Regge Theory in the Modern Era

Recent and ongoing studies from JPAC

Daniel Winney
Bonn. U

QCD @ FAIR
24 June 2025

UNIVERSITÄT BONN

JPAC

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

Regge processes at SIS100

High-energy proton beams at FAIR should give ample phase space to study both baryon and meson systems in **peripheral and central diffractive production reactions** respectively.

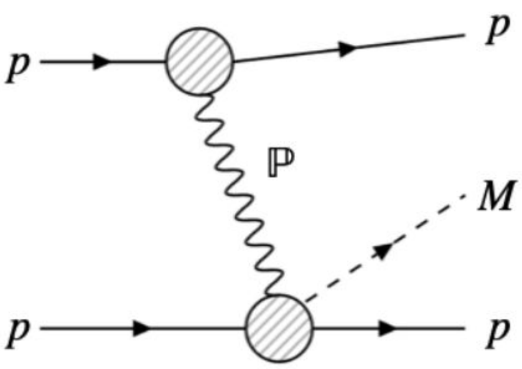
See Sec. 3.4 of the White Paper!

Complementary to ongoing studies at:

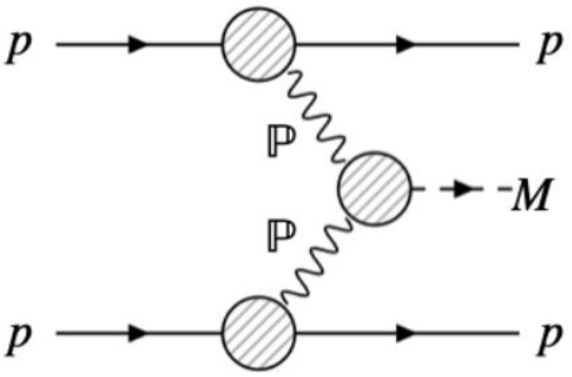
- GlueX at Jefferson Lab with photon beam
- COMPASS at CERN with pion beam

These reactions best understood using Regge theory!

"Single Regge"

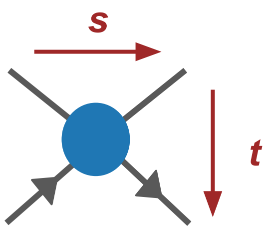


"Double Regge"



T. Regge [Nuovo Cim. 18 (1960) 947-956]

Regge theory 101



PW Expansion → **Sommerfeld-Watson Transform**

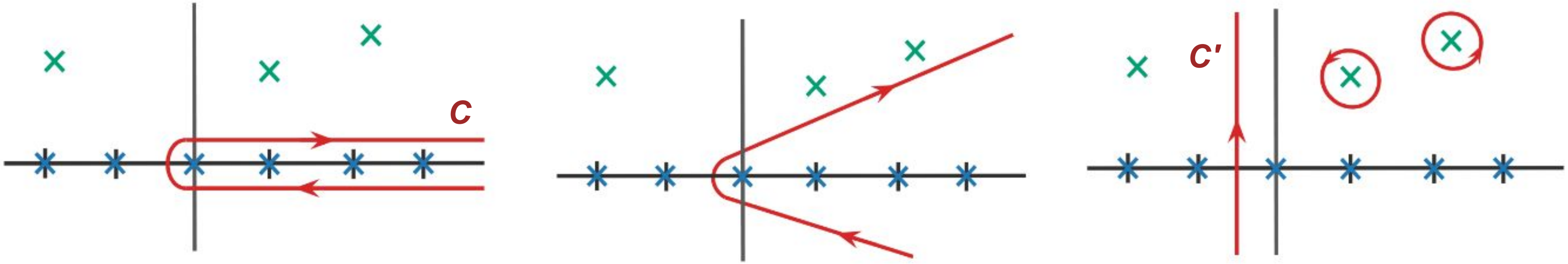
$$F(t,z_t) = \sum_{\ell=0}^{\infty}(2\ell+1)\,P_\ell(z_t)\,f_\ell(t) \longrightarrow \int_C \frac{d\ell}{2i}\,\frac{(2\ell+1)\,P_\ell(-z_t)\,f_\ell(t)}{\sin\pi\ell}$$

$$\beta(t) \equiv \mathrm{Res}\; f_{\ell\to\alpha(t)}(t)$$

Deform contour assuming PW is well-behaved (analytic)

"Background integral" + **"Regge poles"**

$$\hookrightarrow \int_{C'} \frac{d\ell}{2i}\,\frac{(2\ell+1)\,P_\ell(-z_t)\,f_\ell(t)}{\sin\pi\ell} + \sum_i \frac{(2\alpha_i(t)+1)\,P_{\alpha_i(t)}(-z_t)\,\beta_i(t)}{\sin\pi\alpha_i(t)}$$



T. Regge [Nuovo Cim. 18 (1960) 947-956]

Regge theory 101

Scattering amplitudes are analytic in both energy variables → **analytic in angle** (beyond Lehmann ellipse).

$$F(t, z_t) = \int_{C'} \frac{d\ell}{2i} \frac{(2\ell+1)\, P_\ell(-z_t)\, f_\ell(t)}{\sin \pi\ell} + \sum_i \frac{(2\alpha_i(t)+1)\, P_{\alpha_i(t)}(-z_t)\, \beta_i(t)}{\sin \pi\alpha_i(t)}$$

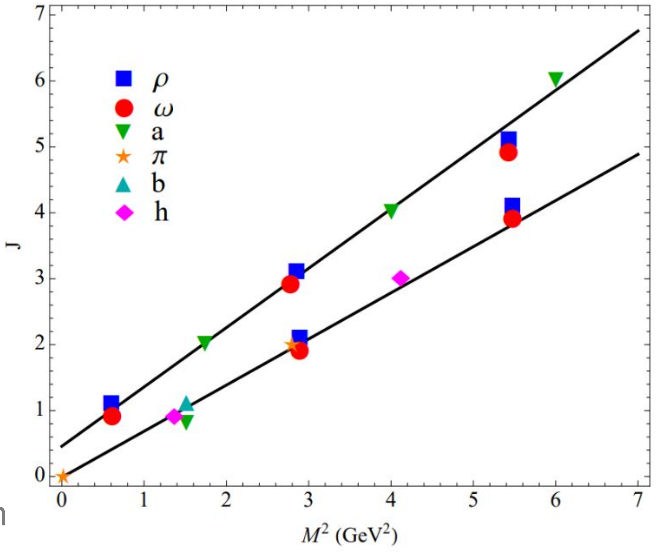
Partial waves must be analytic in angular momentum.

Resonances appear simultaneously as poles in *energy* and *spin!*

$$f_\ell(t) = \int \frac{dz}{2} P_\ell(z)\, F(t,z) = \left(\frac{2\,\alpha(t)+1}{\ell+\alpha(t)+1}\right) \frac{\beta(t)}{\ell-\alpha(t)} + \dots$$

Hadrons lie on Regge trajectories!

