



*EMMI Collaboration Meeting : QCD at FAIR Workshop 2025*

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## EoS and Compact Stars

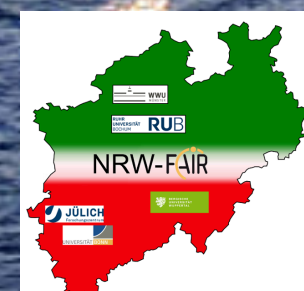
In collaboration with :

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H. Chen & A. Li & Z.H. Li & J. Wei & X.R. Zhou & W. Zuo : China

E. Hiyama & T. Maruyama & N. Yasutake : Japan

T. Rijken : Nijmegen



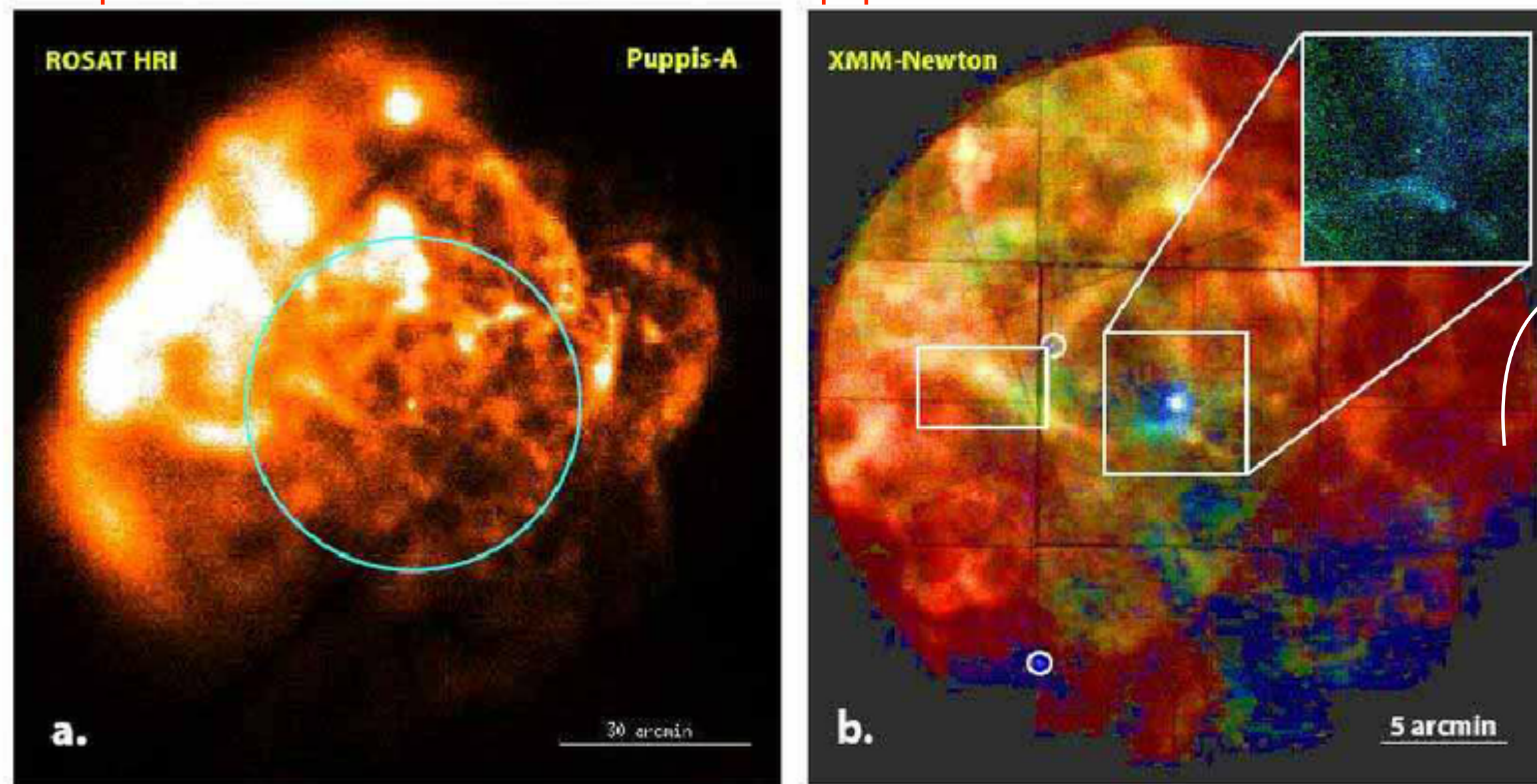
**22-27 June, 2025, Grand Hotel Faraglioni, Aci Trezza (Sicily)**



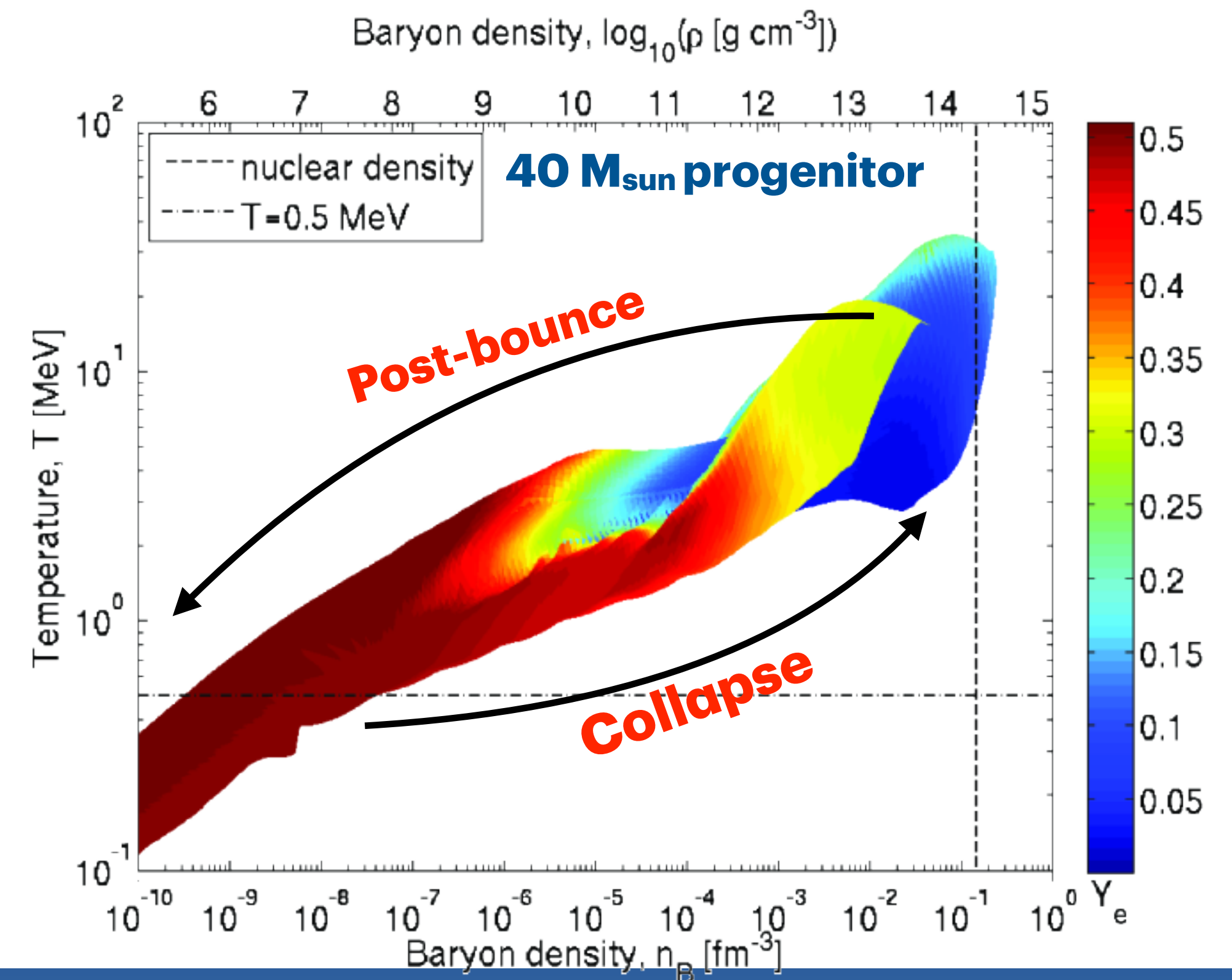
# Dense matter in the Universe : SNe

T. Fischer et al., 2021 ApJS 194 39

## Supernova remnant in Puppis A



Credit: NASA

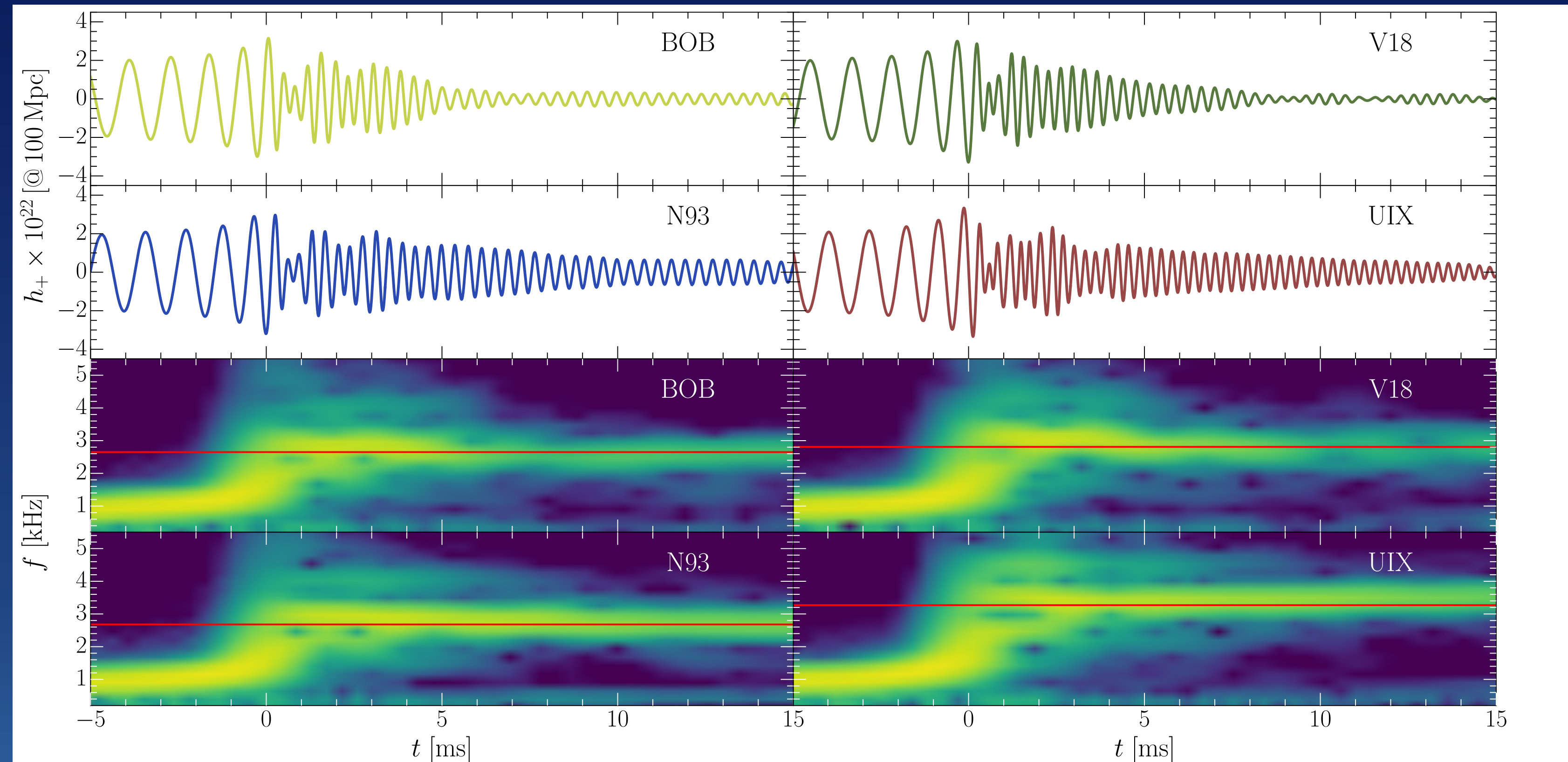


Temperature and density reached during a standard core-collapse supernova simulation at 100 ms post bounce.



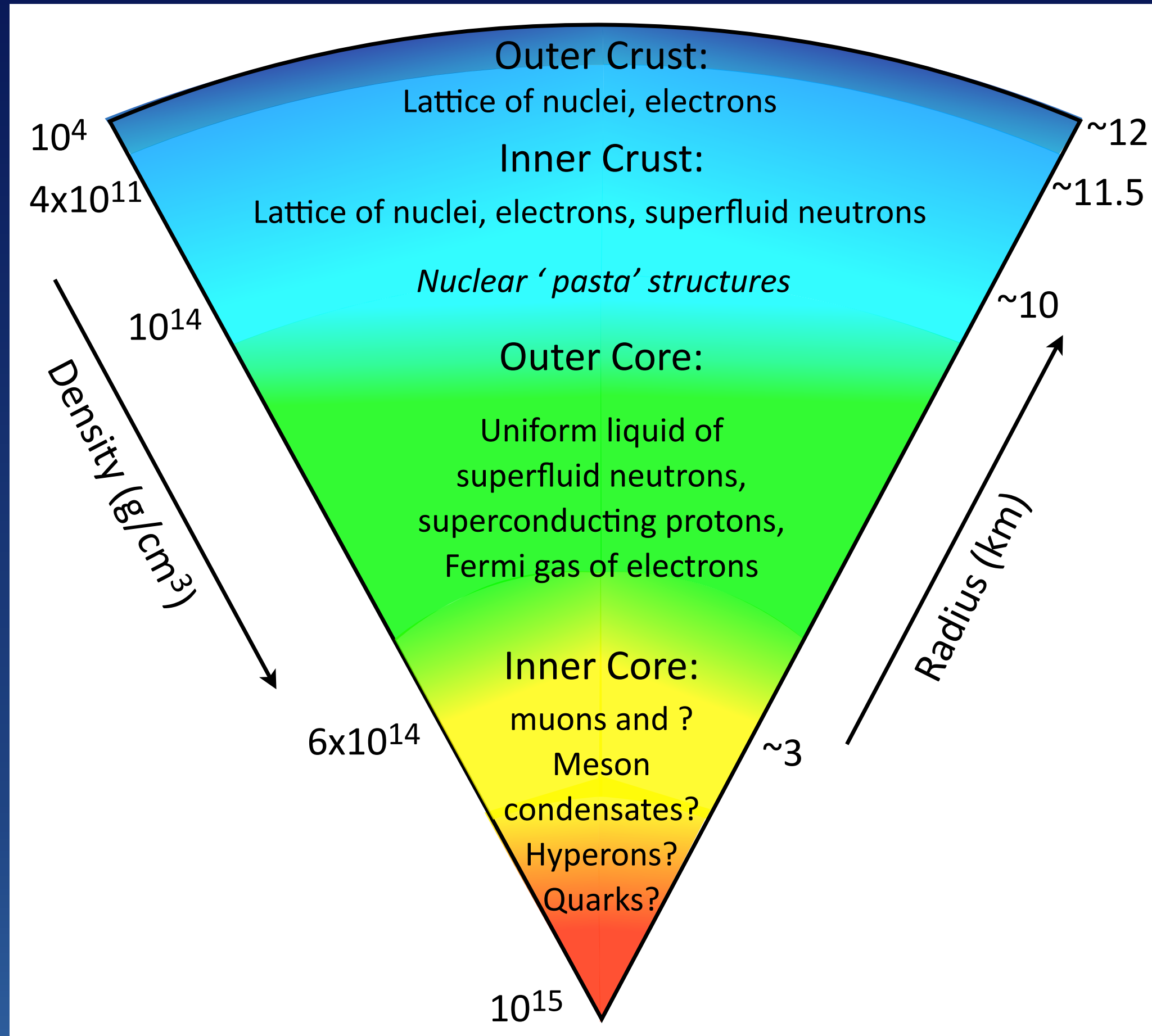
# Dense matter in the Universe : BNS mergers

- (Late) inspiral phase: modification of the gravitational waveform wrt a GR point-particle calculation, due to the star deformation just before merging
- Aftermerger: complex pattern due to the oscillation of the hot supermassive NS before collapsing to BH



A. Figura et al., PRD **103**, 083012 (2021)

# Dense matter in the Universe : NS



**Outer crust** A few hundred m's thick.  $\rho = (10^4 - 10^{11}) \text{g/cm}^3$ . Ions in an electron gas.

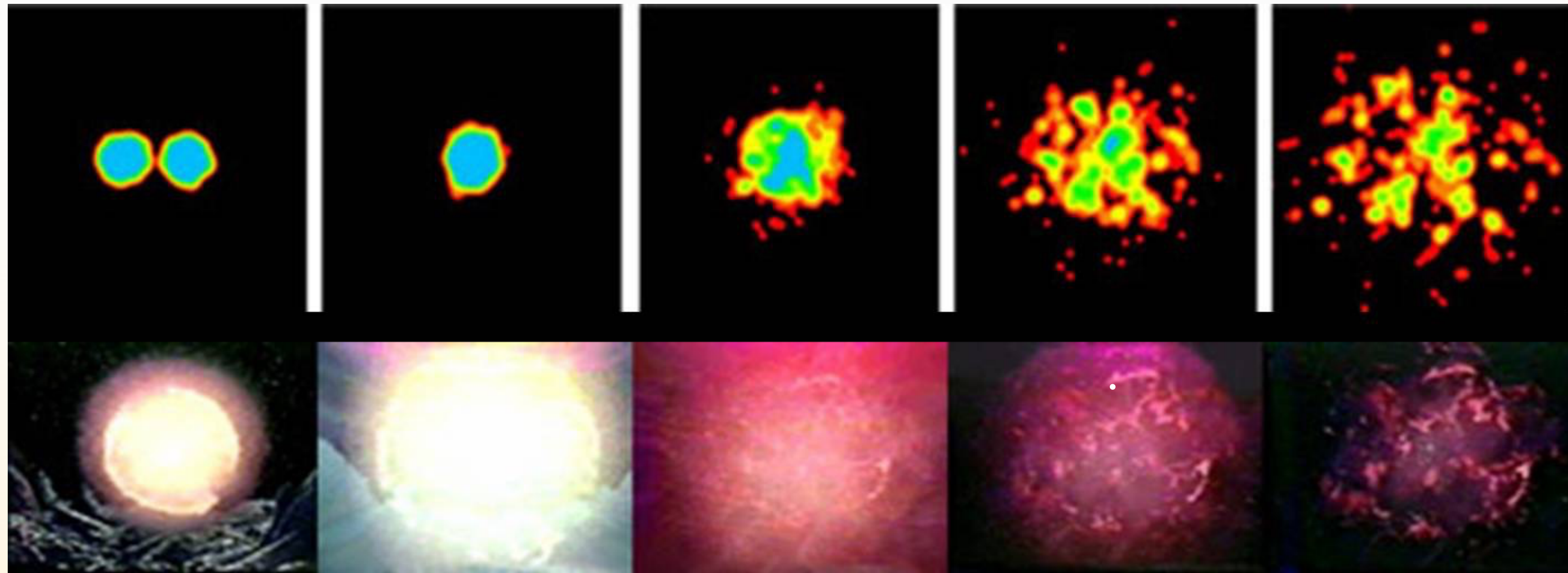
**Inner crust** 1-2 km.  $\rho = 4 \times 10^{11} - 10^{14} \text{g/cm}^3$ . Electrons beta-captured by nuclei  $\rightarrow$  neutron-rich nuclei  $\rightarrow$  drip point. Free neutrons gas. Nuclei melting down and nuclear matter formation from drip up to  $\rho \approx \rho_0/2$  : uniform fluid of  $n, p, e^-$

**Outer core**  $\rho \approx \rho_0/2 - 2\rho_0$ . Asymmetric nuclear matter above saturation. Composition made by neutrons, protons, and leptons. Is it all ?

