SMASH: Equation of State and Fluctuation Observables

Hannah Elfner

November 12th 2025, EMMI Workshop „Collective phenomena Equation of State of dense baryonic matter“

Outline

- Equation of state extraction
  - Degrees of freedom
  - Light nuclei production
  - Treatment of potentials
  - Bayesian analysis for equation of state
  - Comparison to FOPI data
- Fluctuations in the hadronic phase
- Summary and outlook

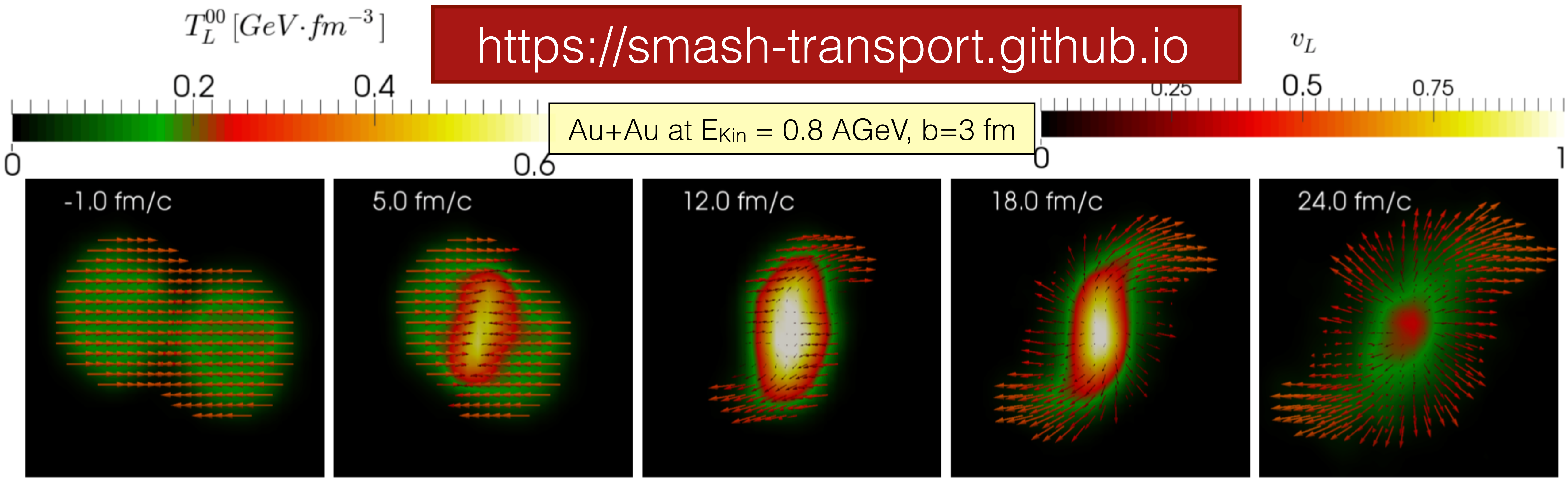
SMASH Transport Approach

SMASH*

J. Weil et al, PRC 94 (2016)

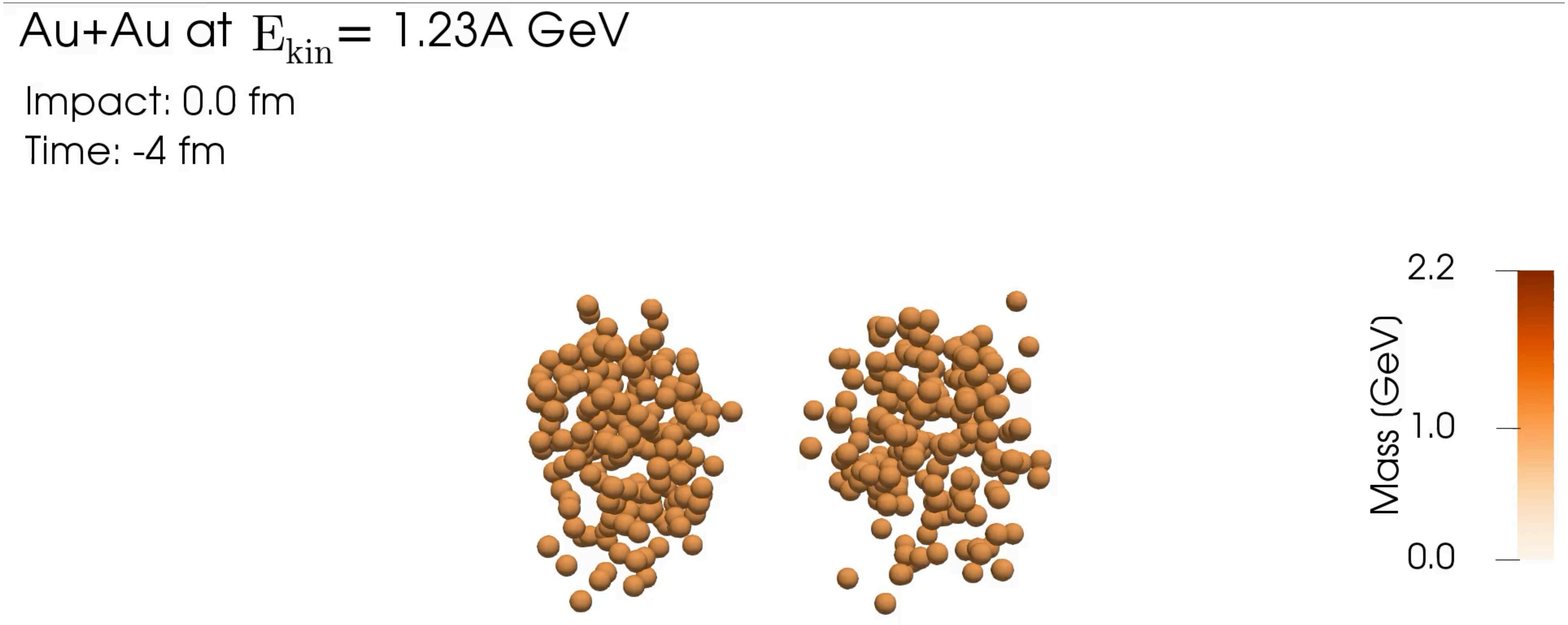
- Hadronic transport approach:
  - Includes all mesons and baryons up to ~2 GeV
  - Binary interactions: Inelastic collisions through resonance/string excitation and decay
  - Infrastructure: C++, Git, Doxygen, ROOT, HepMC, RIVET
  - Used as a library by many groups for afterburner

https://smash-transport.github.io



* Simulating Many Accelerated Strongly-Interacting Hadrons

Example for GSI Energy



- Movies and visualization tutorial available at https://smash-transport.github.io/movies.html

The SMASH Team

- In Frankfurt:
  - Alessandro Sciarra
  - Lucas Constantin
  - Nils Saß
  - Carl Rosenkvist
  - Renan Hirayama
  - Antonio Bozic
  - Robin Sattler
  - Olivia Kolandavelu
  - Manou Engel
  - Sebastian Ostrowski
  - Luis Velez
  - Vincent Pott

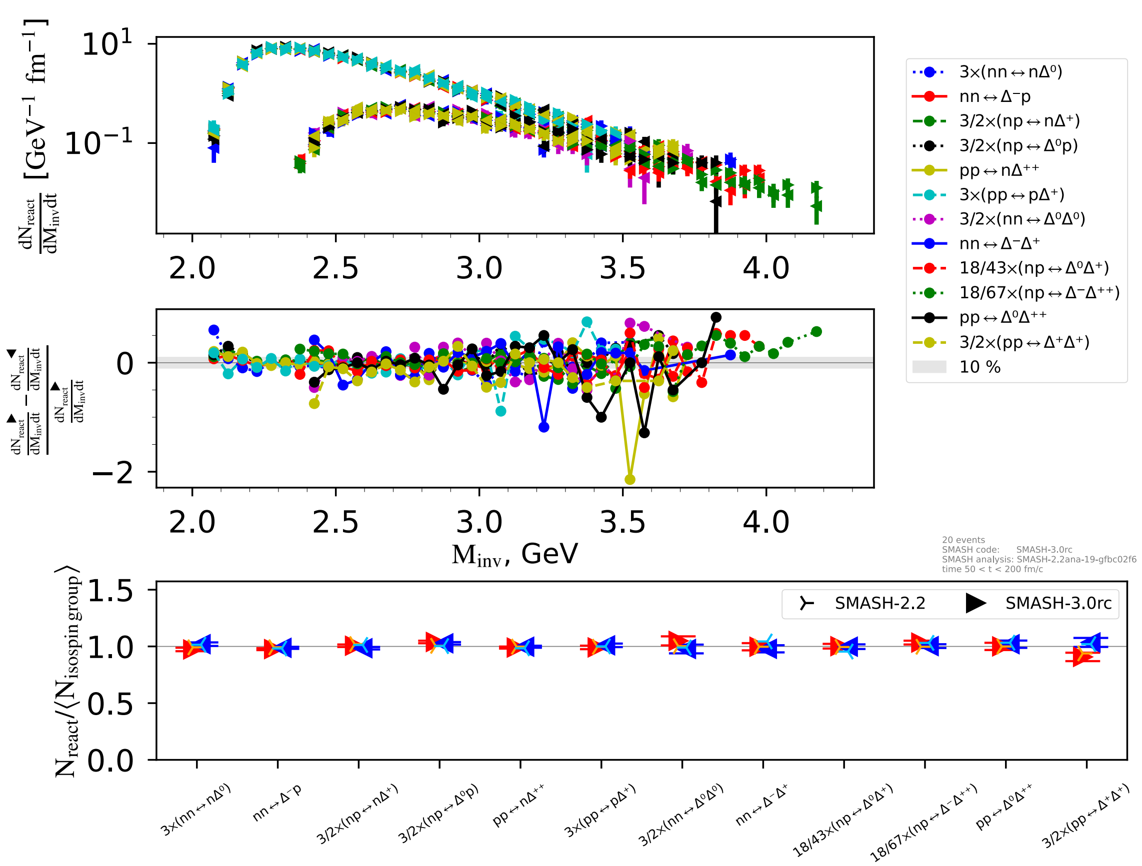
- In US/Spain/Slovakia:
  - Agnieszka Sorensen
  - Hendrik Roch
  - Oscar Garcia-Montero
  - Zuzana Paulinyova

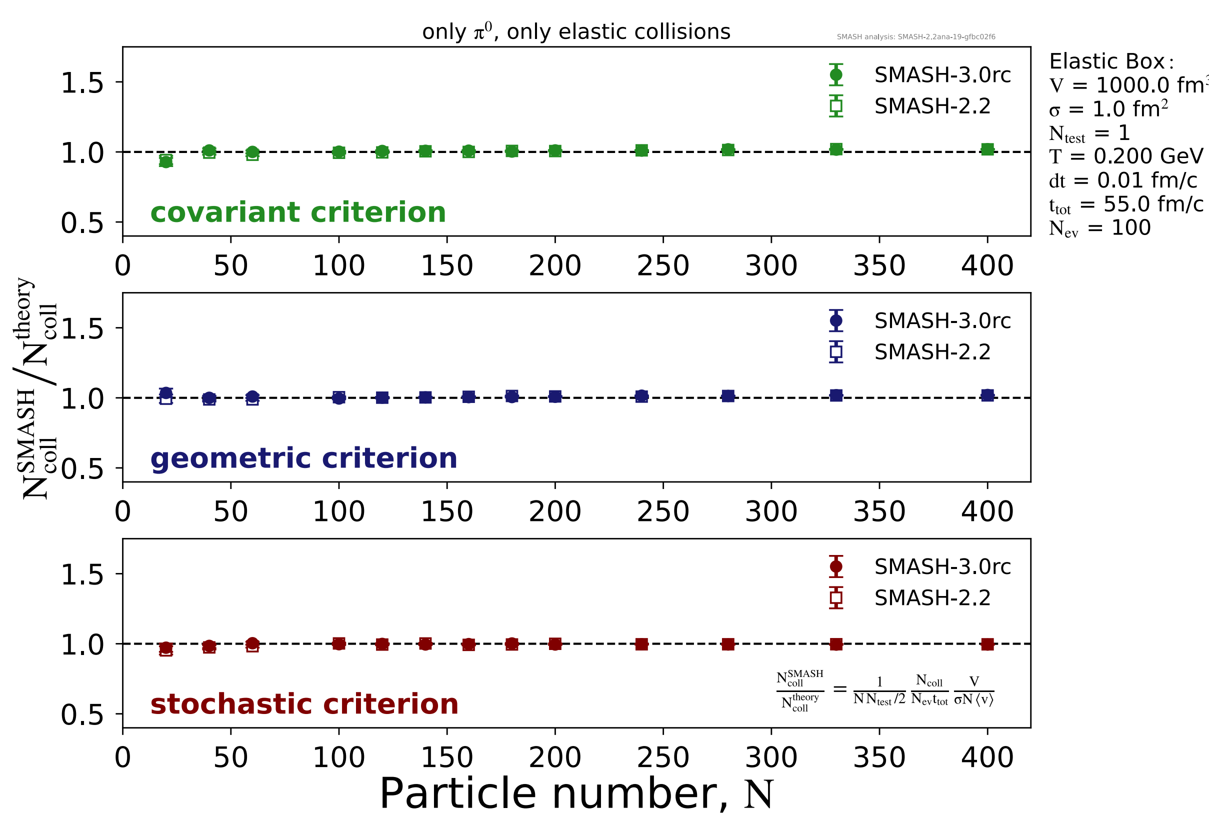
Group excursion in May 2022

Transparent Development

- Stringent rules for software development
  - Each newly implemented feature goes through an internal review process
  - ~100 unit tests, enforced documentation and user guide
- For each new public version physics analysis suite is run (as well open source available)

https://github.com/smash-transport/smash-analysis





Degrees of Freedom

N	Δ	Λ	Σ	Ξ	Ω
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1900}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1905}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1910}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1680}	Δ_{1920}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1930}	Λ_{1810}	Σ_{1940}		
N_{1710}	Δ_{1950}	Λ_{1820}	Σ_{2030}		
N_{1720}		Λ_{1830}	Σ_{2250}		
N_{1875}		Λ_{1890}			
N_{1900}		Λ_{2100}			
N_{1990}		Λ_{2110}			
N_{2060}		Λ_{2350}			
N_{2080}					
N_{2100}					
N_{2120}					
N_{2190}					
N_{2220}					
N_{2250}					

As of SMASH-1.7

Unflavored				Strange
π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
π_{1300}	$f_{0\ 1370}$	$f'_{2\ 1525}$		K^*_{892}
π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
	$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
η_{548}		$f_{2\ 2300}$	$\phi_{3\ 1850}$	K^*_{1410}
η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
η_{1405}		$f_{1\ 1285}$		K^*_{1680}
η_{1475}	ϕ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
	ϕ_{1680}			$K_3^*{}_{1780}$
σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
	$h_{1\ 1170}$			$K_4^*{}_{2045}$
ρ_{776}		$\pi_{1\ 1400}$		
ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
ρ_{1700}				
	$a_{1\ 1260}$	$\eta_{2\ 1645}$		
ω_{783}				
ω_{1420}		$\omega_{3\ 1670}$		
ω_{1650}				

- + corresponding antiparticles
- Perturbative treatment of photons and dileptons
- Isospin symmetry

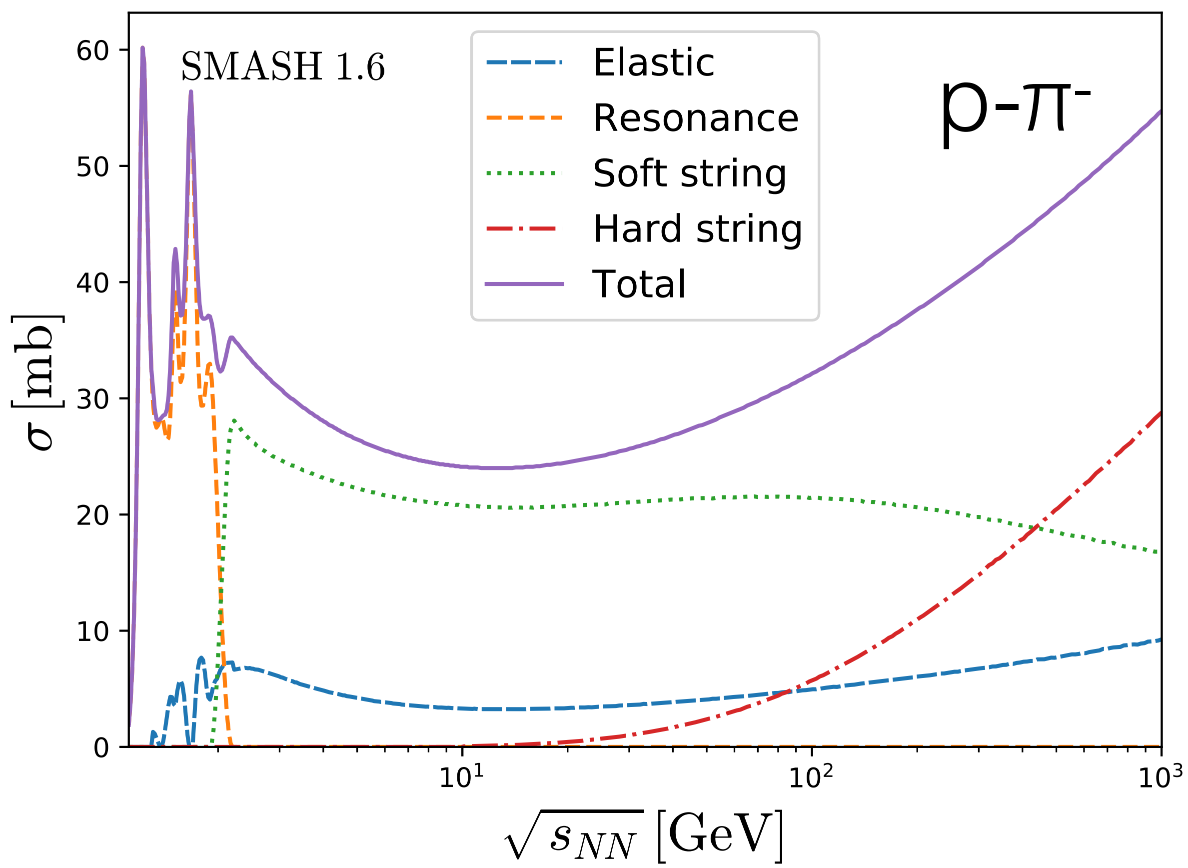
- Mesons and baryons according to particle data group
- Isospin multiplets and anti-particles are included

Cross-Sections

- High energy cross-section is dominated by string excitation and fragmentation while low energy cross-sections are treated via resonances

J. Mohs, S. Ryu and HE, *J.Phys.G* 47 (2020)

- Soft strings
  - Pythia is only employed for fragmentation
  - Single-diffractive, double diffractive and non-diffractive processes
- Hard strings
  - Fully treated by Pythia
  - All species mapped to pions and nucleons

