A combined constraint on EOS, at high densities, by connecting neutron star observations and the data from heavy ion reactions

or: A momentum dependent CMF in UrQMD

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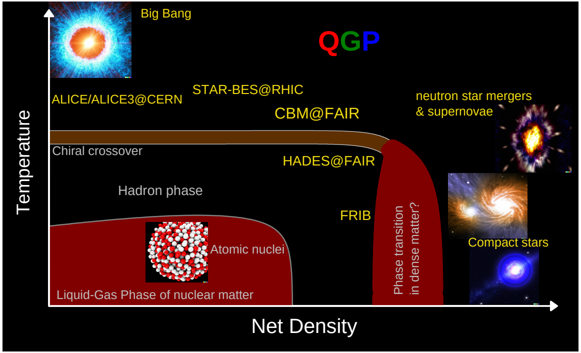
The essence of high energy nuclear physics: The QCD phase diagram



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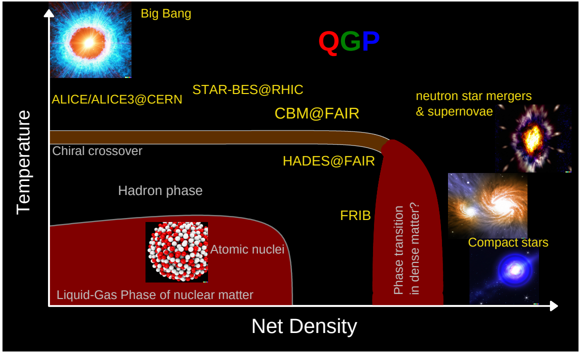

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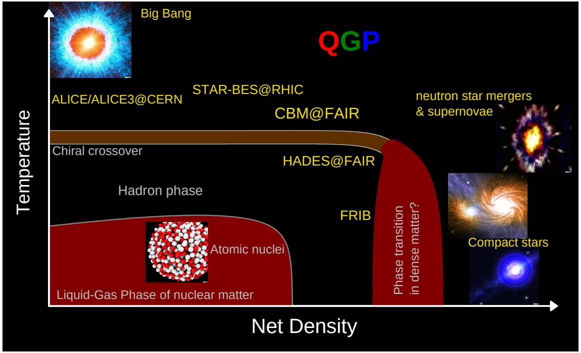

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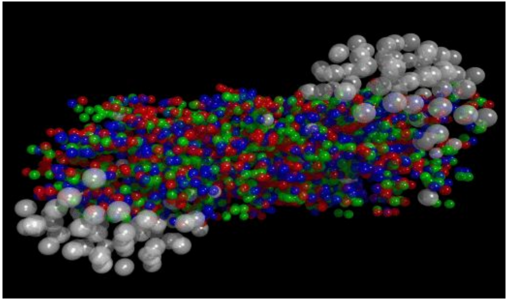
- Results at low density: Crossover is now confirmed.
- Established $T_{cep} \lesssim 120$ MeV.
- High density: room for speculations.
- We have to rely on effective model descriptions of the EoS and experimental data from HIC and astrophysics.

UrQMD for the description

UrQMD is a microscopic transport model

- In cascade mode: Particles follow a straight line until they scatter.
- Only $2 \leftrightarrow 2$, $2 \leftrightarrow 1$, $2 \rightarrow N$ and $1 \rightarrow N$ interactions allowed.
- Resonance excitation and decays according to PDG values + guesstimates.
- EoS resembles a hadron resonance gas.



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In UrQMD the real part of the interaction is implemented by a density dependent potential energy $V(n_B)$.
Once the potential energy is known, the change of momentum of each baryon is calculated as:

$$\dot{\mathbf{p}}_i = -\frac{\partial \langle H \rangle}{\partial \mathbf{r}_i} = -\left(\frac{\partial V_i}{\partial n_i} \cdot \frac{\partial n_i}{\partial \mathbf{r}_i}\right) - \left(\sum_{j \neq i} \frac{\partial V_j}{\partial n_j} \cdot \frac{\partial n_j}{\partial \mathbf{r}_i}\right) , \tag{1}$$

