NEW INSIGHTS INTO THE QCD PHASE DIAGRAM

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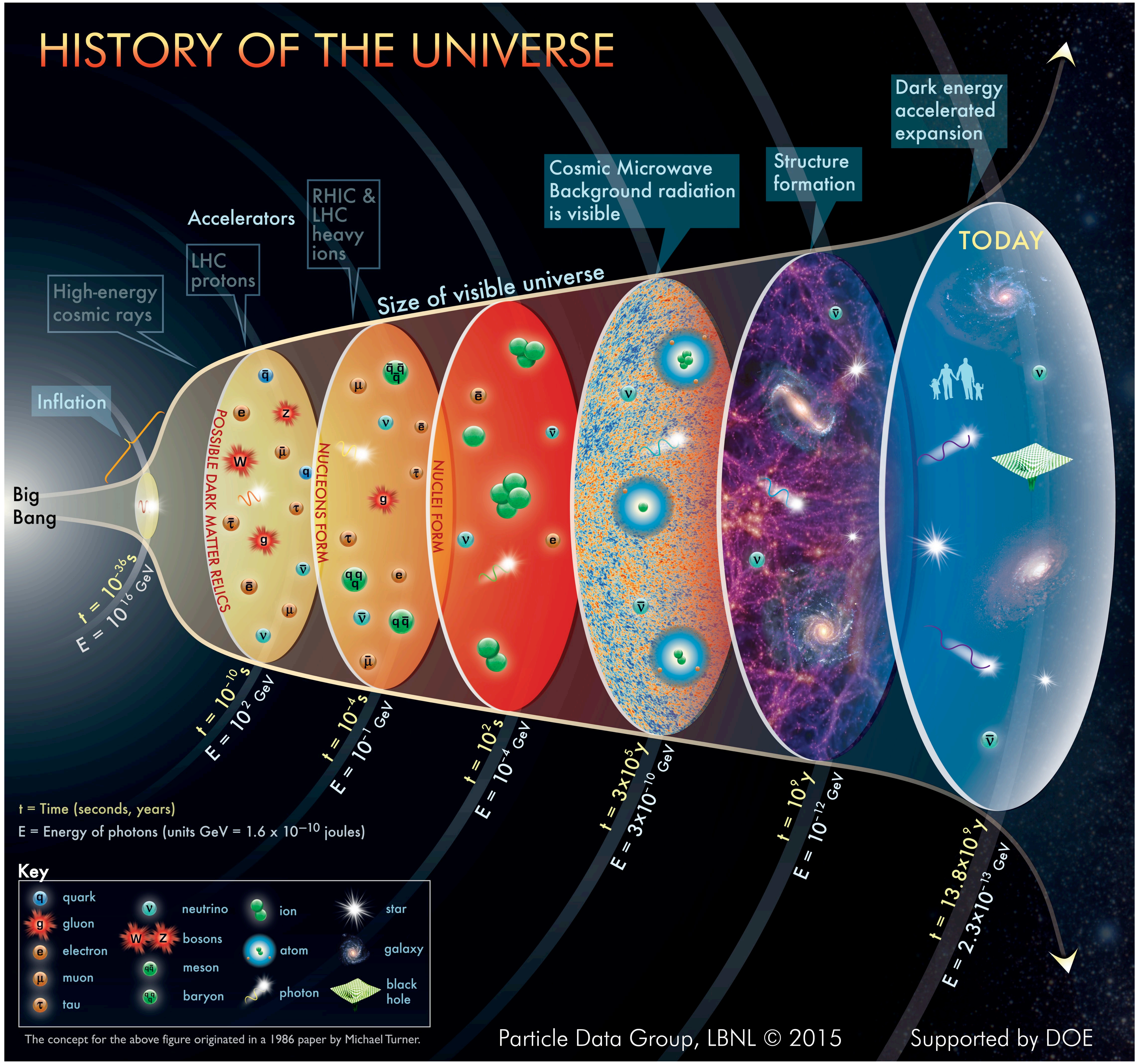
CRC-TR 211
Strong-interaction matter under extreme conditions

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF

DPG SPRING MEETING
KÖLN - 14/03/2025

UNDERSTAND MATTER IN EXTREME CONDITIONS

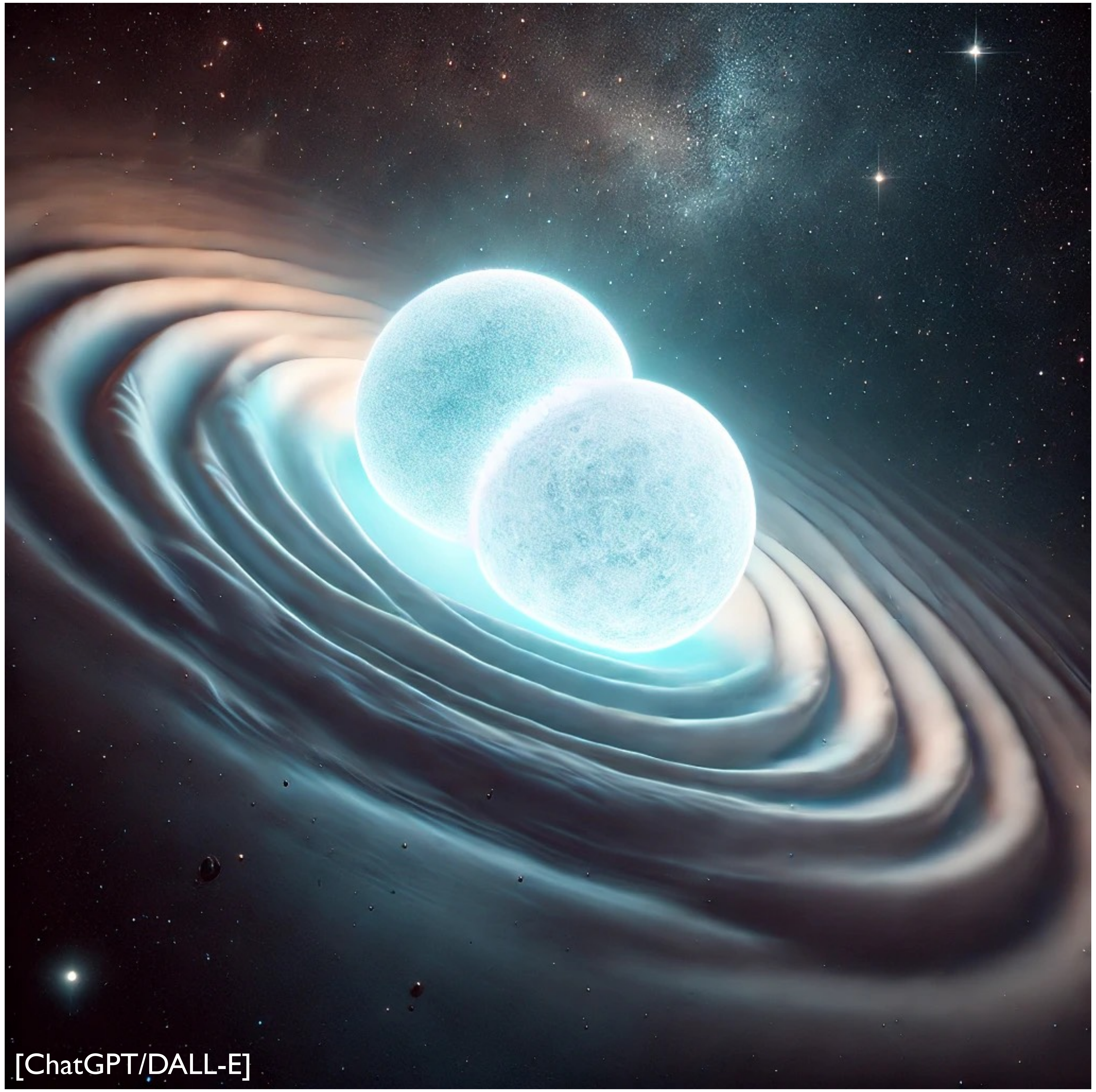


universe before the formation of the CMB
($t \lesssim 3 \times 10^5\,\mathrm{y}, T \gtrsim 1\,\mathrm{eV}$) is invisible to us;
nucleons formed during this time

↓

study hot QCD matter to understand formation of nuclear matter in the universe

UNDERSTAND MATTER IN EXTREME CONDITIONS

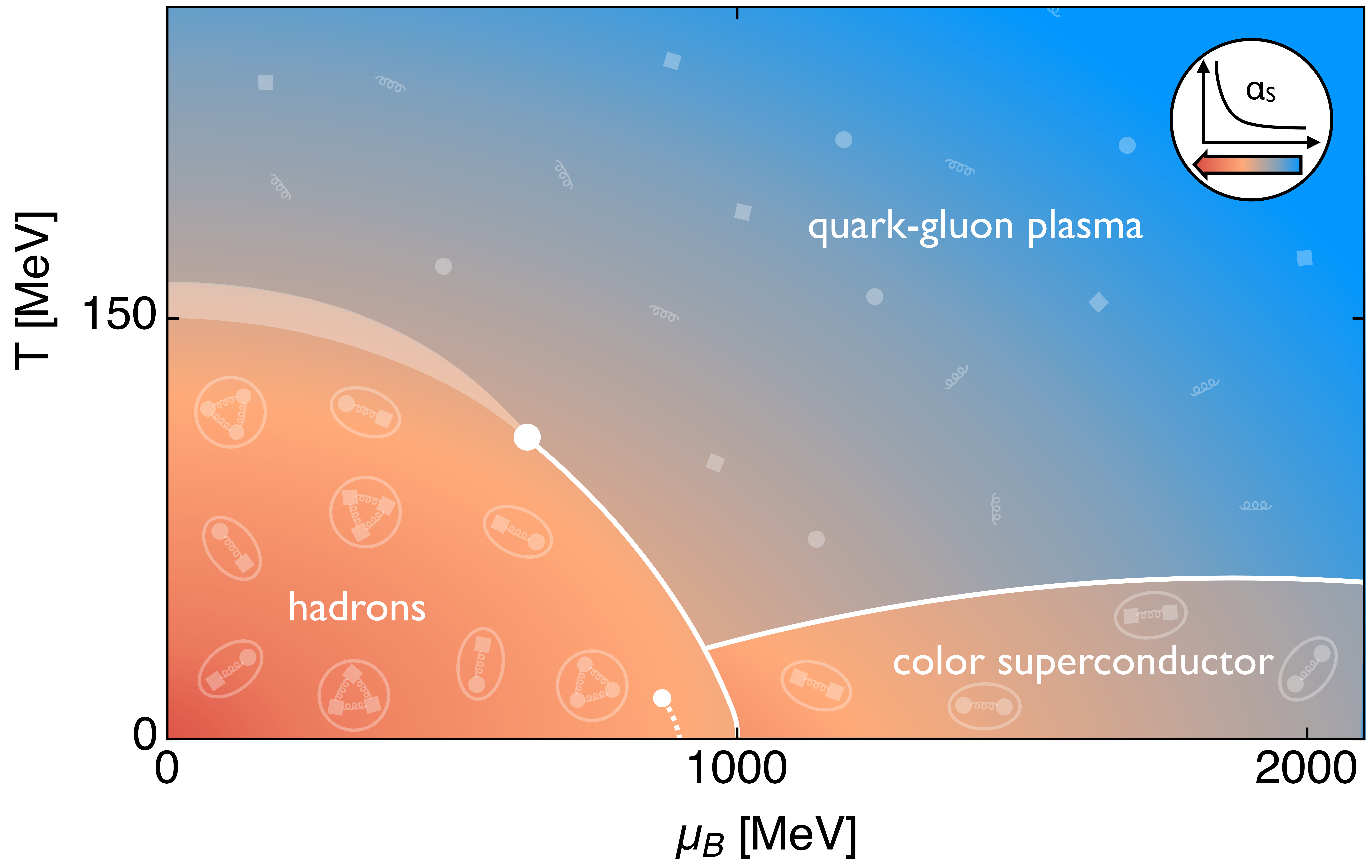


neutron stars can reach densities of several times the nuclear saturation density $(n \approx 5n_0, \mu_B \approx 1.5\,\mathrm{GeV})$

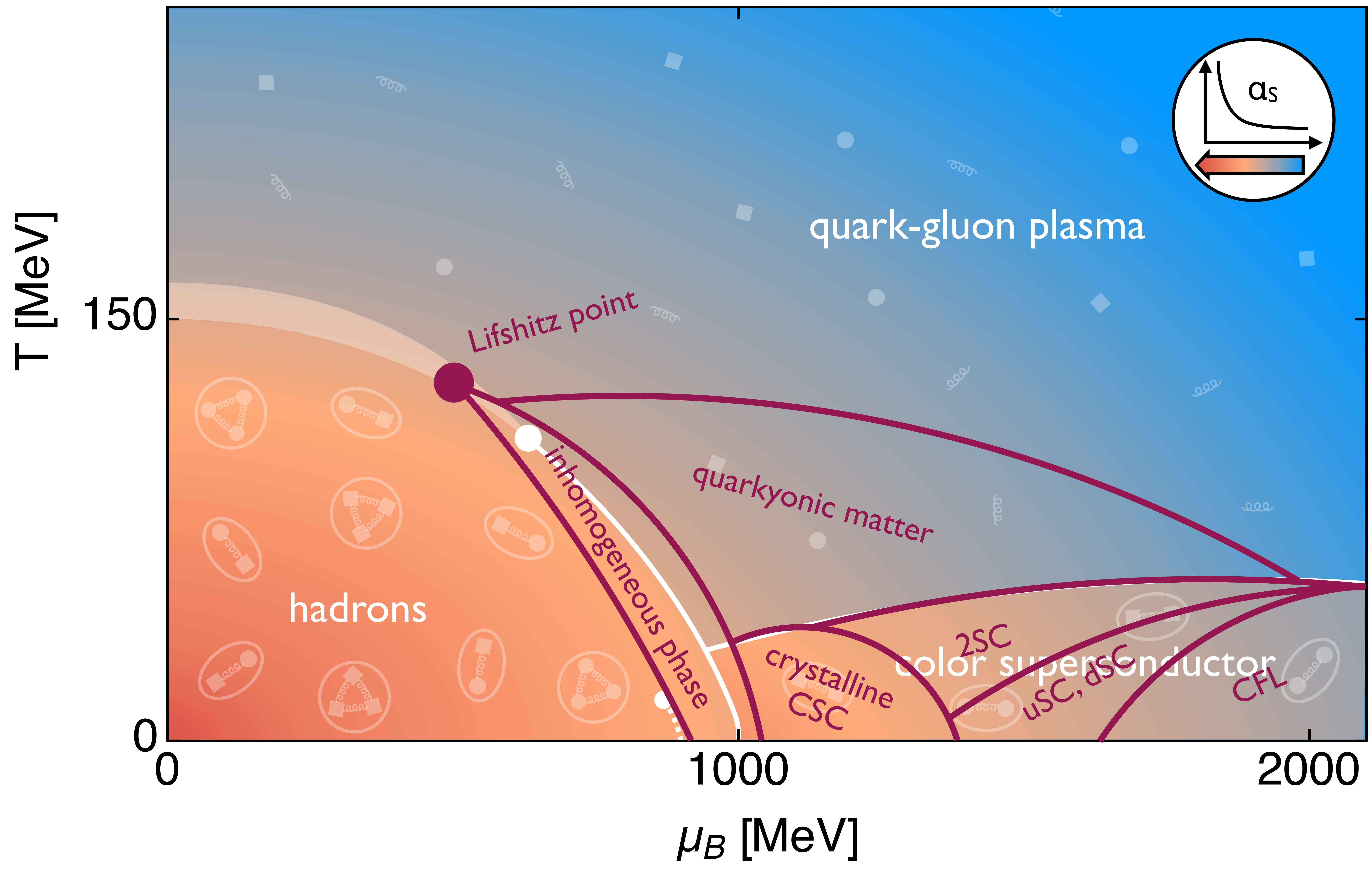
↓

study dense QCD matter to understand neutron stars & their mergers

QCD PHASE DIAGRAM



QCD PHASE DIAGRAM



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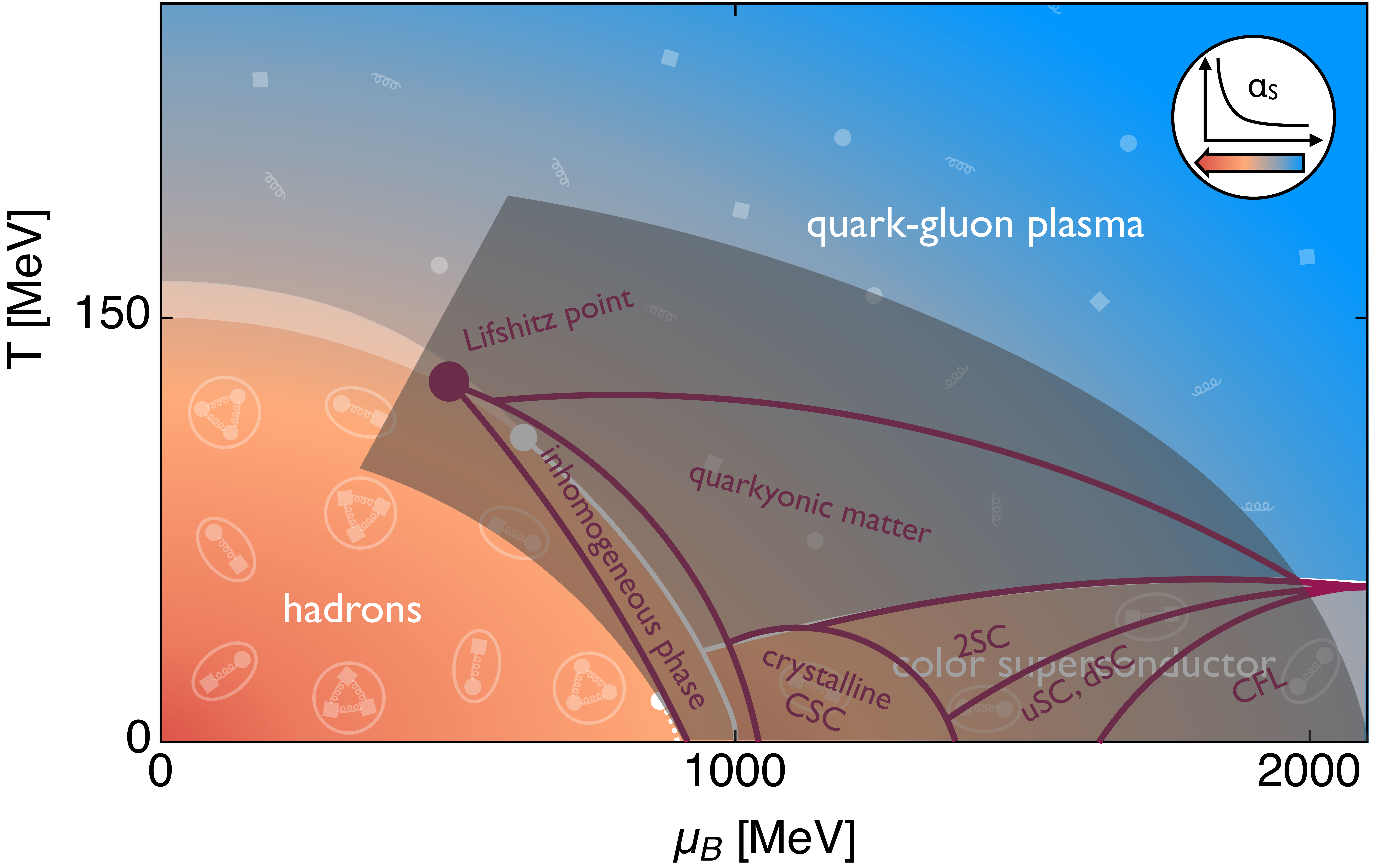