**Inner core**  $\rho \approx 2\rho_0 - (8 - 10)\rho_0$  The most unknown region. "Exotic matter". Hyperons ? Quarks ?



# Relevance of the EoS



1. Heavy ion collisions (small  $N/Z$ , high  $T$ )
2. Supernovae and Neutron Stars  
(high  $N/Z$ , high (small)  $T$  in SN (NS))
3. Binary NS merger and GW emission  
(high density, high  $N/Z$  and  $T$ )

Quite different physical conditions  
in each case !

**A nuclear matter  
theory must be able  
to treat all these  
physical situations.**



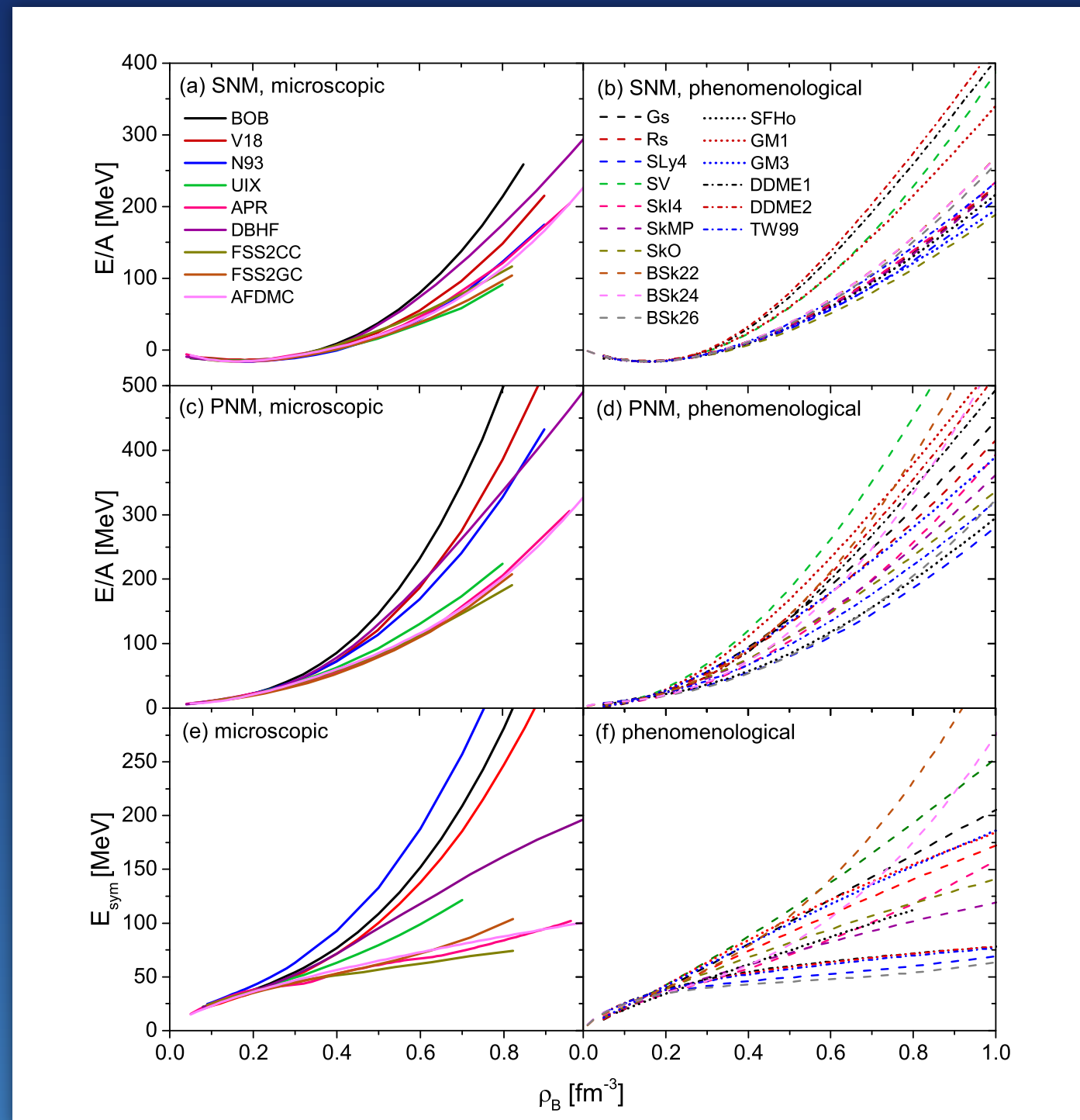
# What do we need?

## An exact theory to deal with

- Strong interactions of particles of different species
- Many-body effects in dense matter

# What do we have?

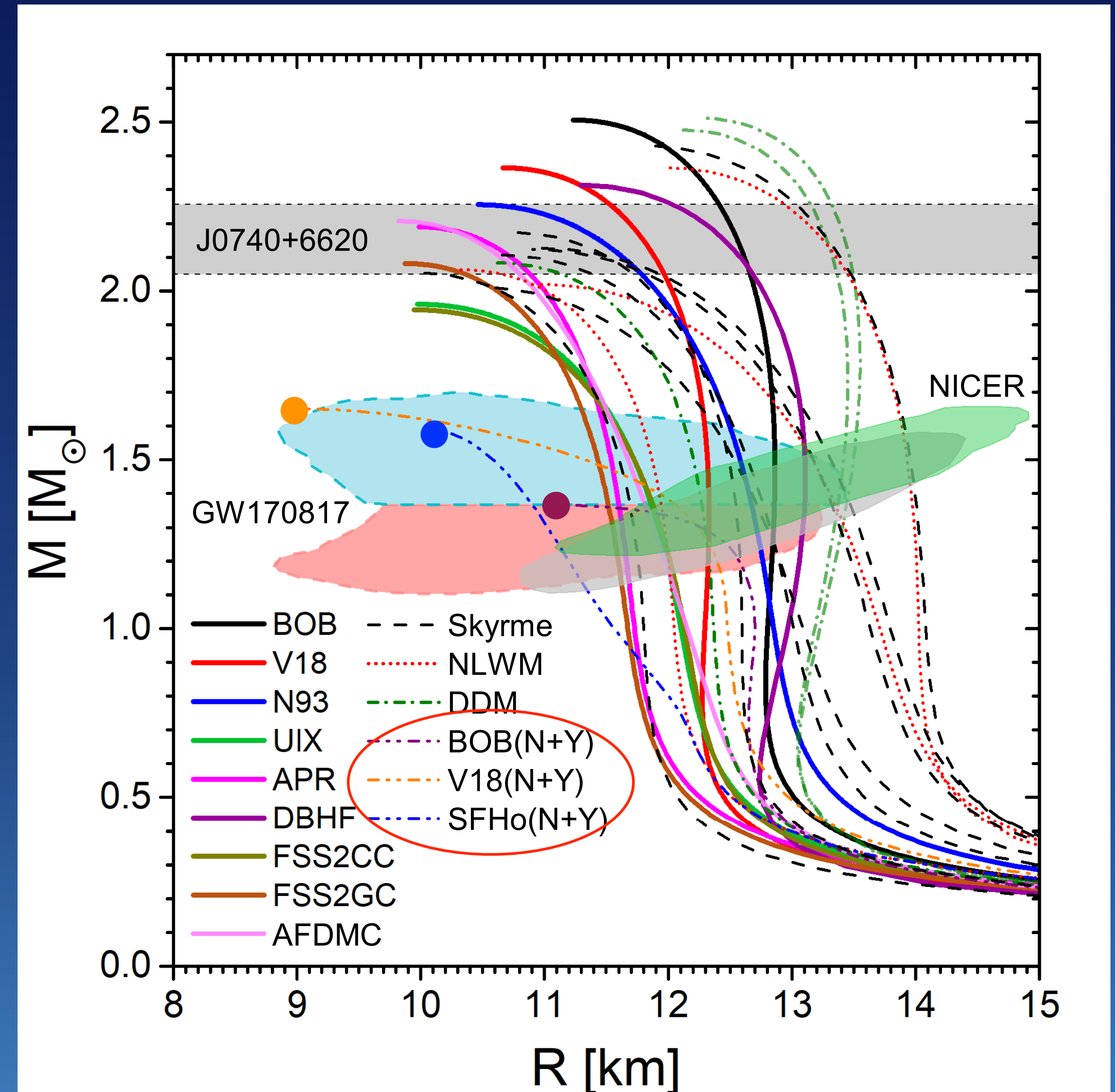
## Many different theoretical models!



Neutron stars and the nuclear equation of state

G.F. Burgio, H.-J. Schulze, I. Vidaña, J.-B. Wei,

PPNP 120 (2021) 103879.







# Tolman–Oppenheimer–Volkoff equations



We consider static spherically symmetric stars.

$$\frac{dP}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2} \left(1 + \frac{P}{\epsilon(r)c^2}\right) \left(1 + \frac{4\pi r^3 P}{m(r)c^2}\right) \left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r)$$

First term on r.h.s. : Newtonian term from hydrostatic equilibrium, with  $\epsilon(r)$  the mass density.

$$\frac{dP}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2}$$

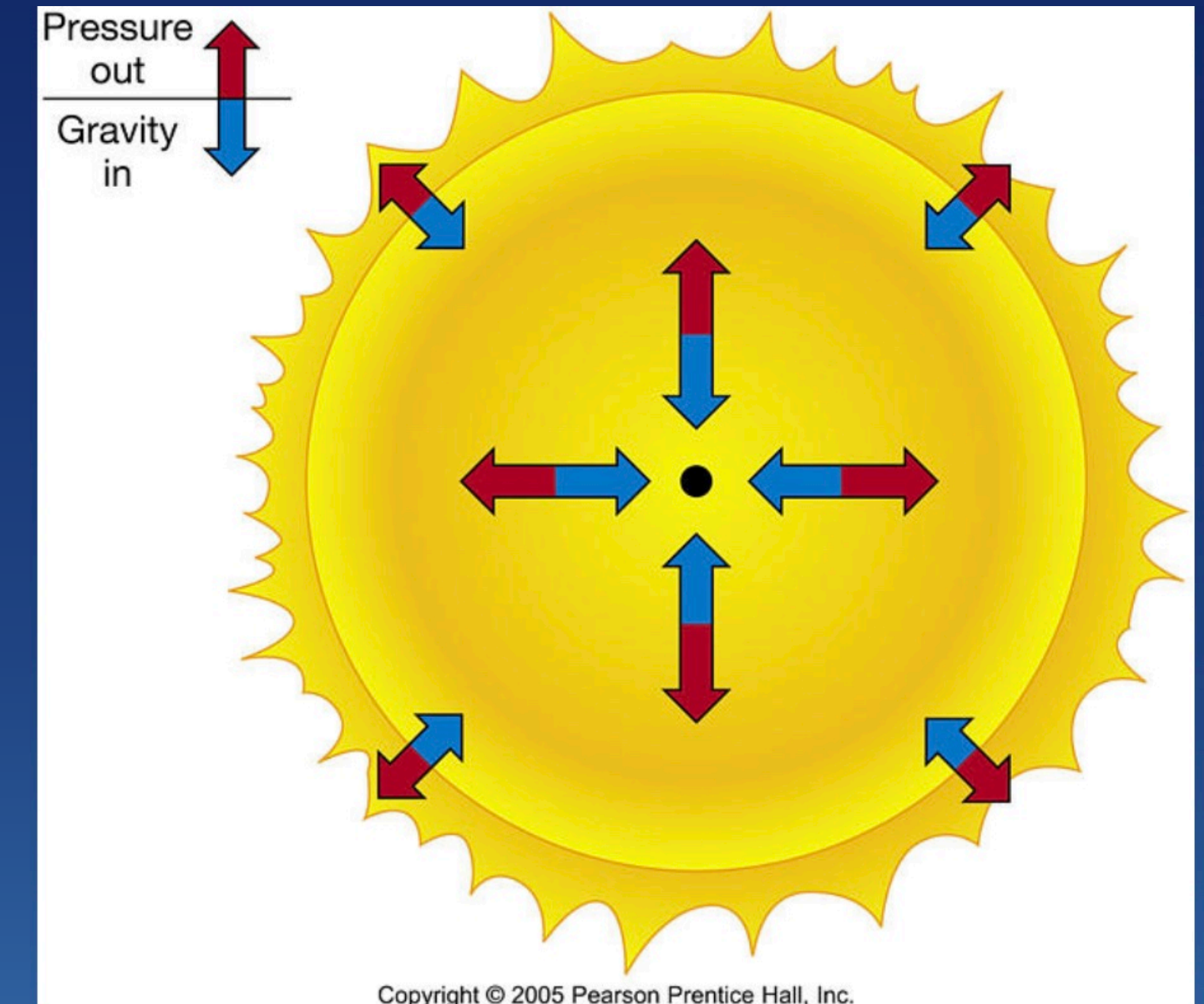
Three correction terms from GR.

- Coupling of gravity to the energy density  $\epsilon(r)$  and the pressure  $P(r)$  of matter.
- Modification of the mass function  $m(r)$  due to the pressure.
- Modification of the radius.



The equation of state is needed to close the system of equations.

$$P = P(\epsilon)$$





# Overview of the strong interaction in dense matter

Hadronic Hamiltonian can, in principle, be derived from the underlying quark-gluon dynamics in QCD.

However, because of the sign problem raised by the non-perturbative character of QCD at low and intermediate energies ( $\alpha_s$  behaviour) one is far from a quantitative understanding of the baryon-baryon interaction from the QCD point of view.