Reggeons emerge from the exchange of *all spins* of the same quantum numbers.



JPAC [Phys. Rev. D 98, 014041 (2018)]

When can we use Regge theory?

$$z_t(s,t) = \frac{s-u}{4\,p^2(t)}$$

TECHNICALLY:

Always. It is a consequence of analyticity, crossing symmetry, and unitarity of scattering (not only in QCD).

IN PRACTICE:

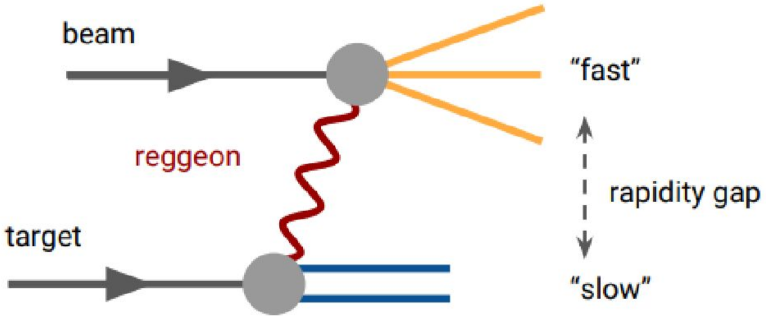
In the **Regge limit** (i.e. large *s*, and small $t \lesssim 0$) $P_\ell(z \to \infty) \propto z^\ell$

"Reggeon exchange"

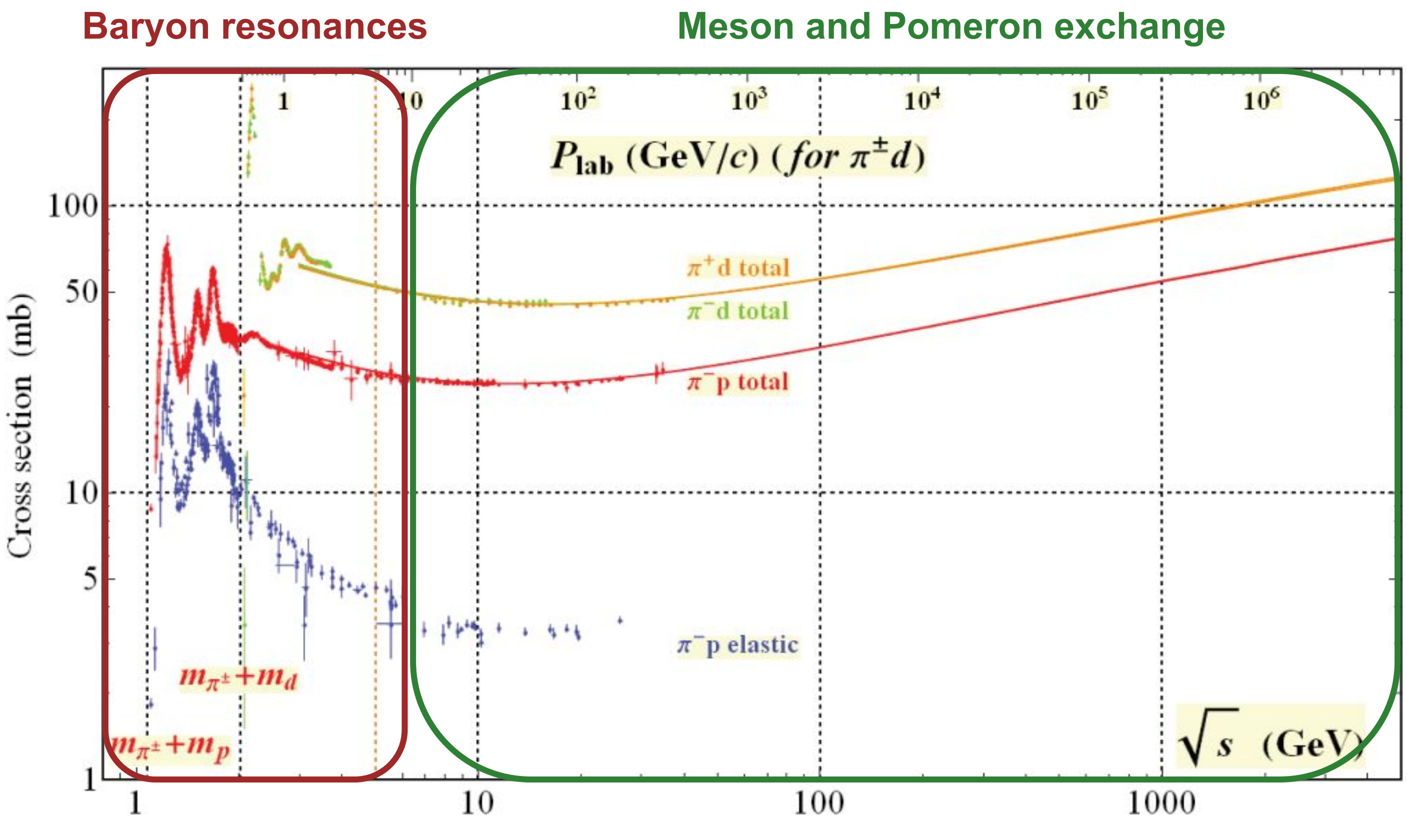
$$F(t,z_t) = \int_{C'} \frac{d\ell}{2i} \frac{(2\ell+1)\,P_\ell(-z_t)\,f_\ell(t)}{\sin \pi\ell} + \sum_i \frac{(2\alpha_i(t)+1)\,P_{\alpha_i(t)}(-z_t)\,\beta_i(t)}{\sin \pi\alpha_i(t)} = \sum_i \beta_i(t) \frac{(2\,\alpha_i(t)+1)}{\sin \pi\alpha_i(t)} \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

Experimentally, rapidity gap separates beam and target fragmentation regions.

Modeling only requires knowing the **crossed-channel exchanges**!



When can we use Regge theory?

