INSTITUTO DE FÍSICA
Universidade Federal Fluminense

Hydrodynamic model of heavy ion collisions

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Erice: INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS, 46th COURSE

What you will see

- ✔ Motivation: why do we use hydrodynamic models?

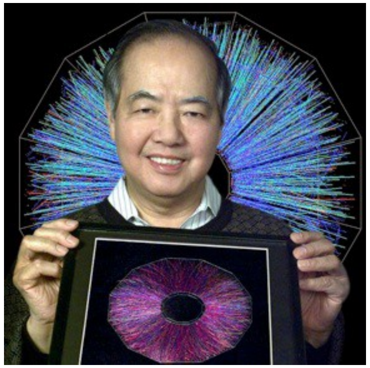
- ✔ Why do we believe in fluid-dynamical models?

- ✔ Relativistic hydrodynamics: fundamentals and ingredients

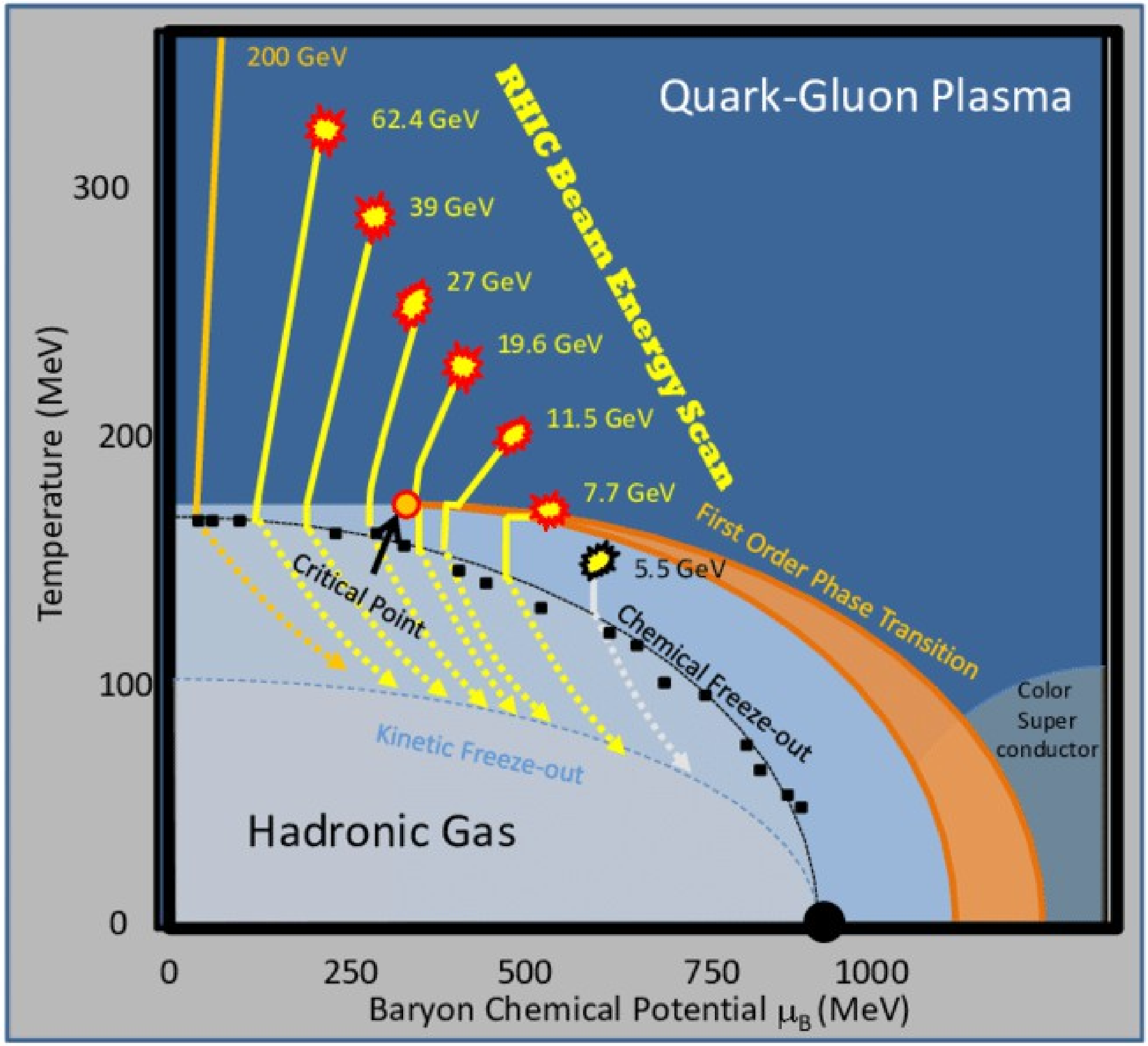
- ✔ Magnetohydrodynamic simulations for heavy ion collisions

- ✔ Final remarks

Goal of Heavy-Ion Collisions: *Produce* and *study* QCD matter near (local) equilibrium



T.D. Lee



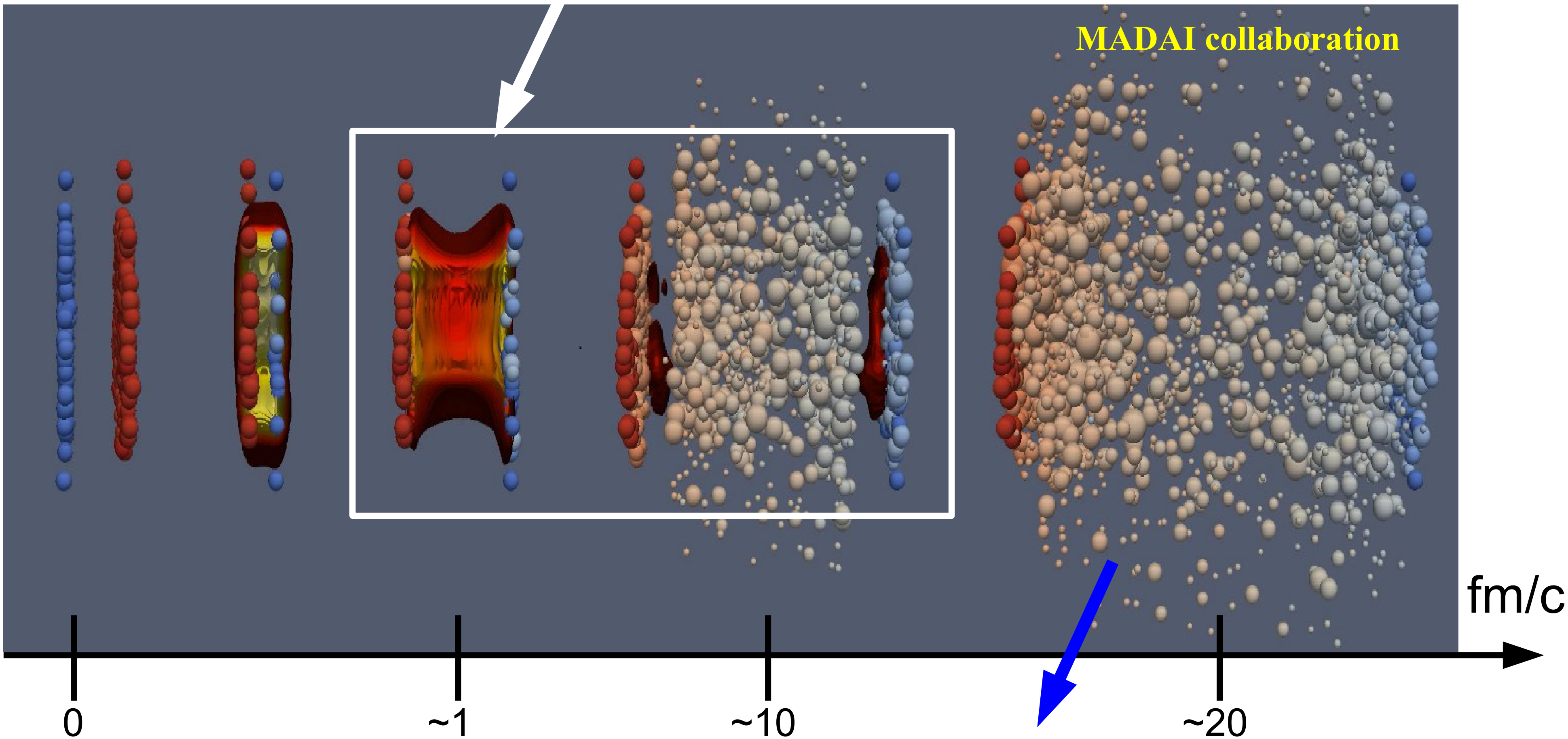
Challenge

- Reach thermodynamic limit experimentally!
- Extract thermodynamic *and* transport properties of QCD matter
- Experiment is not to test QCD, but to understand it

Heavy Ion Collisions

QCD matter is only created *transiently* ~10 fm/c

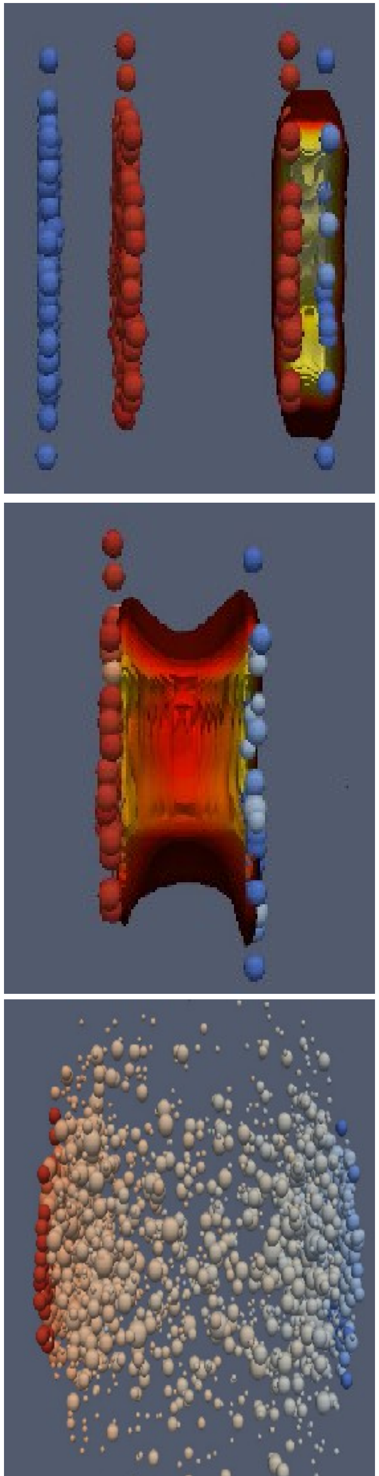
expectation: *fluid-dynamical expansion*



measured: hadrons, leptons, and photons

Properties of matter must be reverse-engineered!

Current theoretical description



1) Initial state and "pre-equilibrium" dynamics:

- description of early-time dynamics and "thermalization"
- initial condition for hydrodynamic evolution

(approach) thermalization (?)

2) Fluid-dynamical expansion of QGP and Hadron Gas

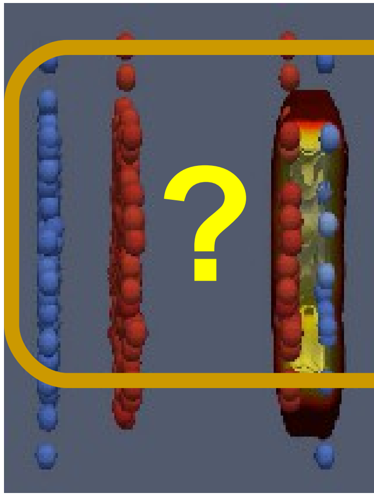
- Phase transition
- Matter described by EoS and transport coefficients
  shear and bulk viscosity, charge diffusion ...

fluid elements converted to particles

3) Transport description of Hadron Gas

- Late stage description using the *hadron resonance gas* model – using cross sections and decay probabilities

Current theoretical description



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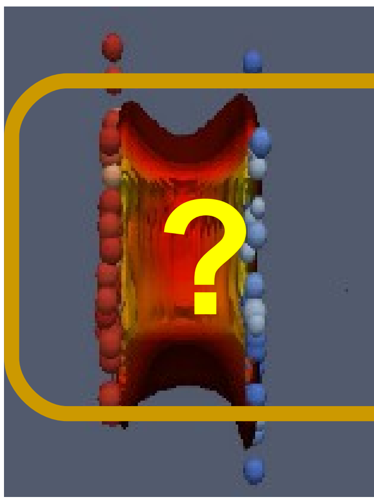
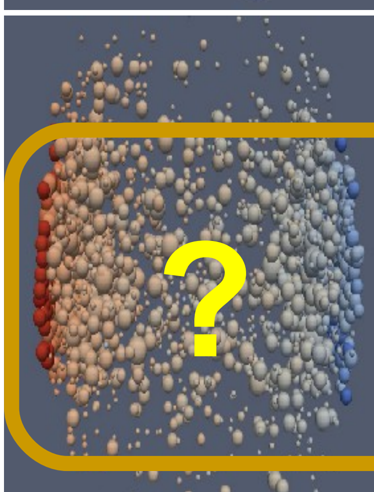


