EMMI Physics Day
July 22nd, 2025

Deutsche Forschungsgemeinschaft
Research Unit FOR 2783

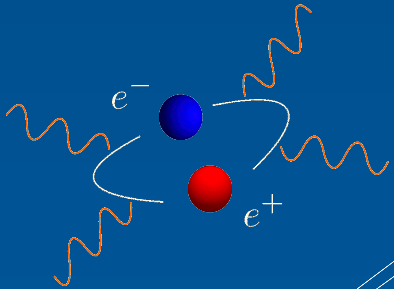
FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

HI JENA
Helmholtz Institute Jena

Towards a Vacuum Birefringence Experiment at the European X-Ray Free-Electron Laser

Felix Karbstein

Helmholtz Institute Jena &
Theoretisch-Physikalisches Institut, FSU Jena



Contents

1 Introduction

Classical electrodynamics (ED): $(\bar{F}^{\mu\nu} = \partial^\mu \bar{A}^\nu - \partial^\nu \bar{A}^\mu;\ \bar{F}^{0i} = E_i,\ \bar{F}^{ij} = \epsilon_{ijk} B_k)$

$\rightarrow$ accurate description of macroscopic electromagnetic fields $\bar{F}^{\mu\nu}$ in laboratory

$$\mathcal{L}_{\rm ED} = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \bar{J}_\mu \bar{A}^\mu$$

[Maxwell: Philos. Trans. R. Soc. **155** (1865)]

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$$\mathcal{L} = \mathcal{L}(\bar{A}^\mu, \bar{F}^{\mu\nu}) \xrightarrow{\text{EOM}} \frac{\partial \mathcal{L}}{\partial \bar{A}_\nu} - 2\partial_\mu \frac{\partial \mathcal{L}}{\partial \bar{F}_{\mu\nu}} = 0$$

$\partial_\mu \bar{F}^{\mu\nu} = -\bar{J}^\nu \quad \simeq$ inhomogeneous Maxwell equations

(homogeneous ones $\partial_\mu {}^\star\bar{F}^{\mu\nu} = 0$ from Bianchi identity of $\bar{F}^{\mu\nu}$)

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$$\downarrow$$

$\partial_\mu \bar{F}^{\mu\nu} = -\bar{J}^\nu \quad \simeq$ inhomogeneous Maxwell equations

$\rightarrow$ linear equations of motion (EOM) $\rightarrow$ superposition principle holds in vacuum

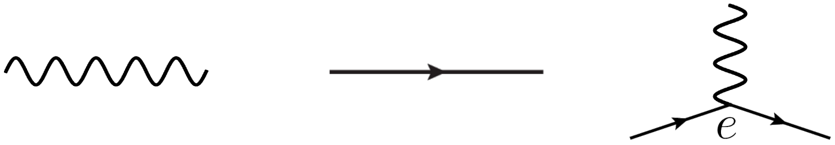
$\leftrightarrow$ our world is quantum $\rightarrow$ there should be corrections (depending on $\hbar$)

1 Introduction

Quantum electrodynamics (QED): $(\hbar = c = 1$; quantum fields A^μ, $\bar\psi$, $\psi)$

$\hat{=}$ relativistic quantum theory of light and matter, quantum field theory

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi + e\bar\psi\gamma_\mu\psi A^\mu$$



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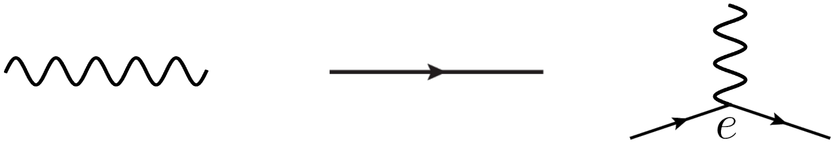
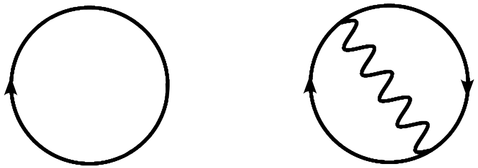


$\rightarrow$ notion of the "quantum vacuum" $\hat{=}$ diagrams without external lines



$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

In this talk:

- What is the true theory of (non-quantized) $\bar{F}^{\mu\nu}$ in the quantum vacuum?

  $\hat{=}$ Heisenberg-Euler (HE) effective theory $\rightarrow$ **Sec. 2**

- How can deviations from classical ED be probed with strong laser fields?

  $\rightarrow$ Towards a Vacuum Birefringence Experiment $\rightarrow$ **Sec. 3**

Recent review: [Fedotov, Ilderton, FK, King, Seipt, Taya, Torgrimsson: Phys. Rep. **1010** (2023)]

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2.1 Heisenberg-Euler effective theory

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Preliminary considerations:

- $\mathcal{L}_{\rm HE} \sim {\rm mass}^4\,, \qquad \mathcal{L}_{\rm HE} \xrightarrow{\hbar\to0} \mathcal{L}_{\rm ED}$
- m is the only dimensionful scale inherited from QED
- gauge invariance: $\mathcal{L}_{\rm HE}(\bar F^{\mu\nu},\partial^\rho \bar F^{\mu\nu},\ldots) \;\sim\; \bar F^2 \times {\rm function}\Bigl(\frac{e\bar F}{m^2},\frac{\partial}{m}\frac{e\bar F}{m^2},\ldots\Bigr)$

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$$\frac{eE_i}{m^2} \qquad\qquad , \qquad \frac{\partial_{\rm x}}{m}$$

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$$\frac{\hbar}{c^3}\frac{eE_i}{m^2} = \frac{E_i}{E_{\mathrm{S}}} = \frac{E_i}{1.3\times 10^{18}\,\mathrm{V/m}}\,, \qquad \frac{\hbar}{c}\frac{\partial_{\mathrm{x}}}{m} = \lambda_{\mathrm{C}}\,\partial_{\mathrm{x}} \sim \frac{3.8\times 10^{-7}\,\mu\mathrm{m}}{\Delta\mathrm{x}}$$

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$\rightarrow$ for "weak & slowly varying fields": $\mathcal{L}_{\rm HE} \rightarrow \mathcal{L}_{\rm ED}$

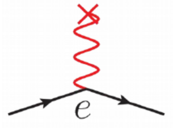
How does $\mathcal{L}_{\rm HE}$ arise (as low-energy EFT) from QED? [Gies, FK: JHEP **3** (2017)]

How does $\mathcal{L}_{\mathrm{HE}}$ arise (as low-energy EFT) from QED?

[Gies, FK: JHEP **3** (2017)],
[FK: 10.22032/dbt.59618 (2024)]

$$\rightarrow \quad \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}_{\mathrm{HE}}} = \Big\langle \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x(-\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \bar{j}_\mu \bar{A}^\mu)} \Big\rangle_{\mathrm{QED}}$$

$$\bar{j}_\mu = e\bar{\psi}\gamma_\mu\psi$$ "tree-level current"



$$\text{with} \quad \langle \ldots \rangle_{\mathrm{QED}} = \frac{\int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi \int \mathcal{D}A \, \ldots \, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}_{\mathrm{QED}}}}{\int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi \int \mathcal{D}A \, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}_{\mathrm{QED}}}}$$

2 Quantum Vacuum + Electromagnetic Fields

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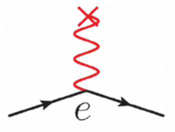

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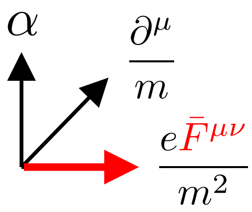
$\mathcal{L}_{\mathrm{HE}}$
- depends on three dimensionless parameters:
- is even in $\bar{F}^{\mu\nu}$ and ∂^μ.



2 Quantum Vacuum + Electromagnetic Fields

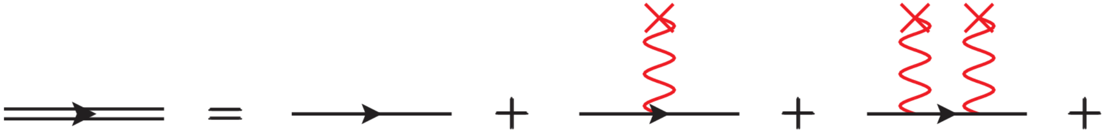
Diagrammatic representation: $\left(\mathcal{L}_{\mathrm{HE}}^{\ell\text{-loop}} \sim \alpha^{\ell-1}\right)$

$$\mathcal{L}_{\mathrm{HE}} = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} +$$

$$+ \cdots$$

$$=: -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \mathcal{L}_{\mathrm{int}}$$

where



$$+ \cdots$$

$\rightarrow$ one-particle irreducible (1PI) and reducible (1PR) contributions

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Diagrammatic representation: $\qquad\qquad (\mathcal{L}_{\rm HE}^{\ell\text{-loop}} \sim \alpha^{\ell-1})$

at $\mathcal{O}(\partial^0)$: [Heisenberg, Euler: Z. Phys. **98** (1936)], [Weisskopf: Mat.-fys. Medd. **14** (1936)], [Schwinger: Phys. Rev. **82** (1951)]

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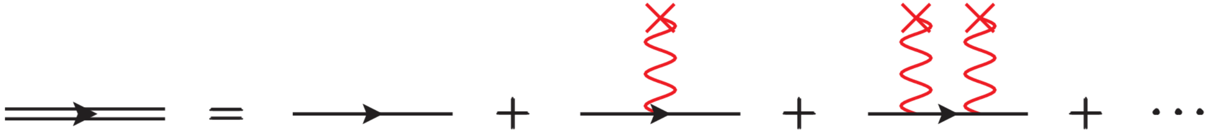

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[Heisenberg, Euler: Z. Phys. **98** (1936)]