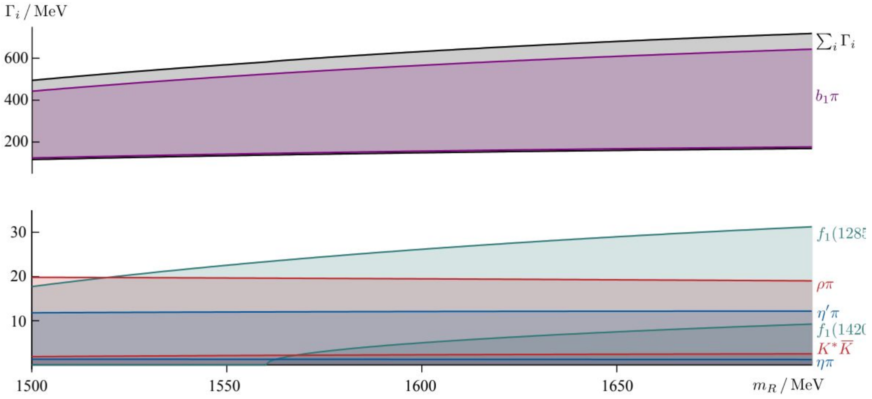


Photoproduction of the $\pi_1(1600)$

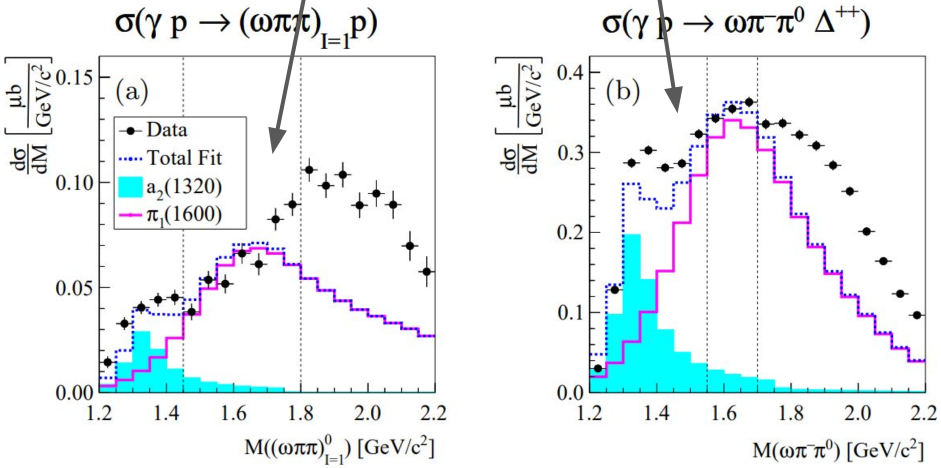
Exotic quantum numbers → **hybrid meson**?

Not yet seen in photoproduction → need better understanding of **production dynamics**!

(Neutral) Pomeron exchange < (Charged) π exchange



HadSpec [Phys.Rev.D 103 (2021) 5, 054502]



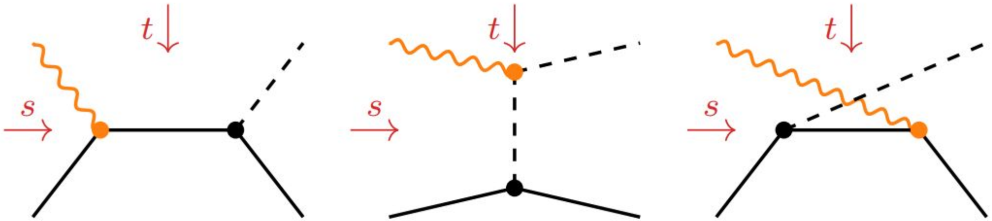
GlueX [Phys.Rev.Lett. 133 (2024) 26, 26]

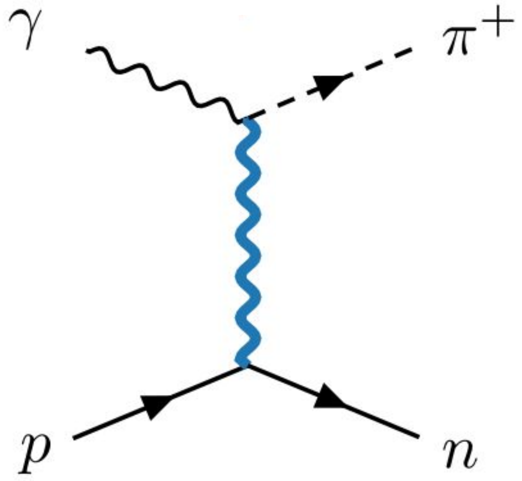
G. Montaña, DW, & JPAC [Phys.Rev.D 110 (2024) 11, 114012]

Understanding π exchange

Simplest charge-exchange process is π **photoproduction.**

Gauge-invariance is easy to implement with Born diagrams…





but what about for a **Reggeized pion**?

$$\langle \lambda_f | T(t, \theta_t) | \lambda_\gamma \lambda_i \rangle = \sum_J (2J+1)\, d^J_{\lambda_\gamma, \lambda_i - \lambda_f}(\theta_t)\, a^J_{\lambda_\gamma \lambda_i \lambda_f}(t)$$

$$|\lambda_\gamma| = 1 \longrightarrow J \geq 1$$

No pion pole?

G. Montaña, DW, & JPAC [Phys.Rev.D 110 (2024) 11, 114012]

Current conservation and the π pole

Construct gauge-invariant, *t*-channel amplitudes with **explicit *J*-dependence** (i..e arbitrary spin exchange)

$$\langle \lambda_f | T^J(t,\theta_t) | \lambda_\gamma \lambda_i \rangle = \sum_{\sigma_J} \frac{V^J_{\lambda_\gamma}(\sigma_J)\, V^J_{\lambda_i \lambda_f}(\sigma_J)}{J - \alpha(t)}$$

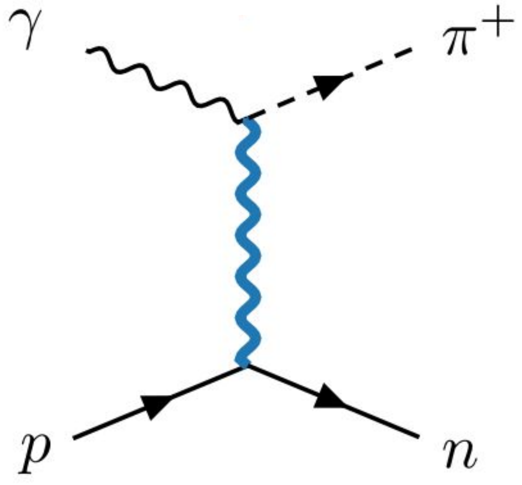
Analytically continue result down to *J = 0*

$$\langle \lambda_f | T(t,\theta_t) | \lambda_\gamma \lambda_i \rangle = \sum_J (2J+1)\, d^J_{\lambda_\gamma, \lambda_i - \lambda_f}(\theta_t)\, a^J_{\lambda_\gamma \lambda_i \lambda_f}(t)$$

$\propto \sqrt{J}$ $\qquad$ $\propto \sqrt{J^{-1}}$

Product is finite at *J* = 0!

$$d^0_{\lambda_\gamma, \lambda_i - \lambda_f}(\theta_t)\, a^0_{\lambda_\gamma \lambda_i \lambda_f}(t) \propto \frac{\delta_{\lambda_i \lambda_f}}{\alpha(t)} \frac{z_t}{z_t^2 - 1} \approx \frac{i \delta_{\lambda_i \lambda_f}}{m_\pi^2 - t}$$



