Importance of tensor interactions in nuclear structure

Understanding nuclear structure with tensor interactions!

Isao Tanihata, RCNP, Osaka University

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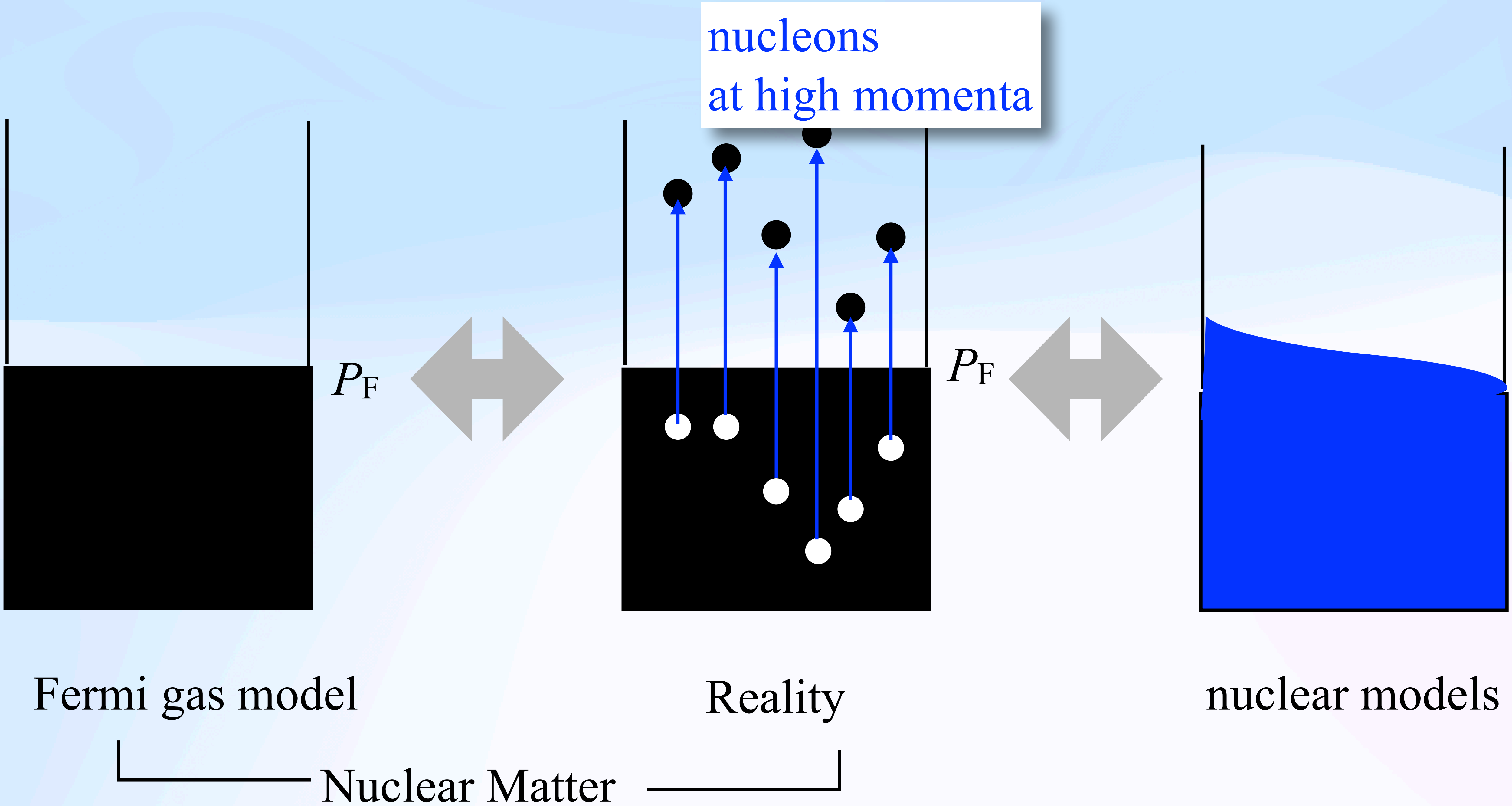
High momentum space in nuclei

- **Nuclear matter**
  - ***E/A = 16 MeV, ρ = 1.6 nucleons/fm³, P_F ~ 1.2 fm⁻¹.***



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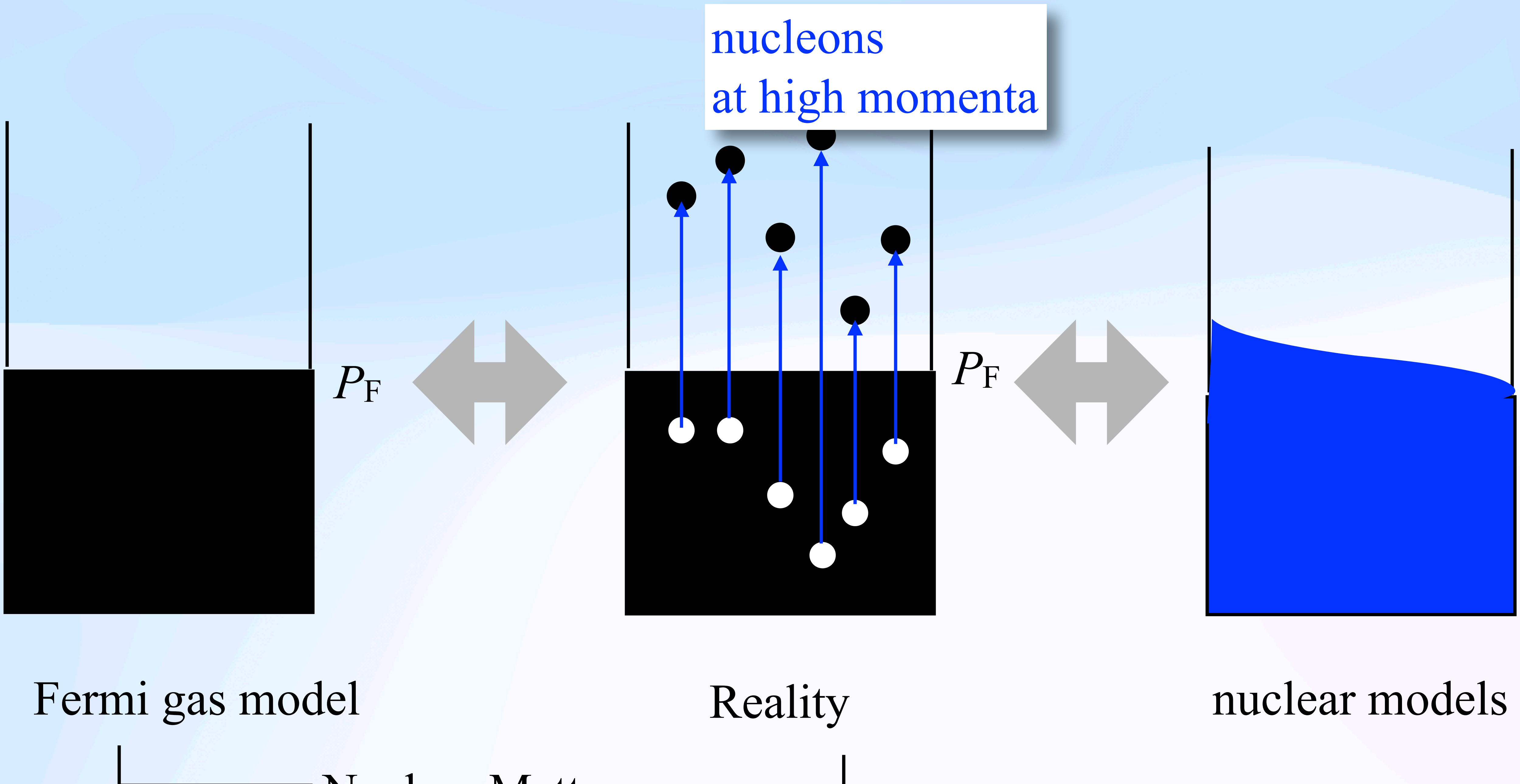
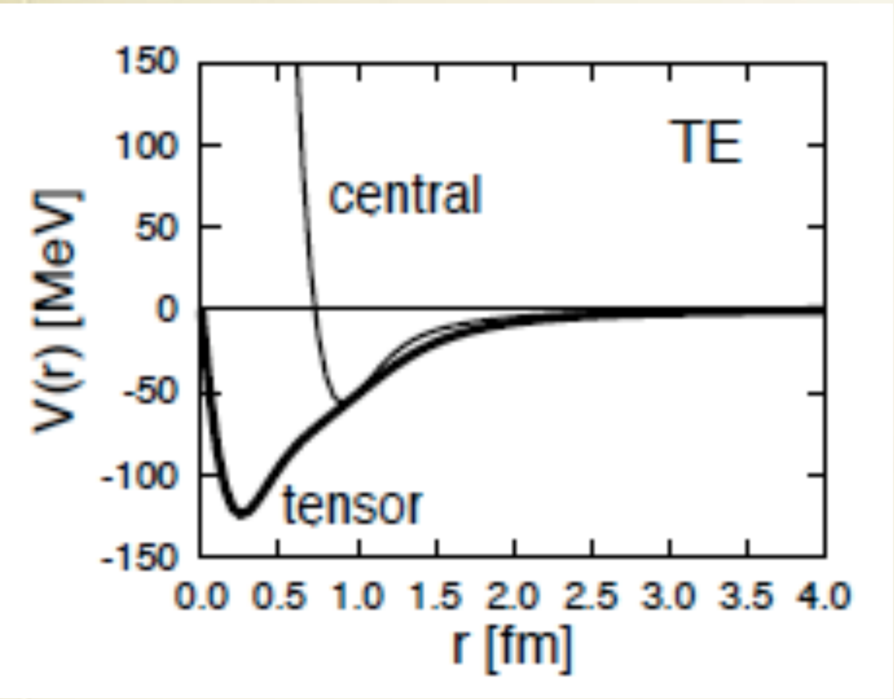
  - ***$E/A = 16$ MeV, $\rho = 1.6$ nucleons/fm^3, $P_F \sim 1.2$ fm^{-1}.***

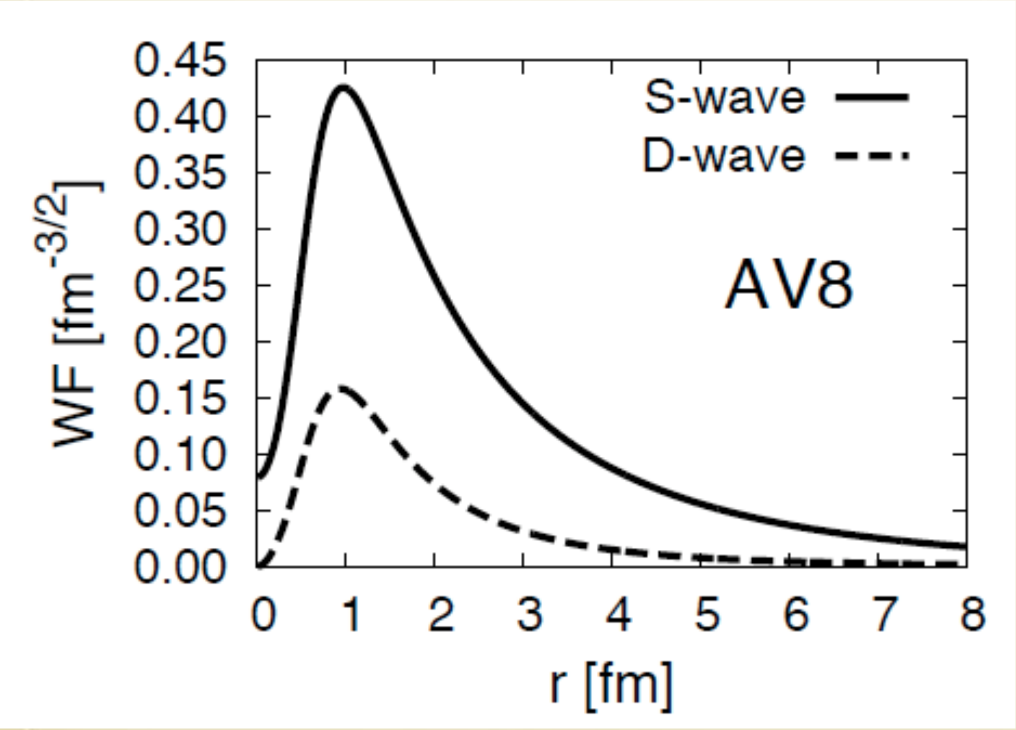


Selection rules apply strongly in nuclei!

In fact

Deuteron is bound by D-wave





S=1 and L=0 or 2

Binding of deuteron (1^+)

Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensor	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(*D*)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

In fact

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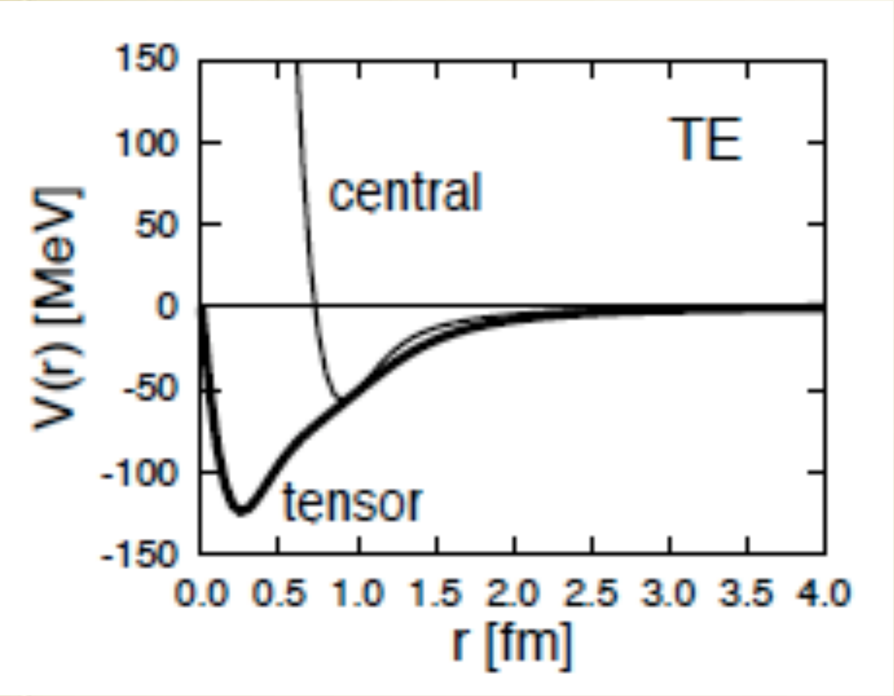

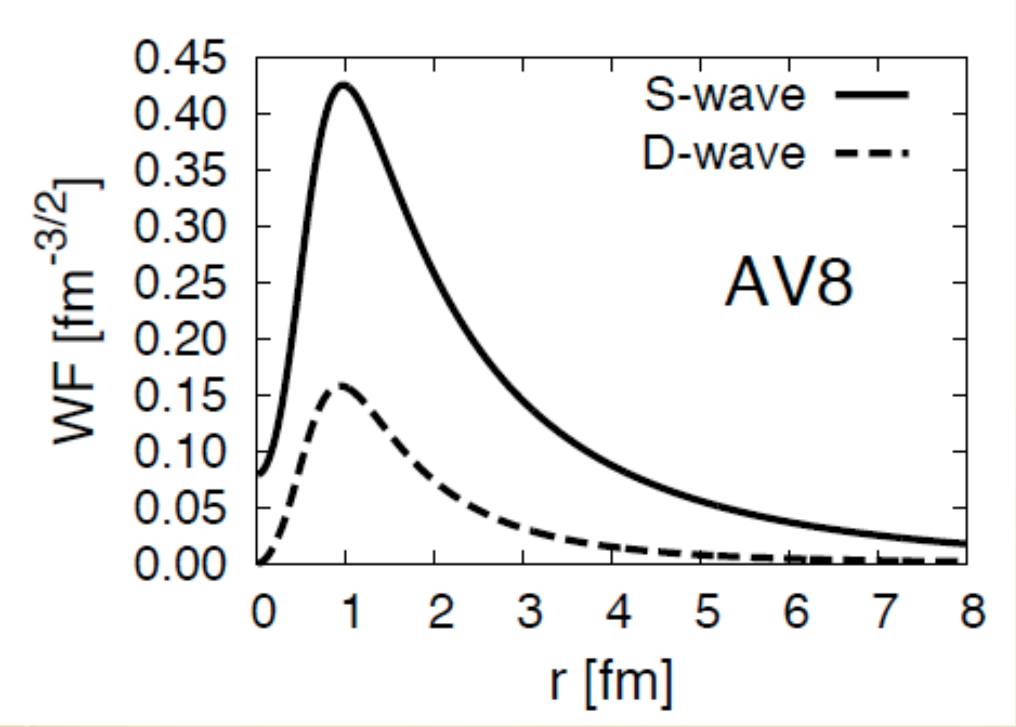

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D wave has a large relative momentum

