Quark Exchange in Tetraquarks

Morgan Kuchta

University of Wrocaw
Institute of Theoretical Physics

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309312@uwr.edu.pl

$\chi_{c1}(3872)$

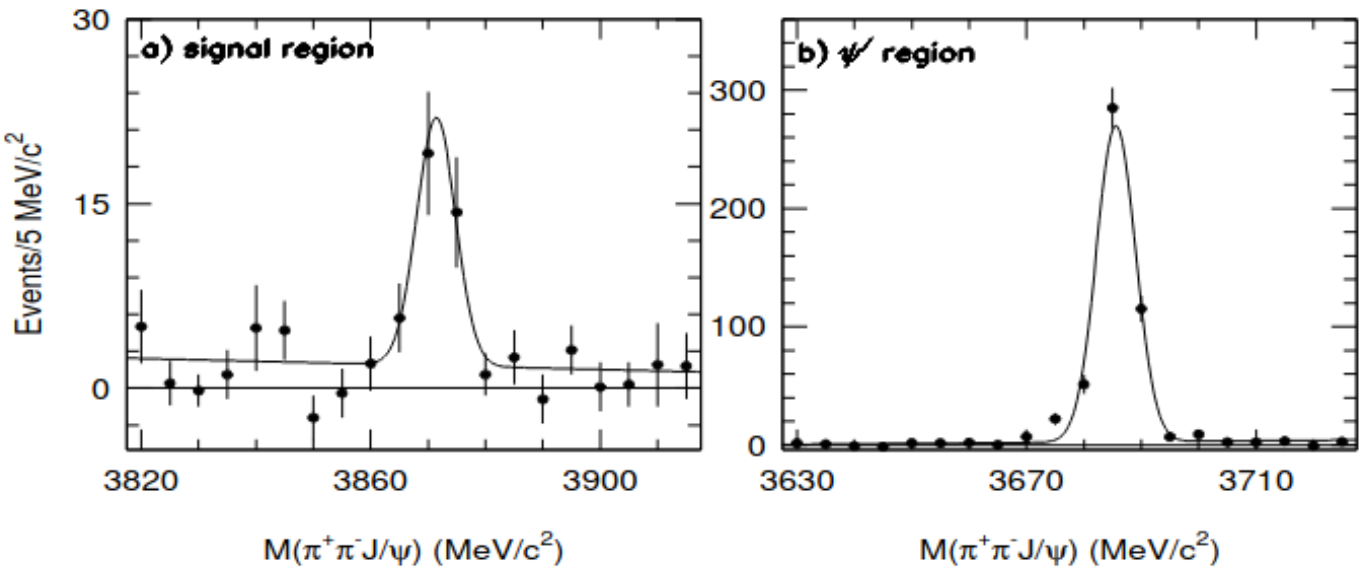
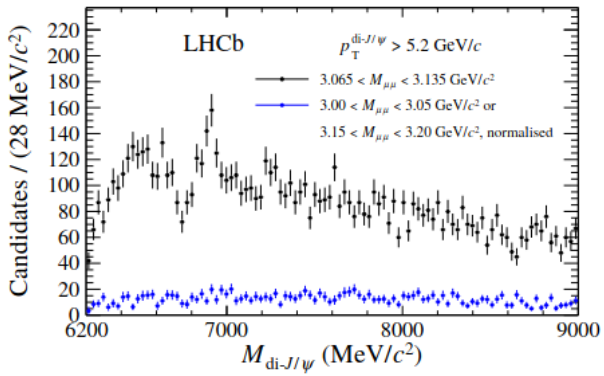


Figure: Resonance $\chi_{c1}(3872)$ previously known as $X(3872)$.
Source: Choi et al. Phys. Rev. Lett. 91, 262001 (2003)