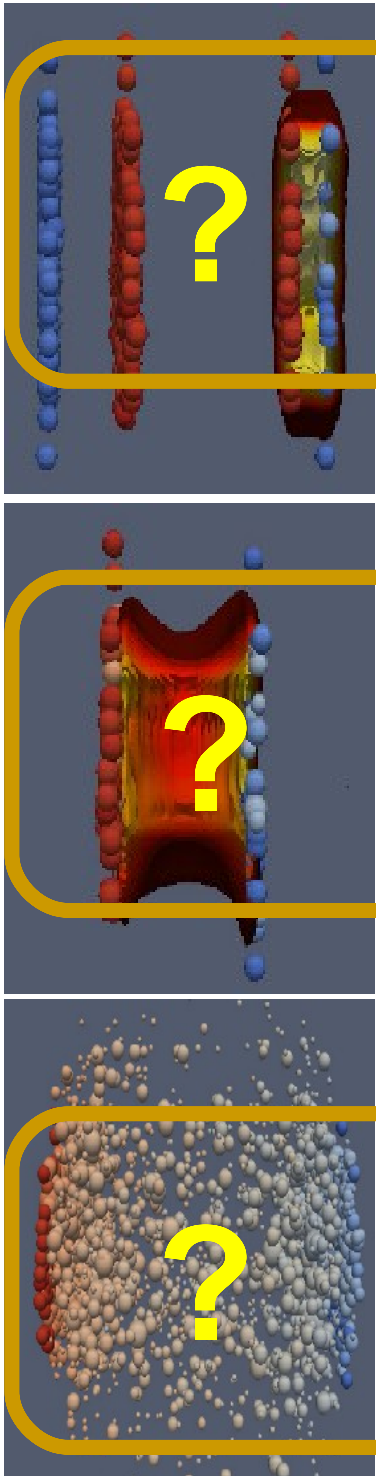
2) Fluid-dynamical expansion of QGP and Hadron Gas
Phase transition
Matter described by EoS and transport coefficients
shear and bulk viscosity, charge diffusion ...

fluid elements converted to particles



3) Transport description of Hadron Gas
Late stage description using the *hadron resonance gas* model – using cross sections and decay probabilities

Current theoretical description



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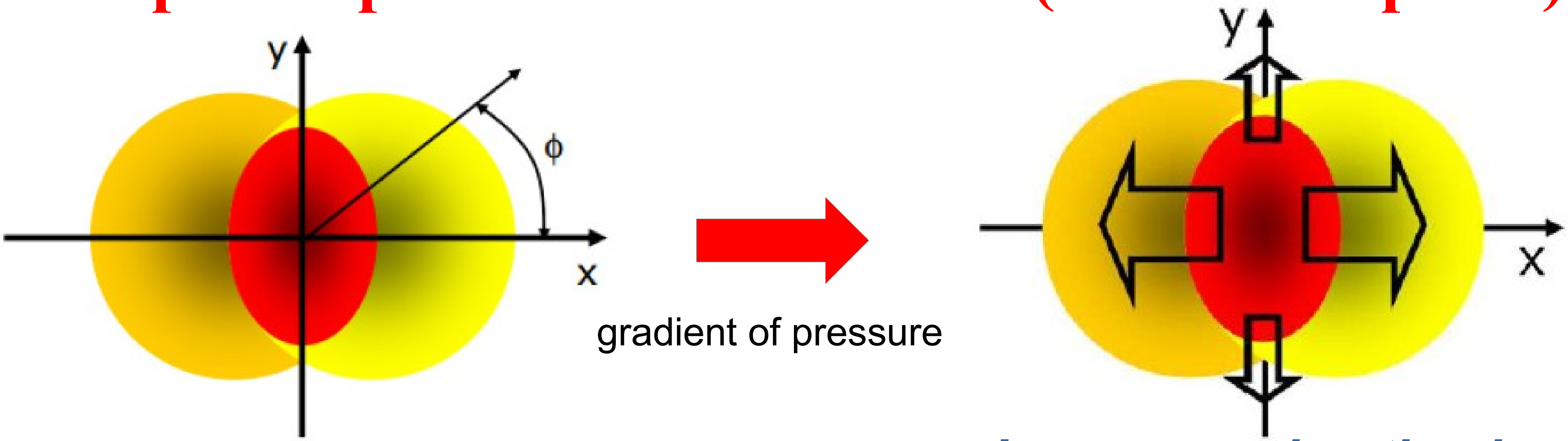
Why do we think fluid-dynamical models work?

Anisotropic flow

- **Simplified picture of the collision (transverse plane)**



Initial elliptical shape in noncentral collisions

Larger acceleration in direction where the fluid is more compressed

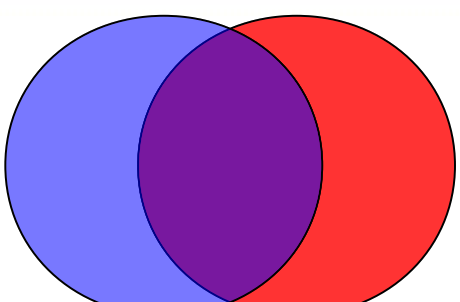
$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_n)$$

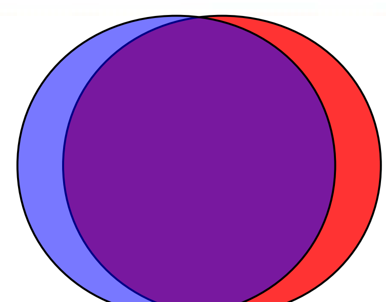
v_2 is large

particles are not emitted isotropically!

