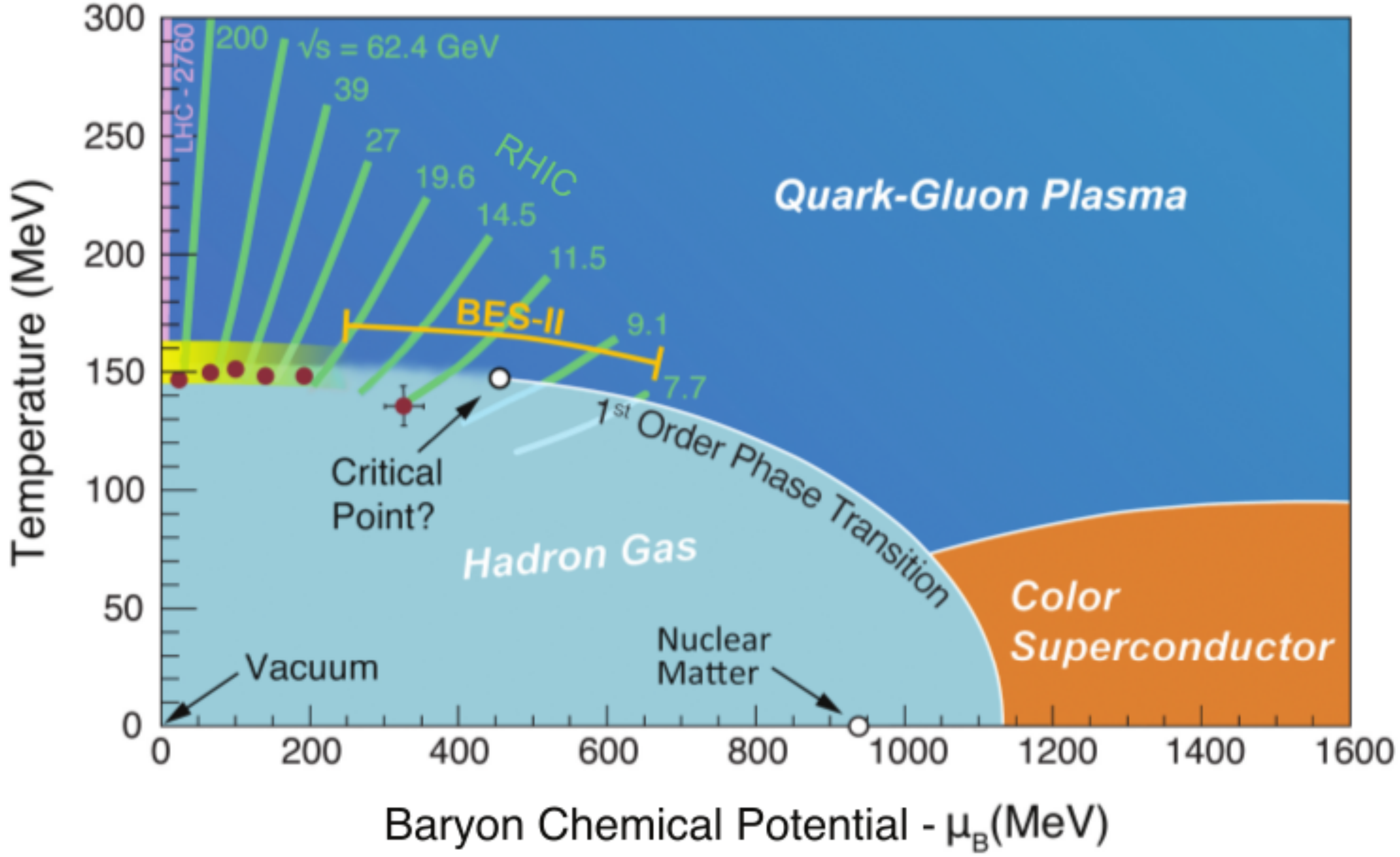
What have we learned from the RHIC beam energy scan about the QCD critical point

"A theory is something nobody believes, except the person who made it.

An experiment is something everybody believes, except the person who made it."

A. Einstein

The phase diagram



Increase chemical potential by lowering the beam energy

In reality, we add baryons (nucleons) from target and projectile to mid-rapidity

What we know about the Phase Diagram

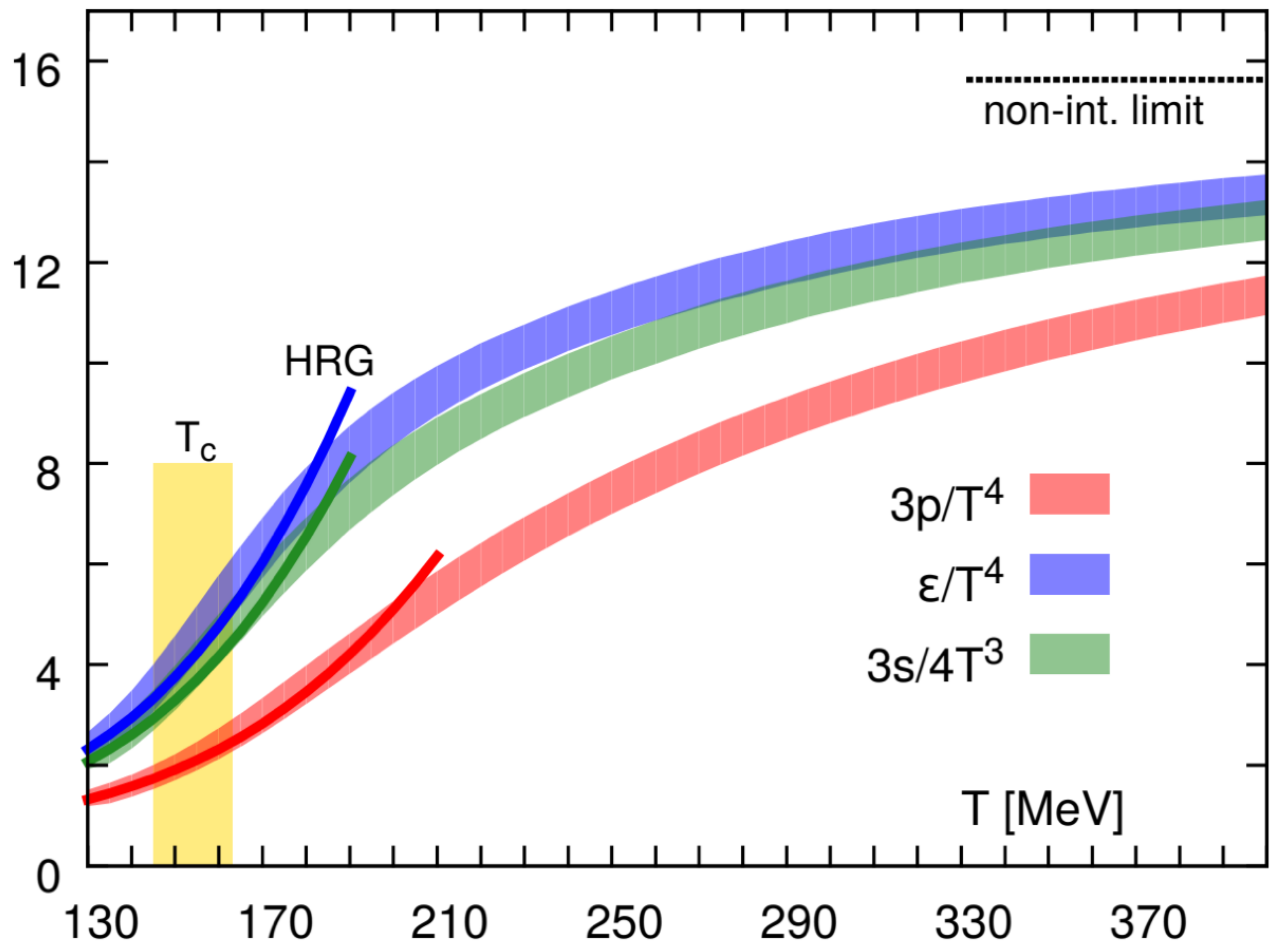
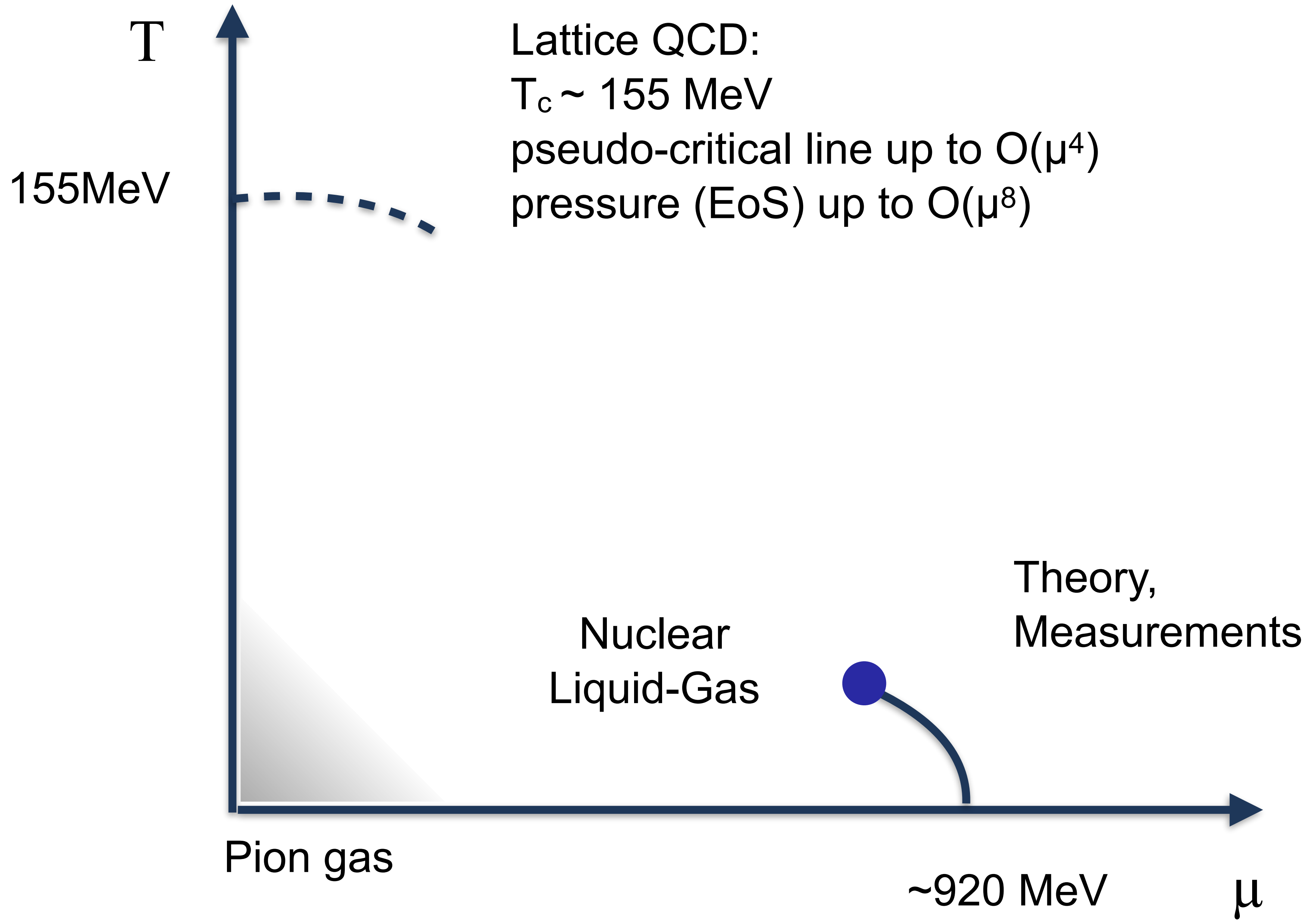
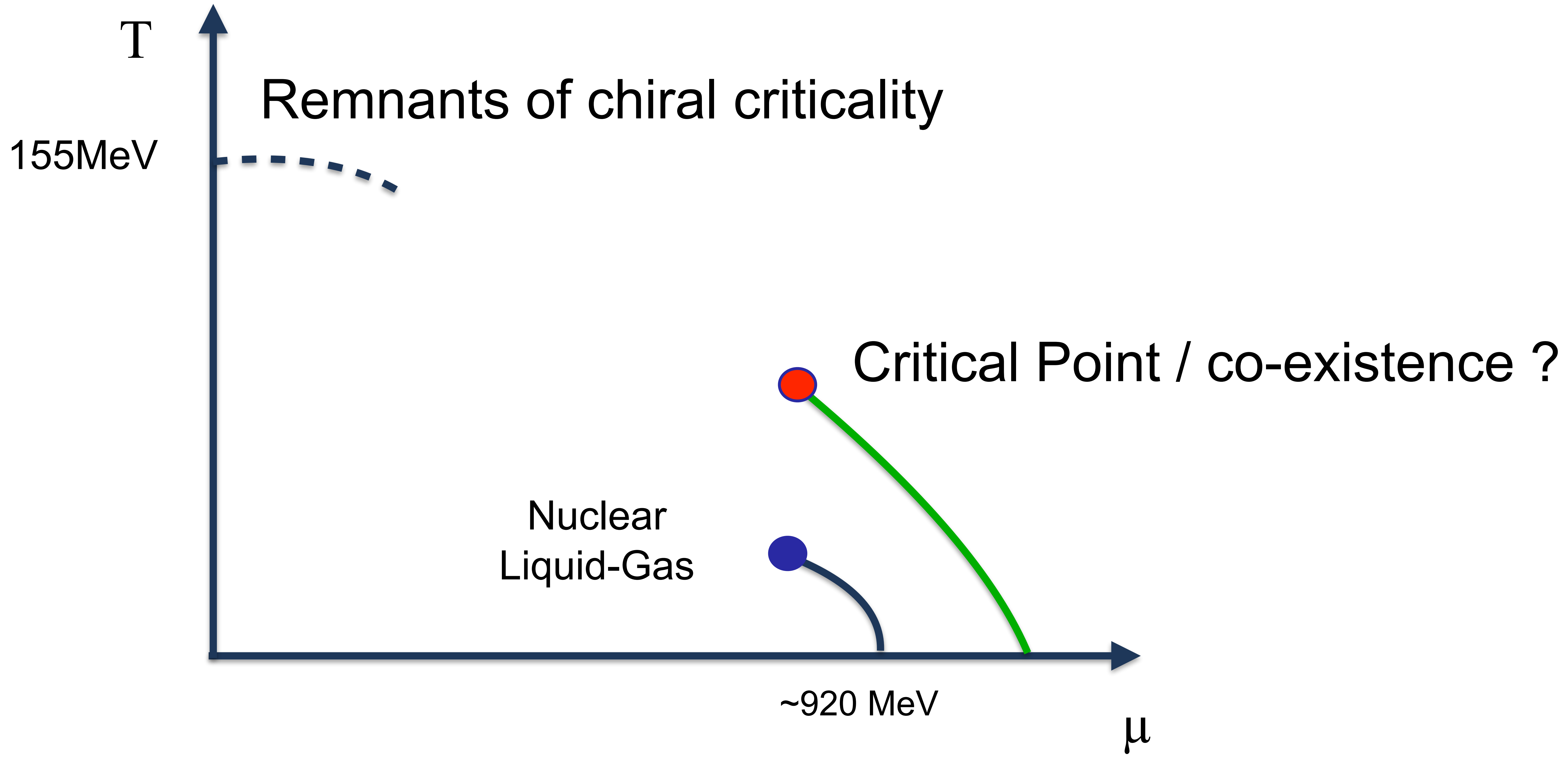


Figure from HotQCD coll., PRD '14



What we are looking for



We are dealing with small system of finite lifetime

NO real singularities!

Recent results on CP

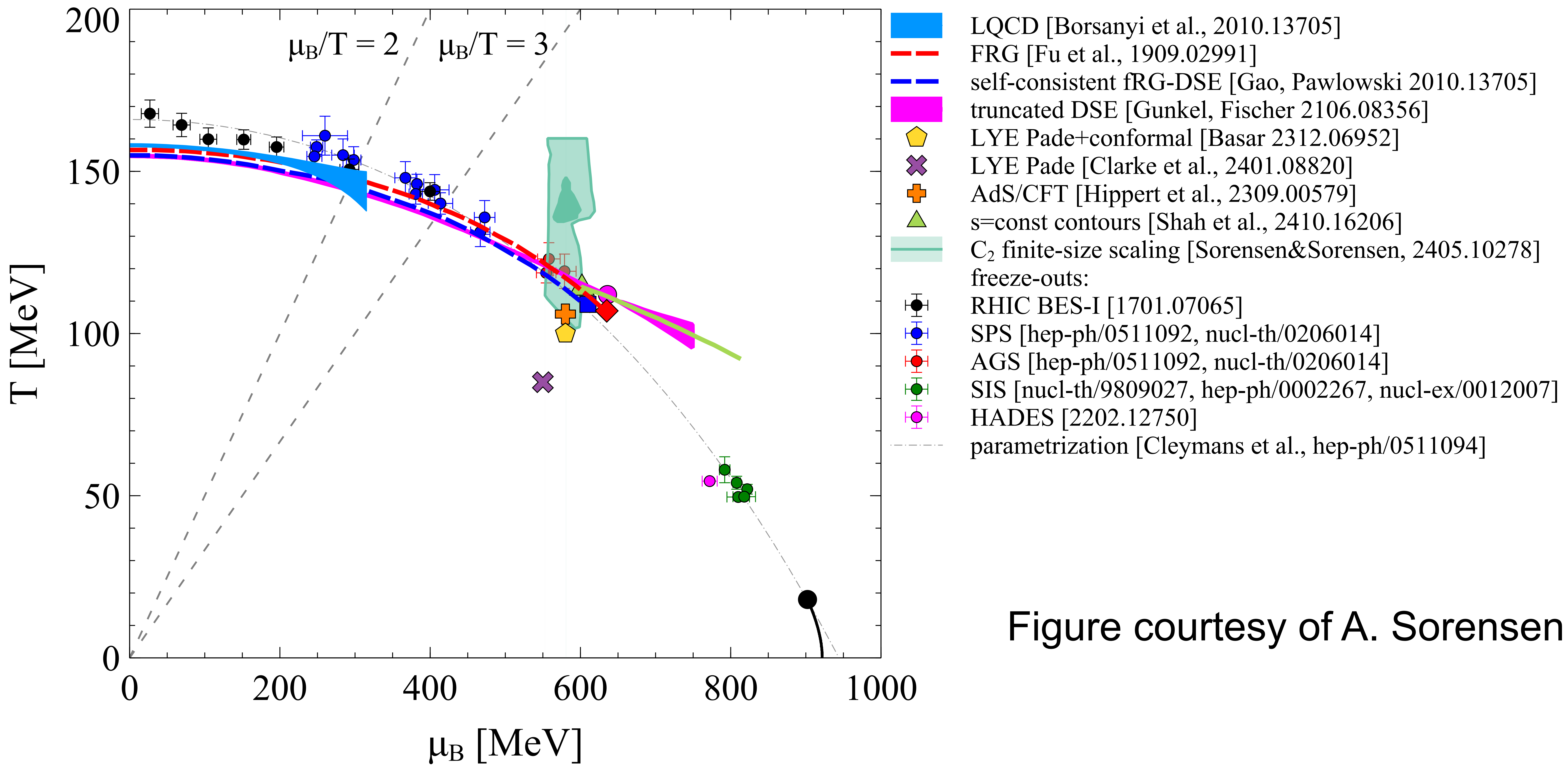
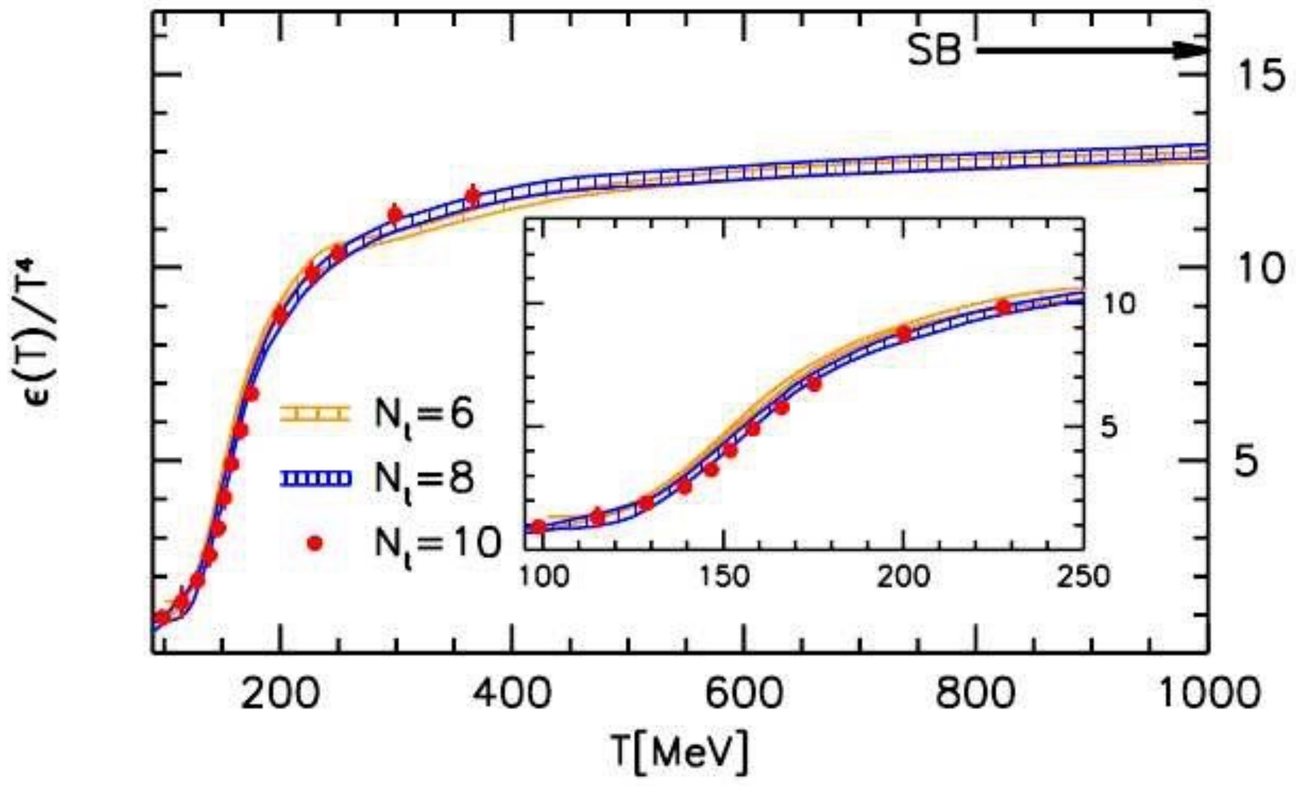
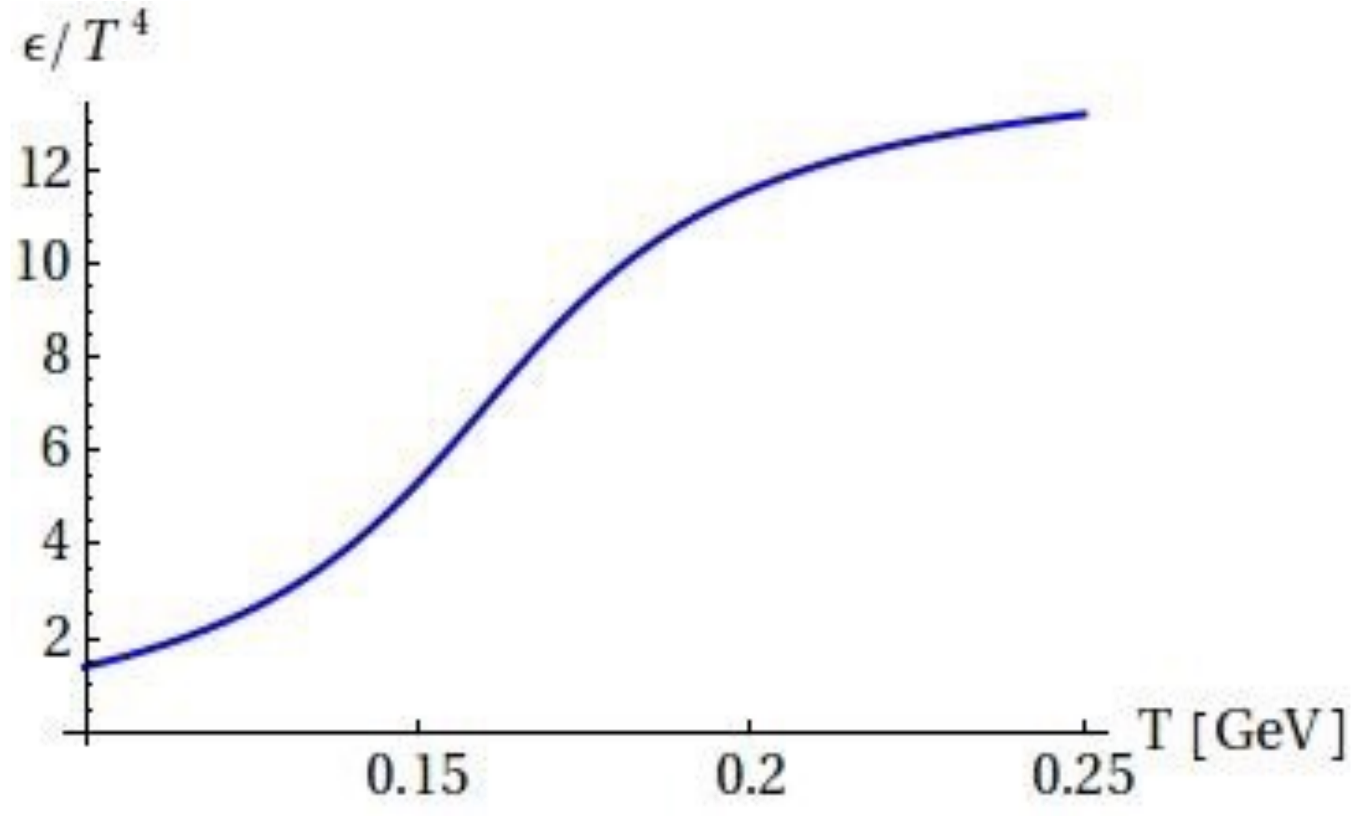


