The phase diagram of physical QCD and neutron stars

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Physical QCD

Strongly interesting matter in colliders have $T, \mu \ll m_K$.

The number of active flavours can be taken as 2 or 3: charm and heavier flavours may affect physics through virtual processes, but not directly.

Most theory is done for $N_f = 2 + 1$, ie, light flavours degenerate in mass, strange mass higher. The real world has $N_f = 1 + 1 + 1$ (all flavours have different masses). It turns out that this has different physics.

Two definitions: **chiral QCD** has vanishing quark masses ($N_f = 2$ or 3), **physical QCD** is $N_f = 1 + 1 + 1$.

Conserved quantities are the energy (E), baryon number (B), charge (Q), and maybe strangeness (S). Conserved quantities define thermodynamics.

Methods for the phase diagram

Effective field theories (EFTs)

Symmetries are used to write EFTs: more accurate than models. The chiral phase transition is often studied using the NJL model; the QCD critical point by the Ising model. EFTs can convert them to accurate tools

Universality and scaling

The breaking of symmetries of QCD have been utilized to make predictions for critical exponents, critical slowing down, and other universal quantities. Examples: the chiral transition is in the universality of an O(4) Heisenberg magnet; the QCD critical point is a gas-liquid critical point.

Thermodynamics

Gibbs' phase rule allows use of continuity in the phase diagram: gives topology of phase diagram. Clausius-Clapeyron equations give further constraints.

Revisiting thermodynamics

Gibbs space has coordinates which are conserved **extensive variables** (E, S, B, etc) and the entropy S. With D conserved quantities, Gibbs space has dimension $D+1$. Thermodynamic **equilibrium states** lie on a convex D dimensional hypersurface given by $E(S, B, \cdots)$.
Phase diagram is drawn in a D dimensional space. Its coordinates are the thermodynamic **intensive variables**: derivatives of E with respect to other extensive variables,

$$T = \frac{\partial E}{\partial S}, \qquad \mu_B = \frac{\partial E}{\partial B}, \qquad \cdots$$

Thermodynamic potential $\Omega(T, \mu_B, \cdots)$ is the Legendre transform of E and a function on the phase diagram.

Example: pure gauge theory has one conserved quantity, E. The equilibrium states lie on a convex curve $E(S)$. There is one intensive thermodynamic variable T. The phase diagram is 1 dimensional, and its free energy is $\Omega(T)$.

Gibbs' phase rule

1st order phase transitions (1°PT) occur when two phases (with potentials Ω_1 and Ω_2) coexist in equilibrium at the same values of the intensive variables. For coexistence one has

$$\Omega_1(T, \mu_B, \cdots) = \Omega_2(T, \mu_B, \cdots).$$

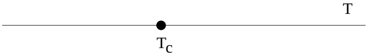
Solutions are of the form $T(\mu_B, \cdots)$. So 1°PT occur along a hypersurface of $D-1$ dimensions. $\Omega_{1,2}$ are convex so the hypersurface cannot be closed. Three phases coexist along the intersection of two 1°PT surfaces: $\Omega_1 = \Omega_2 = \Omega_3$ and therefore are hypersurfaces of $D-2$ dimensions.

These hypersurfaces may extend from $T = 0$–∞ and therefore never end. If they end, then the edges are called critical surfaces and are $D-2$ dimensional. If a three-phase coexistence surface has an end, then the $D-3$ dimensional surface is called a tricritical surface.

Inferences: *No critical point in $D = 1$! No critical line in $D \leq 2$!*

Phase diagram of pure gauge theory

SU(N) pure gauge theory has an order parameter (Polyakov loop) which distinguishes two phases: confined and deconfined. There is one value of T at which these two phases coexist. Agrees with lattice simulations: 1°PT.



Pure gauge SU(2) also has confined and deconfined phases. But it has a second order phase transition! Violation of Gibbs phase rule? Recall: -log(order parameter) is the free energy of medium with a static quark. Quarks have a special symmetry for SU(2) gauge theory. So need to consider extended phase diagram in (T,N). Then line of 1°PT with critical end point! "Predicts" absence of phase transition for photon gas!

Need a "theory parameter" to understand the phase diagram!

Is the full SM needed?

The EW sector is a **chiral theory**: left-handed fermions have a different status than right-handed fermions. Hard to simulate on the lattice.

Standard model at $T \ll M_W$ is in the **Higgsed state**, at $T \gg M_W$ expected to be in the **symmetric state**. Is there a phase transition? Need to understand thermodynamics of a gauge theory coupled to a scalar and left-handed fermions.

The strong CP problem does not seem to have a solution at any scale relevant to QCD.

No extra symmetry

Conserved quantities of full standard model are B, Q, and lepton number L. The section with $L = 0$ has no extra symmetries beyond those of QCD. So, the phase diagram of physical QCD may be understood correctly even if we neglect the EW sector.

Flavour symmetry breaking in QCD

$N_f = 3$ SU(3) flavour symmetry broken to $N_f = 2+1$ when quark masses are not equal: $m_\ell < m_s$. Lattice QCD tunes two quark masses to tune m_π and m_K (isospin averaged mass for each S).