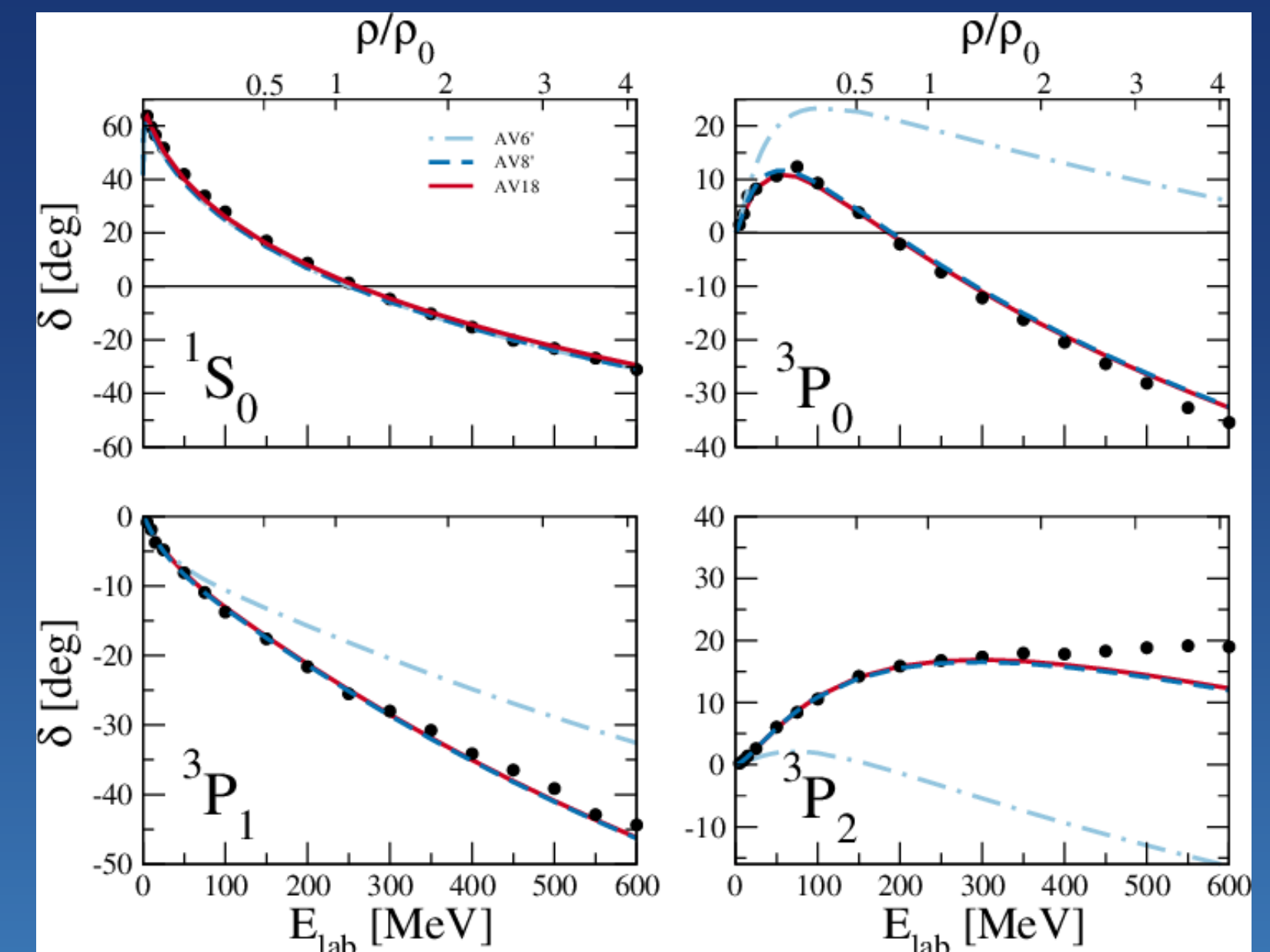
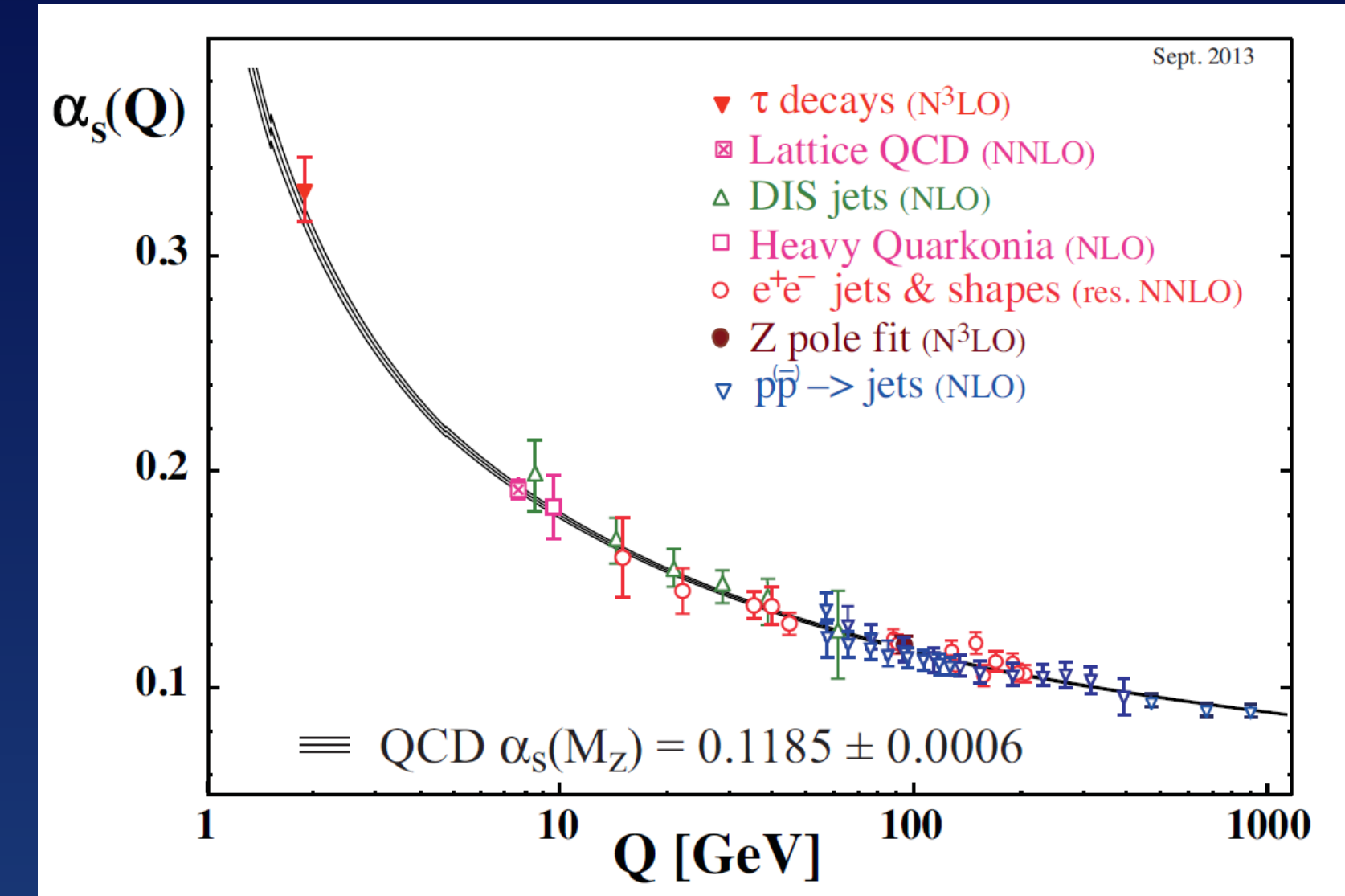
Two different collaborations and strategies : HALQCD & NPLQCD.

\* **HALQCD** investigation of the properties of nuclei and the EoS of nuclear matter. Binding energy per nucleon with a uniform mass-number  $A$  dependence, consistent with the Bethe-Weizsäcker mass formula, but bound at a quark mass corresponding to a pion mass of 469 MeV.

\* **NPLQCD** : in the strangeness sector determination of the binding energies of light hypernuclei. Results for NN, NY and YY interactions.

**Solution** : to adopt models where the hadronic degrees of freedom are the relevant ones. Meson exchange and potential models. Functional form dictated by fundamental symmetries.

Essential requirement : Fit of the NN phase shifts.





# From NN interaction to EoS : Two different philosophies

## Phenomenological vs. ab initio approaches

### Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted to reproduce nuclear observables and compact stars observables.

Independent particles ansatz : the interaction has to be modified

$$E = \langle \psi | \hat{H} | \psi \rangle \approx \langle \mu | \hat{H}_{eff} | \mu \rangle$$

- **Non-relativistic models:** Skyrme and Gogny
- **Relativistic mean-field models (RMF)**
- **SN approximation models** : Liquid Drop models, Thomas-Fermi models, Self-consistent mean field models. NSE models.

### Ab initio approaches

Based on bare NN and NNN realistic interaction which reproduces scattering data and deuteron properties.

- (quasi) exact treatment of nuclear correlations
- the short-range hard core of the NN interaction  $V$  makes any perturbative expansion in terms of  $V$  meaningless and convergence very hard

The EoS is found by solving the complicated many-body problem.

- **Diagrammatic:** (Dirac)-Brueckner-Hartree-Fock, SCGF
- **Variational** : APR, FHNC, LOCV, CBF.
- **Quantum Monte Carlo** : VMC, GFMC, AFDMC.



# Check wrt nuclear physics constraints

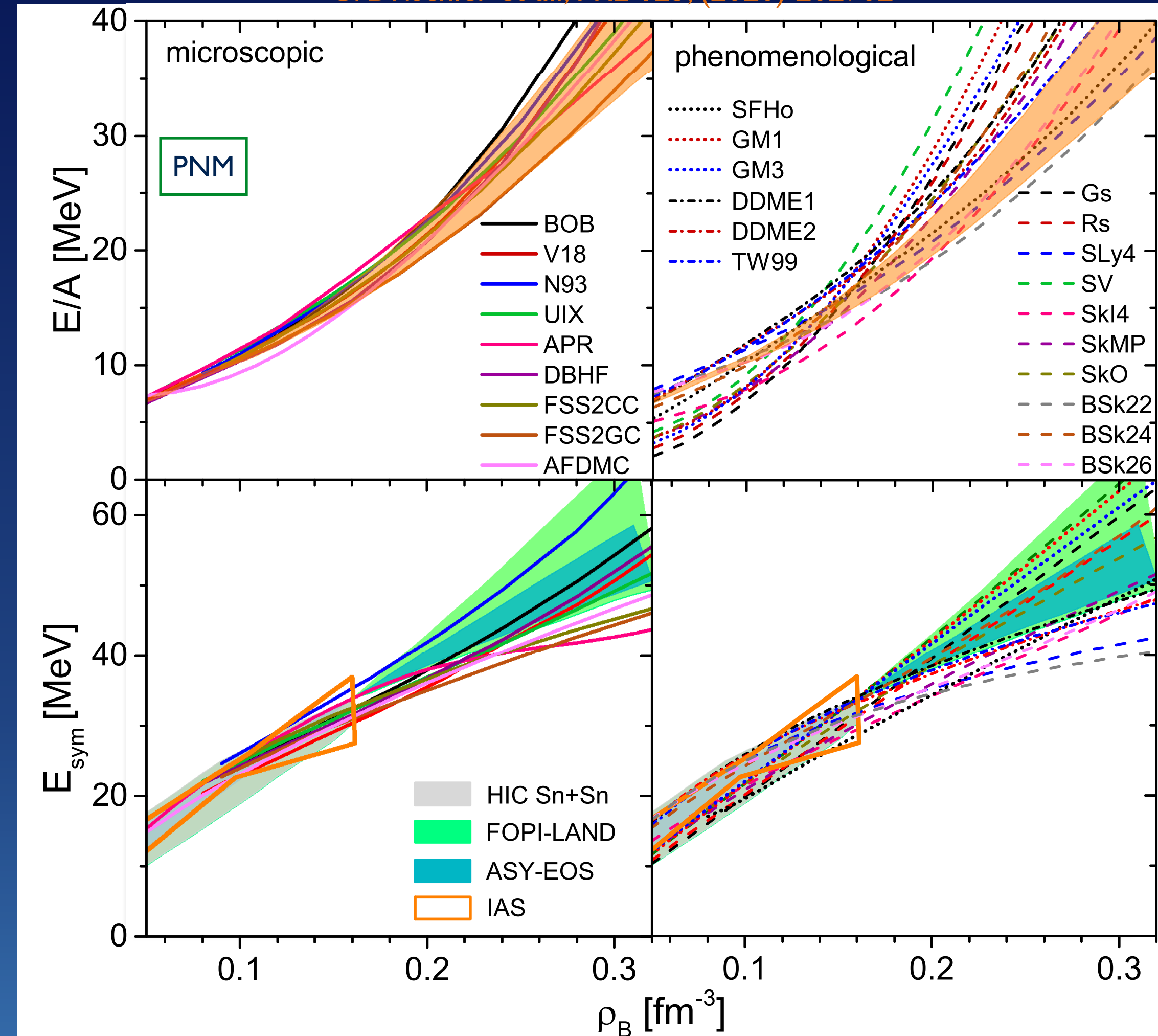
Orange : predictions from the  $\chi$ EFT up to N3LO order  
C. Drischler et al., PRL 125, (2020) 202702

## Microscopic EoS

- BHF with Argonne V18 or Nijmegen 93 2NF and microscopic 3NF (BOB, V18, N93, UIX)
- BHF with FSS2 NN interaction (quark d.o.f. explicitly taken into account)
- Variational APR with Argonne V18 and 3NF of Urbana UIX type
- Relativistic DBHF (Bonn A)
- AFDMC with modified V18

## Phenomenological EoS

- Skyrme forces (Gs, Rs, SLy4, SV etc...)
- Brussels-Montreal group BSk22,24,26
- NLWM (SFHo, GM1,3), RMF models with different parameterizations.
- DDM, RMF model with density dependent coupling constants.





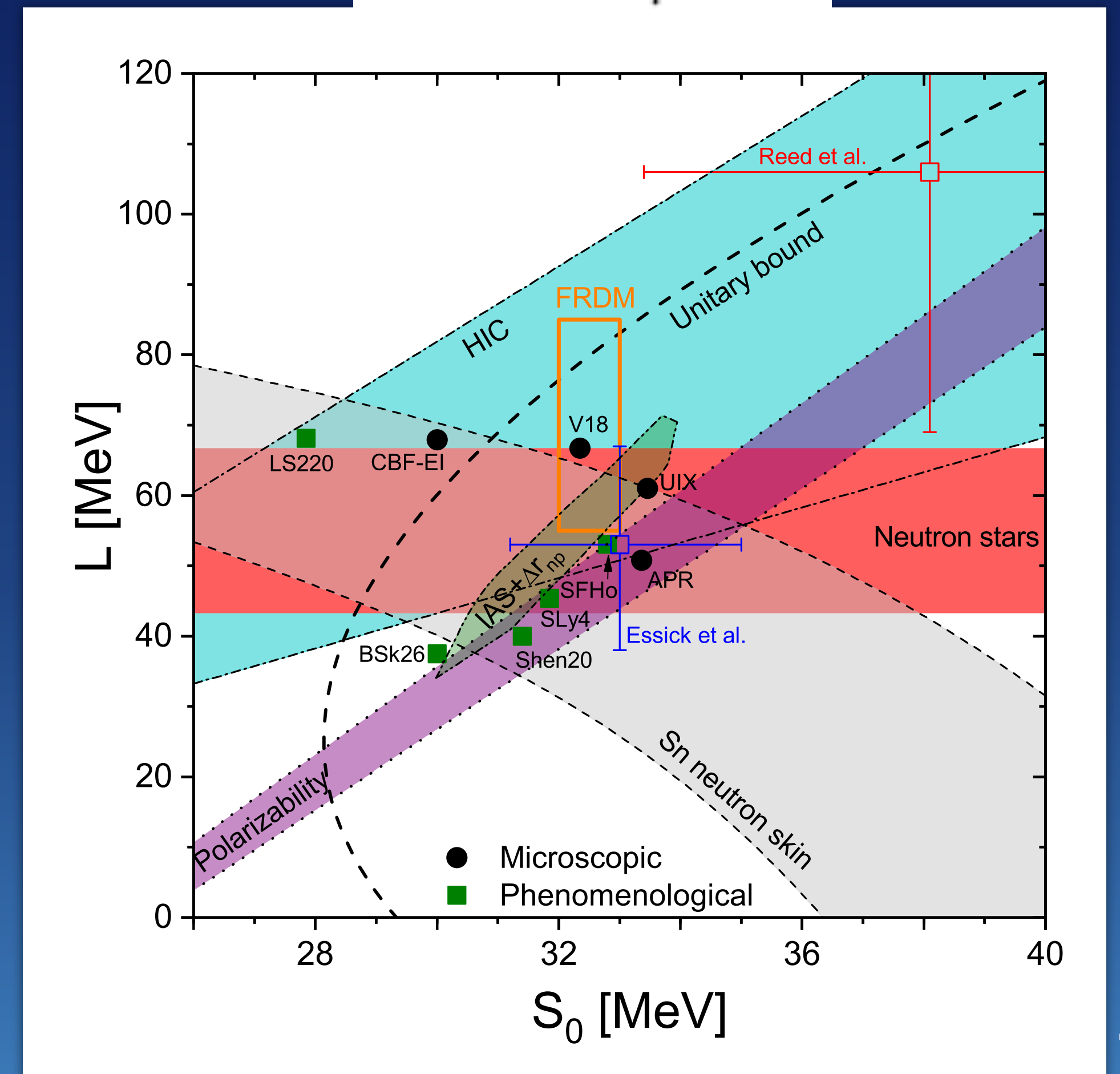
# Constraints from Nuclear Physics Experiments

- $E/A$  from experimentally measured nuclear masses
- $K_0$  from isoscalar giant monopole resonances in heavy nuclei and HiCs  
 $230 \text{ MeV} < K_0 < 270 \text{ MeV}$
- $S_0$  from nuclear masses, isobaric analog state phenomenology, neutron skin thickness and HiCs isospin diffusion  
 $S_0 \sim 30\text{-}32 \text{ MeV}$
- $L$  from dipole resonances, electric dipole polarizability and neutron skin thickness. No overlap region !

Too many uncertainties in the experimental measurements and in the models used for the data interpretation.

