Physics Opportunities with Proton and Pion Beams at GSI/FAIR: Report on Chapter 3

Volker Credé [1] on behalf of the Chapter 3 team

[1] Florida State University, Tallahassee, Florida

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Chapter 3: Tools and Techniques

V. Crede, A. Szczepaniak, C. Fernandez Ramirez, A. Foda, M. Mikhasenko,
D. Mohler, C. Morningstar, A. Pilloni, D. Roenchen, D. Winney, G. Eichmann

Outline of the chapter:

1. 3.1 Partial Wave Analysis (PWA)

   (A. Foda, D. Roenchen)

   - 3.1.1 Reaction Mechanisms for Partial-Wave Analysis

     Brief description of πN, γN, pN, $p\bar{p}$ Reactions

   - 3.1.2 Partial-Wave Analysis Frameworks

K-Matrix Formalism The K-matrix approach ensures the unitarity of scattering amplitudes by defining:

$$T = K(I - iK)^{-1}, \tag{4}$$

where T is the partial wave amplitude matrix that describes the transition probability between initial and final states, and K is the K-matrix encoding transition amplitudes during scattering. This formalism is extensively used in coupled-channel analyses and is particularly effective for investigating resonances like $N(1520)D_{13}$ and $N(1680)F_{15}$ in pion-nucleon scattering. However, the method introduces potential model dependencies due to parameterization choices [54].

Coupled-Channel Approach The coupled-channel formalism evaluates multiple reaction channels simultaneously by solving:

$$T_{ij} = V_{ij} + \sum_k V_{ik} G_k T_{kj}, \tag{5}$$

where T_{ij} is the scattering amplitude for the transition from channel i to j, V_{ij} represents the interaction potential, and G_k is the propagator function for the intermediate channel k. This approach is pivotal for studying inelastic processes and extracting resonance parameters for states like $N(1900)P_{13}$ and $\Delta(1950)F_{37}$. However, it demands extensive experimental input and sophisticated numerical solutions [53].

Dispersion Relations Dispersion relations impose analyticity constraints on scattering amplitudes, linking real and imaginary parts via integral equations:

$$\text{Re}T(s) = \frac{1}{\pi}\mathcal{P}\int \frac{\text{Im}T(s')}{s' - s} ds', \tag{6}$$

where $\mathcal{P}$ denotes the Cauchy principal value. This method is model-independent and integrates causality and analyticity principles. It is particularly useful for analyzing resonances such as $\Delta(1700)D_{33}$, which exhibits significant contributions to $\pi\pi N$ final states. However, precise experimental input is crucial for accurate results.

Chew-Mandelstam K-Matrix An extension of the K-matrix formalism, this framework incorporates an explicit energy-dependent phase-space factor:

$$K(s) = \sum_n \frac{g_n g_n^T}{m_n^2 - s} + C, \tag{7}$$

where $K(s)$ is the energy-dependent K-matrix, g_n represents coupling constants, m_n is the mass of the n-th resonance, and C accounts for background effects. This formalism improves upon the analytic properties of the conventional K-matrix and is well-suited for broad resonances such as $N(1440)P_{11}$, known for its coupling to $\pi\pi N$ final states.

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   - 3.1.3 Advances and Applications of PWA Frameworks ?
   - 3.1.4 PWA approaches for 2-body final states: $\pi N \to MB$

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2. 3.2 Moment expansion of observables
   (L. Bibrzycki, V. Mathieu)

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Moments expansion are typically used in three-particle final state reactions, where standard techniques from two-body production are more challenging to be implemented. *“We will take the photoproduction of two pions on a nucleon target to illustrate our discussion.”*

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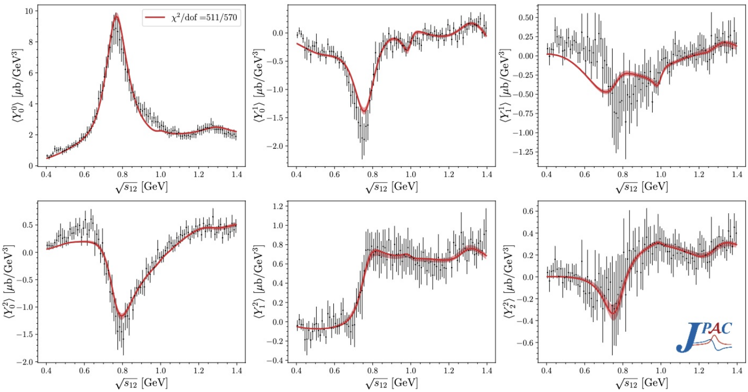

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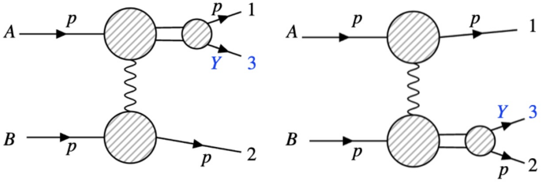

Figure 4: The t-channel exchange mechanisms for exclusive meson production in pp collisions. Left: forward resonance production (top vertex). Right: target resonance excitation (bottom vertex).

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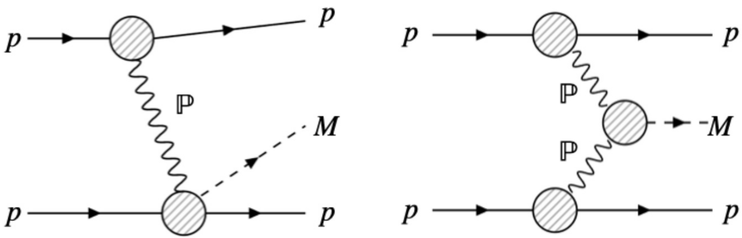

Figure 6: Diagrammatic representation of the meson production through single and (left) and the double (right) Pomeron exchanges.

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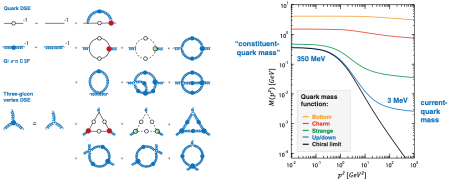

Figure 7: Left: Dyson-Schwinger equations for the quark and gluon propagator and three-gluon vertex. The solid and dashed lines represent quarks and ghosts, the springs are gluons. Two-loop diagrams in the three-gluon DSE are omitted. Right: Quark mass function for different quark flavors obtained from the quark DSE [142].

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➜ All contributions included; references & description of coordinate systems missing.