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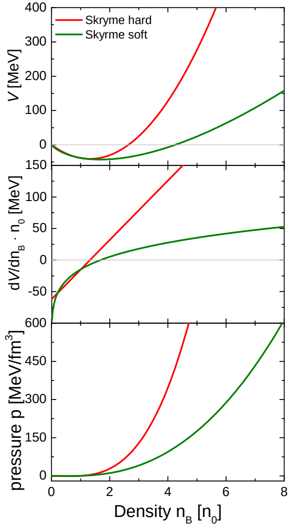
For the potential energy V often a Skyrme model was used that is based on a 2-term expansion in density:

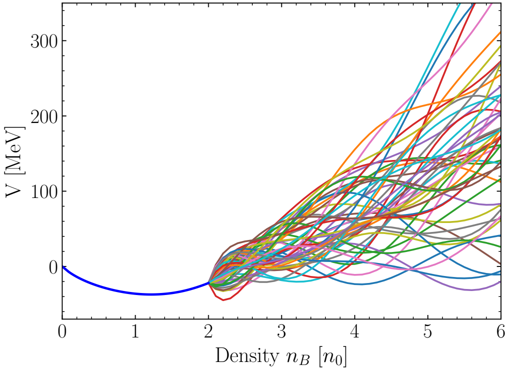
$$U(n_B) = \alpha \cdot n_B + \beta \cdot n_B^\gamma \quad \text{with} \quad U(n_B) = \frac{\partial (n_B \cdot V(n_B))}{\partial n_B} \tag{2}$$

Problem: Once saturation density and binding energy is fixed, only 1 d.o.f. left and EoS likely becomes unphysical. No phase transition possible.



Polynomial extension of the density dependence

- Use UrQMD as described but parameterize $V(n_B)$ with a seventh order polynomial.



M. Omana Kuttan, **JS**, K. Zhou and H. Stöcker,Phys. Rev. Lett. **131**, no.20, 202303 (2023).

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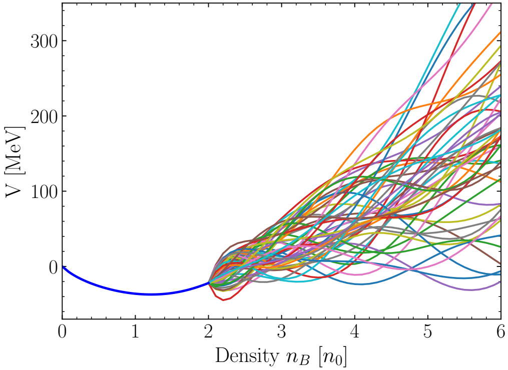
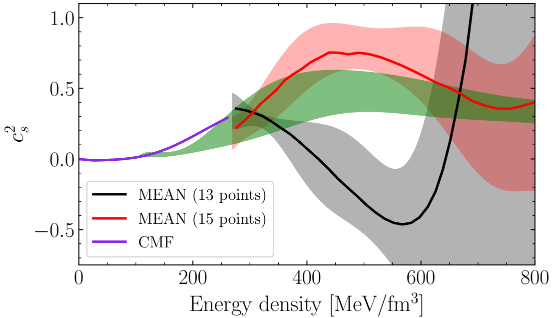


- Using Bayesian inference methods we can try to constrain the EoS parameters from flow data.
- Results depend strongly on the data used.
- If all data on the mean m_T and v_2 are used, constraints are similar to those from astrophysics (NS and BNSM).



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Shortcomings of this approach

- Incorporating nuclear matter requires a large number of polynomial coefficients ($\# > 10$).
- Polynomials are not nicely behaved outside of constraint region.
- Difficult to incorporate constraints like causality, asymptotic limits etc.
- No information on microscopic d.o.f.
- Higher dimensional polynomials get even more messy, e.g. isospin dependence, momentum dependence etc.
- How to deal with constraints from Astro or lattice-QCD?