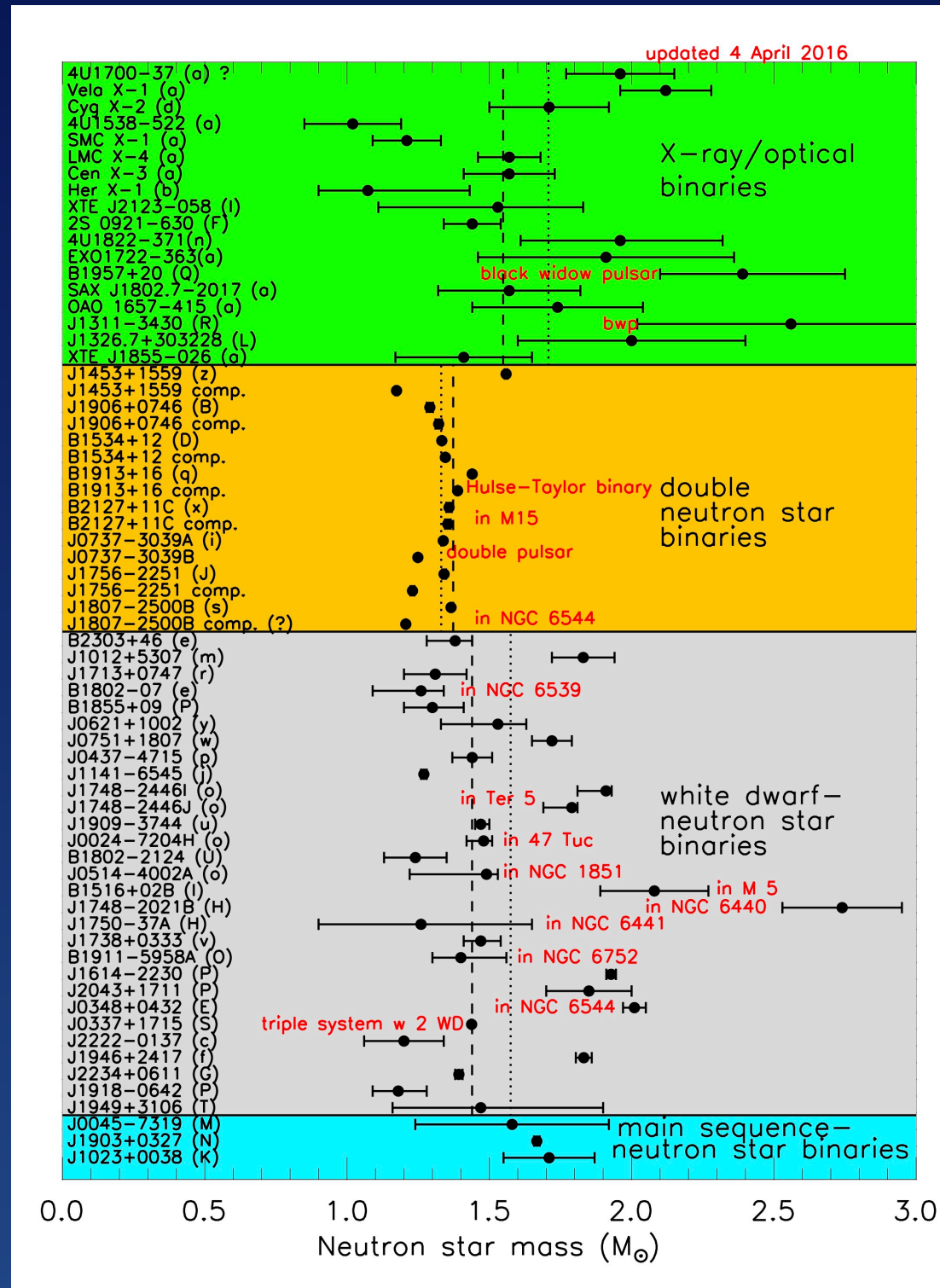
No theoretical model can be ruled out a priori.

$$L \equiv 3\rho_0 \frac{dE_{\text{sym}}}{d\rho}(\rho_0)$$

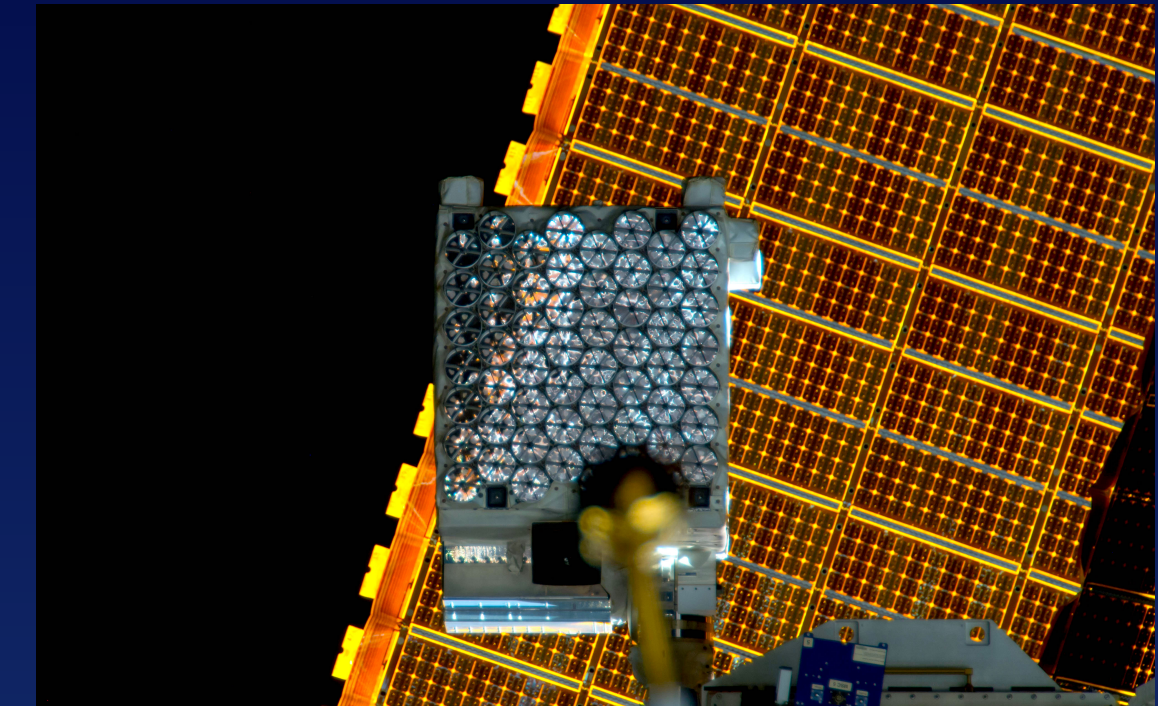




# Check wrt NS observables : M & R



**NICER** : a new technique to measure M & R from rapidly spinning compact stars with a hot spot, based on Doppler effect and GR corrections of the signal.



## PSR J0030+0451

$$M/R = 0.156^{+0.008}_{-0.010}$$

$$R = 13.02^{+1.24}_{-1.06} \text{ km}$$

$$R = 12.71^{+1.14}_{-1.19} \text{ km}$$

## PSR J0740+6620

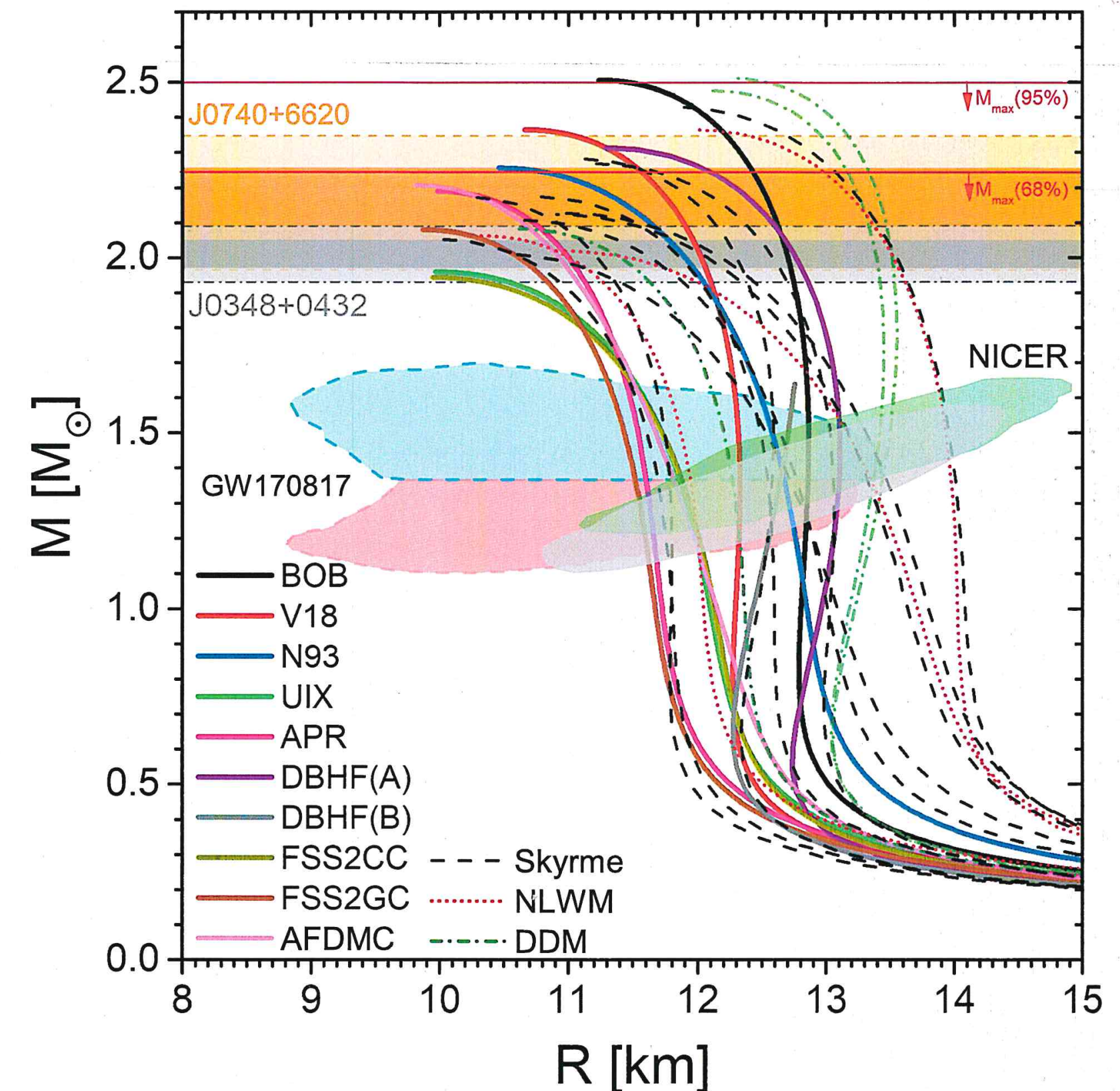
$$M = 2.072^{+0.067}_{-0.066} M_{\odot}$$

$$R = 13.7^{+2.6}_{-1.5} \text{ km}$$

$$R = 12.39^{+1.30}_{-0.98} \text{ km}$$

Miller, Riley, 2019, 2021

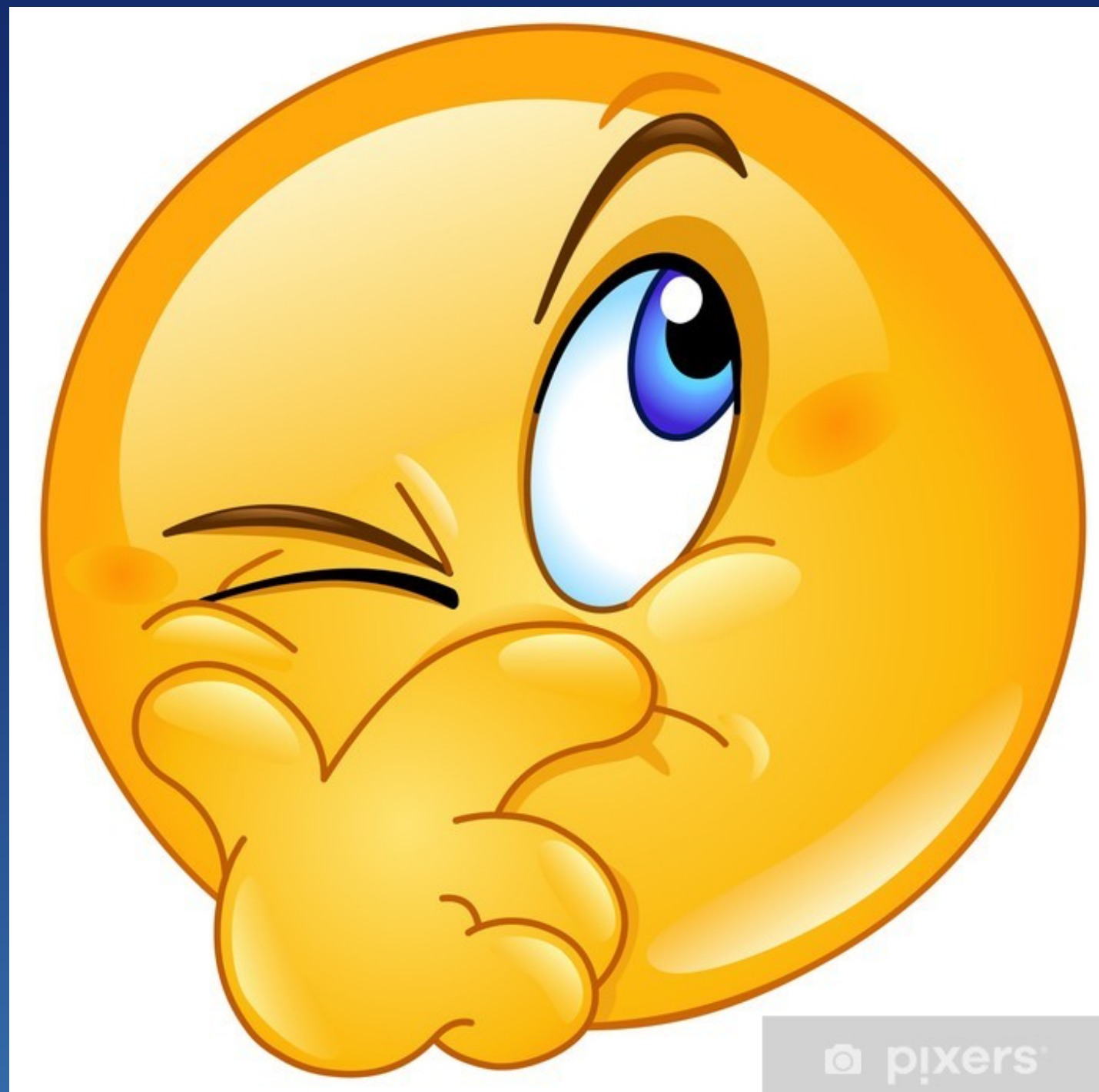
- PSR J1614-2230,  $M = (1.97 \pm 0.04) M_{\odot}$  (P. Demorest et al., Nature, 2010)
- PSR J0348+0432,  $M = (2.01 \pm 0.04) M_{\odot}$  (J. Antoniadis et al., Science, 2013)
- MSP J0740+6620,  $M = (2.14^{+0.2}_{-0.18}) M_{\odot}$  (H. Cromartie et al., Nature Astronomy, 2019)
- PSR J0952-0607,  $M = (2.35 \pm 0.17) M_{\odot}$  (R. Romani et al., ApJ Lett. 2022)





# Neutrons, protons, electrons, muons ... is it all ?

*Observation of  $\sim 2 M_{\odot}$  neutron stars*  
 *$\longleftrightarrow$  strangeness content*



Can hyperons, or strangeness in general, still be present in neutron stars interiors ?

Probably yes, due to the high value of density at the center and the rapid increase of the nucleon chemical potential with density.

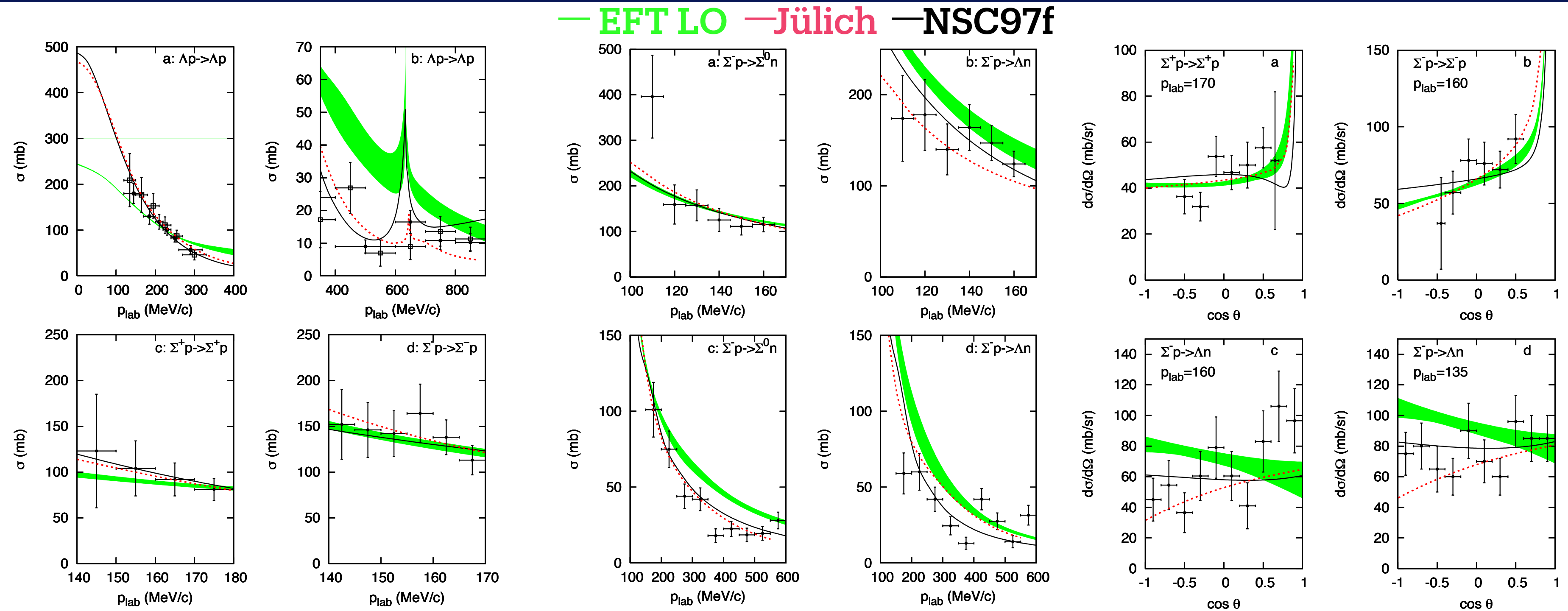
What do we know to include hyperons in the EoS ?

Unfortunately much less than in the nucleonic sector.

Hard to draw strong conclusions given our ignorance of the nucleon-hyperon (NY) and hyperon-hyperon (YY) interaction.



# Available NY Cross Sections Data :



Polinder, Haidenbauer, Meissner, NPA 779, 244 (2006)

- Very few NY scattering data due to hyperon short lifetimes and low intensity beam fluxes
- 35 data points, all from 1960's
- No YY scattering data exist (cf. > 4000 NN data points available !)

Need more data. J-PARC ? FAIR ?



