

Chasing the QCD Critical Point

Closer Than Ever (?)

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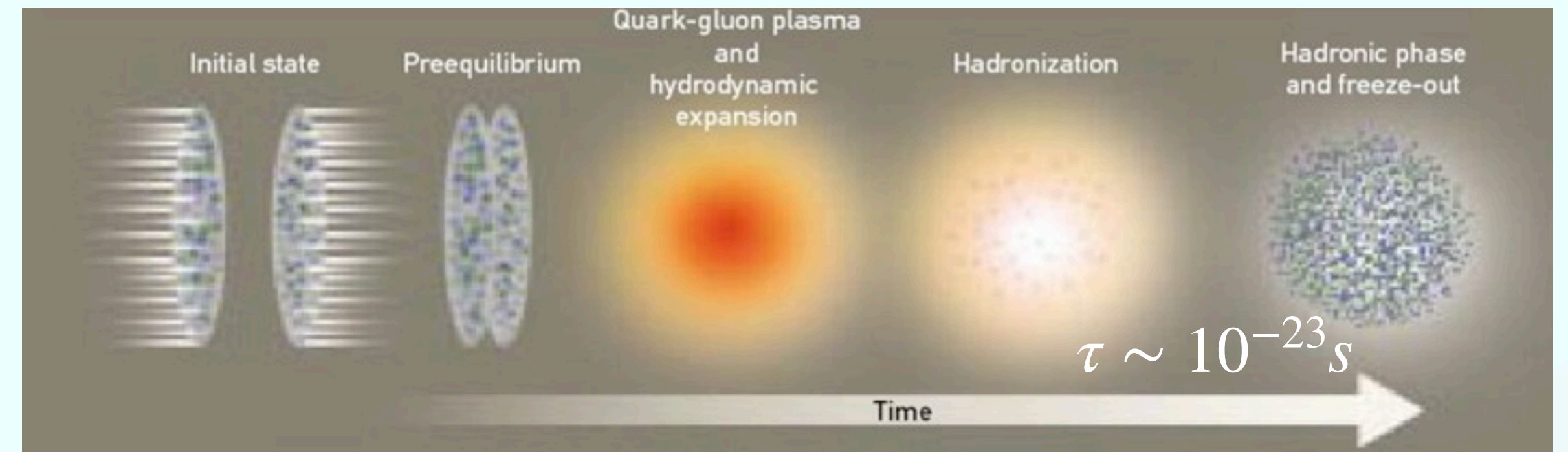
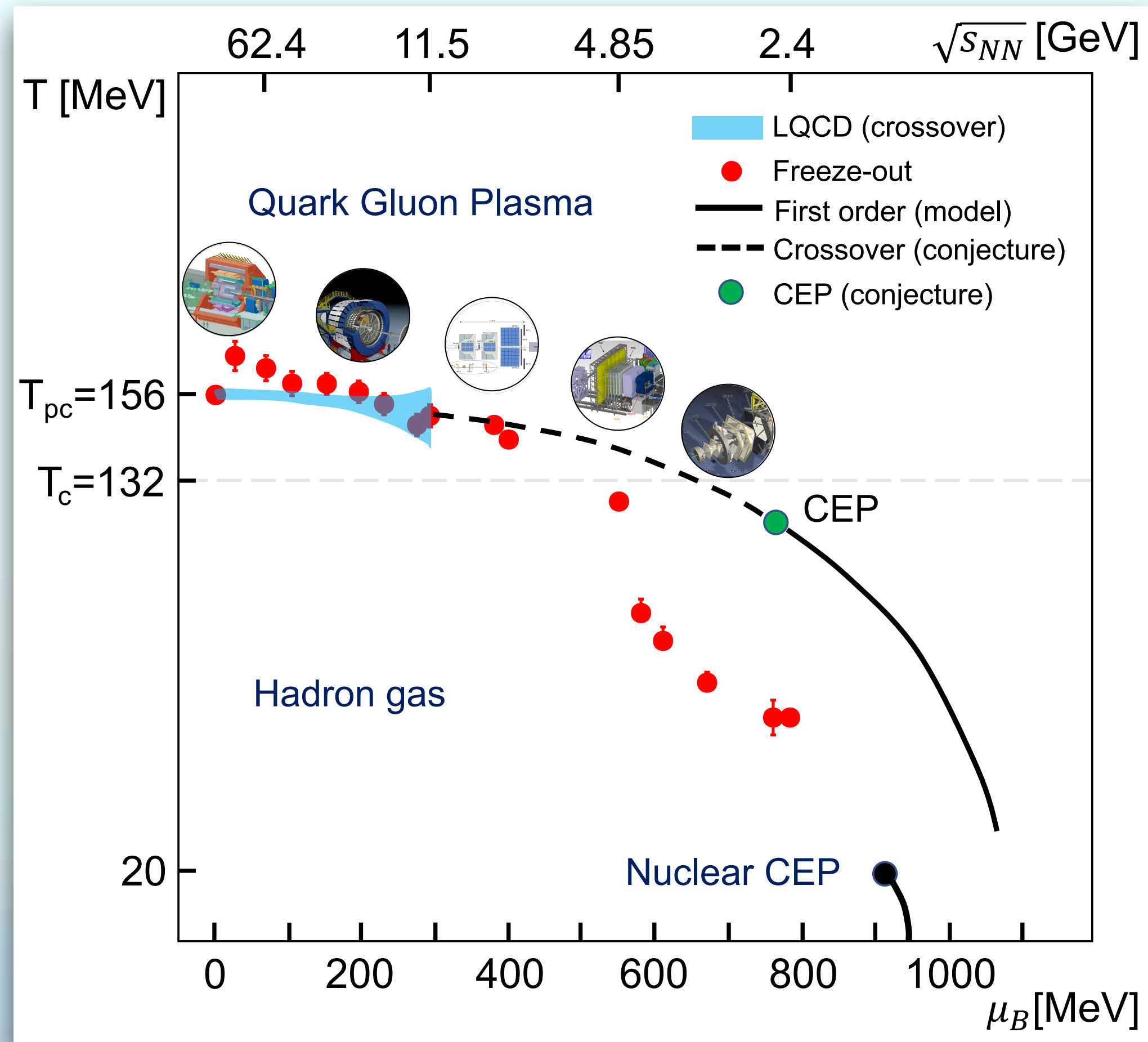
EMMI Workshop

Collective Phenomena and EoS of dense baryonic matter

Nov 10-13, 2025, GSI



Phase diagram of QCD matter



$$T_{fo}^{ALICE} = 156.5 \pm 1.5 \pm 3 \text{ MeV(sys)}$$

$$\mu_B \approx 0, V \approx 5280 \text{ fm}^3 \text{ (in one unit of rapidity)} \quad V_{Pb} \approx 1200 \text{ fm}^3$$

$$T_C^{LQCD} = 156.5 \pm 1.5 \text{ MeV}$$

Freeze-out at the phase boundary!

experiment $\rightarrow$ $\kappa_n(N_B - N_{\bar{B}}) = \frac{\partial^n \ln Z(V, T, \mu_B)}{\partial (\mu_B/T)^n} \equiv VT^3 \hat{\chi}_n^B$ $\leftarrow$ theory

EoS $\downarrow$

- P. Braun-Munzinger, A.R., J. Stachel, 50 years of QCD, EPJ C 83 (2023) 1125
- IQCD: A. Bazavov et al., (HotQCD), PLB 795 (2019) 15-21

Outline

Probing the Equation of State (EoS) with Fluctuations

Experimental Verification of the Crossover Transition

Search for a First-Order Phase Transition and the Critical Point

Summary and Outlook

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Probing the Equation of State (EoS) with Fluctuations

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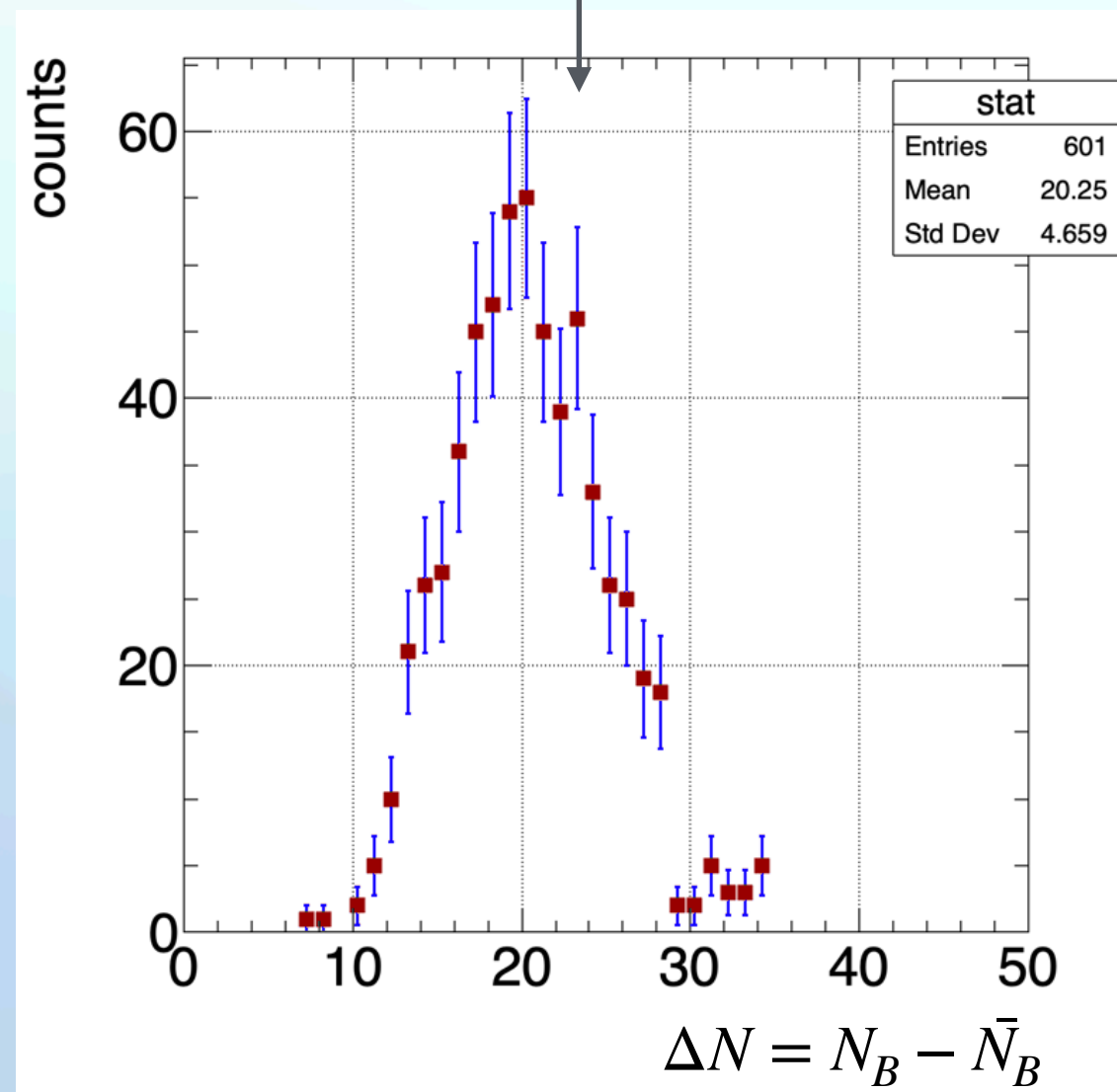
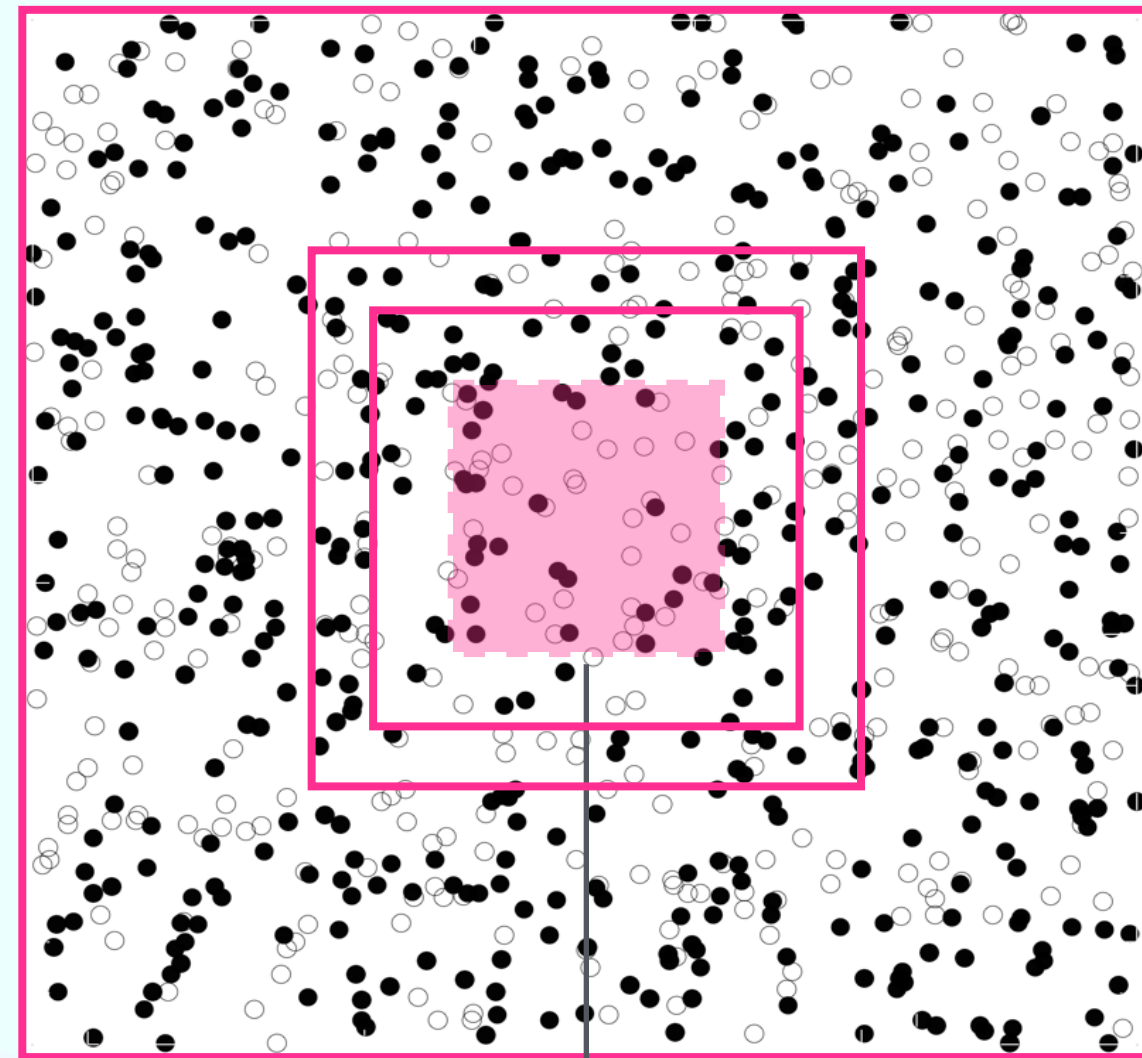
Search for a First-Order Phase Transition and the Critical Point

Summary and Outlook

Probing EoS with event-by-event fluctuations

fluctuations in the Grand Canonical Ensemble (GCE)
for a thermal system at temperature T and volume V

$$\kappa_n(N_B - N_{\bar{B}}) = \frac{\partial^n \ln Z(V, T, \mu_B)}{\partial (\mu_B/T)^n} \equiv VT^3 \hat{\chi}_n^B$$



experiment

E-by-E baryon number conservation
Fluctuating volume

theory

Baryon number conservation on average
Fixed volume

► P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

Quantifying distributions

Regular Cumulants

$$\kappa_1(X) = \langle X \rangle$$

$$\kappa_2(X) = \langle (X - \langle X \rangle)^2 \rangle$$

$$\kappa_3(X) = \langle (X - \langle X \rangle)^3 \rangle$$

$$\kappa_4(X) = \langle (X - \langle X \rangle)^4 \rangle - 3\kappa_2(X)^2$$

Factorial Cumulants

$$C_1(X) = \kappa_1(X) = \langle X \rangle$$

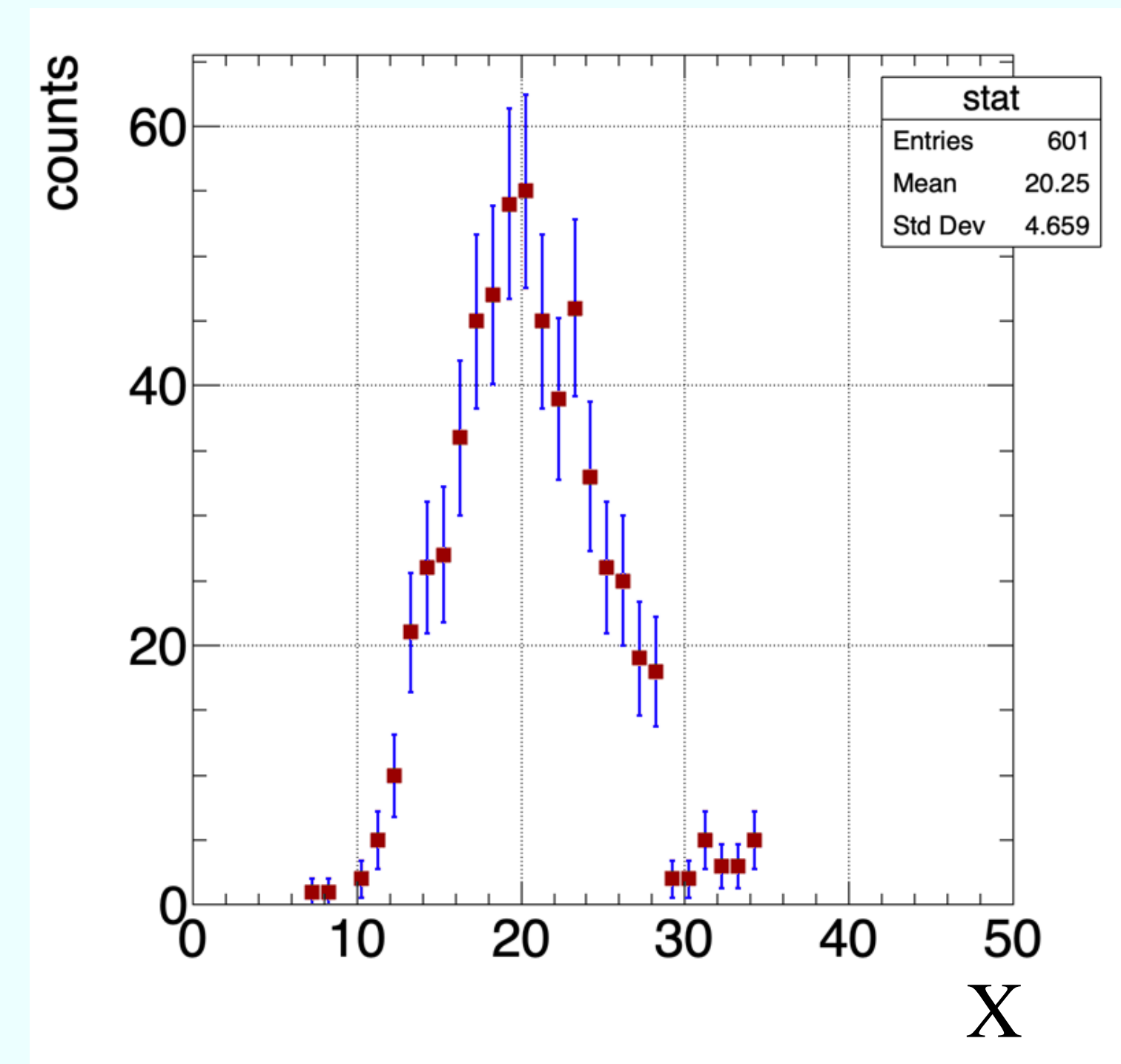
$$C_2(X) = -\kappa_1(X) + \kappa_2(X)$$

$$C_3(X) = 2\kappa_1(X) - \kappa_2(X) + \kappa_3(X)$$

$$C_4(X) = -6\kappa_1(X) + 11\kappa_2(X) - 6\kappa_3(X) + \kappa_4(X)$$

► R. Holzmann, V. Koch, A. R., J. Stroth, Nucl. Phys.A 1050 (2024) 122924

Factorial cumulants isolate genuine correlations



GCE Baselines, **Poisson Limit:** $P(X) = e^{-\lambda} \frac{\lambda^X}{X!}$

$$\kappa_n = \lambda \quad (n \geq 1)$$

$$C_1 = \lambda, \quad C_n = 0 \quad (n \geq 2)$$

Canonical baselines

$$Z_B(V, T) = \sum_{N_B=0}^{\infty} \sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!} \frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}} \right)^{\frac{B}{2}} I_B(2z \sqrt{\lambda_B \lambda_{\bar{B}}})$$

B net baryon number, conserved in each event

► P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel, NPA 1008 (2021) 122141

I_B modified Bessel function of the first kind

$z_B, z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

$\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

$$\frac{\kappa_2(B - \bar{B})}{\langle n_B + n_{\bar{B}} \rangle} = 1 - \frac{\alpha_B \langle n_B \rangle + \alpha_{\bar{B}} \langle n_{\bar{B}} \rangle}{\langle n_B + n_{\bar{B}} \rangle} + (z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle) \frac{(\alpha_B - \alpha_{\bar{B}})^2}{\langle n_B + n_{\bar{B}} \rangle}$$

$\langle N_B \rangle, \langle N_{\bar{B}} \rangle$ - in 4π

canonical suppression

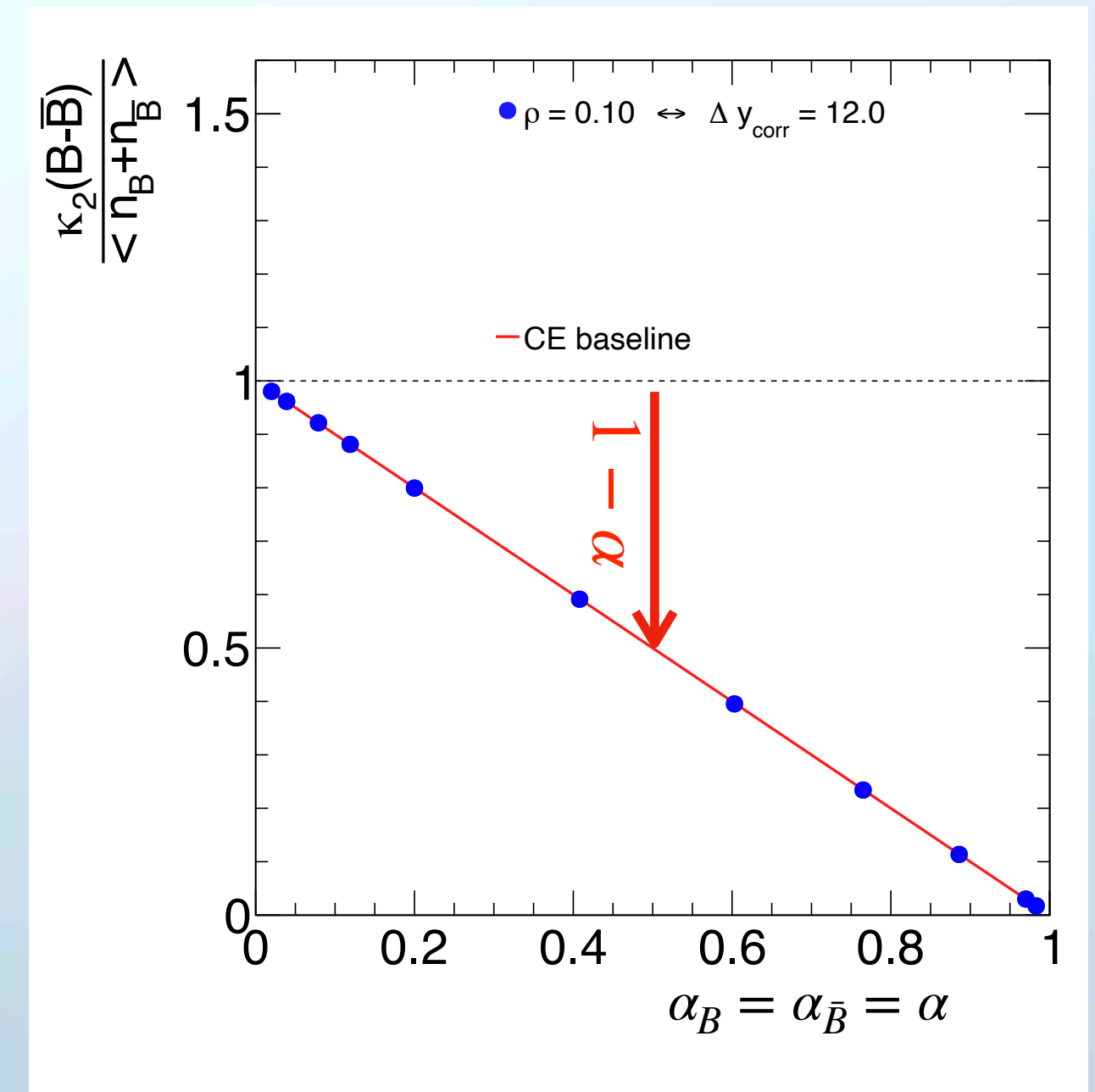
$\langle n_B \rangle, \langle n_{\bar{B}} \rangle$ - inside acceptance

$\alpha_B = \langle n_B \rangle / \langle N_B \rangle$ - acceptance for B

$\alpha_{\bar{B}} = \langle n_{\bar{B}} \rangle / \langle N_{\bar{B}} \rangle$ - acceptance for $\bar{B}$

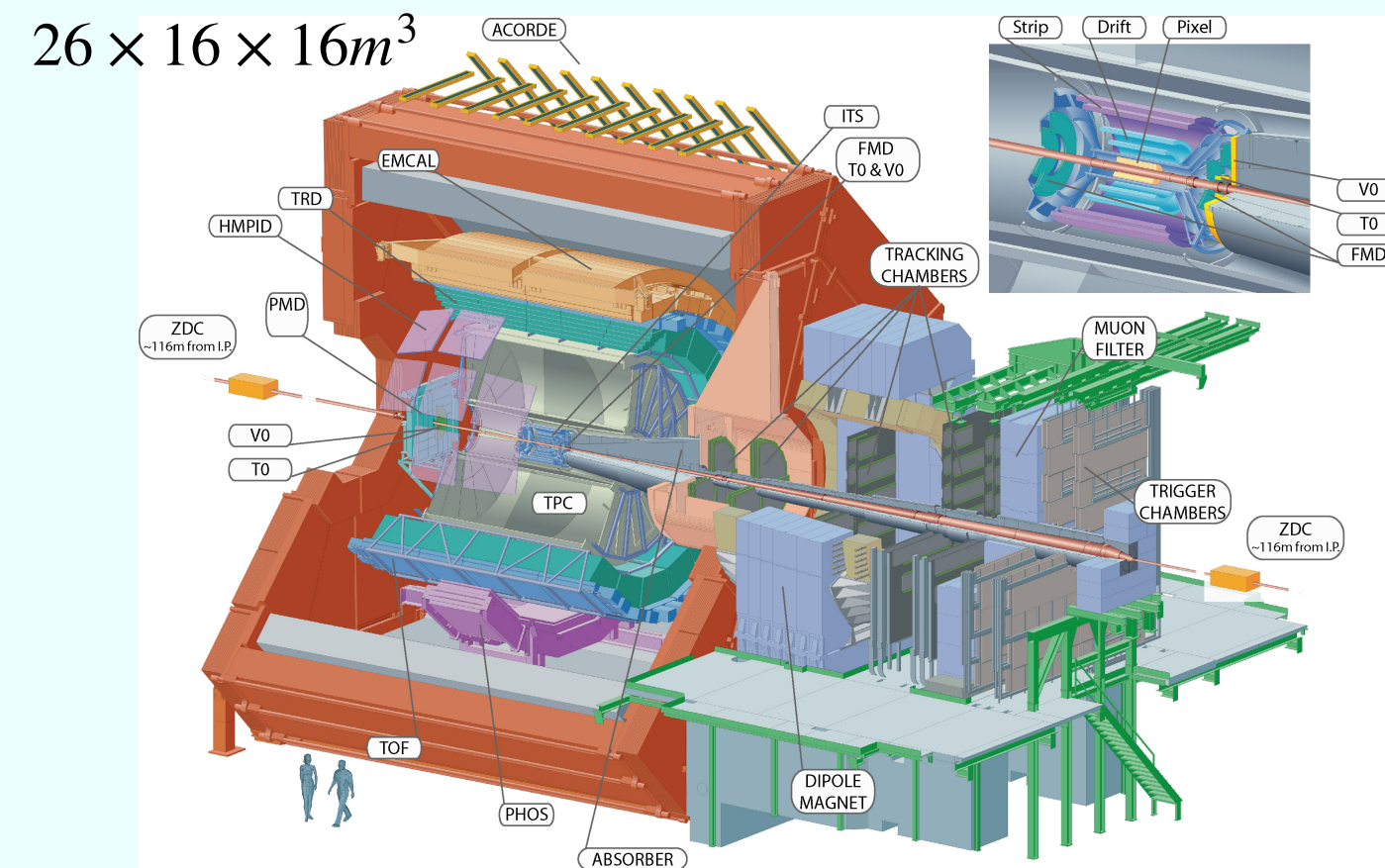
z - single baryon partition function

📌 deviations from unity
📌 induced by conservation laws

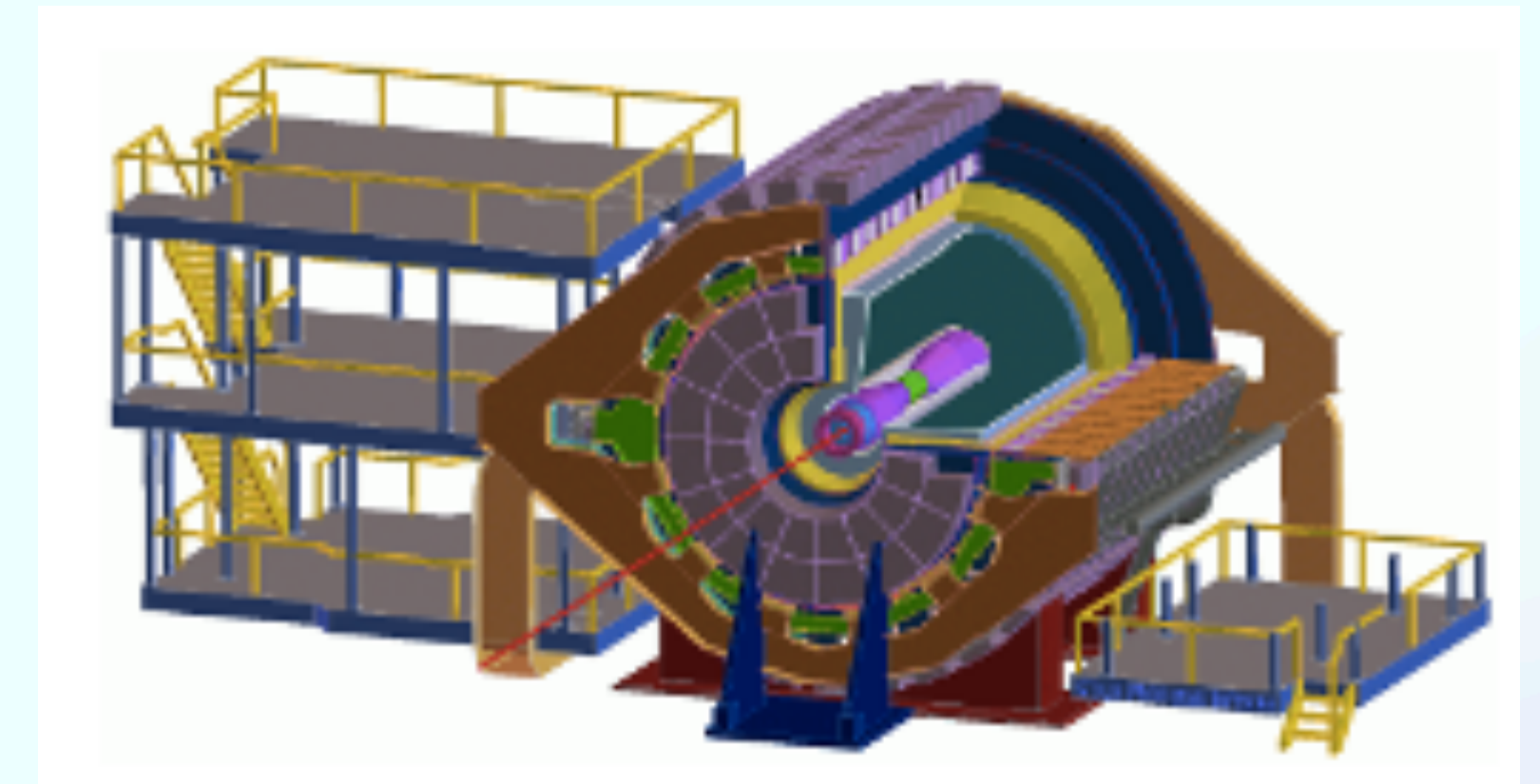


Experiments

ALICE@ CERN LHC (few TeV)



