Dissecting the moat regime at low energies I: Renormalization and the phase structure

Shi Yin

Justus-Liebig-Universität Gießen

Based on: Fabian Rennecke, SY (in preparation)

Erice, 17.09.2025

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Alexander von HUMBOLDT STIFTUNG

CRC–TR 211 Strong-interaction matter under extreme conditions

fQCD

Outline

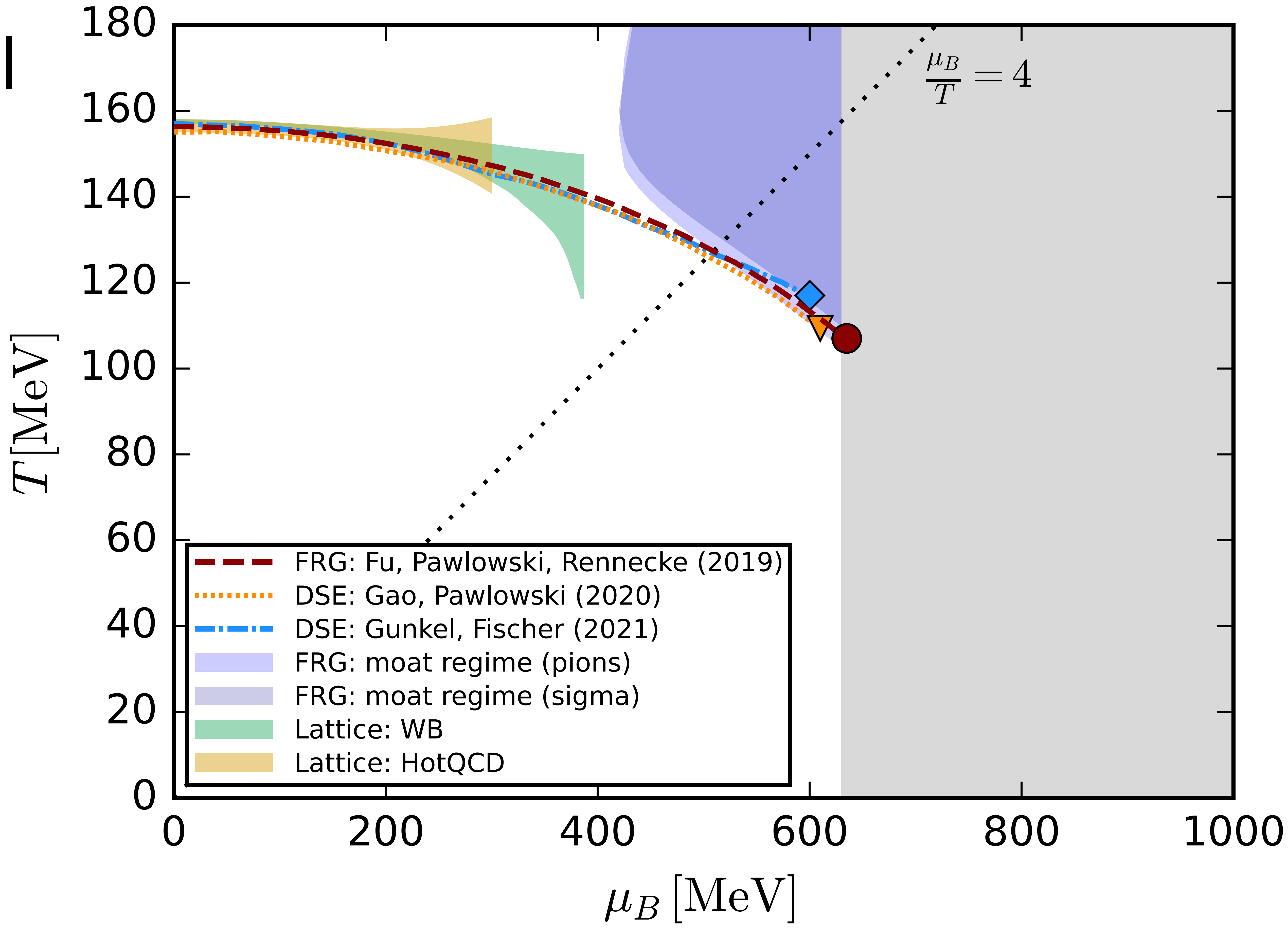
1. Introduction
   - Moat regime on QCD phase diagram
2. Moat regime
   - Random phase approximation
   - Renormalization
   - Yukawa coupling
3. Summary and Outlook

Outline

1. Introduction
    - Moat regime on QCD phase diagram
2. Moat regime
    - Random phase approximation
    - Renormalization
    - Yukawa coupling
3. Summary and Outlook

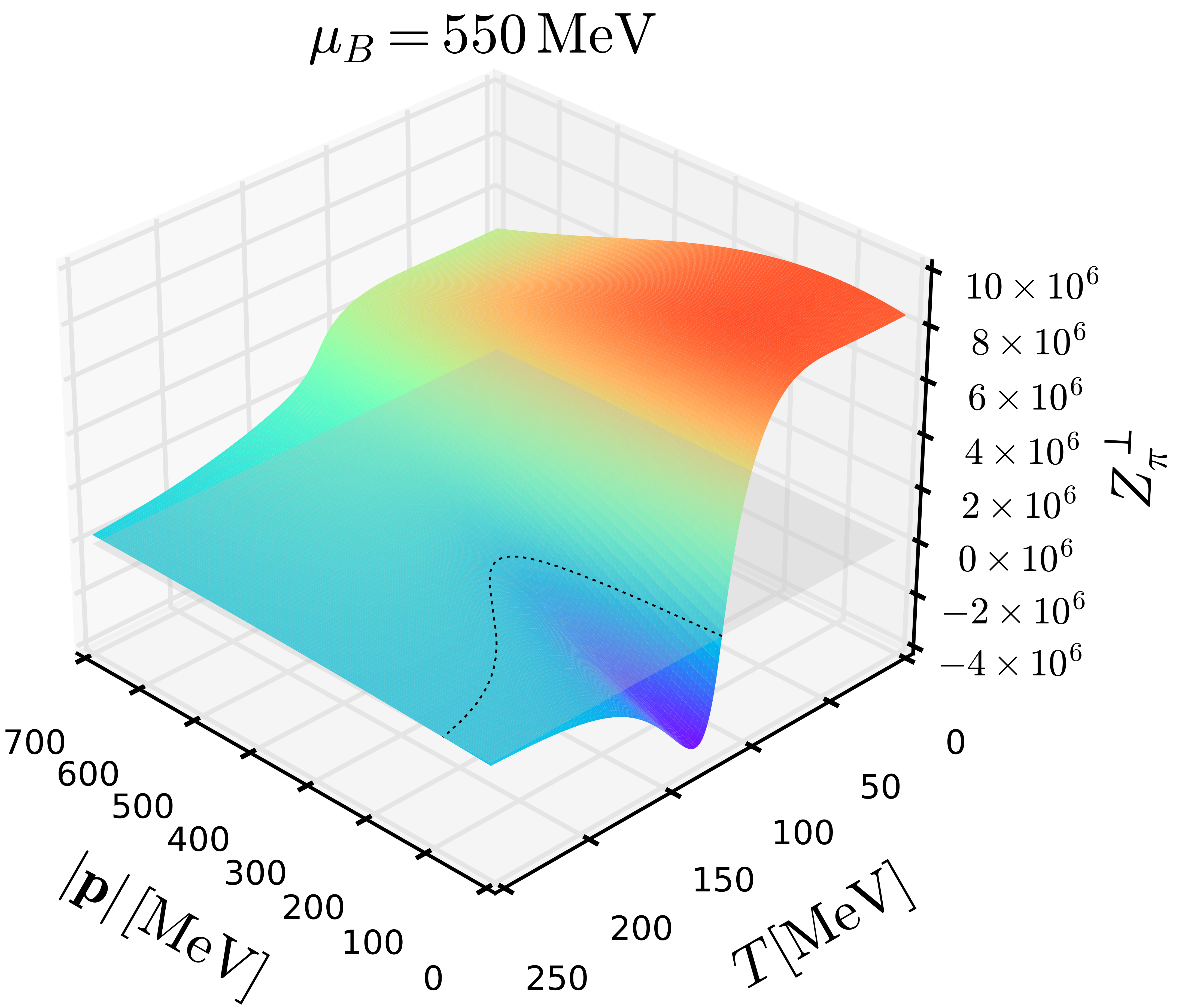
QCD phase diagram

- Lattice QCD (at vanishing chemical potential)
- Dyson-Schwinger Equations (DSE)
- Functional renormalization group (fRG)
- …