The all-charm tetraquark



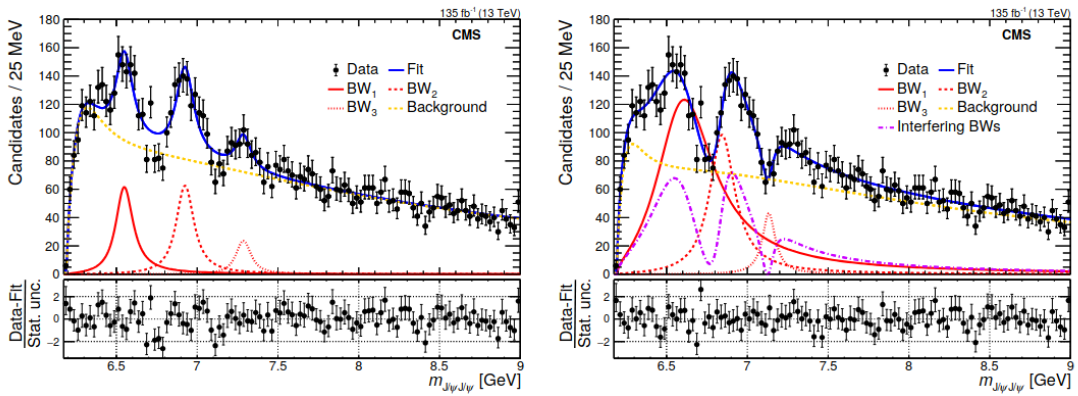


Figure: Comparison of the resonances identified in di-J/ψ spectrum done by LHCb (2019) and CMS (2024).

Tetraquarks - possible structures

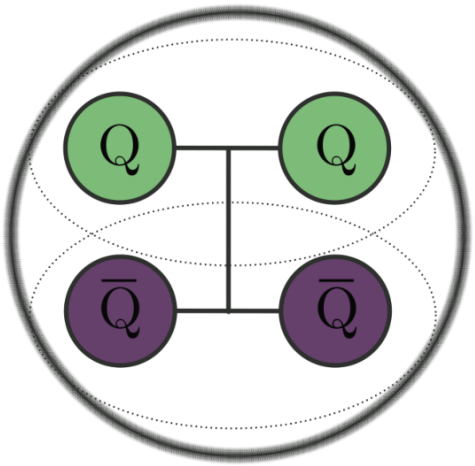


Figure: Tightly bound tetraquark

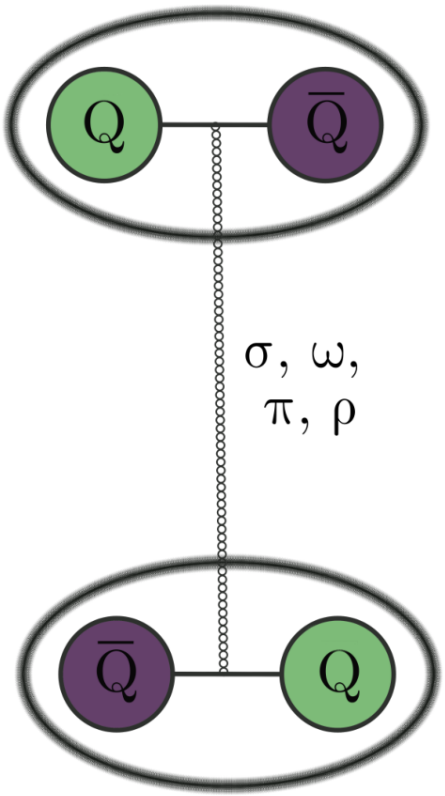


Figure: Meson-meson molecule

Flavor configuration:

$$T_{cc}(6900) = cc\bar{c}\bar{c}$$

$$\chi_{c1}(3872) = c\bar{c}\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

Molecular picture of the tetraquark

Single particle propagator:

$$G_i = G_i^0 + G_i^0 \Sigma_i G_i \tag{1}$$

Two particle propagator:

$$G_{ij} = G_{ij}^0 + G_{ij}^0 K_{ij} G_{ij} = G_{ij}^0 + G_{ij}^0 T_{ij} G_{ij}^0. \tag{2}$$

The T-matrix fulfills BSE in the ladder approximation:

$$T_{ij} = V_{ij} + V_{ij} G_{ij}^0 T_{ij} = \frac{V_{ij}}{1 - \Pi_{ij}^{cl}} \tag{3}$$

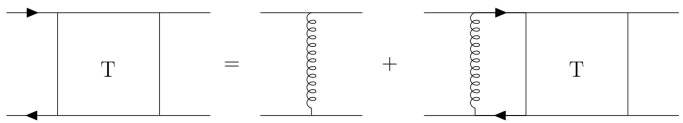


Figure: Two-particle T-matrix.