A possible solution

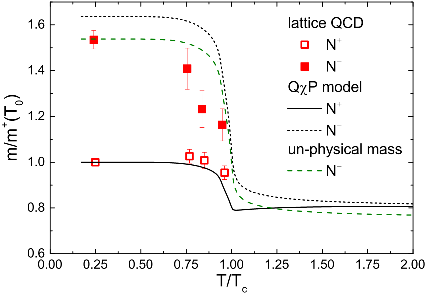
- Use an effective model for QCD thermodynamics (with parameters) instead.
- CMF (Chiral Mean Field model) developed in Frankfurt. Some call it the Swiss knife (or 'Moving Castle') of effective models.
- Best case: may learn something about interactions and QCD and chiral symmetry
- Worst case: get a smart parametrization of the EoS. Easy to implement.

An effective model for everything: the Chiral Mean Field model (CMF)

It's what holds it all together. *One EoS to rule them all.*

Ingredients

- Use a hadronic parity doublet approach for hadronic part.
- Consistent with lattice QCD on effective masses!



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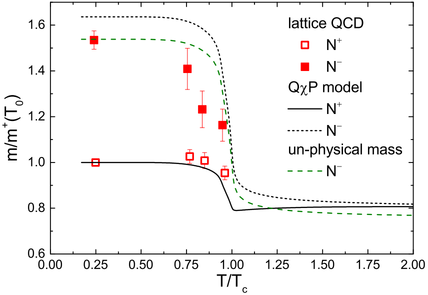

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Parameterizations	L [MeV]
$m_0 = 600$ MeV; $m^*/m = 0.76$; $K = 440$ MeV	48
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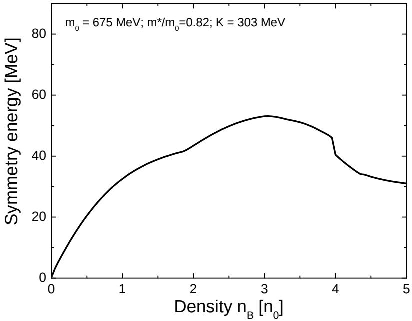

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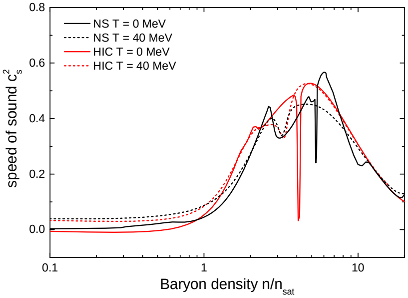

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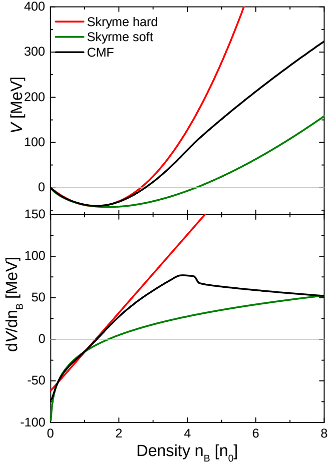
In CMF we can simply use the effective field energy per baryon E_{field}/A calculated from the CMF model:

$$V_{CMF} = E_{\text{field}}/A = E_{\text{CMF}}/A - E_{\text{FFG}}/A \,, \quad (4)$$



JS, A. Motornenko, A. Sorensen, Y. Nara, V. Koch and M. Bleicher, Eur. Phys. J. C **82**, no.10, 911 (2022).

(Why and how) Introducing a momentum dependent potential?

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In CMF can be done straight forward

The definition of the single particle potential is:

$$U_i = E_i^* - E_i \tag{5}$$

where E_i is simply the non-interacting single particle energy $E_i = \sqrt{m_i^2 + p_i^2} - \mu_i$.
and E_i^* is the interacting one $E_i^* = \sqrt{m_i^{*2} + p_i^2} - \mu_i^*$.
It is easy to see that U_i will have a momentum dependent part (from the scalar interactions) and a non momentum dependent part (from the vector interactions). [a]

[a] See also recent work by Yasushi Nara.

Momentum dependence from CMF model

- We can use E_i^* from the **CMF** model to calculate the momentum dependence.
- Remember the parity doubling model + hyperons + Δs.