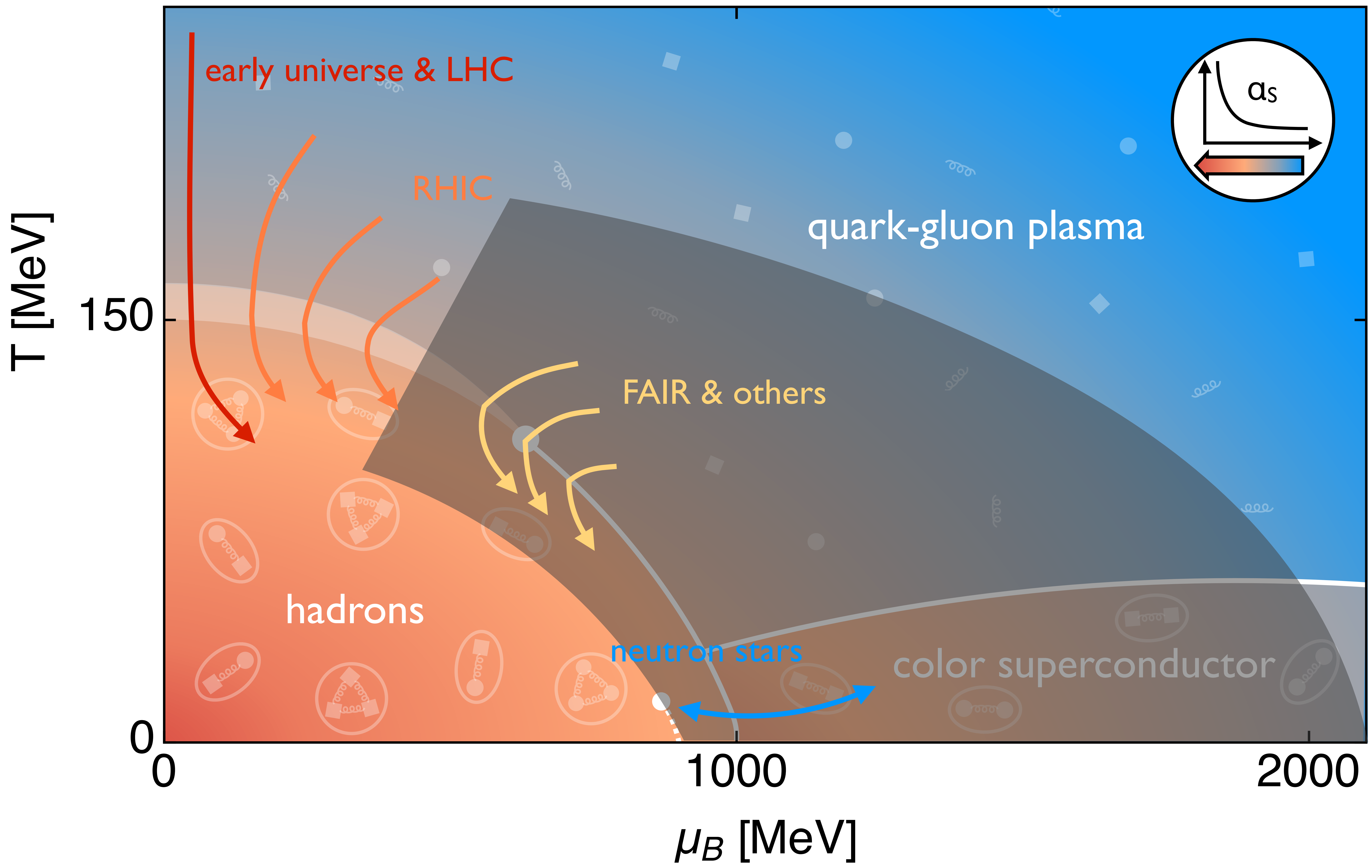
theoretical challenges:

- strong coupling: non-perturbative
- sign problem at finite density: lattice QCD of limited use
- different degrees of freedom in different phases: EFTs of limited use

↓

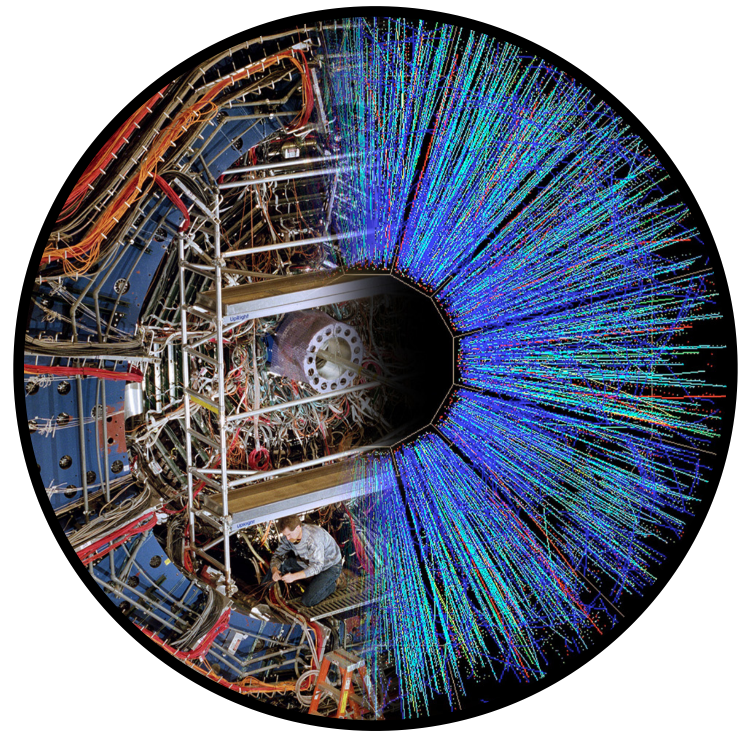
use functional methods

QCD PHASE DIAGRAM

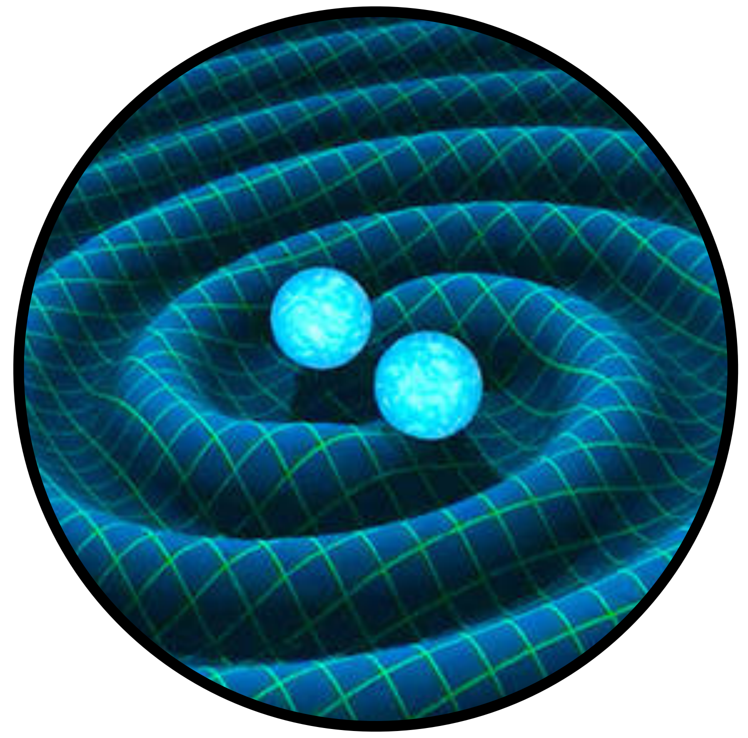


Experiments:

heavy-ion collisions



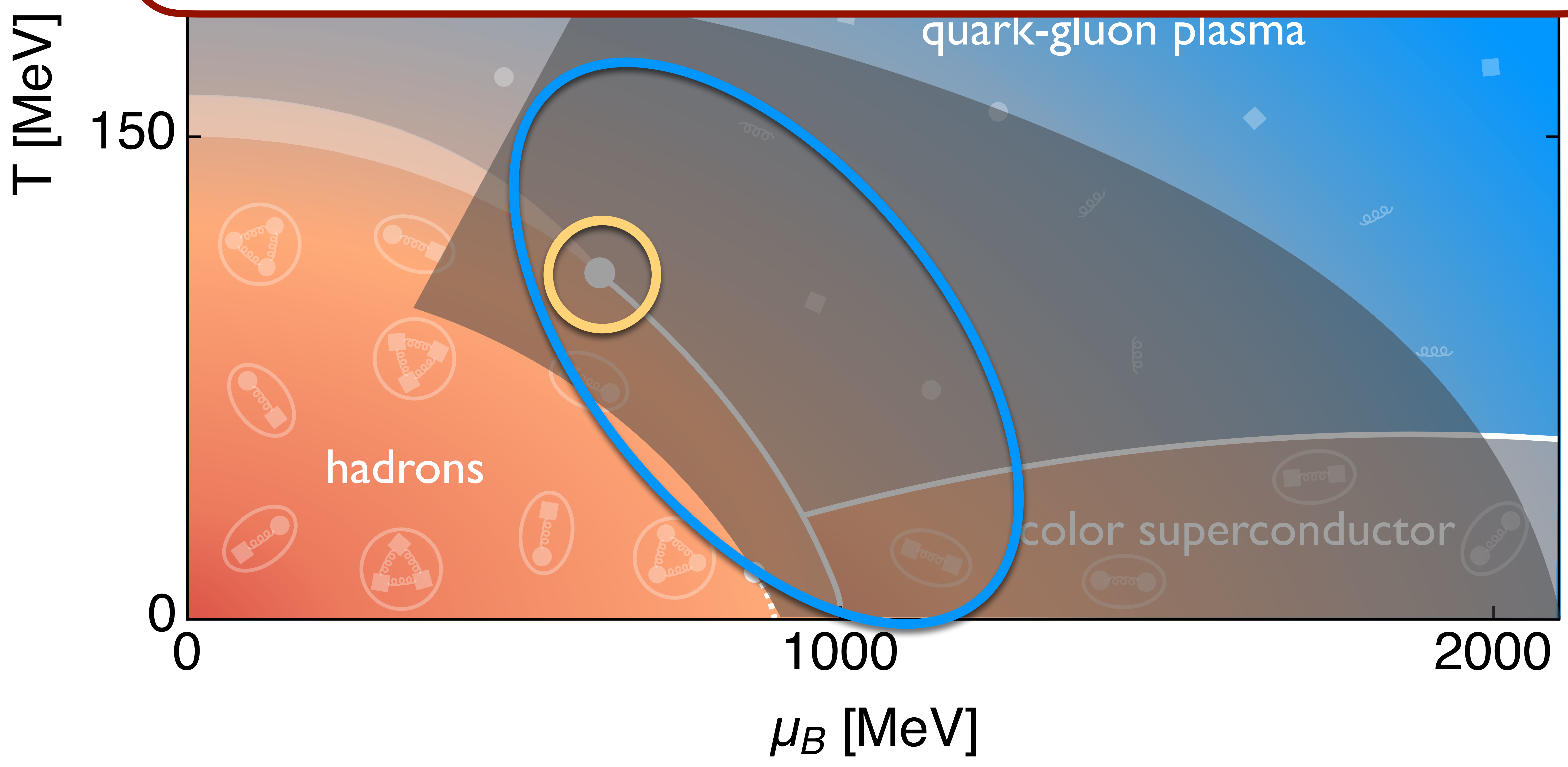
multi-messenger astronomy



QCD PHASE DIAGRAM IN THIS TALK

open questions

- location of phase transitions? → CEP
- what happens at large μ_B? → **the moat regime & pattern formation**



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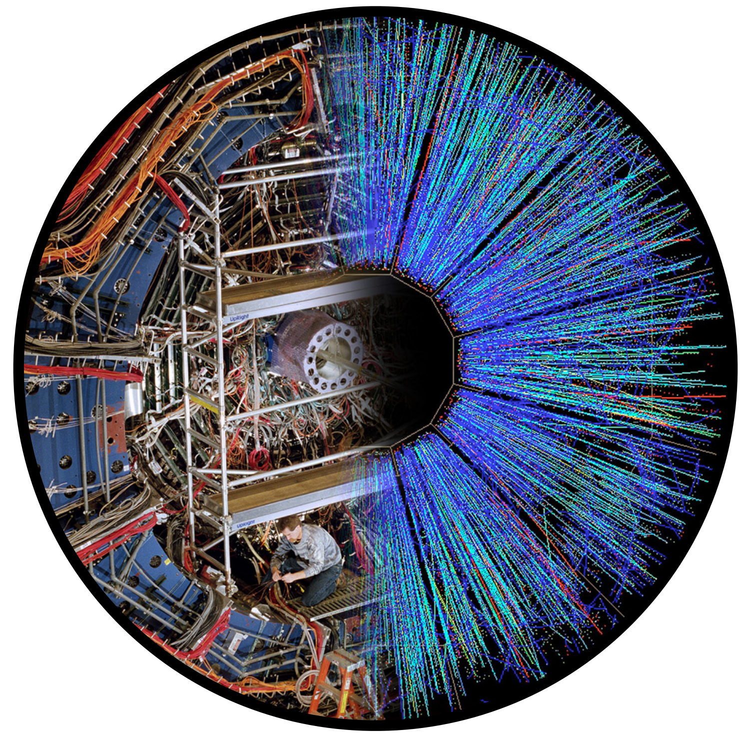

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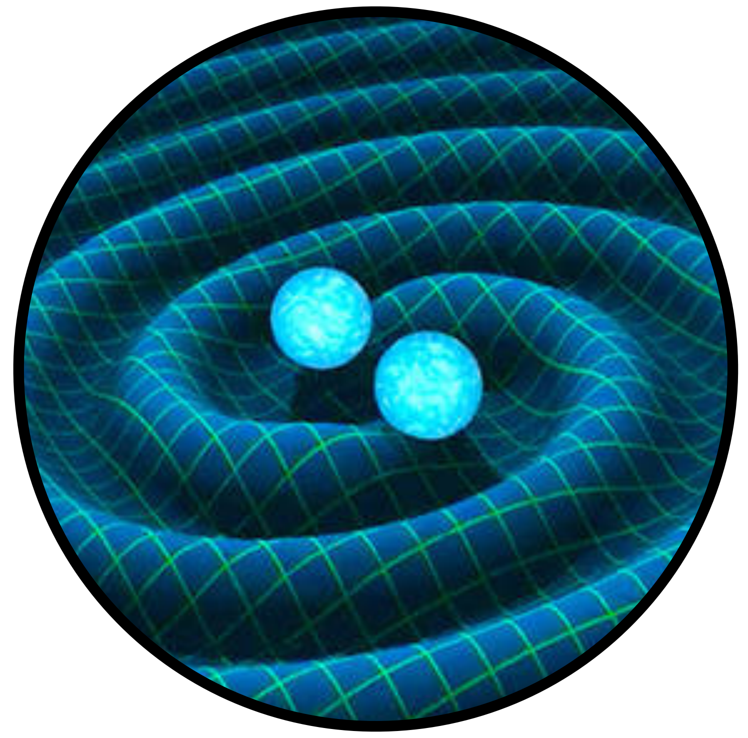

THE QCD CRITICAL POINT

FUNCTIONAL METHODS

The path integral encodes all possible correlation functions of a QFT

$$Z[J] = \int \mathscr{D}\varphi \, e^{iS[\varphi] + i\int_x J(x)\varphi(x)}$$

$$\langle \varphi \cdots \varphi \rangle \sim \frac{\delta}{\delta J} \cdots \frac{\delta}{\delta J} Z[J] \Big|_{J=0}$$

solving a QFT ⇔ knowing all correlation functions

functional methods provide exact relations for correlation functions: DSE & FRG

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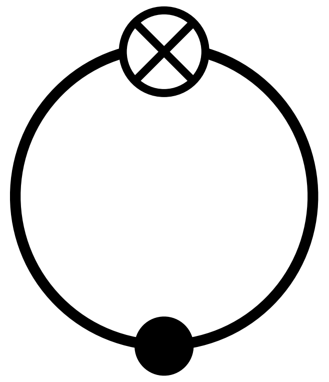
functional methods provide exact relations for correlation functions: DSE & FRG