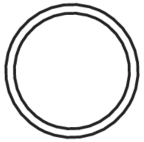
Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}. \quad = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad +$$



$$\left(\begin{array}{l}\mathfrak{E}, \mathfrak{B} \quad \text{Kraft auf das Elektron.} \\ |\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{„137“}\,\frac{e}{(e^2/mc^2)^2} = \text{„Kritische Feldstärke“.}\end{array}\right)$$

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

$$\vec{E}^2 - \vec{B}^2 = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}\,,$$

$$\vec{E}\cdot\vec{B} = -\tfrac{1}{4}F_{\mu\nu}{}^{\star}F^{\mu\nu}$$

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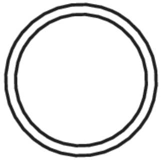
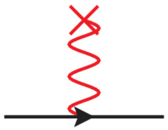
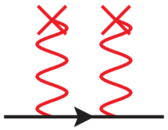
[Gies, FK: JHEP **3** (2017)]

$\mathcal{L}_{\mathrm{HE}} = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} +$  $+$  $+$  $+ \cdots$

[Ritus: JETP **42** (1975)]

$=: -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \mathcal{L}_{\mathrm{int}}$

where  $=$  $+$  $+$  $+ \cdots$

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Recent review: [Fedotov, Ilderton, FK, King, Seipt, Taya, Torgrimsson: Phys. Rep. **1010** (2023)]

3.1 Vacuum emission picture

[FK, Shaisultanov: Phys. Rev. D **91** (2015)]

$\rightarrow$ applied macroscopic electromagnetic fields $\bar{F}^{\mu\nu}$ induce signals

- $(0 \rightarrow 1)$ "vacuum emission" amplitude: $\mathcal{S}_p(\vec{k}) = \big\langle \gamma_p(\vec{k}) \big| \, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{int}}(\bar{F}+f)} \big| 0 \big\rangle$

- differential signal photon number: $\mathrm{d}^3 N_p = \dfrac{\mathrm{d}^3 k}{(2\pi)^3} \big| \mathcal{S}_p(\vec{k}) \big|^2$

3 Towards a Vacuum Birefringence Experiment

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- differential signal photon number: $$\mathrm{d}^3 N_p = \frac{\mathrm{d}^3 k}{(2\pi)^3} \left| \mathcal{S}_p(\vec{k}) \right|^2$$

inhomogeneous strong field $\longrightarrow$ signal photons

- energy $|\vec{k}|$,
- emission direction $\vec{k}/|\vec{k}|$,
- polarization p.

Leading low-energy quantum vacuum nonlinearity:

[Euler, Kockel: Naturwiss. **23** (1935)], [Euler: Ann. Phys. **418** (1936)]

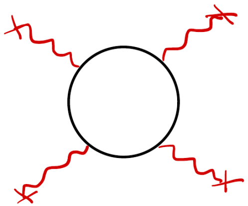
$$\mathcal{L}_{\mathrm{int}} \simeq \frac{m^4}{5760\pi^2}\left(\frac{e}{m^2}\right)^4 \Big[\boldsymbol{a}\,(\bar{F}_{\mu\nu}\bar{F}^{\mu\nu})^2 + \boldsymbol{b}\,(\bar{F}_{\mu\nu}{}^{\star}\bar{F}^{\mu\nu})^2\Big]$$

$\sim (\vec{B}^2 - \vec{E}^2)^2$ $\qquad$ $\sim (\vec{B}\cdot\vec{E})^2$

$\rightarrow$ QED, up to two loops:

$$\boldsymbol{a} = 4\Big(1 + \qquad\qquad\Big)$$

$$\boldsymbol{b} = 7\Big(1 + \qquad\qquad\Big)$$



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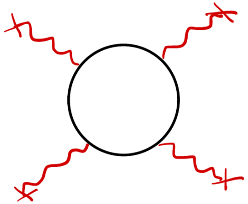
$\sim (\vec{B}^2 - \vec{E}^2)^2$ $\qquad \sim (\vec{B}\cdot\vec{E})^2$

$\rightarrow$ QED, up to two loops:

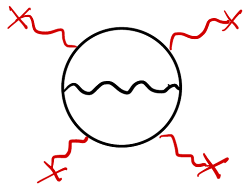
$$\boldsymbol{a} = 4\Big(1 + \frac{40}{9}\frac{\alpha}{\pi} + \ldots\Big)$$

$$\boldsymbol{b} = 7\Big(1 + \frac{1315}{252}\frac{\alpha}{\pi} + \ldots\Big)$$

corrections on 1% level



[Euler: Ann. Phys. **418** (1936)]

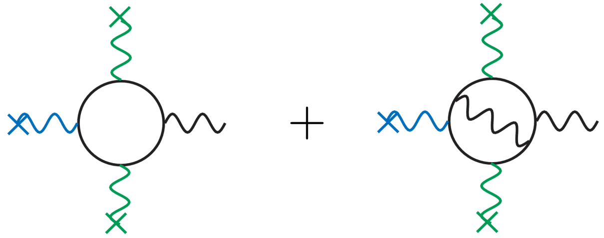


[Ritus: JETP **42** (1975)]

3.2 Laser beam collisions

Consider 2 beam scenario: electromagnetic field $\rightarrow$ 1 pump + 1 probe field

$\rightarrow$ linearize in the probe field:



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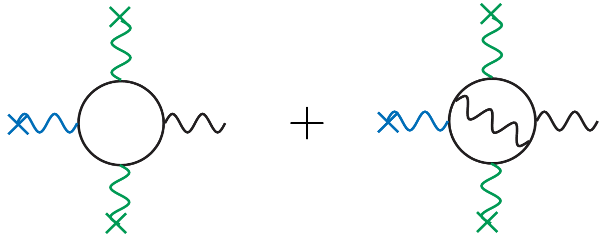


$\rightarrow$ dominant signal component $\hat{=}$ quasi-elastic scattering: [FK, Sundqvist: Phys. Rev. D **94** (2016)]

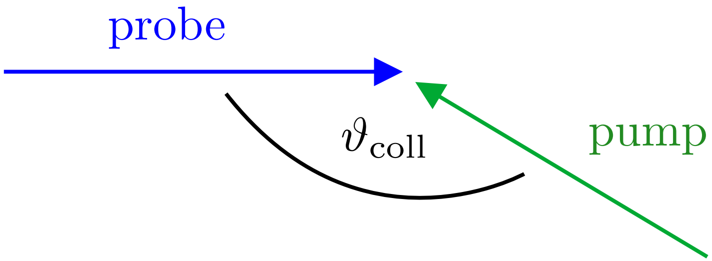
$$\big|\mathcal{S}_p(\vec{k})\big| \sim \left| \int_x \mathrm{e}^{\mathrm{i}(|\vec{k}|t - \vec{k}\cdot\vec{x})}\, I_{\mathrm{pump}}(x)\, \mathcal{E}_{\mathrm{probe}}(x) \right| \quad \rightarrow \quad \mathrm{d}^3 N_p = \frac{\mathrm{d}^3 k}{(2\pi)^3} \big|\mathcal{S}_p(\vec{k})\big|^2$$

$$I_{\mathrm{pump}}(x) := \big\langle \mathcal{E}^2_{\mathrm{pump}}(x) \big\rangle_t$$

3 Towards a Vacuum Birefringence Experiment

Quasi-elastic scattering signal: $(\rightarrow$ mainly in forward cone of probe)

$$N_{\parallel,\perp} \sim c_{\parallel,\perp}(1-\cos\vartheta_{\text{coll}})^4 \left(\frac{I_{0,\text{pump}}}{I_{\text{S}}}\frac{\omega_{\text{probe}}}{mc^2}\right)^2 N_{\text{probe}} \qquad \text{with} \qquad I_{\text{S}} = \left(\frac{m^2}{e}\right)^2 \simeq 10^{29}\,\frac{\text{W}}{\text{cm}^2}$$



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$\rightarrow$ $N_{\parallel,\perp} \sim \mathcal{O}(1\ldots100)/\text{shot}$ for tightly focused petawatt class lasers.

Signal-to-background separation?

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Signal-to-background separation?

$\rightarrow$ measure signal scattered in $\perp$-polarized mode $\simeq$ "vacuum birefringence"

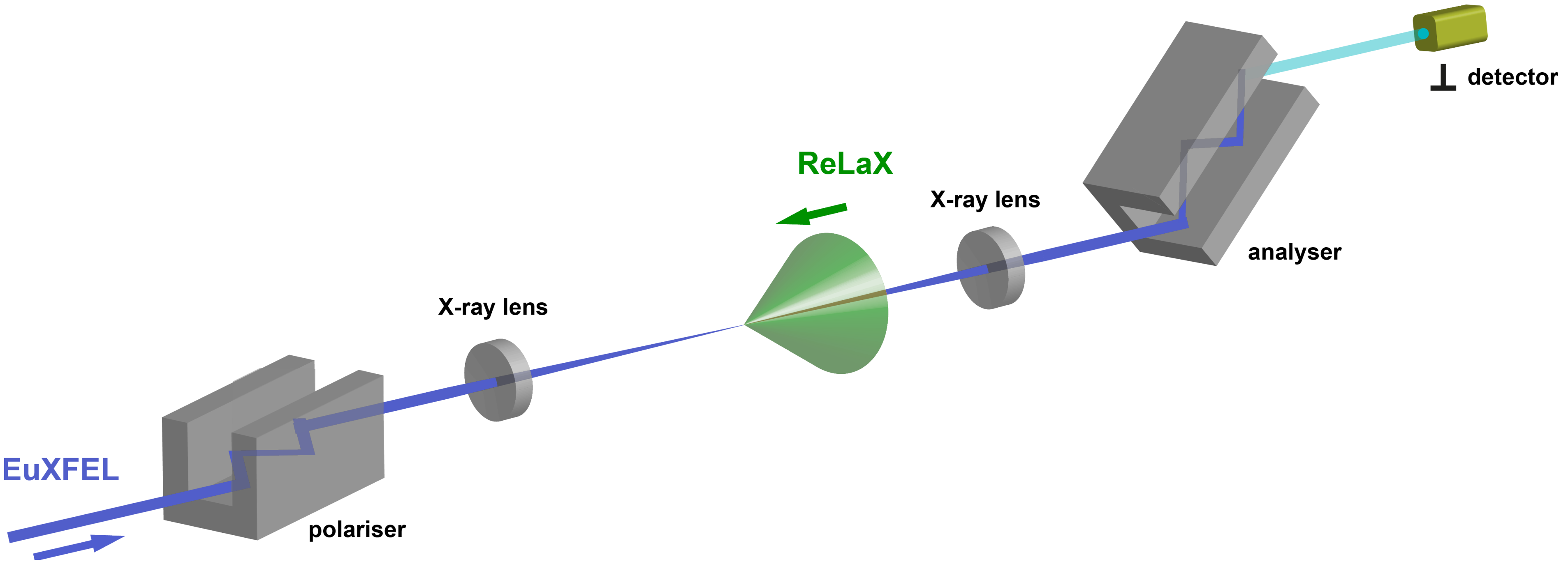
[Aleksandrov, Anselm, Moskalev: JETP **62** (1985)],
[Heinzl, *et al.*: Opt. Commun. **267** (2006)],
[Di Piazza, *et al.*: Phys. Rev. Lett. **97** (2006)], ...