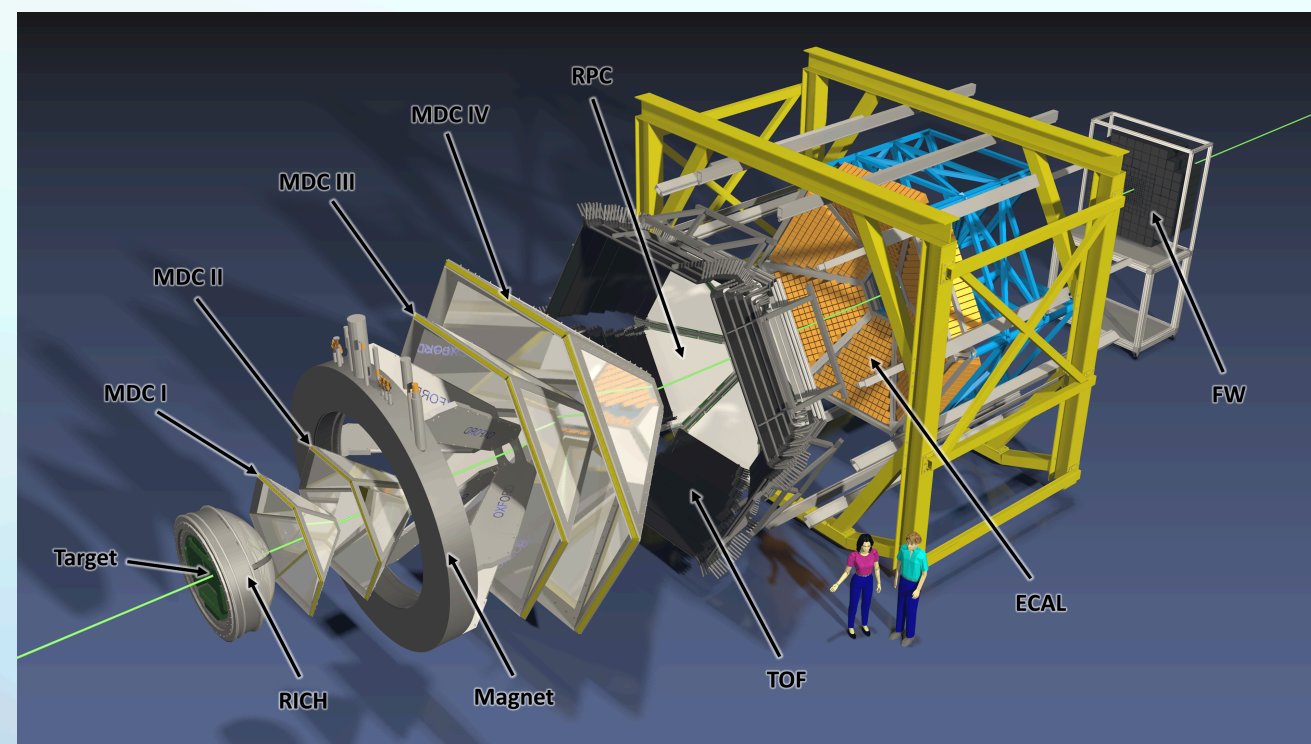
STAR@BNL RHIC (3 - 200 GeV)



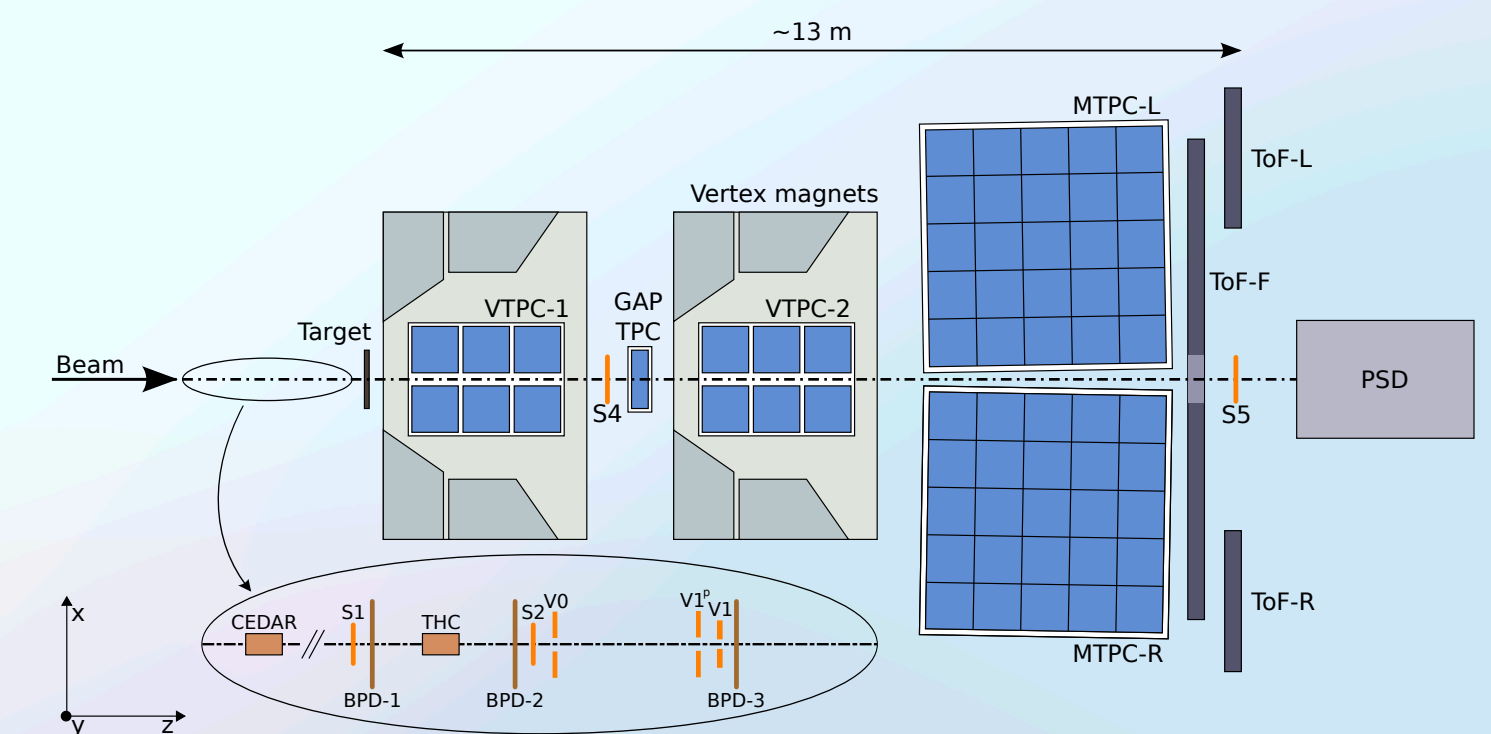
from crossover

towards

Critical End Point



HADES@GSI SIS18 (few GeV)



NA61/SHINE@CERN SPS (5-17 GeV)

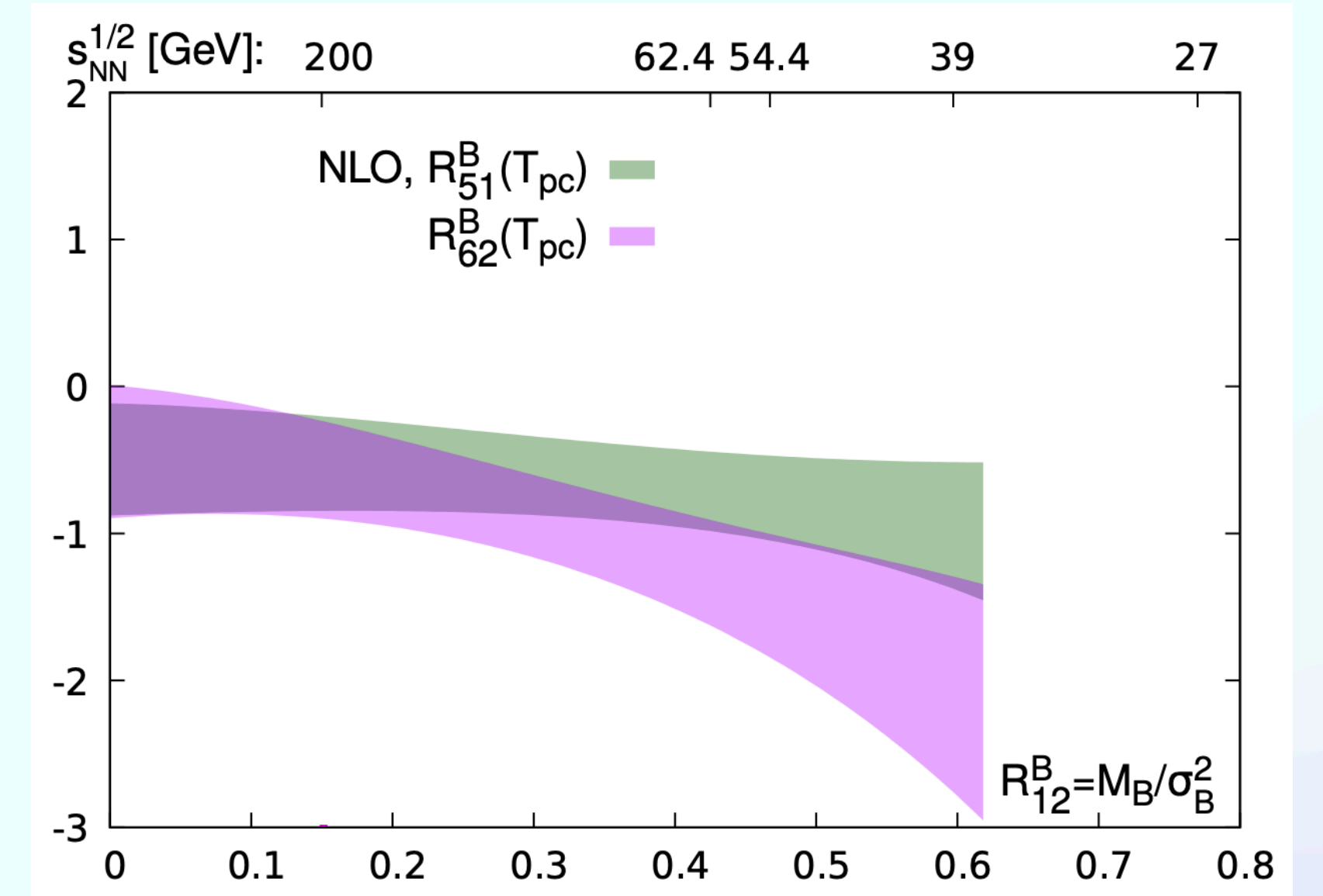
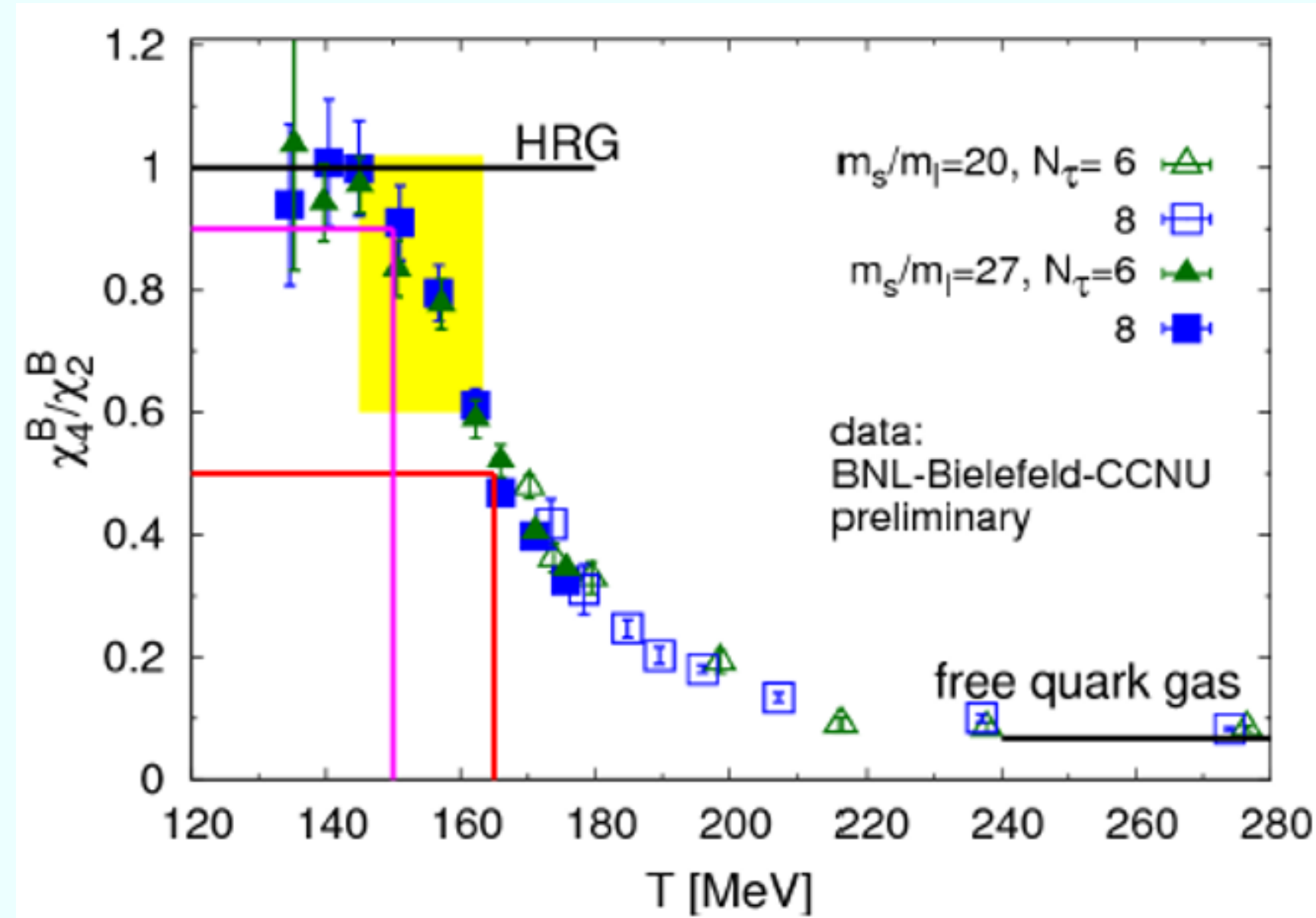
Probing the Equation of State (EoS) with Fluctuations

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Summary and Outlook

Predictions from LQCD



- A. Bazavov et al [HotQCD], PRD 101 (2020) 074502
- A. Bazavov et al., Phys.Rev. D85 (2012) 054503

- χ_4^B/χ_2^B : significant reduction for full QCD compared to HRG in GCE (for $T > 150$ MeV)
- $\chi_{5(6)}^B/\chi_{1(2)}^B$: (progressively) negative sign towards lower energies, **probe for crossover**
- hierarchy of cumulant ratios: $\chi_3^B/\chi_1^B > \chi_4^B/\chi_2^B > \chi_5^B/\chi_1^B > \chi_6^B/\chi_2^B$, **probe for crossover**

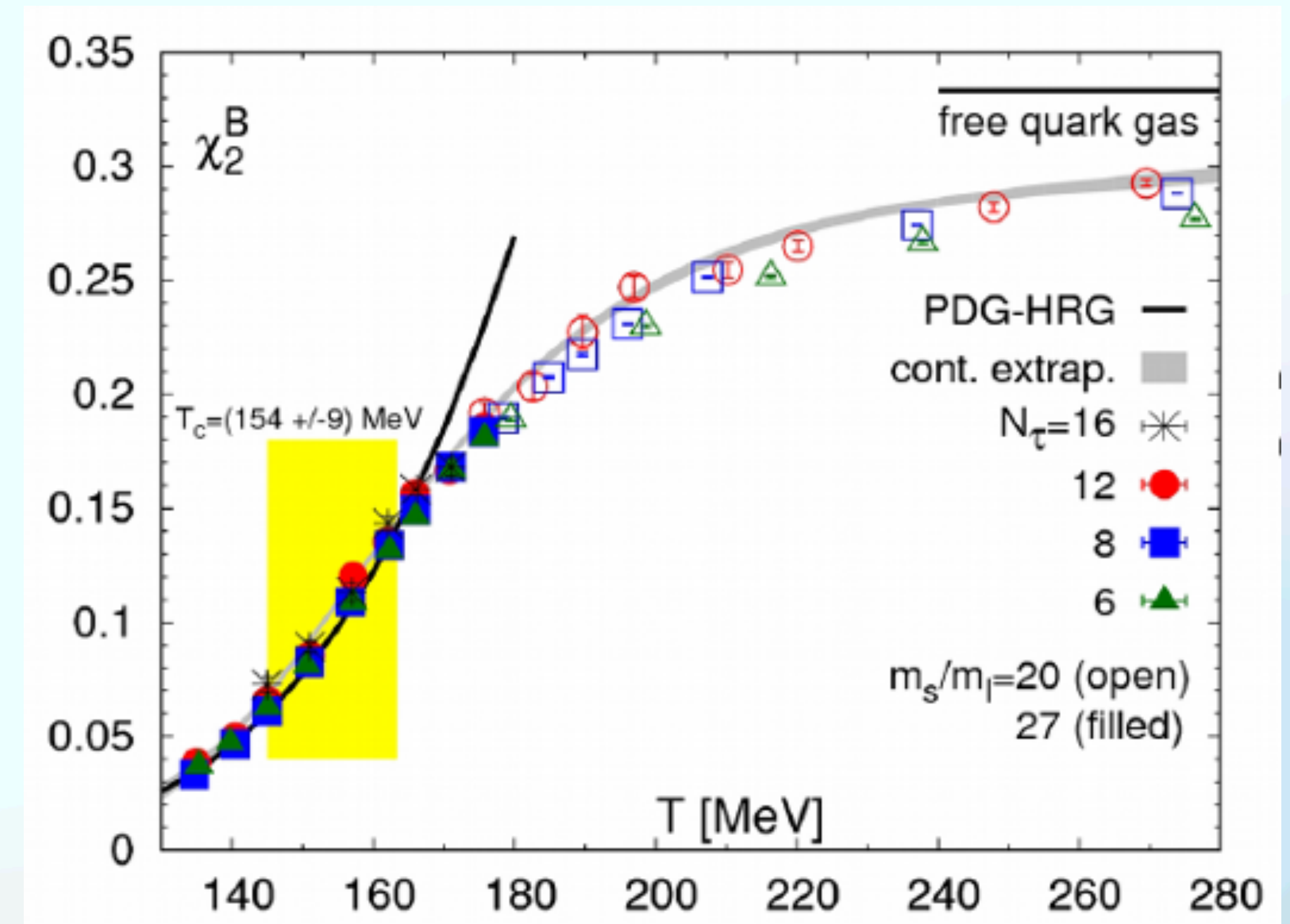
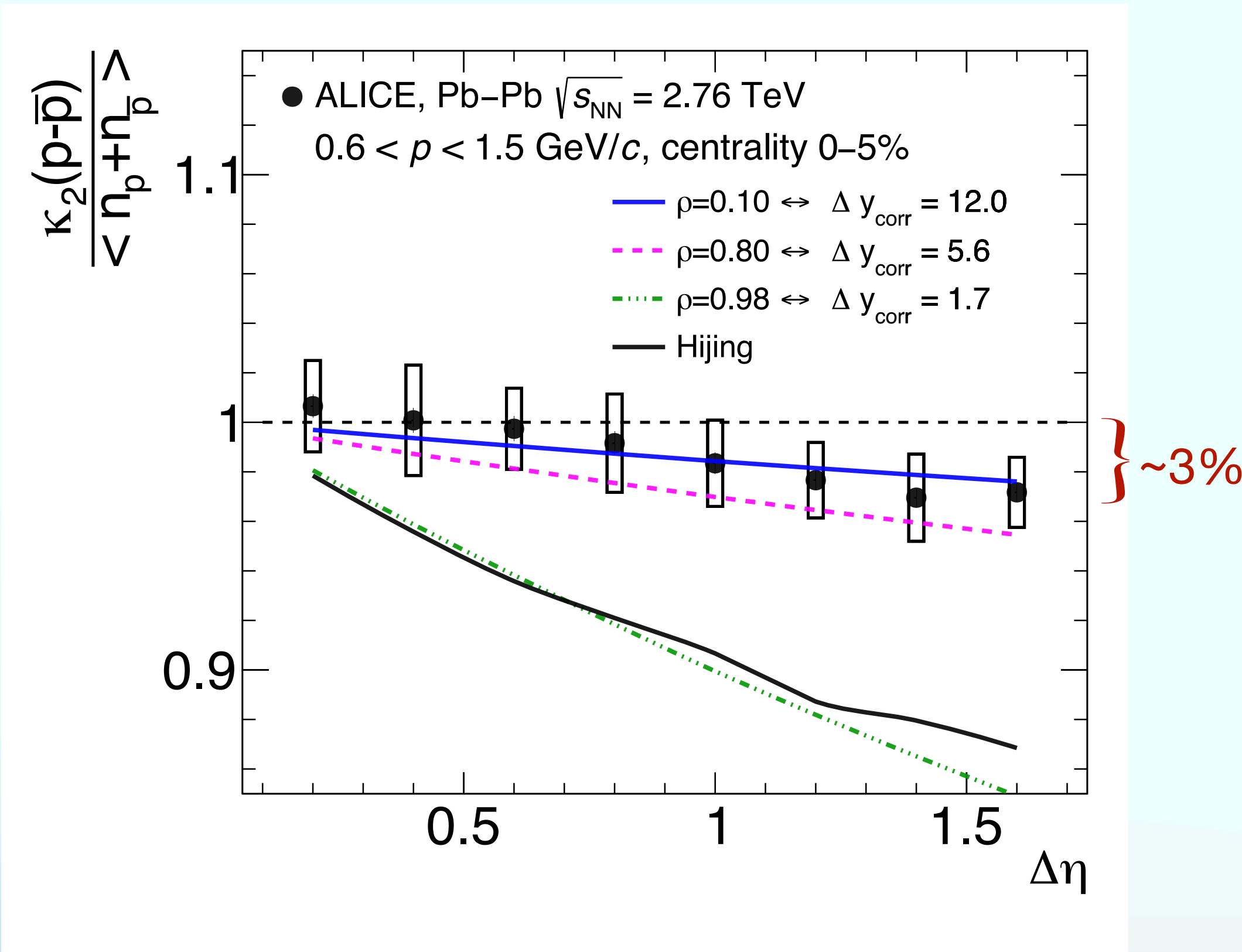
ALICE Results

► P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

► P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, JHEP 08 (2024) 113

► **ALICE:** Phys. Lett. B 807 (2020) 135564, Phys. Lett. B (2022) 137545

► **LQCD:** A. Bazavov et al [HotQCD], PRD 101 (2020) 074502

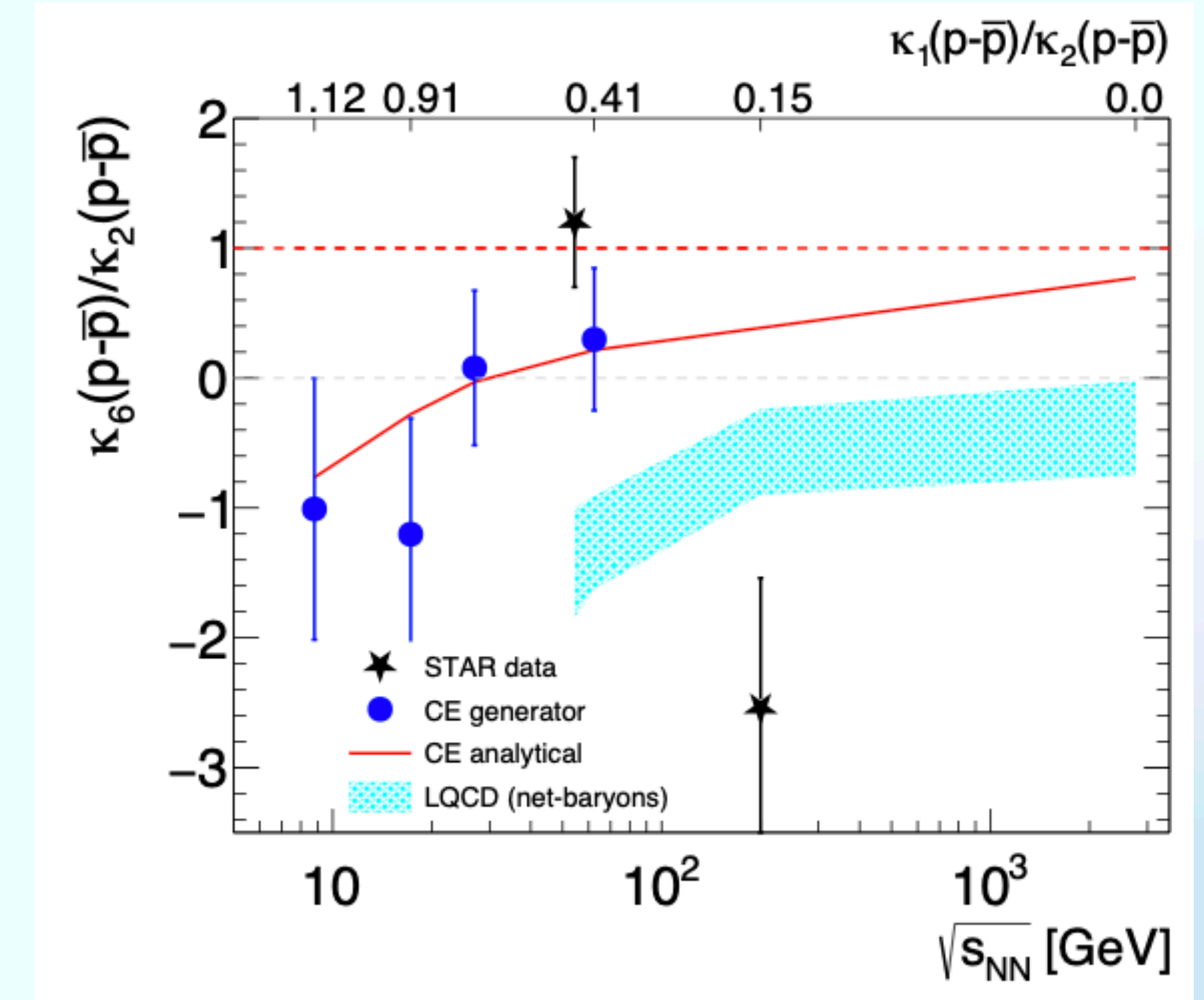
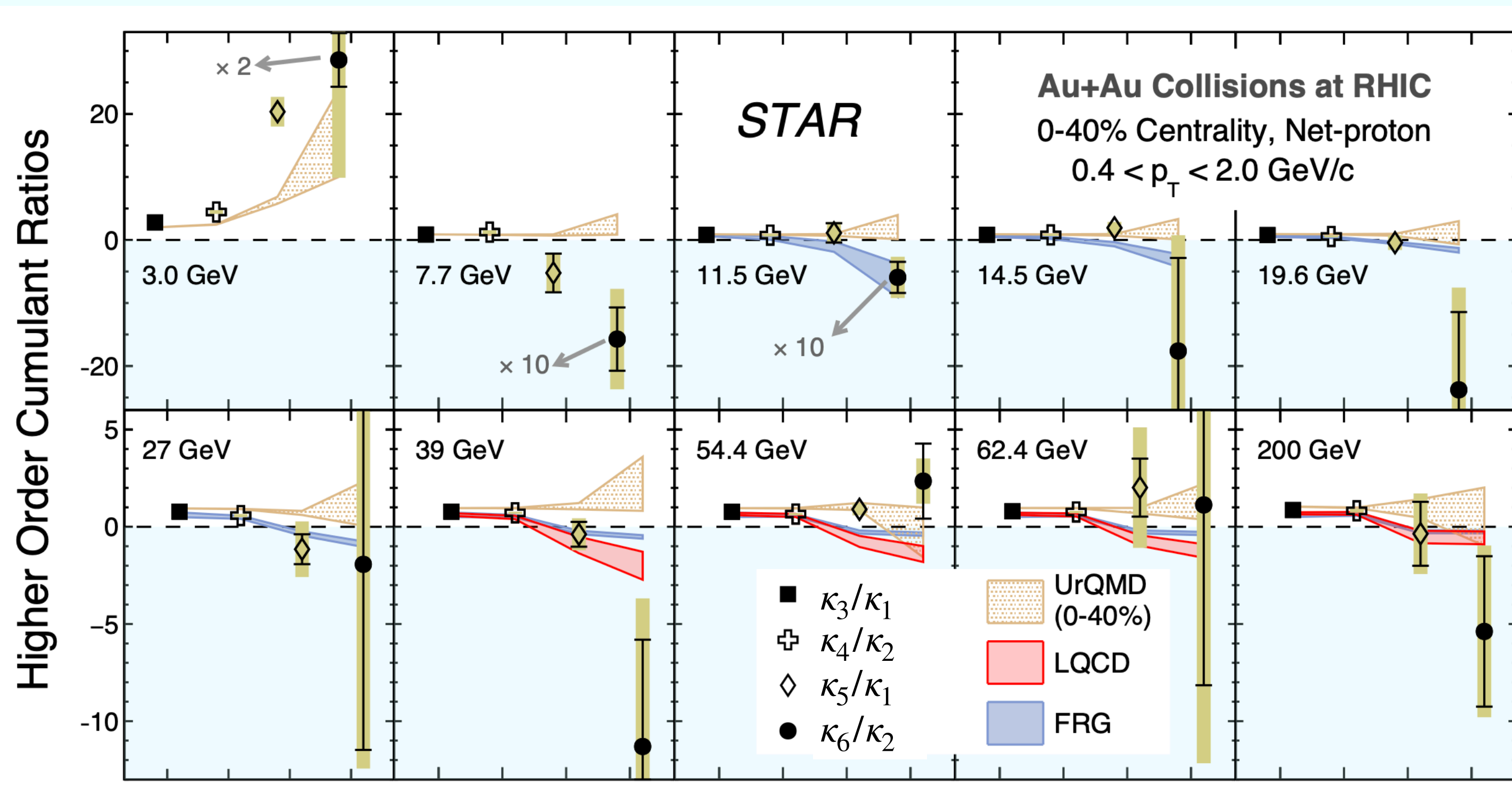


- Alice data: best description with **Global baryon number conservation**
- Agreement with LQCD predictions
- Calls into question baryon production mechanism in Hijing (Lund String Fragmentation)