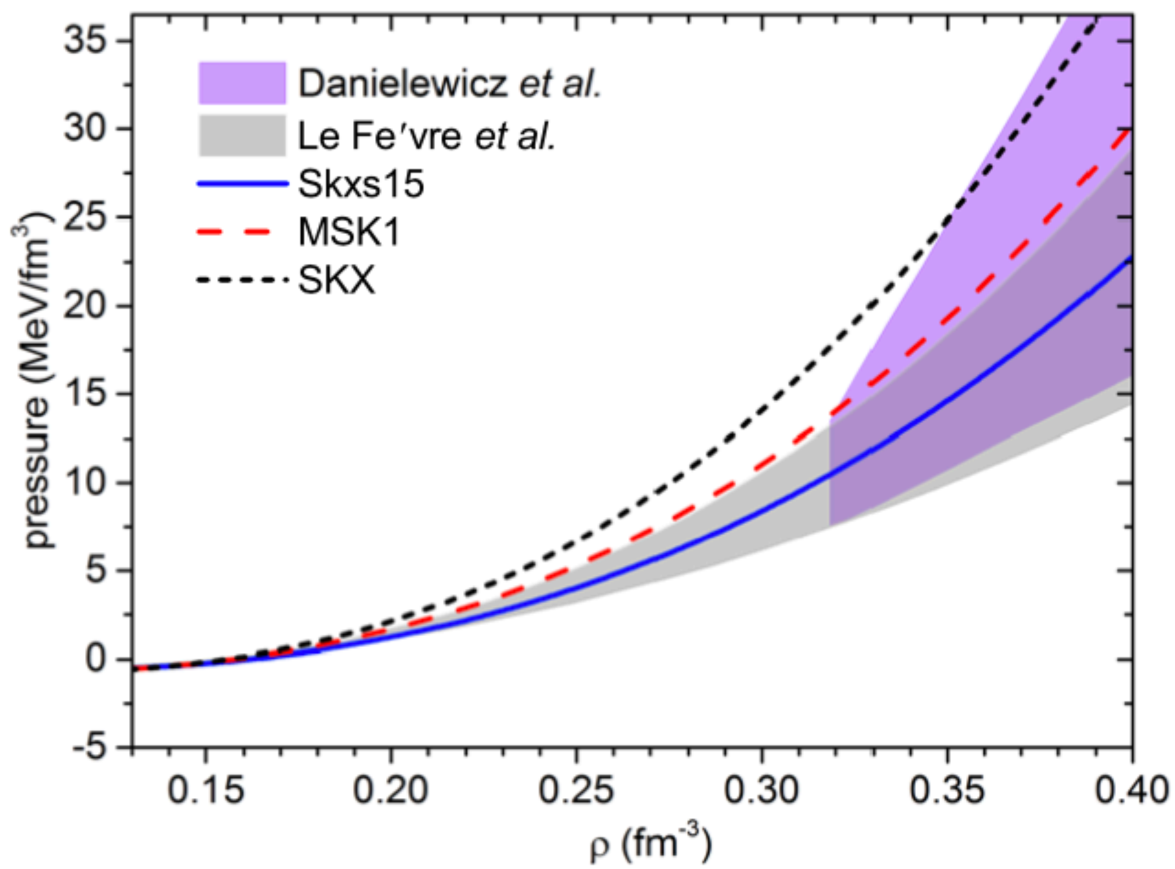


- Note: SMASH-2.0 includes optimised Pythia calls to reduce run-time

Equation of State of Nuclear Matter

Nuclear Matter Equation of State

- ‚Danielewicz constraint' and newer constraints ->varying mean field parameters to fit flow observables best



Y. Wang et al, arXiv:1804.04293
A. Hombach et al, Eur.Phys.J.A 5 (1999)
Show importance of resonances

- Open issues: Results are dependent on details of transport code and in particular mean field properties, e.g. BUU vs QMD, cluster formation, form of the potential…

TMEP collaboration, e.g. Prog.Part.Nucl.Phys. 125 (2022)

Light Clusters

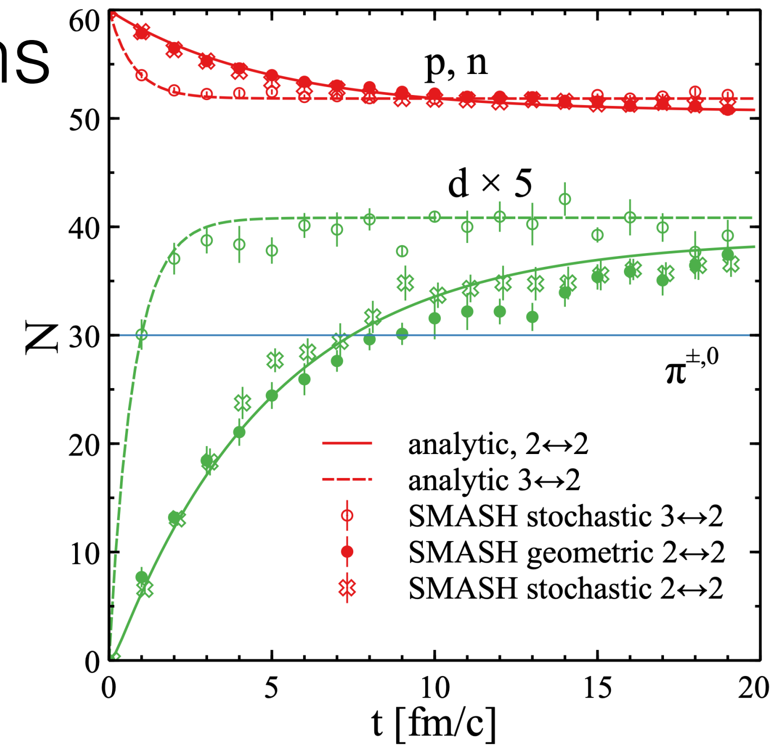
- Microscopic formation and destruction of deuterons
  - Via fictitious deuteron resonance
  - Via explicit 3<->2 reactions

Implemented reactions

$\pi d \leftrightarrow \pi np$ $\quad Nd \leftrightarrow Nnp$

$\pi d \leftrightarrow NN$ $\quad \bar{N}d \leftrightarrow \bar{N}np$

+elastic channels

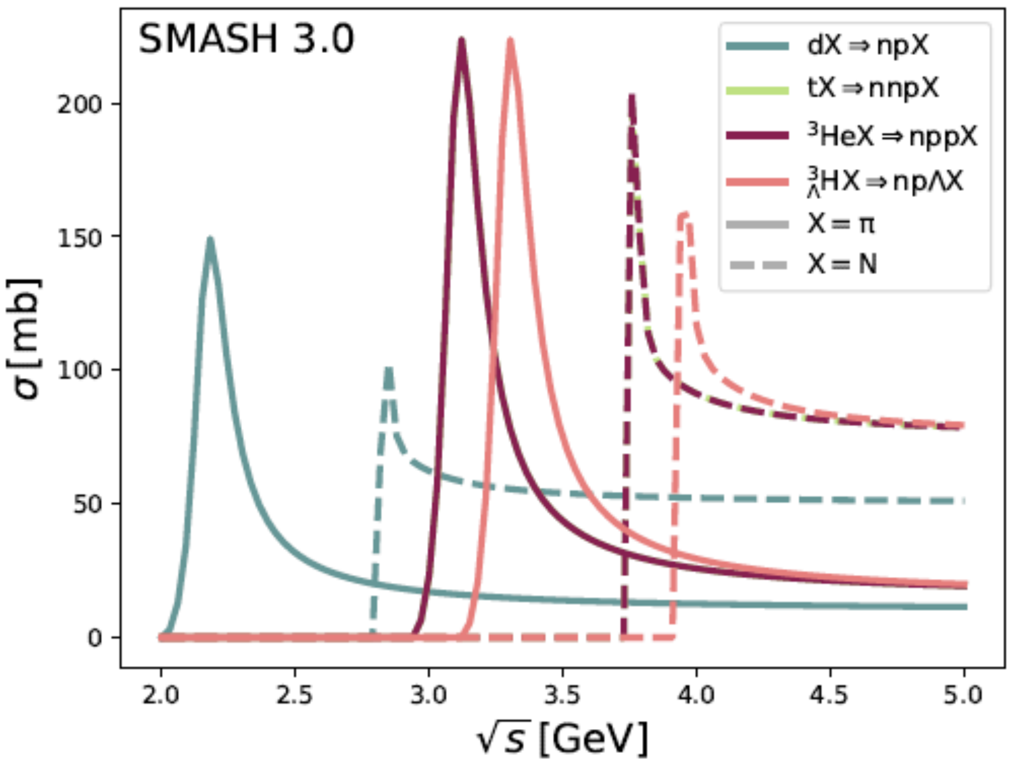


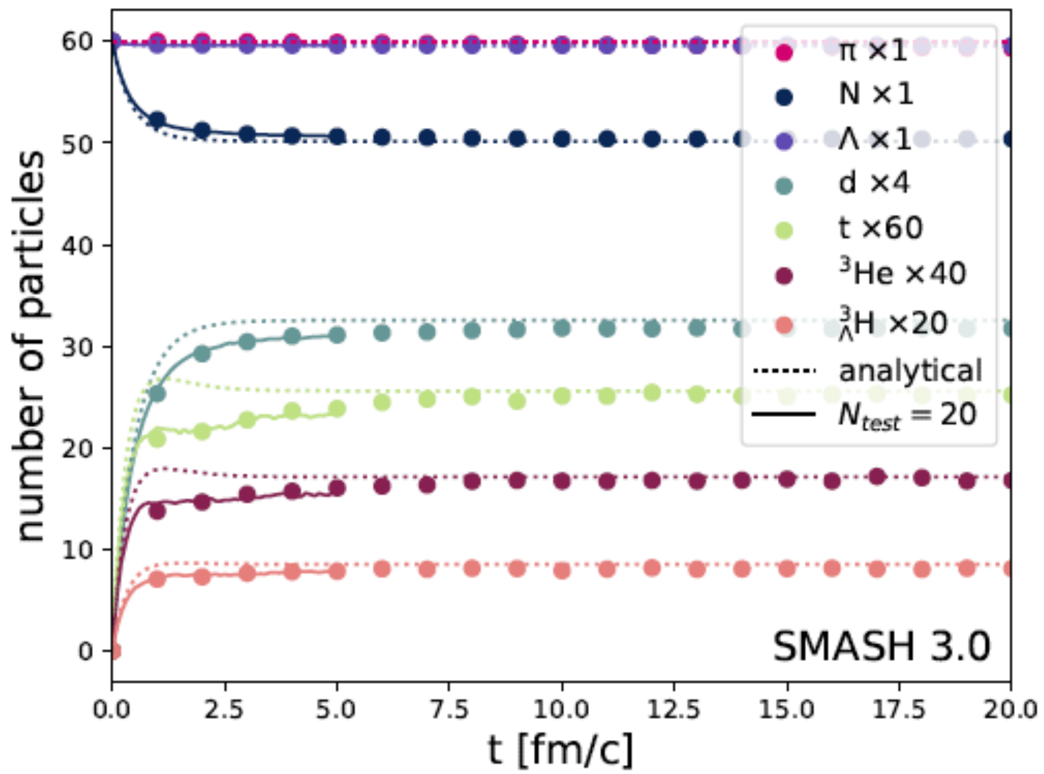
D. Oliinychenko et al, *Phys.Rev.C* 99 (2019)
J. Staudenmaier et al, *Phys.Rev.C* 104 (2021)
M. Ege et al, arXiv:2409.04209

- Extended multi-particle reactions to 3N clusters and Hypertriton, but lots of unknown cross-sections and properties, also approximation as point particle gets worse
- Coalescence-based clustering algorithm
  - Similar to the one used in UrQMD studies by Hillmann et al
  - Collaboration with Spieß et al to improve centrality map including light nuclei

Dynamic Light Cluster Production

- Light cluster are created and destroyed according to the scattering probabilities via stochastic rates
- Multi-particle interactions require test particles and large amount of CPU time





- Rate equation results are matched rather nicely for the time evolution in a box calculation

M. Ege et al, arXiv:2409.04209

Potentials in SMASH

- Simple form of Skyrme and symmetry potential is used
- Coulomb potential can be employed as well
- Densities and their derivatives are required
- BUU method with test particles and parallel ensembles
- Densities are calculated using Gaussian smearing kernel

$$U_{\mathrm{Sk}} = A\left(\frac{\rho_B}{\rho_0}\right) + B\left(\frac{\rho_B}{\rho_0}\right)^{\tau}$$

$$U_{\mathrm{Sym}} = \pm 2S_{\mathrm{pot}}\frac{\rho_{I_3}}{\rho_0}$$

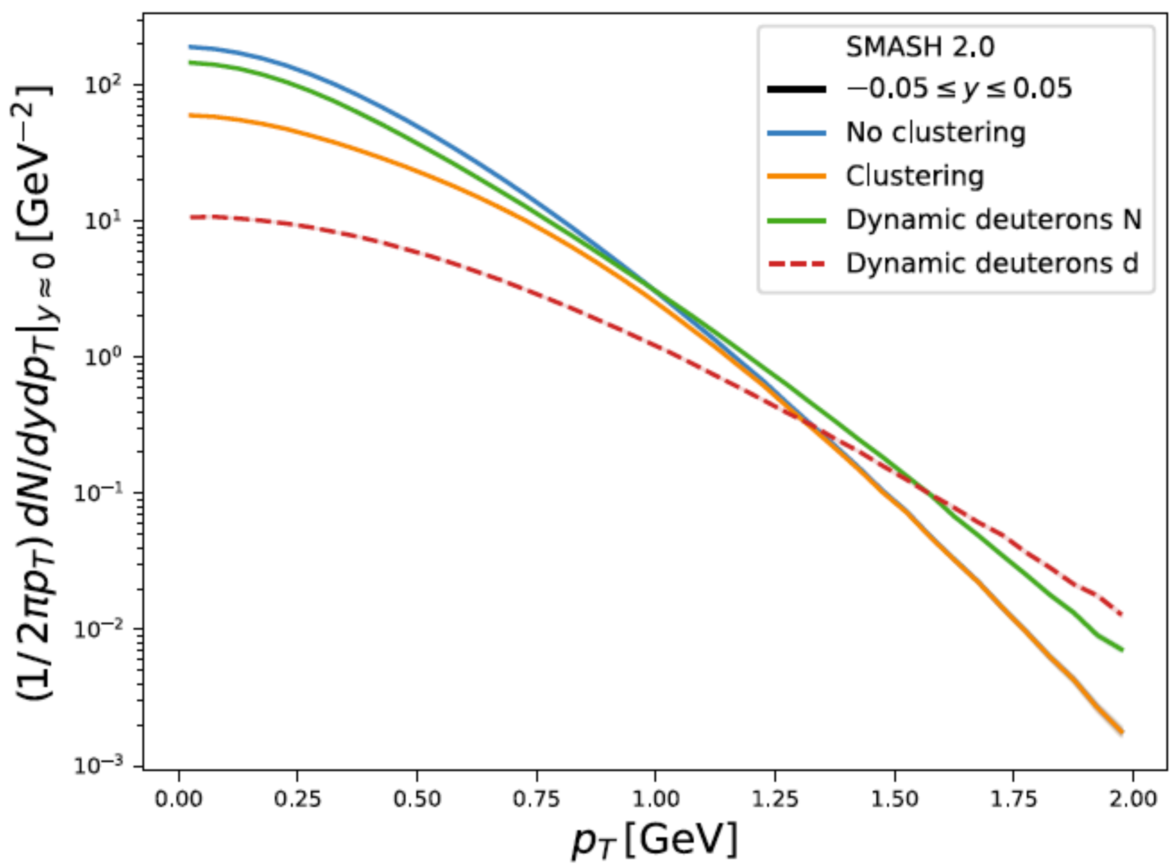
$$f(\mathbf{r},\mathbf{p}) = \frac{1}{N_{\mathrm{test}}}\sum_{i=1}^{N_{\mathrm{test}}} K(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{p}-\mathbf{p}_\mathbf{i})$$

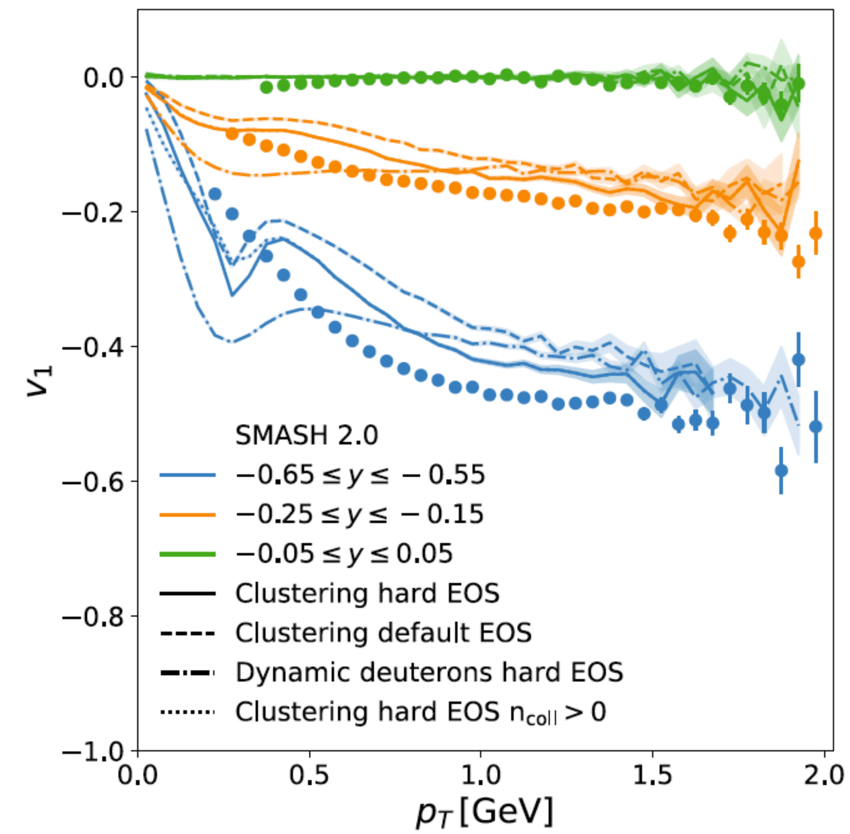
$$K(\mathbf{r}) = (2\pi\sigma^2)^{-\frac{3}{2}}\gamma \exp\left(-\frac{r^2+(\mathbf{r}\cdot\mathbf{u})^2}{2\sigma^2}\right)$$

J. Mohs, S. Spies and HE, arXiv:2409.16927

Initial Study on Potentials

- None of the calculations really fits the data on directed and elliptic flow consistently due to missing momentum dependence