FRG: introduce mass-like regulator R_k that cuts-off field modes with momenta $p^2 \lesssim k^2 \longrightarrow$ flow equation

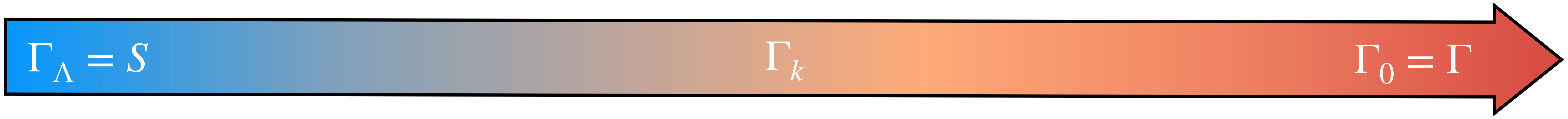
$$\phi = \frac{\delta Z[J]}{\delta J}$$

$$\Gamma[\phi] = \sup_J \left\{ \int_x J(x)\,\phi - \ln Z[J] \right\}$$

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \left[\left(\frac{\delta \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k \right)^{-1} \cdot \partial_k R_k \right] = \frac{1}{2}$$ 

[Wetterich 1993]

realizes **Wilson's RG idea** of successively integrating out quantum fluctuations [Nobel Prize in 1982]



start from "microscopic" QCD action at $\Lambda \gg 1\,\mathrm{GeV}$ (perturbative QCD)

Γ_k incorporates all quantum fluctuations down to scale k

full effective action including all quantum fluctuations

FUNCTIONAL METHODS

- define infinite tower of coupled equations for all correlation functions: truncations necessary
- no sign problem: finite density, real time and complex parameter spaces are all directly accessible
- one/two-loop exact: both intuition and techniques can be leveraged

QCD related reviews:

FRG

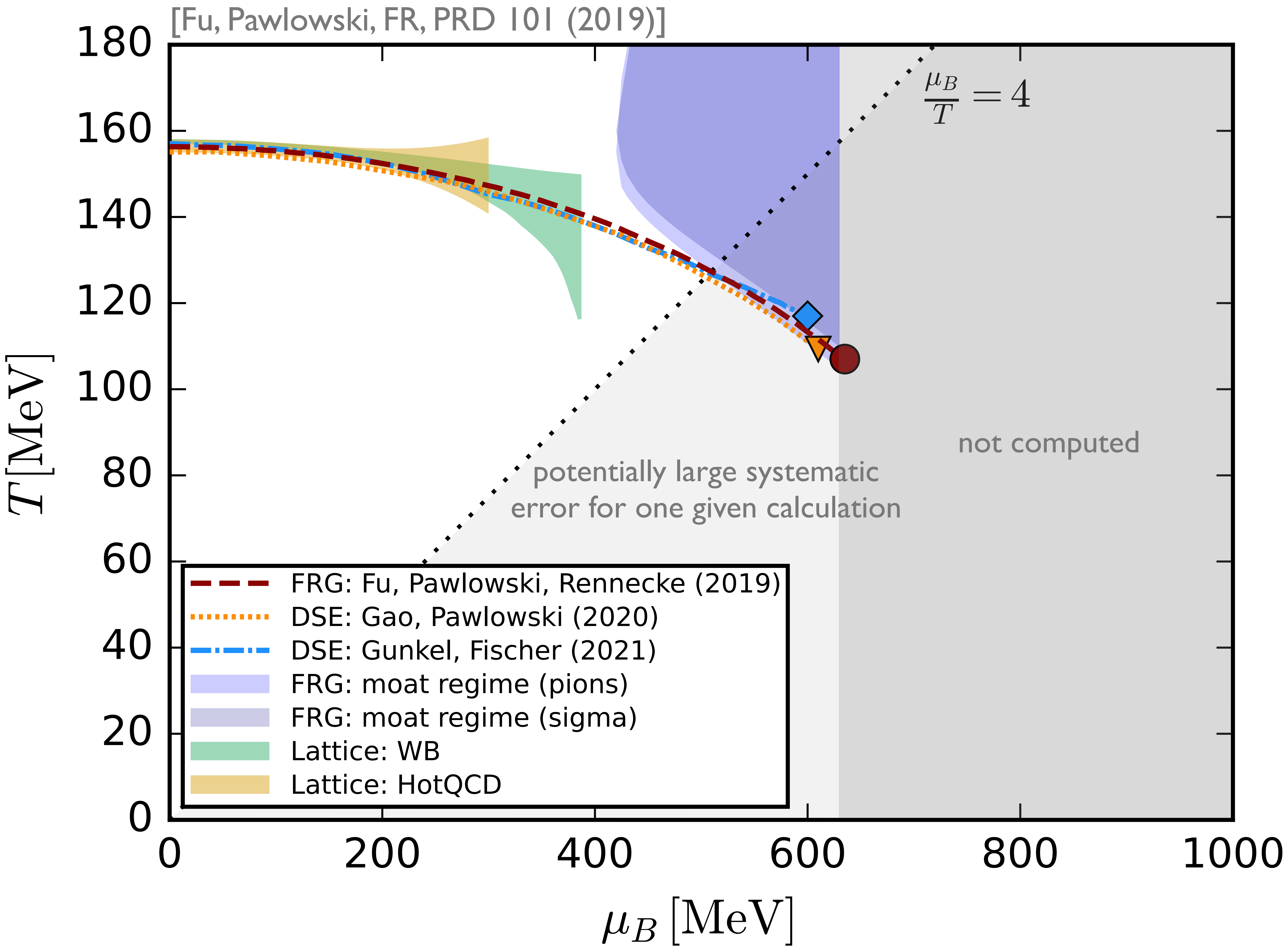
[Pawlowski, arXiv:0512261]
[Gies, arXiv:0611146]
[Rosten, arXiv:1003.1366]
[Braun, arXiv:1108.4449]
[Dupuis at al., arXiv:2006.04853]
[Fu, arXiv:2205.00468]

DSE

[Alkofer, von Smekal, arXiv:0007355]
[Fischer, arXiv:0605173]
[Roberts, Schmidt, arXiv:0005064]
[Eichmann at al, arXiv:1606.09602]
[Fischer, arXiv:1810.12938]
[Huber, arXiv:1808.05227]

QCD PHASE DIAGRAM & THE CEP

Results for the chiral transition from direct computations in QCD



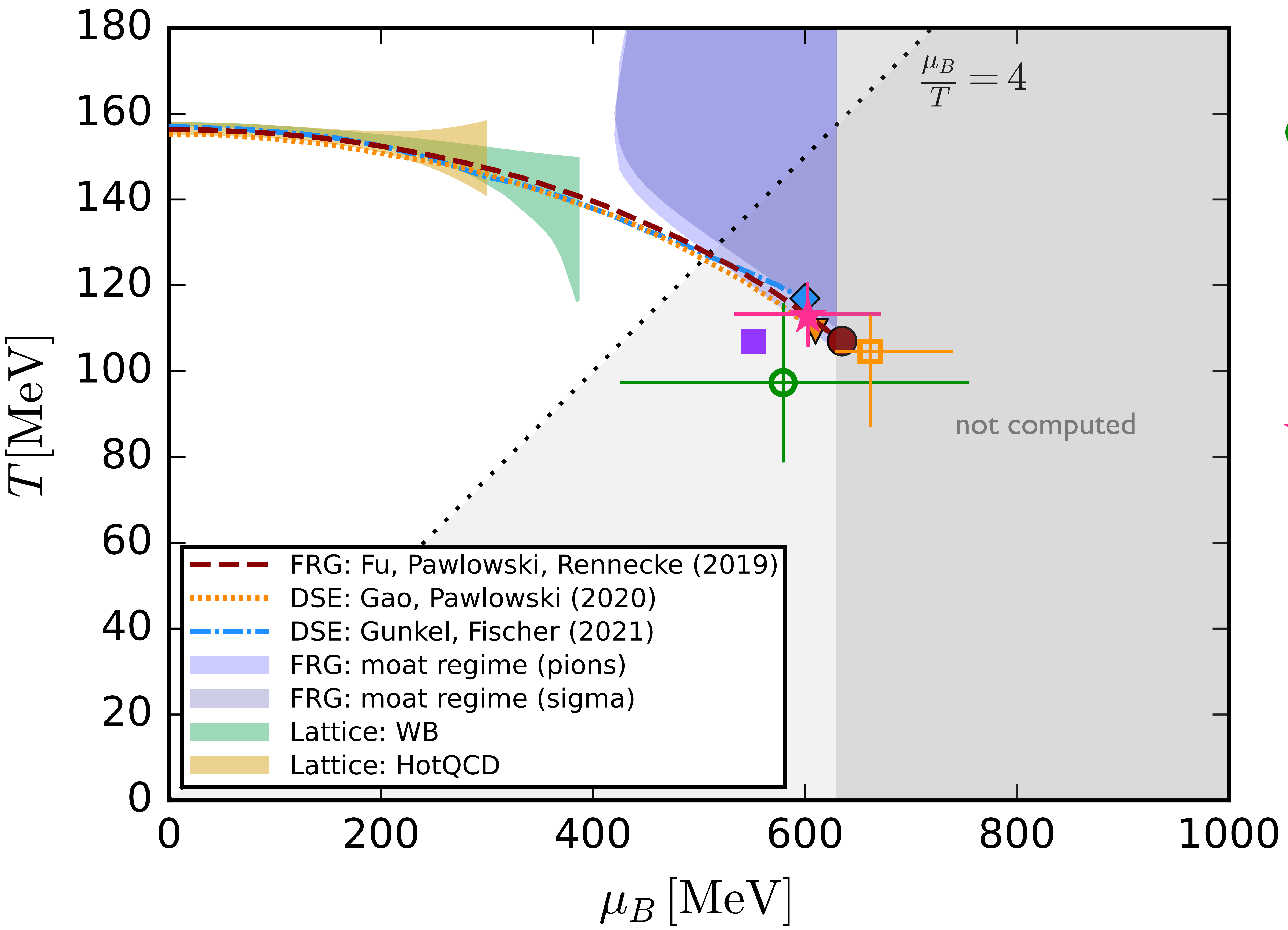
- show only results that agree with lattice data at $\mu_B = 0$
- need to improve/check systematics for $\mu_B/T \gtrsim 4$ (work in progress), but good agreement between different methods and approximations

CEP at $(T, \mu_B) \approx (110, 630)$ MeV

- indications for a new feature: **the moat regime** (more on that later)

QCD PHASE DIAGRAM & THE CEP

FRG & DSE results corroborated by subsequent extrapolations of lattice data



using Yang-Lee edge singularities:

- conformal Padé [Basar, PRC 110 (2024)]
- multi-point Padé [Clarke et al., 2405.10196]

$N_\tau = 6$ results + continuum estimate

using thermodynamics:

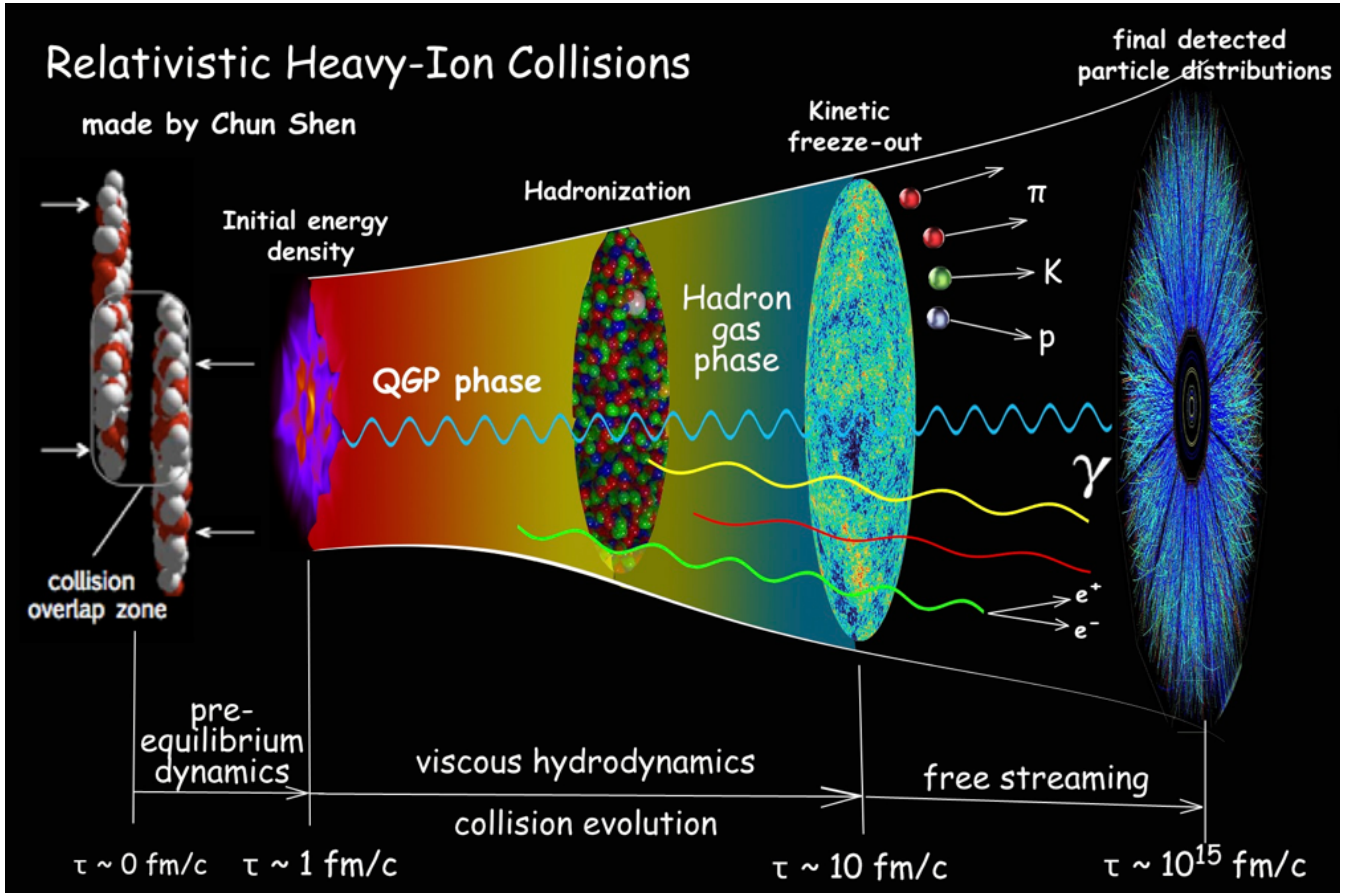
- constant entropy density [Shah et al. 2410.16206]
- holography [Hippert et al., PRD 110 (2023)]

(agrees with [Cai et al., PRD 106 (2022)])

CEP location well constrained by now and it's in **FAIR range!**

$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\mathrm{GeV}$

SEARCH FOR THE CEP: HEAVY-ION COLLISIONS

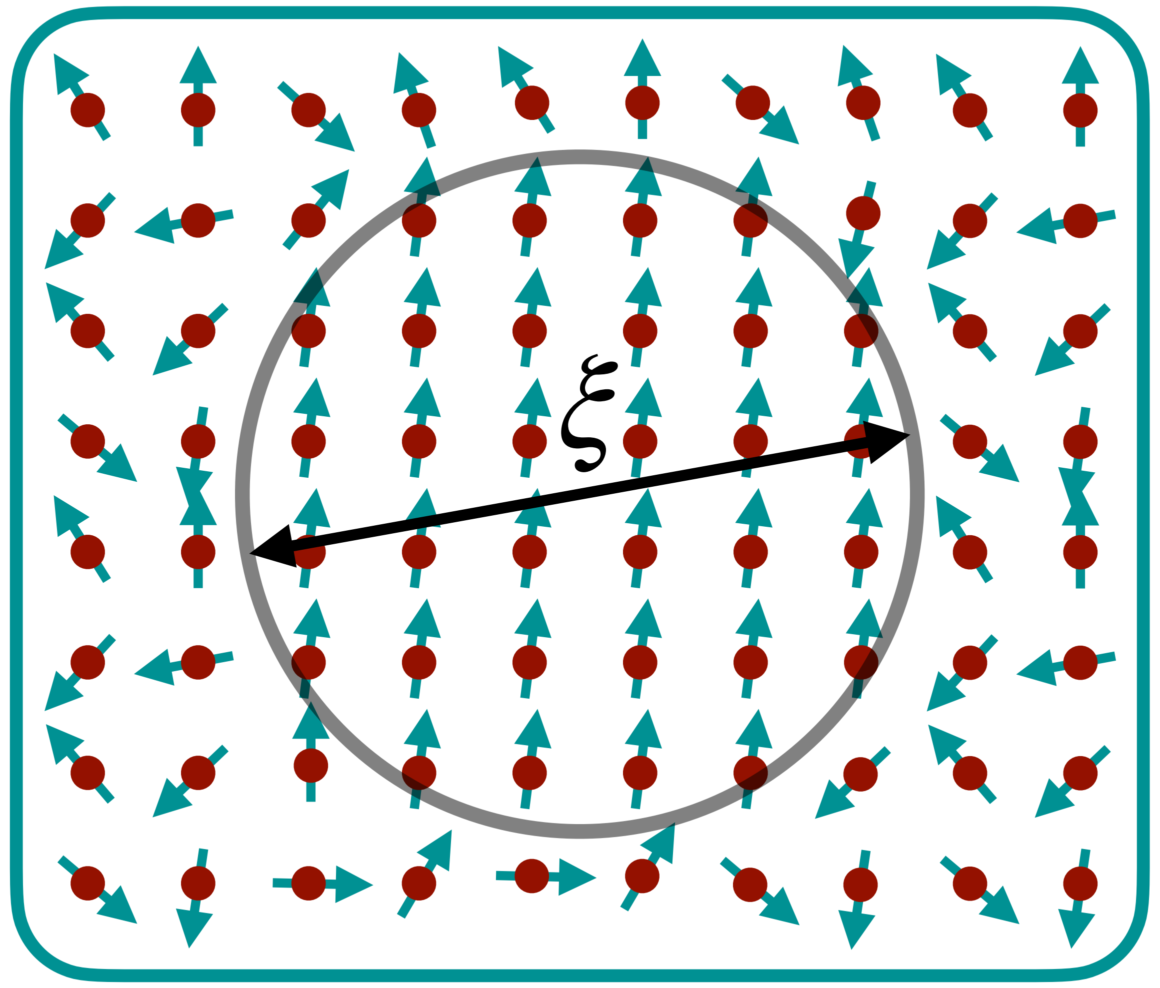


[D. Fehrenz, GSI/FAIR (2024)]

