STATUS OF CHAPTER 4

June 22, 2025 | Christoph Hanhart | IKP/IAS Forschungszentrum Jülich





MOTIVATION - GOAL OF THE CHAPTER

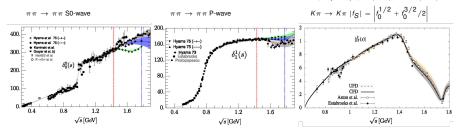
- For various hadron-hadron interactions
 - present the current level of understanding
 - and why they are of interest
- Show how we get important additional insights from FAIR through
 - pion induced reactions (especially inelastic)
 - hypernuclei
 - pp induced production reactions in different kinematics using either
 - dispersion theory
 - femtoscopy





$\pi\pi$, πK , $\bar{K}K$ SCATTERING

R. Pelaez et al., Phys. Rept., 969(2022); EPJC79(2019)1008



- appear in the final state of many decays
- very well known for $\sqrt{s} \le 1.4$ GeV (very sophisticated theory!+data)
- can be included systematically
- can be used to benchmark extraction strategies

note: challenging, since scattering lengths are small



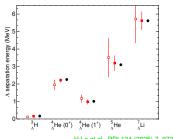


REACTIONS WITH HYPERONS

Hyperon-nucleon and hyperon-hyperon interactions needed

- to understand SU(3) in hadron-hadron dynamics
- get a deeper understanding of hypernuclei (and vice versa)
- for the structure of neutron starts (hyperon puzzle)

chiral NLO and NNLO vs. exp.



H.Le at al., PRL134 (2025) 7, 072502

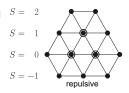
- \implies Hyperon-nucleon interaction dominated by ${}^3\mathcal{S}_1$
- \implies Hypertriton dominated by 1S_0
- ⇒ Three-body forces (@NNLO, effect. 2 para) improve agreement
- ⇒ Higher partial waves not well constrained
- ⇒ Indication for repulsion of 3B forces for large A
- ⇒ Hyperon-hyperon interaction largely unconstrained





REACTIONS WITH CHARM

• π , K, η D meson scattering shows intriguing SU(3) structure (e.g. two poles in πD) similar to $\Lambda(1405)$







• Various non-trivial states predicted for *DN* system (e.g. $\Lambda_c(2595)$ and the $\pi\Sigma_c$ channel)

 $\Longrightarrow \pi \Sigma_c$ and *DN* scattering lengths?

 Phenomenological calculations give attractive Λ_cN interaction with bound state formation expected— HALQCD gives repulsion

To be clarified:

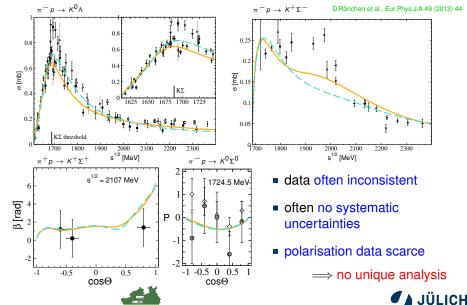
- → How much charm physics can be done at FAIR in the near future?
- ⇒ For which systems can we expect non-trivial result?
- ⇒ In which chapter shall the above be discussed (in what depth)?





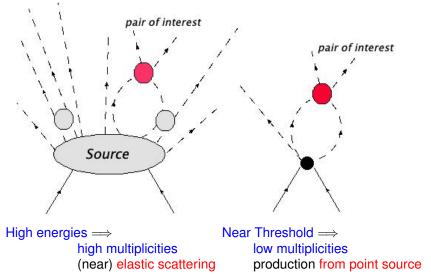
ON PION INDUCED REACTIONS

Member of the Helmholtz Association



Slide 5112

SCATTERING PARAMETERS FROM FSI







DIFFERENT OPTIONS

Production from				
small momentum transfer (femtoscopy)	large momentum transfer (this talk)			
e.g. heavy ion or <i>pp</i> collisions	e.g. meson production in <i>pp</i> coll.			
weak dependence from production	sizeable dep. from production			
uncertainty difficult to quantify	quantify controllable uncertainties			
spin states with known weights	admixture of spin states unknown			

In any case: Two methods with very different systematics





DISPERSION INTEGRAL

For large momentum transfer reactions we find

$$A(s,t,m^2) = \exp\left[\frac{1}{\pi} \int_{m_0^2}^{\infty} \frac{\delta(m'^2)}{m'^2 - m^2 - i0} dm'^2\right] \Phi(s,t,m^2),$$

