Confining density functional for QCD: heavy-ion collisions, neutron stars, mergers and supernovae

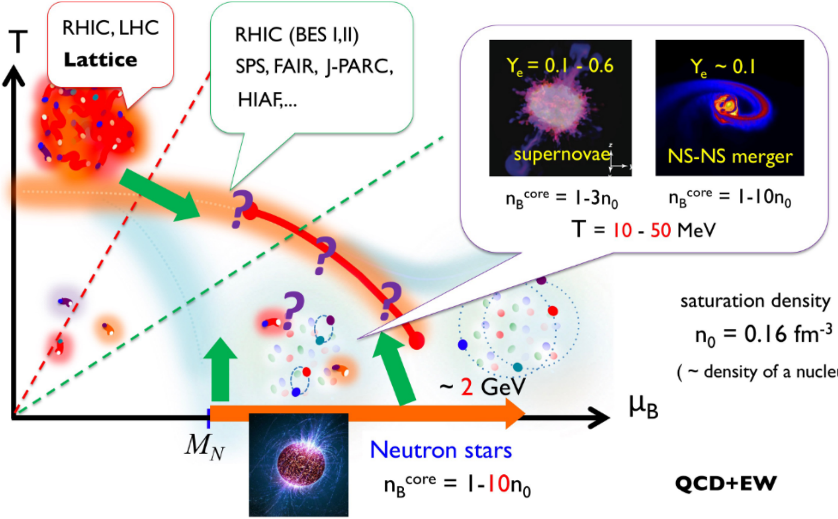
Oleksii Ivanytskyi

based on: PRD 2022, EPJA 2022, EPJA 2024 + update
(with David Blaschke and Gerd Röpke)

46^{th} International School on Nuclear Physics
"QCD under extreme conditions - present and future"

Erice, 16-22 September 2025

QCD phase diagram



saturation density

$n_0 = 0.16\ \mathrm{fm}^{-3}$

(~ density of a nucleus)

QCD+EW

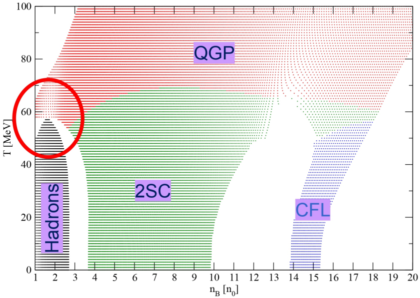
Figure from T. Kojo arXiv:1912.05326 [nucl-th]

False quark dominance in hybrid quark-hadron EoS

- **Hadronic EoS consistent with astro (DDf4) + NJL model**

$$\Downarrow$$

False quark onset already @ T $\simeq$ 60 MeV



- **Hadron decays are energetically favorable**

$$\begin{array}{lcc} M_q \simeq 330 MeV & & M_{meson} > 2M_q \\ M_\omega = 783 MeV & \Rightarrow & quarks\ are\ too\ light \\ M_\rho = 775 MeV & & to\ be\ confined \end{array}$$

Effective quark "confinement" is needed

Confining density functional ($N_f = 2$ for simplicity)

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - m)q + \mathcal{U} + \ldots$$

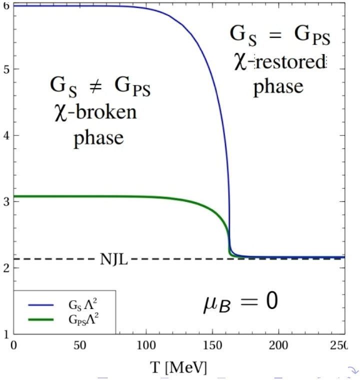
$$\begin{aligned}\mathcal{U} &= D_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^\kappa \\ &= \underbrace{\mathcal{U}_{MF}}_{\mathbf{0^{th}\ order}} + \underbrace{(\overline{q}q - \langle\overline{q}q\rangle)\,\Sigma_S}_{\mathbf{1^{st}\ order}} - \underbrace{G_S\,(\overline{q}q - \langle\overline{q}q\rangle)^2 - G_{PS}\,(\overline{q}i\vec{\tau}\gamma_5 q)^2}_{\mathbf{2^{nd}\ order}} + \ldots\end{aligned}$$

- **Mean-field scalar self-energy**

$$\Sigma_S = \frac{\partial\mathcal{U}_{MF}}{\partial\langle\overline{q}q\rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\overline{q}q\rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