# Including hyperons in BHF approach

$$G_{ab}[W] = V_{ab} + \sum_c \sum_{p,p'} V_{ac} |pp'\rangle \frac{Q_c}{W - E_c + i\epsilon} \langle pp'| G_{cb}[W], \quad (1)$$

where the indices  $a, b, c$  indicate pairs of baryons and the angle-averaged Pauli operator  $Q$  and energy  $E$  determine the propagation of intermediate baryon pairs. In a given nucleon-hyperon channels  $c = (NY)$  one has, for example,

$$E_{(NY)} = m_N + m_Y + \frac{k_N^2}{2m_N} + \frac{k_Y^2}{2m_Y} + U_N(k_N) + U_Y(k_Y). \quad (2)$$

The hyperon single-particle potentials within the continuous choice are given by

$$U_Y(k) = \sum_{N=n,p} U_Y^{(N)}(k) = \text{Re} \sum_{N=n,p} \sum_{k' < k_F^{(N)}} \langle kk' | G_{(NY)(NY)} [E_{(NY)}(k, k')] | kk' \rangle \quad (3)$$

and similar expressions of the form

$$\frac{B}{A} = \frac{\epsilon}{\rho_n + \rho_p + \rho_{\Sigma^-} + \rho_{\Lambda}},$$

$$\epsilon = \sum_{i=n,p,\Sigma^-, \Lambda} \int_0^{k_F^{(i)}} \frac{dk k^2}{\pi^2} \left( m_i + \frac{k^2}{2m_i} + \frac{1}{2} U_i(k) \right) = \epsilon_{NN} + \epsilon_{NY}$$

with

$$\epsilon_{NN} = \sum_{N=n,p} \int_0^{k_F^{(N)}} \frac{dk k^2}{\pi^2} \left( m_N + \frac{k^2}{2m_N} + \frac{1}{2} [U_N^{(n)}(k) + U_N^{(p)}(k)] \right),$$

$$\epsilon_{NY} = \sum_{Y=\Sigma^-, \Lambda} \int_0^{k_F^{(Y)}} \frac{dk k^2}{\pi^2} \left( m_Y + \frac{k^2}{2m_Y} \right) + \sum_{N=n,p} \int_0^{k_F^{(N)}} \frac{dk k^2}{\pi^2} [U_N^{(\Sigma^-)}(k) + U_N^{(\Lambda)}(k)]$$

$$= \sum_{Y=\Sigma^-, \Lambda} \int_0^{k_F^{(Y)}} \frac{dk k^2}{\pi^2} \left( m_Y + \frac{k^2}{2m_Y} + [U_Y^{(n)}(k) + U_Y^{(p)}(k)] \right).$$

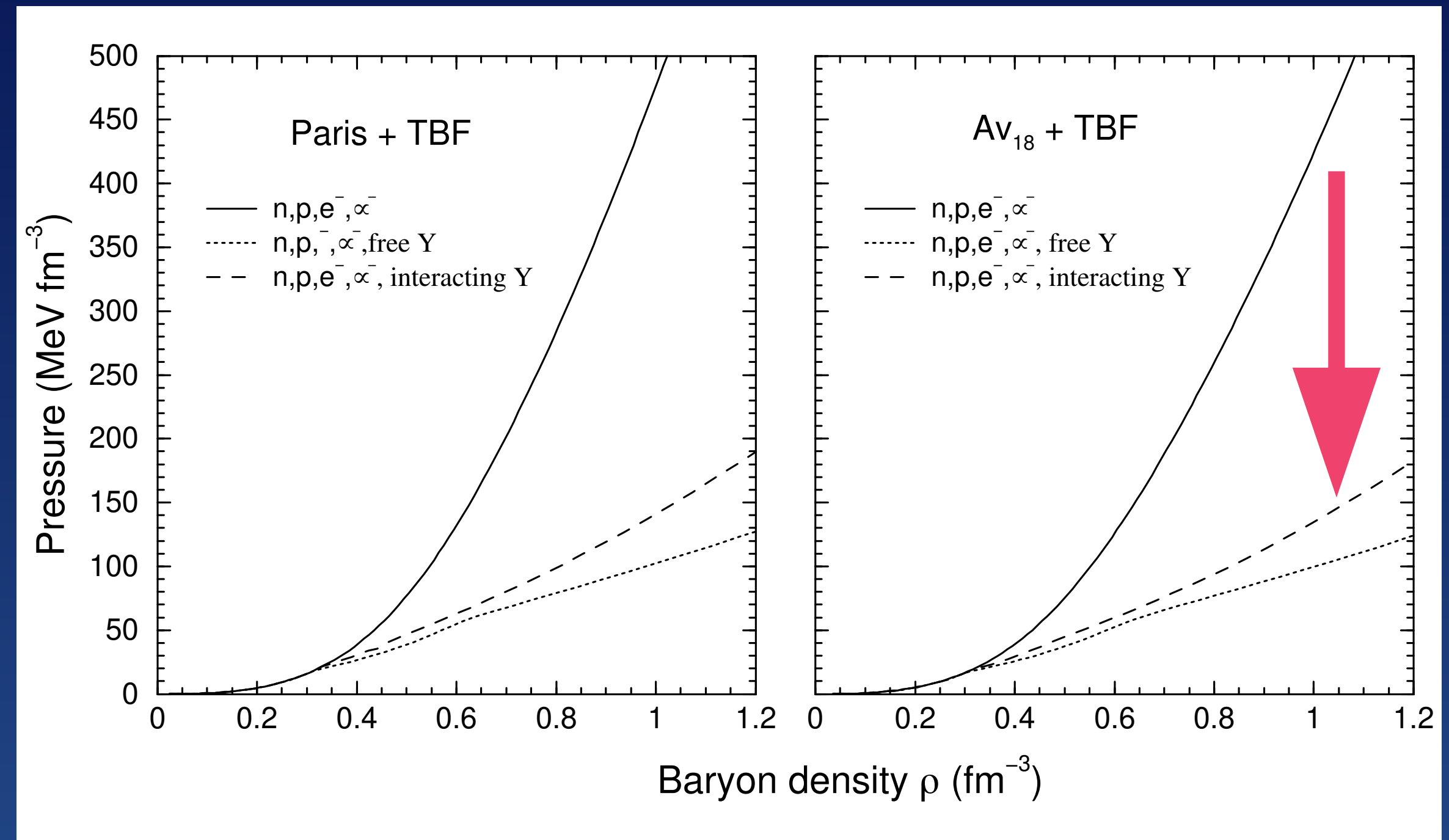
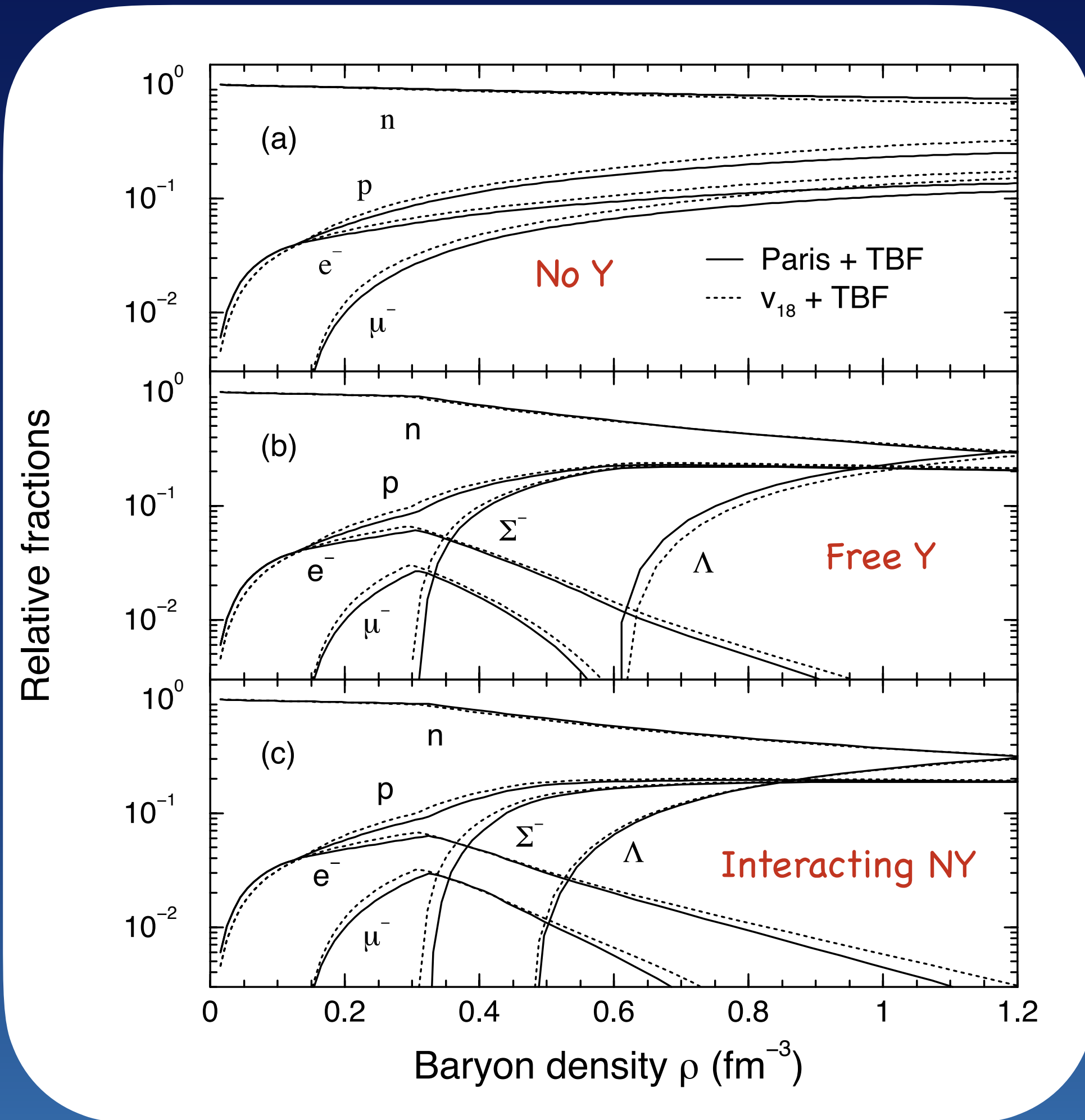
- BHF based on in-medium scattering G-matrix
- NN force and effective NNN interactions only input required

Technical difficulty :  
coupled channel calculation !

- Only NN and NY interactions are included. No YY potentials.
- The nucleons feel direct effects of the other nucleons and the hyperons.
- For the hyperons only nucleonic contributions are included.



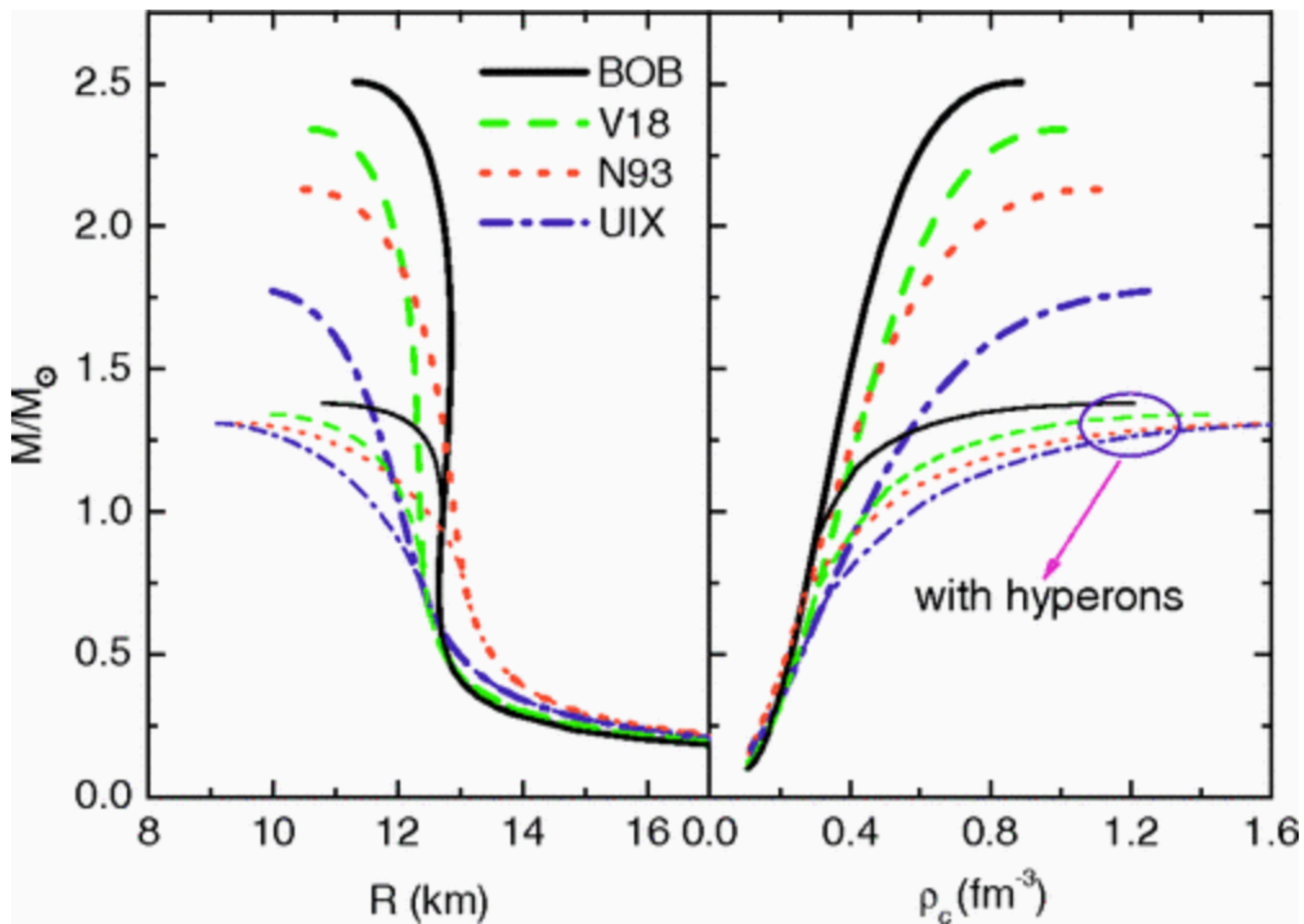
# Composition and EoS of hypernuclear matter



- Hyperon onset occurs at  $\rho \sim (2 - 3)\rho_0$
- Strong softening due to hyperon onset



# Mass-Radius relation with different **NN** interactions



BOB : Bonn B + microscopic TBF

V18 : Argonne  $v_{18}$  + microscopic TBF

N93 : Nijmegen + microscopic TBF

UIX : Argonne  $v_{18}$  + phenom. TBF (Urbana model)

- Only small effect required [ $\delta(B/A) \approx 1 \text{ MeV}$  at  $\rho_0$ ]
- TBF are model dependent, no final theory yet
- Use and compare microscopic and phenomenological TBF...
  - Microscopic TBF of [P. Grangé et al., PRC 40, 1040 \(1989\)](#): Exchange of  $\pi, \rho, \sigma, \omega$  via  $\Delta(1232), R(1440), NN$  Parameters compatible with two-nucleon potential (Paris,  $V_{18}, \dots$ )
  - Urbana IX phenomenological TBF: Only  $2\pi$ -TBF + phenomenological repulsion Fit saturation point

✓ NSC89 NY potential

✓ No (YY, YYY) interaction

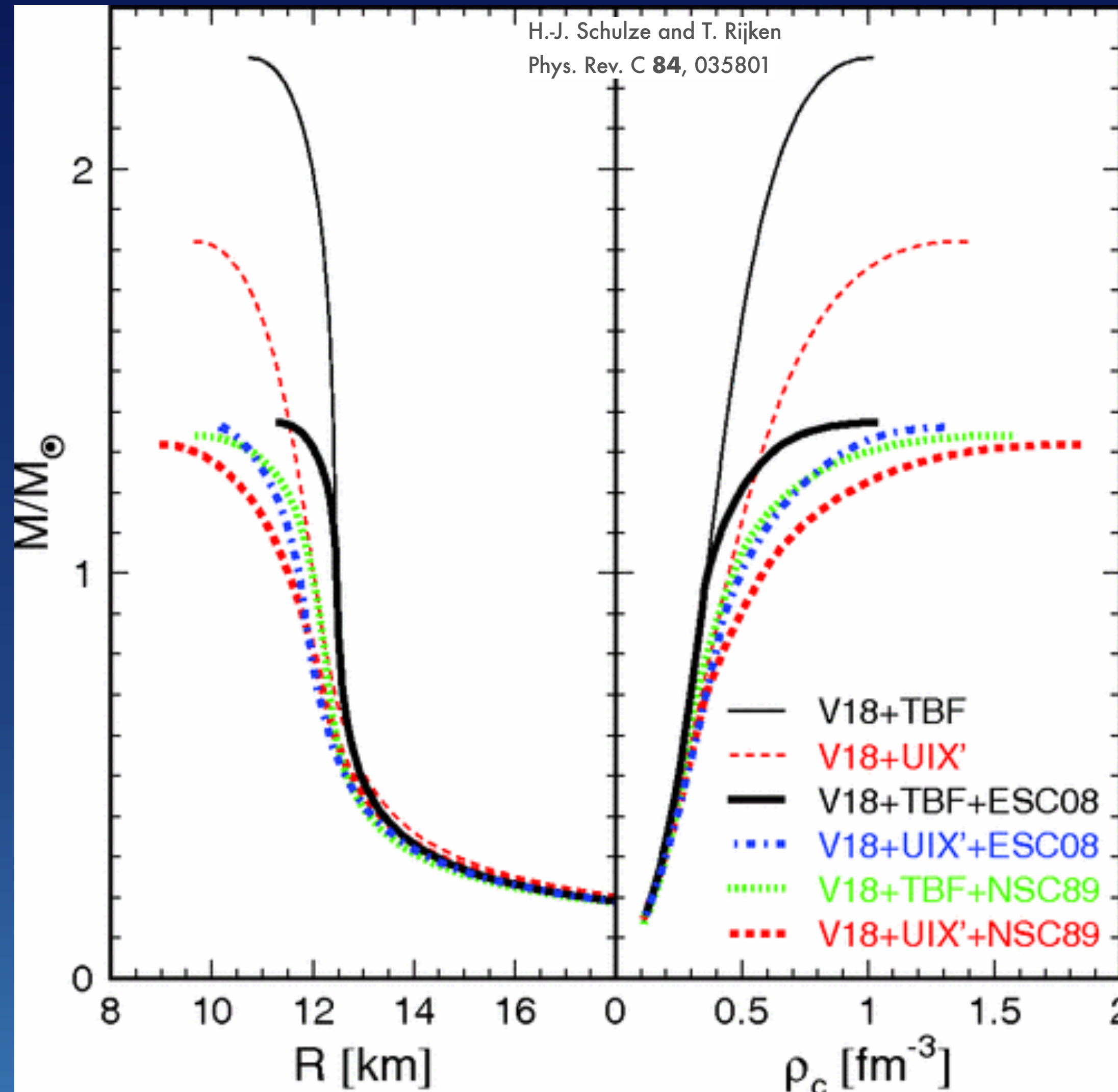
• Large variation of  $M_{\text{max}}$  with the NN interaction

• Softening due to hyperon appearance

• Stiffer EoS  $\longrightarrow$  earlier hyperon onset



# Mass-Radius relation with different *NY* potentials



- Nijmegen ESC08b YN potentials features a more repulsive  $\Sigma$ -N interaction, in contrast to the previously used NSC89, NSC97 YN potentials.
- Smaller  $\Sigma^-$  fraction expected in  $\beta$ -stable matter.
- Maximum mass independent of potentials !