STAR results

► STAR: Phys.Rev.Lett. 127 (2021) 26, 262301
Phys.Rev.Lett. 130 (2023) 8, 082301

► P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141



- first attempt to measure the cross-over signals via fluctuations of **net-protons**
- reverse ordering at 3 GeV (driven by volume fluctuations of genuine physics?)
- no anticipated ordering at 54.4 GeV !
- no consistent trend between 54.4 and 200 GeV data (both are negative in LQCD)

calls for higher statistics for firm conclusions

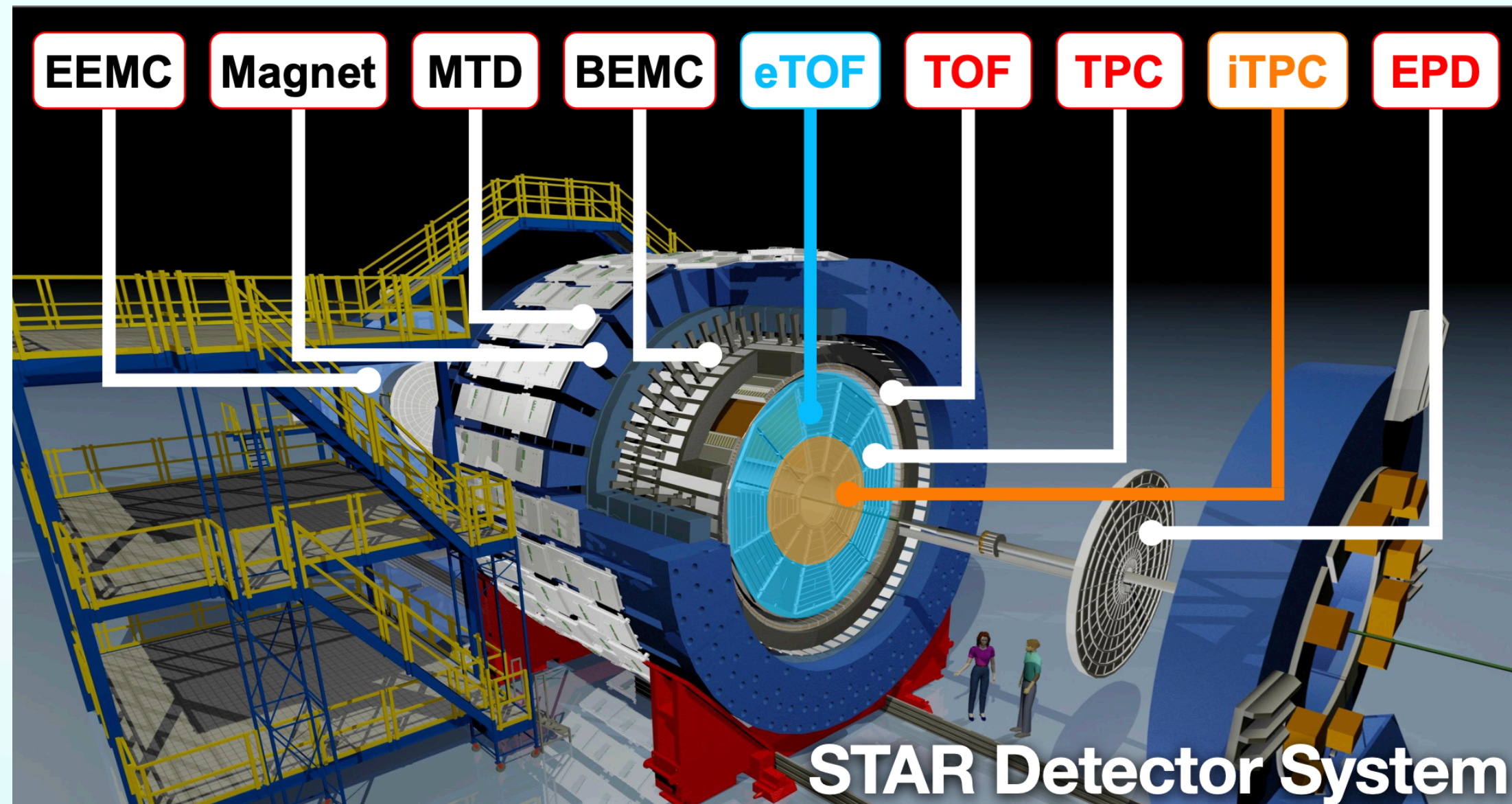
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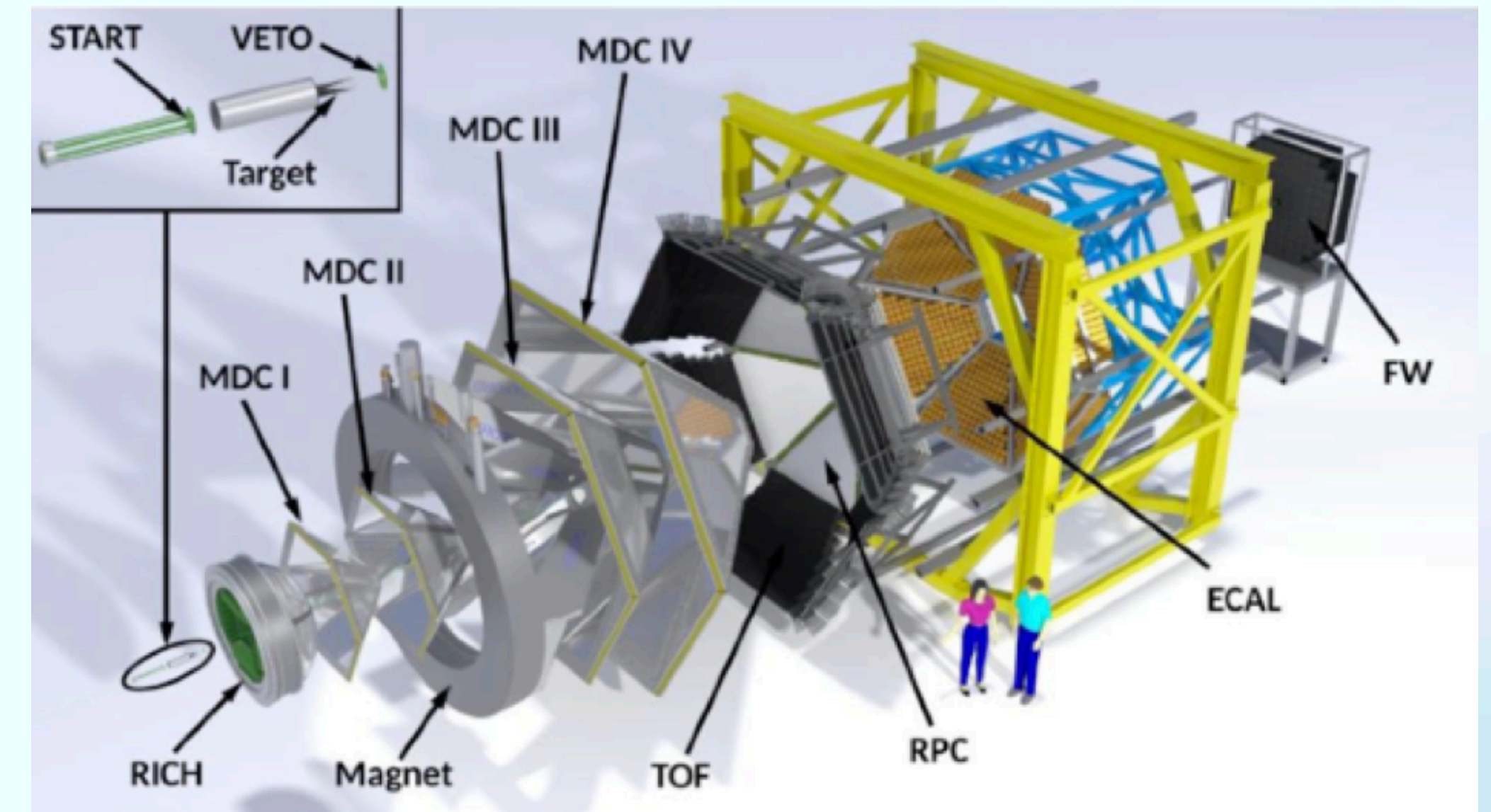
Search for a First-Order Phase Transition and the Critical Point

Summary and Outlook

STAR@BNL/RHIC



HADES@GSI/SIS18



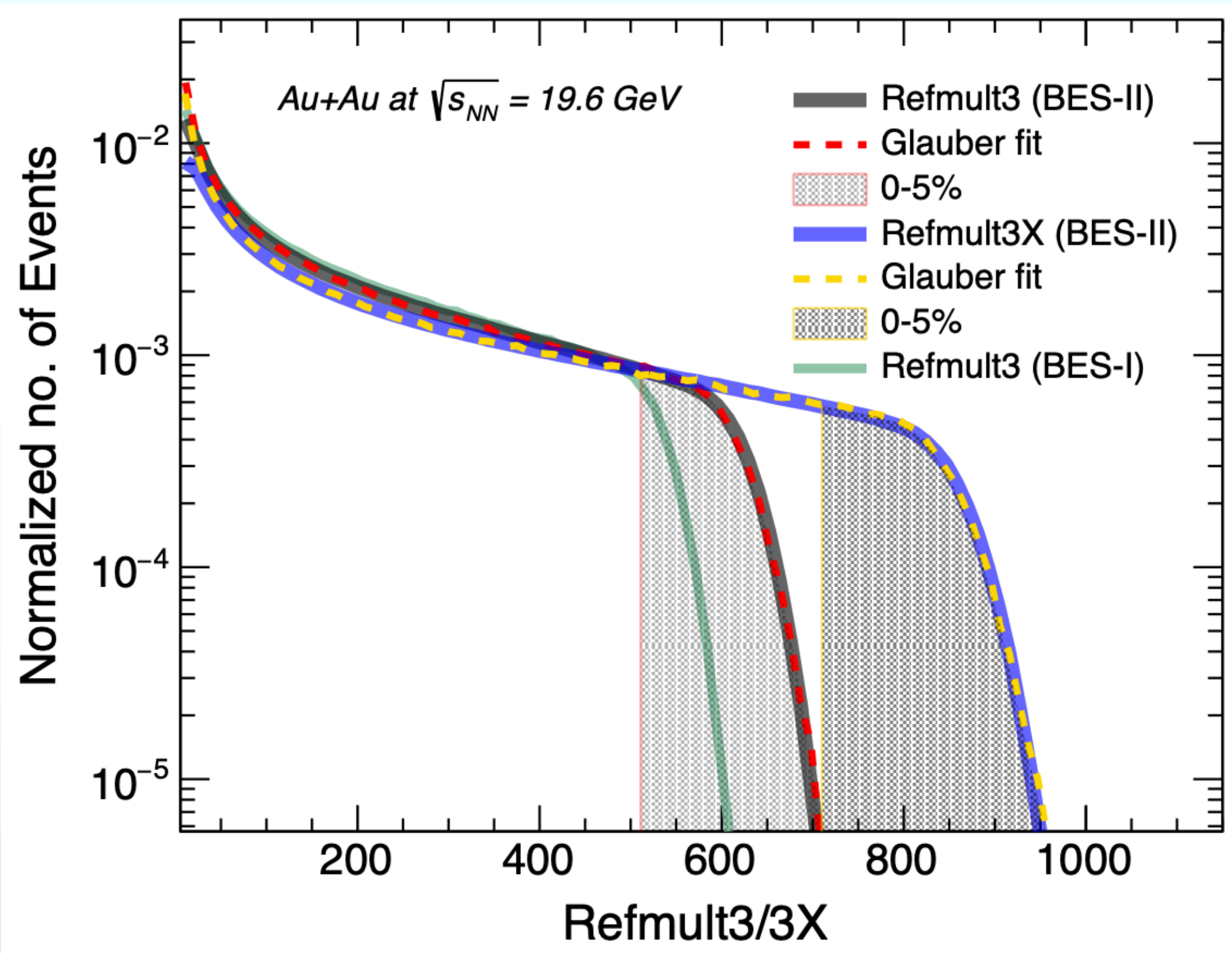
In this presentation

STAR BESII: Au-Au@ $\sqrt{s_{NN}} = 7.7, 9.2, 11.5, 14.6, 17.3, 19.6, 27$ GeV and **STAR FXT:** Au-Au@ 3 GeV

HADES: Ag-Ag@ $\sqrt{s_{NN}} = 2.55$ GeV

Centrality Selection in STAR

► A. Pandav, CPOD 2024



► **BESI, Refmult3:**

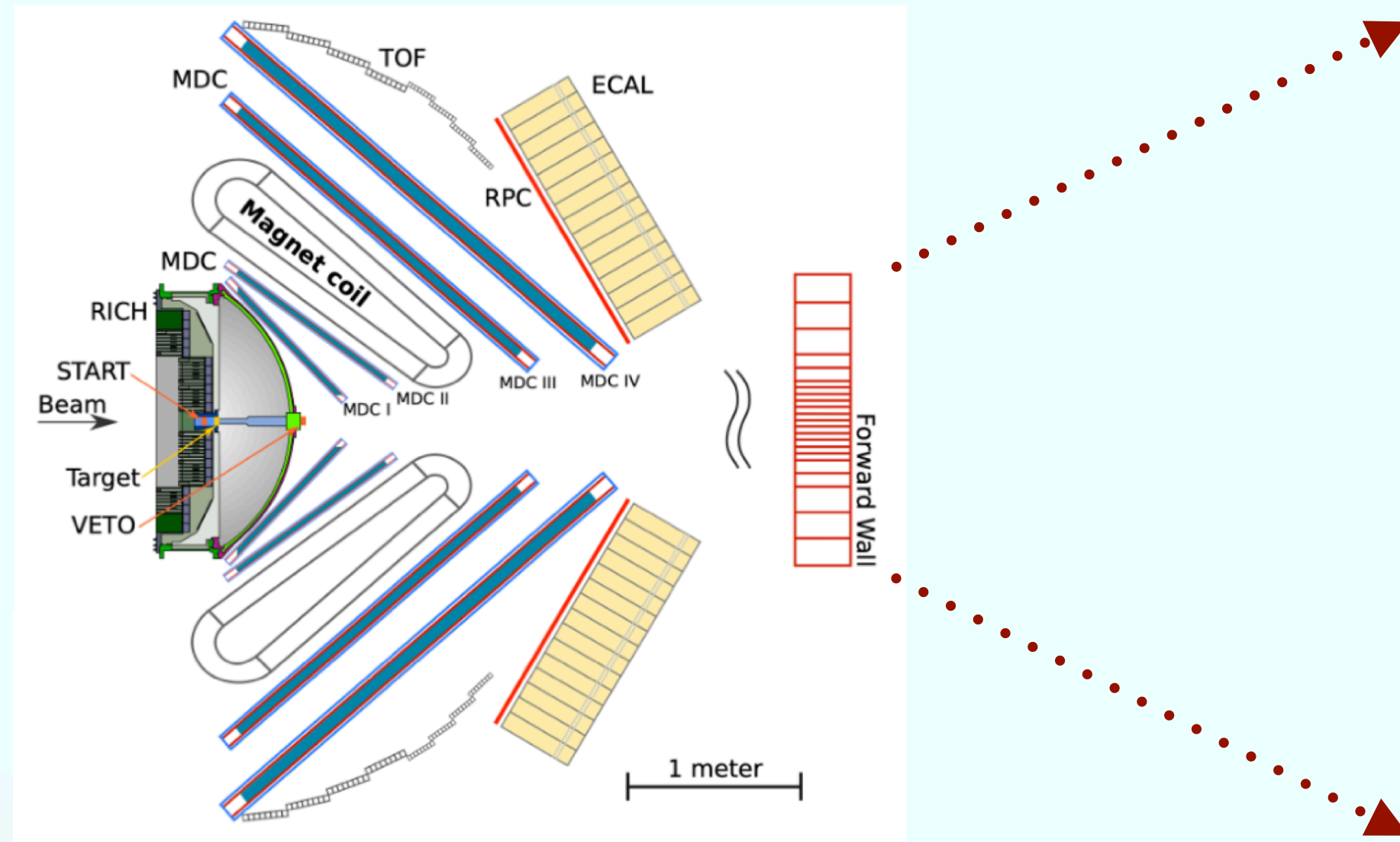
- Charged-particle multiplicity **excluding protons**, measured within $|\eta| < 1.0$

► **BESII, Refmult3X:**

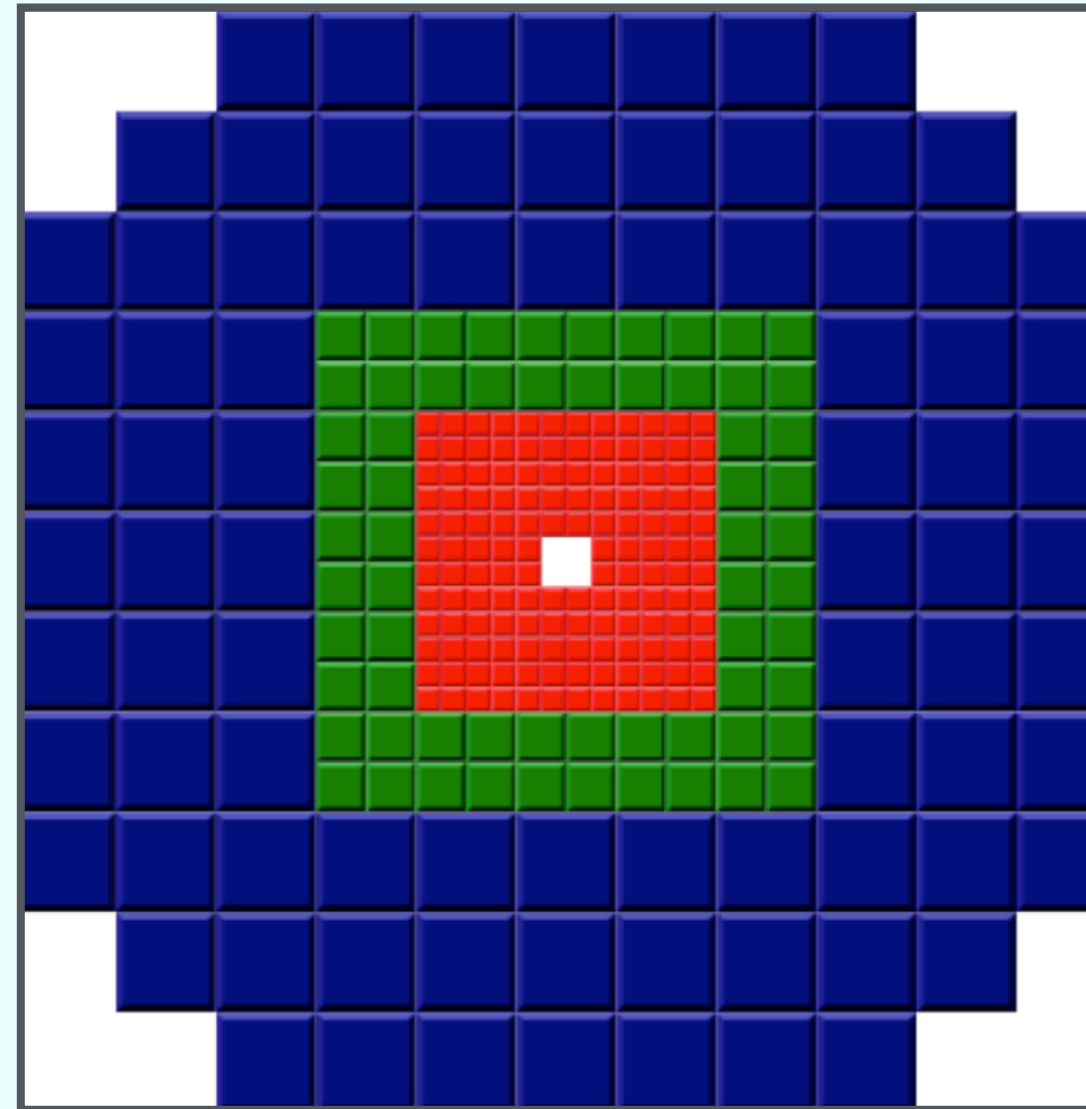
- Charged-particle multiplicity **excluding protons**, measured within $|\eta| < 1.6$
 - made possible owing to iTPC upgrade

Centrality Selection in HADES

cross section (1/6 of HADES)



Forward Wall



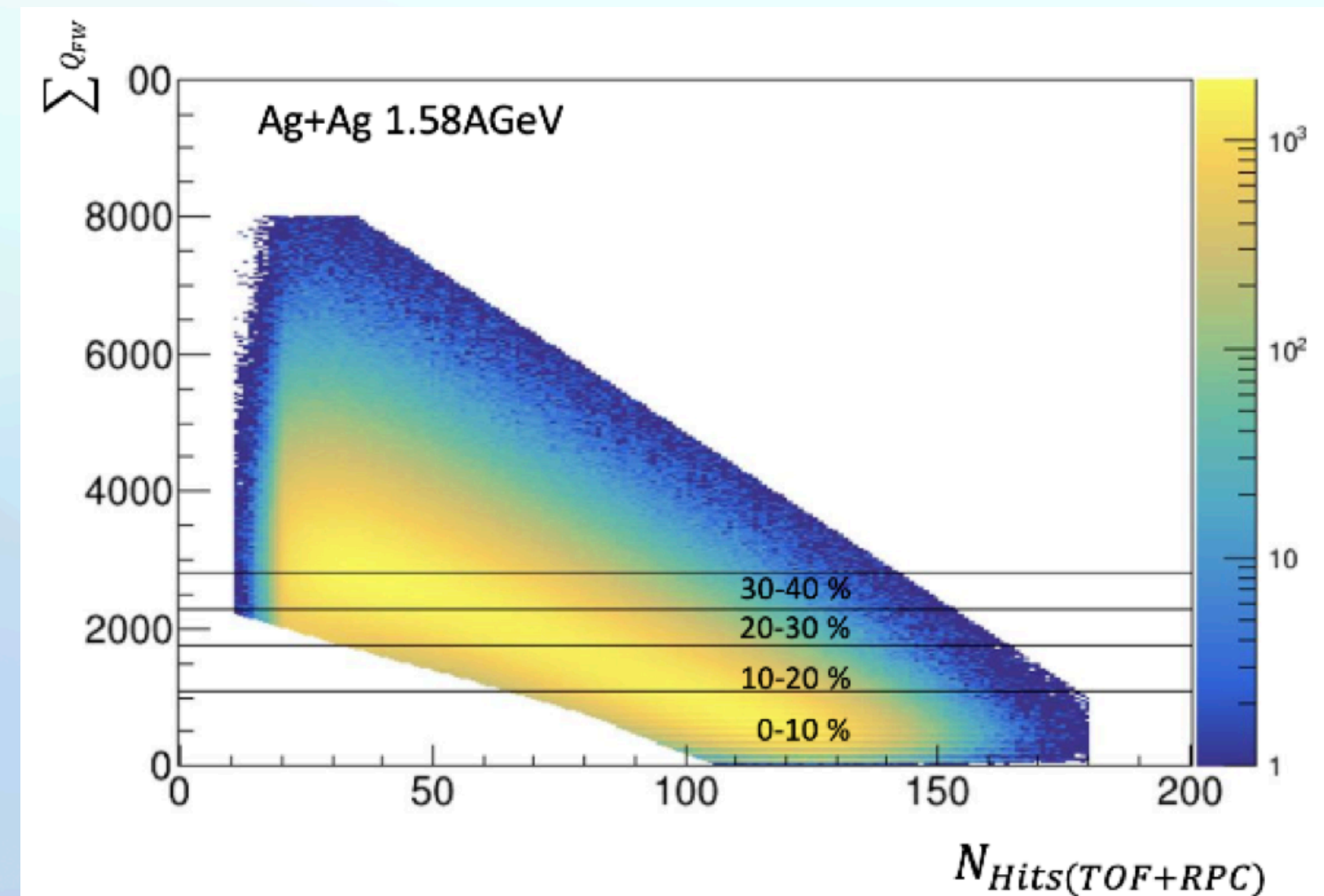
Made of plastic scintillator modules

three concentric zones

4x4, 8x8, 16x16 cm² tiles

Polar angle coverage: 0.34° - 7.4°

Analysis coverage: 18° - 85°



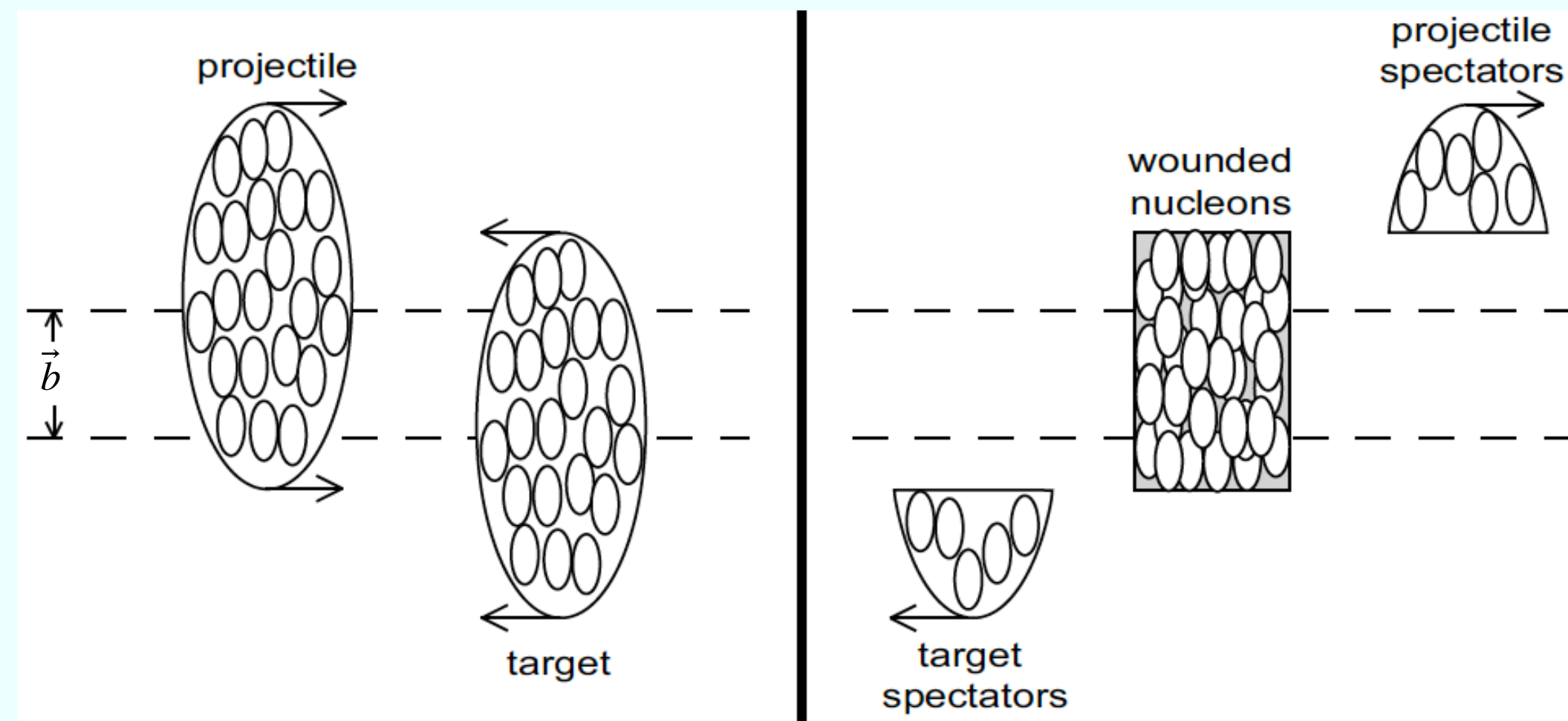
Centrality is determined using forward-going particles (ΣQ_{FW})

reduces **auto-correlations** with mid-rapidity protons

Essential for a **reliable** fluctuation analysis

► HADES: Marvin Naborth, Quark Matter 2025

Volume Fluctuations



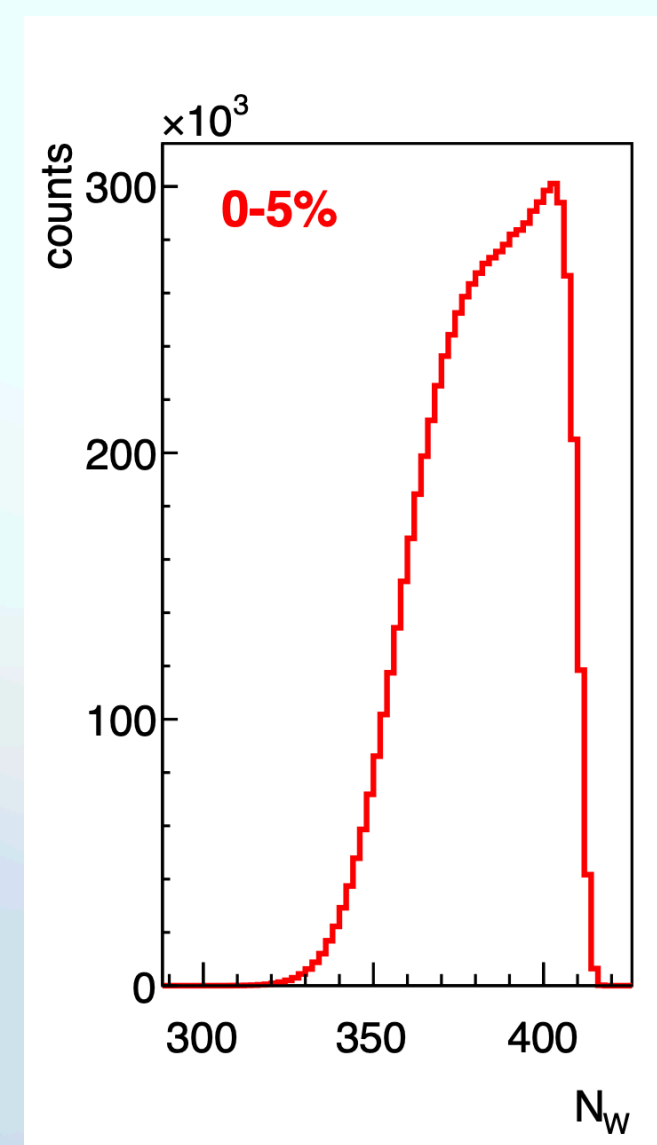
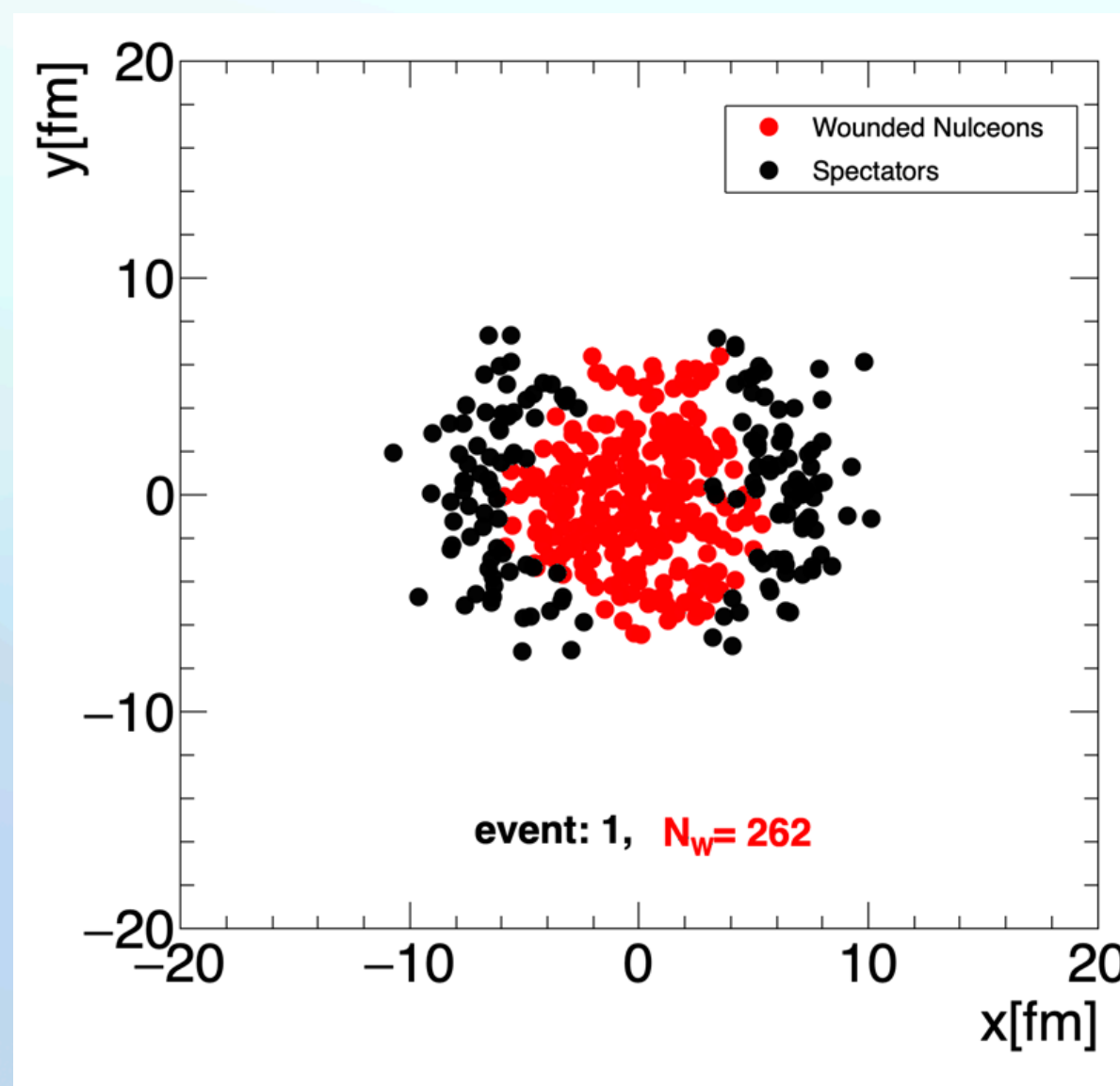
A. Bialas, and M. Bleszynski, W. Czyz, Nucl. Phys. B111 (1976) 461

In this paper we propose to describe the nucleus-nucleus collisions in terms of the number of “wounded” nucleons (w) i.e. the number of nucleons which underwent at least one inelastic collisions in this process.