→ **imprints of phase structure at freeze-out?**

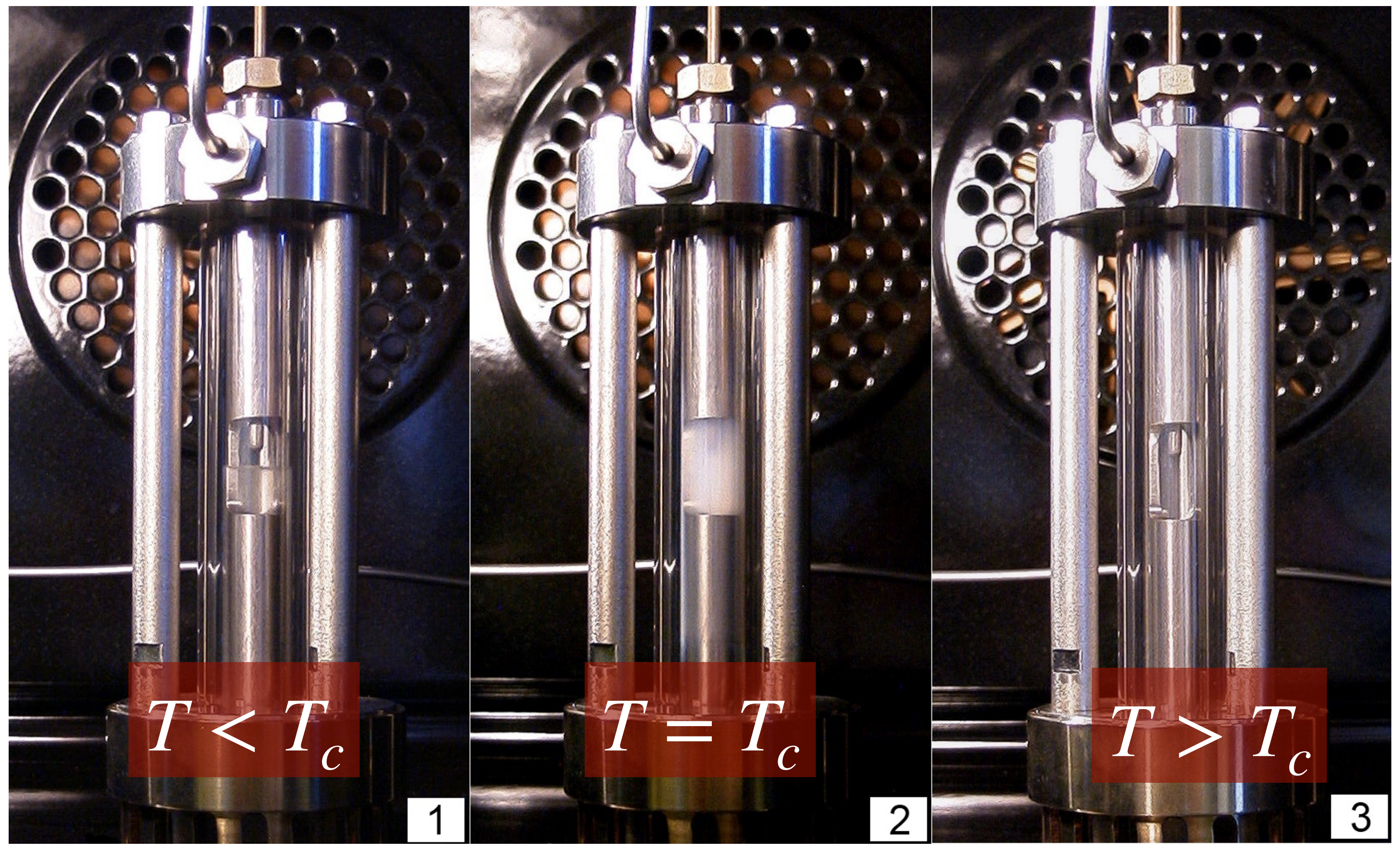
CRITICAL PHENOMENA & UNIVERSALITY

correlation length



2nd order transition:
$\xi \to \infty$

fluctuations on all length scales

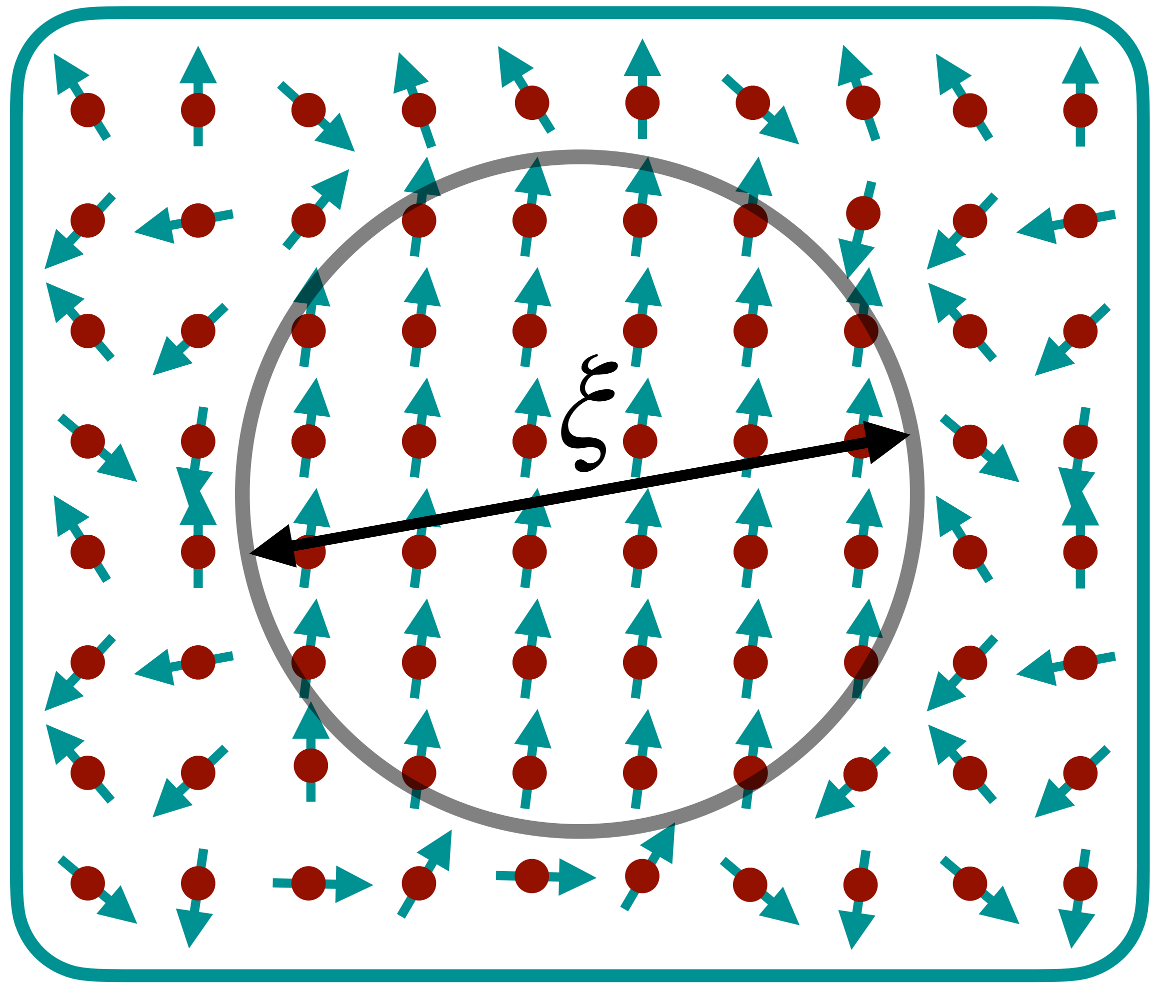


critical opalescence of ethane [Wikipedia]

Near the critical point the system is scale invariant and microscopic details are irrelevant

CRITICAL PHENOMENA & UNIVERSALITY

correlation length



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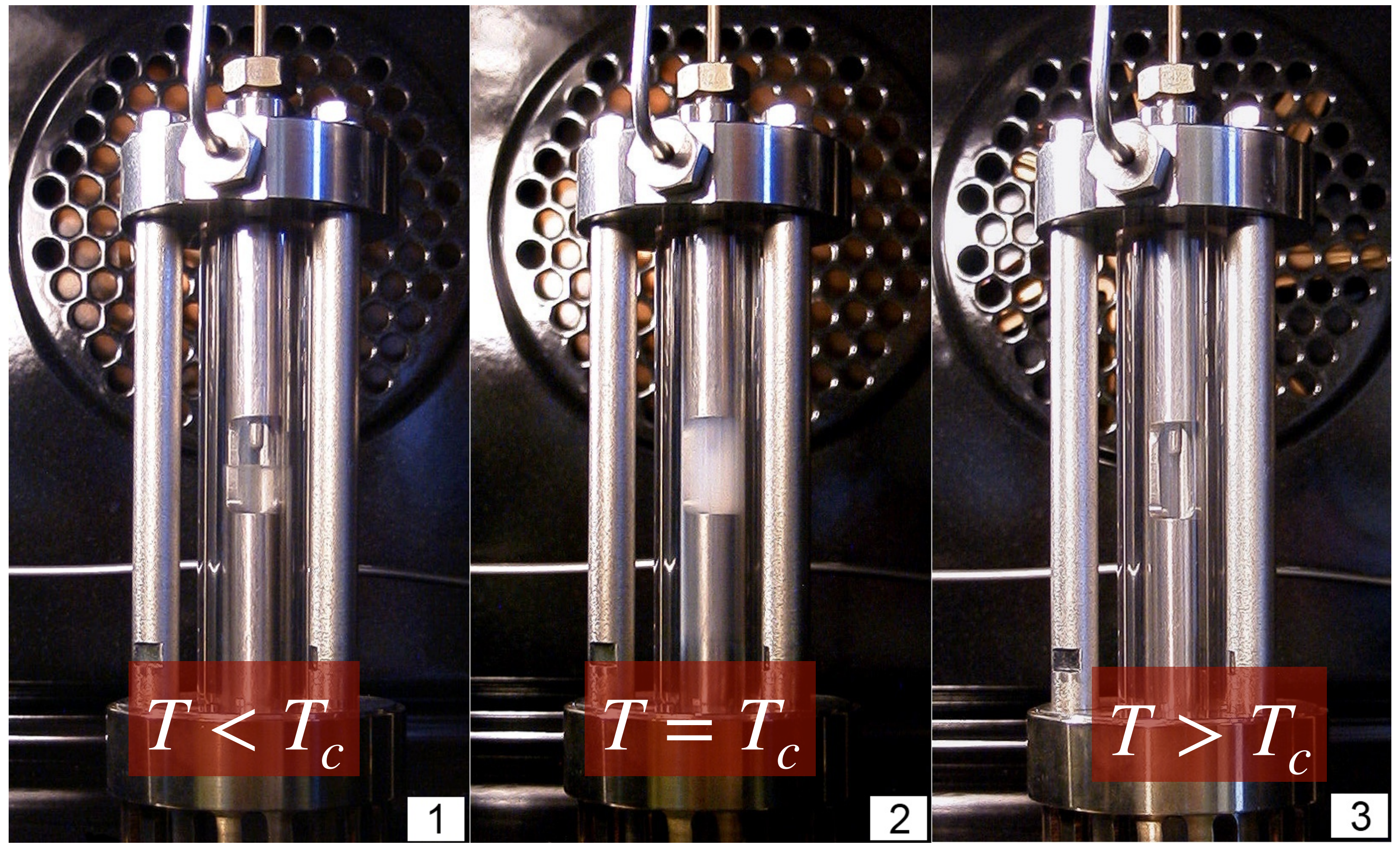
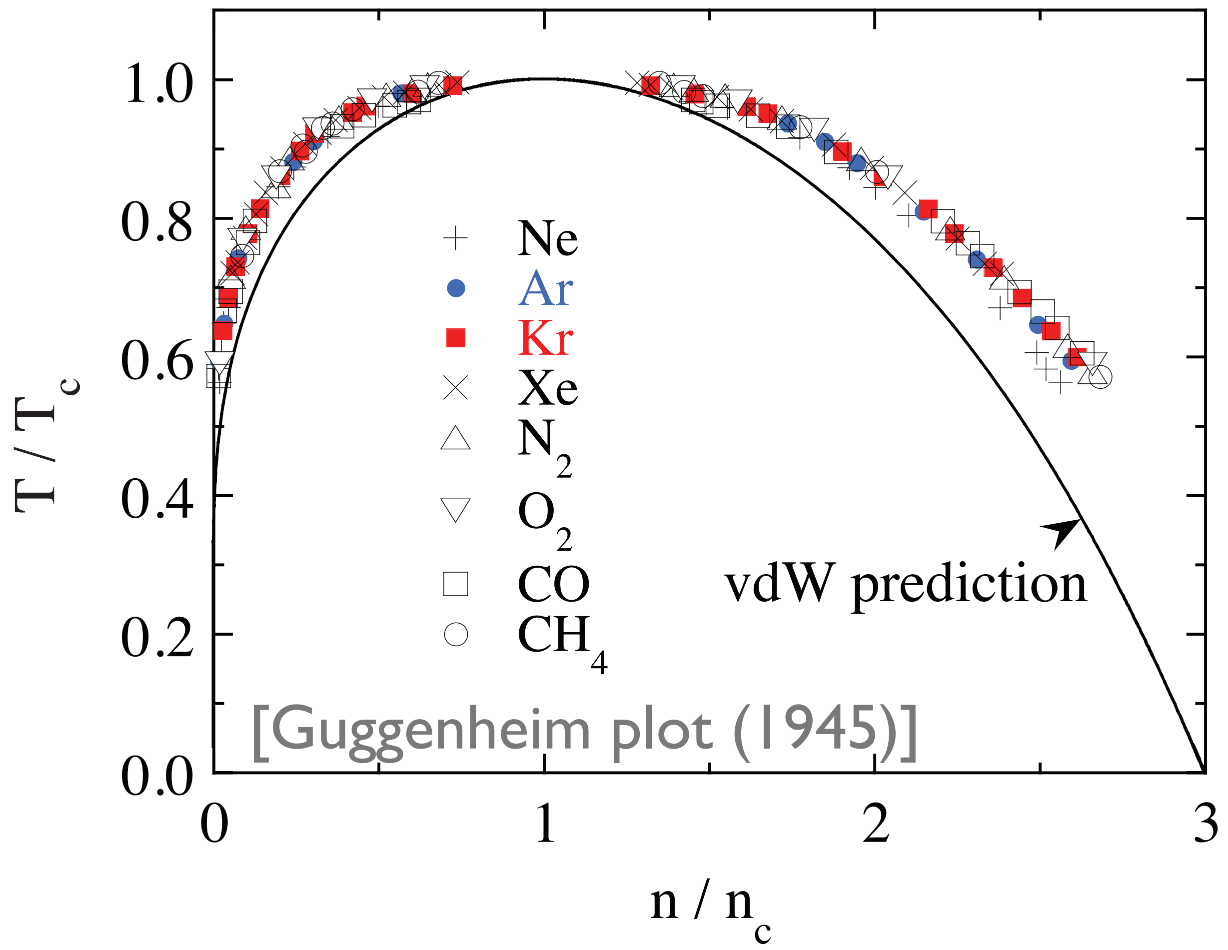


