Constraining Color Superconducting Low-energy Model from First Principle QCD

Ugo Mire

based on [Hosein Gholami, UM, Fabian Rennecke, Bernd-Jochen Schaefer, Shi Yin; in preparation]

International School of Nuclear Physics

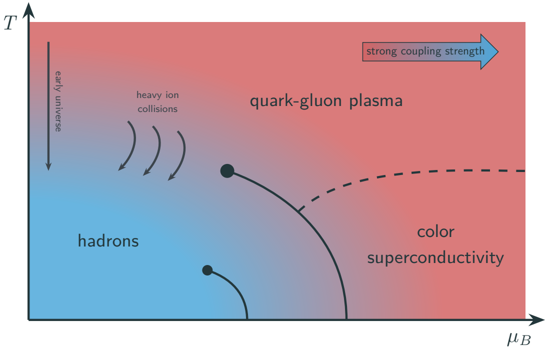
Erice - September 17, 2025

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

CRC-TR 211 Strong-interaction matter under extreme conditions

Motivation



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heavy ion collisions
strong coupling strength
quark-gluon plasma
hadrons
color superconductivity
μ_B

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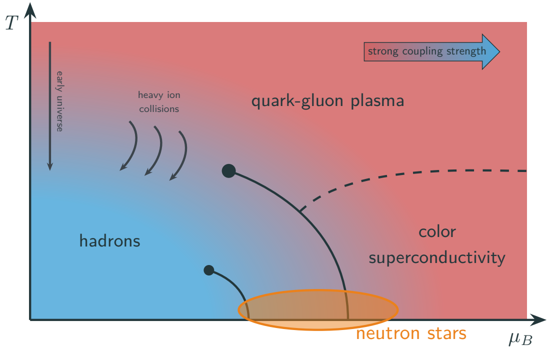

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Color superconductivity

Cooper instability: Fermi-surface is unstable in presence of attractive interaction

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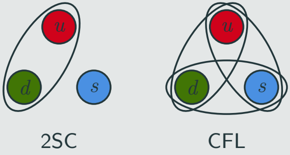
Difference with standard superconductivity?

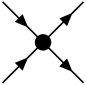
Quark field indices

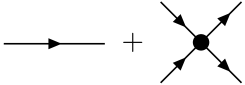
- Dirac index
- Flavor index (up, down, strange)
- Color index (red, green, blue)

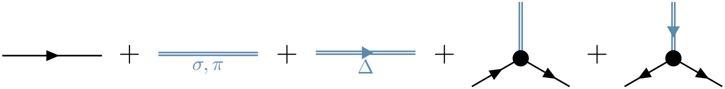
$\rightarrow$ different pairings are possible (2SC, CFL, CSL, ...)



Effective model for color superconductivity

$S_{\mathrm{NJL}} =$  $+$ 

$S_{\mathrm{NJL}} =$ 

$S_{\mathrm{QMD}} =$ 

Quark-Meson-Diquark model

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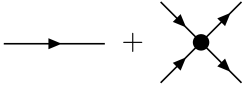

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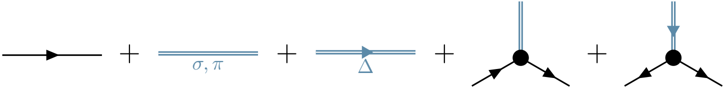

Quark-Meson-Diquark model

Renormalization Conditions

Usually in vacuum $T = \mu = 0$:

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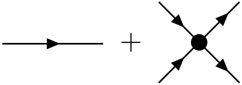
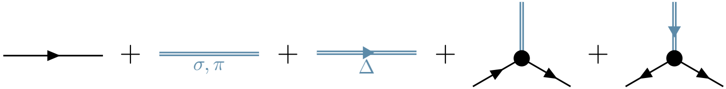
$S_{\mathrm{NJL}} =$ 

$S_{\mathrm{QMD}} =$ 

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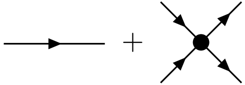
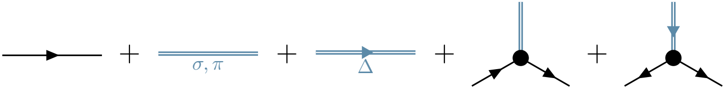
$S_{\mathrm{NJL}} =$ 

