HOT AND DENSE QCD FROM FUNCTIONAL METHODS

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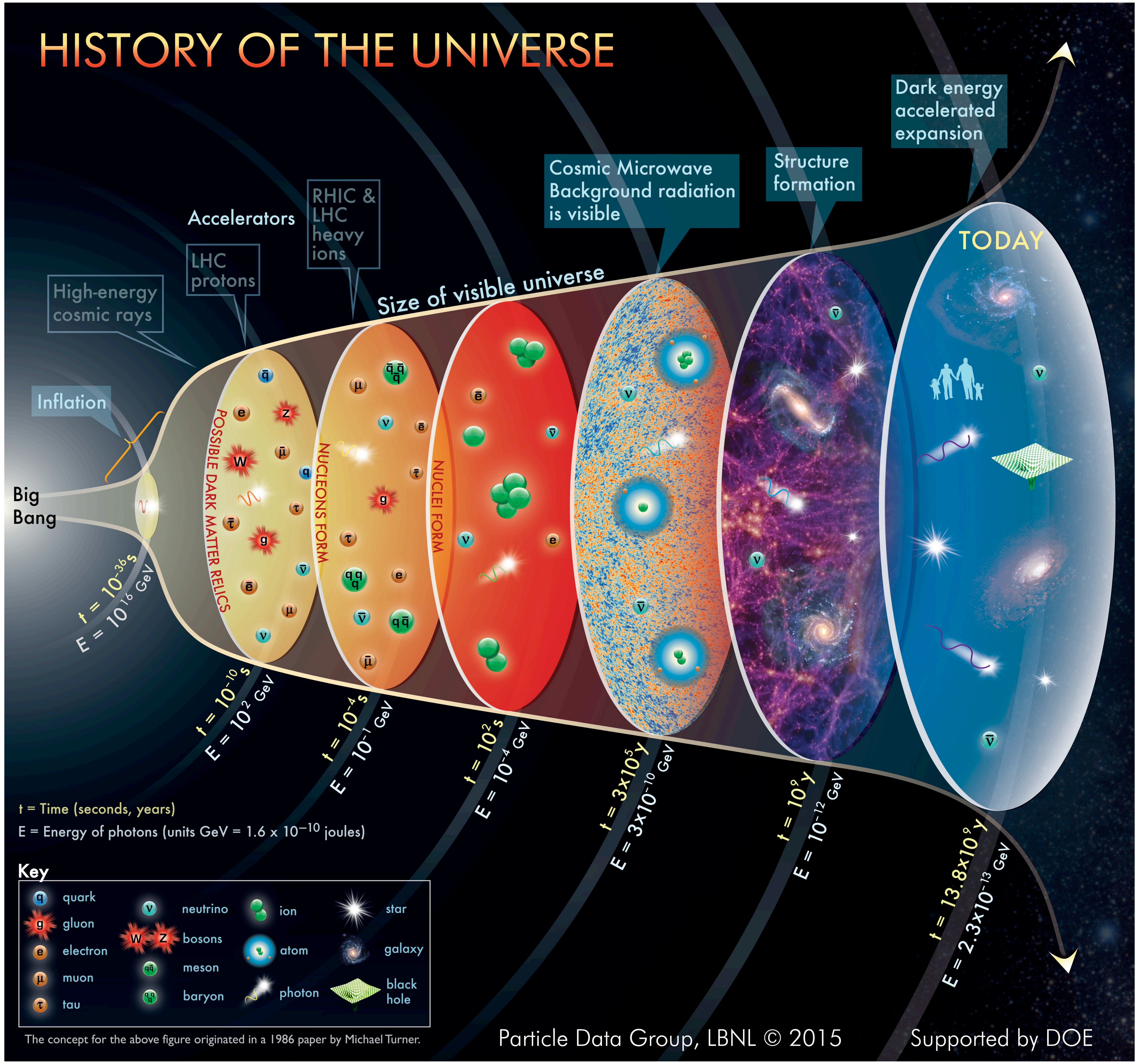
ERICE - 17/09/2025