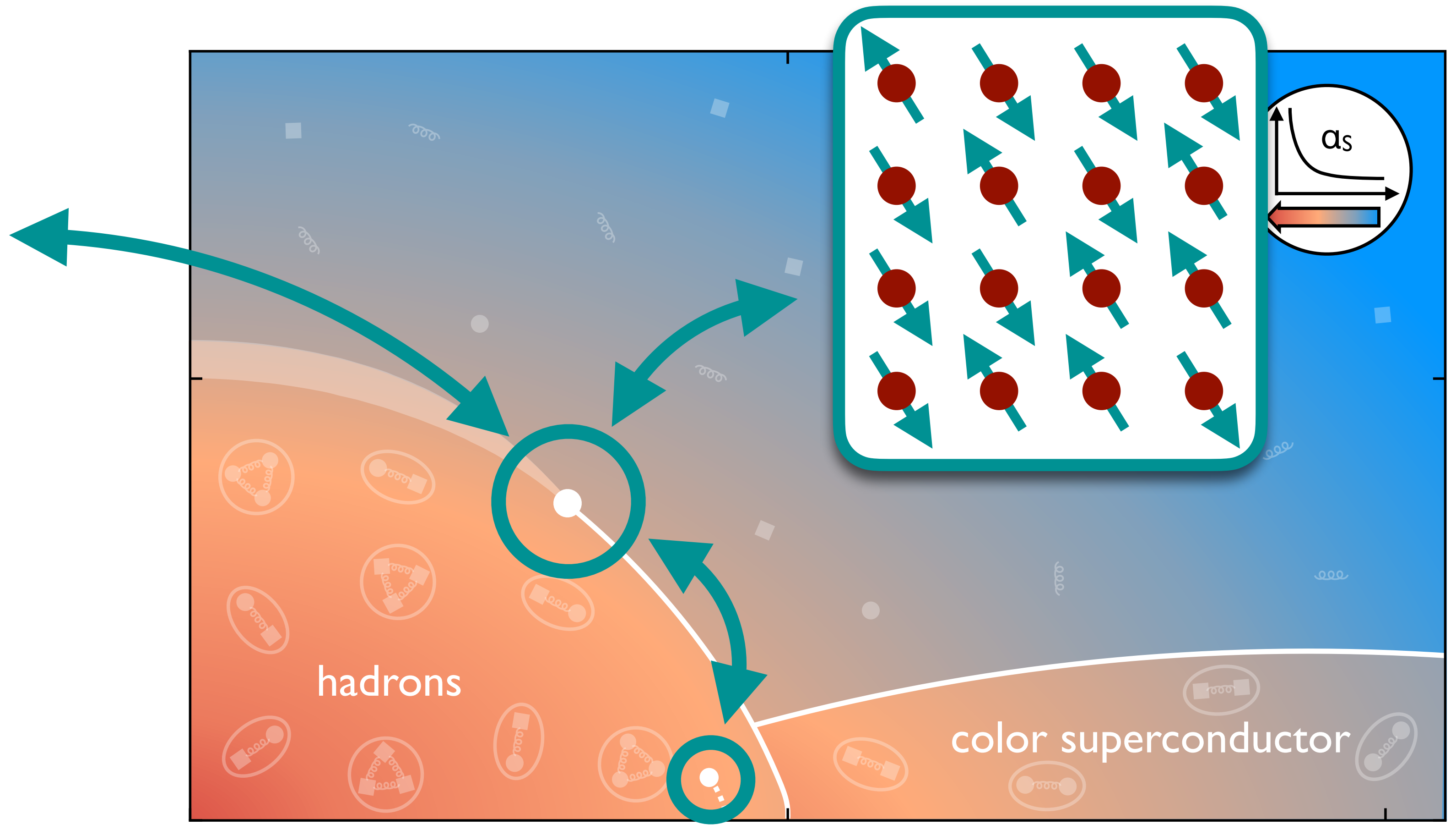
critical opalescence of ethane [Wikipedia]

Near the critical point the system is scale invariant and microscopic details are irrelevant

Universality: main features of the system are described by universal critical exponents, e.g., $\xi \sim (T - T_c)^{-\nu}$

example:
liquid-gas transition
=
3d Ising
=
QCD CEP



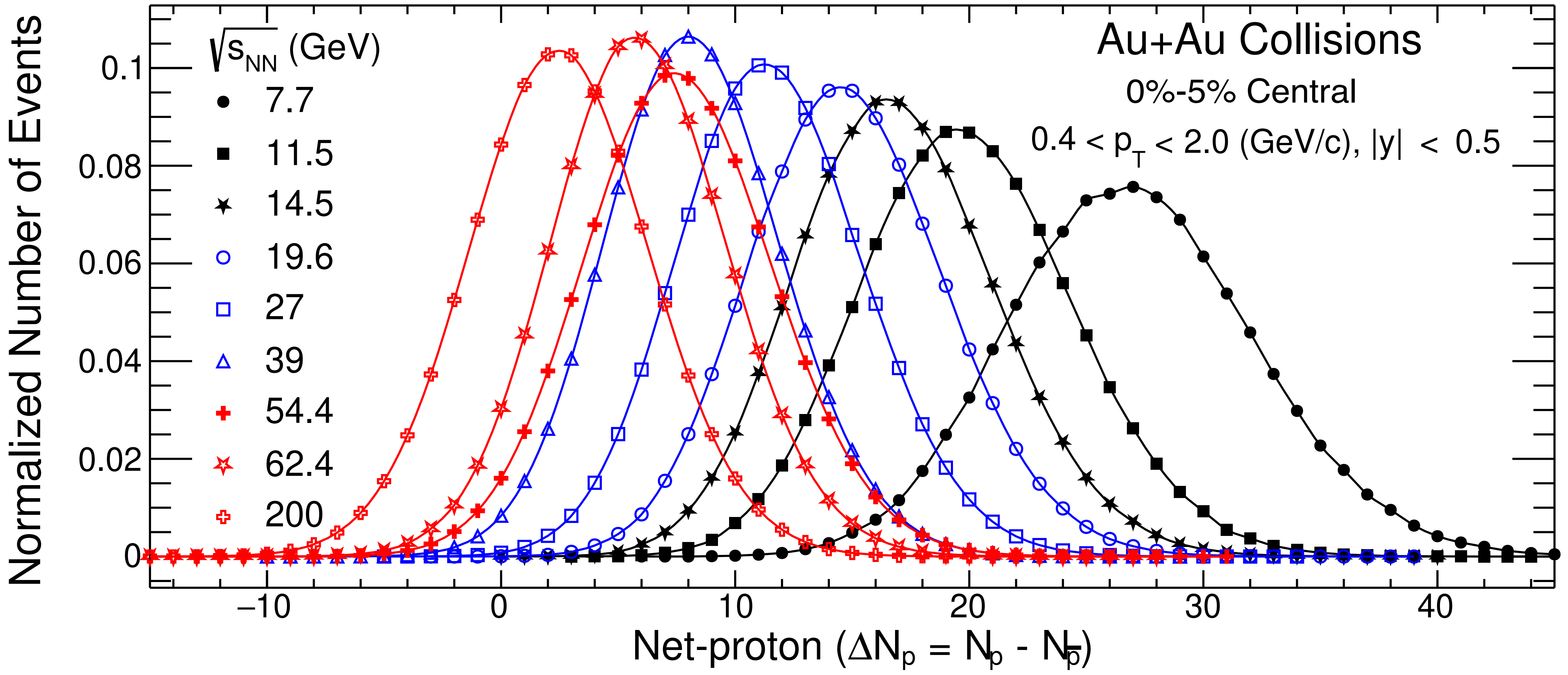


CAN WE OBSERVE SCALING NEAR THE CEP?

experiment: heavy-ion collisions

- measure net-proton distributions $P(N_P)$

[STAR (2021)]



- net-proton susceptibilities from the distribution

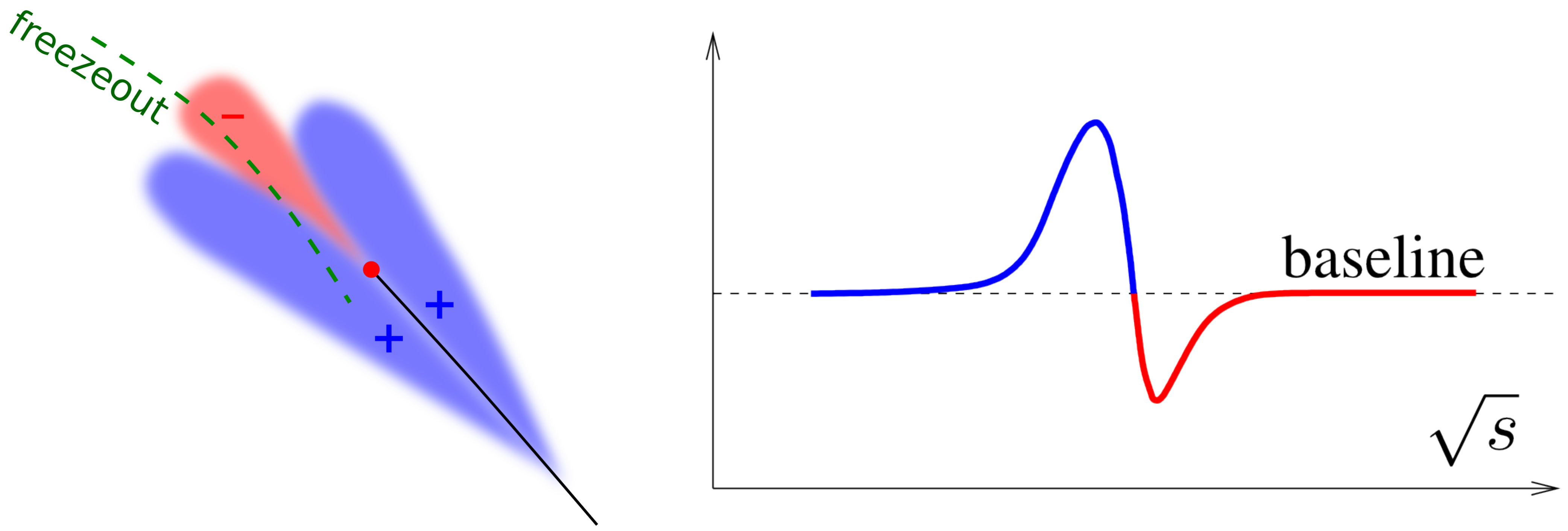
$$\chi_n^P \sim \sum_{N_P} \left[\left(N_P - \langle N_P \rangle \right)^n + \cdots \right] P(N_P)$$

theory

- net-baryon susceptibilities from the pressure

$$\chi_n^B = T^{n-4} \frac{\partial^n p}{\partial \mu_B^n}$$

- χ_n scale near CEP, e.g., $\chi_4 \sim \xi^7$
- scaling near the CEP: non-monotonic beam-energy dependence of kurtosis $\sim R_{42}^B = \chi_4/\chi_2$



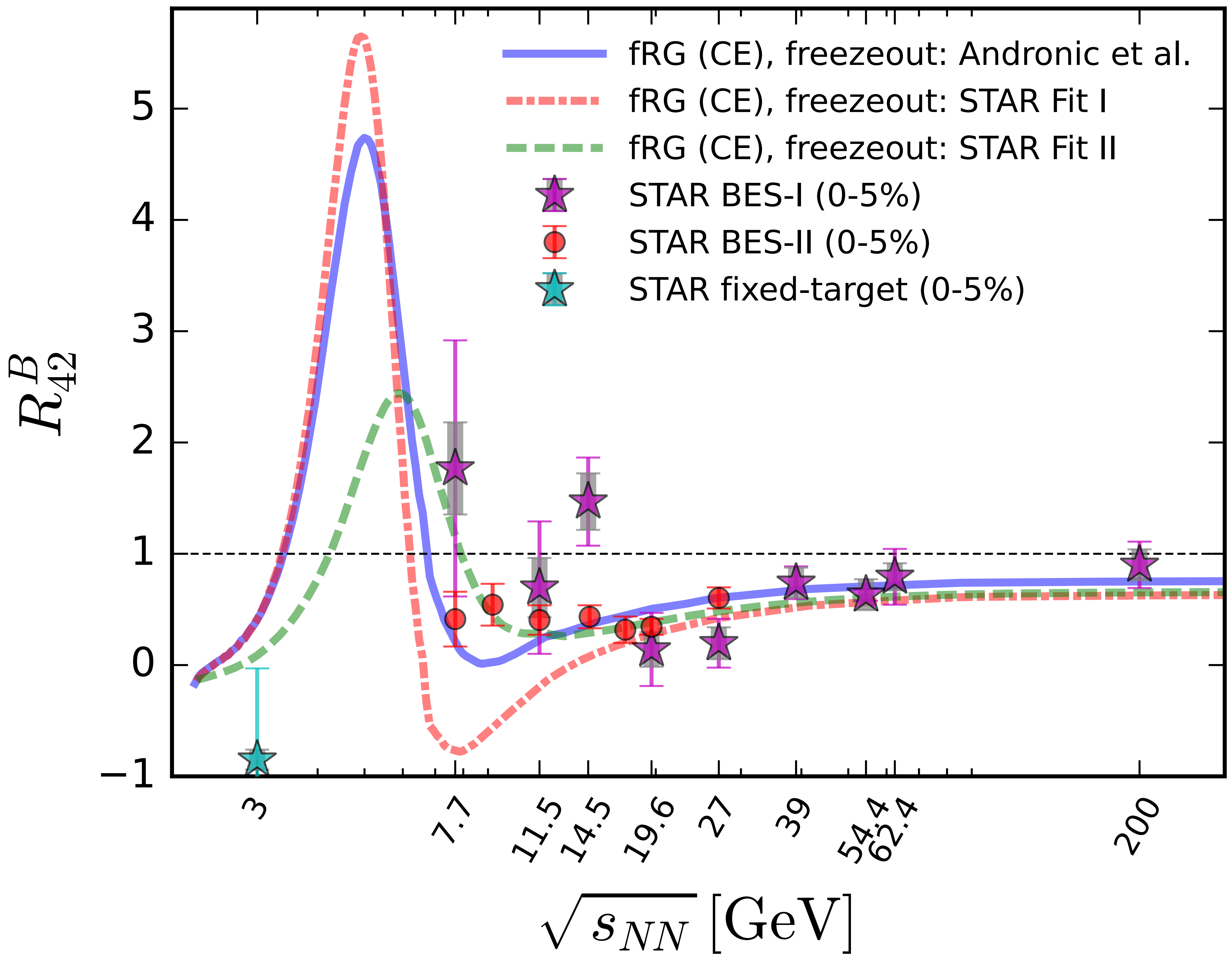
[Stephanov, PRL 102 (2009)]

→ measurements can be sensitive to critical fluctuations, but there are **many caveats and subtleties!**

RIPPLES OF THE CEP

[Fu, Luo, Pawlowski, FR, Yin, PRD 111 (2023)]

FRG: low-energy model with QCD and HIC input



- pronounced non-monotonicity at low beam-energies, but no scaling

→ criticality not necessary for non-monotonic $\sqrt{s_{NN}}$ dependence of R_{42}

- CEP location encoded in the peak height
- no sign of the CEP in experimental data yet

→
- **need data between $\sqrt{s_{NN}} = 3 - 8\,\mathrm{GeV}$: FAIR**
- **need better observables (smoking gun)?**

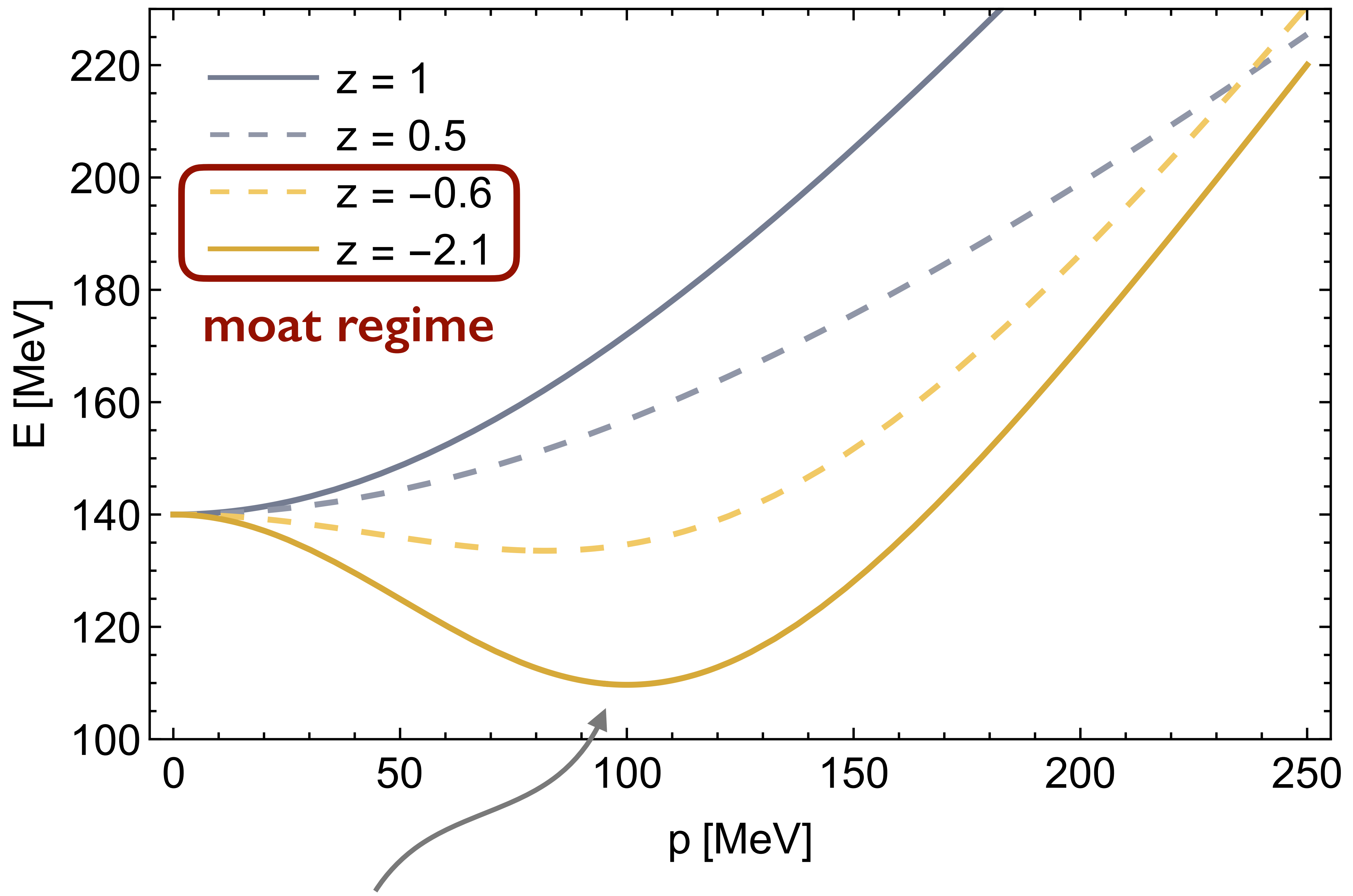
THE MOAT REGIME

THE MOAT REGIME

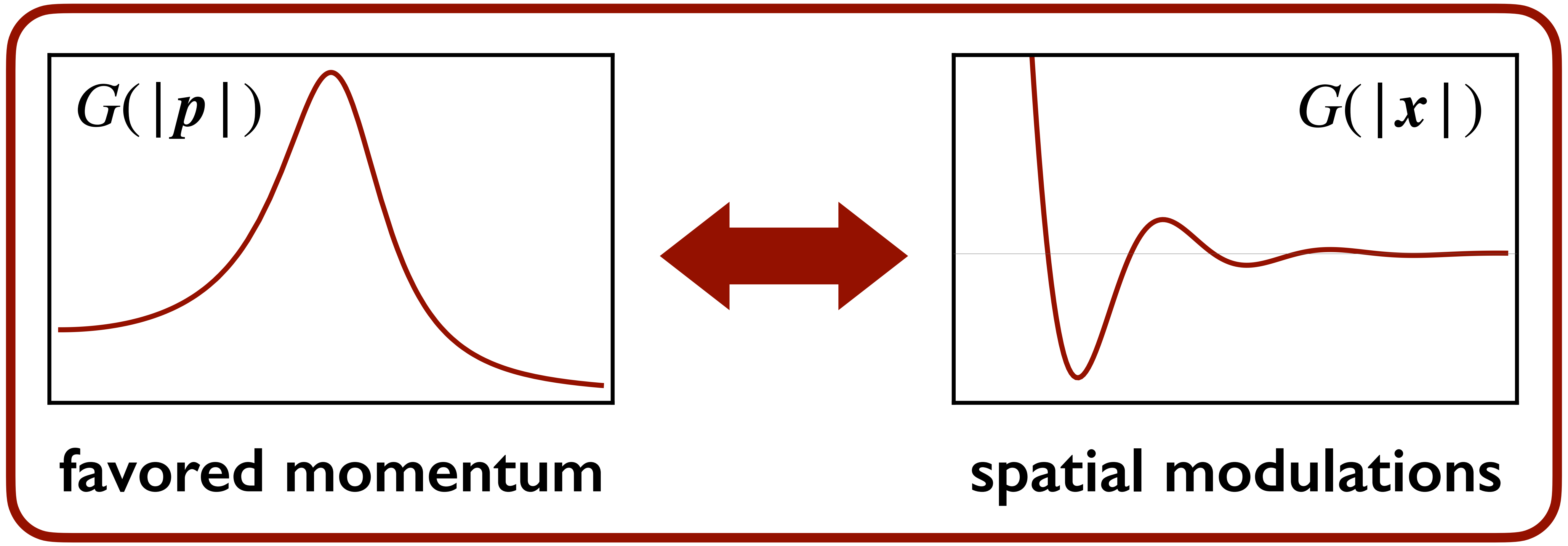
[Pisarski, FR, PRL 127 (2021)]