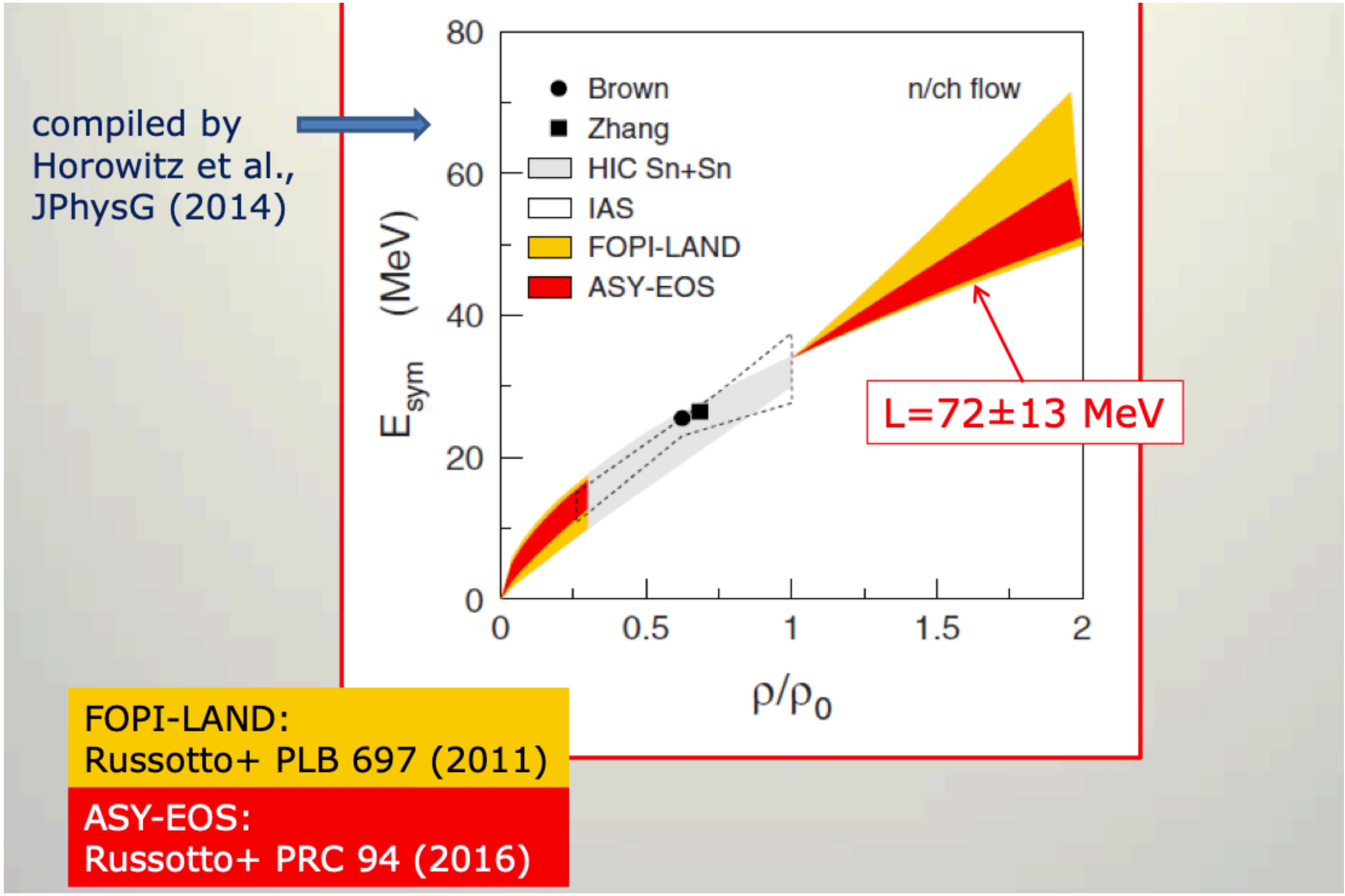


- Main conclusion: 1.0<pT<1.5 GeV is less influenced by cluster production

J. Mohs et al, PRC 105 (2022)

Symmetry Energy

- Extracted from flow of neutrons

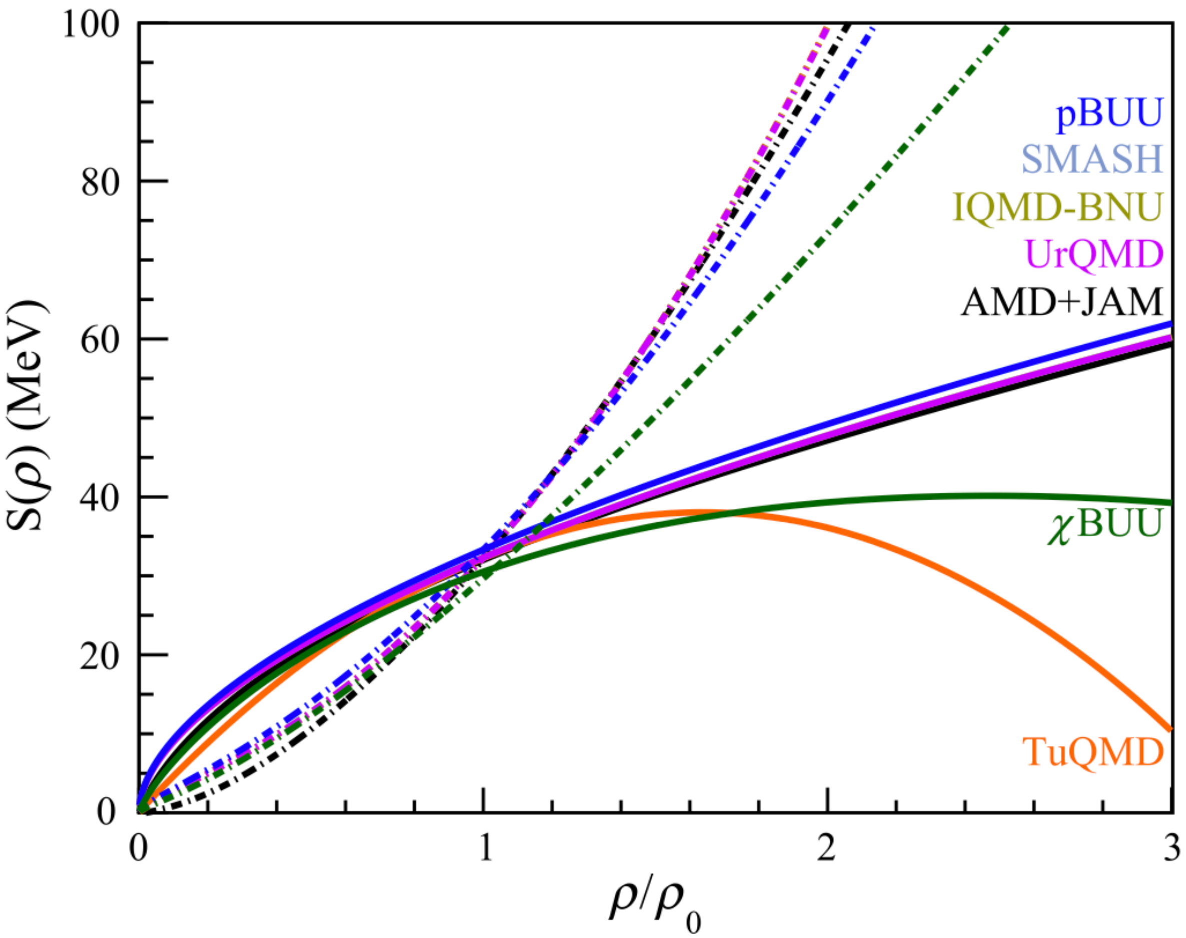


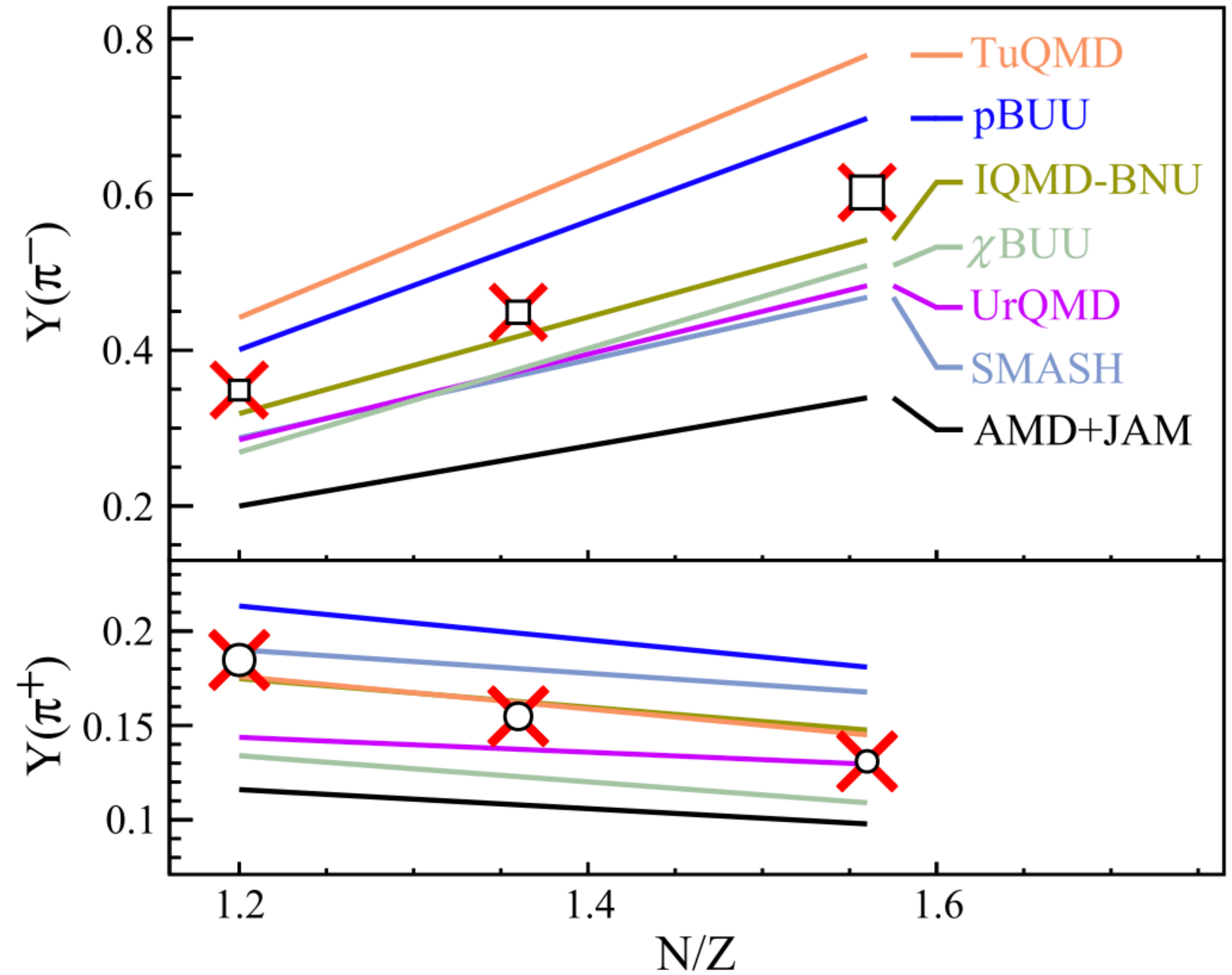
- Crucial to extrapolate to neutron-rich matter (stars)

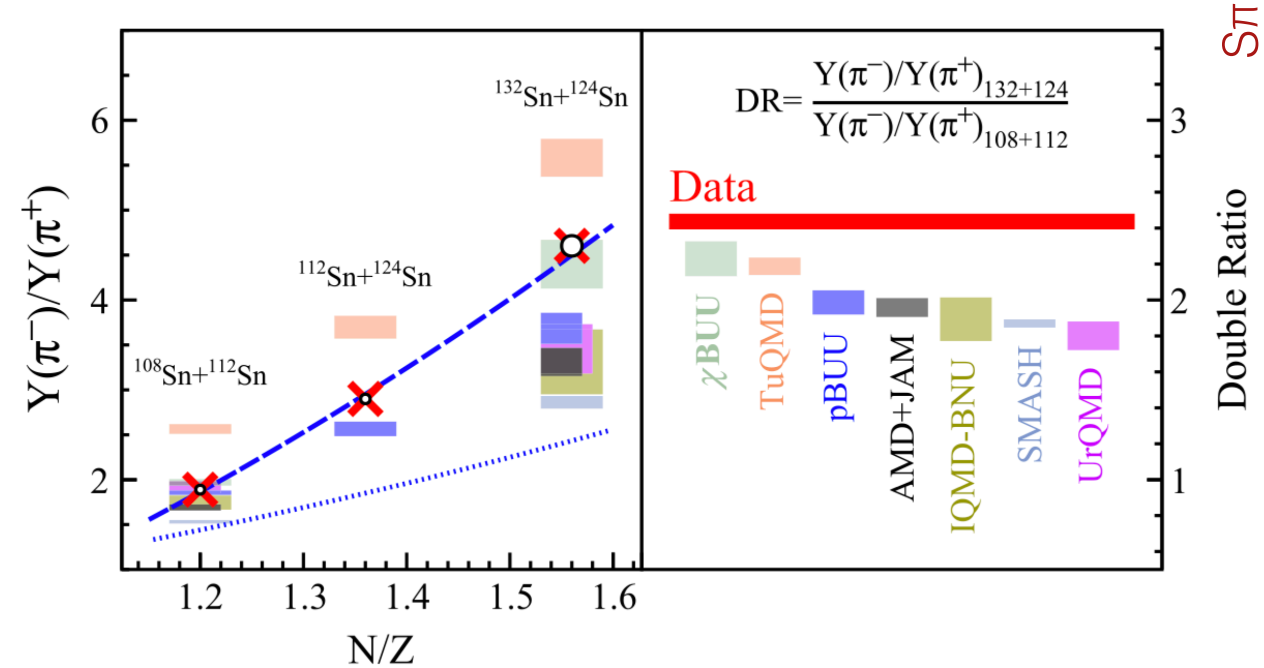
Slide from W. Trautmann, https://indico.mitp.uni-mainz.de/event/188/contributions/1057/attachments/851/910/trautmann_bormio2019.pdf

Symmetry Energy

- The symmetry energy is crucial when moving from heavy ions to neutron stars







SπRIT and TMEP collaboration, PLB, 813 (2021)

- Pion production in tin isotopes indicates large model dependence

Momentum Dependence

- Nuclear potential should include momentum dependence
- Implemented following Welke et al.

G. M. Welke et al. Phys.Rev.C 38 (1988)
Used in GiBUU: O. Buss et al. Phys.Rept. 512 (2012)

$$U(\mathbf{r},\mathbf{p}) = A\frac{\rho(\mathbf{r})}{\rho_0} + B\left(\frac{\rho(\mathbf{r})}{\rho_0}\right)^{\tau} + \frac{2C}{\rho_0} g \int \frac{d^3p'}{(2\pi)^3} \frac{f(\mathbf{r},\mathbf{p}')}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2}$$

Skyrme Potential

Momentum-dependent part

- Integral simplified by assuming cold nuclear matter

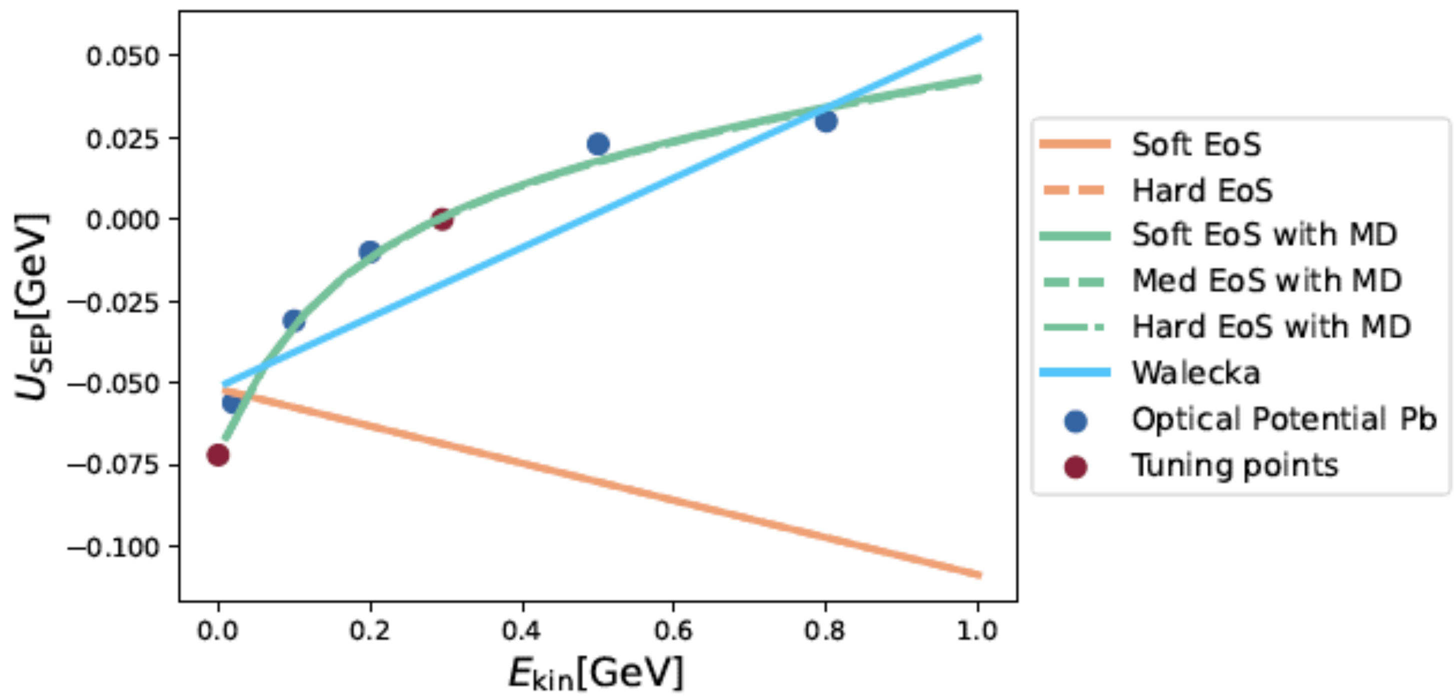
$$f(\vec{r},\vec{p}) = \Theta(p_F - p)$$

- Single particle energy evaluated in local rest frame for equation of motion $\dot{\vec{p}} = -\nabla E$

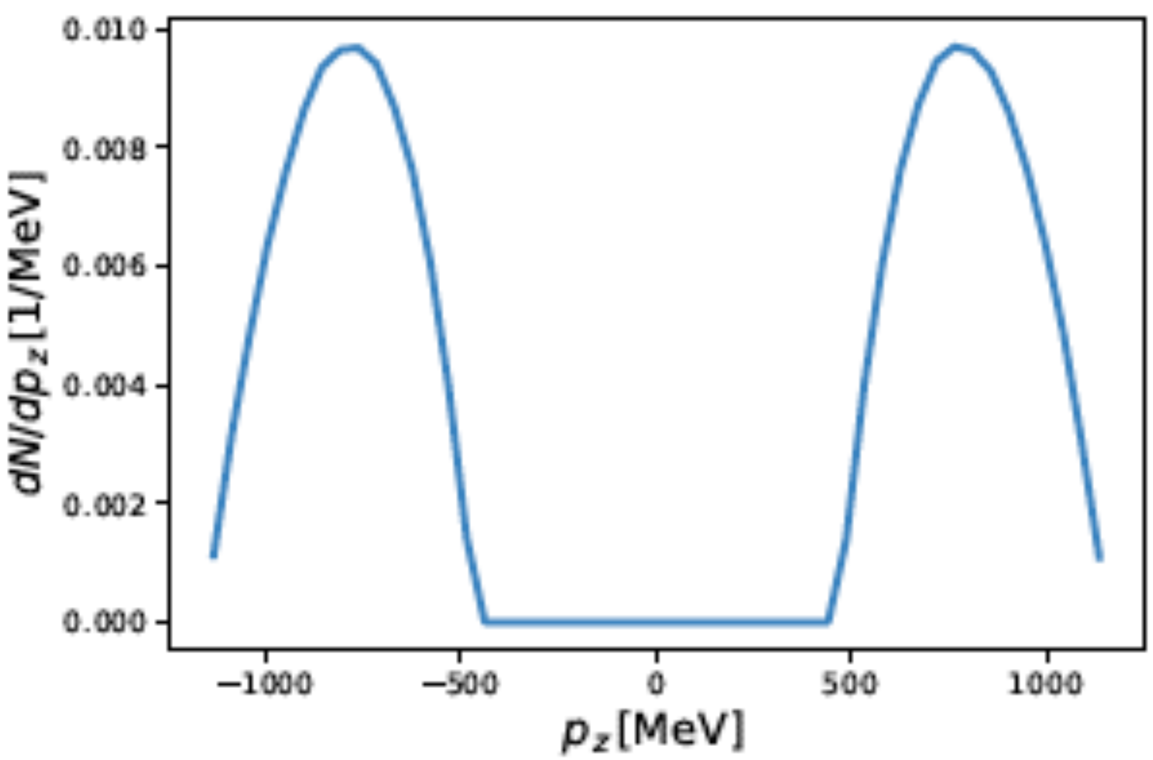
J. Mohs, S. Spies and HE, PRC 112 (2025)