UNDERSTAND MATTER IN EXTREME CONDITIONS

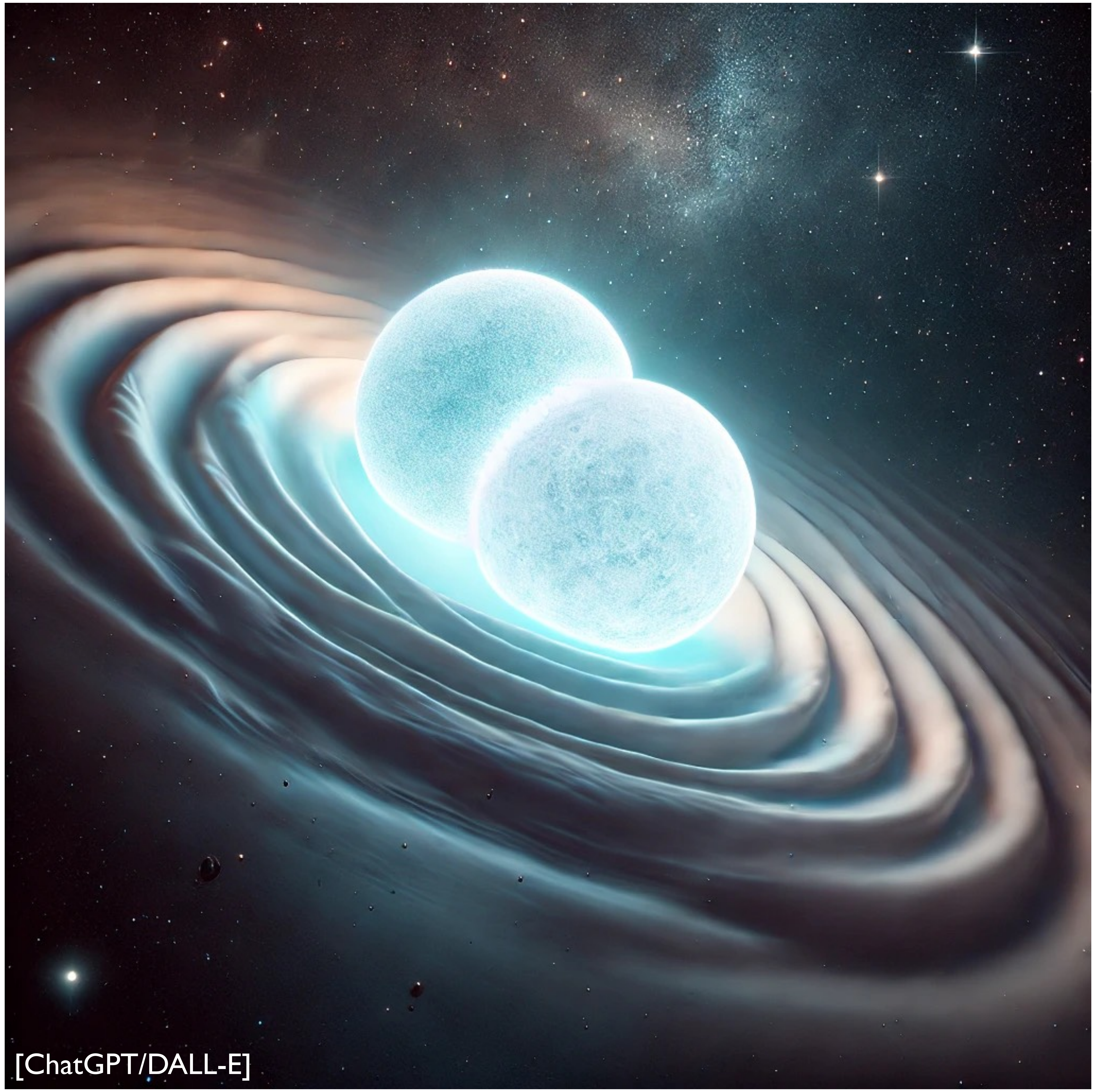


universe before the formation of the CMB ($t \lesssim 3 \times 10^5\,\mathrm{y}, T \gtrsim 1\,\mathrm{eV}$) is invisible to us; nucleons formed during this time

↓

study hot QCD matter to understand formation of nuclear matter in the universe

UNDERSTAND MATTER IN EXTREME CONDITIONS



neutron stars can reach densities of several times the nuclear saturation density ($n \approx 5n_0, \mu_B \approx 1.5\,\mathrm{GeV}$)

↓

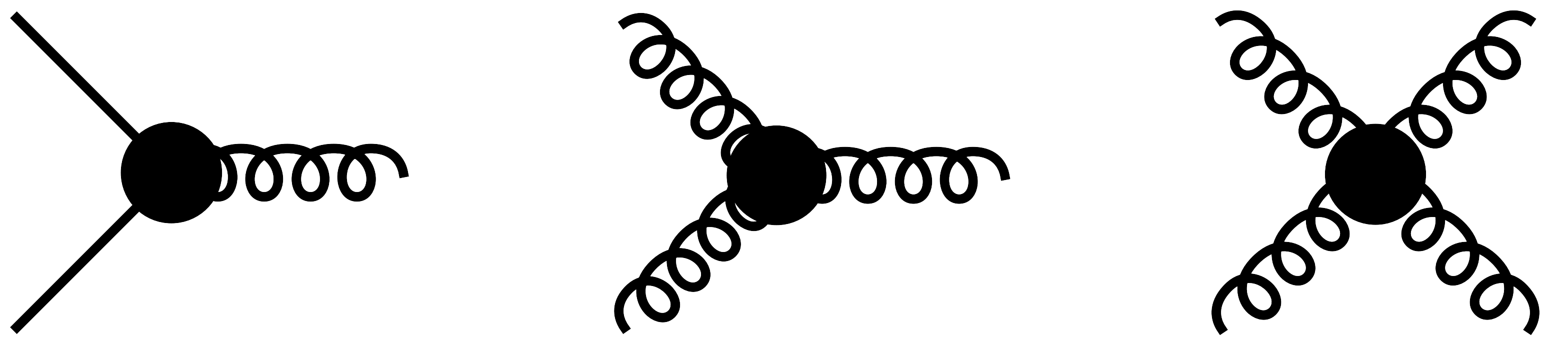
study dense QCD matter to understand neutron stars & their mergers

QCD

$SU(N_c = 3)$ gauge theory with $N_f = 6$ flavors of quarks

$$\mathscr{L}_{\text{QCD}} = \bar{q}\left(\gamma_\mu D_\mu - m_q\right) q - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