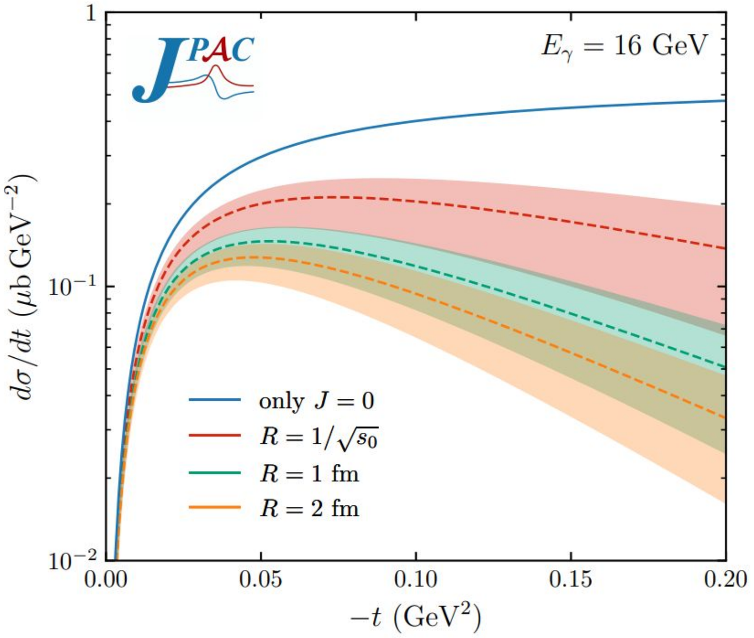
Extra angular piece comes from nucleon exchanges and is constant in Regge limit!

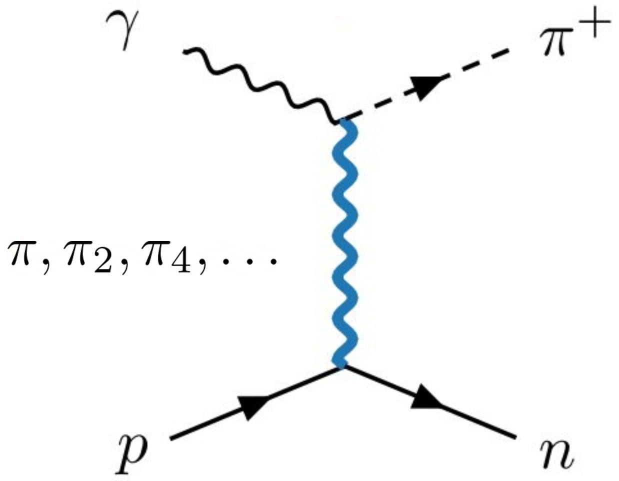
G. Montaña, DW, & JPAC [Phys.Rev.D 110 (2024) 11, 114012]

Reggeization of π exchange

Summing over all *J* to Reggeizes the pion:

$$\langle \lambda_f | T_{\text{Regge}}(t, \theta_t) | \lambda_\gamma \lambda_i \rangle = \sum_{J=0}^{\infty} \langle \lambda_f | T^J(t, \theta_t) | \lambda_\gamma \lambda_i \rangle$$





Investigate subleading effects of background integral terms, governed by the **closest singularities in left-half plane**, i.e. with $J \leq -1$.

More "first principles" approach to Regge amplitudes.

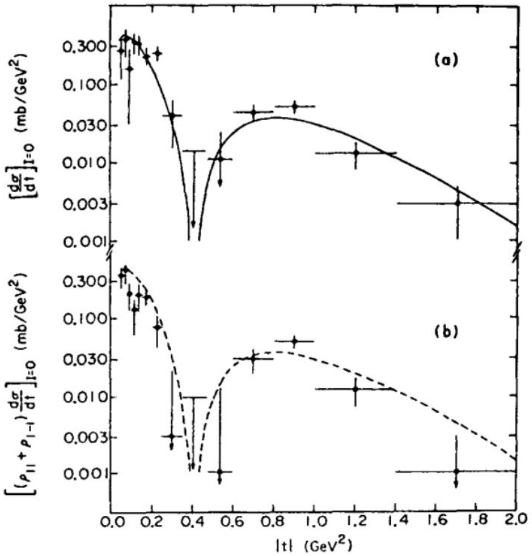
Other meson exchanges

Access to different trajectories and their inference in charge-exchange $\gamma\pi\to\pi\Delta$ reactions.

Unnatural exchanges (π and b_1) : small t

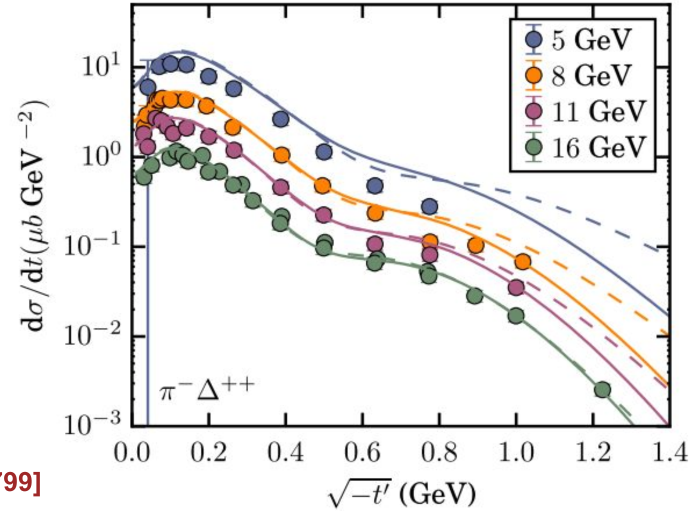
Natural exchanges (ϱ and a_2) : large t

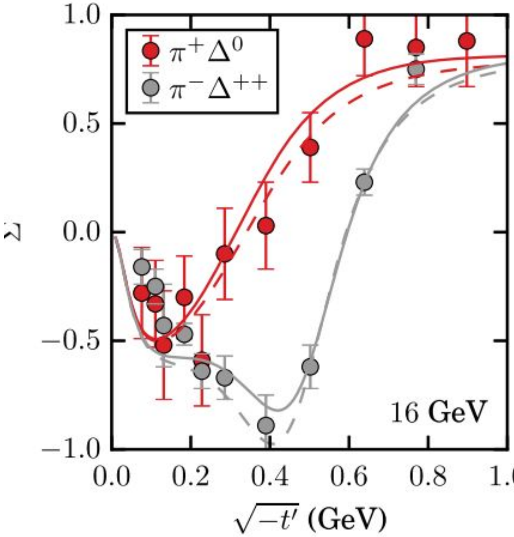
Regge dips in ϱ photoproduction



Gordon et al. [Phys. Rev. D 8 (1973) 779–799]