hydrodynamic response to the initial geometry

It is possible to "estimate" the centrality

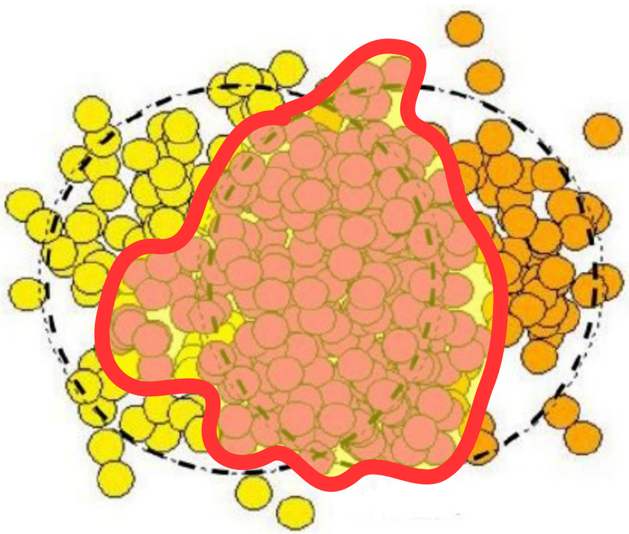


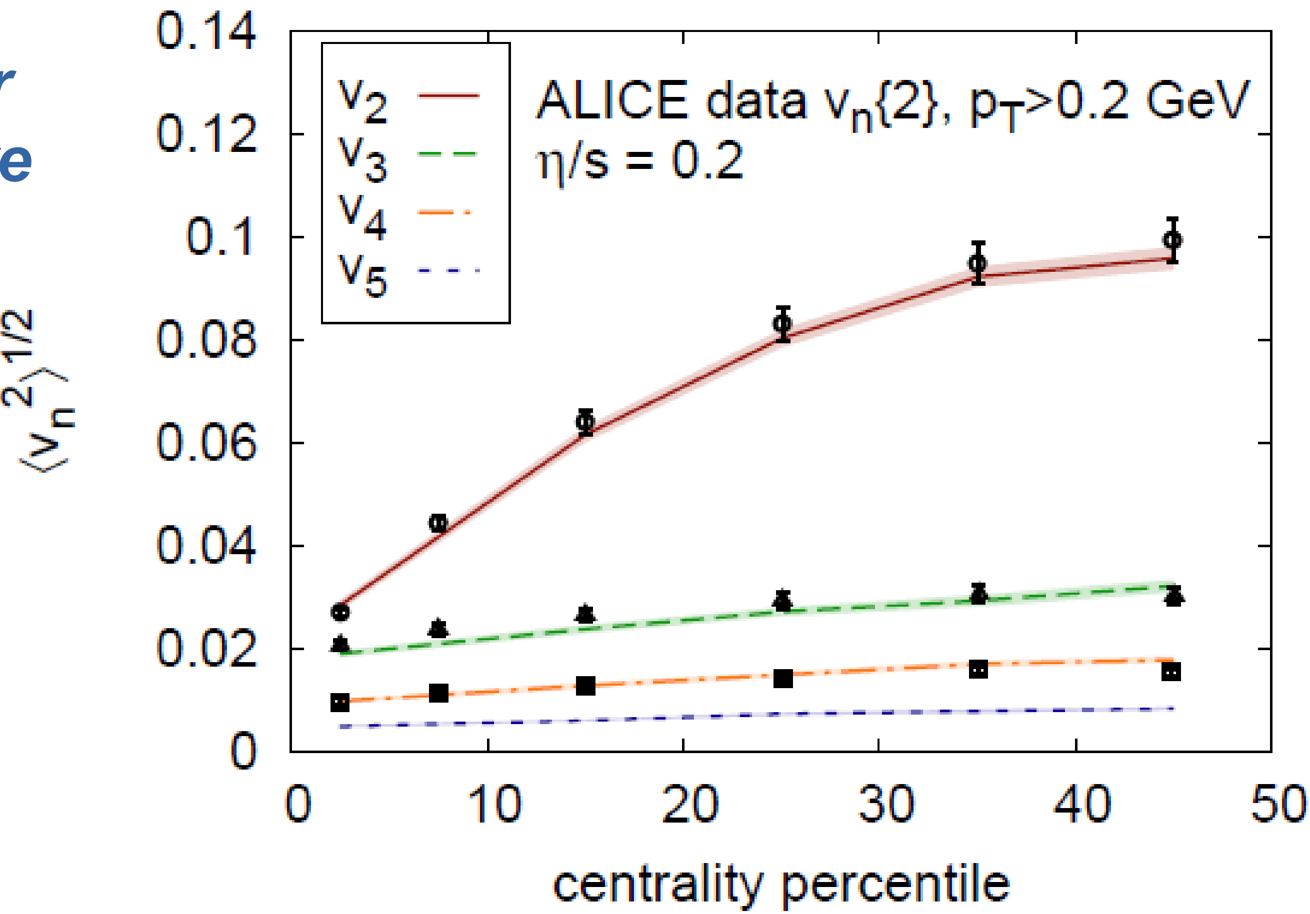


Hydrodynamic response to initial state can be observed experimentally

Gale *et al*, PRL 110 (2013) no.1, 012302

Quantum effects render the initial geometry more complicated





non-trivial test of fluid-dynamical models

Hydrodynamic response to initial state can be observed experimentally

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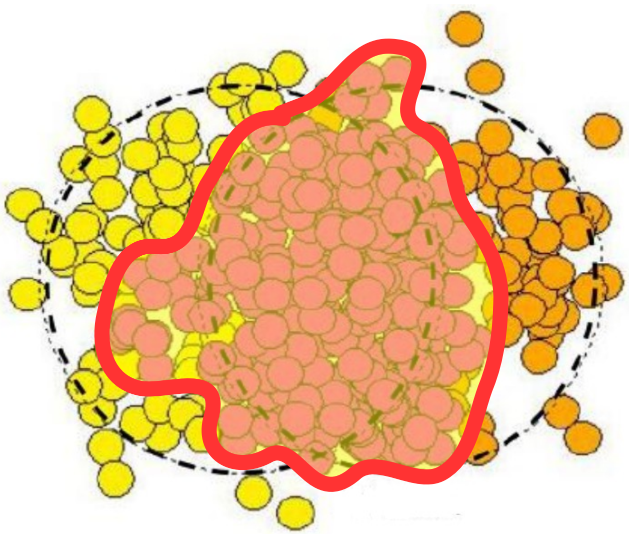
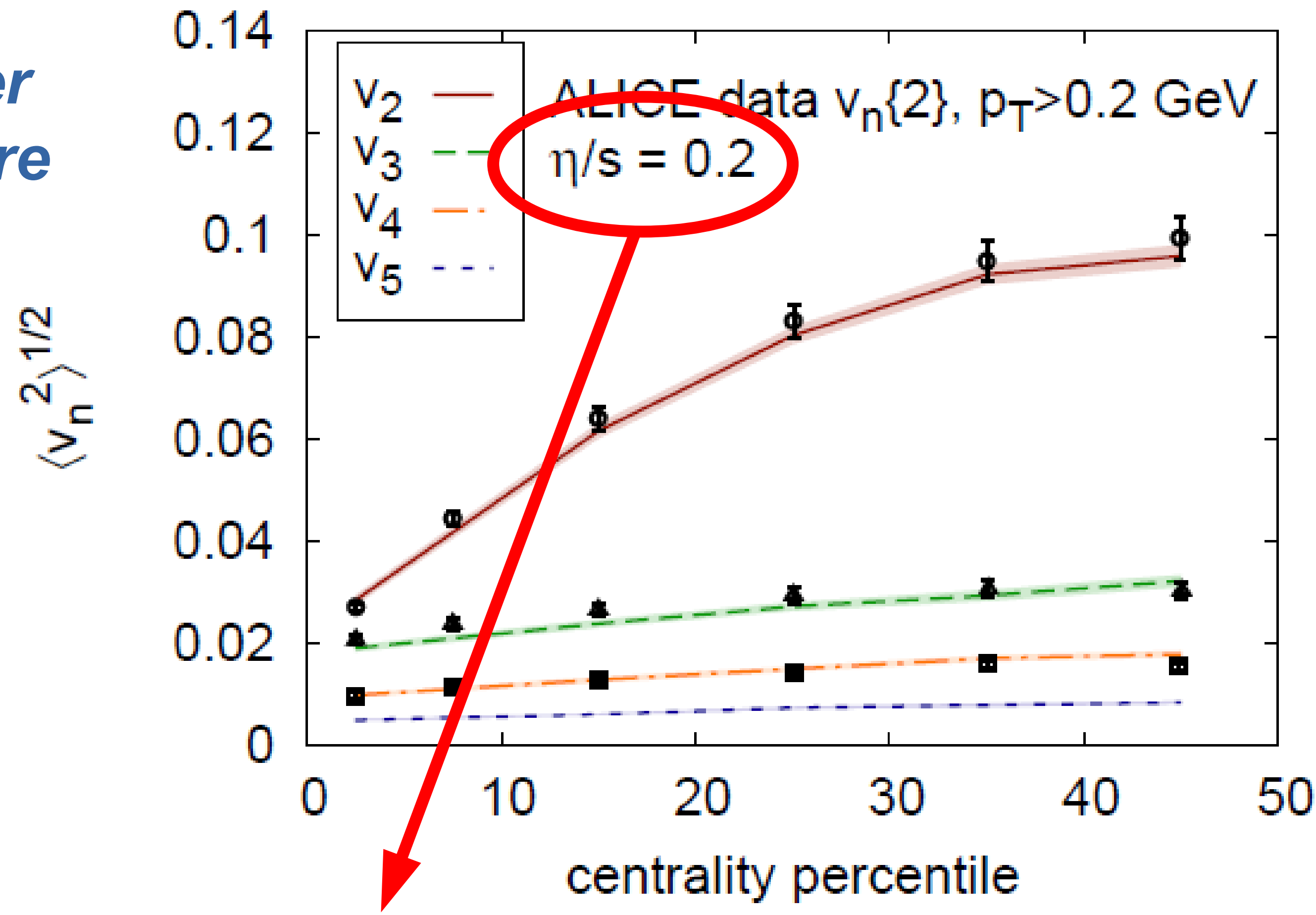
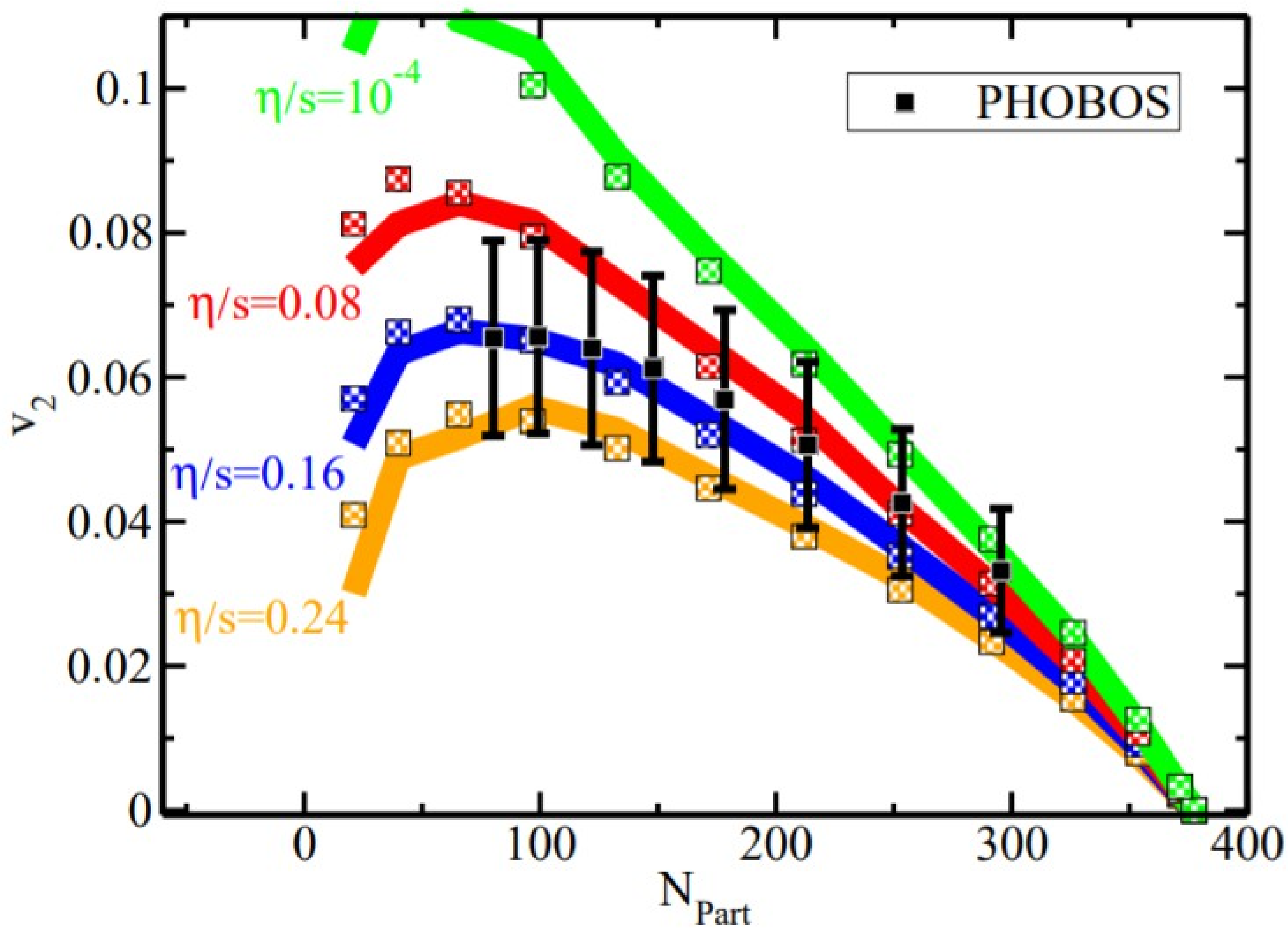




Value of shear viscosity extracted

Extraction of shear viscosity: an example

M. Luzum and P. Romatschke, PRC78 (2008) 034915



Momentum anisotropy *decreases* with *increasing* viscosity

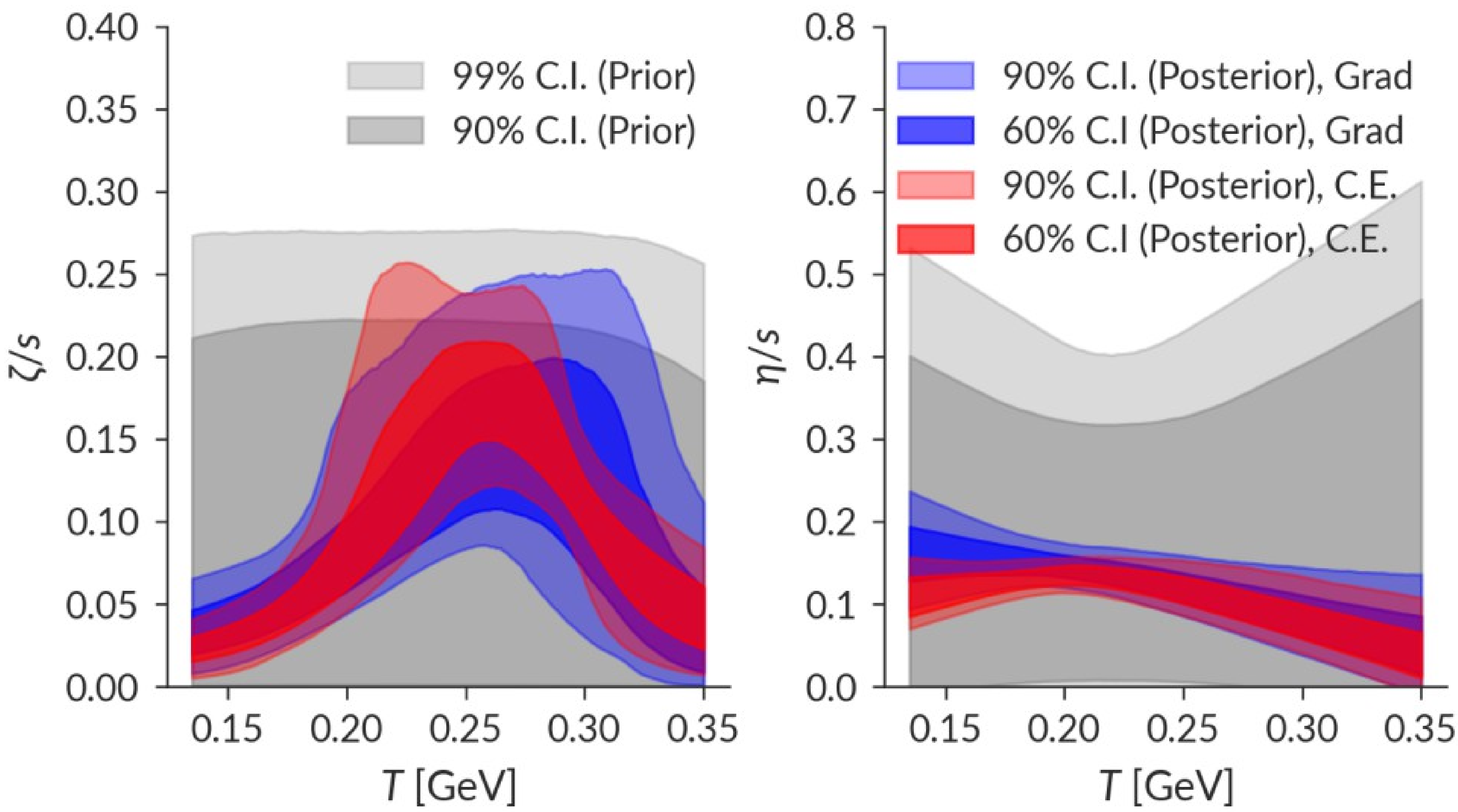
Nowadays: *systematic model-to-data comparison with Global fits*

J. Bernhard, J. Moreland & Steffen A. Bass, Nature Physics , 1–5 (2019)

Systematic model-to-data comparison – Bayesian Analysis

IPGlasma+MUSIC+UrQMD

Heffernan *et al*, Phys.Rev.C 109 (2024) 6, 065207

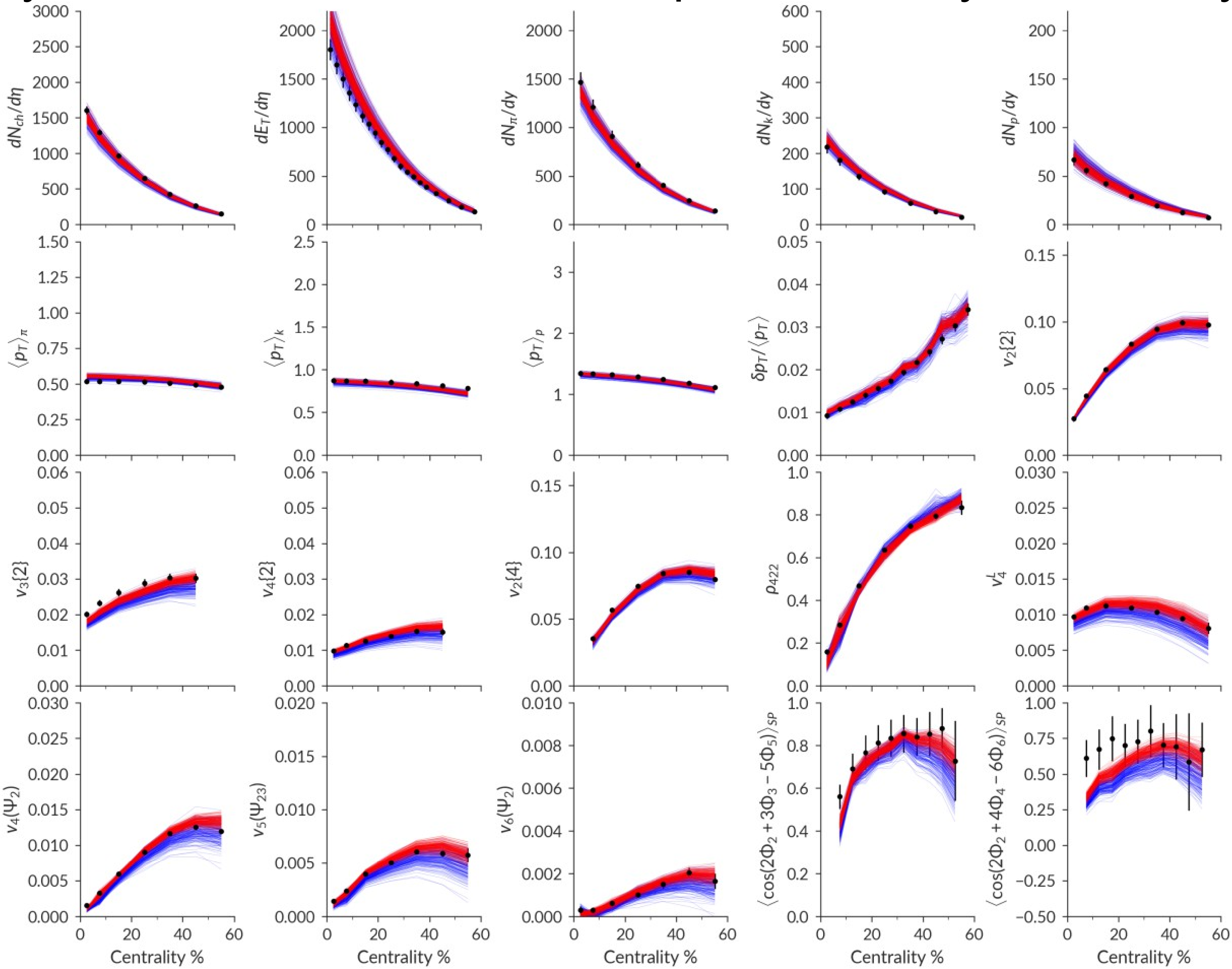


large bulk viscosity around "Tc"!

Almost perfect fluid!