$S_{\mathrm{QMD}} =$ 

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Problem: no experimental data available for diquarks

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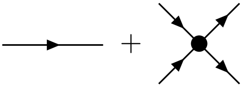
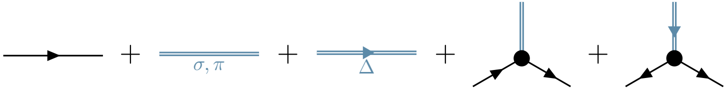
$S_{\mathrm{NJL}} =$ 

$S_{\mathrm{QMD}} =$ 

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Problem: no experimental data available for diquarks

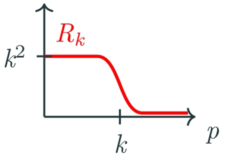
Goal: compute diquark effective potential from first principle QCD

The functional renormalization group

Main idea: successively integrate out fluctuations in the spirit of Wilson's RG.

$$Z_k[J] = \int \mathcal{D}\Phi\, e^{-S[\Phi]-\Delta S_k[\Phi]-\int J\Phi}$$

with $\quad \Delta S_k[\Phi] = \int_p \Phi(p) R_k(p) \Phi(-p)$



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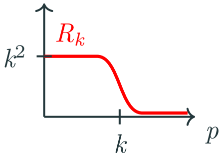

Formulated on the effective action: Γ_k

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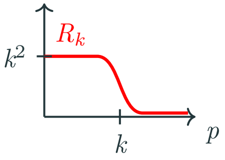
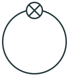


Formulated on the effective action: Γ_k

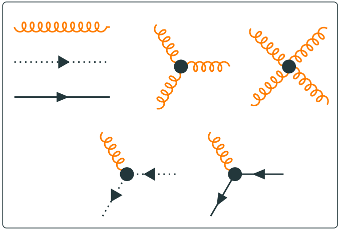
integrate from $k = \Lambda$ to $k = 0$ with

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)}[\Phi] + R_k\right)^{-1} \partial_t R_k\right] = \frac{1}{2}$$



General strategy for QCD

quark-gluon sector



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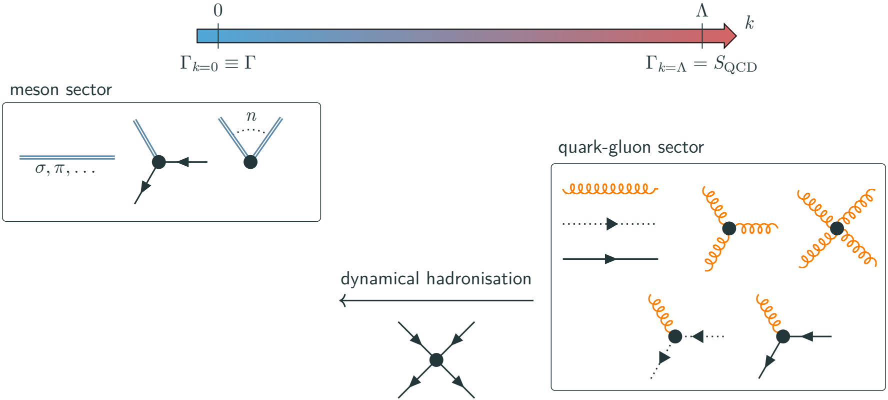
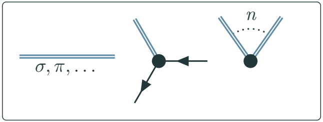


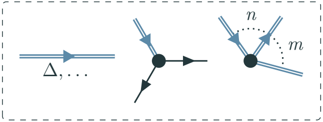
[Braun, Fister, Pawlowski, Rennecke; 1412.1045]

[Fu, Pawlowski, Rennecke; 1909.02991]

meson sector

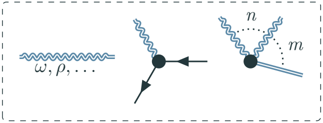


diquark sector



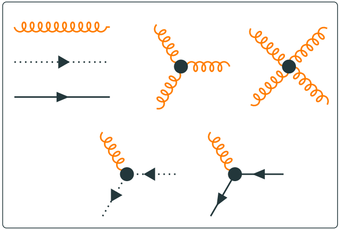
[Braun, Schallmo; 2106.04198]

vector sector

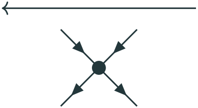


[Rennecke; 1504.03585]

quark-gluon sector



dynamical hadronisation



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More details

[Goertz, Pastor-Gutiérrez, Pawlowski; 2412.12254]