- large momentum transfer $\rightarrow \Phi$ is at most weakly m^2 dependent.
 - ⇒ included into uncertainty estimate
- The FSI effect in terms of the (elastic) scattering phaseshift.
 - \implies large m' region into uncertainty estimate
- Equation can be inverted

$$\delta_{S}(m^{2}) = -\frac{1}{2\pi} \int_{m_{0}^{2}}^{m_{max}^{2}} \frac{dm'^{2}}{m'^{2} - m^{2}} \sqrt{\frac{(m_{max}^{2} - m^{2})(m^{2} - m_{0}^{2})}{(m_{max}^{2} - m'^{2})(m'^{2} - m_{0}^{2})}} \log \left\{ \frac{1}{p'} \left(\frac{d^{2}\sigma_{S}}{dm'^{2}dt} \right) \right\}$$



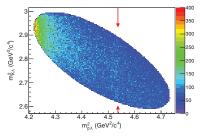


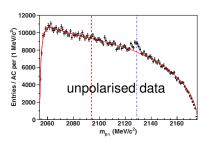
F. Hauenstein et al. ICOSY-TOFI, PRC95(2017)034001

S=1 and S=0 possible in final state with unknown relative weight

Unpolarised data give access only to effective scattering length $a_{\rm eff}$

Gasparyan, Haidenbauer, CH PRC72(2005)034006





Procedure: Fit m_{ph} spectrum with $\exp\{C_0 + C_1/(m_{ph}^2 - C_2)\}$, then

$$a_{\text{eff}} = \frac{C_1}{2} \sqrt{\frac{m_0^2 (m_{max}^2 - m_0^2)}{m_p m_{\Lambda} (m_{max}^2 - C_2) (m_0^2 - C_2)^3}} \Longrightarrow -1.38^{+0.04}_{-0.05 \text{ stat.}} \pm 0.22_{\text{syst.}} \pm 0.3_{\text{theo.}}$$



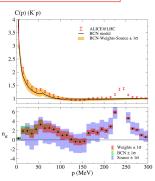


FEMTOSCOPY

Object studied: Pair-correlation functions (CF):

$$CF(q) = 1 + \underbrace{\int S(\vec{r}) \left(|\Psi(q, \vec{r})|^2 - |j_0(q, r)|^2 \right) dr^3}_{\text{theory: Koonin-Pratt-formula}} = \underbrace{\frac{Same(q)}{\textit{Mixed}(q)}}_{\text{experiment}}$$

- Strong final state interactions leave significant imprints in CFs
- Many corrections need to be applied to data to access relevant piece
- Data for various systems with various sources
- Accuracy of extracted scattering parameters difficult to quantify



P. Encarnación et al., PRD111(2025) 114013





ACCURACY?

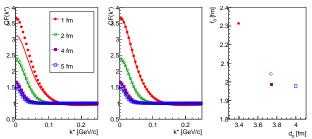
- Source function universal?
- Sensitivity to short range wave function? (strong model dependence)

Lednicky-Lyuboshitz formula may address the issue

E. Epelbaum, et al., arXiv:2504.08631

$$CF(q) = 1 + \frac{|f(q)|^2}{2R^2}F_0(d_0) + \frac{2\text{Re}(f(q))}{\sqrt{\pi}R}F_1(2qR) - \frac{\text{Im}(f(q))}{R}F_2(2qR) + ...$$

Comparison Usmani pot. vs. LL formula: $\Delta a \gtrsim 0.3$ fm??, Δd_0 unclear



Moreover: Relativistic treatment and different isospins may be necessary





SUMMARY: STATUS CHAPTER 4

Overall: Chapter 4 is in a good shape!

Thanks a lot to all contributors!

- Still a lot to be learned esp. about hyperons and their interactions
- Need to discuss what can be done for charmed systems
- Pion induced reactions will help to refine e.g. hyperon-kaon interactions and the N^*/Δ^* spectrum
- Dispersive method for FSI extraction needs to be exploited further (e.g. $\Sigma^{+}\Sigma^{+}$ channel)
- Femtoscopy: Here the most still needs to be done for chapter 4
 - What is the accuracy of the extraction?
 - Can we control it parametrically?
 - Is the source really universal? πK from pp: R = 0.4 0.7 fm

M. Albaladeio, et al., PLB866(2025)139552

What should be discussed in the whitepaper?