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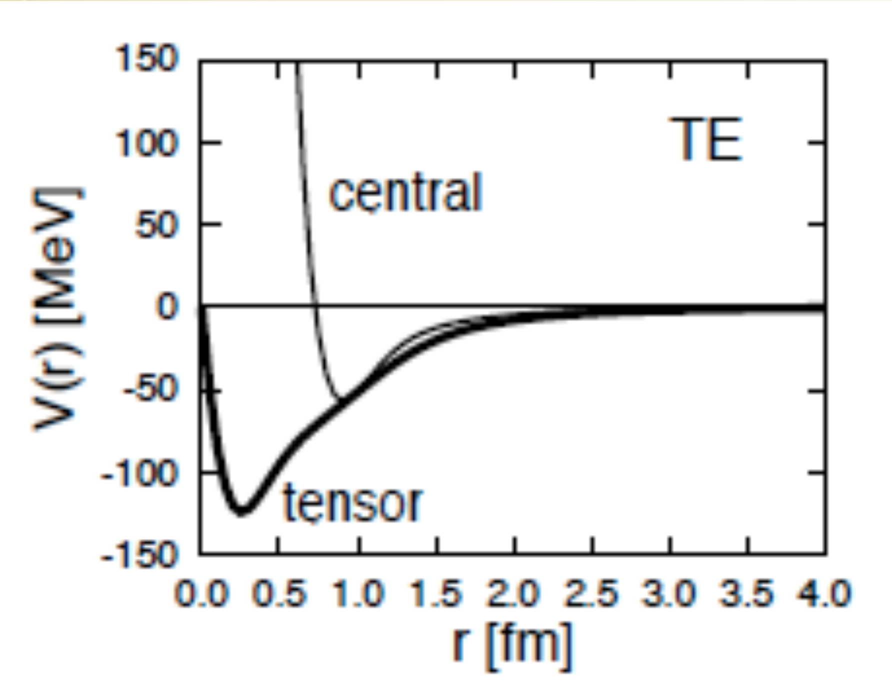

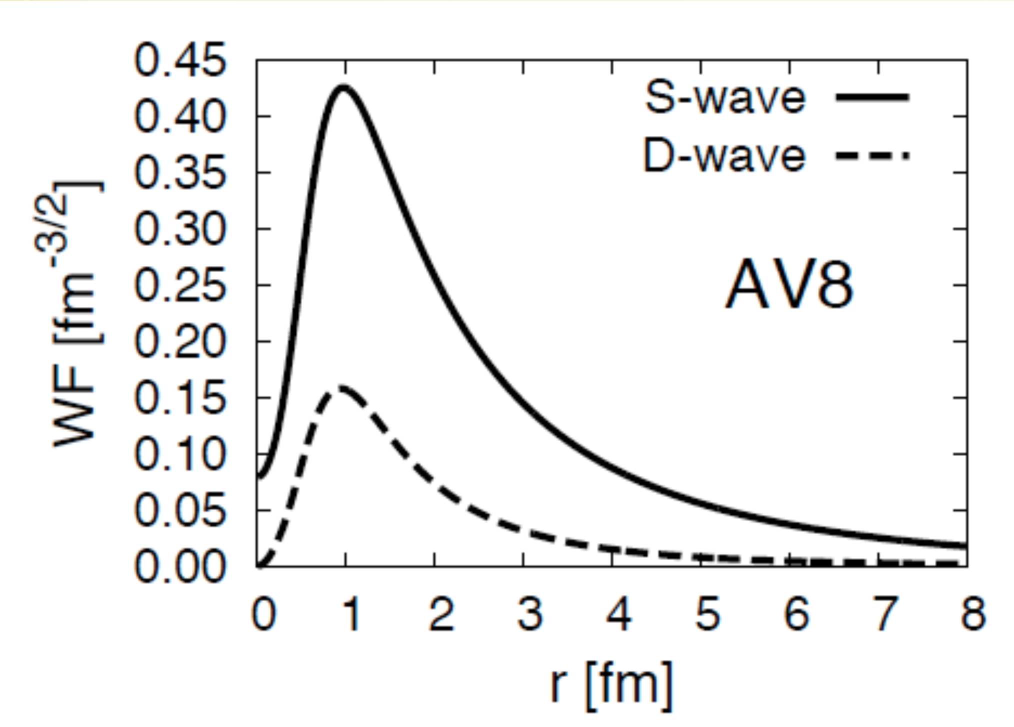

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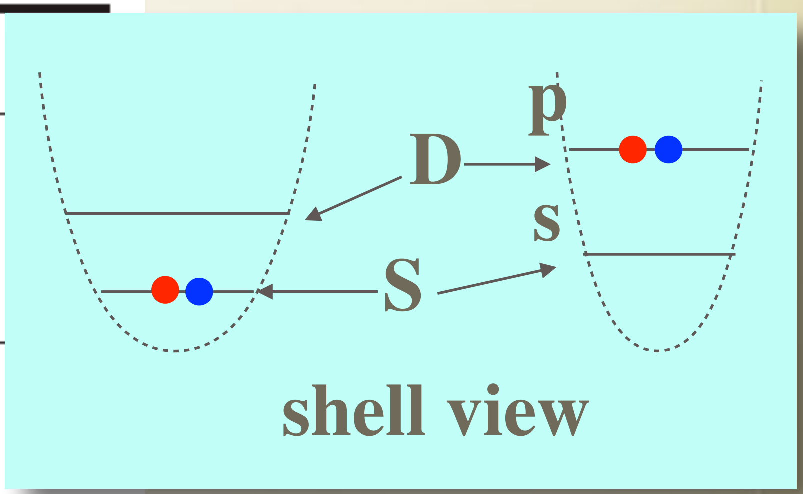
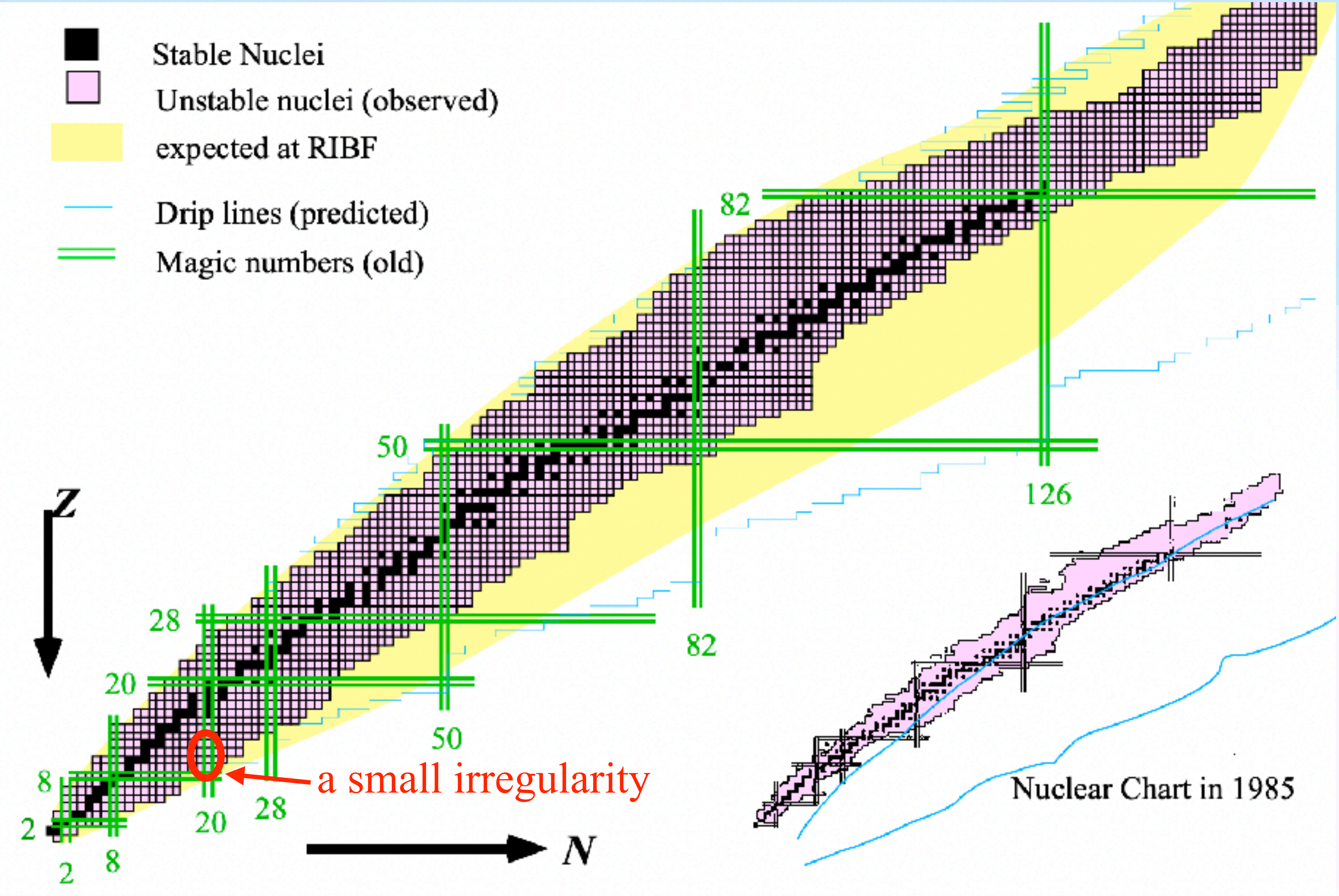


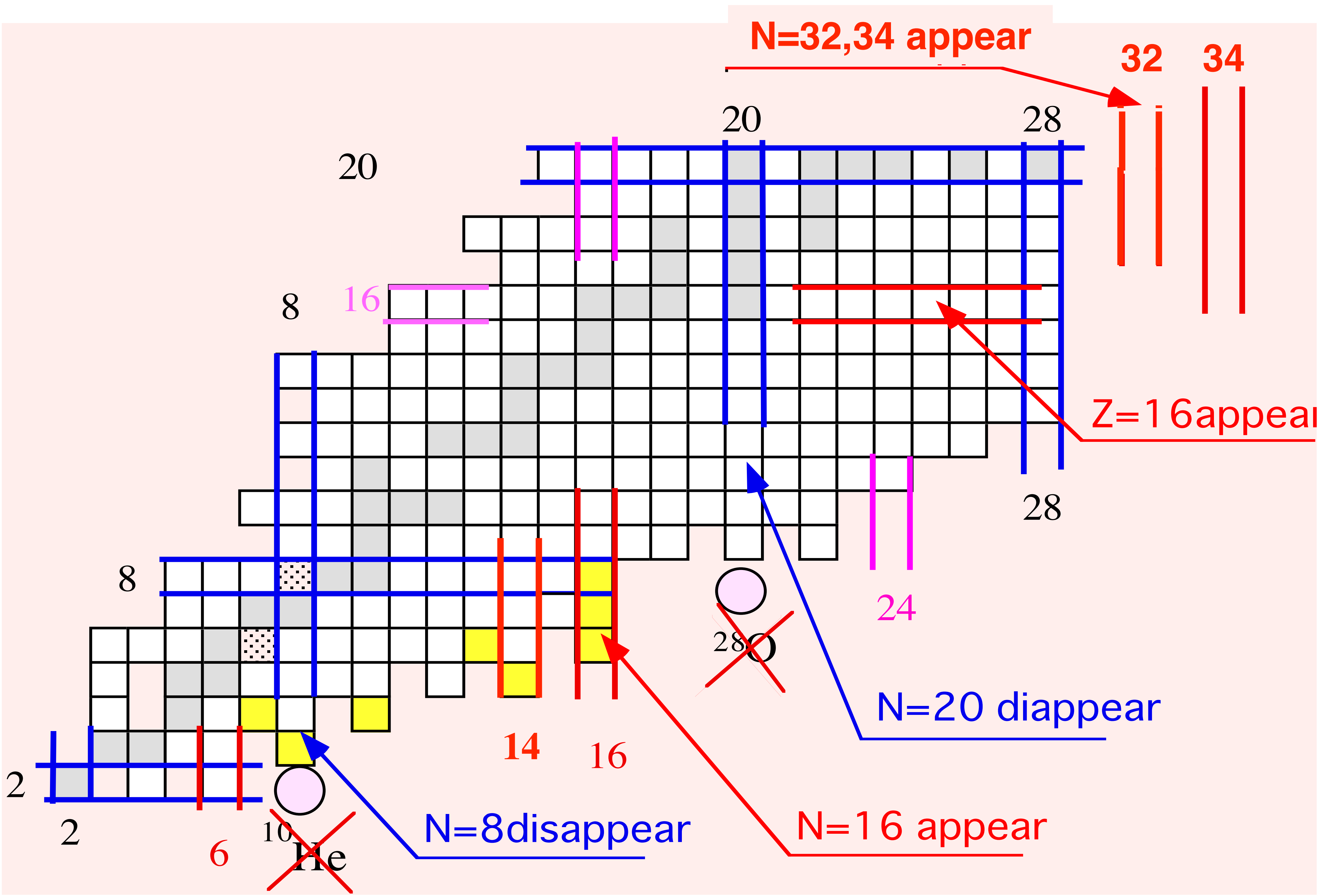
D wave has a large relative momentum

Beautiful magic numbers

Until ~1990



Studies with Radioactive Beams changed the Map of Nuclei



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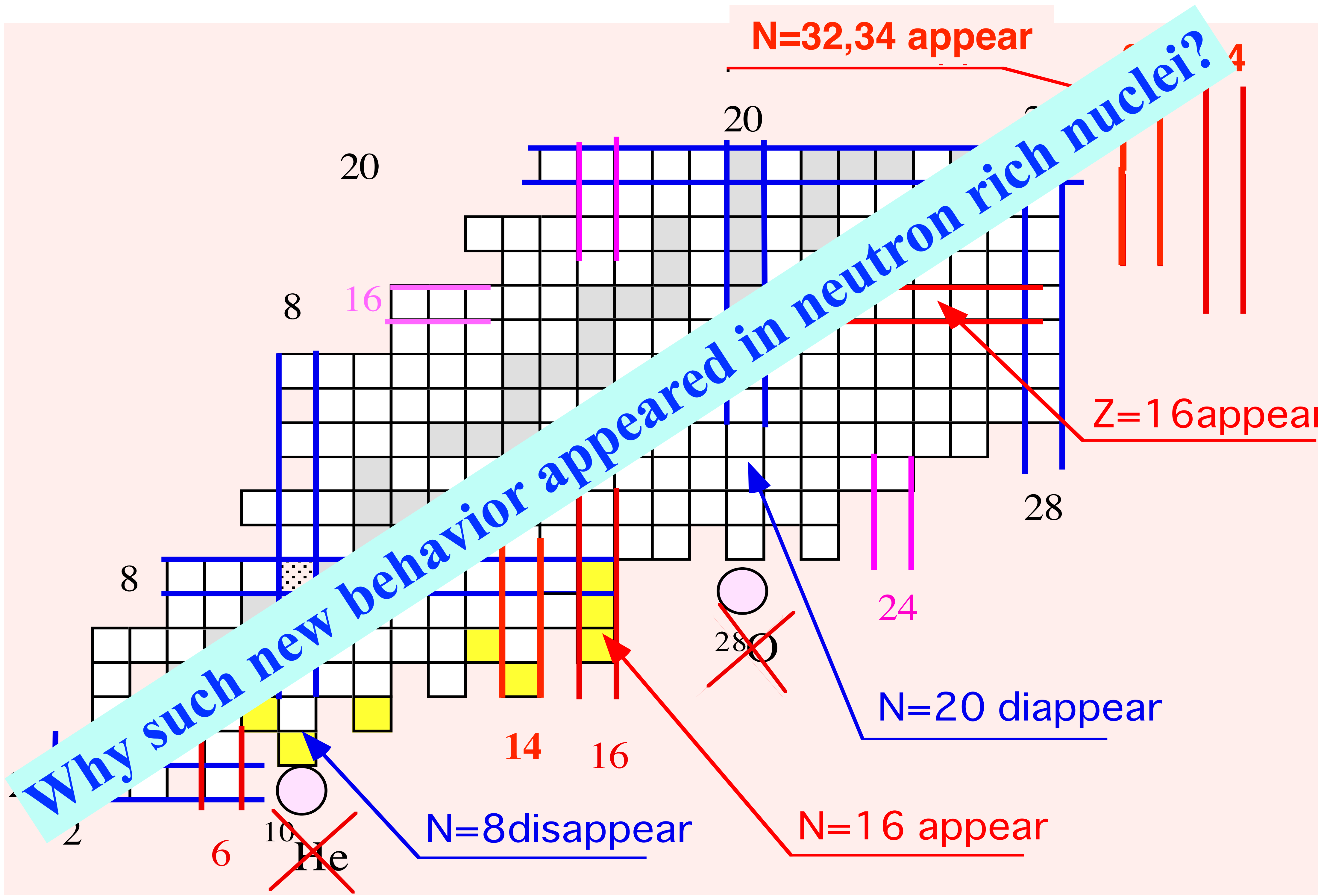
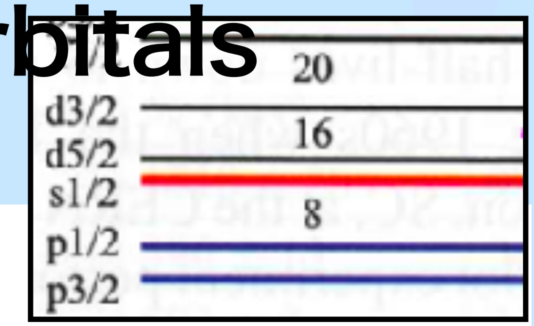


Known importance of tensor force in nuclei

Tanihata, H. Toki, T. Kajino, Handbook of nuclear physics, Springer Nature 2023, Section IX.

- Deuteron is not bound without tensor forces. S+D waves.
- Essential role for binding light nuclei.
- Isosceles magnetic moments of mirror nuclei.
- Spin-orbit coupling
- **Inversion of (1/2$^+$) state in ^{11}Be ground state** and $s_{1/2}$ and $p_{1/2}$ mixing in a neutron halo nucleus ^{11}Li.
- Disappearance of traditional magic numbers and appearance of new magic numbers in nuclei far from the stability line.
- **High-momentum components of nucleons in nuclei.**

Mystery of s-wave behavior in ^{11}Be and ^{11}Li orbitals



- **^{11}Be ground state is $1/2^+$ instead of $1/2^-$.**
  - *I. Talmi and I. Unna, PRL 4 (1960) 469.*

    They found that this change can be understood as shell model behavior due to single and two particle residual interactions. However no realistic calculations has been done.
  - *"Coral nucleon forces" by C. Foreseen, P. Navratilova, and W. E. Ormand, UCRL-JRNL-208555 (2004)* **.**

    does not succeed to make $1/2^+$ ground state.
  - *"First principle calculations using chiral 2-,3-nucleon interactions with continuum effects" by A. Calci, P. Navrátil, PRL 117 (2016) 242501.*
    - A tiny bit inversion of $1/2^+$.

