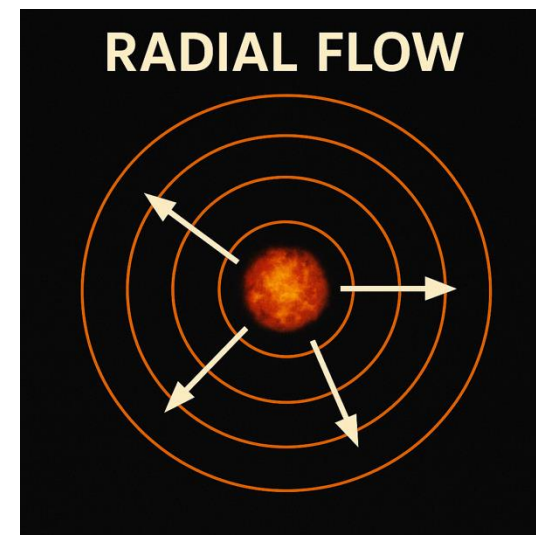




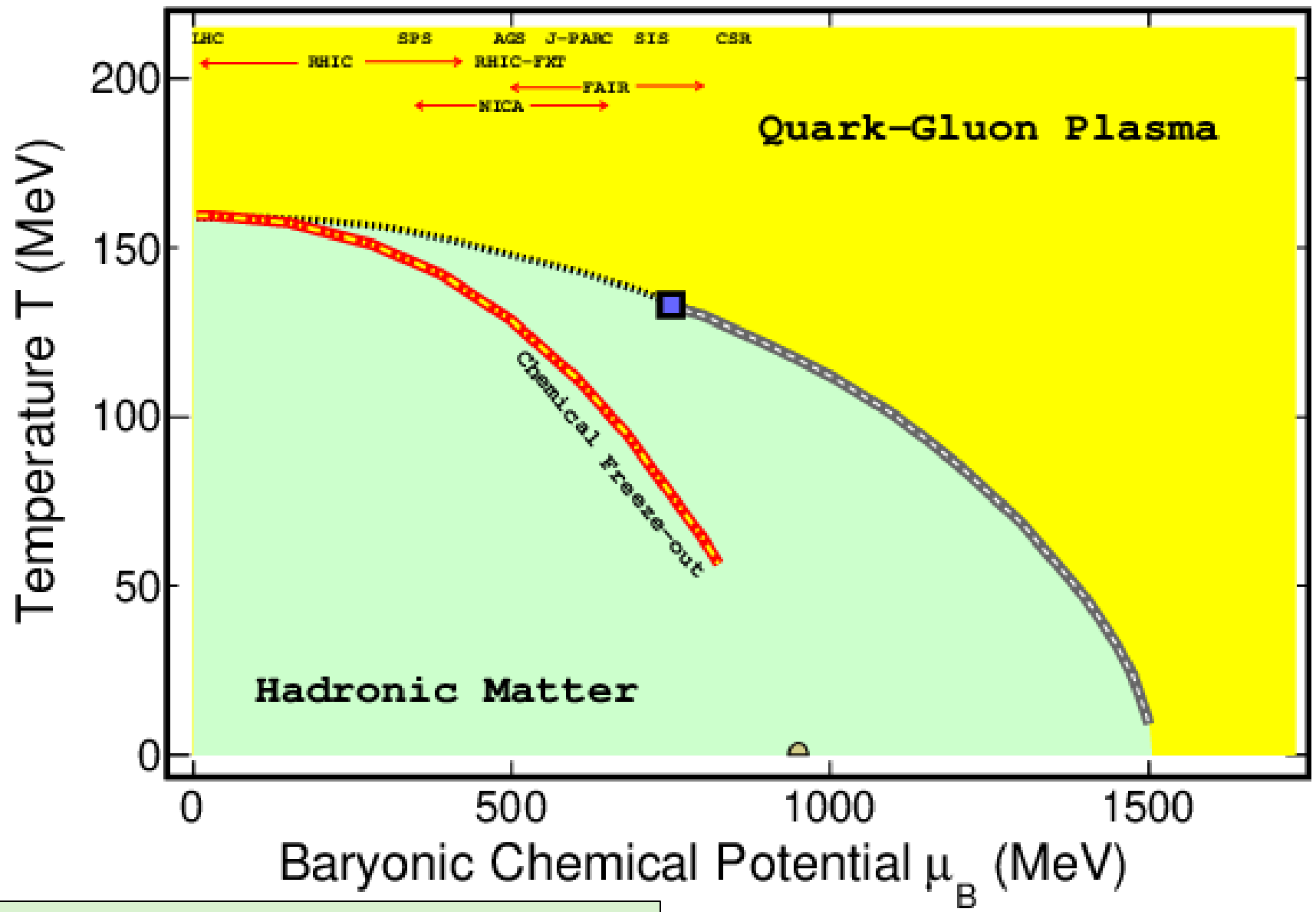
*INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS, 46th Course  
QCD under extreme conditions - present and future  
Erice, Sicily, September 16 – 22, 2025*

A new observable of  
radial flow: Probing  
isotropic expansion of  
QCD matter

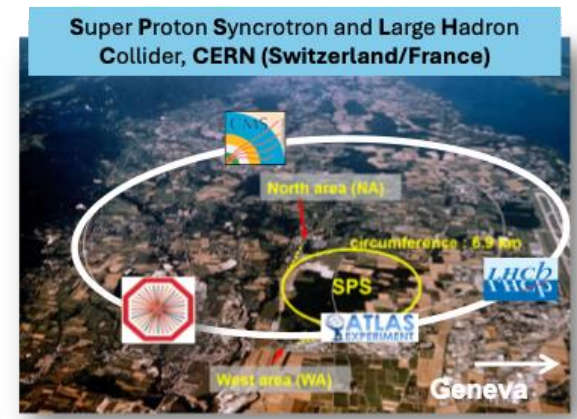
Bedanga Mohanty  
NISER/CERN



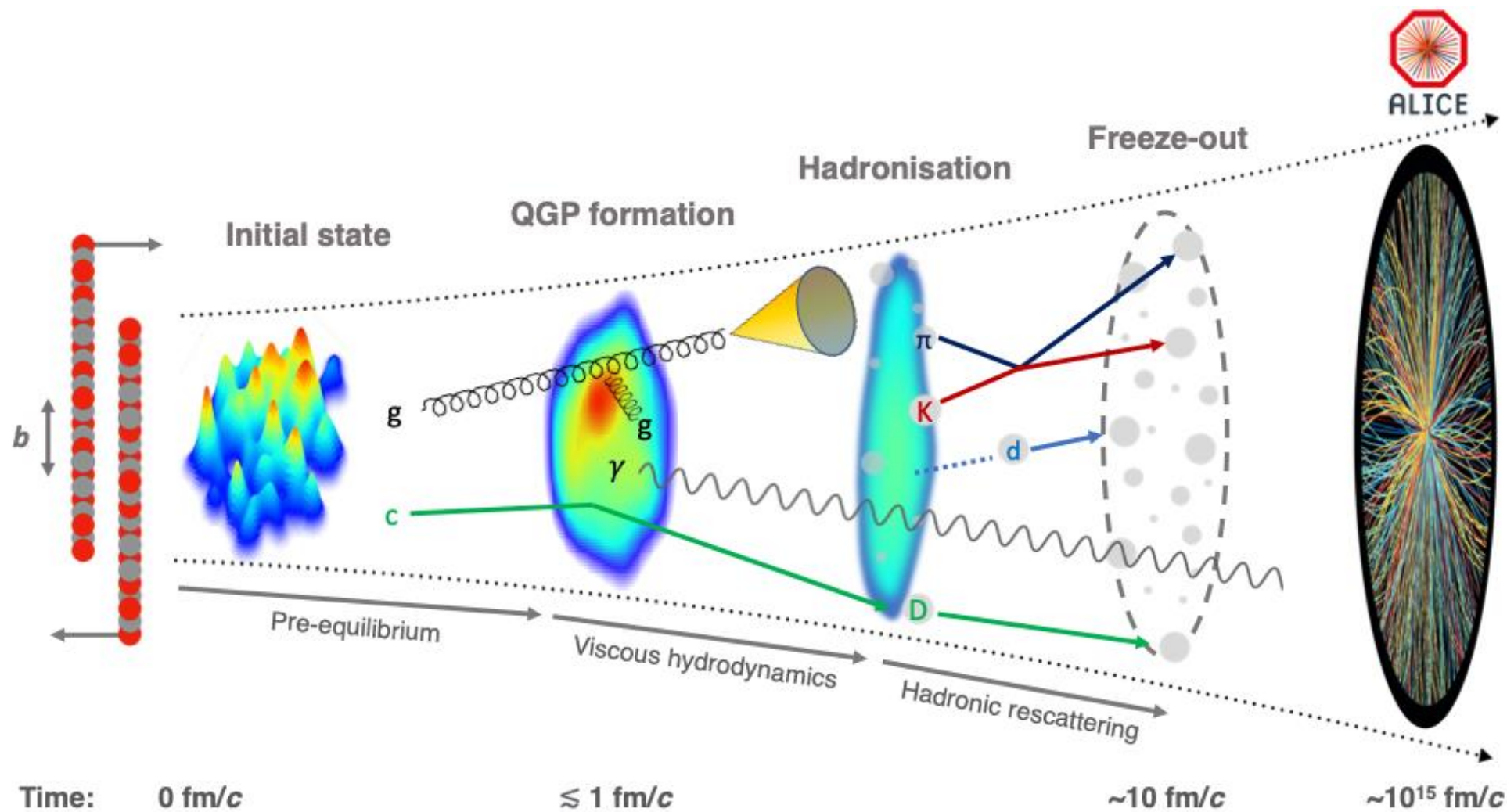
# Relativistic heavy-ion collisions - goals



Understanding the phases of QCD



# Relativistic heavy-ion collisions – time evolution

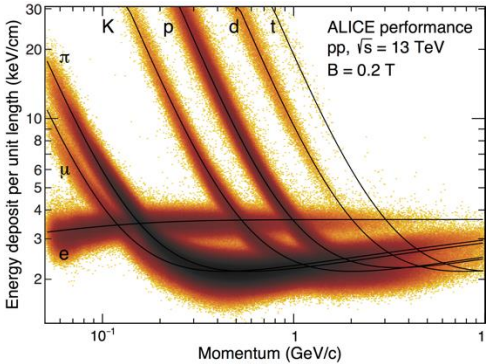
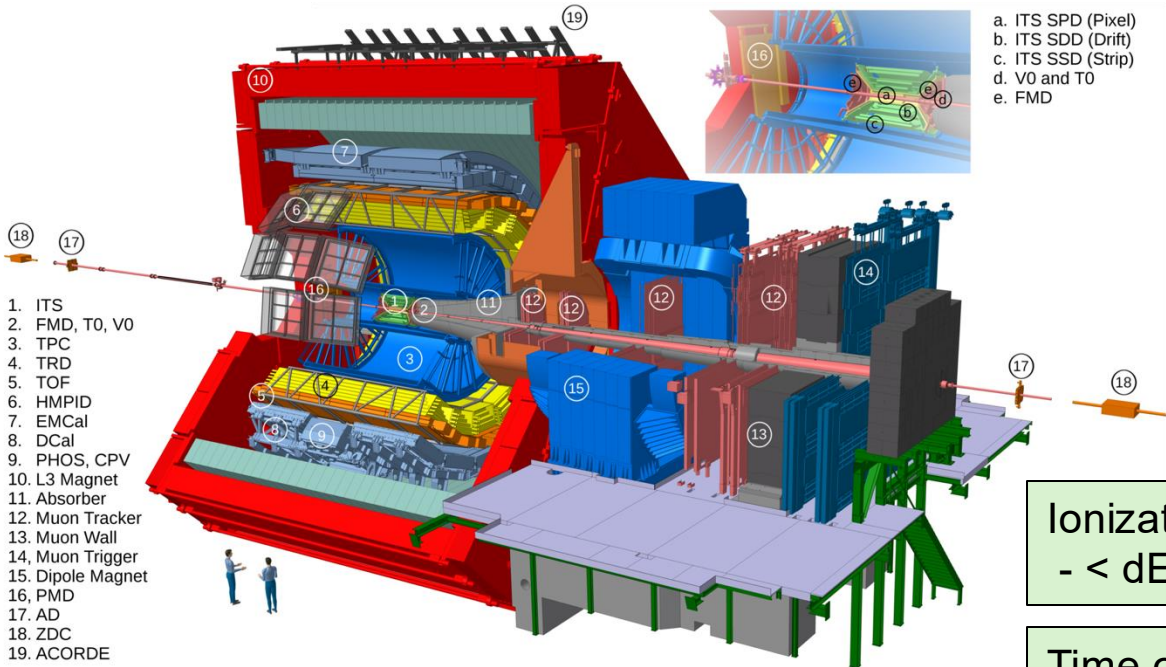


# Relativistic heavy-ion collisions – experiment

ALICE: Eur.Phys.J.C 84 (2024) 8, 813

What we identify

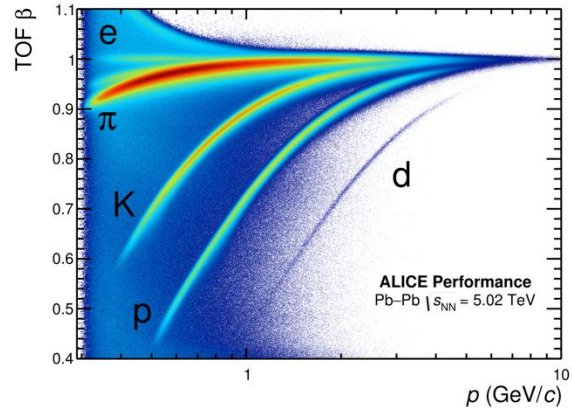
How we identify



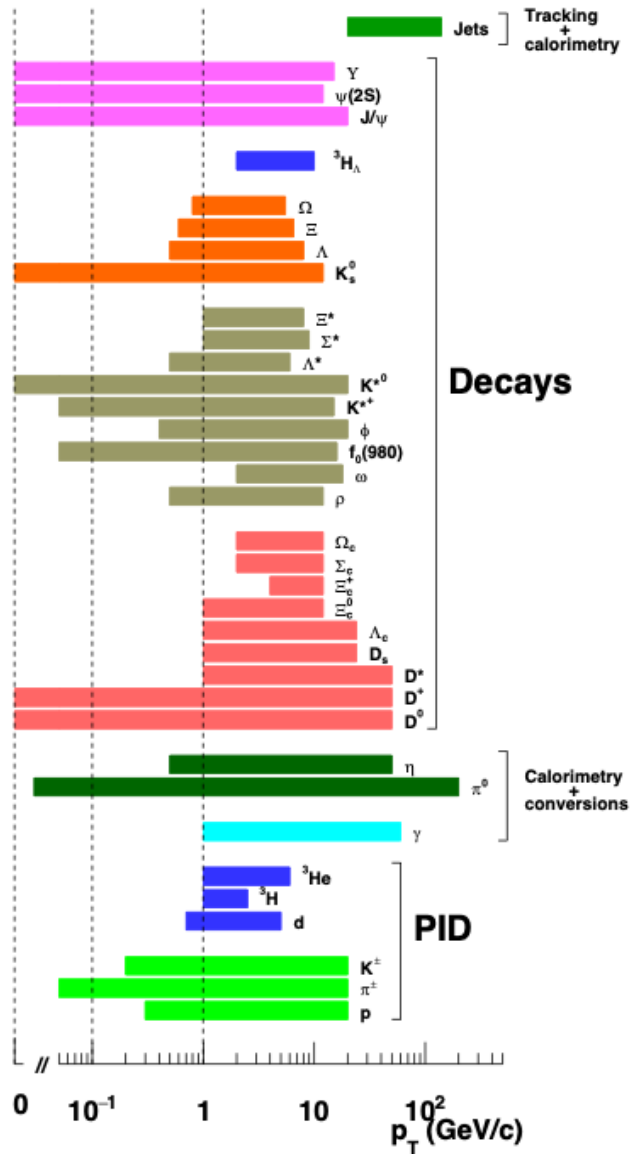
Ionization energy loss:  
 $- \langle dE/dx \rangle \sim A / \beta^2 = A (1 + m^2 / p^2)$

Time of flight:  
 $\langle \tau \rangle = L / \beta = L (1 + m^2 / p^2)^{1/2}$

What we measure (generally):  
1. Position  
2. Momentum  
3. Energy  
4. Multiplicity  
Rest are derived.