$M_{\text{max}}$  too low ( $< 1.4 M_\odot$ ) !



# Possible effects of the $YY$ potentials

ESC08 model : inclusion of the ( $S = -2$ )  $\Lambda\Lambda$ ,  $N\Xi$ ,  $\Sigma\Lambda$ ,  $\Sigma\Sigma$  channels

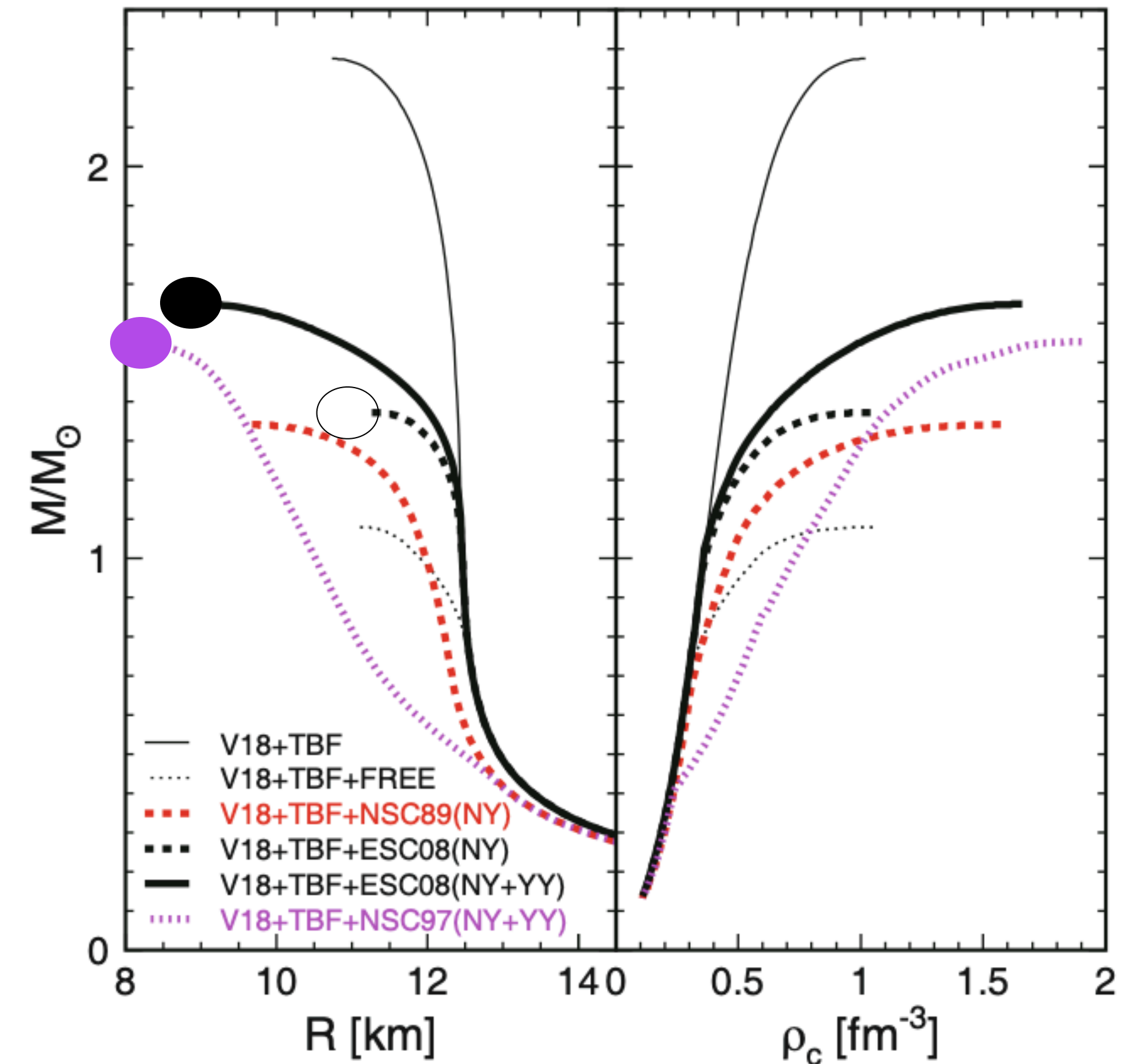
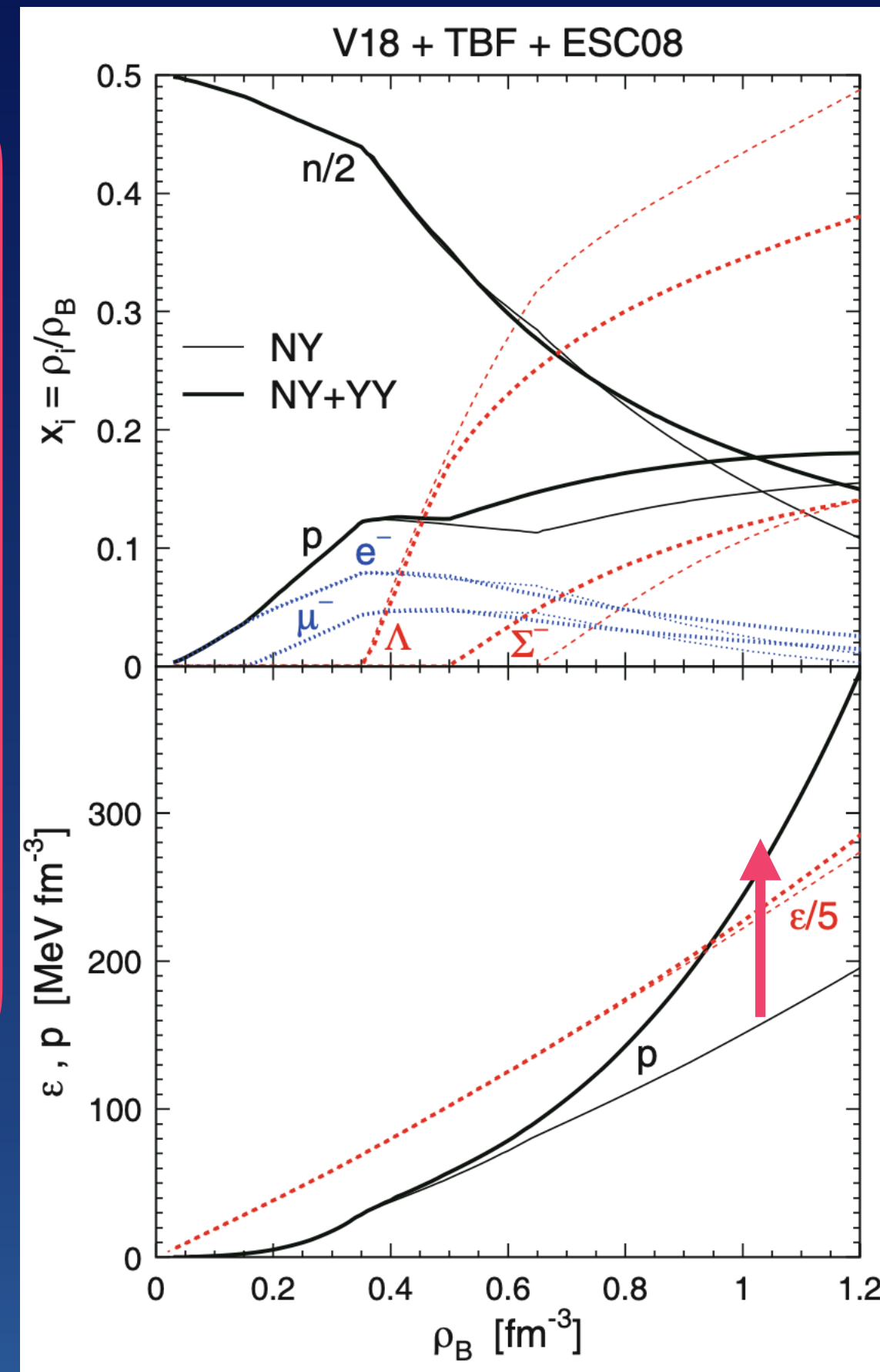
Th. A. Rijken & H. -J. Schulze, EPJA 52, 21 (2016)

•  $\Lambda\Lambda$ ,  $\Sigma\Sigma$ -repulsive

•  $\Lambda\Sigma$ -attractive

• Stiffer EoS

• Expected larger  $M_{\max}$




$M_{\max}$  increase to about  $1.7 M_\odot$

**Proof for quark matter inside NS ?**





# Hyperon puzzle

Hyperons  too soft EoS not compatible with measured NS masses.  
CAVEAT : the presence of hyperons in the NS core seems to be unavoidable !



## *Possible solutions of the hyperon puzzle*

- One excludes hyperons in the nuclear models by hand, thus ignoring experimental data from hypernuclei.
- One pushes up the critical density for the onset of hyperon formation in neutron star matter beyond the maximum density in neutron stars. Arbitrary !

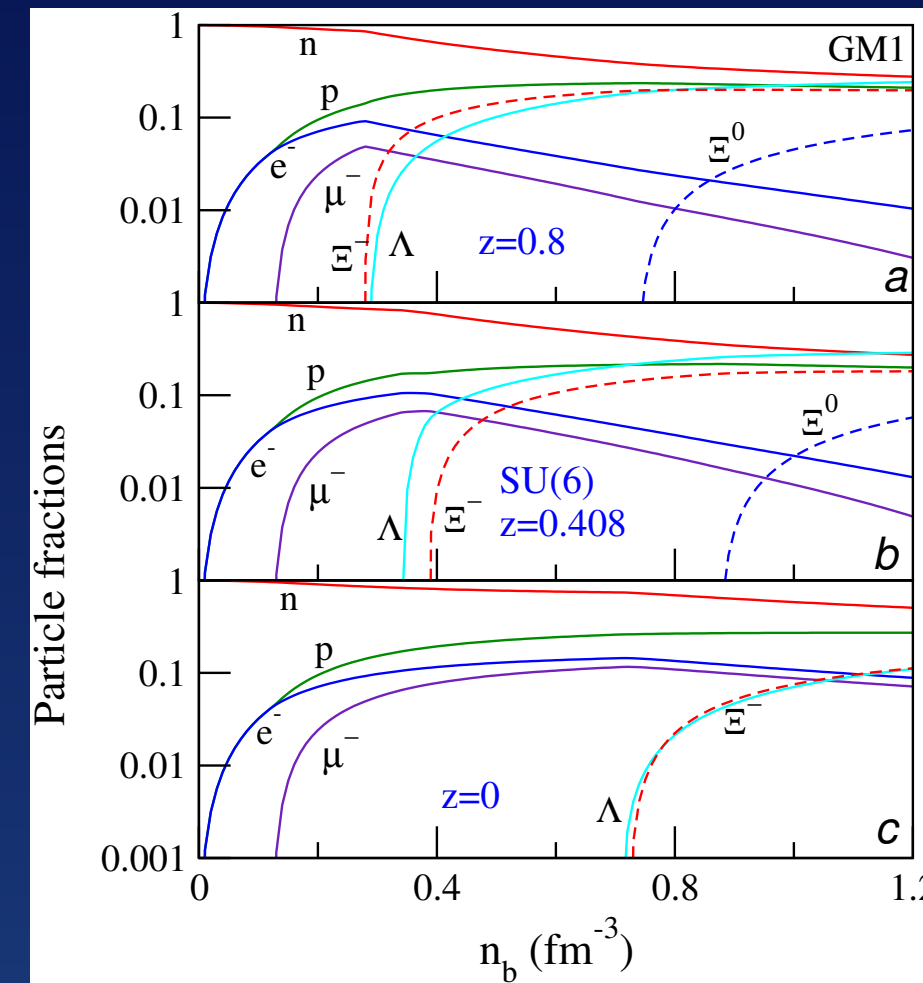


# Solution I: $YY$ and $NY$ vector meson repulsion

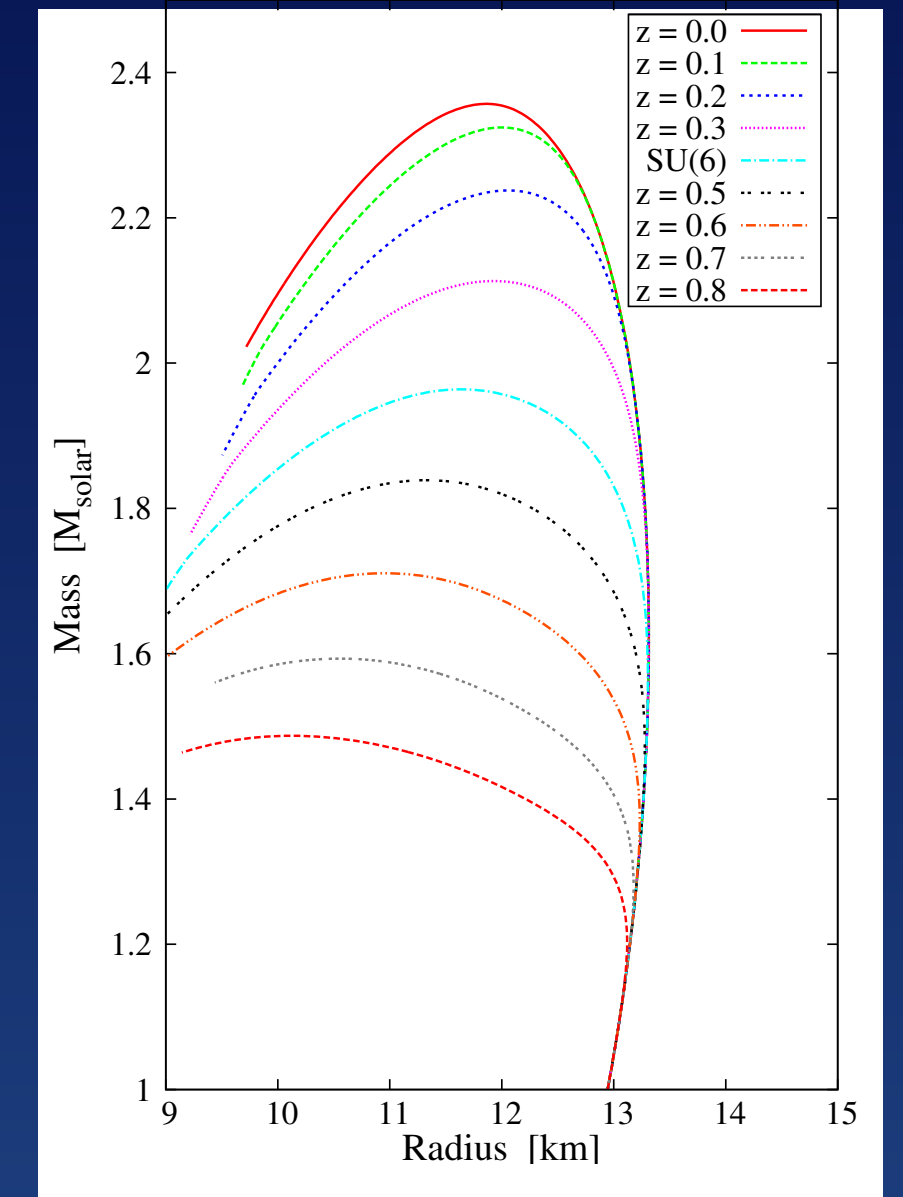
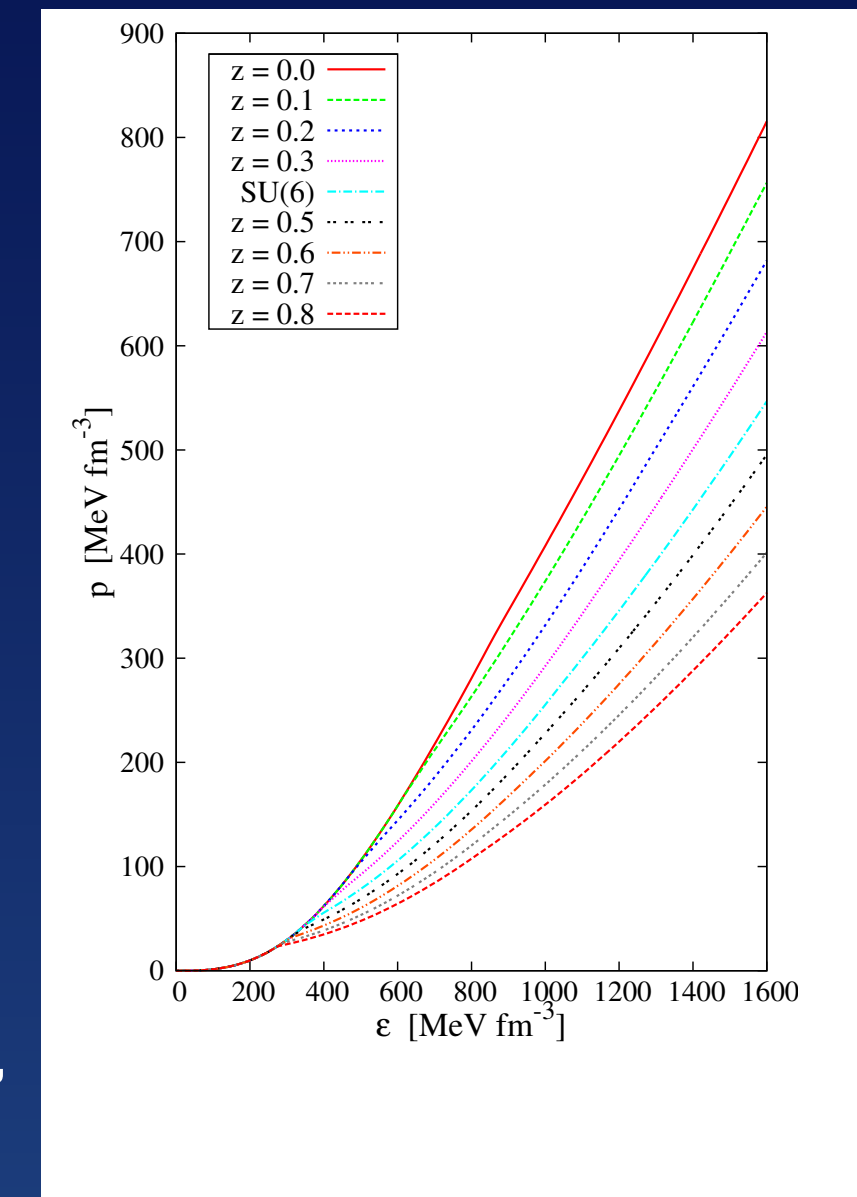
## Mainly studied in RMF models

- Baryon-baryon interaction mediated by the exchange of scalar ( $\sigma$ ), vector ( $\omega$ ) and isovector ( $\rho$ ) mesons.
- Hyperon-hyperon interaction incorporated through additional strange scalar ( $\sigma^*$ ) and vector ( $\phi$ ) mesons.
- Shift of the hyperon onset to higher density.