Figure courtesy of A. Sorensen

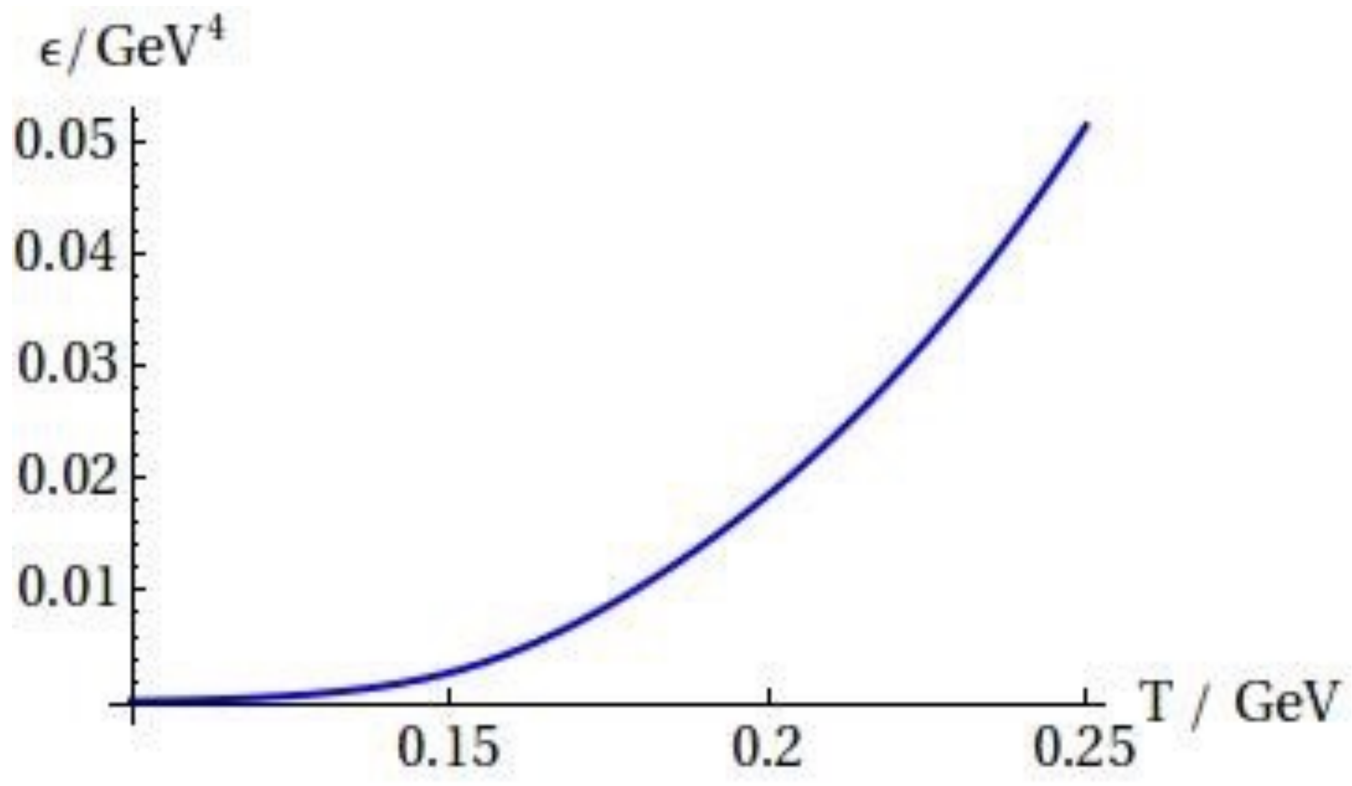
Cumulants and Phase structure

S. Borsanyi et al, JHEP 1011 (2010) 077





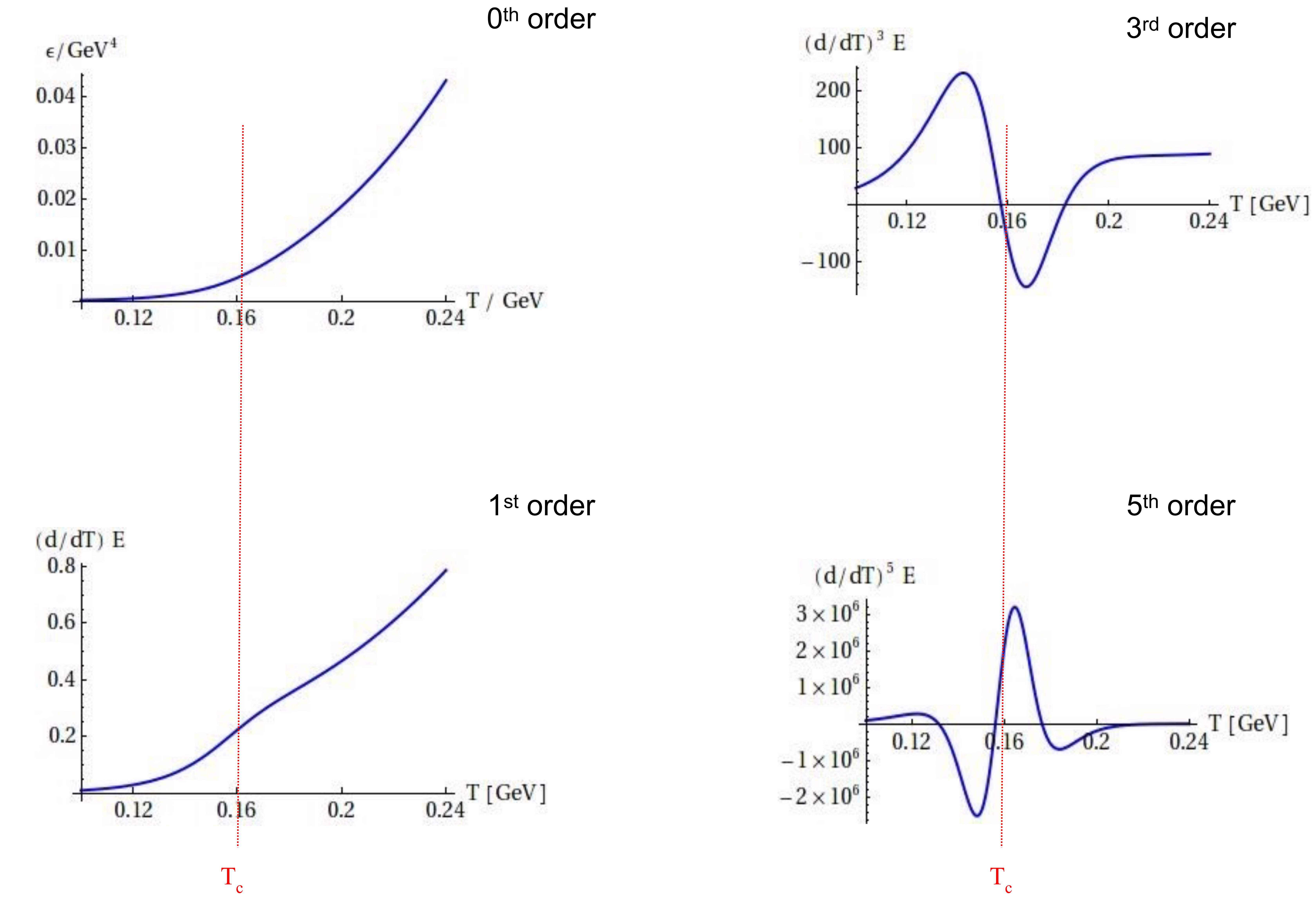
What we always see....



What it really means....

"T_c" ~ 155 MeV

Derivatives



How to measure derivatives

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Cumulants of (Baryon) Number

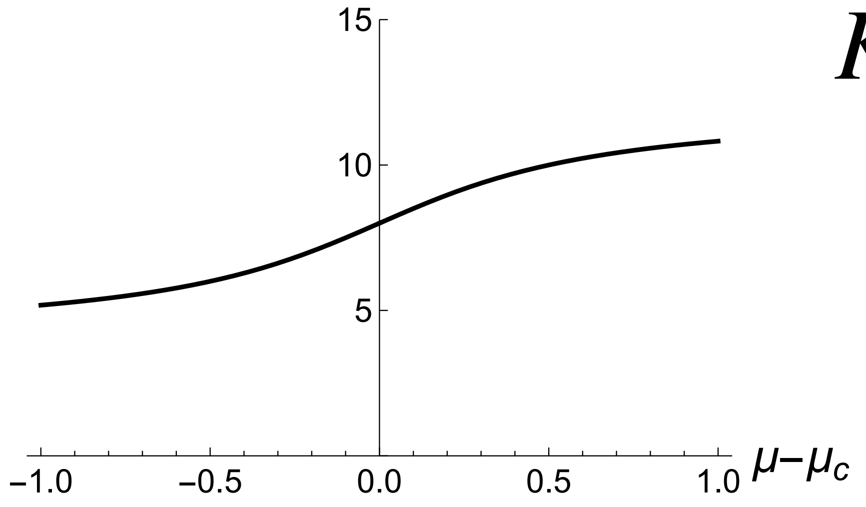
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,,\;\; K_2 = \langle N - \langle N \rangle \rangle^2\,,\;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

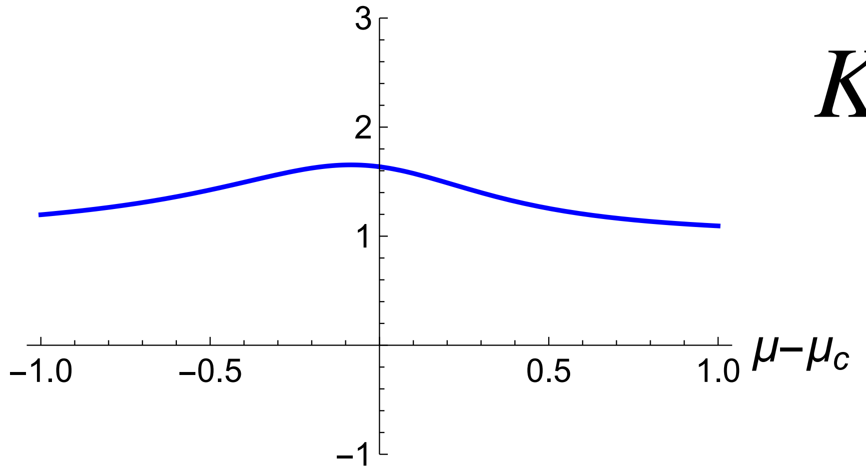
Cumulants scale with volume (extensive): $K_n \sim V$

Volume not well controlled in heavy ion collisions

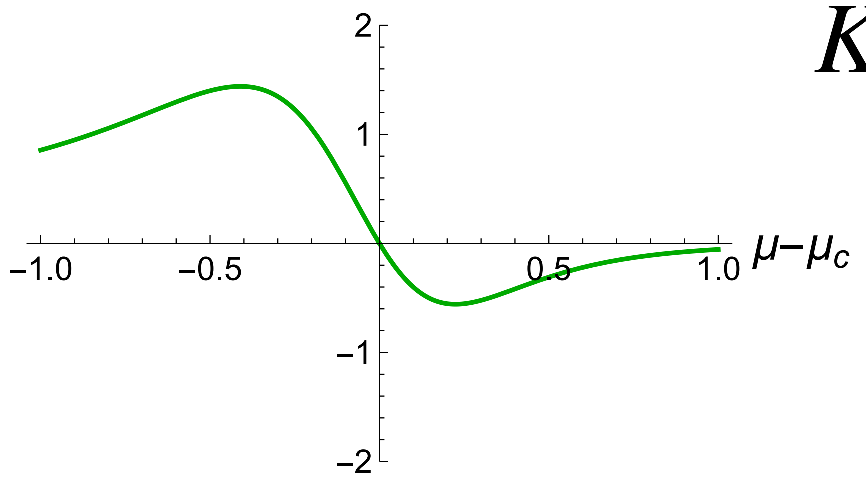
Cumulant Ratios: $\frac{K_2}{\langle N \rangle},\;\; \frac{K_3}{K_2},\;\; \frac{K_4}{K_2}$



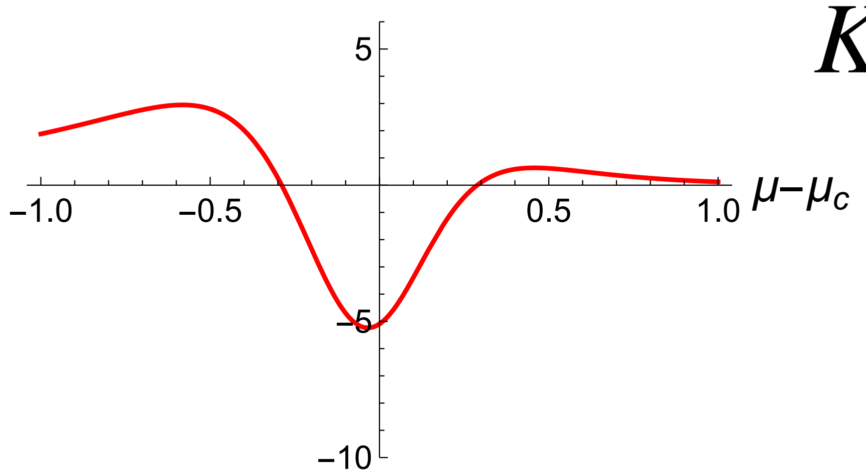
$K_1 = \langle N \rangle$



$K_2 = \frac{\partial}{\partial(\mu/T)} K_1$

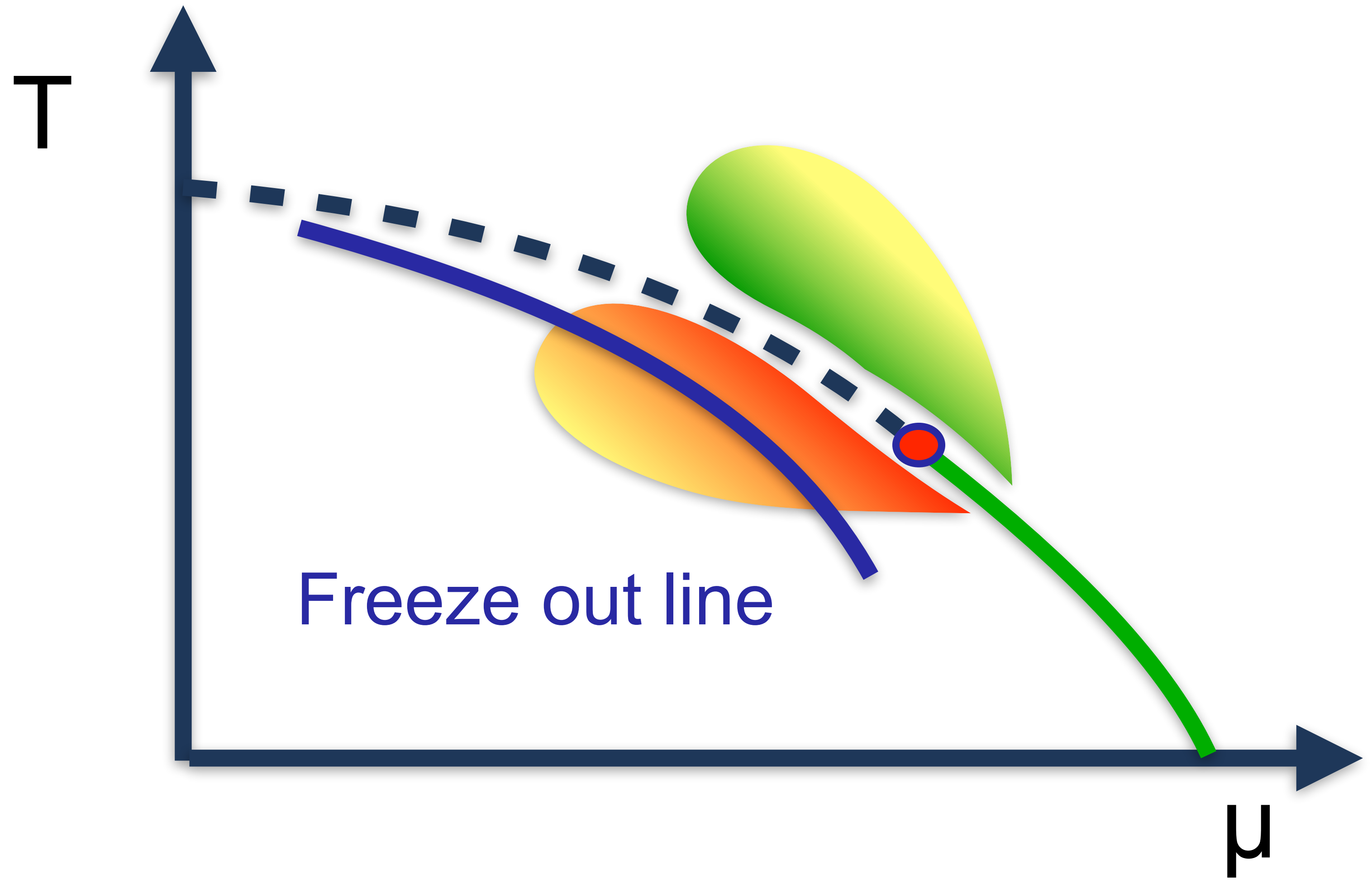


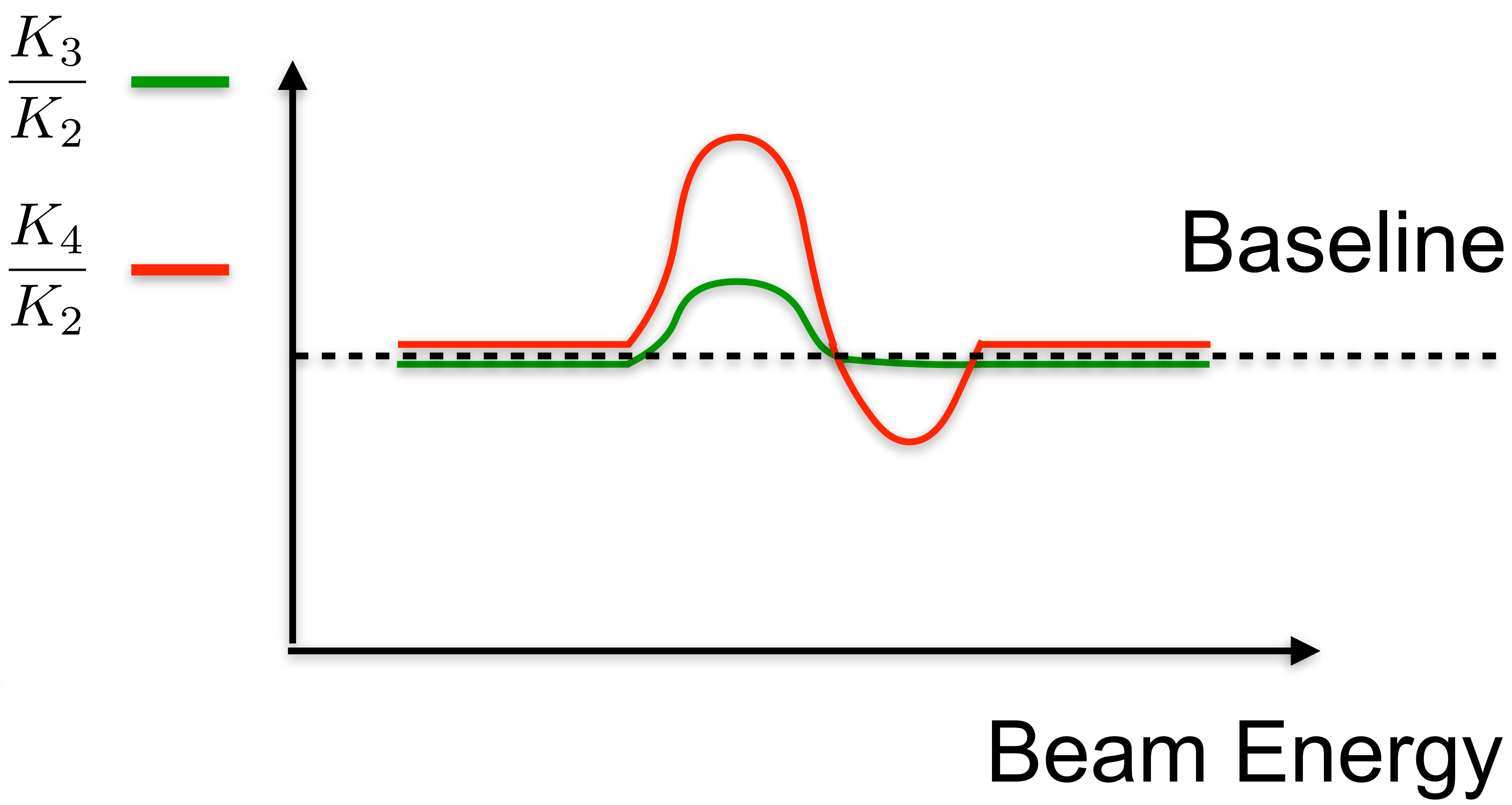
$K_3 = \frac{\partial}{\partial(\mu/T)} K_2$

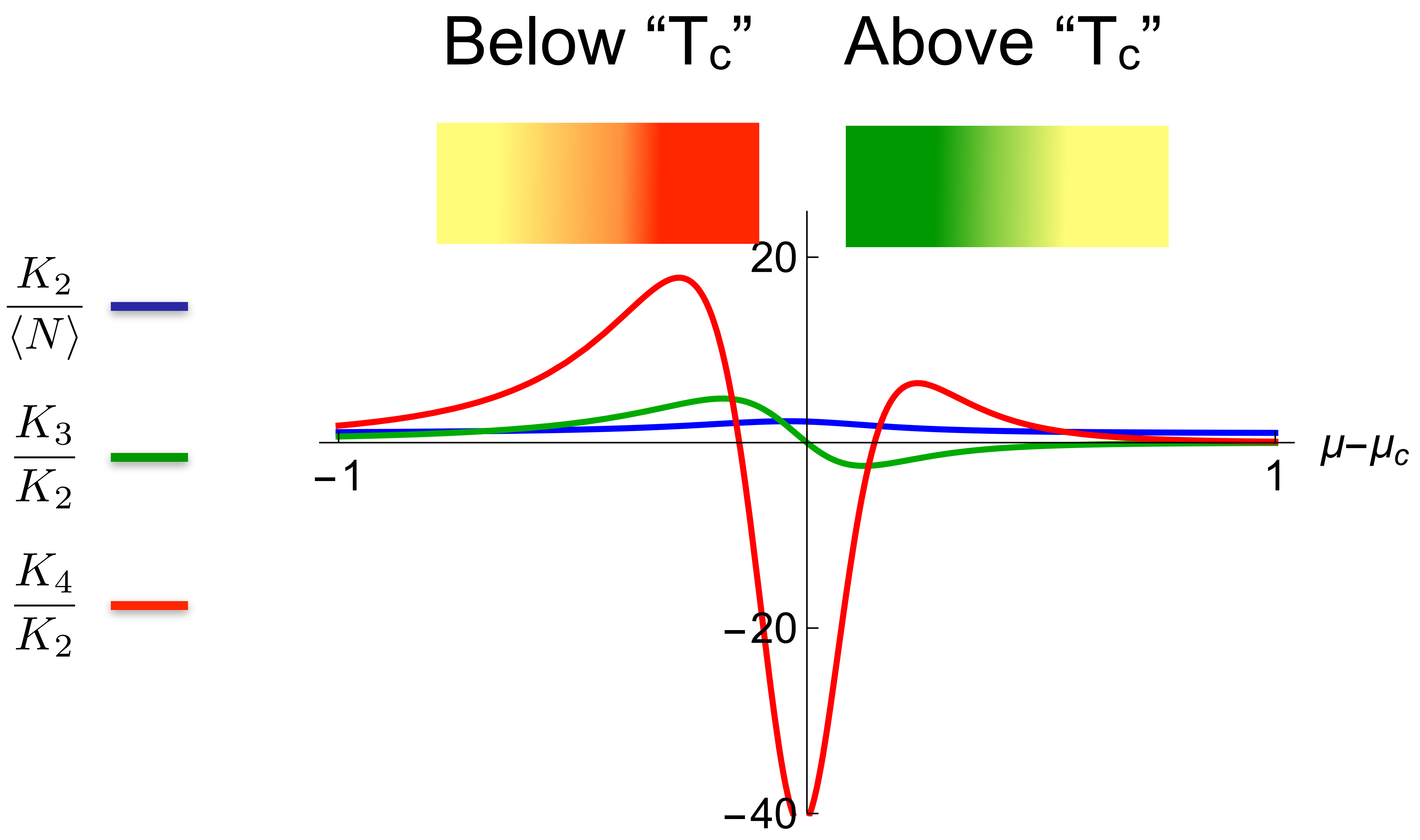


$K_4 = \frac{\partial}{\partial(\mu/T)} K_3$

What to expect?