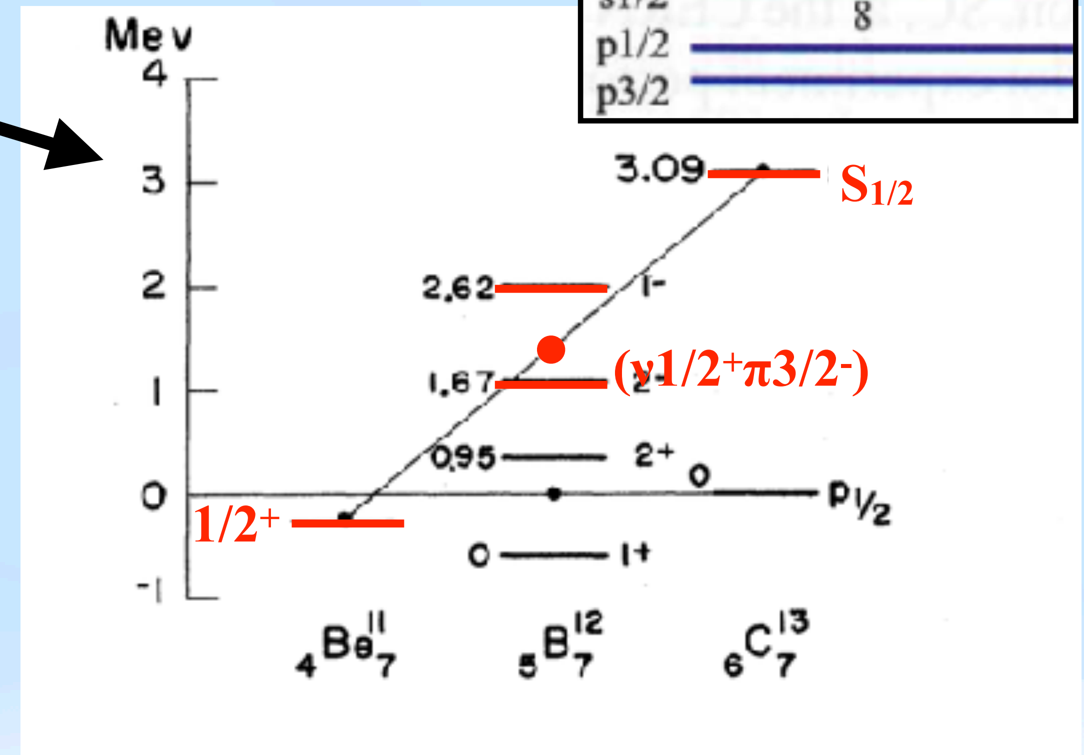
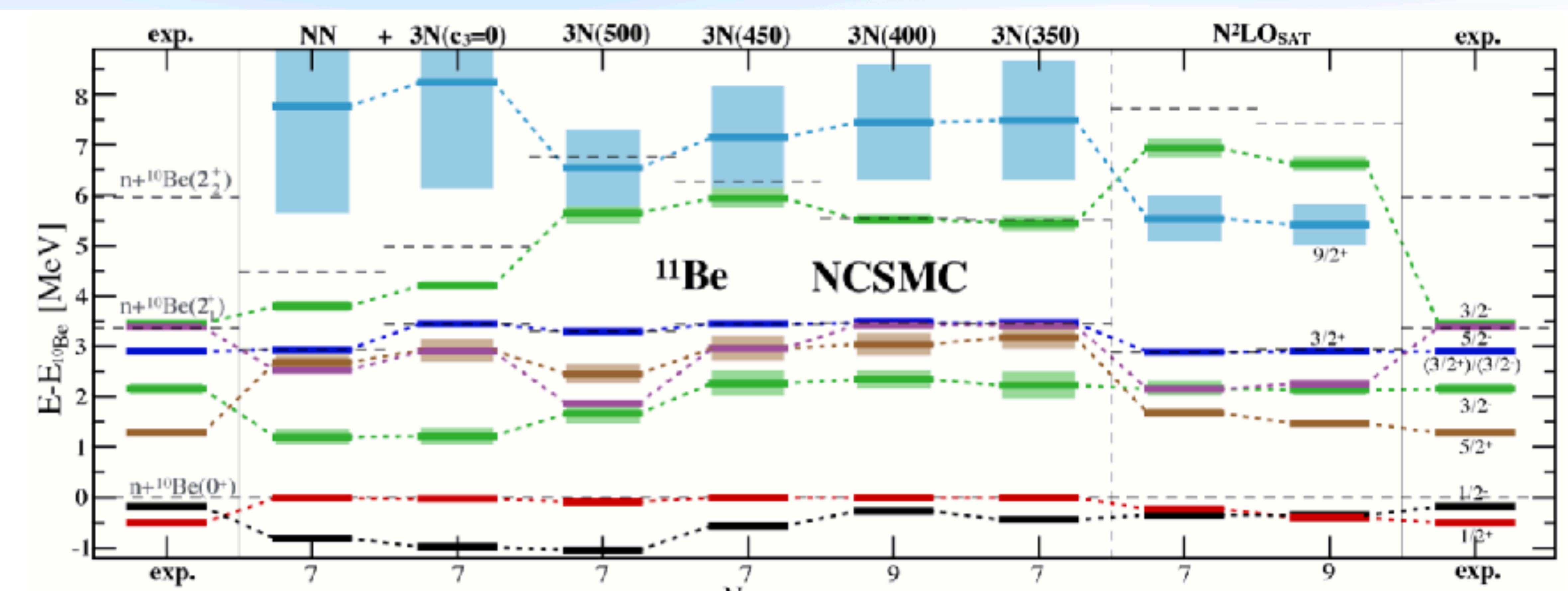


FIG. 1. Competition between $s_{1/2}$ and $p_{1/2}$ levels.



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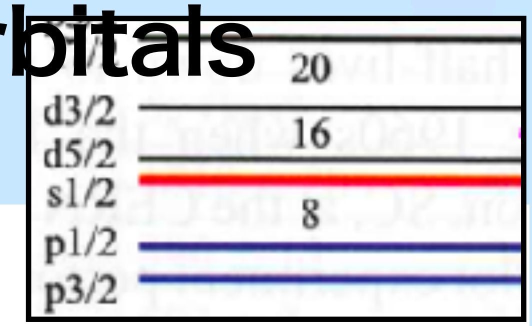

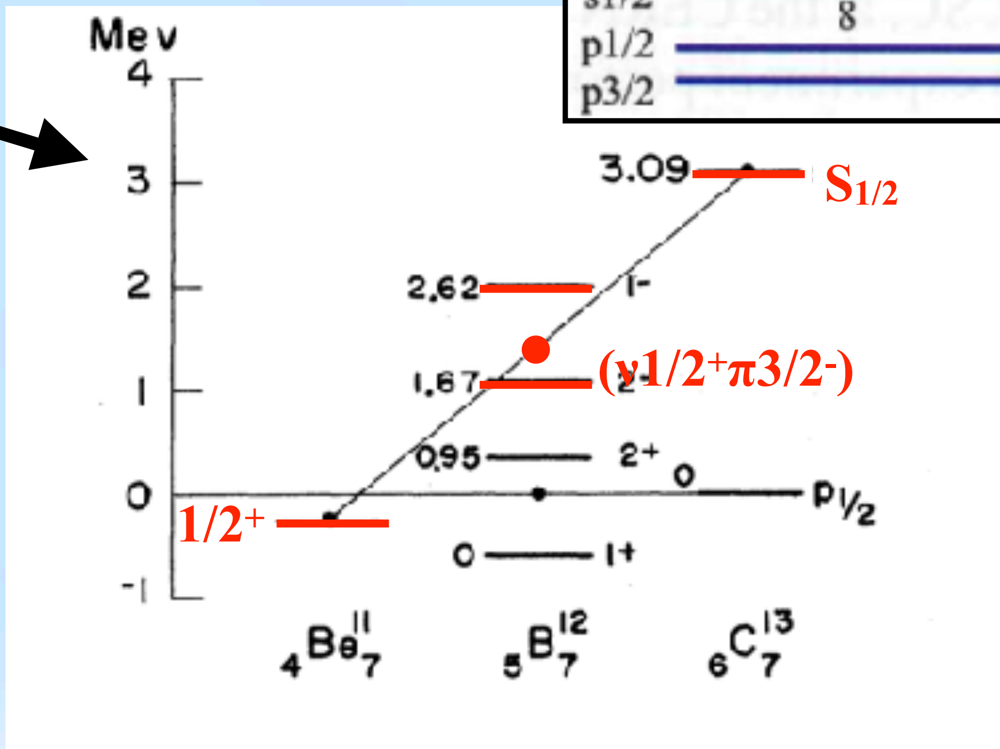

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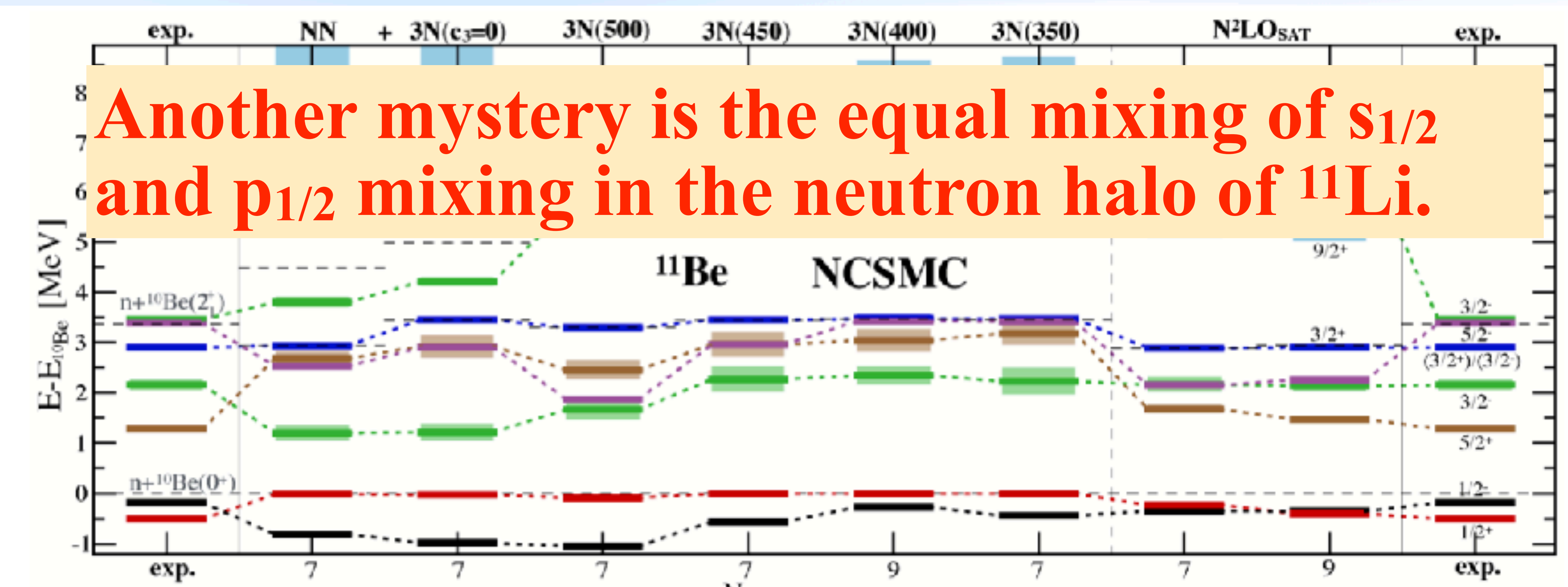


Another mystery is the equal mixing of $s_{1/2}$ and $p_{1/2}$ mixing in the neutron halo of ^{11}Li.

Tensor interactions

- **80% of binding energy of nuclei are from pion exchange interactions.**
  - ***C. Pieper and R. B. Wiringa, Ann. Rev. Null. Part. Sci. 51 (2001), VMC+GFMC.***
- **Pion interaction includes strong tensor interaction.**

$$\frac{\langle \Psi | V_{\pi} | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$

Tensor interactions from a view of Shell Structure

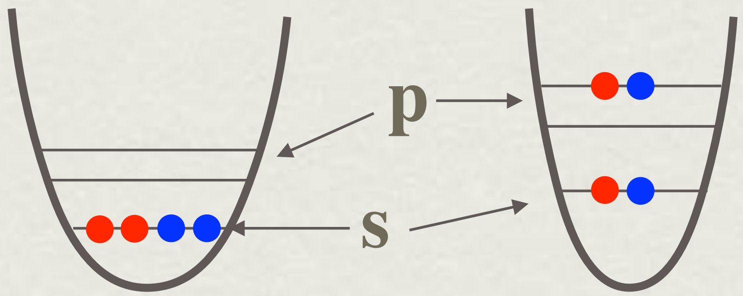
Tensor Optimized Shell Model (TOSM)

Myo, Toki, Ikeda, Kato, Sugimoto, PTP 117 (2006)

0p-0h + 2p-2h

$$\Phi(^4\mathrm{He}) = \Sigma_i\, C_i\, \psi_i(\{b_\alpha\}) \;=\; C_1\,(0s)^4 + C_2\,(0s)^2(\overline{0p}_{1/2})^2 + \cdots$$

size parameter: $b_{0s} \neq b_{\overline{0p}}$



Energy variation

$$H = \sum_{i=1}^{A} t_i - T_G + \sum_{i<j}^{A} v_{ij},$$

$$v_{ij} = v_{ij}^{C} + v_{ij}^{T} + v_{ij}^{LS} + v_{ij}^{Clmb},$$

$$\delta \frac{\langle\Phi|\, H\, |\Phi\rangle}{\langle\Phi|\Phi\rangle} = 0 \quad \Rightarrow \quad \frac{\partial\langle H - E\rangle}{\partial b_\alpha} = 0\,, \quad \frac{\partial\langle H - E\rangle}{\partial C_i} = 0.$$

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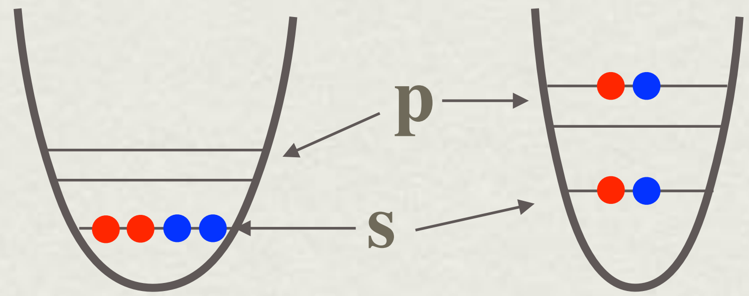

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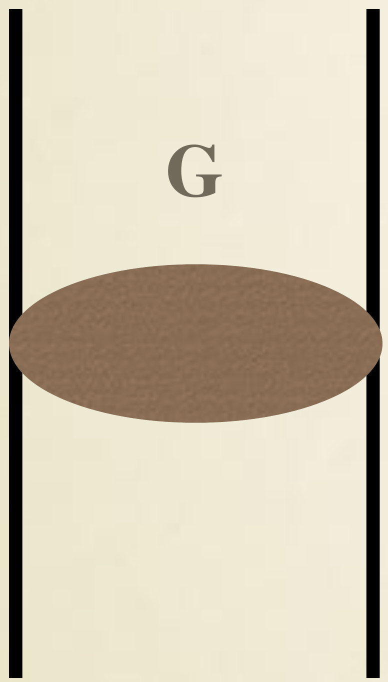
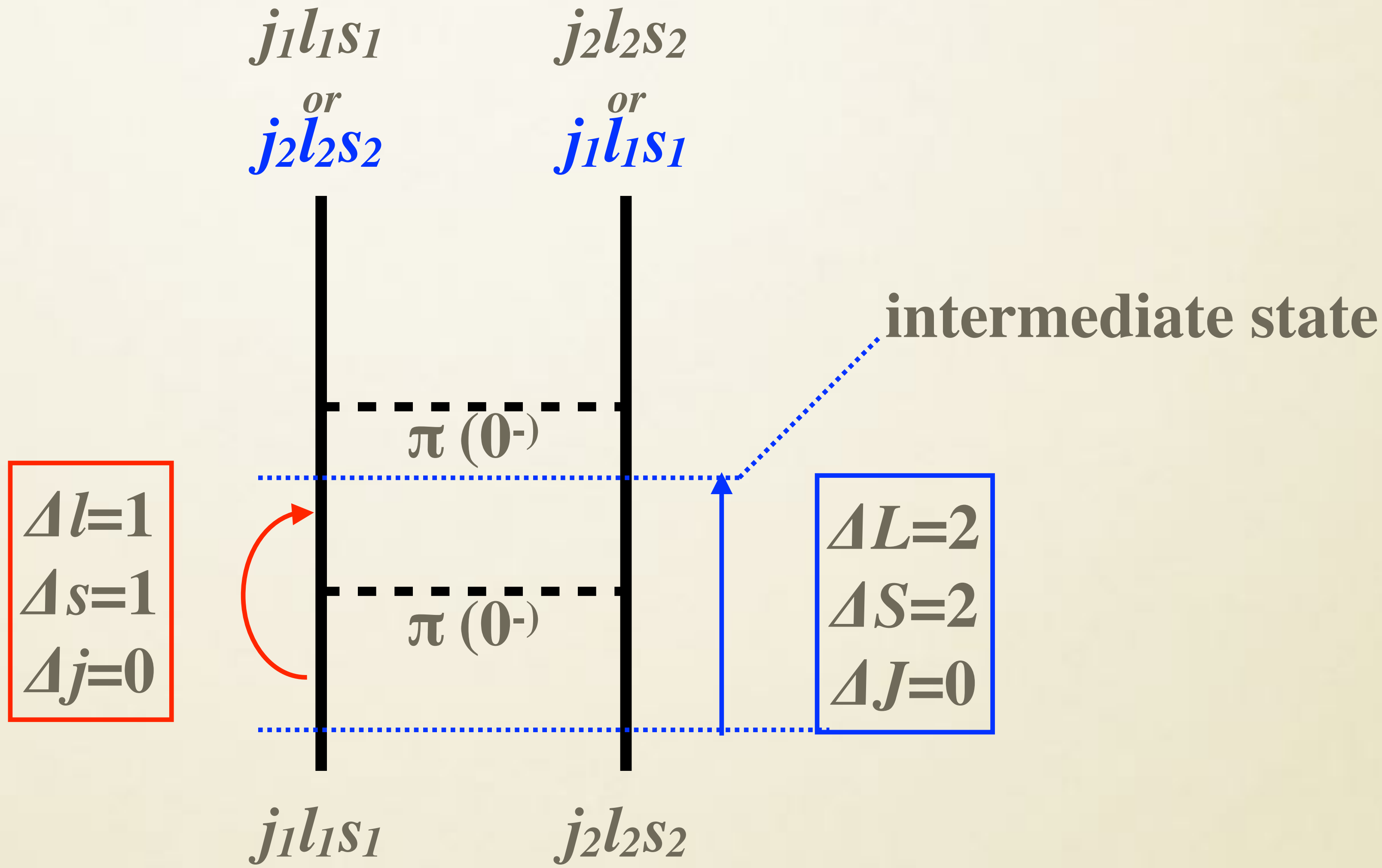
It is not a perturbation model!