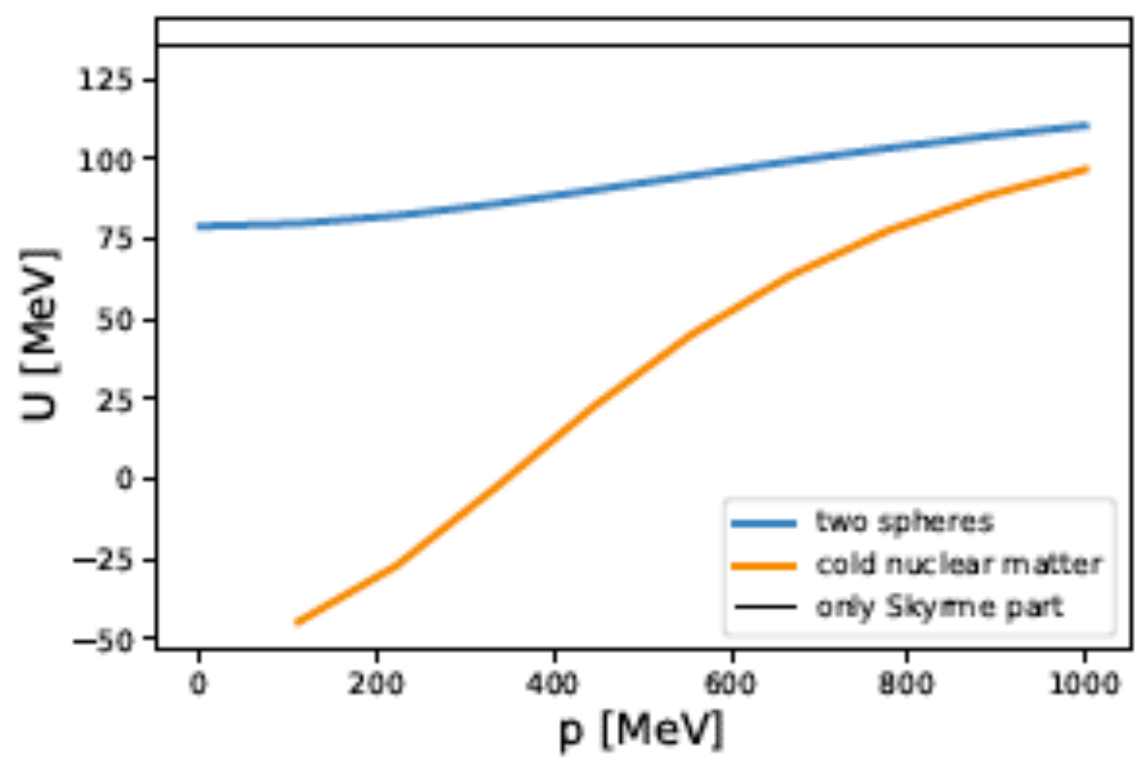
Fixing Momentum Dependence

J. Mohs, PhD thesis, 2025



- Momentum dependence is tuned to reproduce optical potential data
- Cold nuclear matter assumption influences results



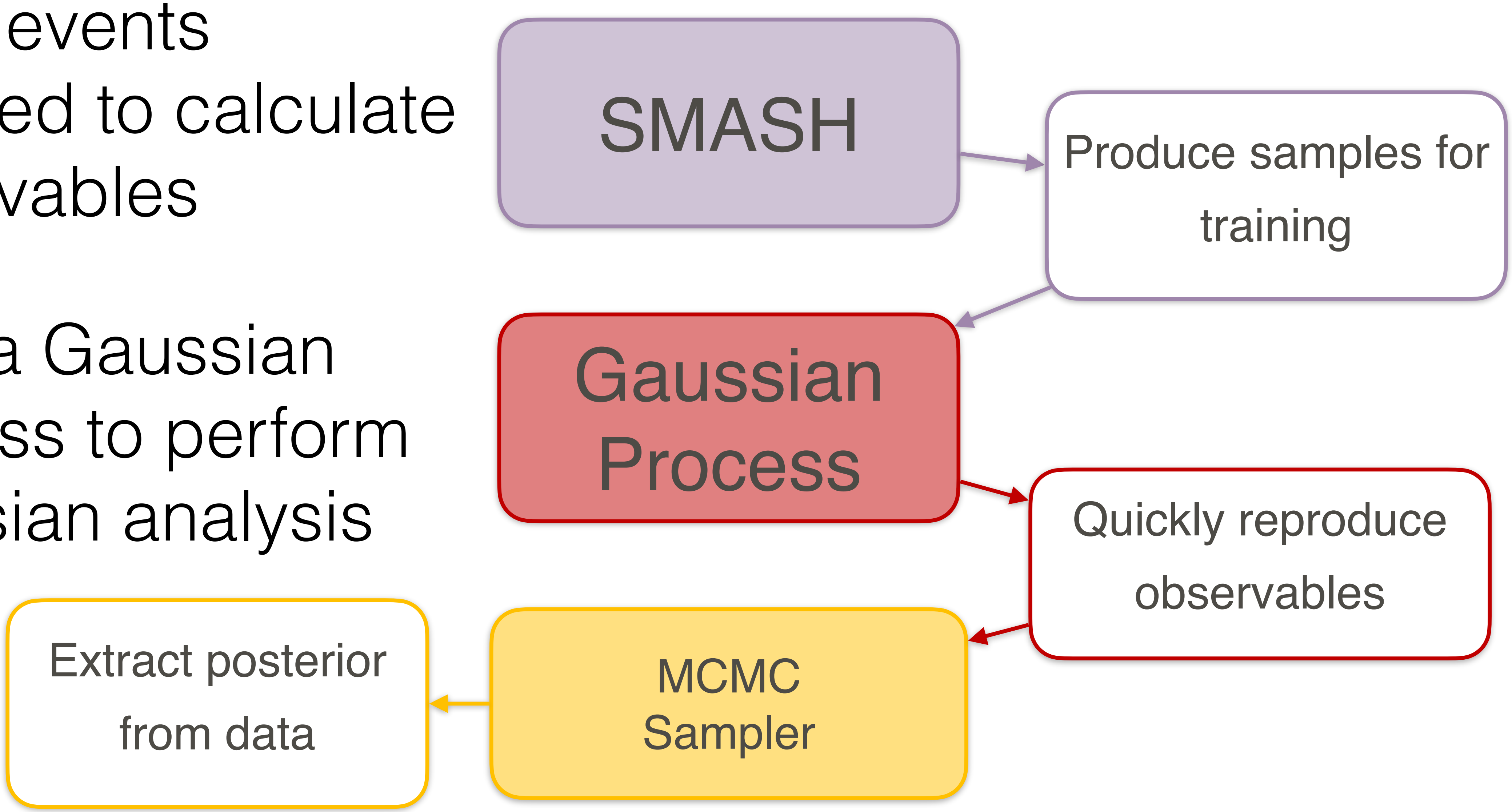


Bayesian Analysis Structure

- Comparison of SMASH calculations with different mean field settings to HADES flow data for protons and deuterons in 4 centrality classes

- Many events required to calculate observables

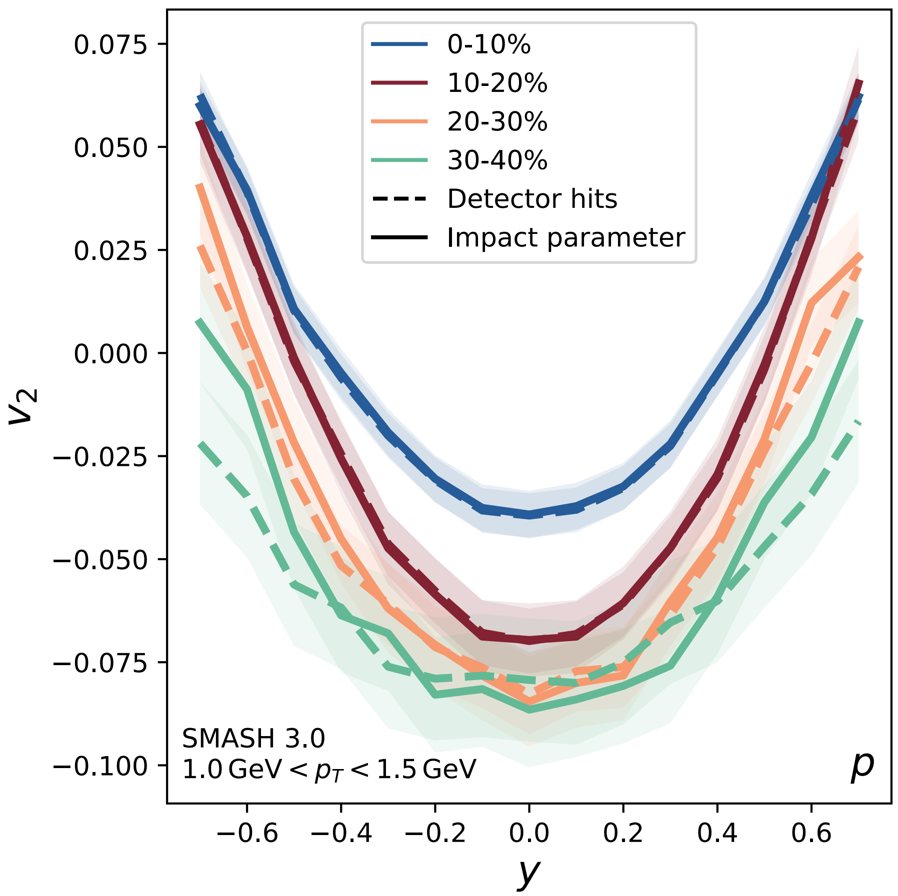
- Train a Gaussian process to perform Bayesian analysis

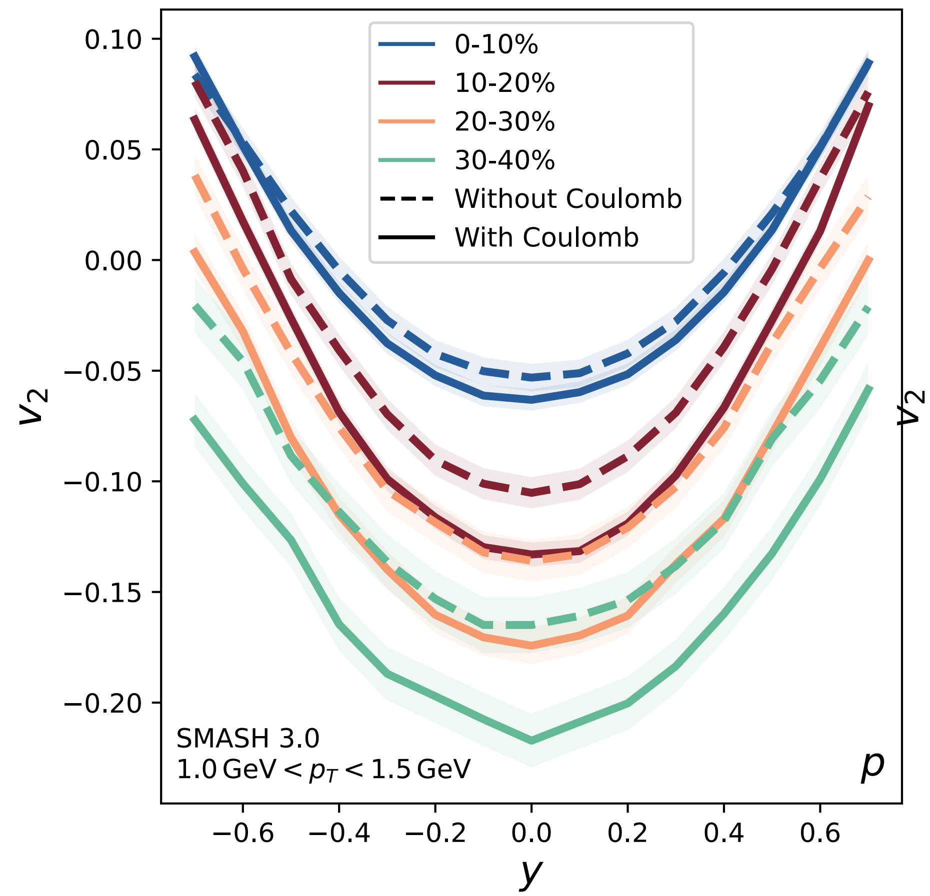


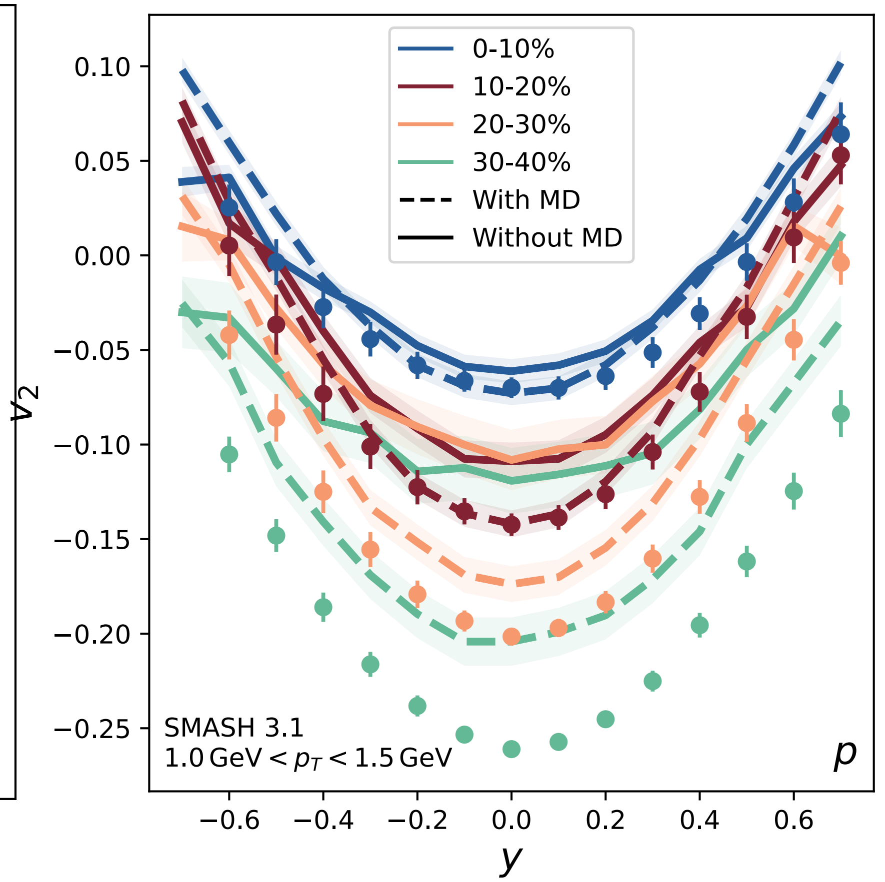
J. Mohs, S. Spies and HE, PRC 112 (2025)

Validation of Approach

- Bayesian analysis of HADES flow data versus SMASH with momentum dependent mean field J. Mohs, S. Spies, HE, PRC 112 (2025)





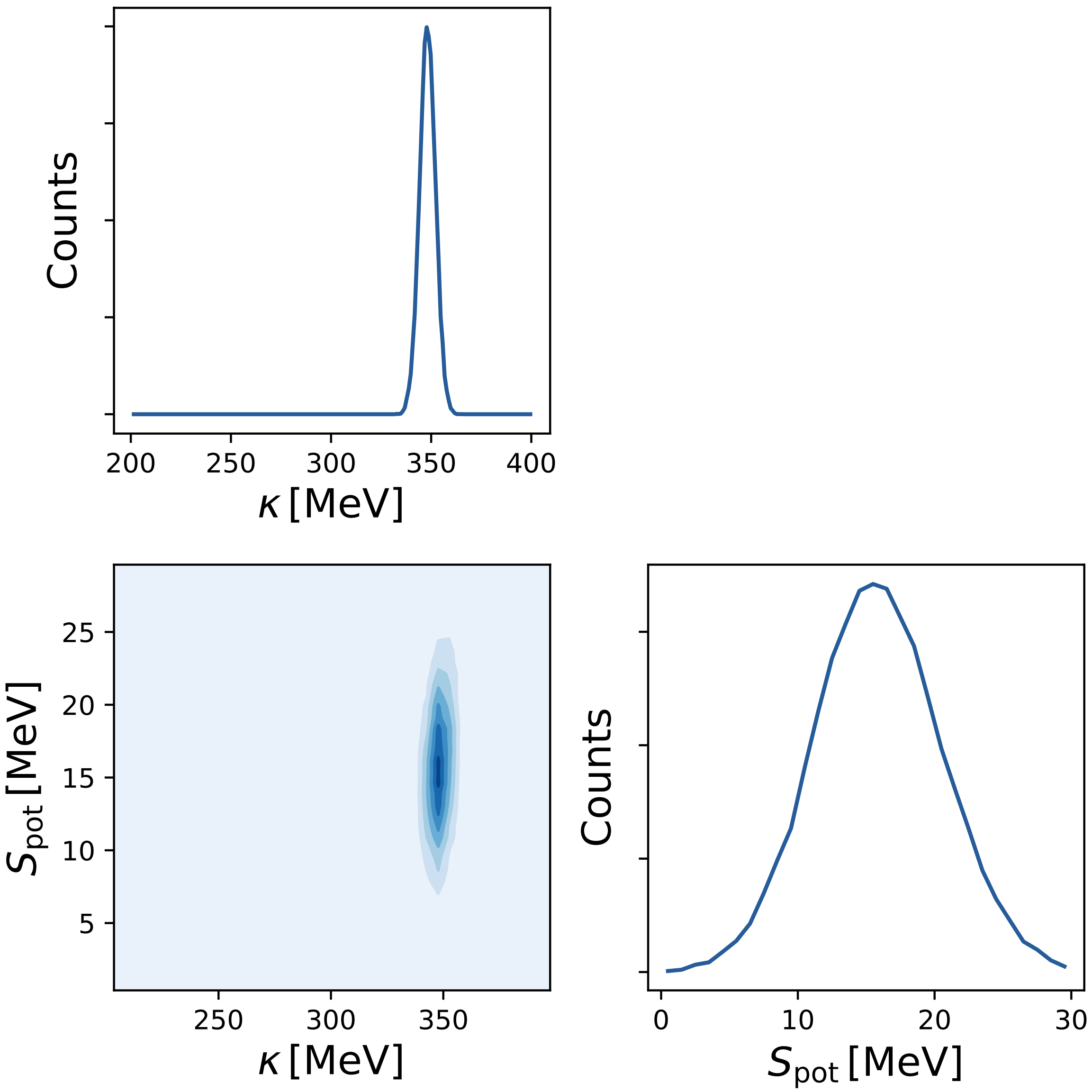


- Centrality selection does not matter at mid rapidity
- Coulomb potential is important and is included
- Momentum dependence crucial for elliptic flow strength

Posterior Distribution

- Result of analysis based on proton and deuteron flow in 4 centrality classes

- Preliminary results:
  - $\kappa = 348.2^{+4.0}_{-3.9}\,\mathrm{MeV}$
  - $S_{\mathrm{Pot}} = 15.8^{+4.7}_{-4.6}\,\mathrm{MeV}$