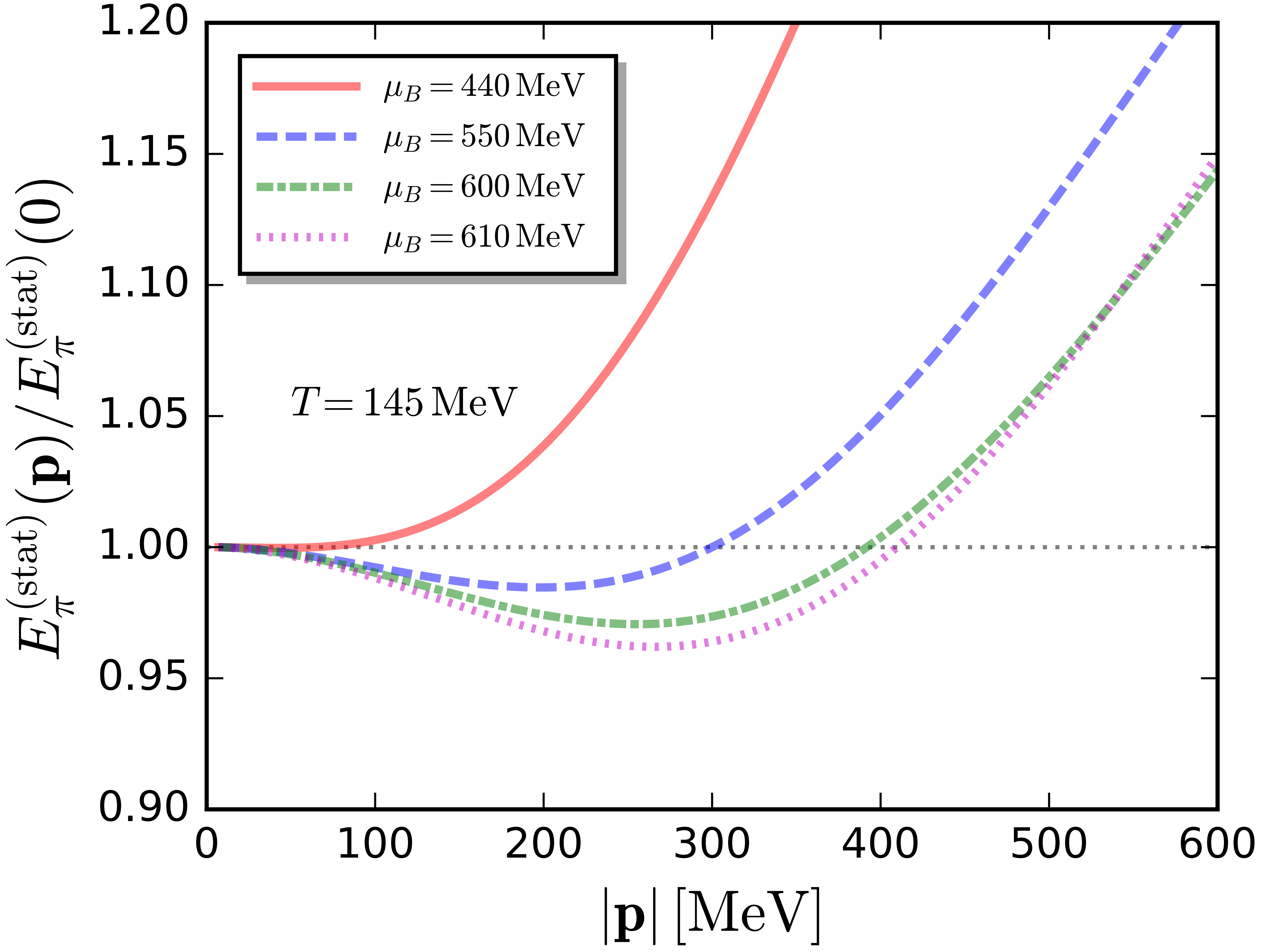


fQCD 2025

Negative meson wave function renormalization



$$E_\pi^{(stat)}(\mathbf{p}) = \sqrt{Z_\pi^\perp\,\mathbf{p}^2 + m_\pi^2}$$



Fu, Pawlowski, Pisarski, Rennecke, Wen, SY, arXiv:2412.15949

Outline

1. Introduction
    - Moat regime on QCD phase diagram
2. Moat regime
    - Random phase approximation
    - Renormalization
    - Yukawa coupling
3. Summary and Outlook

Quark-Meson model within RPA

Lagrangian:

$$\mathcal{L}[\phi, q, \bar{q}] = \bar{q}\big[\gamma_\mu \partial_\mu - \gamma_0 \hat{\mu}\big] q + \frac{1}{2}(\partial_\mu \phi)^2$$
$$+ h\,\bar{q}(T^0 \sigma + i\gamma_5 \mathbf{T}\cdot\boldsymbol{\pi})q + U(\rho) - c\sigma$$
$$+ \mathcal{L}_{\rm ct}$$

Counter term

$$V(\rho) = V_{\rm vac}(\rho) + V_{\rm thermal}(\rho)$$

$$V_{\rm vac}^{\overline{\rm MS}}(\rho) = -\frac{N_c N_f m_f^4}{8\pi^2} \ln\Big(\frac{m_f}{M}\Big)$$

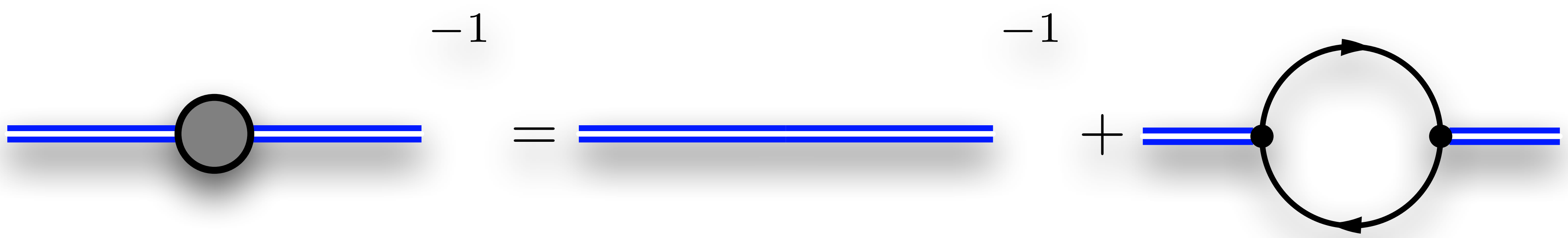
Renormalization scale

$$V_{\rm thermal}(\rho) = \frac{N_c N_f}{3\pi^2}\int_0^\infty dq\, \frac{q^4}{E_q}$$
$$\times \Big[\big(n_F(E_q;T,\mu) + n_F(E_q;T,-\mu)\big) + \underbrace{\nu\rho - \frac{\lambda}{2}\rho^2}_{\text{Bare potential}}\Big]$$

$$\sigma_0 = 92\,\mathrm{MeV}$$
$$\bar{m}_\pi^{\rm vac} = 136\,\mathrm{MeV}$$
$$\bar{m}_\sigma^{\rm vac} = 480\,\mathrm{MeV}$$
$$m_f^{\rm vac} = 300\,\mathrm{MeV}$$

- Random phase approximation
- Renormalization
- Yukawa coupling

Quark-Meson model within RPA



Pion two-point function:

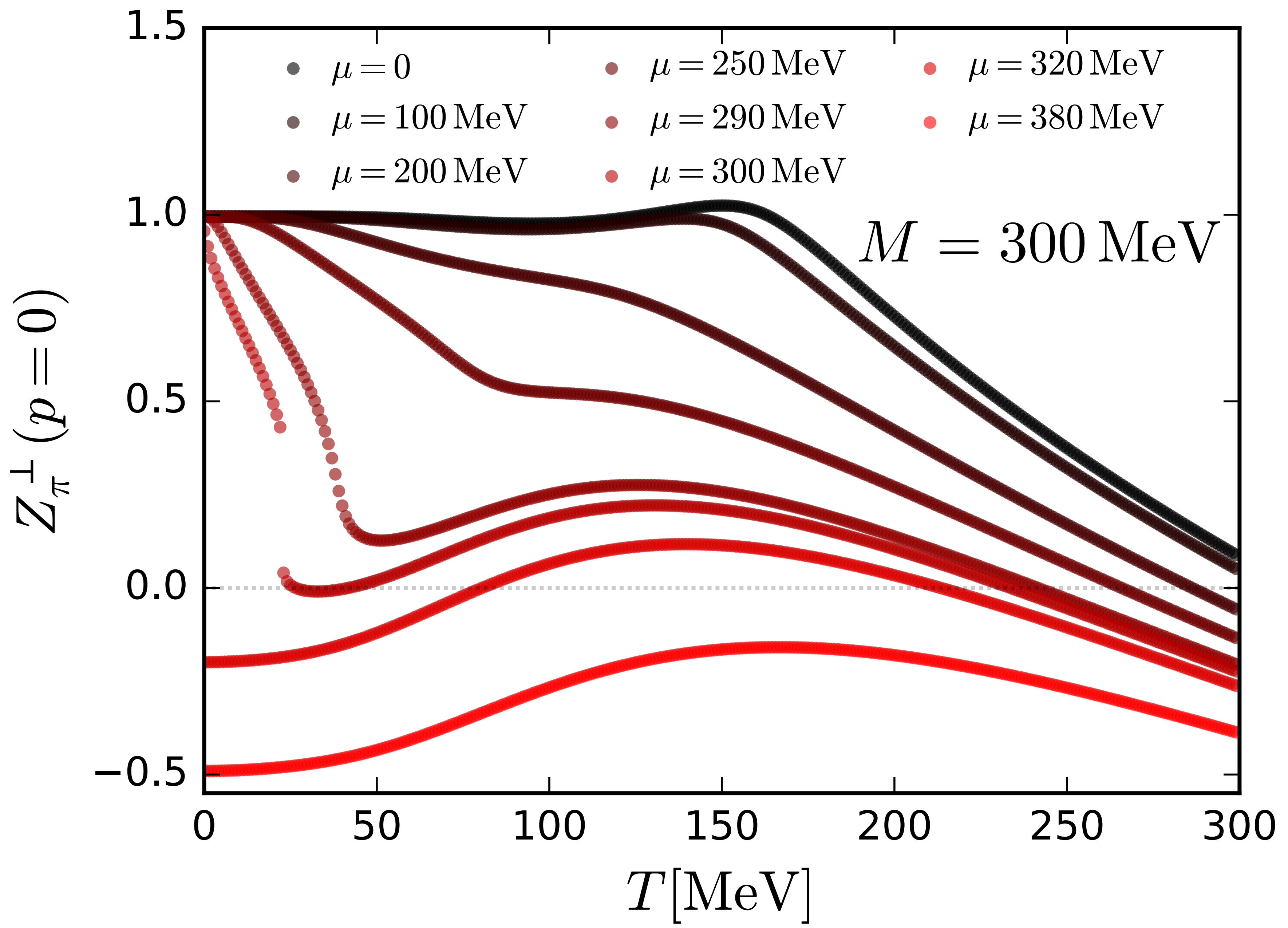
$$\Sigma_\pi(p^2;T,\mu) = p^2 + m_\pi^2 + \Pi^\pi_{\mathrm{RPA}}(p^2;T,\mu)$$