$\rightarrow$ linearly polarized beams (rel. pol. ϕ):

$$c_{\|} = \left[(\boldsymbol{a}+\boldsymbol{b}) + (\boldsymbol{a}-\boldsymbol{b})\cos(2\phi)\right]^2, \qquad c_{\perp} = \left[(\boldsymbol{a}-\boldsymbol{b})\sin(2\phi)\right]^2$$

3 Towards a Vacuum Birefringence Experiment

Schematic experimental setup with XFEL probe:



Very hard to detect $\perp$-signal with standard high-intensity laser + XFEL beams.

$\leftrightarrow$ polarization purity $\mathcal{P} = \mathcal{O}(10^{-11})$ @ $\omega_{\rm probe} = \mathcal{O}(10)\,{\rm keV}$ achieved experimentally

[Schulze, *et al.*: Phys. Rev. Res. **4** (2022)]

$$\rightarrow \quad N_{\rm bgr} = \mathcal{P} N_{\rm probe}$$

Various reasons (including):

- only lasers providing $\lesssim 0.5$ petawatt installed at XFELs
- standard Be lenses deteriorate polarization purity
- losses in optimal elements, bandwidth missmatch XFEL $\leftrightarrow$ polarizers, etc.

$\rightarrow$ signals are typically background dominated [Mosman, FK: Phys. Rev. D **104** (2021)]

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$\leftrightarrow$ Flagship Experiment of

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ExtreMe Matter Institute EMMI

EMMI Collaboration Meeting

Towards a Vacuum Birefringence Experiment at the Helmholtz International Beamline for Extreme Fields

Jena, Germany, October 26 - 28, 2023

The recent inauguration of the Helmholtz International Beamline for Extreme Fields – a worldwide unique combination of an XFEL and an intense laser – at the European XFEL, advances in high-precision x-ray polarimetry, refinements of prospective discovery scenarios and progress in their accurate theoretical modeling have set the stage for performing an actual discovery experiment right now. The goal of this EMMI Collaboration Meeting is to reach agreement on the most promising experimental setup and to work out a roadmap towards the first detection of this effect in a concerted effort of theory and experiment.

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Towards a Vacuum Birefringence Experiment at the Helmholtz International Beamline for Extreme Fields

(Letter of Intent of the BIREF@HIBEF Collaboration)

N. Ahmadiniaz[1], C. Bähtz[1], A. Benediktovitch[2], C. Bömer[2], L. Bocklage[2], T. E. Cowan[1,3], J. Edwards[4], S. Evans[1], S. Franchino Viñas[1], H. Gies[5,6], S. Göde[7], J. Görs[5], J. Grenzer[1], U. Hernandez Acosta[1,8], T. Heinzl[4], P. Hilz[6], W. Hippler[5], L. G. Huang[1], O. Humphries[7], F. Karbstein[5,6,9], P. Khademi[6], B. King[4], T. Kluge[1], C. Kohlfürst[1], D. Krebs[2], A. Laso-García[1], R. Lötzsch[5], A. J. Macleod[10], B. Marx-Glowna[6,9], E. A. Mosman[5,6], M. Nakatsutsumi[7], G. G. Paulus[5], S. V. Rahul[7], L. Randolph[7], R. Röhlsberger[2,5,6,9], N. Rohringer[2], A. Sävert[6], S. Sadashivaiah[6], R. Sauerbrey[1,3], H.-P. Schlenvoigt[1], S. M. Schmidt[1,3], U. Schramm[1,3], R. Schützhold[1,3], J.-P. Schwinkendorf[7], D. Seipt[5,6,9], M. Šmíd[1], T. Stöhlker[5,6,9], T. Toncian[1], M. Valialshchikov[6], A. Wipf[5], U. Zastrau[7], M. Zepf[5,6,9]

[1]Helmholtz-Zentrum Dresden-Rossendorf, Bautzner Landstraße 400, 01328 Dresden, Germany.
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[3]Technische Universität Dresden, 01062 Dresden, Germany.
[4]Centre for Mathematical Sciences, University of Plymouth, Plymouth, PL4 8AA, UK.
[5]Department of Physics and Astronomy, Friedrich-Schiller-Universität Jena, 07743 Jena, Germany.
[6]Helmholtz Institute Jena, Fröbelstieg 3, 07743 Jena, Germany.
[7]European XFEL GmbH, Holzkoppel 4, 22869 Schenefeld, Germany.
[8]Center for Advanced Systems Understanding, Untermarkt 20, 02826 Görlitz, Germany.
[9]GSI Helmholtzzentrum für Schwerionenforschung, Planckstraße 1, 64291 Darmstadt, Germany.
[10]ELI Beamlines Facility, The Extreme Light Infrastructure ERIC, Dolní Břežany, Czech Republic.

Abstract

Quantum field theory predicts a nonlinear response of the vacuum to strong electromagnetic fields of macroscopic extent. This fundamental tenet has remained experimentally challenging and is yet to be tested in the laboratory. A particularly distinct signature of the resulting optical activity of the quantum vacuum is vacuum birefringence. This offers an excellent opportunity for a precision test of nonlinear quantum electrodynamics in an uncharted parameter regime. Recently, the operation of the high-intensity laser ReLaX provided by the Helmholtz International Beamline for Extreme Fields (HIBEF) has been inaugurated at the High Energy Density (HED) scientific instrument of the European XFEL. We make the case that this worldwide unique combination of an x-ray free-electron laser and an ultra-intense near-infrared laser together with recent advances in high-precision x-ray polarimetry, refinements of prospective discovery scenarios, and progress in their accurate theoretical modelling have set the stage for performing an actual discovery experiment of quantum vacuum nonlinearity.

3 Towards a Vacuum Birefringence Experiment

Possible scenarios:

[Letter of Intent: High Power Laser Sci. Eng. **13**, e7 (2025)]

(1) conventional vacuum birefringence scenario

[Aleksandrov, Anselm, Moskalev: JETP **62** (1985)], [Heinzl, *et al.*: Opt. Commun. **267** (2006)], ...

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(2) dark-field approach

[FK, Mosman: Phys. Rev. D **101** (2020)], [FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]

[Peatross, *et al.*: Opt. Lett. **19** (1994)], [Zepf, *et al.*: Phys. Rev. E **58** (1998)]

2 beams

(3) four-wave-mixing processes

[King, Hu, Shen: Phys. Rev. A **98** (2018)], [Ahmadiniaz, *et al.*: Phys. Rev. D **108** (2023)], ...

[Moulin, *et al.*: Opt. Commun. **164** (1999)], [Bernard, *et al.*: Eur. Phys. J. D **10** (2000)], ...

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(1) conventional vacuum birefringence scenario $\rightarrow N_\perp \sim (\boldsymbol{a}-\boldsymbol{b})^2$

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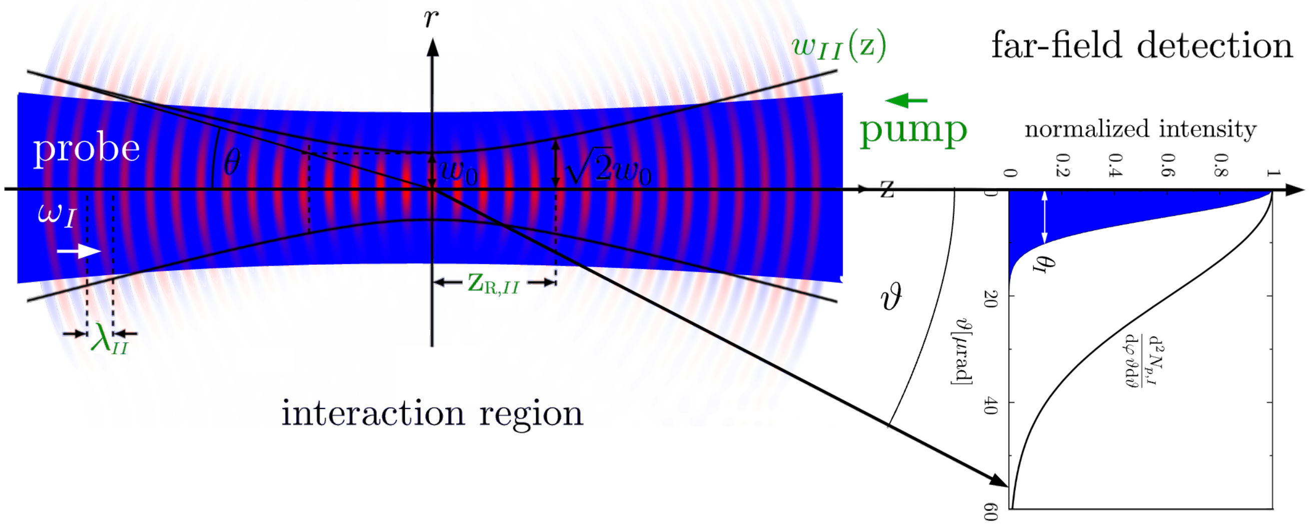
Signal enhancement strategies (in high-intensity laser + XFEL collisions)?