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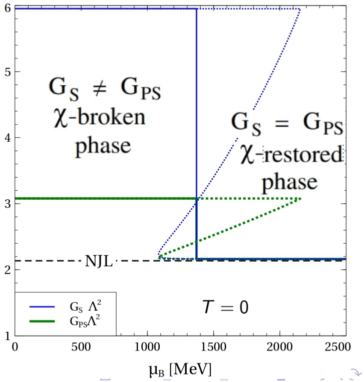


Comparison to NJL model

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \underbrace{(m+\Sigma_S)}_{\text{effective mass } m^*})q + G_S(\bar{q}q)^2 + G_{PS}\left(\bar{q}i\vec{\tau}\gamma_5 q\right)^2 + \ldots$$

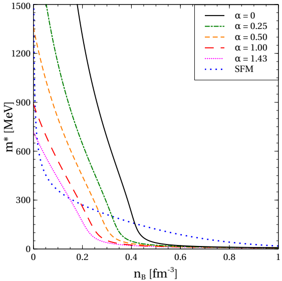
- **Similarities:**
  - current-current interaction
  - (pseudo)scalar, vector, diquark, ... channels
  - the same vacuum phenomenology
- **Differences:**
  - $\kappa = 1/3 \neq \kappa_{NJL} = 1$
  - high m^* at low T, $\mu \Rightarrow$ **"confinement"**

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2\kappa D_0\langle\bar{q}q\rangle_0}{(\alpha\langle\bar{q}q\rangle_0^2)^{1-\kappa}} \xrightarrow{\alpha\to 0} \infty$$

  - medium dependent couplings :

$$\begin{array}{lll} \text{low } T,\ \mu, \Rightarrow & G_S \neq G_{PS} & \Rightarrow \chi\text{-broken} \\ \text{high } T,\ \mu, \Rightarrow & G_S = G_{PS} & \Rightarrow \chi\text{-symmetric} \end{array}$$

T = 0

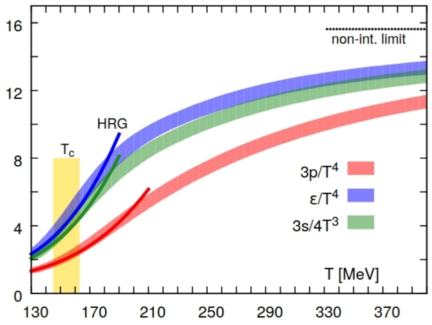


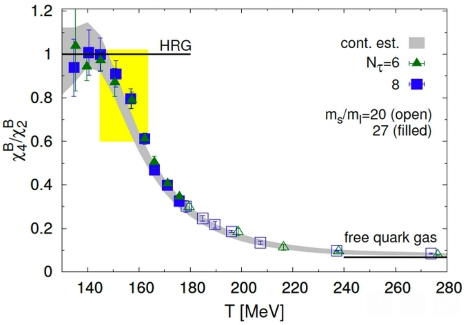
- **Dimensionality**

$$\begin{array}{l} [\mathcal{U}] = energy^4 \\ [\bar{q}q] = energy^3 \end{array} \quad\Rightarrow\quad [D_0]_{\varkappa=1/3} = energy^2 = \left[\begin{array}{c} string \\ tension \end{array}\right]$$

self energy = [string tension]×separation ⇒ "confinement"?

Smooth chiral crossover, ...



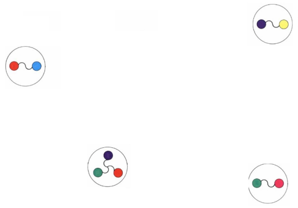


Bazavov et al., JHEP (2010)

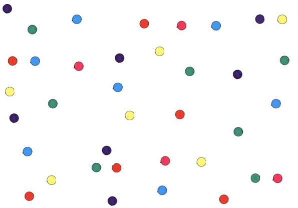
Bazavov et al., PRD (2009)

..., catastrophic rearrangement of wave function

Hadronic gas

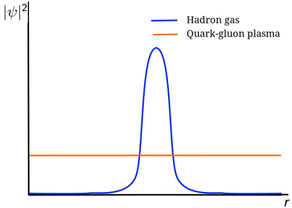


Quark-gluon plasma

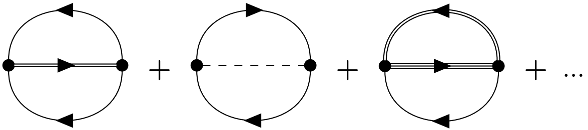


Can a catasyrophic rearrangement of ψ lead to a smooth thermodynamics?