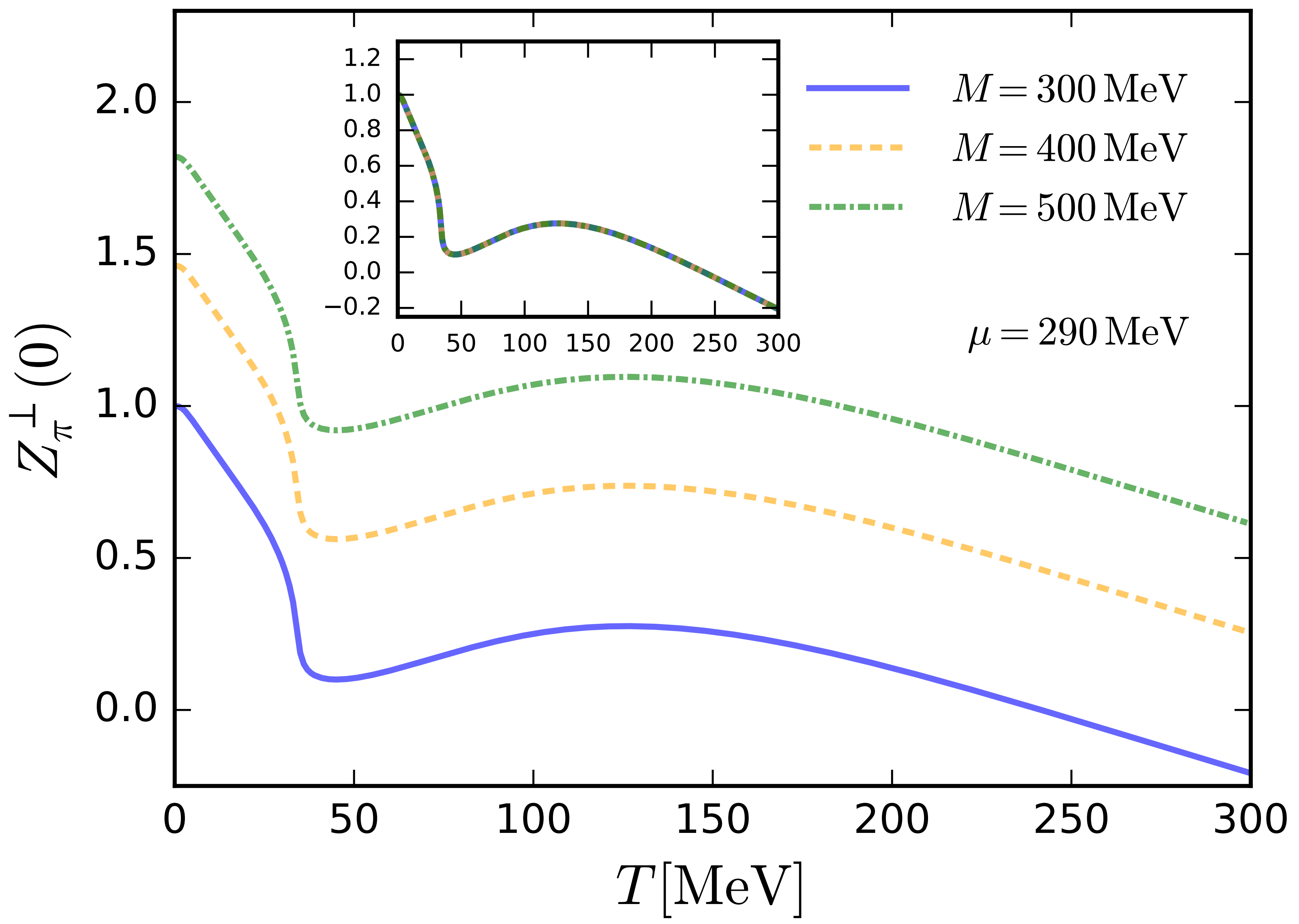
Pion wave function renormalization:

$$Z^\perp_\pi(p=0;T,\mu) = \frac{\partial}{\partial \boldsymbol{p}^2}\Sigma_\pi(p_0=0,\boldsymbol{p};T,\mu)\Big|_{\boldsymbol{p}^2=0}$$

$\overline{MS}$:

$$Z^{\perp,\mathrm{re}}_{\pi,\mathrm{vac}}(0) = 1 - \frac{h^2 N_c}{8\pi^2}\left[\bar{C} + \ln\left(\frac{m_f}{M}\right)\right]$$





Renormalization

Definitions of mass: $$\Sigma_\pi(p_0=0,\mathbf{p}=-i\,m_\pi^{\text{screening}})=0$$

$$\Sigma_\pi(p_0=-i\,m_\pi^{\text{pole}},\mathbf{p}=0)=0$$

$$m_\pi^{2,\text{curvature}}=\partial_\rho V(\rho)$$

Approximation: $$m_\pi^{2,\text{screening}}=\frac{m_\pi^{2,\text{curvature}}}{Z_\pi^{\perp}(p=0)}\qquad m_\pi^{2,\text{pole}}=\frac{m_\pi^{2,\text{curvature}}}{Z_\pi^{\parallel}(p=0)}$$

Apply a renormalization condition

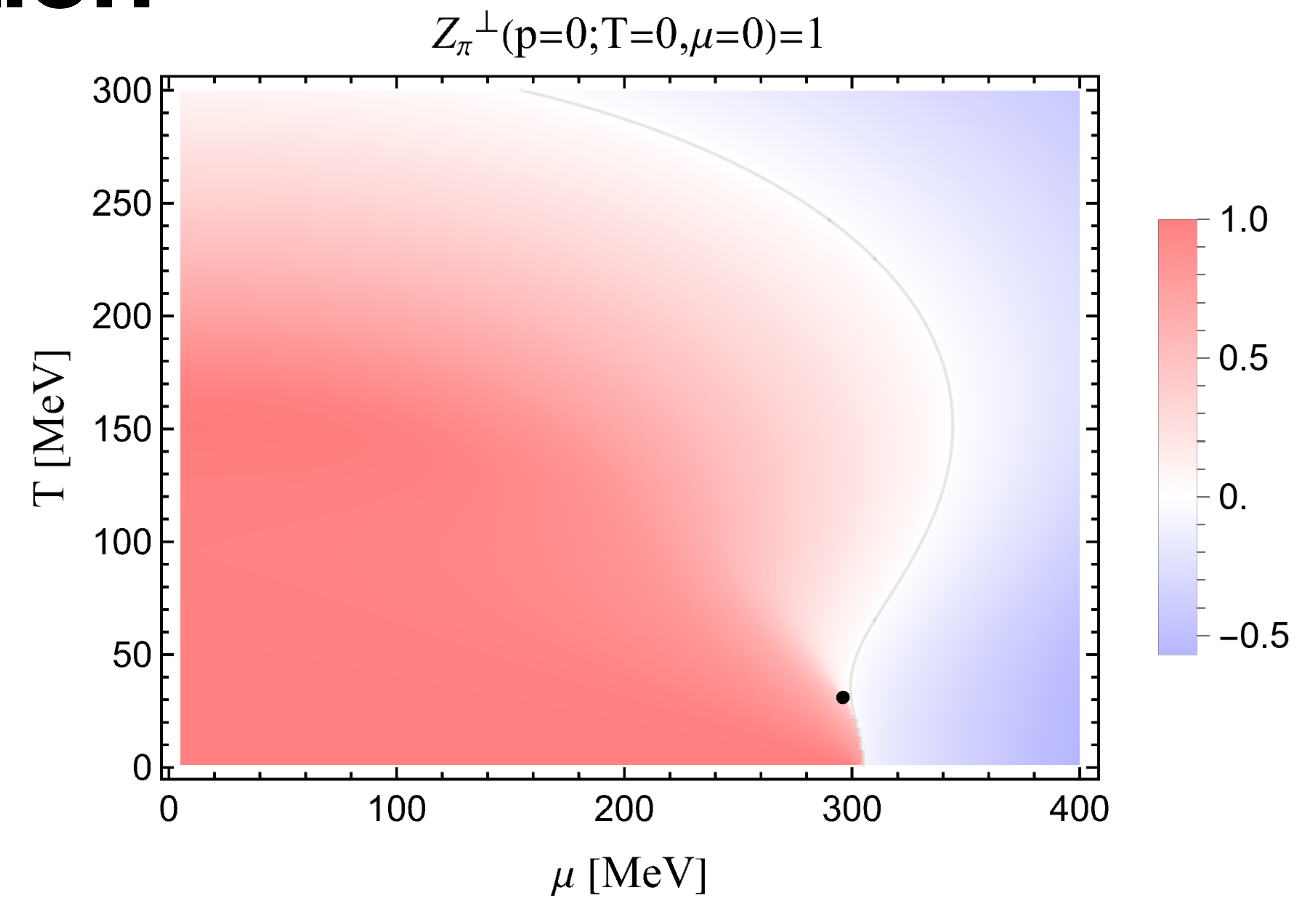
for wave function renormalization:

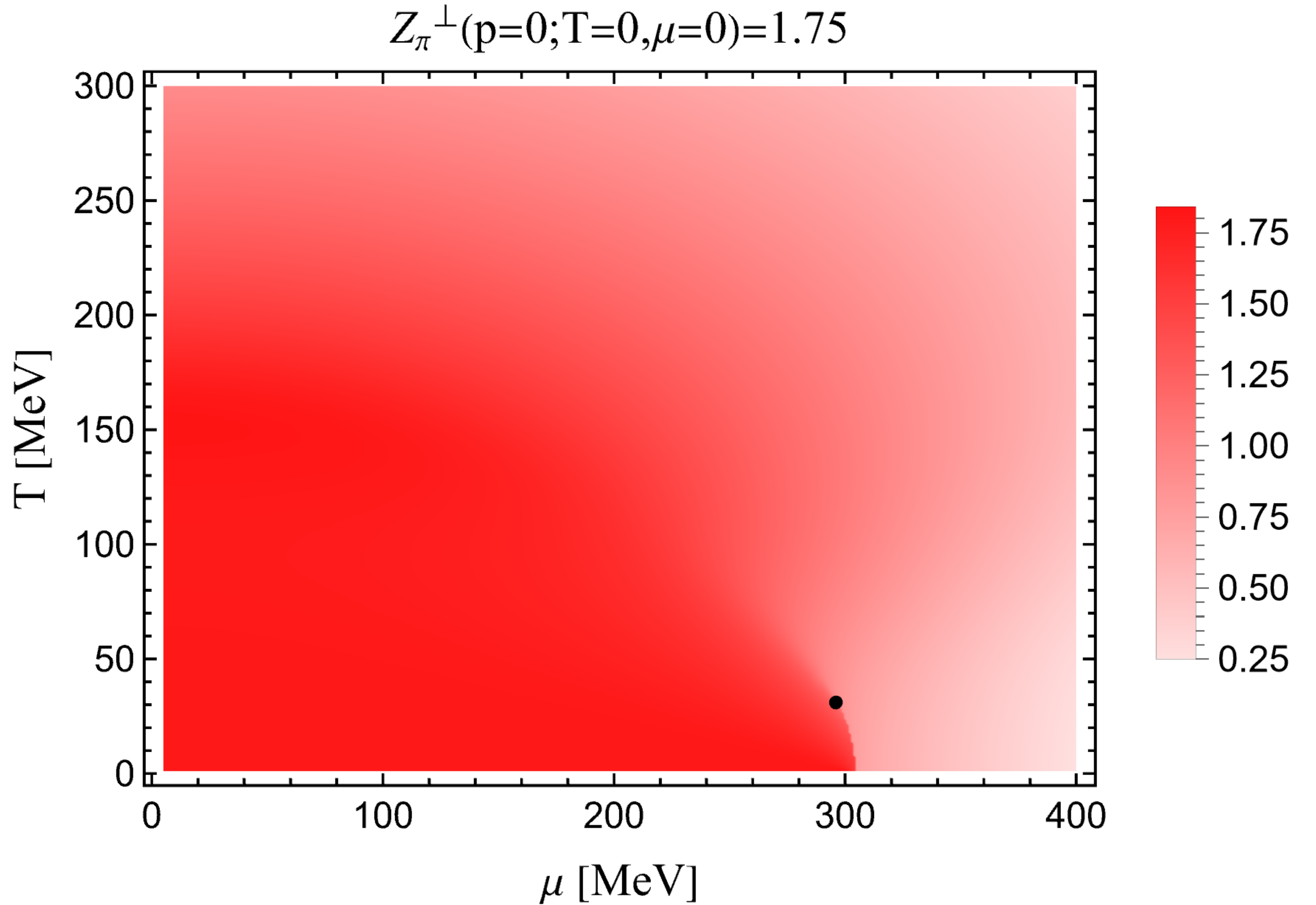
$$m_\pi^{2,\text{screening}}=m_\pi^{2,\text{pole}}=m_\pi^{2,\text{curvature}}$$

$$\boxed{Z_\pi^{\perp}(p=0;T=0,\mu=0)=1}$$

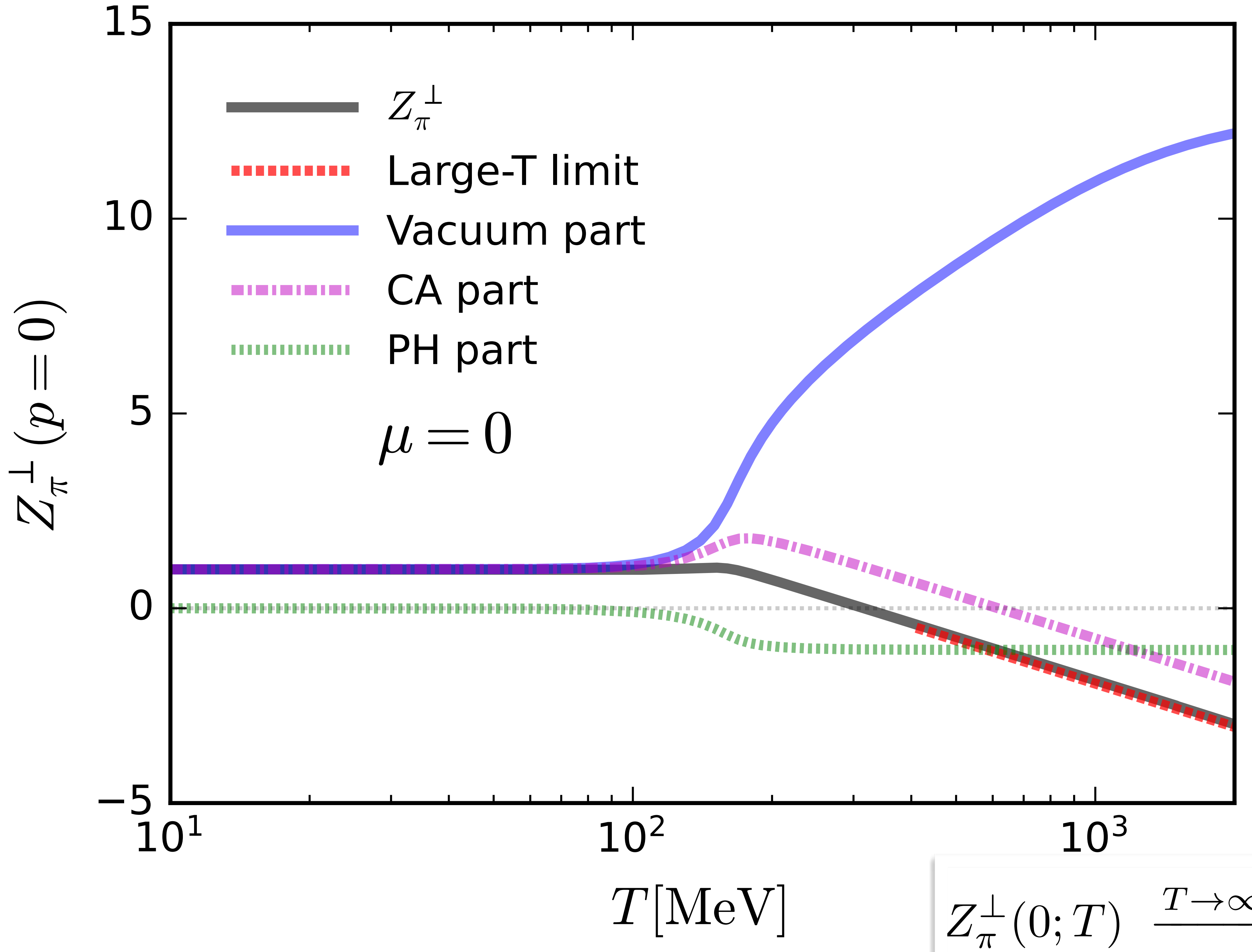
$$Z_{\pi,\text{vac}}^{\perp,\text{re}}(0)=1-\frac{h^2N_c}{8\pi^2}\left[\bar{C}+\ln\left(\frac{m_f}{M}\right)\right]$$

