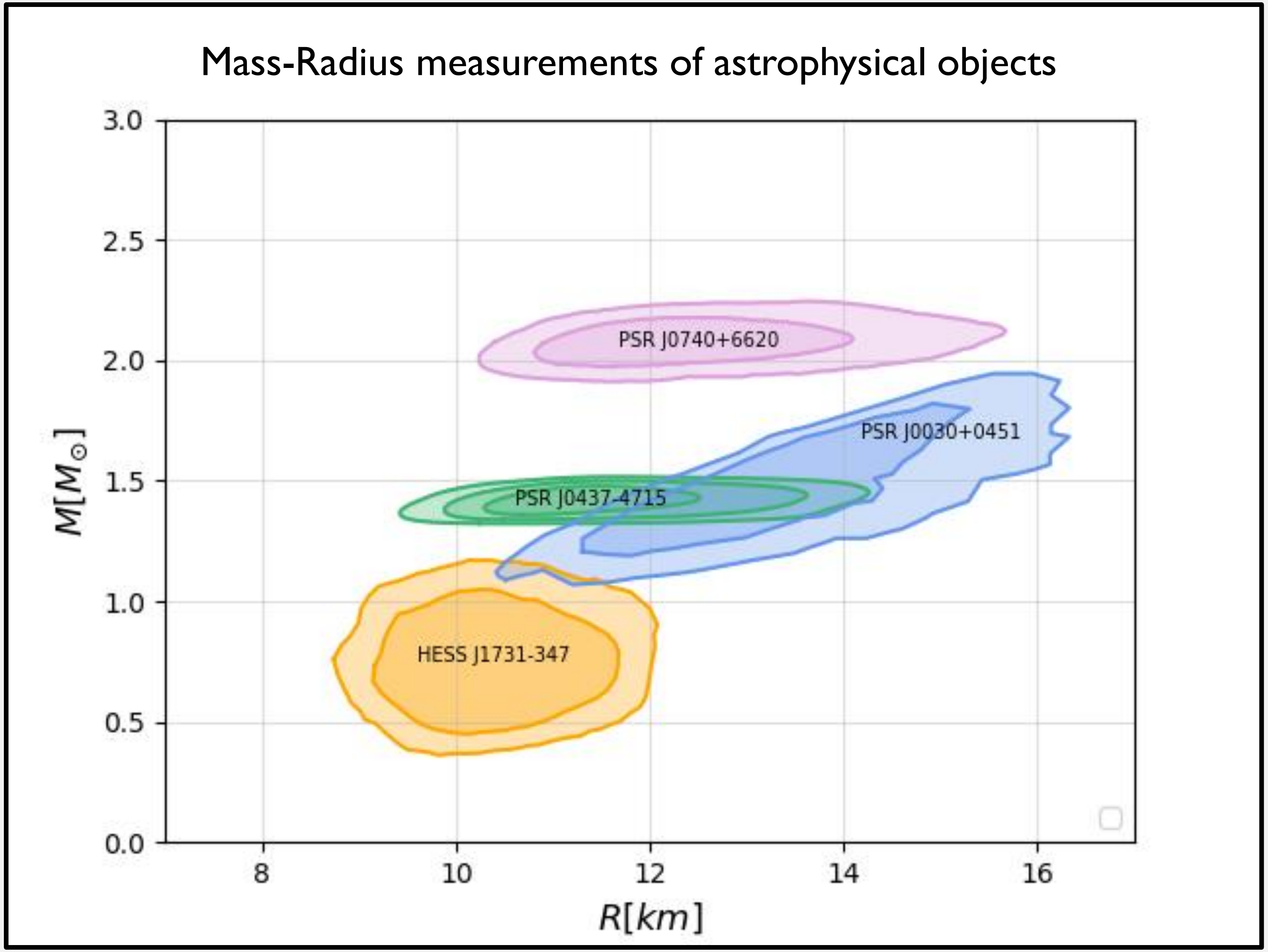


https://www.esa.int/ESA_Multimedia/Images/2024/03/What_is_a_neutron_star

Neutron Stars are the densest objects in the universe which we can physically describe ***and*** *observe!*

WHY STUDY MINIMAL MASSES OF PROTO-NEUTRON STARS?

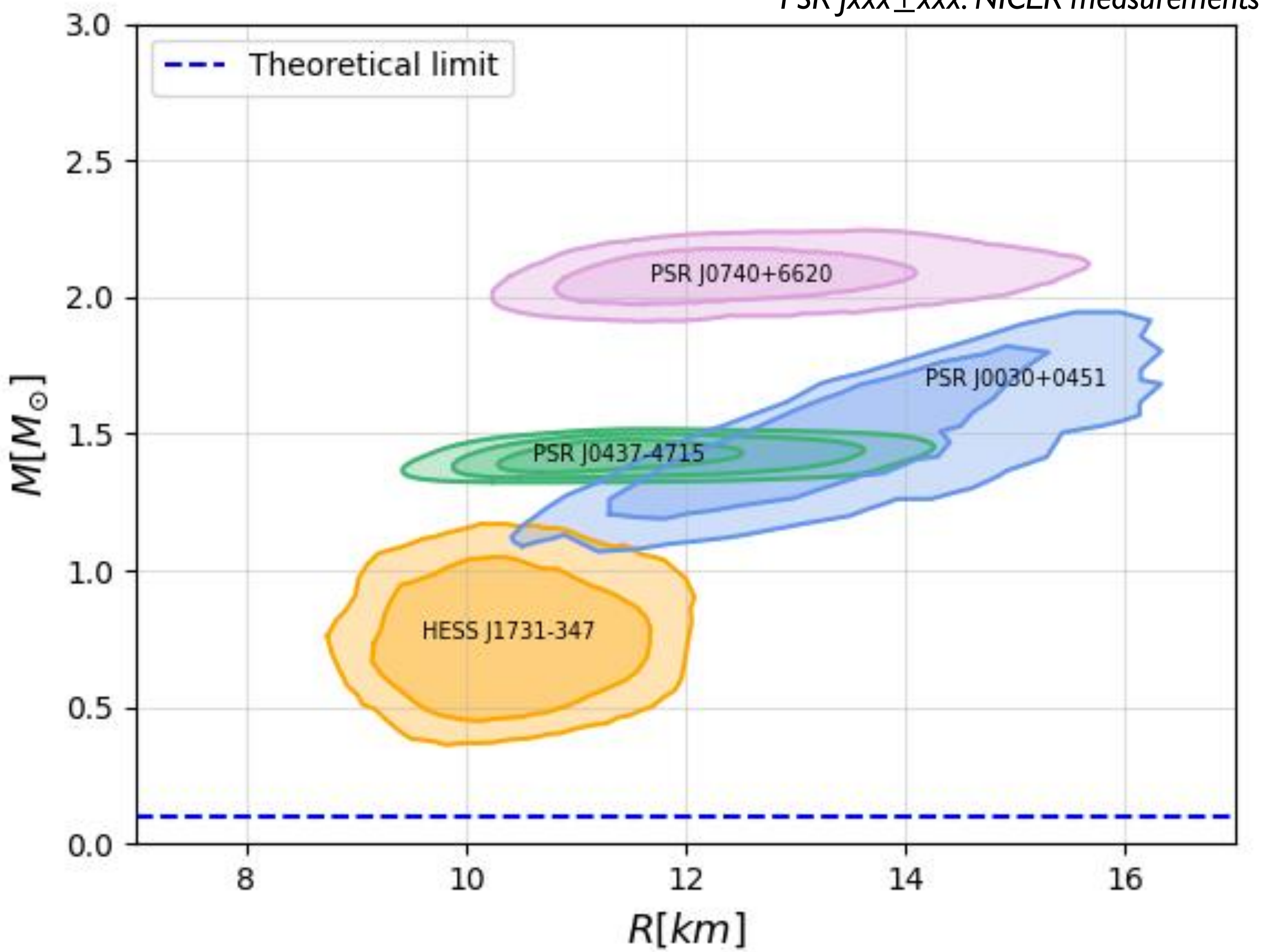


WHY STUDY MINIMAL MASSES OF PROTO-NEUTRON STARS?

Theory:

Lower mass limit of a cold neutron star at $M_{min} \sim 0.1 M_{\odot}$

PSR Jxxx±xxx: NICER measurements



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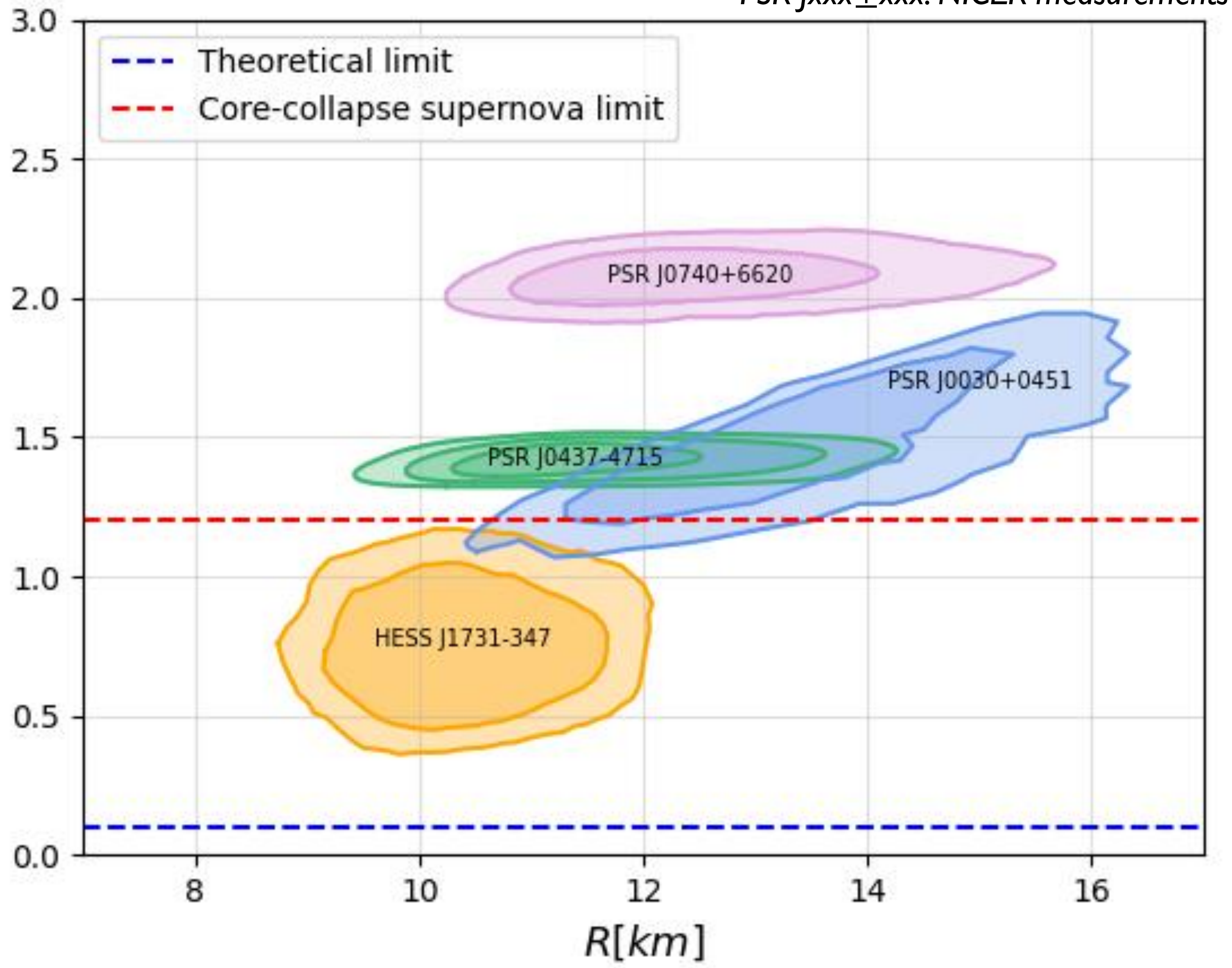
Lower mass limit of a cold neutron star at $M_{min} \sim 0.1 M_{\odot}$

Core-collapse supernova simulations:

- Hot PNS (proto-neutron star) with $M_{min} \sim 1.2 M_{\odot}$ is formed
- Cools down to cold PNS

HESS J1731-347 too light!