Bednarek et al '12; Oertel et al '15; Maslov et al '15.



Weissenborn, Chatterjee, Schaffner-Bielich,  
Phys. Rev. C 85, 065802 (2012)



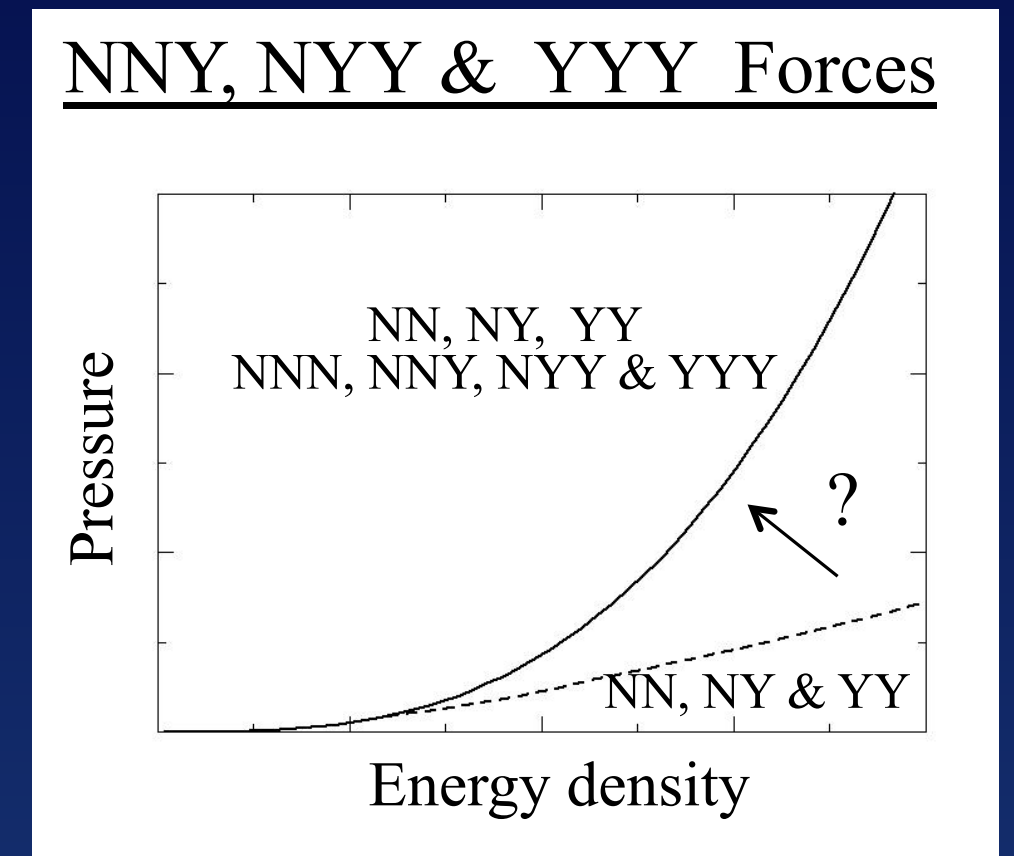
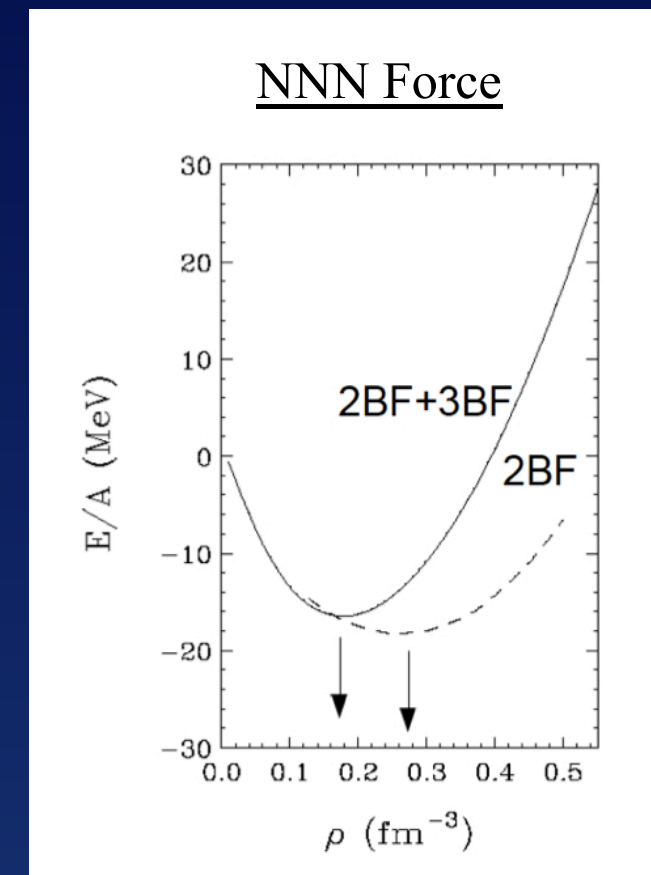
Some of these models are able to reconcile the presence of hyperons with the  $2M_\odot$  limit, they contain several free parameters which very often are arbitrarily chosen.

Validity of these models is questionable.

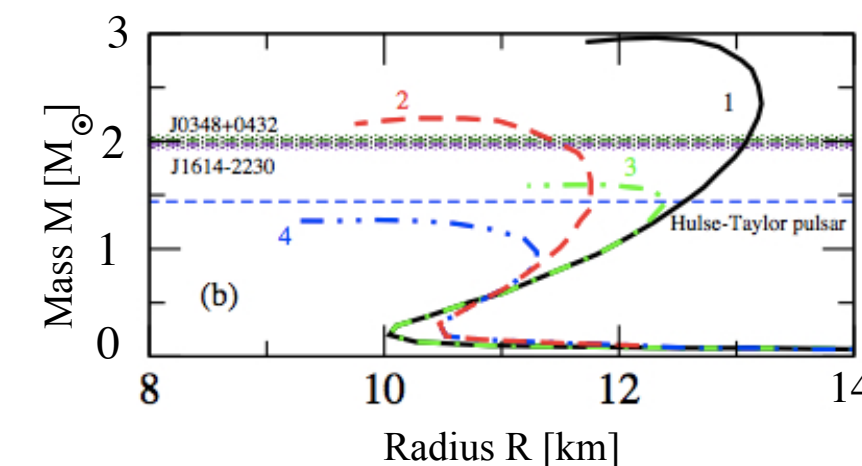


# Solution II : Hyperonic TBF

- Importance of TBF in Nuclear Physics
- Correct saturation point in microscopic approaches
- Can hyperonic TBF solve the puzzle ?
- No general consensus regarding the results.
- Hyperonic 3-body forces in  $\chi$ EFT might solve the hyperon puzzle ??



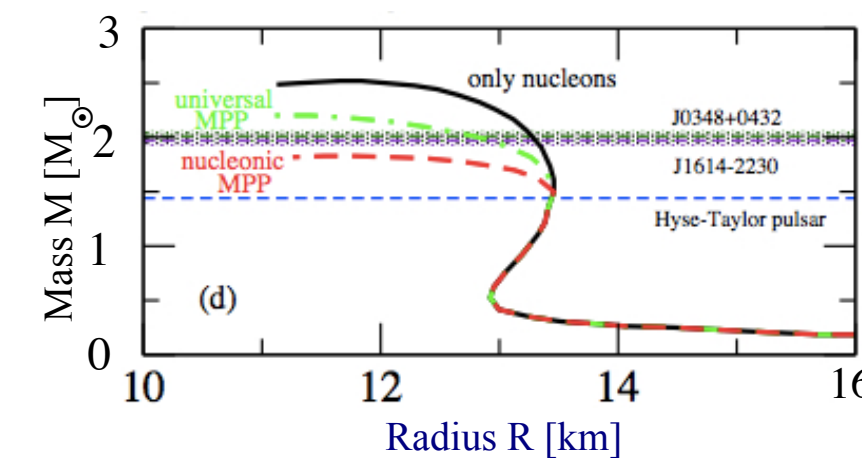
The results are contradictory



I. V. et al. (2011)

BHF with NN+YN+phenomenological YTBF. Different strength of YTBF including the case of universal TBF

$$1.27 < M_{\max} < 1.6 M_{\odot}$$



Yamamoto et al. (2015)

BHF with NN+YN+universal repulsive TBF (multipomeron exchange mechanism)

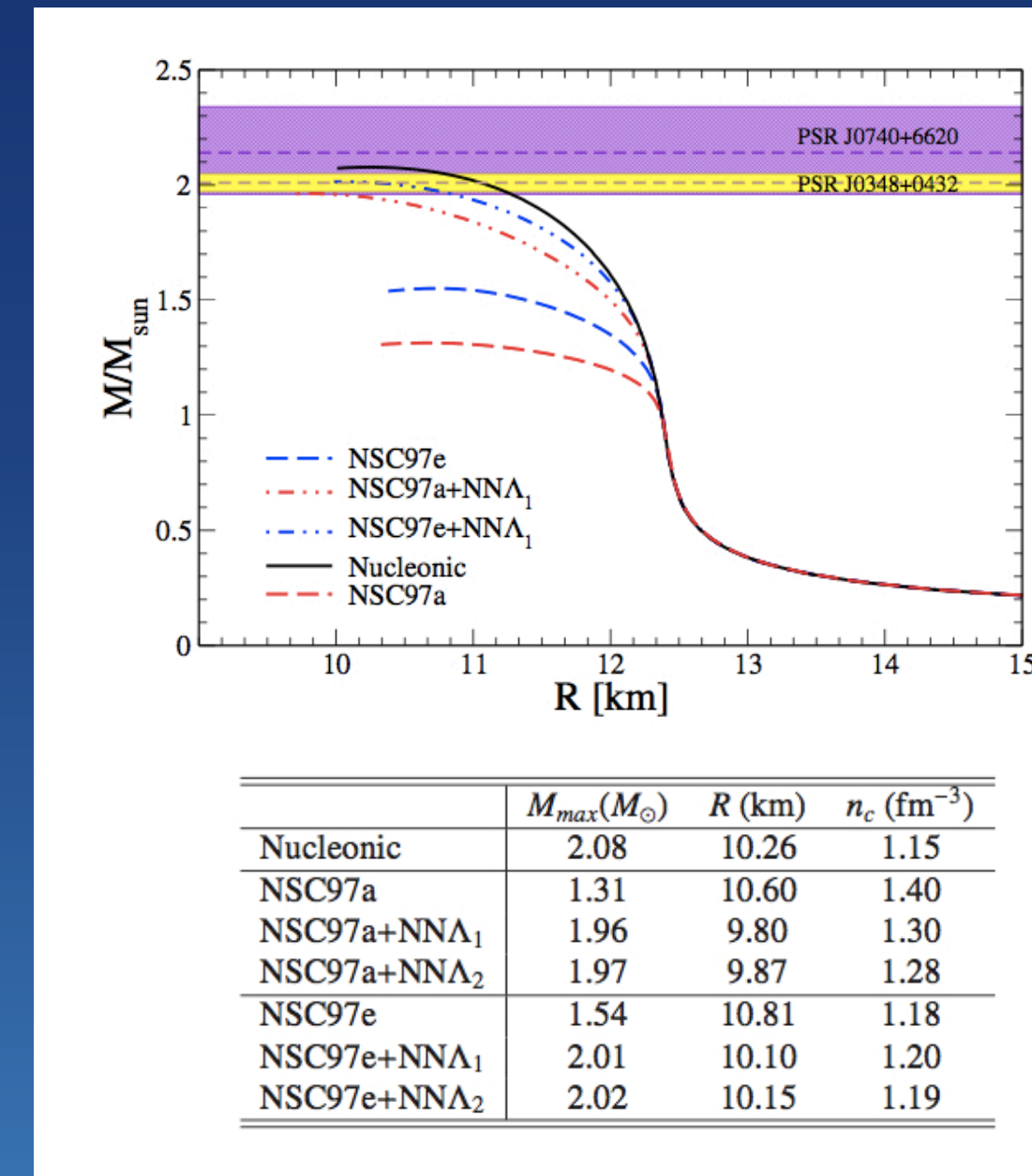
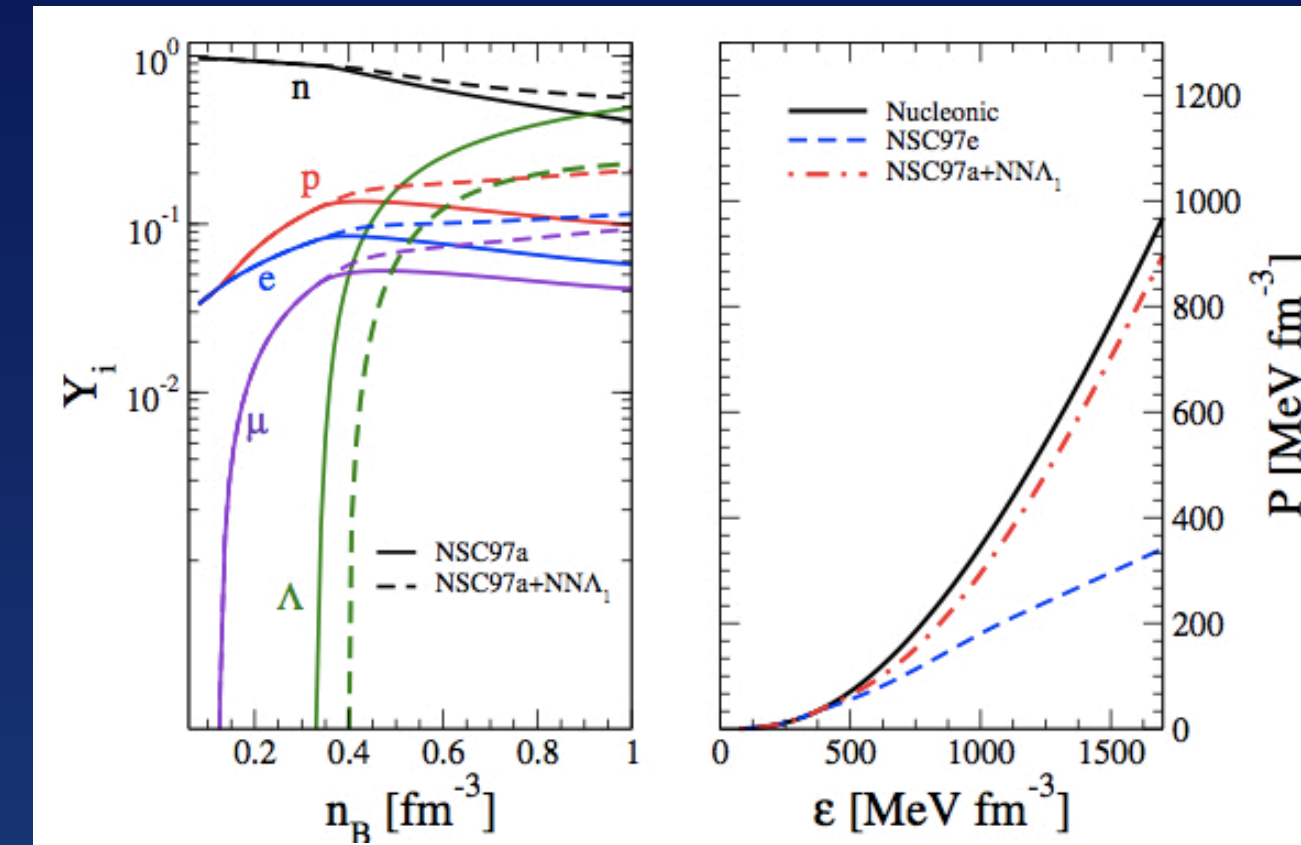
$$M_{\max} > 2 M_{\odot}$$



# Solution III : the $NN\Lambda$ chiral forces

- Preliminary exploratory work by Logoteta, Vidana & Bombaci on the role of  $NN\Lambda$  interaction. Eur. Phys. J. A (2019) 55:207.
- The chiral  $NN\Lambda$  shifts the onset of  $\Lambda$  to slightly larger densities and largely reduces the concentration, thus stiffening the EoS.
- Maximum mass “almost” compatible with the  $2 M_{\odot}$  limit, but other hyperons should be included.