We establish the quark exchange - do properly account for the Pauli blocking.

Quark exchange

Figure: Quark exchage in the four quark system. Source:D. Blaschke and G. Ropke, https://doi.org/10.1016/0370-2693(93)90269-N

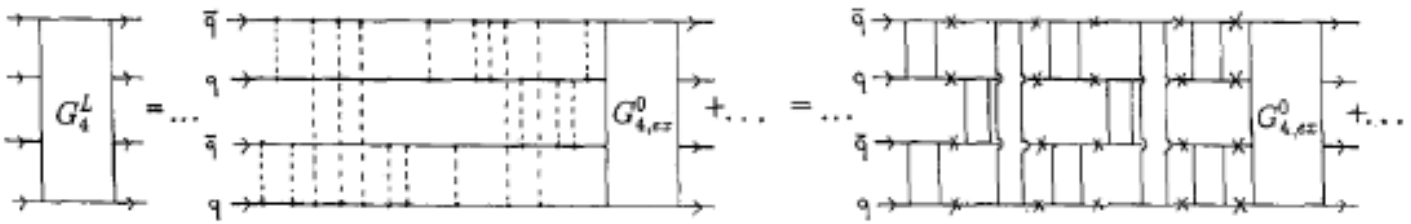
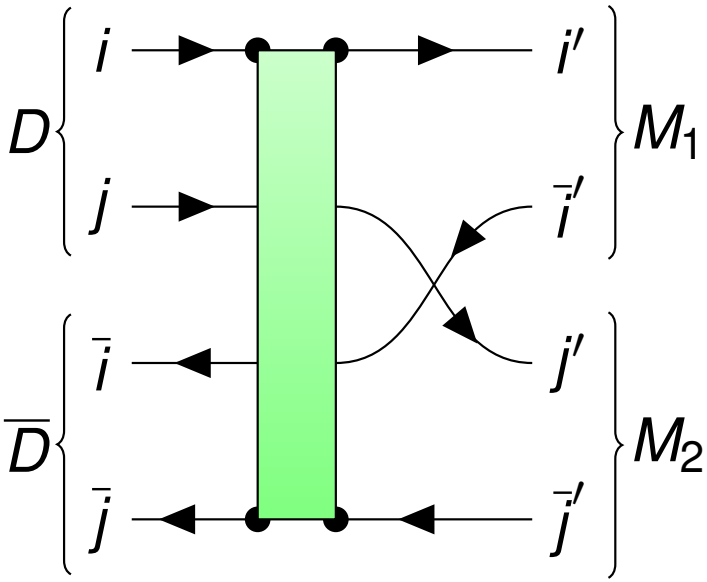
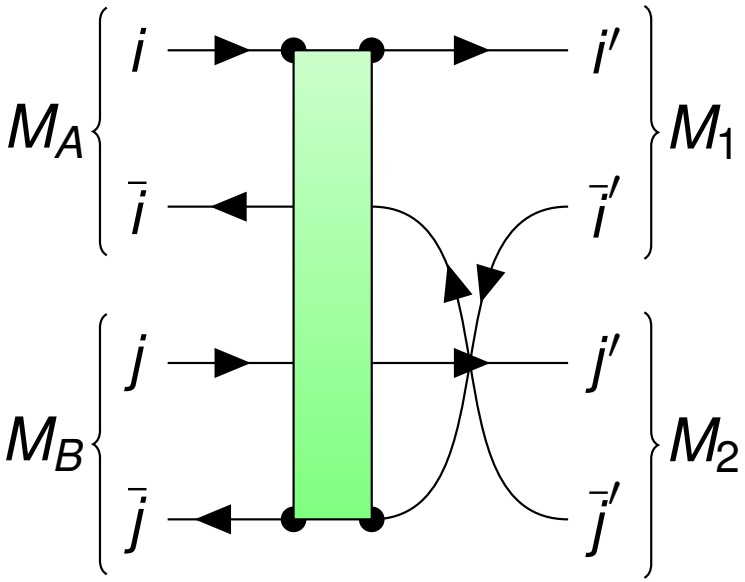