Stephanov, arXiv:1104.1627

Cumulants and Factorial cumulants

Cumulants:

generating function: $g(t) = \ln\left[\sum_n P(n) e^{n\,t}\right]$

$$\kappa_n = \frac{\partial^n}{\partial t^n} g(t)\Big|_{t=0}$$

Gaussian: $\kappa_n = 0; \quad n \geq 2$

Factorial Cumulants (no anti-particles):

generating function: $g_F(z) = \ln\left[\sum_n P(n) z^n\right]$

$$FC_n = \frac{\partial^n}{\partial z^n} g_F(z)\Big|_{z=1}$$

Poisson: $FC_n = 0; \quad n \geq 1$

Relations : $g(t) = g_F(e^t)$

$$FC_n = \sum_{k=1}^{n} s(n,k)\kappa_k; \qquad \kappa_n = \sum_{k=1}^{n} S(n,k) FC_k$$

$s(n,k)$; $S(n,k)$ Sterling numbers 1st and 2nd kind

For example: $FC_1 = \kappa_1 = \langle n \rangle;$ $\quad FC_2 = \kappa_2 - \kappa_1;$ $\quad FC_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1;$

Same singularity structure!

Factorial cumulants and correlation functions

$$\rho_1(p) = \frac{dN}{dp}; \quad \rho_2(p_1, p_2) = \frac{d^2N}{dp_1 dp_2}; \ \cdots$$

Two particle density: $\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2)$

Three particle density: $\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)C_2(p_2, p_3) + \rho_1(p_2)C_2(p_1, p_3)$

$$+\rho_1(p_3)C_2(p_1, p_2) + C_3(p_1, p_2, p_3)$$

$C_n(p_1, \cdots, p_n)$ n-particle genuine correlations functions

Factorial cumulants are integrals over correlation functions: $FC_n = \int_{\text{acceptance}} dp_1 \cdots dp_n \, C(p_1, \cdots, p_n)$

Poisson: $FC_n = 0; \quad n \geq 1;$ in contrast with $\kappa_n = \langle n \rangle$

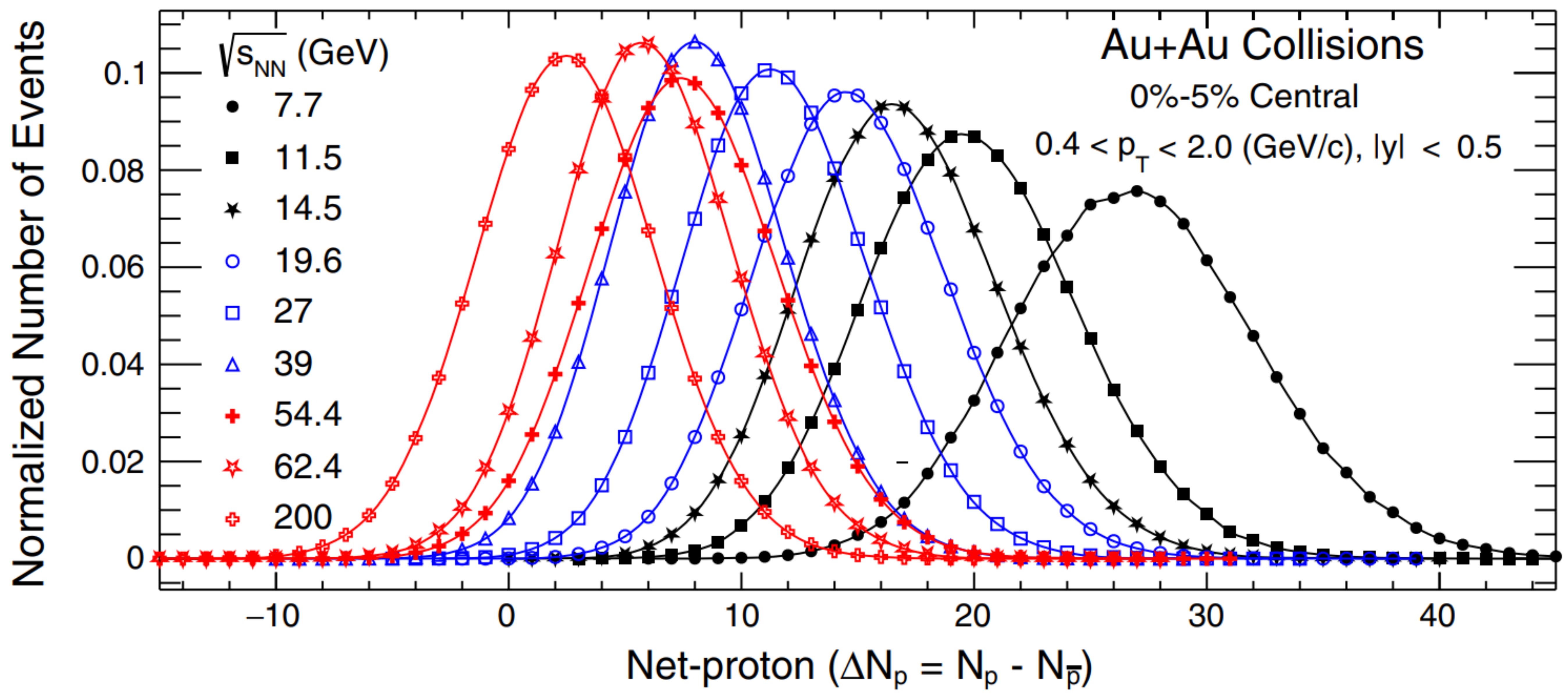
Measuring cumulants (derivatives)

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \sum_N P(N)(N - \langle N \rangle)^2$$

$$K_3 = \langle N - \langle N \rangle \rangle^3 = \sum_N P(N)(N - \langle N \rangle)^3$$

$$P(N) = \frac{N_{events}(N)}{N_{events}(total)}$$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



Compare Data with Lattice QCD and other field theoretical models

Experiment

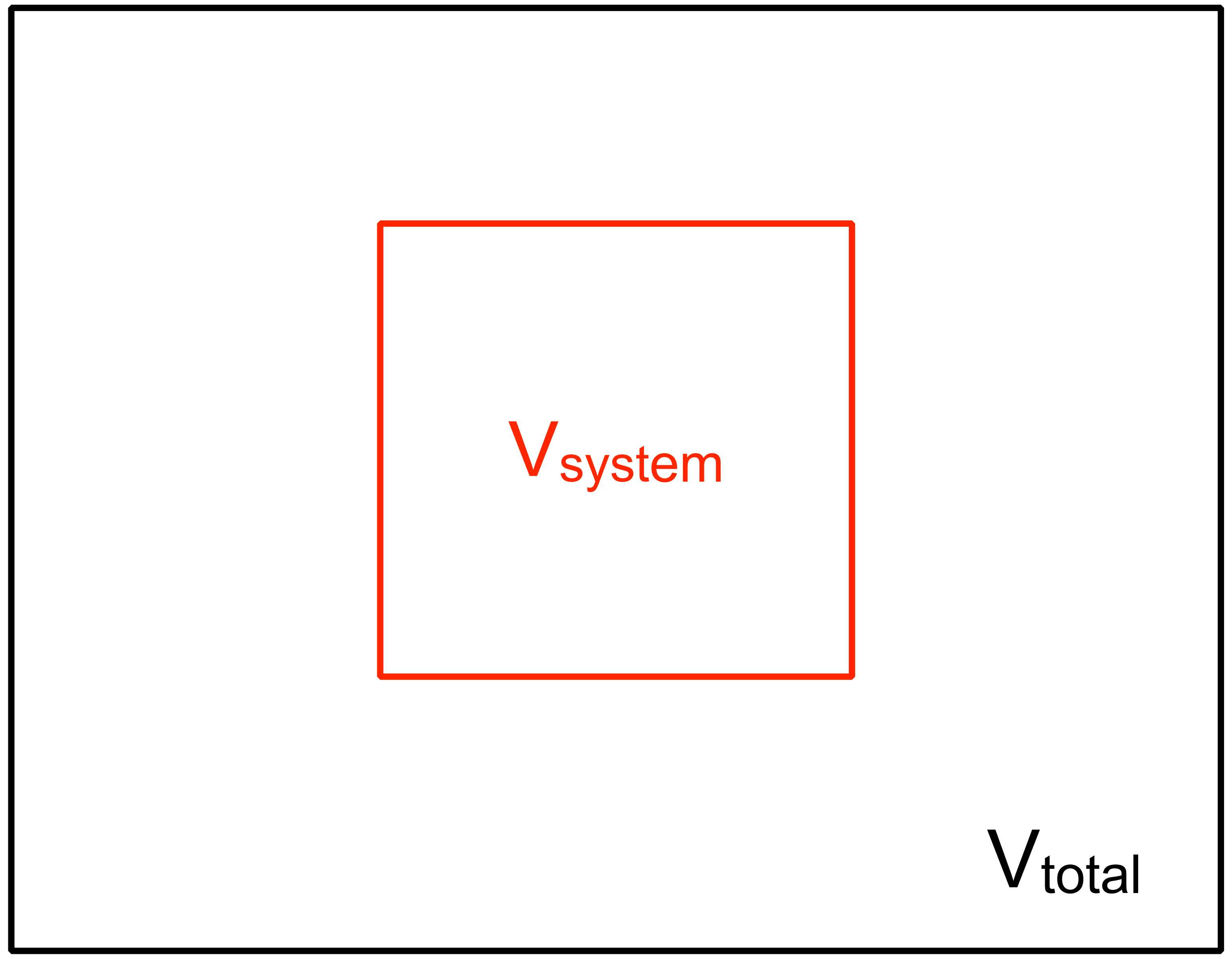
- Baryon number conserved globally
  - Solution (V. Vovchenko et al, arXiv 2003.13905, arXiv:2007.03850)
- Experiment measures protons only
- Volume is not fixed in experiment
  - Possible solution (Rustamov et al, 2211.14849, Holzmann et al, 2403.03598)
- Momentum cuts
- expansion, time evolution
- Detector fluctuates (efficiency etc…)

Lattice, FRG etc

- Baryon number (and other charges) conserved only on average (grand canonical ensemble)
- "measures" ALL baryons
- Volume is fixed
- Includes all momenta
- static system, no expansion

Need a dynamical model

Grand canonical ensemble

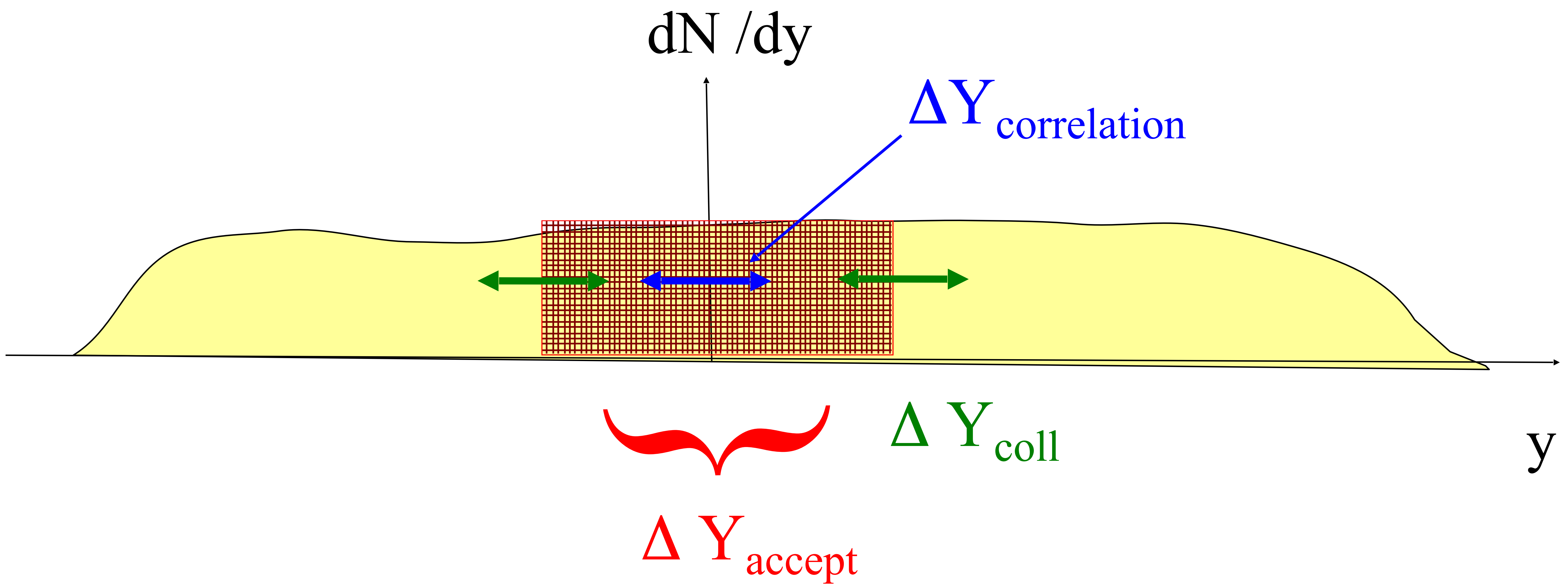


$$V_{total} \to \infty$$
$$V_{system} \to \infty$$

$$\frac{V_{system}}{V_{total}} \to 0$$

In coordinate space!!!!

How to make a grand-canonical ensemble in experiment



Conditions for "charge" fluctuations:

- $\Delta Y_{\text{corrrelation}} << \Delta Y_{\text{accept}}$ **(catch the physics)**
- $\Delta Y_{\text{total}} >> \Delta Y_{\text{accept}} >> \Delta Y_{\text{coll}}$ **(keep the physics and minimize charge conservation effect)**

Volume fluctuations

(factorial) cumulants are extensive: $\kappa_n \sim V$

Cumulant ratios do not depend on AVERAGE volume, BUT are affected by FLUCTUATIONS of the volume

For example:

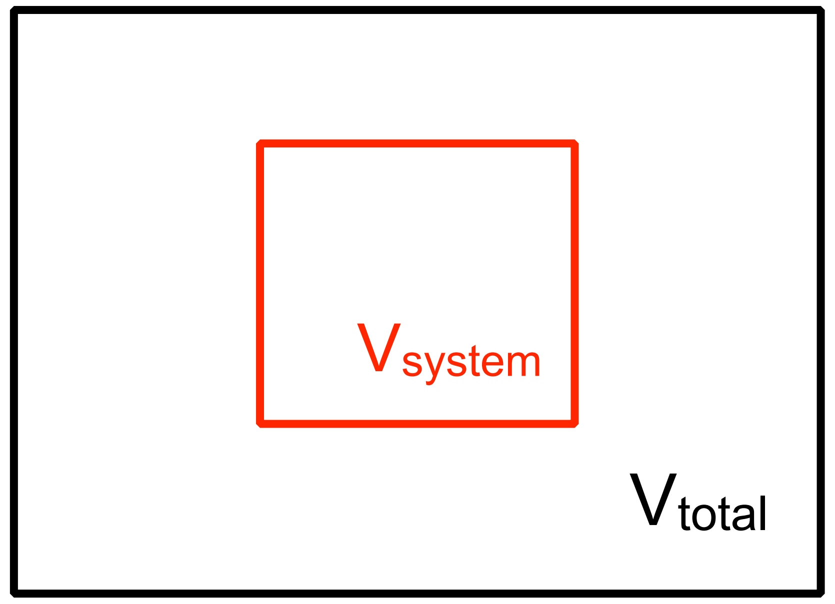
$$N = \rho V; \;\; \rho = \text{density}$$

$$\delta N = \bar{V}\delta\rho + \bar{\rho}\delta V$$

Volume fluctuations NOT wanted

κ_2 at fixed volume. This is what we want!

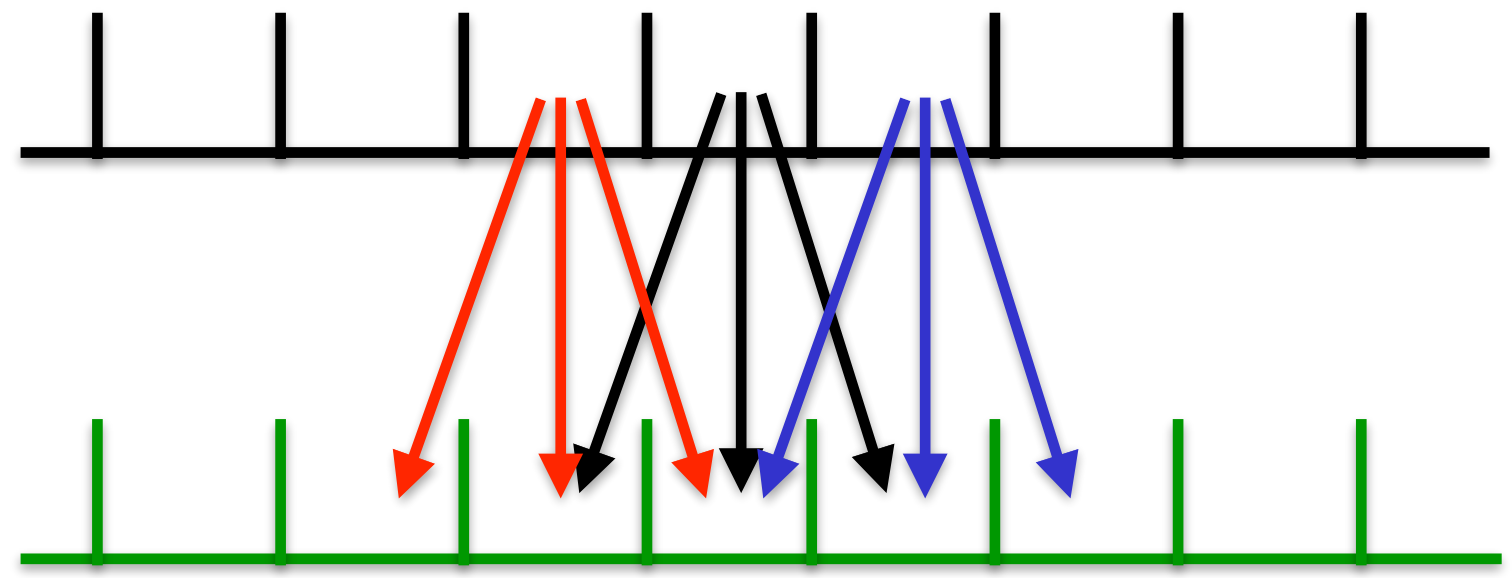
$$\kappa_2 = \langle (N - \langle N \rangle)^2 \rangle = \langle (\delta N)^2 \rangle = \bar{\rho}^2 (\delta V)^2 + \bar{V}^2 (\delta\rho)^2$$



Cumulant ratios in “experiment” ($\mu_B \sim 0$)

(without volume fluctuations)

Thermal smearing

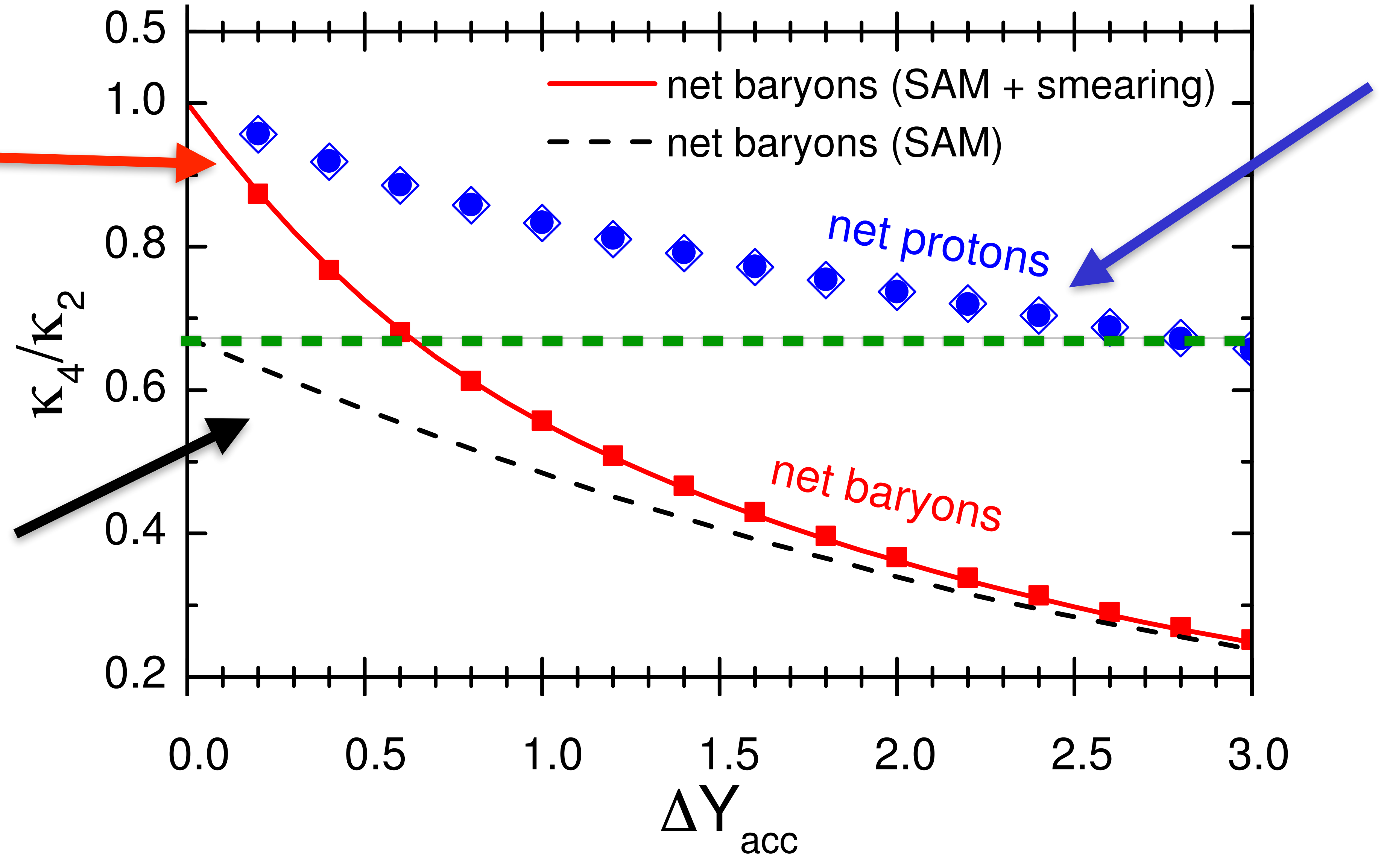


η, spacial rapidity

Y, momentum space rapidity

Lattice +
baryon conservation
+ thermal smearing

Lattice +
baryon conservation



What is REALLY measured

- net baryons
- net protons
- net protons (Kitazawa-Asakawa)
- Lattice result

Cumulants Lattice vs Experiment
Obvious conclusion!