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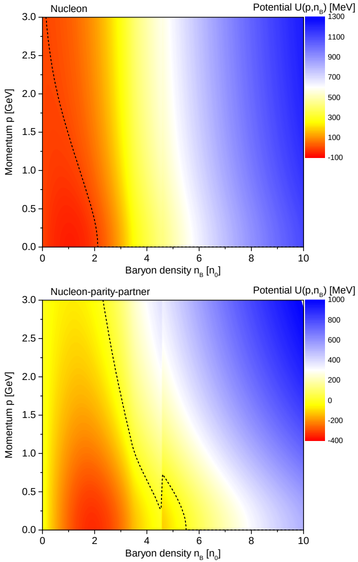
- Remember the parity doubling model + hyperons + Δs.
- We have all the potentials as function of density + momentum

We can calculate the potential energy:

$$V(n_B, p) = \frac{1}{n_B} \int_0^{n_B} U(n_B, p) dn_B \Big|_{p=\text{const.}}$$



The strategy

- Fix the CMF parameters (scalar and vector interactions, bare mass,...) using neutron star observables.
- Simulate heavy ion reactions and compare to data from HADES (Then FOPI and STAR).

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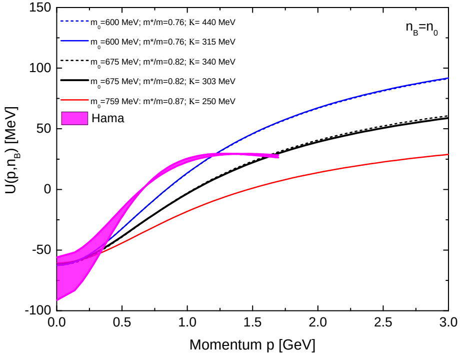

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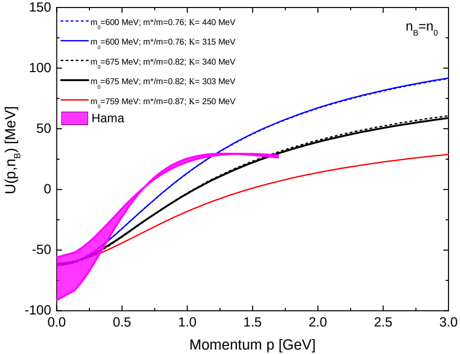

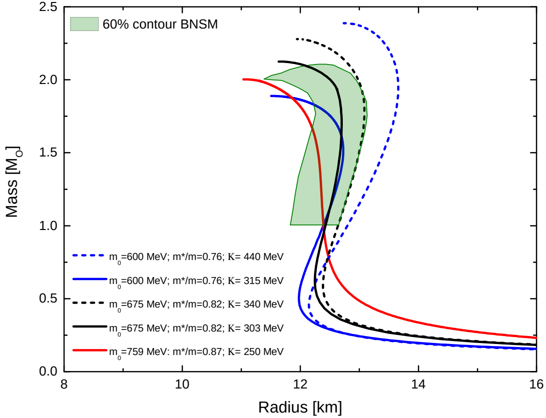

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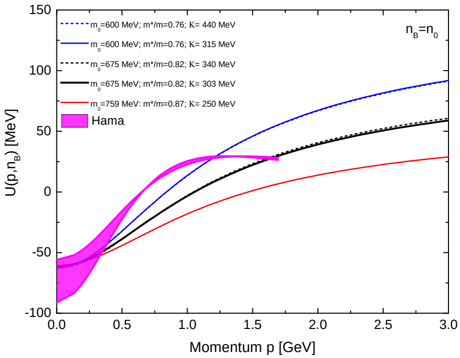
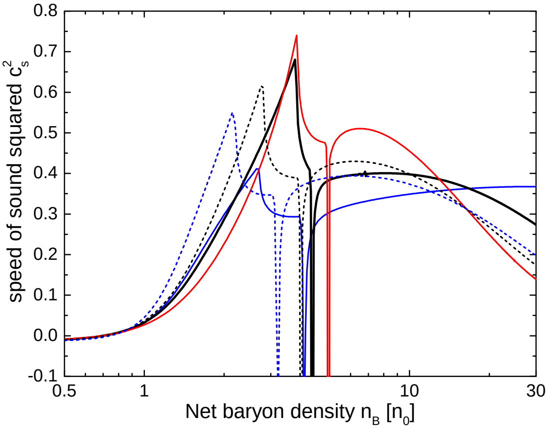
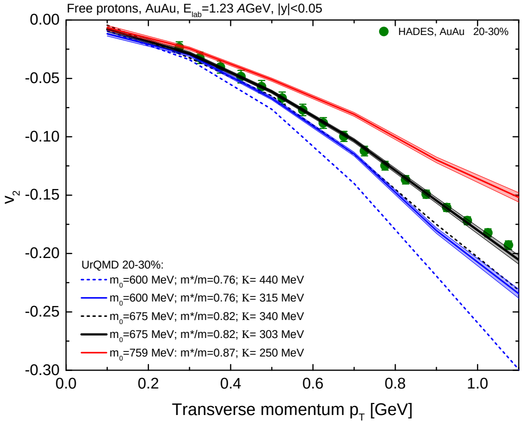