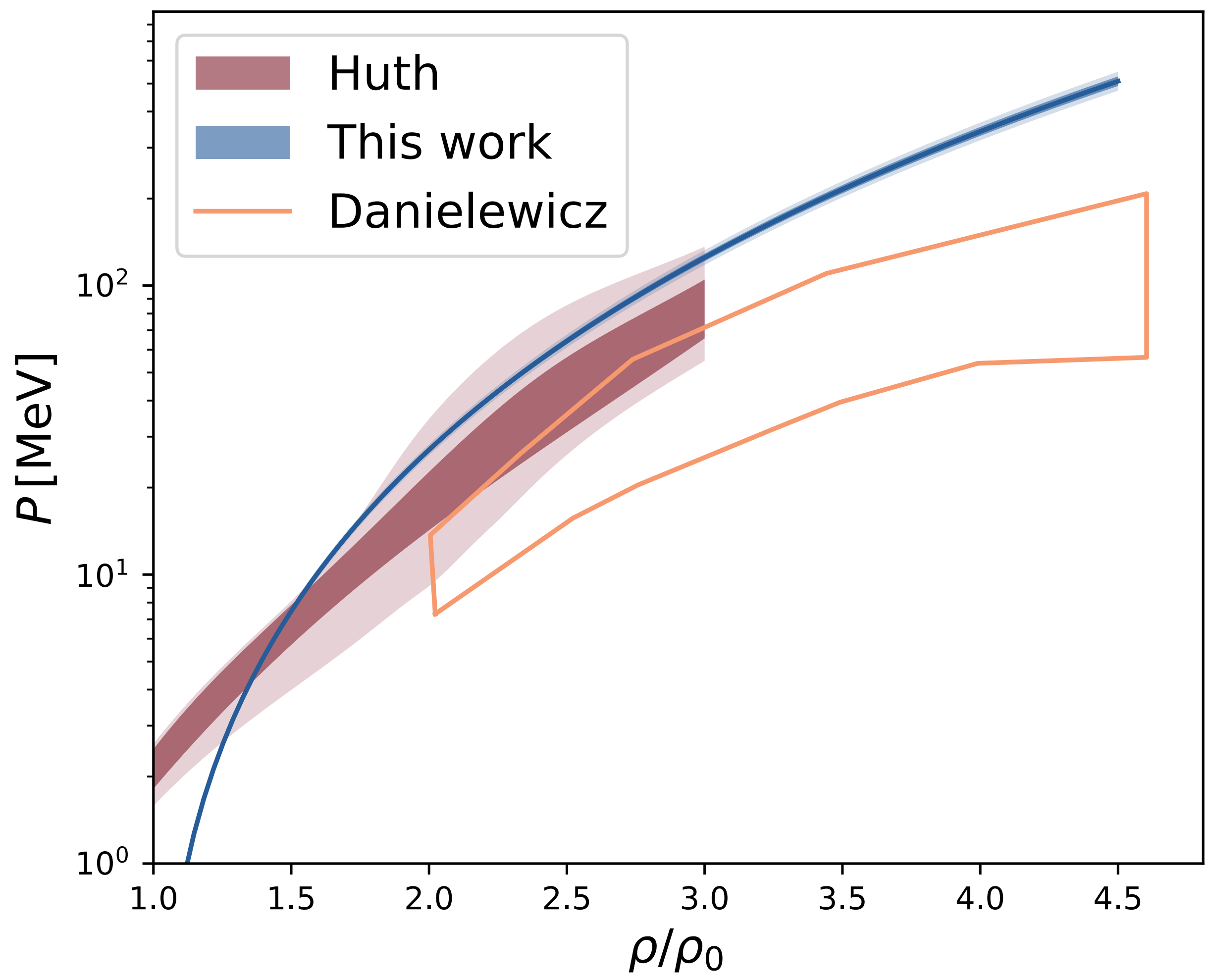
- Symmetry potential is consistent with other prior studies

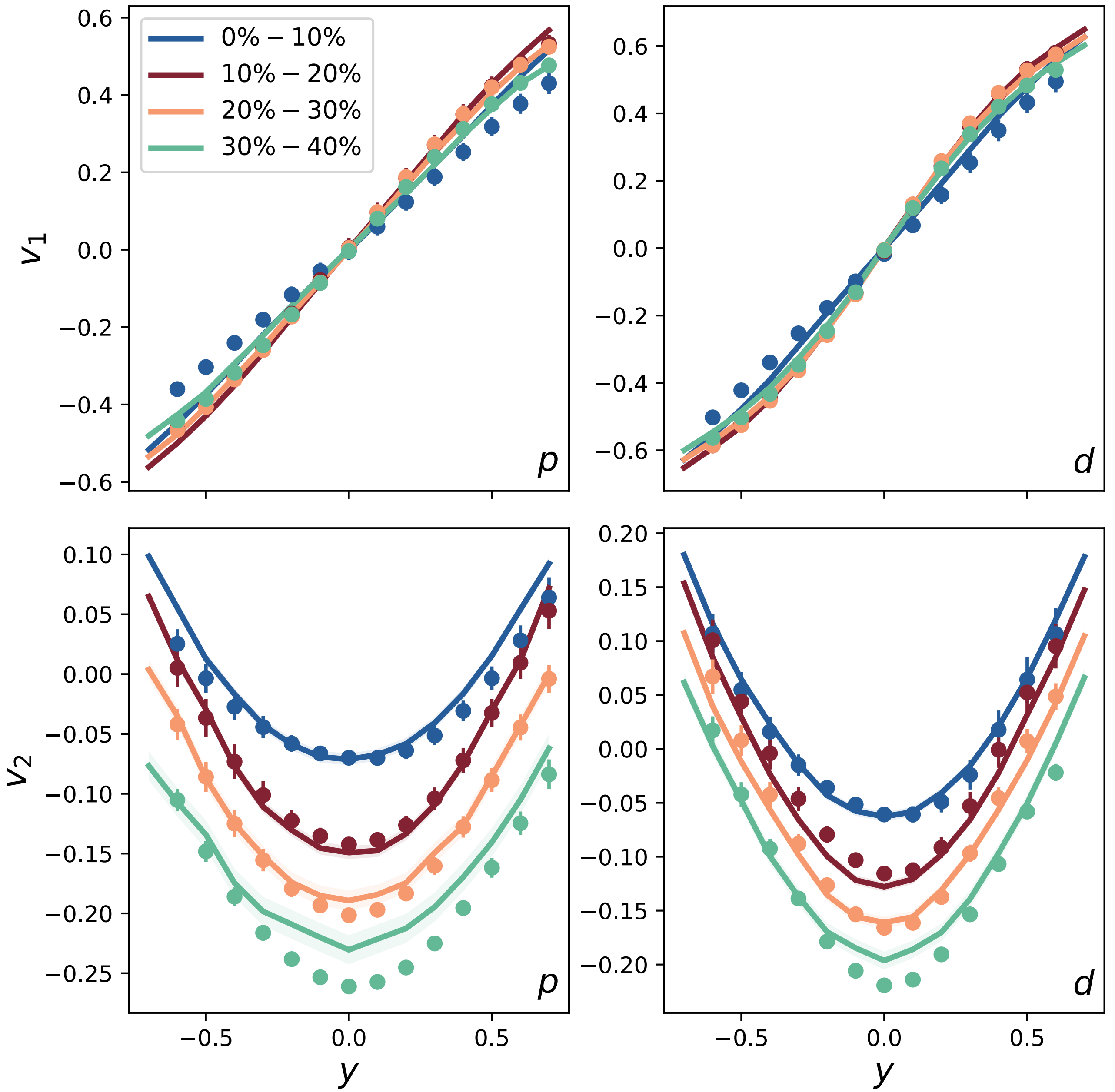
- Incompressibility is relatively large

J. Mohs, S. Spies, HE, PRC 112 (2025)

Maximum Posterior

- Comparing the maximum of the posterior distribution to data



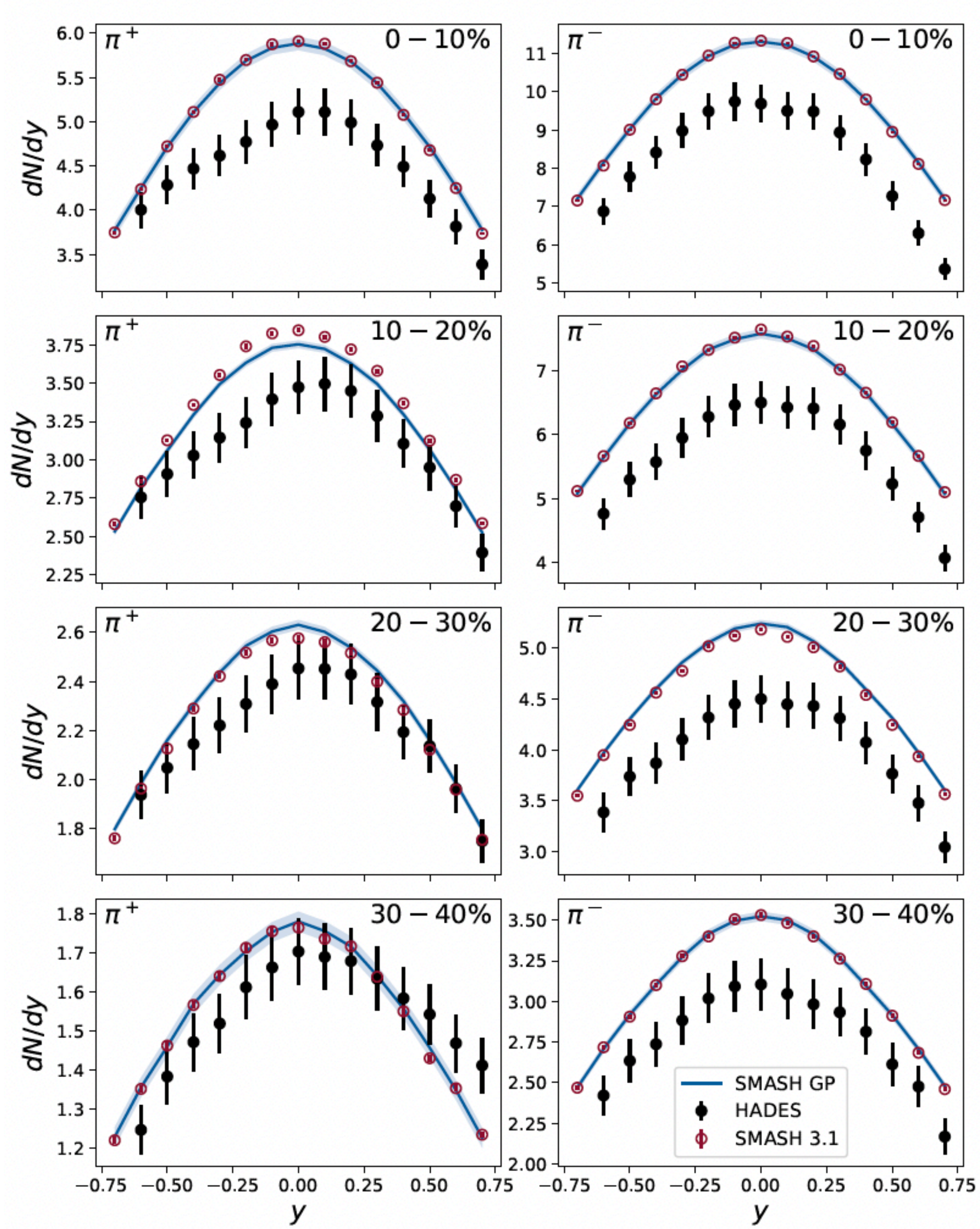


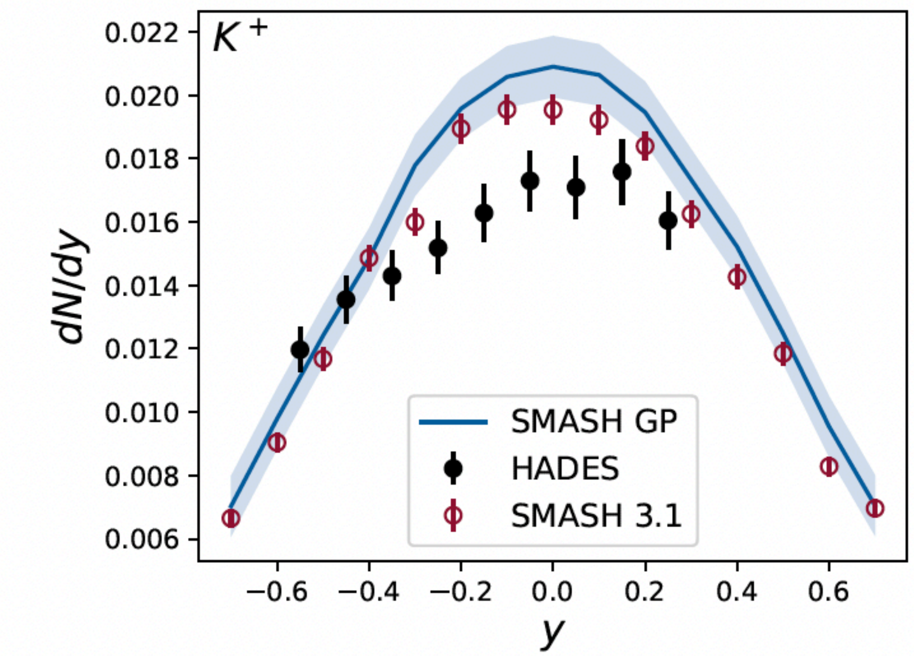
J. Mohs, S. Spies, HE, PRC 112 (2025)

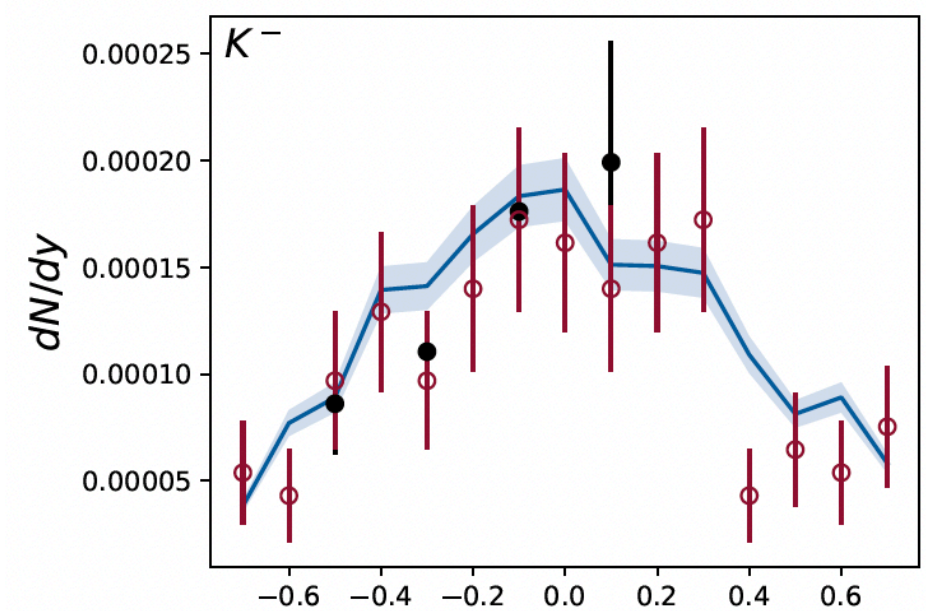
- Not all data perfectly reproduced but good agreement overall
  The pion and kaon yields also agree with HADES measurements

Particle Production

- Validation: Result of Gaussian process coincides with direct SMASH-3.1 calculation





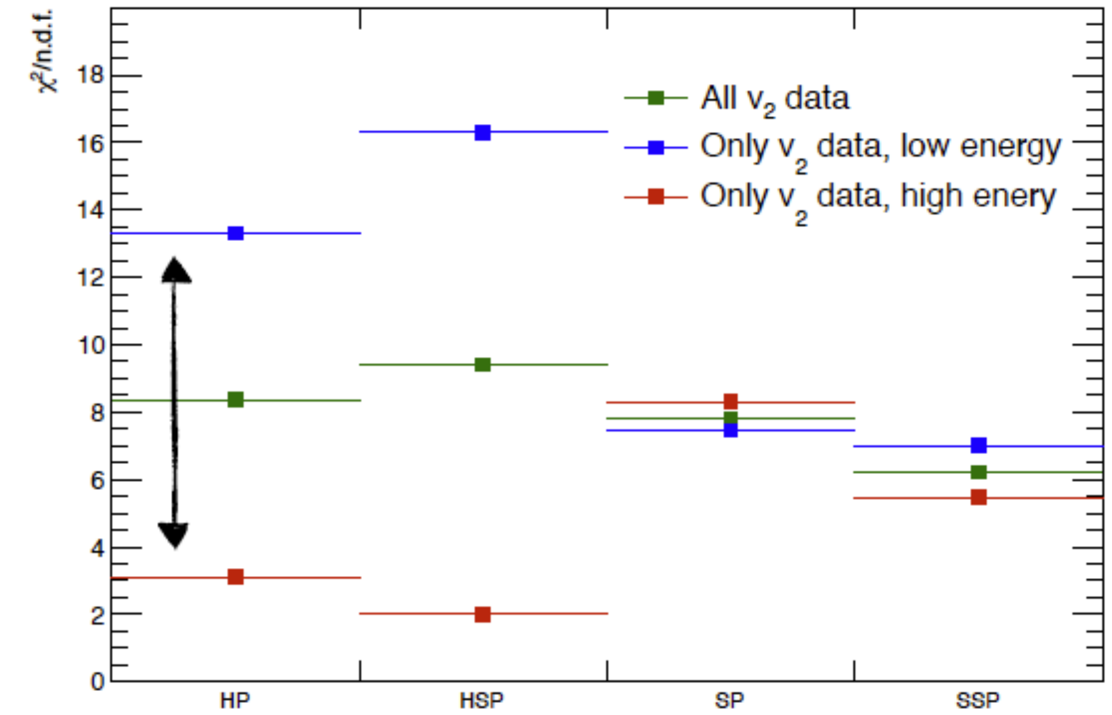


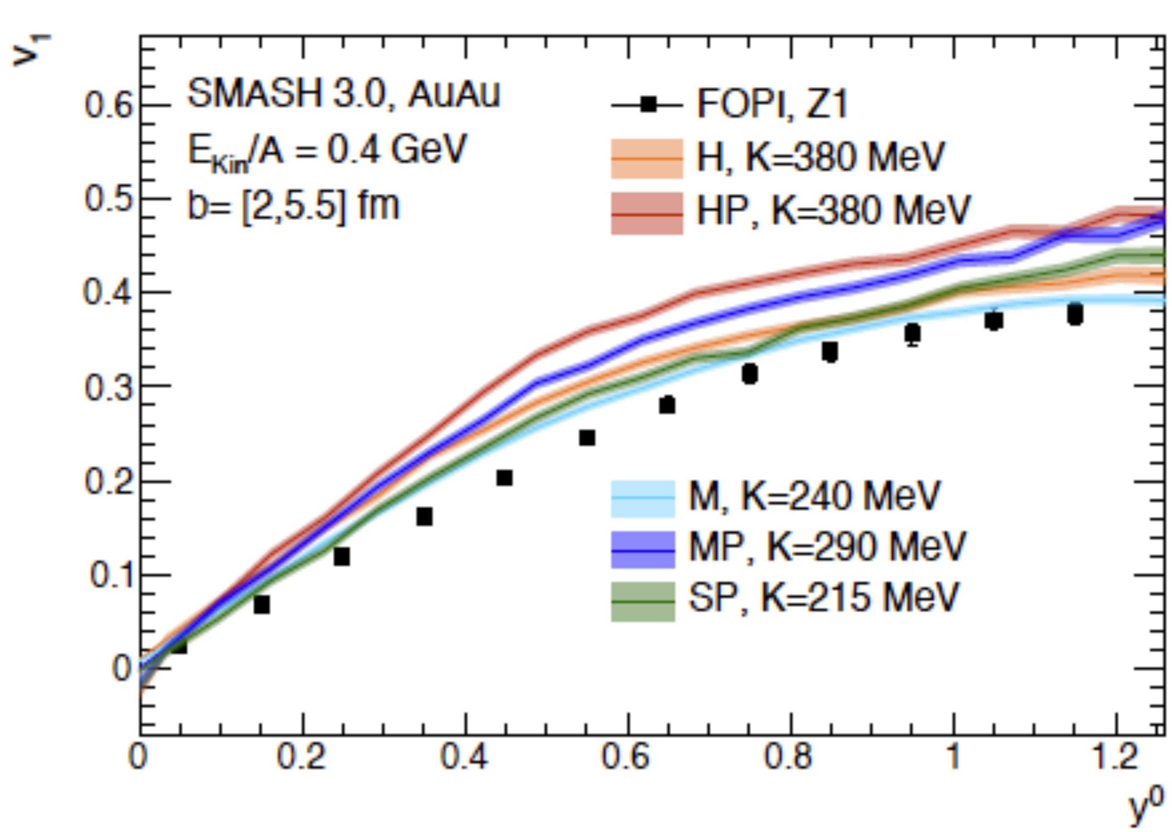
J. Mohs, S. Spies, HE, PRC 112 (2025)