Selection rule of the tensor interaction

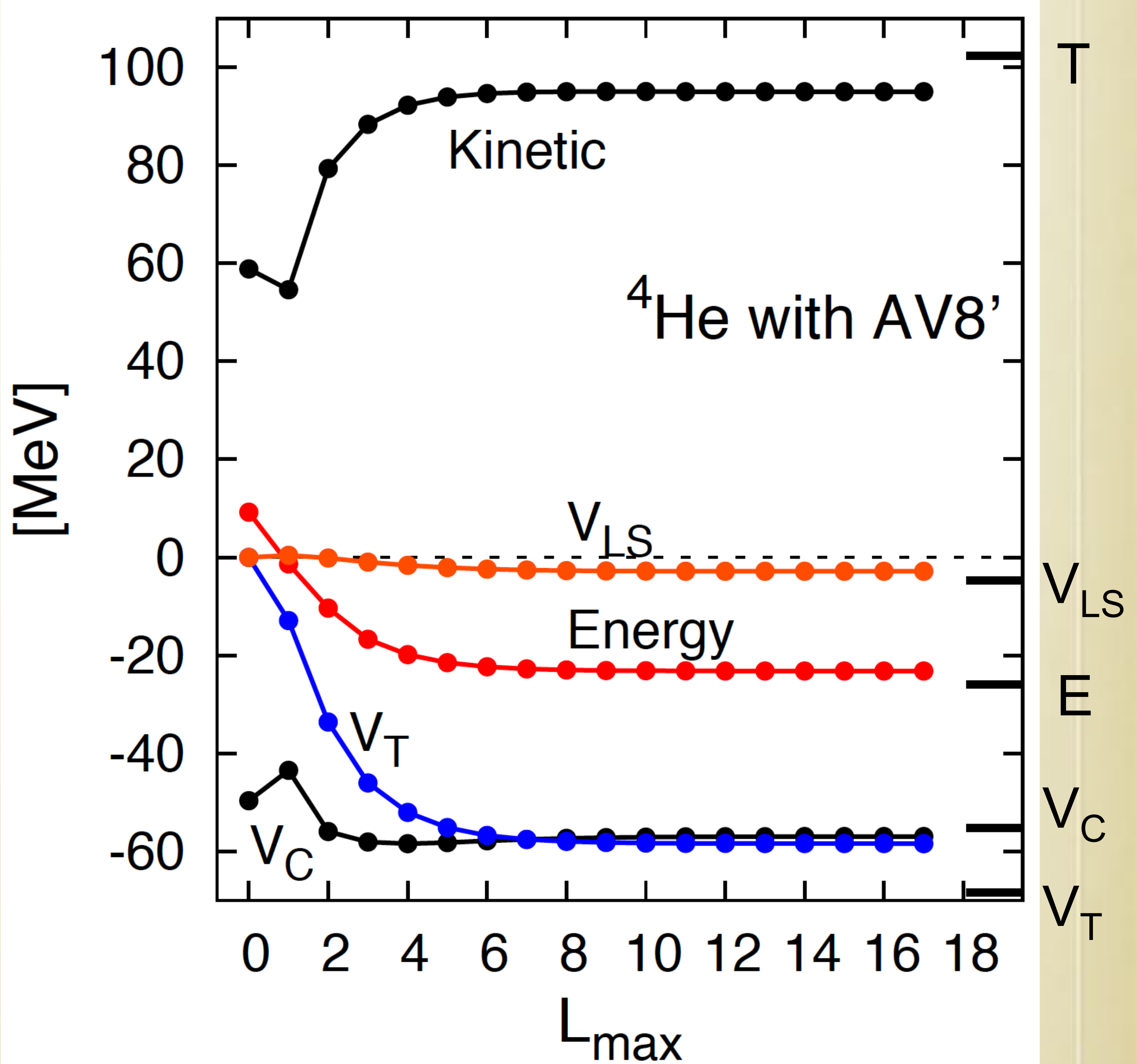




Tensor interaction in nucleus ^{4}He

- V_T contribute from higher *l* orbitals and convergence is slow.
- 2p-2h excitations of p-n pair under *ΔS*=2, *ΔL*=2 provide tensor energies.

- Tensor interactions give ~60 MeV of potential energy.
- **Remember** *l*=1 excitation already gives ~14 MeV of potential energy.
- Higher *l* contribute more but…



Tensor Optimized Shell Model by T. Myo, H. Toki and K. Ikeda, Progr. Theor. Phys. **121** 511 (2009)

The most important 2p-2h configuration

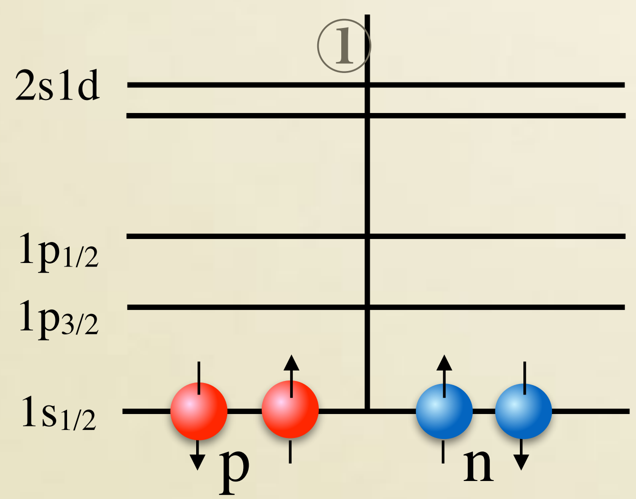
Configurations up to *l*=1

T. Myo, K. Kato, and K. Ikeda, PTP 113, (2005) 763.

$$\Psi(^4\mathrm{He}) = \sum_{i=1}^{6} a_i\, \Phi_i\ ,$$

$$\begin{aligned}
\Phi_1 &= (0s_{1/2})^4_{00}\ ; \\
\Phi_2 &= \left[(0s_{1/2})^2_{01}, (0p_{1/2})^2_{01}\right]_{00}\ ; \\
\Phi_3 &= \left[(0s_{1/2})^2_{10}, (0p_{1/2})^2_{10}\right]_{00}\ ; \\
\Phi_4 &= \left[(0s_{1/2})^2_{01}, (0p_{3/2})^2_{01}\right]_{00}\ ; \\
\Phi_5 &= \left[(0s_{1/2})^2_{10}, (0p_{3/2})^2_{10}\right]_{00}\ ; \\
\Phi_6 &= \left[(0s_{1/2})^2_{10}, \left[(0p_{1/2})(0p_{3/2})\right]_{10}\right]_{00}
\end{aligned}$$

^{4}He



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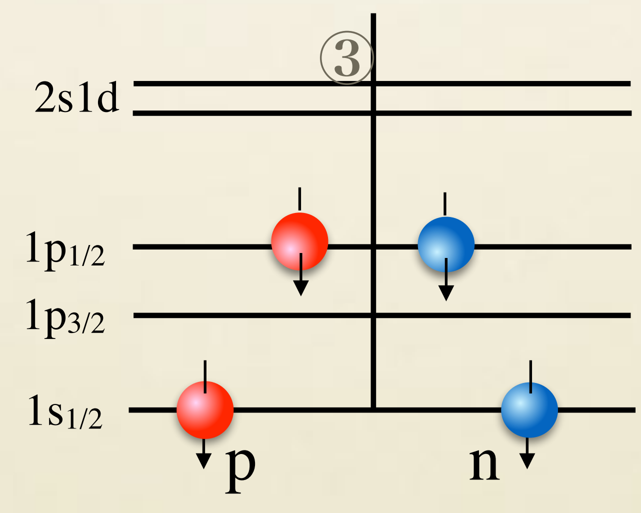
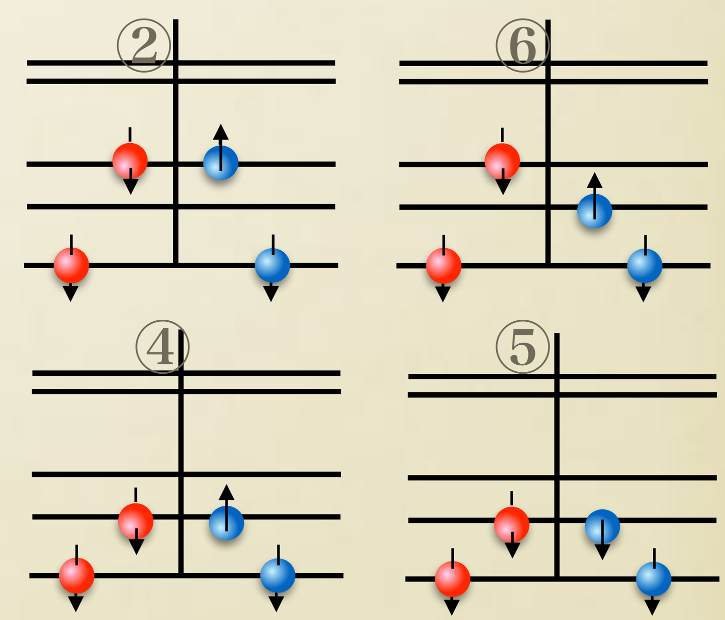


~10%



The most important 2p-2h configuration

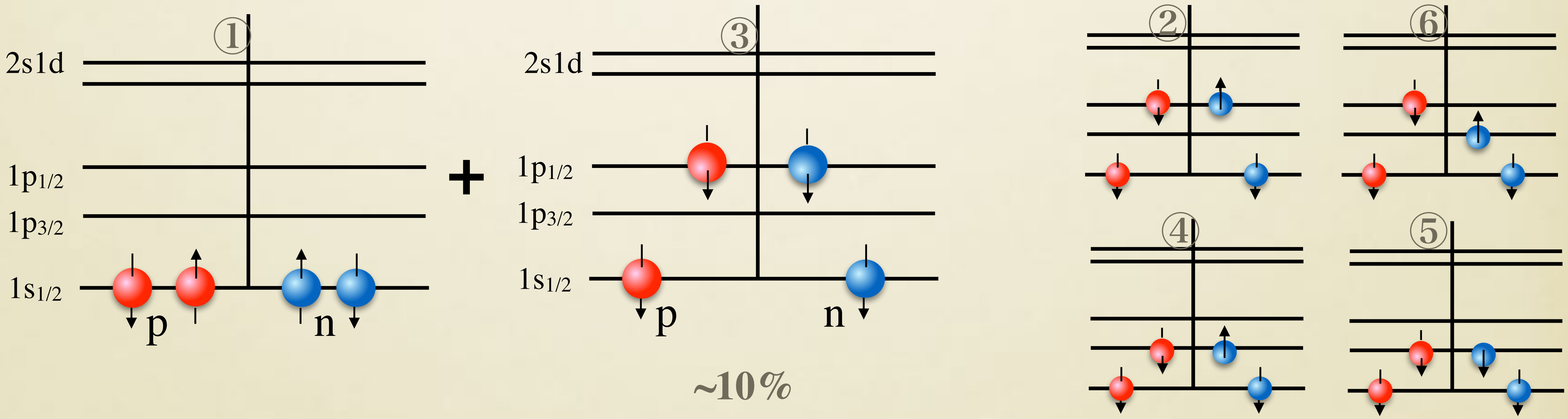
Configurations up to l=1

T. Myo, K. Kato, and K. Ikeda, PTP 113, (2005) 763.

$$\Psi(^4\mathrm{He}) = \sum_{i=1}^{6} a_i \, \Phi_i \; ,$$

Configuration	V_T [MeV]
$\Phi_1 = (0s_{1/2})^4_{00}$;	
$\Phi_2 = \left[(0s_{1/2})^2_{01}, (0p_{1/2})^2_{01}\right]_{00}$;	0.37
$\Phi_3 = \left[(0s_{1/2})^2_{10}, (0p_{1/2})^2_{10}\right]_{00}$;	14.49
$\Phi_4 = \left[(0s_{1/2})^2_{01}, (0p_{3/2})^2_{01}\right]_{00}$;	0.19
$\Phi_5 = \left[(0s_{1/2})^2_{10}, (0p_{3/2})^2_{10}\right]_{00}$;	1.67
$\Phi_6 = \left[(0s_{1/2})^2_{10}, \left[(0p_{1/2})(0p_{3/2})\right]_{10}\right]_{00}$	0.09

^{4}He



Highest spin orbital ($j_>$) in a major shell is not used for the tensor interaction.
An example is $1p_{3/2}$ orbital in ^{4}He and deuteron.

$$\Psi(^4\mathrm{He}) = \sum_{i=1}^{6} a_i \Phi_i \ ,$$

V_T [MeV]

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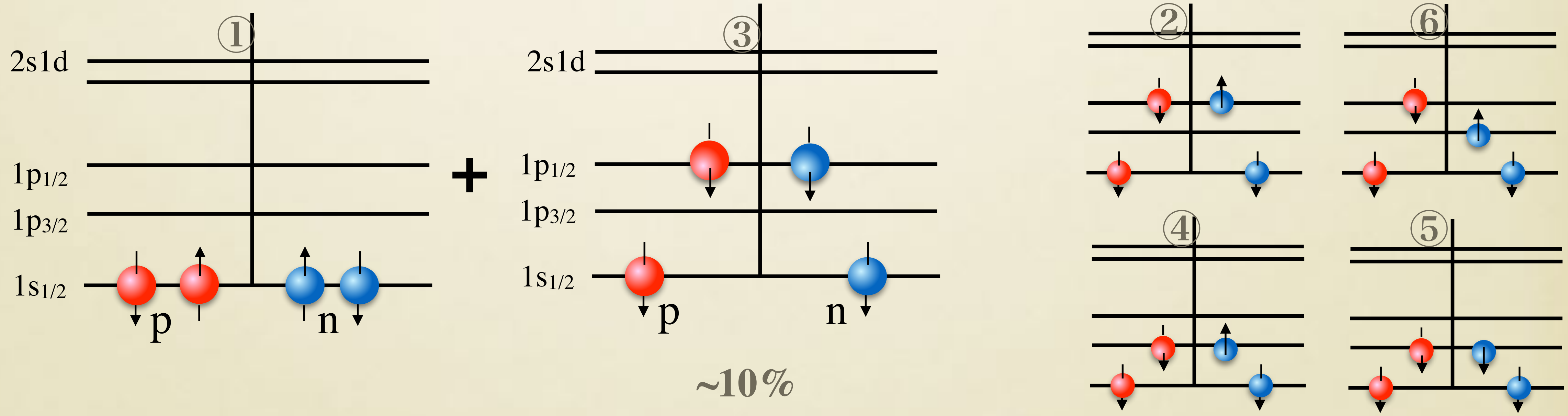
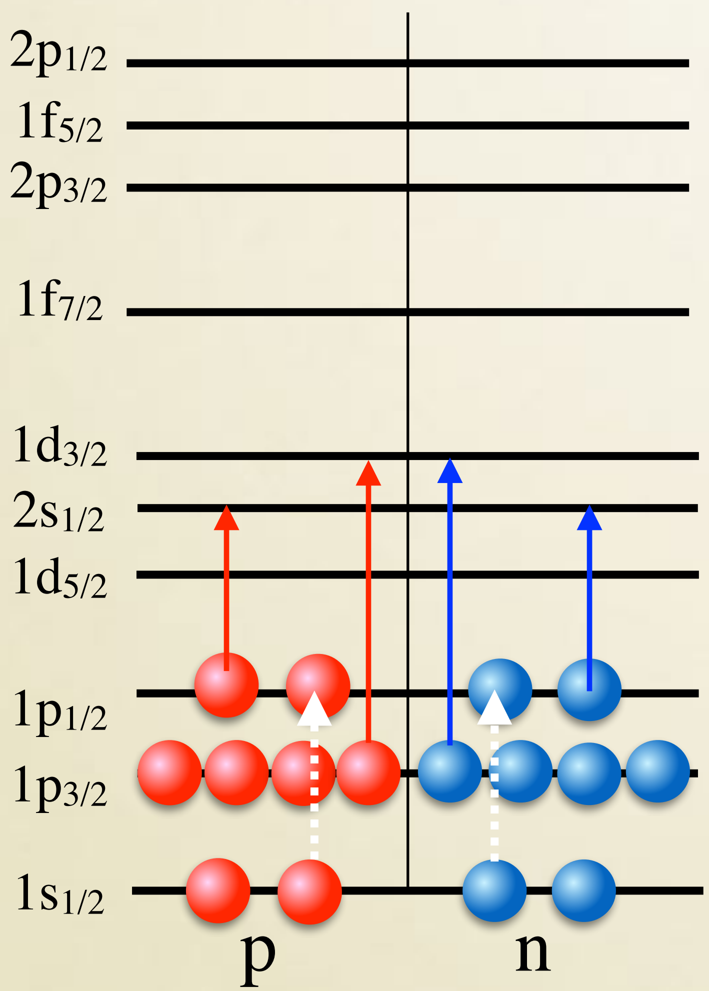
^{4}He



In more general p-n pairs from $(nlj)^2$ configuration to $(n+1,l+1,j)^2$ or $(n+1,l-1,j)^2$

What is the difference between stable and neutron rich nuclei?