Gauge sector:

Zero-momentum expansion around the gauge fixed classical tensor structure, including running gluon mass

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Dynamical Hadronisation

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- QCD flow generates four-quark interactions

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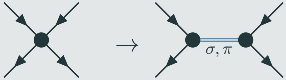
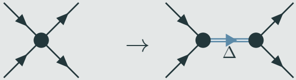
Dynamical Hadronisation

[Gies, Wetterich; hep-th/0107221]

- QCD flow generates four-quark interactions
- Hubbard-Stratanovich transformation: four-quark interaction can be mapped to boson exchange
- Perform the Hubbard-Stratanovich transformation at each k-step $\rightarrow$ continuously generates the low-energy model

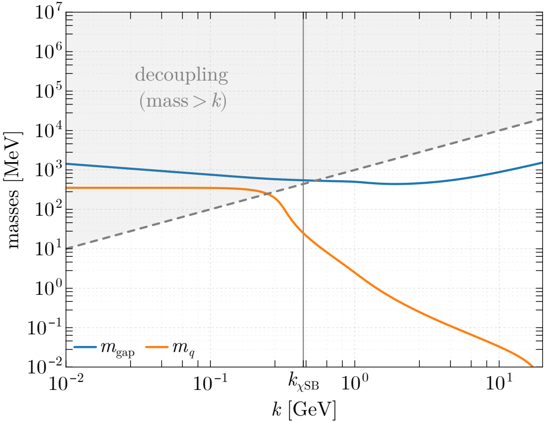


scalar-pseudoscalar



color superconducting

Masses & successive decoupling



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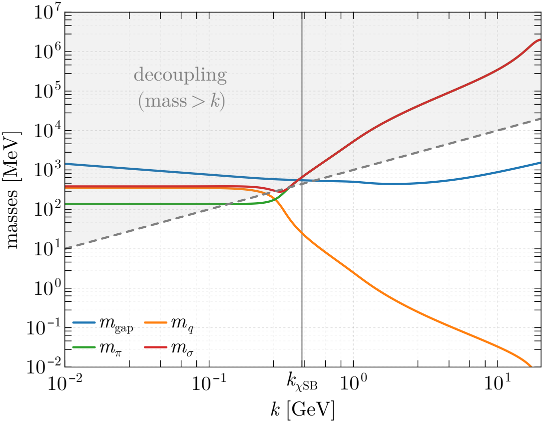

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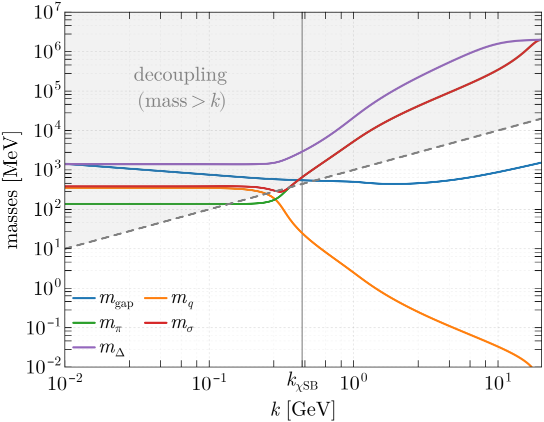

Connection to low-energy models

IR set of correlations for vanishing momenta $\rightarrow$ can be fitted by a low-energy model:

QCD

(all momenta vanishing)

$$\langle \pi \bar{q} q \rangle \sim 5.3$$

$$\langle \Delta \bar{q} \bar{q} \rangle \sim 4.3$$

$$\langle \sigma\sigma \rangle \sim (382\,\mathrm{MeV})^2$$

$$\langle \Delta\Delta \rangle \sim (1392\,\mathrm{MeV})^2$$

$$\langle \sigma\sigma\Delta\Delta \rangle \sim 7.5$$

$$\ldots$$

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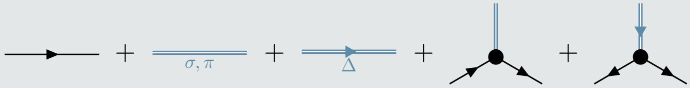
$\dots$	$\longrightarrow$	$\dots$