[FK, Gies, Reuter, Zepf: Phys. Rev. D **92** (2015)],
[FK, Sundqvist: Phys. Rev. D **94** (2016)],
[FK: Phys. Rev. D **98** (2018)]

(a) Diffraction assisted measurement:



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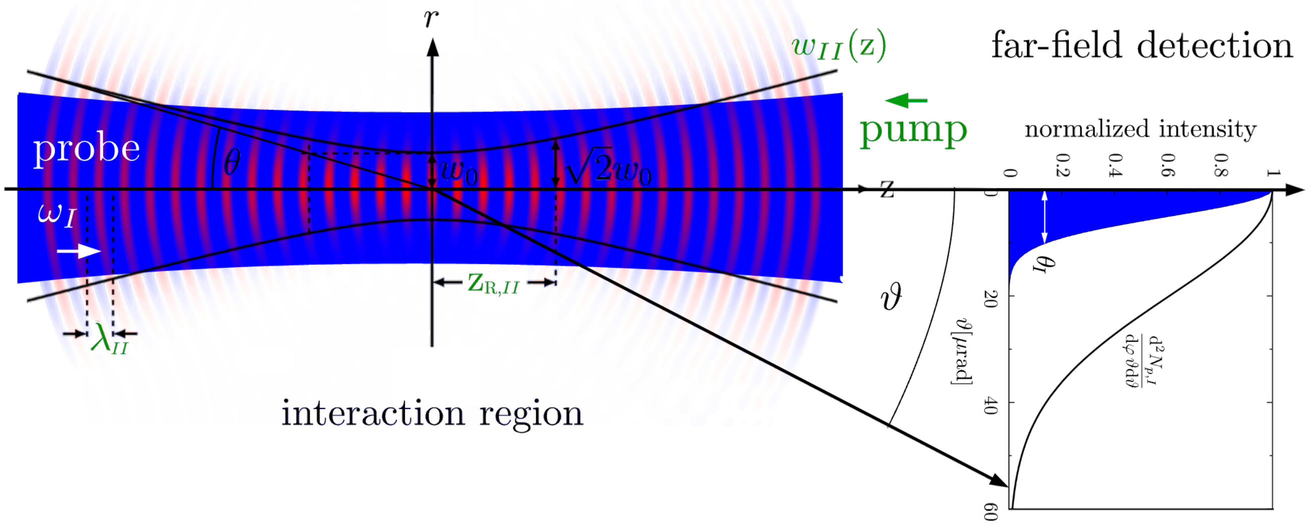
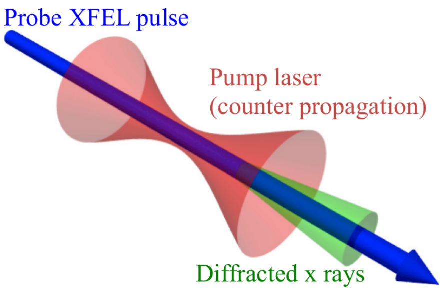




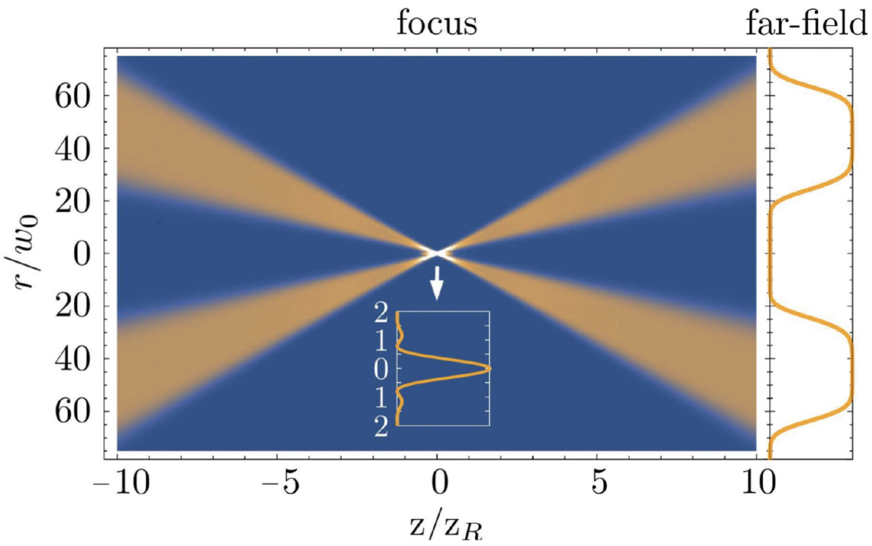
→ attempts to measure $N_{\parallel}$ with this schema at SACLA, Japan

[Inada, *et al.*: Appl. Sci. **7** (2017)],
[Seino, *et al.*: PTEP **7** (2020)]

(b) Annular probe beam / dark-field approach:

[FK, Mosman: Phys. Rev. D **101** (2020)],
[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]

$\rightarrow$ modify the probe field to make the forward signal experimentally accessible:



[Peatross, *et al.*: Opt. Lett. **19** (1994)],
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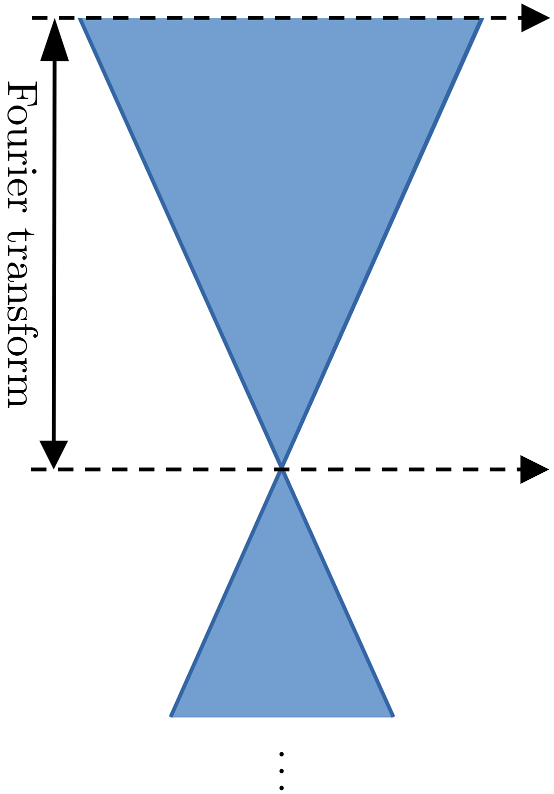
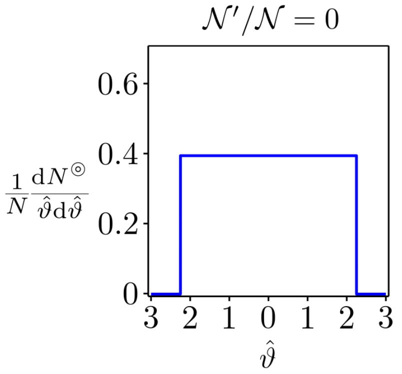
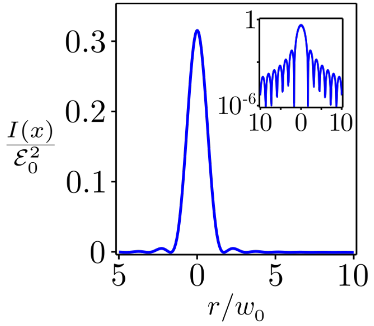
$\leftrightarrow$ self-consistent beam model needed to confirm that this works, & if so, how good



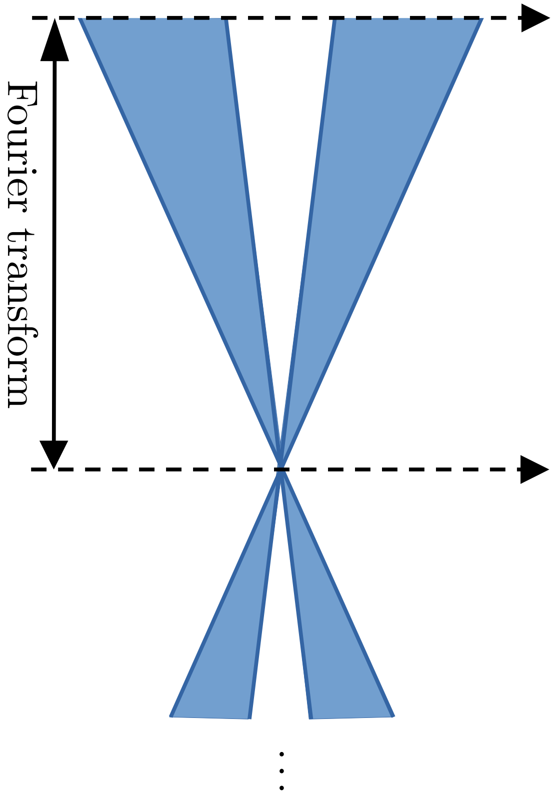
Far-field intensity profiles:



Focus intensity profiles:



3 Towards a Vacuum Birefringence Experiment



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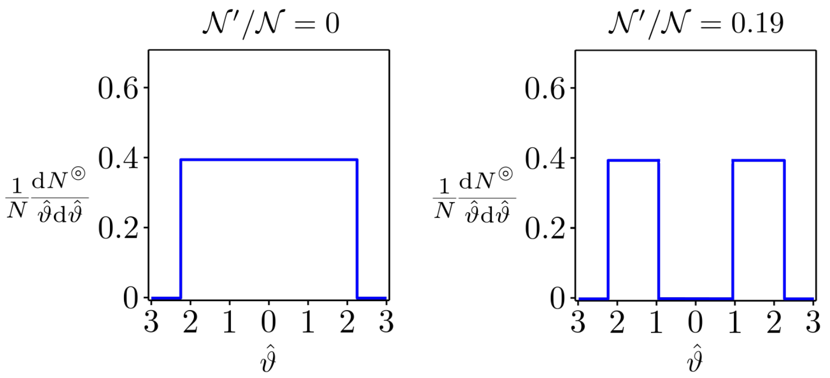

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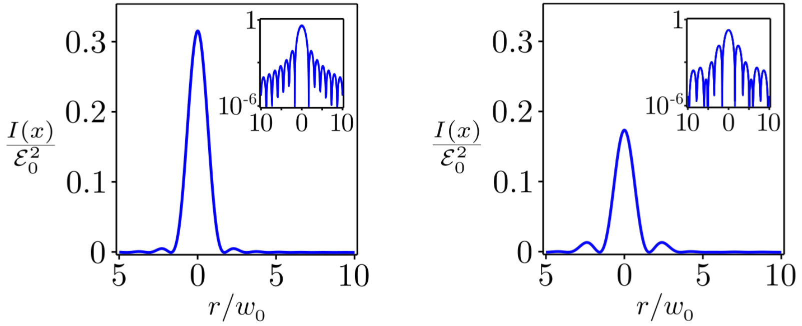

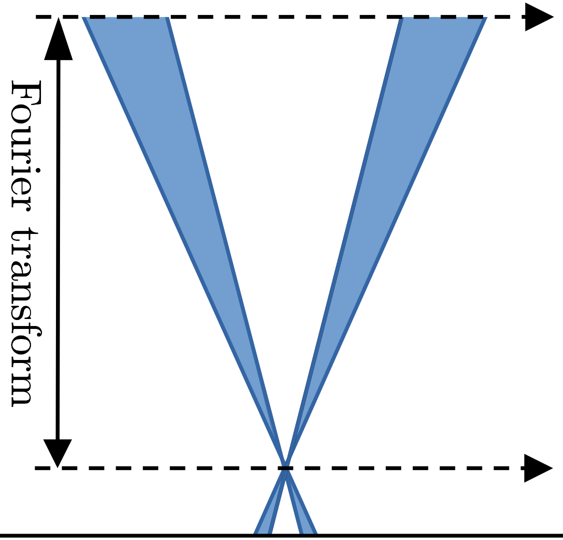

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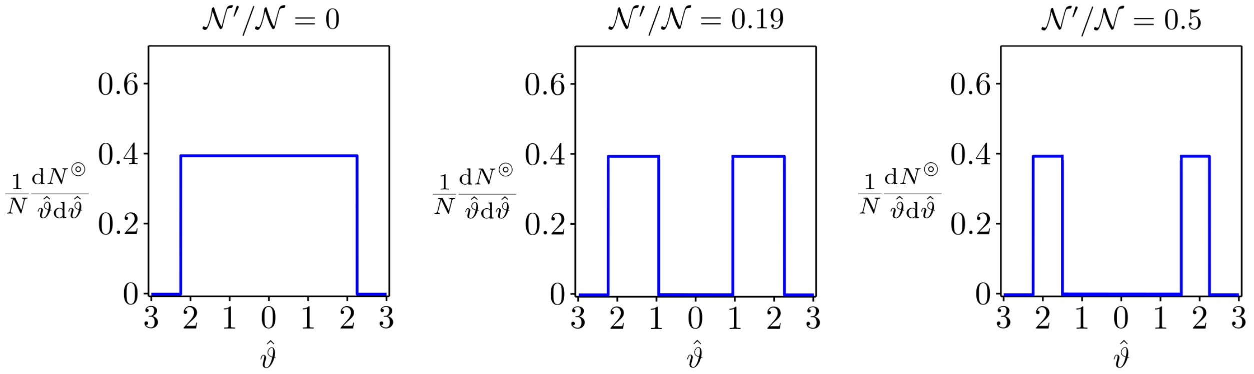

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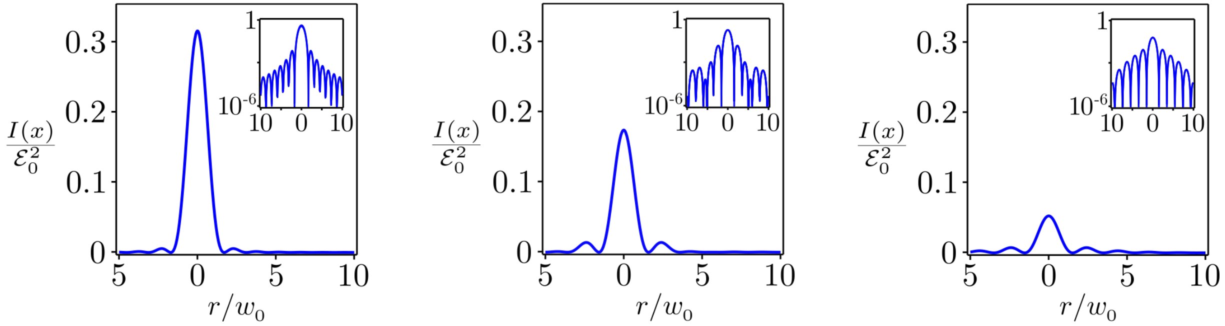

→ in the focus deviations from Gaussian far-field profile are encoded in Airy pattern.

3 Towards a Vacuum Birefringence Experiment

- dark-field probe $\rightarrow$ perspective of background free detection of $\parallel$ and $\perp$ signals

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- focus on counter-propagating pump and probe beams + rotational symmetry:

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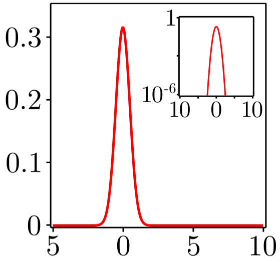
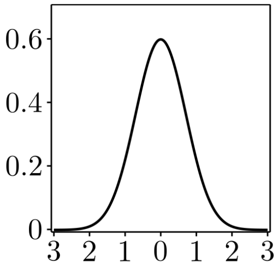
$=$ 

$\rightarrow$ signal has 

profile

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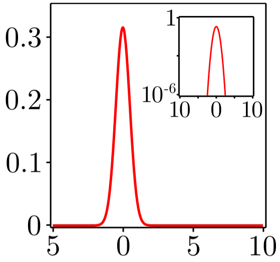
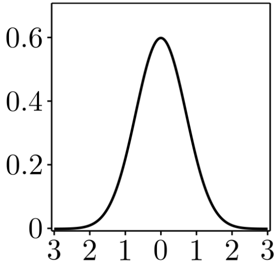
$=$ 

$\rightarrow$ signal has 

profile

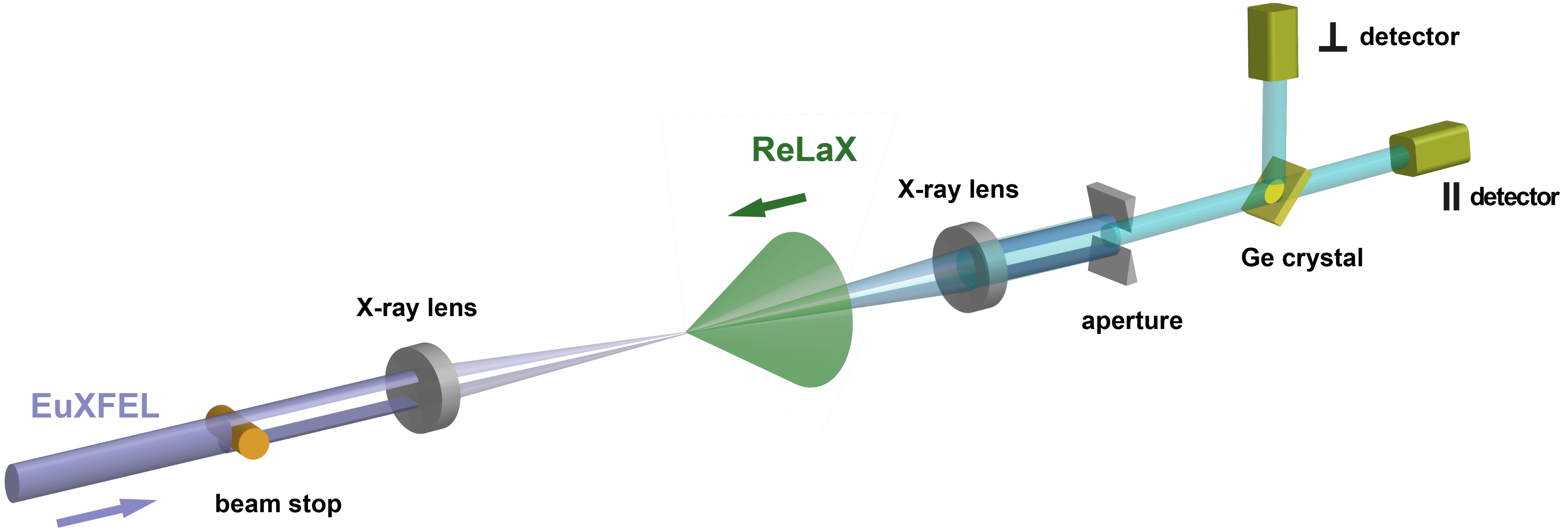
$$\rightarrow \ \text{laser fluctuation independent ratio} \quad \left.\frac{N^{\bullet}_{\perp,\rm sig}}{N^{\bullet}_{\parallel,\rm sig}}\right|_{\phi=\frac{\pi}{4}} \simeq \left(\frac{\boldsymbol{a}-\boldsymbol{b}}{\boldsymbol{a}+\boldsymbol{b}}\right)^2 \xrightarrow{\rm QED} \frac{9}{121}\left(1+\frac{\alpha}{\pi}\frac{260}{99}+\cdots\right)$$

3 Towards a Vacuum Birefringence Experiment

Shadow quality $\mathcal{S} = N^{\bullet}_{\rm bgr}/N_{\rm probe}$?

[FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]
[Letter of Intent: High Power Laser Sci. Eng. **13**, e7 (2025)]

$\rightarrow$ envisioned implementation:



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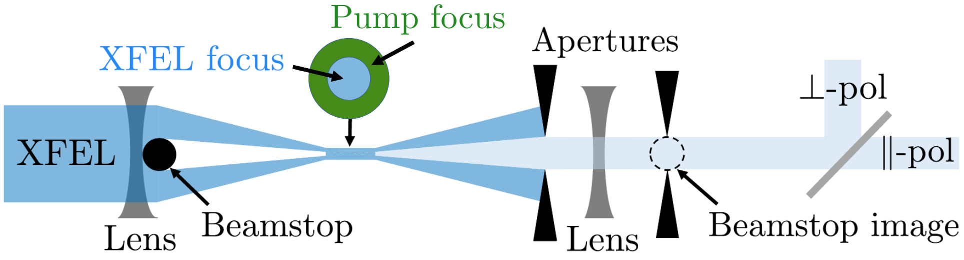
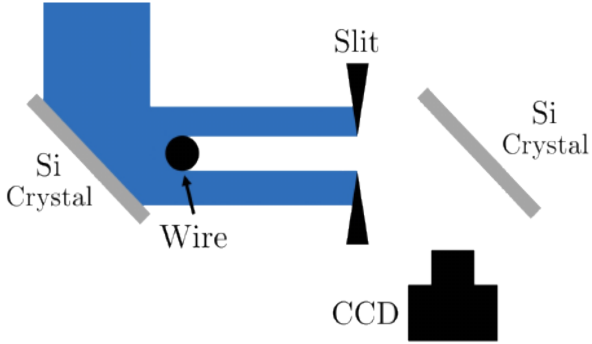
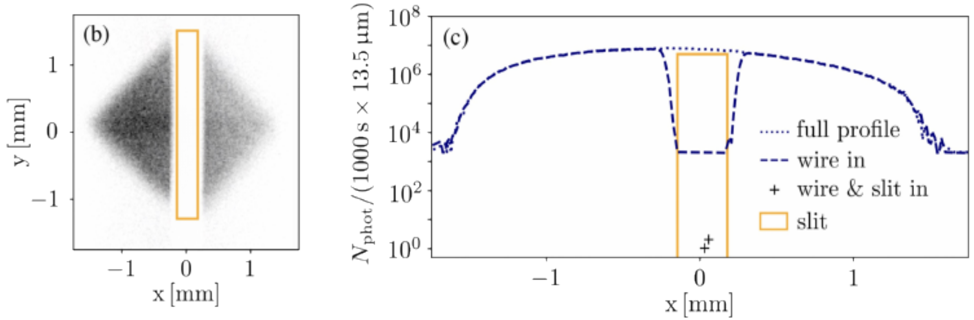


$\rightarrow$ proof-of-concept experiment @ x-ray tube: [FK, Ullmann, Mosman, Zepf: Phys. Rev. Lett. **129** (2022)]





$\rightarrow \; \mathcal{S} \simeq 3 \times 10^{-8}$

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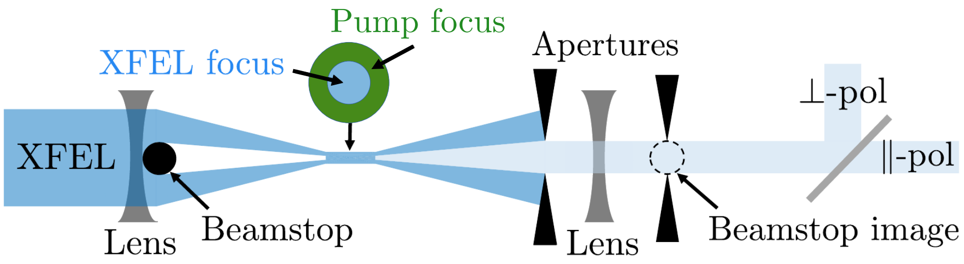
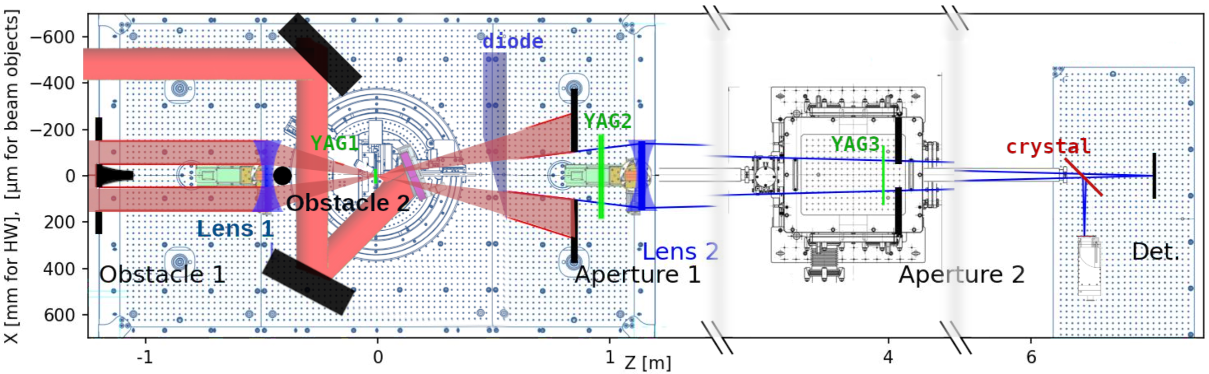
→ envisioned implementation:



→ proof-of-concept experiment @ EuXFEL:

[HIBEF Priority Access: Proposal **5438** (March, 2024)
& Proposal **6436** (September, 2024)]



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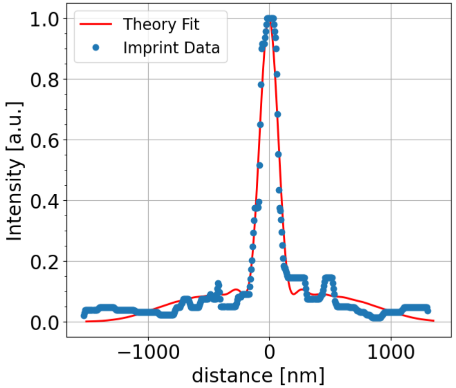

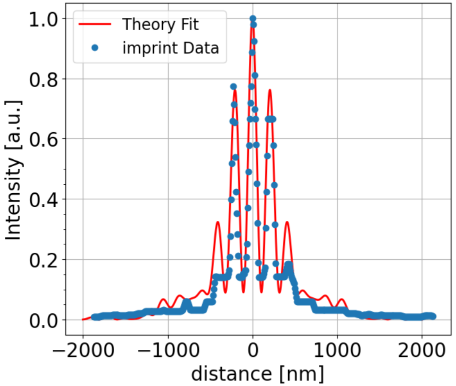

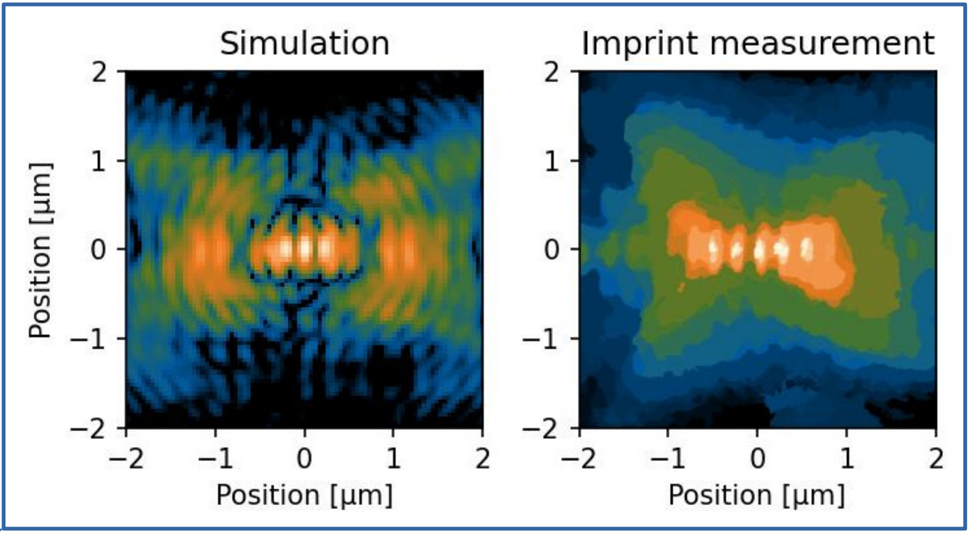

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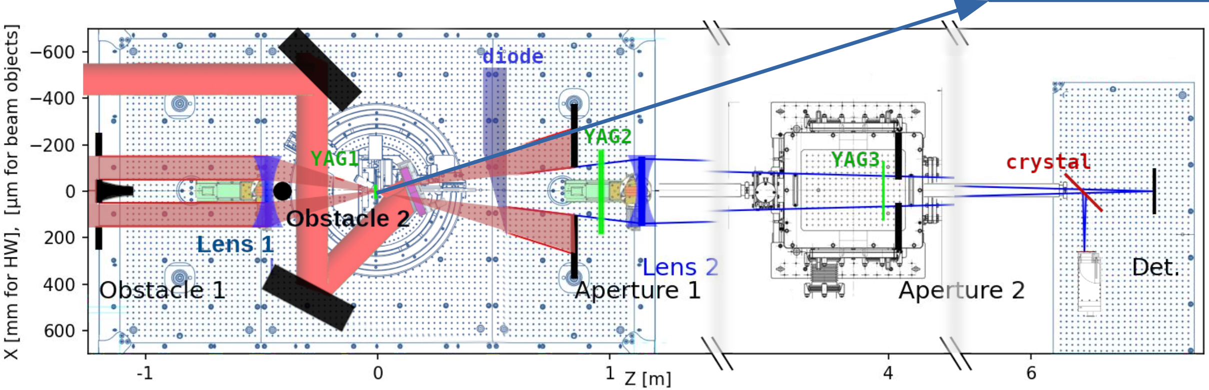