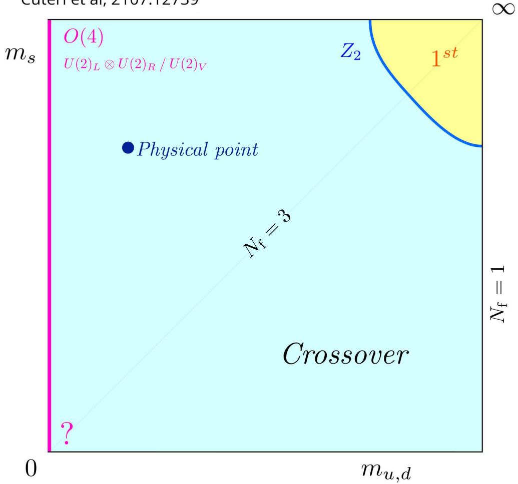
Columbia plot tracks nature of phase transition for $\mu_B = \mu_Q = 0$.

$N_f = 2+1$ SU(2) flavour symmetry broken to $N_f = 1+1+1$ when $\Delta m = m_d - m_u$ is non-zero. Remnant symmetry is U(1) generated by τ_3 (λ_3). Same symmetry also broken by α_{EM}. Lattice QCD tunes Δm so that $m_{\pi^0}/m_{\pi^\pm}$ is close to physical.

Gell-Mann Nishijima relation $I_3 = Q - (B+S)/2$ can be used to relate $dI_3 = dQ$. This implies equality of charge and isospin chemical potential $\mu_Q = \mu_I$.

The Columbia plot on the lattice

Cuteri et al, 2107.12739



The crossover

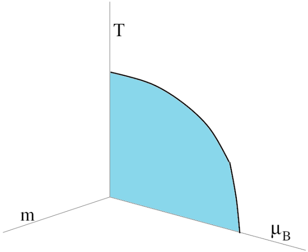
The free energy has no discontinuity or singularity. Second derivative, chiral susceptibility χ_m, has finite maximum. Location of maximum defines T_{co}. Other definitions of crossover could be different: non-zero width of crossover: ΔT (goes to zero when $m = 0$, all T_{co} go to T_c). Switching on μ_B does not change chiral symmetry, so critical line:

$$T_c(\mu_B) = T_c \left[1 - \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c} \right)^4 + \cdots \right]$$

Two sets of results available for $N_f = 2 + 1$ and realistic hadron masses:

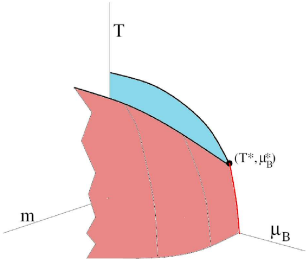
	T_{co} (MeV)	ΔT (MeV)	κ_2	κ_4
HotQCD 2017	156.5 (1.5)		0.015 (4)	-0.001 (3)
BHJW 2020	158.0 (0.6)	15 (1)	0.0153 (18)	0.00032 (67)

The phase diagram in T-μ_B-m



Coexistence surface for chiral QCD has boundary $O(4)$ critical line;

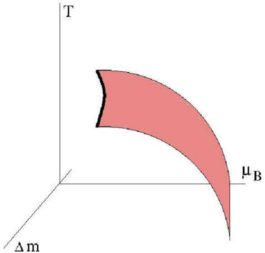
The phase diagram in T-μ_B-m



Berges and Rajagopal; Halasz et al; 1998. HotQCD 2017, BHJW 2020

Coexistence surface for chiral QCD has boundary $O(4)$ critical line; Hadron-quark coexistence surface has boundary Ising critical line. Two surfaces intersect along triple line; its end point (T^*, μ^*) is a tricritical point. QCD critical point (T^E, μ^E) is the intersection of the Ising line with the physical m plane. Lattice: $\mu^E/T^E \simeq 2$ (**Mumbai 2017**), 4–5 **Clarke et al 2024**), $T^E < 100$ MeV (**Adam et al 2025**).