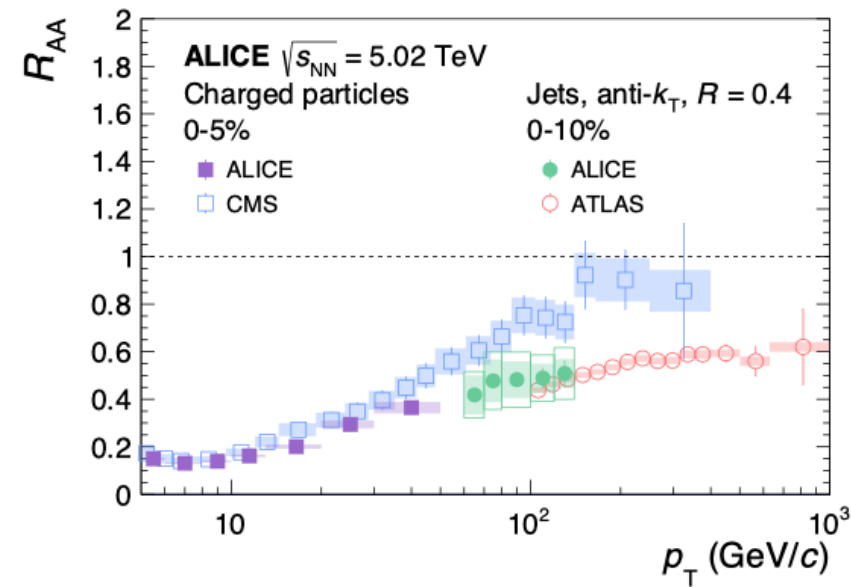
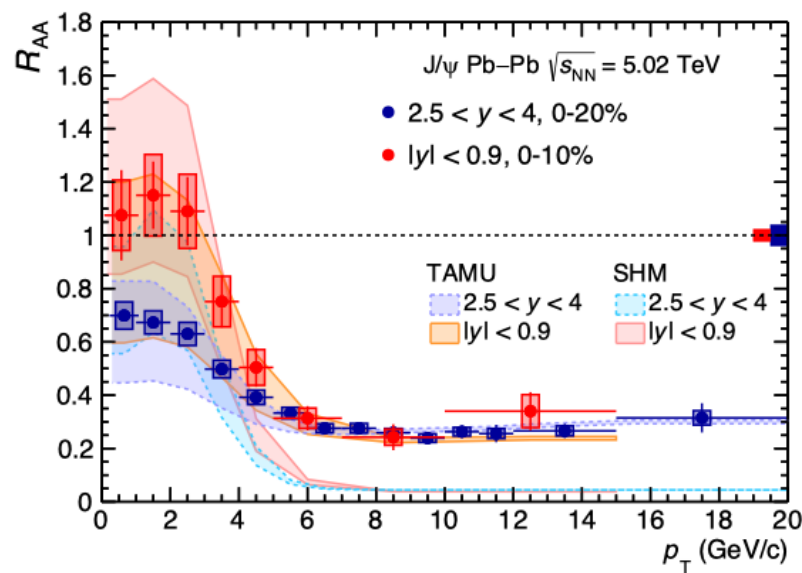
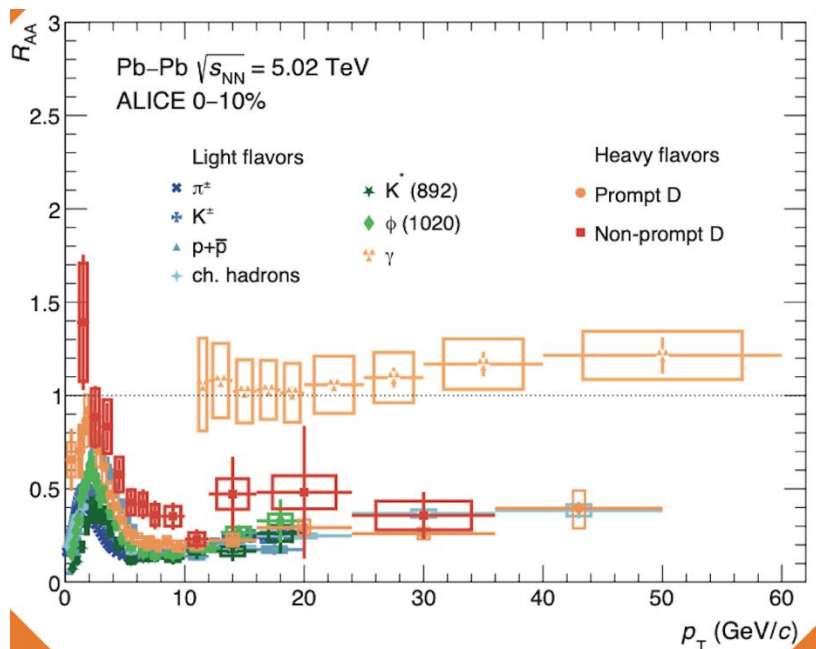


$$m = \sqrt{\left(\sum_i E^i\right)^2 - \left(\sum_i \vec{p}^i\right)^2}$$



# Relativistic heavy-ion collisions – Quark Gluon Plasma

ALICE: Eur.Phys.J.C 84 (2024) 8, 813



$$R_{AA}(p_T) = \frac{1}{\langle T_{AA} \rangle} \frac{dN_{AA}(p_T)/dp_T}{d\sigma_{pp}(p_T)/dp_T}$$

This state of matter is not describable in terms of ordinary color-neutral hadrons, because there is no known self-consistent theory of matter composed of ordinary hadrons at the measured densities.

The most economical description of this matter is in terms of the underlying quark and gluon degrees of freedom

# Collectivity and plasma

## QED Plasma Physics

1. A system of charged particles is called a **plasma** when collective, long-range interactions dominate over independent, binary collisions.
2. Key parameter: number of particles in a Debye sphere. If  $N_D \gg 1$ , many particles interact collectively  $\rightarrow$  true plasma. If  $N_D \sim 1$ , the system behaves more like a neutral gas.
3. Collective modes (e.g., plasma oscillations, screening, instabilities) are the defining feature.
4. In *electromagnetic plasmas*, **collectivity is one of the essential criterion** distinguishing plasma from an ordinary gas.

## Quark–Gluon Plasma (QGP)

1. The QGP is not just a collection of free quarks and gluons — **it must exhibit collective behavior** driven by color interactions.
2. Without collectivity, one could not distinguish QGP from a dilute gas of free quarks/gluons produced incoherently.

## Collectivity Matters in Defining a Plasma

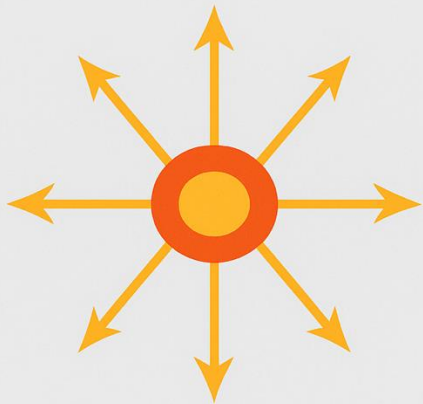
1. **Necessary condition:** Collectivity ensures that interactions are not just local but correlated across many particles.
2. **Defines plasma state:** Both in electromagnetic and QCD contexts, plasma means *collective modes dominate*.
3. **Diagnostic power:** **Observing collective behavior** (oscillations in EM plasma, hydrodynamic flow in QGP) **is one way how we confirm plasma formation in practice.**

# Different kinds of flow in heavy-ion collisions

## Radial Flow

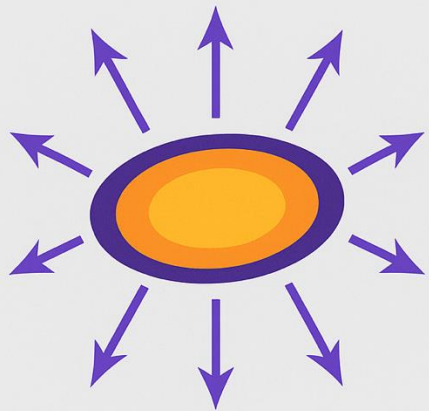
1. Isotropic expansion due to pressure gradients.
2. Pushes heavier hadrons to higher  $p_T$ .
3. Seen in transverse momentum spectra.

### Radial Flow



Isotropic outward expansion in the transverse plane

### Anisotropic Flow



Direction-dependent expansion due to asymmetries in initial overlap geometry

## Anisotropic Flow (Fourier Harmonics)

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\phi - \Psi_n)]$$

### 1. $v_1$ :Directed Flow

Sideward deflection, sensitive to early dynamics & EoS.

### 2. $v_2$ :Elliptic Flow

Almond-shaped geometry  $\rightarrow$  strong QGP signal.

Hallmark of nearly perfect fluid.

### 3. $v_3$ :Triangular Flow

Originates from event-by-event fluctuations.

Reveals initial-state granularity.

### 4. Higher Harmonics ( $v_4, v_5, \dots$ )

Nonlinear mode coupling, constrain viscosity.

# Flow and transport properties

1. Flow observables are the *experimental bridge* between initial geometry and transport properties.
2. By measuring radial + anisotropic flows across energies, centralities, and particle species, we can extract QGP's key transport coefficients:
  - a) **Shear viscosity** ( $\eta/s$ )
  - b) **Bulk viscosity** ( $\zeta/s$ )
  - c) **Diffusion coefficients** (light & heavy quarks, baryon number, etc.)
  - d) **Conductivity**

This is why flow is often called a **viscometer of the QGP**.

**Shear viscosity** ( $\eta$ )  $\rightarrow$  velocity gradients (momentum transport). Suppresses anisotropic flow  $v_n$

$$\pi_{\text{shear}}^{ij} = -\eta (\partial^i u^j + \partial^j u^i - \frac{2}{3} \delta^{ij} \nabla \cdot u)$$

**Bulk viscosity** ( $\zeta$ )  $\rightarrow$  expansion/compression gradients (pressure/volume). Controls isotropic (radial) flow.

$$\pi_{\text{bulk}}^{ij} = -\zeta \delta^{ij} (\nabla \cdot u)$$

**Diffusion** (**D**)  $\rightarrow$  chemical potential gradients (charge transport).  $D_s$  controls how heavy quarks couple to the medium.

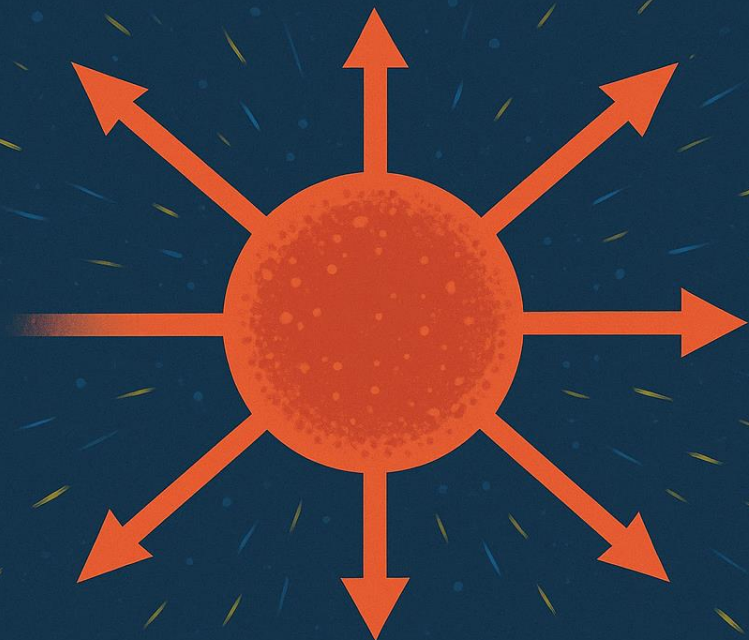
$$j_a^i = -D_{ab} \nabla^i \left( \frac{\mu_b}{T} \right)$$

**Conductivity** ( $\sigma$ )  $\rightarrow$  electromagnetic field gradients. electrical conductivity affects photon and dilepton emission.


$$j^i = \sigma E^i$$

# RADIAL FLOW


## IN A HEAVY-ION COLLISION



### 1 SPEED OF SOUND

 higher  $c_s$  →  
 →stronger  
 radial flow

### 2 BULK VISCOSITY

 larger  $\zeta$   
 weaker  
 radial flow

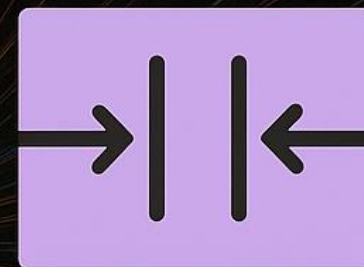
# RADIAL FLOW

## DEPENDS ON



### SPEED OF SOUND

FASTER  
PRESSURE  
WAVES



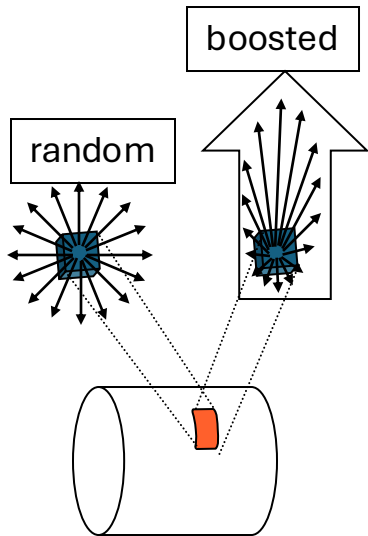
### BULK VISCOSITY

RESISTANCE  
TO EXPANSION

# Standard ways to estimate radial flow in experiments

E. Schnedermann, J. Sollfrank, and U. W. Heinz, Phys. Rev. C 48, 2462 (1993).

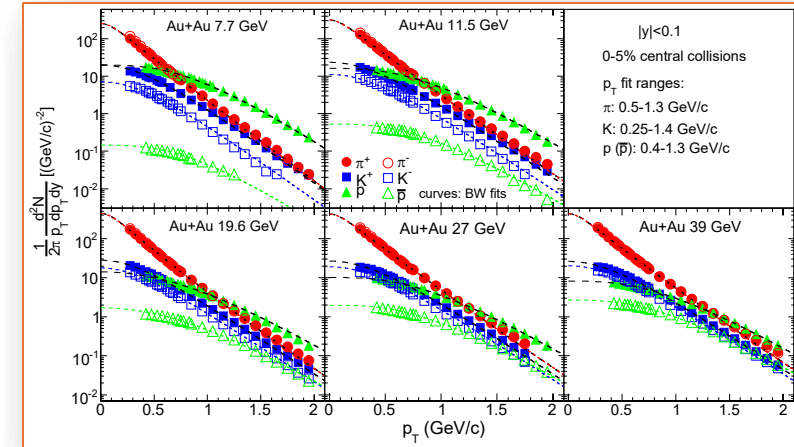
## Blast Wave Model



$$E \frac{d^3 N}{dp^3} \propto \int_{\sigma} e^{-(u^\mu p_\mu)/T_{fo}} p d\sigma_\mu$$

$$\frac{dN}{m_T dm_T} \propto \int_0^R r dr m_T K_1 \left[ \frac{m_T \cosh \rho}{T_{fo}} \right] I_0 \left[ \frac{p_T \sinh \rho}{T_{fo}} \right]$$

$$\rho = \tanh^{-1} \beta_T \quad \beta_T = \beta_s \frac{r}{R} \quad \alpha = 0.5, 1, 2$$