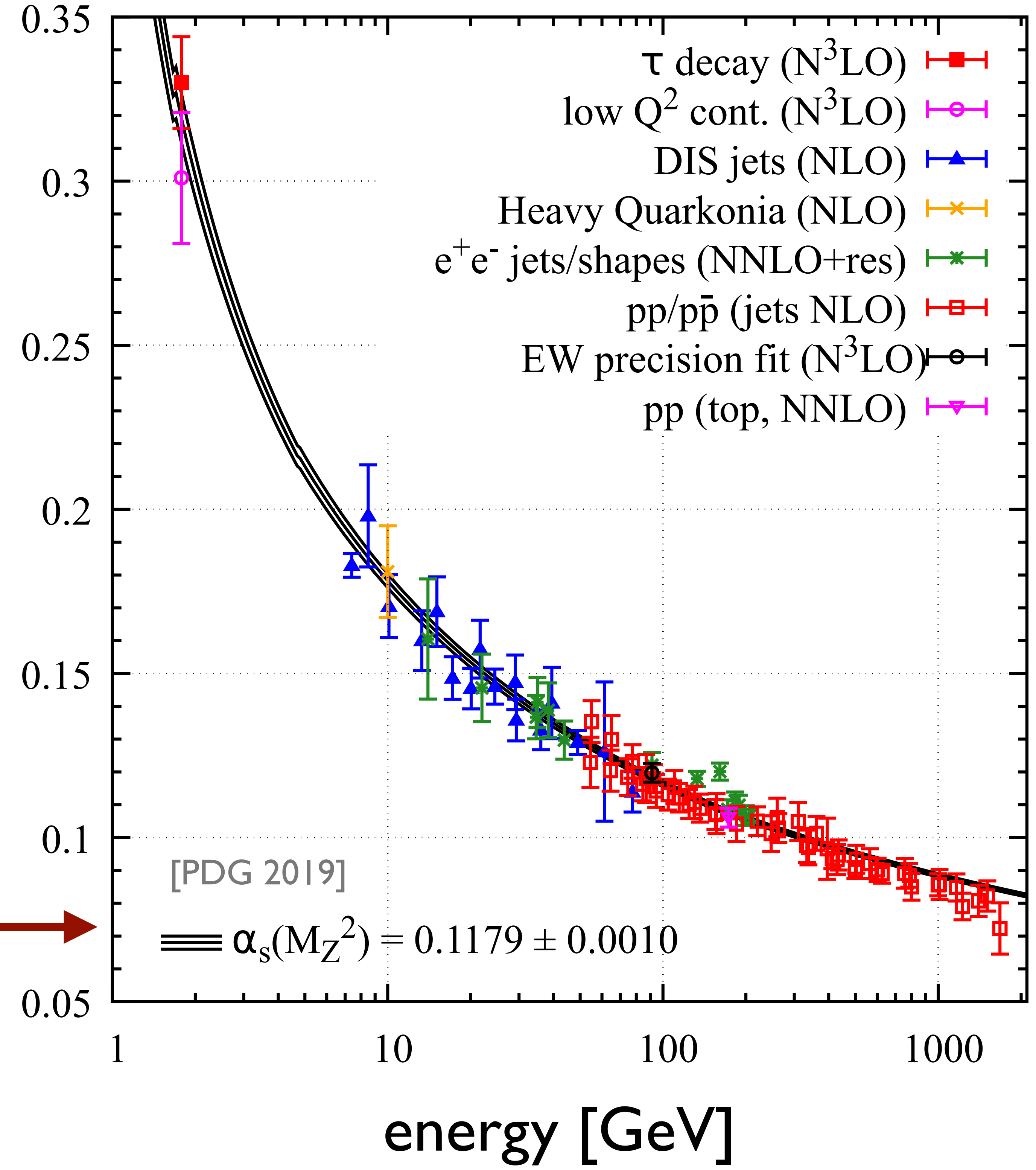
microscopic interactions:



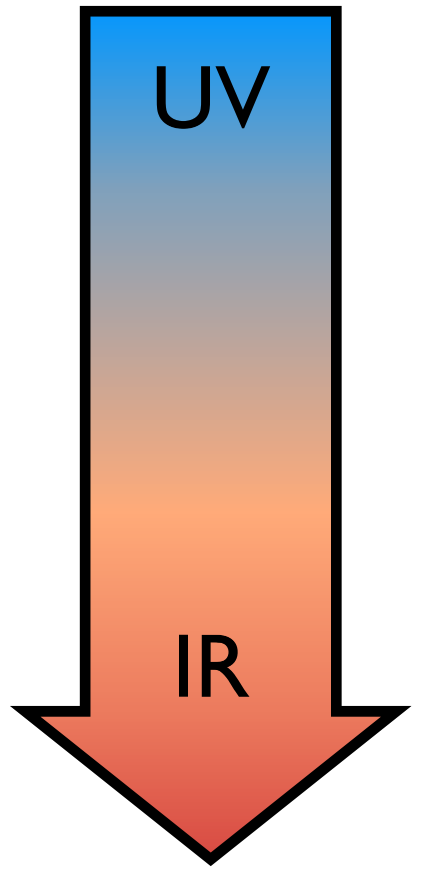
- innocent looking, but still not understood to a large extent
- key feature: **asymptotic freedom**

$$\alpha_s = \frac{g^2}{4\pi}$$

$$10 \times \frac{1}{137} \rightarrow$$



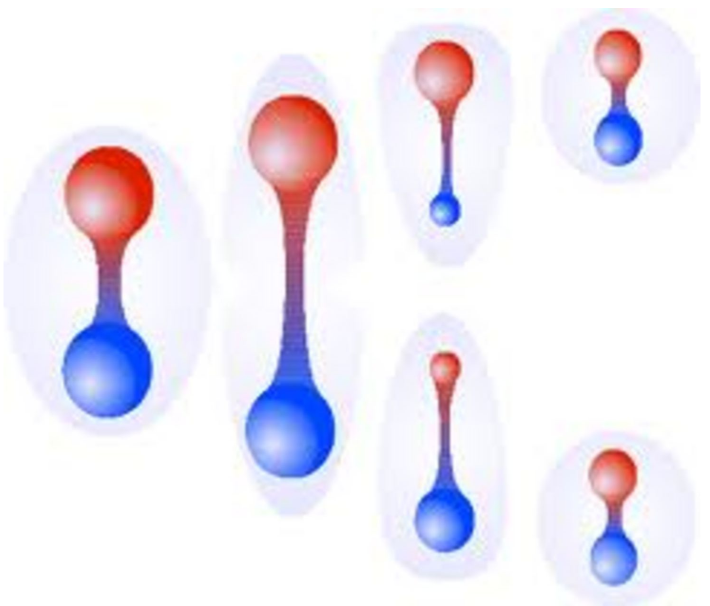
perturbative physics
quarks & gluons



non-perturbative
many-body physics
bound states & condensates

- confinement:

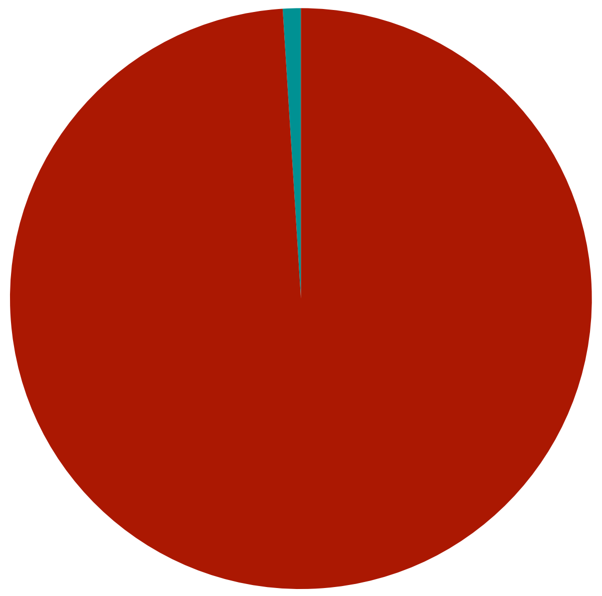
Millennium
Prize Problem



- origin of mass: chiral symmetry breaking & scale anomaly

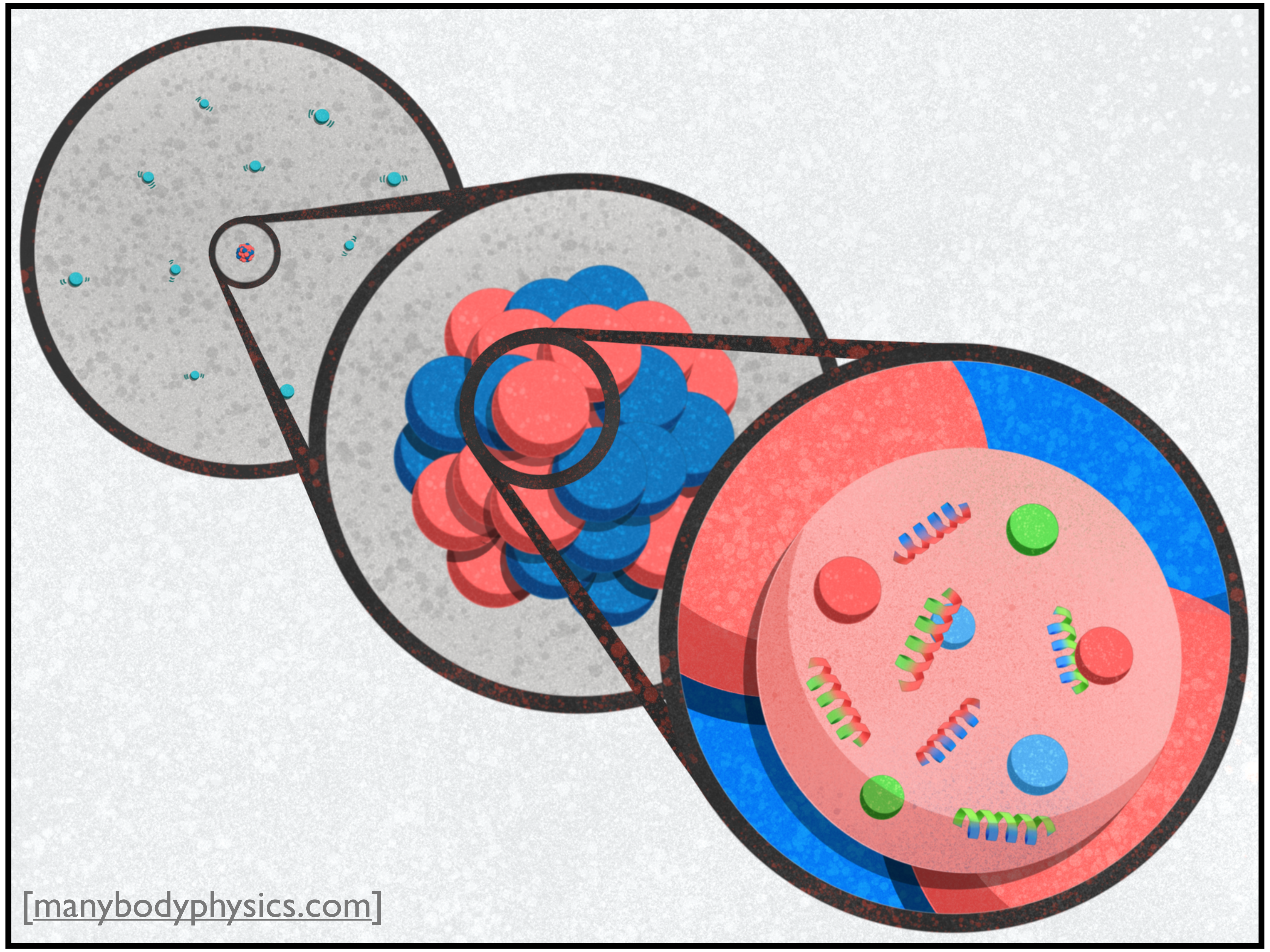
chiral symmetry: left- and right-handed fermions are independent (do not mix)

nucleon mass:



THE PHYSICS OF SCALES

different degrees of freedom at different energy scales



$\lesssim 1$ eV: matter

≈ 1 keV: atoms

≈ 1 MeV: nuclei

≈ 100 MeV: nucleons/pions (hadrons)

$\gtrsim 1$ GeV: quarks & gluons

more general: constituents vs. bound states/condensates/collective excitations

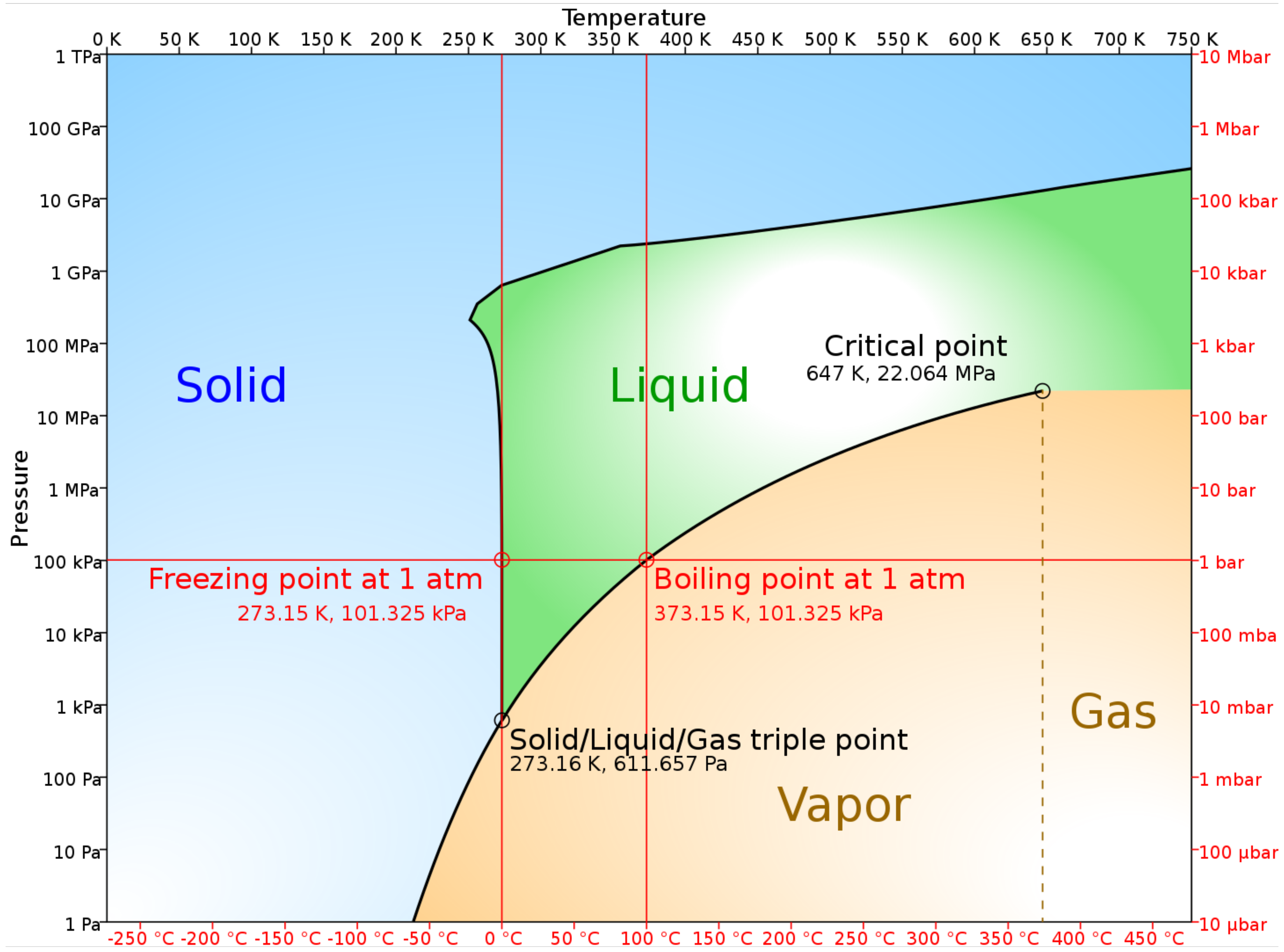
THE QCD PHASE DIAGRAM