Figure: Quark exchage in the four quark system. Source: D. Blaschke and G. Ropke, https://doi.org/10.1016/0370-2693(93)90269-N

Exchange diagrams



(a) Diquark-antidiquark to meson-meson transition.



(b) Mesonic-Mesonic to meson-meson transition.

Figure: Exchange diagram with the interaction between the constituents included.

Capture and transfer diagrams

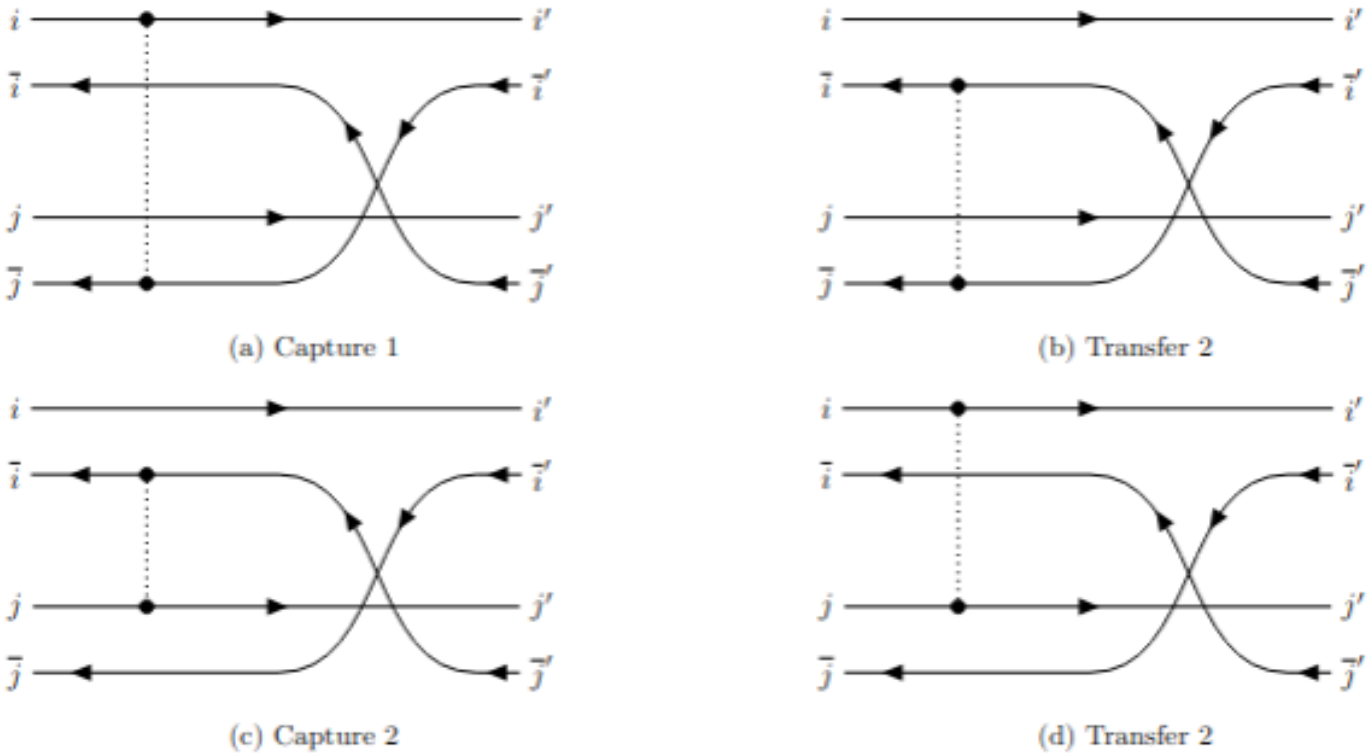


Fig. 2: Capture and transfer diagrams.