Wounded nucleons, N_W

- Nucleons which collided at least once inelastically

Detailed application: P. Braun-Munzinger, A.R., J. Stachel, NPA 960 (2017) 114



Correction methods for wounded nucleon/volume fluctuations

V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

P. Braun-Munzinger, A.R., J. Stachel, NPA 960 (2017) 114

HADES: Event mixing technique

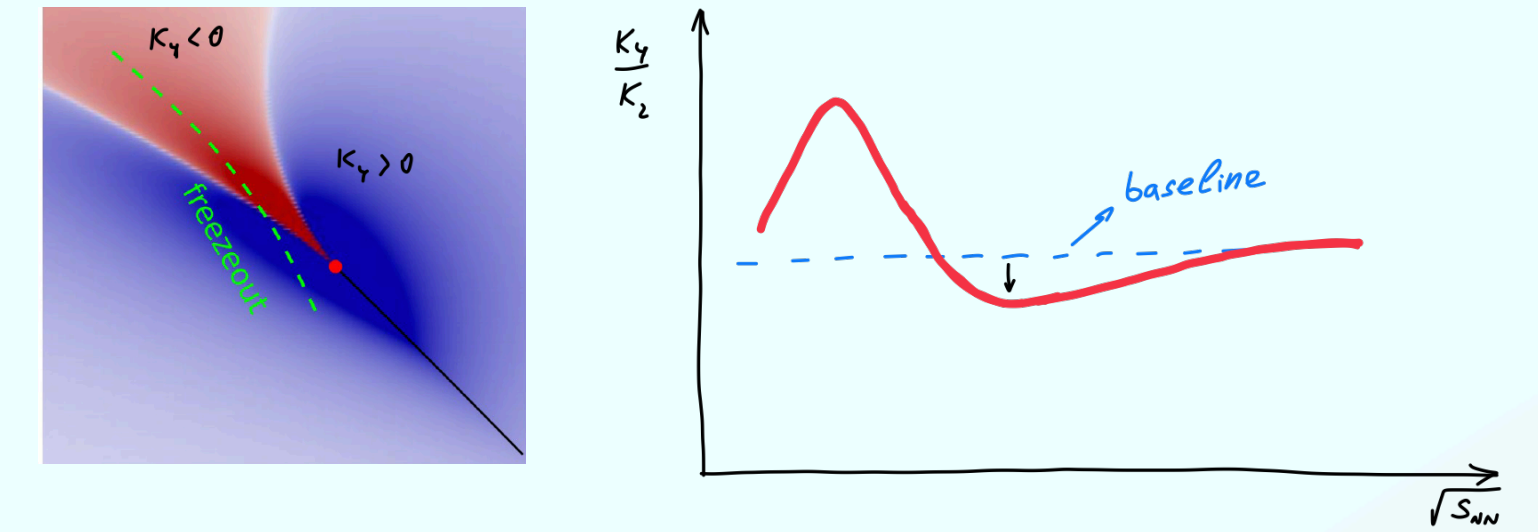
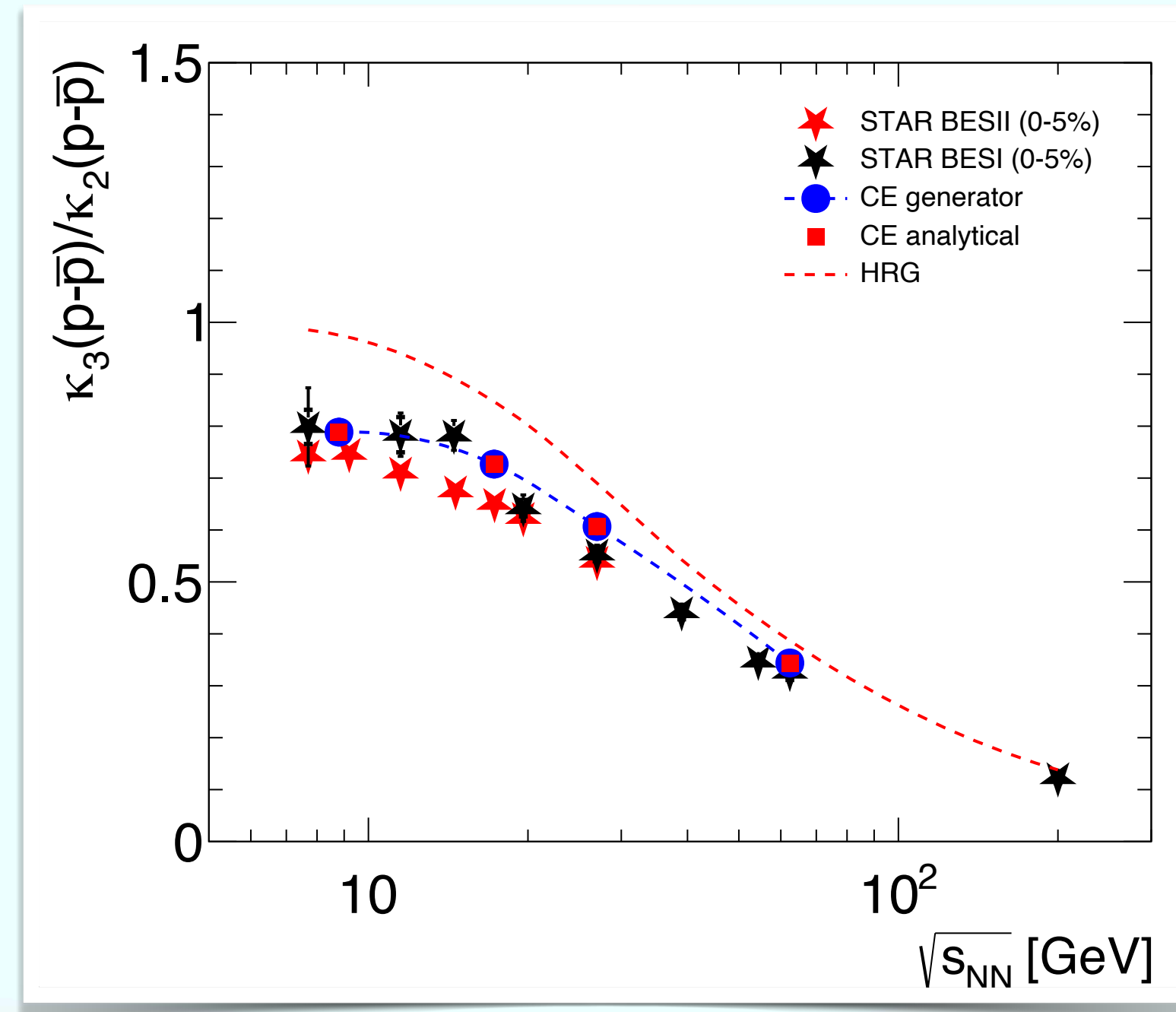
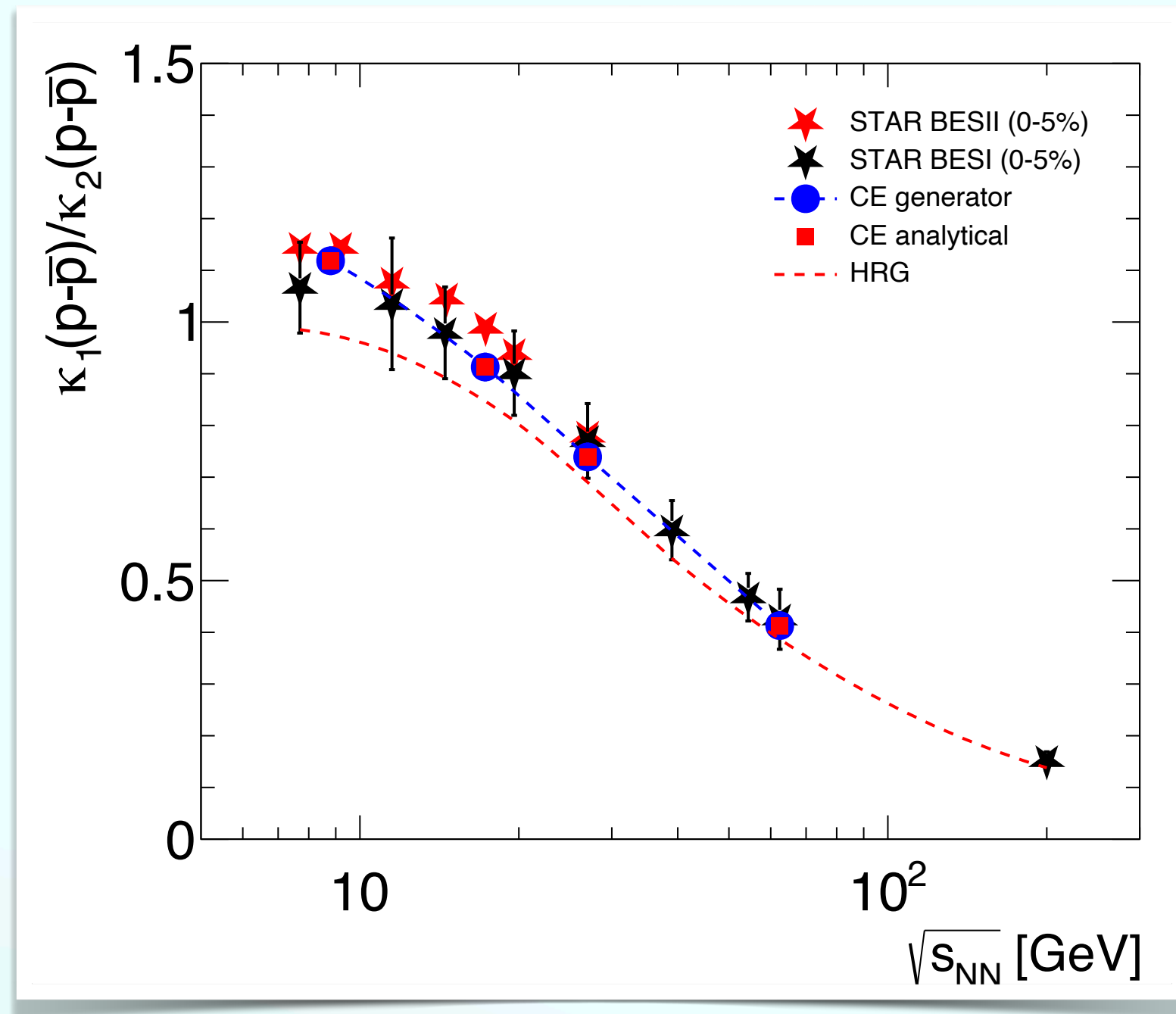
A.R., R. Holzmann, J. Stroth, NPA 1034 (2023) 122641

V. Koch, R. Holzmann, A.R., J. Stroth, Nucl.Phys.A 1050 (2024) 122924

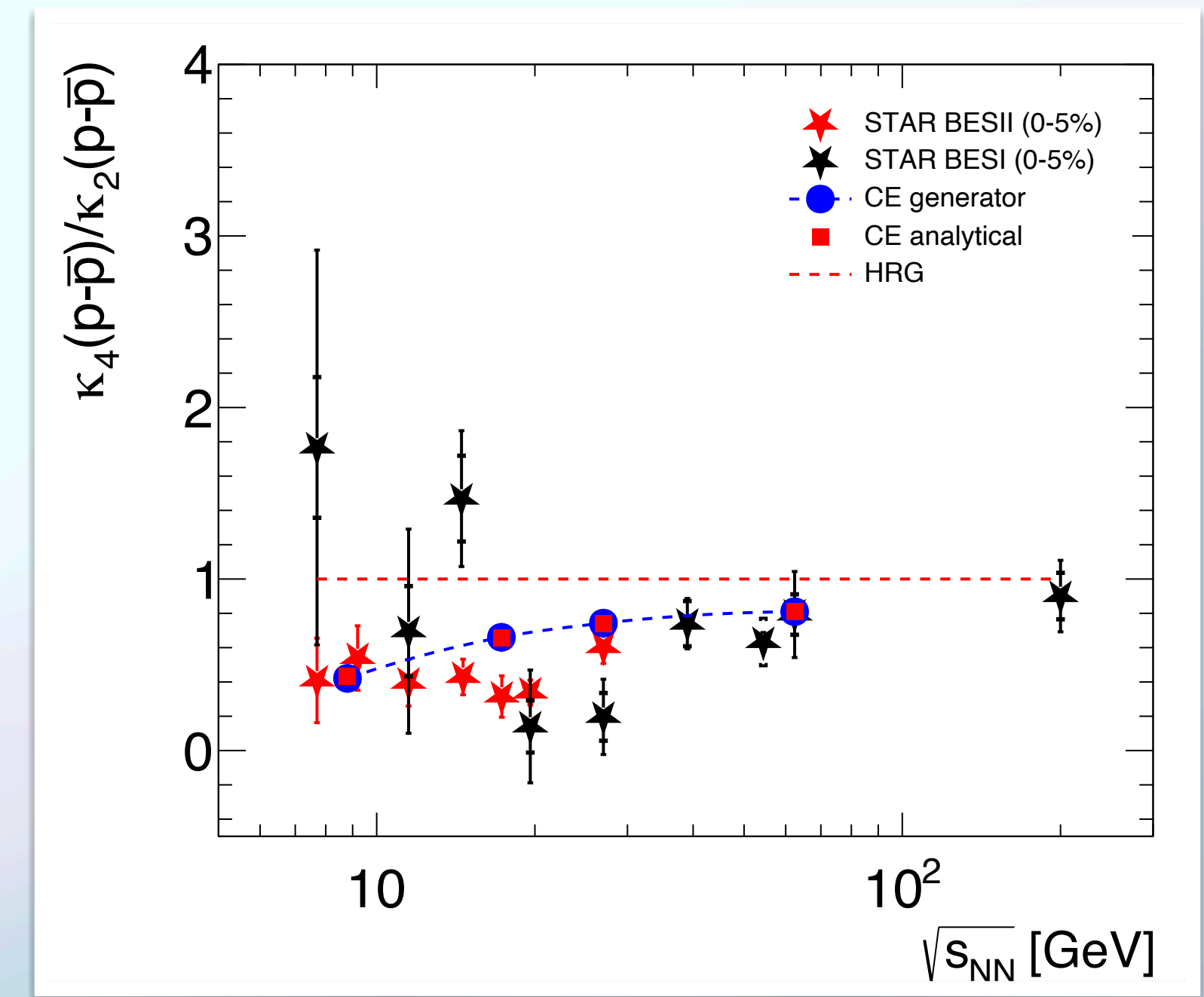
STAR: Centrality Bin Width Correction

X. Luo, J. Xu, B. Mohanty, and N. Xu, J.Phys. G40 (2013) 105104

Canonical baselines for cumulants vs. STAR data



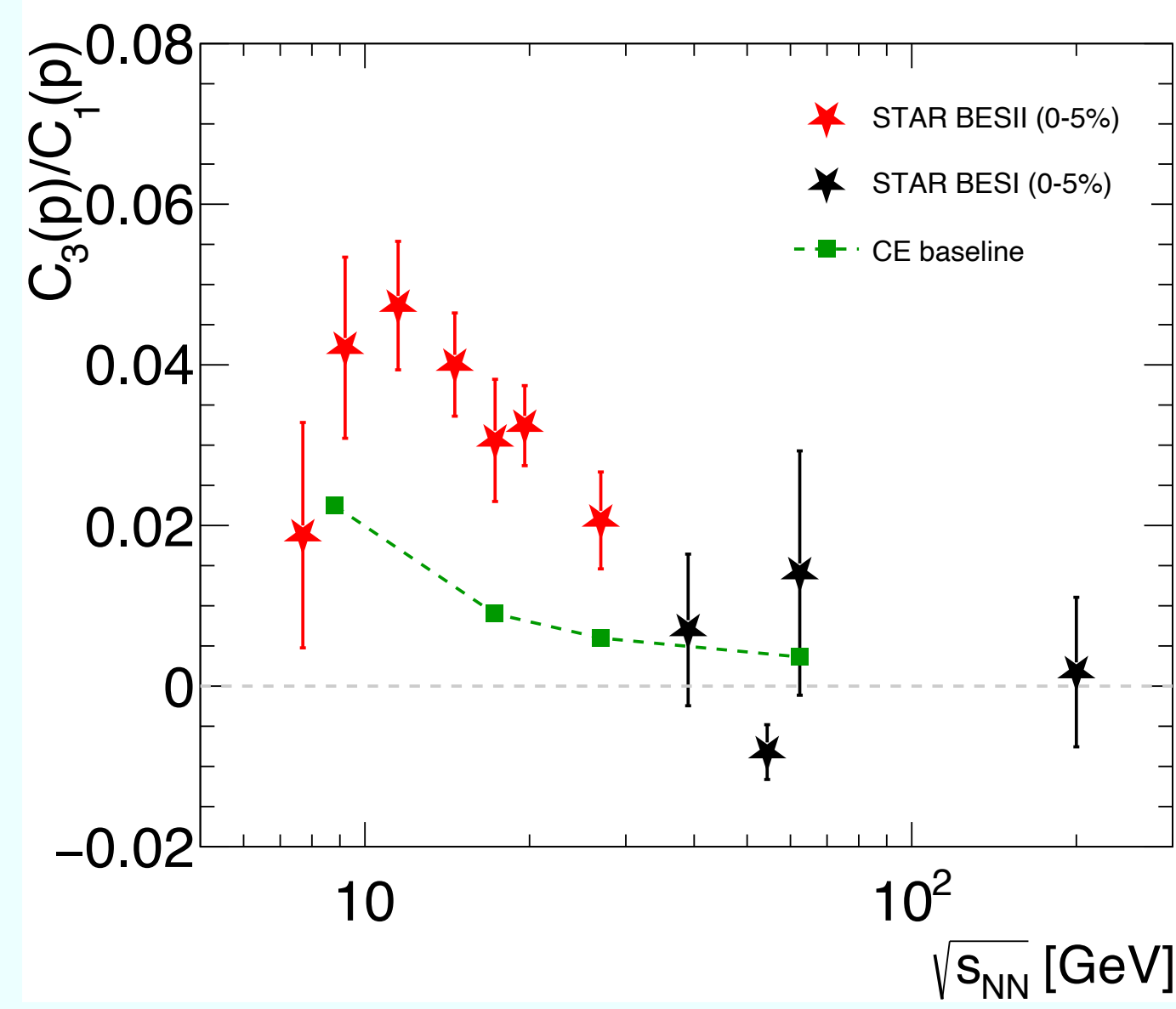
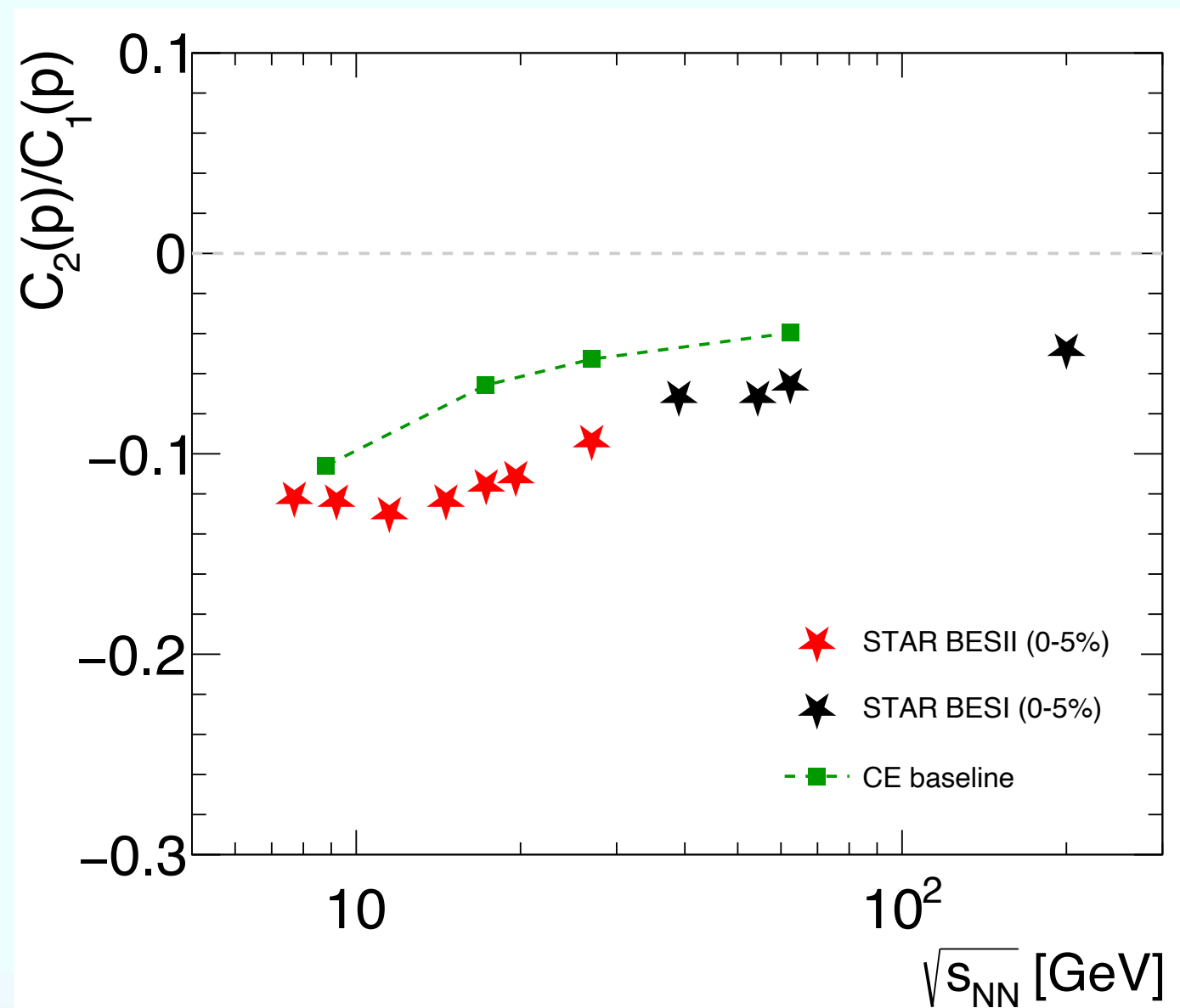
M. Stephanov, PRL102.032301(2009)
PRL107.052301(2011)



- STAR BESII : Phys.Rev.Lett. 126 (2021) 9, 092301
- STAR BESII: arXiv:2504.00817 [nucl-ex]
- CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

The STAR BESII data are not consistent with the the CE baseline

Factorial cumulants



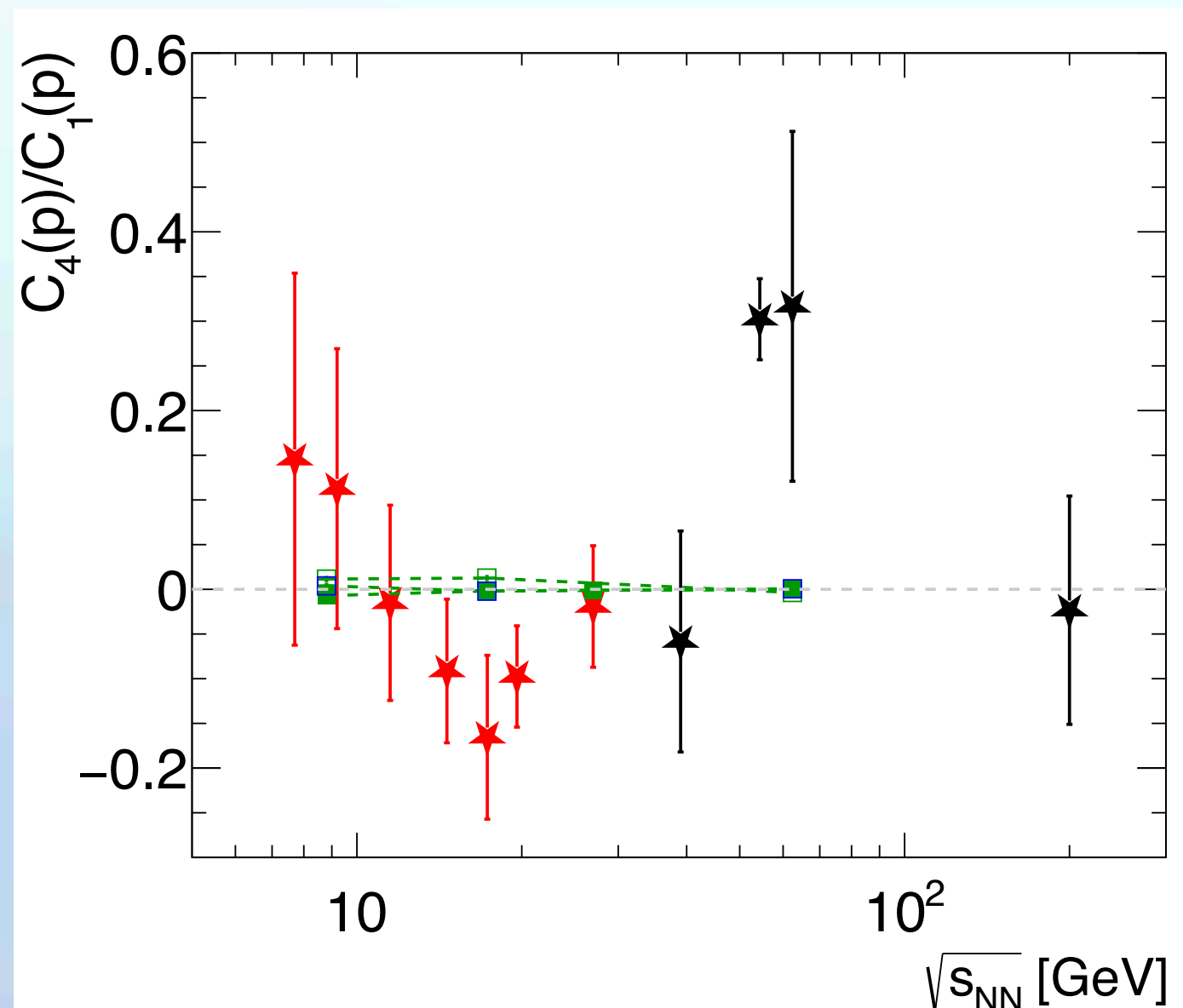
$$C_1(p) = \kappa_1(p) = \langle n_p \rangle$$

$$C_2(p) = -\kappa_1(p) + \kappa_2(p)$$

$$C_3(p) = 2\kappa_1(p) - 3\kappa_2(p) + \kappa_3(p)$$

$$C_4(p) = -6\kappa_1(p) + 11\kappa_2(p) - 6\kappa_3(p) + \kappa_4(p)$$

► R. Holzmam, V. Koch, A. R., J. Stroth, 2403.03598 (2024)



The STAR BESII data are not consistent with the the CE baseline

► STAR BESI : Phys. Rev. Lett. 126 (2021) 9, 092301

► STAR BESII: arXiv:2504.00817 [nucl-ex]

► CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

Modeling Correlations: Attractive and Repulsive

Attractive correlations

➤ Via cost function

$$\Delta_n = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

$$\rho = \frac{\langle y_1 y_2 \rangle - \langle y_1 \rangle \langle y_2 \rangle}{\sigma_{y_1} \sigma_{y_2}}$$

ρ - desired correlation coefficient

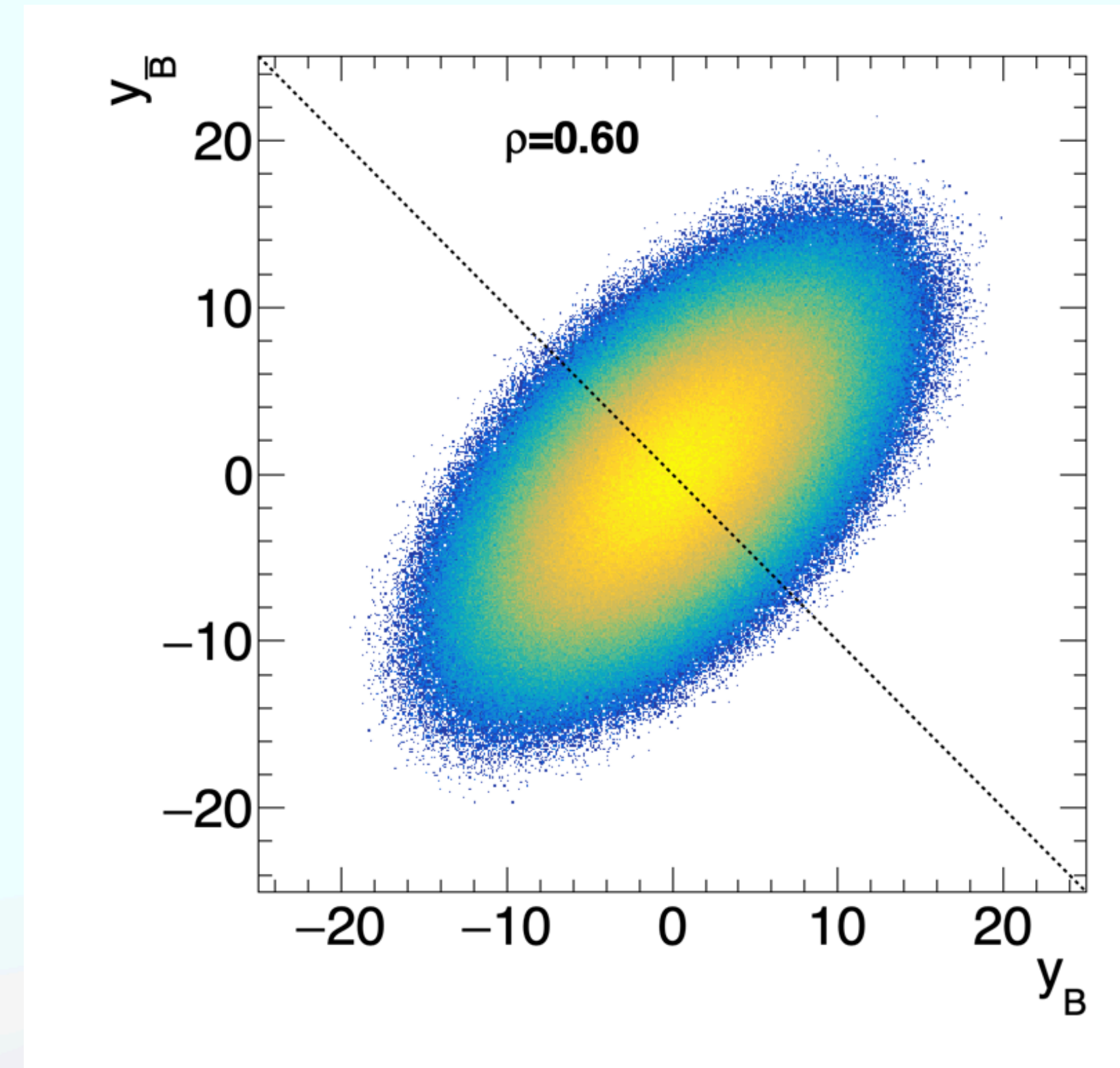
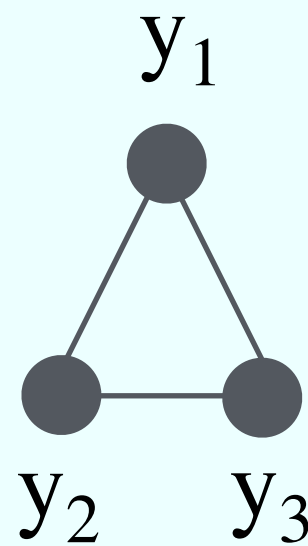
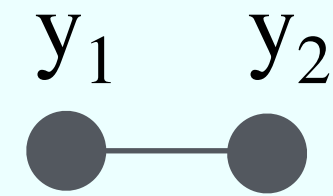
n - iteration step

► P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel,
JHEP 08 (2024) 113

➤ Via confining potential

$$E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$

► B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]



Modeling Correlations: Attractive and Repulsive

Attractive correlations

➤ Via cost function

$$\Delta_n = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

$$\rho = \frac{\langle y_1 y_2 \rangle - \langle y_1 \rangle \langle y_2 \rangle}{\sigma_{y_1} \sigma_{y_2}}$$