Best fitting: $K = 303$ MeV, $E_{sym} = 31$ MeV and $L = 53$ MeV.

Momentum dependence effects: flow from HADES

- Momentum dependence of elliptic flow shows the momentum dependence of the potentials.
- Black line seems to work best for flow.



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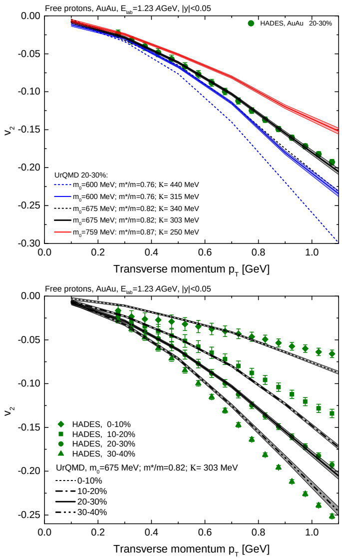
- Other centralities not so perfect but still quite good.



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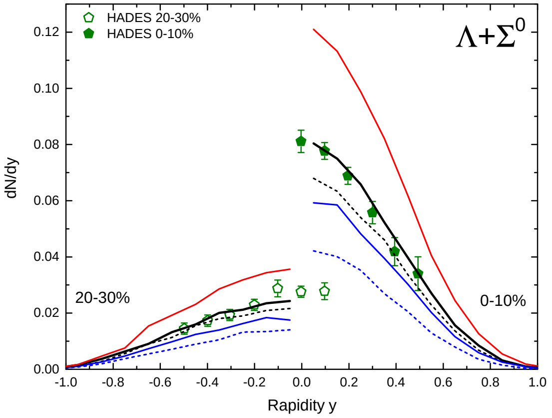
- Multiplicities of produced hadrons are sensitive to the momentum dependence.
- Near threshold production of different hadronic species may be used to constrain the momentum dependence independently of the EoS.



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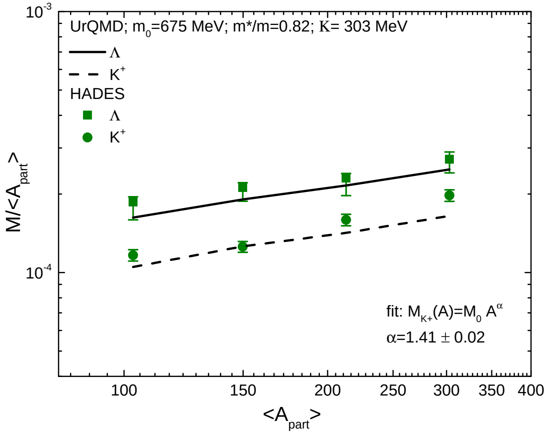
- Kaon centrality dependence consistent with HADES and KAOS data: $\alpha = 1.41$.