(static) boson dispersion is minimal at nonzero momentum

$$\sqrt{1/G(p_0=0,\boldsymbol{p}^2)} = \quad E(\mathbf{p}^2) = \sqrt{Z(\mathbf{p}^2)\,\mathbf{p}^2 + m^2} \approx \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p}^6) + m^2}$$



"gain energy by going faster"

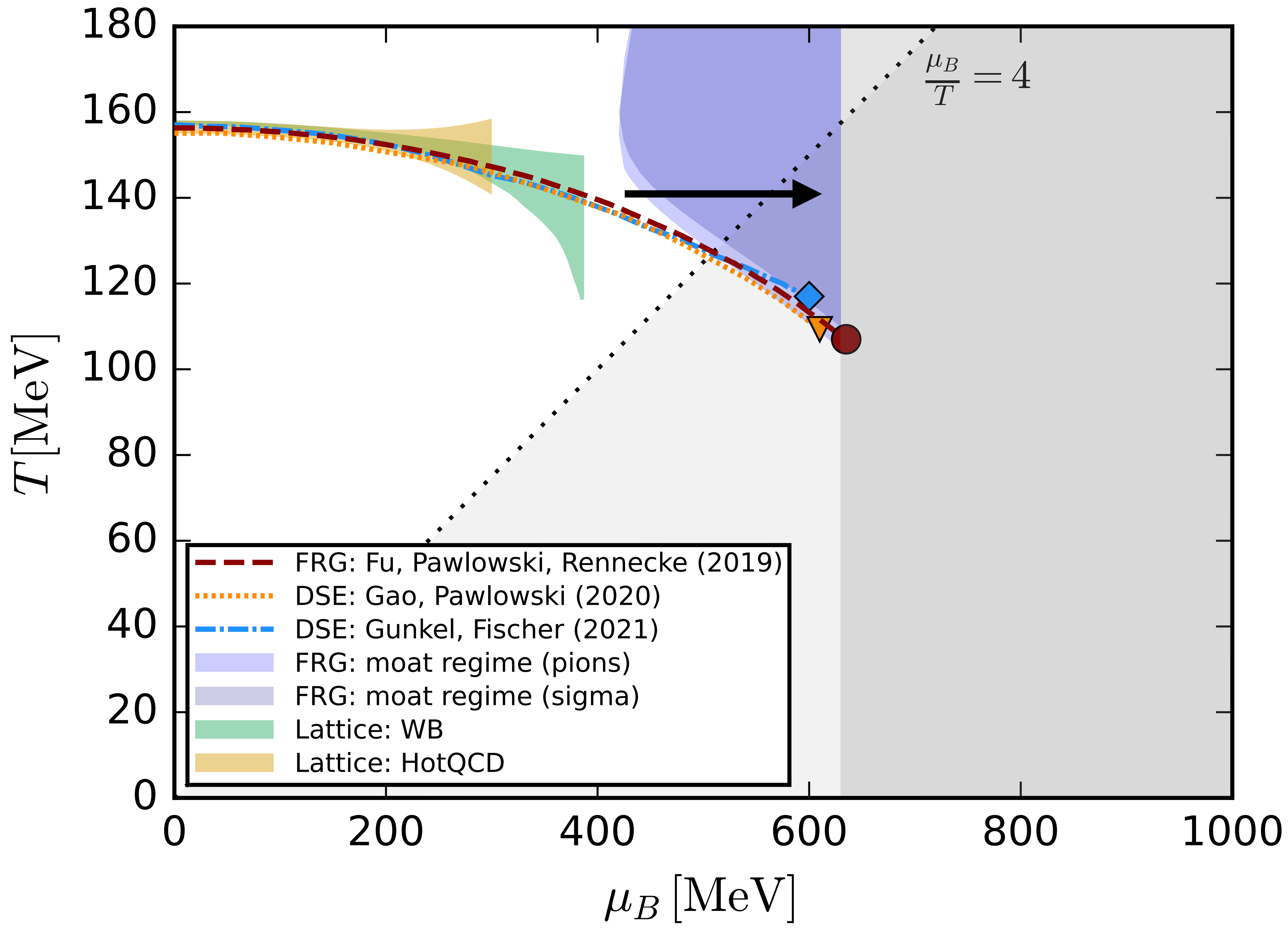


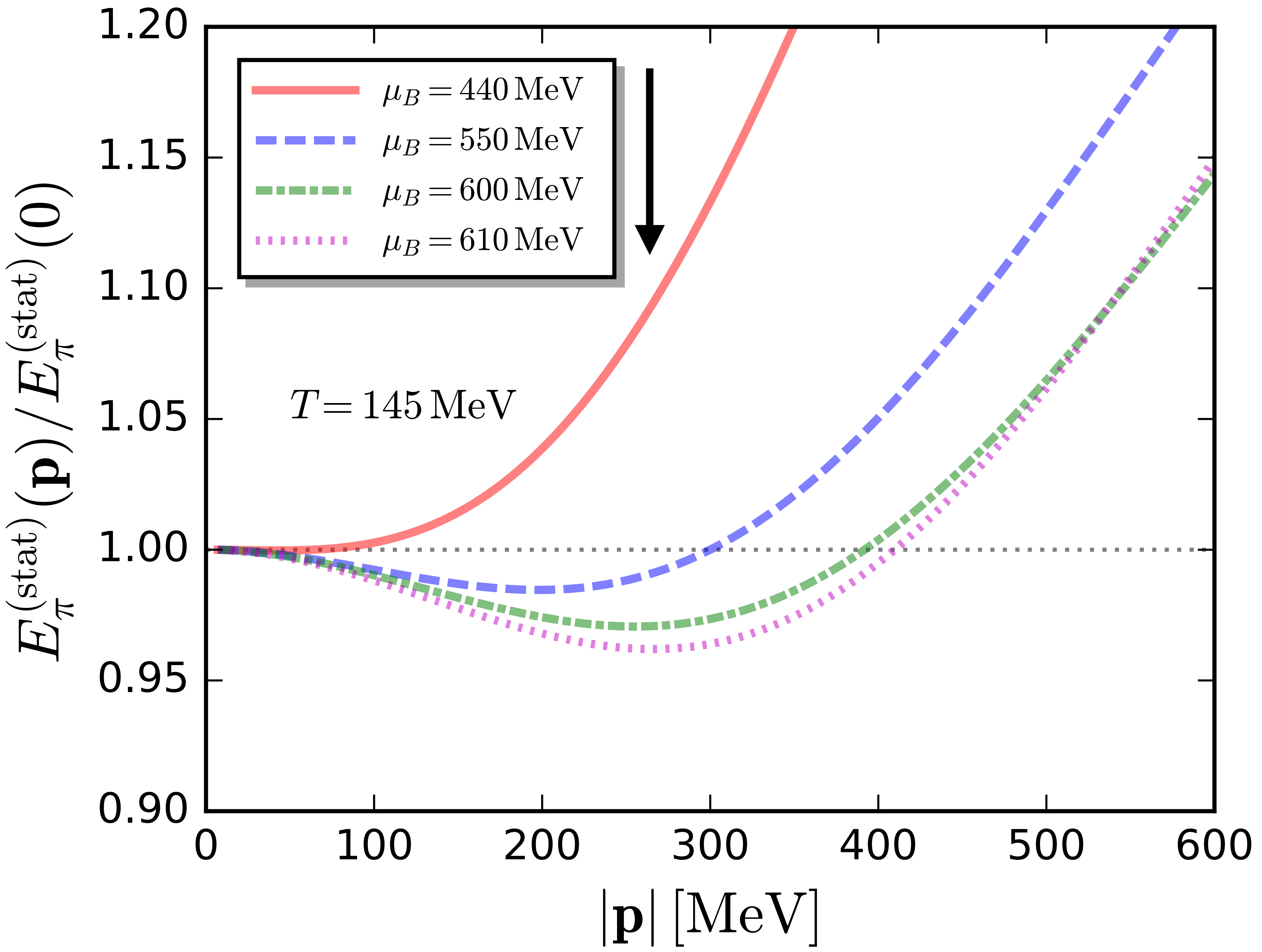
THE MOAT REGIMES IN QCD

[Fu, Pawlowski, Pisarski, FR, Wen, Yin, 2412.15949]

- moat appears in static mesonic dispersion; regime appears to depend slightly on the species
- the lighter the meson, the stronger the signal → pions are good probes

cf. [Töpfel, Pawlowski, Braun, 2412.16059]



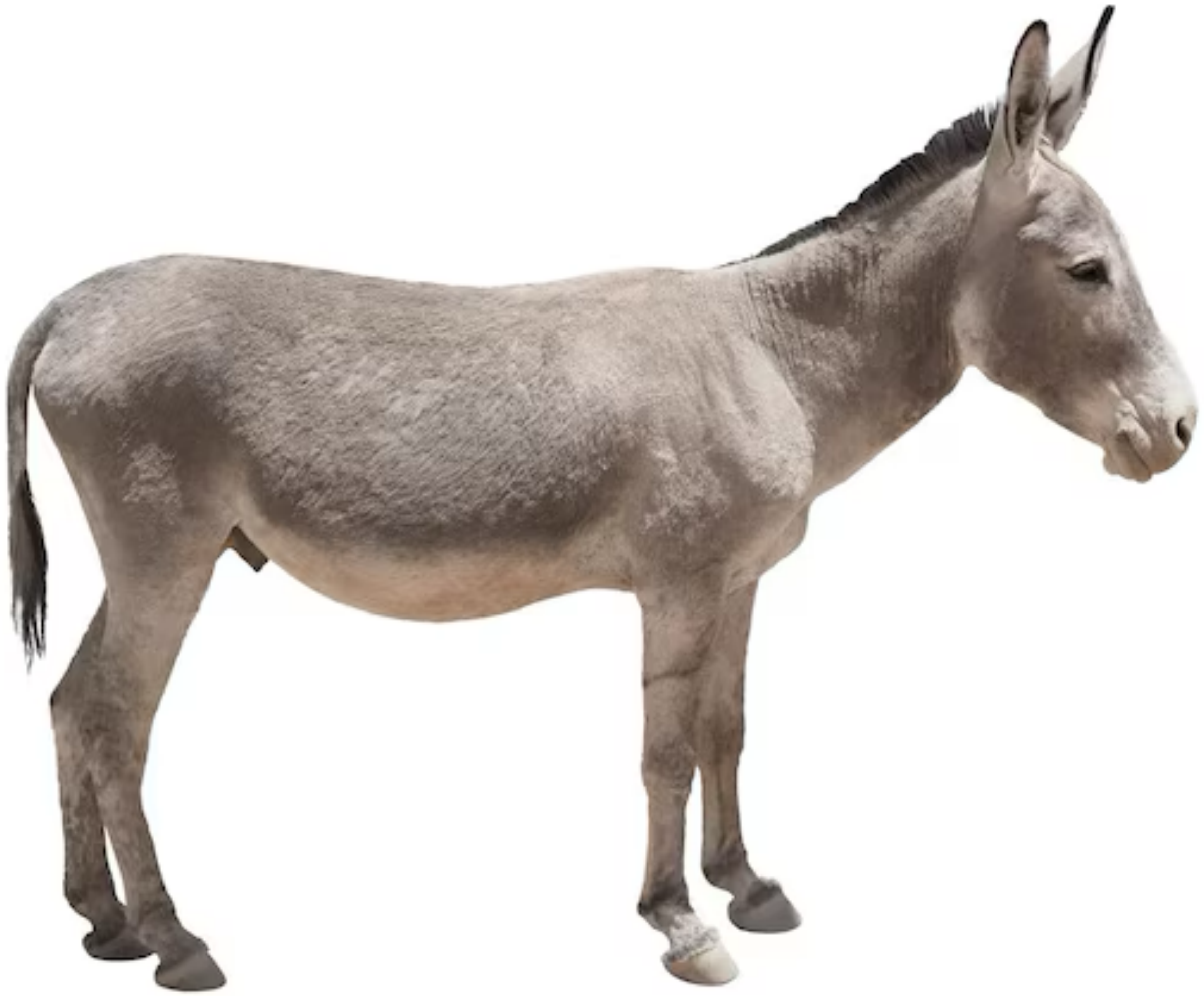


→ evidence for spatial modulations in the phase diagram

MOATS & PATTERNS

Moat regime indicates pattern formation in dense QCD matter

QCD at small μ_B



QCD at large μ_B

MOATS & PATTERNS - A TOY MODEL

ϕ^4 theory with p^4-term:

$$\mathscr{H} = \frac{Z}{2}(\nabla\phi)^2 + \frac{W}{2}(\nabla^2\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

preliminary results

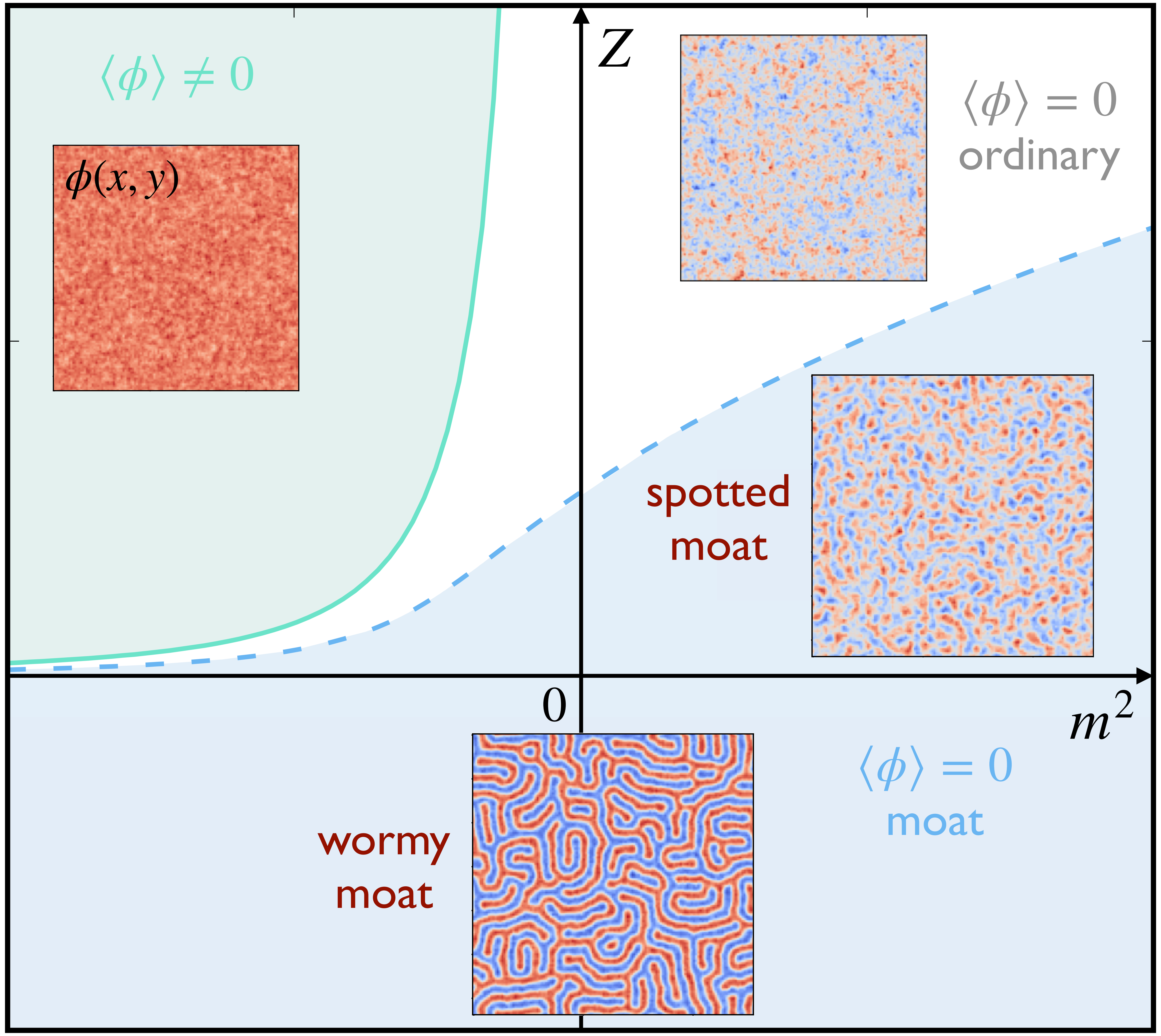
simulation on a 2+1d lattice: different patterns emerge, depending on choice of bare parameters Z and m^2