Less pronounced dips in $\pi\Delta$





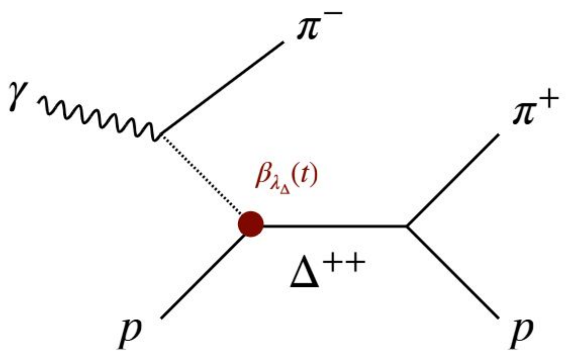
V. Shastry & JPAC [in preparation]

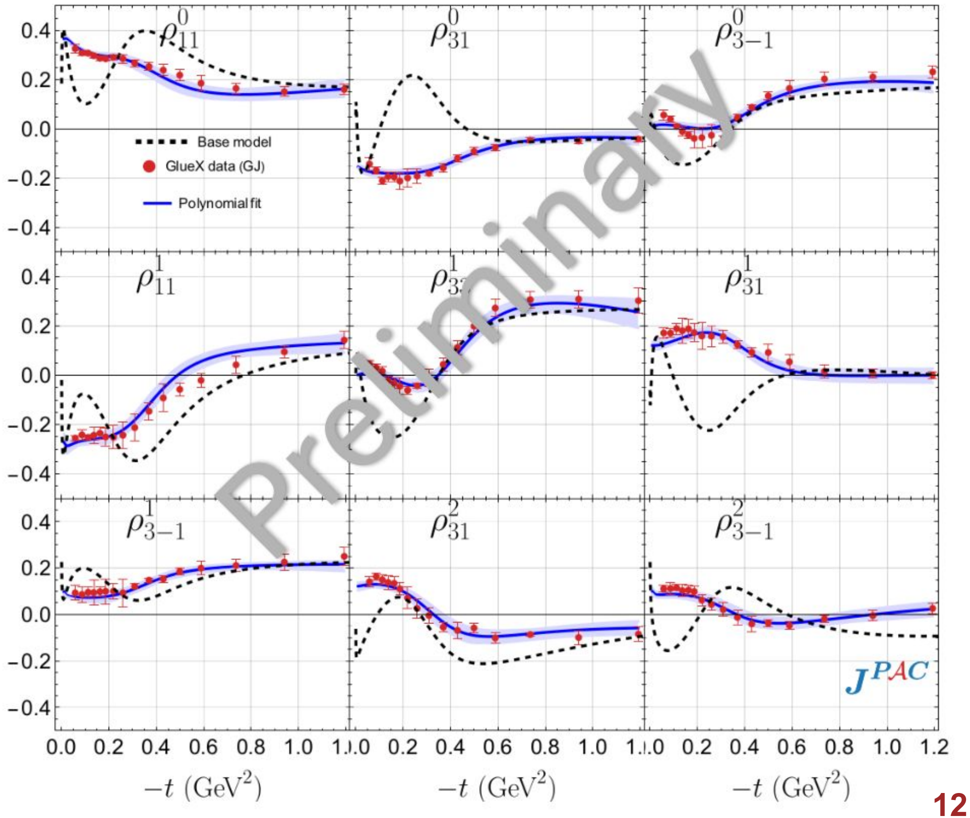
SDMEs of Δ baryon

New high-statistics SDME measurements from GlueX give very stringent constraints on couplings.

GlueX [Phys.Lett.B 863 (2025) 139368]

Want to identify **hierarchy of exchanges**, test exchange degeneracy and pion absorption effects.





Ł Bibrzycki et al. & JPAC [Phys.Rev.D 111 (2025) 1, 014002]

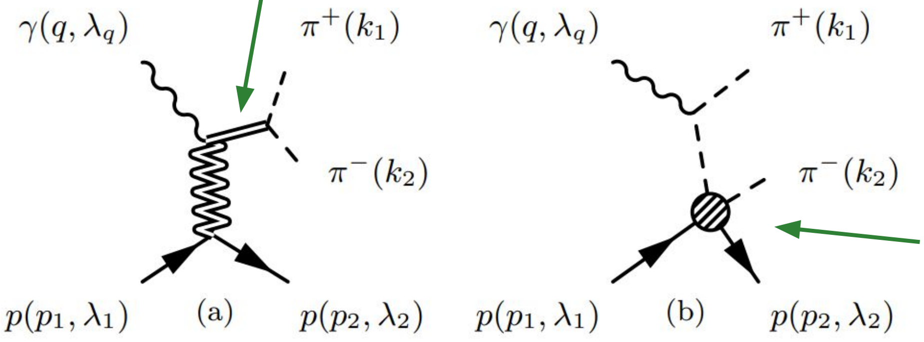
What about the Pomeron?

Interplay between meson and Pomeron exchanges can be studied by looking at 2π photoproduction!

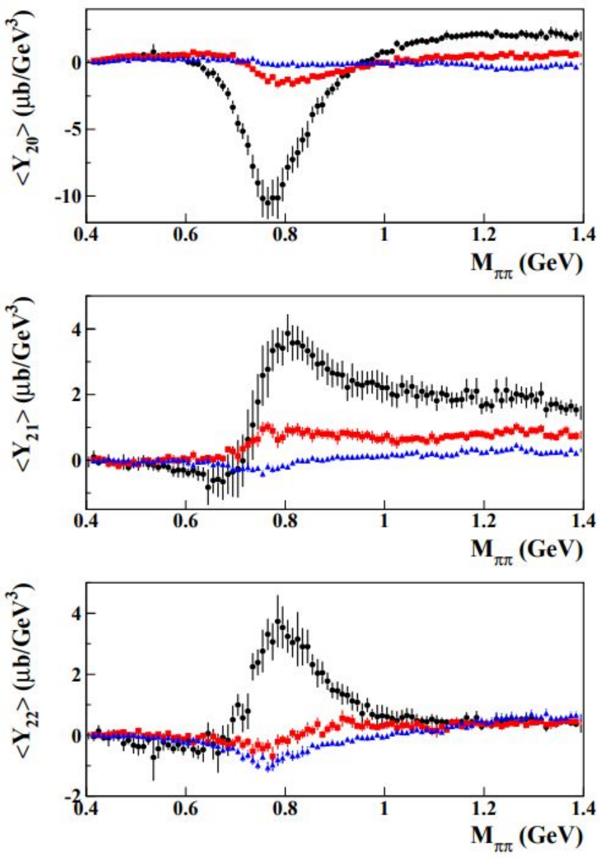
Meson resonances
$\varrho(770)$
$f_0(500)$, $f_0(980)$, $f_0(1370)$
$f_2(1270)$

5 dimensional analysis!