- Cumulants from Lattice and cumulants from measured (net)-protons SHOULD NOT agree if they study the SAME system

- IF they agree: Experiment and Lattice study DIFFERENT systems!

- Exception: Both Lattice and experiment find Poisson statistics, i.e. system without any correlation a.k.a a boring system. In this case we simply cannot tell

Compare Data with Lattice QCD and other field theoretical models

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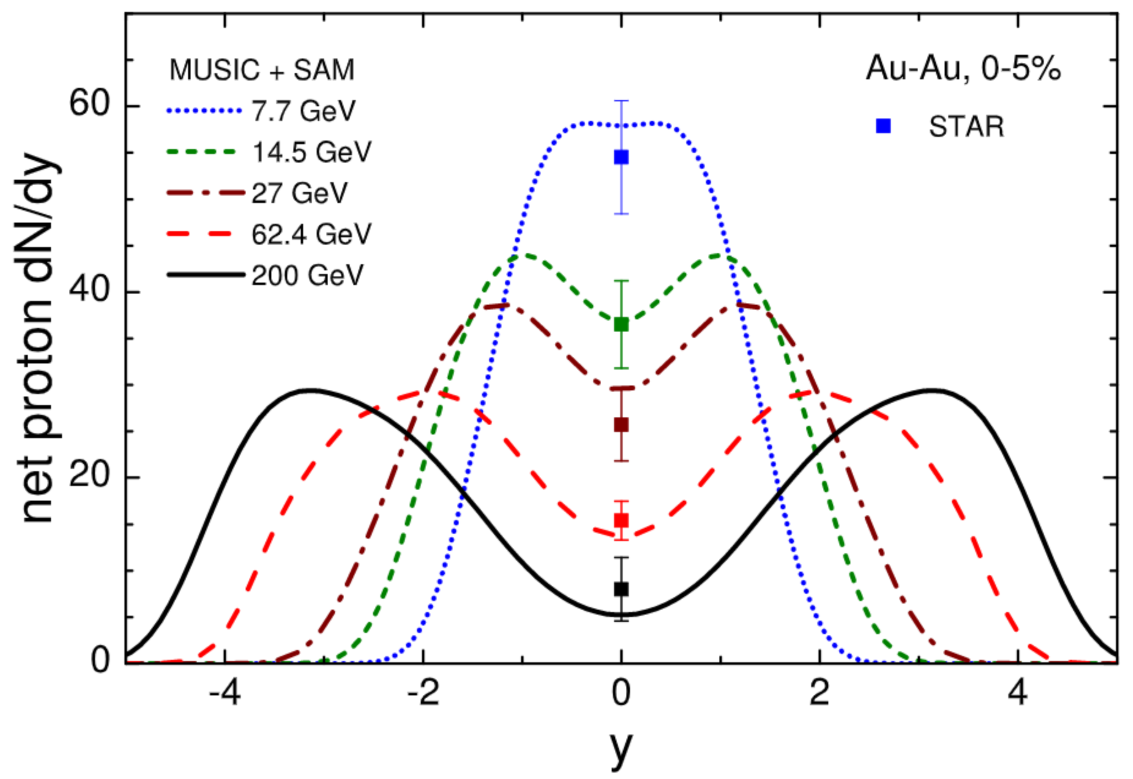
Need a dynamical model

Calculation of non-critical contributions at RHIC-BES

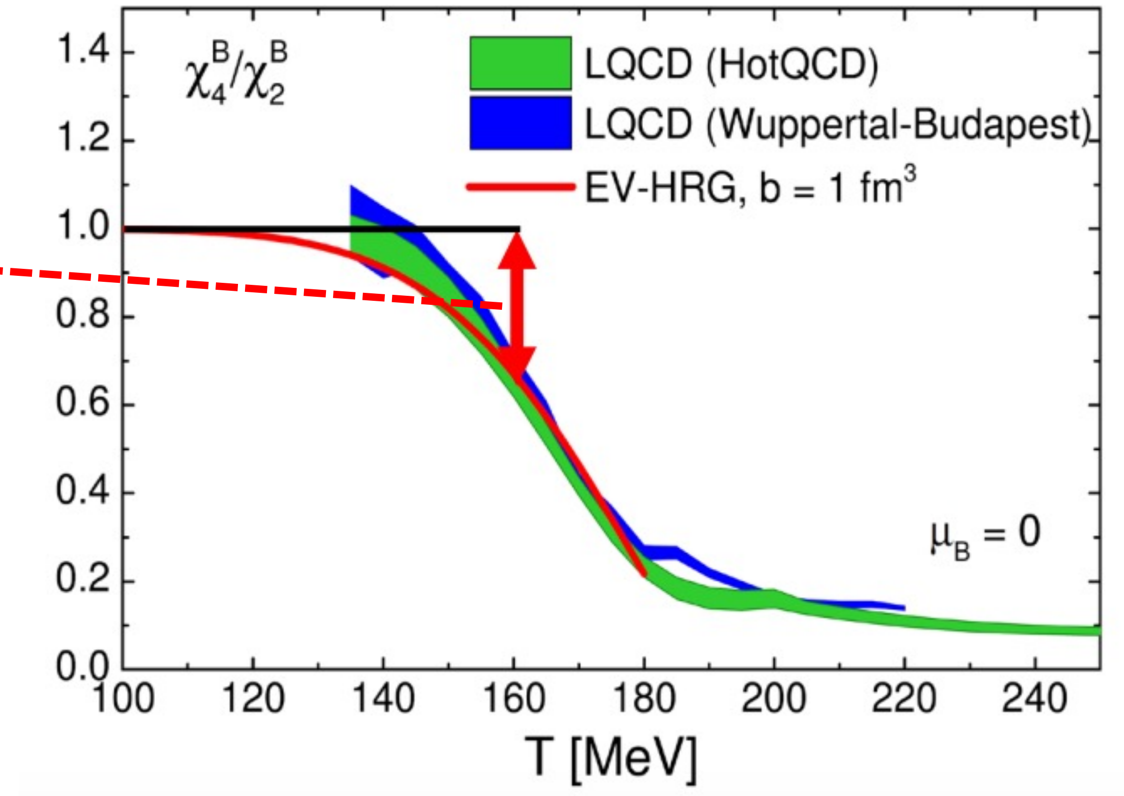
UNIVERSITY OF HOUSTON

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3



- Non-critical contributions are computed at particlization
  - QCD-like baryon number distribution (χ_n^B) via **excluded volume** b = 1 fm^3 [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - **Exact global baryon conservation*** (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)] https://github.com/vlvovch/fist-sampler



- Absent: critical point, local conservation, initial-state/volume fluctuations, hadronic phase

*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro
Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

22

Results (Prediction) for proton cumulants

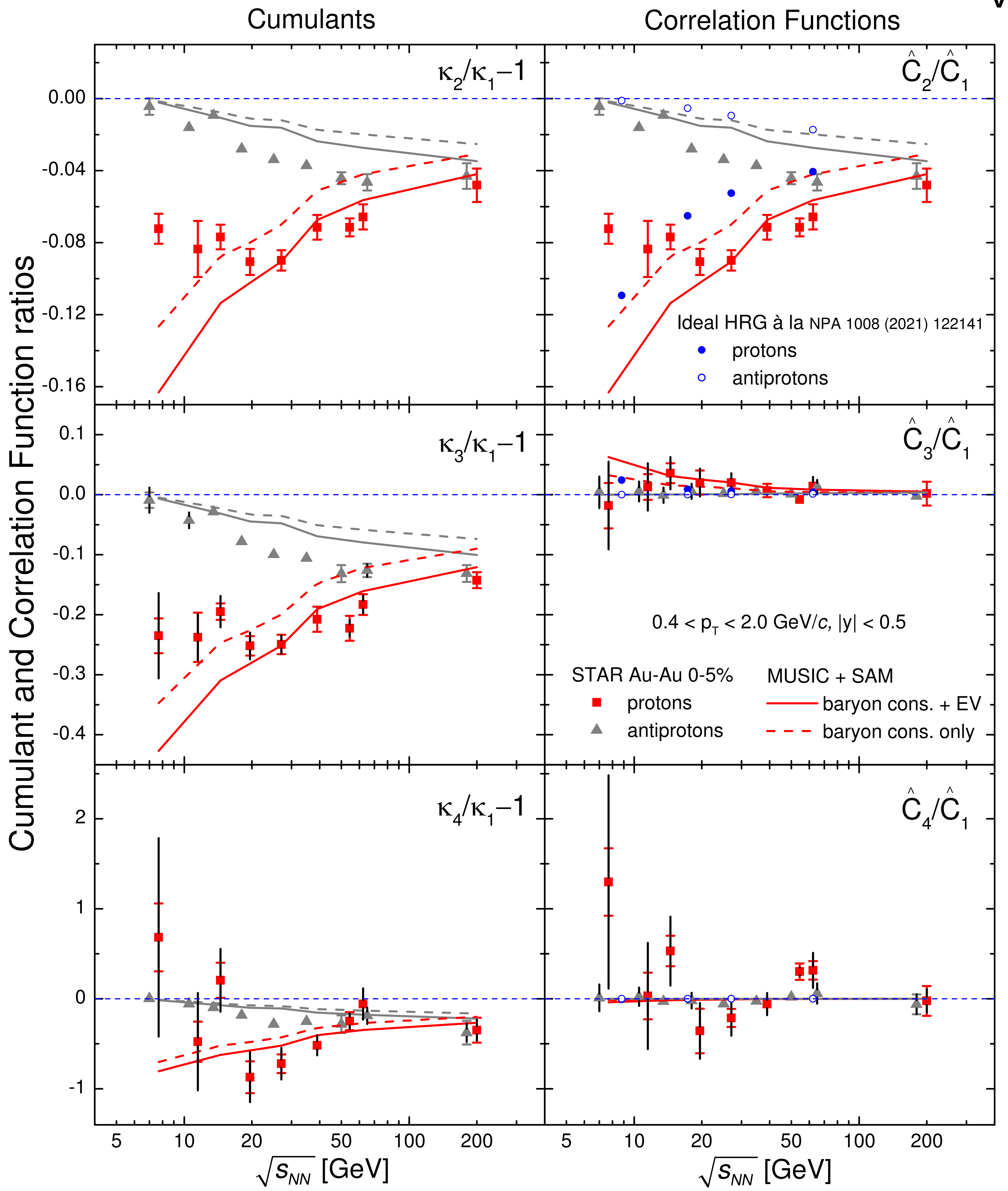
Vovchenko, Shen, VK, 2107.00163

- Viscous hydro
- EOS tuned to LQCD
- Correct for global charge conservation
- Protons NOT baryons

- Baseline!
  No critical point or phase transition

- No volume fluctuations

See also: Braun-Munzinger et al, NPA 1008 (2021) 122141



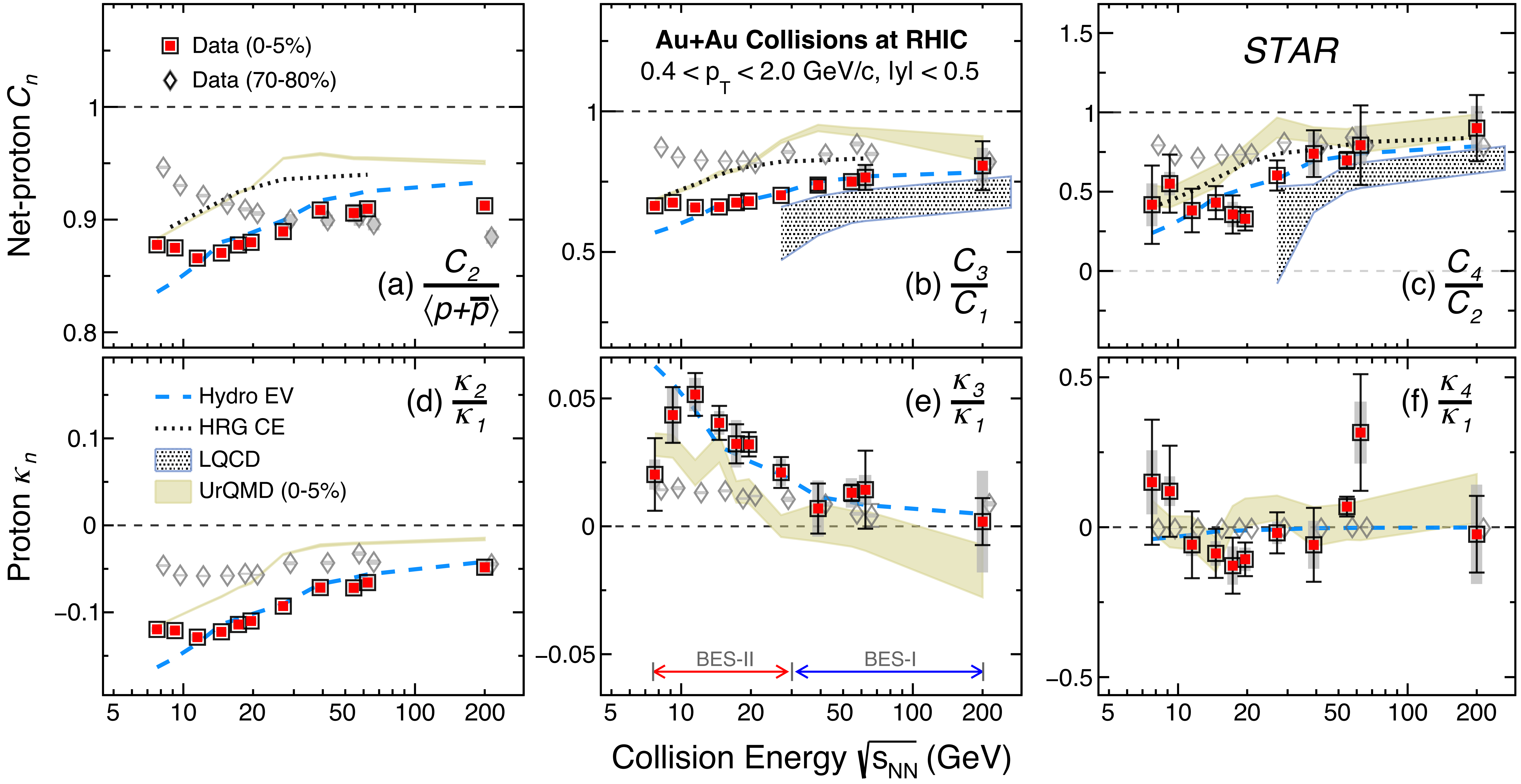
STAR BESI data

STAR BESII data

arXiv:2504.00817

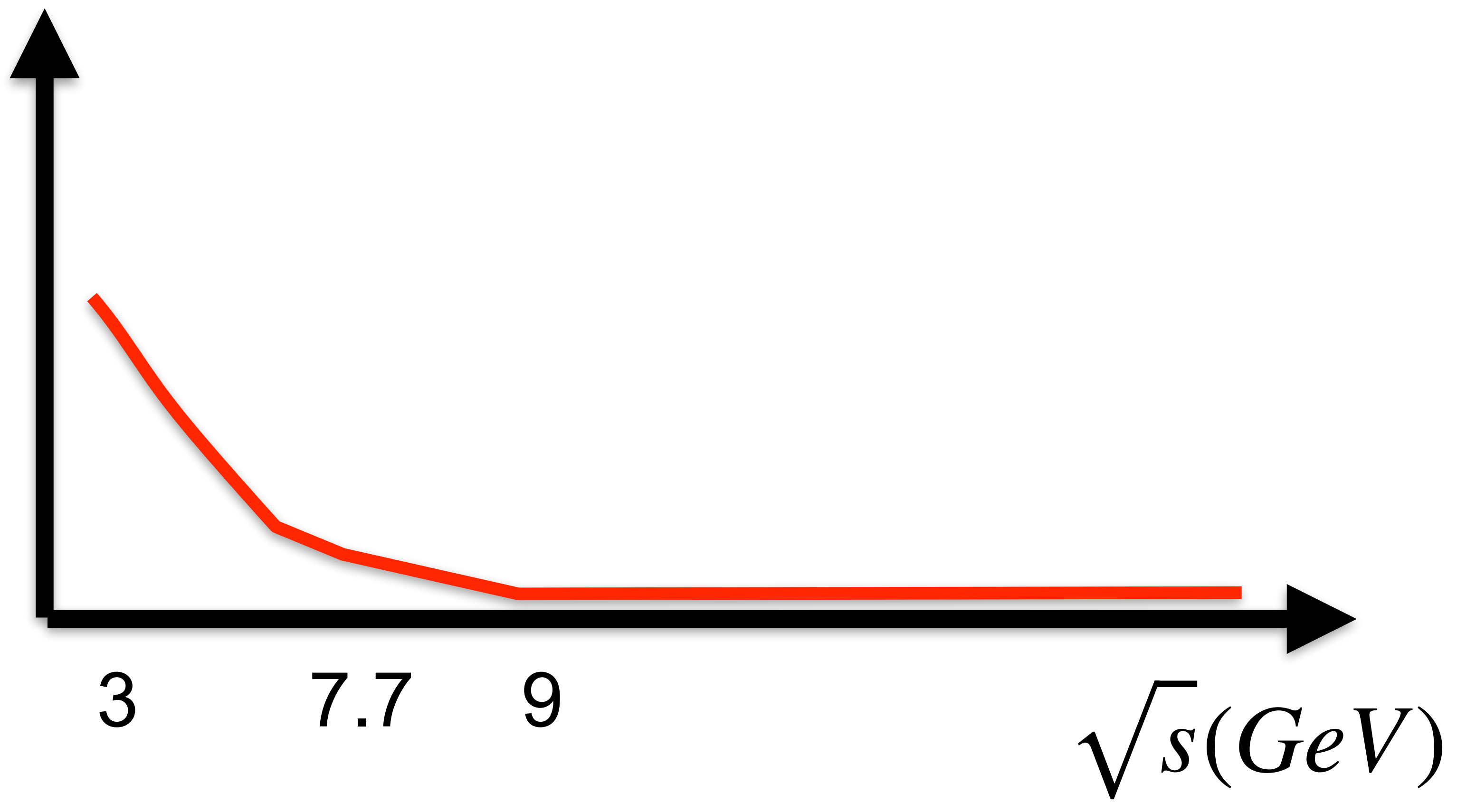
Cumulants

Factorial Cumulants (FC)

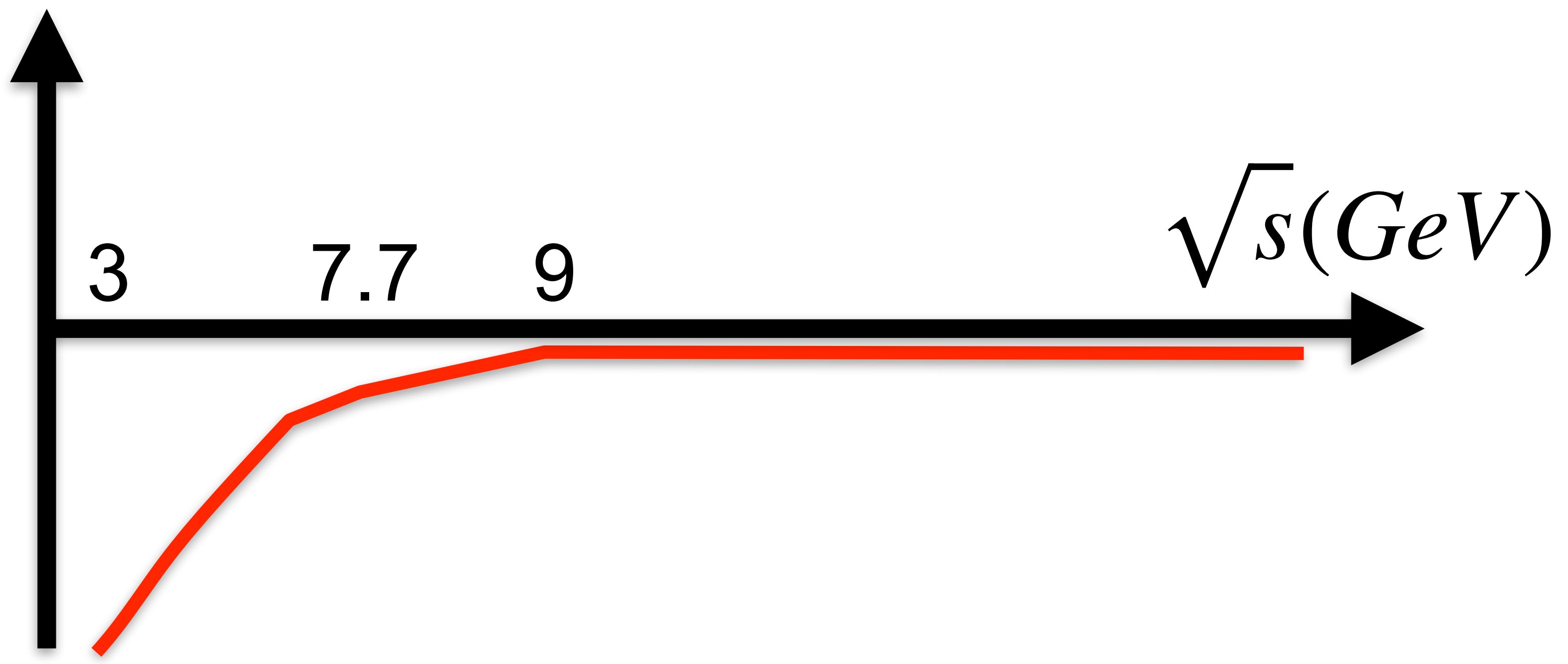


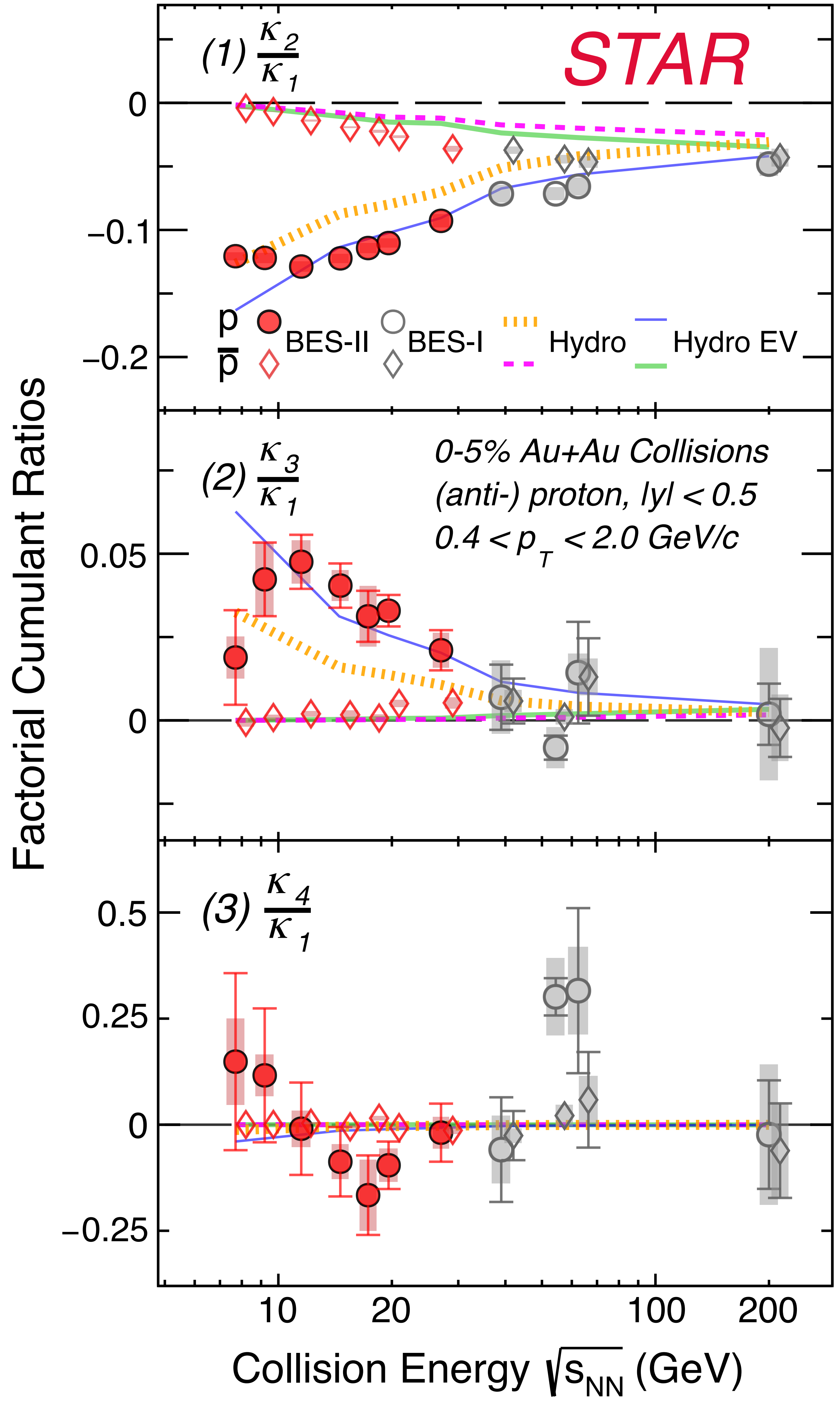
The "signal" (relative to baseline)