A unified quark-hadron approach is needed



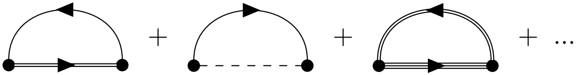
Cluster decomposition in two-loop approximation

$\varphi =$ 

G. Baym, L.P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, Phys. Rev. 127, 1391 (1962)

- **Two-loop self-energies & Dyson-Schwinger propagators**

$$\Pi_n = \frac{\delta\varphi}{\delta S_n}, \quad (S_n)^{-1} = (S_n^{free})^{-1} - \Pi_n, \quad n = 1, 2, 3, \ldots$$

quarks: $\Pi_1 =$ 

Dyson-Schwinger problem requires solving all S_n simultaneously

- **Mean-field approximation for quark propagators**

The Dyson-Schwinger problem reduces to a subsequent solving S_n using $S_{m<n}$

$$\Omega = \Omega_{\textbf{quarks}} + \underbrace{\mathcal{U}_\chi - \langle \overline{q}\hat{\Sigma} q\rangle}_{\textbf{condensates}} + \mathcal{U}_\Phi + \underbrace{\Omega_{\textbf{hadrons}} + \Omega_{\textbf{colored clusters}}}_{\textbf{multiquark clusters}}$$

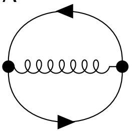
- **Quarks** coupled to Φ
  - Non-perturbative states at low momenta $k < \Lambda$

$$\Omega^{k<\Lambda}_{quarks} = -\frac{1}{\beta V} Tr \ln(\beta S^{-1}_{quarks})$$

S_{quarks} - quark propagator @ mean-field

  - Perturbative states at high momenta $k > \Lambda$

$$\Omega^{k>\Lambda}_{quarks} = \frac{1}{2\beta V}$$



J.I. Kapusta, Finite Temperature Field Theory, Cambridge (1989)

- **Gluons** - Polyakov loop potential $\mathcal{U}_\Phi$

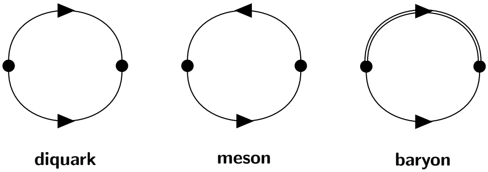
P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

- **Hadrons** - 62 mesons, 60+60 (anti)baryons states with $M < 2.6$ GeV
- **Colored multiquark states** - diquarks, tetraquarks, pentaquarks coupled to Φ

Generalized Beth-Uhlenbeck approach for hadrons

- **Large size clusters as correlations of the smaller size ones $\Rightarrow$ propagators**

polarization loops :



- **Phase shift of multiquark clusters**

$$S_n = |S_n| e^{i\delta_n} \quad \Rightarrow \quad \delta_n = \Im \ln S_n$$

(parameterized based on the results of calculation for pions)

- **Generalized Beth-Uhlenbeck formula**

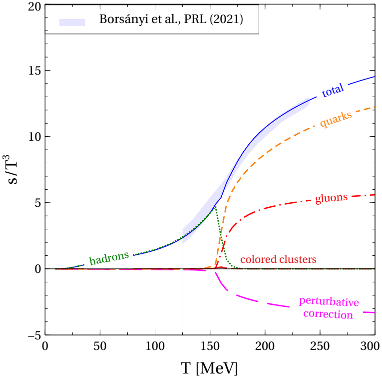
$$\Omega_n = \frac{d_n}{\kappa_n} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} (tr_D)^{2-\kappa_n} \ln(\beta^{\kappa_n} S_n^{-1}) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}$$

$\kappa_n = 1$ - fermions, $\kappa_n = 2$ - bosons

G. Röpke, N.U. Bastian, D. Blaschke, T. Klähn, S. Typel, H. Wolter, NPA 897 (2013)

$$s = -\frac{\partial \Omega}{\partial T}$$

- **Low T**
  1. hadron dominance
- **High T**
  1. quark-gluon dominance
  2. negative perturbative contribution
- **Colored multiquark states ($\mu_B = 0$ only)**
  1. suppressed by the Polyakov loop at high T
  2. suppressed by high mass at high T



Chiral condensate

$$\langle \bar{f} f \rangle = -\frac{\partial \Omega}{\partial m_f} = \underbrace{2N_c \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{M_f}{E_f}(f_f^+ + f_f^- - 1)}_{\textbf{quarks}}$$

$$+ \underbrace{\sum_{n>1} \frac{d_n \sigma_n^f}{m_f} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{\pi} \frac{M_n}{\omega} (f_n^+ + f_n^-) \sin^2 \delta_n \frac{\partial \delta_n}{\partial \omega}}_{\textbf{multiquark clusters}}$$