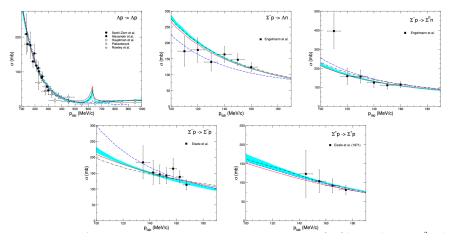


Back-up slides





HYPERON-NUCLEON INTERACTION



M.Döring et al., arxiv:2505.02745





πK FROM pp

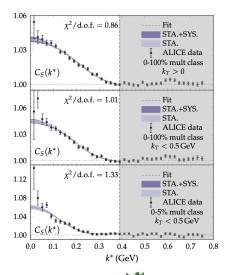


	Table	1: Parameters as	and χ^2 /d.o.f. of the	o.f. of the fits in Fig. 1.	
		0-100% m. cl.	0-100% m. cl.	0-5% m. cl.	
		$k_T > 0$	$k_T < 0.5\mathrm{GeV}$	$k_T < 0.5\mathrm{GeV}$	
	$\chi^2/\text{d.o.f.}$	0.86	1.01	1.33	
	R(fm)	0.36(3)(3)	0.41(3)(3)	0.68(5)(3)	
	λ	0.19(2)(4)	0.29(4)(5)	0.80(14)(13)	
	$(N-1)\times 10^2$	0.80(8)(7)	0.85(8)(6)	0.97(6)(4)	

- ALICE analysis gave different K*(700) masses per setting
- These fits consistent with πK scattering data
- Relativistic corrections sizeable
- Both isospin 1/2 and 3/2 important

For πK: M. Albaladejo, et al., PLB866(2025)139552





THREE STRATEGIES

Three different strategies to proceed:

Assume that phaseshifts are given by effective range expansion;

$$p' \operatorname{ctg}(\delta(m^2)) = 1/a + (1/2)rp'^2 \text{ (sign!)} \Rightarrow A(m^2) = \frac{(p'^2 + \alpha^2)r/2}{1/a + (r/2)p'^2 - ip'} \Phi(m^2) ,$$

$$\alpha = 1/r(1 + \sqrt{1 + 2r/a})$$
 (Jost–function method)

Sibirtsev et al. (1996, 2004), Shyam et al. (2001), ...

2 Ignore numerator (Watson method)

Goldberger, Watson 1964

Invert the equation and express phaseshifts through observables

Geshkenbein (1969)

and restrict integration range! (Integral)

Gasparyan et al. (2004)ff

This is the most systematic approach in line with goal





SCATTERING LENGTH

It is possible to invert the Omnes-function:

Geshkenbein (1969), Gasparyan et al. (2004)

$$\delta_{\mathcal{S}}(m^2) = -\frac{1}{2\pi} \int_{m_0^2}^{m_{\text{max}}^2} \frac{dm'^2}{m'^2 - m^2} \sqrt{\frac{(m_{\text{max}}^2 - m^2)(m^2 - m_0^2)}{(m_{\text{max}}^2 - m'^2)(m'^2 - m_0^2)}} \log \left\{ \frac{1}{p'} \left(\frac{d^2 \sigma_{\mathcal{S}}}{dm'^2 dt} \right) \right\}$$

with $\lim_{m^2 \to m_0^2} \delta_S(s) = a_S p(s)$ and S denoting a specified spin state

we chose: $\epsilon_{max} = m_{max} - m_0 \simeq 1/(2\mu a^2) \approx 40$ MeV for $a \sim 1$ fm

Estimates for uncertainties: $\delta a^{(th)} = \delta a^{(lhc)} + \delta a^{m_{max}} \sim 0.3$ fm since

$$|\delta a^{m_{max}}| = \frac{2}{\pi p'_{max}} \left| \int_0^\infty \frac{\delta(y) dy}{(1 + y^2)^{(3/2)}} \right| \le \frac{2}{\pi p'_{max}} |\delta_{max}| \sim 0.2 \text{ fm}$$

 $\delta a^{(lhc)} \sim (p'_{max}/p^2) \sim 0.05 \text{ fm}$

using $\delta_{max}=0.4$ rad (for ΛN - see next pages) and $p'\sim 1/a$





We want to test the dispersive method

⇒ Can only be done if we know true parameters

Our strategy:

- Generate pseudo data for $d\sigma/dm^2$ for various models for ΛN (and NN)
- Extract scattering length.
- Compare to exact value.