STAR: PRC 96 (2017) 044904

$$T \cong T_F + \frac{1}{2} m \langle v_T \rangle^2$$

apparent temperature      true temperature      transv. flow velocity

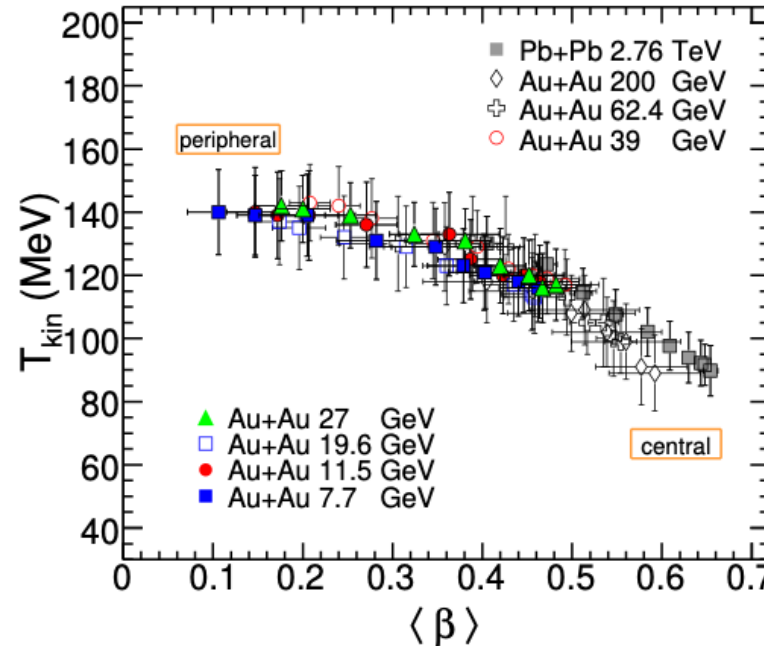
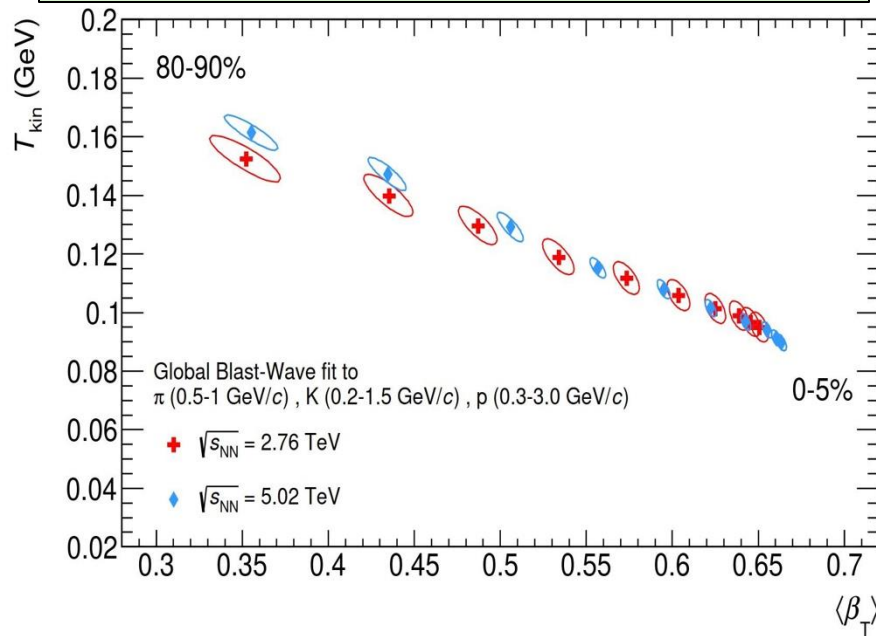
Source is **assumed** to be:

- Locally thermally equilibrated
- Boosted in radial direction

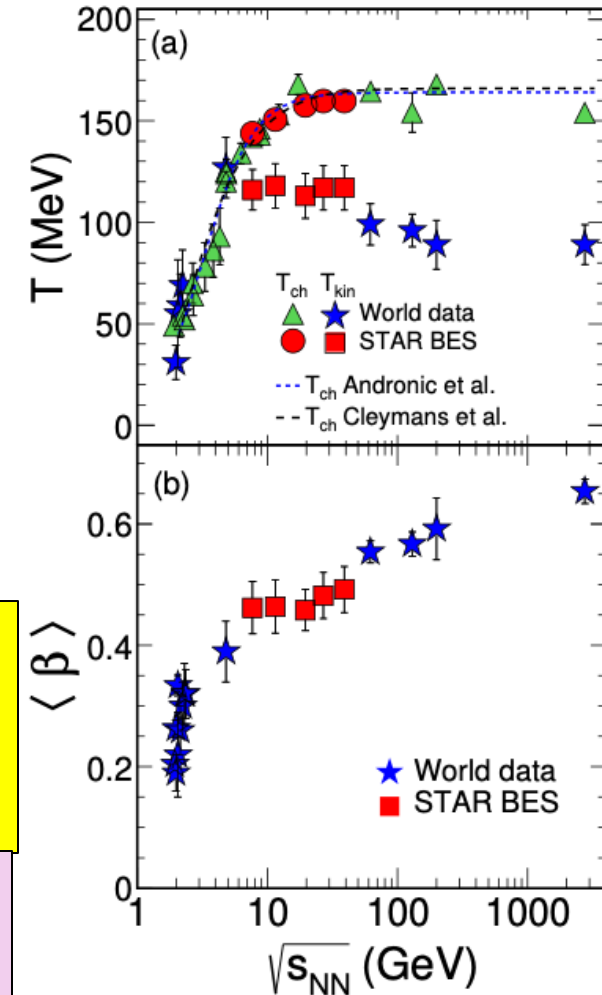
**Characterized** by:  
Thermal temperature  $T_{fo}$   
and velocity parameter  $\langle \beta_T \rangle$

# Radial flow at LHC and RHIC

ALICE: *Phys. Rev. C* 101, 044907 (2020)



STAR: arXiv: 1701.07065



1. Radial flow ( $p_T$  integrated) obtained from momentum distributions.
2. Temperature and radial flow anti-correlated.
3. Radial flow increases with collision energy and collision centrality.

- A. No scope to study differential radial flow.
- B. Observations seen in anisotropic flow cannot be explored.
- C. Non-flow estimates not available or not sure how to suppress.
- D. No concept of radial flow fluctuations event-by-event.

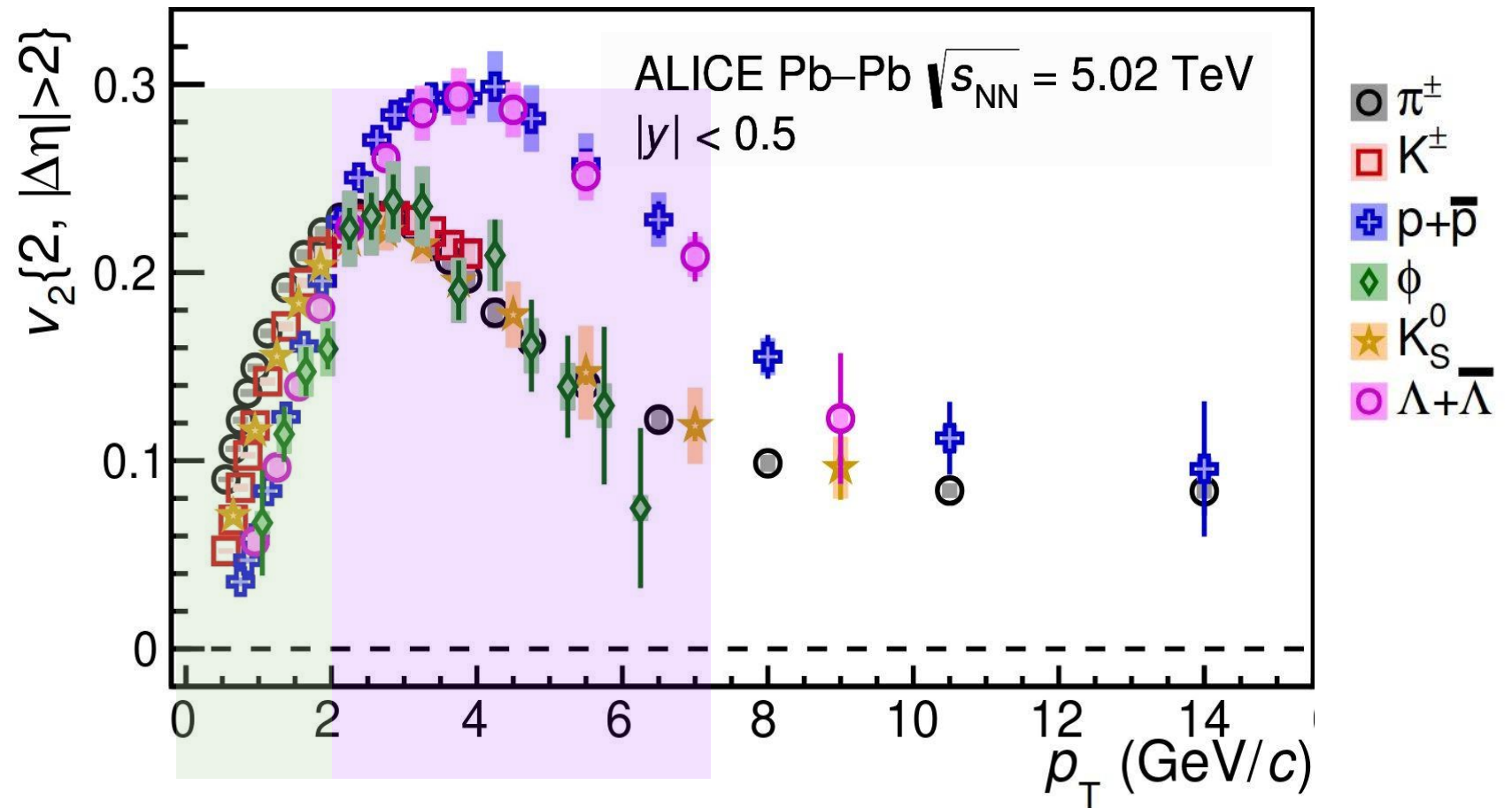
# Anisotropic flow observations

$$v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Low- $p_T$ : Mass ordering  
(hydrodynamics)

Intermediate- $p_T$ :  
Baryon-meson  
splitting (quark  
recombination)

ALICE: JHEP09 (2018) 006



Does isotropic expansion/radial flow show these features ?

Was not possible till now as  $p_T$  differential radial flow observable did not exist.

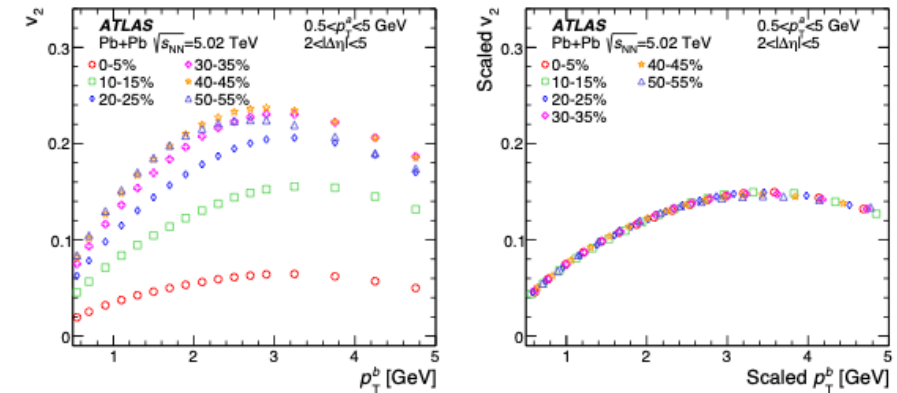
# Anisotropic flow observations

$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

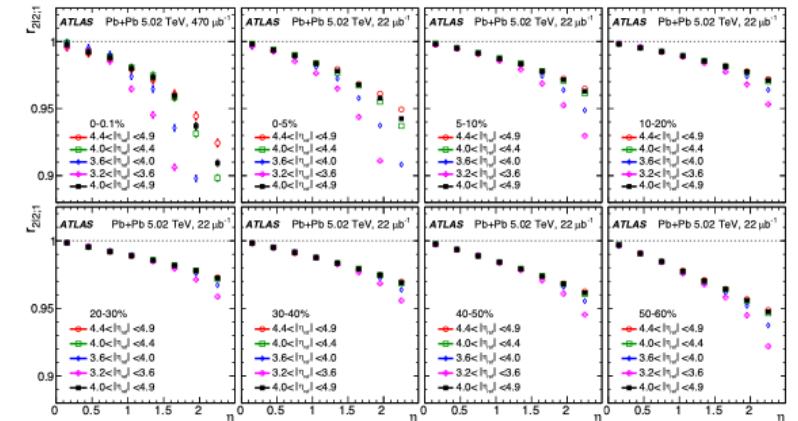
## Collective origins of anisotropic flow ( $v_2$ )

1. Centrality-independent scaling (not just artifact of initial geometry)
2. Factorization in transverse momentum (consistent with a collective flow field rather than local non-flow)
3. Long-range correlations in pseudorapidity (excludes short-range non-flow sources (like jets or resonance decays))

ATLAS: Eur. Phys. J. C 78 (2018) 997



ATLAS: Eur. Phys. J. C 78 (2018) 142



Does isotropic expansion/radial flow show these features ?

Was not possible till now as  $p_T$  differential radial flow observable did not exist.

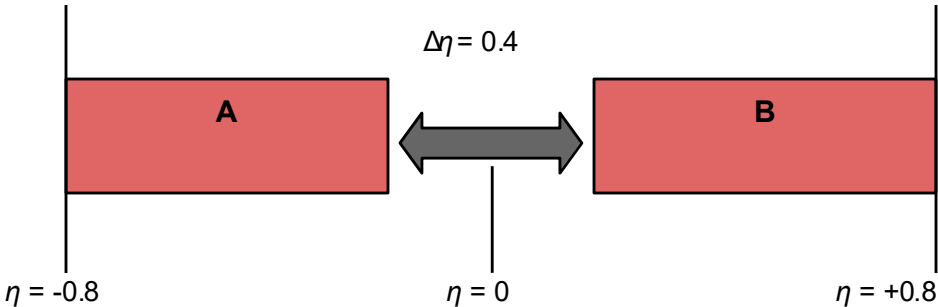
# New radial flow observable

B. Schenke, et al., Phys. Rev. C 102, 034905 (2020)  
T. Parida, et al., Phys. Lett. B 857 (2024) 138985

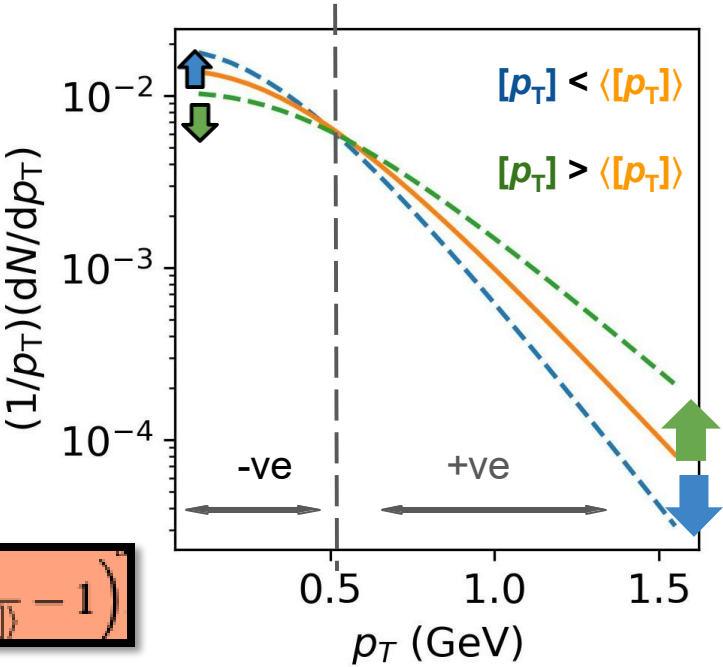
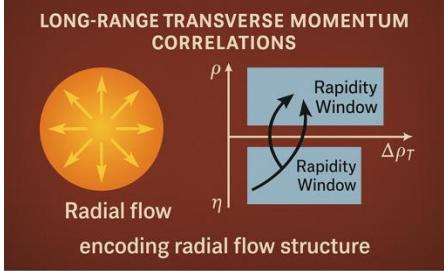
$$v_0(p_T) = \frac{\langle f_A(p_T)[p_T]_B \rangle - \langle f_A(p_T) \rangle \langle [p_T]_B \rangle}{\langle f_A(p_T) \rangle \sigma_{[p_T]}}$$
$$\sigma_{[p_T]} = \sqrt{\langle [p_T]_A [p_T]_B \rangle - \langle [p_T]_A \rangle \langle [p_T]_B \rangle}$$

$f_A(p_T) \rightarrow$  Transverse momentum distribution  
 $[p_T] \rightarrow$  Mean transverse momentum of an event  
 $\langle [p_T] \rangle \rightarrow$  Mean transverse momentum averaged over events

Normalized covariance between the fraction of particles in a  $p_T$ -bin and the event-wise mean  $p_T$ , evaluated using a pseudorapidity ( $\eta$ ) gap.