- **σ-factor**
  1. σ_π^f, σ_K^f – defined from the GMOR relations
  2. other multiquark clusters

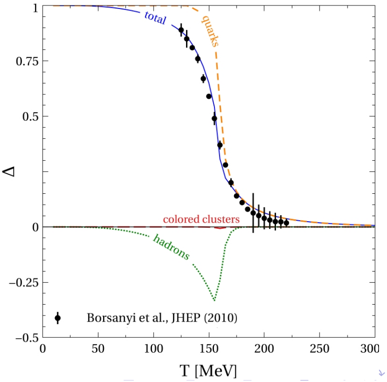
$$\sigma_n^f = m_f \frac{\partial M_n}{\partial m_f} = m_f N_n^f$$

J. Jankowski, D. Blaschke, M. Spalinski, PRD 87, 10 (2013)

- **Scaled chiral condensate**

$$\Delta = \frac{m_s \langle \bar{l} l \rangle - m_l \langle \bar{s} s \rangle}{m_s \langle \bar{l} l \rangle_0 - m_l \langle \bar{s} s \rangle_0}$$

Almost constant quark term below T_c
Hadrons are necessary to reproduce the lQCD data



Cumulants and composition

$$\frac{p}{T^4} = \sum_n \frac{1}{n!} \left(\frac{\mu_B}{T}\right)^n \chi_n^B, \quad \chi_n^B = T^{n-4} \frac{\partial^4 p}{\partial \mu_B^4}$$

- **Boltzmann hadron gas**

$$\frac{\chi_4^B}{\chi_2^B} = 1$$

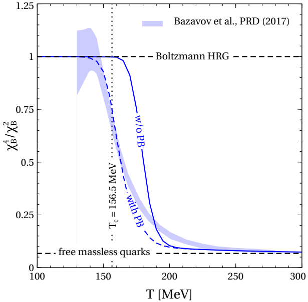
- **Free massless quarks**

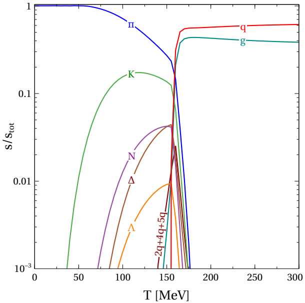
$$\frac{\chi_4^B}{\chi_2^B} = \frac{1}{N_c^2} \cdot \frac{6}{\pi^2}$$

- χ_4^B/χ_2^B **probes the quark-to-baryon ratio?**

Only asymptotically

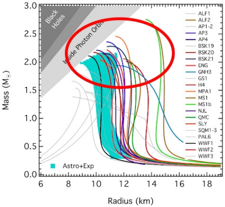
Evidences a repulsive interaction among baryons





Density functional for color-superconducting quark matter

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U}$$



- **Vector repulsion**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2$$

  - needed to reach $2M_\odot$
  - motivated by non-perturbative gluon exchange

- **Diquark pairing**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q} i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

  - motivated by Cooper theorem
  - color superconductivity

- **Mean-field thermodynamic potential**

$$\Omega = -\frac{1}{2\beta V} Tr\ln(\beta\hat{S}^{-1}) - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D} + \mathcal{U}_{MF} - \langle\overline{q}q\rangle\Sigma_S$$

$$\hat{S}^{-1} = \begin{pmatrix} \hat{\mathcal{S}}_+^{-1} & i\Delta_A\gamma_5\tau_2\lambda_A \\ i\Delta_A^*\gamma_5\tau_2\lambda_A & \hat{\mathcal{S}}_-^{-1} \end{pmatrix}, \quad \hat{\mathcal{S}}_\pm^{-1} = i\partial\!\!\!/ - m^* - \sigma - i\gamma^5\vec{\pi}\cdot\vec{\tau} \pm (\gamma_0\hat{\mu} + \omega\!\!\!/)$$