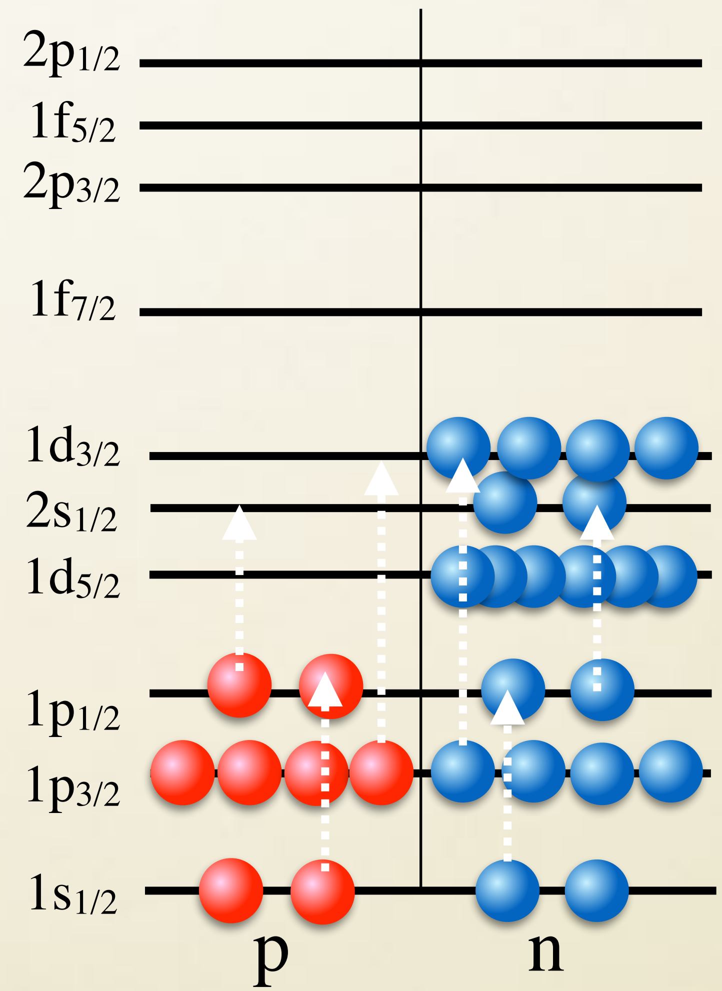
Symmetric nuclei



^{16}O

Blocking and Opening occur simultaneously.

Neutron rich nuclei

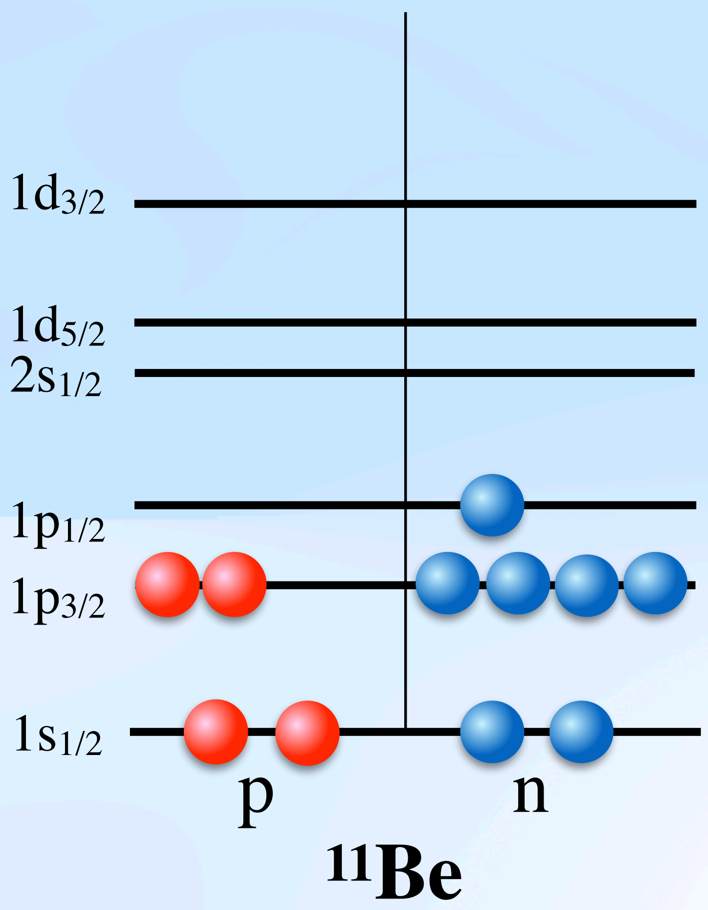


^{28}O

Only tensor blocking occurs.

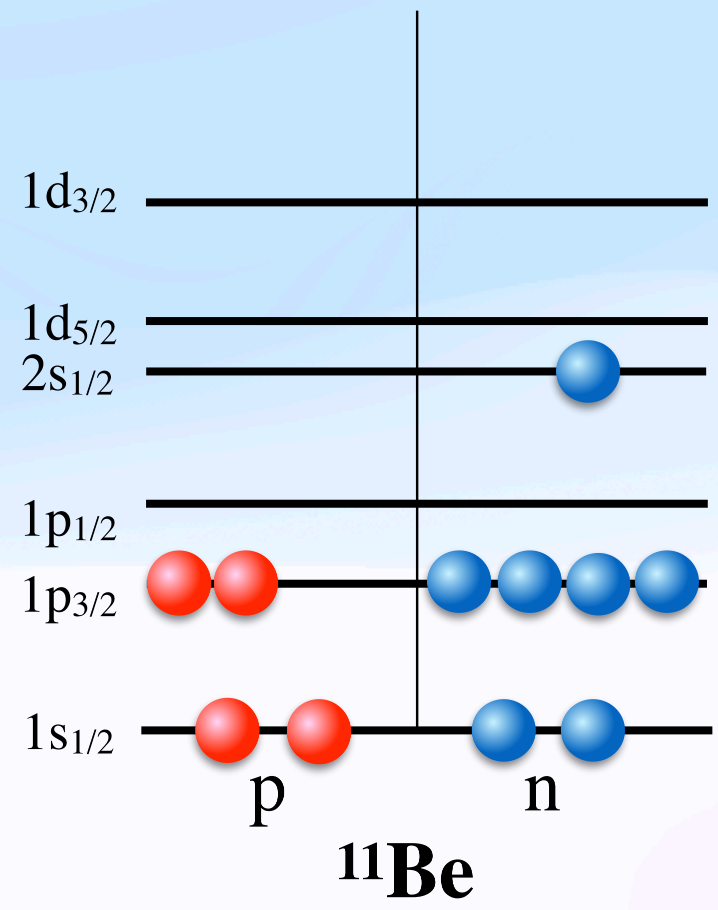
Explanation of $s_{1/2}$ anomaly

Conf. A



or

Conf. B



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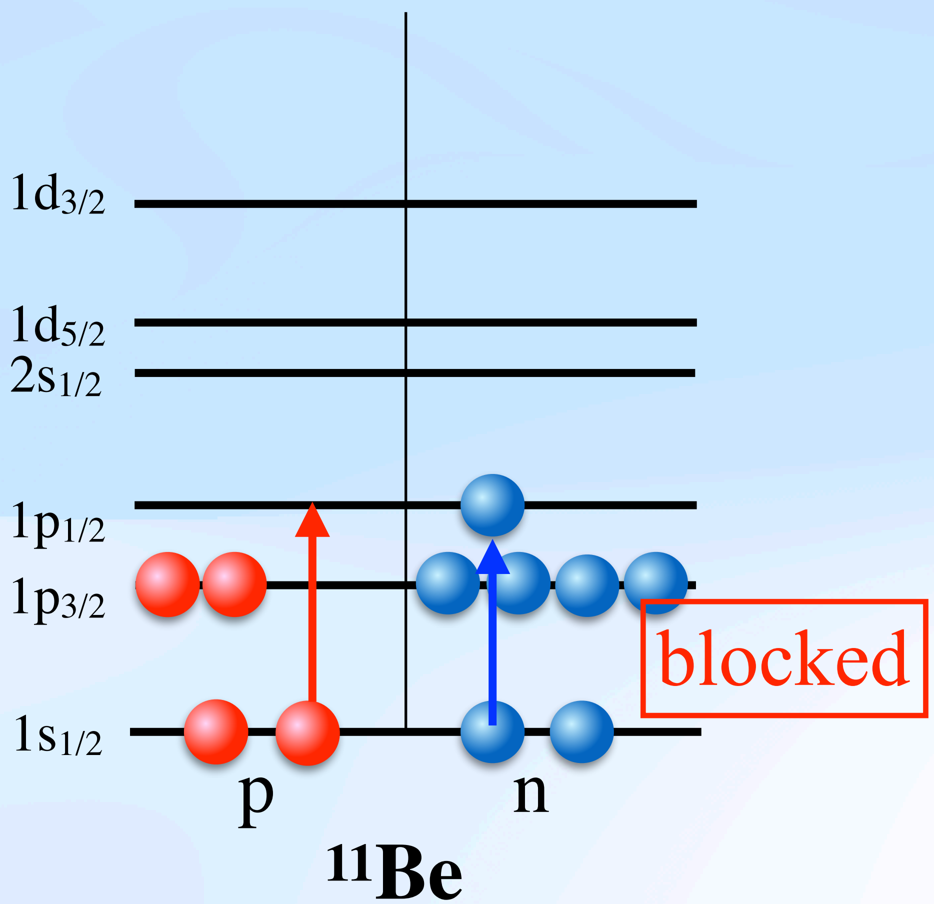

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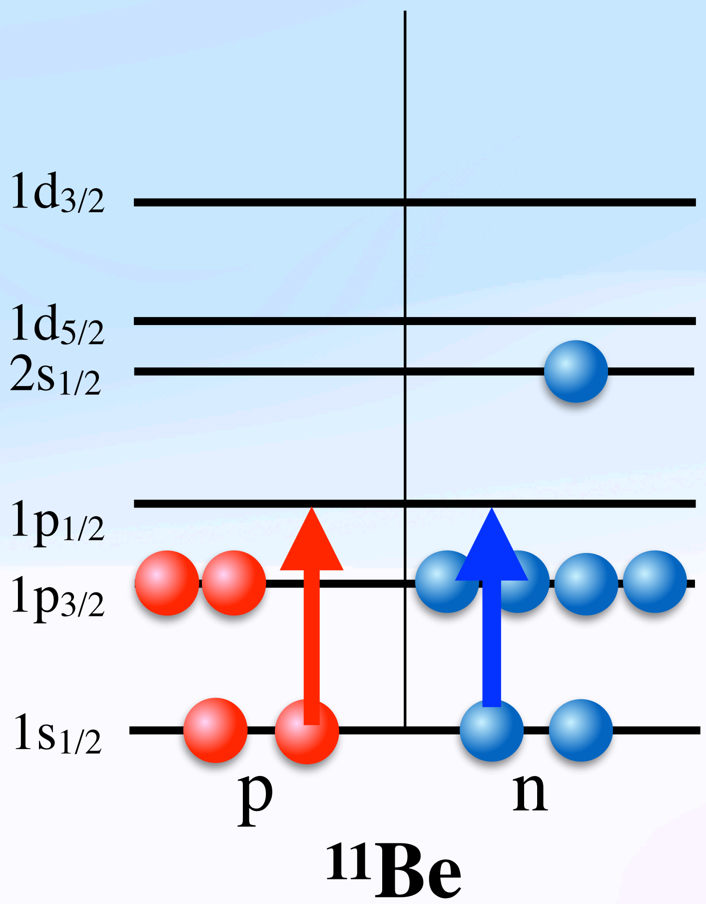
Conf. B



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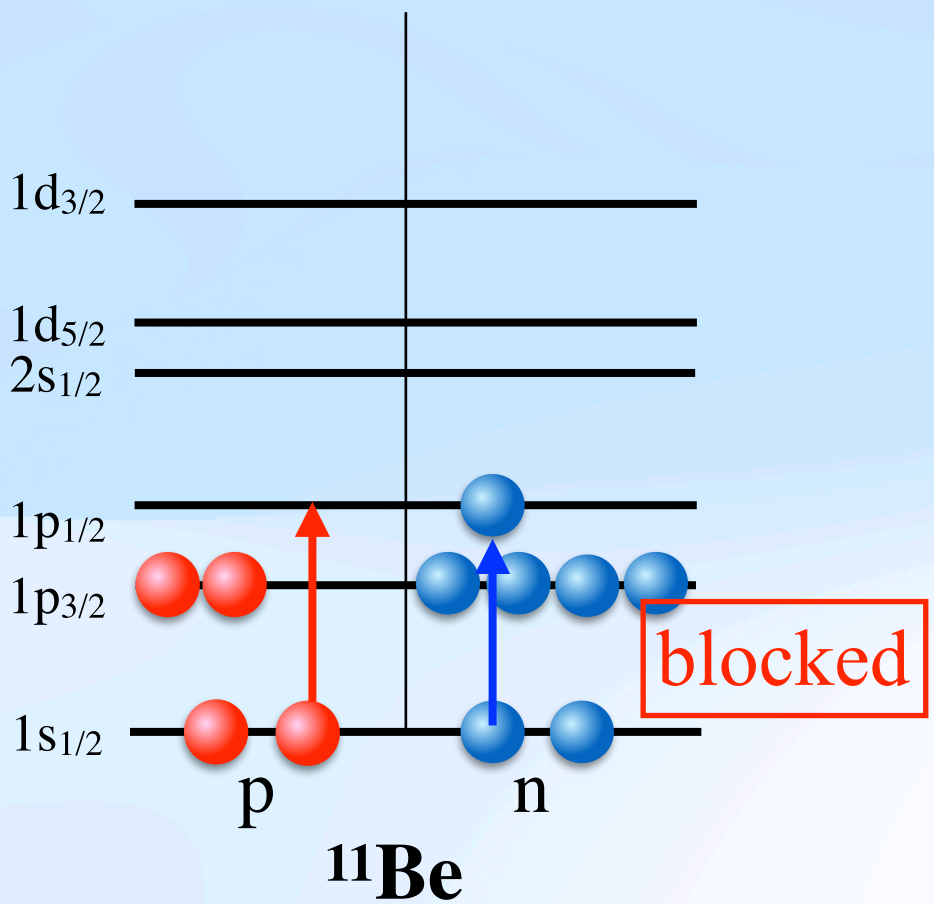

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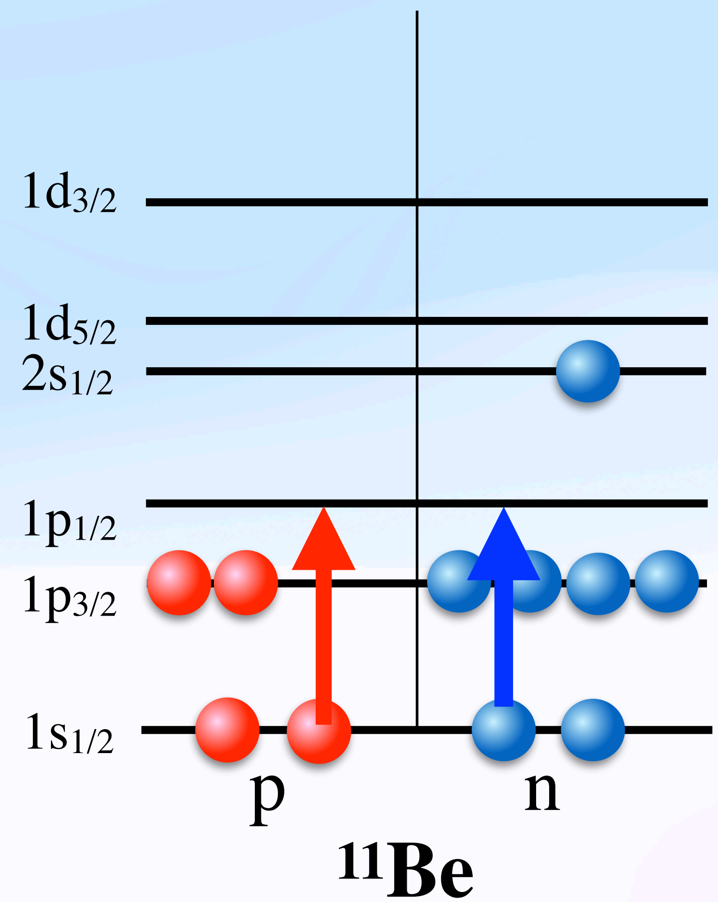