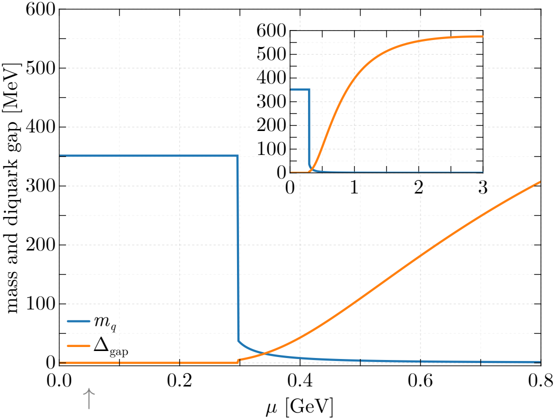
Quark-meson-diquark model ($N_f = 2$)

$S_{\mathrm{QMD}} =$



with renormalized mean-field approximation [Hosein Gholami, Lennart Kurth, UM, Michael Buballa, Bernd-Jochen Schaefer; 2505.22542]

Results for low-energy model

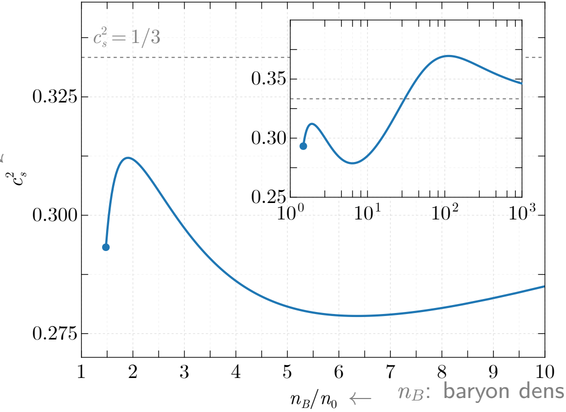


m_q: quark mass

$\Delta_{\rm gap}$: color superconducting gap

Results for low-energy model

speed of sound
$c_s^2 = \frac{\partial p}{\partial \epsilon}$



n_B: baryon density
n_0: nuclear saturation density

- $c_s^2 < 3$ for astrophysically relevant densities

Summary & Outlook

Astrophysics applications

Add ω and ρ vector meson to renormalized model and QCD flow
$\longrightarrow$ astrophysical prediction with no external parameters

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Add ω and ρ vector meson to renormalized model and QCD flow
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- Lack of first principle constrain on color superconducting models
- Simple QCD truncation $\rightarrow$ constrain diquark effective potential
- First application to low-energy model $\rightarrow$ $c_s^2 < 1/3$ for astrophysically relevant densities?

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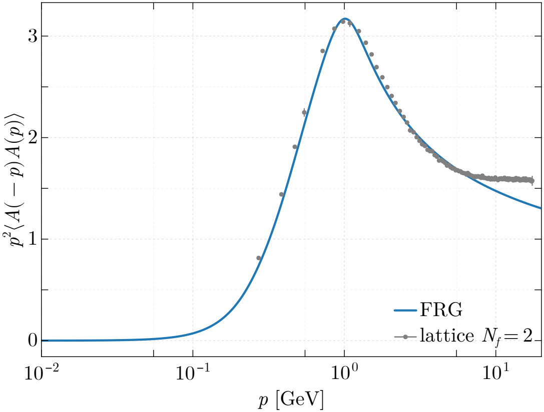
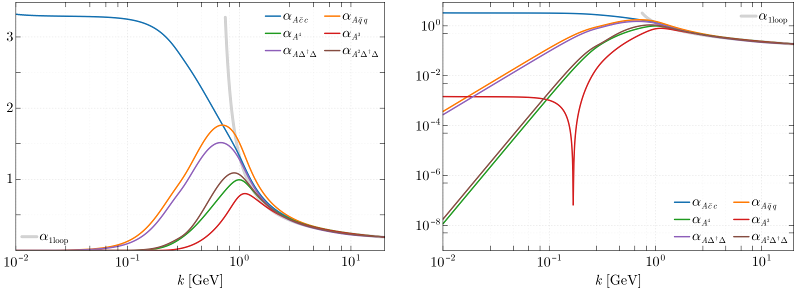
Thank you!