PSR Jxxx±xxx: NICER measurements



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- Hot PNS (proto-neutron star) with $M_{min} \sim 1.2 M_{\odot}$ is formed
- Cools down to cold PNS

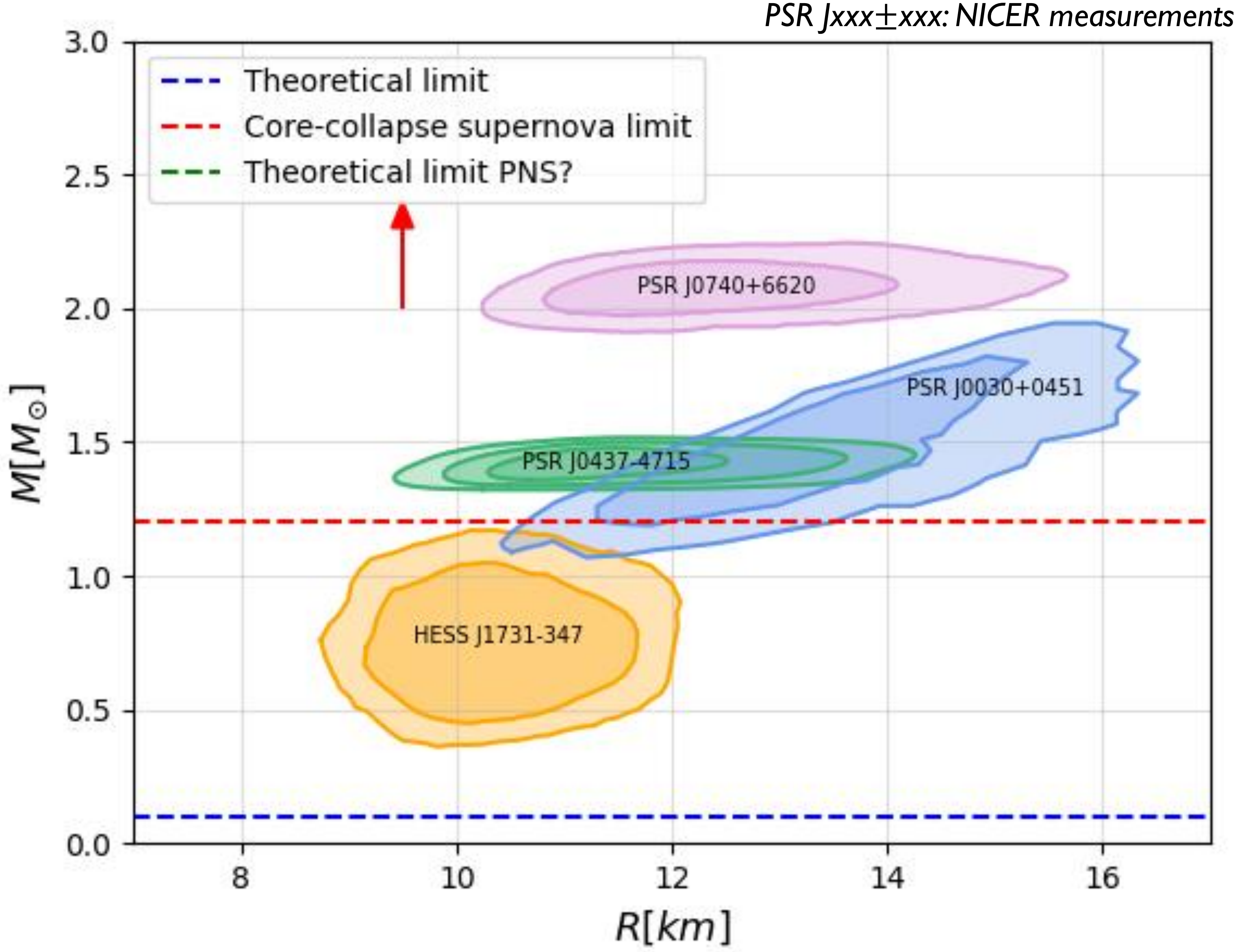
HESS J1731-347 too light!

<u>Theory for proto-neutron star:</u>

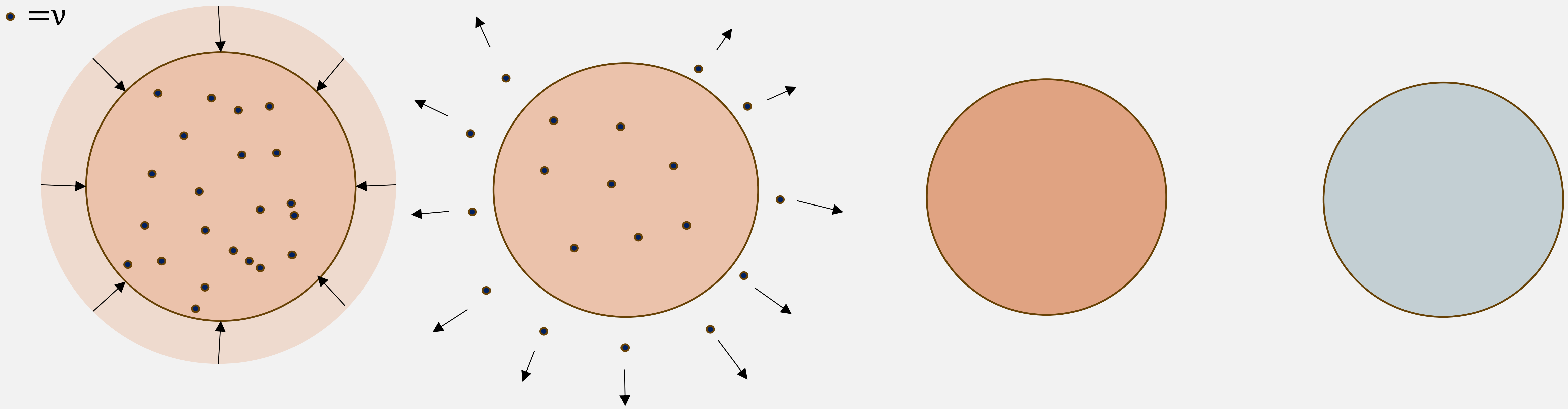
- Mass constraint of hot PNS gives constraint on mass of cold PNS!

Is HESS J1731-347 still a potential neutron star candidate?

PSR Jxxx±xxx: NICER measurements



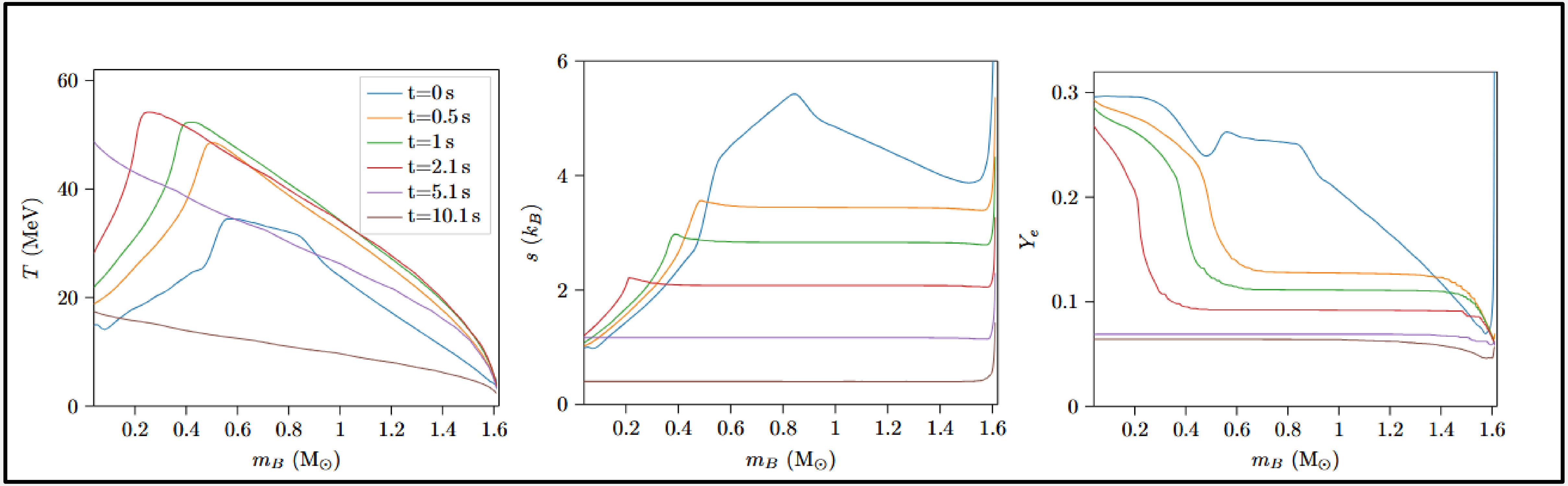
PROTO-NEUTRON STAR EVOLUTION



Collapse	**Deleptonization**	**Cooling**	**Cold Neutron Star**
t = 0s	t = 0.5-1s	t = 15s	t = 10^6yr
$s = 1$	$s = 1 - 3$	$s = 2$	$s = 0$
$Y_L = 0.4$	$Y_L = 0 - 0.4$	$Y_\nu = 0$	$Y_\nu = 0$

Entropy per baryon: $s = \frac{S}{A}$ Lepton fraction: $Y_L = \frac{(n_e - n_{e^+}) + (n_{\nu_e} - n_{\bar{\nu}_e})}{(n_n - n_{\bar{n}}) + (n_p - n_{\bar{p}})}$