Exchanges considered:
ϱ / ω, f_2 / a_2, Pomeron, π



Baryon resonances From SAID



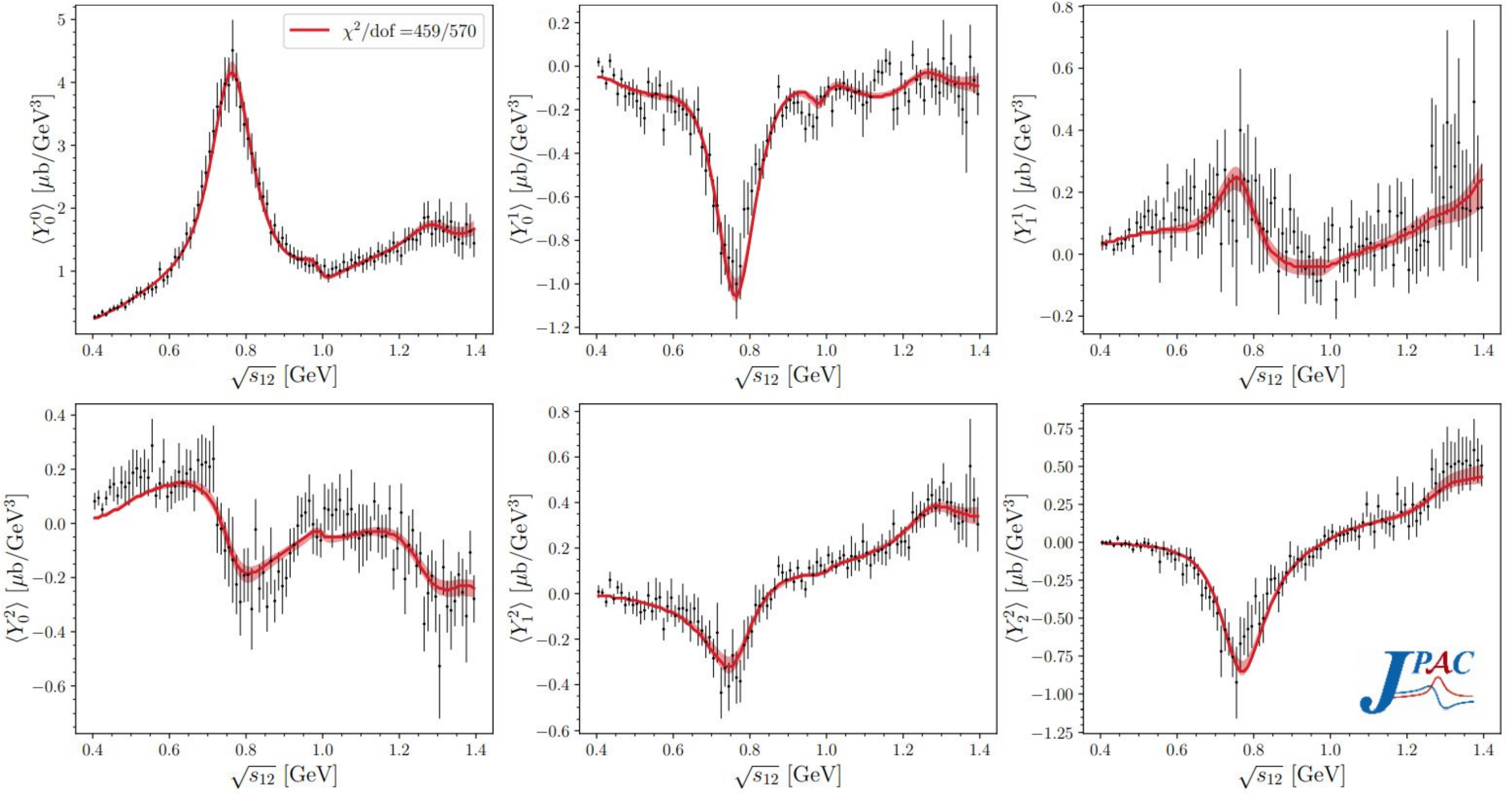
CLAS [Phys.Rev.D 80 (2009) 072005]

Ł Bibrzycki et al. & JPAC [Phys.Rev.D 111 (2025) 1, 014002]

Angular moment analysis

Good description of lowest angular moments *L = 0, 1, 2* and all *M* and for all bins of −*t* < 1 GeV²

−*t* = 0.95 GeV²

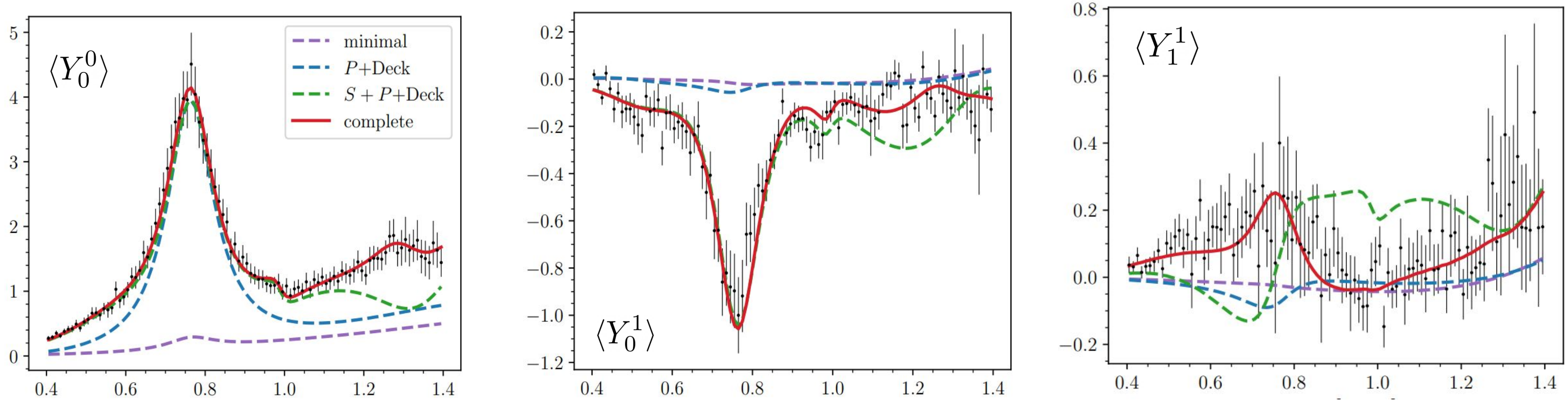


Ł Bibrzycki et al. & JPAC [Phys.Rev.D 111 (2025) 1, 014002]

Angular moment analysis

Pomeron-induced ϱ production and Deck contributions are not enough to describe any nontrivial moments! Complete model is proof-of-concept for Regge description of 2π photoproduction.