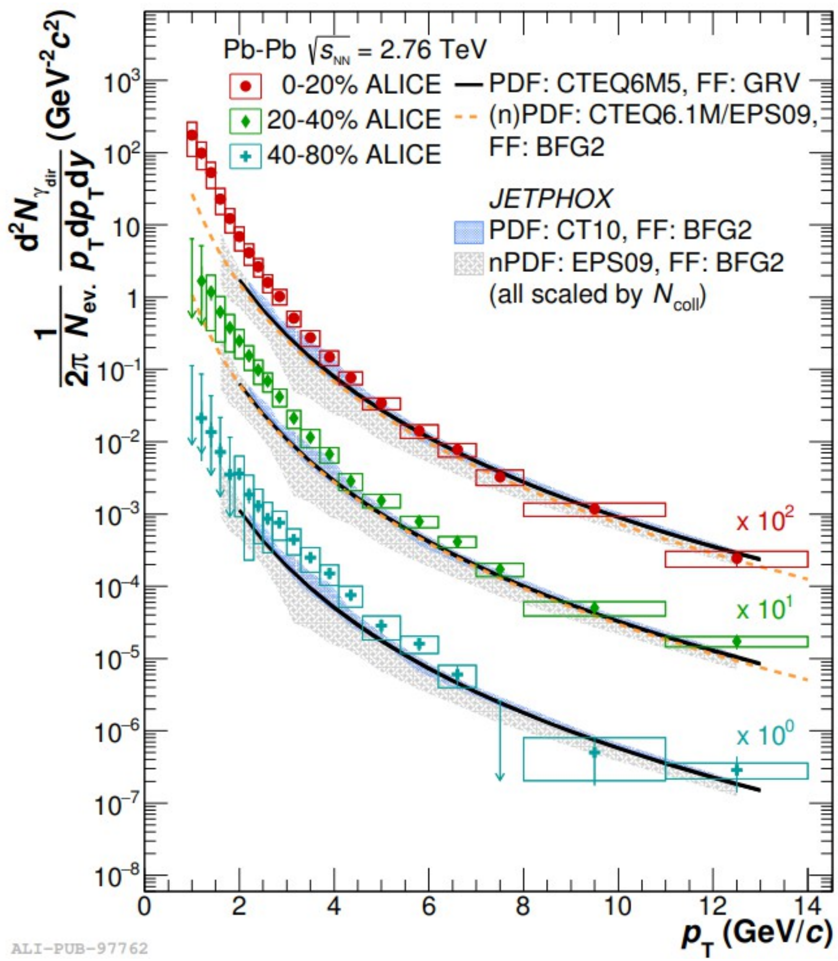
$\eta/s \sim 0.1$

Systematic model-to-data comparison – Bayesian Analysis



Significant evidence of a medium

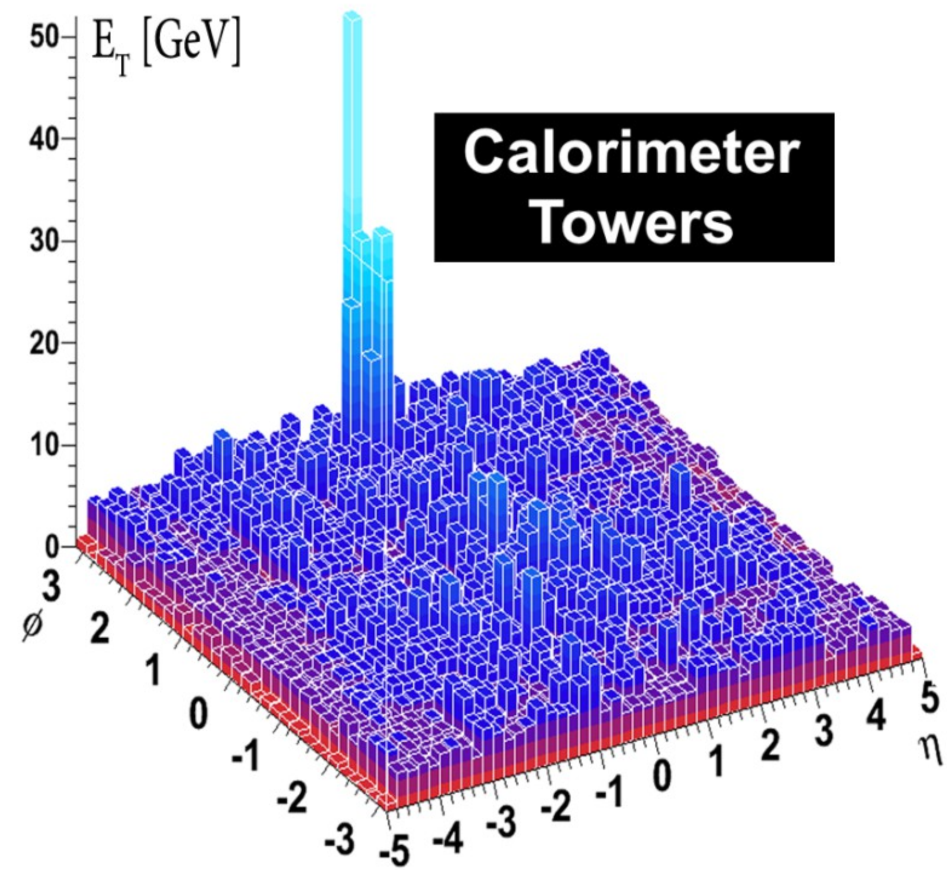
thermal photons



Excess of direct photons at low pt – thermal radiation

ALICE, Phys.Lett.B 754 (2016) 235-248

Jet quenching



Jets are stopped by the medium

ATLAS, Phys.Rev.Lett. 105 (2010) 252303

Relativistic fluid dynamics

Effective theory describing the dynamics
of a system over long-times and long-distances

Conservation laws
+
Equation of state
+
simple constitutive relations

Basics of fluid dynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net charge conservation

$$\partial_\mu N_s^\mu = 0 \quad \text{strangeness}$$

$$\partial_\mu N_e^\mu = 0 \quad \text{electric charge}$$

$$\partial_\mu N_b^\mu = 0 \quad \text{Baryon number}$$

Tensor decomposition

$$N_q^\mu = n_q u^\mu + n_q^\mu$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

net-charge diffusion 4-current (n_q^μ)

Bulk viscous pressure (Π)

Shear stress tensor ($\pi^{\mu\nu}$)

Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Basics of fluid dynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Net charge conservation

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$$\partial_\mu N_b^\mu = 0$$ **Baryon number**

Tensor decomposition

$$N_q^\mu = n_q u^\mu + n_q^\mu$$

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???

net-charge diffusion 4-current (n_q^μ)

Bulk viscous pressure (Π)

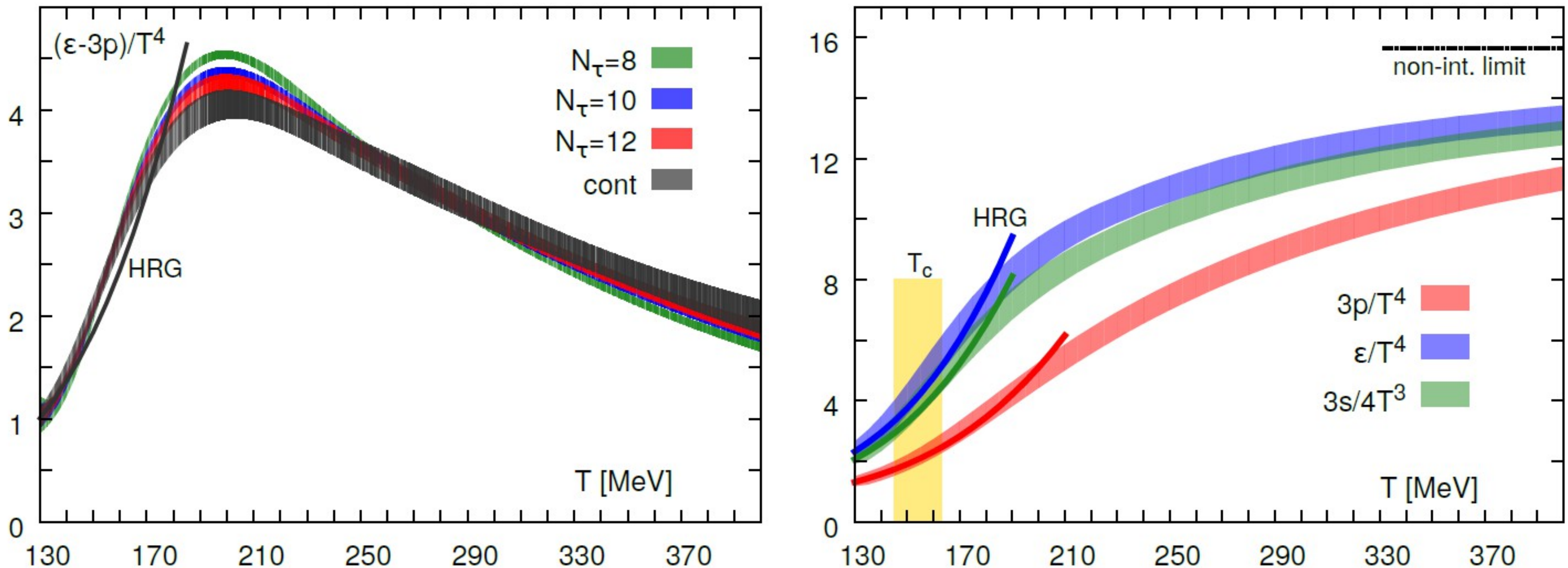
Shear stress tensor ($\pi^{\mu\nu}$)

Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

EoS for QCD matter – $\mu_B = 0$

Lattice QCD: entropy density increases near T = 180 MeV

quark and gluon degrees of freedom become manifest



PRD 90, 094503 (2014)

- QCD predicts a *cross-over*

Below ~ 150 MeV: matches a hadron resonance gas

EoS at finite μ

<u>*Thermodynamic pressure:*</u> $P_0 \quad = \quad P_0\left(T, \mu_b, \mu_e, \mu_s\right)$

Taylor expansion up to 4th order:

$$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \underbrace{\frac{\chi_{l,m,n}^{B,Q,S}}{l!m!n!}}_{\text{lQCD}} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

- matched to *hadron resonance gas* model at small T
- matched to Stefan-Boltzmann limit at large T

- Prescription employed by:

Monnai, Schenke, Shen, PRC 100, 024907 (2019)
Noronha-Hostler, Parotto, Ratti, Stafford, PRC 100, 064910 (2019)

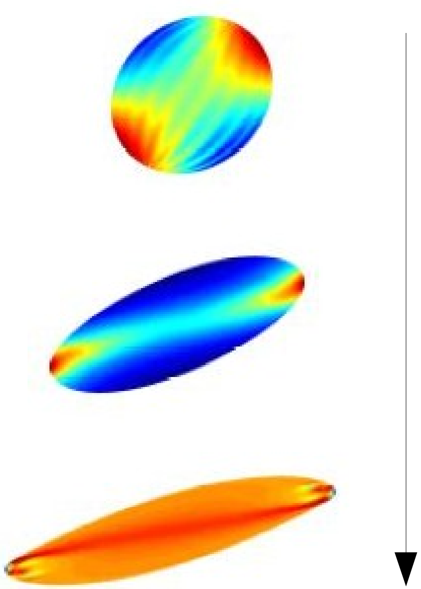
Critical point?

Relativistic Navier-Stokes theory

Shear Viscosity

(Resistance to deformation)

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$



$$\eta\left(T,\mu_q\right)$$

Bulk Viscosity

(Resistance to expansion)

$$\Pi = -\zeta\nabla_\mu u^\mu$$

$$\zeta\left(T,\mu_q\right)$$

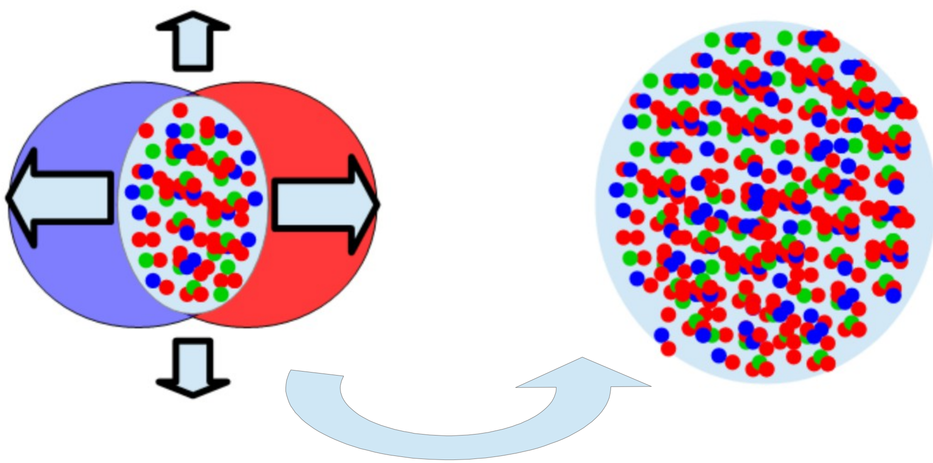
Net-Charge Diffusion

$$n_q^\mu = \kappa_q \nabla^\mu \frac{\mu_q}{T}$$

$$\kappa_q\left(T,\mu_q\right)$$

Shear Viscosity (Resistance to deformation)

Important for heavy ion collisions



Change from elliptical to spherical shape

- *Shear viscosity will resist this process*
- *Shear viscosity reduces the flow coefficients*

Intuition from kinetic theory (gas)

density

$$\eta = \frac{1}{3} n \, p \, \ell_{\mathrm{mfp}}$$

mean free path

average momentum

- *Proportional to scattering rate*

small η/s

↕

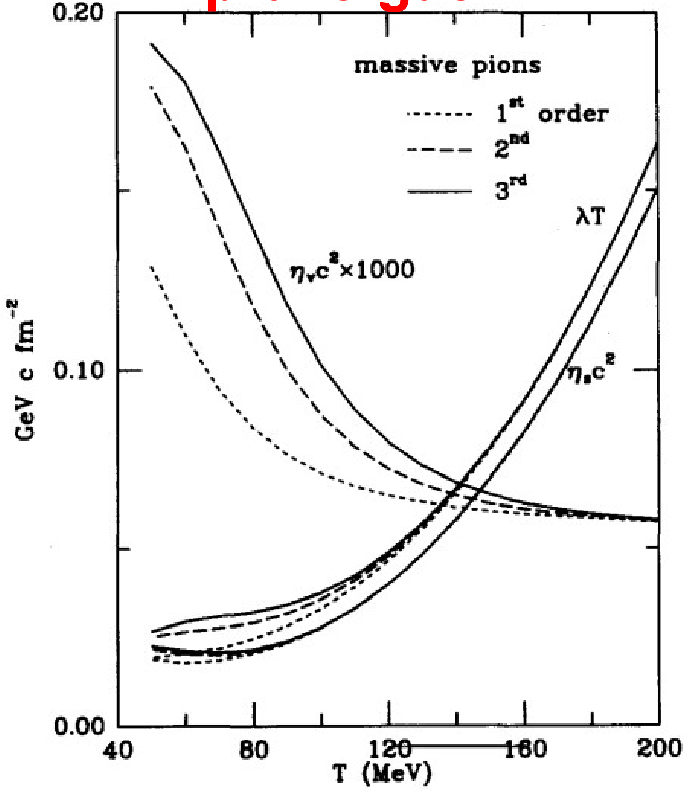
"strong" interaction

(this does not work for liquids)

What about bulk viscosity?