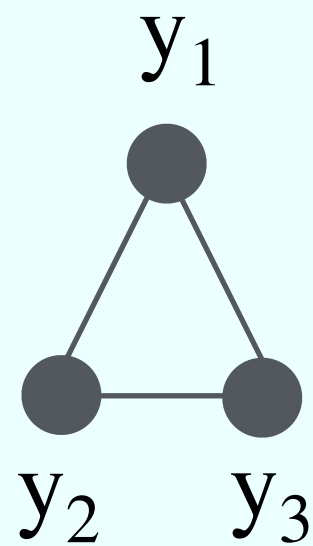
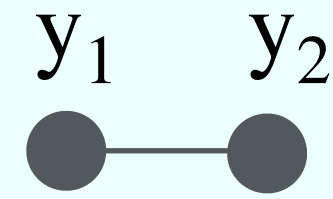
ρ - desired correlation coefficient

n - iteration step

► P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel,
JHEP 08 (2024) 113

➤ Via confining potential

$$E_a(y_1, y_2) = \alpha_a |y_1 - y_2|^{\beta_a}$$



Repulsive correlations

➤ Buckingham-type potential

$$E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|/\beta_r}$$

multi-particle correlations
are also considered

► B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]

Metropolis + Simulated annealing
acceptance probability

forces low-energy states to dominate

$$P(y_1, y_2) \sim \frac{e^{-E(y_1, y_2)}}{Z}$$

A note on notation

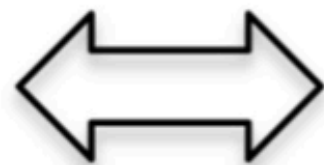
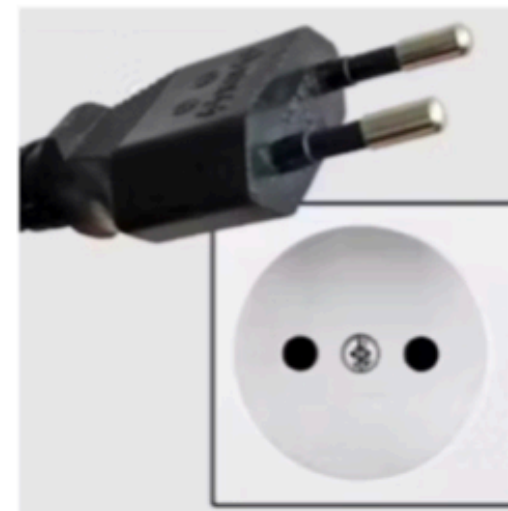
STAR

Cumulants (C)
Factorial cumulants (κ)



Others

Cumulants (κ)
Factorial cumulants (C)



M. Arslanok, QM 2025

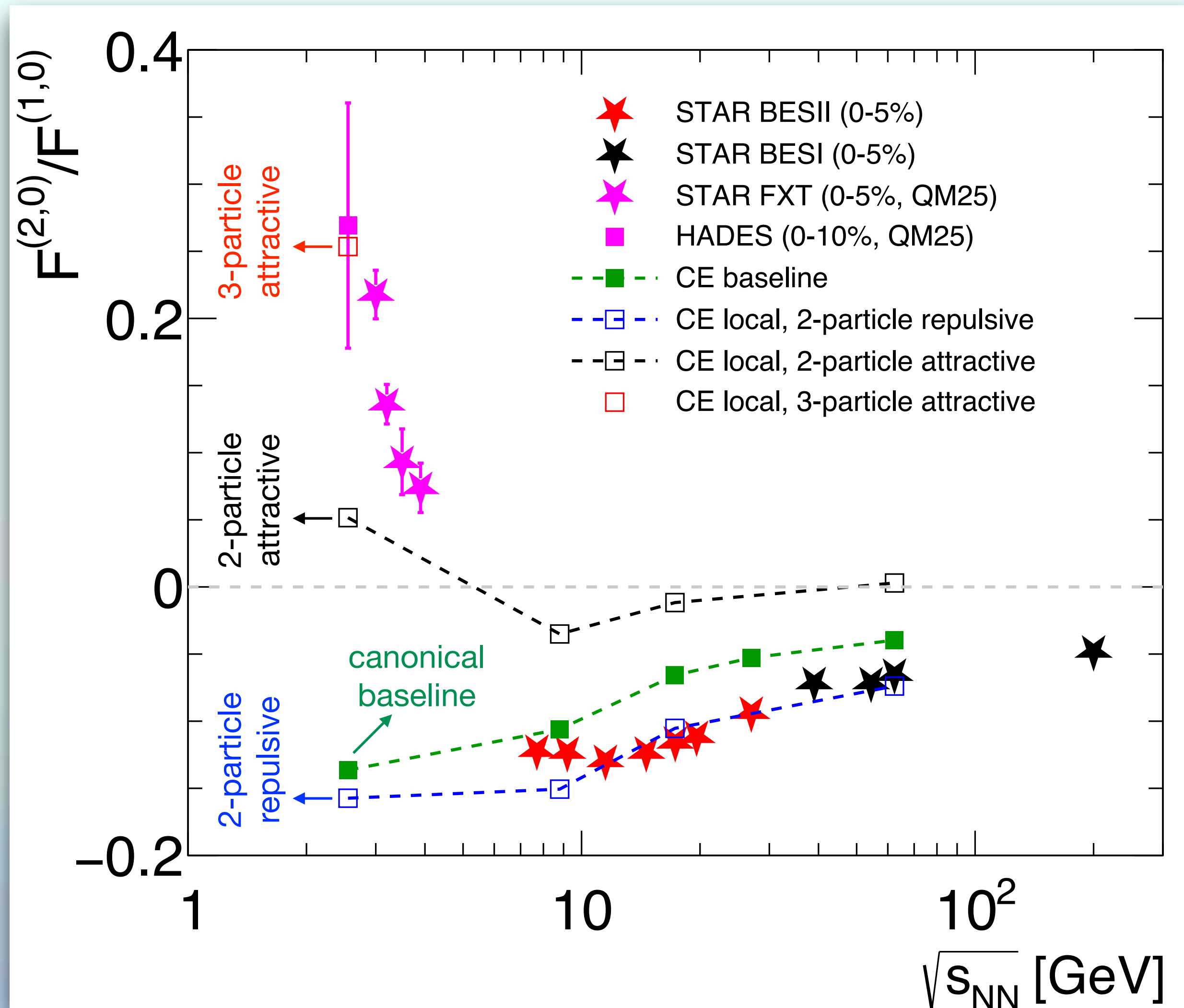
A note on notation

STAR Cumulants (C) Factorial cumulants (κ) 	Others Cumulants (κ) Factorial cumulants (C) 	Cumulants (κ) Factorial cumulants (FC) 	Cumulants (κ) Factorial cumulants ($F^{(n,0)}$) 	Cumulants (κ) Factorial cumulants ($\hat{C}$) 
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M. Arslan, QM 2025

► V. Vovchenko, Seattle Meeting

$F^{(2,0)}/F^{(1,0)}$ - fixing model parameters



STAR BES I-II: Au-Au, $-0.5 < y < 0.5$, $0.4 < p_t < 2$ GeV/c

STAR FXT: Au-Au, $-0.5 < y < 0$, $0.4 < p_t < 2$ GeV/c

HADES: Ag-Ag, $-0.4 < y < 0.4$, $0.4 < p_t < 1.6$ GeV/c

$$E_a(y_1, y_2) = \alpha_a |y_1 - y_2| \quad E_r(y_1, y_2) = \alpha_r e^{-|y_1 - y_2|}$$

Using **HADES data**, α_a is fixed

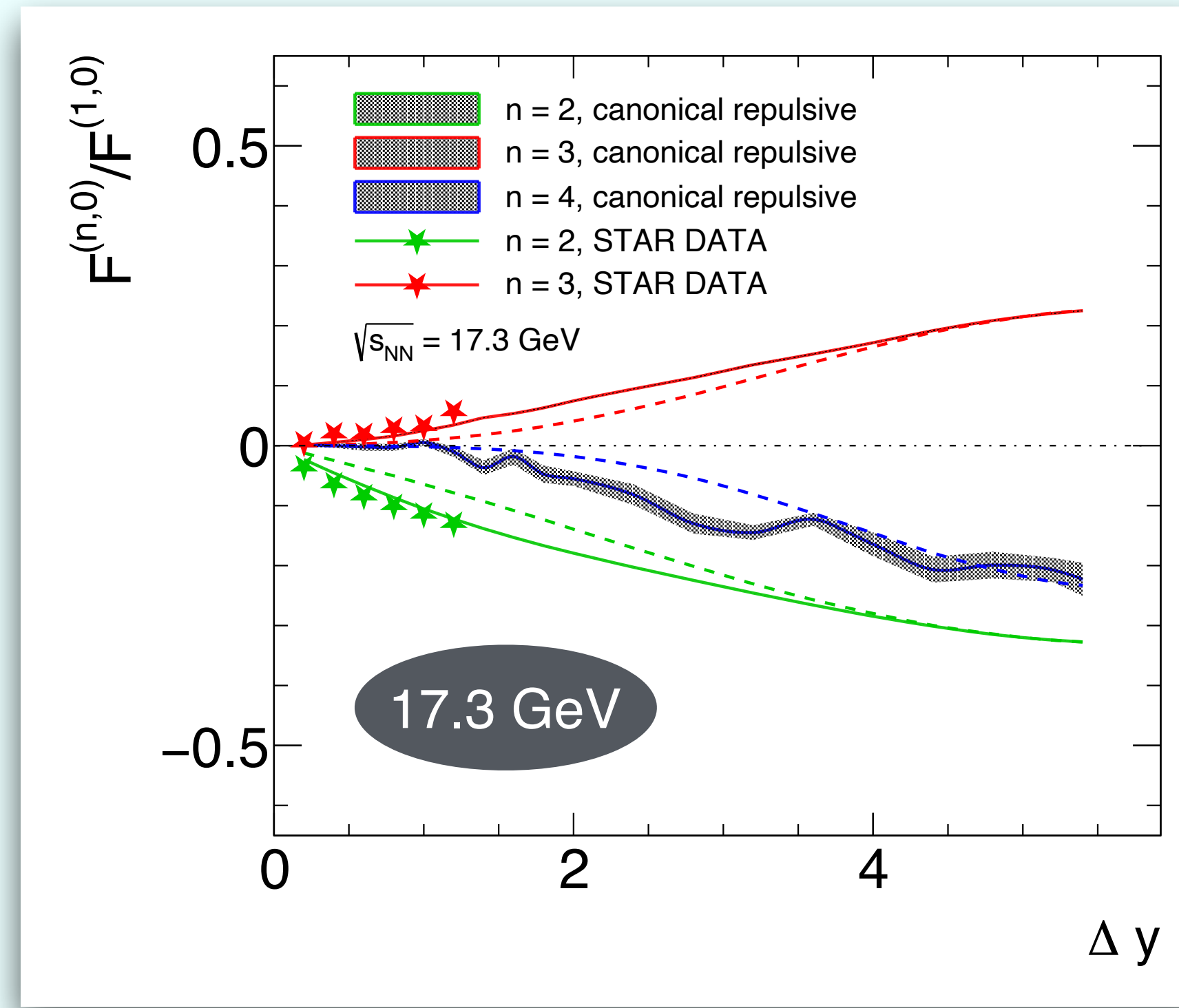
Using STAR data at 17.3 GeV α_r is fixed

Same α_r is used for all other STAR energies

- STAR BESI: Phys. Rev. Lett. 126 (2021) 9, 092301
- STAR BESII: arXiv:2504.00817 [nucl-ex]
- STAR FXT: Zachary Sweger, Quark Matter 2025

► B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]

STAR BES-II: Predictions for Repulsive Correlations



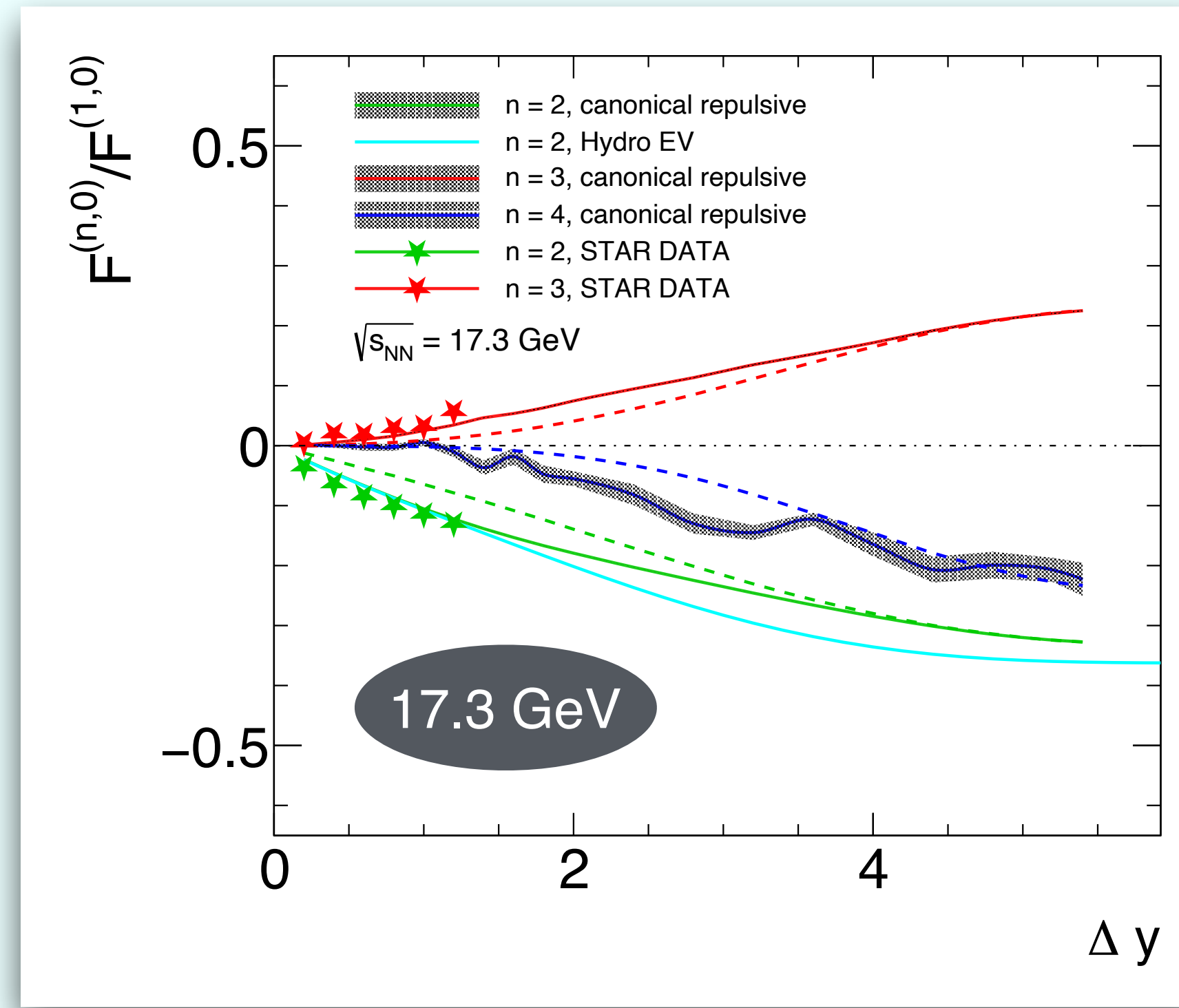
Dashed lines: Canonical baselines

STAR BES-II data → favors 2-particle repulsive interactions

Canonical baselines alone fail to capture the observed correlations

- Calculations: B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]
- STAR: Yige Huang, Quark Matter 2025

STAR BES-II: Predictions for Repulsive Correlations



Dashed lines: Canonical baselines

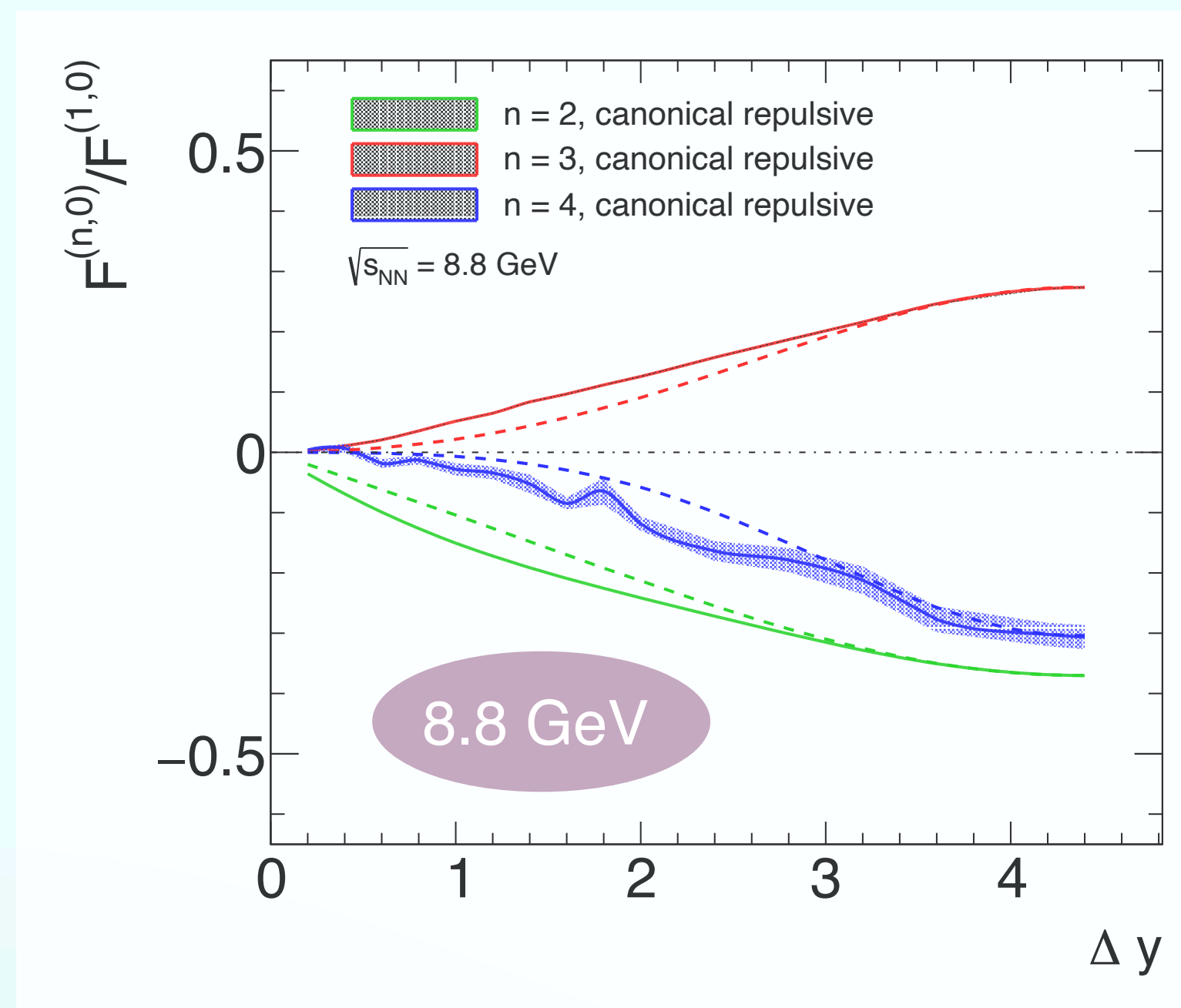
- Calculations: B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th], **Repulsion in momentum space**
- STAR: Yige Huang, Quark Matter 2025

Consistent with using hydrodynamics + excluded-volume HRG model. Repulsion in coordinate space

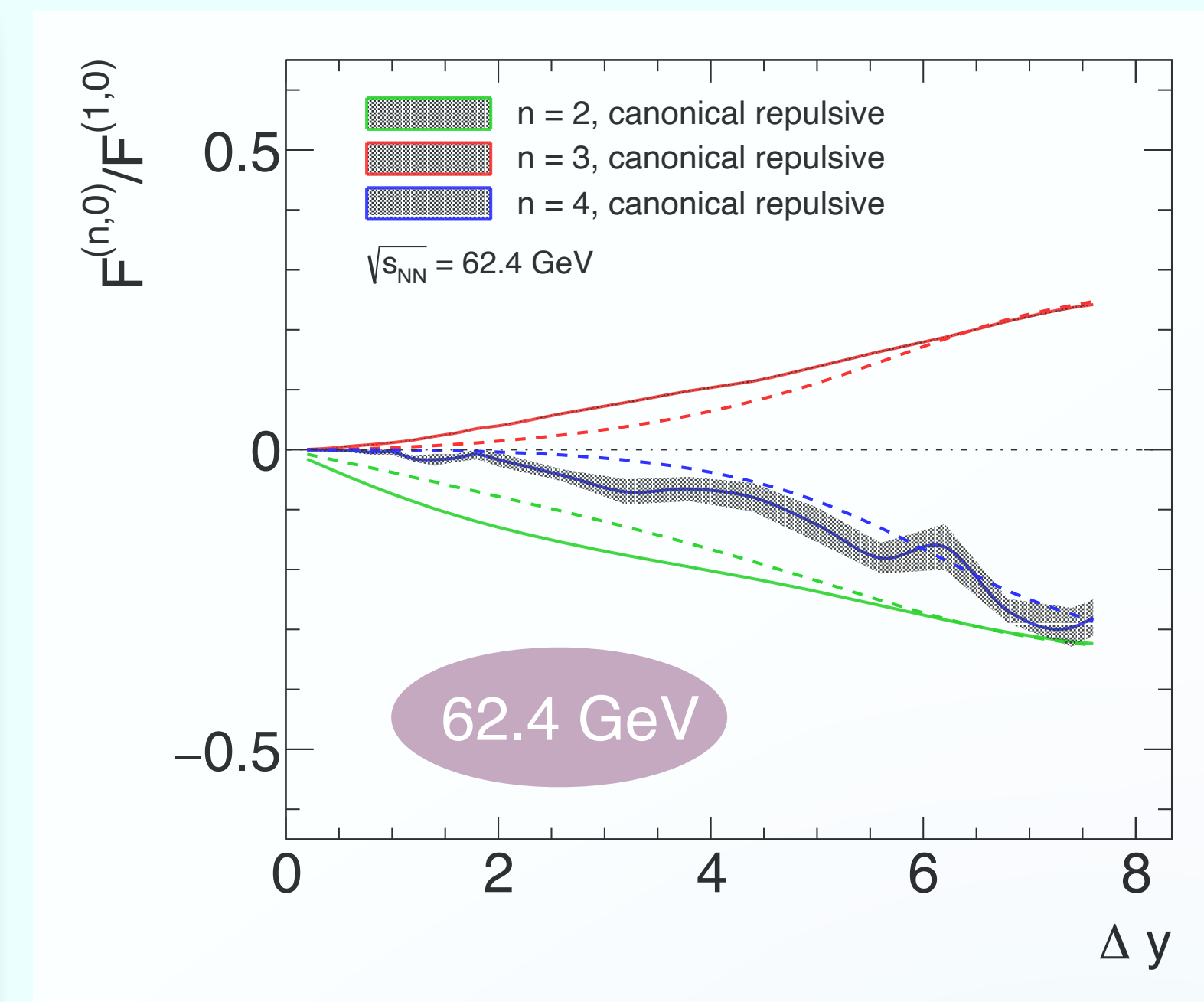
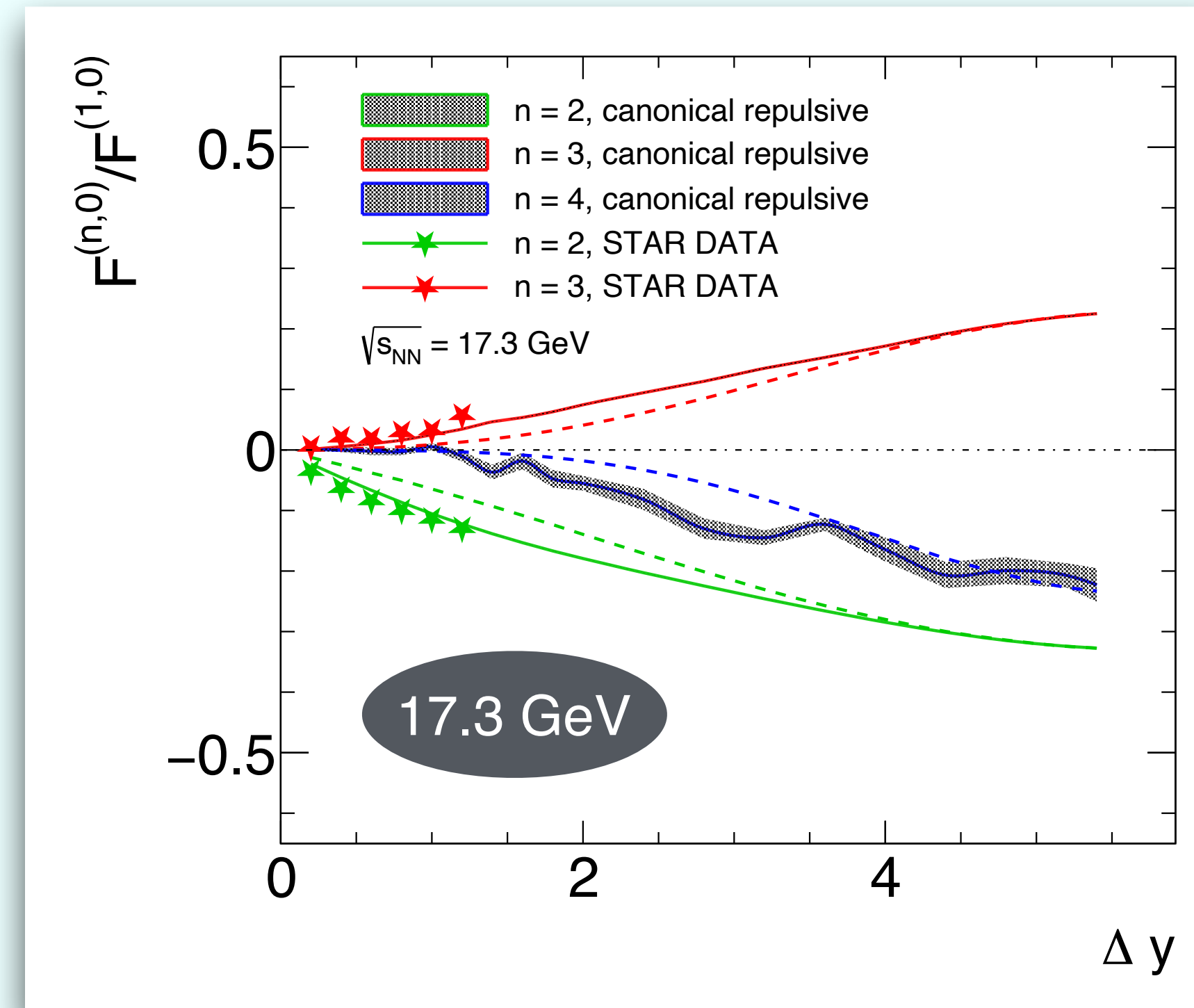
- V. Vovchenko et. al, Phys. Rev. C 105, 014904 (2022)

► **Note: The results of Hydro + EV at 17.3 GeV are obtained by interpolation between the two neighboring energies, 14.5 and 19.6 GeV**