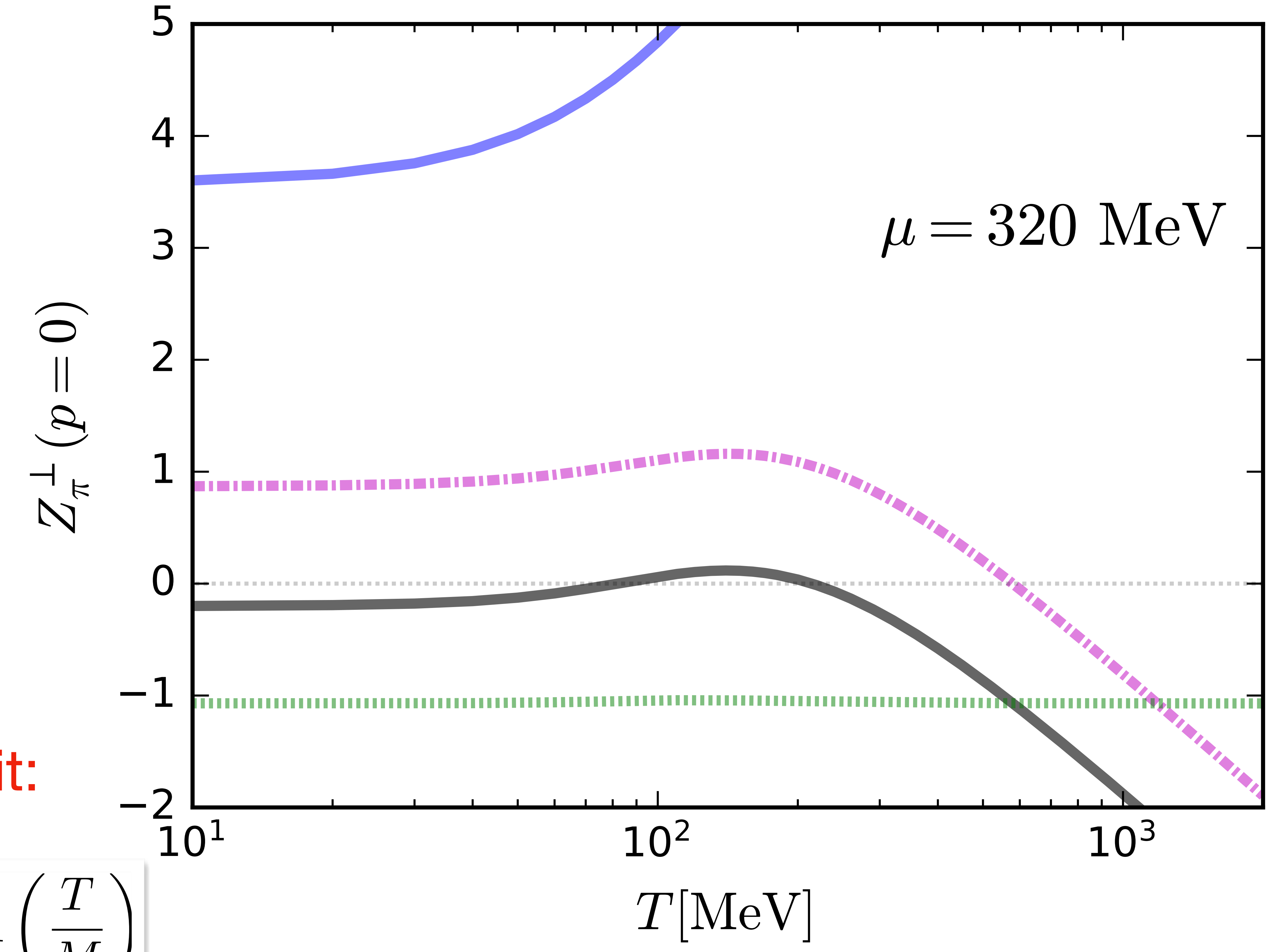
$$\bar{C}=\ln(M/m_f^{\text{vac}})$$





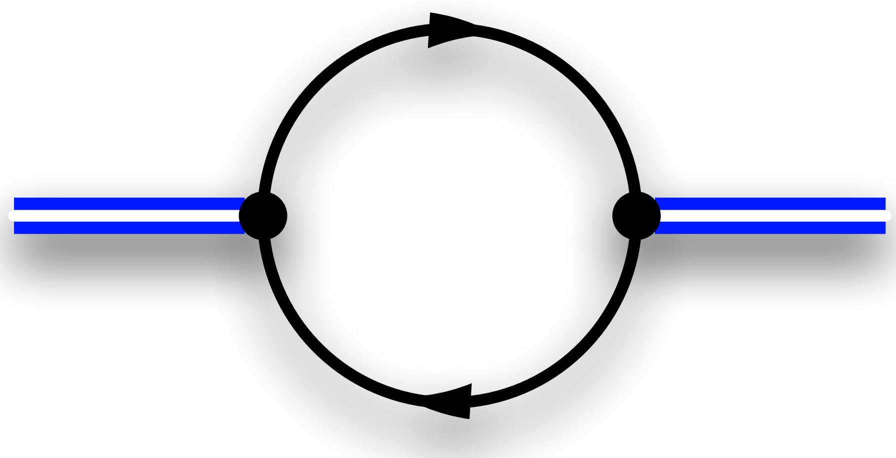
CA contribution and PH fluctuations





High T limit:

$$Z_\pi^\perp(0;T) \xrightarrow{T\to\infty} -\frac{h^2 N_c}{8\pi^2}\ln\left(\frac{T}{M}\right)$$



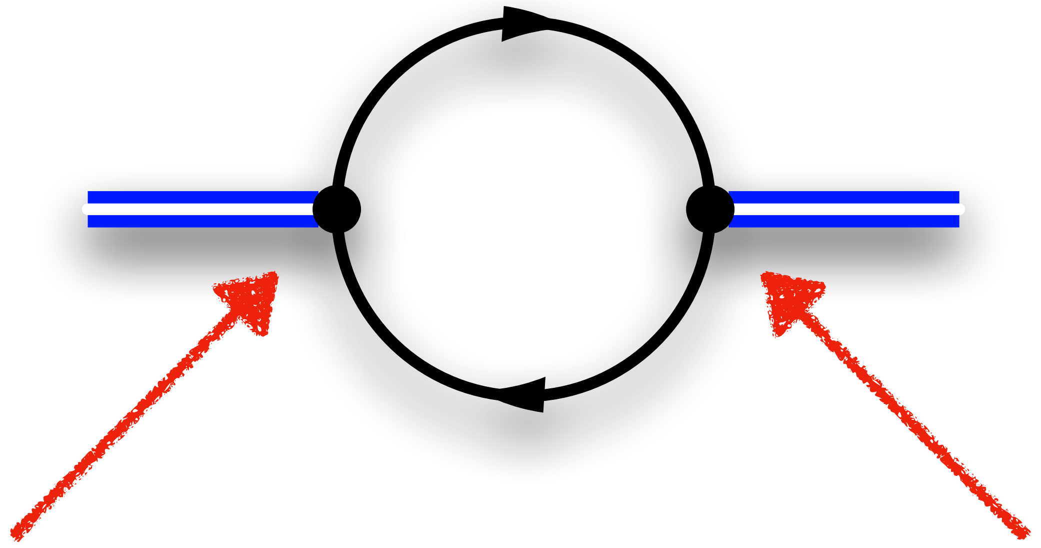
Partical creation and annihilation:

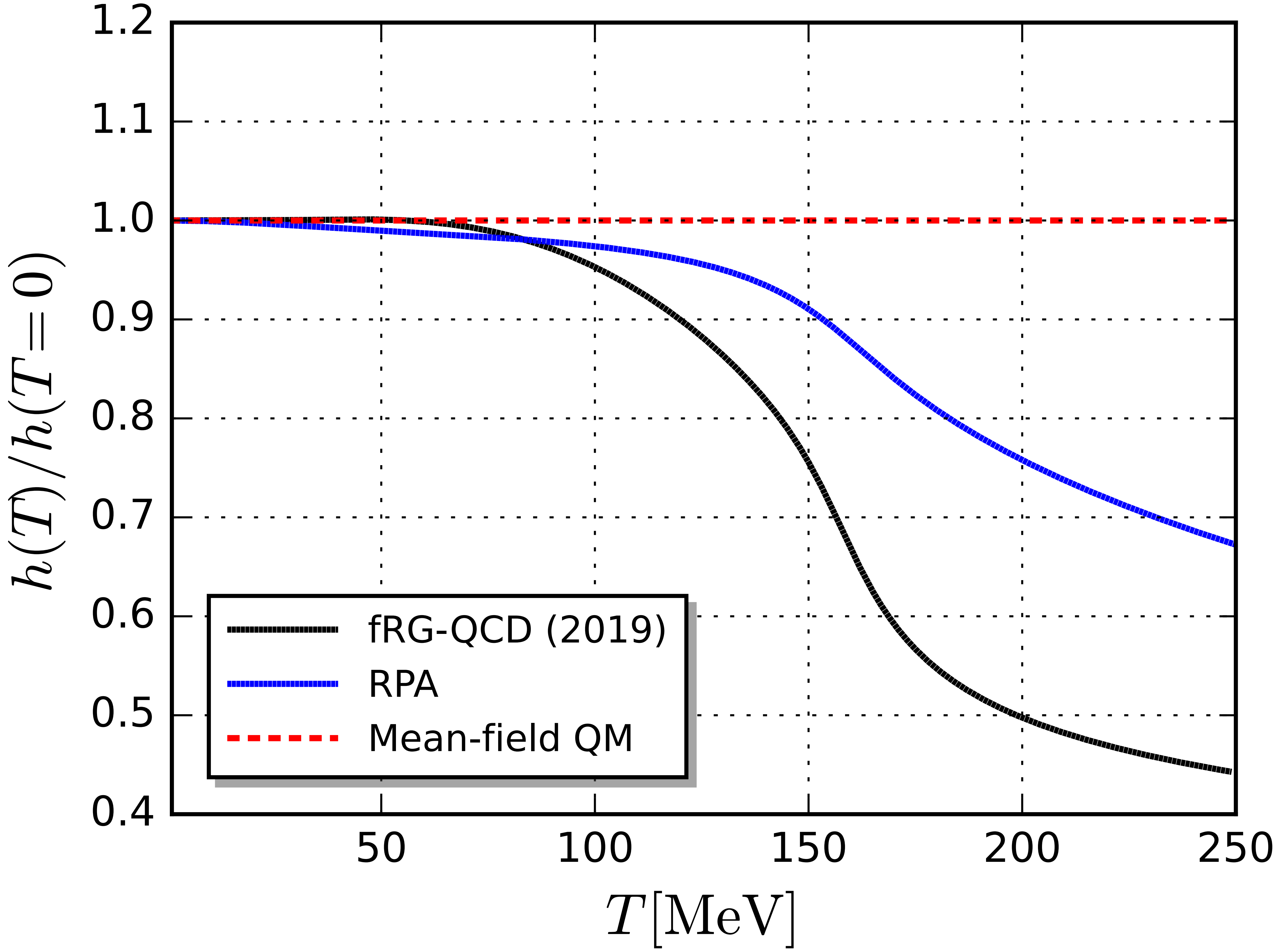
$$\mathcal{F}\mathcal{F}^{-,\mathrm{CA}}_{(1,1)}(p_0,\boldsymbol{p},\boldsymbol{q};m_f,T,\mu) = \frac{1}{4E_qE_{q-p}} \times \left\{ \frac{n_F(E_{q-p};T,\mu)+n_F(E_q;T,-\mu)}{ip_0-E_q-E_{q-p}} + \frac{-n_F(E_q;T,\mu)-n_F(E_{q-p};T,-\mu)}{ip_0+E_q+E_{q-p}} \right\}$$

Partical-Hole fluctuation:

$$\mathcal{F}\mathcal{F}^{-,\mathrm{PH}}_{(1,1)}(p_0,\boldsymbol{p},\boldsymbol{q};m_f,T,\mu) = \frac{1}{4E_qE_{q-p}} \times \left\{ \frac{-n_F(E_q;T,-\mu)+n_F(E_{q-p};T,-\mu)}{ip_0-E_q+E_{q-p}} + \frac{n_F(E_q;T,\mu)-n_F(E_{q-p};T,\mu)}{ip_0+E_q-E_{q-p}} \right\}$$

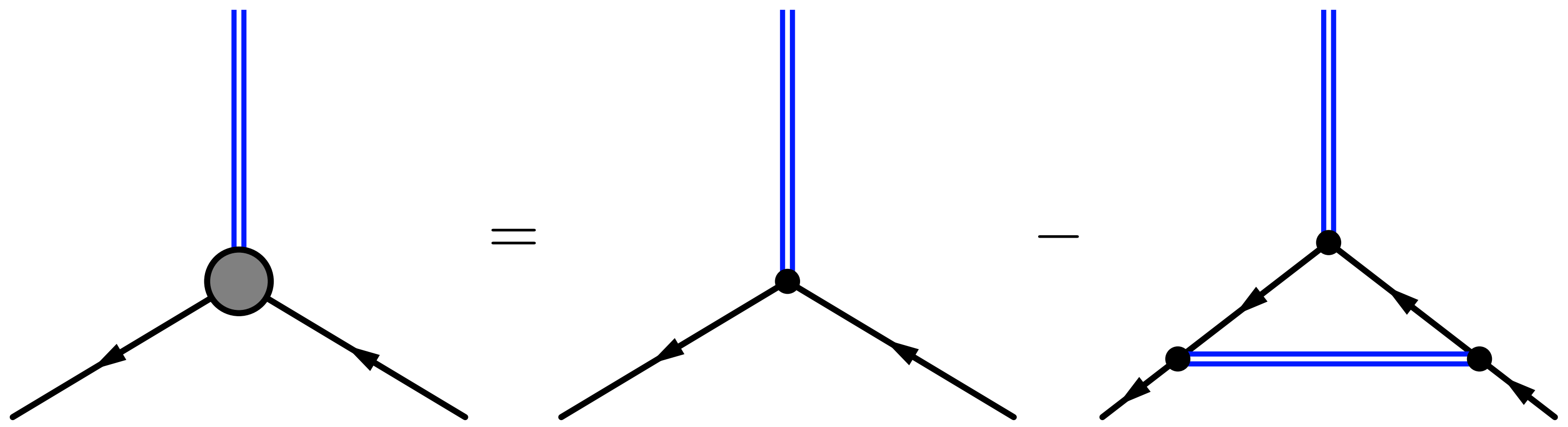
Yukawa coupling



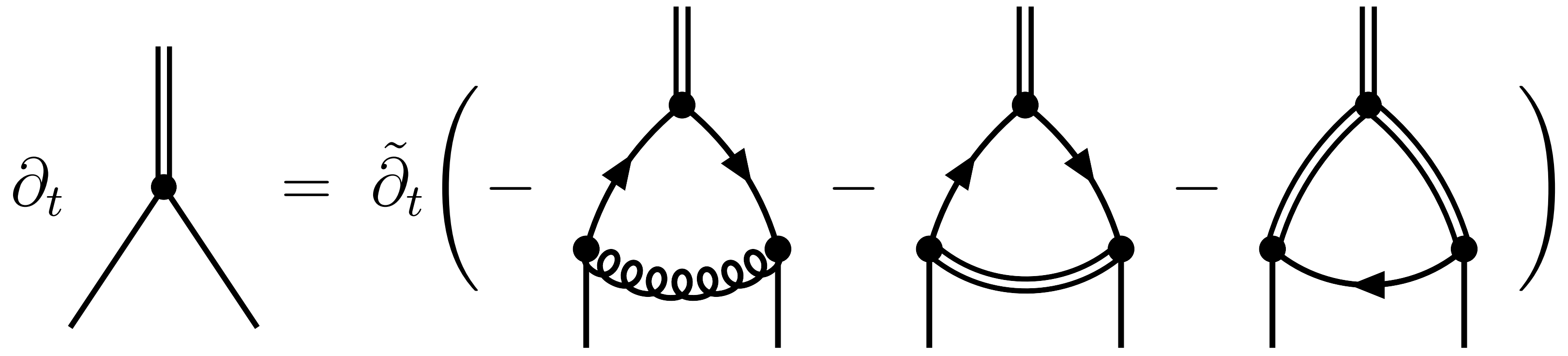


Mean-field QM: $h = 6.5$

RPA QM:

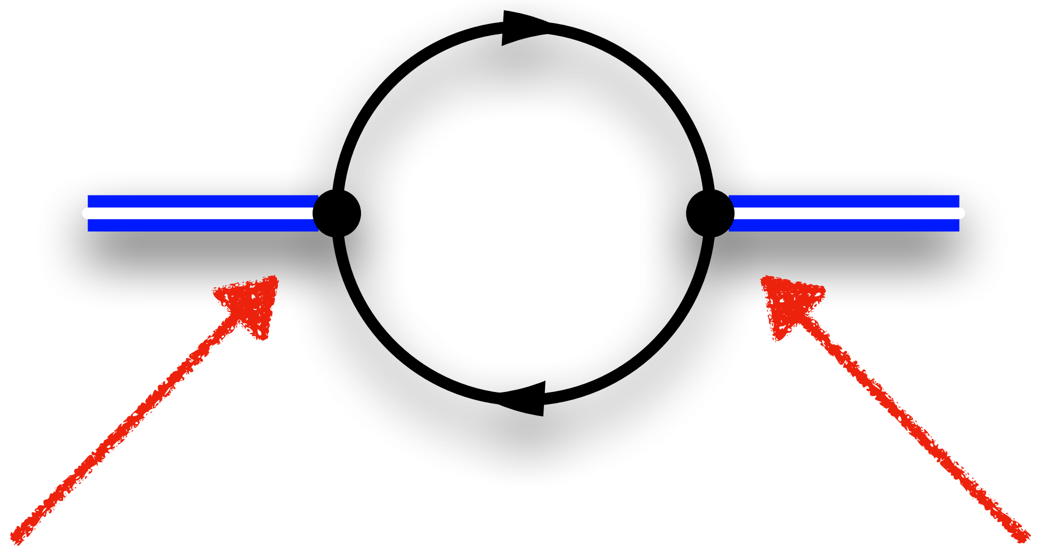


fRG-QCD:



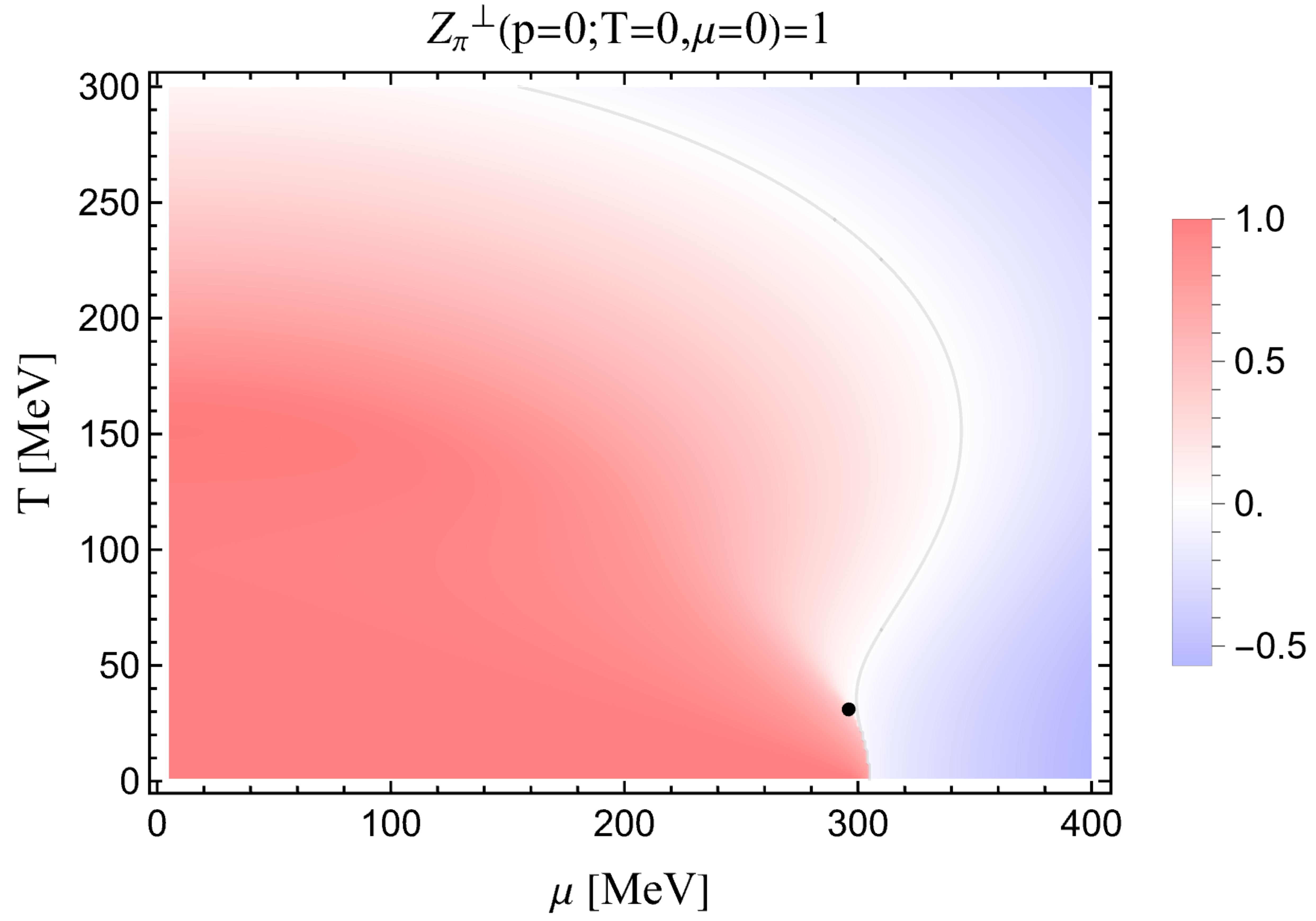
- Random phase approximation
- Renormalization
- Yukawa coupling

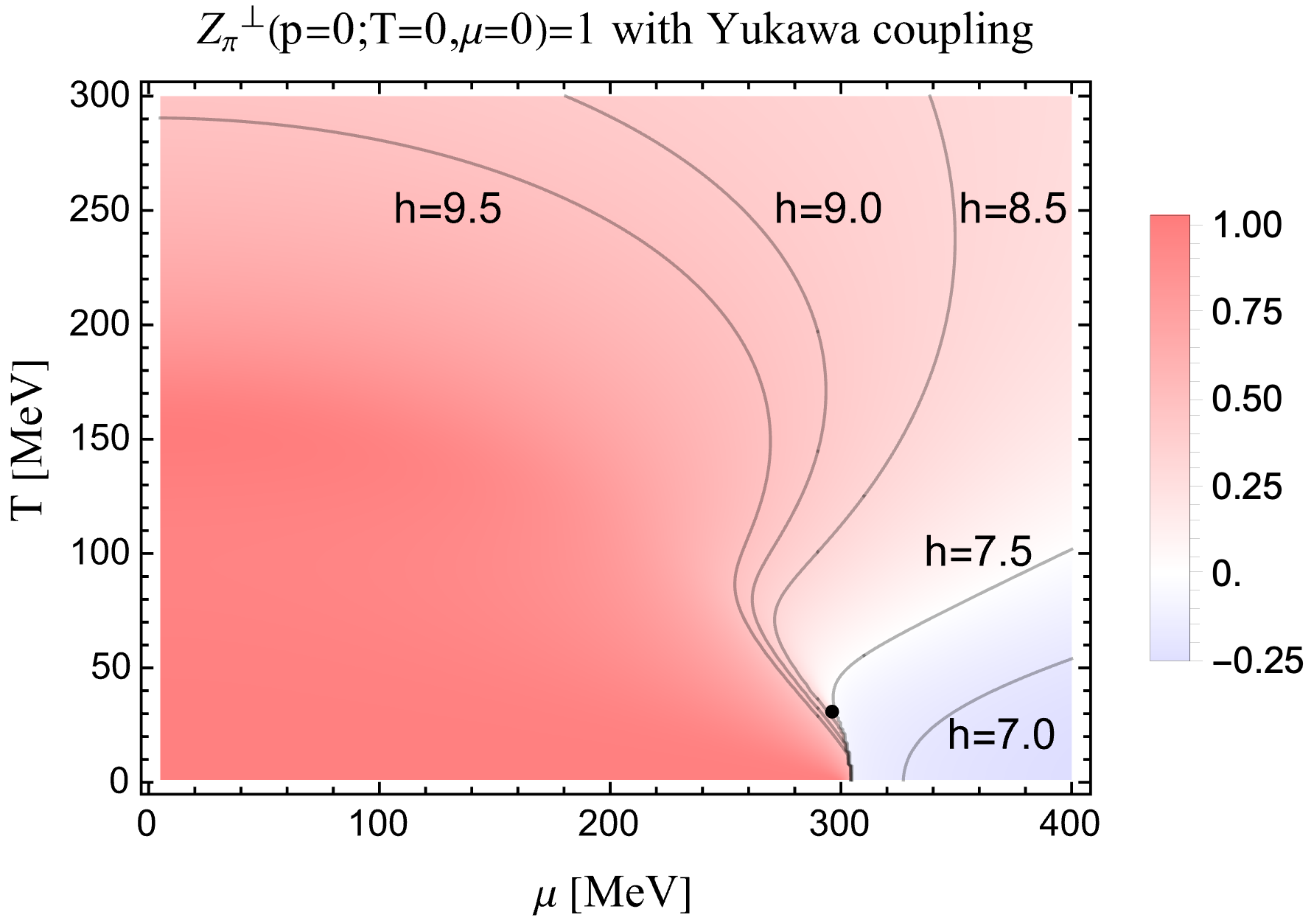
Yukawa coupling



Constant $h = 6.5$

RPA Yukawa coupling





Outline

1. Introduction
   - Moat regime on QCD phase diagram
2. Moat regime
   - Random phase approximation
   - Renormalization
   - Yukawa coupling
3. Summary and Outlook