$\rightarrow$ simulations $\hat{=}$ measurements

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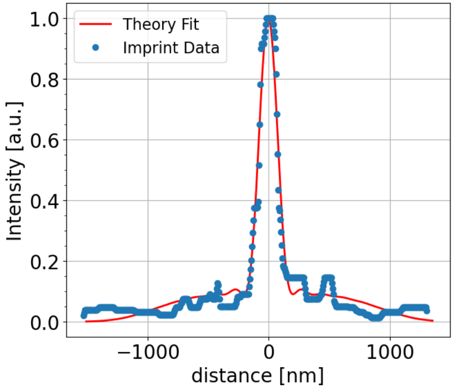
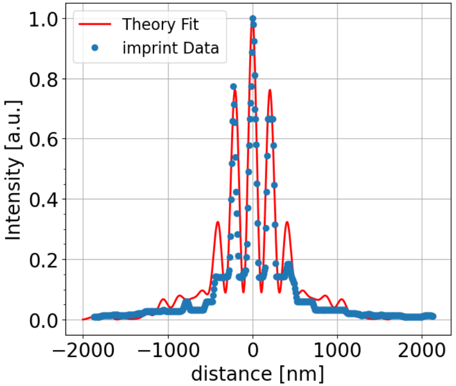
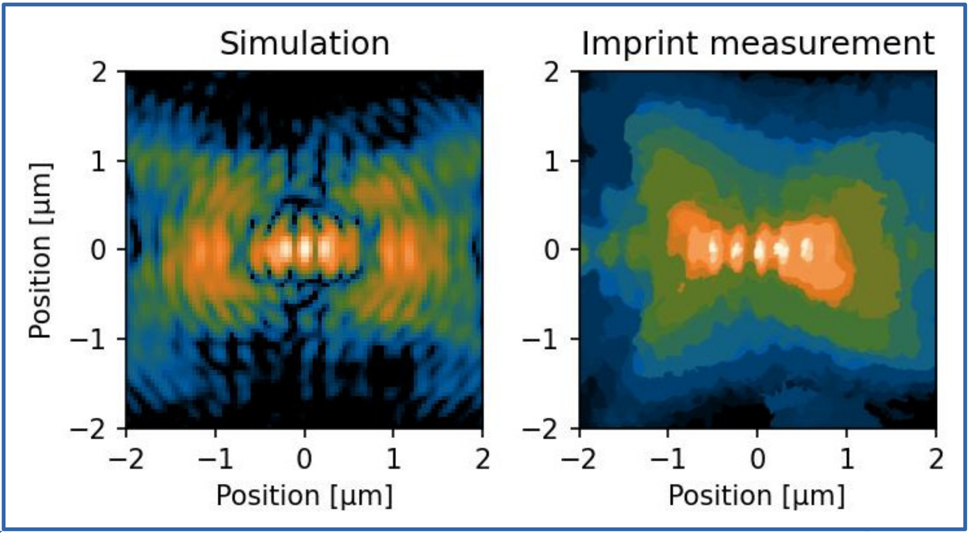
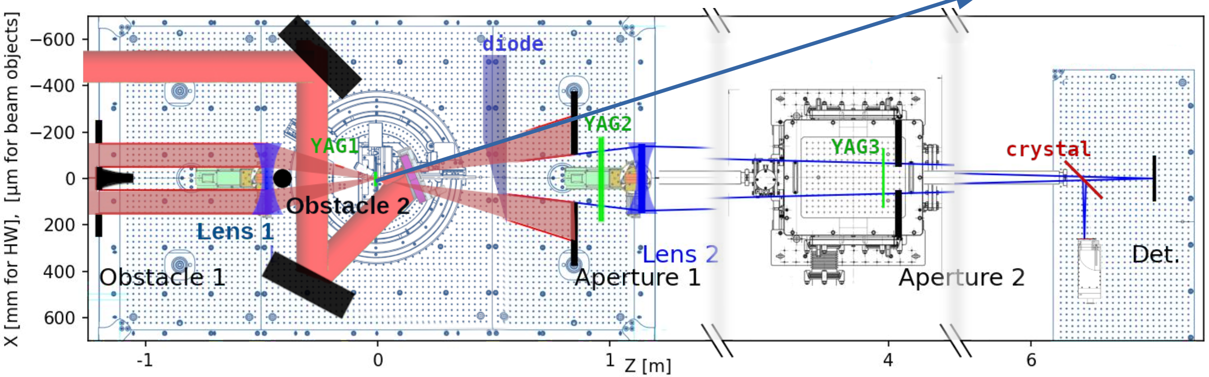






$\rightarrow$ proof-of-concept experiment @ EuXFEL:



$\rightarrow$ simulations $\hat{=}$ measurements

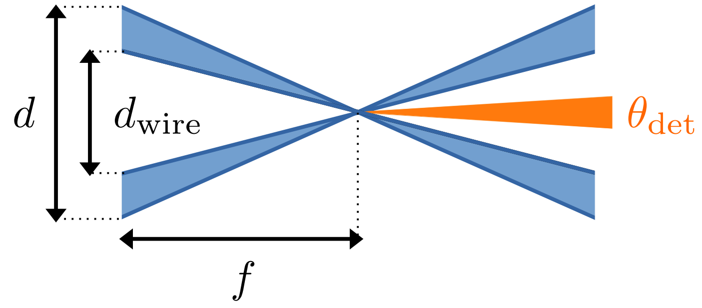
$$\boxed{\begin{array}{l} \rightarrow \; \mathcal{S}_{\parallel} \simeq 6.8 \times 10^{-12} \\ \rightarrow \; \mathcal{S}_{\perp} \simeq 3.9 \times 10^{-16} \end{array}}$$

[Šmíd, *et al.*: arXiv:2506.11649 (2025)]

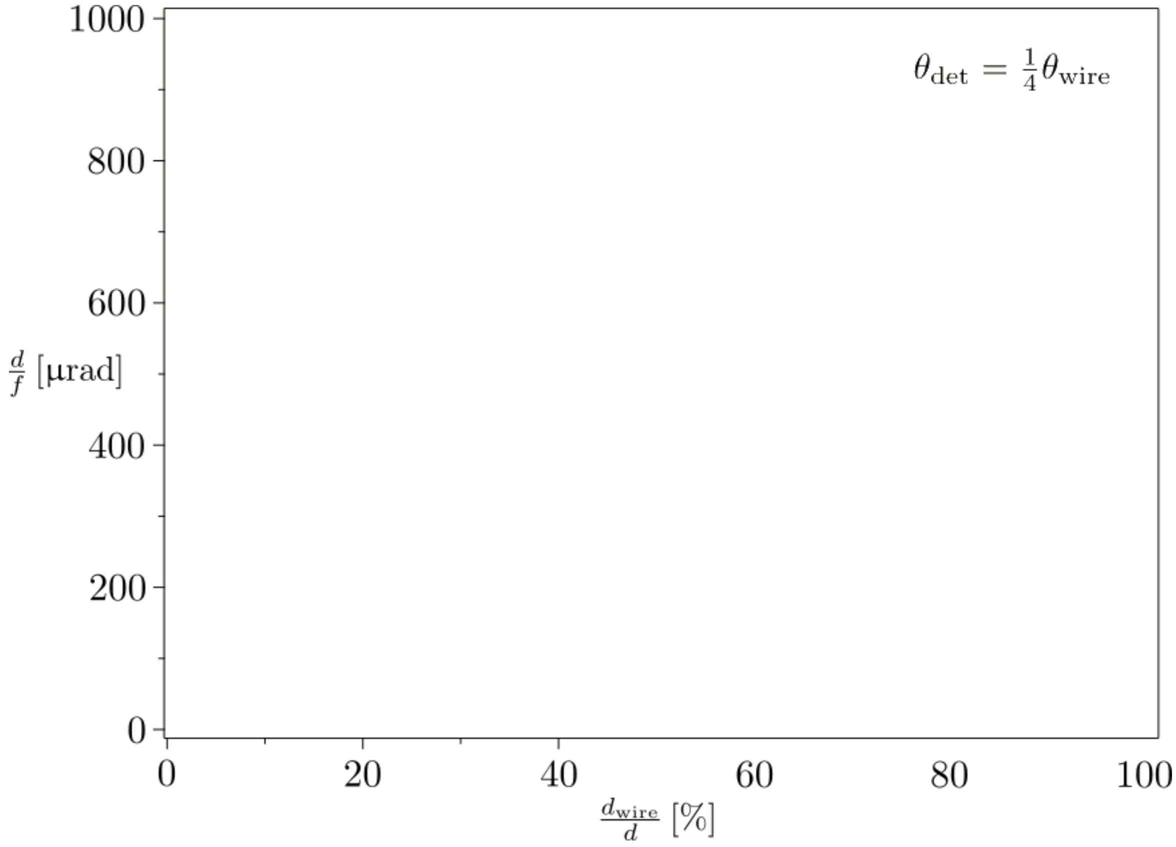
3 Towards a Vacuum Birefringence Experiment