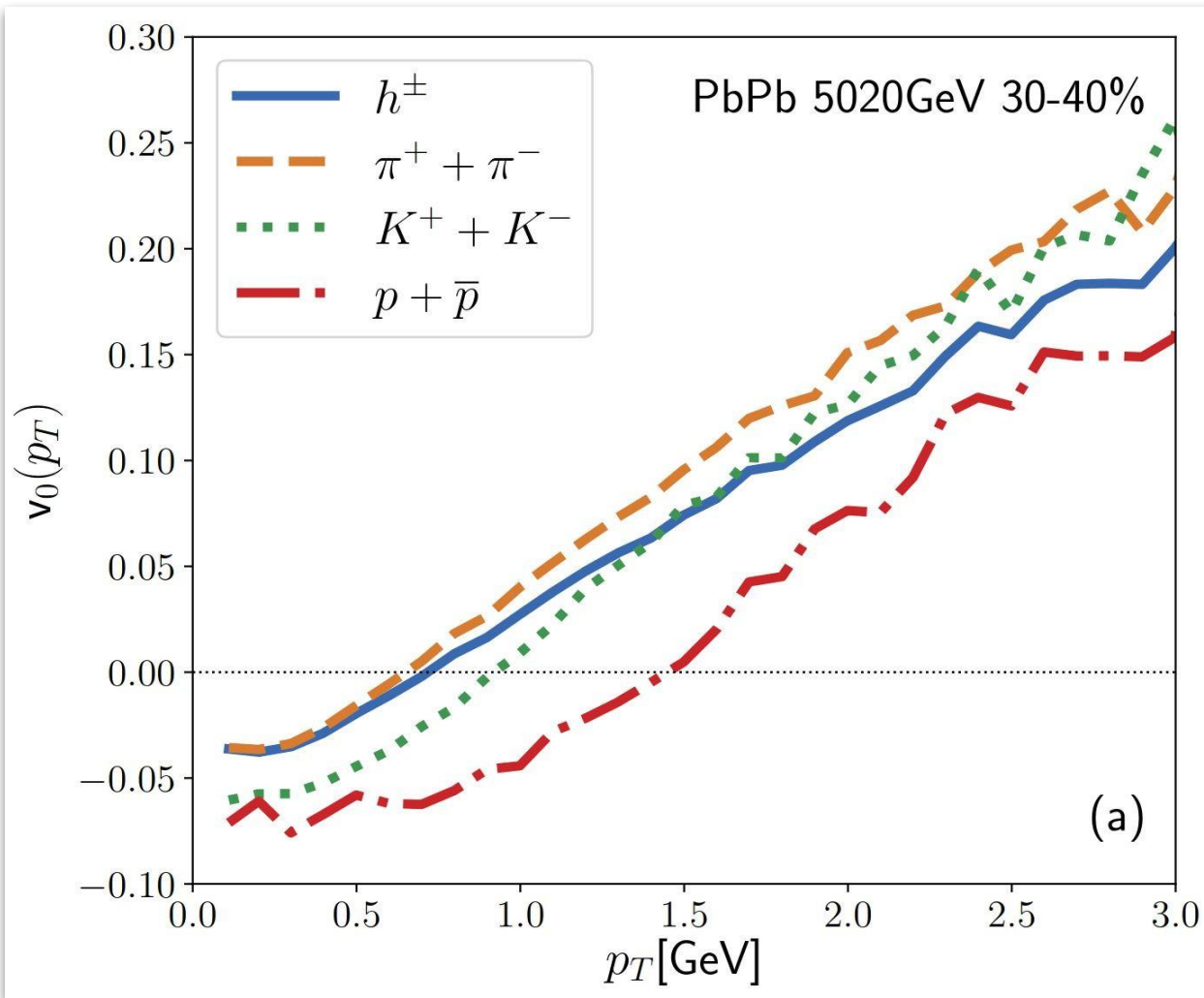
$$v_0(p_T) \approx 2 \frac{\sigma_{[p_T]}}{\langle [p_T] \rangle} \left( \frac{p_T}{\langle [p_T] \rangle} - 1 \right)$$



- 1.  $V_0(p_T)$  increases with  $p_T$ .
- 2. Changes sign around mean  $p_T$ .
- 3. Zero without fluctuations.

- 1. Correlating transverse momentum and multiplicity with rapidity gap.
- 2. Rapidity gap suppresses short range correlations/non-flow.
- 3. Analysis in similar footing as for anisotropic flow; allows differential studies

# New radial flow Observable – hydro-theory



IP Glasma + MUSIC + UrQMD

T. Parida, et al., Phys. Lett. B  
857 (2024) 138985

$$v_0(p_T) \propto \frac{\sigma_{[p_T]}}{\langle [p_T] \rangle} (p_T - m_T v)$$

$v \rightarrow$  fluid velocity

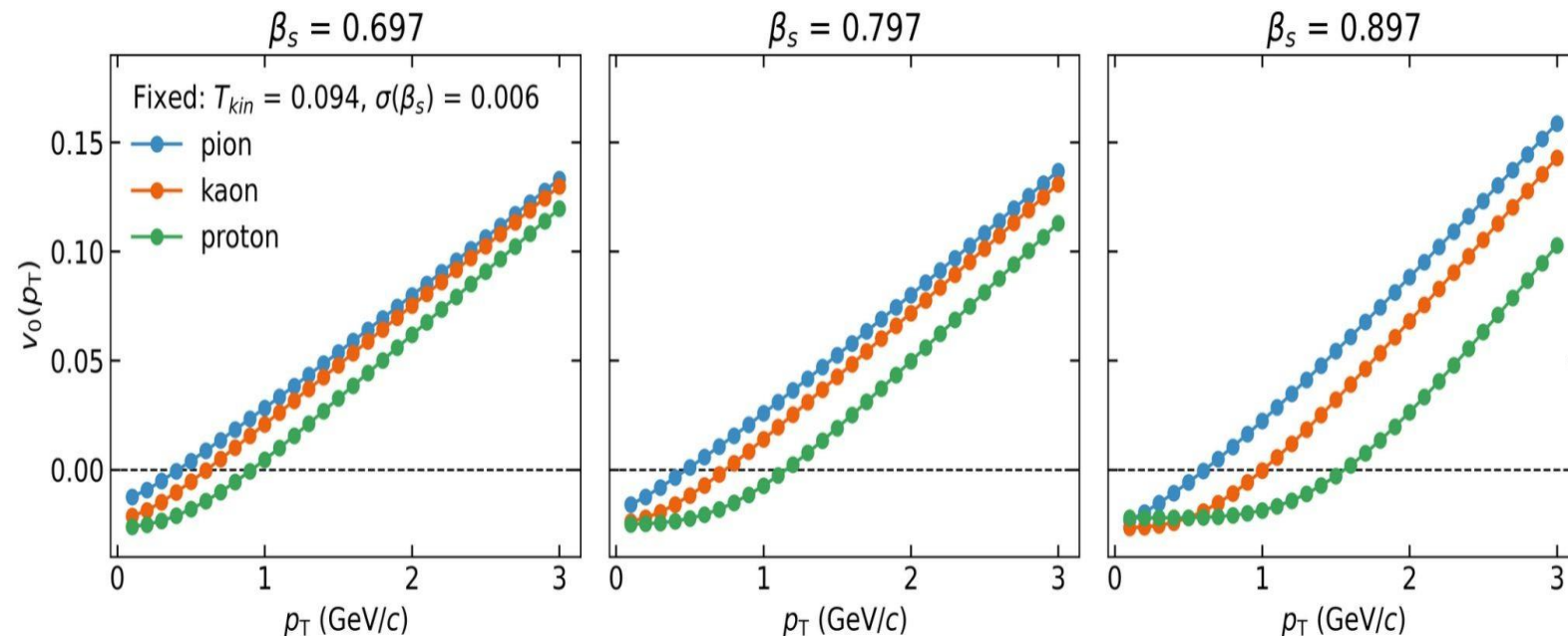
1.  $V_0(p_T)$  increases with  $p_T$
2. Changes sign around mean  $p_T$
3. Zero without fluctuations
4. Mass ordering at low  $p_T$

B. Schenke, et al., Phys. Rev. C 102, 034905 (2020)

# Understanding the observable: linking old to new

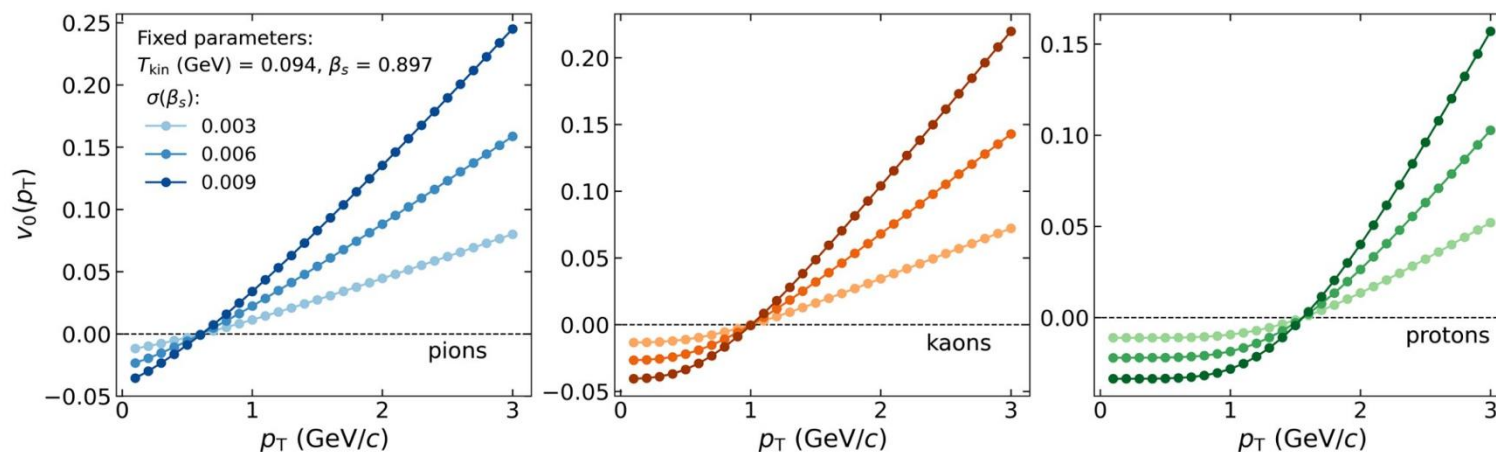
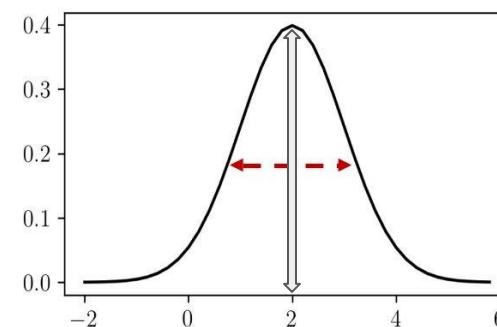
## – via Blast Wave model

S. Saha et al.,  
Phys. Rev. C 112, 024902 (2025)



Higher the radial flow velocity ( $\beta_s$ ), stronger the mass ordering in  $v_0(p_T)$

Fluctuation in  $T_{kin}$  and  $\beta_s$



Without fluctuations in radial flow  $V_0(p_T) \sim 0$

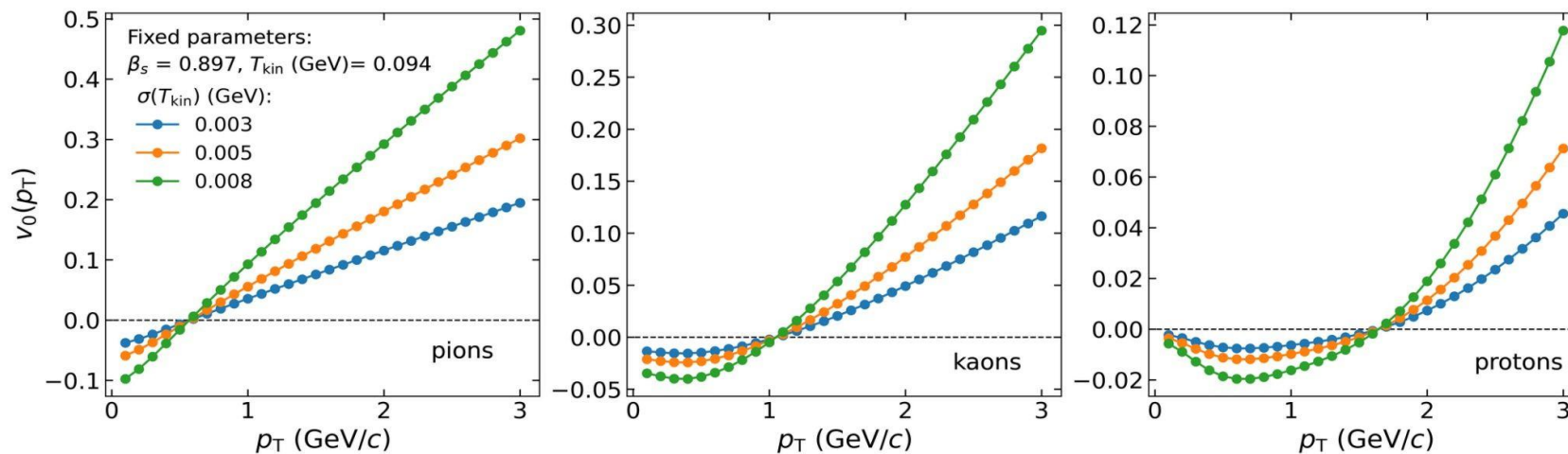
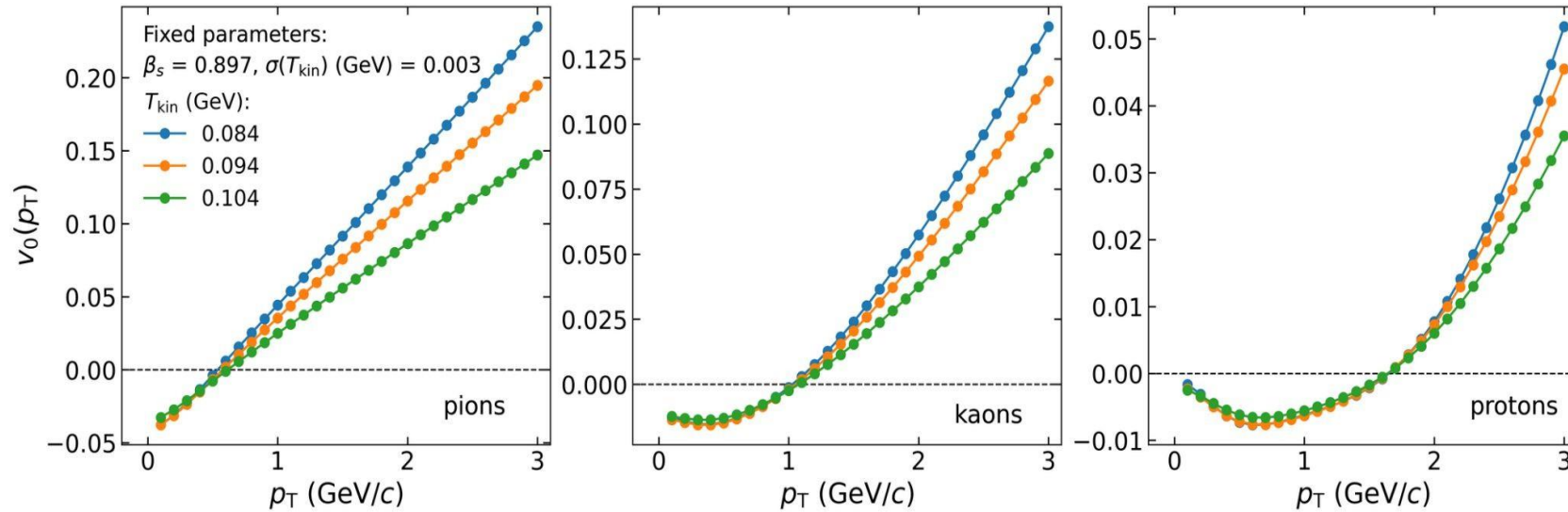
Fluctuations in  $\beta_s$  increases the slope of  $v_0(p_T)$