STAR BES-II: Predictions for Repulsive Correlations



predictions



predictions

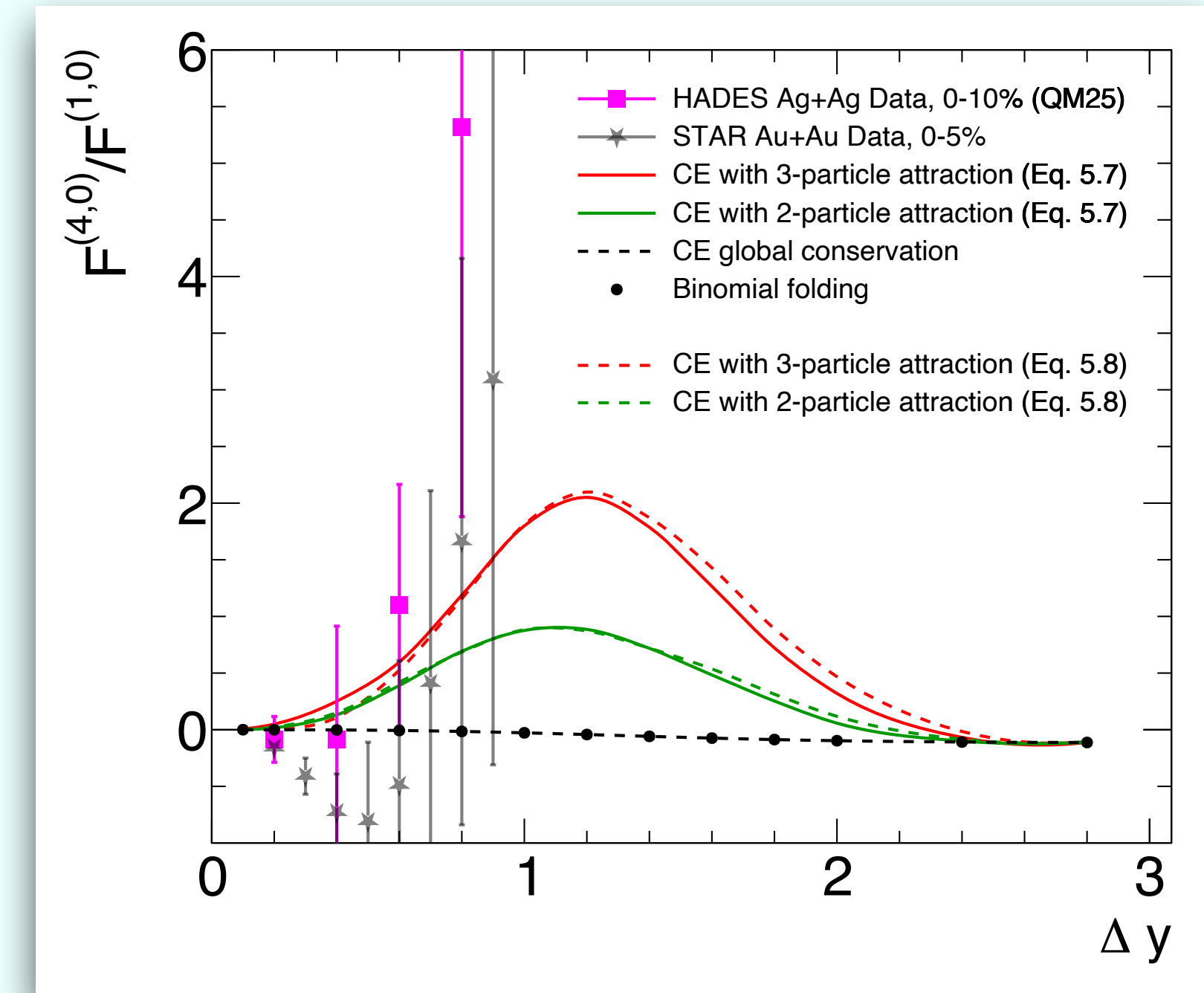
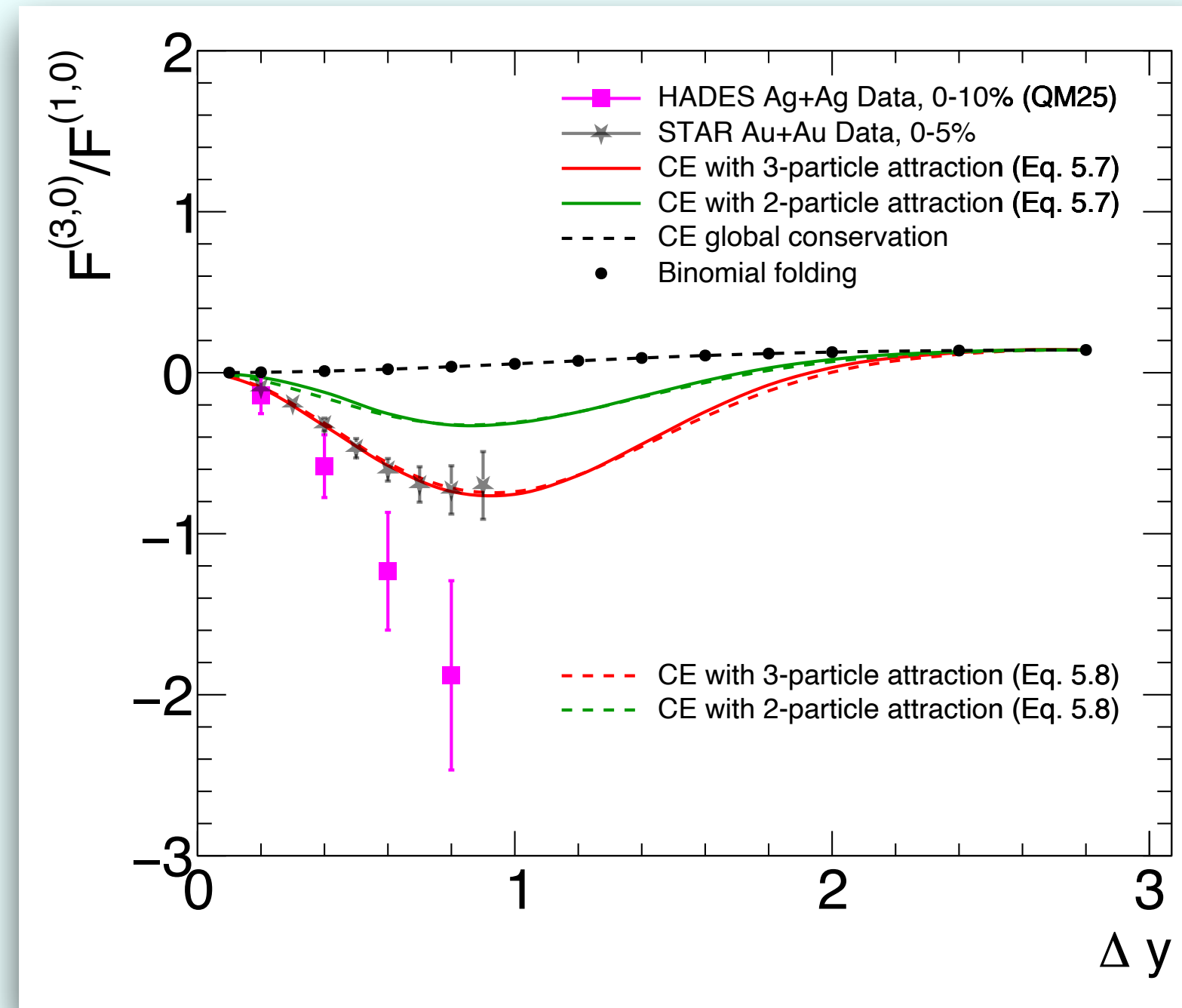
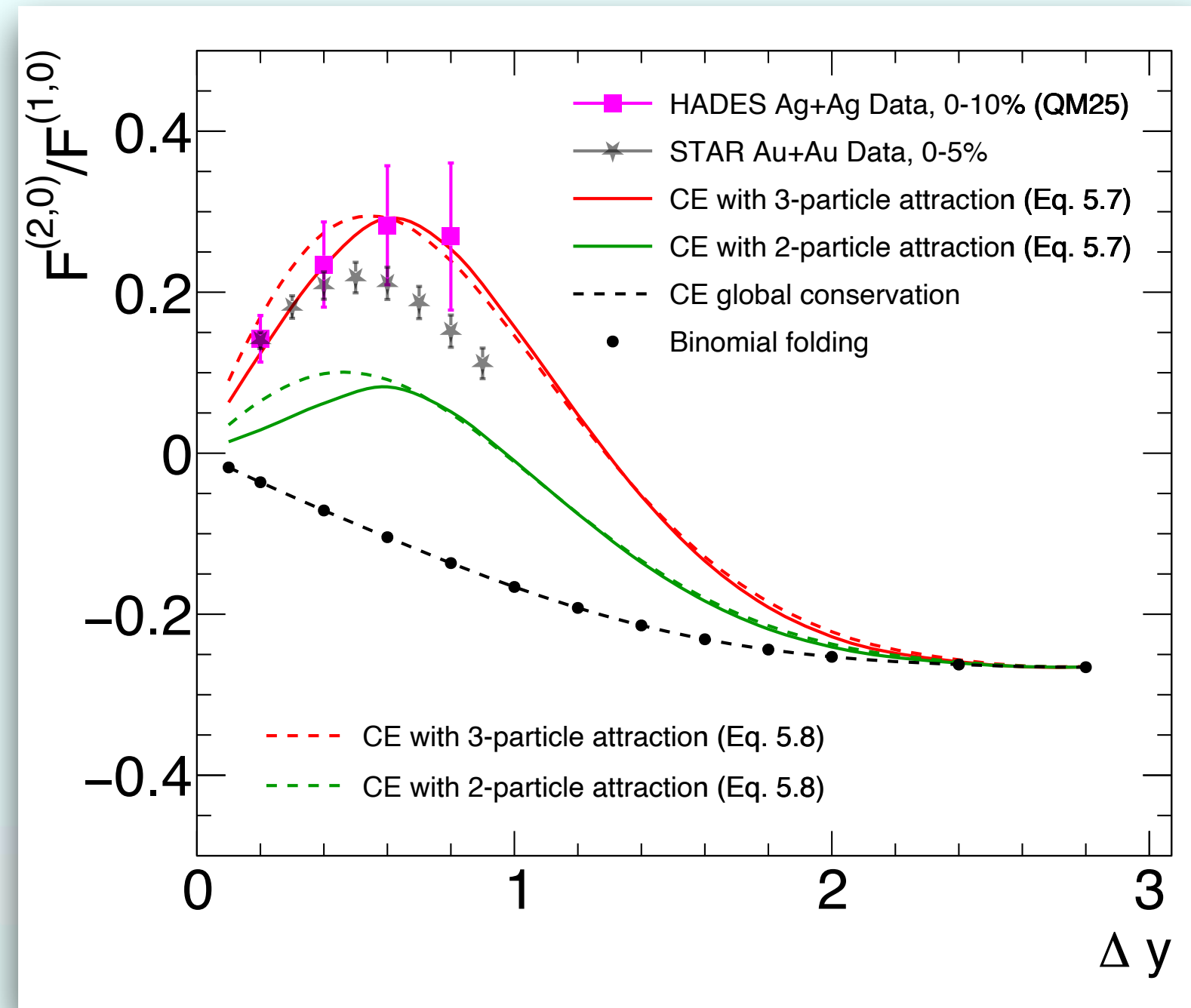
Dashed lines: Canonical baselines

STAR BES-II data $\rightarrow$ favors 2-particle repulsive interactions

Canonical baselines alone fail to capture the observed correlations

- Calculations: B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]
- STAR: Yige Huang, Quark Matter 2025

HADES: Predictions for Attractive Correlations



Dashed lines: Canonical baselines

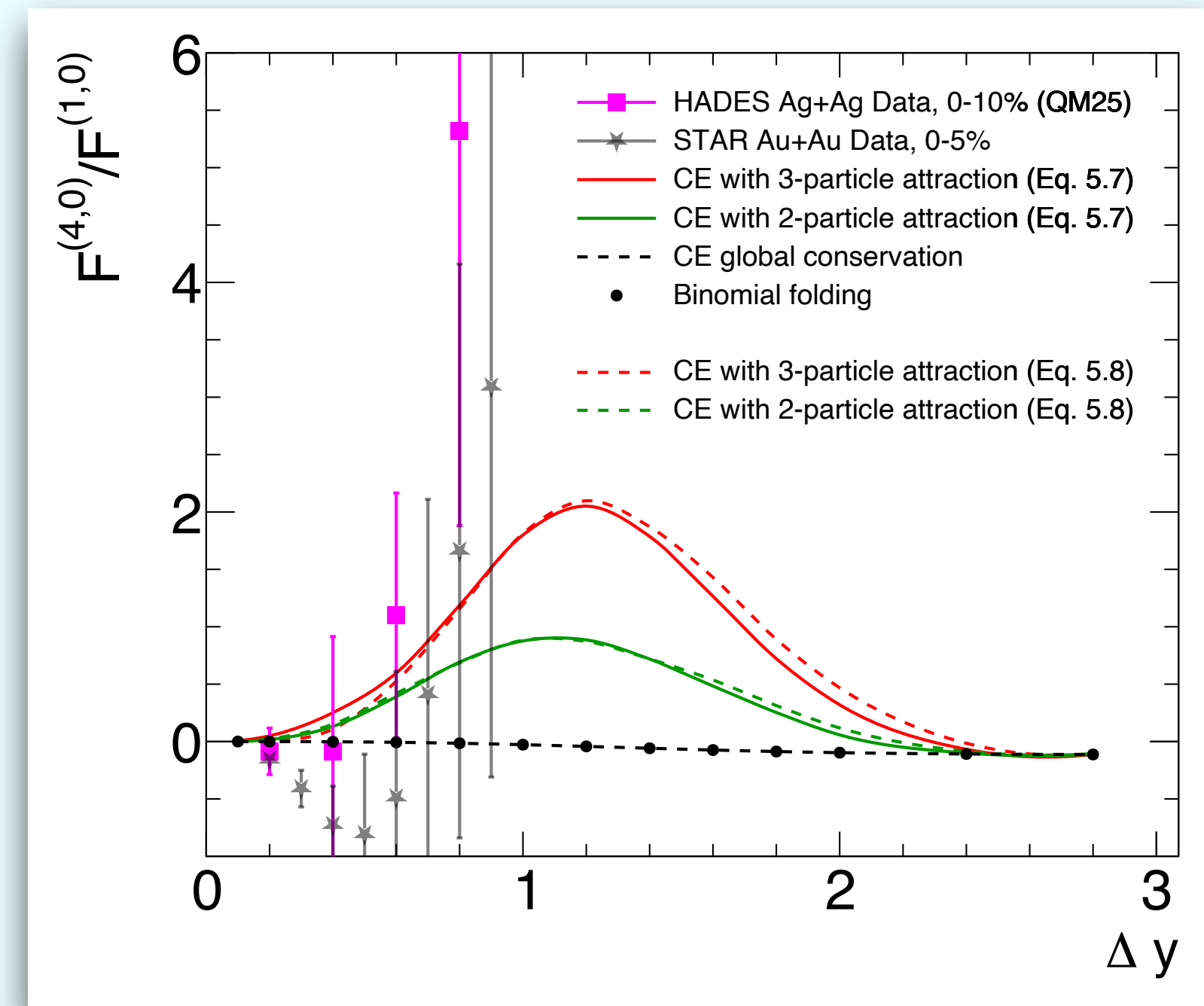
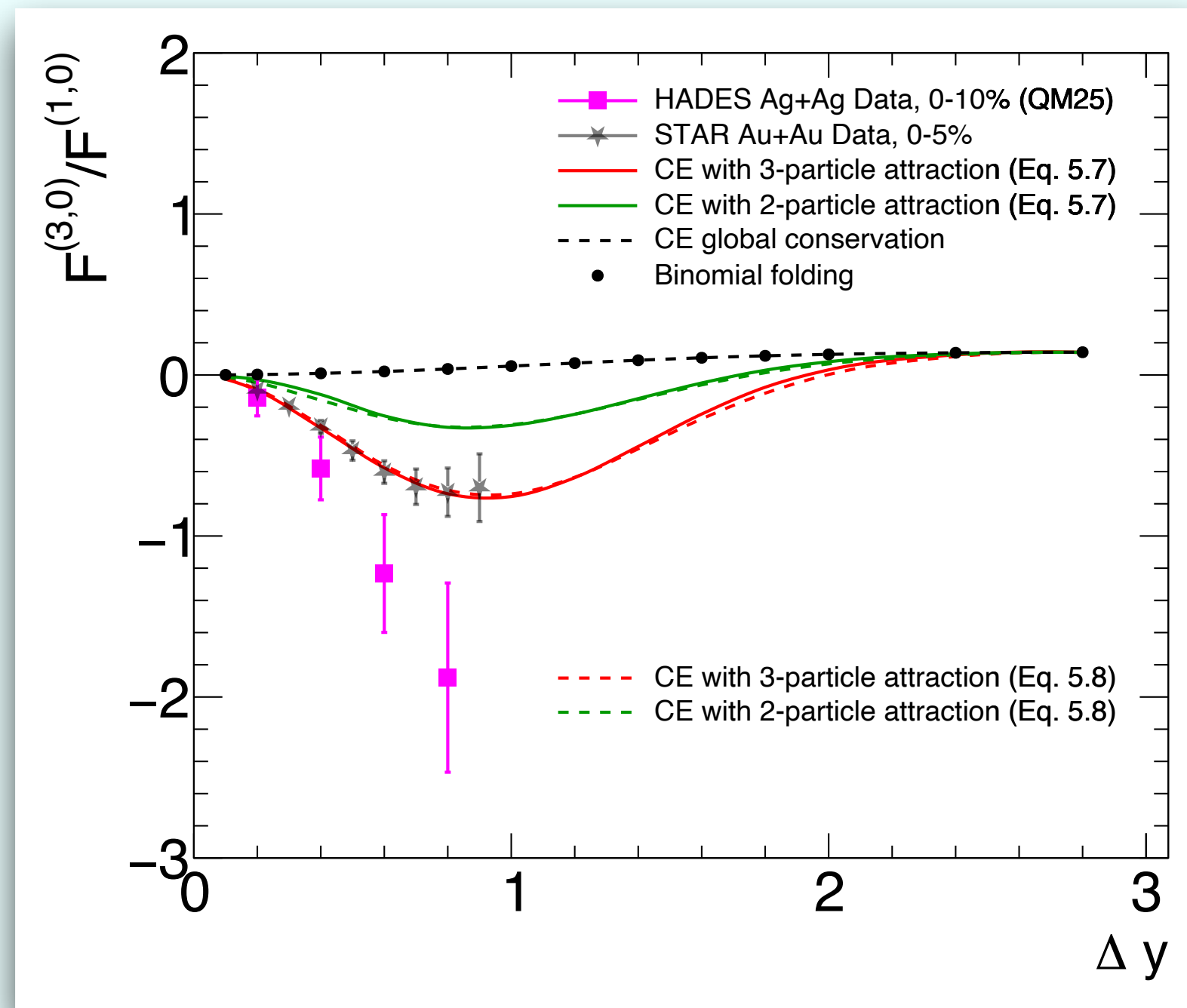
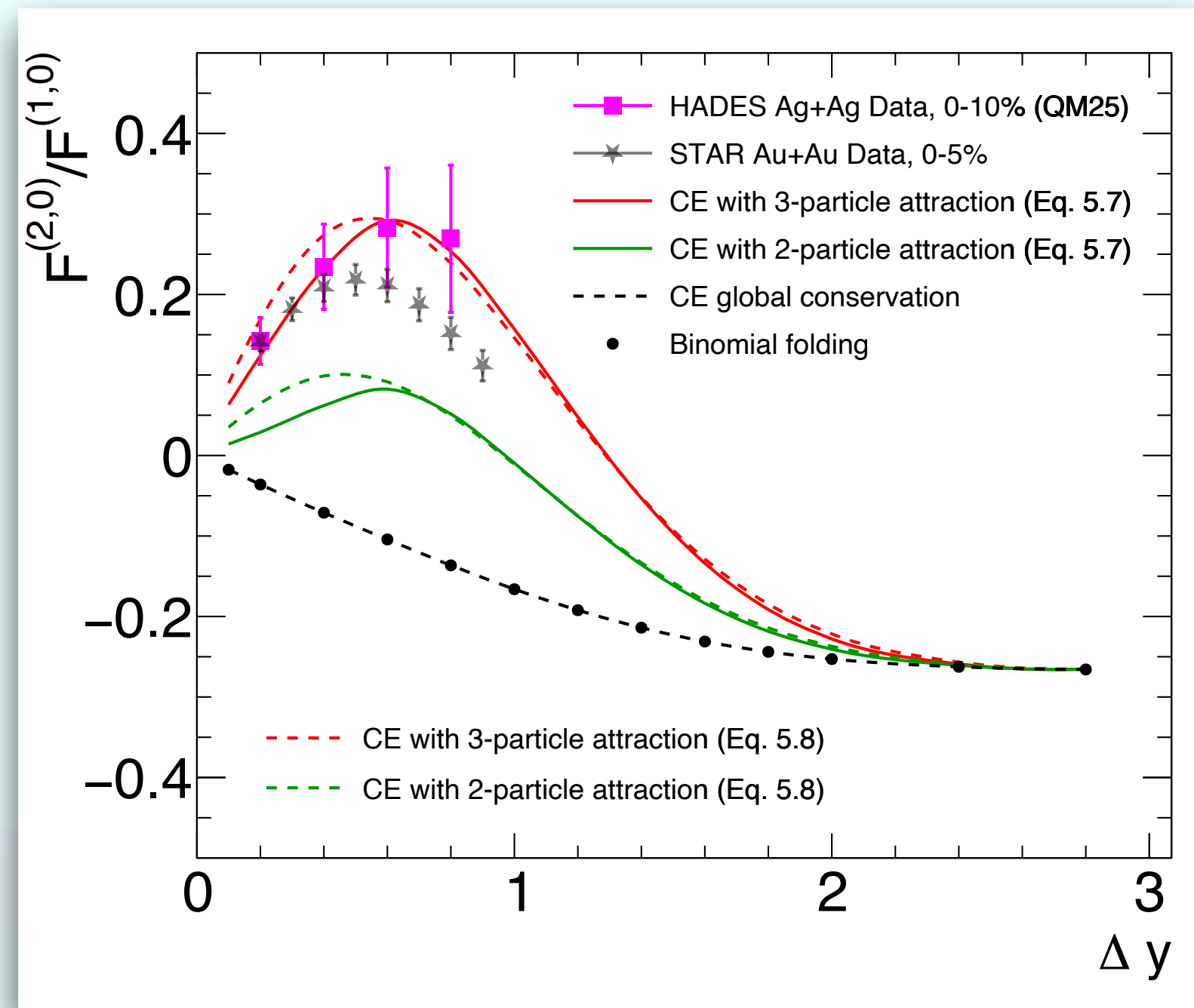
HADES data (with Fuzzy Logic) → prefers 3-particle attractive interactions

Canonical baselines alone fail to capture the observed correlations

► Calculations: B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]

► HADES: Marvin Nabroth, Quark Matter 2025

HADES & STAR-FXT Data



Dashed lines: Canonical baselines

HADES Ag–Ag @ 2.5 GeV and STAR FXT Au–Au @ 3 GeV show the same trend

- Calculations: B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]
- HADES: Marvin Nabroth, Quark Matter 2025
- STAR: Phys.Rev.C 107 (2023) 2, 024908

HADES: Ag-Ag, $\langle N_W \rangle \approx 170$

$-0.4 < y < 0.4$

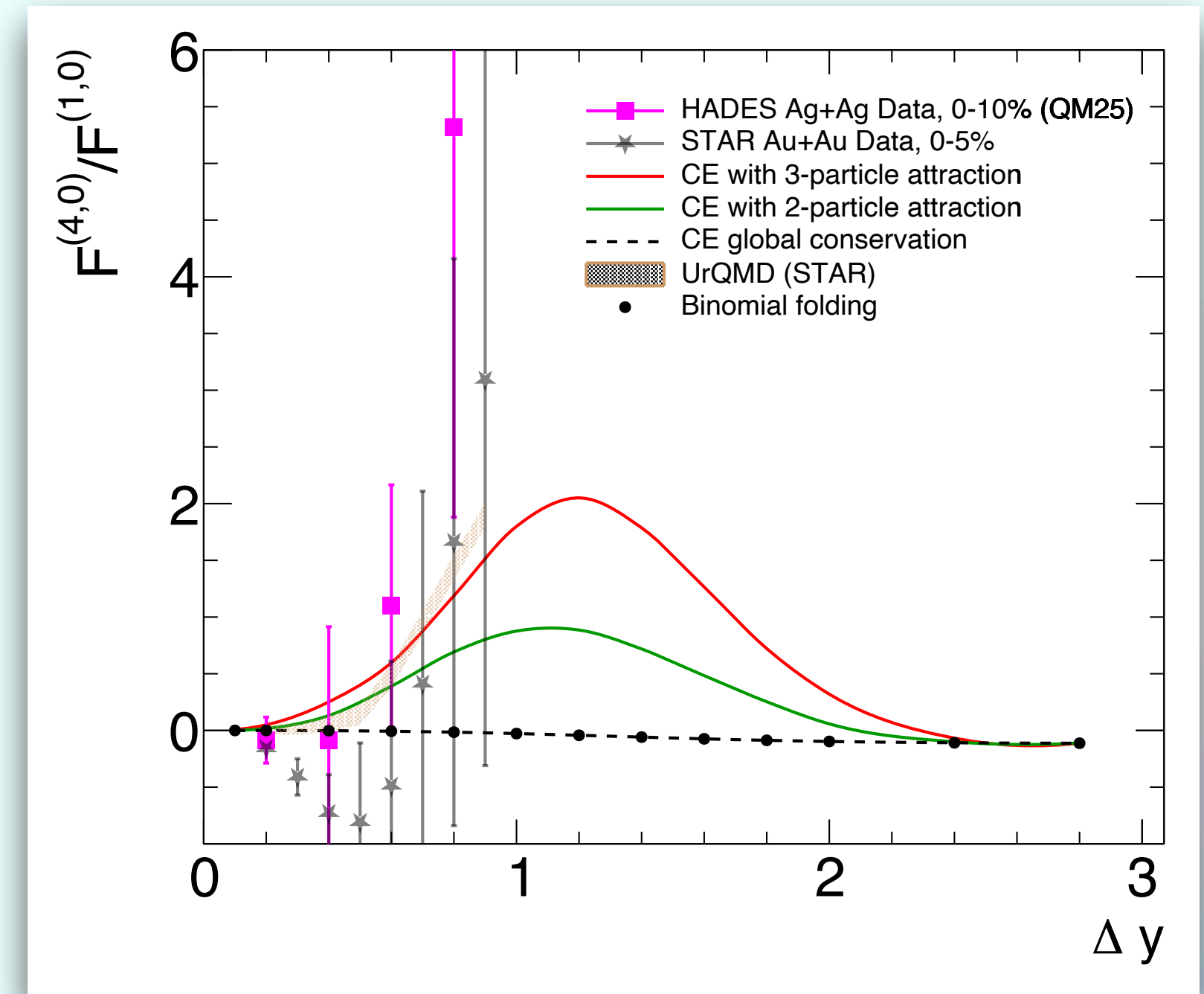
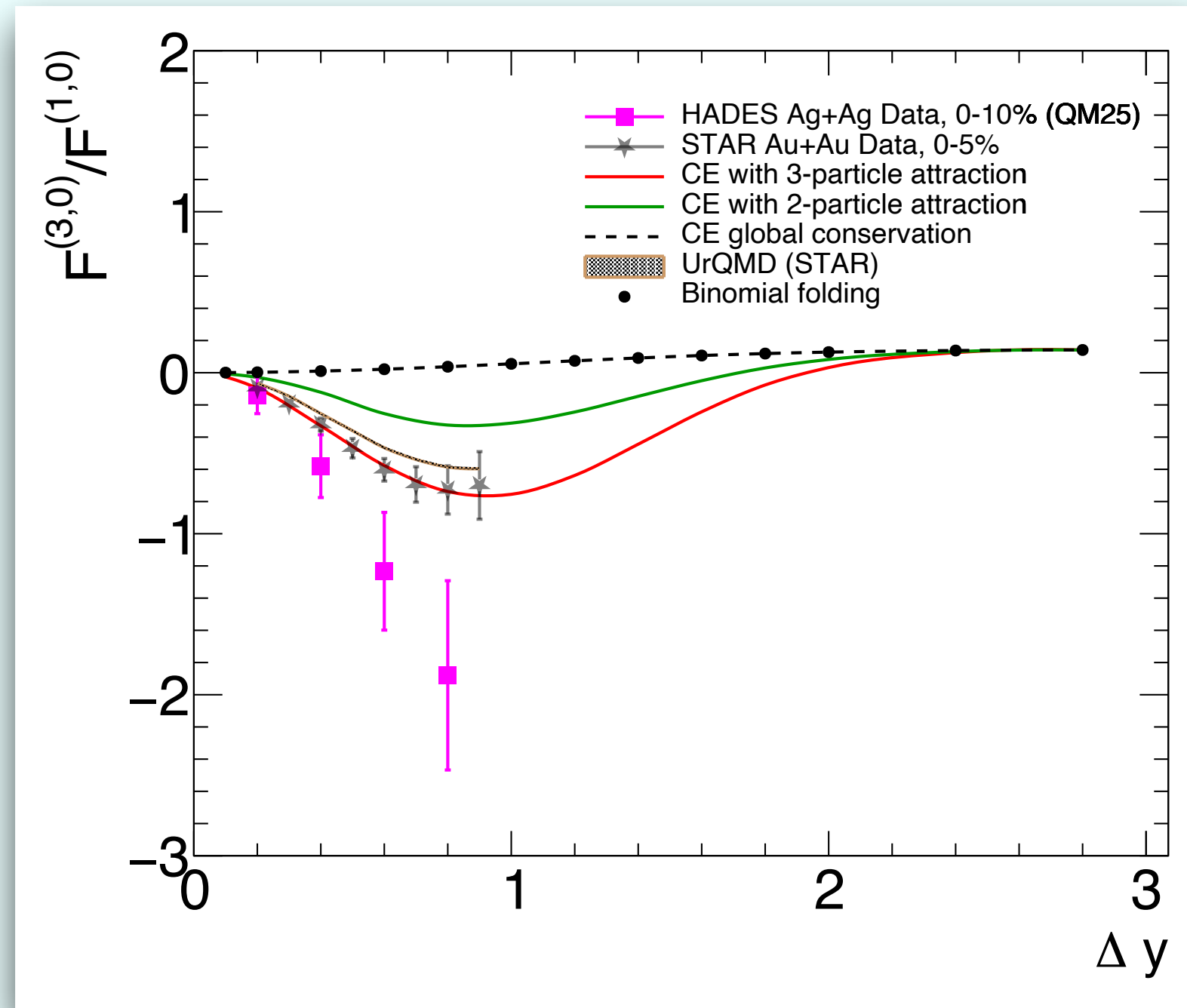
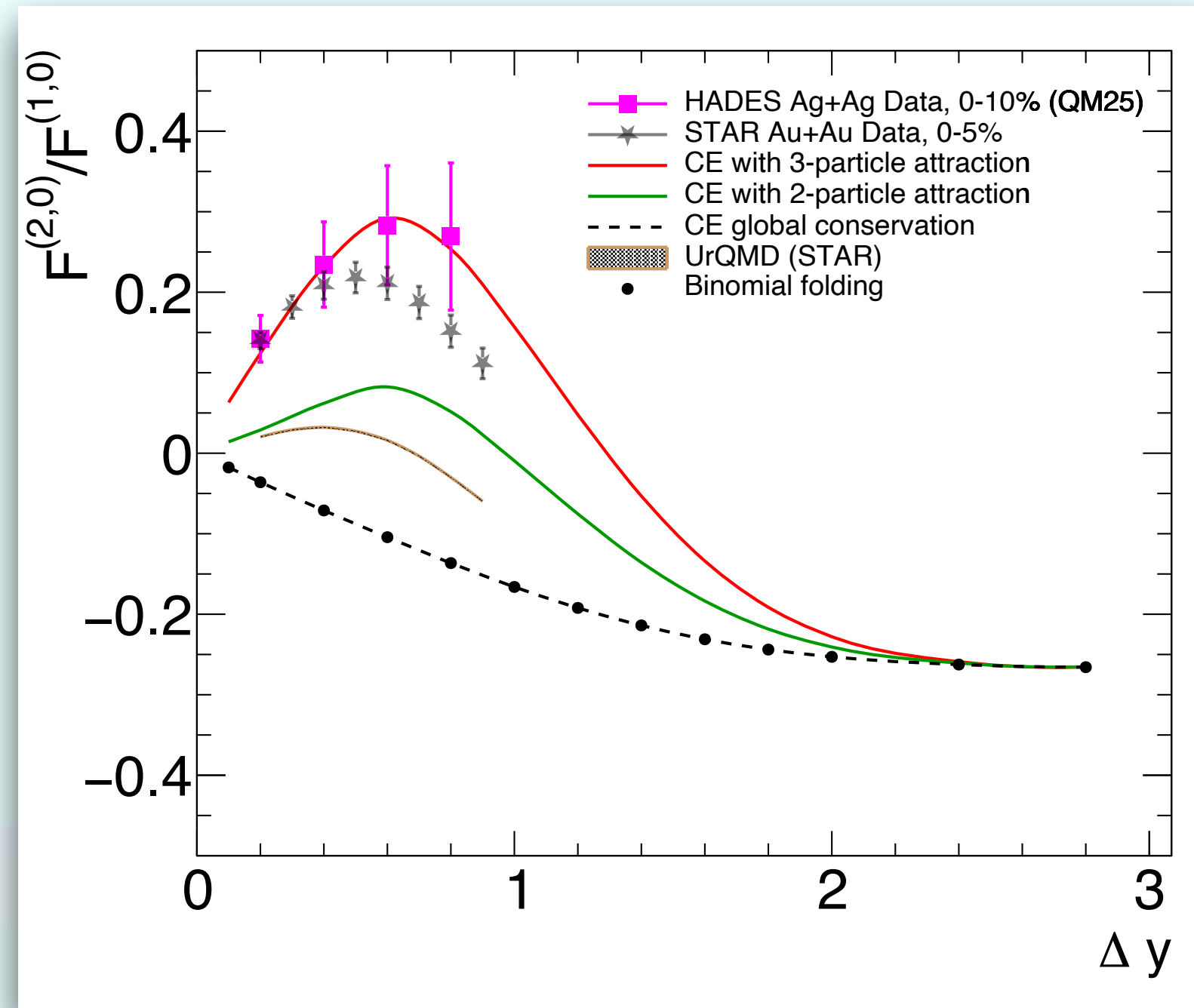
$0.4 < p_t < 1.6$ GeV/c

STAR FXT: Au-Au, $\langle N_W \rangle \approx 320$

$-0.5 < y < 0$

$0.4 < p_t < 2$ GeV/c

HADES & STAR-FXT Data vs. UrQMD



Dashed lines: Canonical baselines

UrQMD result for STAR data (no mean field)