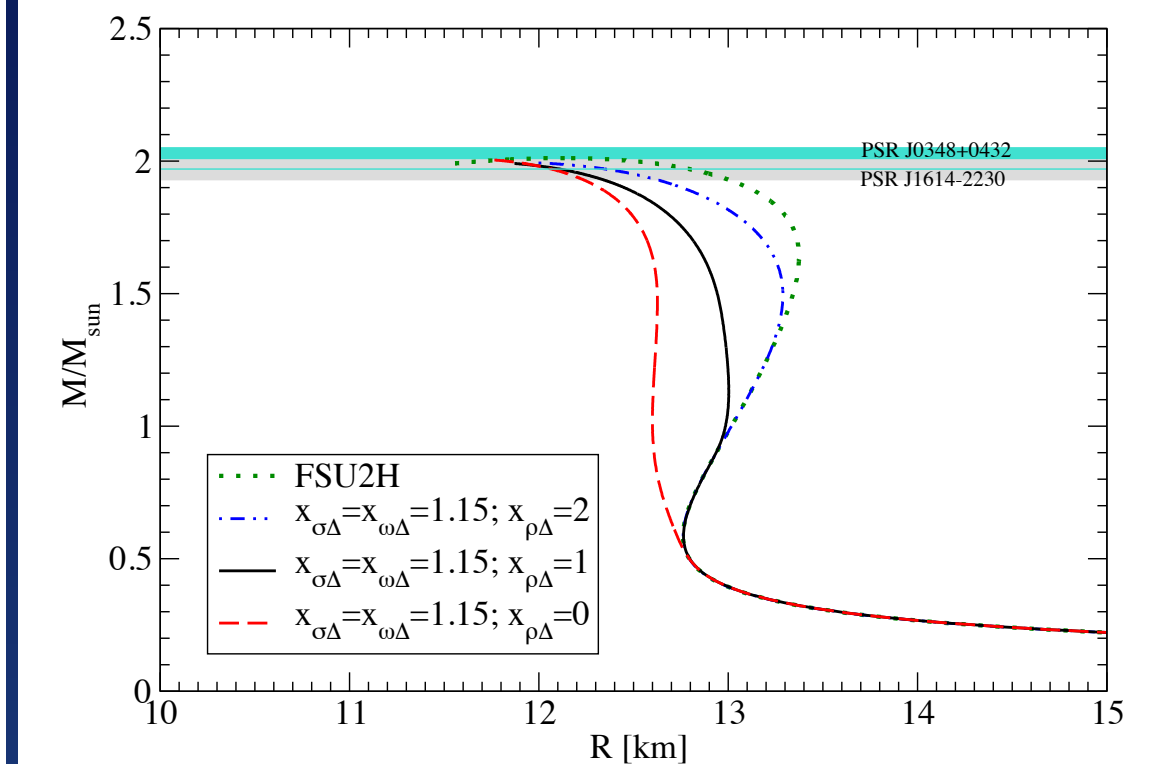
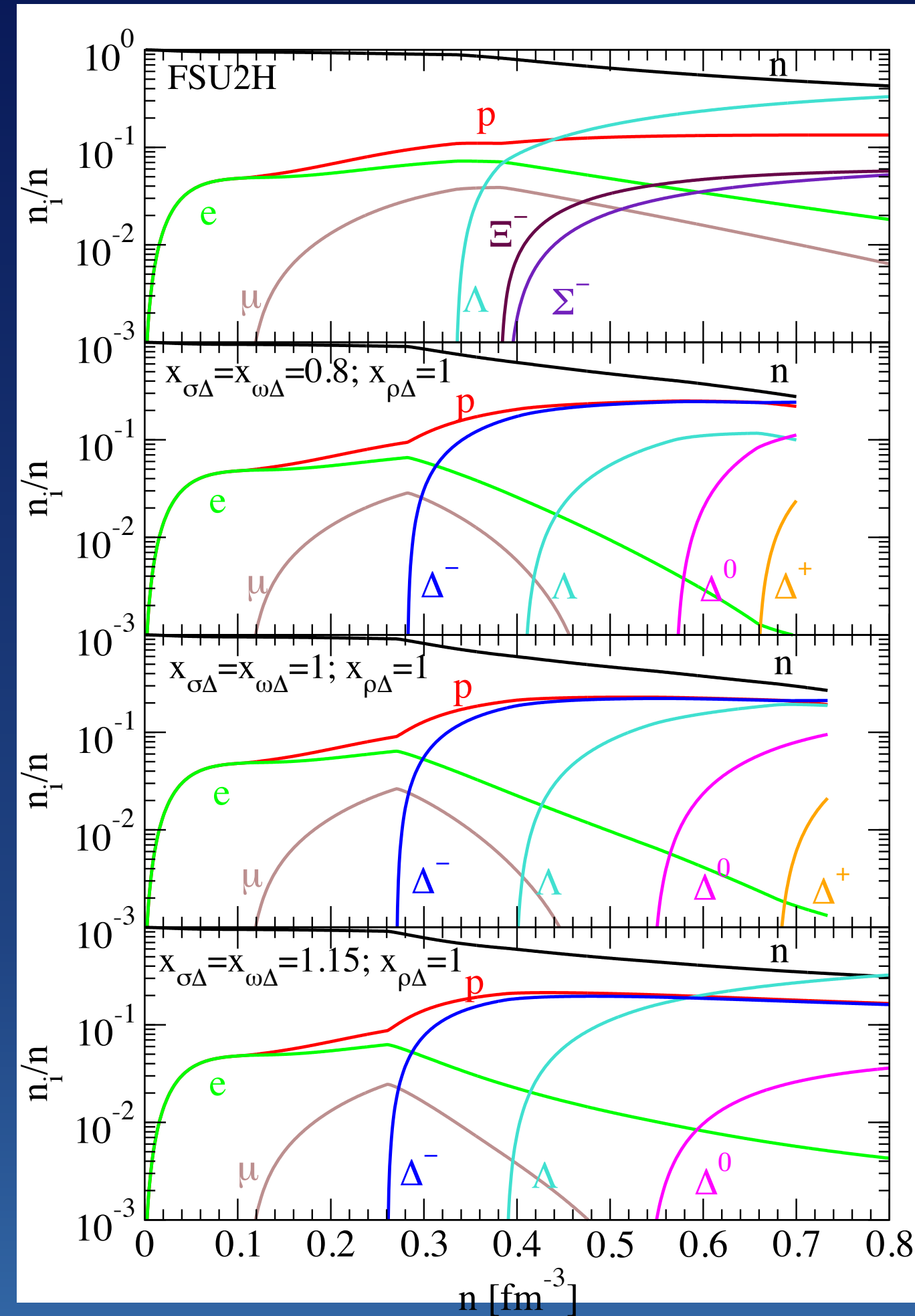
Hyperon puzzle cannot be considered as solved !





# Solution IV : Appearance of $\Delta$ -isobars

- Mainly in RMF models.
- Couplings of the  $\Delta$  isobars with the meson fields  $x_{\sigma\Delta}$ ,  $x_{\omega\Delta}$ ,  $x_{\rho\Delta}$  are poorly constrained due to the limited experimental data.
- Imposing an attractive  $\Delta$ -nucleus potential of a few tenths of MeV thus containing the range.
- Push of the Y onset to higher densities, even disappearing !
- The 2Mo limit might be reached for some choices of the meson fields.



Ribes, Ramos, Tolos,  
Gonzalez-Boquera,  
Centelles, ApJ (2019)

Drago et al '14 '15, Jie Li et al '19...

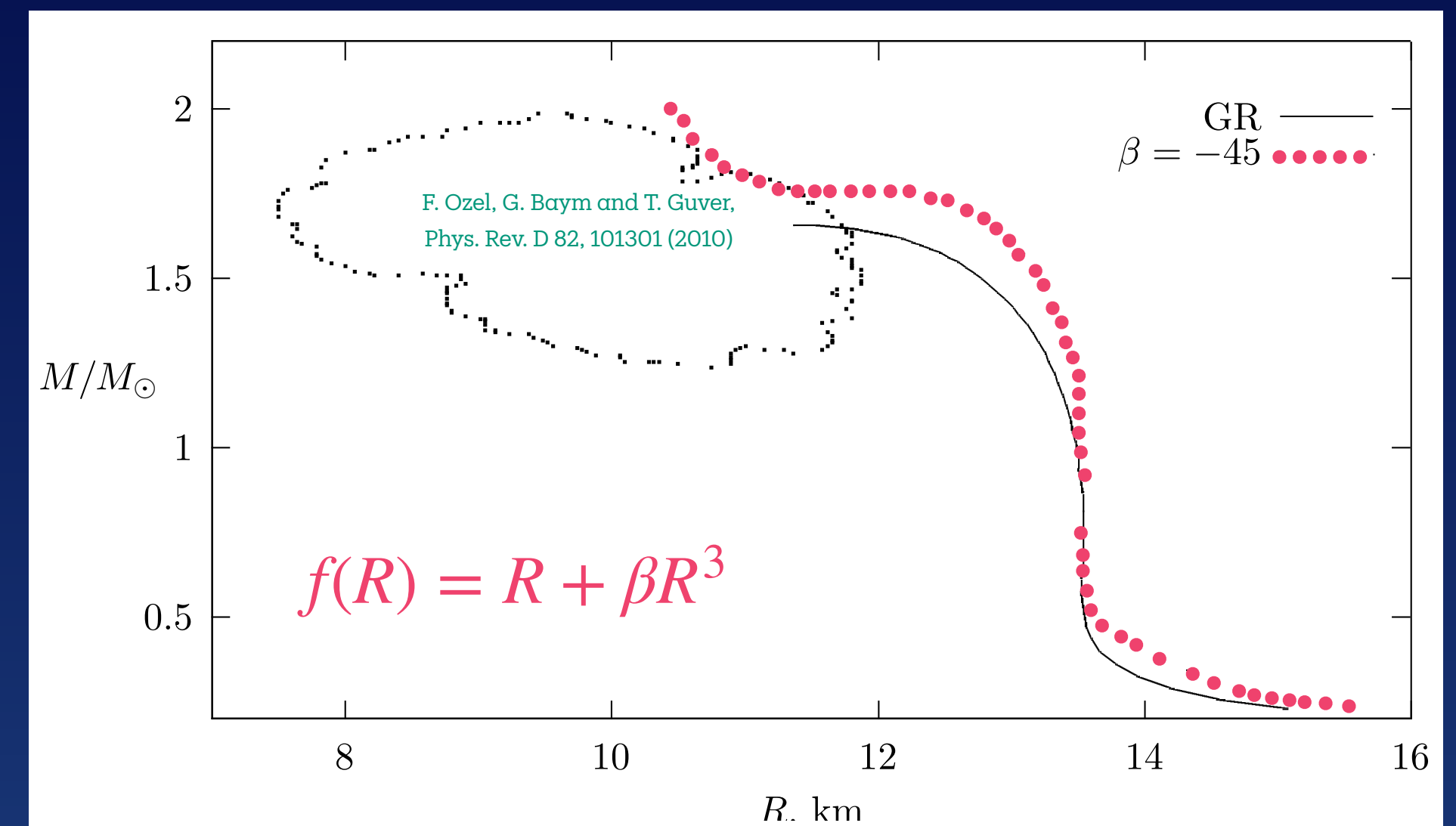


# Unconventional Solutions

## Solution V : Modified gravity

- Modified TOV eqs. in  $f(R)$  gravity.
- It depends on the chosen  $f(R)$ .

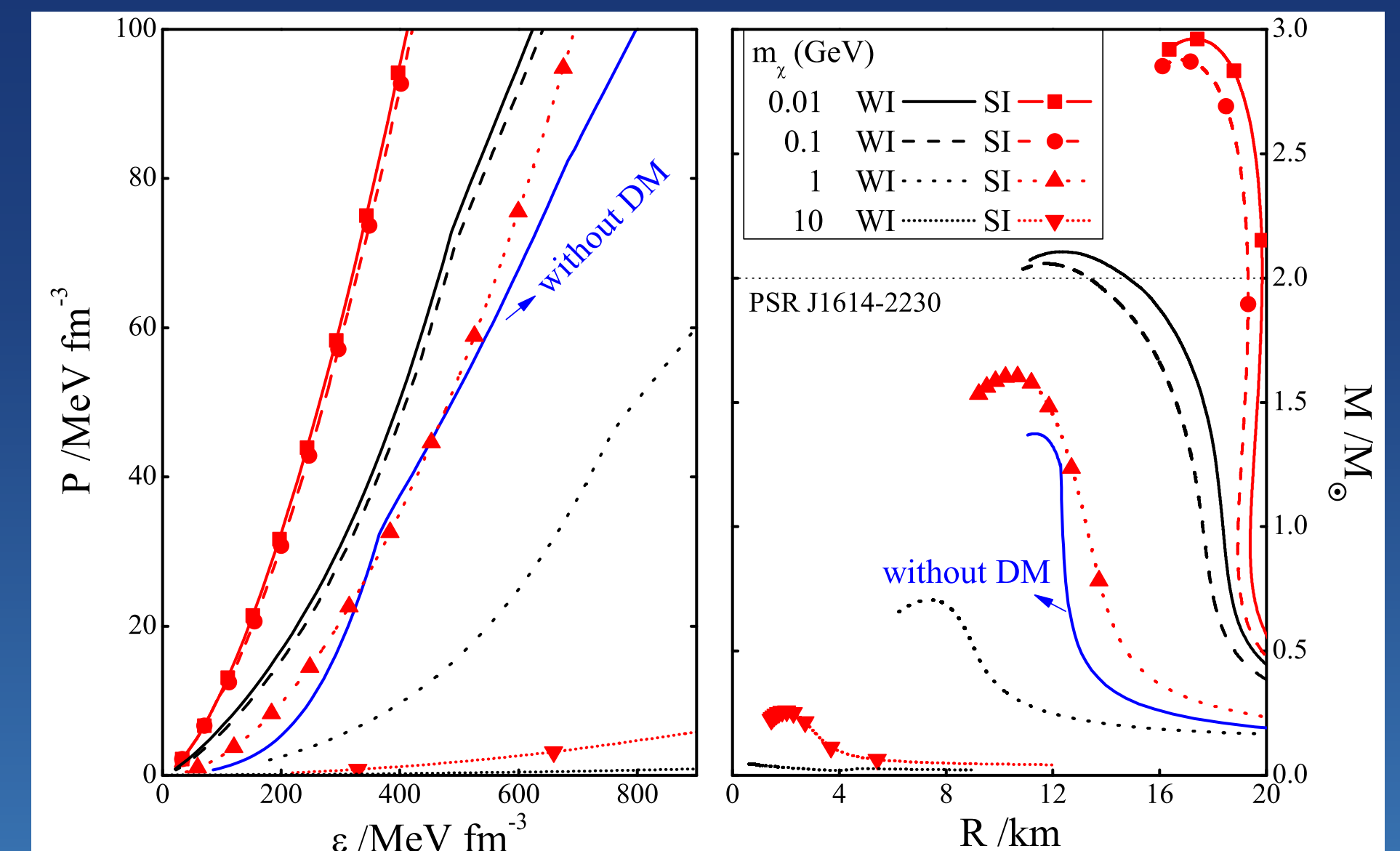
Astashenok, Capozziello, Odintsov '10, '11, '14 ...



## Solution VI : DM-admixed NS

- DM particle nature and mass
- DM interacting strength
- DM fraction in NS interior
- Large parameter space to explore
- Possible solution of the hyperon puzzle but strong parameter dependence.

Li, Huang, Xu, Astroparticle Physics 37 (2012) 70–74



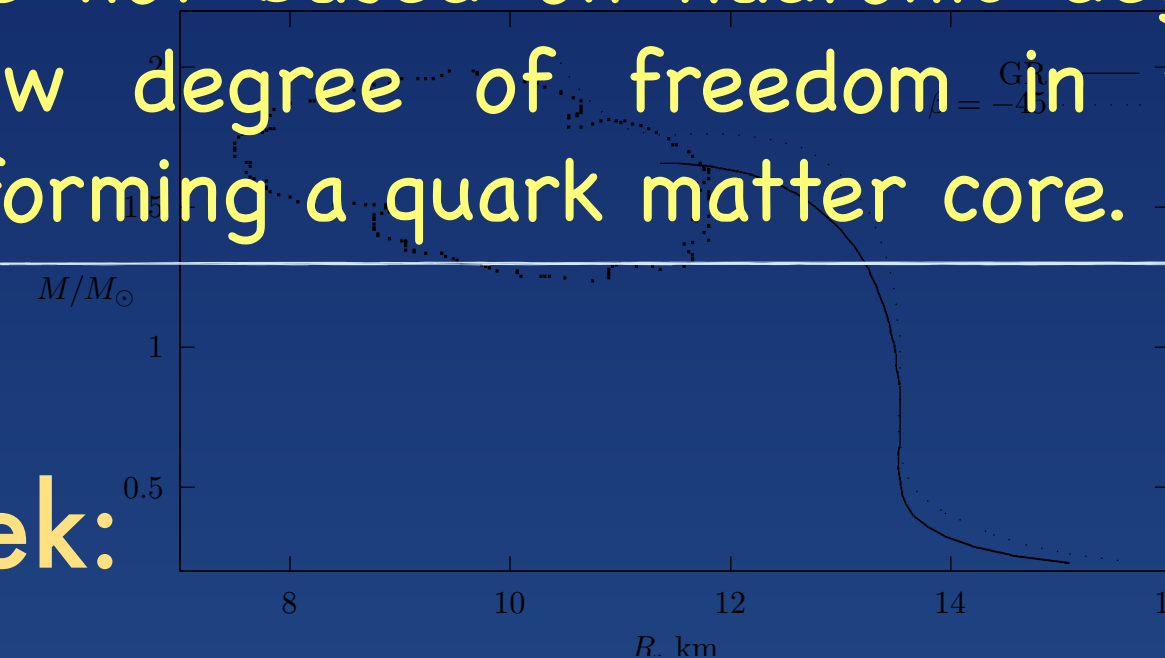


# Solution VII : Quark matter core

Ribes et al '19

- A particular striking solution : Hyperons appear but before they can destabilize the neutron star a new phase appears at high density with a stiff EOS supporting a  $2M_{\odot}$  compact star. That new phase would be not based on hadronic degrees of freedom, nucleons, and hyperons, but on a new degree of freedom in the form of the constituents of hadrons, that is, quarks, forming a quark matter core.

As better expressed by F. Wilczek:



*“The behaviour of QCD at large net baryon density (and low temperature) is also of obvious interest. It answers yet another childlike question : What will happen when you keep squeezing things harder and harder ?”*

*(Wilczek, Phys. Today, August 2000.)*



# Conclusions

- On the  $NN + NNN + NY$  level, the prediction of very low NS maximum masses is rather robust.
- Reliable  $YY$ ,  $YNN$ ,  $YYN$ ,  $YYY$  forces are not available (probably in the future ?).
- However, any single less repulsive channel will keep the maximum mass low.
- Only simultaneous repulsion in all relevant  $YY$ ,  $YNN$ ,... channels could substantially increase the maximum mass.

Need quark matter to reach higher masses of hybrid stars !

A big theoretical challenge for the future.