Next is to **compute SDMEs** for any of the mesons or baryons as a function of *s*, *t*, and 2π mass!

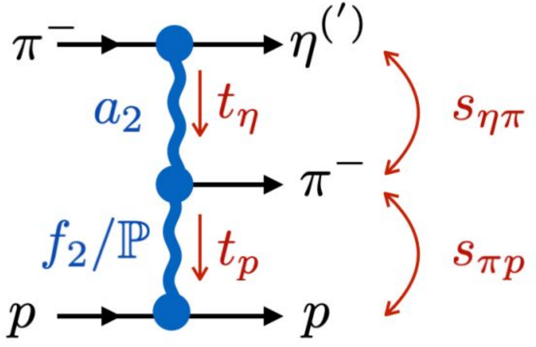


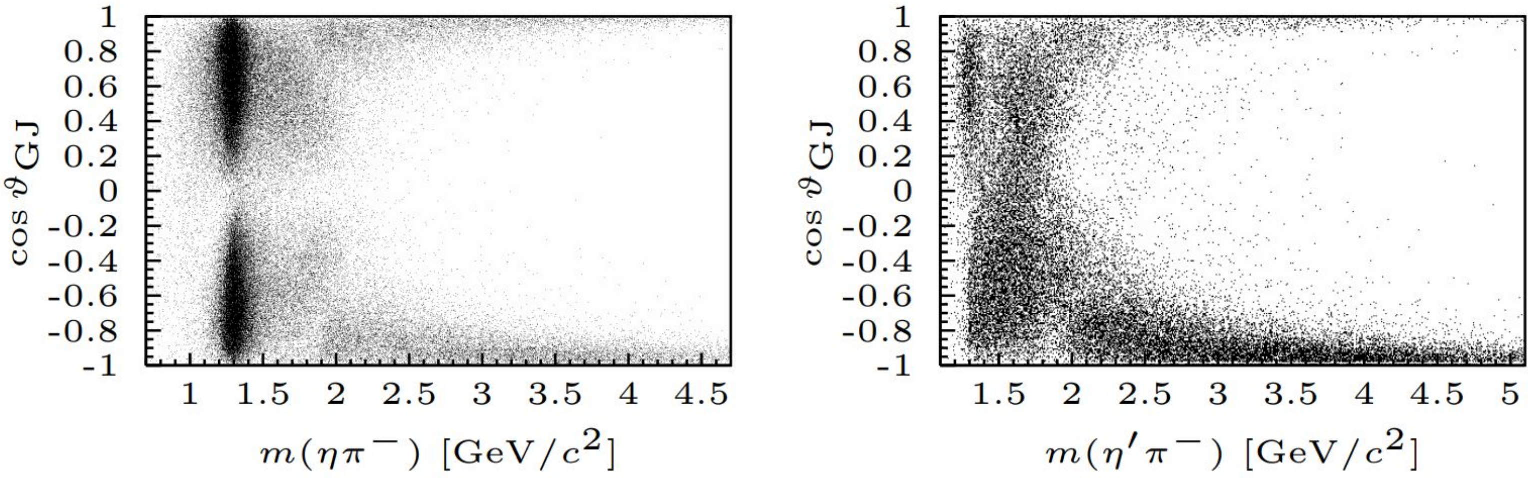
Ł Bibrzycki, et al. & JPAC [Eur.Phys.J.C 81 (2021) 10, 915]

Double Regge production of $\eta\pi$

In 2 → 3 when both two-body subsystems have large invariant masses, they both Reggeize (two rapidity gaps)!

Forward-backward asymmetry proportional to interference of odd and even partial waves → **sensitive to exotic *P*-waves**, i.e. the $\pi_1(1600)$





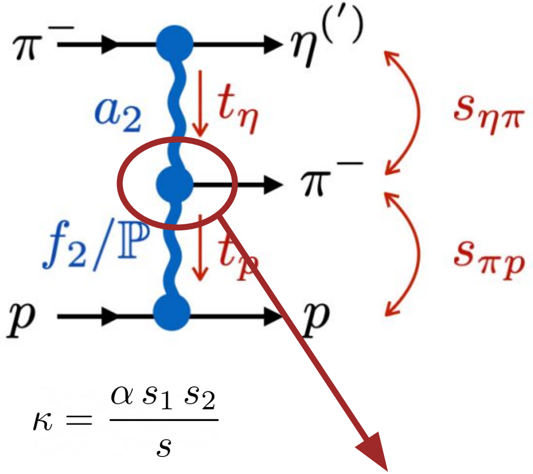
COMPASS [Phys.Lett.B 740 (2015) 303-311]

Ł Bibrzycki, et al. & JPAC [Eur.Phys.J.C 81 (2021) 10, 915]

Double Regge models

Regge theory predicts the energy dependence, but details of Reggeon-Reggeon-particle vertex largely unexplored.

Theory still very underdeveloped, need to **benchmark and develop better models**!



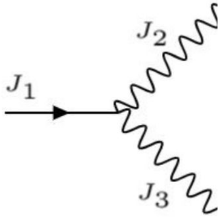
$$\kappa = \frac{\alpha\, s_1\, s_2}{s}$$

$$T(\alpha_1, \alpha_2; s_1, s_2) = K \times \left[\Gamma(1-\alpha_1)\, \xi_1\, (\alpha'\, s)^{\alpha_1}\right] \left[\Gamma(1-\alpha_2)\, \xi_{21}\, (\alpha'\, s_2)^{\alpha_2-\alpha_1}\right] V(\alpha_1, \alpha_2; \kappa)$$

Top exchange **Bottom exchange** **Middle coupling**

Shimada et al. [Nucl.Phys.B 142 (1978) 344-364]

$$V(\alpha_1, \alpha_2; \kappa) = \frac{\Gamma(\alpha_1 - \alpha_2)}{\Gamma(1-\alpha_2)}\, {}_1F_1(1-\alpha_1, 1-\alpha_1+\alpha_2; -\kappa)$$



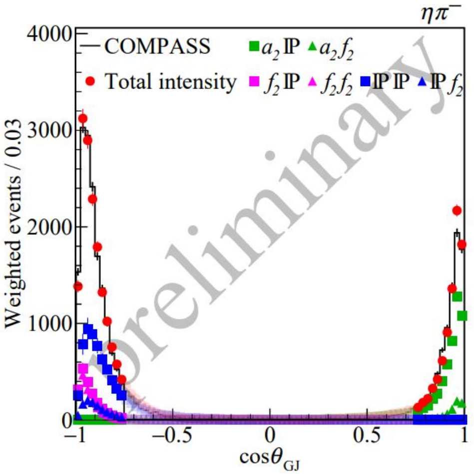
$$\propto \frac{1}{J_2!\, J_3!}$$

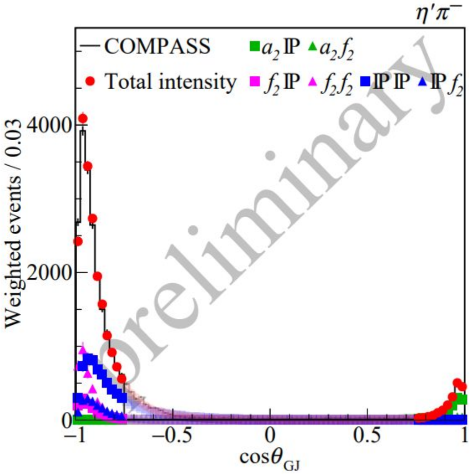
Ł Bibrzycki, et al. & JPAC [Eur.Phys.J.C 81 (2021) 10, 915]