$\frac{FC_2}{FC_1}$ - baseline



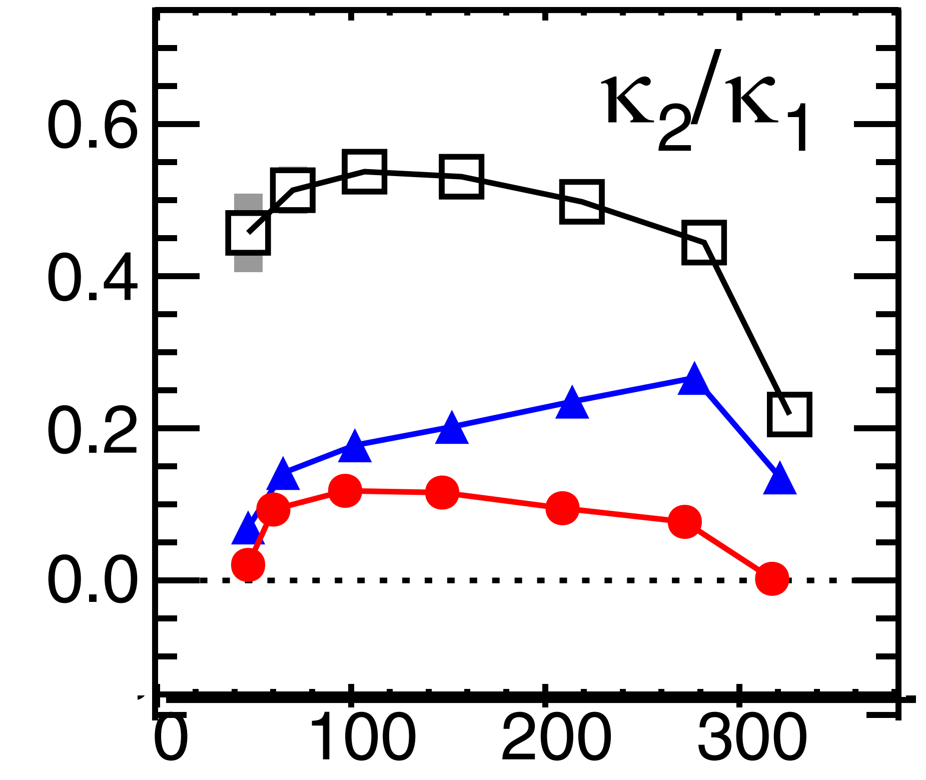
$\frac{FC_3}{FC_1}$ - baseline

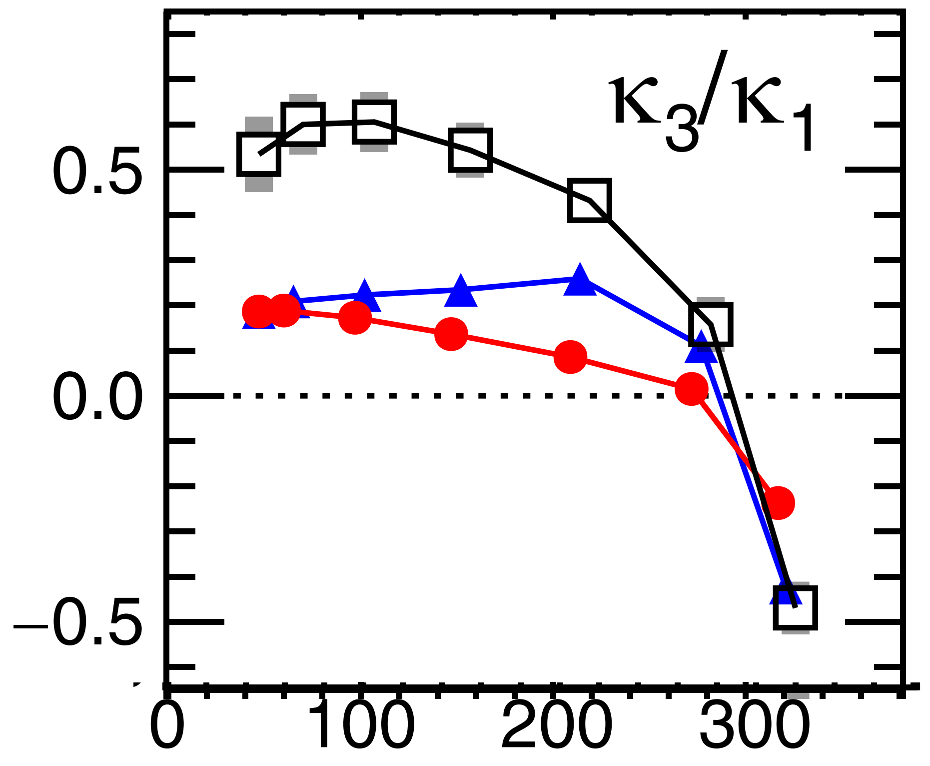




STAR 2209.11940

$\sqrt{s} = 3GeV$



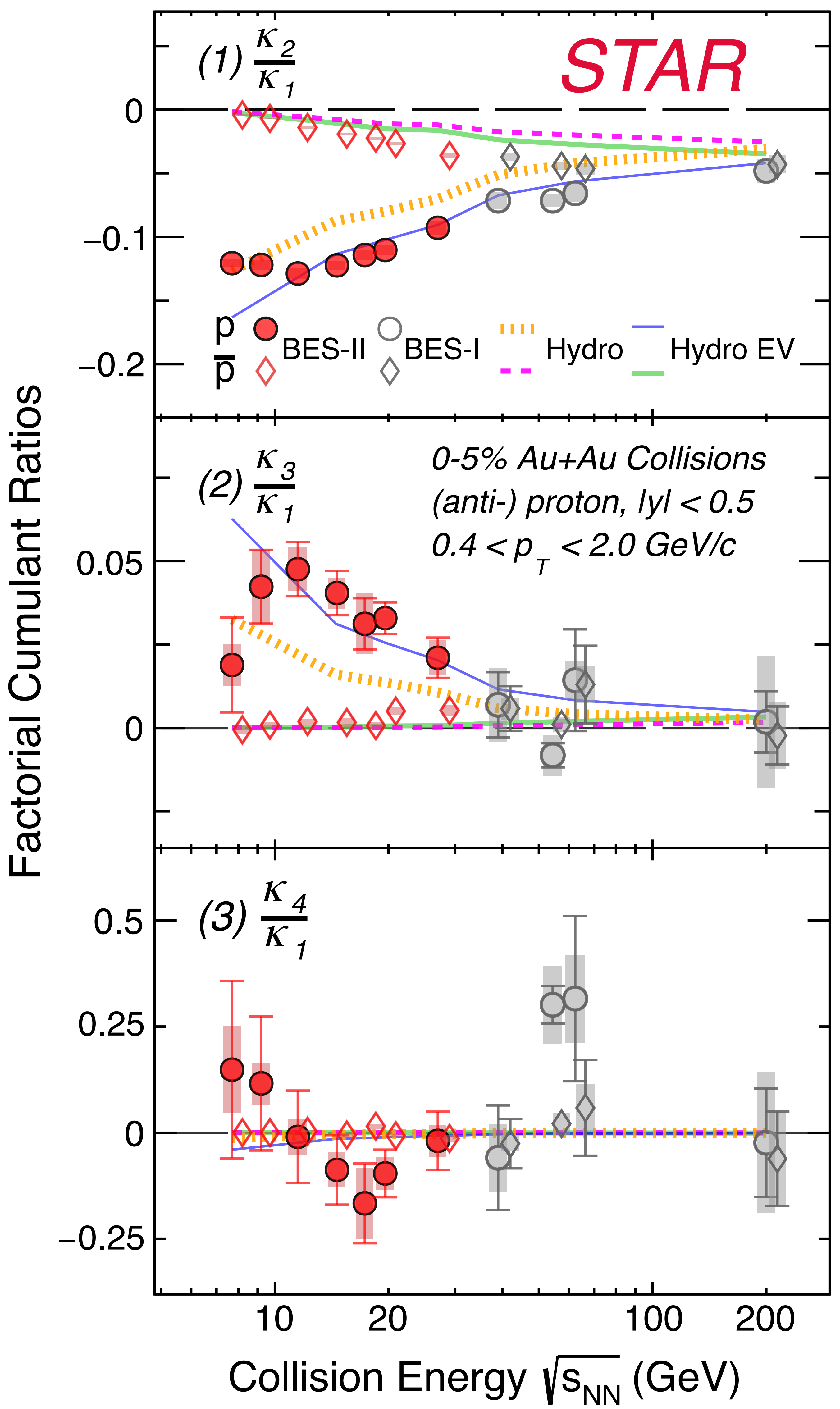


CAVEAT: 3 GeV has acceptance $-0.5 \leq Y \leq 0$

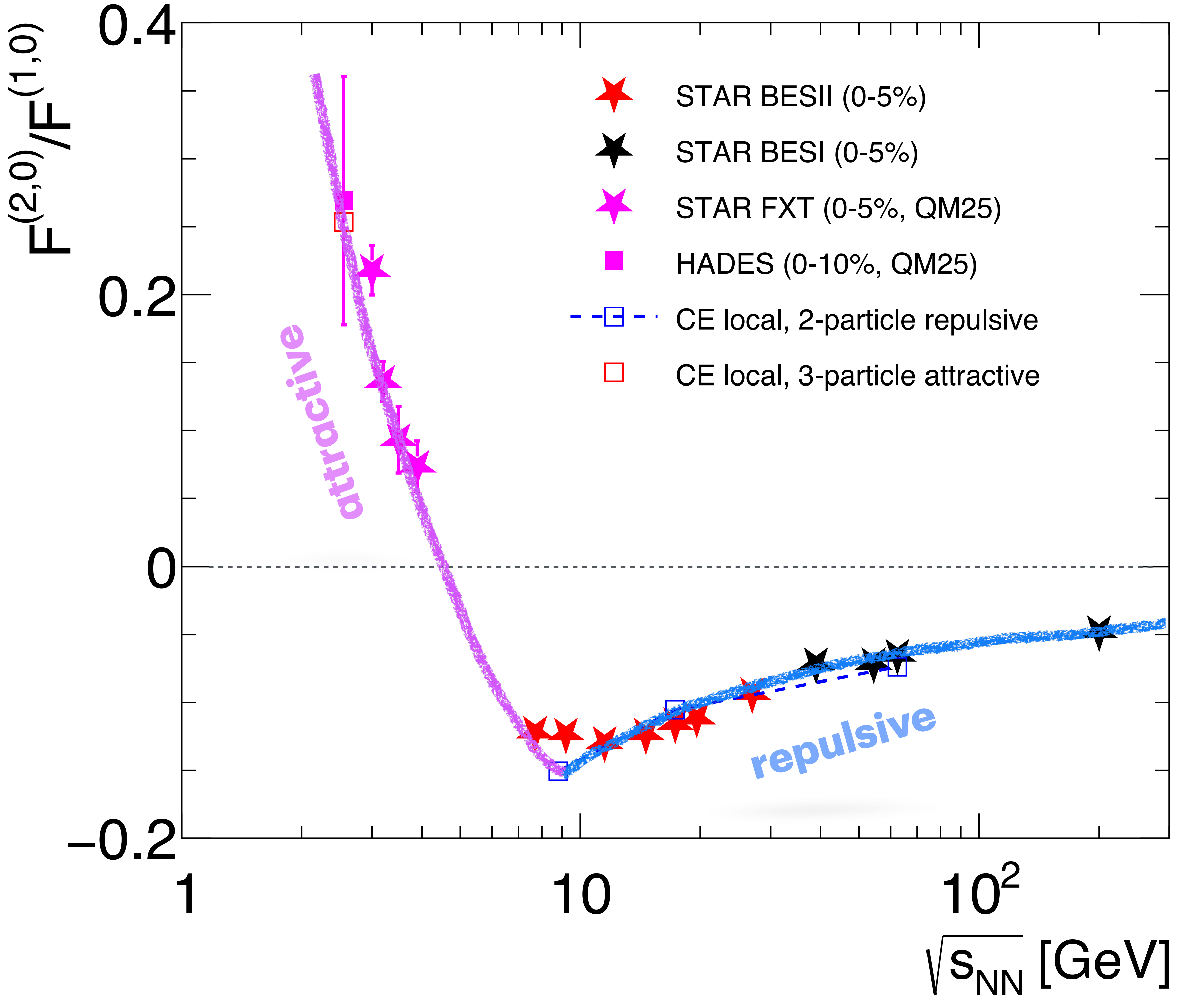
New STAR data (BESII)

Looks like we need some kind of attraction!

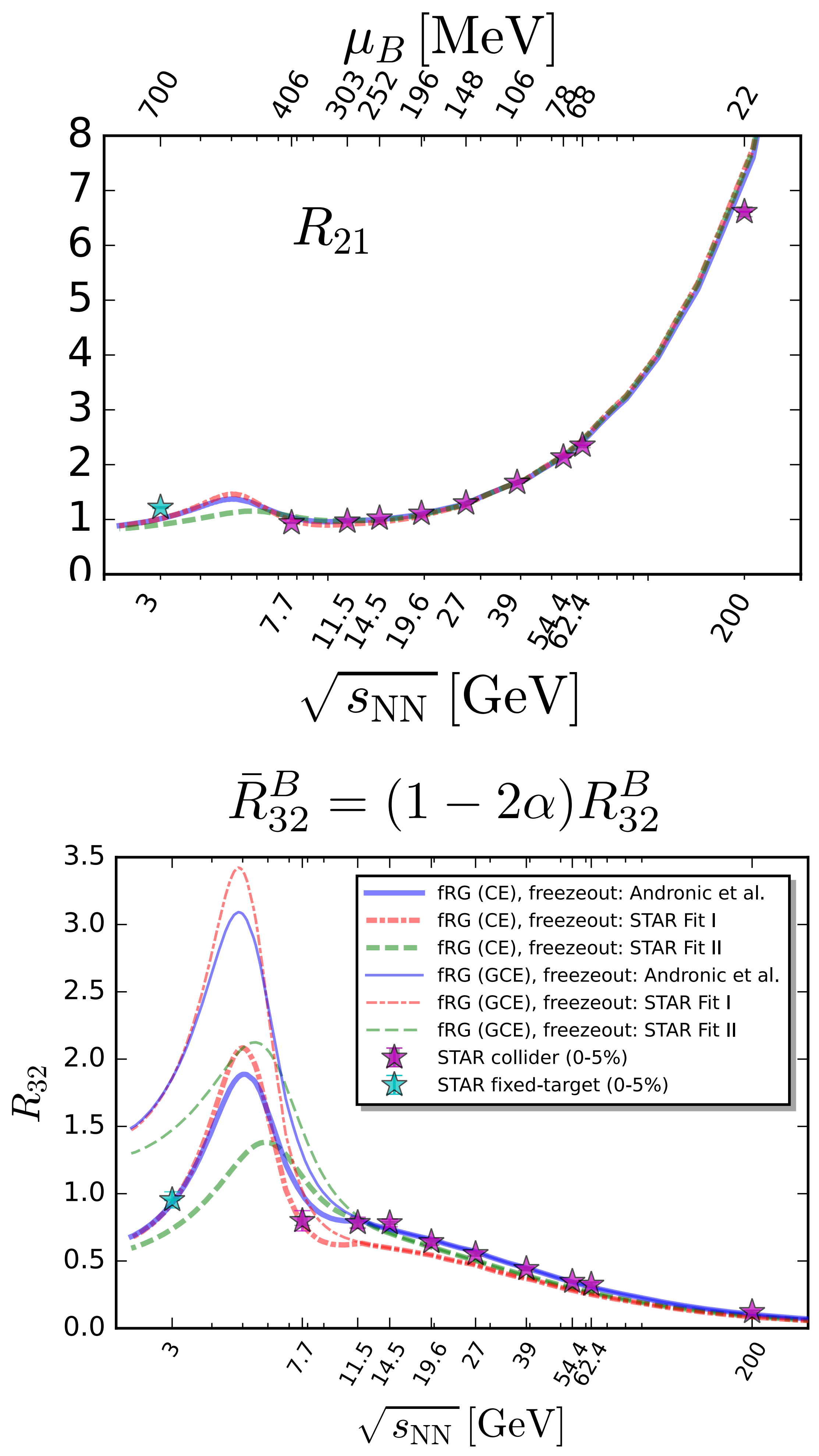
Factorial cumulants



Attraction does the trick



Friman, Redlich, Rustamov, arXiv: 2508.18879

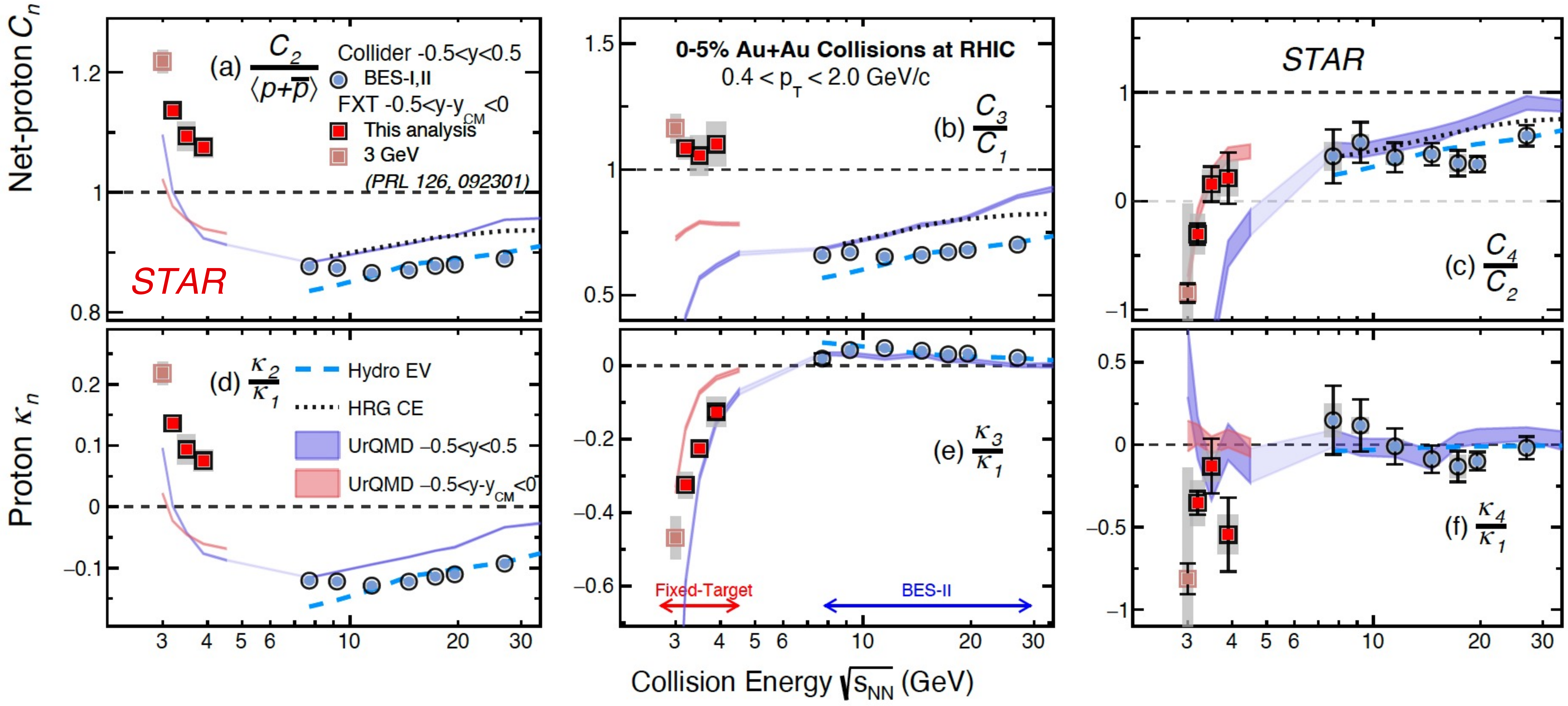


Fu, Luo, Pawlowski, Rennecke, Yin arXiv:2308.15508

Time to celebrate !