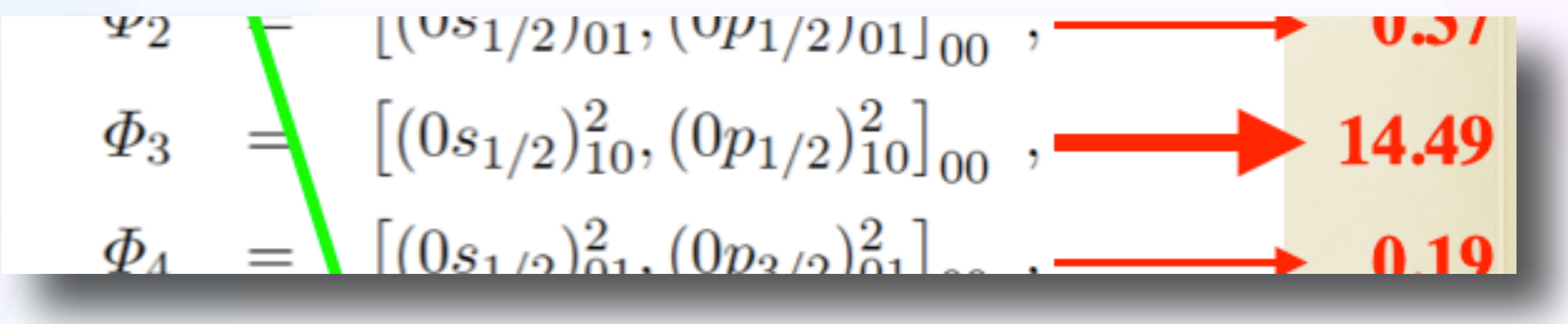

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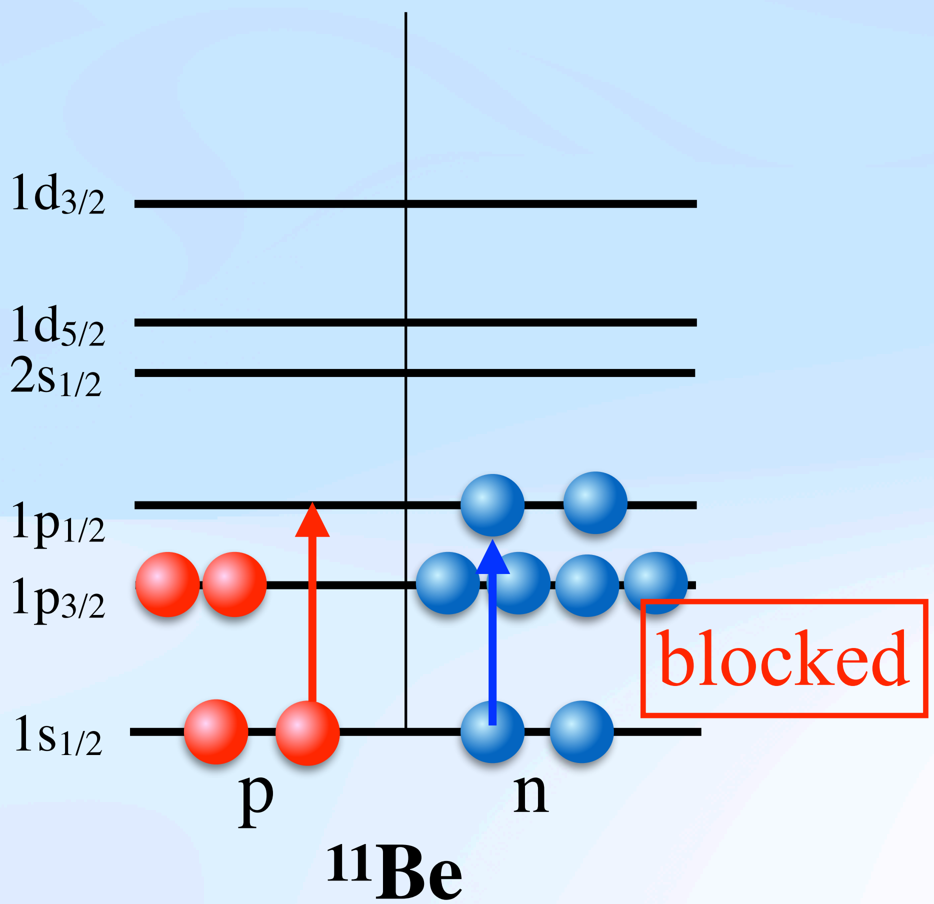
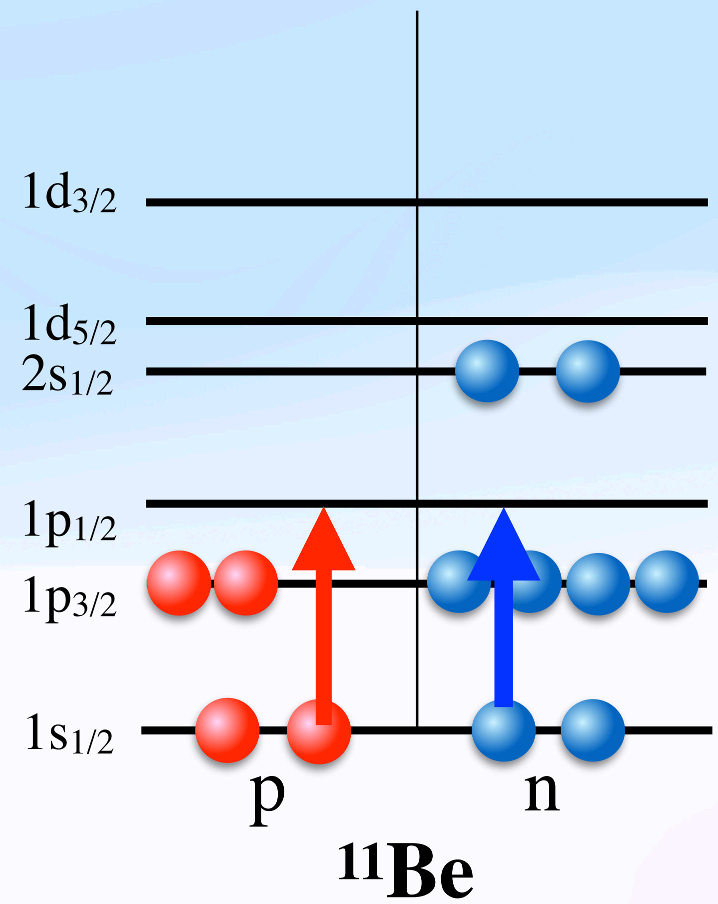
Conf. A

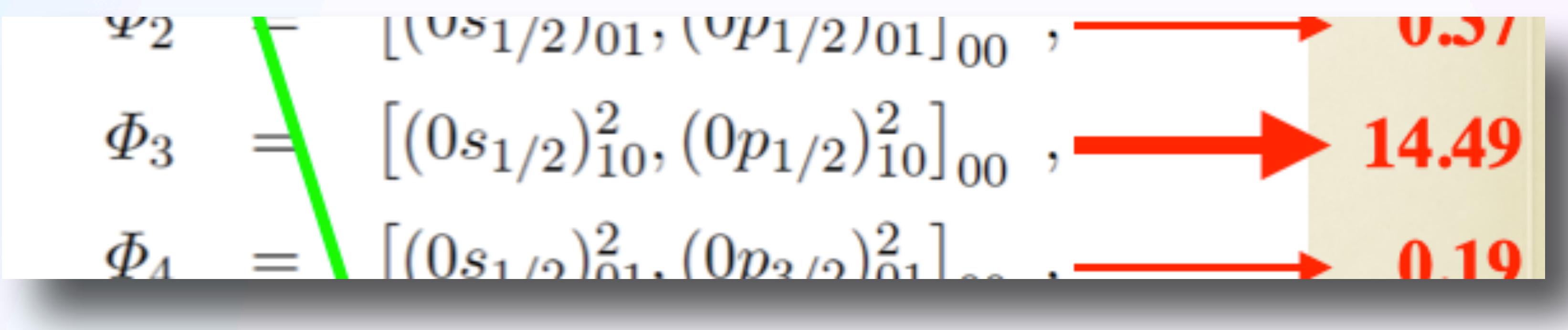


^{11}Li

or

Conf. B





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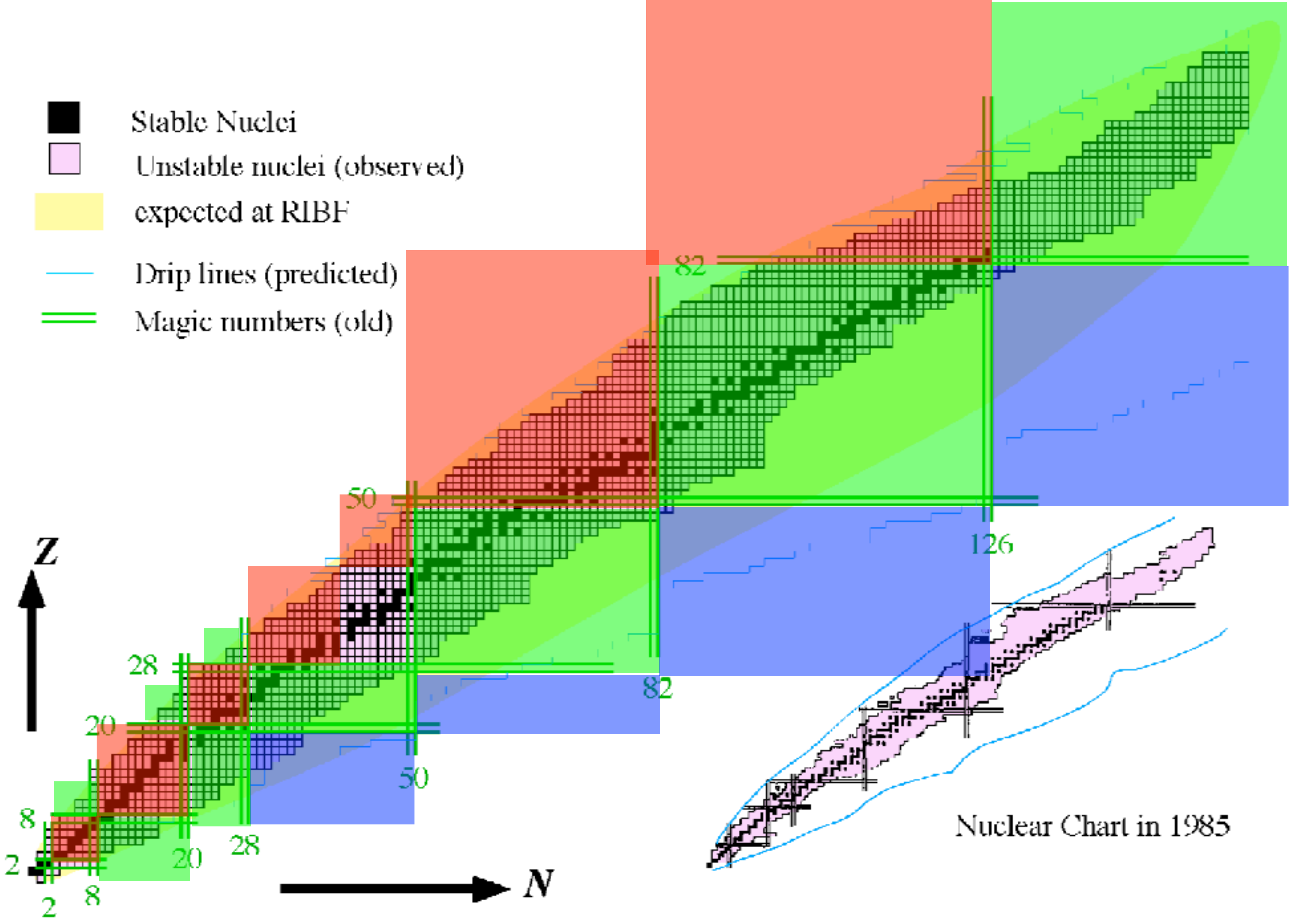
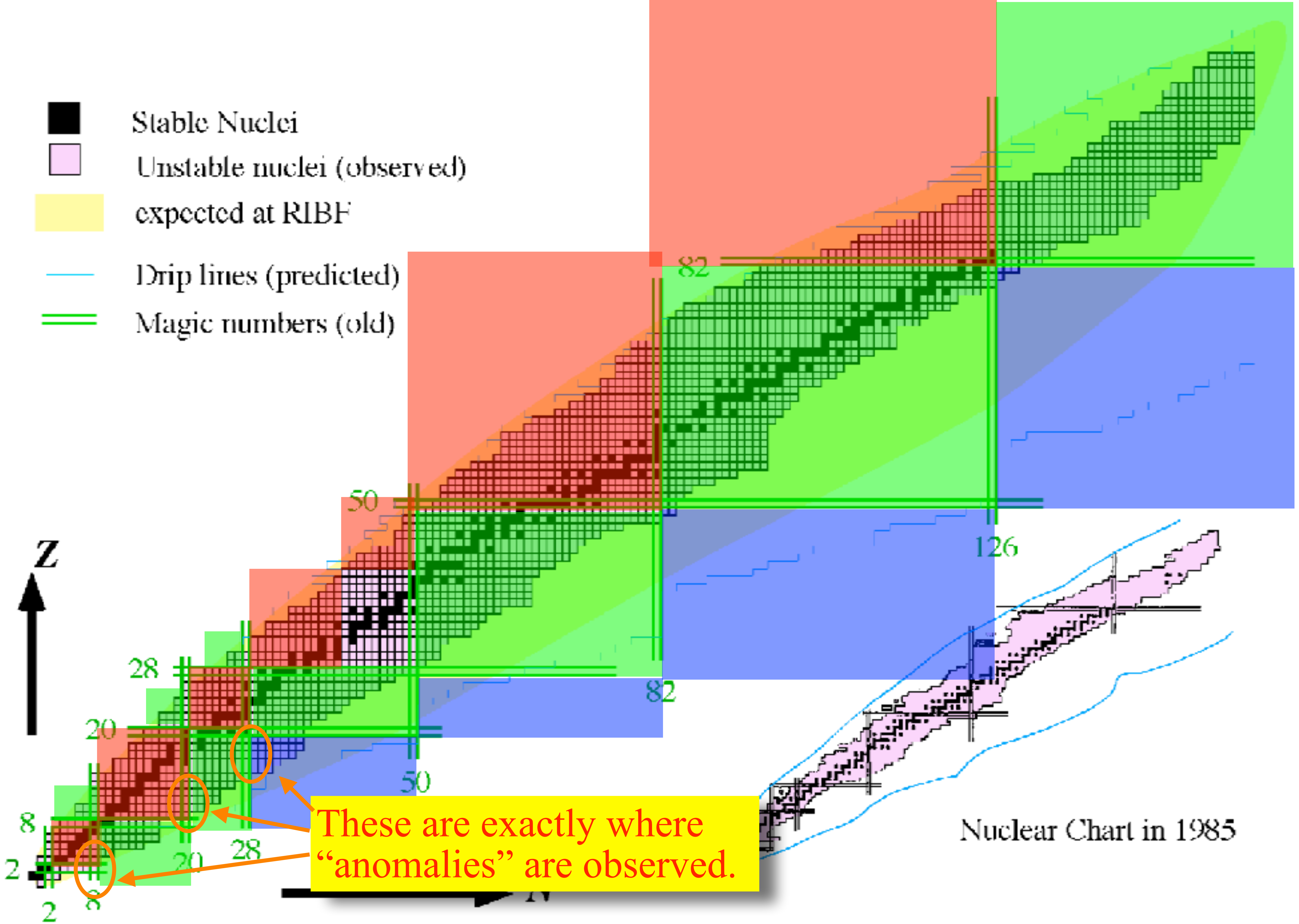

Stable Nuclei
Unstable nuclei (observed)
expected at RIBF
Drip lines (predicted)
Magic numbers (old)
Z
N
2
8
20
28
50
82
126
Nuclear Chart in 1985



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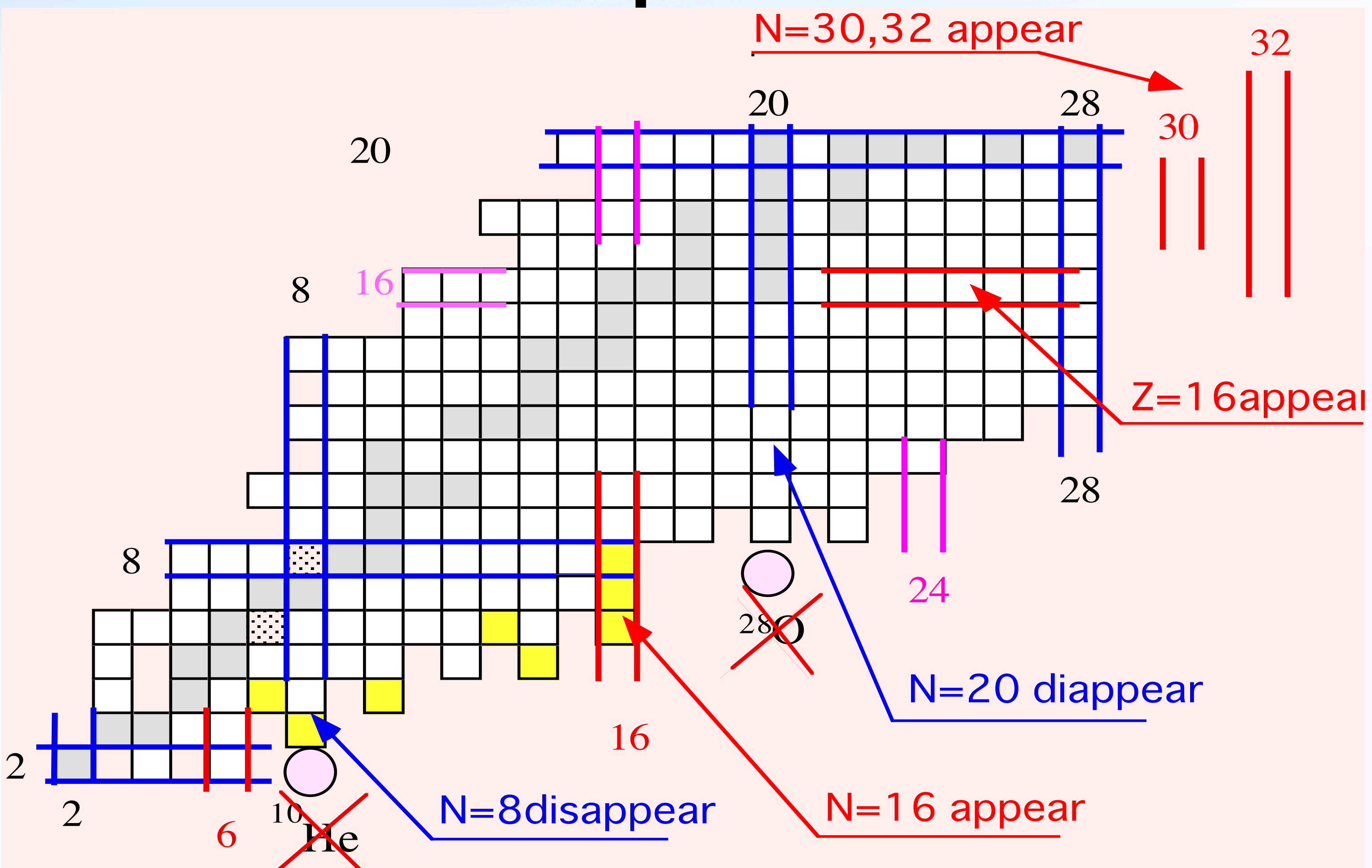
50
82
126
These are exactly where "anomalies" are observed.
Nuclear Chart in 1985