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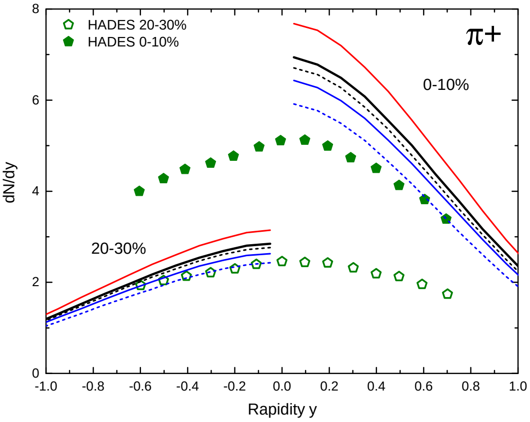

Momentum dependence effects: Pion Multiplicities

- Multiplicities of produced hadrons are sensitive to the momentum dependence.
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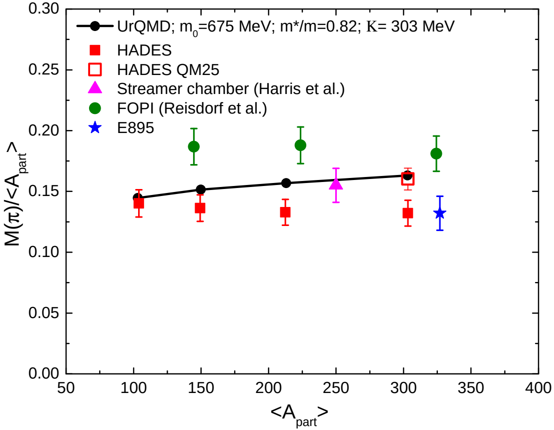
- Comparison with other data shows better agreement.
- New re-analysis of HADES data shown at QM25 indicates consistency with FOPI and our calculation.



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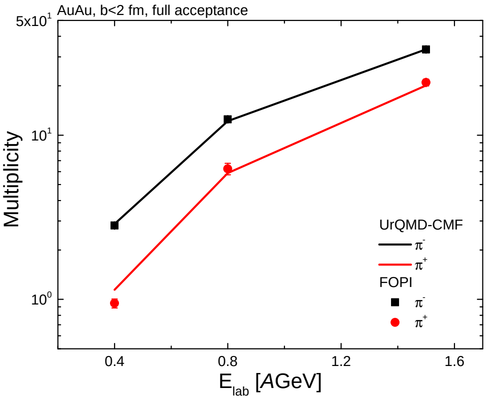

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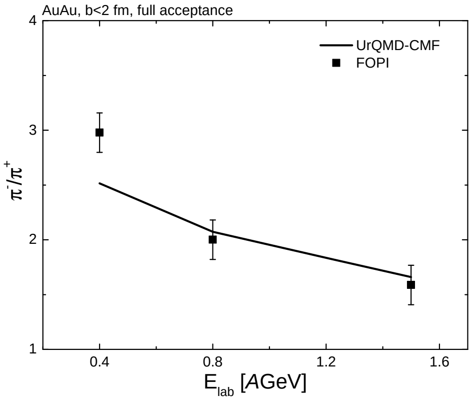

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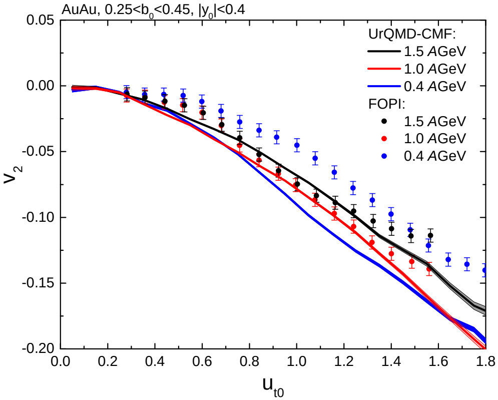

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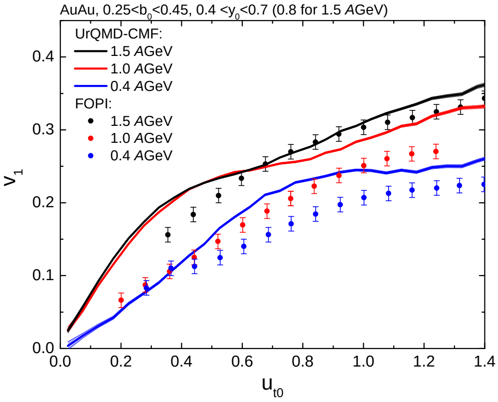