# Understanding the observable: linking old to new

## – via Blast Wave model

S. Saha et al.,  
Phys. Rev. C 112, 024902 (2025)

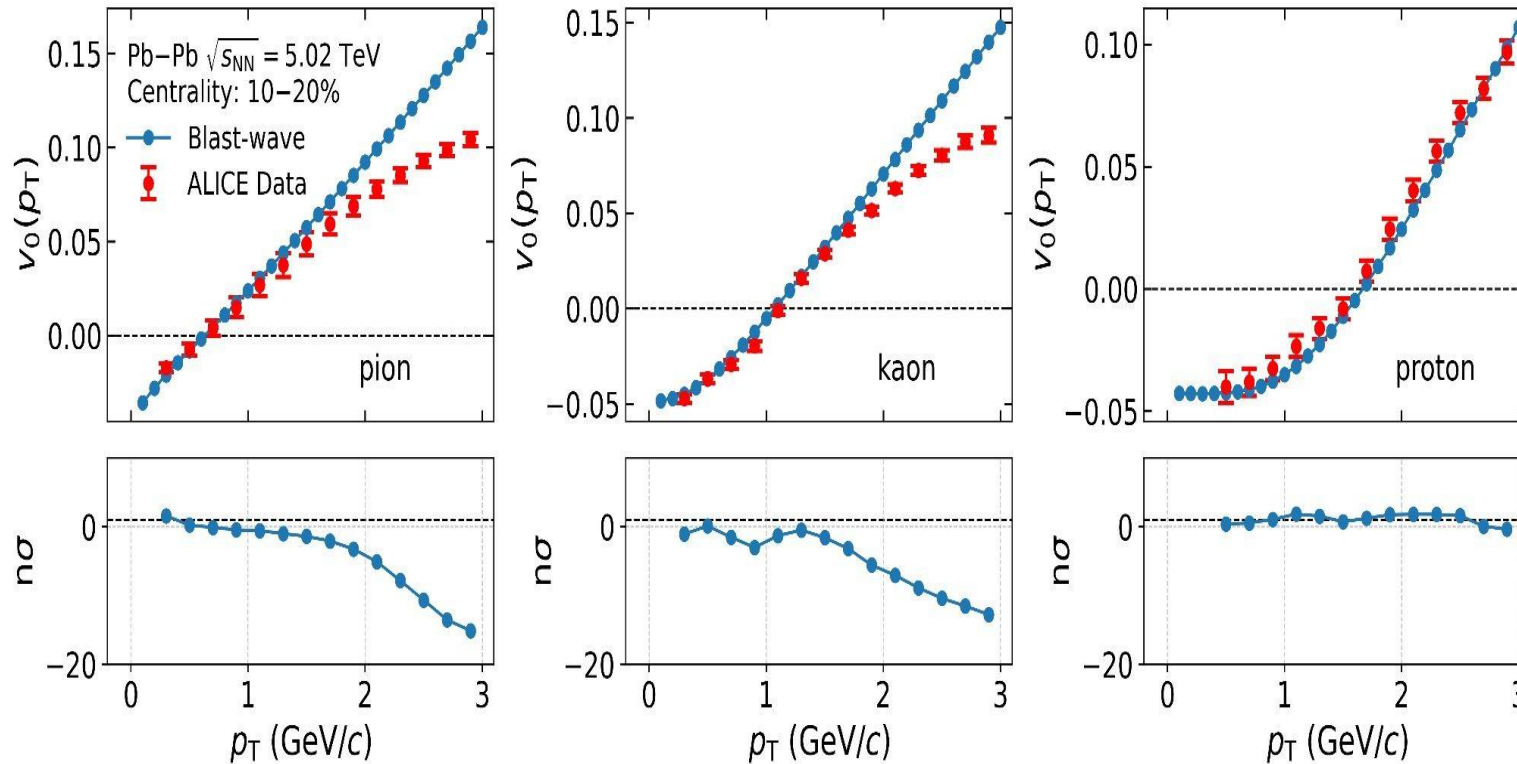
Higher the freeze-out temperature smaller is the slope of  $v_0(p_T)$



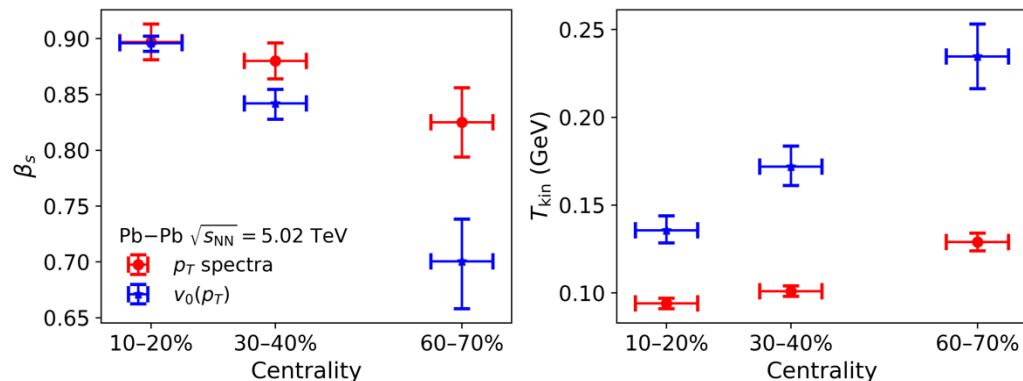
Fluctuations in temperature increases  $v_0(p_T)$

# Understanding the observable: linking old to new – via Blast Wave model

S. Saha et al.,  
Phys. Rev. C 112, 024902 (2025)



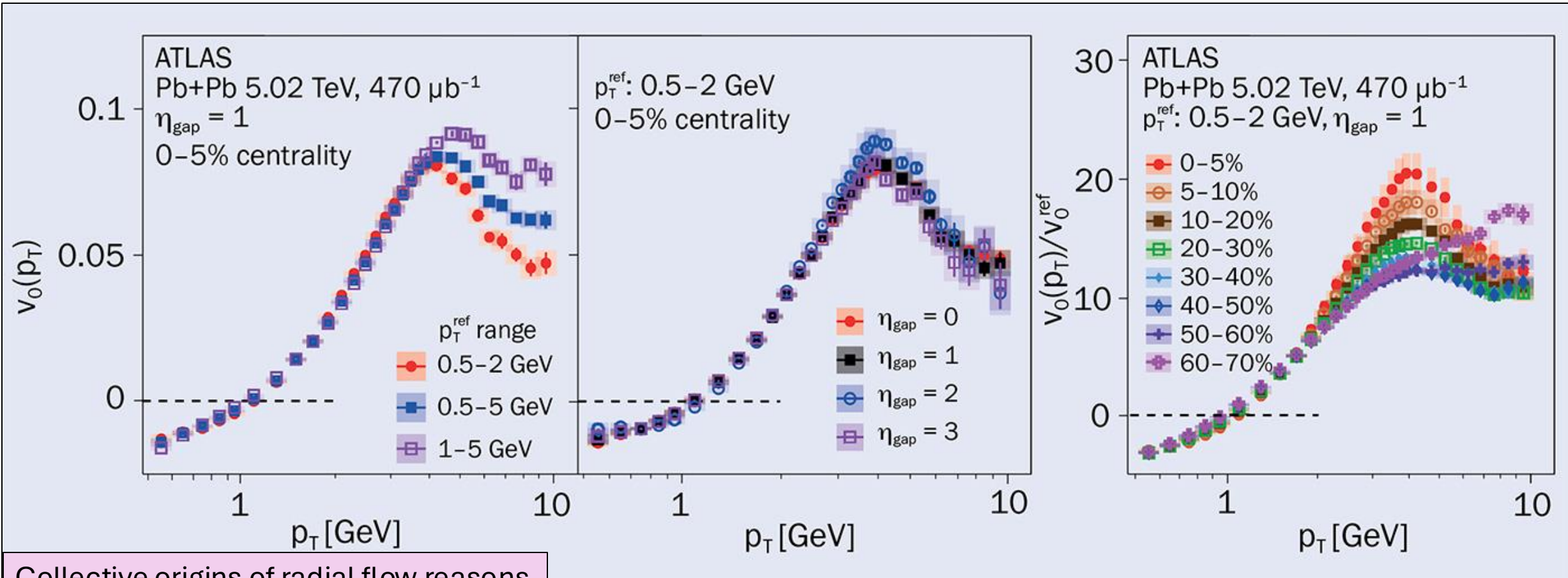
Bayesian analysis of  $V_0(p_T)$  data:  
 $\beta_s = 0.893 \pm 0.005$   
 $\sigma(\beta_s) = 0.0056 \pm 0.0004$   
 and  
 $T_{\text{kin}} = 86.3 \pm 5.3 \text{ MeV}$   
 $\sigma(T_{\text{kin}}) = 0.2 \pm 0.1 \text{ MeV}$



Comparison between radial flow velocity from  $V_0(p_T)$  and momentum spectra

# Collectivity properties (ATLAS)

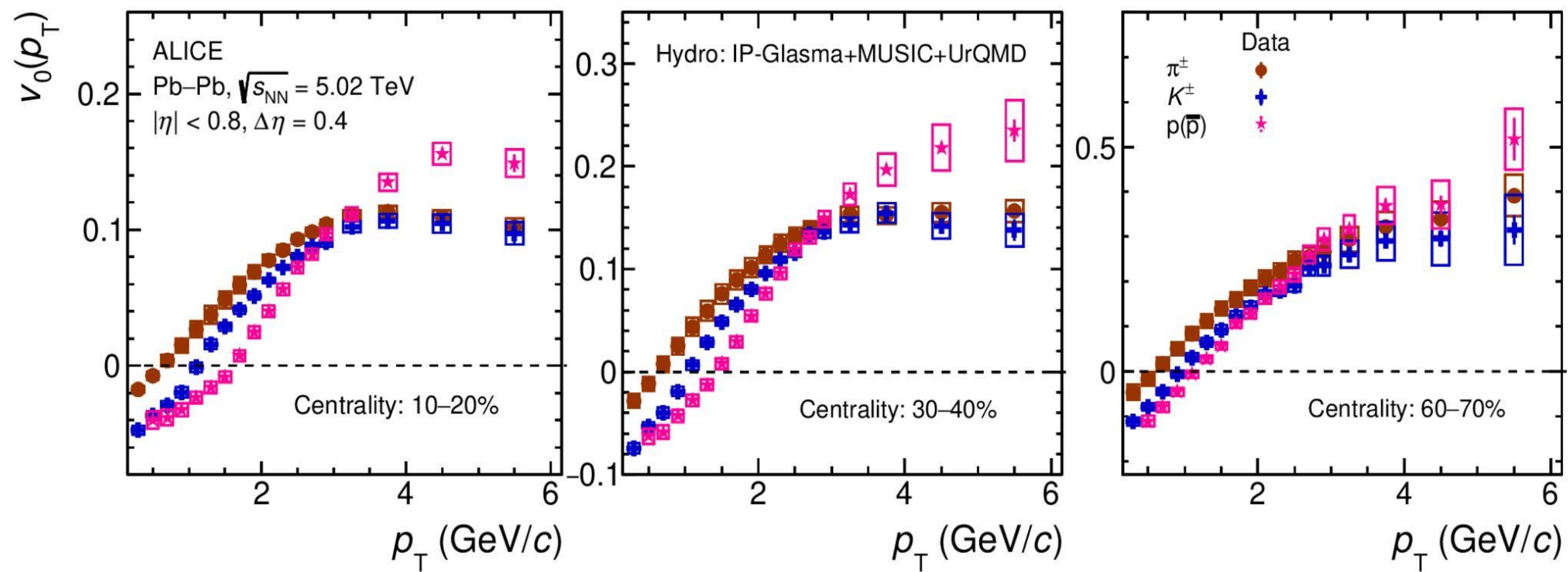
ATLAS: arXiv:2503.24125



Collective origins of radial flow reasons

1. Long-range correlations in 3-units of pseudorapidity (middle): not due to non-flow effects like resonance decays or jets
2. Factorization in transverse momentum (left):  $\langle \delta n(p_T) \delta [p_T] \rangle$  factorizes into  $v_0(p_T) v_0$
3. Centrality-independent shape (right): not just an artifact of initial geometry

# Momentum dependence: mass and baryon-meson ordering

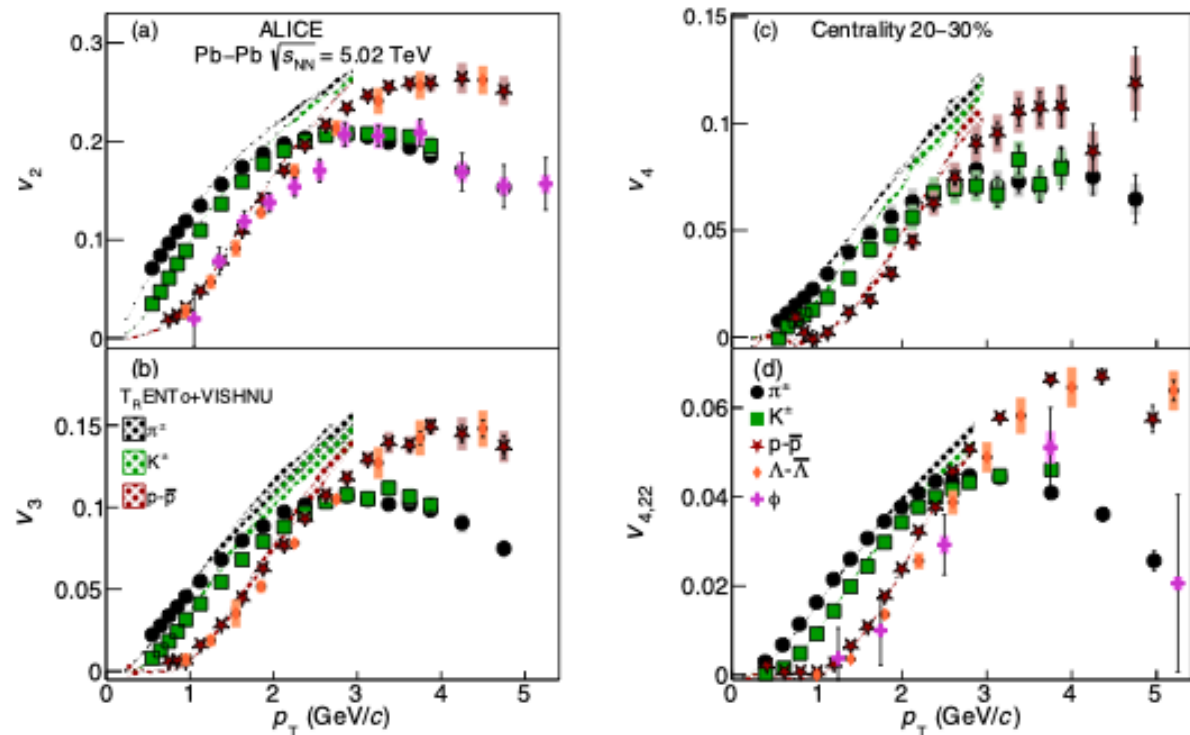


1. Low transverse momentum : Mass ordering - consistent with hydrodynamic expectations.
2. Intermediate transverse momentum: Baryon meson grouping – consistent with coalescence/recombination picture.