PROTO-NEUTRON STAR SIMULATIONS



Pascal et al., Mon. Not. R. Astron. Soc. 511, 1, 356–370 (2022)

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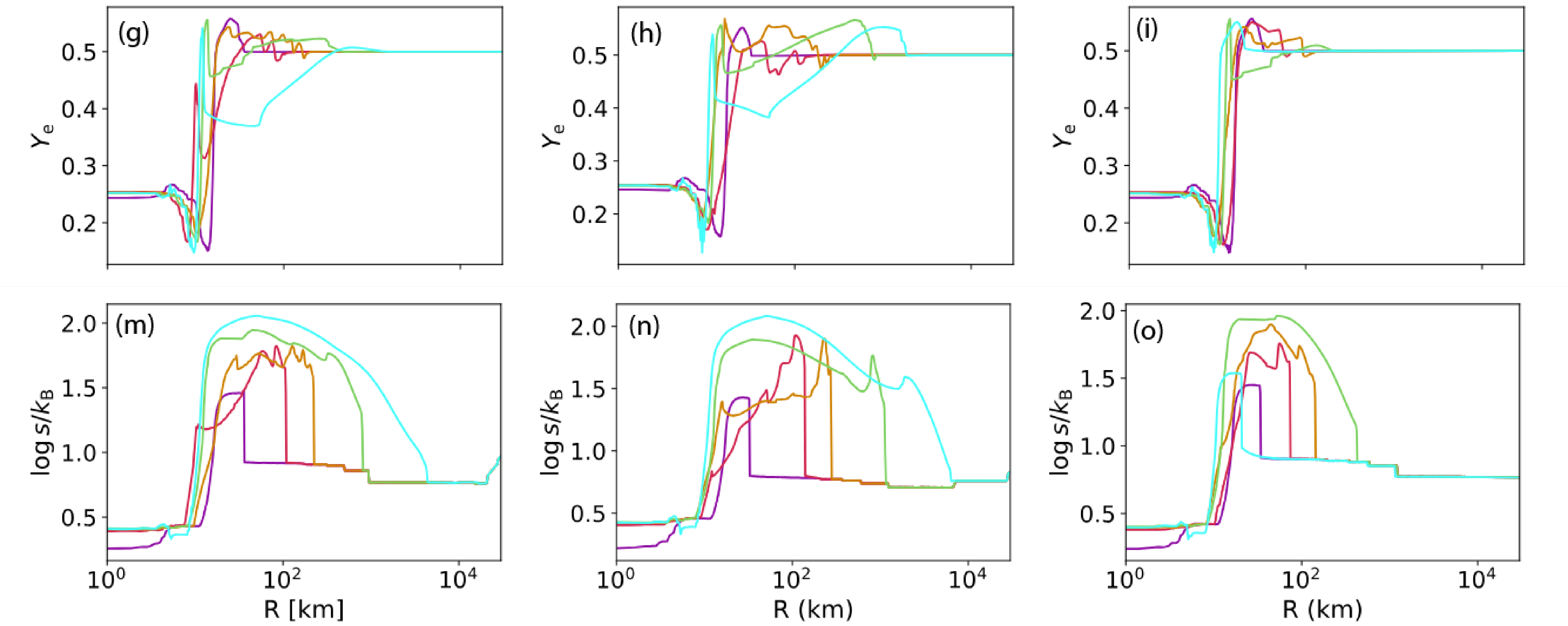
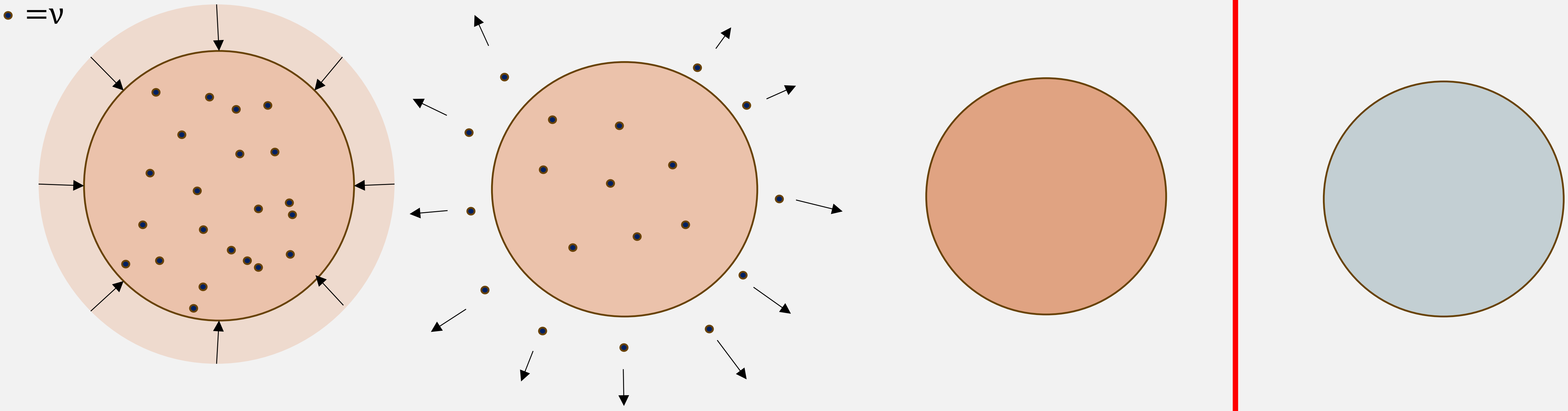


Jakobus et al., Mon. Not. R. Astron. Soc. 516 (2022)

PROTO-NEUTRON STAR EVOLUTION



Collapse

t = 0s

$s = 1$
$Y_L = 0.4$

Deleptonization

t = 0.5-1s

$s = 1 - 3$
$Y_L = 0 - 0.4$

Cooling

t = 15s

$s = 2$
$Y_\nu = 0$

Cold Neutron Star

t = 10^6yr

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Entropy per baryon: $s = \frac{S}{A}$

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WHY STUDY IT WITH CHIRAL EFT?

Low masses of neutron stars → Low central densities → Framework which describes matter at low densities

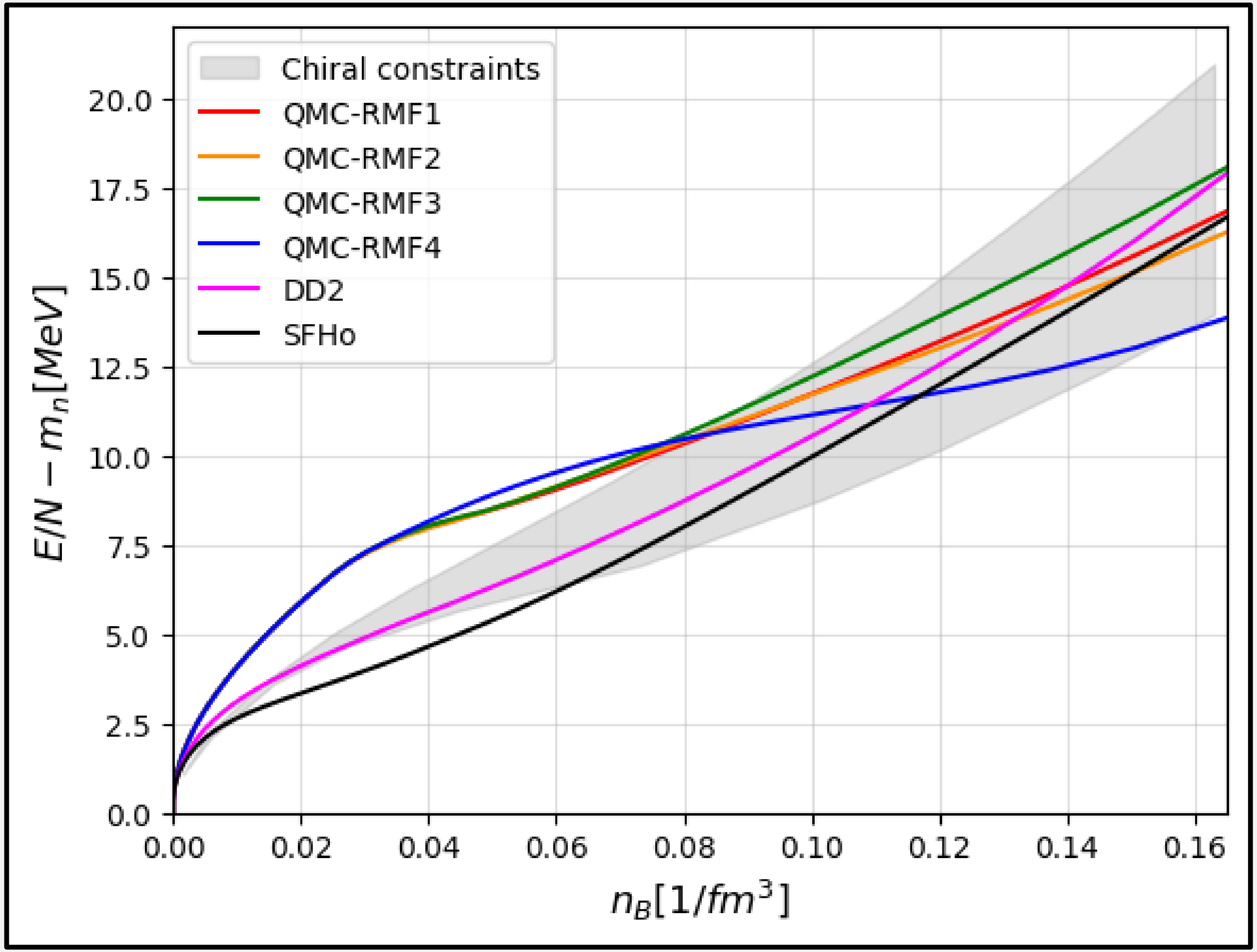
Chiral EFT

Description of nucleon interaction at low energies	Based on chiral symmetry and chiral symmetry breaking	Power counting scheme
Relevant degrees of freedom are nucleons and pions	Pion as goldstone boson	Expansion in $\left(\frac{p}{\Lambda_\chi}\right)^\nu$

EQUATIONS OF STATE

QMC-RMF *(Alford et al., Phys. Scripta 98, 125302 (2023))*
DD2 *(Typel et al., Phys. Rev. C 81, 015803 (2010))*
SFHO*(Steiner, Hempel & Fischer, Astrophys. J. 774, 17 (2013))*

- Fulfill chiral EFT constraints for the binding energy at T = 0
- Relativistic Mean Field Models



Chiral constraints taken from: *Krüger et al., Phys. Rev. C 88, 025802 (2013)*

TOLMAN-OPPENHEIMER-VOLKOFF EQUATIONS

Structure of a static, spherically symmetric star in General Relativity *

- Mass conservation

mass

energy density

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

radius

- Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\varepsilon\left(1+\frac{P}{\varepsilon c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

pressure

*R. C. Tolman, Phys. Rev. **55**, 364 (1939);
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Input: Pressure and Energy Density