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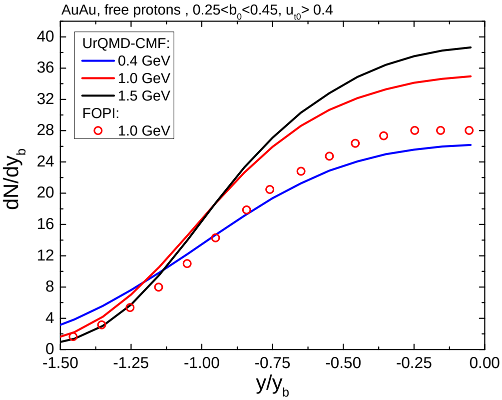

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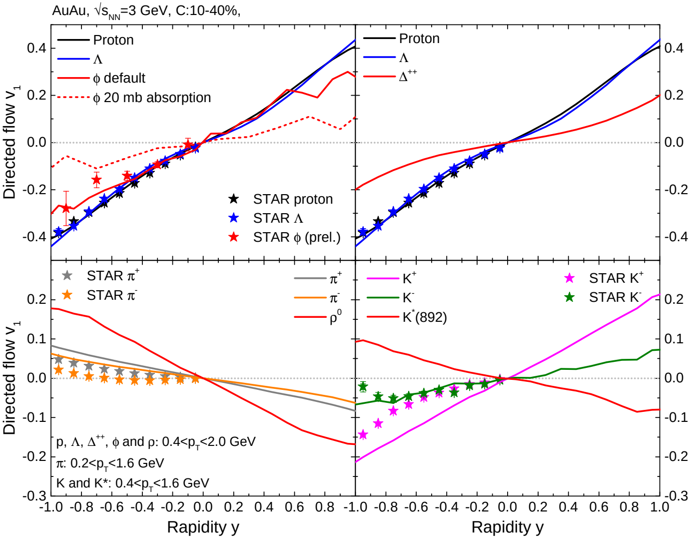
- Free proton number also overestimated.
- Due to difficulties in separating spectator from participant nucleons $\rightarrow$ 'bumps' at low momenta.
- Worse for lower beam energies where spectators are closer to the fireball in rapidity.

Comparison to STAR data



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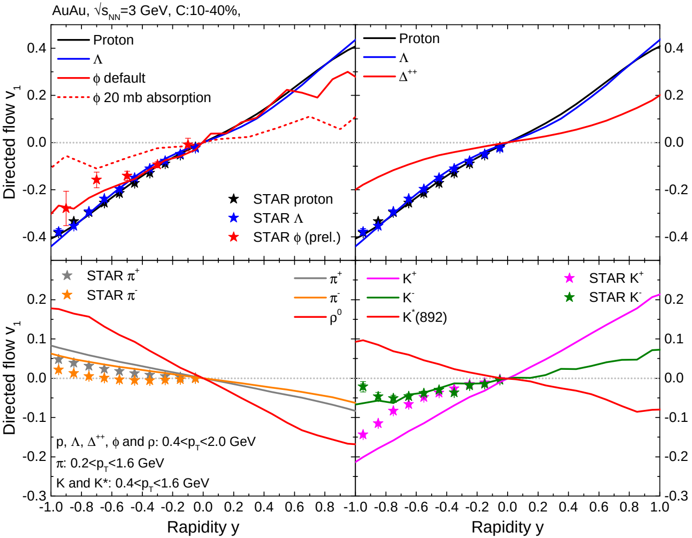

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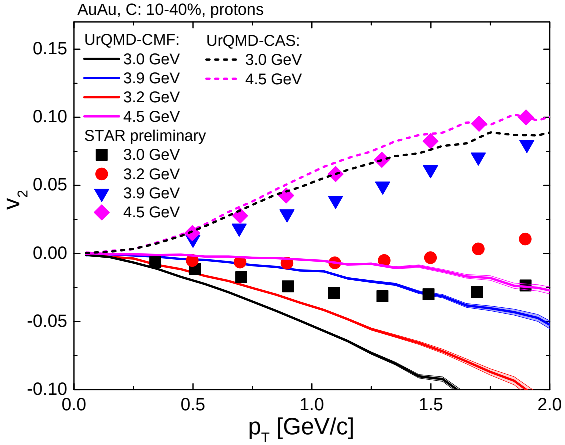