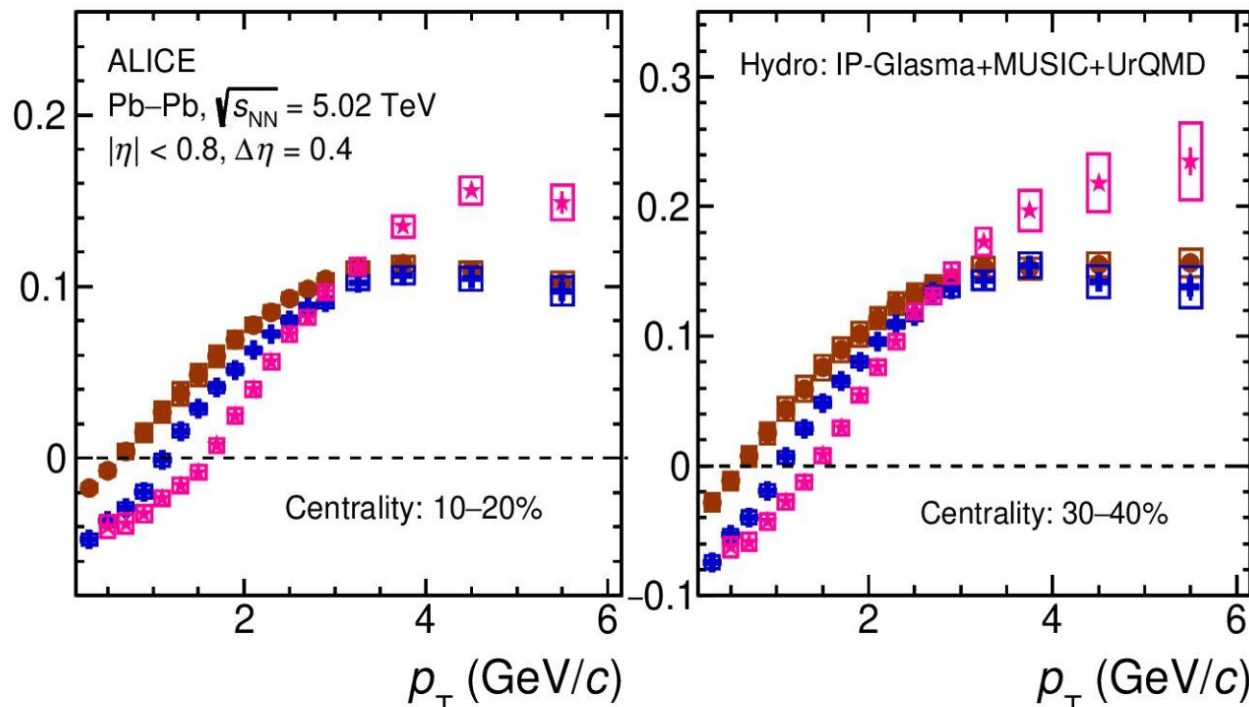
# Momentum dependence: radial flow vs. elliptic flow

ALICE: Eur.Phys.J.C 84 (2024) 8, 813

ALICE: arXiv:2504.04796



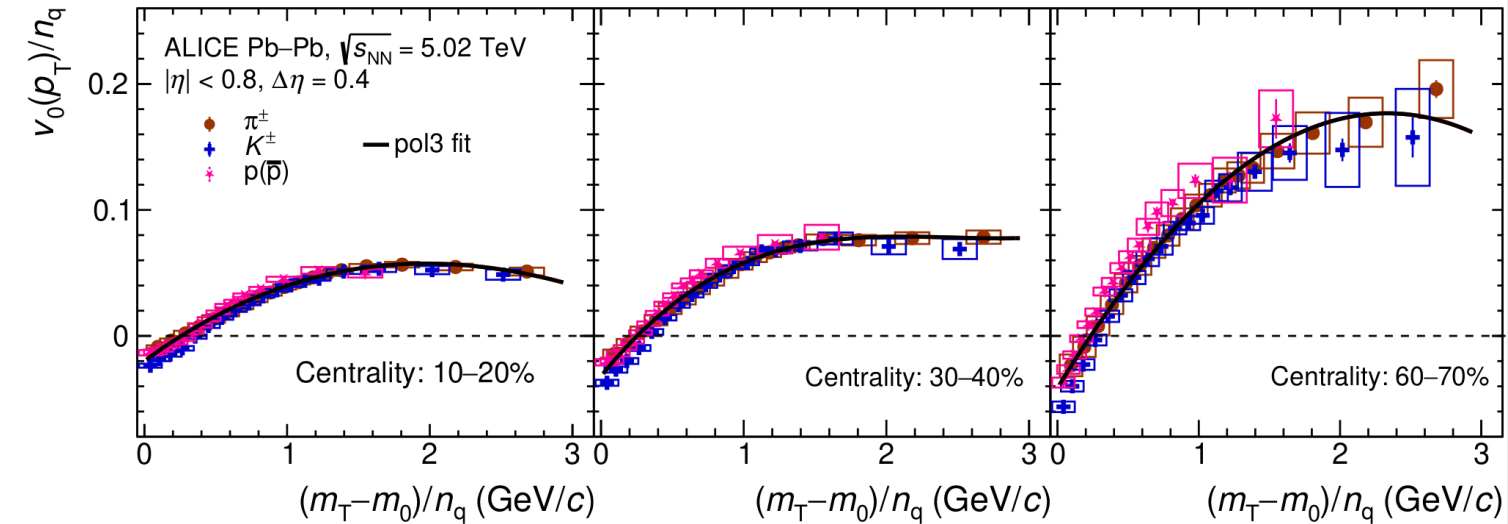
$v_0(p_T)$



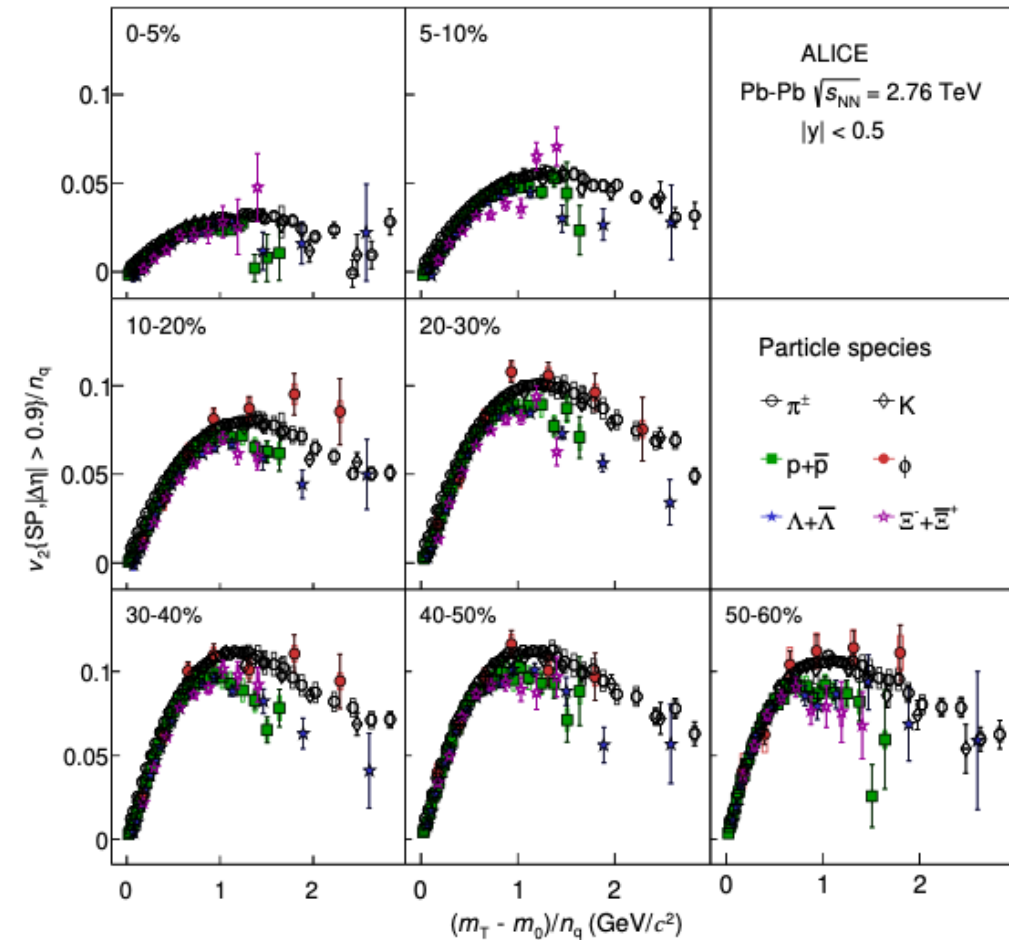
Qualitatively similar features seen for anisotropic flow and isotropic flow.

# Number of Constituent Quark (NCQ) scaling

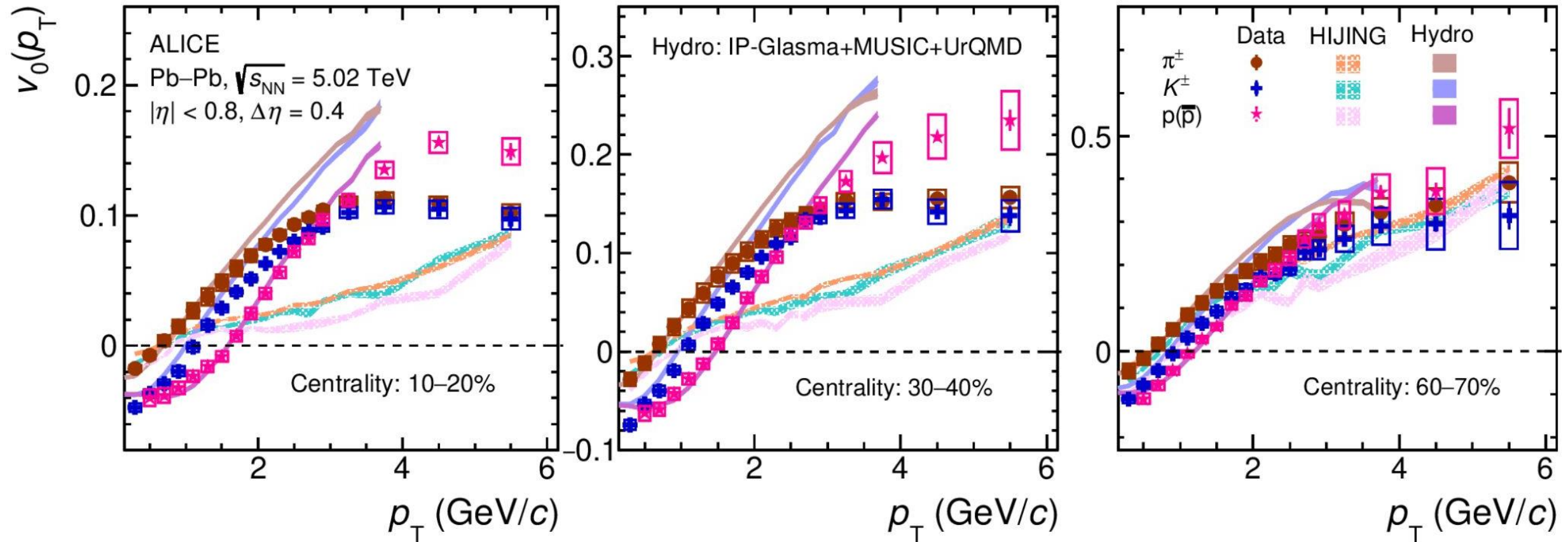
ALICE: JHEP 06 (2015) 190



NCQ scaling observed for radial flow. Features similar between anisotropic flow and isotropic flow.

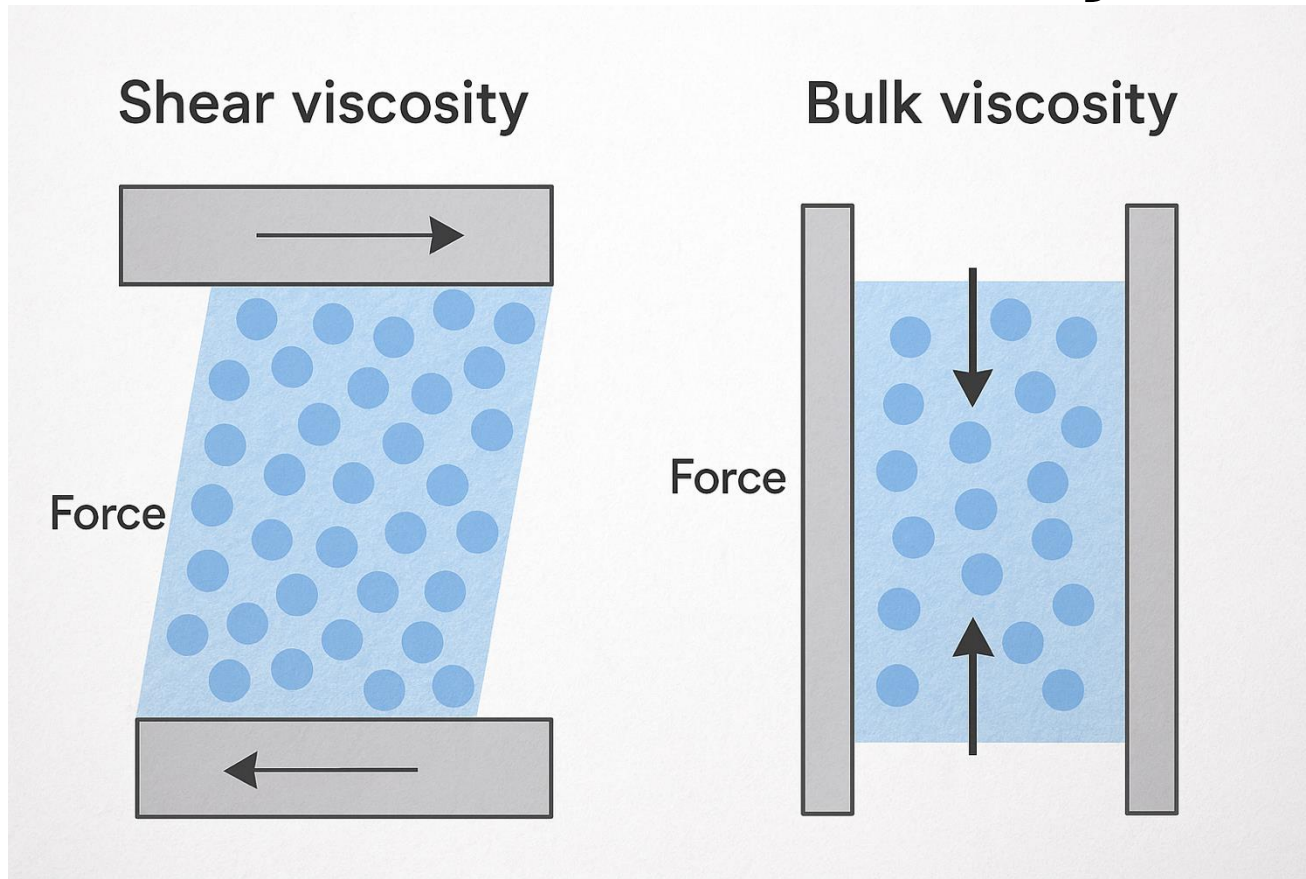


# Momentum dependence: hydrodynamics vs. non-hydrodynamics expectations



1. Hydrodynamic model: Quantitatively captures the features of data at low transverse momentum and across various centrality
2. Non-hydrodynamic model (HIJING) : Fails to capture quantitatively and qualitatively the features of data for central collisions.