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Input: Pressure and Energy Density → **Output: Mass and Radius**

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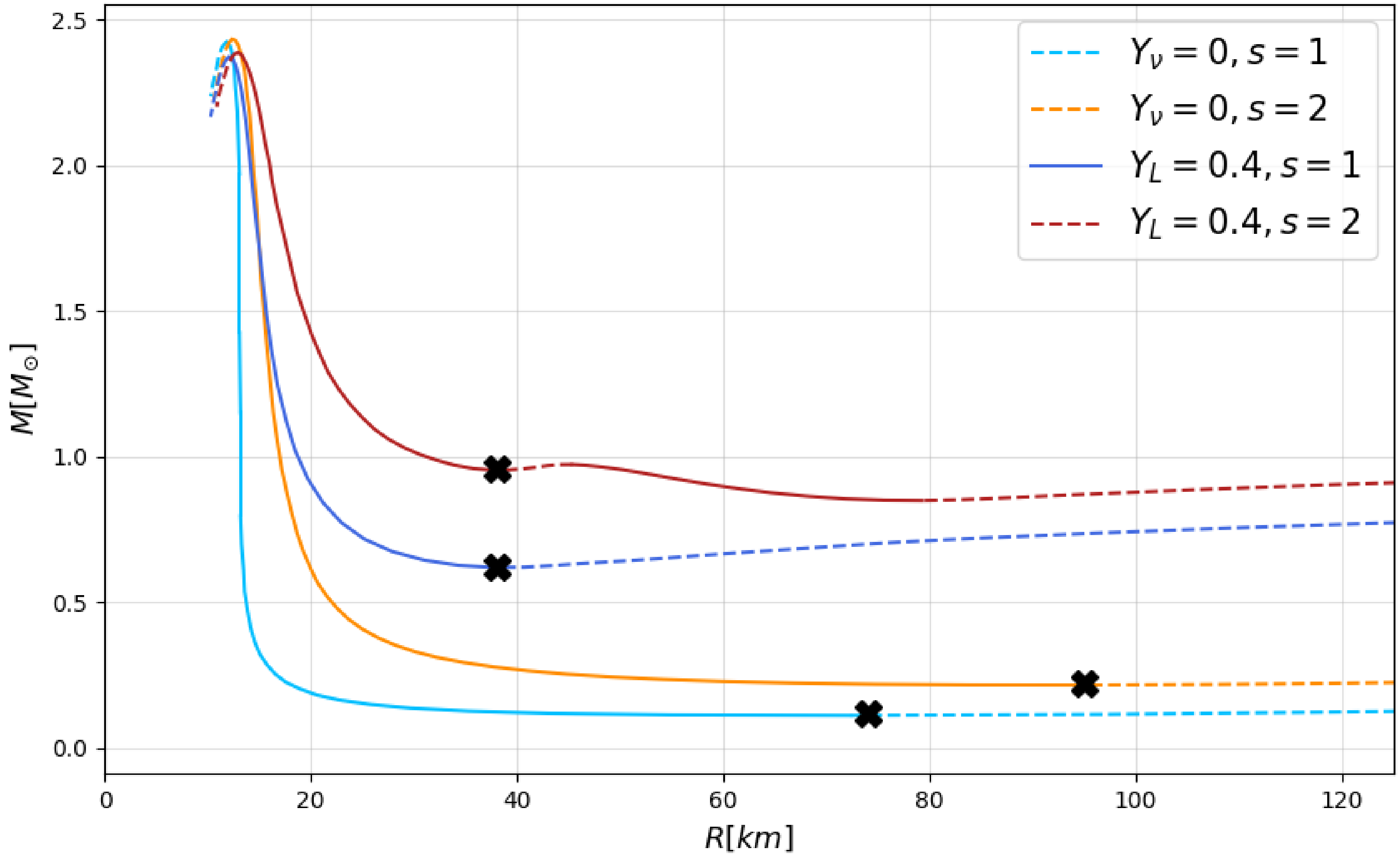
RESULTS – MASS-RADIUS CURVES



Solid: Stable
Dashed: Unstable

✖ = *minimum mass configuration*

RESULTS – MASS-RADIUS CURVES

$s \uparrow \rightarrow M_{min} \uparrow$

Additional thermal pressure!

Solid: Stable
Dashed: Unstable

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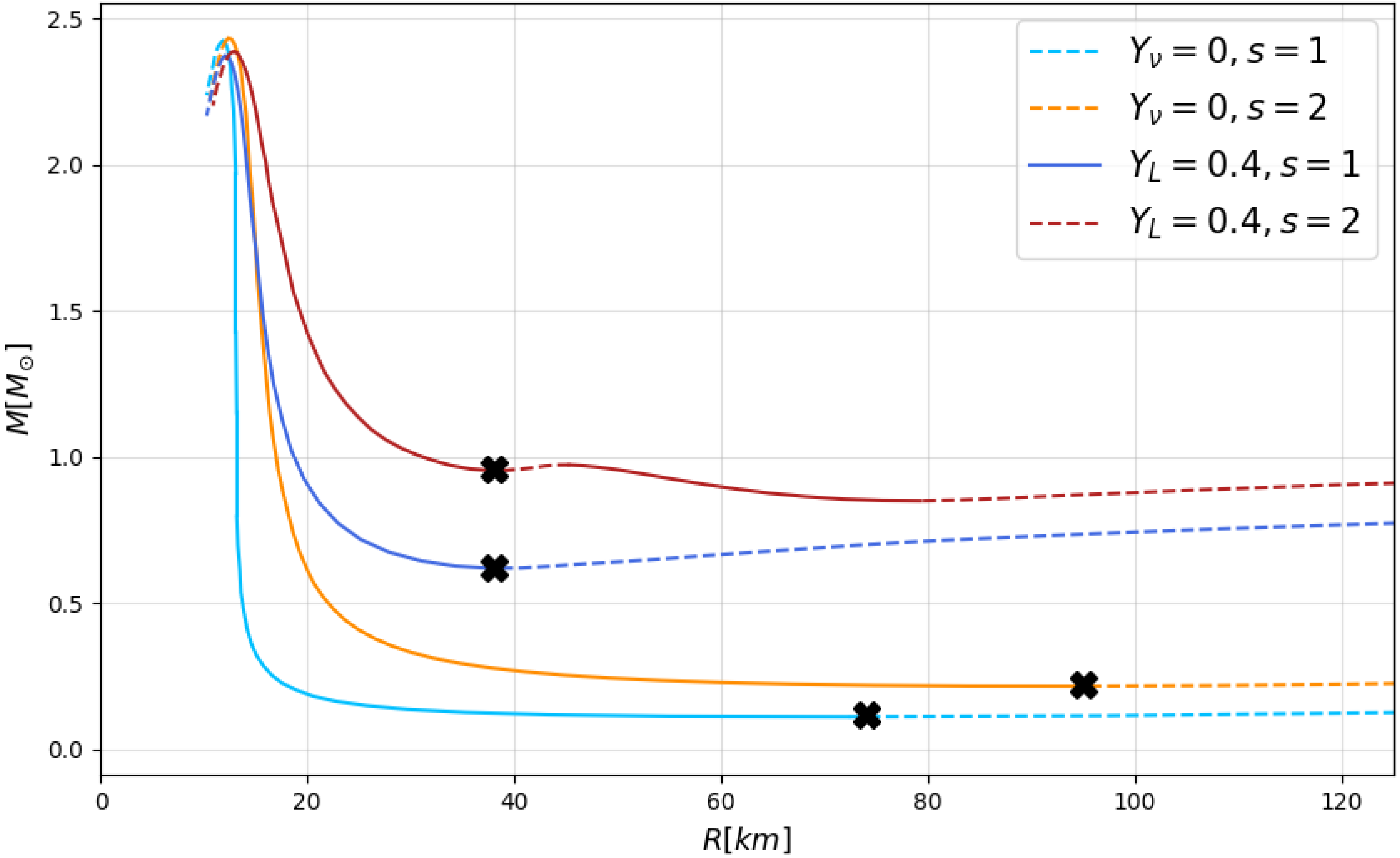

RESULTS – MASS-RADIUS CURVES

$s \uparrow \rightarrow M_{min} \uparrow$

Additional thermal pressure!

$Y_L \uparrow \rightarrow M_{min} \uparrow$

Additional lepton pressure!

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Dashed: Unstable

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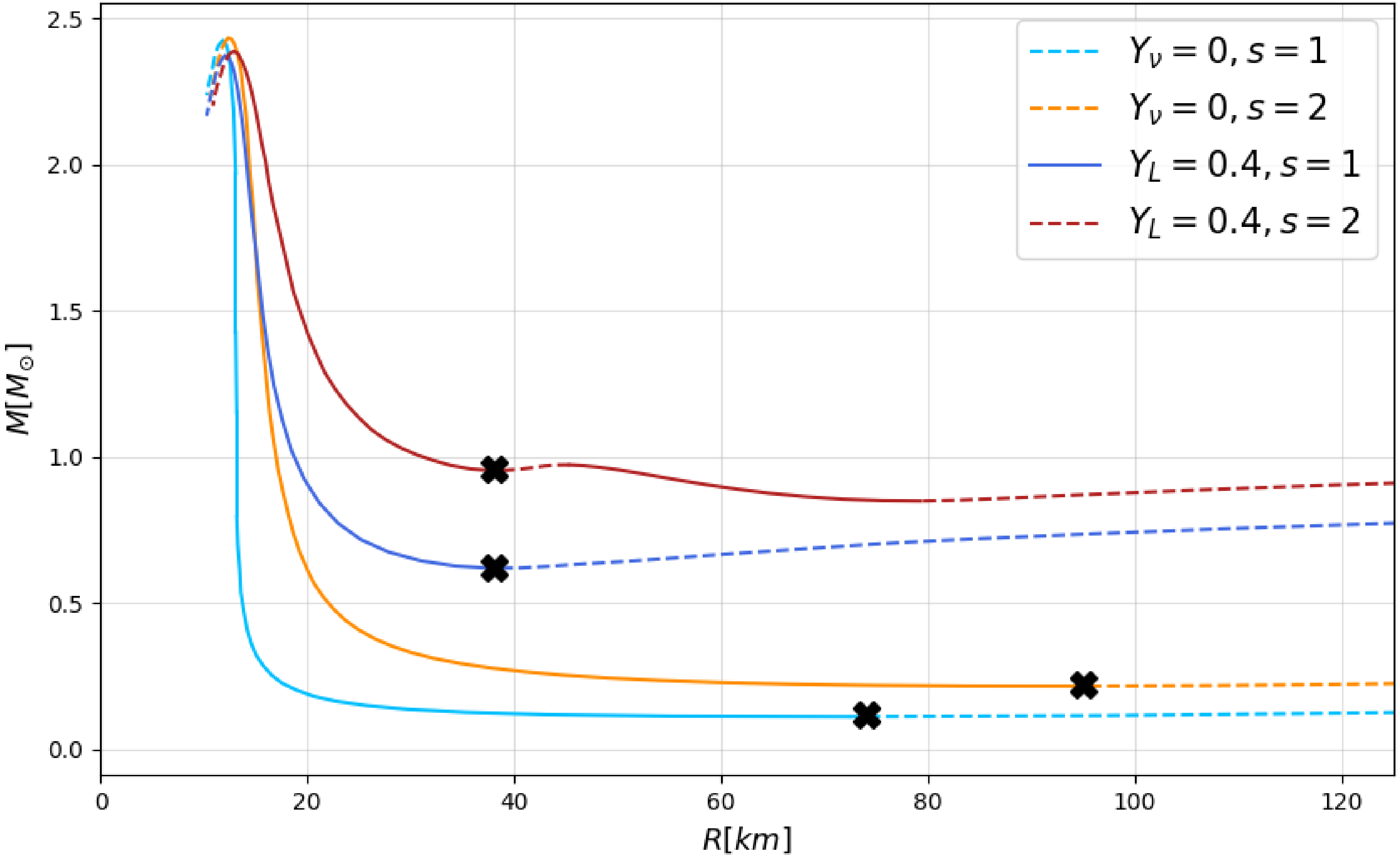