The energy scale can be set by external parameters

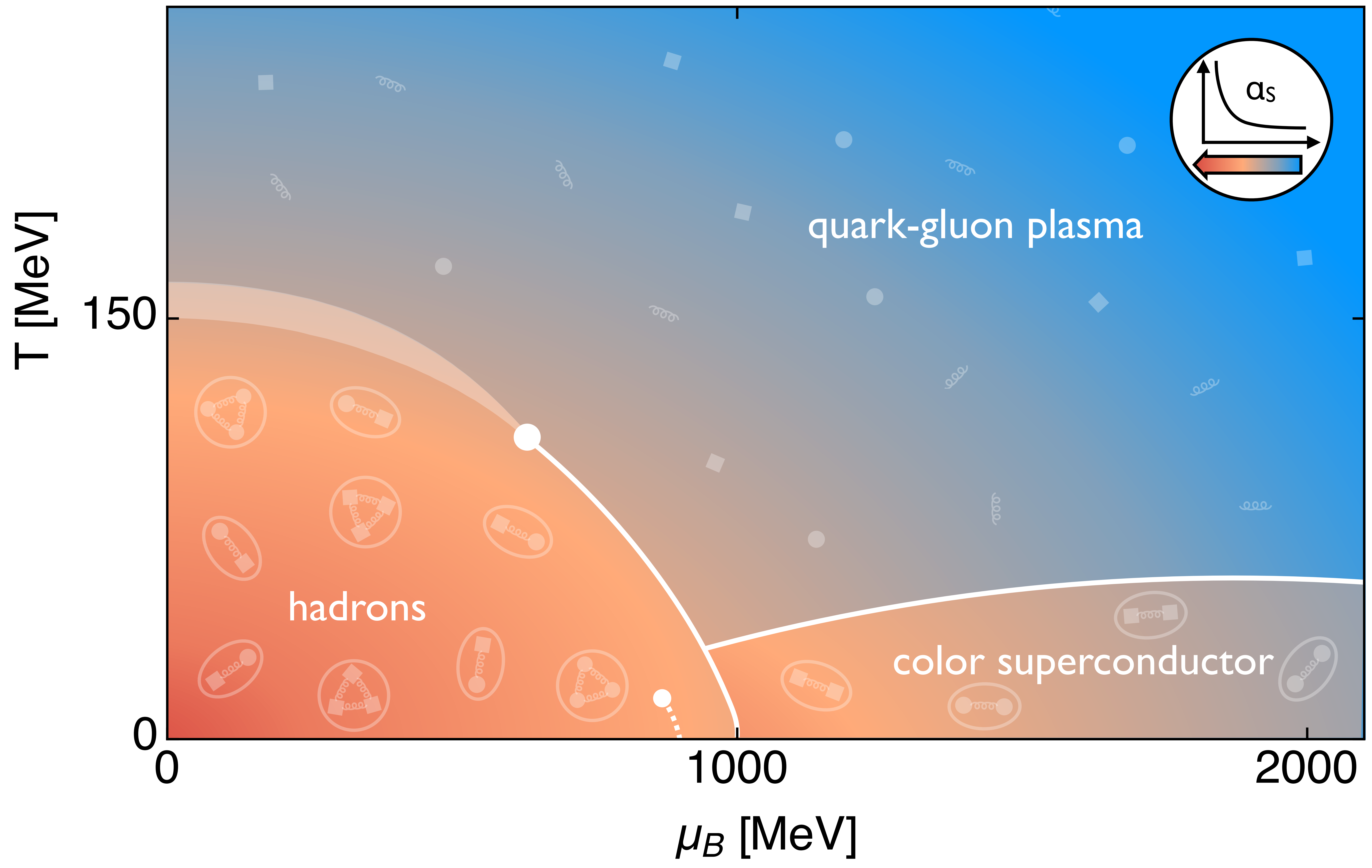
- temperature
- various chemical potentials/densities
- magnetic field
- angular momentum

→ phase diagram

Example: T-P diagram of water



QCD PHASE DIAGRAM



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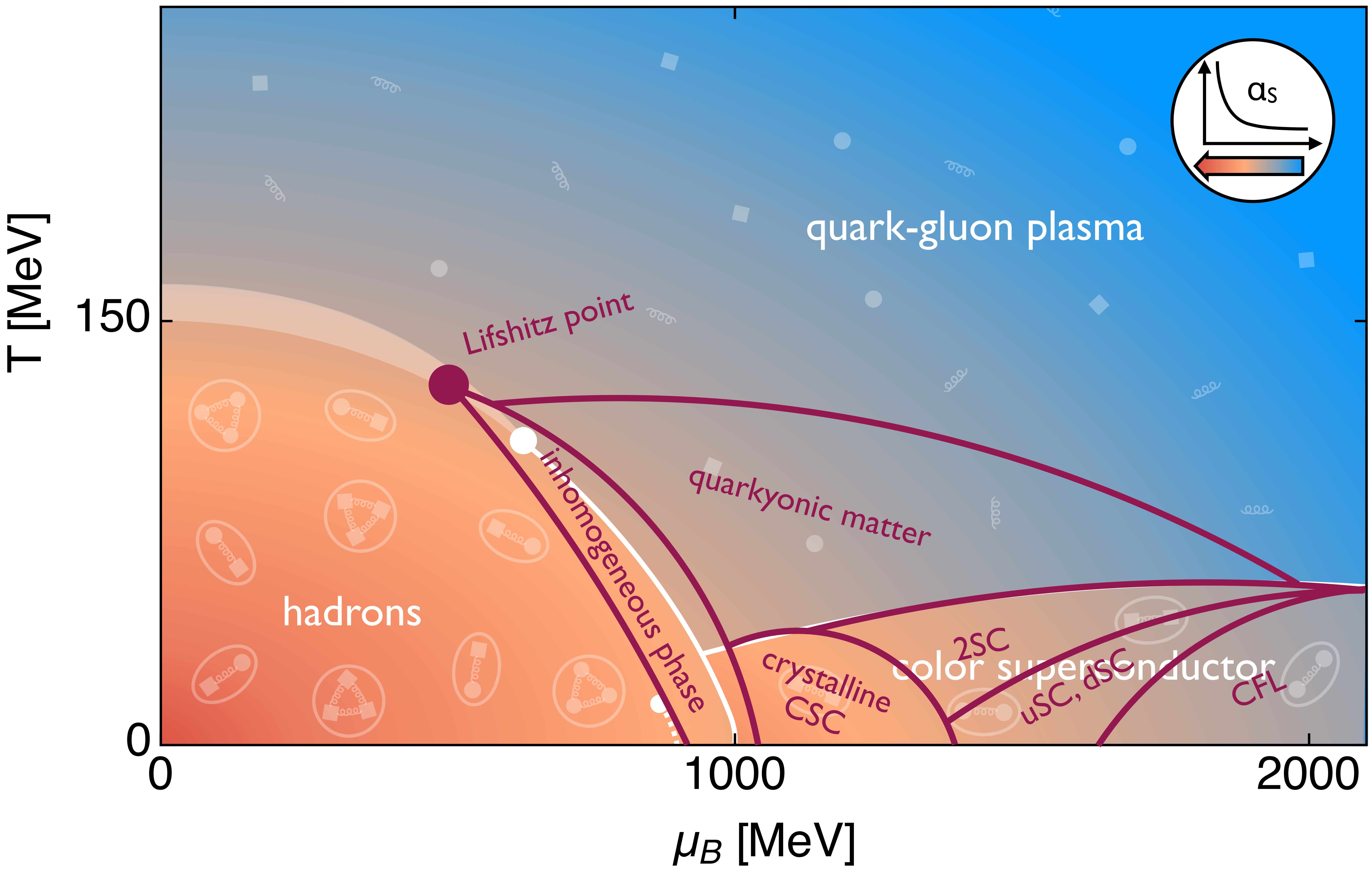