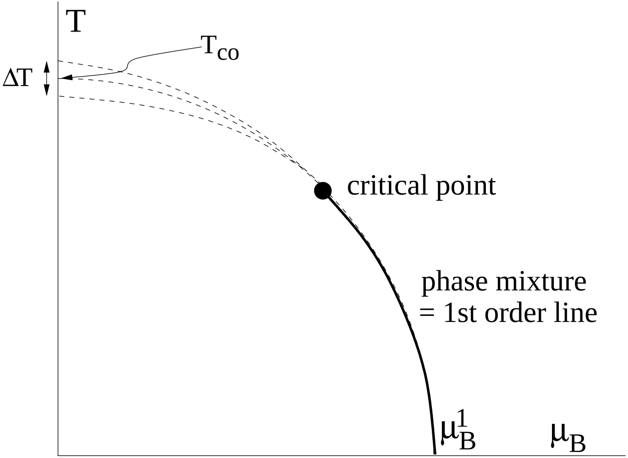
The phase diagram in T-μ_B-Δm



$N_f = 1 + 1 + 1$: generic Δm

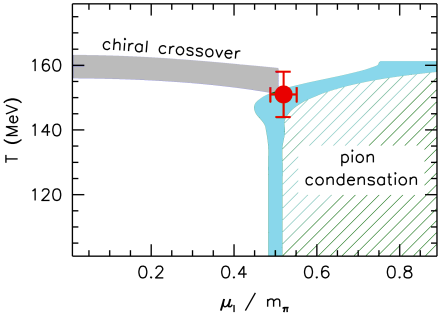
When $\Delta m \neq 0$ SU(2) flavour breaks to U(1). Hadron multiplets only labelled by I_3: so π^0 mixes with isoscalar. Flavour singlet quark condensate mildly rotated. Phase diagram similar to $N_f = 2 + 1$: if 1°PT line with critical end point, then the same for realistic Δm.
(**SG and Sharma, in progress**)

Crossover, critical point, 1st order transition



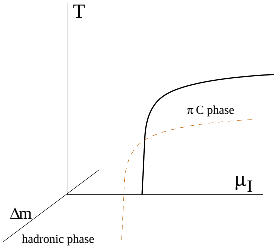
For $N_f = 2 + 1$: $T^E = 105^{+8}_{-18}$ MeV and $\mu_B^E = 422^{+80}_{-35}$ MeV. Curvature gives $\mu_B^1 = 1280 \pm 170$ or 1280 ± 75 MeV.

The phase diagram in T-μ_I ($N_f = 2+1$)



μ_I breaks SU(2) flavour to $U_{\tau_3}(1)$. Charged pion condensation sets in at $\mu_I = m_\pi/2$ at $T = 0$; O(2) critical line nearly vertical till hadron-quark crossover, then nearly horizontal (**Brandt et al 2017**). For $\mu_I > 1500$ MeV and $T \simeq 0$ possible crossover between pion condensate and colour superconducting states (**Abbott et al 2025**).

The phase diagram in T-μ_I-Δm

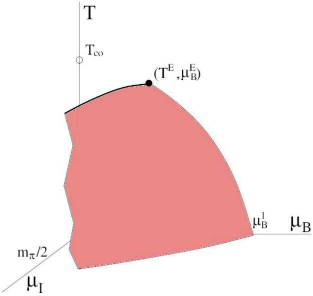


$N_f = 1 + 1 + 1$: sign problem arises with μ_I

Same symmetry breaking by Δm and μ_I, so for $\Delta m \neq 0$ must be crossover. Critical line for $\Delta m = 0$ without 1°PT violates Gibbs' phase rule. But 1°PT for symmetry breaking in the remnant $U_{\tau_3}(1)$: fictitious direction.

No pion condensing phase transition in physical QCD: possibly a crossover, and at larger μ_I another crossover to CSC.

The phase diagram in physical QCD



$$\mu_B^1 \approx 1200 \text{ MeV}.$$

Sign problem — everywhere except $T = 0$ line.
Hadron-quark coexistence surface ends in Ising critical line. Crossover between hadron and pion condensed phases “inside” this surface.
Cut-away view: colour superconductivity involved in outward development of phase diagram.

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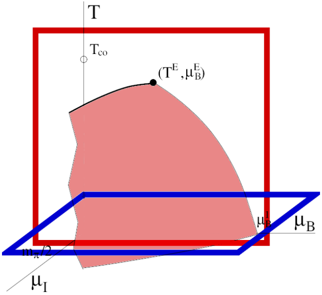

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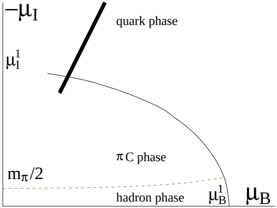
The phase diagram for NSs



NSs charge neutral and stable against β-decay: almost all baryons are neutrons. So $\mu_B \simeq \mu_n$ and $\mu_I \simeq -2\mu_n$. Density of core between nuclear saturation density and two to three times larger.

Lattice strongly implies quark matter cores, CSC possible, no pion condensation. Lattice implies $c_s > 1/\sqrt{3}$ quite normal.

But lattice is a rough guide in region with large μ_B and μ_I due to sign problem. Needs study in well-tuned models or EFTs.

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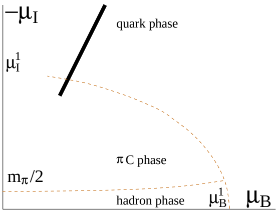

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Summary

Physical QCD

Sign problem everywhere in the phase diagram. The phase diagram of physical QCD contains at most one surface of 1°PT (hadronic to quark), and one critical line. Based on symmetries, lattice and EFTs, supplemented by Gibbs phase rule.

Possible alternative: no 1°PT, only crossovers.

Neutron Stars

- If NS cores are entirely in the hadronic phase, then there could be a 1°PT between hadron and quark phases. In binary NS collisions, local heating could cause transition through it, leading to an explosion. Can this lead to GRB?
- If there are no 1°PT, still there may be quark cores and CSC phases in NSs, but without interface tension or bubbles. No exotic explanations for energy generation.