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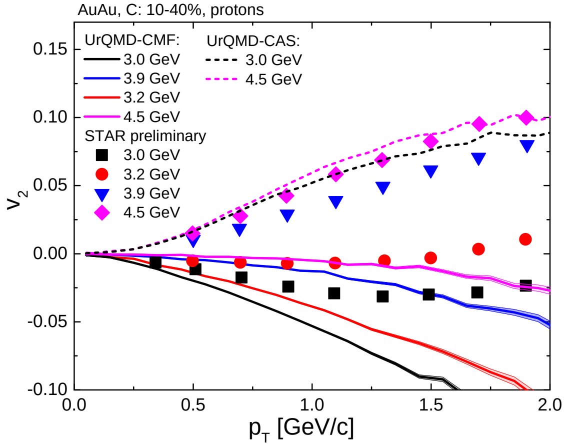

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- New physics? Relativistic description? Momentum dependence at large momenta?

The main takeaway

A consistent description of neutron star and heavy ion data is possible.

Future challenges and ways to approach them

1. The role of the Δ at finite temperature and in the isospin dependence.
2. Relativistic effects.
3. How to combine all that in a quantitative statistical analysis (or inference).
4. How do (past, current and future) experiments contribute to the EoS?

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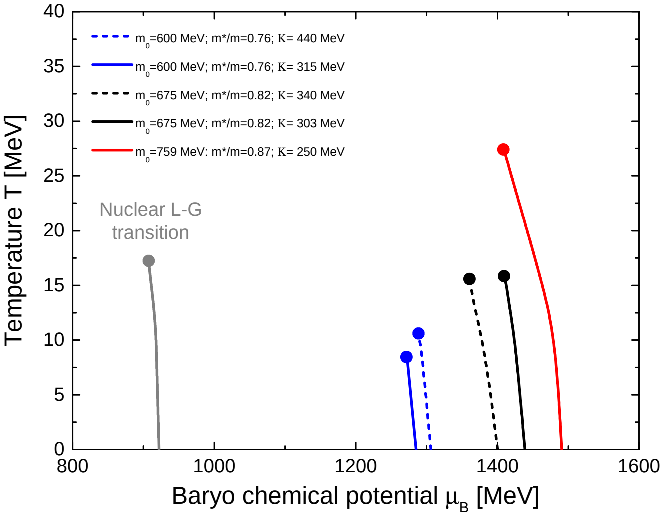

The value of a good and versatile EoS model!

- Sensitivity to fluctuations and correlations in the nucleus - nuclear structure.
- Which observables should be used to connect the isospin dependence in HIC to GW observables?
- Pions depend on Δ-interaction which do not appear in cold NS.
- We use classical Hamiltonian dynamics. Clearly wrong. But how wrong?
- Proper relativistic QMD description is difficult to achieve (no interaction theorem).
- How can the finite T EoS be implemented?
- Interaction length scale at high density? Density dependence of the QMD-range parameter?
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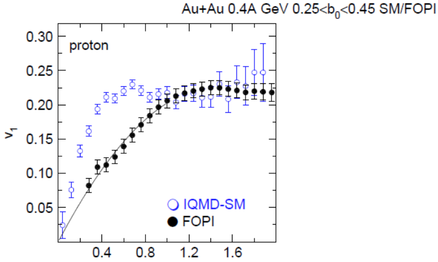
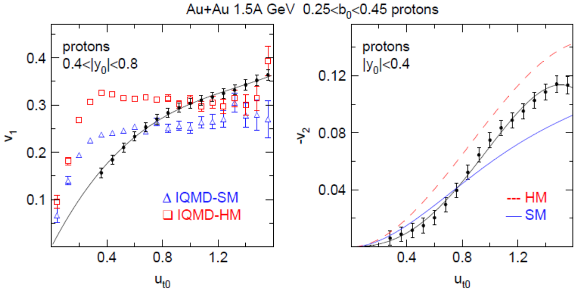
Comparing FOPI data with other models





Best fitting EoS did not do very well in the past.
W. Reisdorf *et al.* [FOPI], Nucl. Phys. A **876**, 1-60 (2012)