ReLaX
- $W = 4.8\,\mathrm{J}$
- $\tau_{\mathrm{FWHM}} = 30\,\mathrm{fs}$
- $w_{\mathrm{FWHM}} = 1.3\,\mu\mathrm{m}$

EuXFEL
- $N = 2\times10^{11}$
- $\omega = 8766\,\mathrm{eV}$
- $\tau_{\mathrm{FWHM}} = 25\,\mathrm{fs}$



$$\boxed{N^{\max}_{\perp,\mathrm{sig}} = 3/\mathrm{h}\,,\ N^{\max}_{\parallel,\mathrm{sig}} = 40/\mathrm{h}}$$



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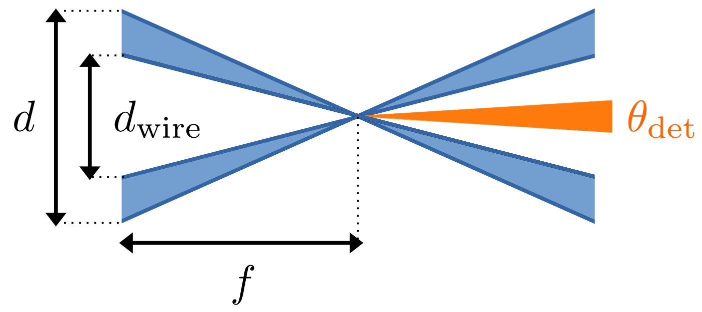
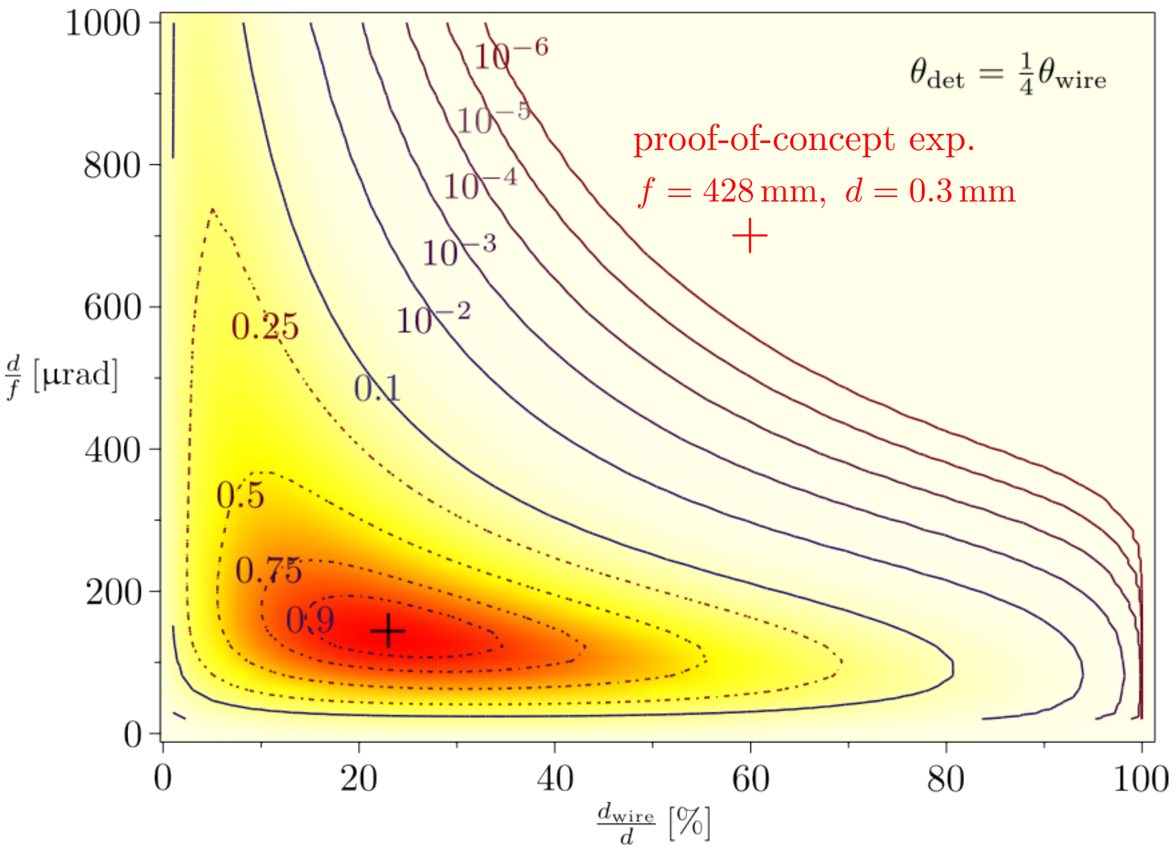


$$\boxed{N^{\max}_{\perp,\mathrm{sig}} = 3/\mathrm{h}\,,\ N^{\max}_{\parallel,\mathrm{sig}} = 40/\mathrm{h}}$$



ReLaX
- $W = 4.8\,\mathrm{J}$
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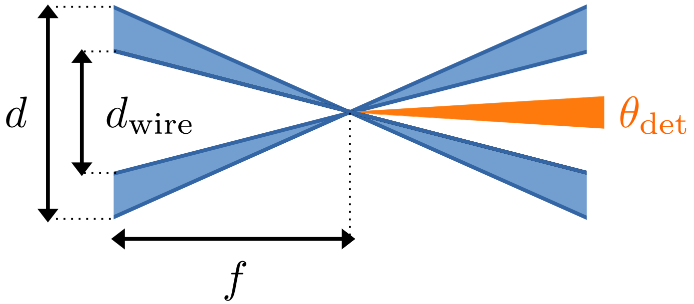
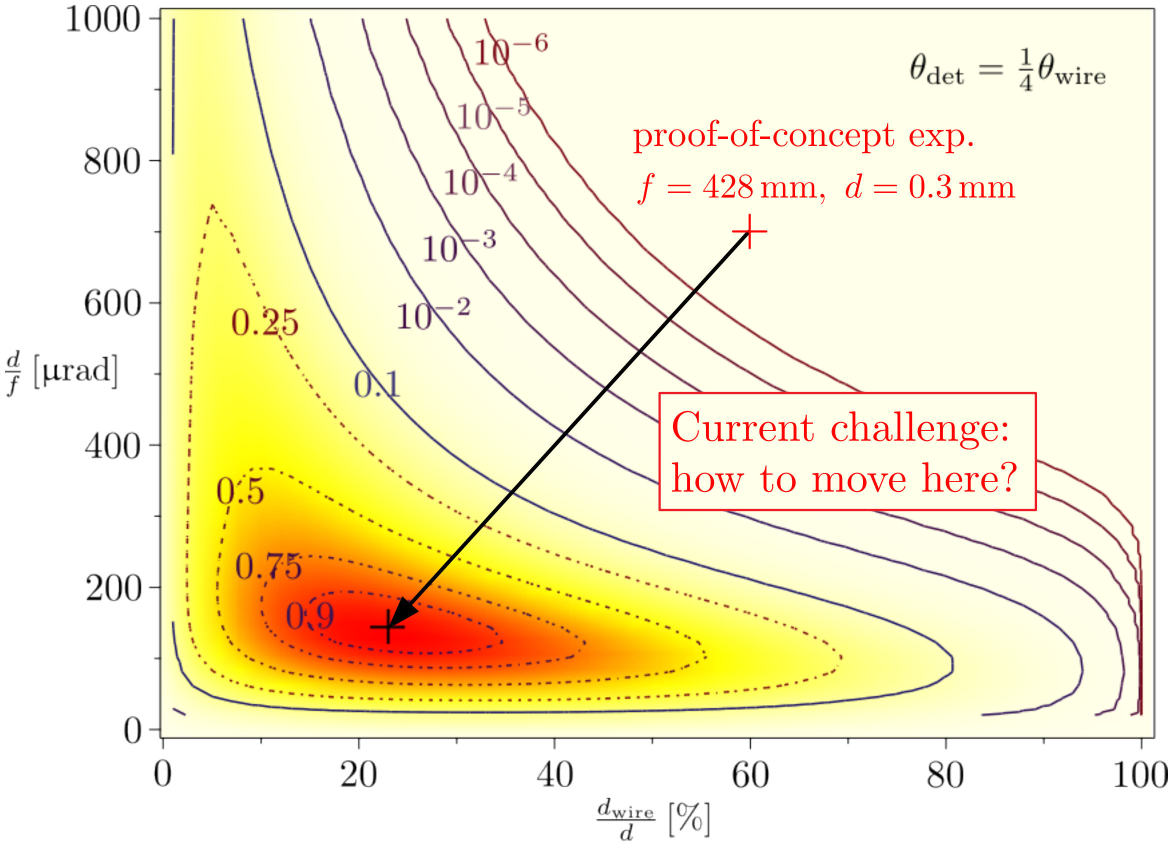


$$\boxed{N_{\perp,\mathrm{sig}}^{\mathrm{max}} = 3/\mathrm{h}\,,\ N_{\parallel,\mathrm{sig}}^{\mathrm{max}} = 40/\mathrm{h}}$$



4 Conclusions

In this talk:

→ **Sec. 2** fundamental aspects:

- definition & properties of Heisenberg-Euler effective theory

→ **Sec. 3** phenomenological consequences:

- photonic quantum vacuum signals
- issue of signal-to-background separation
- dark-field approach → towards an actual experiment @ EuXFEL

Deutsche Forschungsgemeinschaft
Research Unit FOR 2783

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

HI JENA
Helmholtz Institute Jena

Thank you very much for the invitation and for your attention!

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