Maybe not 😕

Zachary Sweger (for STAR) QM 2025



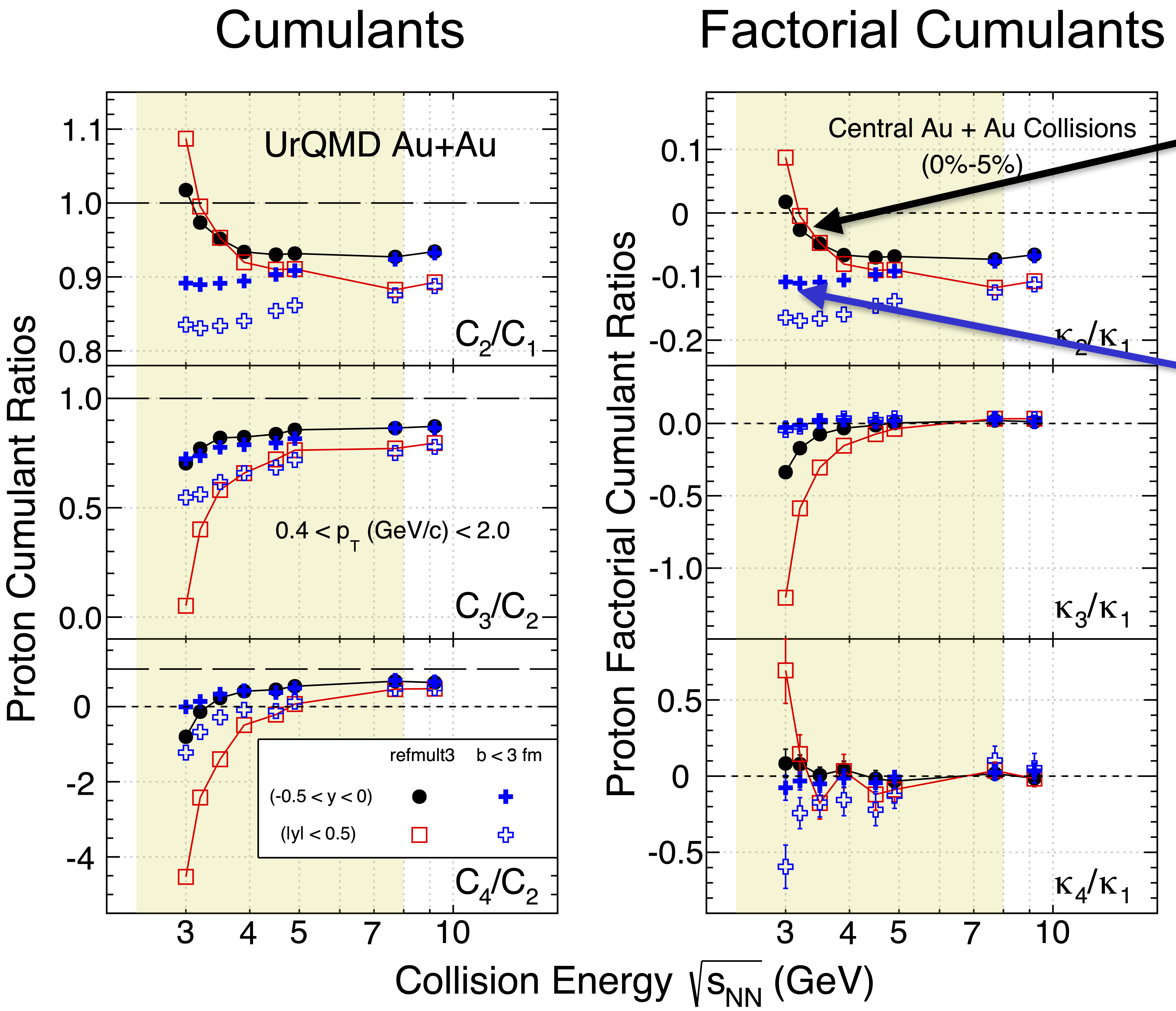
Cumulants

Factorial Cumulants

UrQMD (WITHOUT mean field) get energy dependence qualitatively right!!!

The (possible) culprit

X. Zhang, Y. Zhang, X. Luo, N. Xu, arXiv: 2506.18832



Fluctuating impact parameter STAR centrality selection

Fixed impact parameter (3=3 fm) minimal volume fluctuations.

N.B.: Centrality Bin Width Corrections applied to both

Possible culprit:
volume fluctuations

Test for Baseline

A. Bzdak, V. Vovchenko, VK, arXiv:2503.16405

Both global charge conservation and volume fluctuations are long range correlations

Factorial cumulant: $C_n = \int_{\Delta_Y} dy_1 \cdots \int_{\Delta_Y} dy_n C(y_1, \cdots, y_n)$

$C(y_1, \cdots, y_n)$: n-particle correlations function

Long range correlations: $C(y_1, \cdots, y_n) = const$ within ΔY

$\Rightarrow C_n \sim (\Delta Y)^n$

$\Rightarrow \frac{C_n}{C_1{}^n} = const$ as function of ΔY

and

$\left.\frac{C_2}{C_1{}^2}\right|_{\text{protons}} = \left.\frac{C_2}{C_1{}^2}\right|_{\text{antiprotons}}$ as function of ΔY

Test of Baseline

Both GLOBAL charge conservation and volume fluctuations introduce only

LONG Range correlations in rapidity (larger than acceptance)

Baryon number conservation

Within acceptance:

$$\alpha = \frac{\langle N \rangle_{\Delta Y}}{\langle N + \bar{N} \rangle_{4\pi}} \qquad \bar{\alpha} = \frac{\langle \bar{N} \rangle_{\Delta Y}}{\langle N + \bar{N} \rangle_{4\pi}}$$

$$P(n, \bar{n}) = \sum_{N, \bar{N}} B(n, N; \alpha) B(\bar{n}, \bar{N}, \bar{\alpha}) P(N, \bar{N})$$

$P(N, \bar{N})$ Distribution of protons (N) and anti-protons subject to global baryon number conservation

$B(n, N, \alpha)$ Binomial distribution with Bernoulli prob α

Factorial cumulants:

$$C_k(n; \Delta Y) = \alpha^k C_k(N, 4\pi)$$

$$C_k(\bar{n}; \Delta Y) = \bar{\alpha}^k C_k(\bar{N}, 4\pi)$$

analogous to “efficiency” corrections

$\Rightarrow \dfrac{C_k}{C_1{}^k} = const$ as function of ΔY for both protons and anti protons

Include volume fluctuations

Holzmann et al. 2403.03598

$$C_1[N] = \langle N_w \rangle C_1[n] = \langle N_w \rangle \langle n \rangle = \langle N \rangle ,$$

$$C_2[N] = \bar{C}_2[N] + \langle N \rangle^2 \frac{\kappa_2[N_w]}{\langle N_w \rangle^2},$$

$$C_3[N] = \bar{C}_3[N] + 3 \langle N \rangle \bar{C}_2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \langle N \rangle^3 \frac{\kappa_3[N_w]}{\langle N_w \rangle^3},$$

$$C_4[N] = \bar{C}_4[N] + 4 \langle N \rangle \bar{C}_3[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 3\bar{C}_2^2[N] \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + 6 \langle N \rangle^2 \bar{C}_2[N] \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} + \langle N \rangle^4 \frac{\kappa_4[N_w]}{\langle N_w \rangle^4}.$$

Since $\bar{C}_k \sim \alpha^k \Rightarrow C_k \sim \alpha^k$

$\bar{C}_k$: Factorial cumulant WITHOUT volume fluctuations

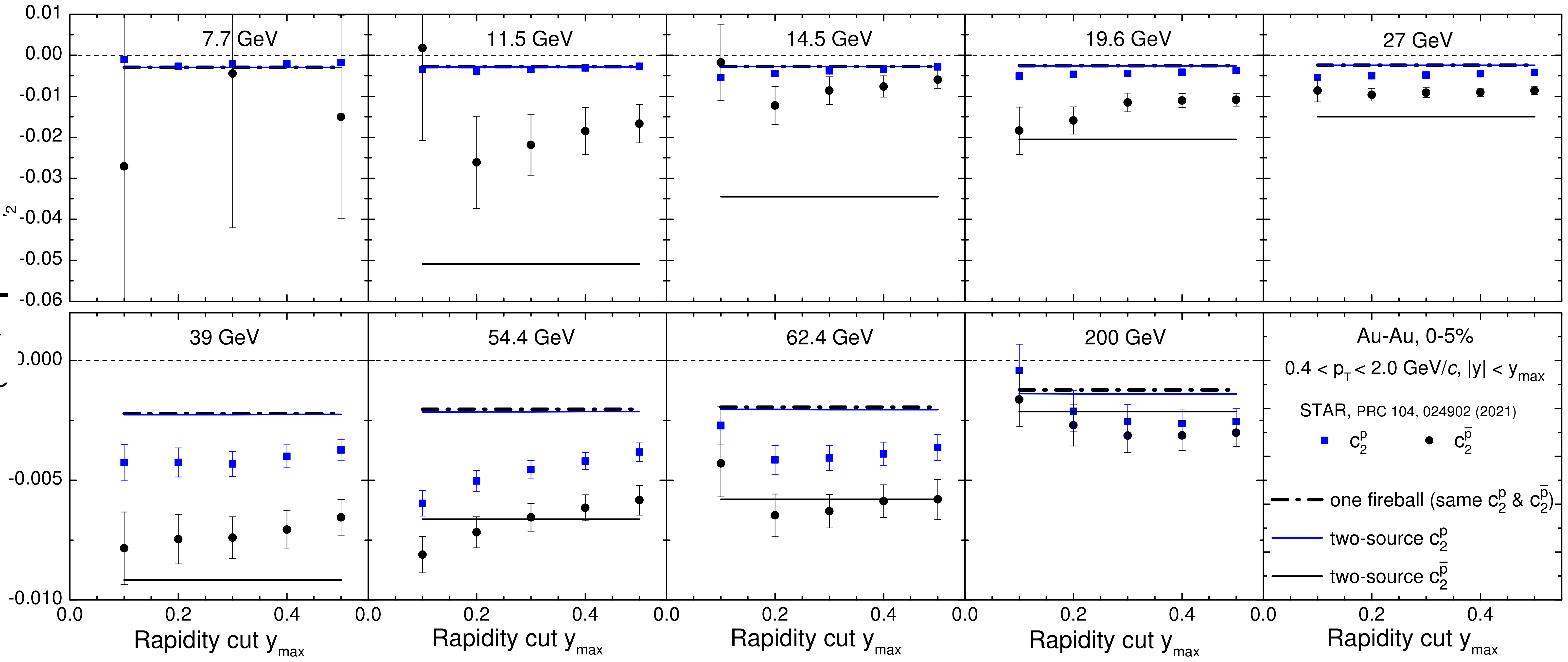
C_k : Factorial cumulant WITH volume fluctuations

If $\dfrac{C_k}{C_1{}^k} \neq const$ as function of ΔY: Some other (short range) physics is at play as well (Example: excluded volume)

Test of baseline BES-I data

- $\frac{FC_2}{FC_1^2}$ more or less constant
- $\frac{FC_2[p]}{FC_1^2[p]} \neq \frac{FC_2[\bar{p}]}{FC_1^2[\bar{p}]}$
- baseline OK for protons
- No good for anti-protons ??
- How will it look with BES II data?

$\frac{FC_2}{FC_1^2}$



Two component model

2 sources: stopped and produced particles

- All anti-protons are produced
- protons come from produced and stopped sources

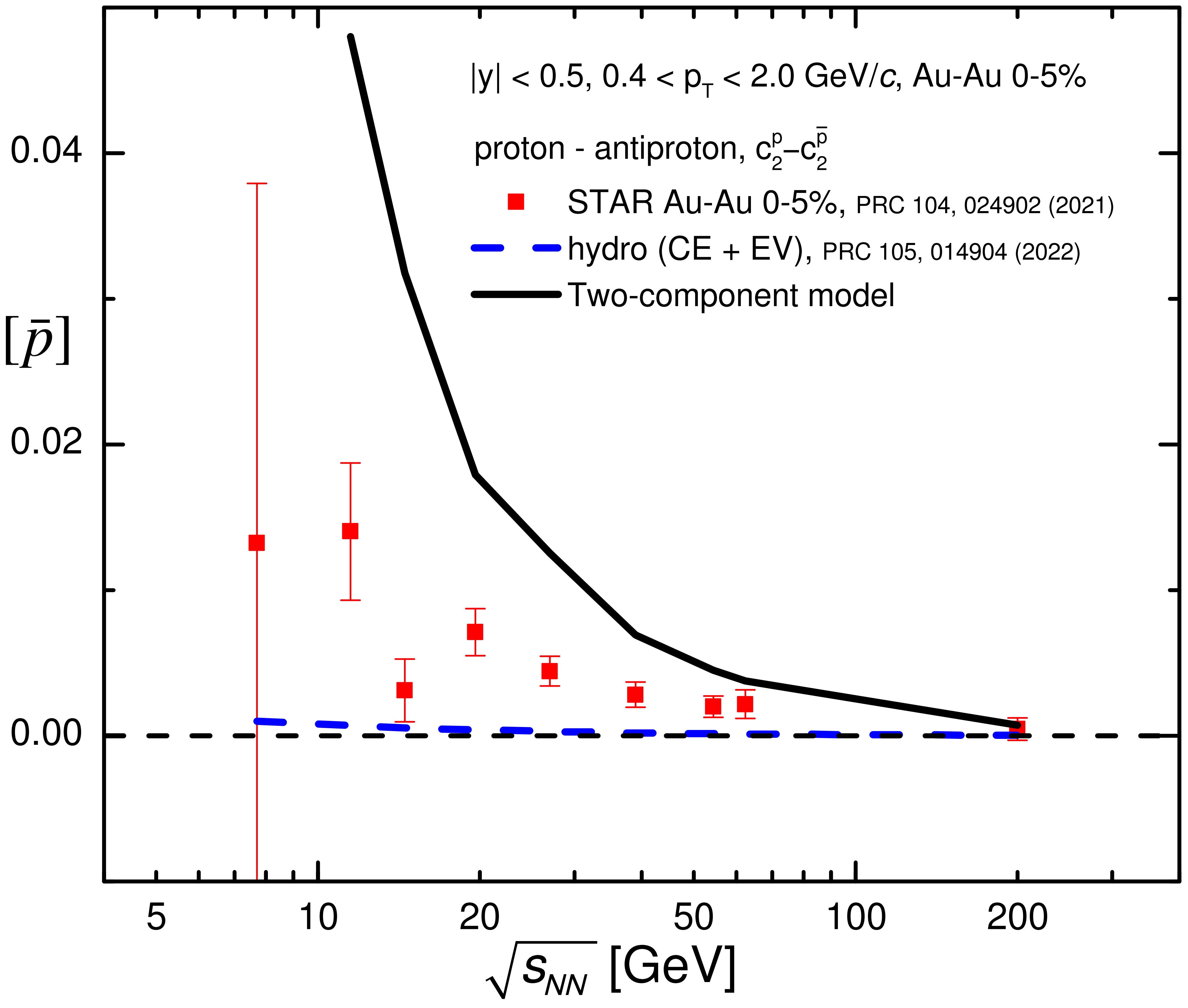
$$N_p(\text{produced}) = \mathrm{N}_{\bar{\mathrm{p}}}$$

$$N_p = N_p(\text{stopped}) + \mathrm{N}_{\bar{\mathrm{p}}}$$

- Produced source: Thermal with zero net baryon number $\langle B - \bar{B} \rangle = 0$

- Stopped source: Follows binomial distribution

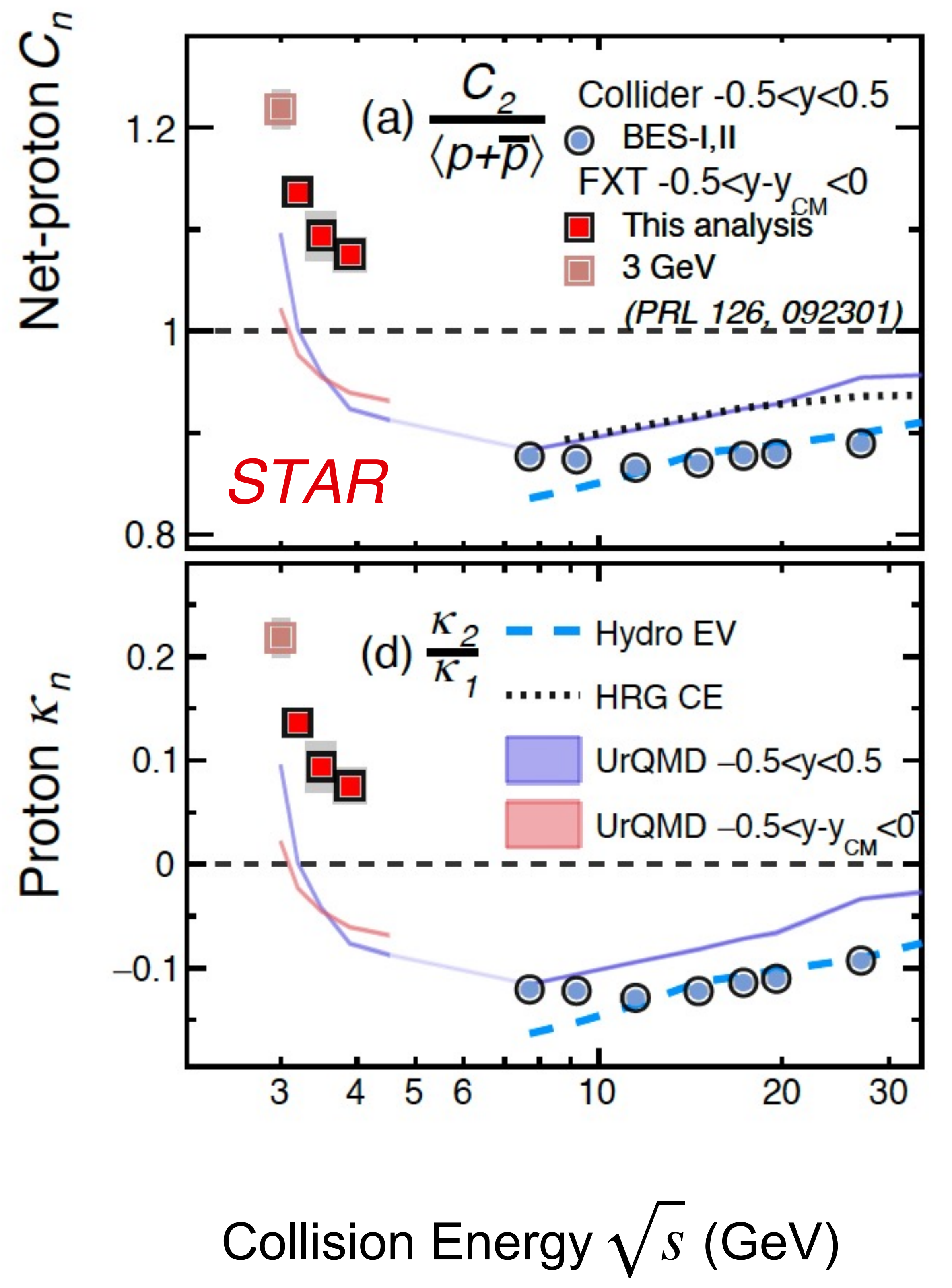
$$\frac{FC_2}{FC_1^2}[p] - \frac{FC_2}{FC_1^2}[\bar{p}]$$

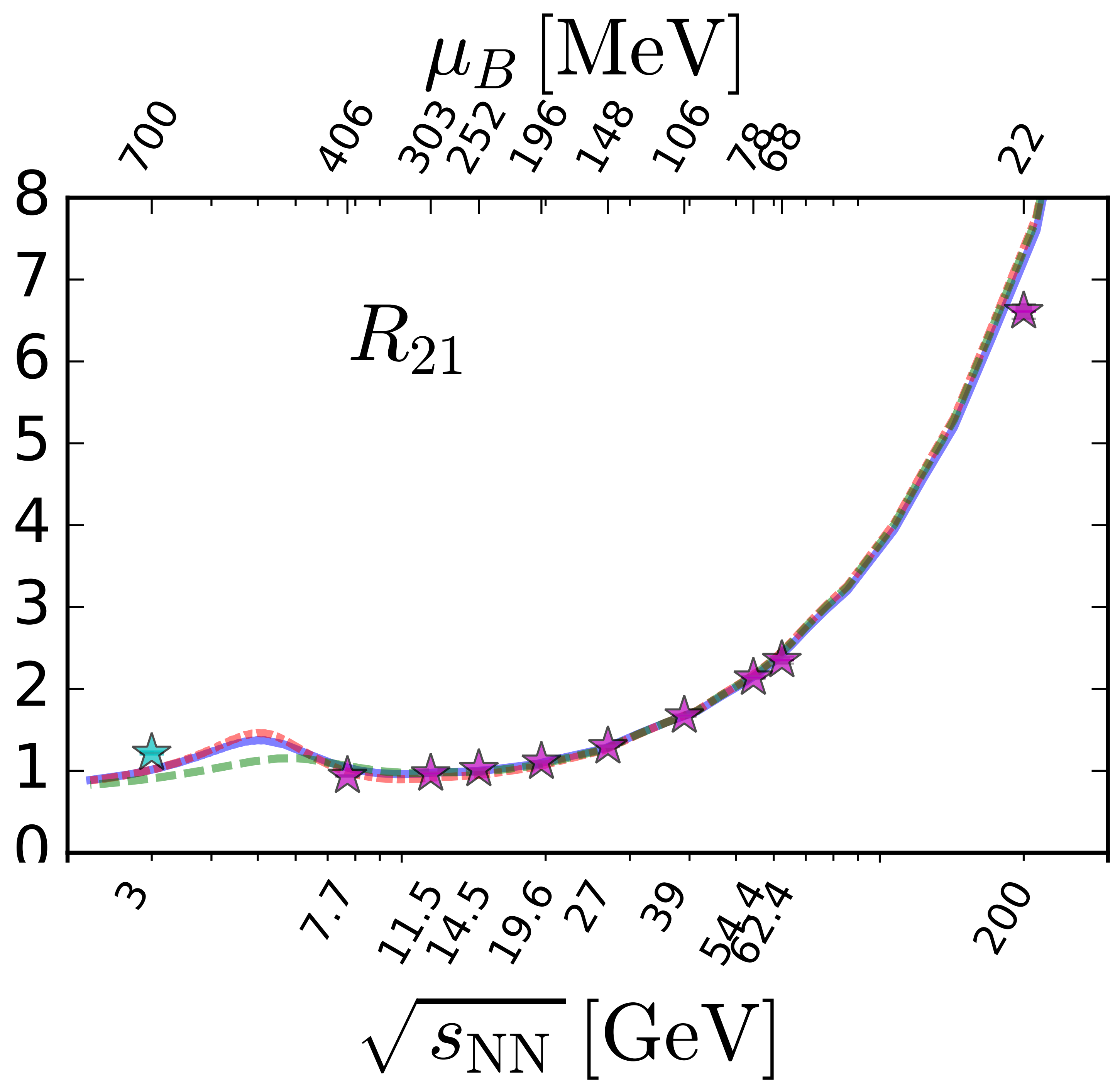


Summary

- STAR has delivered on the BESII data! Congratulations!
  - cannot hide behind error bars anymore
- Baseline with baryon number conservation and repulsive interaction tuned to LQCD agrees with data down to $\sqrt{s} \sim 10\,\mathrm{GeV}$
- Data below $\sqrt{s} \sim 10\,\mathrm{GeV}$ seem to require some kind of "attraction"
- HOWEVER, UrQMD get the trend in the energy dependence right. Volume fluctuations a low energy?
- Possible test of a baseline involving baryon number conservation and volume fluctuations via ratio of factorial cumulants $\frac{FC_n}{FC_1^N}$
- Anti protons from BES I are NOT understood. BESII comparison needed. Two source model?.

Data for symmetric $[-\Delta Y, \Delta Y]$ below $\sqrt{s} < 7.7\,\mathrm{GeV}$ desperately needed!