# Shear viscosity vs. Bulk viscosity



Shear viscosity: tangential force  
(sliding layers).

Bulk viscosity: normal force  
(compression/ expansion).

Dilute gas,  $\eta = (1/3) npl$ .  
*Uncertainty principle*  $pl \gtrsim \hbar$ .  
 Entropy density,  $s \sim k_B n$ ,  
 Lower bound to  $\eta/s \gtrsim \frac{\hbar}{k_B}$ .

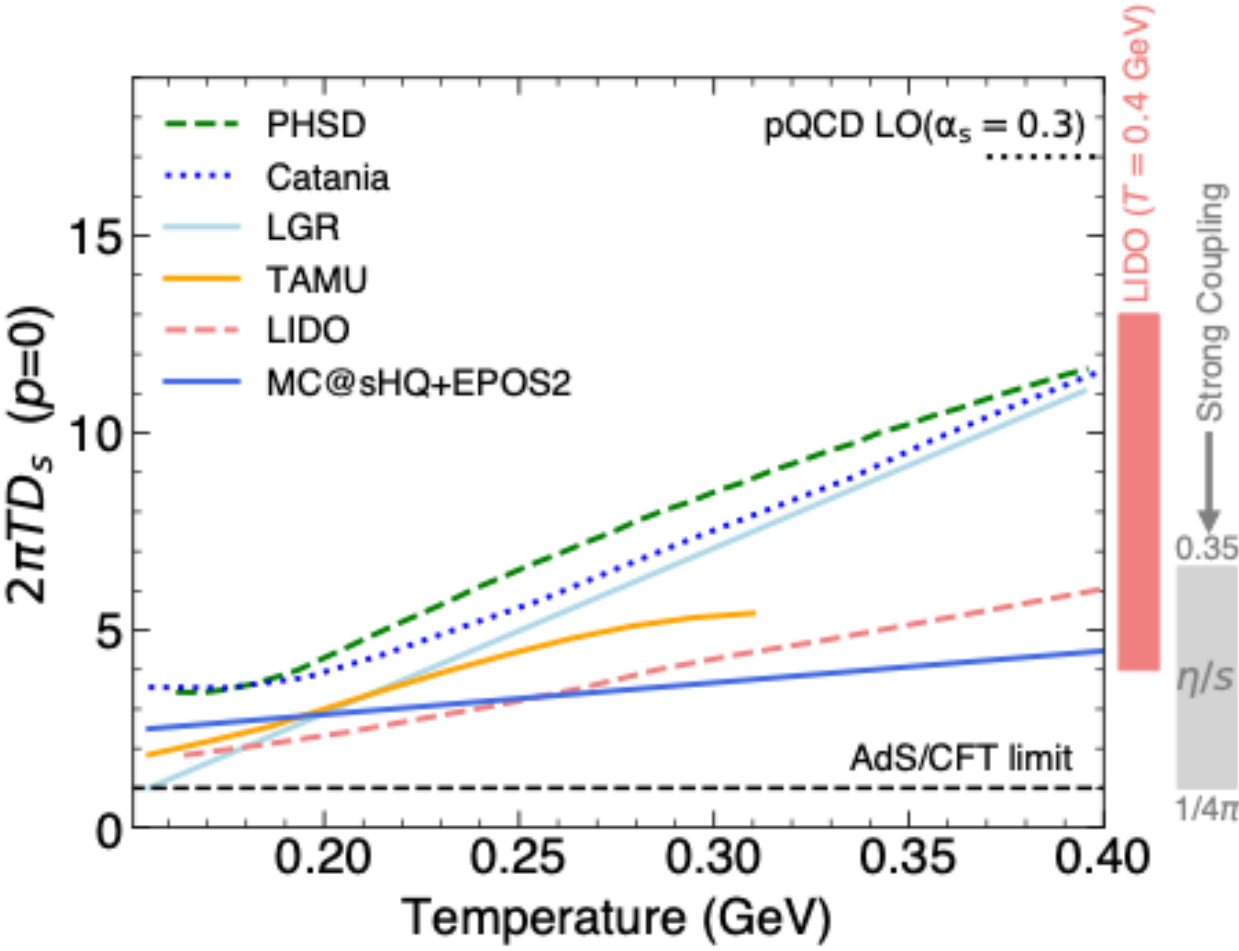
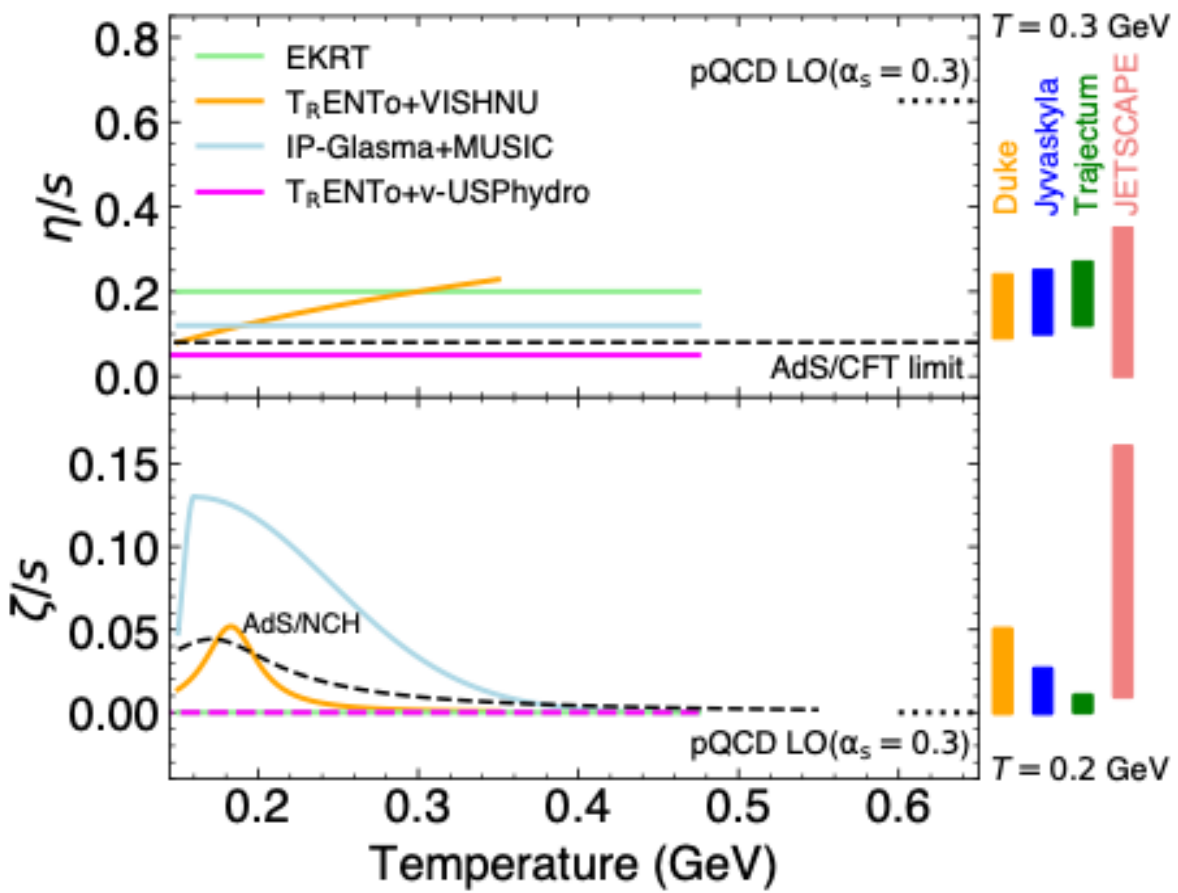
Kovtun, Son, and Starinets  
 (KSS bound)  $\eta/s \geq \frac{\hbar}{4\pi k_B} = 1/4\pi$ .

Natural principles  
 kinematic viscosity  $\eta/s \geq 1/4\pi$ .

P. Kovtun, D. T. Son, A. O. Starinets,  
*Phys.Rev.Lett.* 94 (2005) 111601

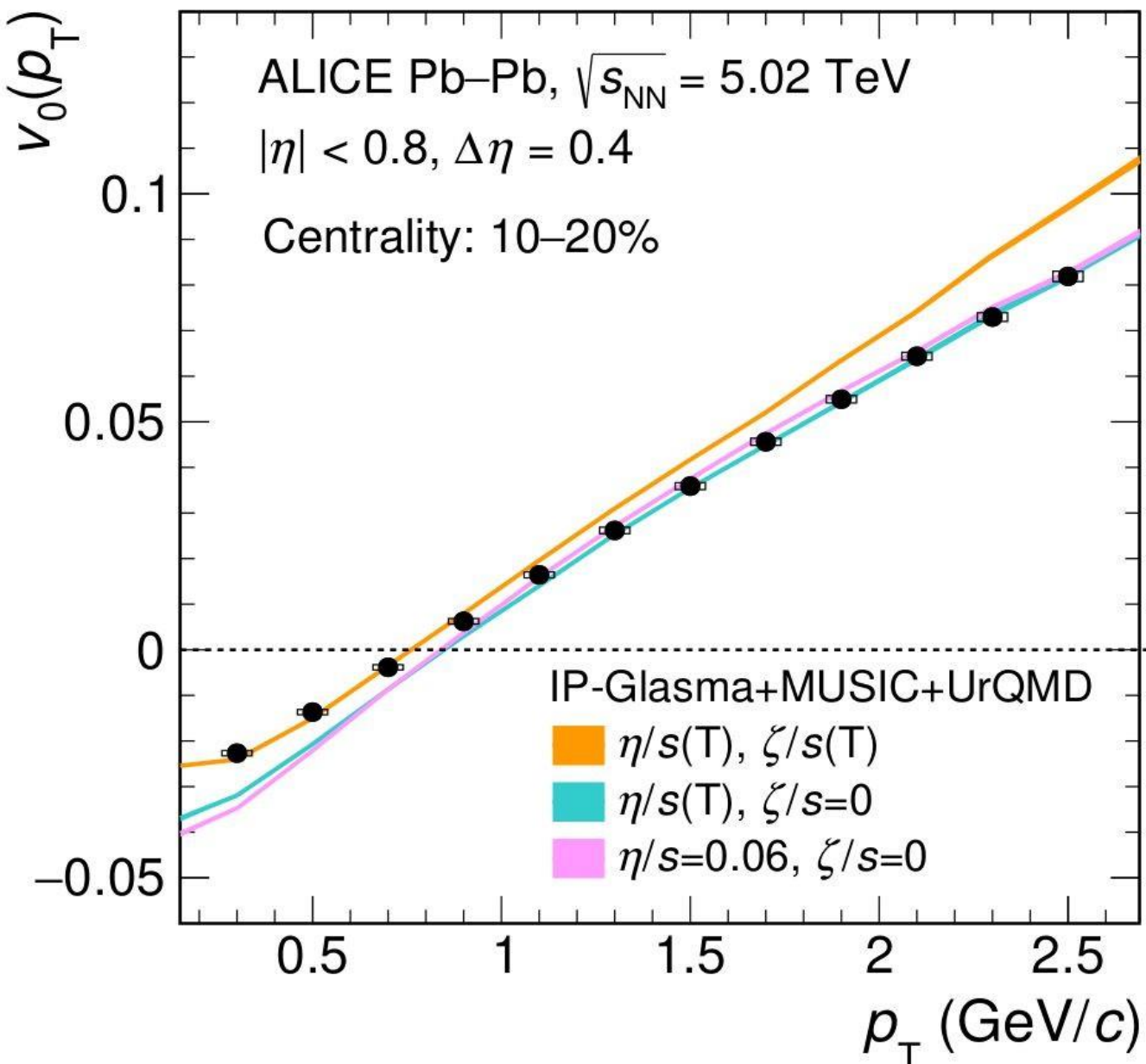
# Transport properties of medium in heavy-ion collisions