Hybrid quark-hadron EoS of NSs

- **Charge neutrality:** electrons
- β**-equilibrium:**

$$\mu_d = \mu_u + \mu_e$$

- **Hadron EoS:** DD2

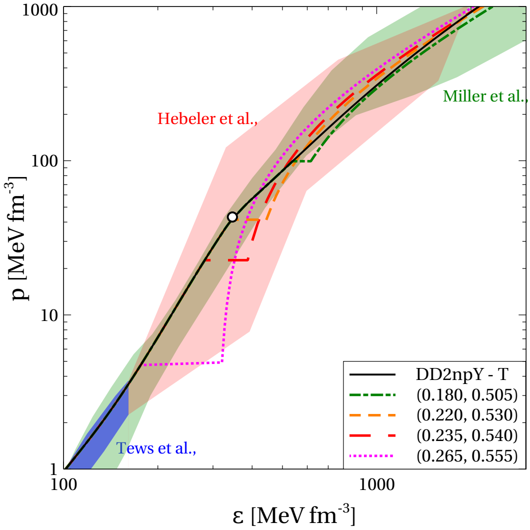
S. Typel et al., PRC 81, 015803 (2010)

- **Maxwell construction:**

$$p_q(\mu_B^c) = p_h(\mu_B^c)$$

- **Model labeling:**

$$(\eta_V, \eta_D), \quad \eta_{V,D} \equiv \frac{G_{V,D}}{G_S|_{T=\mu=0}}$$



O. Ivanytskyi and D. Blaschke, Phys. Rev. D 105, 114042 (2022)

Mass radius diagram of NSs

- **Hydrostatic equilibrium**

$$\frac{dp}{dr} = -\frac{Gm\epsilon}{r^2}\frac{\left(1+\frac{p}{\epsilon}\right)\left(1+\frac{4\pi r^3 p}{m}\right)}{\left(1-\frac{2Gm}{r}\right)}$$

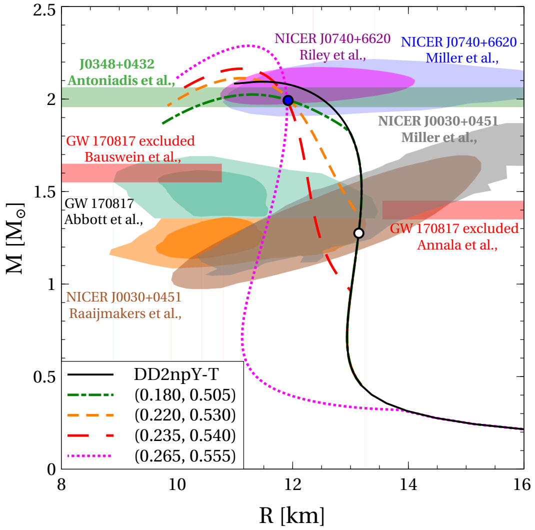
$$\frac{dm}{dr} = 4\pi r^2 \epsilon$$

- **Cold matter EoS**

$$\begin{cases} p = p(\mu_B) \\ \epsilon = \mu_B \frac{\partial p}{\partial \mu_B} - p \end{cases} \Rightarrow p = p(\epsilon)$$