Note: any working method should work for any realistic model

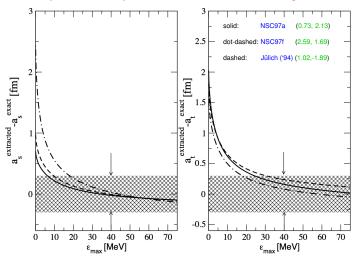
We use S = 0 & S = 1 for YN from (where in green: scattering lengths in fm) NSC97a (0.73,2.13), NSC97f (2.59,1.69), Jülich ('94) (1.02,-1.89)





TESTING THE DISPERSIVE METHOD

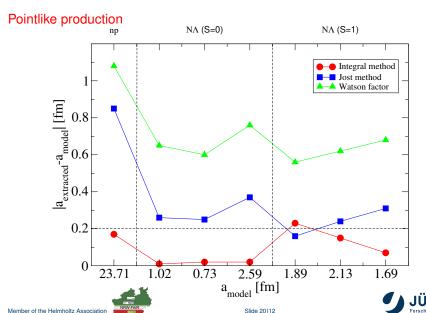
Calculations for production operator with π and K exchange!





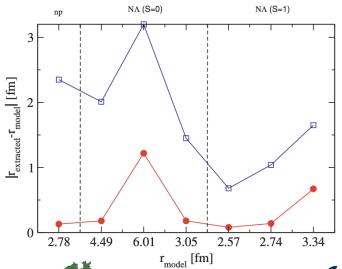


COMPARISON OF METHODS



EFFECTIVE RANGE

 \implies enters as $a^2((2/3)a - r_e)$





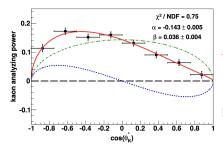


SPIN TRIPLET $a_{\Lambda p}$ **FROM** $\vec{p}p \rightarrow pK\Lambda$

F. Hauenstein et al. ICOSY-TOFI, PRC95(2017)034001

Gasparvan, Haidenbauer, CH PRC72(2005)034006

Spin=1 can be isolated from analysing power



$$A_{0y}\sigma_0 = -\frac{1}{4}k^2\beta\sin(2\theta)\cos(\phi) + \sin(\theta)\cos(\phi)(\text{spin triplet only}),$$

Needs for each m_{ph} bin angular dist.

Procedure: Fit m_{ph} spectrum with $\exp\{C_0 + C_1/(m_{ph}^2 - C_2)\}$, then

$$a_t = \frac{C_1}{2} \sqrt{\frac{m_0^2(m_{max}^2 - m_0^2)}{m_0 m_\Lambda(m_{max}^2 - C_2)(m_0^2 - C_2)^3}} \Longrightarrow -2.55^{+0.72}_{-1.39} {
m_{stat.}} \pm 0.6_{
m syst.} \pm 0.3_{
m theo.} {
m fm}$$

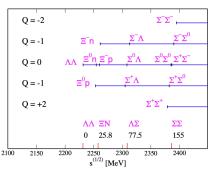
Combined analysis of femtoscopy and scattering: $-1.4 \pm 0.2 \pm 0.?_{\rm theo.}$ fm

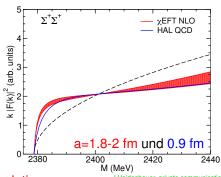
see Mihaylov, Haidenbauer, Sarti PLB850(2024)138550



THE S = -2 SYSTEMS

Moving to S = -2





Needs high statistics and high resolution

- J.Haidenbauer, private communication
- $\Sigma^{\pm}\Sigma^{\pm}$ and $\Lambda\Lambda$ are identical fermions \Longrightarrow *S*-wave must be spin zero

 Note: repulsive Coulomb interaction can be accounted for

Gasparyan, CH, Haidenbauer, Phys.Rev.C 72 (2005) 034006

■ Threshold difference $\Lambda\Lambda$ - ΞN rather small





OPPORTUNITIES WITH PROTONS AT SIS100

The high initial energy ($\sqrt{s_{\rm max}} = 7.5 \text{ GeV}$) promises access to

- $pp \rightarrow ppJ/\psi$ and the pJ/ψ interaction $\sqrt{s} > 5$ GeV \implies discovery channel of $\bar{c}c$ pentaguarks & role of $\Lambda_c D^{(*)}$ channels
- $pp \rightarrow p\Sigma_c^{(*)}\bar{D}^{(*)}$ and the $\Sigma_c^{(*)}\bar{D}^{(*)}$ interaction $\sqrt{s} > 5.6 \text{ GeV}$ \implies formation of $\bar{c}c$ pentaguarks
- $pp \to \bar{K}^0 \bar{K}^0 \Sigma^+ \Sigma^+$ and the $\Sigma^+ \Sigma^+$ interaction (S=0 only!) $\sqrt{s} > 3.4$ GeV \Longrightarrow closely SU(3) related to pp scattering
- certainly many more

Note: Measurements need to be well above threshold, but not too high ...

Challenge: Needs high resolution for small relative momenta and high statistics