Significantly below the data for $F^{(2,0)}/F^{(1,0)}$

Much better description for $F^{(3,0)}/F^{(1,0)}$ and $F^{(4,0)}/F^{(1,0)}$ 🤔

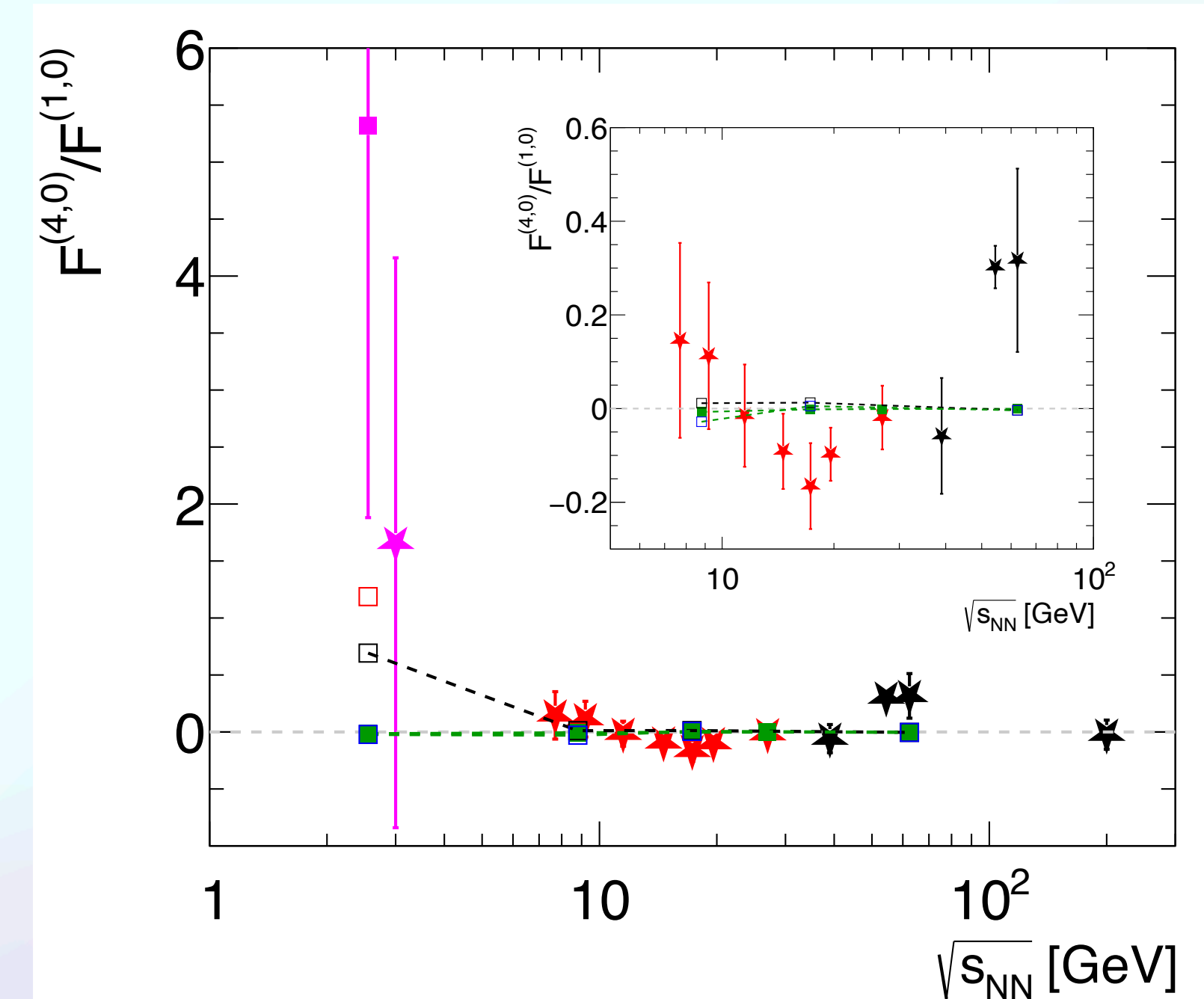
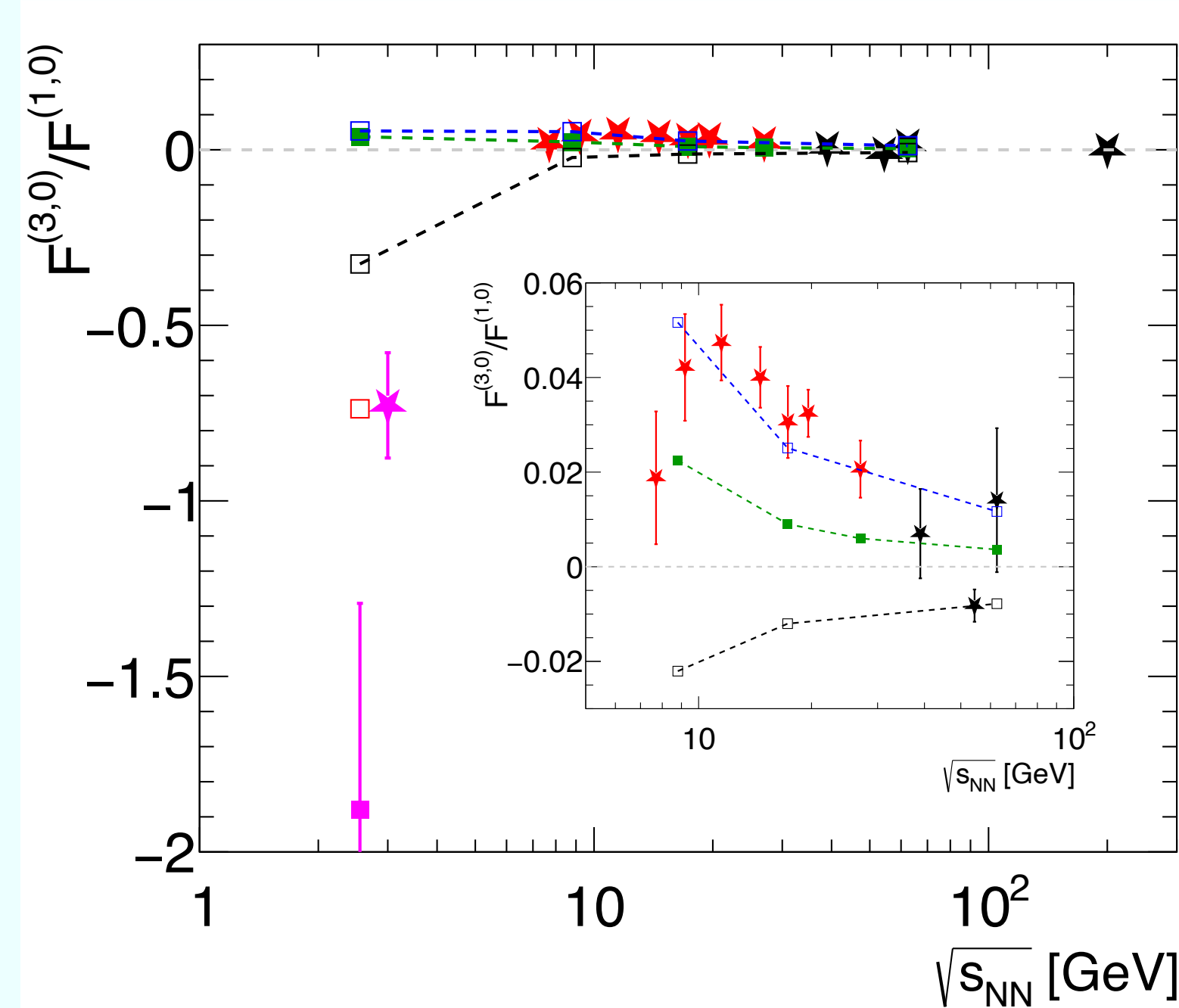
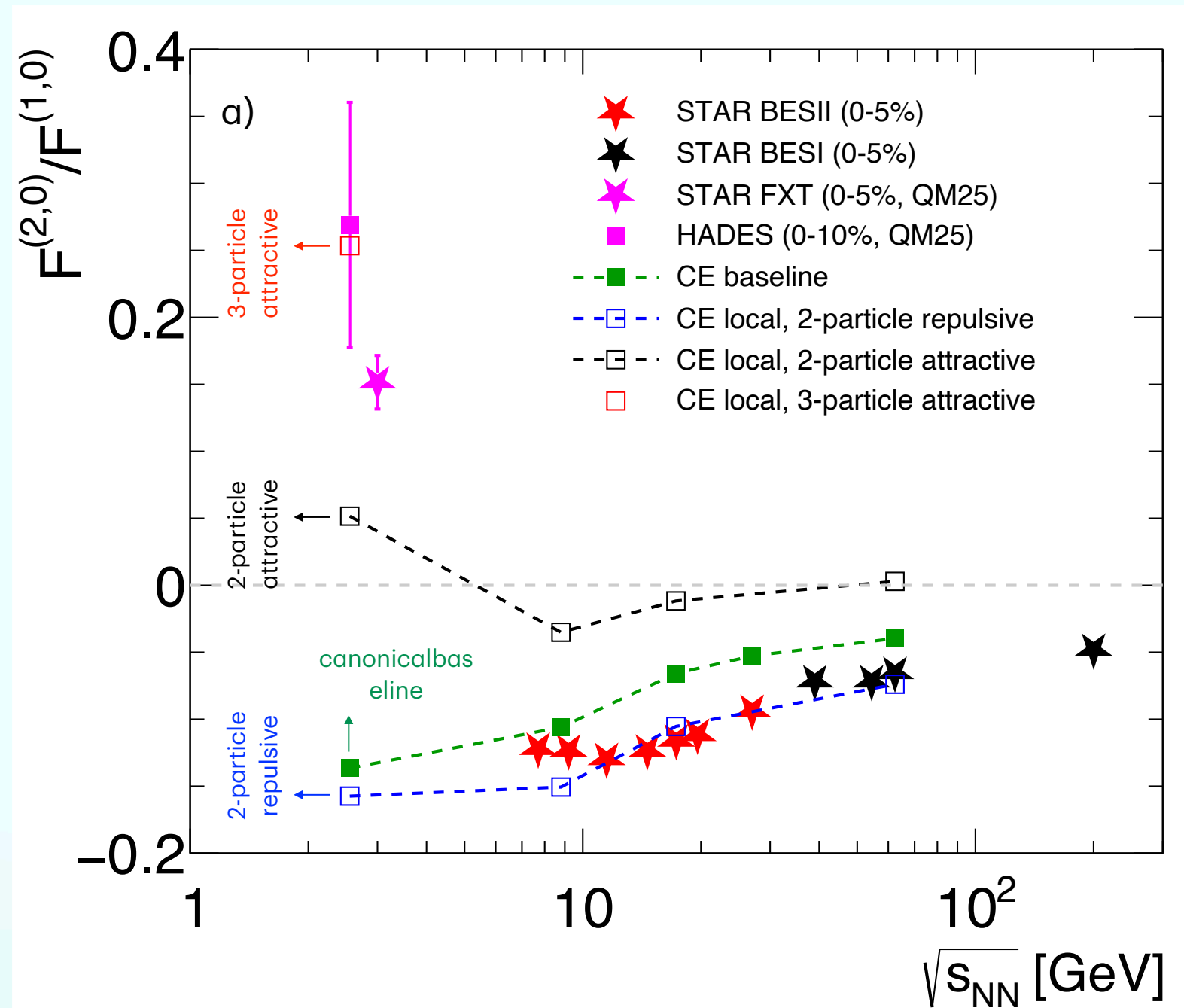
$$F^{(2,0)}(p) = -\kappa_1(p) + \kappa_2(p)$$

$$F^{(3,0)}(p) = 2\kappa_1(p) - 3\kappa_2(p) + \kappa_3(p)$$

$$F^{(4,0)}(p) = -6\kappa_1(p) + 11\kappa_2(p) - 6\kappa_3(p) + \kappa_4(p)$$

Important: What are the asymptotic values at large Δy from UrQMD?

Repulsive $\rightarrow$ Attractive, all cumulant ratios



The smoking gun

Repulsive $\rightarrow$ Attractive From High- to Low-Energies

- B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]
- D. DeMartini and E. Shuryak, Phys. Rev. C 104, 024908 (2021)

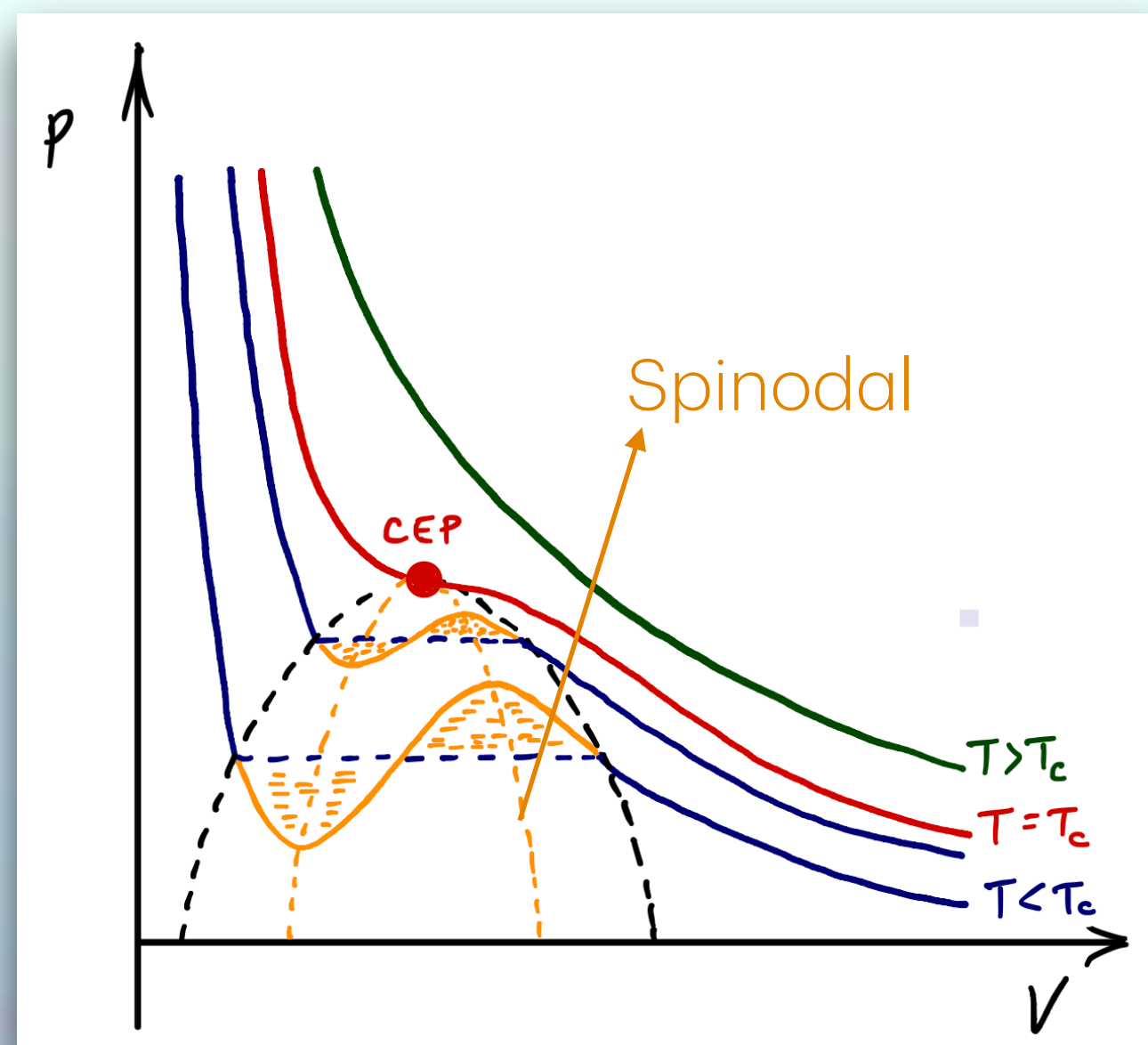
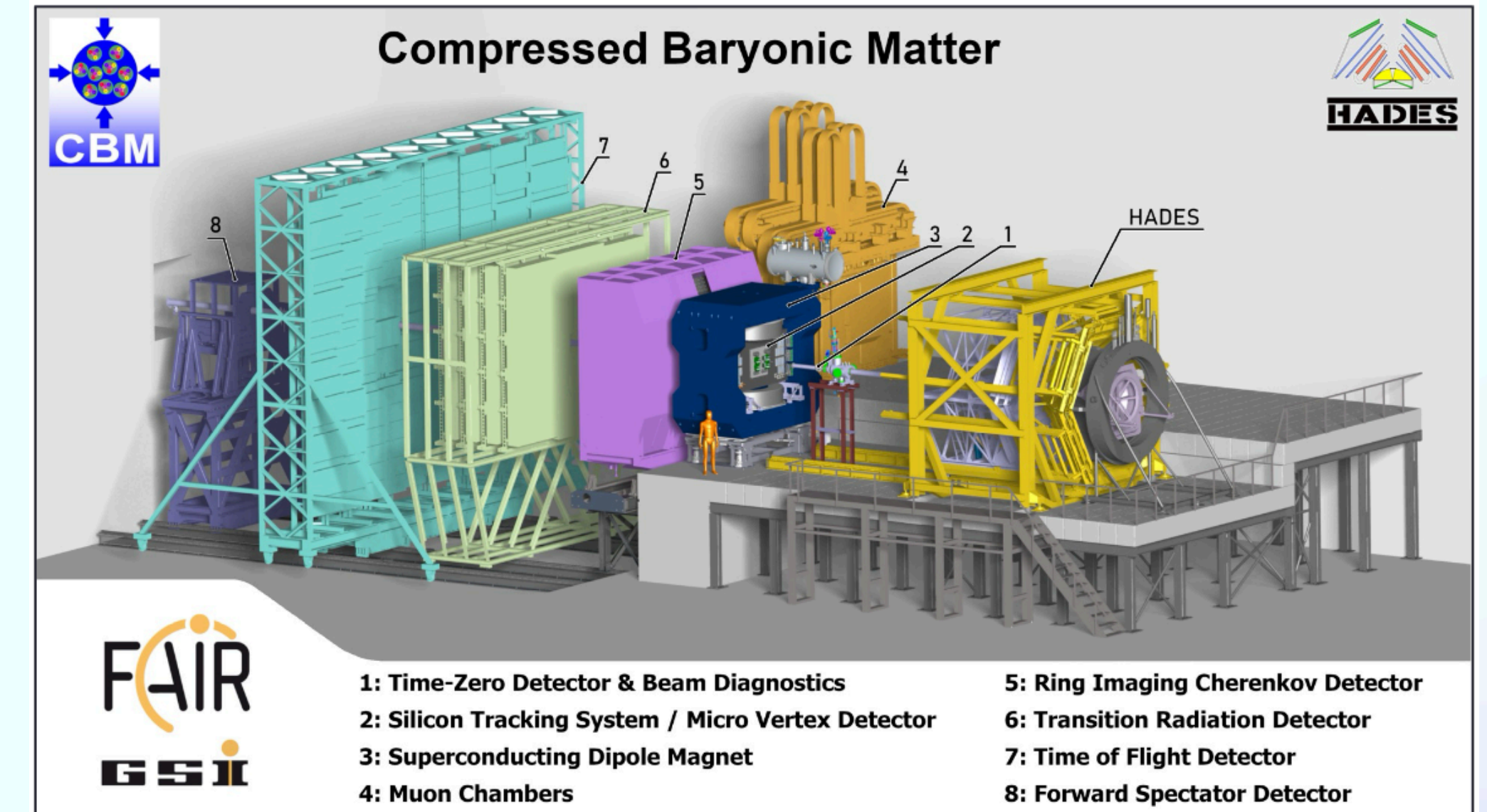
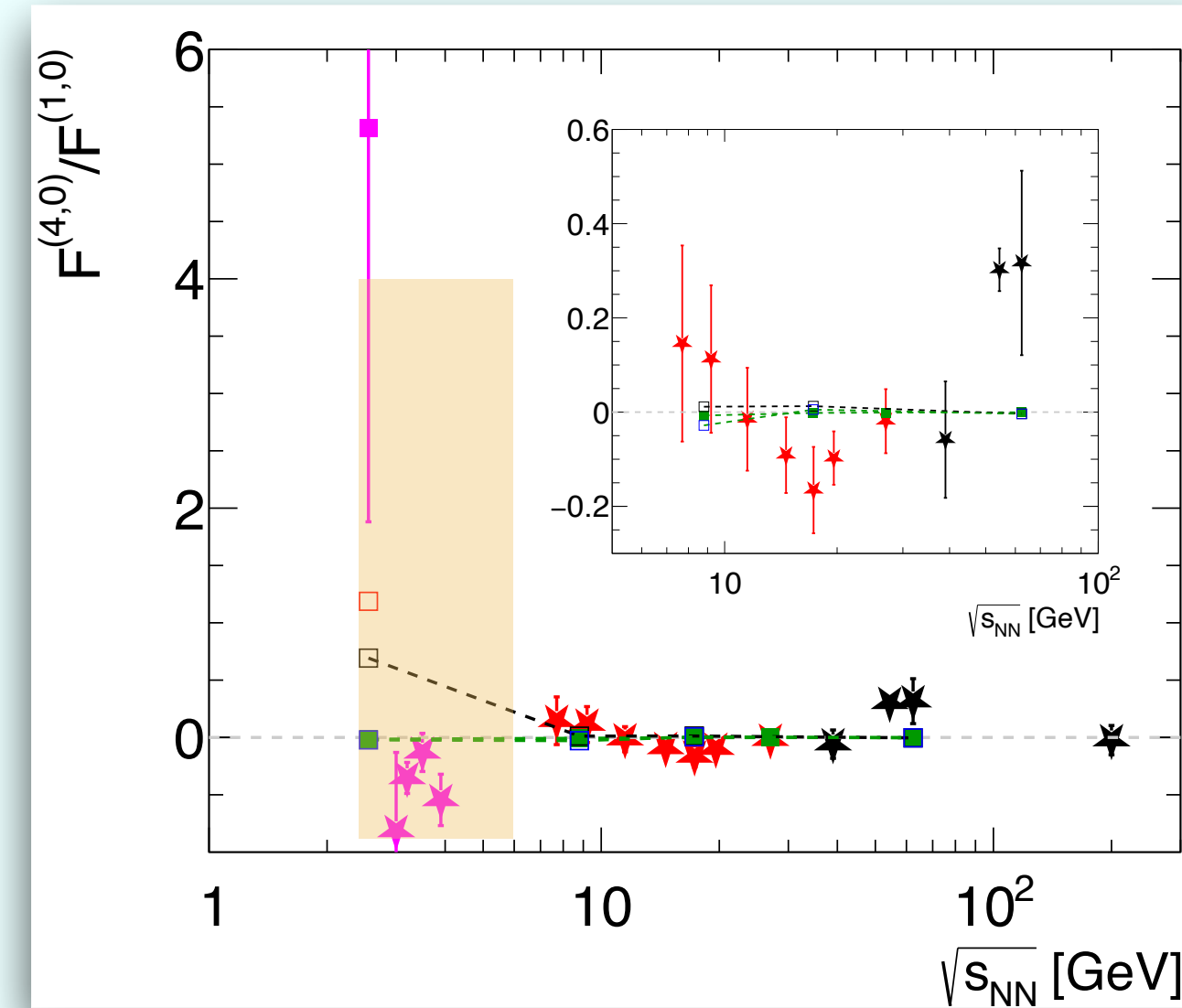
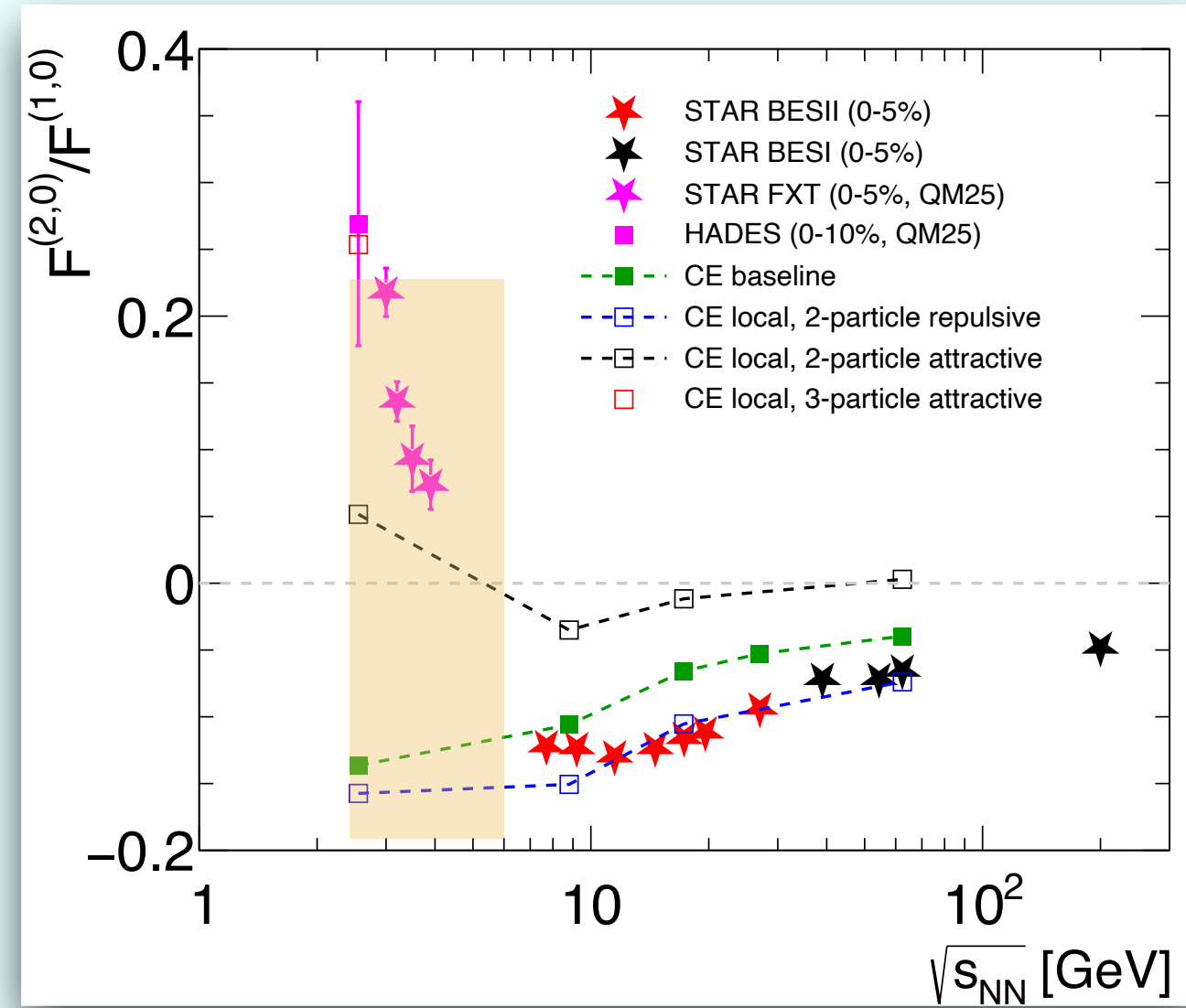
- STAR BESI: Phys. Rev. Lett. 126 (2021) 9, 092301
- STAR BESII: arXiv:2504.00817 [nucl-ex]
- STAR FXT: Zachary Sweger, Quark Matter 2025

Summary

Fluctuations in Particle Numbers: A Window into the QCD Phase Structure

- ▶ **Canonical Ensemble**: provides accurate baselines for cumulants of arbitrary order (incl. multi-particle correlations)
- ▶ **Fuzzy Logic Methods**: modern analysis tools developed and applied to experimental data
- ▶ **Key Finding**: canonical baselines **alone** cannot reproduce measurements
- ▶ **Role of Interactions**:
 - ▶ High energies (STAR): predominantly **repulsive**
 - ▶ Low energies (HADES & STAR): predominantly **attractive**
- ▶ **Consistency**: HADES Ag–Ag @ 2.5 GeV and STAR FXT Au–Au @ 3 GeV show the same trend
- ▶ **Implication**: The observed pattern could be **a tell-tale sign** of a first-order phase transition in strongly interacting QCD matter, analogous to the van der Waals liquid–gas phase transition

Outlook



- Systematic measurements of fluctuations stemming from critical point
- Measuring fluctuations induced by spinodal decomposition
- Search for cluster formation

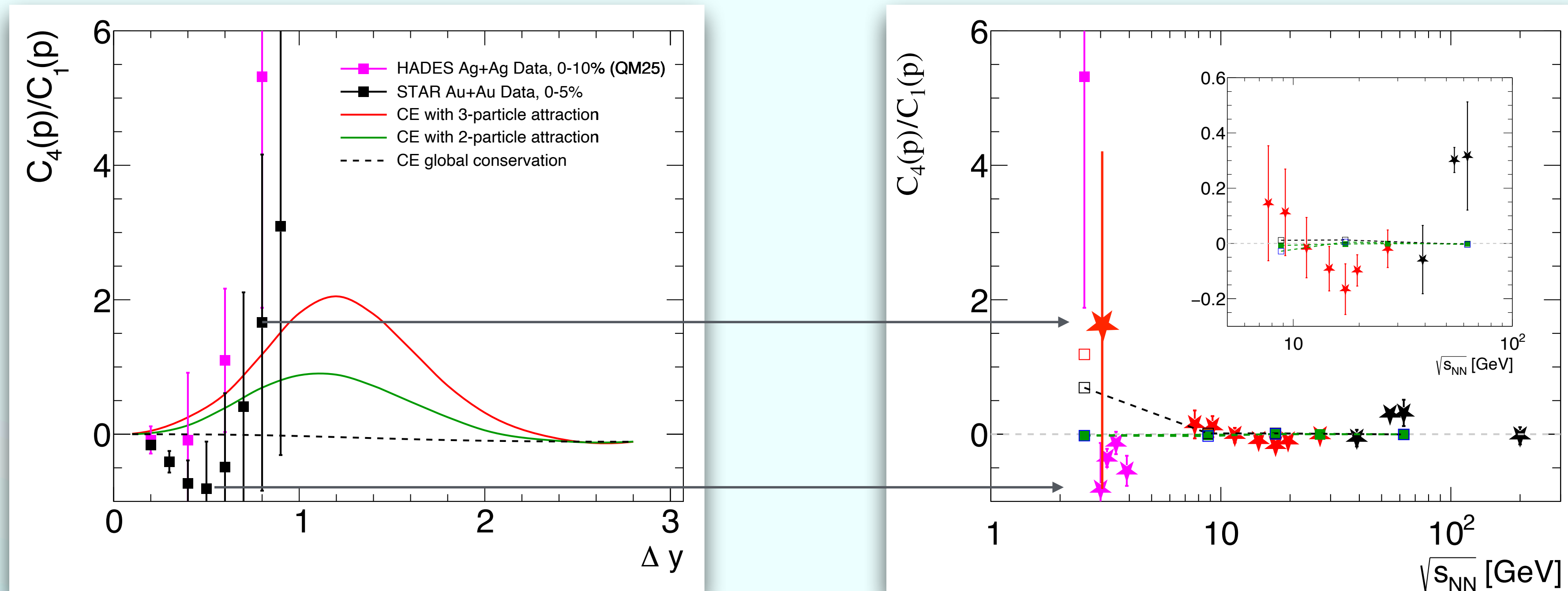
► P. Braun-Munzinger, K. Redlich, A. R., J. Stachel, JHEP 08 (2024) 113

► C. Sasaki, B. Friman, K. Redlich, Phys.Rev.D 77 (2008) 034024

**THANKS FOR YOUR
ATTENTION**



The Crucial Role of Acceptance Selection, example for C_4/C_1



Baselines in STAR FXT: To Be Calculated Within Matching Acceptance

► B. Friman, A. R., K. Redlich, e-Print: 2508.18879 [nucl-th]

