Physics Opportunities with Proton and Pion Beams at GSI/FAIR: Report on Chapter 3

The Chapter 3 Team

EMMI Collaboration Meeting

QCD at FAIR Workshop 2025

Sicily, Italy

6/25/2025

Chapter 3: Tools and Techniques

V. Crede, A. Szczepaniak, C. Fernandez Ramirez, A. Foda, M. Mikhasenko,
D. Mohler, C. Morningstar, A. Pilloni, D. Roenchen, D. Winney, G. Eichmann, M. Mai

Outline of the chapter:

1. 3.1 Partial Wave Analysis (PWA)
   (A. Foda, D. Roenchen)
2. 3.2 Moment expansion of observables
   (L. Bibrzycki, V. Mathieu)
3. 3.3 Spin alignment for multibody processes
   (M. Mikhasenko)
4. 3.4 Regge approaches in pp collisions
   (D. Winney, C. Fernández-Ramírez, A. Pilloni)
5. 3.5 Lattice QCD
   (C. Morningstar and D. Mohler)
6. 3.6 Functional methods
   (G. Eichmann)

➜ All contributions included; references & description of coordinate systems missing.

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7. 3.7 Effective Field Theories (from Chapter 4), 3.7.X Quark-Level EFT (N. Brambilla)

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1. 3.1 Partial Wave Analysis (PWA)
   (A. Foda, D. Roenchen)
   - 3.1.1 Reaction Mechanisms for Partial-Wave Analysis
     Brief description of πN, γN, pN, $p\bar{p}$ Reactions
   - 3.1.2 Partial-Wave Analysis Frameworks ➜ Needs work!!

K-Matrix Formalism The K-matrix approach ensures the unitarity of scattering amplitudes by defining:

$$T = K(I - iK)^{-1}, \tag{4}$$

where T is the partial wave amplitude matrix that describes the transition probability between initial and final states, and K is the K-matrix encoding transition amplitudes during scattering. This formalism is extensively used in coupled-channel analyses and is particularly effective for investigating resonances like $N(1520)D_{13}$ and $N(1680)F_{15}$ in pion-nucleon scattering. However, the method introduces potential model dependencies due to parameterization choices [54].

Coupled-Channel Approach The coupled-channel formalism evaluates multiple reaction channels simultaneously by solving:

$$T_{ij} = V_{ij} + \sum_k V_{ik} G_k T_{kj}, \tag{5}$$

where T_{ij} is the scattering amplitude for the transition from channel i to j, V_{ij} represents the interaction potential, and G_k is the propagator function for the intermediate channel k. This approach is pivotal for studying inelastic processes and extracting resonance parameters for states like $N(1900)P_{13}$ and $\Delta(1950)F_{37}$. However, it demands extensive experimental input and sophisticated numerical solutions [53].

Dispersion Relations Dispersion relations impose analyticity constraints on scattering amplitudes, linking real and imaginary parts via integral equations:

$$\mathrm{Re}T(s) = \frac{1}{\pi}\mathcal{P}\int \frac{\mathrm{Im}T(s')}{s' - s} ds', \tag{6}$$

where $\mathcal{P}$ denotes the Cauchy principal value. This method is model-independent and integrates causality and analyticity principles. It is particularly useful for analyzing resonances such as $\Delta(1700)D_{33}$, which exhibits significant contributions to $\pi\pi N$ final states. However, precise experimental input is crucial for accurate results.

Chew-Mandelstam K-Matrix An extension of the K-matrix formalism, this framework incorporates an explicit energy-dependent phase-space factor:

$$K(s) = \sum_n \frac{g_n g_n^T}{m_n^2 - s} + C, \tag{7}$$

where $K(s)$ is the energy-dependent K-matrix, g_n represents coupling constants, m_n is the mass of the n-th resonance, and C accounts for background effects. This formalism improves upon the analytic properties of the conventional K-matrix and is well-suited for broad resonances such as $N(1440)P_{11}$, known for its coupling to $\pi\pi N$ final states.

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   3.1.2 Definition of Coordinate Systems

   3.1.3 Partial-Wave Analysis Frameworks

   ~~3.1.3 Advances and Applications of PWA Frameworks~~ ?

   3.1.4 PWA approaches for 2-body final states: $\pi N \to MB$
   (Received comments from C. Hanhart)