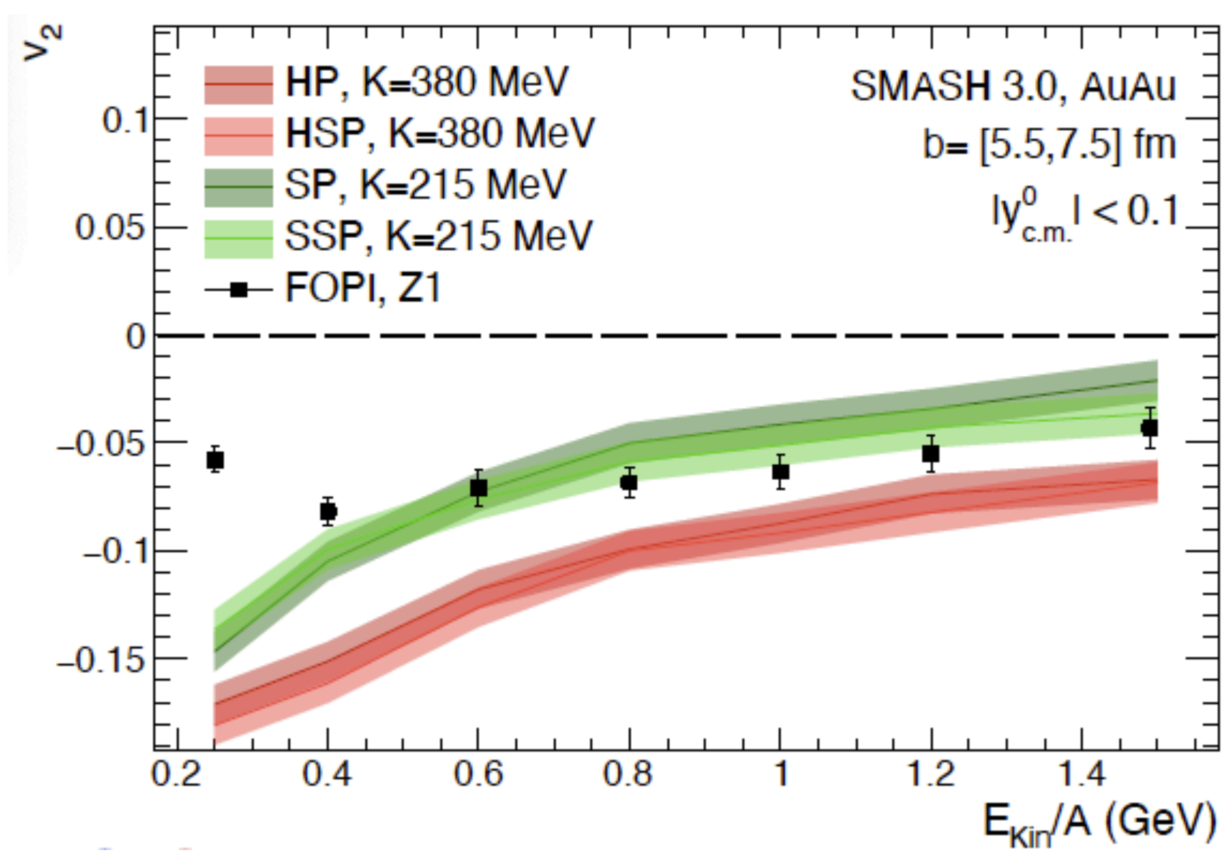
Comparison to FOPI data

- v_1 requires soft momentum dependent EoS while v_2 asks for hard momentum dependent EoS, transition as a function of energy

L. Tarasovicova, J. Mohs, A. Andronic, HE and K.-H. Kampert EPJA 60 (2024), FAIR-NRW network



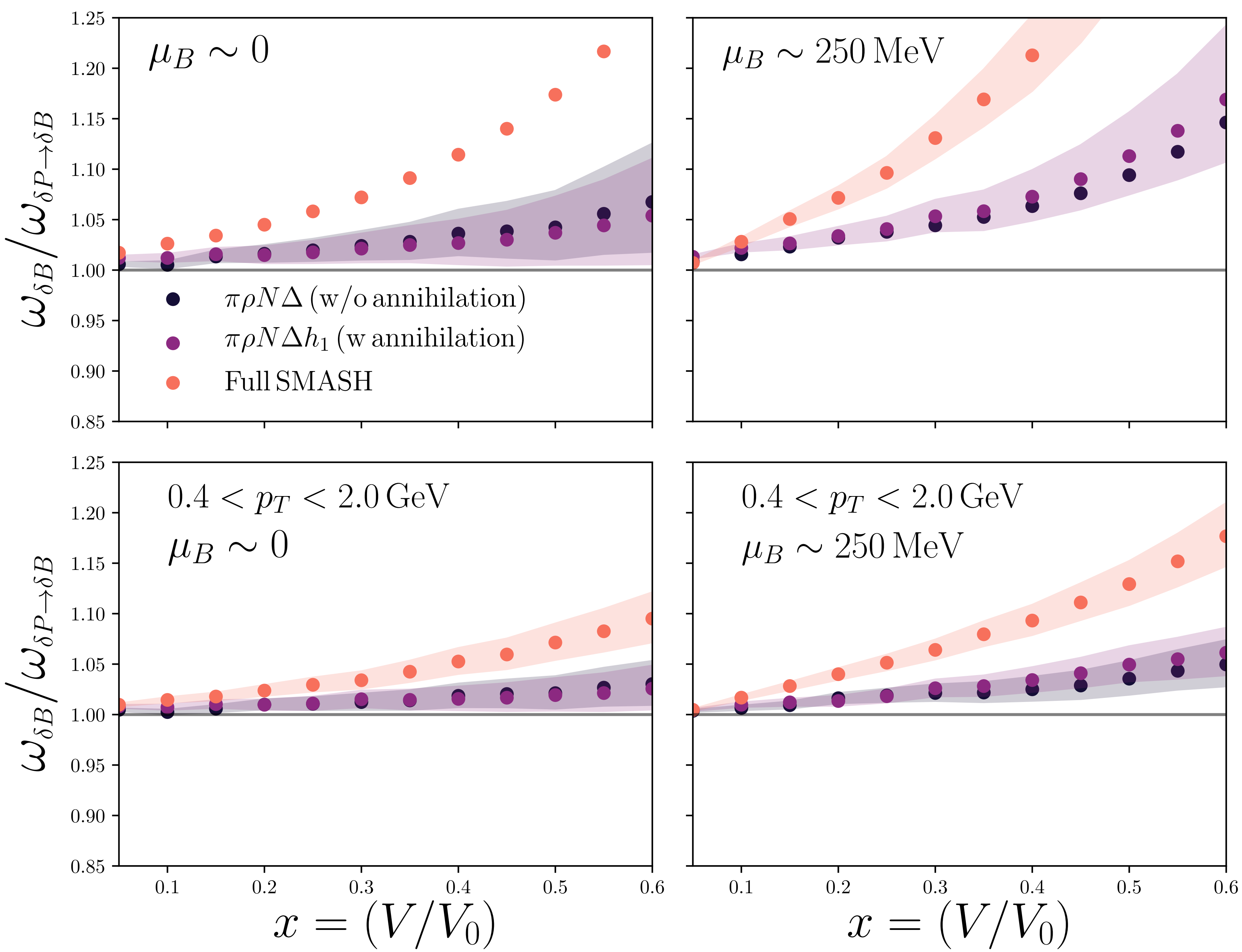




Fluctuation Observables

Mapping Protons to Baryons

- Usually net protons serve as a proxy for net baryons
- Test in a box calculation how well the proposed unfolding works:

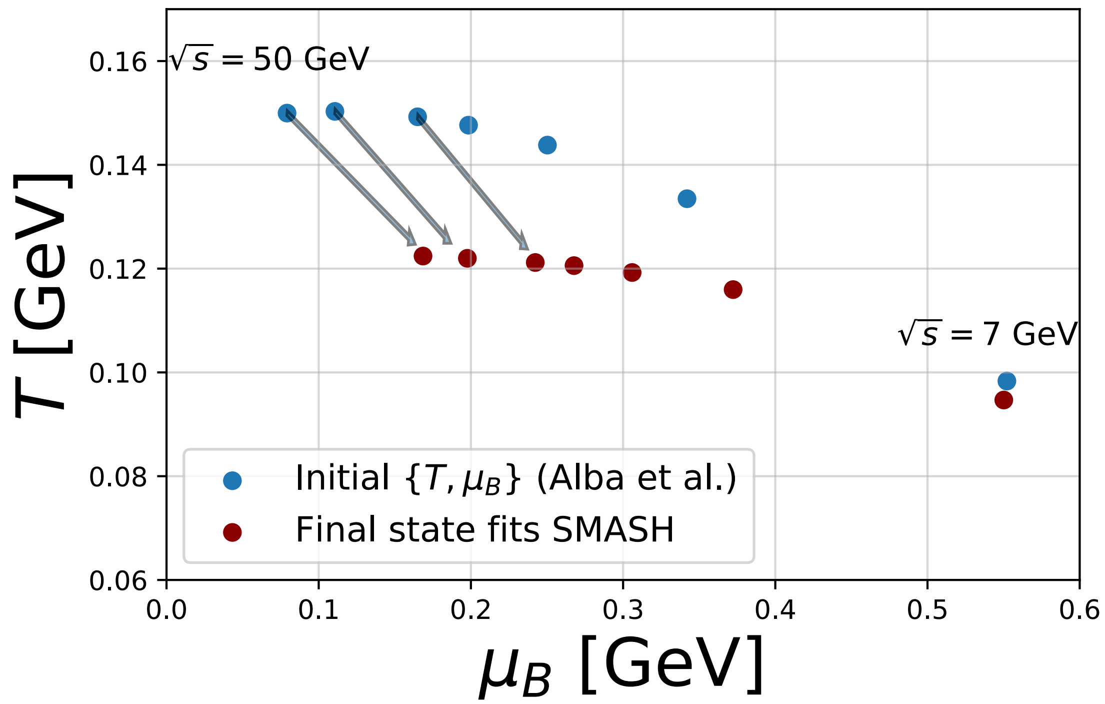


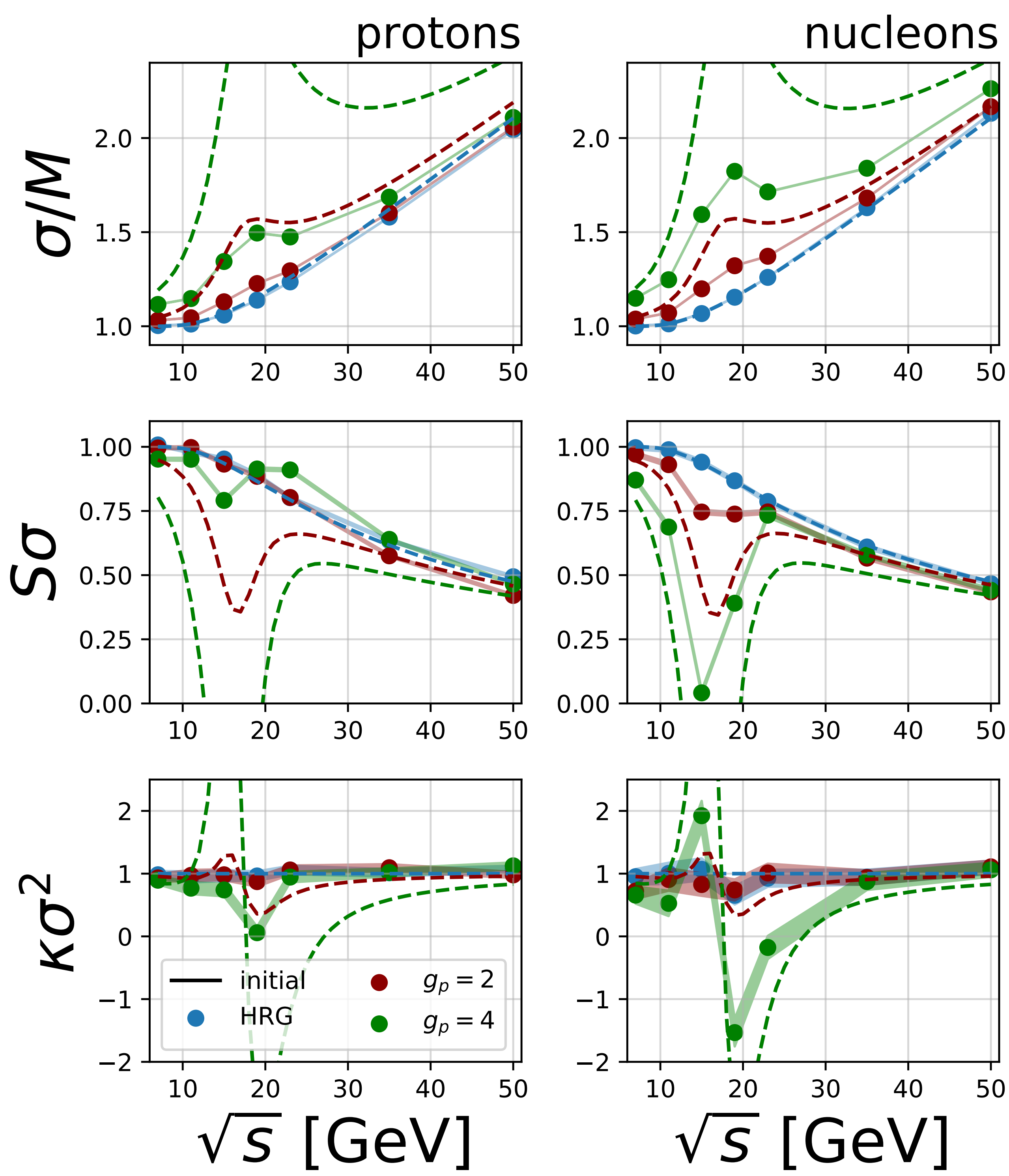
J. Hammelmann et al., *Phys.Rev.C* 107 (2023)

Survival of Critical Fluctuations

J. Hammelmann et al, PRC 110 (2024)

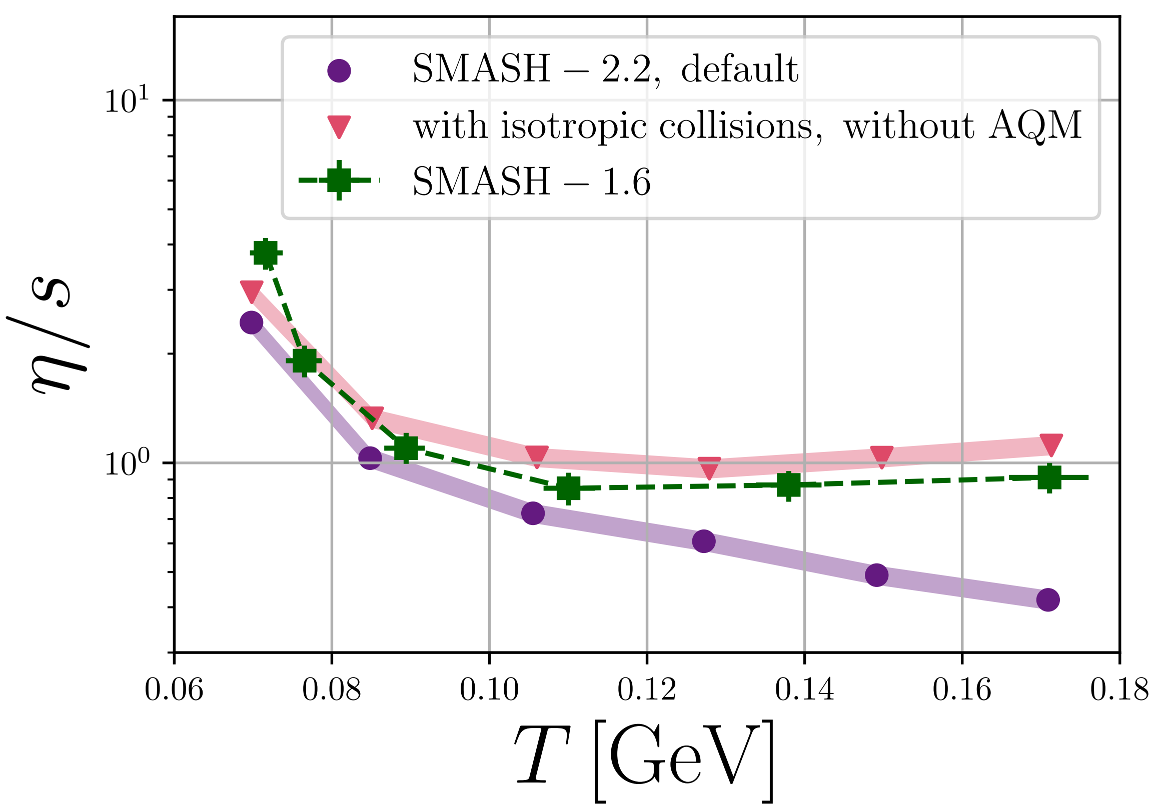
- Plugging critical fluctuations in an expanding sphere with full SMASH hadronic dynamics
- Signal survives, if coupling strong enough

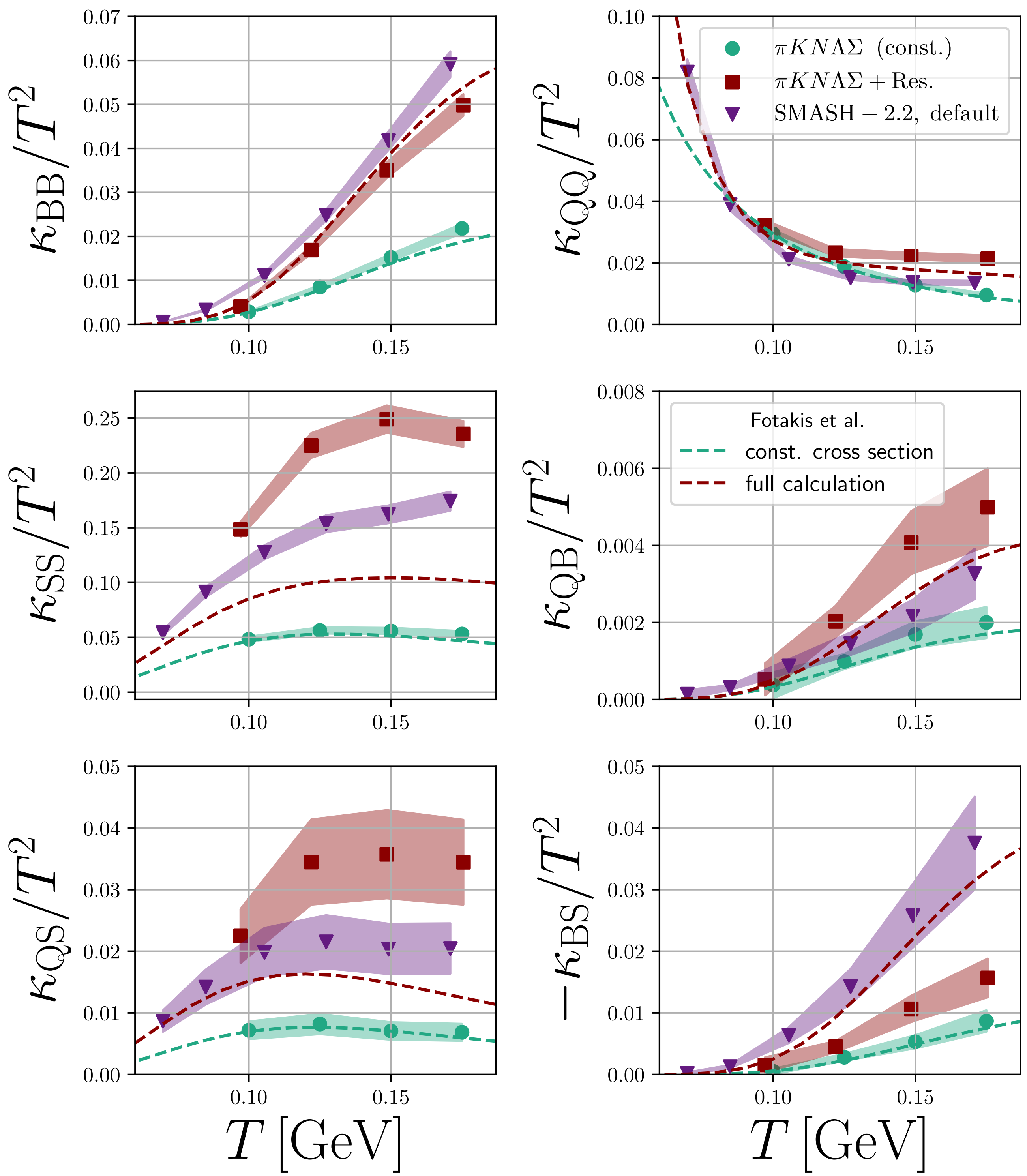




Transport Coefficients

- Shear viscosity as a function of temperature
- (Cross-) conductivities for all charges B, S, Q as a function of temperature