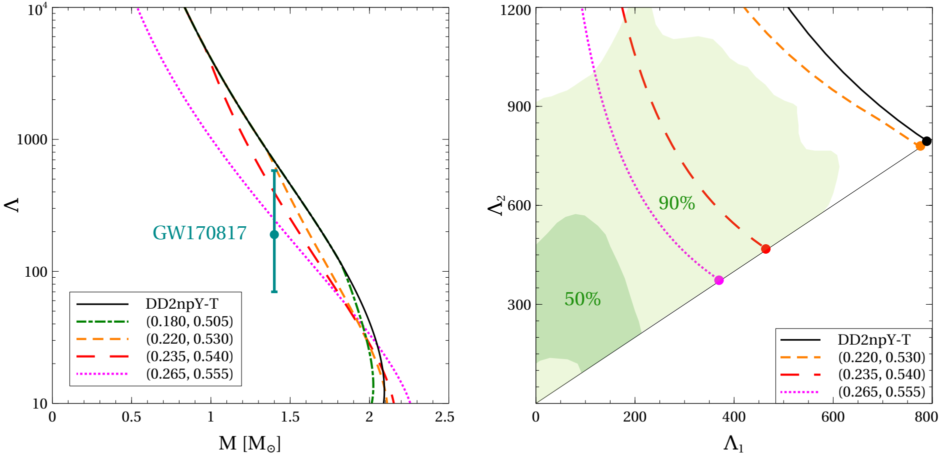
- **Total radius R and mass M**

$$r = R \quad \Rightarrow \begin{cases} p = 0 \\ m = M \end{cases}$$



O. Ivanytskyi and D. Blaschke, Phys. Rev. D 105, 114042 (2022)

Tidal deformability of NSs



O. Ivanytskyi and D. Blaschke, Phys. Rev. D 105, 114042 (2022)

Phase diagram

- **Normal quark matter**

  2 spin $\times$ 2 flavor $\times$ 3 color $=12$

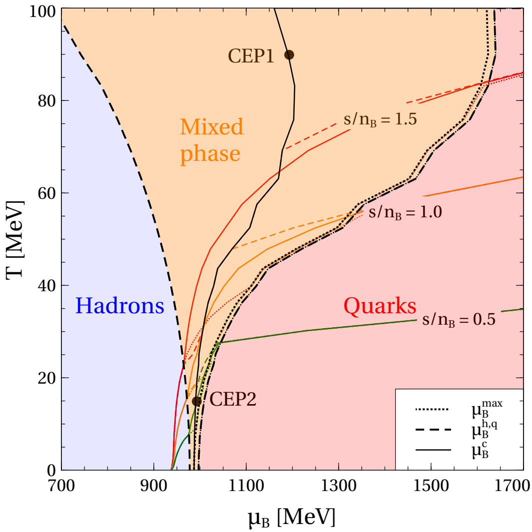
- **2SC quark matter**

  2 spin $\times$ 2 flavor $\times$ 1 color$+1=5$

Quark pairing reduces number of quark states

$\Downarrow$

requires higher T along adiabat



Conclusions

- A phenomenological "confinement" mechanism
- A unified EoS of strongly interacting matter based on a cluster decomposition approach
- Agreement with the lattice QCD data on entropy density and chiral condensate

  (EPJ A 60 (2024) for baryon density)
- Sudden switching between partonic and hadronic degrees of freedom
- Agreement with the astrophysical data on neutron stars

Confining density functional with Polyakov loop @ $N_f = 3$

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ + gA\!\!\!/ - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = diag(m_u, m_d, m_s)$$

A_μ - homogeneous static gluon field in the Polyakov gauge

- **Density functional**

$$\mathcal{U}_\chi = D_0 \left[(1+\alpha)\langle\hat{\mathcal{O}}\rangle_0 - \hat{\mathcal{O}}\right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2}\sum_{a=0,8}\left[(\overline{q}\tau_a q)^2 + (\overline{q}i\gamma_5\tau_a q)^2\right]$$

D. Blaschke, O. Ivanytskyi, M. Shahrbaf, 2202.05061 [nucl-th]

- **Polyakov loop potential**

$$\Phi = \frac{1}{N_c} Tr_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b\log M_H + \frac{1}{2}c\left(\Phi^3 + \bar{\Phi}^3\right) + d\left(\bar{\Phi}\Phi\right)^2$$

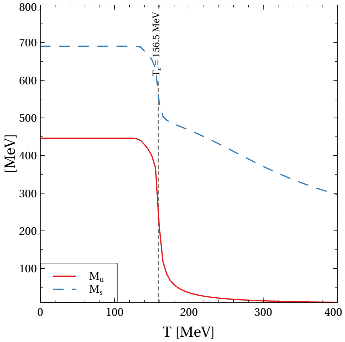
T-dependence of $a, b, c,\ d$ is fitted to the pure SU(3) gauge lattice data

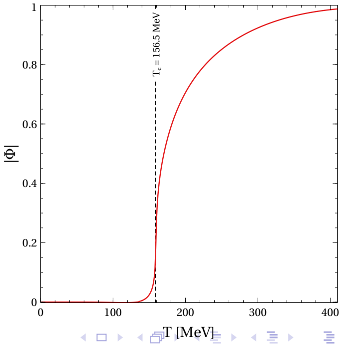
P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

- **Fitting vacuum phenomenology**

$$\begin{cases} M_\pi = 140\ MeV \\ F_\pi = 93\ MeV \\ M_K = 494\ MeV \\ F_K = 112\ MeV \\ T_c = 156.5\ MeV \end{cases} \Rightarrow \quad \begin{array}{l} m_u = m_d = 4.4\ MeV \\ m_s = 134.8\ MeV \\ \Lambda = 636.1\ MeV \\ \sqrt{D_0} = 729.6\ MeV \\ \alpha = 1.44 \end{array}$$

- **Effective masses and Polyakov loop**





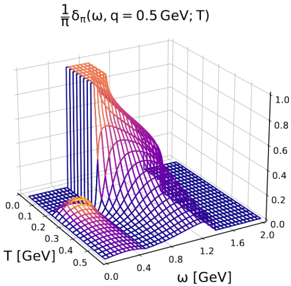
Phase shifts of multiquark states

- **Microscopic calculations for pions**

  K. Maslov, D. Blaschke, PRD 107, 094010 (2023)

  1. discontinuous jump at the on-shell energy below the dissociation temperature
  2. continuous growth at small energies above the dissociation temperature
  3. continuous fall above the decay threshold
  4. vanishing at high energies (Levonson's theorem)