ALICE: Eur.Phys.J.C 84 (2024) 8, 813

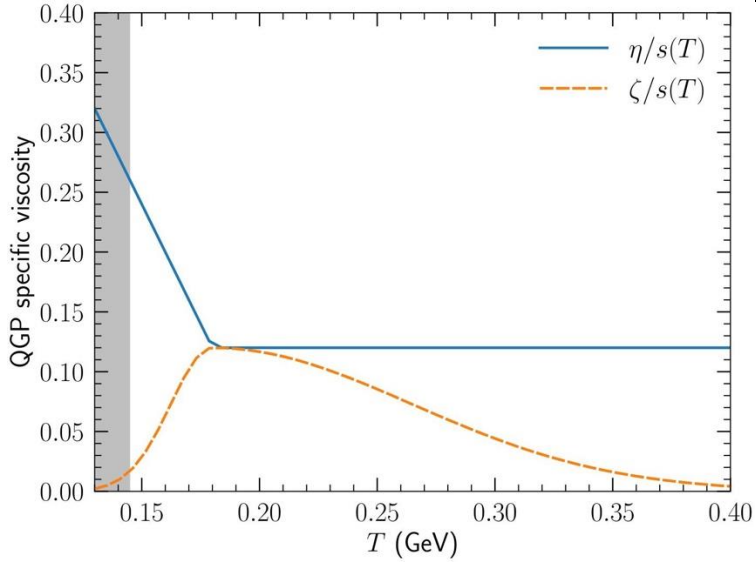


Bulk and diffusion coefficient needs to be better constrained

# $V_0(p_T)$ : Sensitivity to $\eta/s$ vs. $\zeta/s$

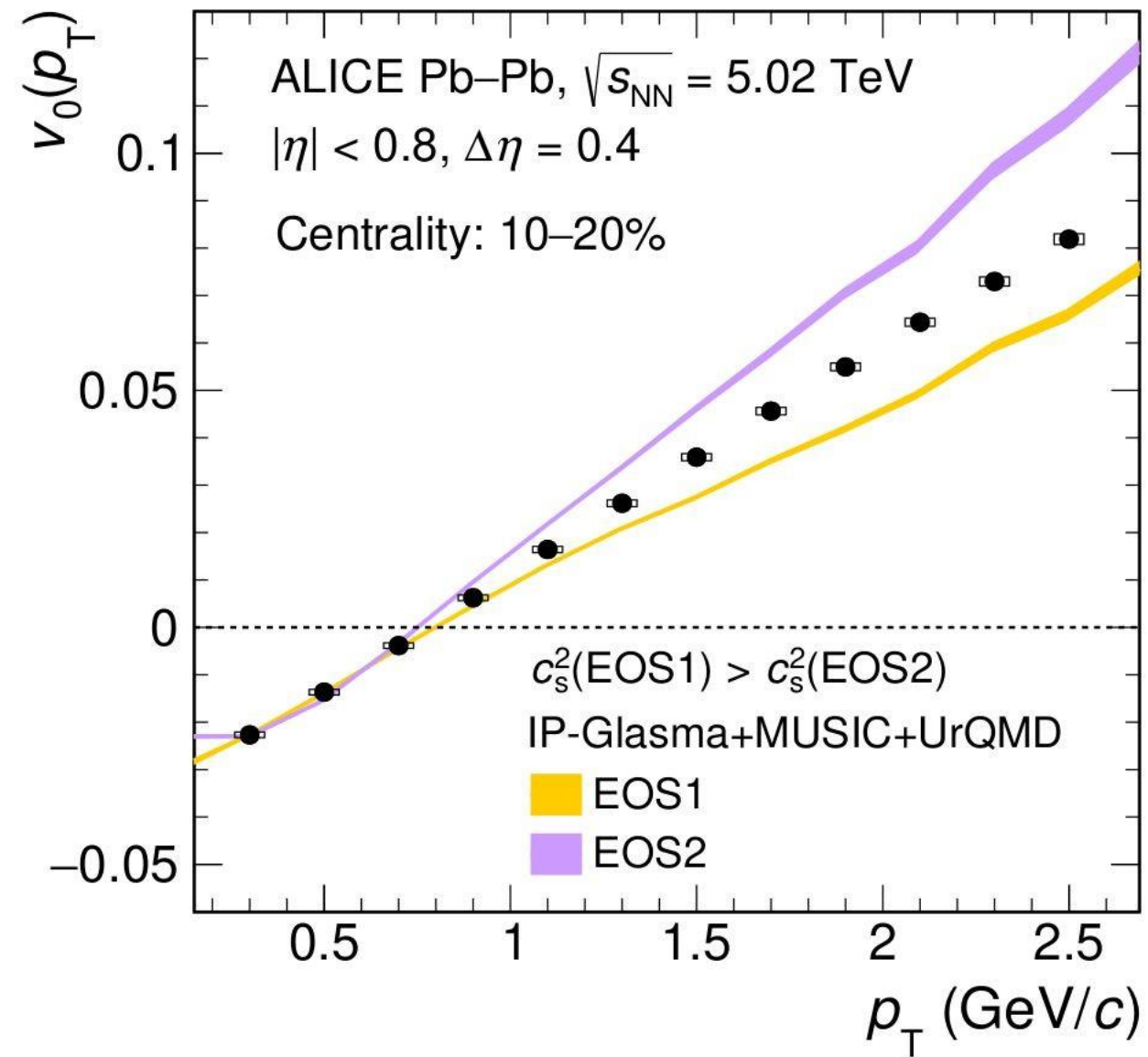


H. Mäntysaari, et al.,  
Phys. Rev. C 110, 054913 (2024)

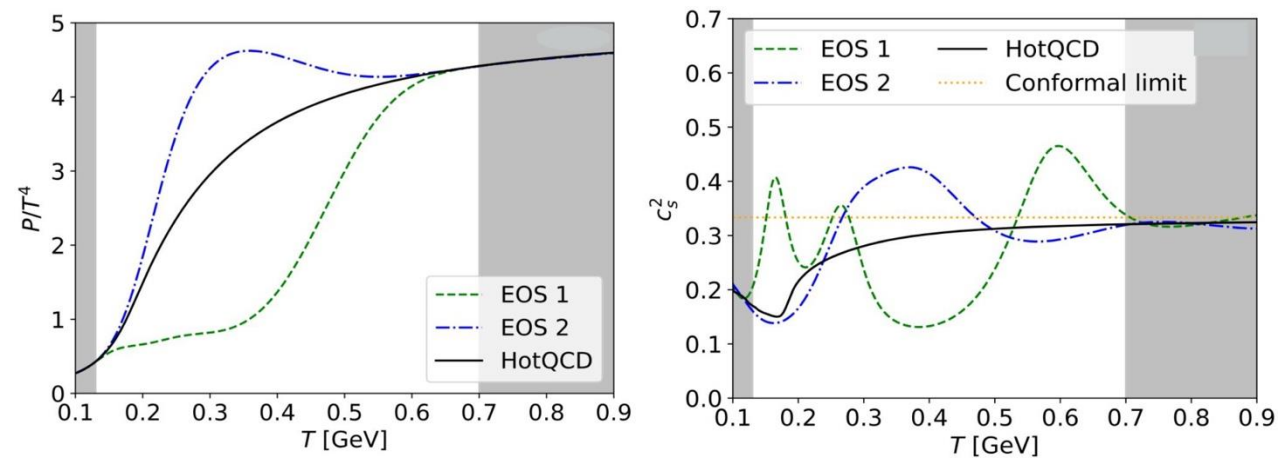


1. Sensitive to changes in bulk viscosity ( $\zeta/s$ )
2. Largely insensitive to changes in shear viscosity ( $\eta/s$ )

# $V_0(p_T)$ : Sensitivity to speed of sound (EOS)

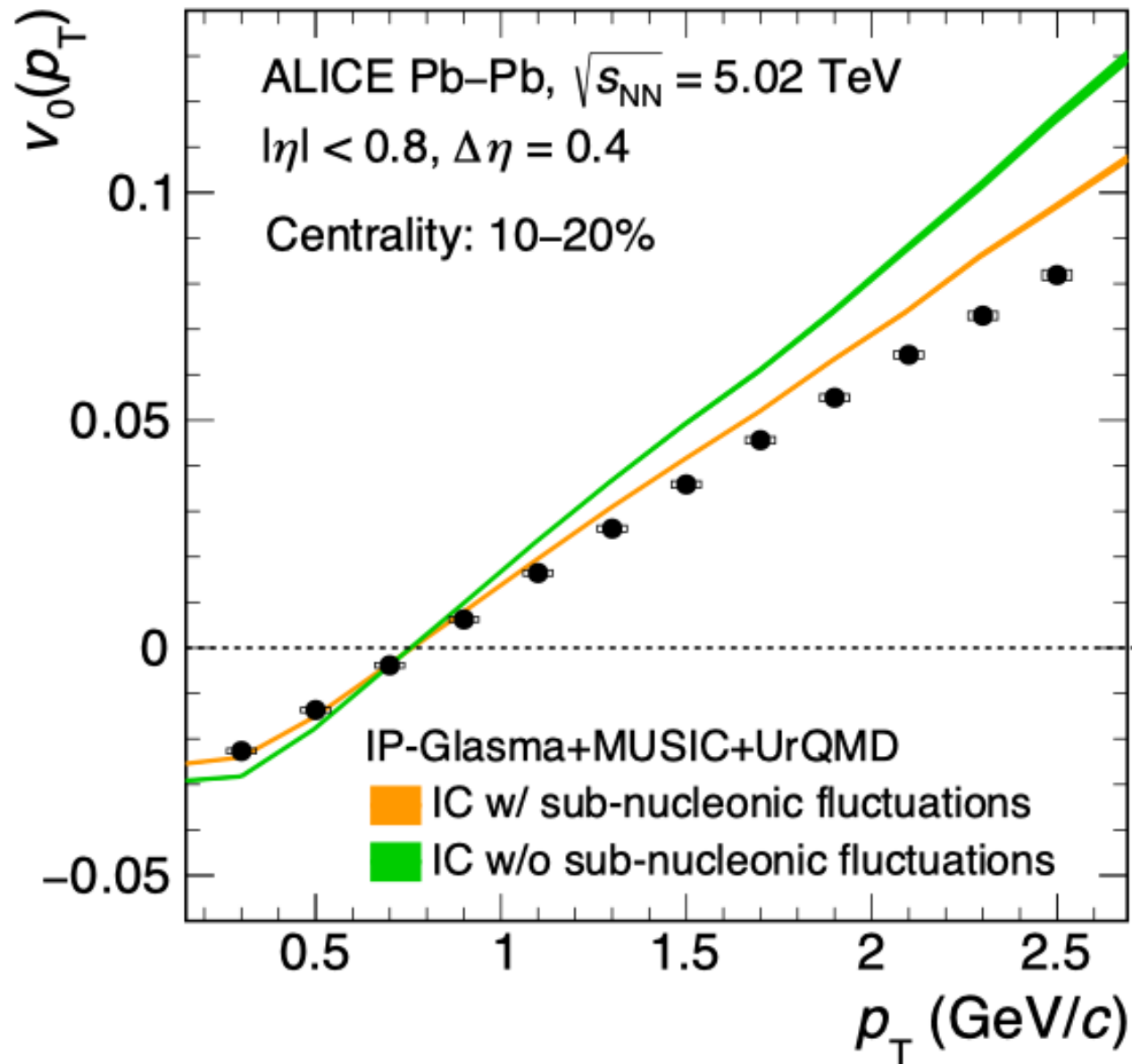


J. Gong, et al.,  
Phys. Rev. C 111, 044912 (2025)

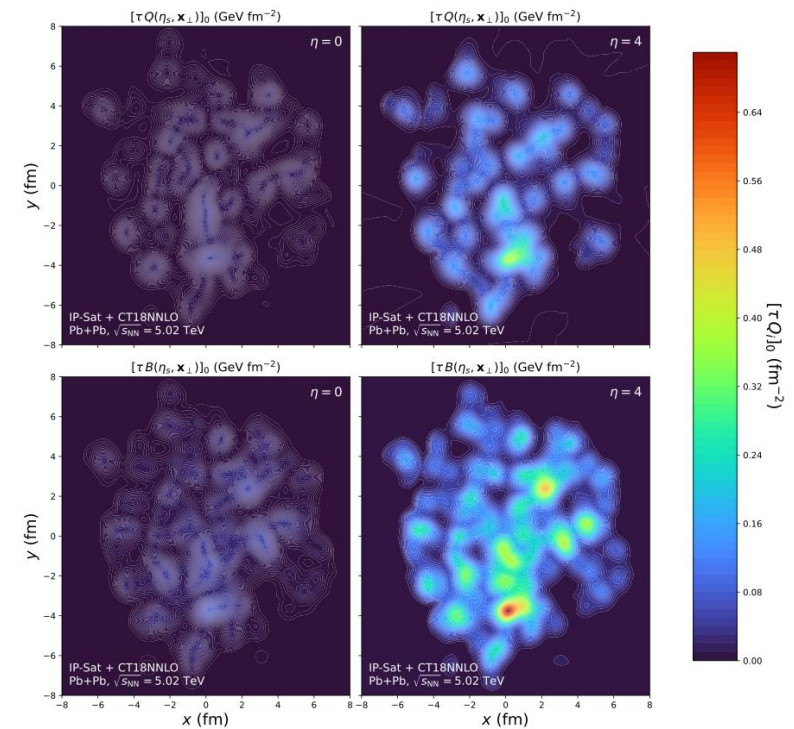


Sensitive to changes in  
speed of sound in the  
medium

# $V_0(p_T)$ : Sensitivity to initial conditions



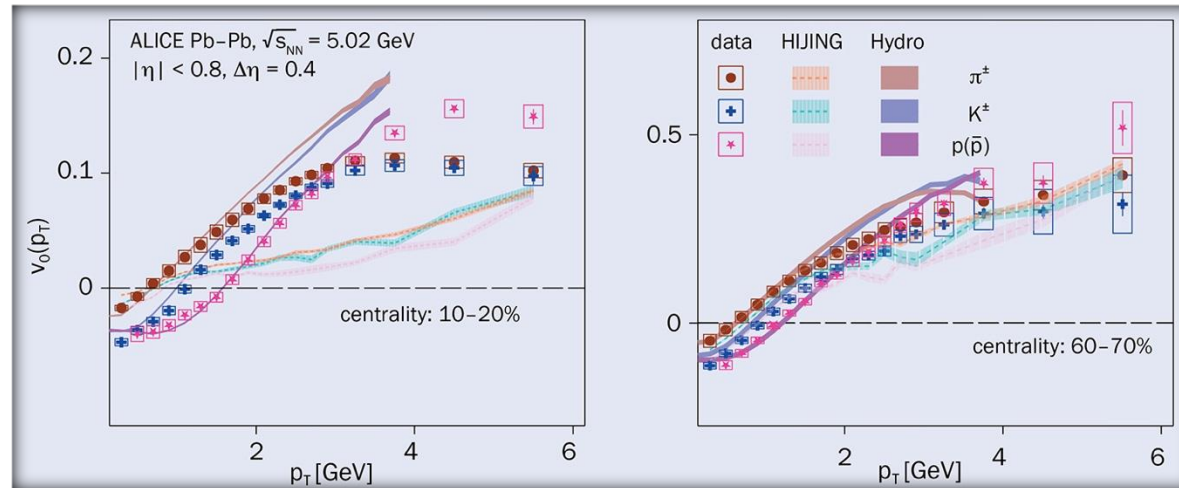
H. Mäntysaari, et al.,  
 Phys. Rev. C 110, 054913 (2024)



Sensitive to changes in  
 initial conditions.

# Summary

1. New momentum differential observable for radial flow / measuring isotropic expansion of medium in heavy-ion collisions.
2. Exhibits hydrodynamic features – momentum and mass dependence.



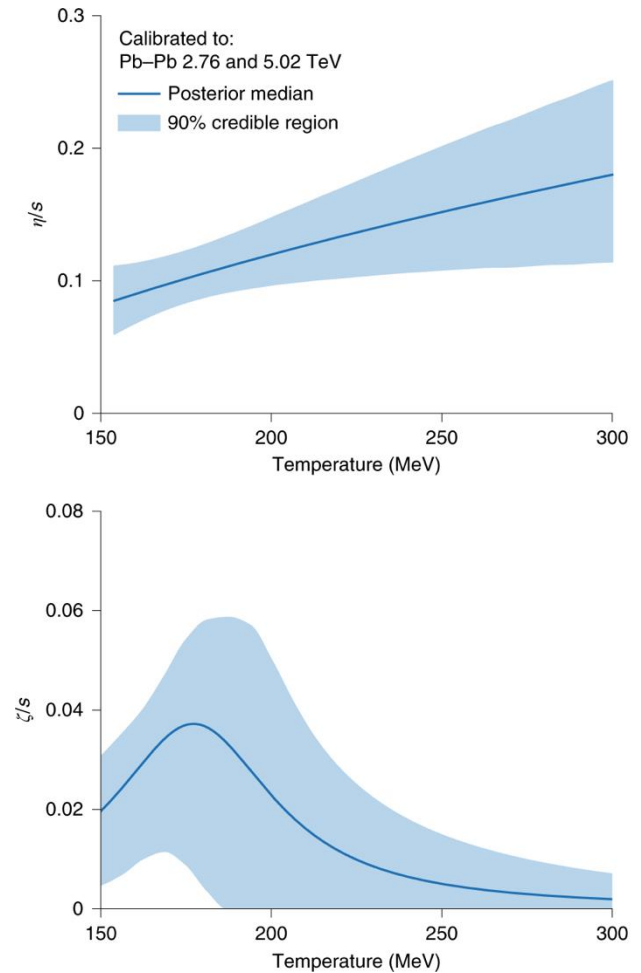
CERN Courier, August 2025

3. NCQ scaling similar to those seen for anisotropic flow observed for radial flow.
4. Sensitive to bulk viscosity and not shear viscosity – constraining  $\zeta/s$ .
5. Sensitive to speed of sound of the medium or the equation of state.
6. Sensitive to initial conditions in heavy-ion collisions.

# Outlook

1. New domain has opened up – radial flow can be now investigated with similar rigor as anisotropic flow.
2. From LHC perspective – study small vs. large system
3. Time evolution of radial flow in heavy-ion collisions – heavy – strange – light quark carrying hadrons. Exploit their production and freeze-out time scales
4. Excellent observable to be included in Bayesian analysis to extract transport properties and speed of sound/EOS.

Bernhard, J.E., et. al., *Nat. Phys.* 15, 1113–1117 (2019)

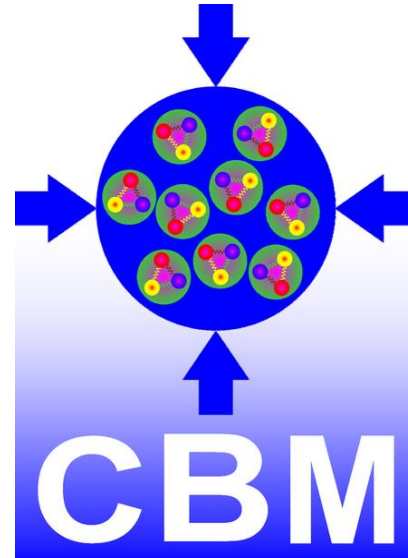


*Differential measurements of radial flow offers a new tool to probe this fluid-like expansion in detail, establishing its collective origin and complementing decades of studies of anisotropic flow.*

# Acknowledgements



ALICE



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