Figure: Capture and transfer diagrams for the meson-meson interaction. Based on the work by T. Barnes, E. S. Swanson Phys. Rev. D 46, 131 (1992).

Pauli blocking

How we implement it

The total energy of the cluster takes a form of:

$$E = E^0 + E^{Pauli}$$

$$E^{Pauli} = E^{Pauli,0} + E^{Pauli,1} + \dots$$

with the contribution to the Pauli blocking being:

$$E^{Pauli,1} = \frac{1}{N}\langle\psi(a_1\bar{b}_1)\psi(a_2\bar{b}_2)|$$
$$(E(a_1) + E(a_2) + E(\bar{b}_1) + E(\bar{b}_2) - E^0)$$
$$|-\psi(a_2\bar{b}_1)\psi(a_1\bar{b}_2) - \psi(a_1\bar{b}_2)\psi(a_2\bar{b}_1)\rangle$$

$$E^{Pauli,2} = \frac{1}{N}\langle\psi(a_1\bar{b}_1)\psi(a_2\bar{b}_2)$$
$$|(E(a_1) + E(a_2) + E(\bar{b}_1) + E(\bar{b}_2) - E^0)$$
$$|\psi(a_2\bar{b}_1)\psi(a_1\bar{b}_2)\rangle.$$

Separable potential

T-matrix for separable potentials:

$$T(a,b;a',b',z_\nu) = A \sum_{a,b,a',b'} \omega(ab)\tilde{\omega}(a'b') \left[1-\omega^2(p_{a,b}) \frac{1-f(a)-f(b)}{z_\nu-\epsilon_a-\epsilon_b}\right] \quad (4)$$

$$V(p_1,p_2,p_1'p_2') = A\omega(p_1p_2)\tilde{\omega}(p'1p_2')\delta_{p_1+p_2,p_1'+p_2'} \quad (5)$$

Coordinates

$$P_{a,b} = \frac{a+b}{2}; p_{a,b} = |a-b|$$

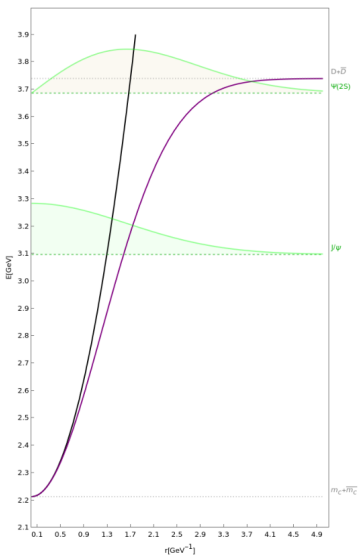


Figure: Gaussian potential with applied for charmonia.

Obtaining solutions

The T-matrix takes on complex values:

$$T(a,b;a',b',\tilde{E}) = A\omega(ab)\tilde{\omega}(a'b')\left[\frac{1+\Re I(\tilde{E})}{1+I(\tilde{E})} - \frac{i\Im I(\tilde{E})}{1+I(\tilde{E})}\right]^{-1} \quad (6)$$

with the

$$I(z_\nu) = (1 + A\int \frac{d^3p}{2\pi}\frac{\omega(ab)\tilde{\omega}(a'b')}{z_\nu - \epsilon_{a,b}}). \quad (7)$$

- The energy shift caused by the Pauli blocking in all-charm tetraquark system: $E^{Pauli} \approx 0.64$ GeV.