[Harhoff, FR, Riedel, Schlichting (in preparation)]
3+1d simulations: [Valgushev, Winstel, APP B 17 (2024)]
related system: [Schindler et al., PRD 102 (2019)]

NB: in reaction-diffusion systems these are known as **Turing patterns**

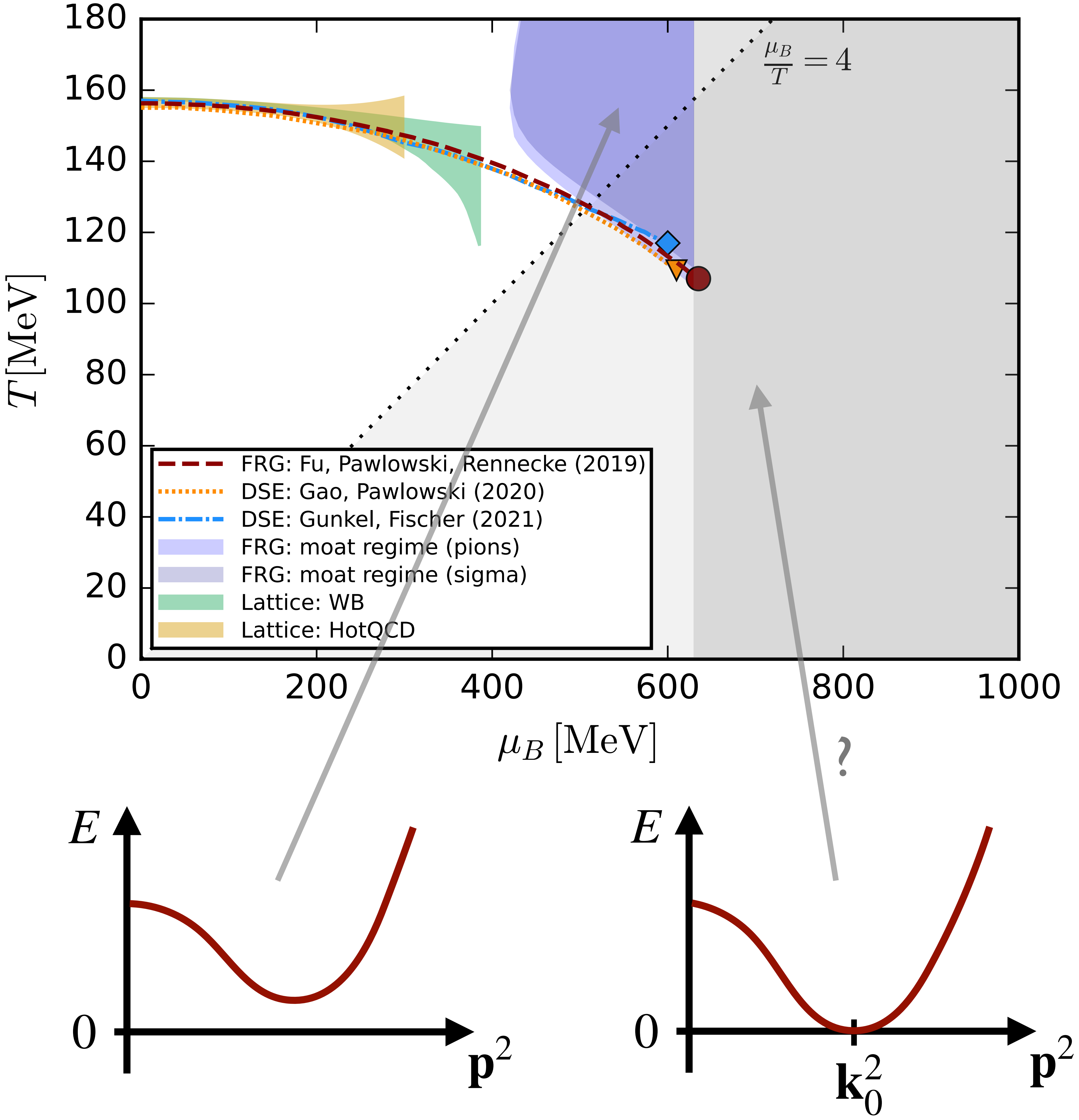
adapted phase diagram from large-N limit

[Pisarski, Tsvelik, Valgushev (2020)]



IMPLICATIONS OF THE MOAT

The energy gap might close:



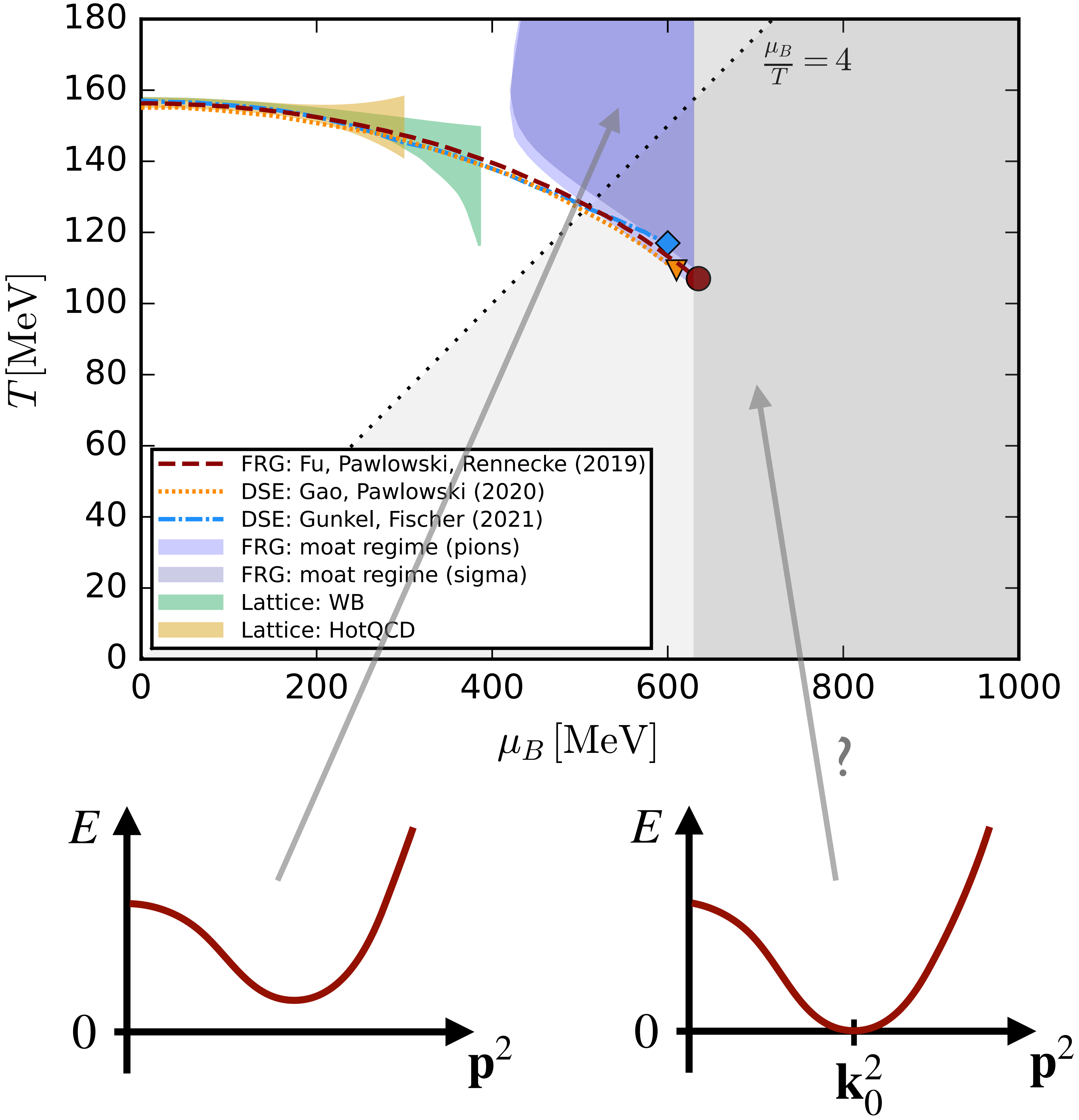
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

IMPLICATIONS OF THE MOAT

The energy gap might close:

DSE: [Motta, Buballa, Fischer, *PRD* 108 (2023) & 2411.02285)]
FRG: [Fu, Pawlowski, Pisarski, FR, Wen, Yin, 2412.15949]



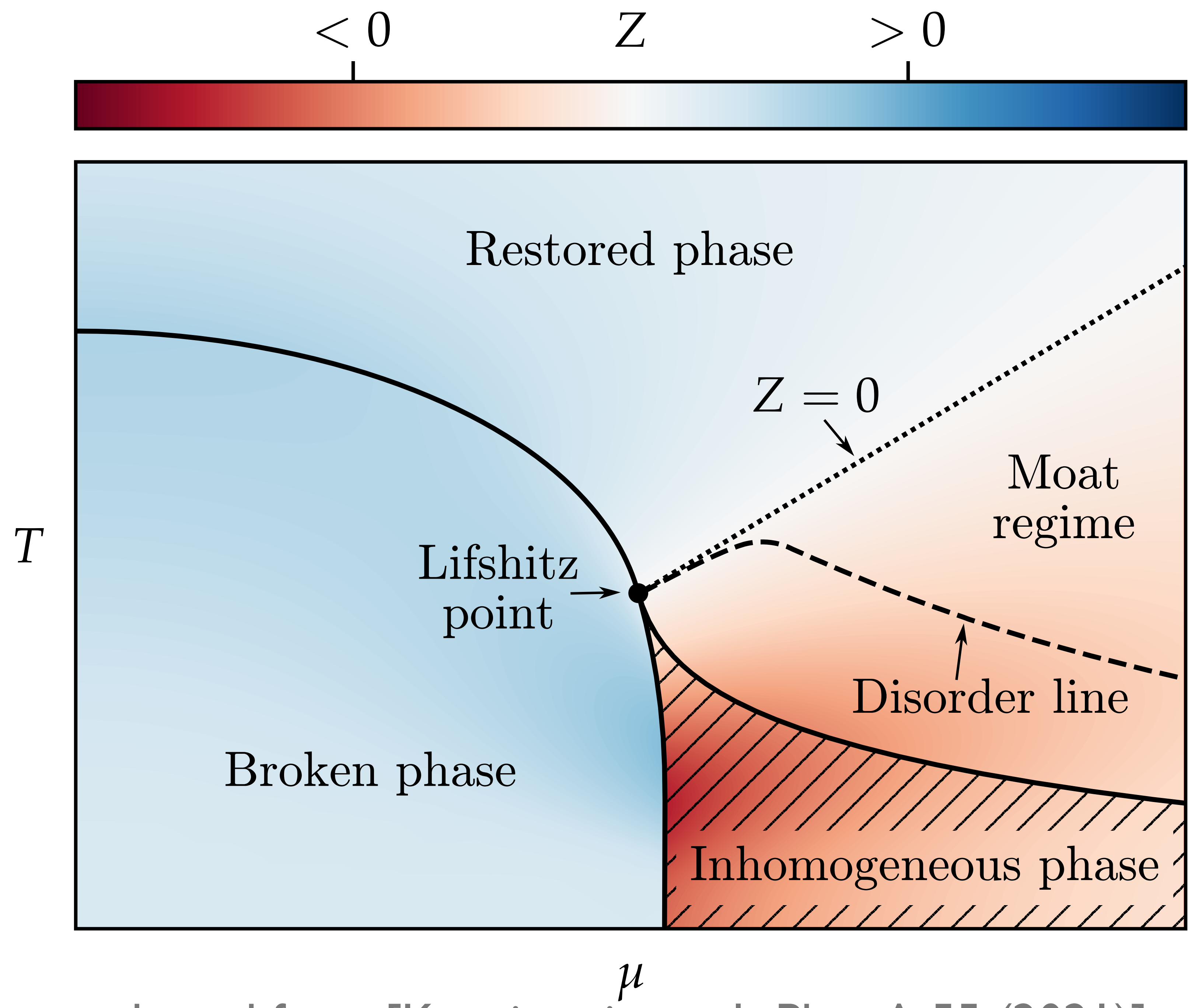
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an inhomogeneous condensate

common feature of low-energy models,

[Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel, 2410.22418]



adapted from [Koenigstein et al., Phys. A 55 (2021)]

- many types of inhomogeneous phases possible (crystals, liquid crystals, ... depends on which spatial symmetries are broken)
- in any case, they are always accompanied by a moat regime

SEARCH FOR MOATS IN HICS

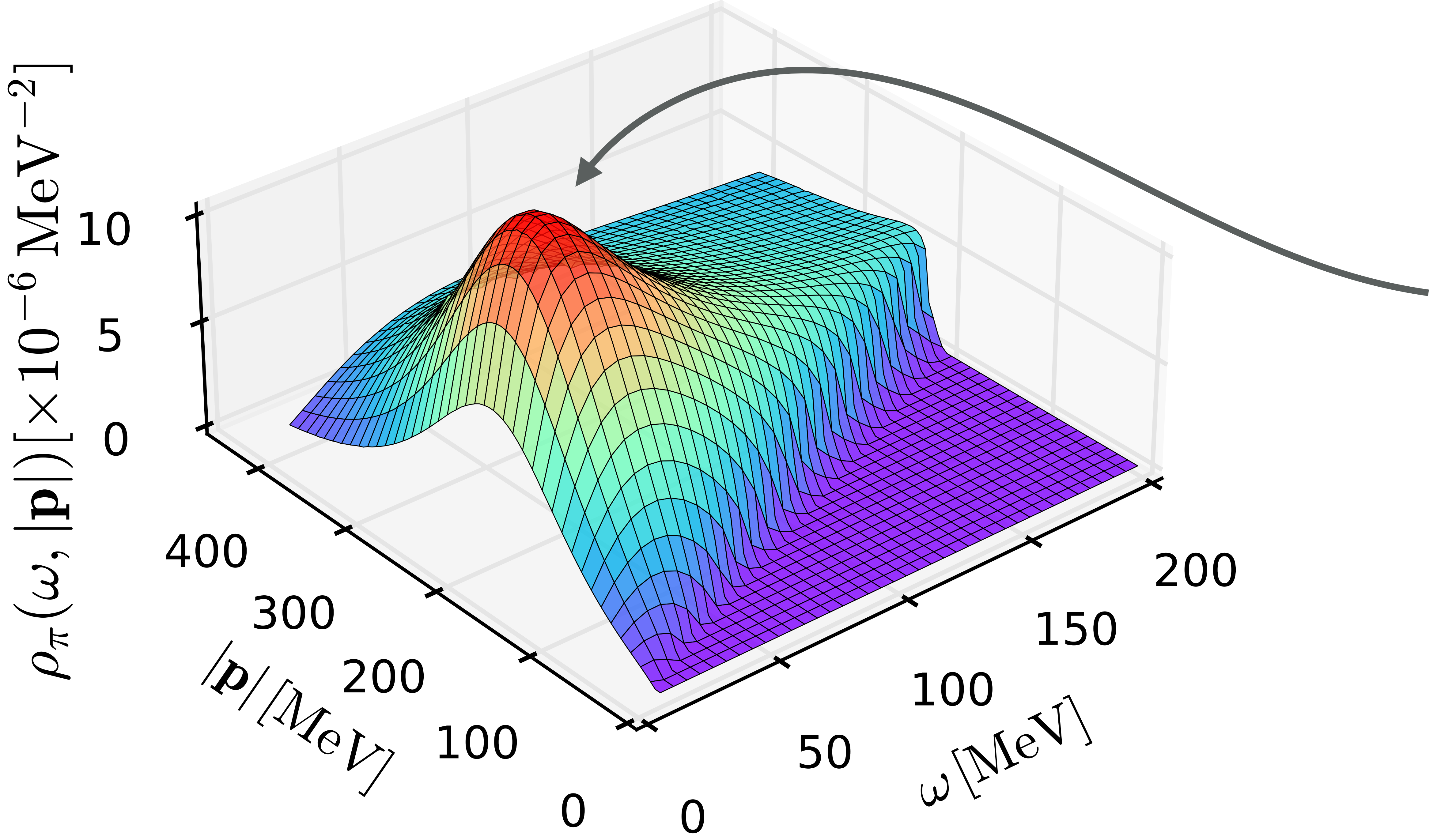
[Pisarski, FR, PRL 127 (2021)]

intuitive idea

Characteristic feature of moats: particles with minimal energy at nonzero momentum

$\Longrightarrow$ **modified particle production at nonzero momentum**

- description of particle production requires knowledge of real-time correlation functions
- directly accessible with the FRG [Floerchinger, JHEP 1025 (2012); Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014), ...]



pion spectral function in the moat regime

quasi-particle like peak in the spacelike region:

the "moaton"