Summary and Outlook

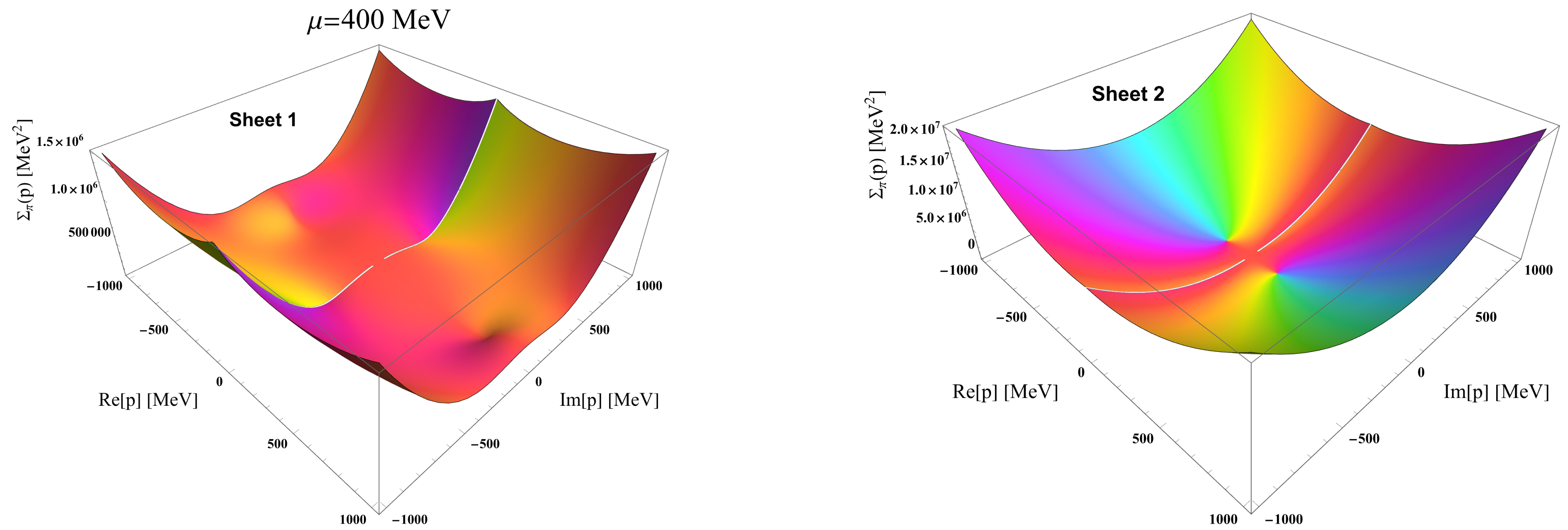
- We find a moat regime in the Quark-Meson model within RPA computation
- The moat regime is related to the choice of renormalization condition
- The interaction between quarks and mesons (Yukawa coupling) can influence the moat regime

Summary and Outlook

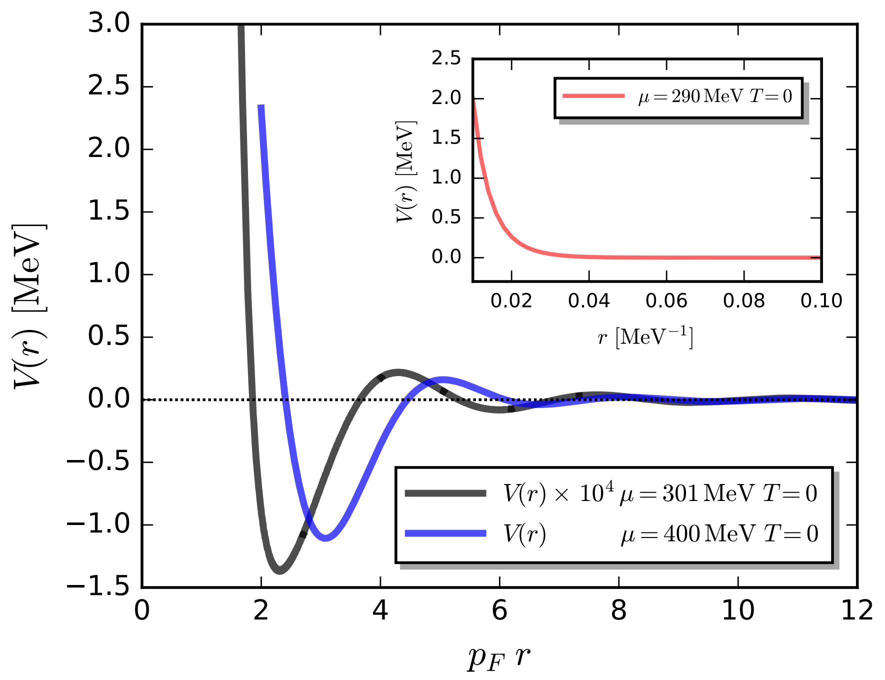
- We find a moat regime in the Quark-Meson model within RPA computation
- The moat regime is related to the choice of renormalization condition
- The interaction between quarks and mesons (Yukawa coupling) can influence the moat regime

Dissecting the moat regime at low energies II :

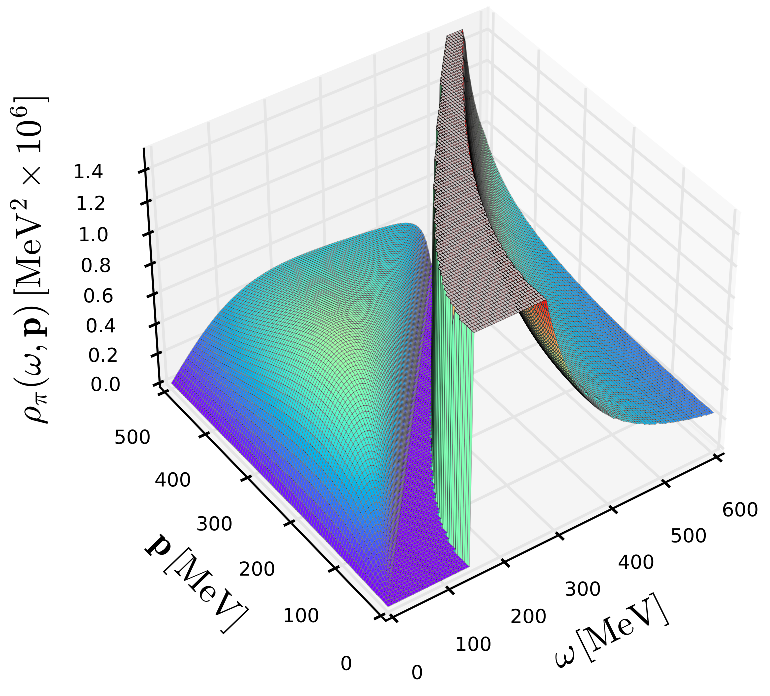
- Investigate the pion two-point function at complex spatial momentum plane

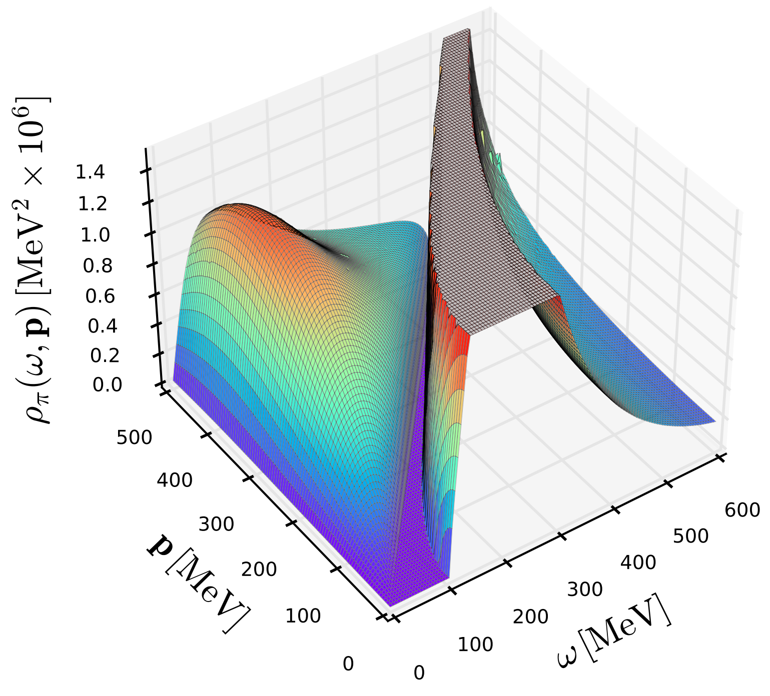


- Influence of moat on the Friedel Oscillations



- Influence of moat on pion spectral function





- Inhomogeneous instability