  M. Wellner, Am. J. Phys. 32, 787–789 (1964)

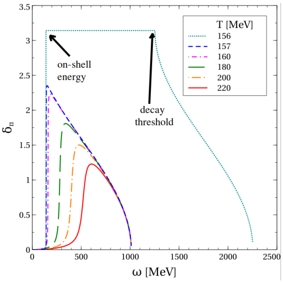


- **Parametric model of $\delta = \delta(\mathbf{T}, \omega)$**

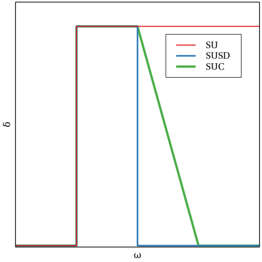
  D. Blaschke, M. Cierniak, O. Ivanytskyi, G. Röpke, EPJ A 60 (2024)

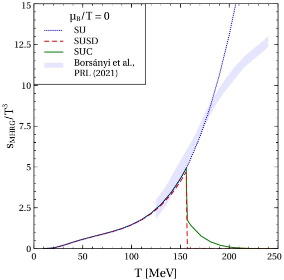
  1. parametric expression reproduces all the properties of the microscopic calculations
  2. T-dependence of the hadron masses & widths agree with the microscopic calculations
  3. requires hadron decay threshold given by quark masses $M_{u,d,s}$

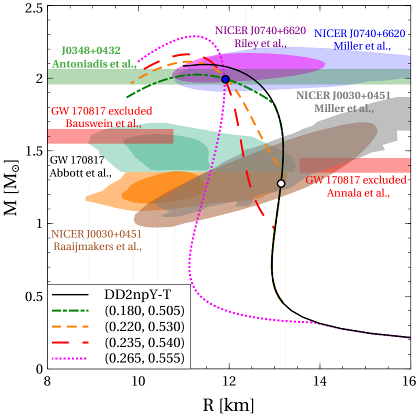
$$M_h^{\mathrm{Th}} = N_h^u M_u + N_h^d M_d + N_h^s M_s$$

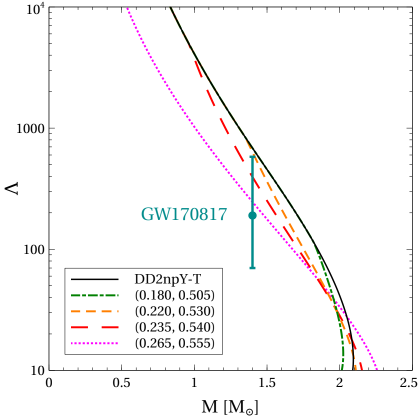


- **Step-up (SU)**
  1. is generated by the pole of $S_{n>1}$
  2. corresponds to a bound multiquark state
  3. is present only below the dissociation temperature
  4. generates a HRG-like term in Ω
- **Step-down (SUSD)**
  1. rough account of the decay threshold
  2. partially/totally compensates HRG-like term in Ω
- **Continuum (SUC)**
  1. corresponds to a scattering multiquark state
  2. partially compensates HRG-like term in Ω









O.Ivanytskyi, D. Blaschke, PRD 105, 114042 (2022)

Agreement with the observational constraints

on mass-radius relation and tidal deformability of neutron stars

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ + gA\!\!\!/ - \hat{m})q - \mathcal{U}_\chi - \mathcal{U}_\Phi, \quad \hat{m} = diag(m_u, m_d, m_s)$$

A_μ - homogeneous static gluon field in the Polyakov gauge

- **Density functional**

$$\mathcal{U}_\chi = D_0 \left[(1+\alpha)\langle\hat{\mathcal{O}}\rangle_0 - \hat{\mathcal{O}}\right]^{1/3}, \quad \hat{\mathcal{O}} = \frac{1}{2}\sum_{a=0,8}\left[(\overline{q}\tau_a q)^2 + (\overline{q}i\gamma_5\tau_a q)^2\right]$$