J. Hammelmann, J. Staudenmaier and HE, arXiv:2307.15606

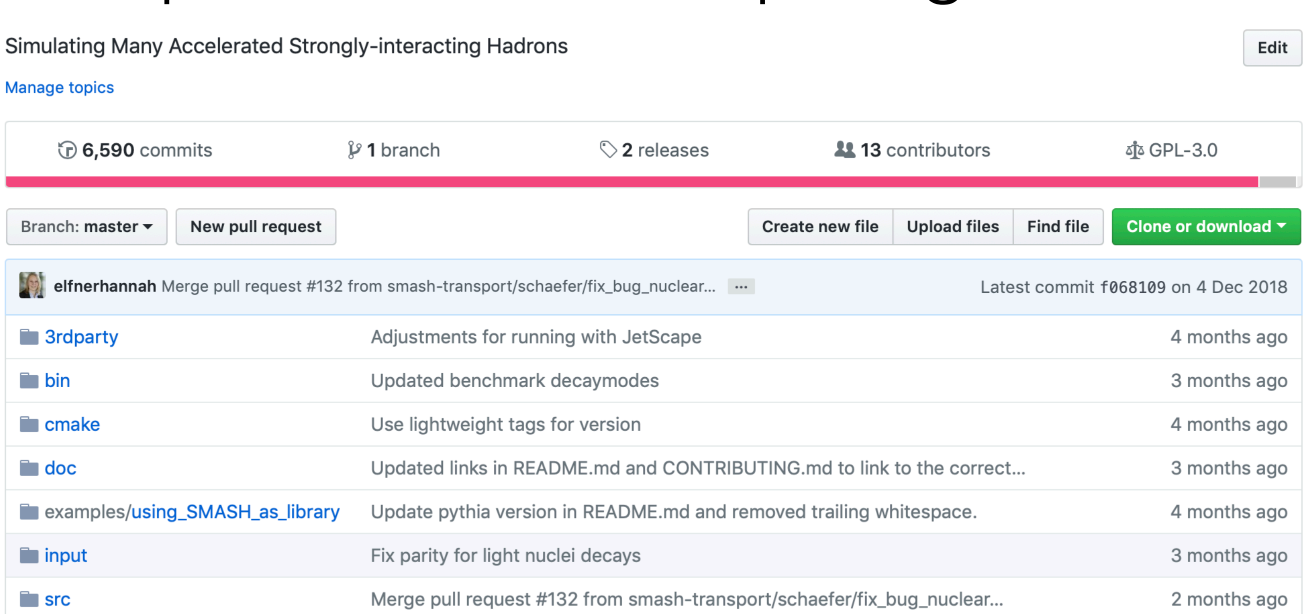
Summary

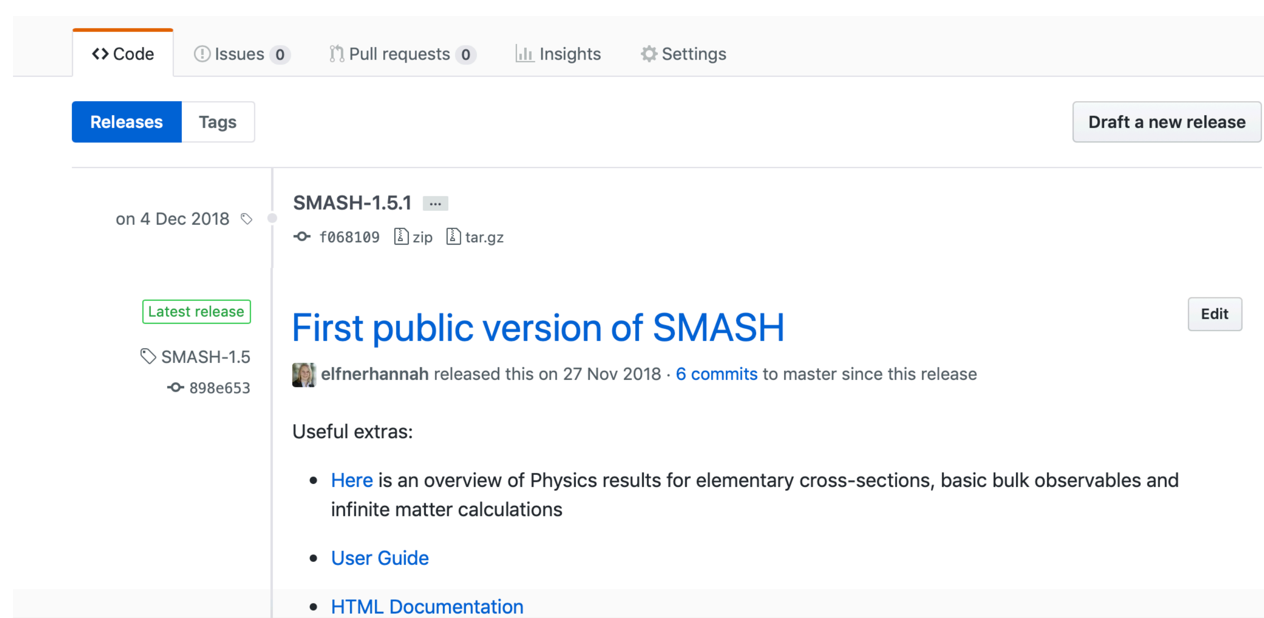
- SMASH and SMASH-vHLLE hybrid approach are available to the public and there is support
  - Bulk observables are in reasonable agreement with experimental data
  - SMASH with momentum dependent nuclear potentials has been compared to FOPI and HADES flow data
    - -> constraints for equation of state
  - Fluctuation observables survive rescattering, if coupling is strong enough
- To resolve systematic uncertainties in EoS extraction from heavy-ion data
  - Comparison of BUU and QMD calculation with the same degrees of freedom and cross-sections
  - Multi-model Bayesian analysis

Open Source Strategy

- Visit the webpage to find publications and link to SMASH-3.2 results https://smash-transport.github.io
- Download the code at https://github.com/smash-transport/smash
- Checkout the Analysis Suite at https://github.com/smash-transport/smash-analysis
- Find user guide and documentation at https://github.com/smash-transport/smash/releases
- Animations and Visualization Tutorial under https://smash-transport.github.io/movies.html

SMASH-3.1 has HepMC and RIVET





Backup

Issues

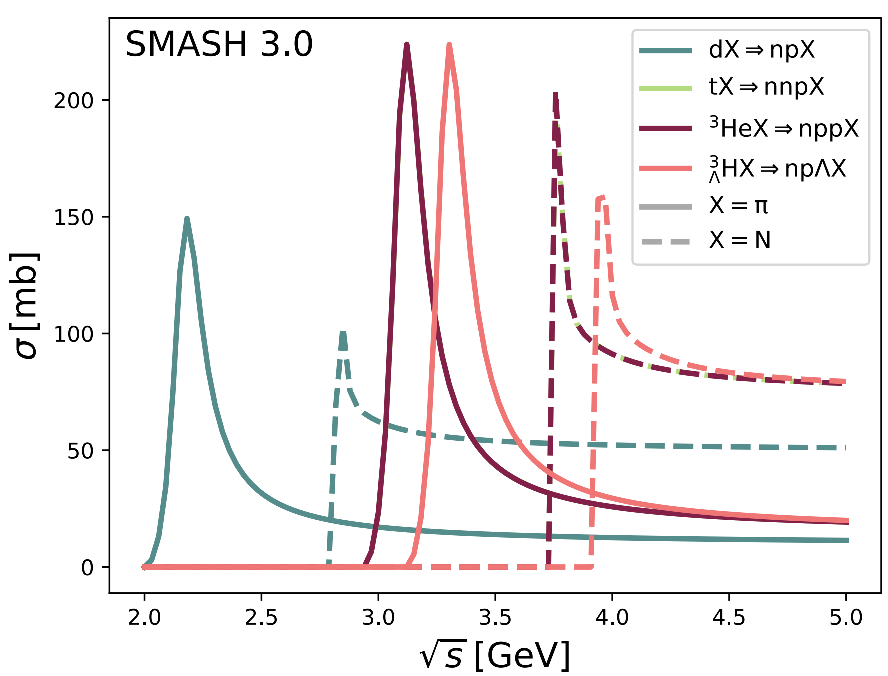
- Challenging to describe multiple data sets within one approach consistently (FOPI, HADES, v1/v2, protons/deuterons/pions)
- Degrees of freedom matter and resonance dynamics influences mean field parameters
- Medium-modified cross-sections
- Influence of Coulomb potential
- Clustering and light nuclei production
- Spectators and mixing of forward and central dynamics
- Bayesian analysis to see how well data can constrain EoS/mean field
  - Add future data to study sensitivities
  - Assess systematic uncertainties in modeling

Production of Light Nuclei

Production of Light Nuclei

- Production of deuteron, triton, helium3, hypertriton via multi-particle interactions

- Hypersurface from hydrodynamics calculations:
  - Sample particles directly (similar to statistical hadronization)
  - Run through SMASH as hadronic afterburner and apply coalescence
  - Run SMASH with dynamic reactions for light nuclei



$d\pi \leftrightarrow NN$

$dX \leftrightarrow npX$

$^3HeX \leftrightarrow nppX$

$tX \leftrightarrow nnpX$

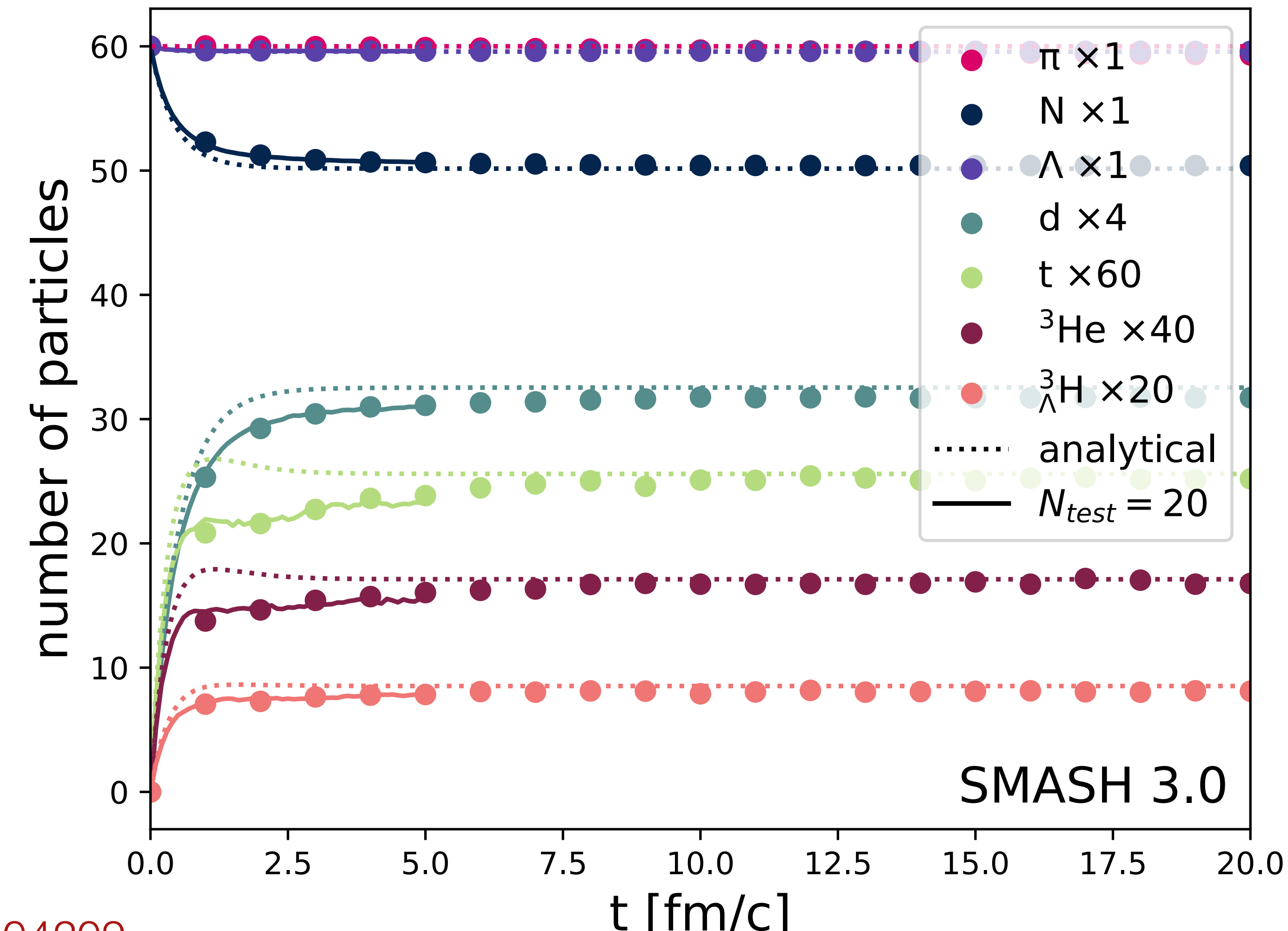
$^3_\Lambda HX \leftrightarrow np\Lambda X$

X=nucleon or pion

M. Ege, J. Mohs, J. Staudenmaier and HE, arXiv: 2409.04209

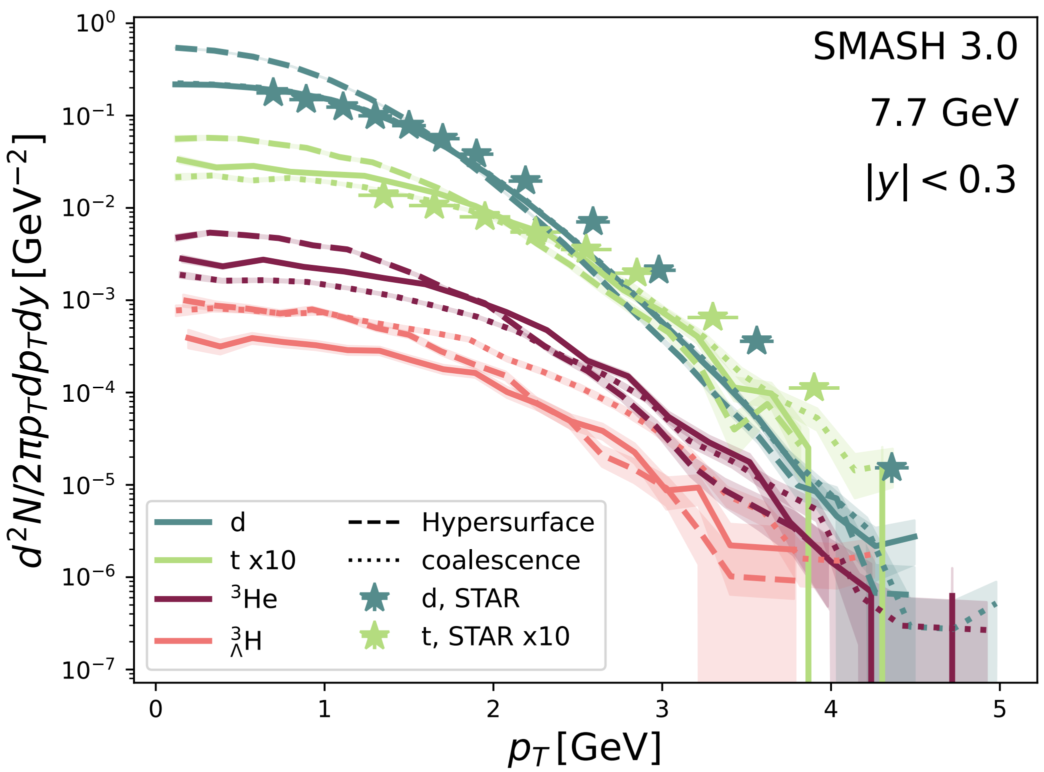
Rate Equations

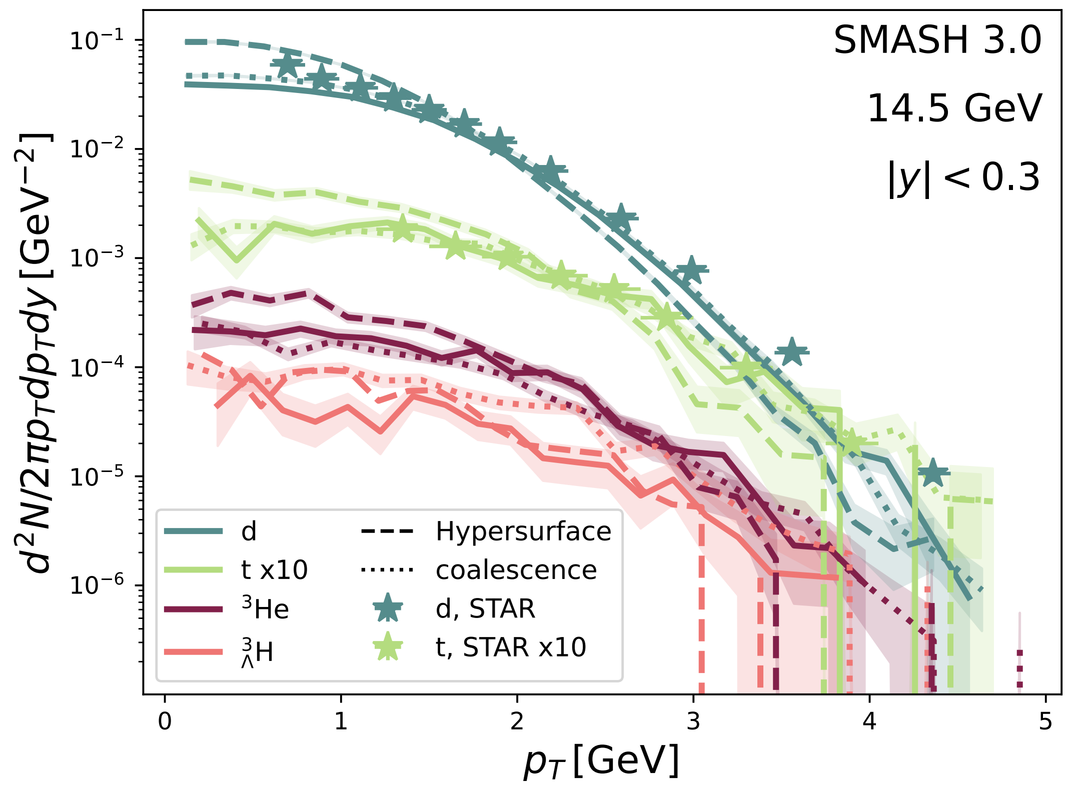
- The stochastic collision criterion allows for multi-particle reactions
- Validation against rate equation result in a box
- The final yields are in very good agreement, while the equilibration process is even faster in the numeric solution
- 10 testparticles are sufficient