RESULTS – MASS-RADIUS CURVES

$s \uparrow \rightarrow M_{min} \uparrow$

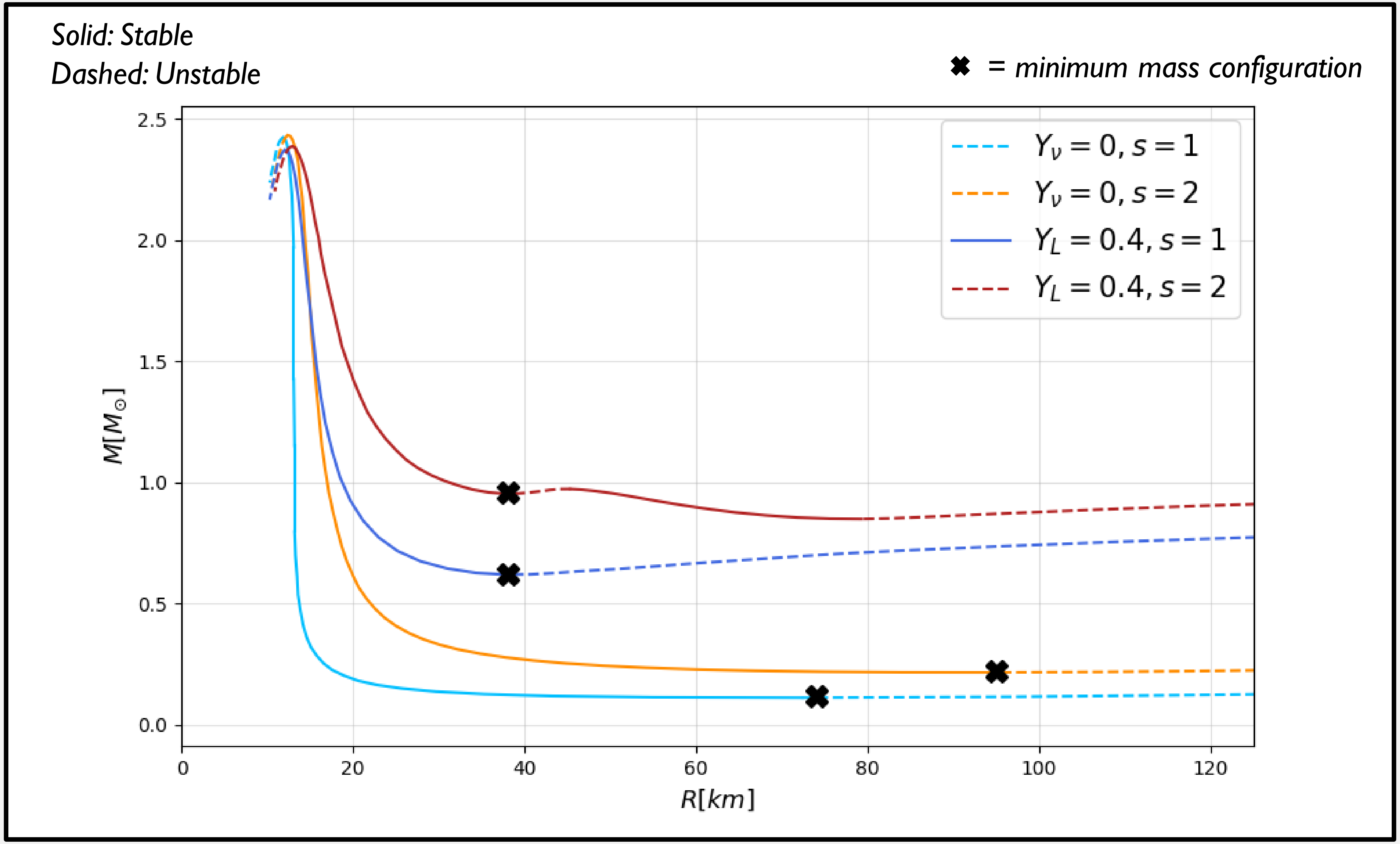
Additional thermal pressure!

$Y_L \uparrow \rightarrow M_{min} \uparrow$

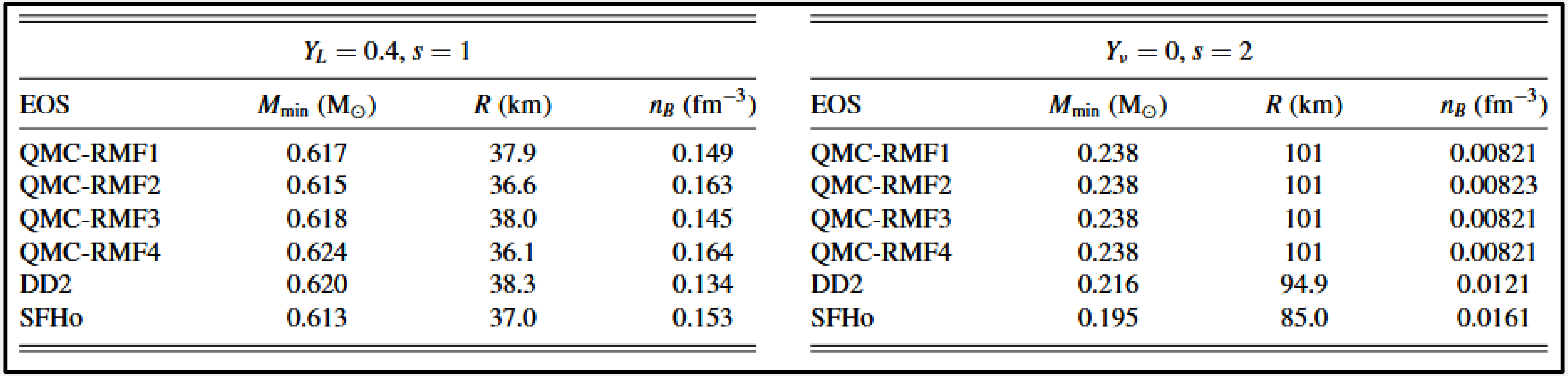
Additional lepton pressure!

$Y_L \uparrow \rightarrow M_{max} \downarrow$

Less neutron pressure!



MINIMAL MASSES, RADII AND CENTRAL DENSITIES



	$Y_L = 0.4, s = 1$		
EOS	$M_{\min}$ ($M_\odot$)	R (km)	n_B (fm^{-3})
QMC-RMF1	0.617	37.9	0.149
QMC-RMF2	0.615	36.6	0.163
QMC-RMF3	0.618	38.0	0.145
QMC-RMF4	0.624	36.1	0.164
DD2	0.620	38.3	0.134
SFHo	0.613	37.0	0.153

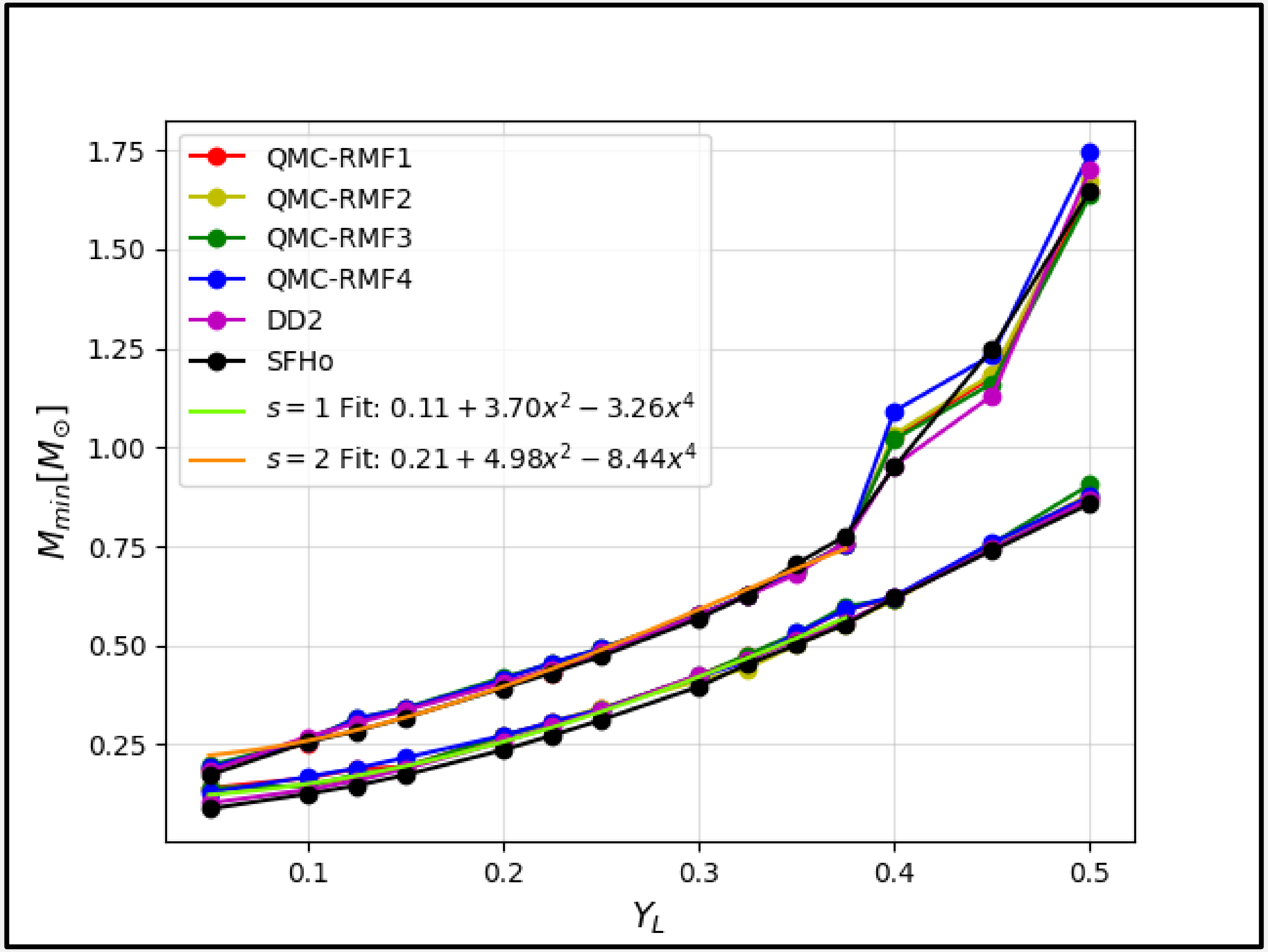
	$Y_\nu = 0, s = 2$		
EOS	$M_{\min}$ ($M_\odot$)	R (km)	n_B (fm^{-3})
QMC-RMF1	0.238	101	0.00821
QMC-RMF2	0.238	101	0.00823
QMC-RMF3	0.238	101	0.00821
QMC-RMF4	0.238	101	0.00821
DD2	0.216	94.9	0.0121
SFHo	0.195	85.0	0.0161