Thank You

Sources

- A. Ali, L. Maiani, and A. D. Polosa, Multiquark Hadrons. Cambridge University Press, 2019, isbn: 978-1-316-76146-5, 978-1-107-17158-9, 9781-316-77419-9. doi: 10.1017/9781316761465.
- D. Blaschke and G. Ropke, Quark exchange contribution to the effective meson meson interaction potential
- C. Pena and D. Blaschke, $X(3872)$ as a $D - \bar{D}^*$ molecule bound by quark exchange forces,
- D. Gromes, Bethe-Salpeter Equation With Confining Kernel: Correct Nonrelativistic Limit and the Constant to Be Added to the Linear Potential
- M. Kuchta, All-charm tetraquark mass and possible quantum numbers of $X(6900)$

Backup

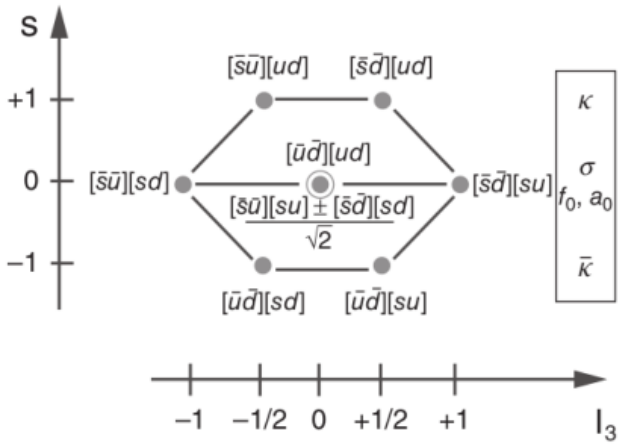


Figure: Light tetraquark candidates. Source: A. Ali, L. Maiani, and A. D. Polosa, 2019

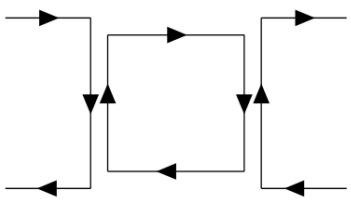


Figure: The basis of the meson exchange mechanism.

Backup

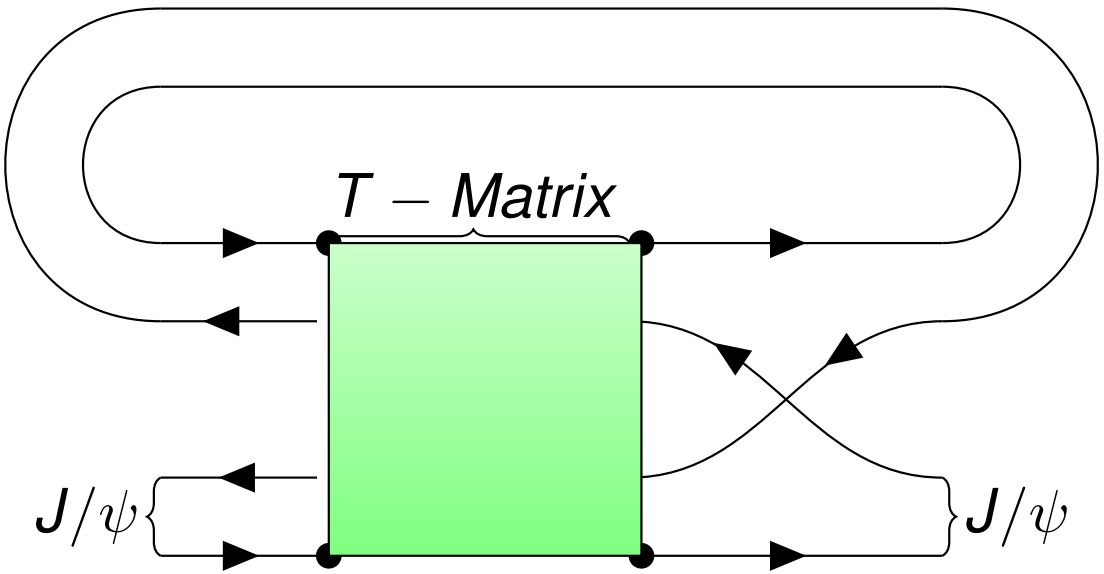


Figure: Self energy of the meson.