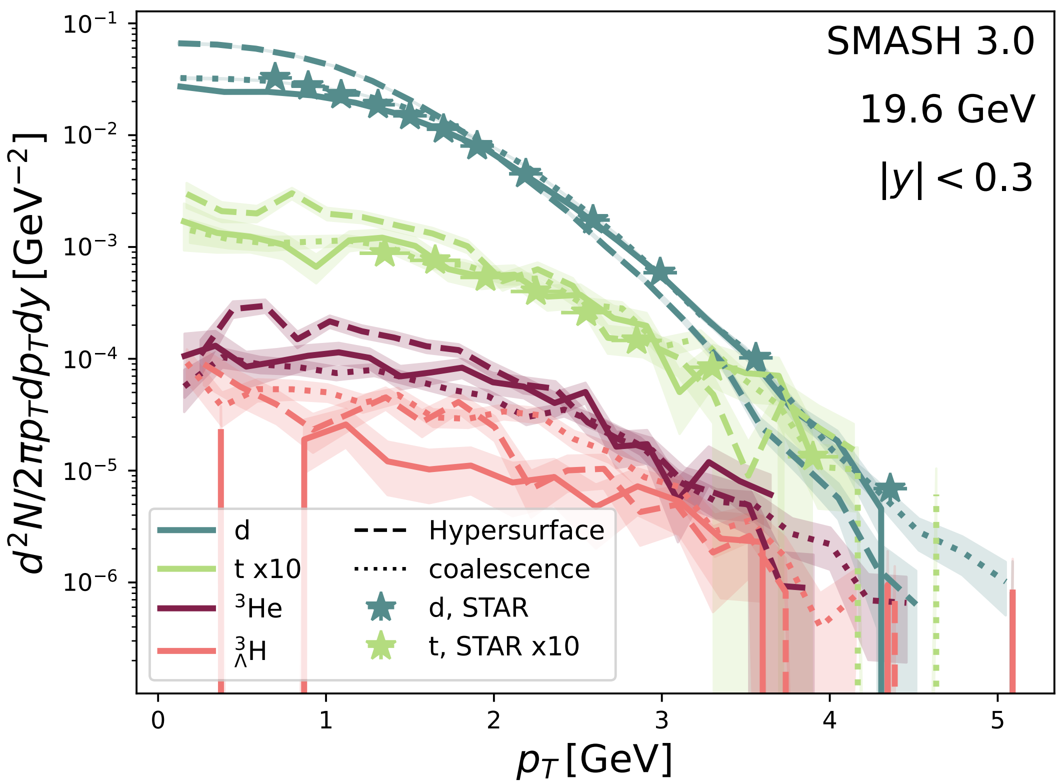


M. Ege, J. Mohs, J. Staudenmaier and HE, arXiv: 2409.04209

Transverse Momentum Spectra



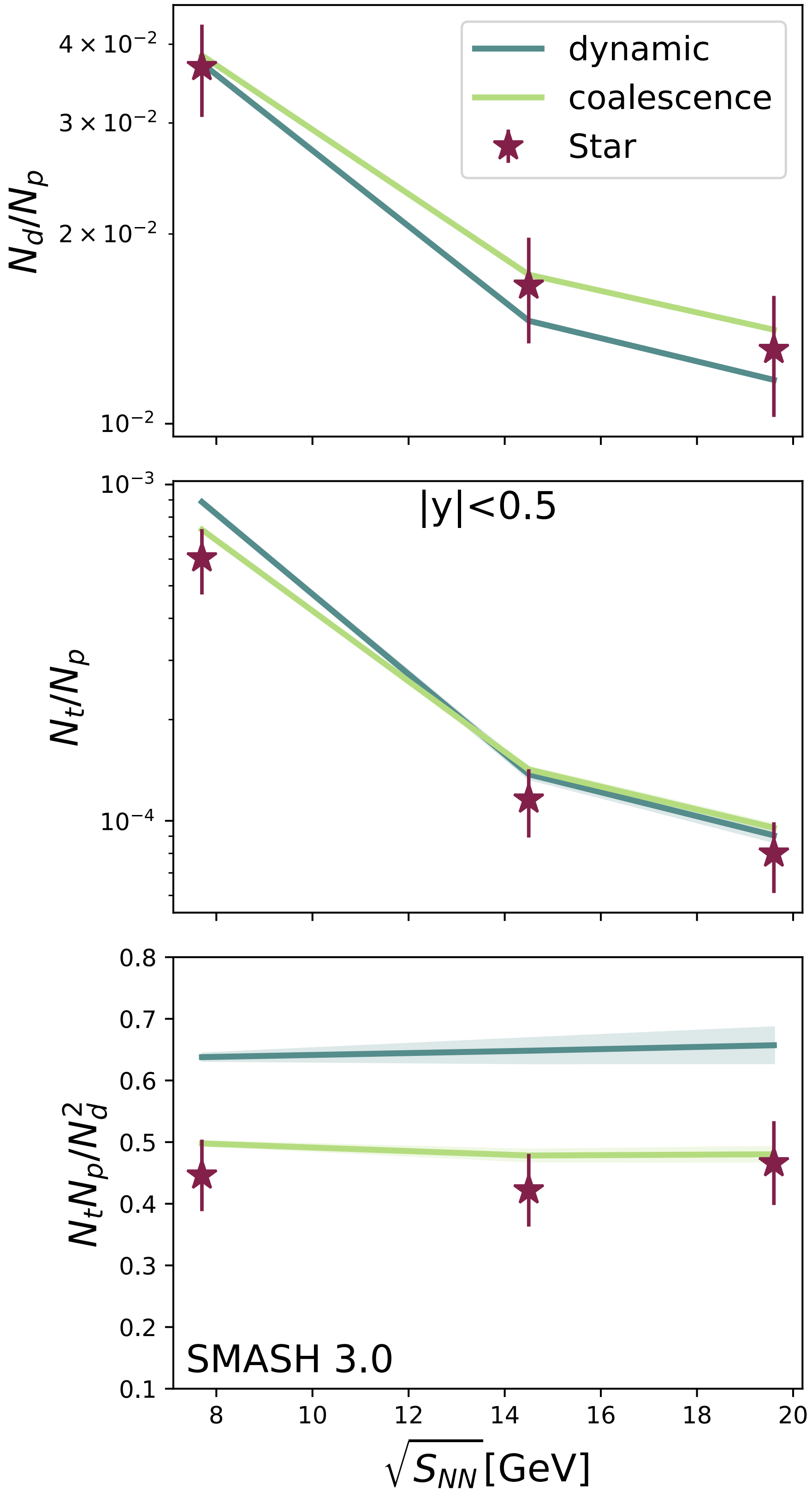


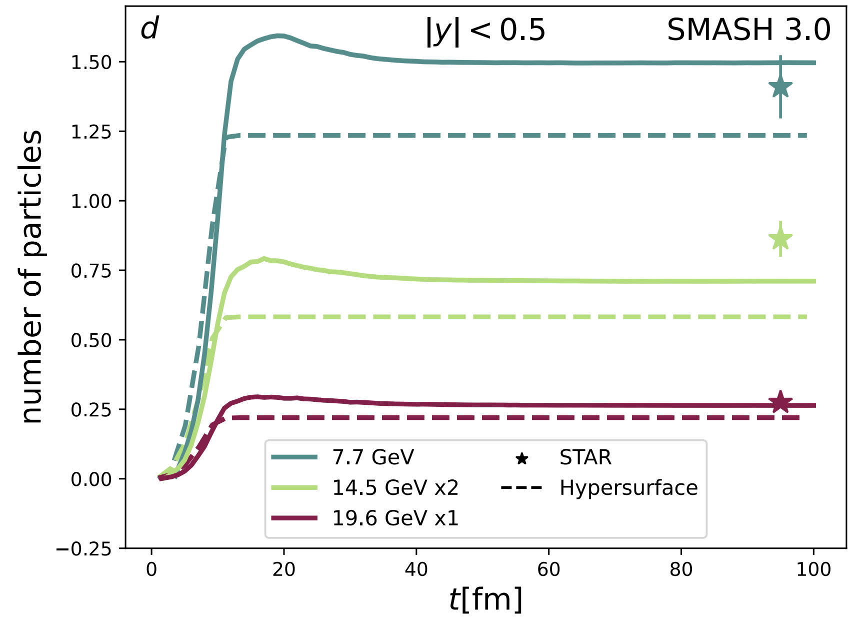


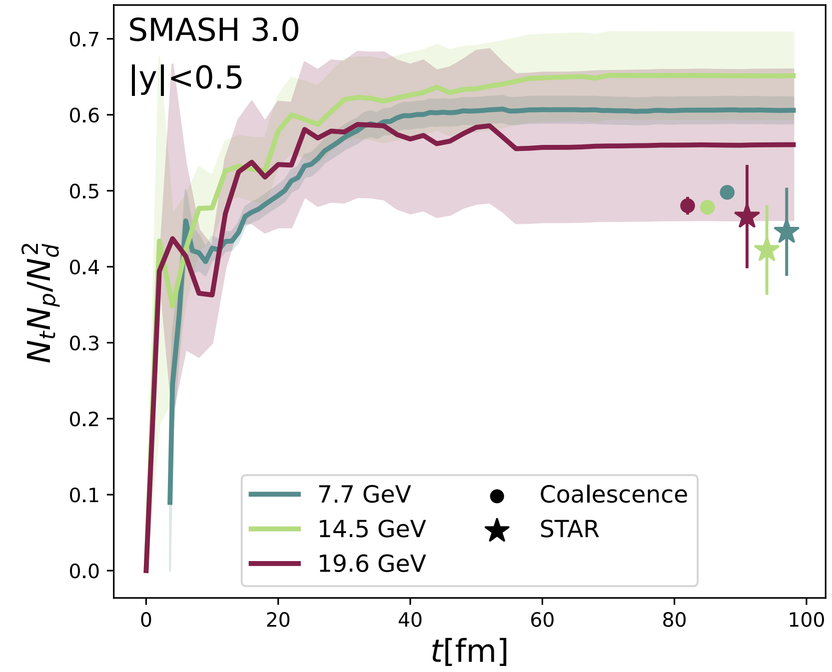
- Hadronic rescattering is necessary to describe shape of the spectra at low momentum
- Small differences between coalescence and dynamic processes

M. Ege, J. Mohs, J. Staudenmaier and HE, arXiv: 2409.04209

Yields and Evolution



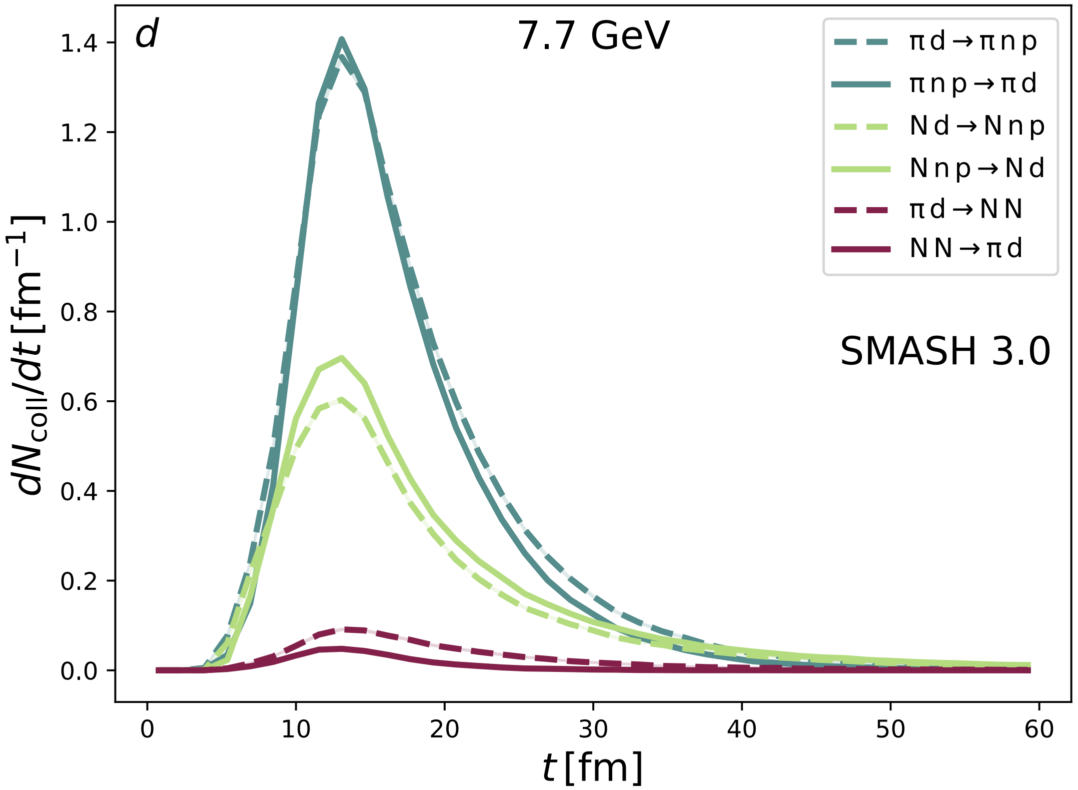


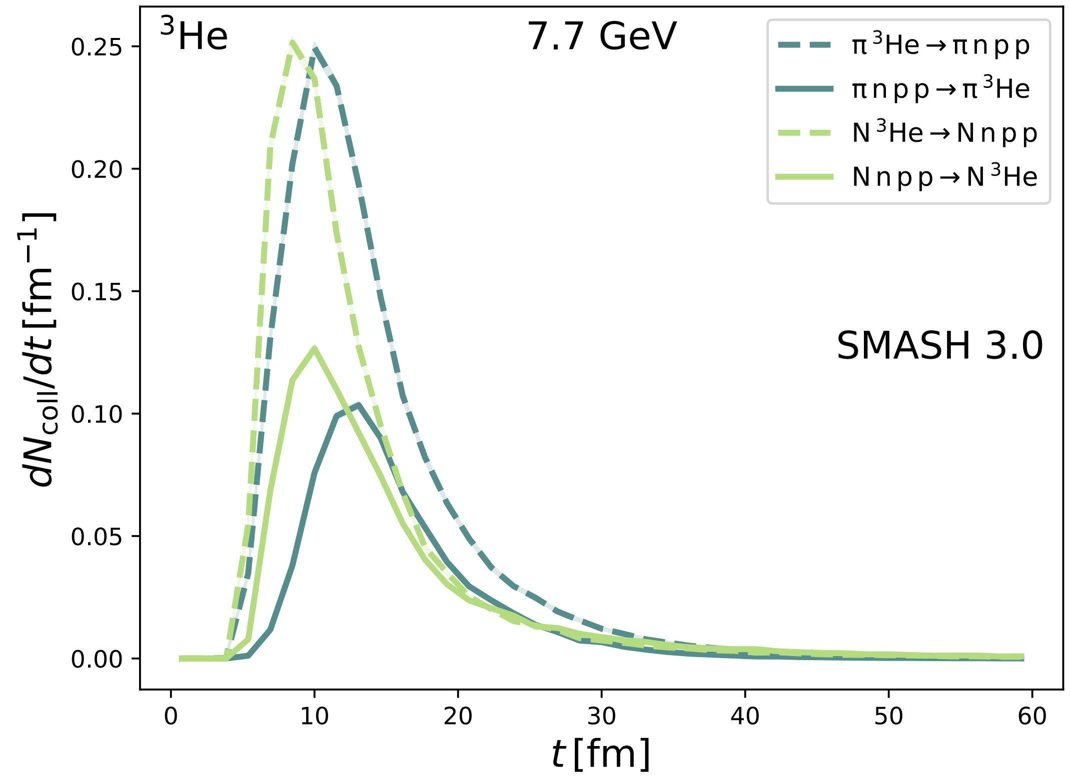


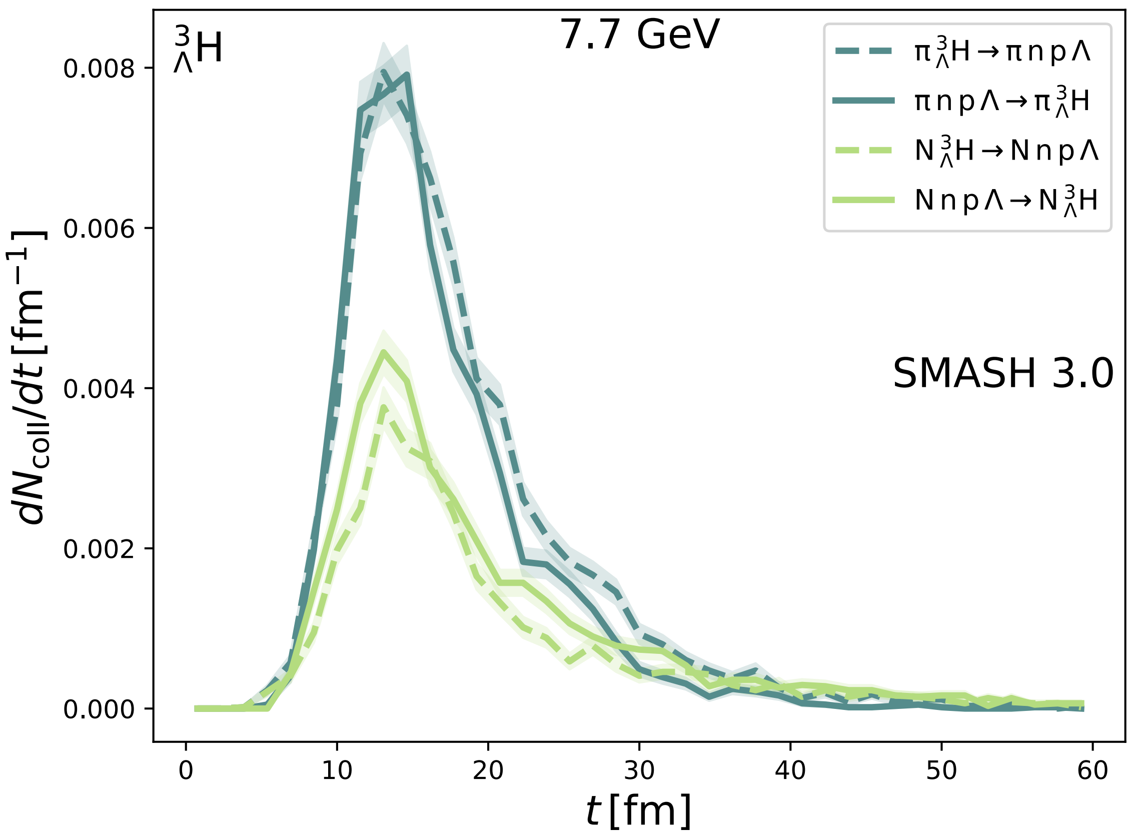
- Single ratios are well described, while double ratio slightly deviates
- Non-trivial time evolution cancels in double ratio

M. Ege, J. Mohs, J. Staudenmaier and HE, arXiv: 2409.04209

Dynamic Production







- Many collisions are happening in the afterburner stage
- Production and destruction of light nuclei largely balance each other

M. Ege, J. Mohs, J. Staudenmaier and HE, arXiv: 2409.04209