- Captured neutrinos before the explosion result in: **Bigger minimum mass, smaller radius, higher densities**
  → more compact

- Loss of neutrinos after the explosion result in: **Smaller minimum mass, bigger radius, lower densities**
  →less compact

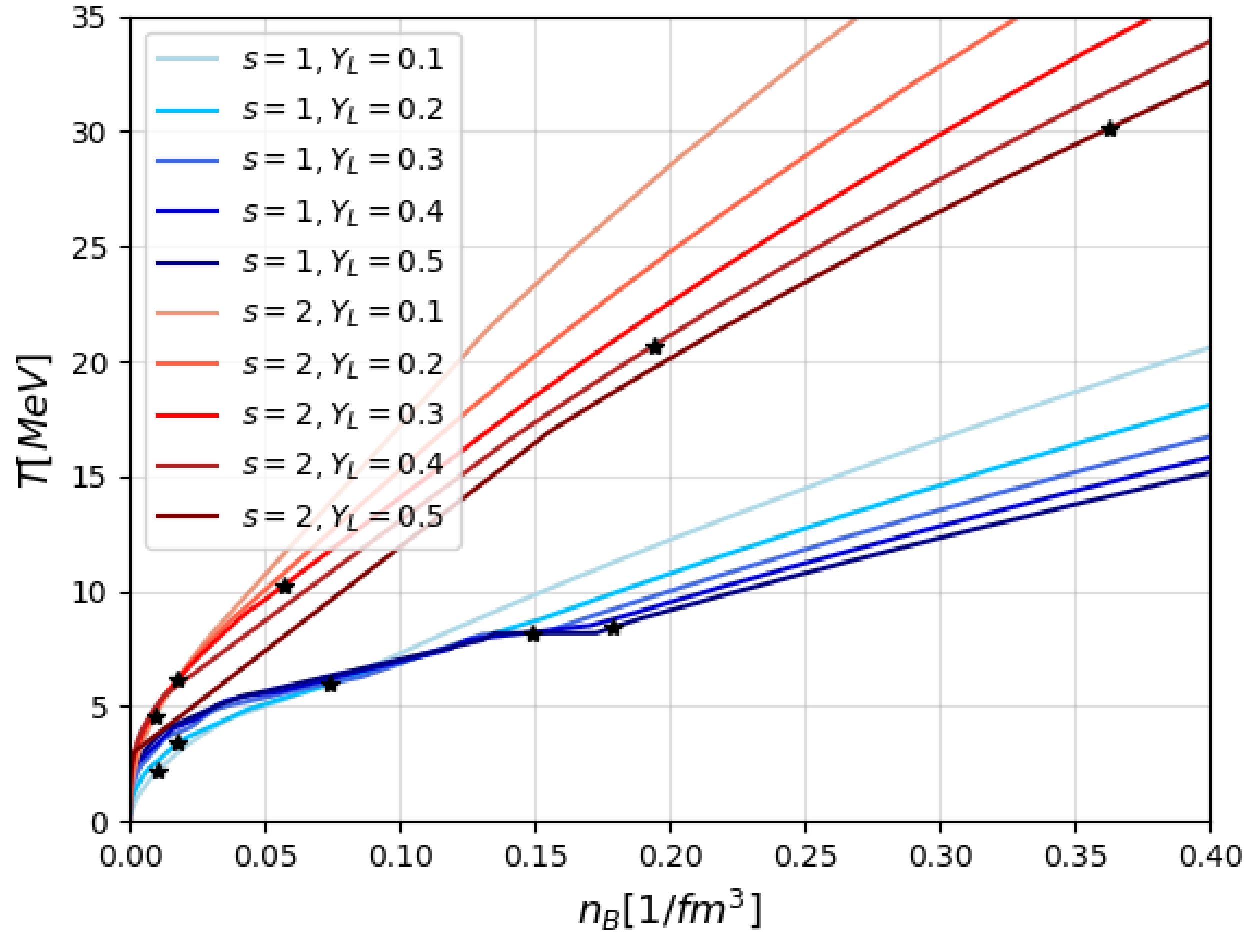
RESULTS – MINIMAL MASSES FOR DIFFERENT LEPTON FRACTIONS

- Increasing minimal mass with increasing lepton fraction
- Higher minimal masses for higher entropy
- Non-linear dependency
- EoS show same behaviour → Universal relation for EoS fulfilling chiral EFT
- Common fit for all EoS up to $Y_L \leq 0.375$

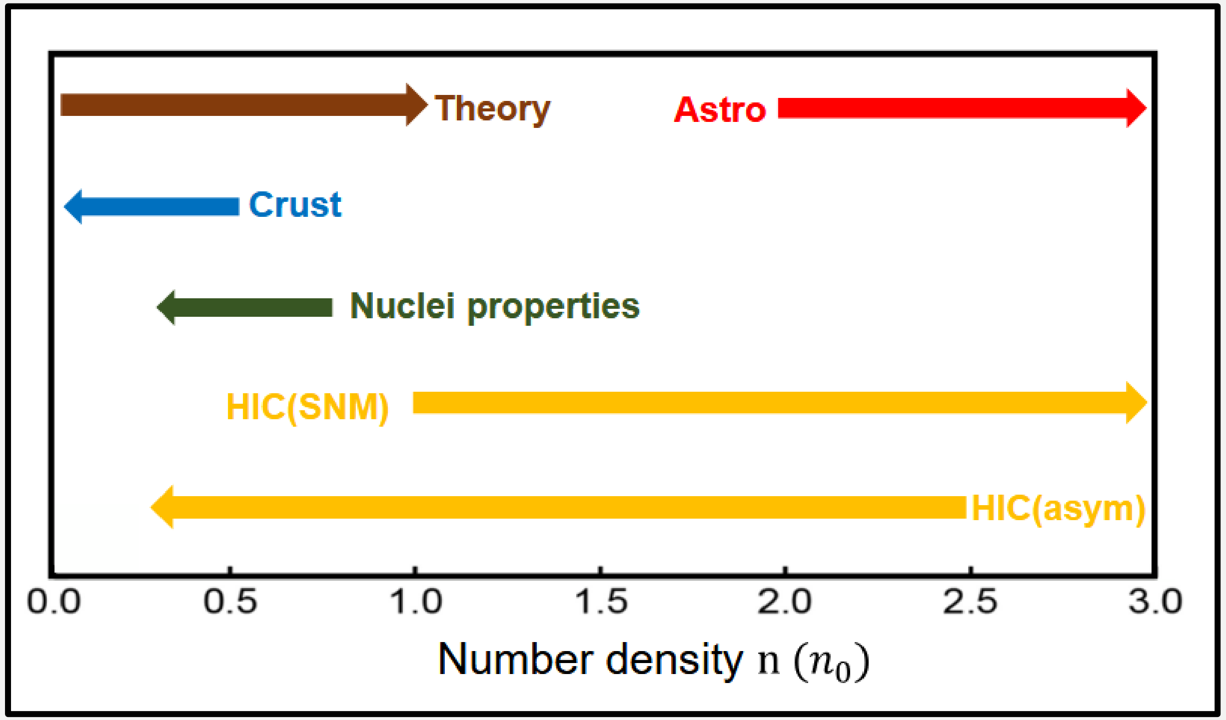


RESULTS - ISENTROPES

- High Y_L and high s → high n_B
- Increase in temperature and density for $(Y_L = 0.4, s = 2)$ and $(Y_L = 0.5, s = 2)$ is huge
- At those densities we are not in chiral band anymore
  → noticable differences in M_{min}