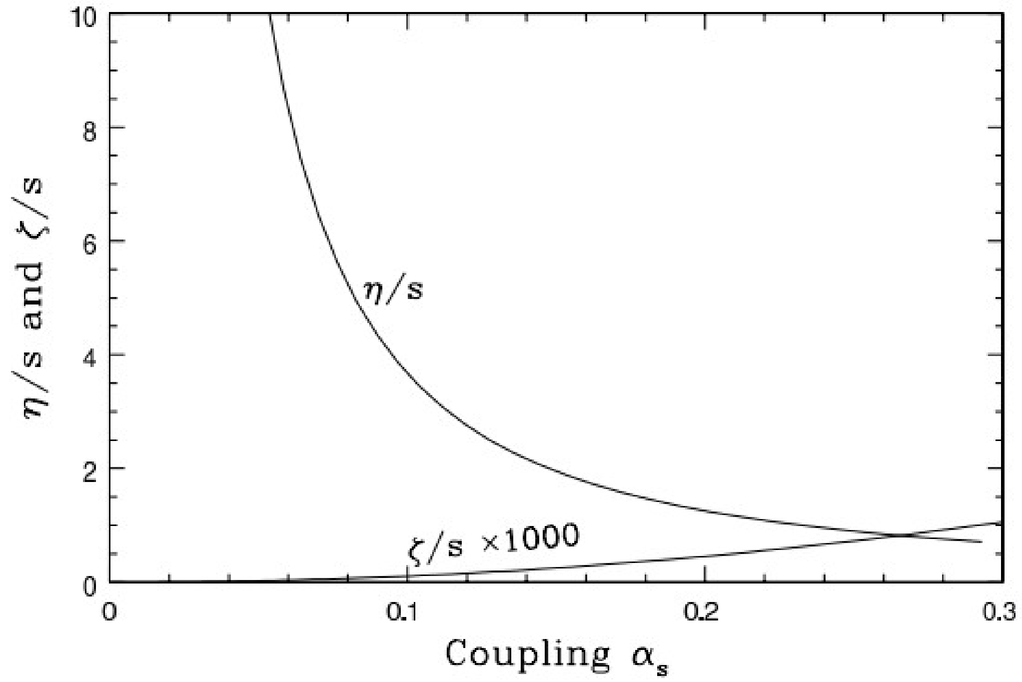
- should be important for highly explosive systems

pions gas



Prakash *et al*, Phys. Rept. 227, 321 (1993)

pQCD



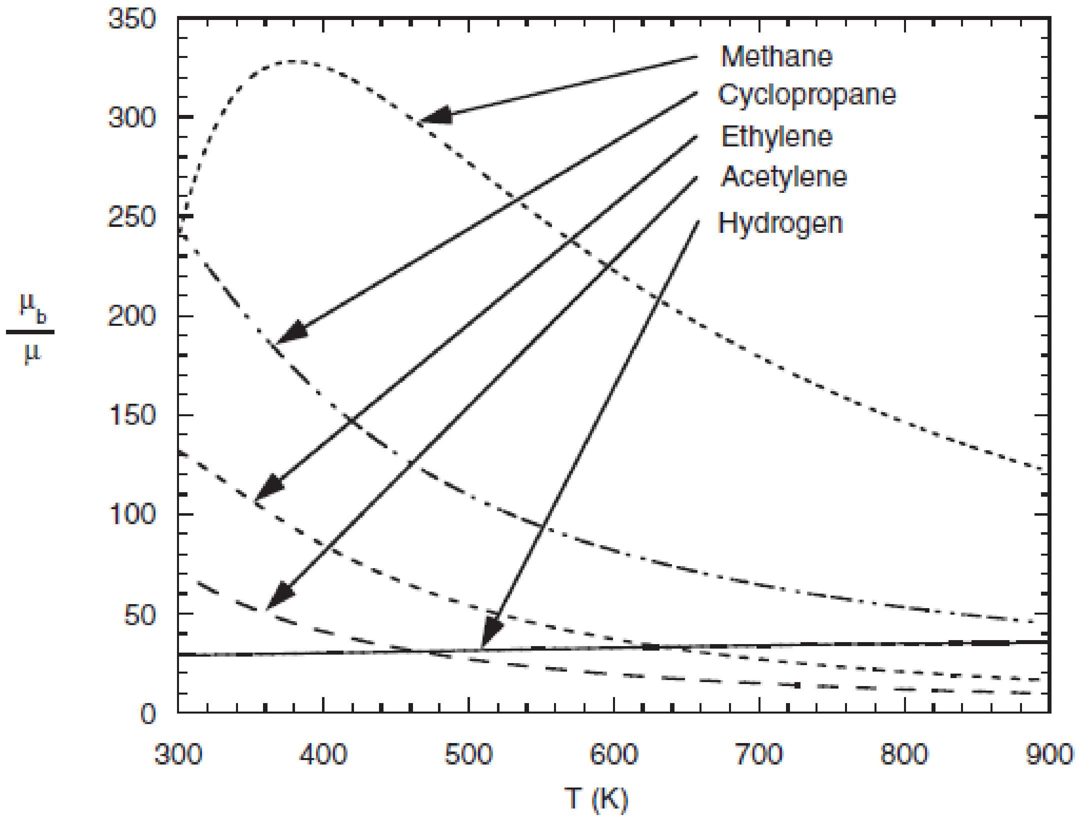
Arnold *et al*, PRD 74, 085021 (2006)

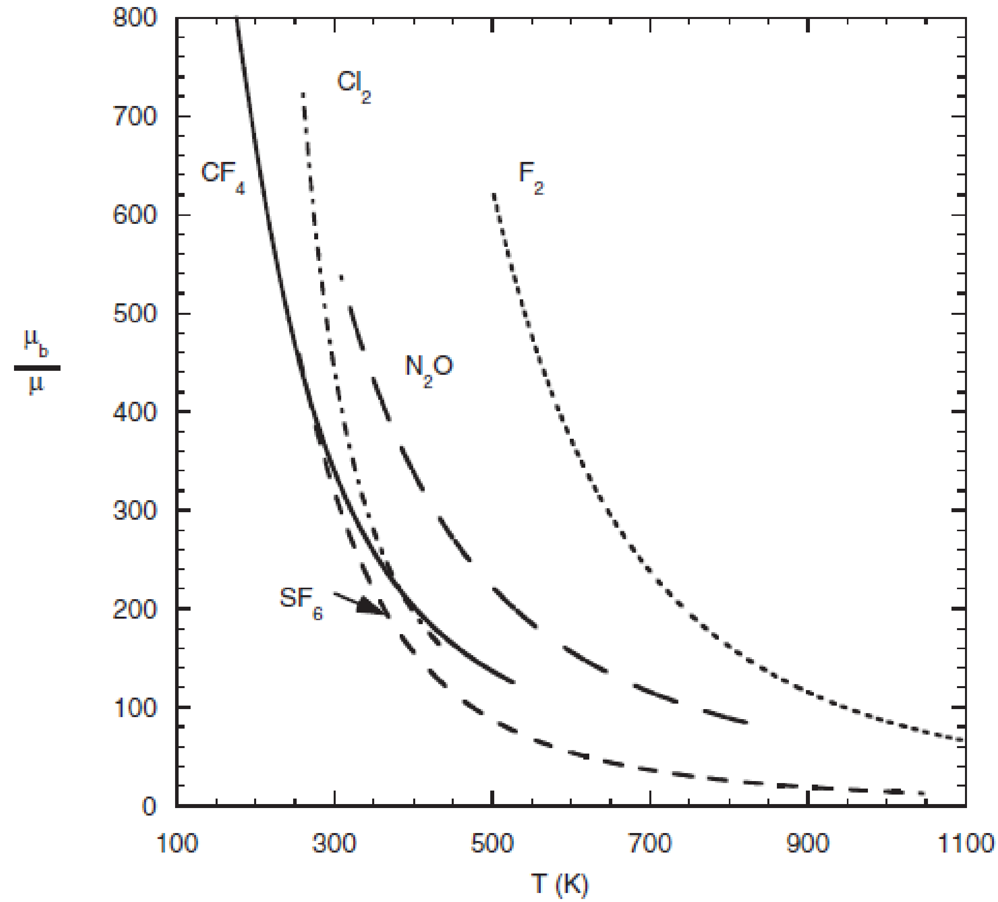
Bulk viscosity ~1000 times smaller than shear

What about near Tc?

What about bulk viscosity?

Bulk to shear viscosity ratio for several gases





M. Cramer, Phys. of fluids 24, 066102 (2012)

Bulk viscosity is large when molecules have excited states (vibration and rotation): $\zeta \sim \tau_{\text{excitation}}$ ~~$\zeta \sim \tau_{\text{mean free-time}}$~~

What about bulk viscosity?

Bulk viscosity is large when moleculas have internal degrees of freedom: can vibrate or rotate

i.e., bulk viscosity is large when kinetic energy is transfered to internal energy: production of excited states

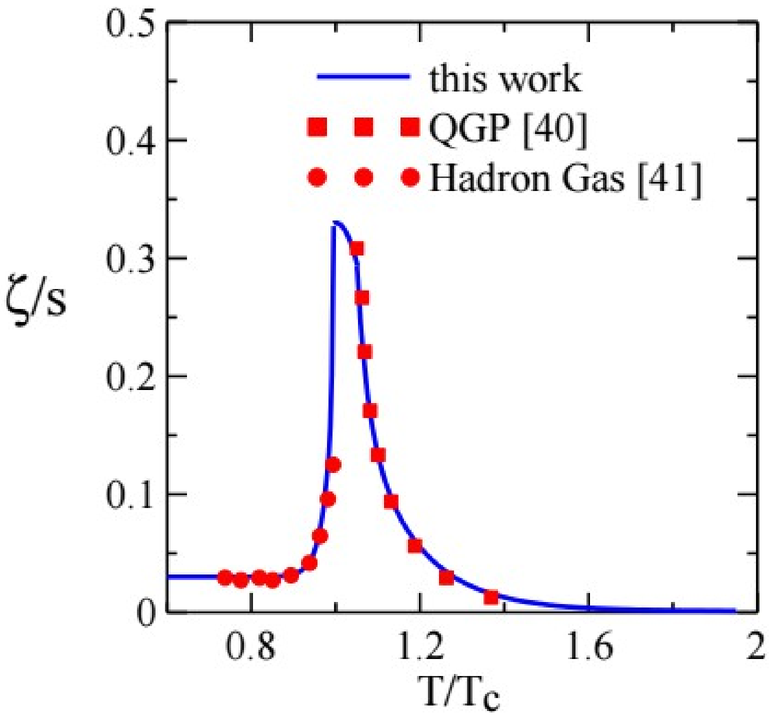
In QCD, we expect many resonances to be formed at the cross-over region: magnitude of bulk viscosity is related to the lifetime of these excited states

Importance of bulk viscosity

IP-Glasma + MUSIC + UrQMD

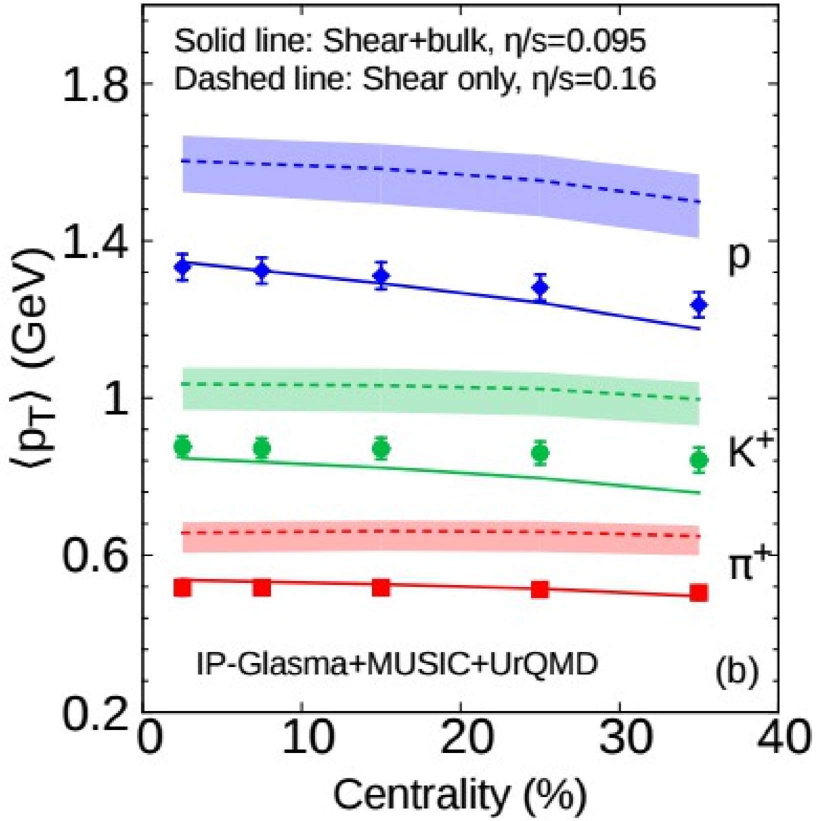
Ryu *et al*, PRL 115, 132301 (2015)

LHC $\eta/s=0.095$



IP-Glasma initial conditions lead to high mean pT

Bulk viscosity reduces mean pT



Value of shear viscosity extracted changes significantly

$\eta/s=0.16 \longrightarrow 0.095$

Relativistic Navier-Stokes theory

Shear Viscosity

(Resistance to deformation)

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$

Bulk Viscosity

(Resistance to expansion)

$$\Pi = -\zeta\nabla_\mu u^\mu$$

Net-Charge Diffusion

$$n_q^\mu = \kappa_q \nabla^\mu \frac{\mu_q}{T}$$

- Equations violate causality and display unphysical instabilities

Global equilibrium state is linearly unstable

Hiscock and Lindblom, Phys.Rev.D 31 (1985) 725-733

- Relativistic Navier-Stokes theory cannot be used to model relativistic fluids

- This is the reason why it took the field so long to add dissipation into the models

Why acausal and unstable?