CBM is perfectly positioned to do so

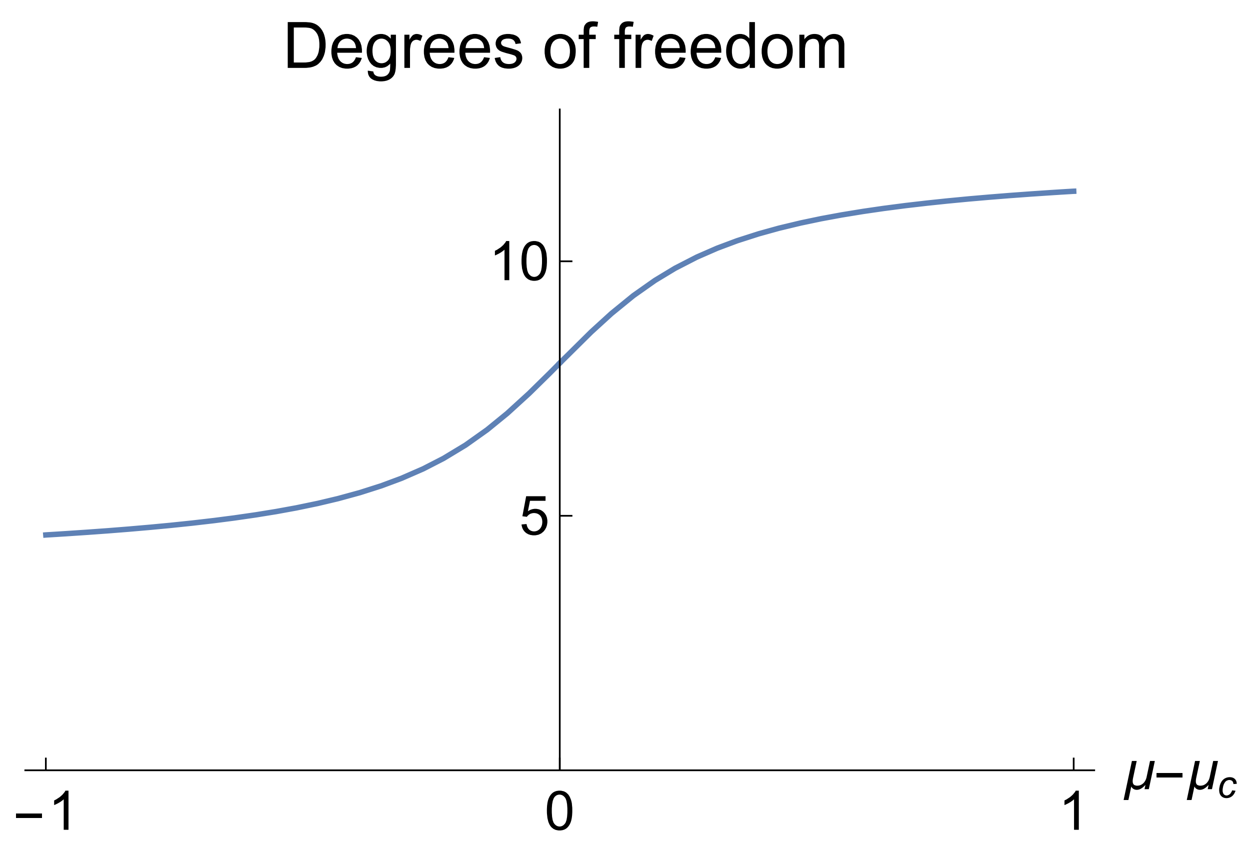
Meanwhile: understand what UrQMD in STAR acceptance does

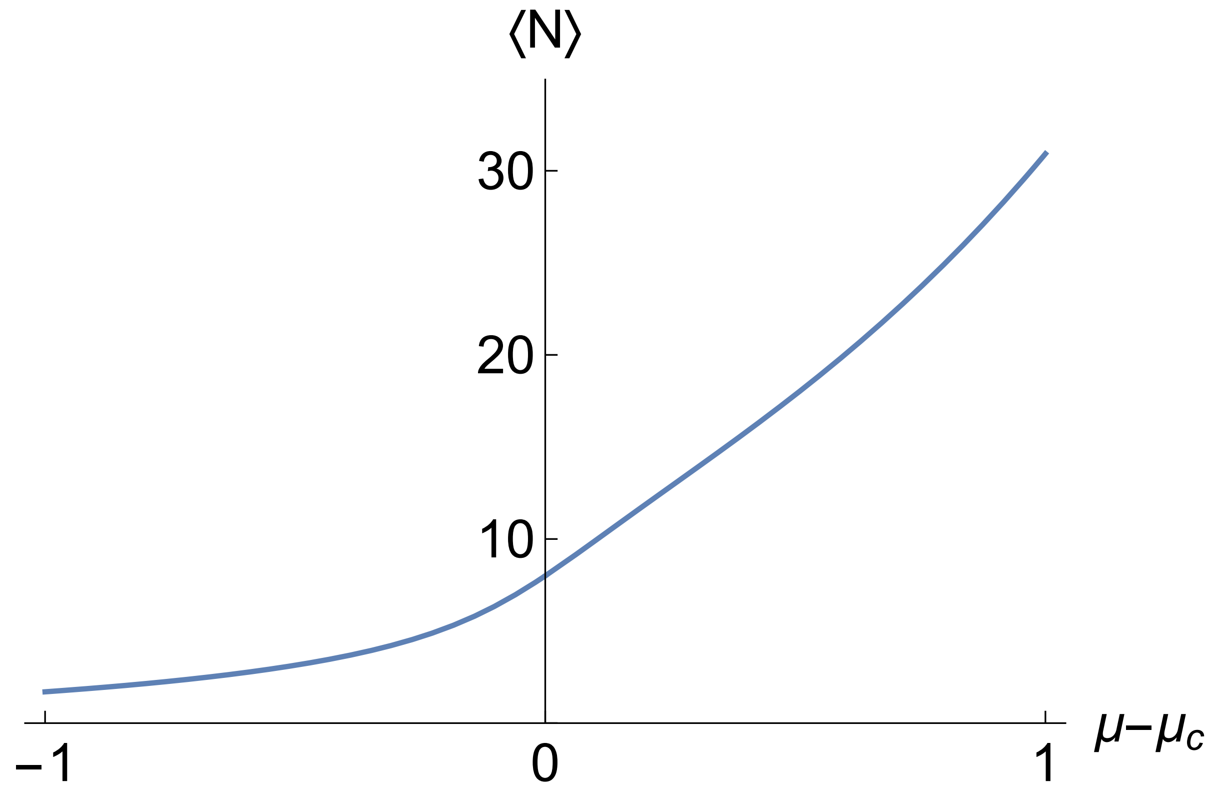
Backup

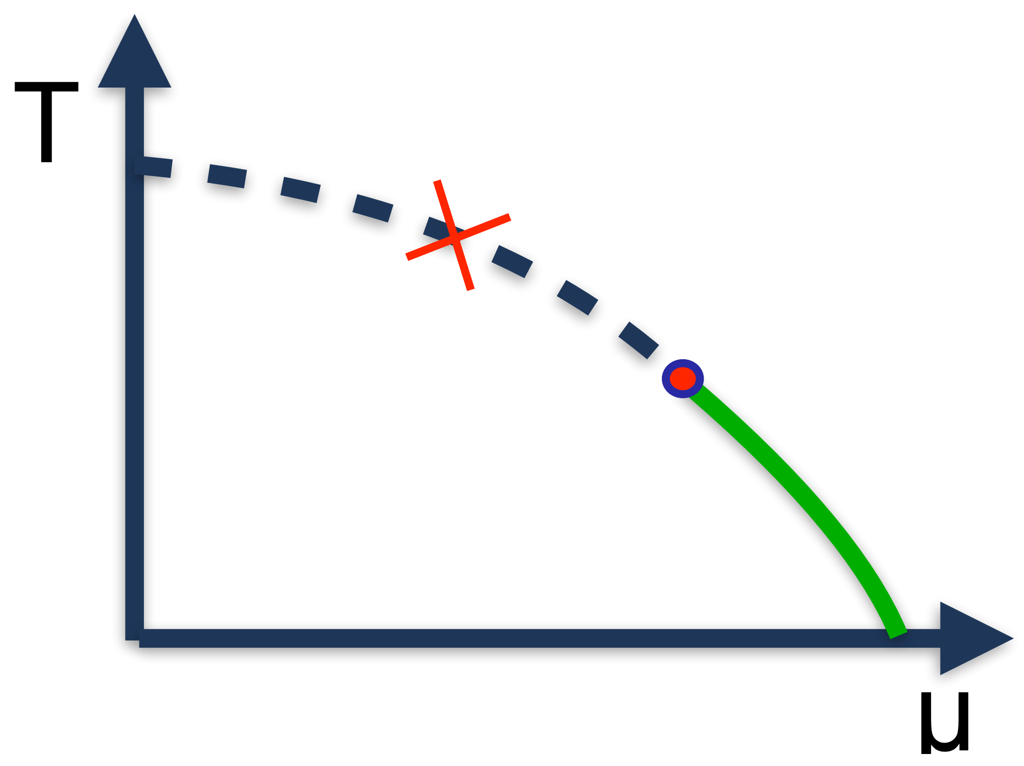
Simple model

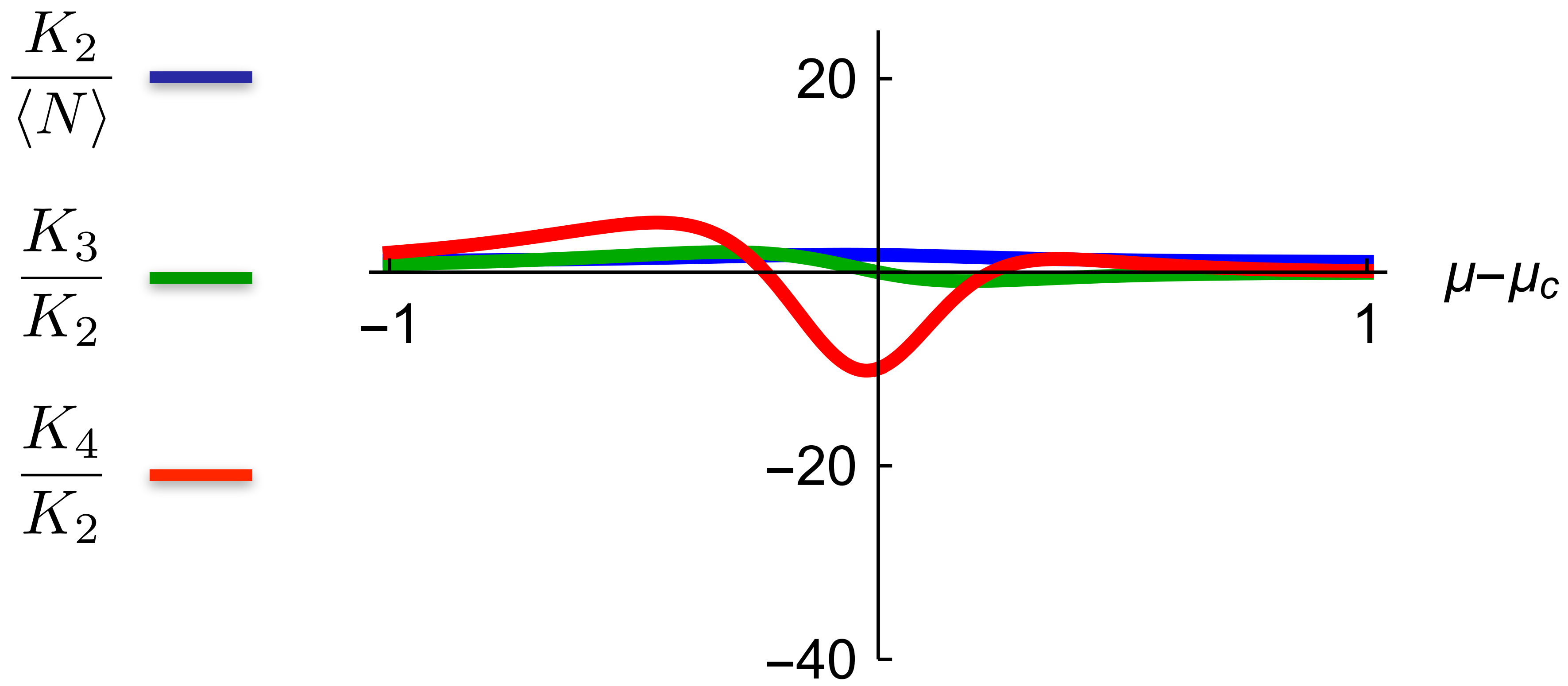
Change degrees of freedom
at phase transition

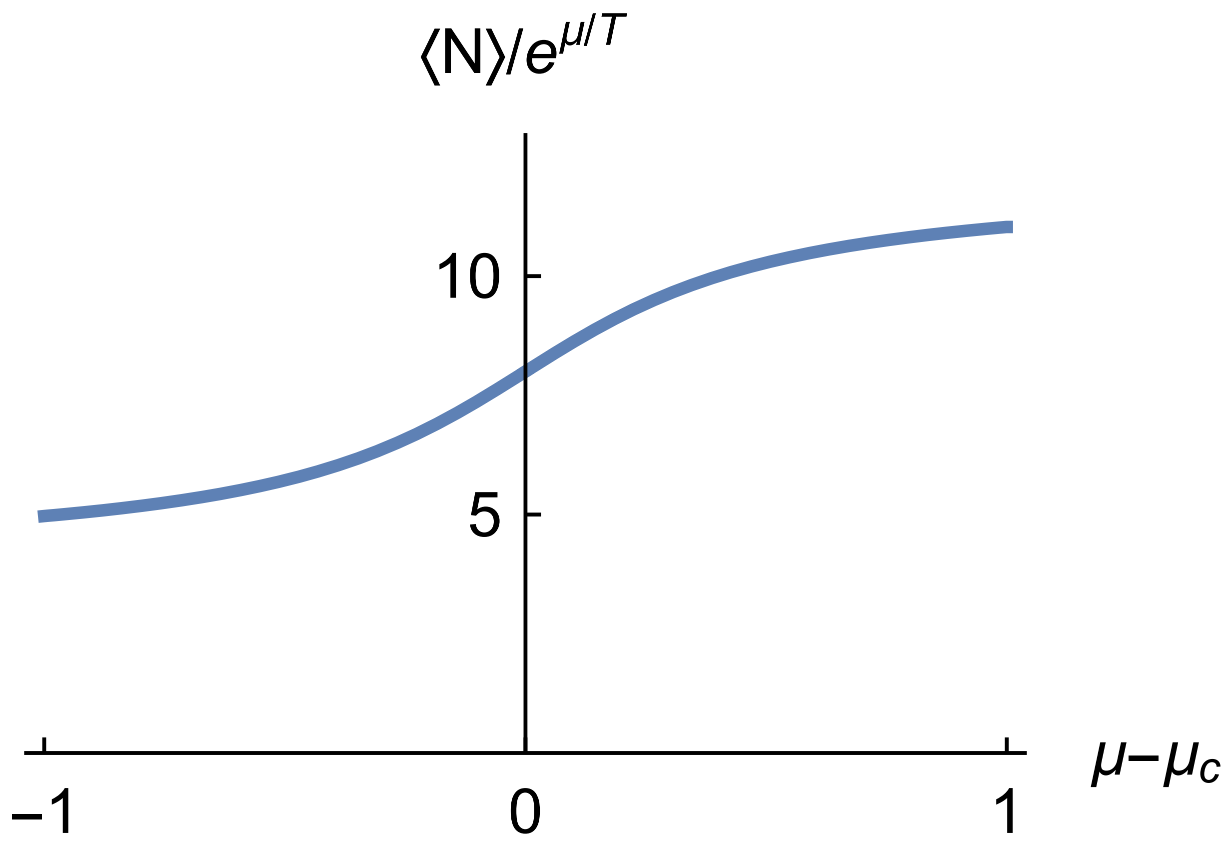
$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$