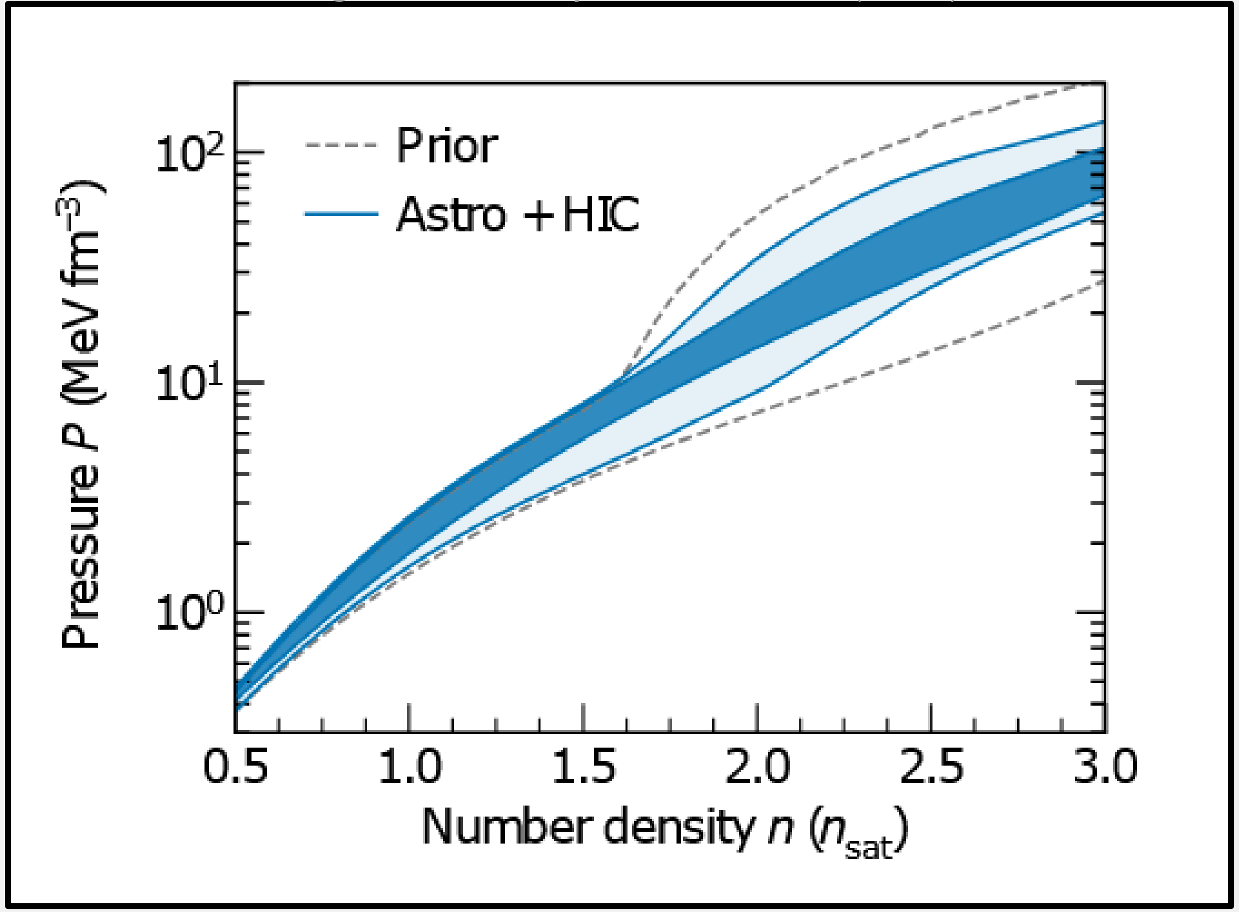


CONSTRAINTS FROM HIC AND ASTRO



Heavy-Ion Collisions and astrophysical constraints both describe matter at $2.0 - 3.0n_0$

Sorensen et al., Prog. Part. Nucl. Phys. 134, 104080 (2024)



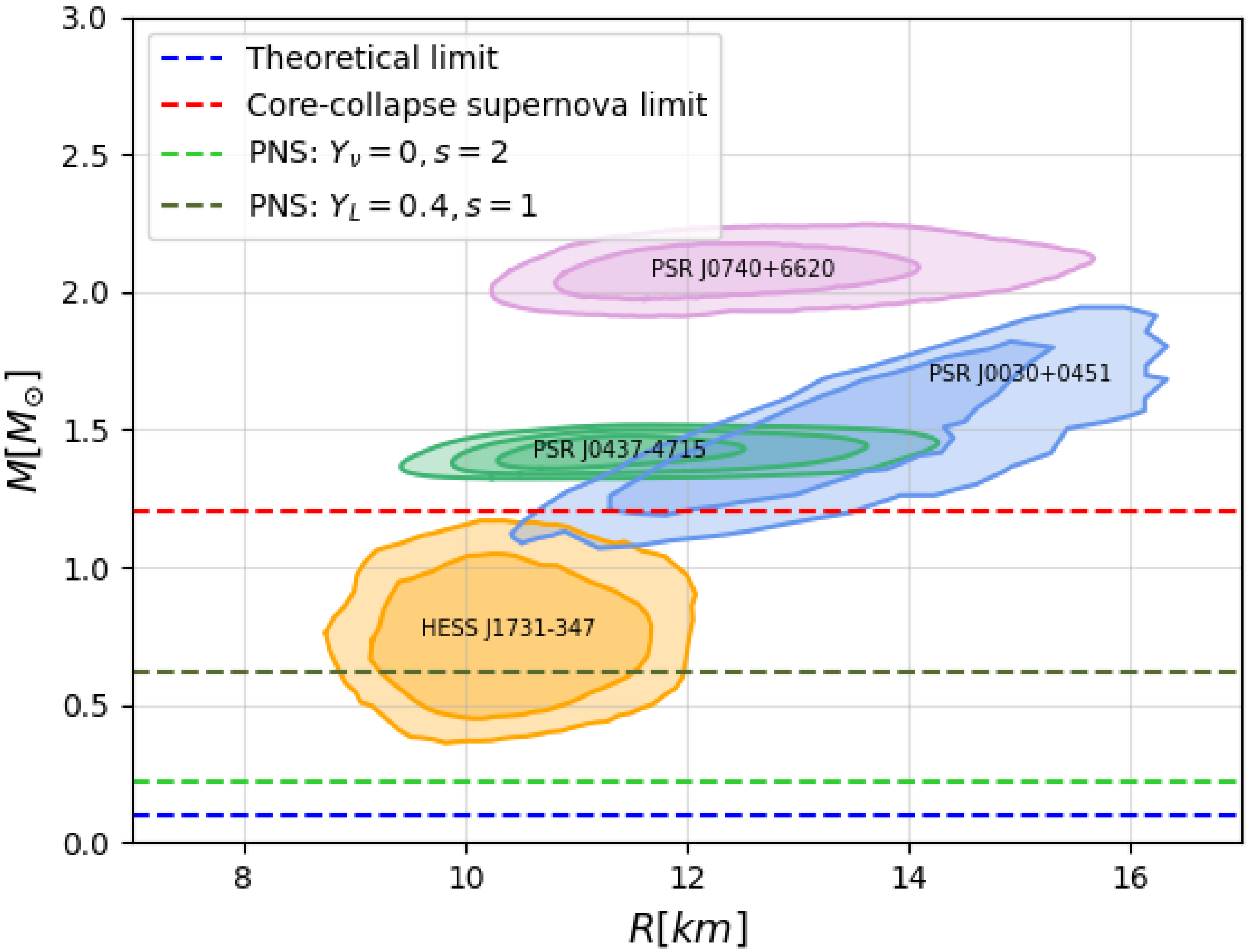
Combination of Heavy-Ion Collisions with astrophysical constraints narrow down pressure region

- PNS: $T \approx 10 - 50$MeV & $Y_p \approx 0.1 - 0.4$
- HIC create similar conditions after collision

Heavy-Ion Collisions probe regions in temperature and proton fraction that are found in Proto-Neutron Star matter and can therefore help to constrain PNS properties!

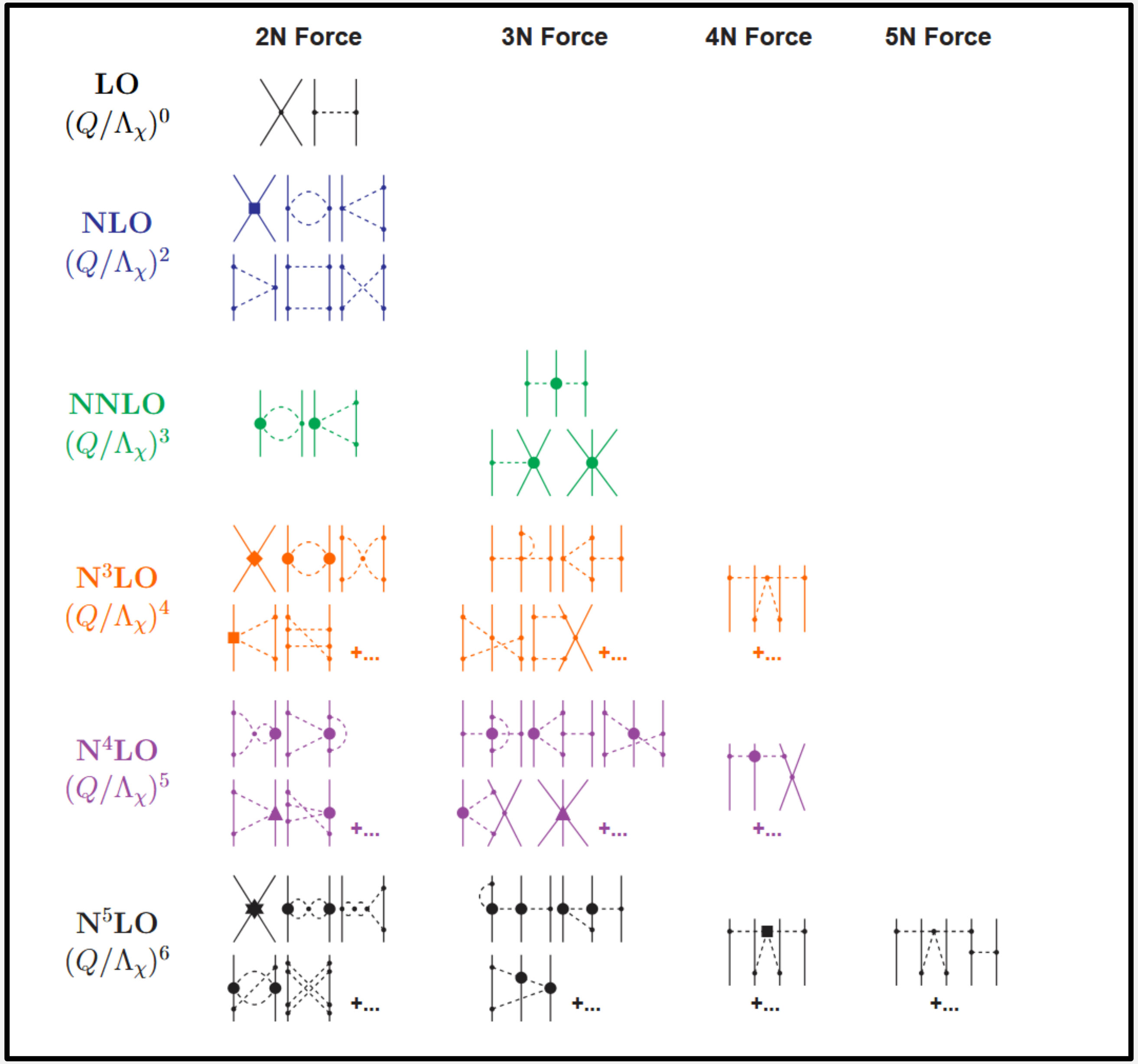
SUMMARY

- Minimal mass of proto-neutron stars with chirally constrained equations of state:
  - $t = 0$: $Y_L = 0.4$ & $s = 1$ → $M_{min} = 0.62 M_{\odot}$
  - $t = 15s$: $Y_\nu = 0$ & $s = 2$ → $M_{min} = 0.22 M_{\odot}$
- Chiral EFT gives very good estimate on M_{min} of PNS!
- Information from HIC could help constrain the equation of state for M_{max} of PNS!