- At large wave numbers, dispersion relations of Navier-Stokes have the typical form associated to the diffusion equation

$$\omega = iDk^2 \qquad \partial_t T = D\,\nabla^2 T\,,$$

acausal

- If we perform a (1D) Lorentz boost

$$\begin{pmatrix}\omega\\ k\end{pmatrix} = \begin{pmatrix}\gamma & -\gamma V\\ -\gamma V & \gamma\end{pmatrix}\begin{pmatrix}\omega'\\ k'\end{pmatrix} = \begin{pmatrix}\gamma\omega' - \gamma V k'\\ -\gamma V\omega' + \gamma k'\end{pmatrix}$$

$$\omega = iDk^2 \;\longrightarrow\; \omega' - Vk' = i\gamma D\left(k' - V\omega'\right)^2$$

- An unstable mode appears: $\omega'(k'=0) = -\dfrac{i}{\gamma D V^2}\,,$

What we solve is not "traditional" fluid dynamics

Israel and Stewart, Annals Phys. 118 (1979) 341-372

$$\tau_\Pi D\Pi + \Pi = -\zeta\theta + \cdots ,$$
$$\tau_n \Delta^\mu_\alpha D n^\alpha + n^\mu = \kappa_n \nabla^\mu \alpha + \cdots ,$$
$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots .$$

expansion rate

$$\theta = \nabla_\mu u^\mu$$

shear tensor

$$\sigma^{\mu\nu} = \nabla^{\langle\mu} u^{\nu\rangle}$$

- relaxes to Navier-Stokes theory (asymptotic solution)

- **causality:** relaxation times cannot be arbitrarily small

$$\frac{\zeta}{(\varepsilon_0 + p_0)\tau_\Pi} + \frac{4}{3}\frac{\eta}{(\varepsilon_0 + p_0)\tau_\pi} \le 1 - c_s^2 ,$$

Complete equations: second order hydrodynamics

Denicol *et al* PRD 85 (2012) 114047

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} - \beta_\Pi \theta - \ell_{\Pi n} \partial \cdot n - \tau_{\Pi n} n \cdot \dot{u} - \delta_{\Pi\Pi} \Pi \theta - \lambda_{\Pi n} n \cdot \nabla \alpha_0 + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \,, \quad (20)$$

bulk

$$\dot{n}^{\langle\mu\rangle} = -\frac{n^\mu}{\tau_n} + \beta_n \nabla^\mu \alpha_0 - n_\nu \omega^{\nu\mu} - \delta_{nn} n^\mu \theta - \ell_{n\Pi} \nabla^\mu \Pi + \ell_{n\pi} \Delta^{\mu\nu} \partial_\lambda \pi^\lambda_\nu + \tau_{n\Pi} \Pi \dot{u}^\mu - \tau_{n\pi} \pi^\mu_\nu \dot{u}^\nu - \lambda_{nn} n^\nu \sigma^\mu_\nu + \lambda_{n\Pi} \Pi \nabla^\mu \alpha_0 - \lambda_{n\pi} \pi^{\mu\nu} \nabla_\nu \alpha_0 \,, \quad (21)$$

diffusion

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi \sigma^{\mu\nu} + 2\pi^{\langle\mu}_\alpha \omega^{\nu\rangle\alpha} - \tau_{\pi n} n^{\langle\mu} \dot{u}^{\nu\rangle} + \ell_{\pi n} \nabla^{\langle\mu} n^{\nu\rangle} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi^{\langle\mu}_\alpha \sigma^{\nu\rangle\alpha} + \lambda_{\pi n} n^{\langle\mu} \nabla^{\nu\rangle} \alpha_0 + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \,. \quad (22)$$

shear

- derived from kinetic theory
- often called DNMR equations

Equations of motion employed - DNMR

Denicol *et al*, Phys.Rev.D 85 (2012) 114047

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \ ,$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu} .$$

Transport coefficients – ultrarelativistic gas

$z = m/T$

Phys.Rev.C 90 (2014) 2, 024912

$$\frac{\eta}{\tau_\pi} = \frac{\varepsilon_0 + P_0}{5} + \mathcal{O}\left(z^2\right),$$

$$\frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3} + \mathcal{O}\left(z^2\right) \ ,$$

$$\frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7} + \mathcal{O}\left(z^2\right) \ ,$$

$$\frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5} + \mathcal{O}\left(z^2 \ln z\right) \ ,$$

$$\frac{\zeta}{\tau_\Pi} = 15\left(\frac{1}{3} - c_s^2\right)^2 (\varepsilon_0 + P_0) + \mathcal{O}\left(z^5\right)$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3} + \mathcal{O}\left(z^2 \ln z\right) \ ,$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(z^4\right) \ ,$$

Applied to heavy ion collisions ...
(MUSIC)

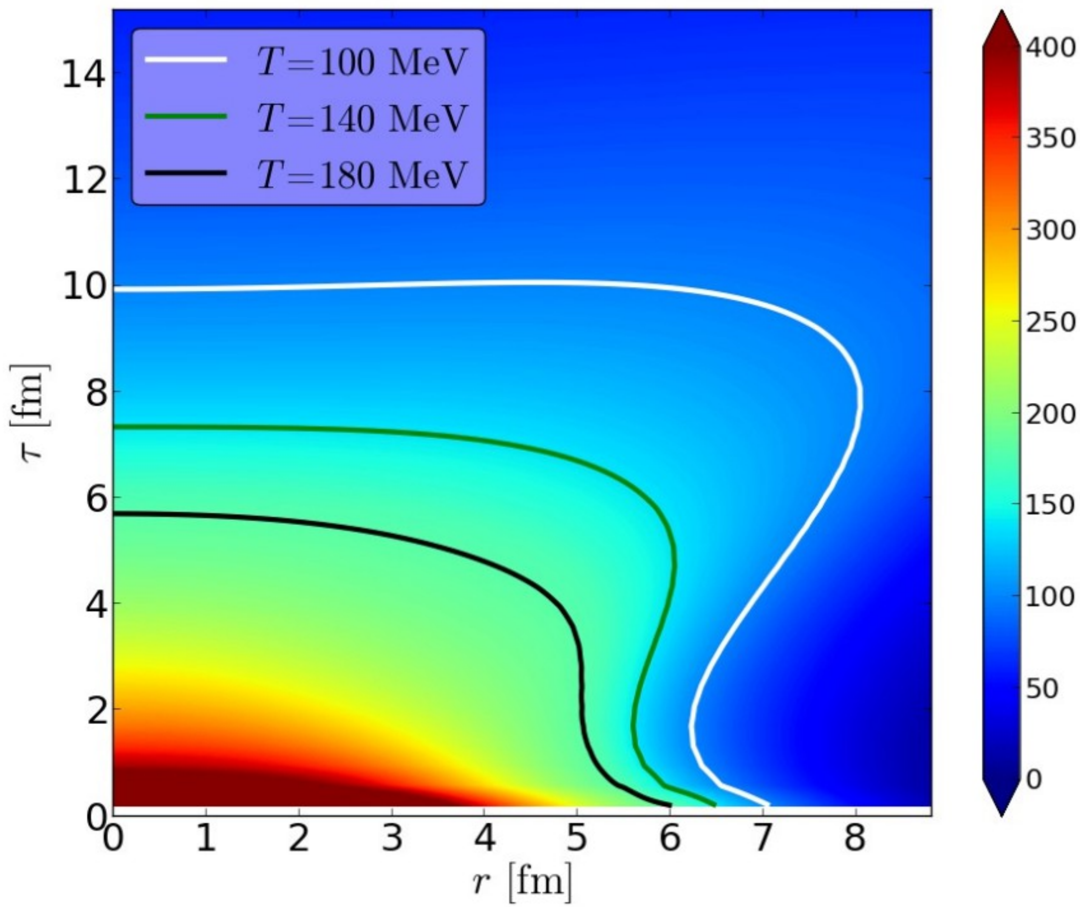
Particle emission

Fluid elements → **Particles**

Assumption: hadrons are emitted from a hypersurface

constant temperature

(smooth) central collision



Particle emission

Fluid elements → **Particles**

Assumption: hadrons are emitted from a hypersurface

constant temperature

- Cooper-Frye procedure

$$\frac{d^2 N_i}{d^2 p_T dy} = \frac{g_i}{(2\pi)^3} \int_\Sigma p_i^\mu d\sigma_\mu (f_0^i + \delta f^i)$$

Freeze-out Hypersurface

Non-equilibrium single-particle momentum distribution

Common choice: "Democratic ansatz"

$$\delta f_i = f_{0i} \frac{p_i^\mu p_i^\nu \pi_{\mu\nu}}{T^2 (e + P)}.$$

Teaney PRC68 034913, Monnai&Hirano PRC80 054906

Particle emission

Fluid elements → **Particles**

Remark: we have to solve a very complicated problem!

Energy-momentum tensor
hadron resonance gas

$$T^{\mu\nu}(x^\lambda)$$

difficult →

← easy

Momentum distribution
of hadrons

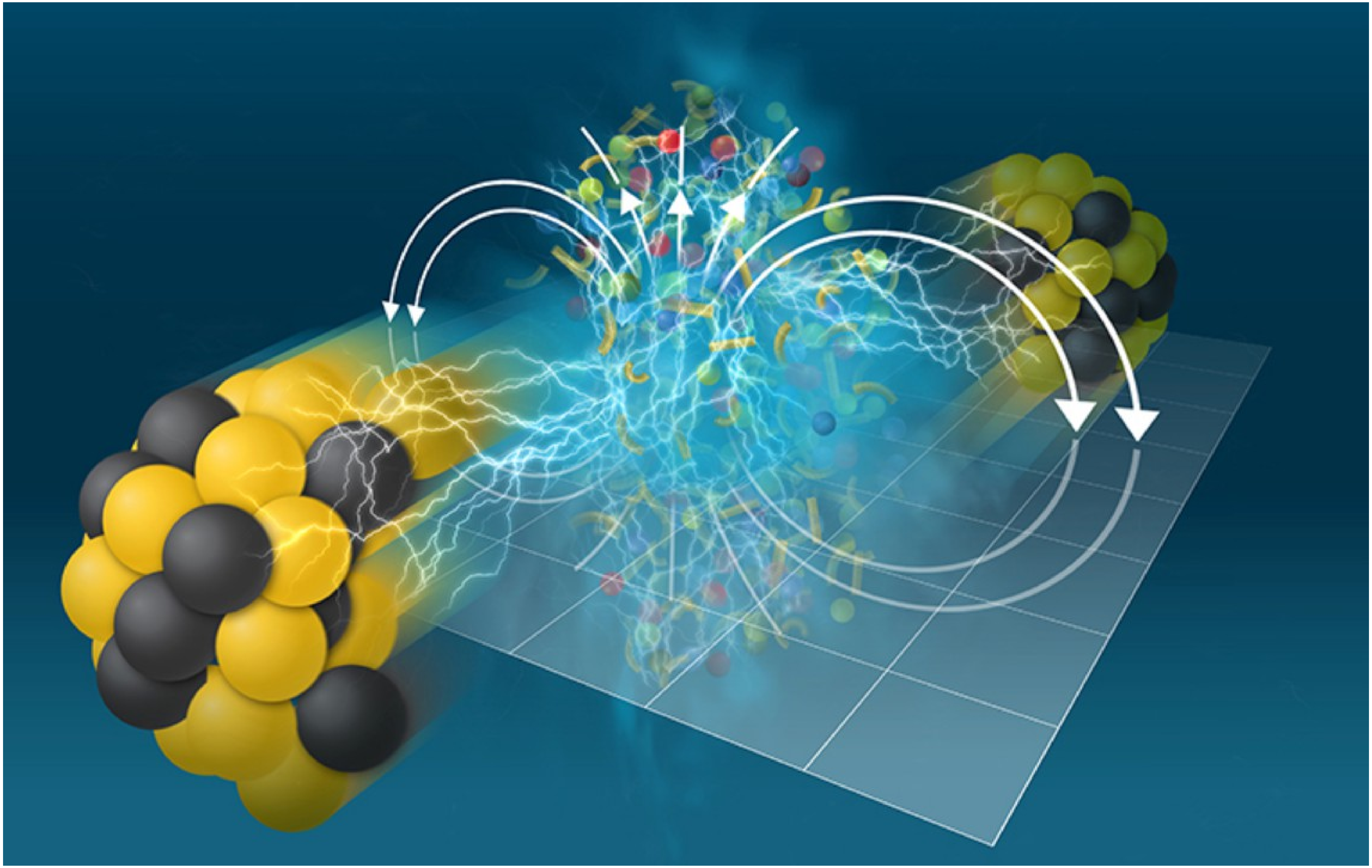
$$f(x^\lambda, p^\lambda)$$

Assumption: system is very close to equilibrium ...

Assumption: hydrodynamics and transport match ...

Assumption: Relaxation time approximation ...