D. Blaschke, O. Ivanytskyi, M. Shahrbaf, 2202.05061 [nucl-th]

- **Polyakov loop potential**

$$\Phi = \frac{1}{N_c} Tr_c \exp(i\beta A_0), \quad M_H = 1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2$$

$$\frac{\mathcal{U}_\Phi}{T^4} = -\frac{1}{2}a\bar{\Phi}\Phi + b\log M_H + \frac{1}{2}c\left(\Phi^3 + \bar{\Phi}^3\right) + d\left(\bar{\Phi}\Phi\right)^2$$

T-dependence of a, b, c, d is fitted to the pure SU(3) gauge lattice data

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich, C. Sasaki, Phys. Rev. D 88, 074502 (2013)

Expansion around mean-field solution @ $N_f = 3$

$$\mathcal{U}_\chi = \underbrace{\mathcal{U}_\chi^{MF}}_{\textbf{0}^{\textbf{th}}\ \textbf{order}} + \underbrace{\overline{q}\hat{\Sigma}q - \langle\overline{q}\hat{\Sigma}q\rangle}_{\textbf{1}^{\textbf{st}}\ \textbf{order}}$$

$$\underbrace{-\sum_{f,f'}\left(\overline{f}f - \langle\overline{f}f\rangle\right) G_S^{ff'}\left(\overline{f'}f' - \langle\overline{f'}f'\rangle\right) - G_{PS}\sum_f\left(\overline{f}i\gamma_5 f\right)^2}_{\textbf{2}^{\textbf{nd}}\ \textbf{order}} + \ldots$$

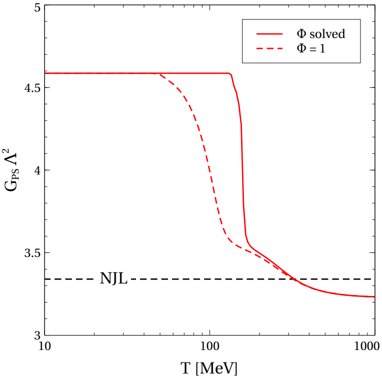
- **Mean-field scalar self-energy**

$$\hat{\Sigma} = diag(\Sigma_u, \Sigma_d, \Sigma_s), \quad \Sigma_f = \frac{\partial\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle}$$

- **Effective medium dependent couplings**

$$G_S^{ff'} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}f\rangle\partial\langle\overline{f'}f\rangle}$$

$$G_{PS} = -\frac{1}{2}\frac{\partial^2\mathcal{U}_\chi^{MF}}{\partial\langle\overline{f}i\gamma_5 f\rangle^2}$$

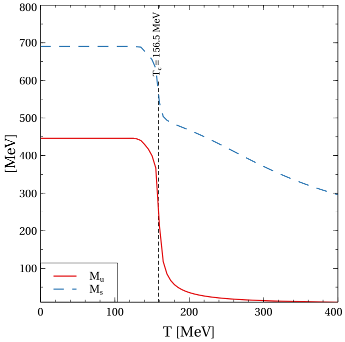


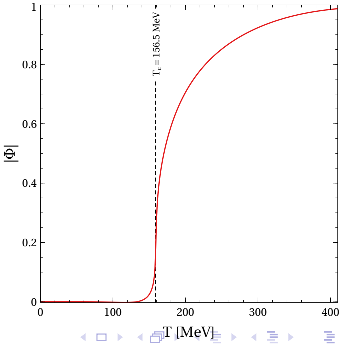
RDF setup

- **Fitting vacuum phenomenology**

$$\begin{cases} M_\pi = 140 \ MeV \\ F_\pi = 93 \ MeV \\ M_K = 494 \ MeV \\ F_K = 112 \ MeV \\ T_c = 156.5 \ MeV \end{cases} \Rightarrow \quad \begin{array}{l} m_u = m_d = 4.4 \ MeV \\ m_s = 134.8 \ MeV \\ \Lambda = 636.1 \ MeV \\ \sqrt{D_0} = 729.6 \ MeV \\ \alpha = 1.44 \end{array}$$

- **Effective masses and Polyakov loop**





Mass-spectrum

$$M_{n>1} = M_{n>1}^{vacuum} + A(T - T_c)\theta(T - T_c)$$

$$\Gamma_{n>1} = B\theta(T - T_c)$$

$T_c = 156.6$ MeV, A, B - fitted to lQCD

- **Low T (χ-broken matter)**
  1. heavy quarks
  2. stable multiquark clusters
  3. constant mass of multiquark states
  4. zero width of multiquark states

- **High T (χ-symmetric matter)**
  1. light quarks
  2. unstable multiquark clusters
  3. growing mass of multiquark states
  4. growing width of multiquark states