Backup Slides

Gluon propagator

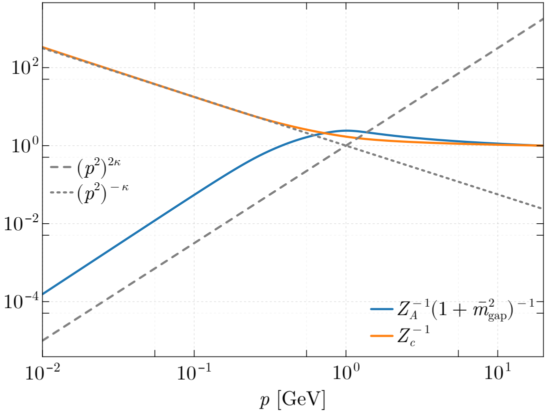


Flow of the strong coupling avatars



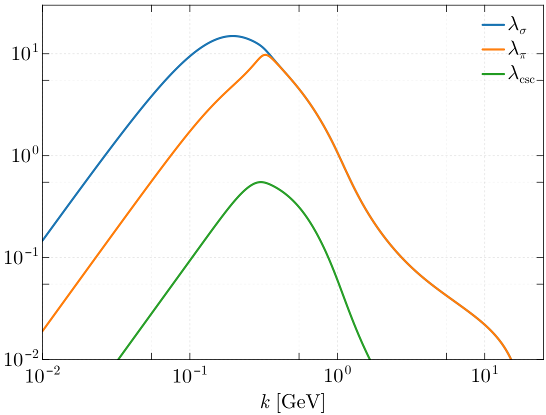
$$\alpha = \frac{g_s^4}{4\pi}$$

Scaling solution

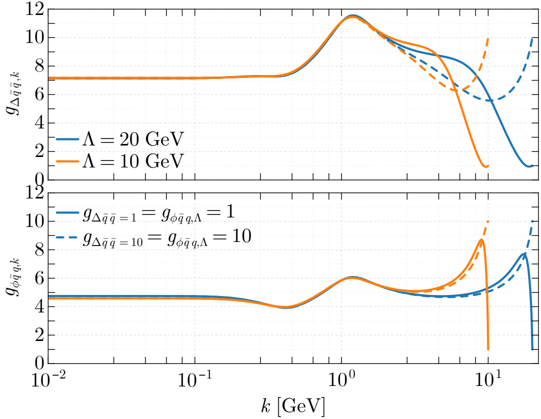


$\kappa = 0.625$: scaling exponent

Effective 4q interactions



Invariance on initial conditions



Quark-meson-diquark model action

$$S[\bar{q}, q, \phi, \Delta, \Delta^*] = \int_0^\beta dx_4 \int d^3x \Big\{ \bar{q} \left[Z_q \left(\partial\!\!\!/ - \mu\gamma_4 \right) + g_\phi \left(\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\tau} \right) \right] q$$
$$+ \frac{1}{2} g_\Delta \left(\Delta_a \bar{q} \gamma_5 \tau_2 i\epsilon_a C \bar{q}^\top - \Delta_a^* q^\top C \gamma_5 \tau_2 i\epsilon_a q \right)$$
$$+ \frac{Z_\phi}{2} (\partial_\mu \phi)(\partial_\mu \phi) + Z_\Delta (\partial_\mu + 2\mu\delta_{\mu 4}) \Delta_a^* (\partial_\mu - 2\mu\delta_{\mu 4}) \Delta_a$$
$$+ U(\phi^2, |\Delta|^2) - c\sigma \Big\}$$

with

$$U(\phi^2, |\Delta|^2) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda_\phi \phi^4$$
$$+ m_\Delta^2 |\Delta|^2 + \lambda_\Delta |\Delta|^4 + \frac{1}{2} \lambda_{\mathsf{mix}} \phi^2 |\Delta|^2$$

and

$$\phi^2 = \sigma^2 + \vec{\pi}^2$$