Large Eletromagnetic fields



https://www.bnl.gov/newsroom/

Large Eletromagnetic fields

Spectators (protons) produce *intense* and *shortlived* eletromagnetic fields

eletromagnetic fields induced by the medium can increase the life time of this effect

Gursoy&Kharzeev&Rajagopal
Phys.Rev.C 89 (2014) 5, 054905

This can only be addressed within a framework of relativistic magnetohydrodynamics

Magnetohydrodynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = -F^{\nu\lambda} N_\lambda$$

Net electric-charge conservation

$$\partial_\mu N^\mu = 0$$

energy-momentum exchanged with EM fields

Faraday tensor: $F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$

Electric field (E) **Magnetic field** (B)

Maxwell's equations

$$\partial_\mu F^{\mu\nu} = N^\nu,$$
$$\partial_\mu \tilde{F}^{\mu\nu} = 0,$$

$$\tilde{F}^{\mu\nu} = B^\mu u^\nu - B^\nu u^\mu - \epsilon^{\mu\nu\alpha\beta} u_\alpha E_\beta$$

Fields due to spectators are added as an external source

Kharzeev, McLarren, Warringa, Nucl.Phys.A 803 (2008) 227-253

Magnetohydrodynamics

Dissipative equations (simplified)

Viscous contributions

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - \frac{4}{3}\tau_\pi\pi^{\mu\nu}\nabla_\lambda u^\lambda$$

$$\tau_\Pi\dot{\Pi} + \Pi = -\zeta\nabla_\lambda u^\lambda - \tau_\Pi\Pi\nabla_\lambda u^\lambda$$

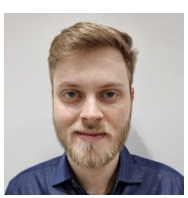
- no effects due to EM fields included

charge 4-diffusion current

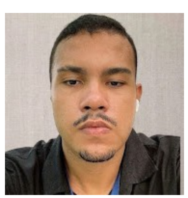
$$\tau_e\dot{n}^\mu + n^\mu = \underbrace{\kappa_e\nabla^\mu\frac{\mu_e}{T}}_{\text{negligible at high energies}} - \tau_e n_e^\mu\nabla_\lambda u^\lambda + \underbrace{\sigma_e E^\mu}_{\text{Ohms Law}}$$

negligible at high energies **Ohms Law**

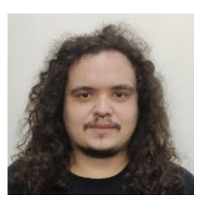
These equations were implemented in a new simulation code: PRAIA