Tensor blocking explains not only the behavior of $s_{1/2}$ orbital but also explains,

- **Why doubly magic number nuclei ^{10}He, ^{28}O is unbound,**
- **Why neutron magic numbers 8, 20 disappears in neutron-rich nuclei,**
- **Appearance of new magic numbers 6, 14, 16, 32, 34,**
- **Why drip line suddenly extend a lot in F isotopes.**



I. Tanihata, J. of Physics. CS vol.1643 (2020) 012109.

High momentum component in ground states of nuclei
= Shell dependent contributions =

- **The existence of high-momentum components in nucleons is observed in e-scattering and proton scattering.**
  - ***J. Arrington, N. Somin, and A. Schmidt, Ann. Rev. Nucl. Part. Sci. 72 (2022) 307.***

- **High momentum pairs are there in ground states of nuclei.**
- **High-momentum pairs are important to provide a large amount of binding energy through tensor interactions.**

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In general this effect is considered just to cut the shell model space restricting only low momentum nucleons. Say Depletion of spectroscopic factors.

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Ground state properties are strongly affected by High-Momentum Nucleons.

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- **High momentum pairs are there in ground states of n**
- **High-momentum pairs are important to provi... of binding energy through tensor interactio...**

Can we see enhancement of high-momentum nucleons in 2p-2h excitations?

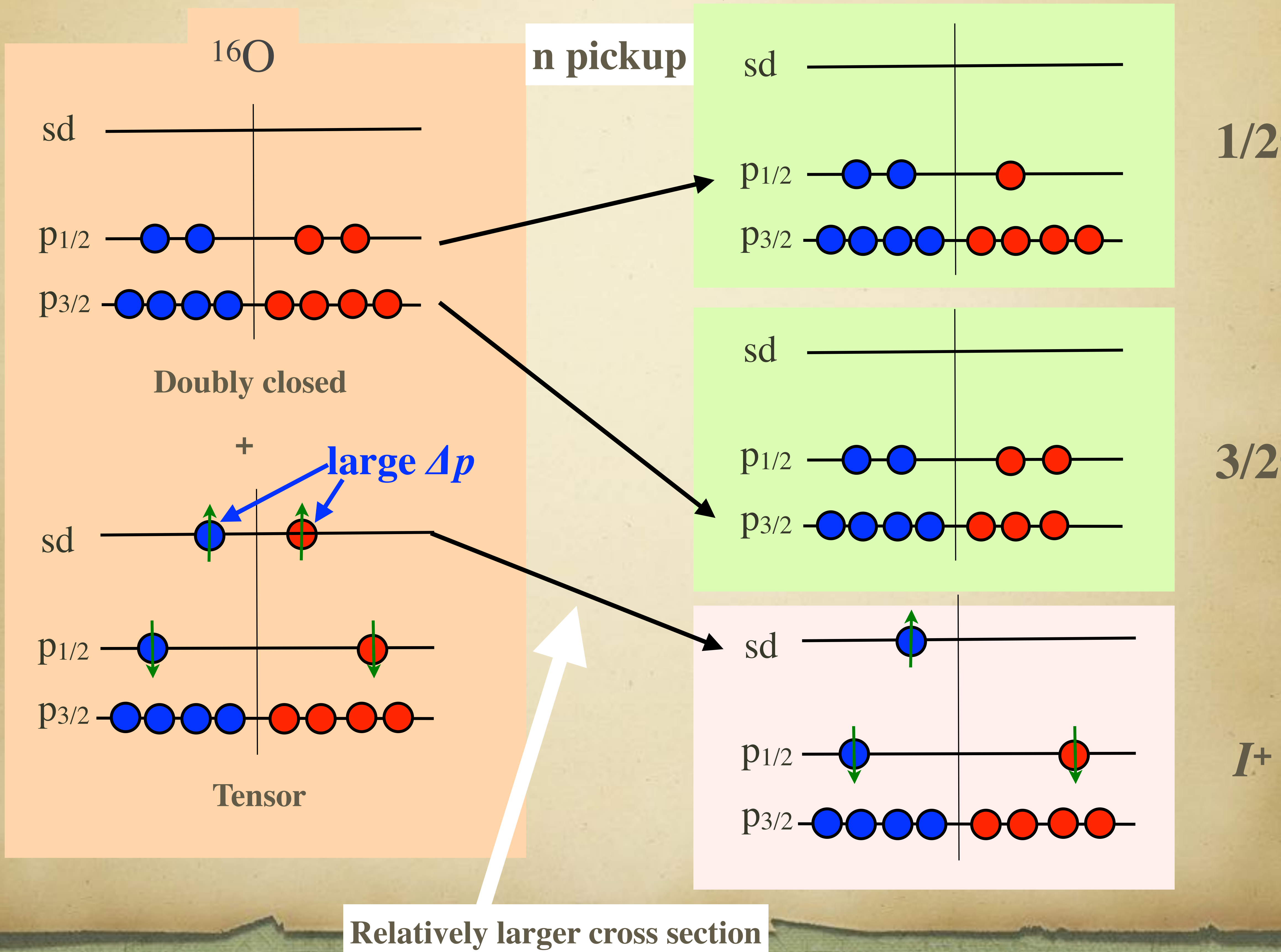
... properties are strongly affected ... y High-Momentum Nucleons.

- **Because of the strong selection rules, tensor interactions affect orbitals differently based on their quantum numbers.**

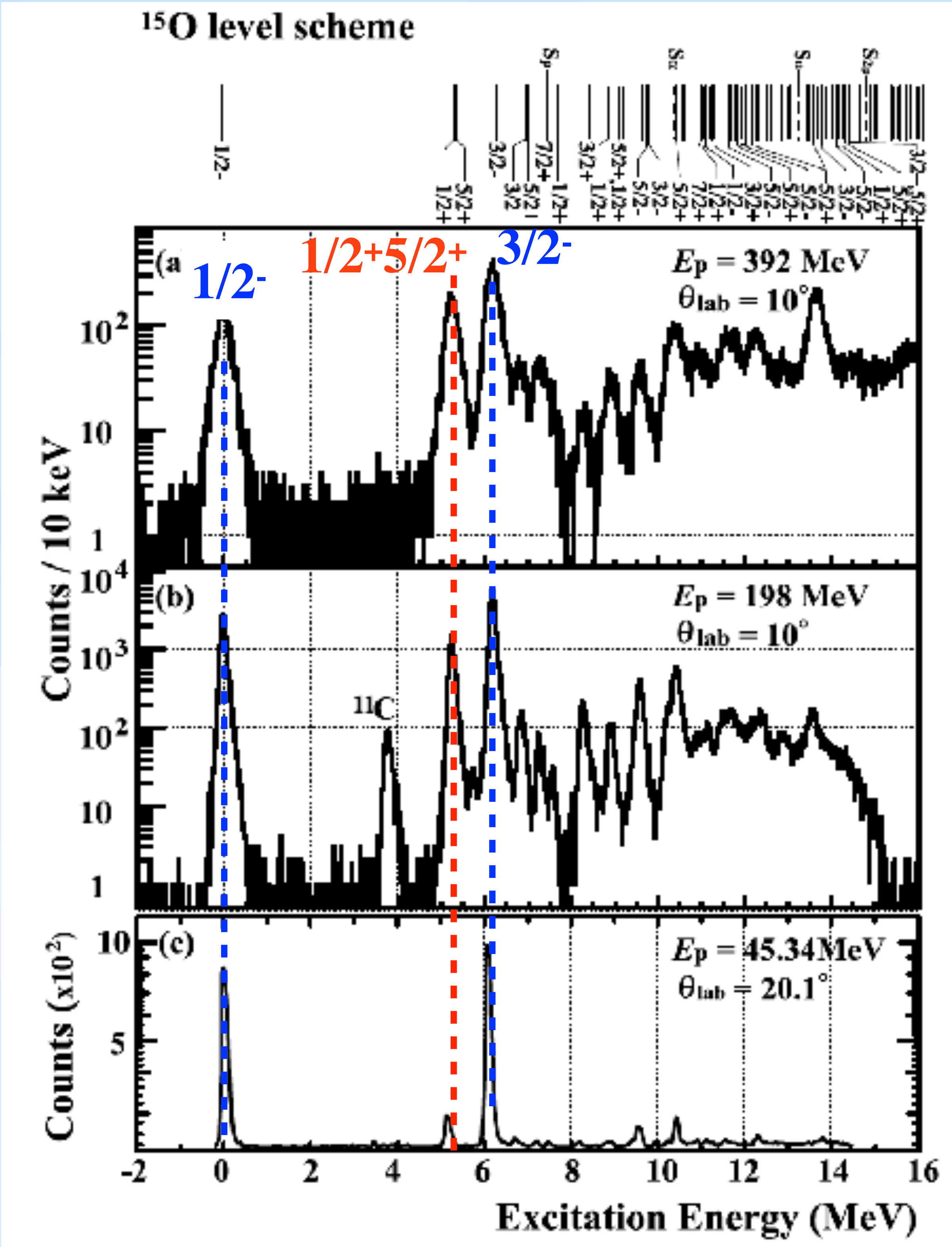
- **State by state differences of the tensor contributions make exotic behaviors of nuclear structure.**

- **It can be observed as state by state difference of high-momentum components.**

^{16}O and (p,d) reaction



Energy dependence of the cross sections



Study the ratio of the positive parity/negative parity final states

H.-J. Ong et al., Phys. Lett. B 725, 277-281 (2013)

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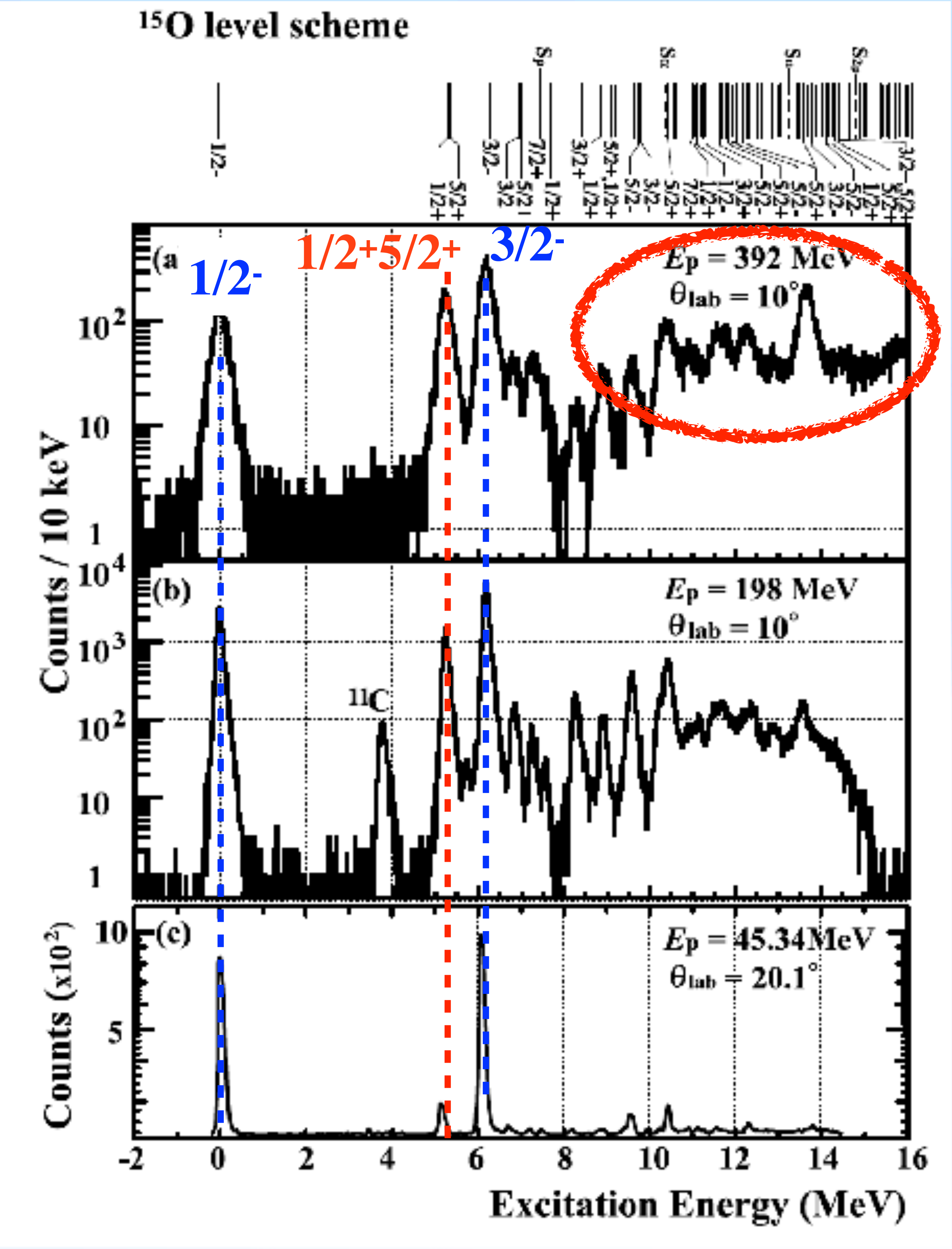