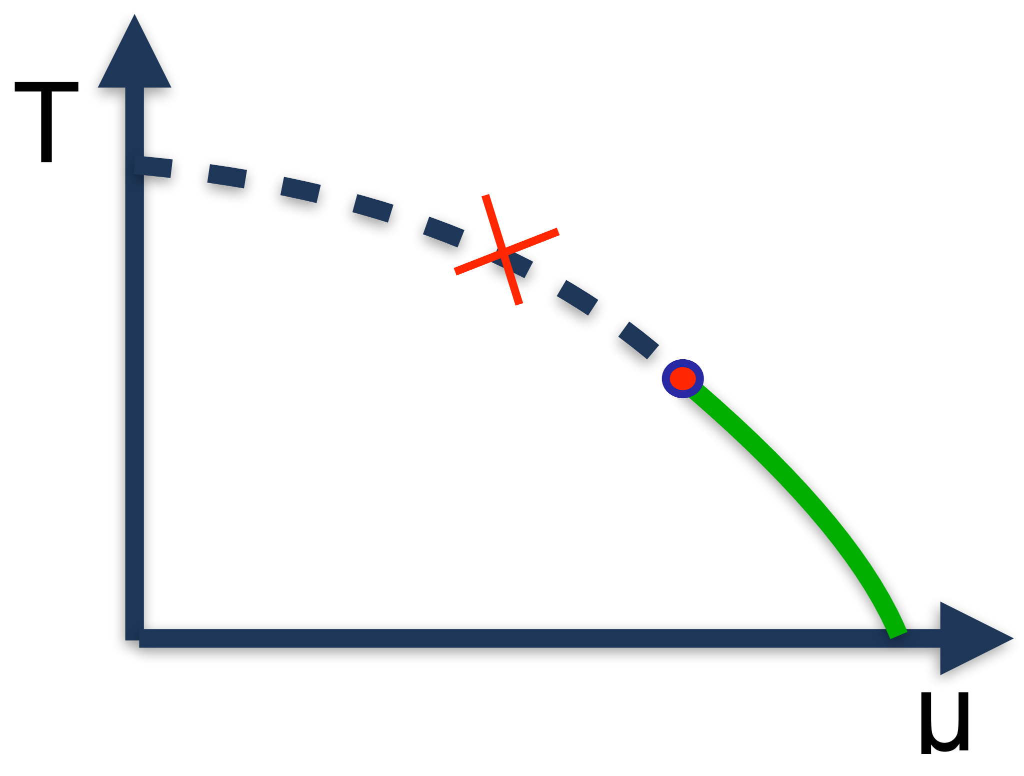


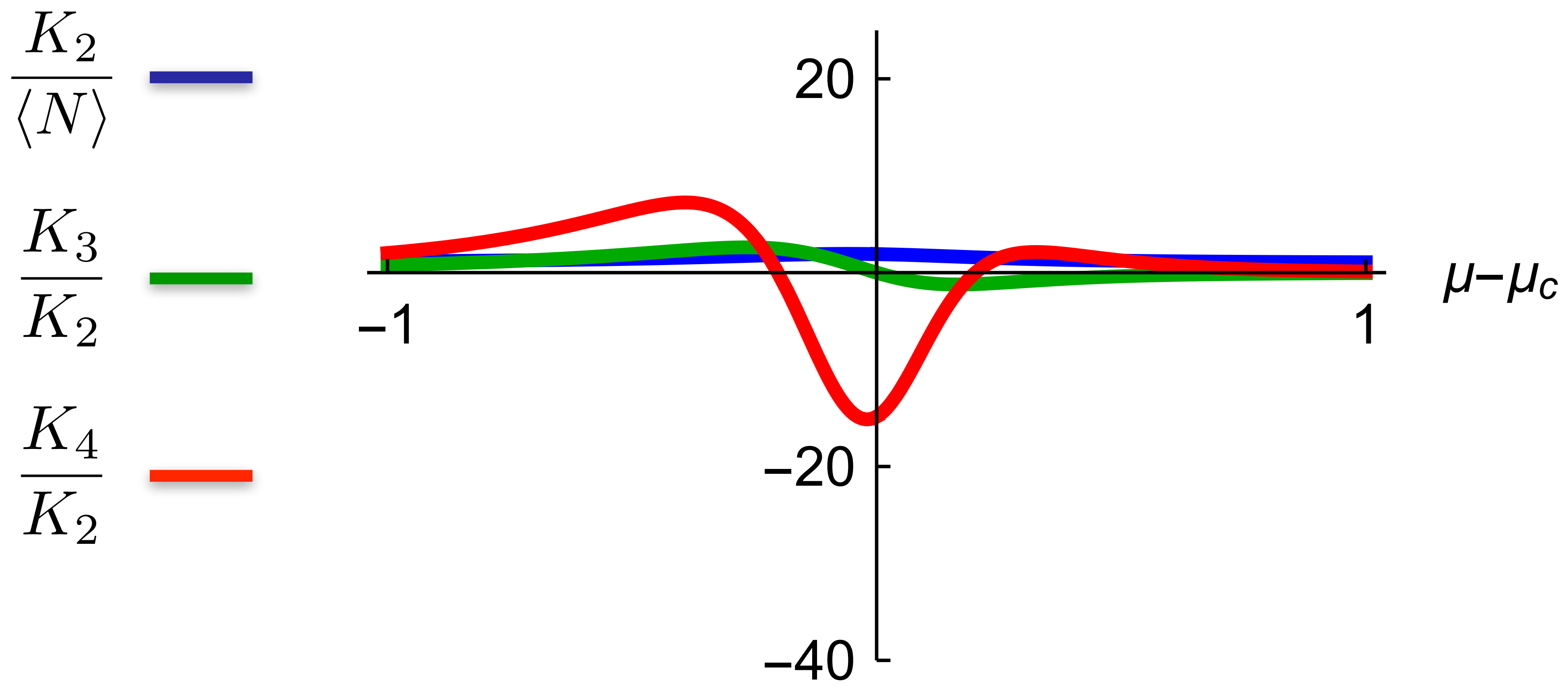



T
μ


$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
μ−μc


⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc


T
μ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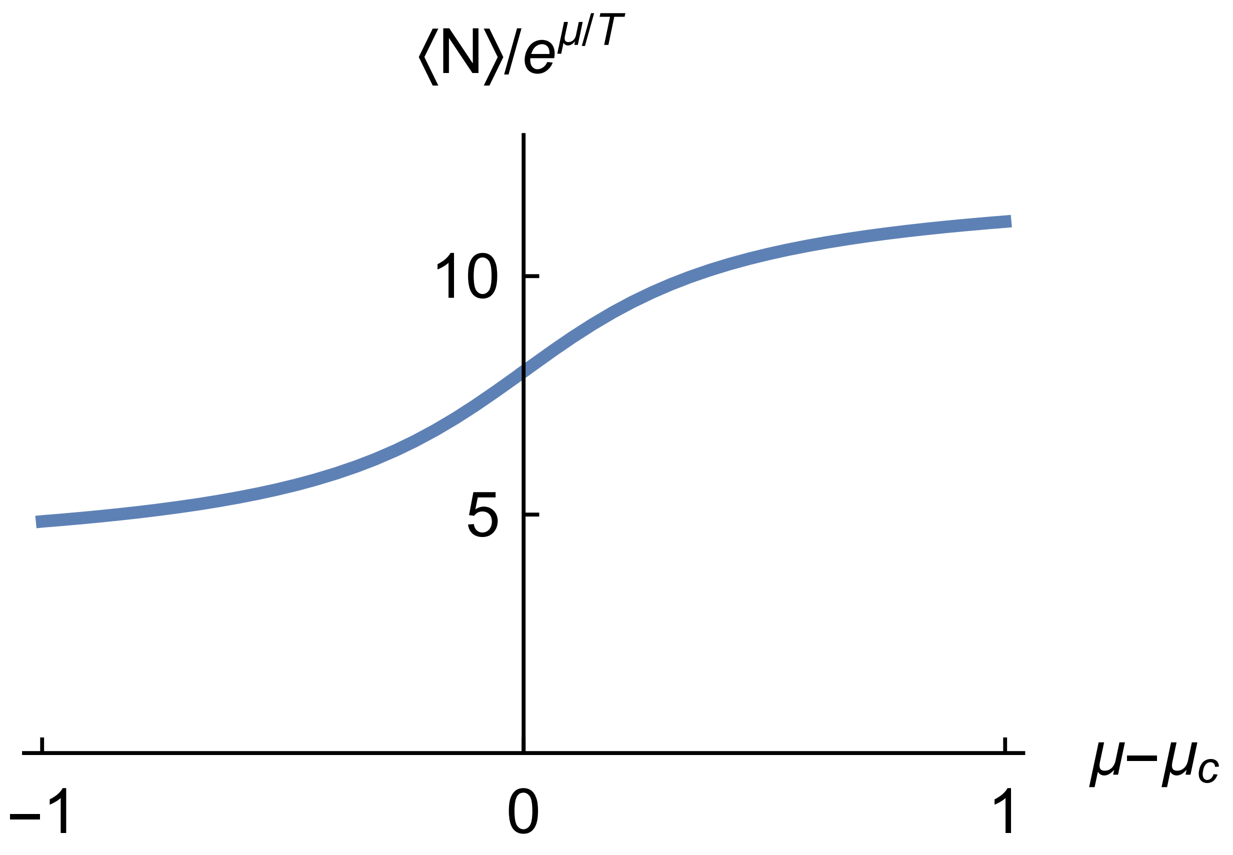
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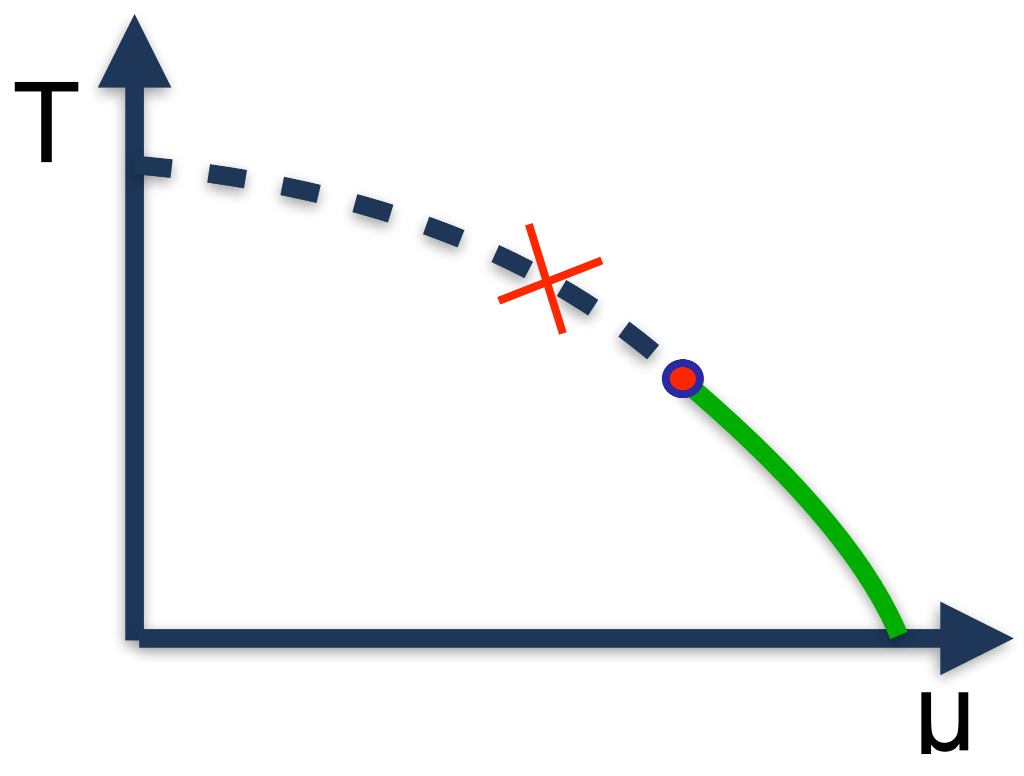
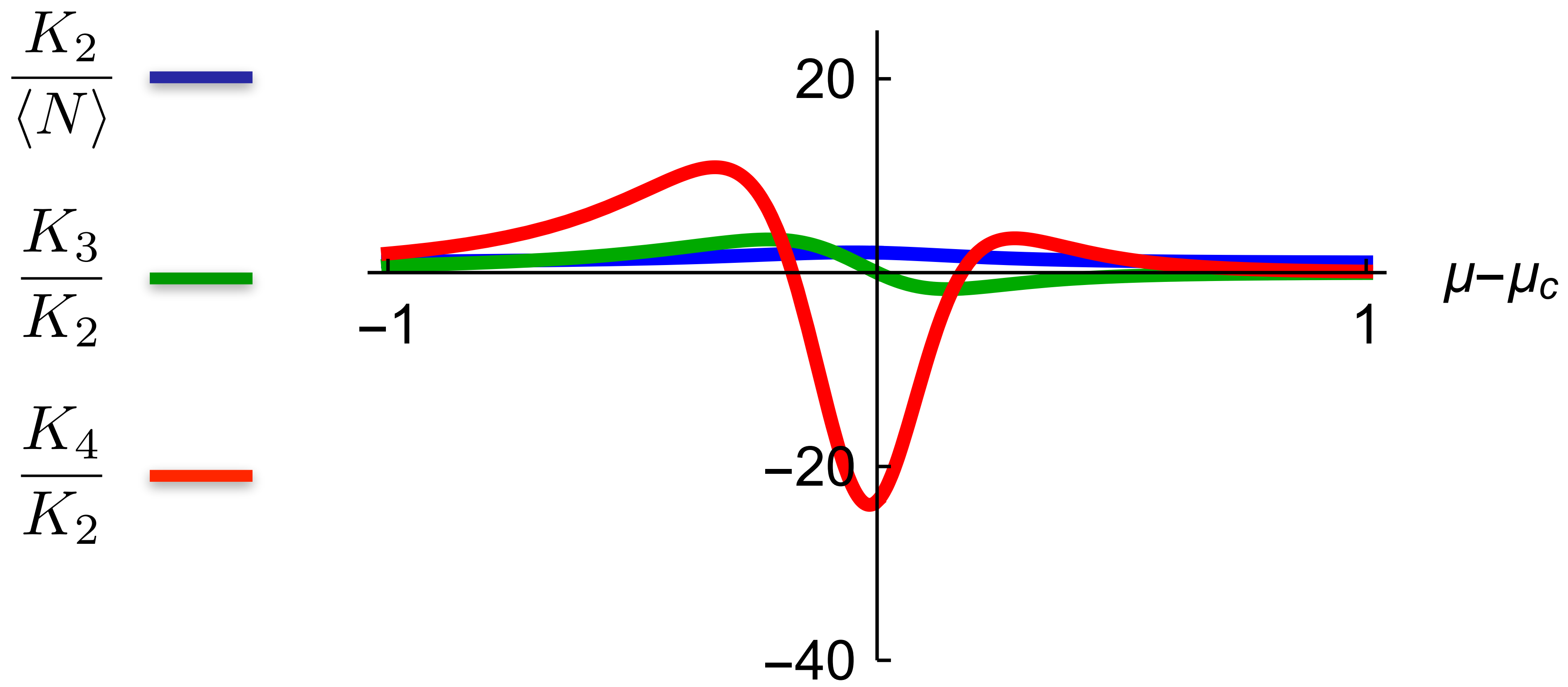

T
μ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-20
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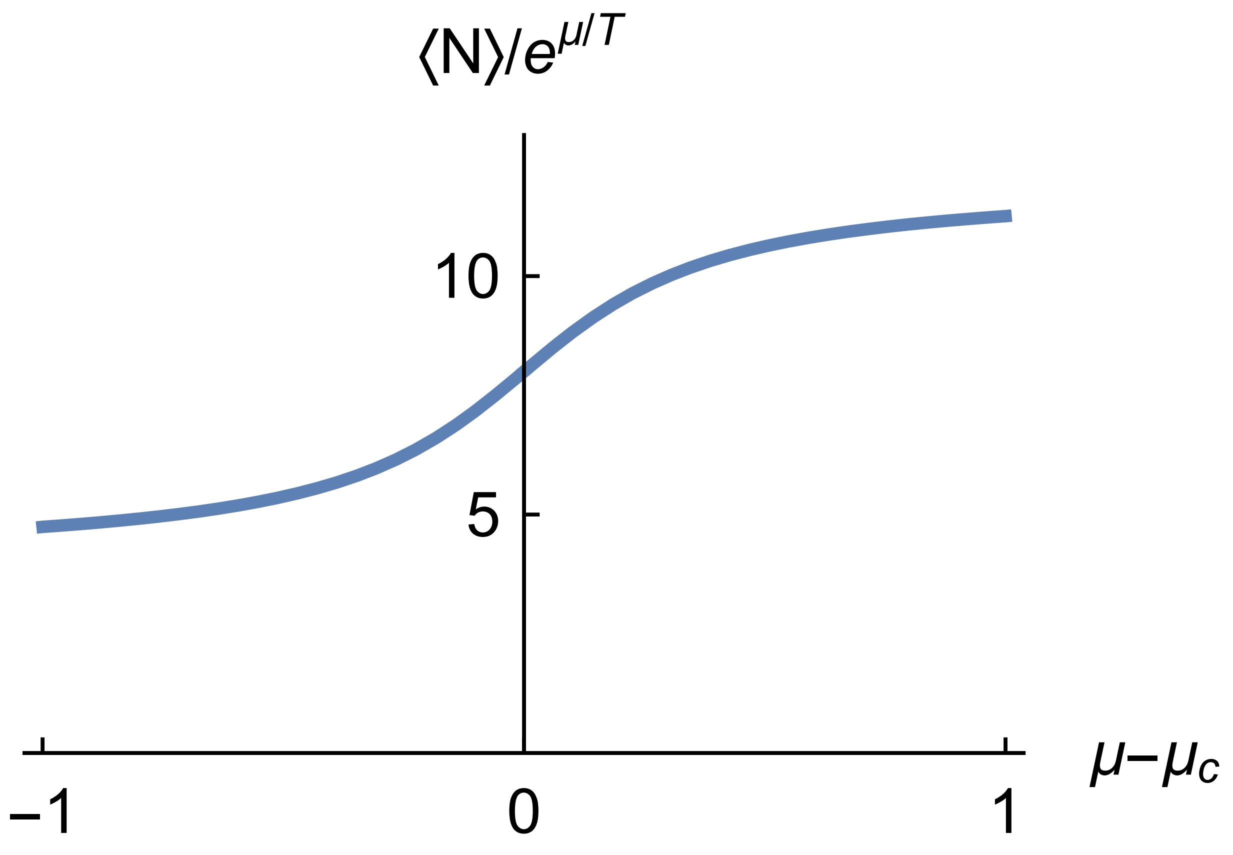
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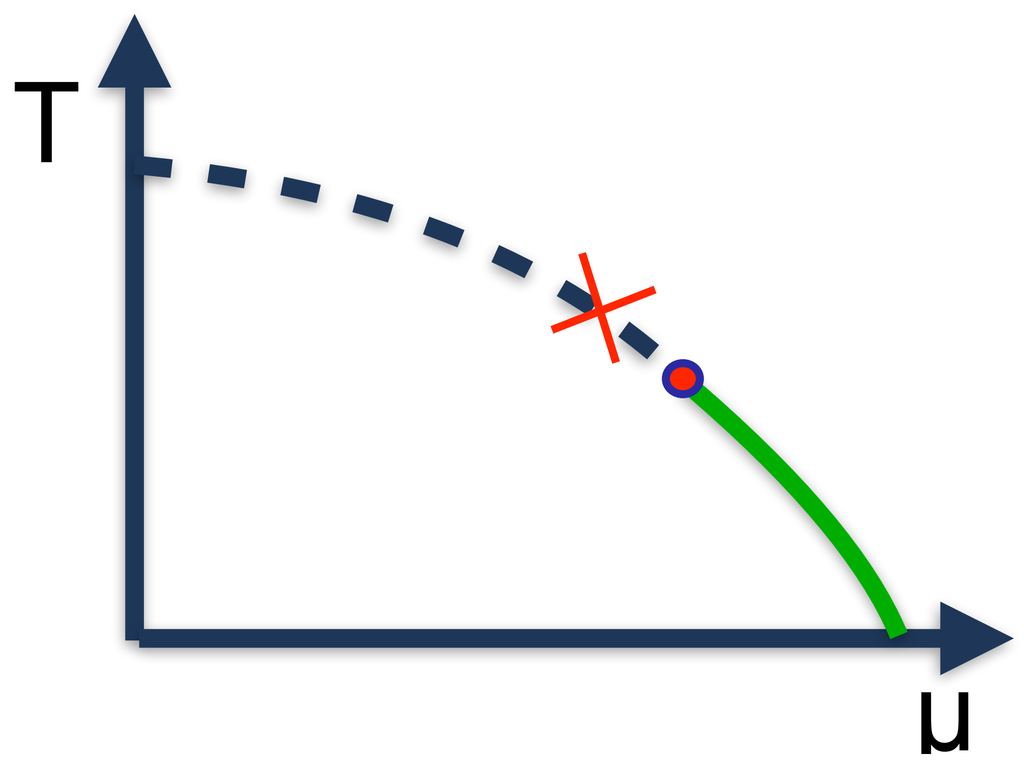
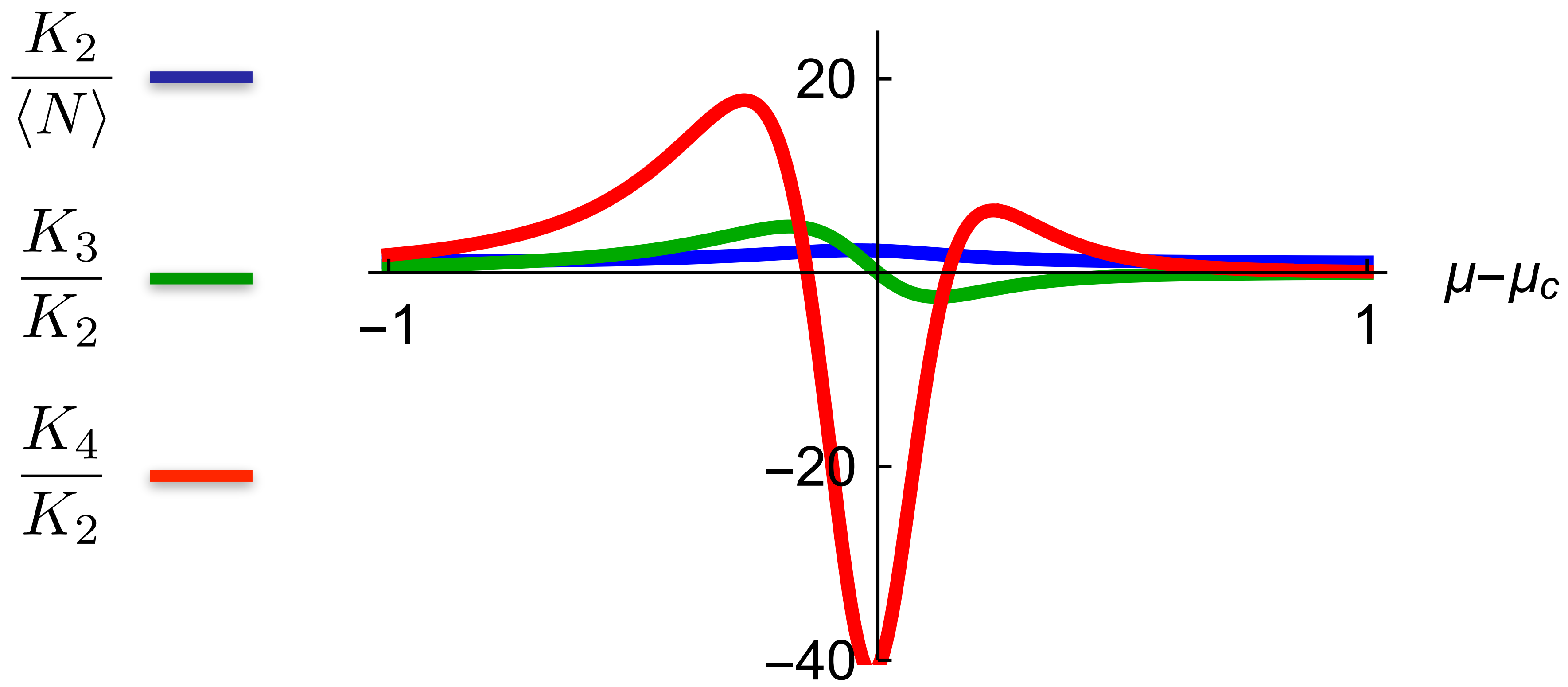
10
5
-1
0
1
μ−μc


T
μ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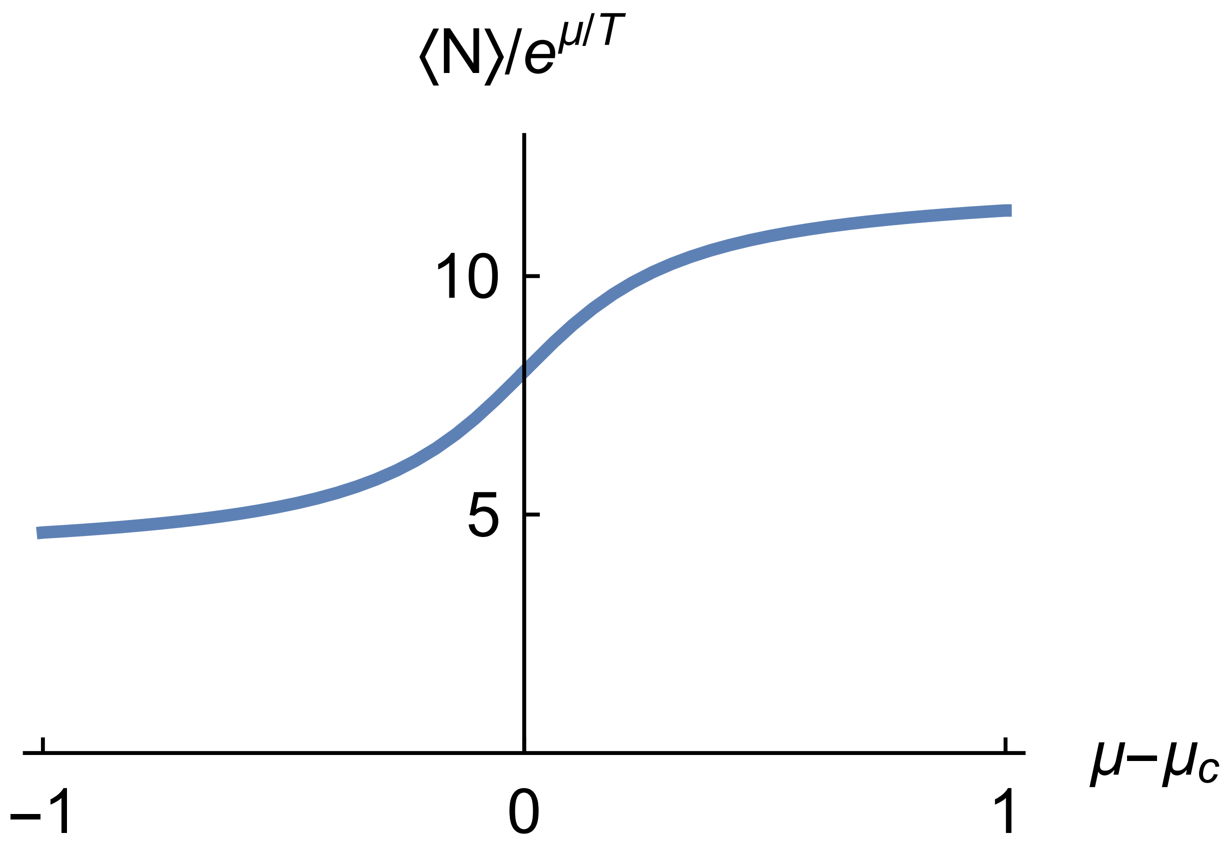
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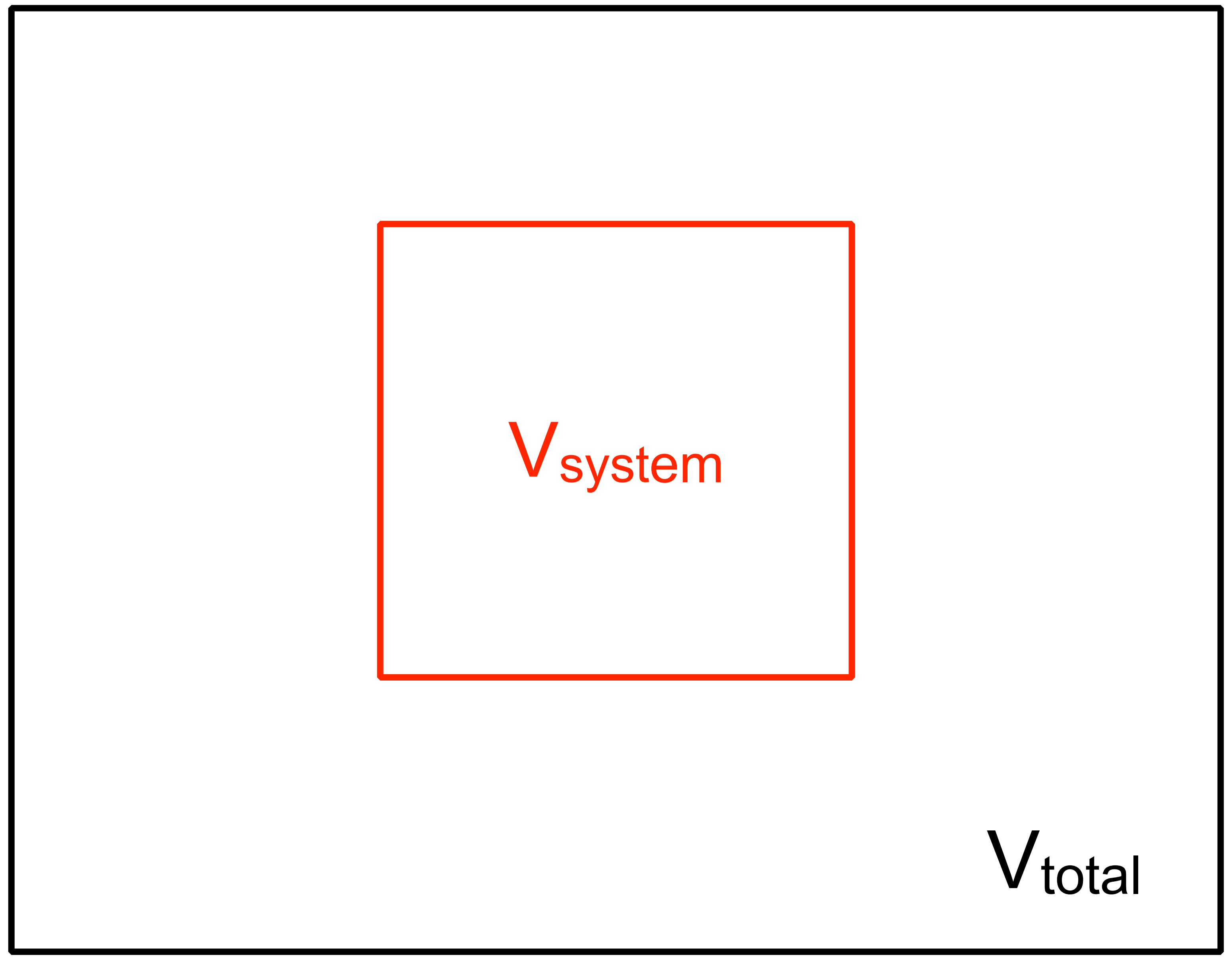
10
5
−1
0
1
μ−μc

Grand canonical ensemble



$$V_{total} \to \infty$$
$$V_{system} \to \infty$$

$$\frac{V_{system}}{V_{total}} \to 0$$

In coordinate space!!!!

Lattice:
$V_{total} \to \infty$
grand-canonical ensemble
Coordinate space

Experiment:
V_{total} finite!
$V_{system} \ll V_{total}$ (hopefully)
effect of global charge conservation
Momentum Space