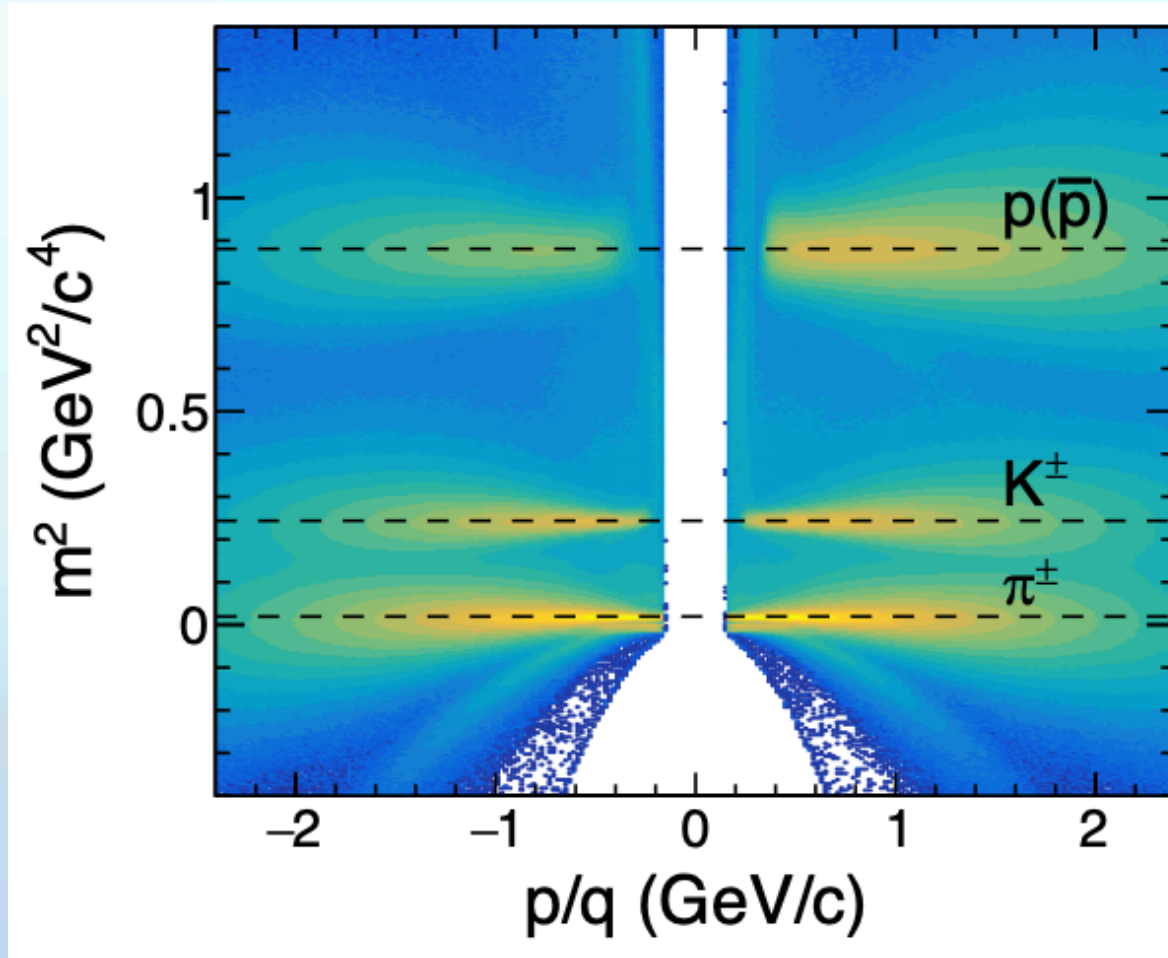
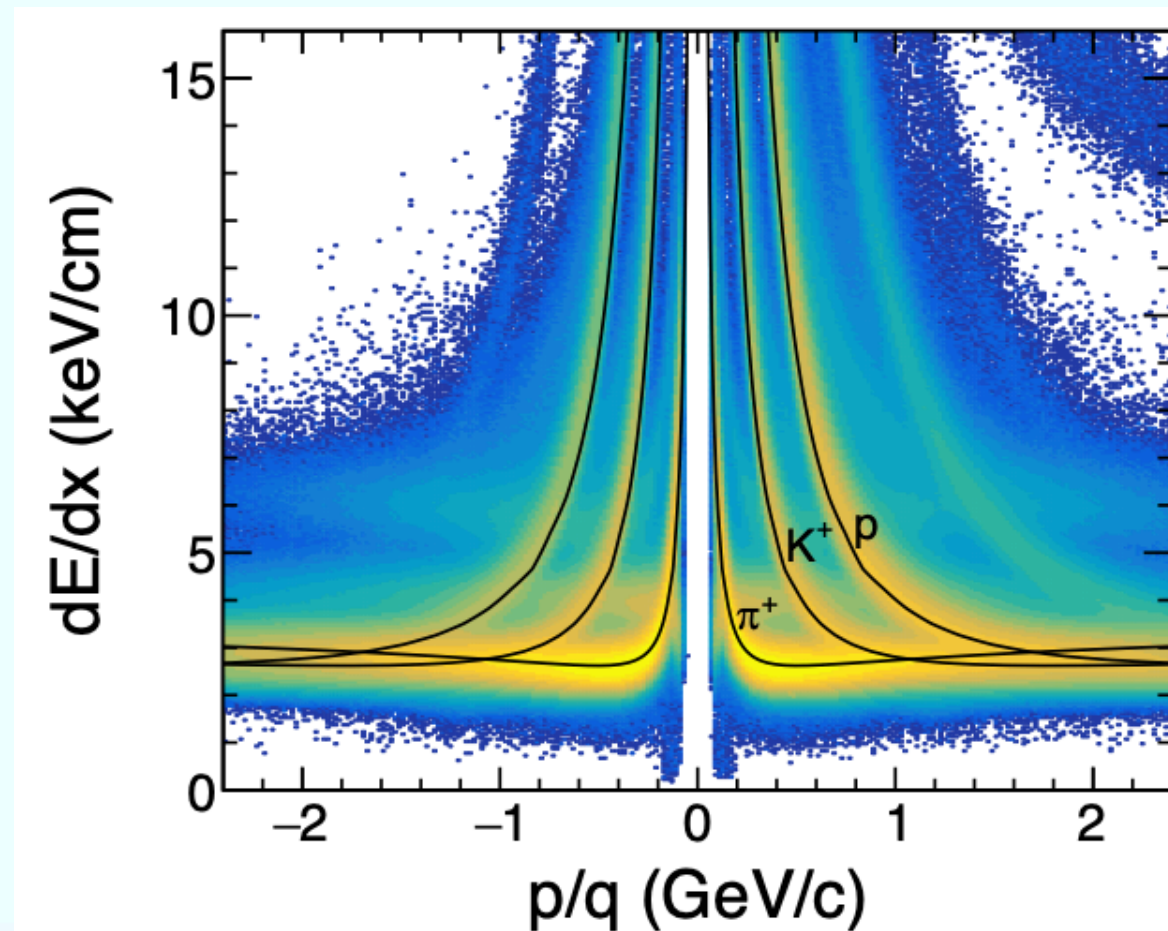
► STAR BES I: Phys. Rev. Lett. 126 (2021) 9, 092301

► STAR BES II: arXiv:2504.00817 [nucl-ex]

► STAR FXT: Zachary Sweger, Quark Matter 2025

Particle identification (PID)

STAR, Au-Au@19.6 GeV



STAR: PRL 126 (2021) 9, 092301

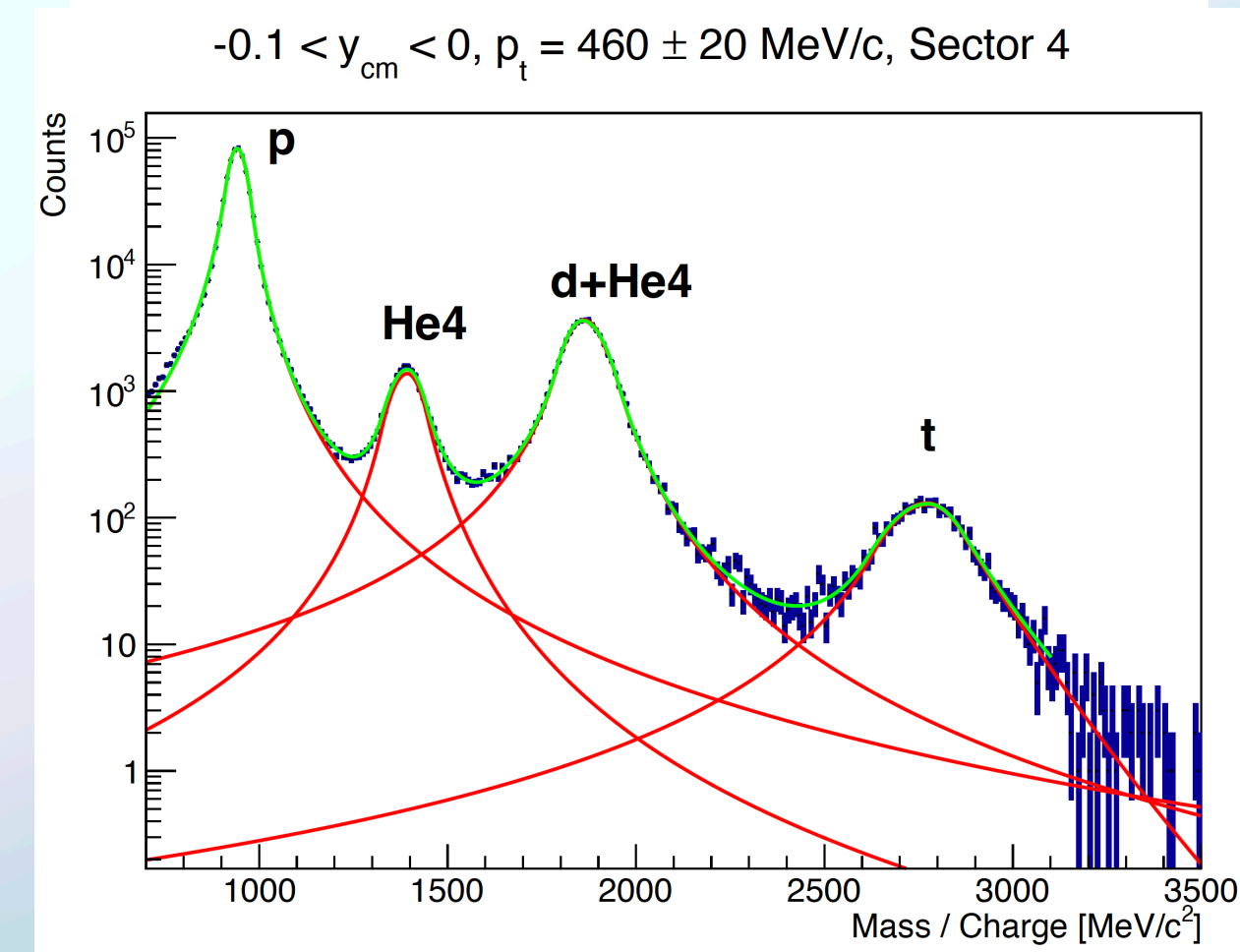
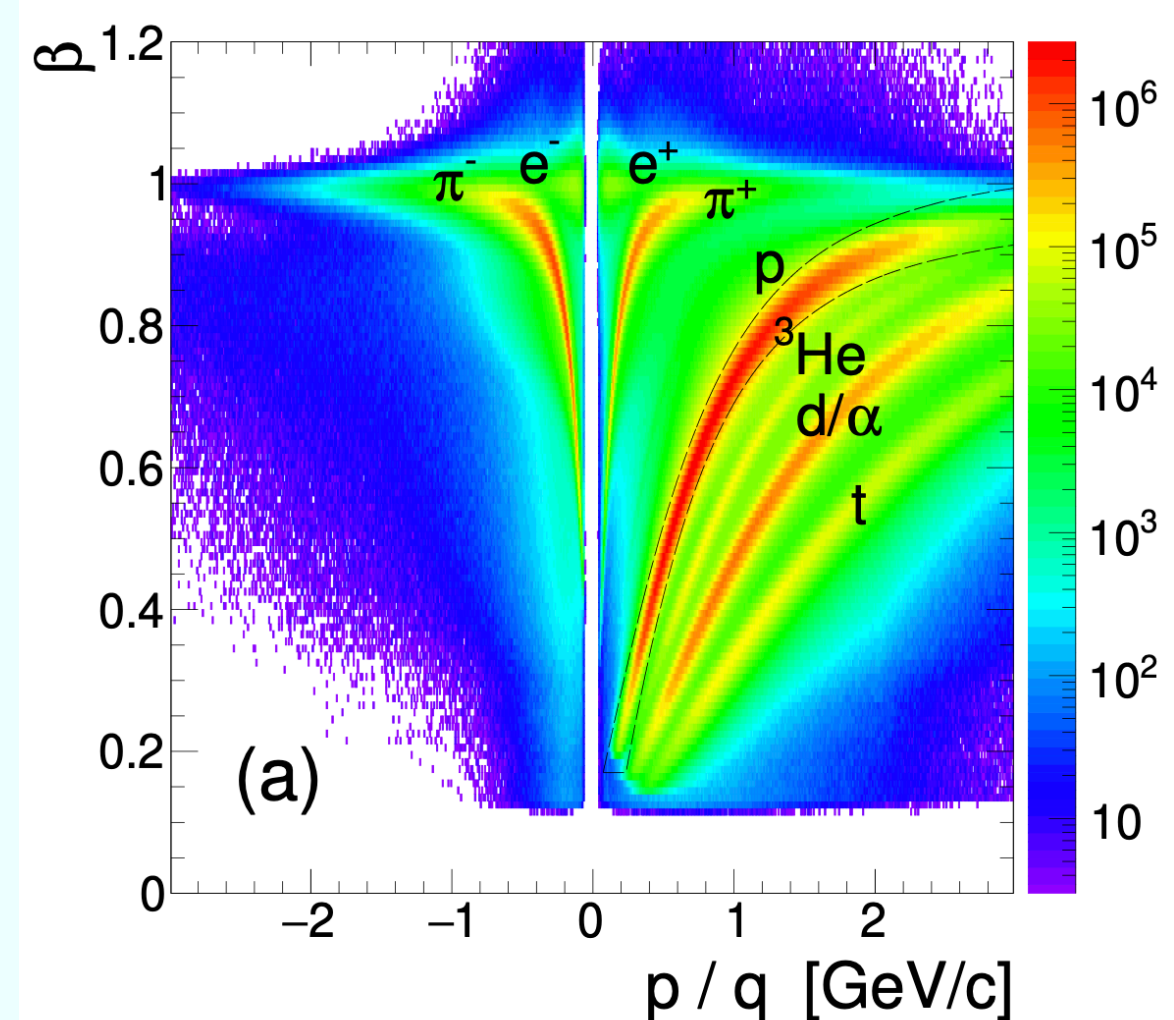
$$n_{\sigma,p} = \frac{1}{\sigma_R} \ln \left(\frac{dE/dx}{\langle dE/dx \rangle_p^{BB}} \right)$$

σ_R - dE/dx resolution $\sim 7.5\%$

Only TPC
 $p_T \leq 0.8$ GeV/c
 $|n_{\sigma,p}| < 2$

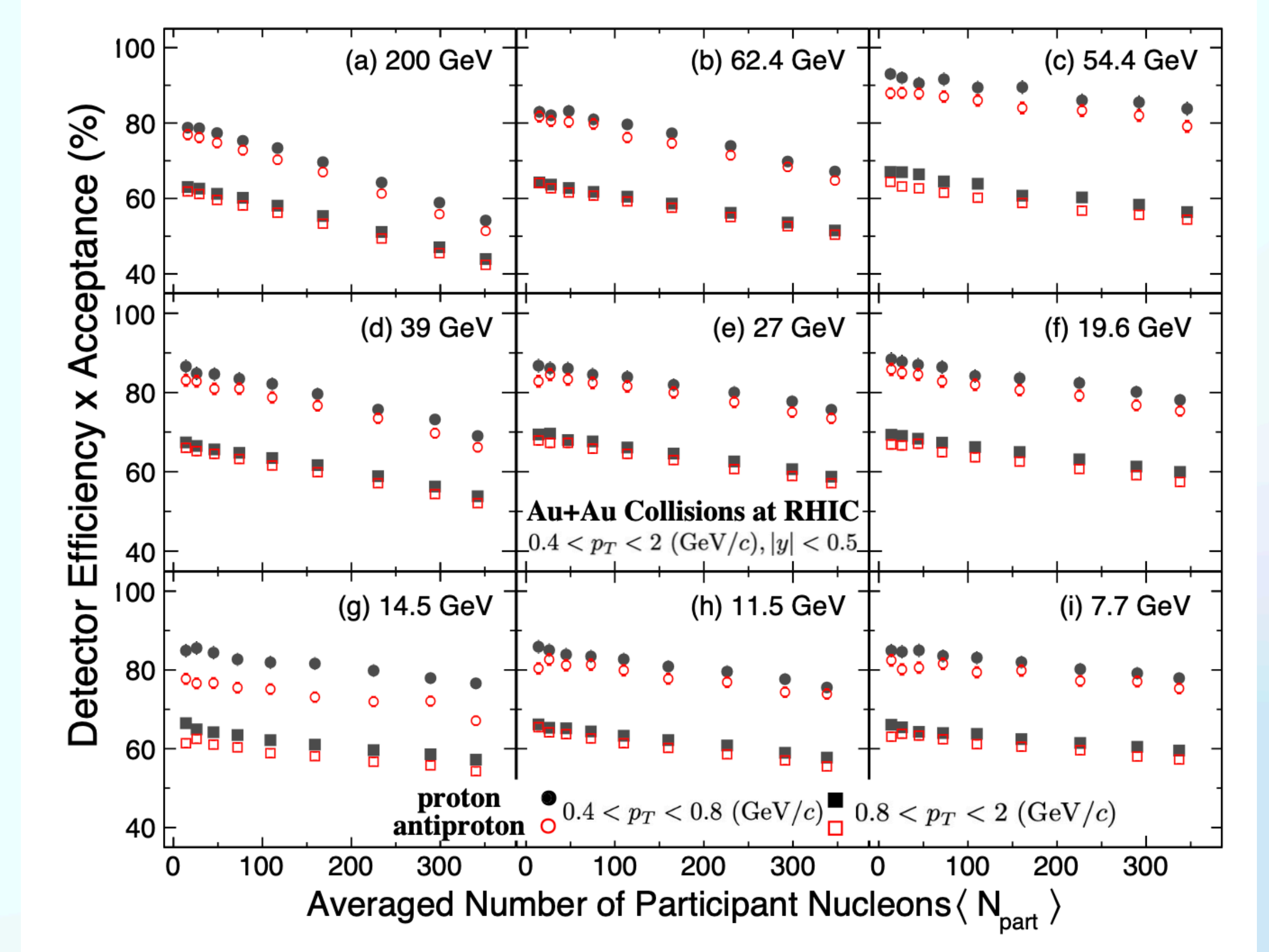
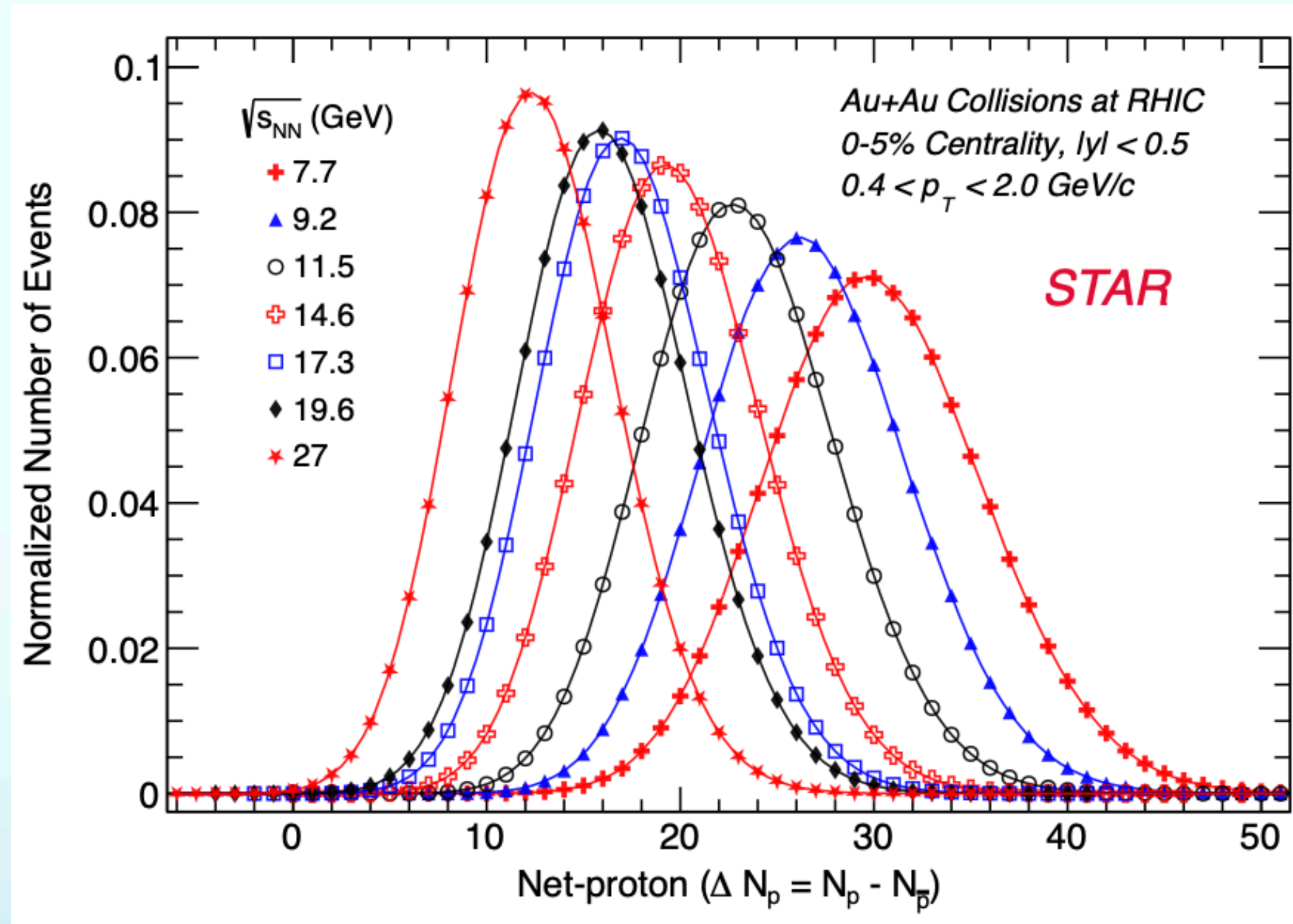
TPC+TOF
 $0.8 < p_T \leq 2$ GeV/c
 $|n_{\sigma,p}| < 2$
 $0.6 < m^2 < 1.2$ GeV²/c⁴

HADES, Au-Au@2.4 GeV



drift chambers+TOF

Net-proton distributions measured by STAR



► STAR BES1 : Phys.Rev.Lett. 126 (2021) 9, 092301

Reduced efficiency for TPC+TOF

Potential danger for suppressing critical fluctuations

Identity Method (Fuzzy logic)

Input for this event: 3 pions **1** **2** **3**, 2 kaons **4** **5**

Probabilities that a given measurement x_i is pion or Kaon

$$\omega_{\pi}^i = \frac{\rho_{\pi}(x_i)}{\rho_{\pi}(x_i) + \rho_K(x_i)}$$

$$\omega_K^i = \frac{\rho_K(x_i)}{\rho_{\pi}(x_i) + \rho_K(x_i)}$$

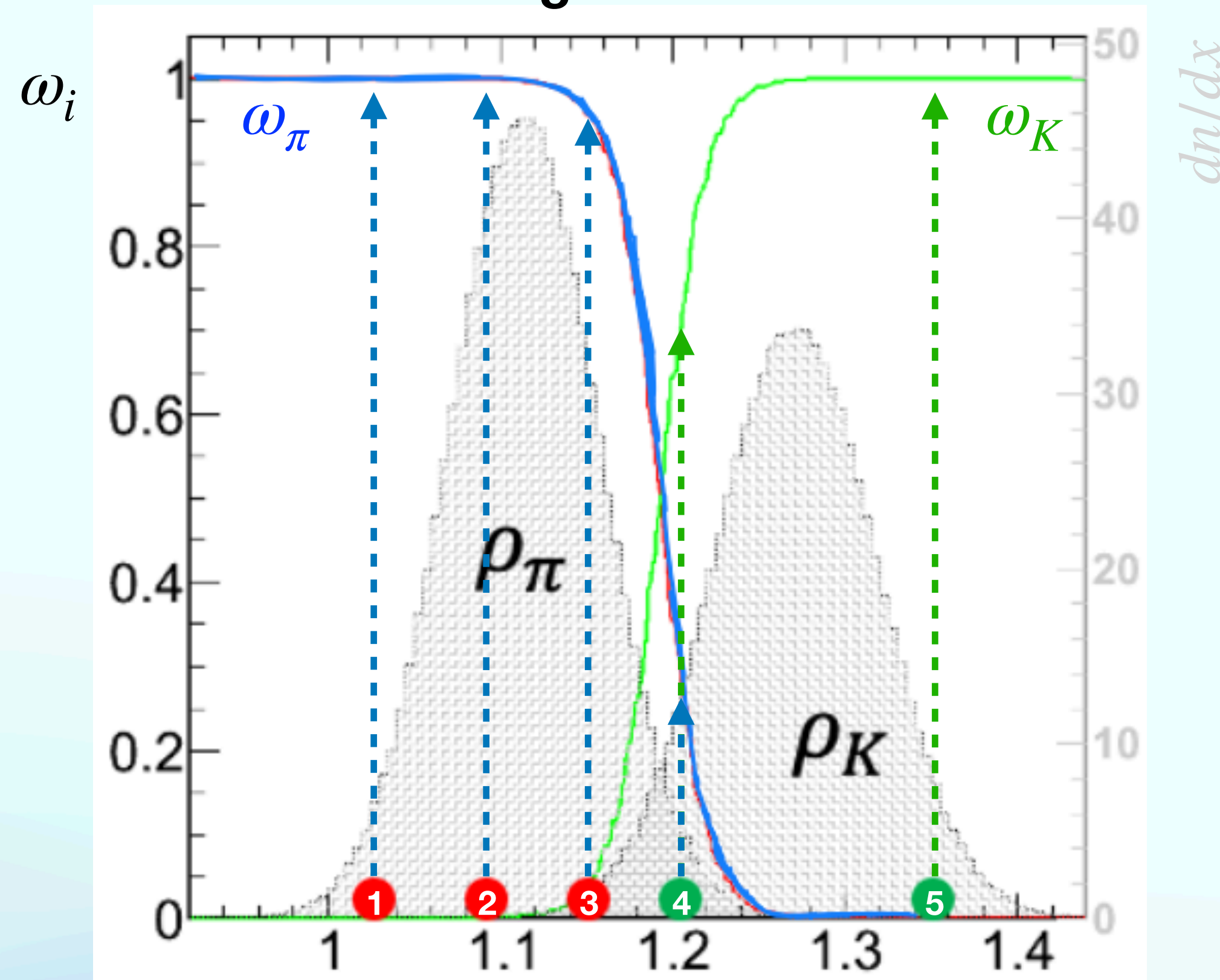
New Idea: Introducing proxies for particle numbers

$$W_{\pi} = \sum_{i=1}^{i=5} \omega_{\pi}(x_i)$$

$$W_K = \sum_{i=1}^{i=5} \omega_K(x_i)$$

Fuzzification

single event



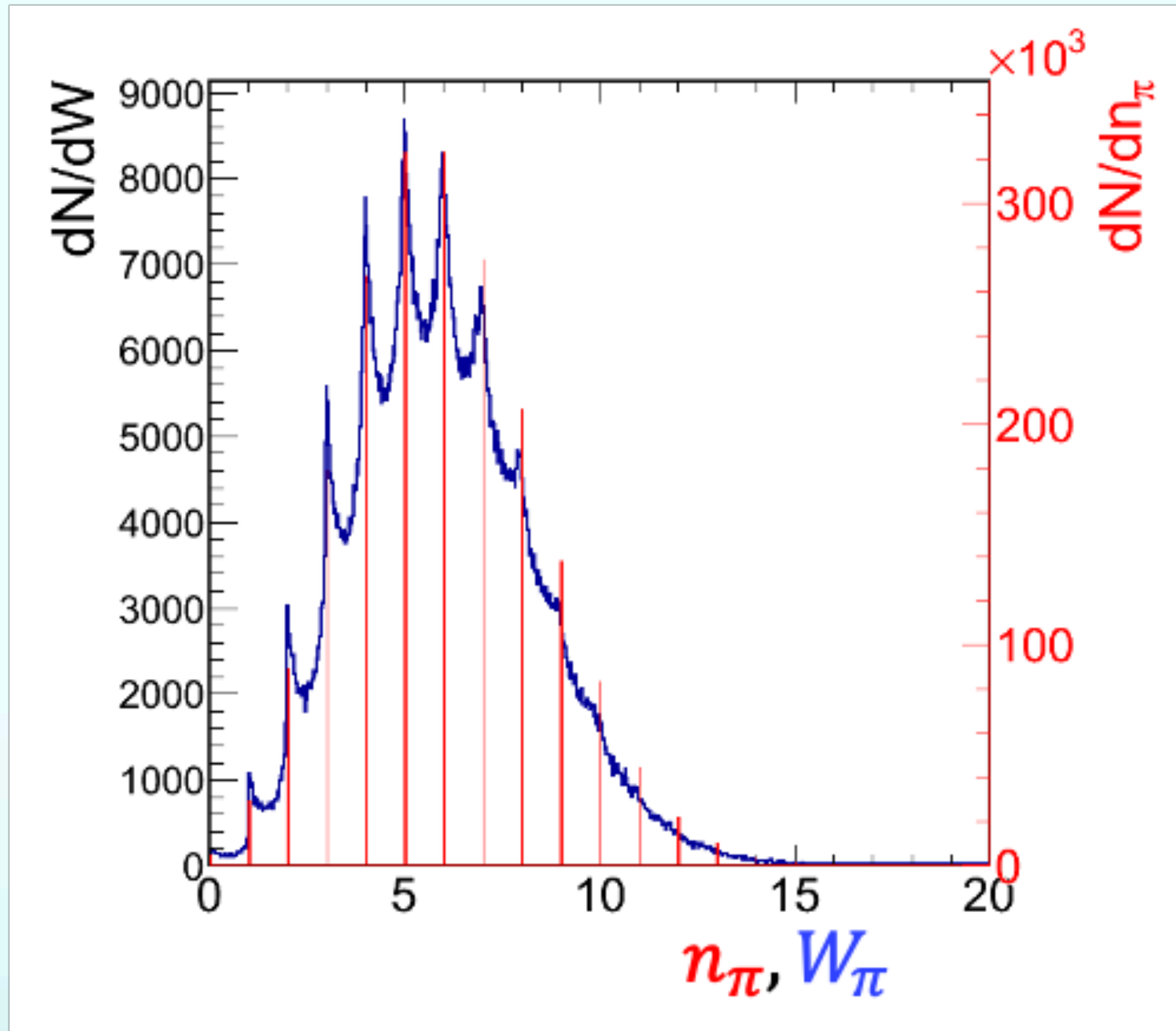
x for PID

- M. I. Gorenstein, *Phys.Rev.C* 84 (2011) 024902
- A. Rustamov, M. I. Gorenstein, *Phys.Rev.C* 86 (2012) 044906
- M. Arslanok, A. R., *Nucl. Instrum. Meth.A* 946 (2019) 162622
- A. Rustamov, *Phys.Rev.C* 110 (2024) 6, 064910

$n_{\pi}, n_K \rightarrow W_{\pi}, W_K$: from integer to floating particle numbers

Fuzzy logic

moment-generating function for an incomplete (W) distribution



$$M_W(t_1, \dots, t_n) = \sum_{N_1, \dots, N_n=0}^{\infty} P(n_1, \dots, n_n) \prod_{i=1}^n e^{h_i(t_1, \dots, t_n) n_i}$$

$$h_i(t_1, \dots, t_n) = \ln \left[\int_{-\infty}^{+\infty} e^{\sum_{j=1}^n t_j \omega_j(x)} \mathcal{P}_i(x) dx \right], \quad \mathcal{P}_i(x) \equiv \frac{\rho_i(x)}{\langle N_i \rangle}$$

$$\langle W_a^n \rangle = \frac{\partial^n M_W(t_a)}{\partial t_a^n} \quad \langle W_a^n W_b^m \rangle = \frac{\partial^{n+m} M_W(t_a, t_b)}{\partial t_a^n \partial t_b^m}$$

$$\begin{pmatrix} \langle W_\pi^2 \rangle \\ \langle W_K^2 \rangle \\ \langle W_\pi W_K \rangle \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} \langle n_\pi^2 \rangle \\ \langle n_K^2 \rangle \\ \langle n_\pi n_K \rangle \end{pmatrix}$$

De-Fuzzification

Applied in: NA49, NA61, ALICE, HADES

Fuzzy logic: arbitrary moments

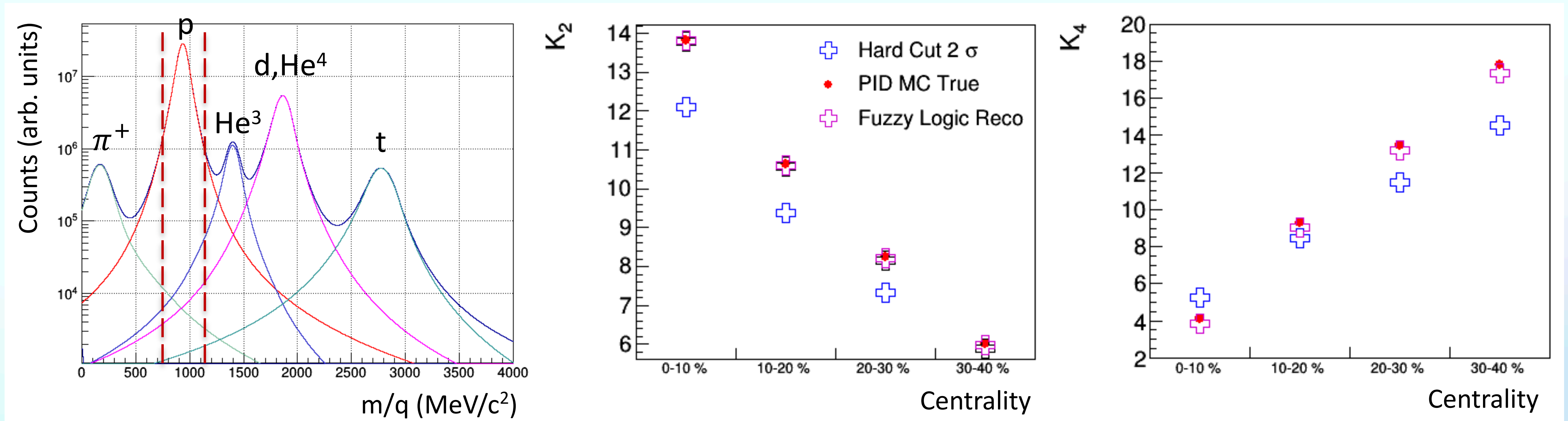
- A. Rustamov, Phys.Rev.C 110 (2024) 6, 064910

Identity Method: 2nd and 3rd moments

- M. I. Gorenstein, PRC 84, 024902 (2011)
- A.R., M. I. Gorenstein, PRC 86, 044906 (2012)
- M. Arslanok, A.R., NIM A946, 162622 (2019)

Fuzzy logic vs. hard cut

► HADES: Marvin Nabroth, Quark Matter 2025



The results obtained using Fuzzy Logic successfully reproduce the Monte Carlo (MC) true values

Achieving the right balance between purity and efficiency is crucial in cut-based analyses

A new package developed in C++ TFuzzyLogic

► A. R., Phys.Rev.C 110 (2024) 6, 064910

► M. Nabroth, A.R., NIM, in preparation