Study the ratio of the positive parity/negative parity final states

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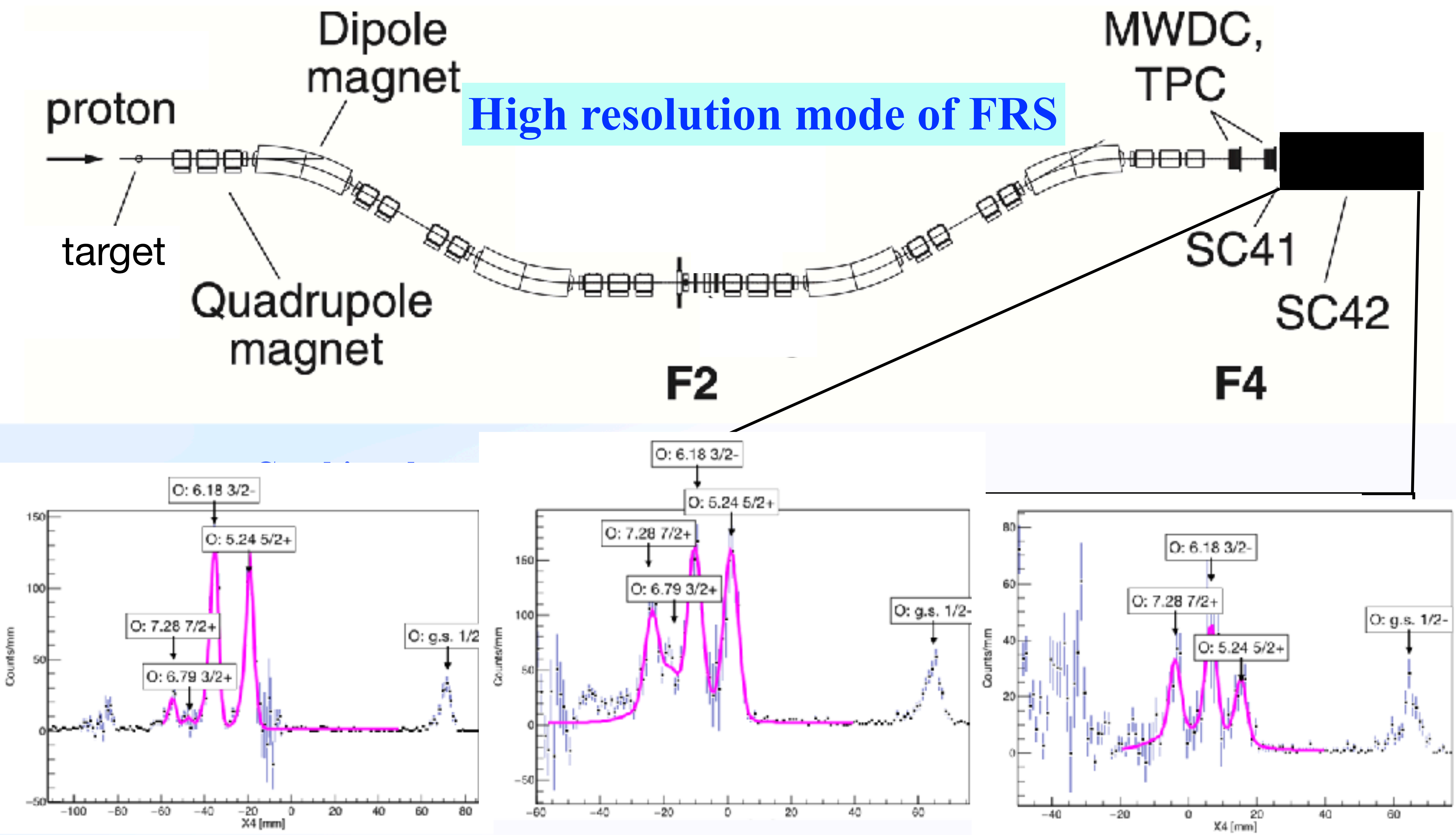
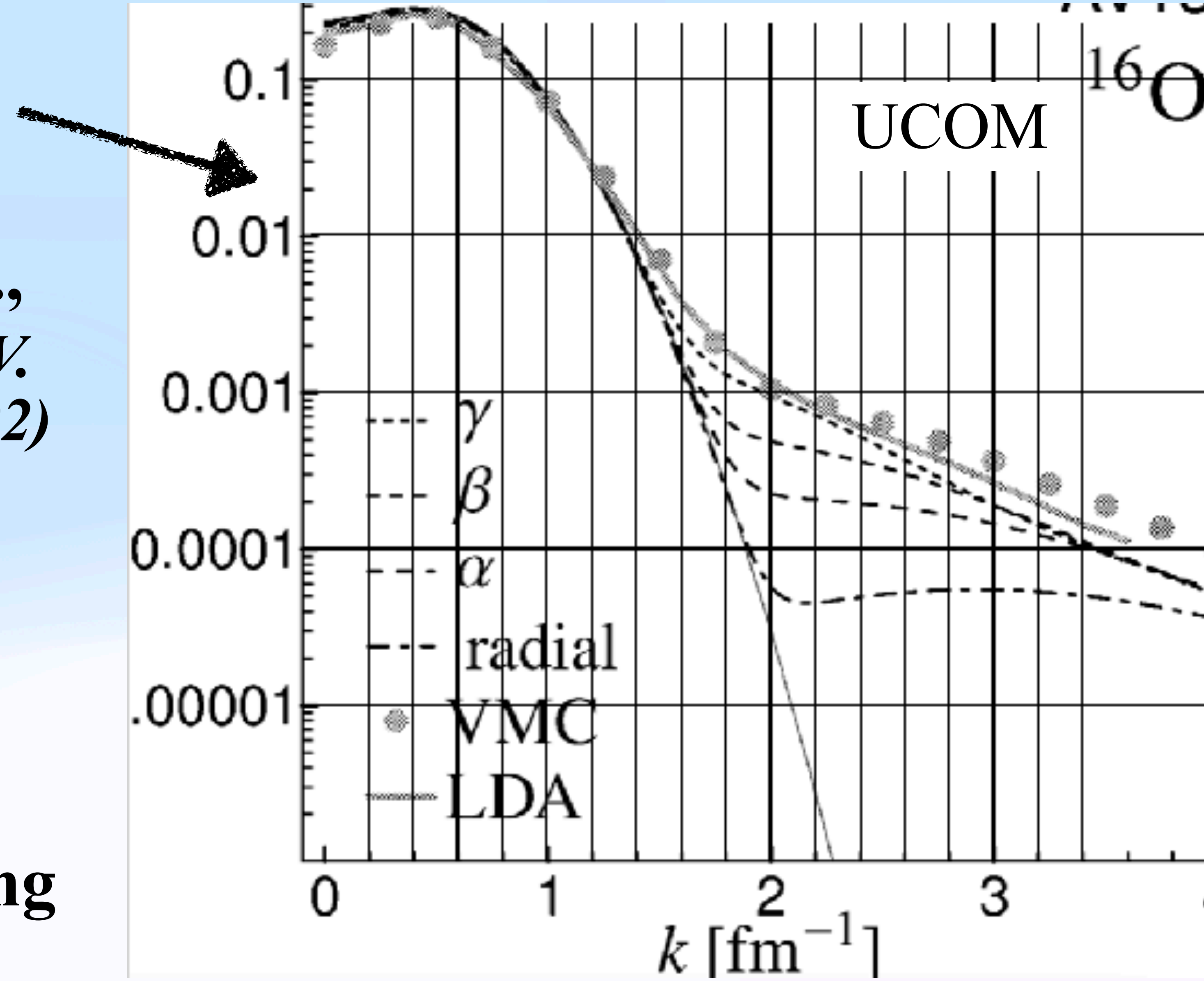
$^{16}O(p, d)^{15}O$ at $\theta_d=0°$ at GSI/FRS



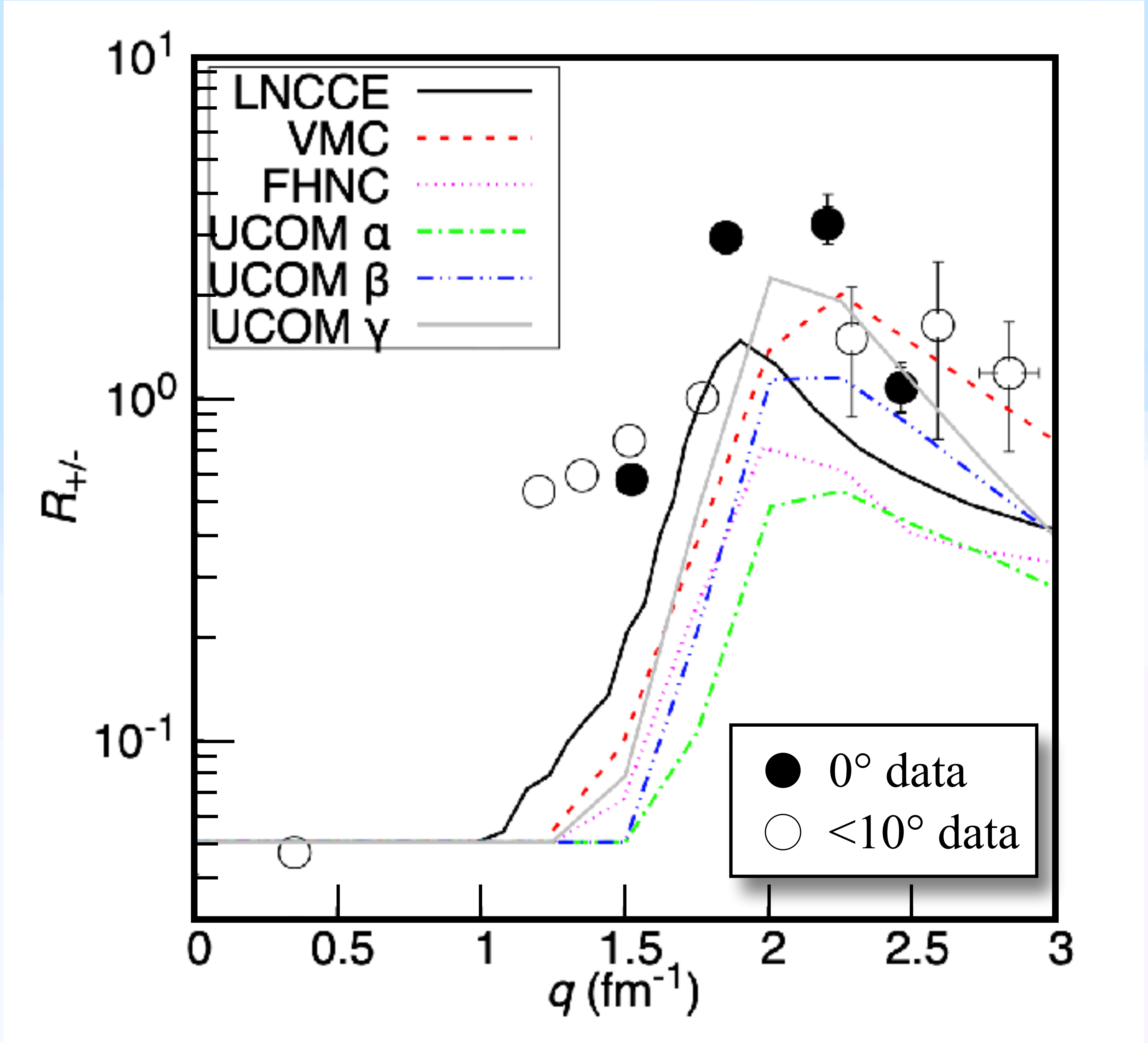
Theoretical model predictions

- **UCOM calculation,**
  - ***T. Neff and H. Feldmeier, Nucl. Phys. A 713, 311 (2003).***
- **Variational Monte Carlo calc.,**
  - ***S.C. Pieper, R. B. Wiringa, and V. R. Pandharipande, PRC 46 (1992) 1741.***
- **Fermi hyper netted chain integral equation method,**
  - ***A. Fabrocini and G. Go PRC 63 (2001) 044319.***
- **Linked and number conserving cluster expansion method,**
  - ***M. Alvioli, C.C.D. Atti, and H. Morita, PRL 100 (2008) 162503.***



Comparison with models

- All models predict peak near 2 fm^{-1}.
- Data also show a peak near 2 fm^{-1}.
- The height of the peak is strongly depends on the assumptions to obtain the ratio from model momentum amplitudes.

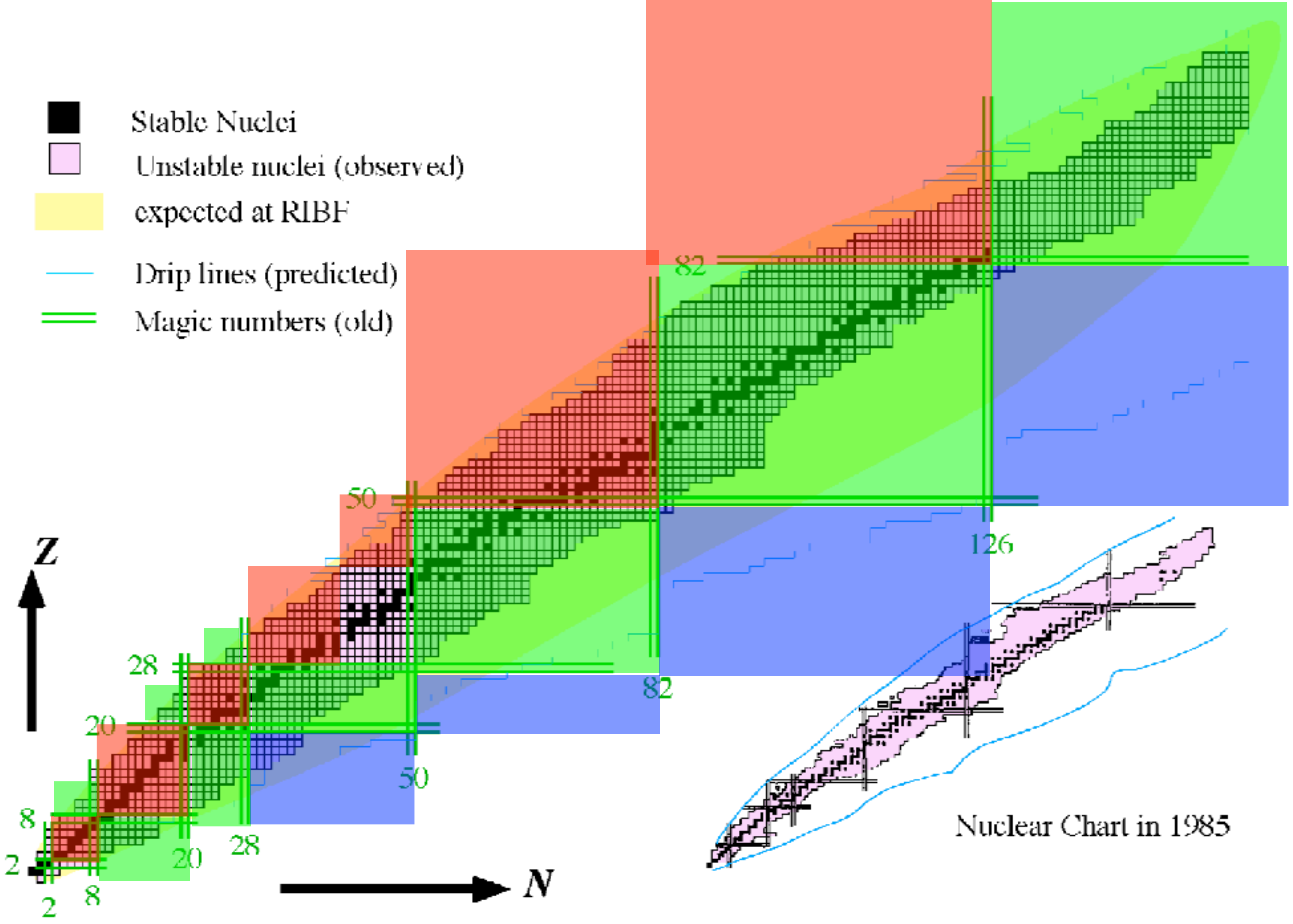


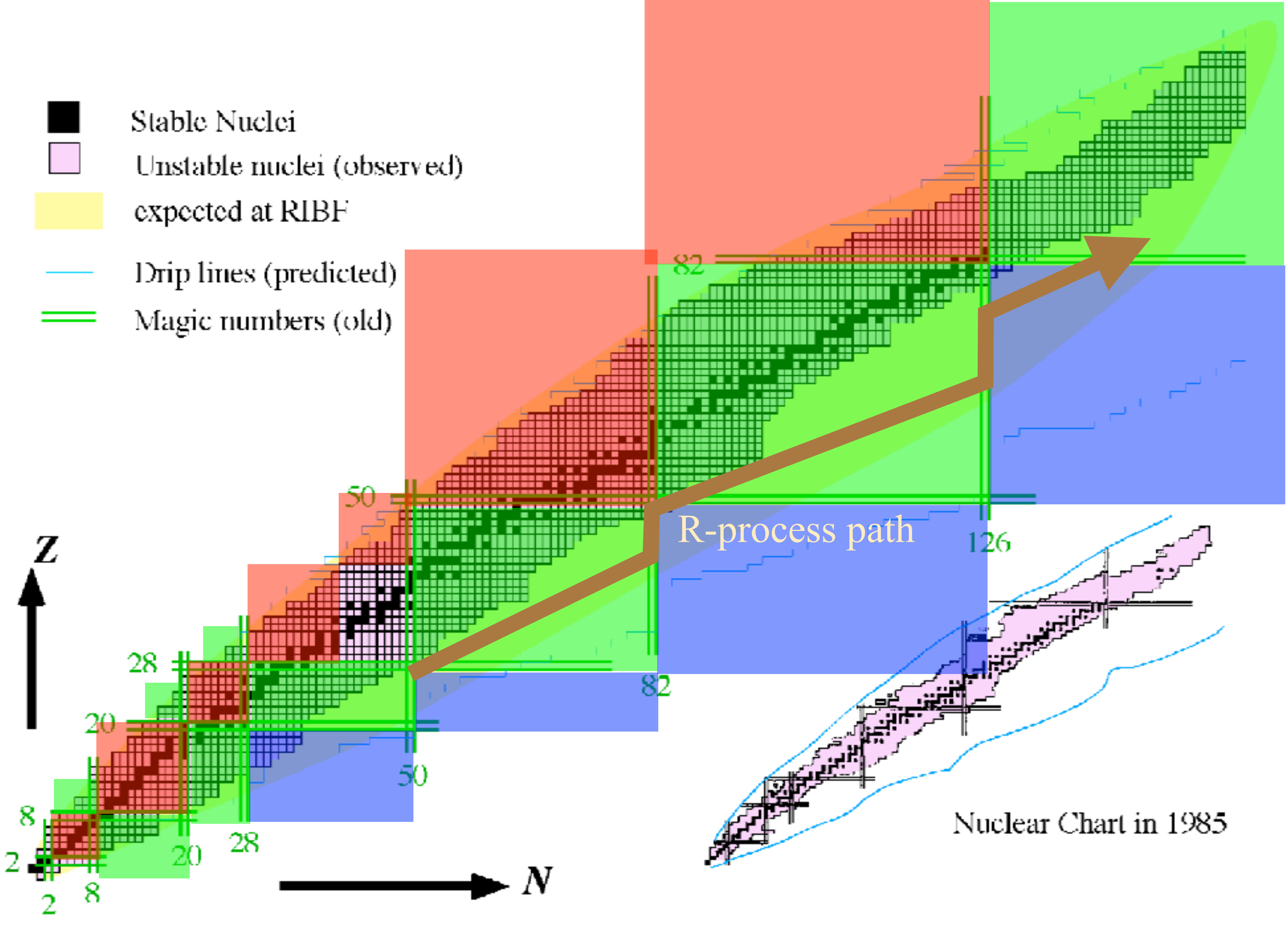
Summary

- **Tensor interactions strongly affect the properties of nuclei at ground state and other low lying states.**
- **Blocking of the 2p-2h excitations (Tensor blocking) change the orbitals of g.s. and low-lying states of nuclei.**
- **It gives sudden change of orbitals at above $j_>$ orbitals and alter magic numbers.**
- **Those effects are due to high-momentum nucleons in nuclei which contribute strongly to the tensor interactions.**
- **High-momentum neutrons in a 2p-2h state has been observed.**

We need,

- **nuclear-structure theory that includes tensor interactions and high-momentum nucleons to understand nuclei near and far from the stability line simultaneously.**
- **reaction theory that can treat (p, d) reactions well at higher energies.**


Stable Nuclei
Unstable nuclei (observed)
expected at RIBF
Drip lines (predicted)
Magic numbers (old)
Z
N
2
8
20
28
50
82
126
Nuclear Chart in 1985



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R-process path
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