BACKUP

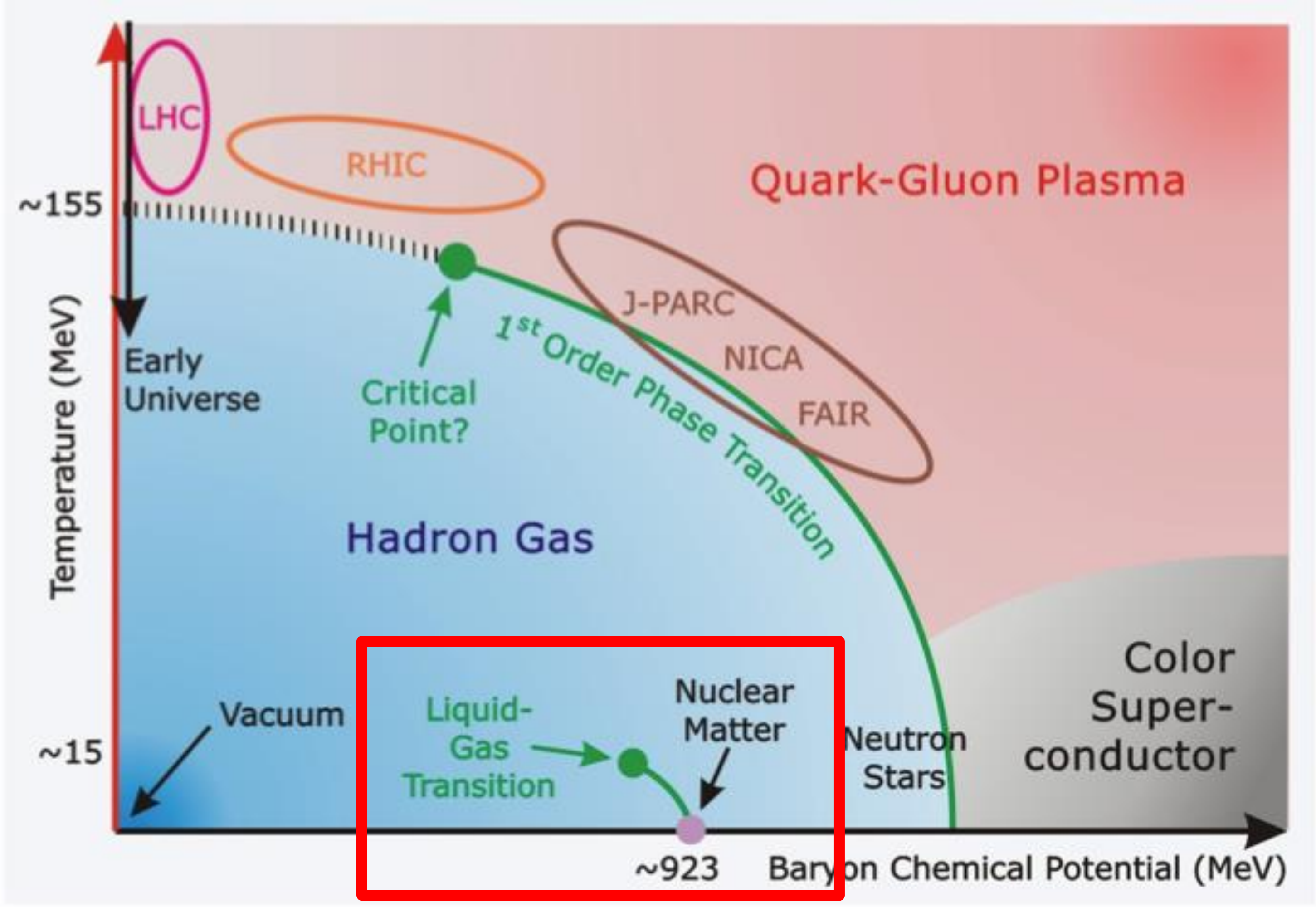
CHIRAL HIRARCHY



Machleidt & Sammarruca (2016), Phys. Scripta 91, 083007

BRIEF EXCURSION: LIQUID-GAS PHASE TRANSITION

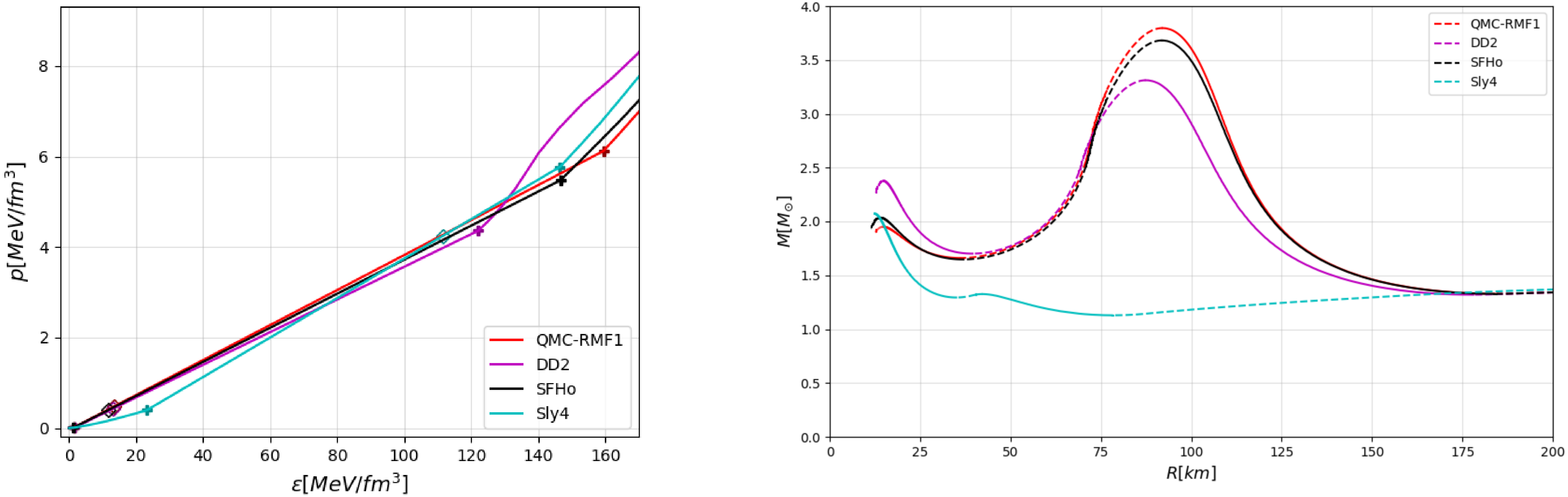
- First order phase transition between
  - A liquid of nucleons at higher densities
  - A gas of nuclei at lower densities
- Two globally conserved quantities in PNS: Y_L and N_B → Gibbs construction
- Mixture of both phases at the crust-core transition in proto-neutron stars
- What we do: Linear interpolation
- What is physically right? Gibbs construction



https://www.gsi.de/work/forschung/theorie/theory-new/hot-and-dense-qcd

IMPACT OF LIQUID GAS PHASE TRANSITION FOR HIGH LEPTON FRACTION

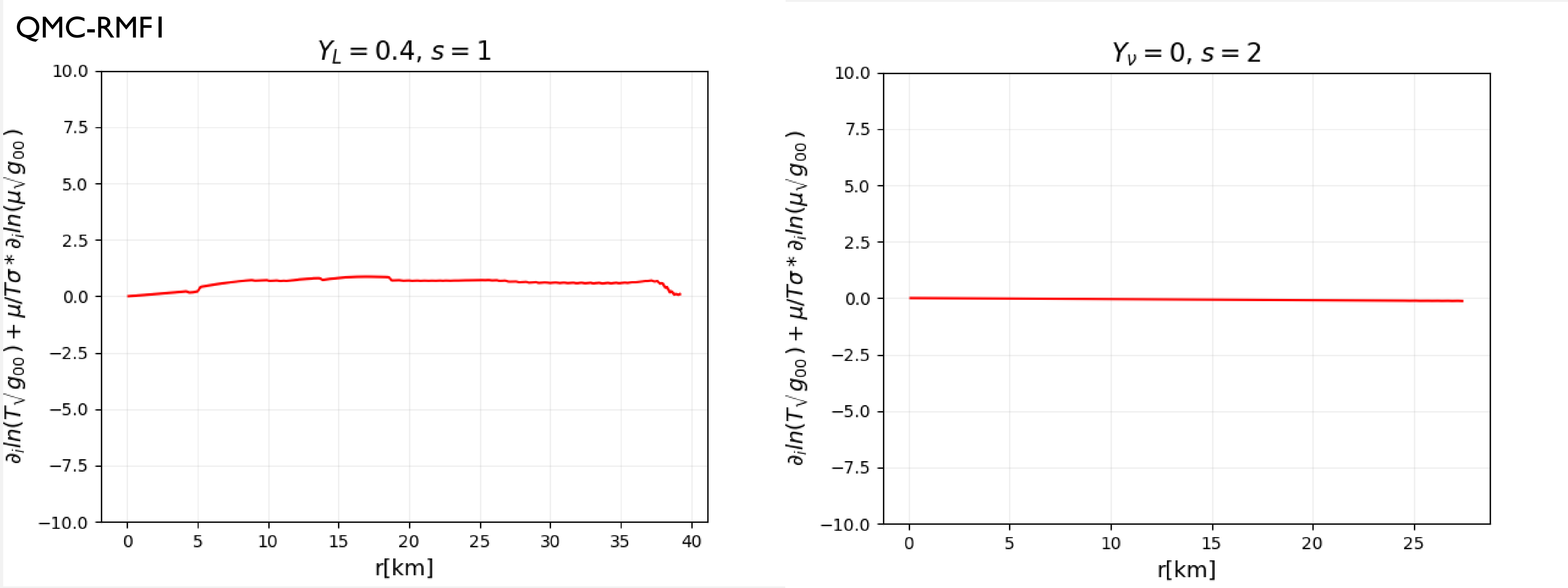
$Y_L = 0.5$



- High Y_L → Huge gap between crust and core, which needs to be constructed
- Leads to a huge second maximum in the mass-radius curve

TOLMAN-EHRENFEST-KLEIN LAW

$$\partial_i \ln(T\sqrt{g_{00}}) + \frac{\mu}{T\sigma}\partial_i ln(\mu\sqrt{g_{00}}) = 0^*$$

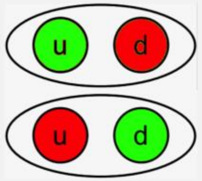


R. C. Tolman & P. Ehrenfest, Phys. Rev. **36, 1791 (1930);*
*O. Klein, Rev. Mod. Phys. **21**, 531 (1949)*

OUTLOOK

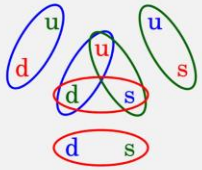
- Proto-neutron stars with color-superconducting matter
  - Which color-superconducting phases can we reach in proto-neutron stars?
  - Which maximal masses do we get for the same conditions (same lepton fraction and entropy per baryon)?

2SC



Intermediate $\mu \lesssim M_s$
1 finite gap parameter
Δ_{ud}

Color-flavor-locking (CFL)



Large $\mu \gg M_s$
3 finite gap parameters
$\Delta_{ud}, \Delta_{us}, \Delta_{ds}$