Kevin Pala



Vinicius Franção



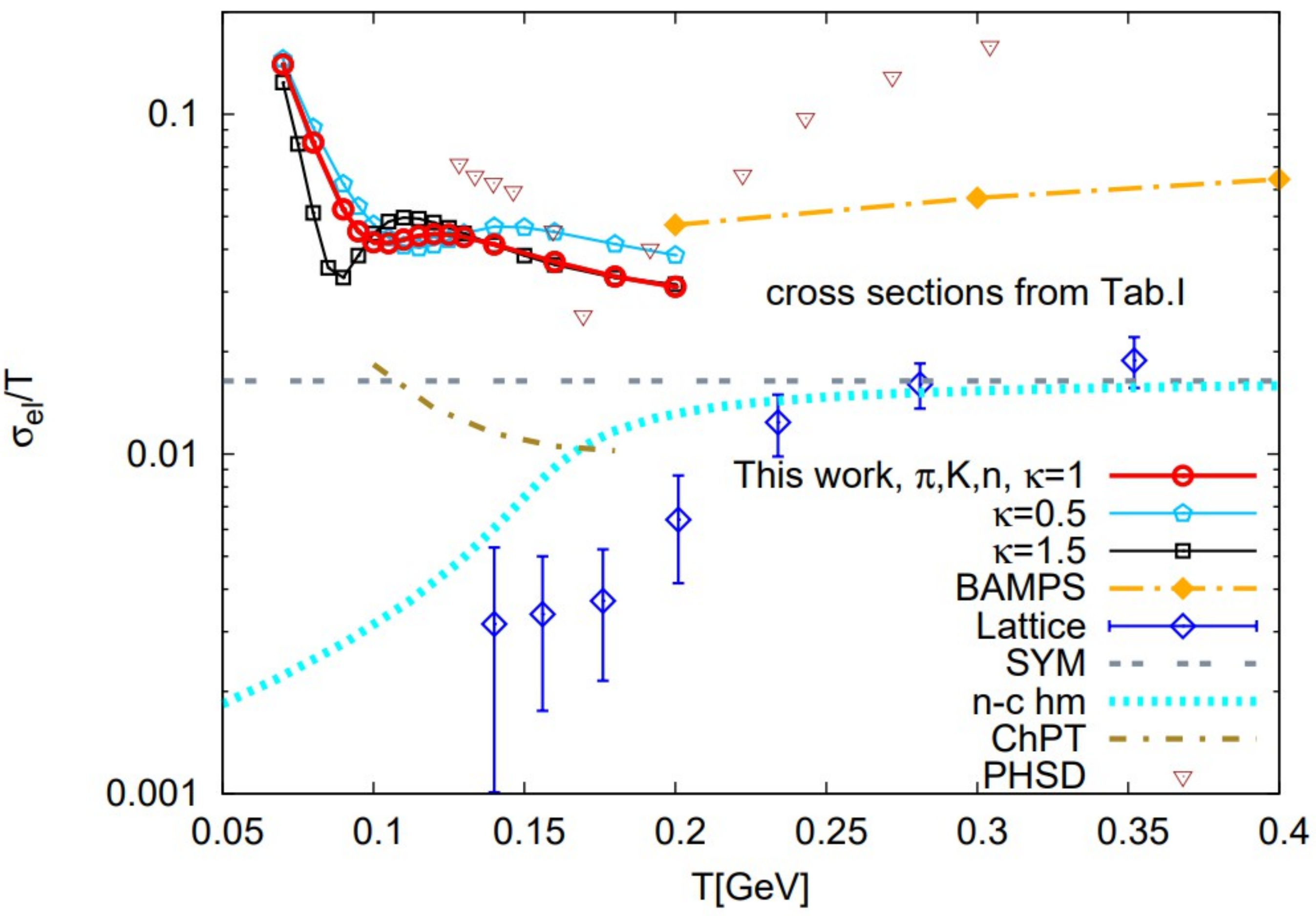
Jonatan Sola

Smooth Particle Hydrodynamics algorithm

Magnetohydrodynamics

Electric conductivity

Greif&Greiner&DenicolPhys.Rev.D 93 (2016) 9, 096012



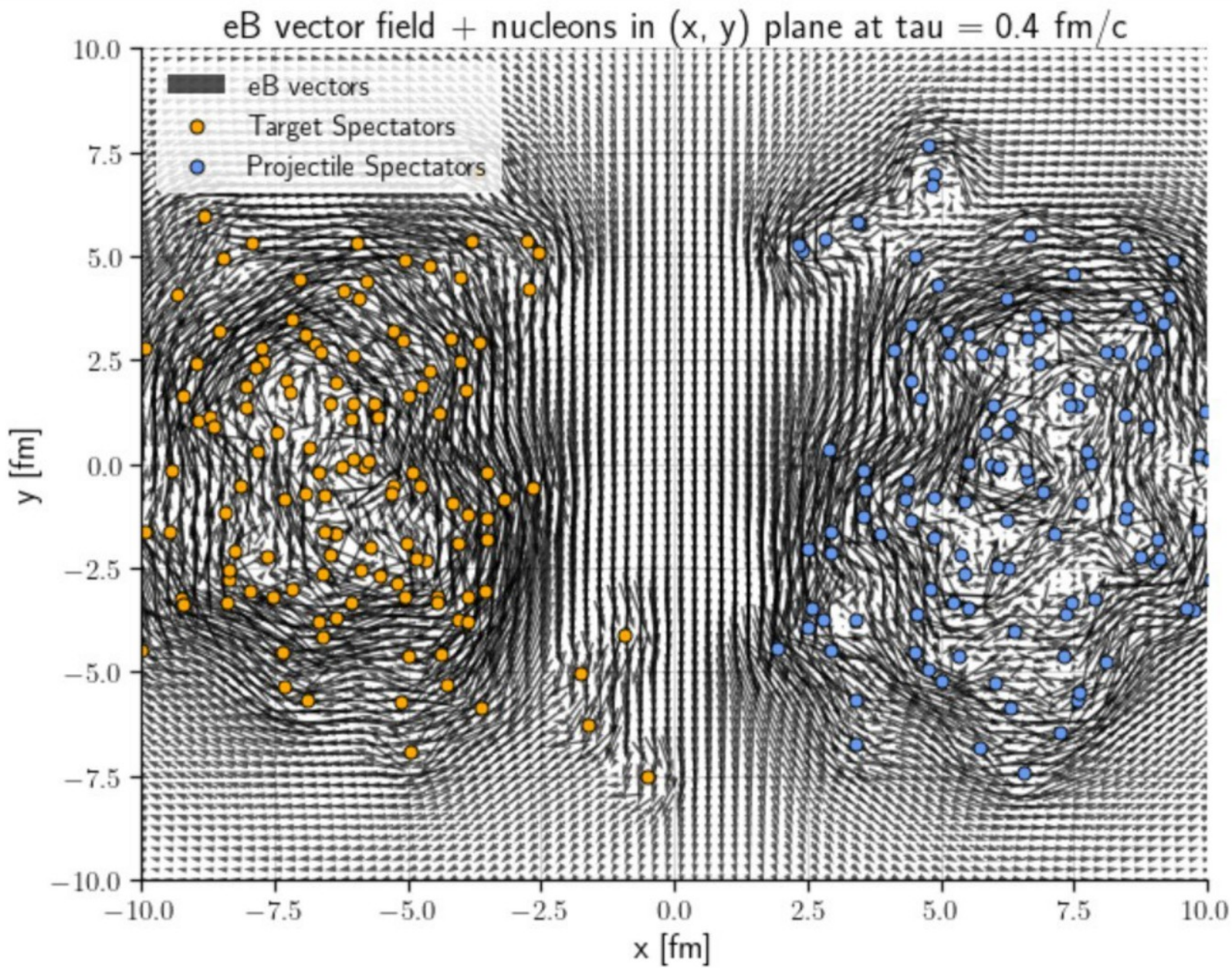
$\sigma_e \sim 0.001\text{-}0.1\ T$

Spectator's magnetic field

AMPT+PRAIA, $\tau_0 = 0.4$ fm

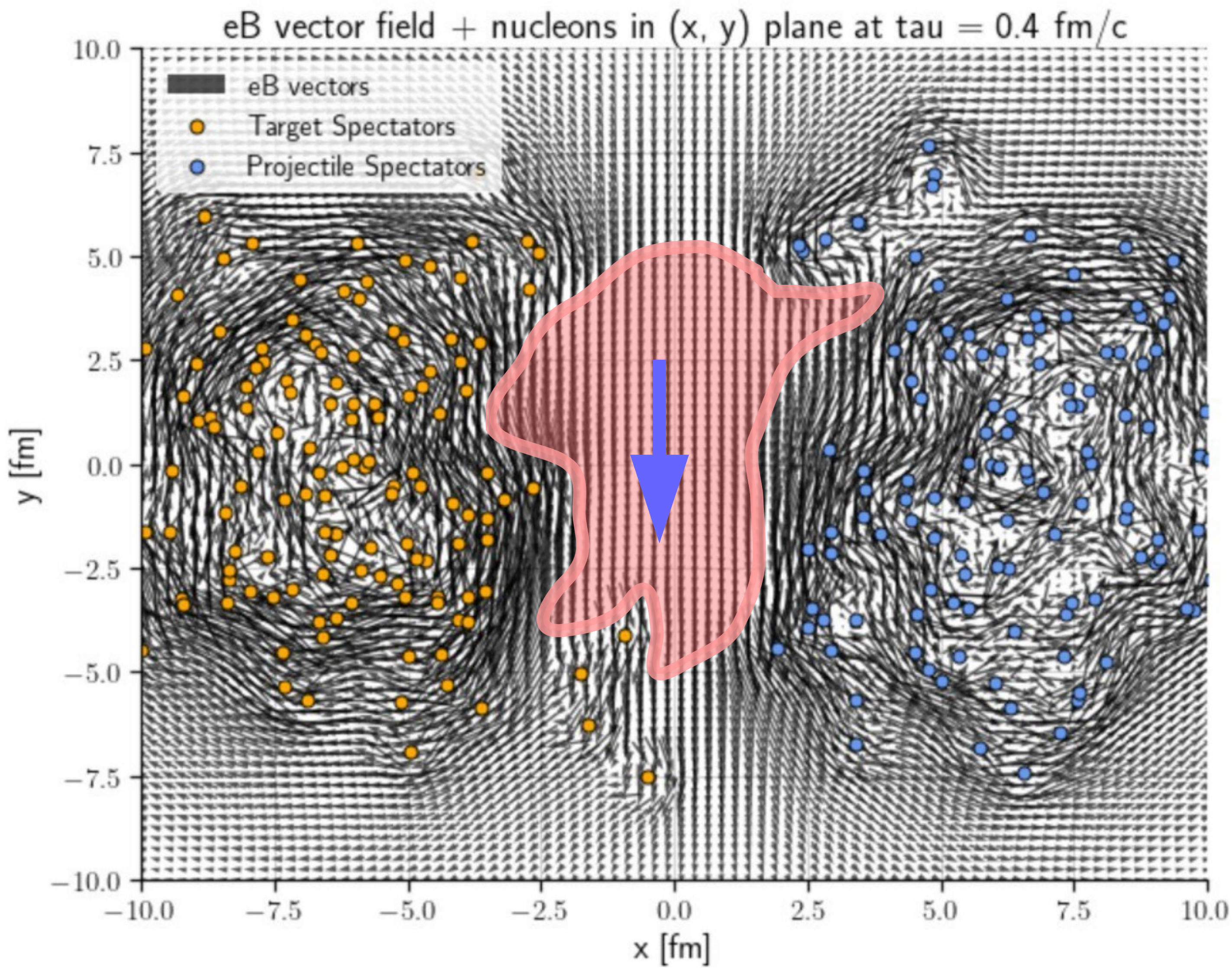
30-40%

PRELIMINARY



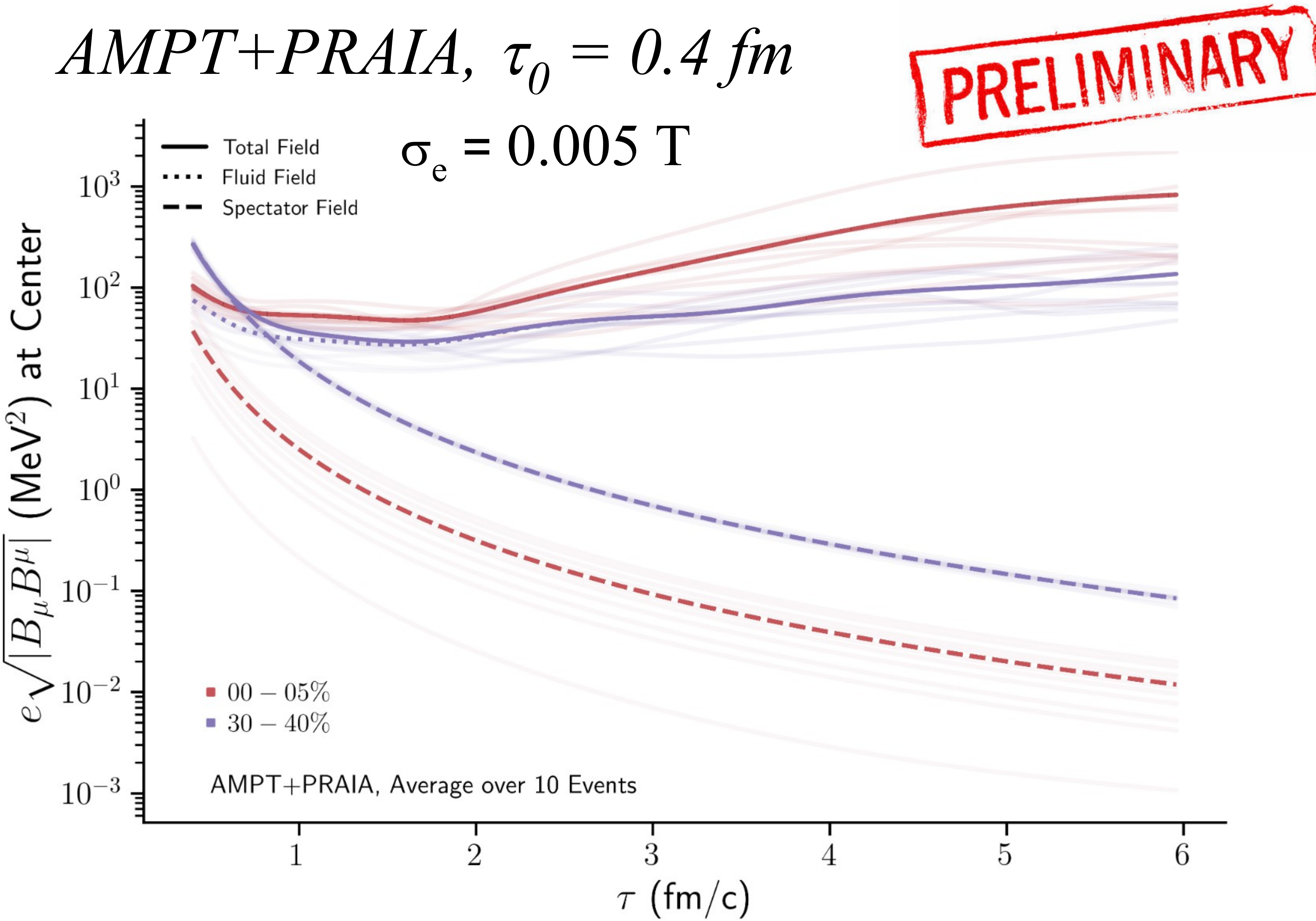
Spectator's magnetic field

AMPT+PRAIA, $\tau_0 = 0.4\ fm$
30-40%



3D+1 Event-by-event Simulations

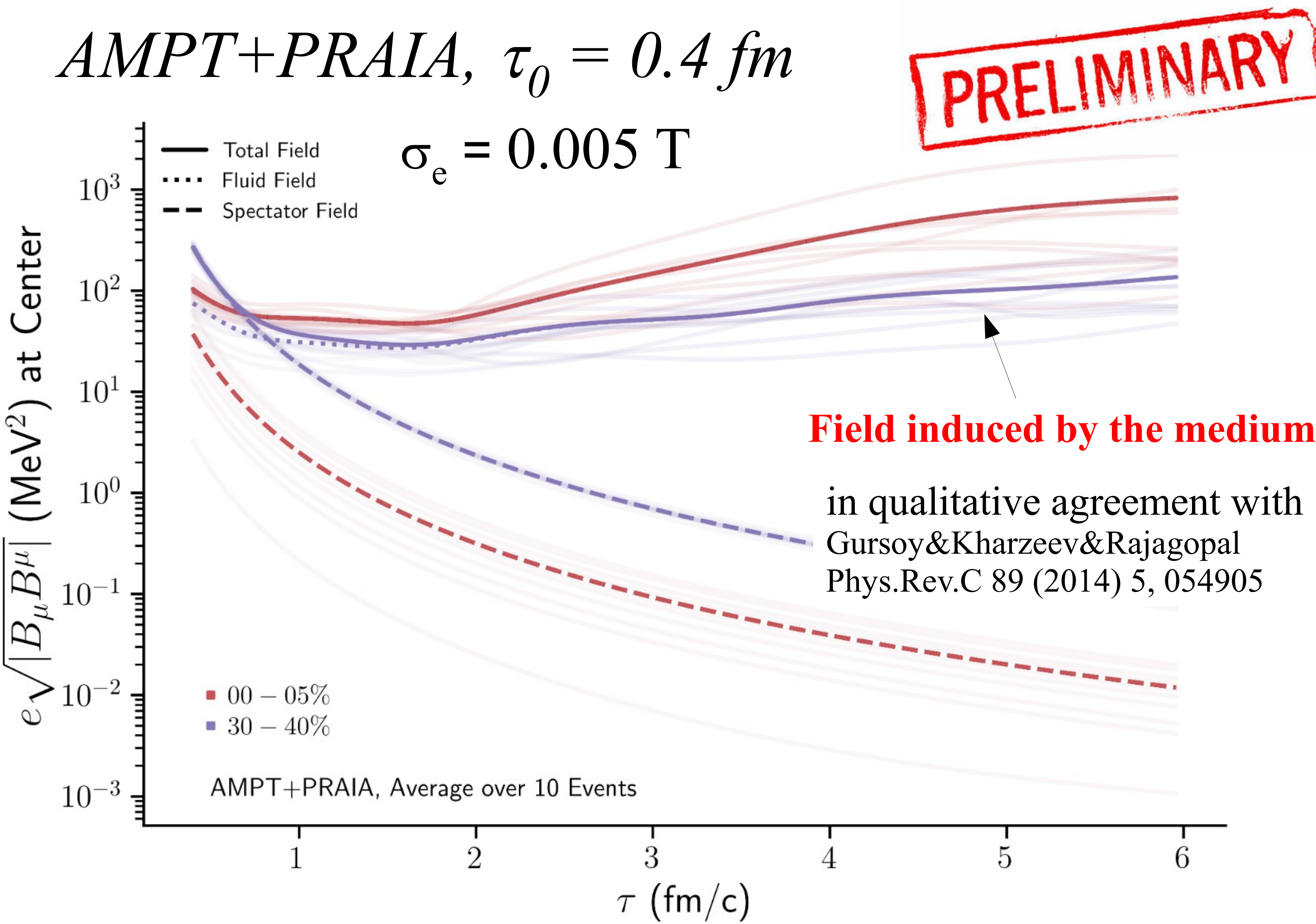
AMPT+PRAIA, $\tau_0 = 0.4$ *fm*



3D+1 Event-by-event Simulations

AMPT+PRAIA, $\tau_0 = 0.4\ fm$

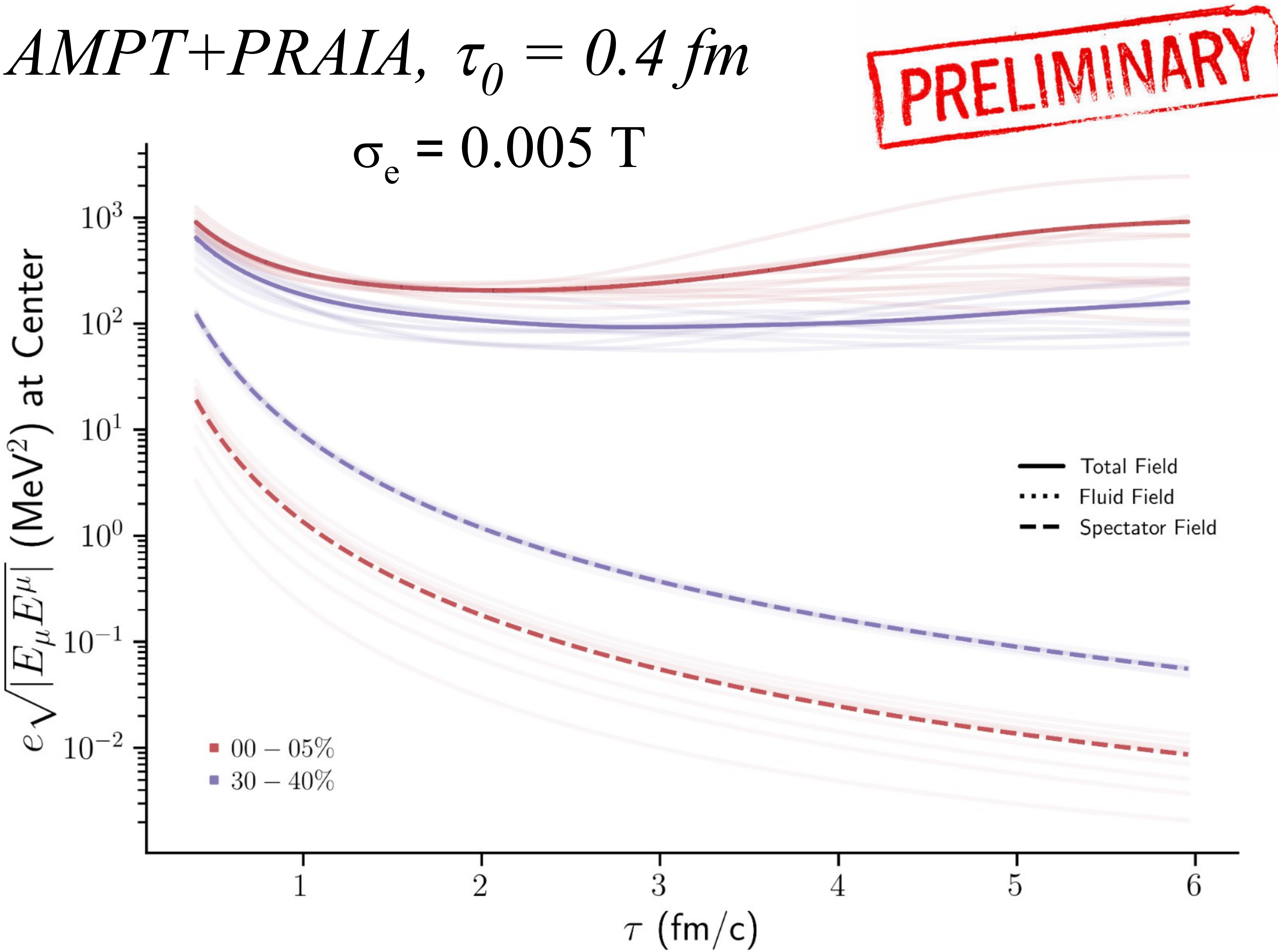
$\sigma_e = 0.005$ T



in qualitative agreement with Gursoy&Kharzeev&Rajagopal Phys.Rev.C 89 (2014) 5, 054905

3D+1 Event-by-event Simulations

AMPT+PRAIA, τ_0 = 0.4 fm



Future directions

- Field generated by participants? Effect on observables?

- Chiral magnetic effect

Chiral current:

$$\partial_\mu N_5^\mu = -\frac{C}{2\pi^2} E_\mu B^\mu$$

$$N_5^\mu = n_5 u^\mu + n_5^\mu$$

Unstable?

Dissipative currents:

$$\tau_e \dot{n}^\mu + n^\mu = \kappa_e \nabla^\mu \frac{\mu_e}{T} - \tau_e n_e^\mu \nabla_\lambda u^\lambda + \sigma_e E^\mu + C \mu_5 B^\mu$$

$$\tau_5 \dot{n}_5^\mu + n_5^\mu = \kappa_5 \nabla^\mu \frac{\mu_5}{T} - \tau_5 n_5^\mu \nabla_\lambda u^\lambda + C \mu_e B^\mu$$

Final remarks

Discussed *some* basic aspects of fluid-dynamical models

- Simulations are *essential* to interpret the data and extract the properties of QCD matter

Properties of matter must be reverse-engineered!

- Models are very complicated and include a lot of physics

Nowadays, we study heavy ion collisions themselves, as a proxy to develop our understanding of QCD

Final remarks – a few challanges

Hydrodynamics with spin degrees of freedom?

- New theories being constructed.
- May be crucial to describe data.

Hydrodynamics near critical point?

- New theories are required ...
- Inclusion of fluctuations.

Relativistic Magnetohydrodynamics?

- Causal theories of magnetohydrodynamics must be constructed