[Fu, Pawlowski, Pisarski, FR, Wen, Yin, 2412.15949]

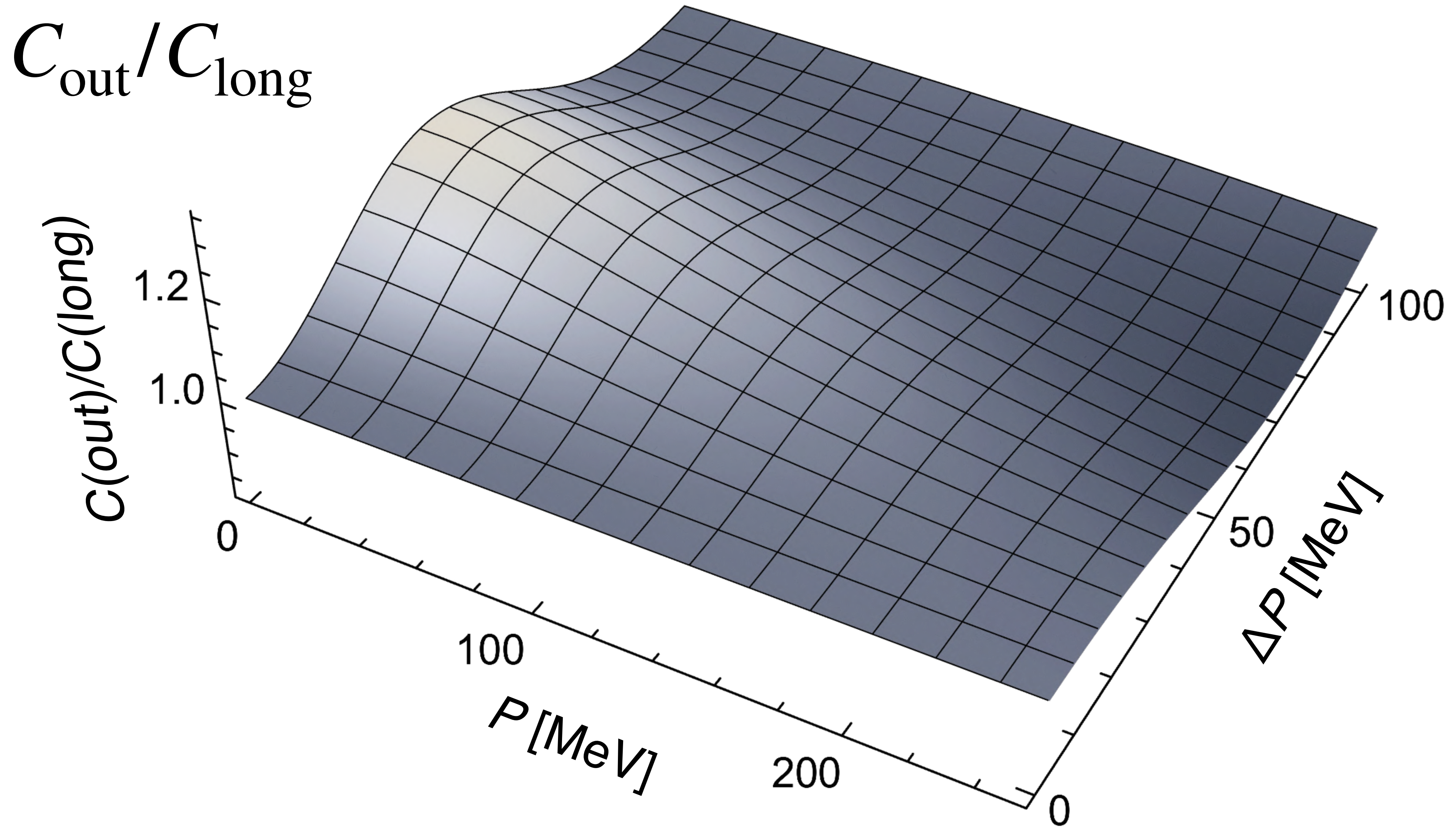
HANBURY-BROWN-TWISS CORRELATIONS

[FR, Pisarski, Rischke, PRD 107 (2023)]

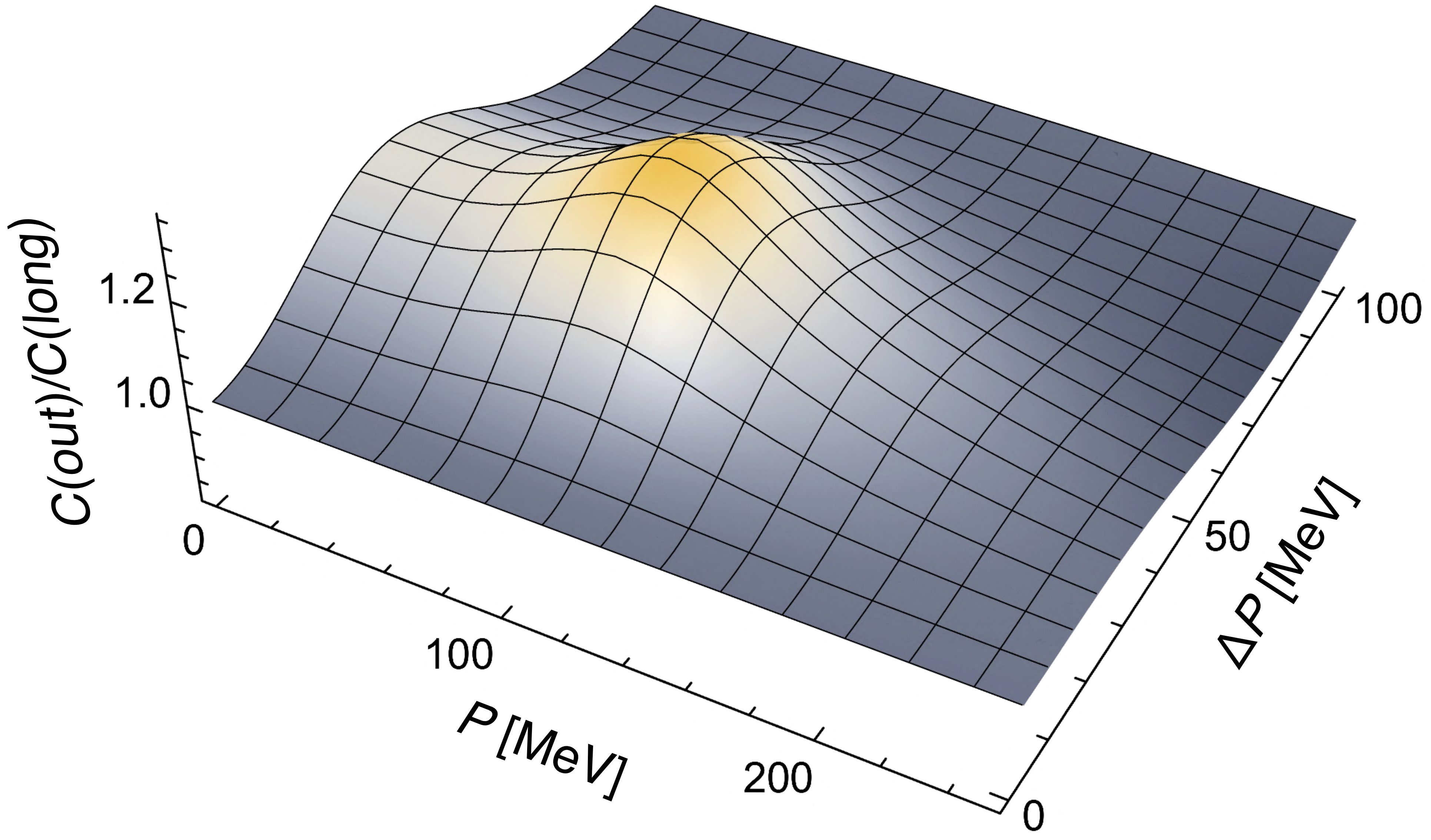
Two-pion correlations generated by interference: HBT correlations

$$C(\mathbf{P}, \Delta\mathbf{P}) \sim \int_{X,P_0} e^{-i\Delta P\cdot X} f(X,P)\, \rho(X,P)$$

normal:



moat:



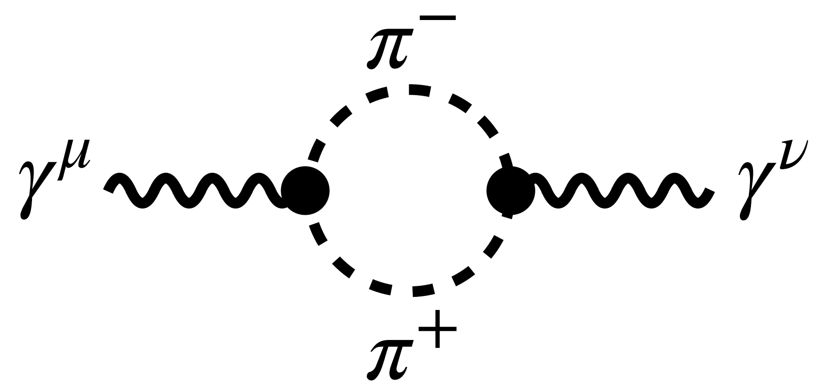
→ characteristic peak at nonzero average pair momentum

DILEPTON PRODUCTION

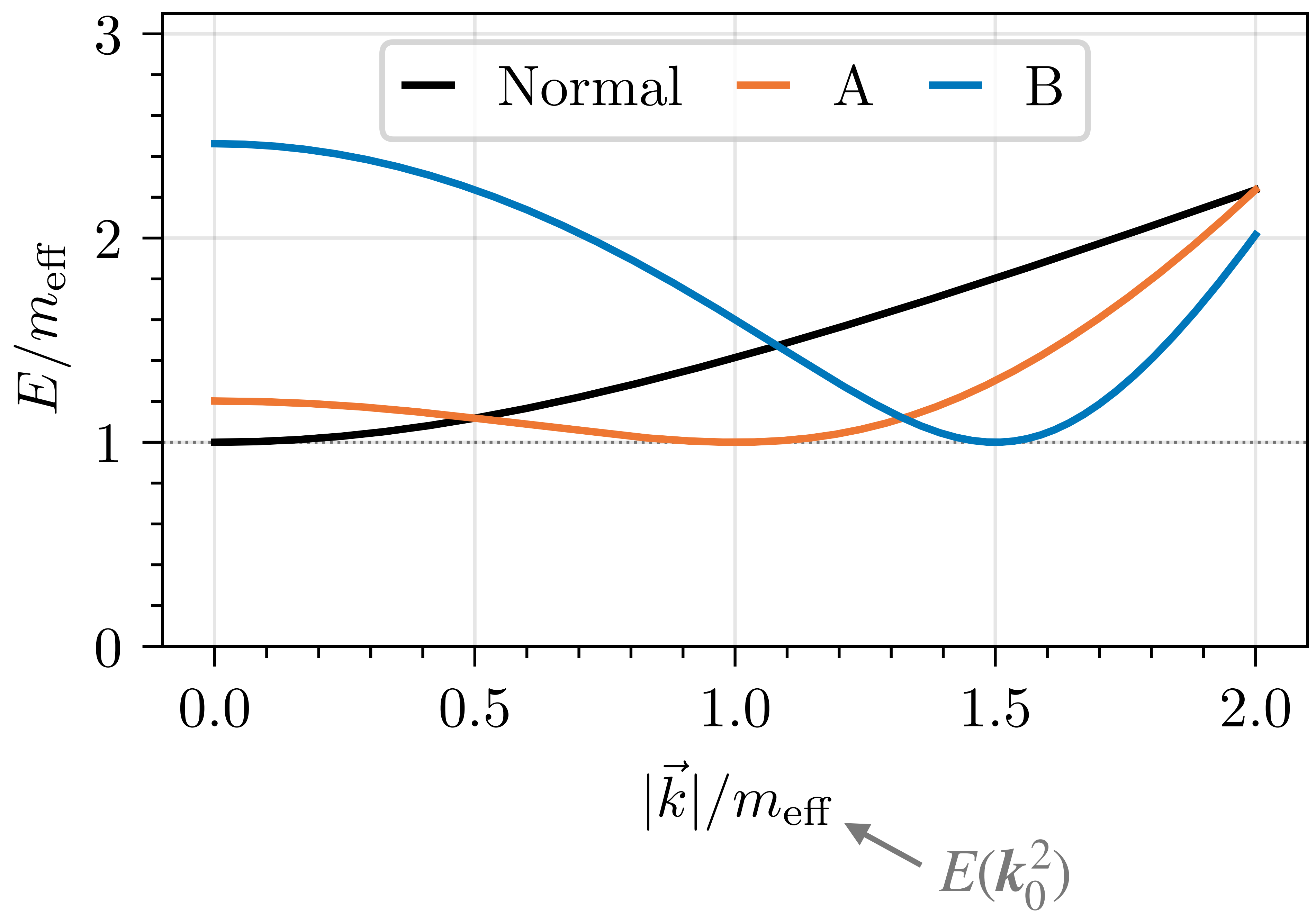
[Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel, 2410.22418]

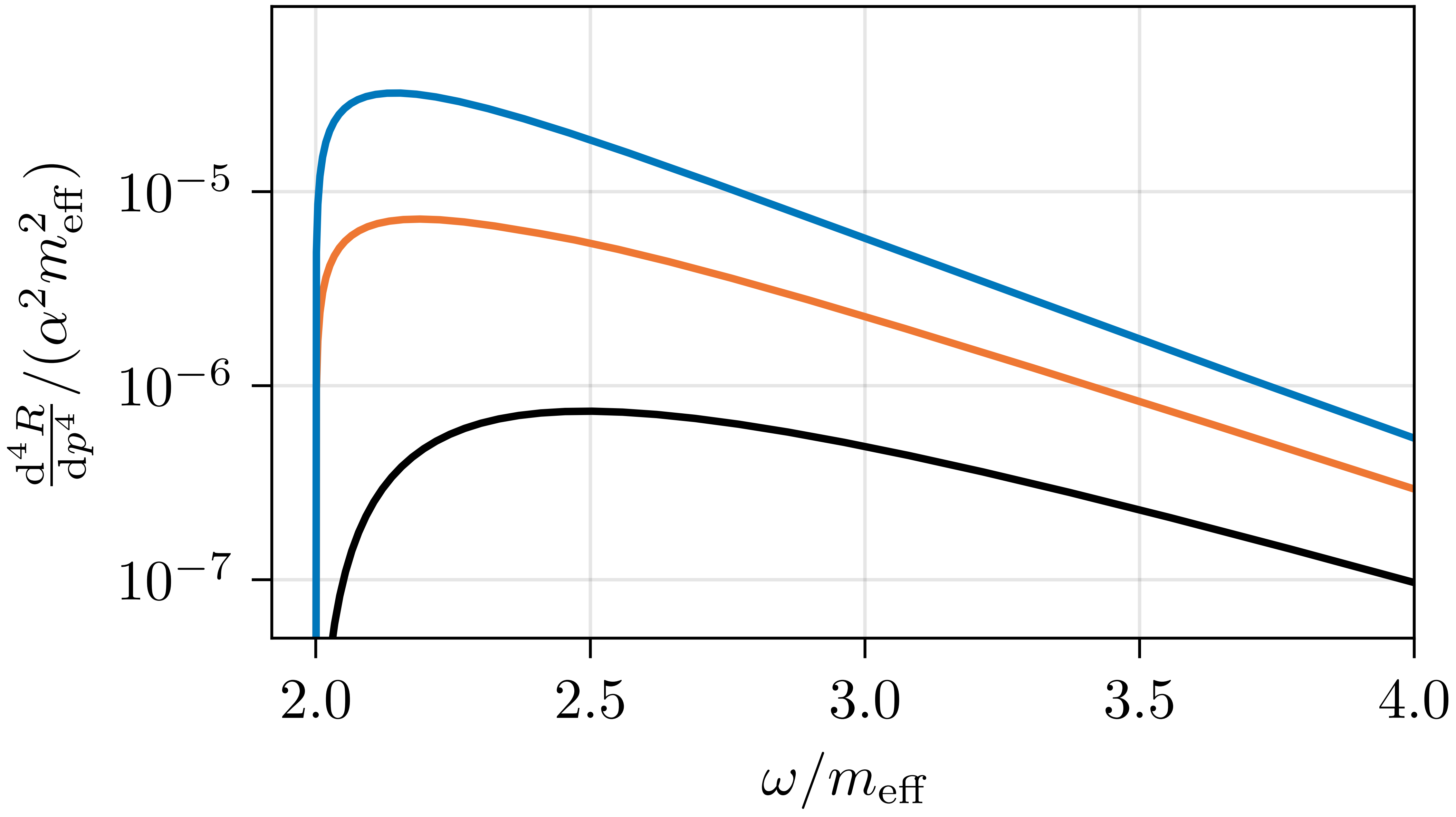
Contribution to the dilepton rate from pions/moatons:

$$\frac{d^4R}{dp^4} \sim \mathrm{Im}\,\Pi^{\mu\nu}(p) \qquad \Pi^{\mu\nu} =$$



For back-to-back dileptons $(\boldsymbol{p} = 0)$





→ enhanced dilepton rate from "moaton threshold"

SUMMARY

functional methods provide new insights into the QCD phase diagram

determination of the CEP location directly in QCD from first principles

- different calculations with different systematics show good agreement
- subsequent confirmed by extrapolations of lattice data
- smoking-gun signals for CEP in HICs still missing

reveal new feature of the phase diagram at large μ_B: the moat regime

- indicates the formation of patterns/crystalline structures
- also natural consequence of emergent CK symmetry in finite-density QCD

[Schindler, Schindler, Medina, Ogilvie, PRD 102 (2019)]
[Haensch, FR, von Smekal, PRD 110 (2023)]
[Winstel, PRD 110 (2024)]

- can leave signatures in particle correlations and the dilepton rate