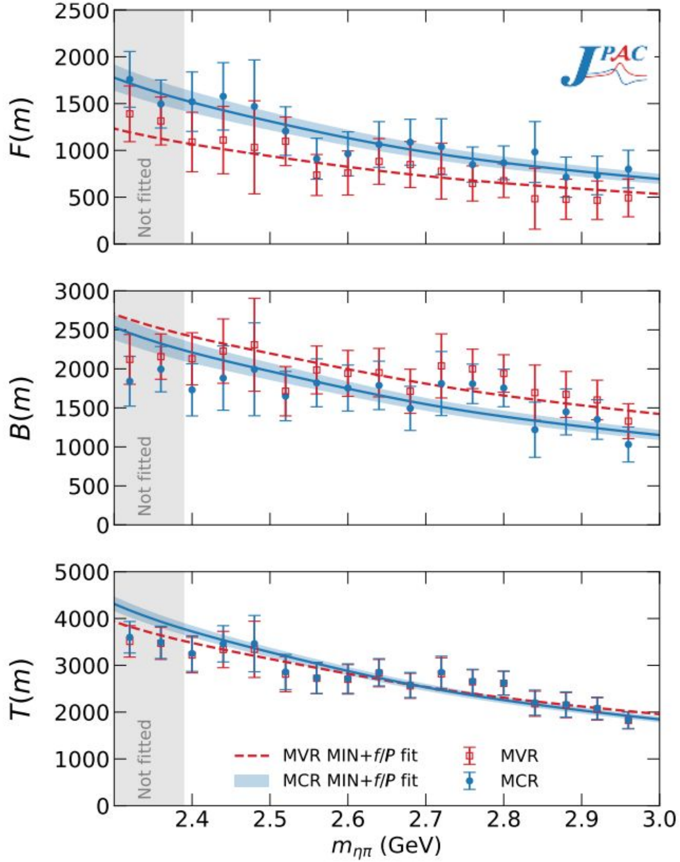
Analysis of COMPASS data

Shimada model seems to be sufficient to describe COMPASS intensity for both $\eta\pi$ and $\eta'\pi$ $\rightarrow$ identify leading top and bottom exchanges!

Detailed analysis from **event-by-event fits** are ongoing!







H. Pekeler for COMPASS [Talk at HADRON2025 in Osaka, JP]

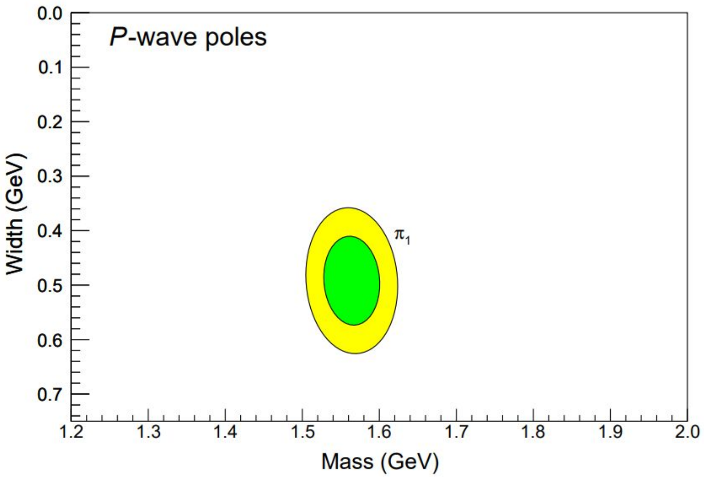
Application to GlueX data

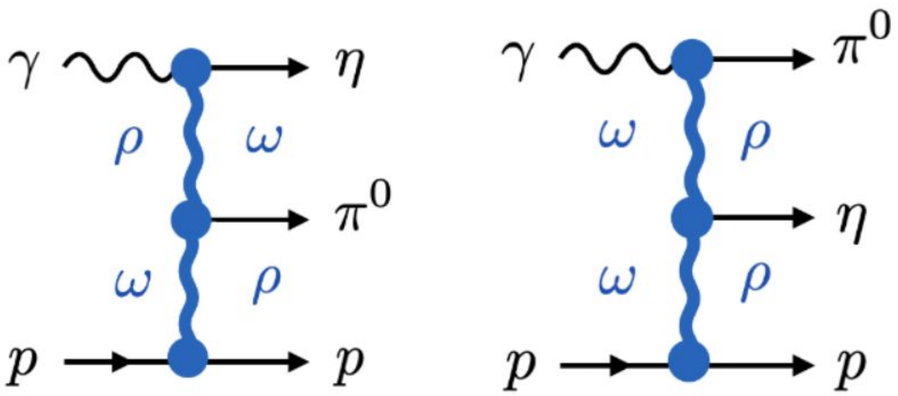
Compare with similar production at GlueX → photon beam means **different exchanges** (no Pomeron).

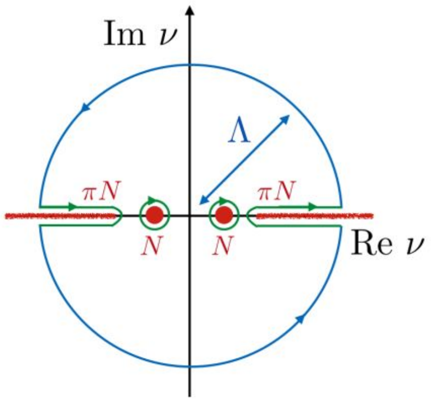
Need to better quantify boundary of “DR region” in terms of finite kinematics.

Develop **dispersion relations** relating DR amplitudes to resonant partial-waves (i.e. FESRs for 2 → 3)

A Rodas & JPAC [Phys.Rev.Lett. 122 (2019) 4, 042002]







Summary

Regge theory offers economical tool to parameterize and understand **high-energy production reactions**.

Many ongoing studies with **current facilities in photon and pion beams**, relevant for light meson spectroscopy, in particular light hybrid candidates!

Opportunity for analogous spectroscopy of light meson and baryon systems with **proton beams at SIS100**.