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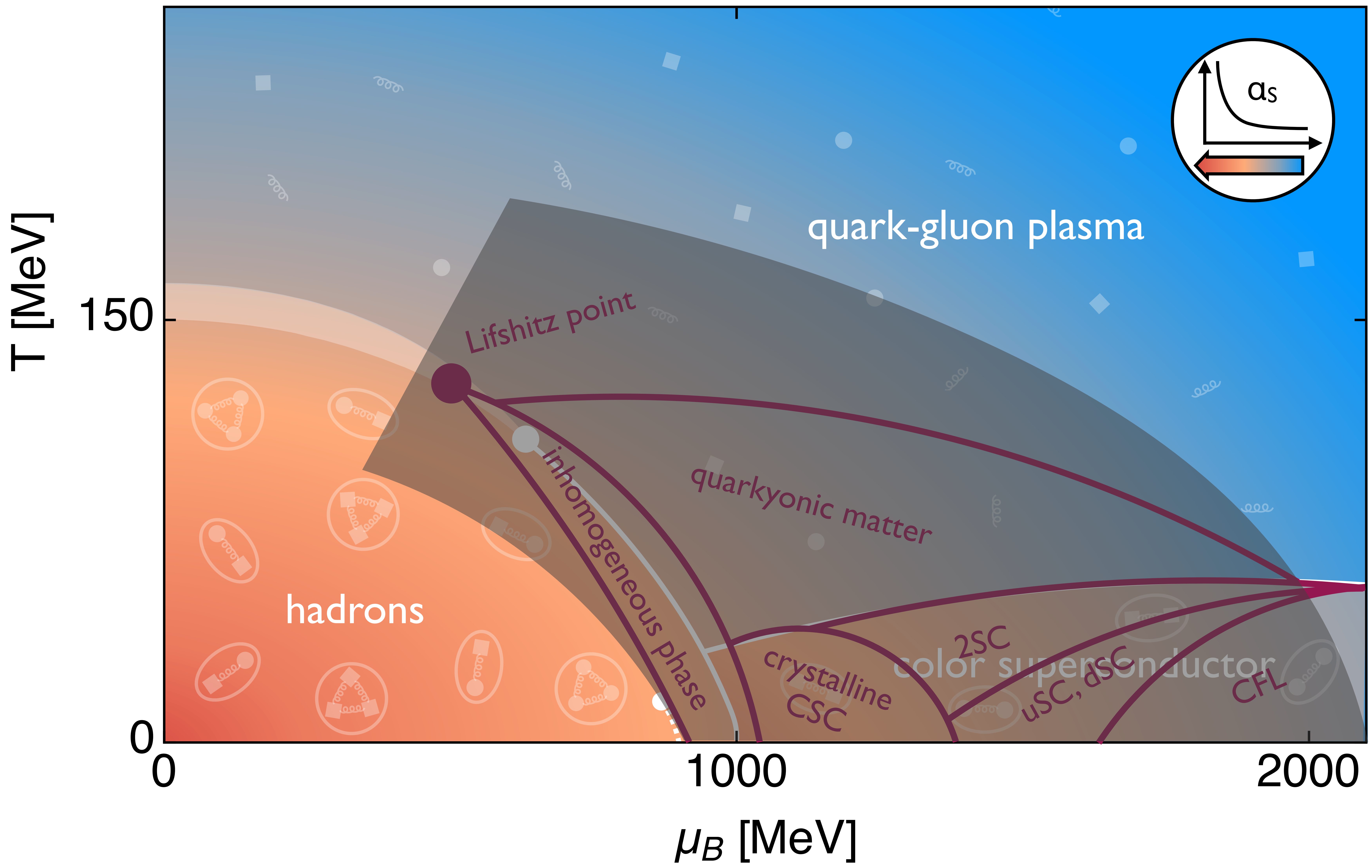


theoretical challenges:

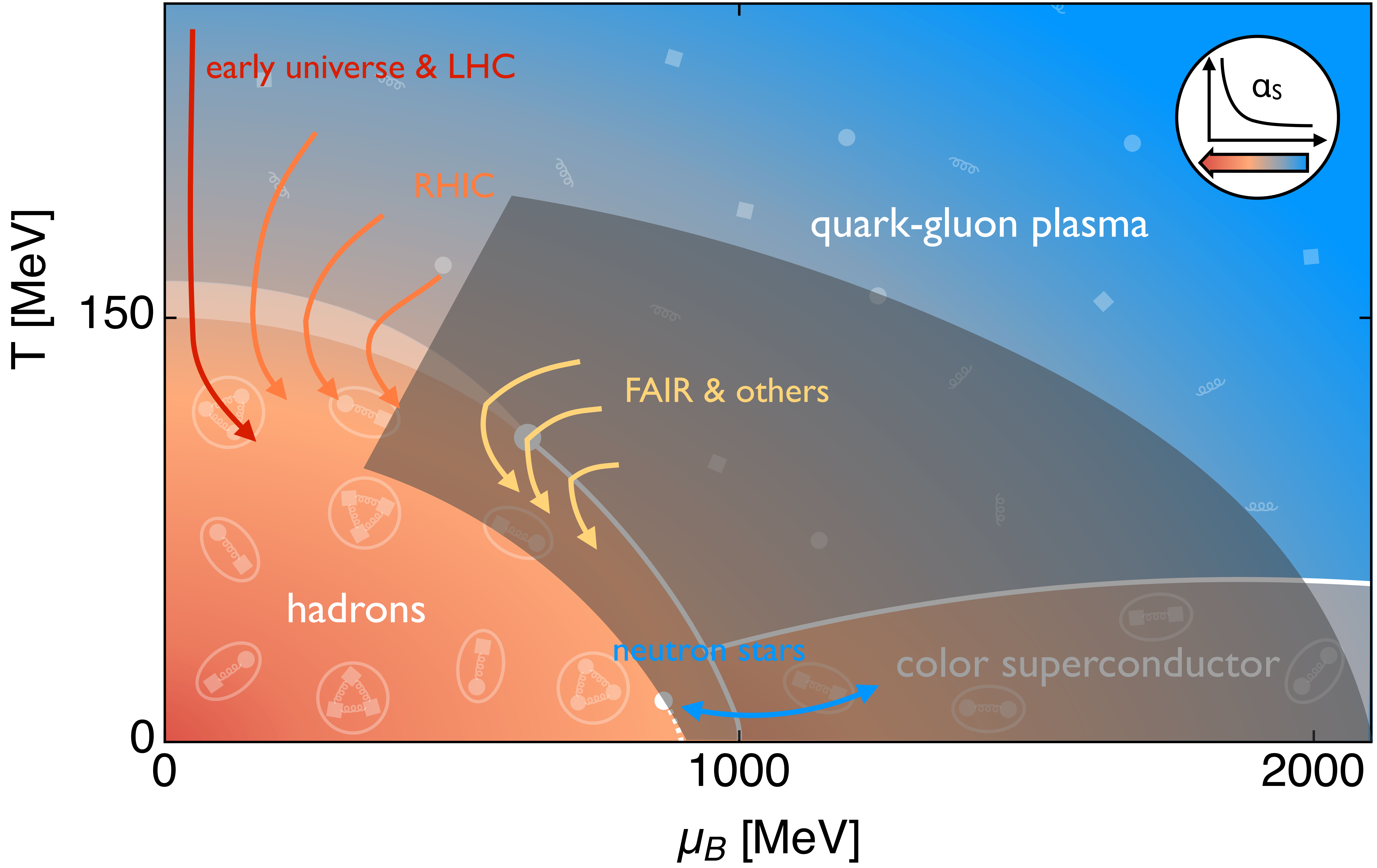
- strong coupling: non-perturbative
- sign problem at finite density: lattice QCD of limited use
- different degrees of freedom in different phases: EFTs of limited use

use functional methods*

(or wait for quantum computers)

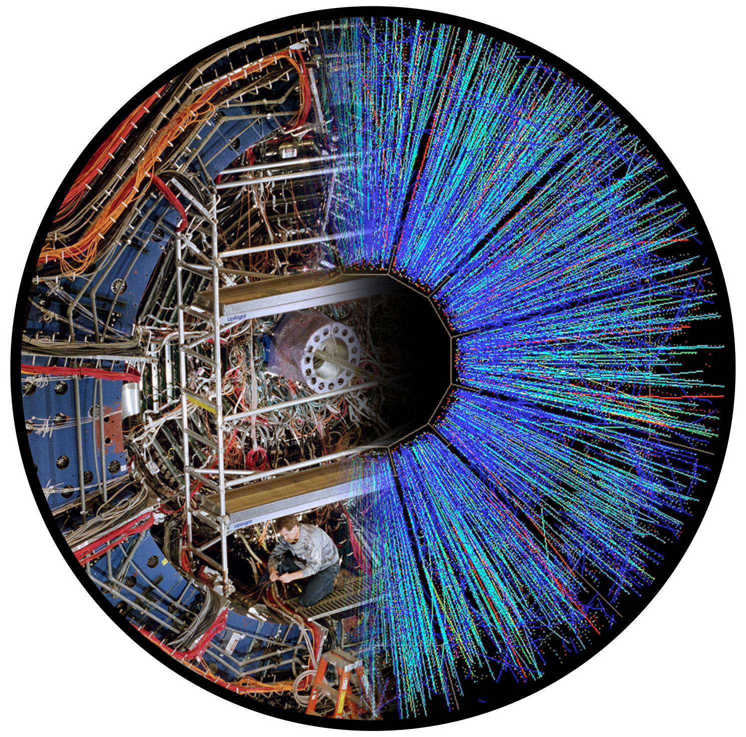
* functional renormalization group (FRG) & Dyson-Schwinger equations (DSE)

QCD PHASE DIAGRAM

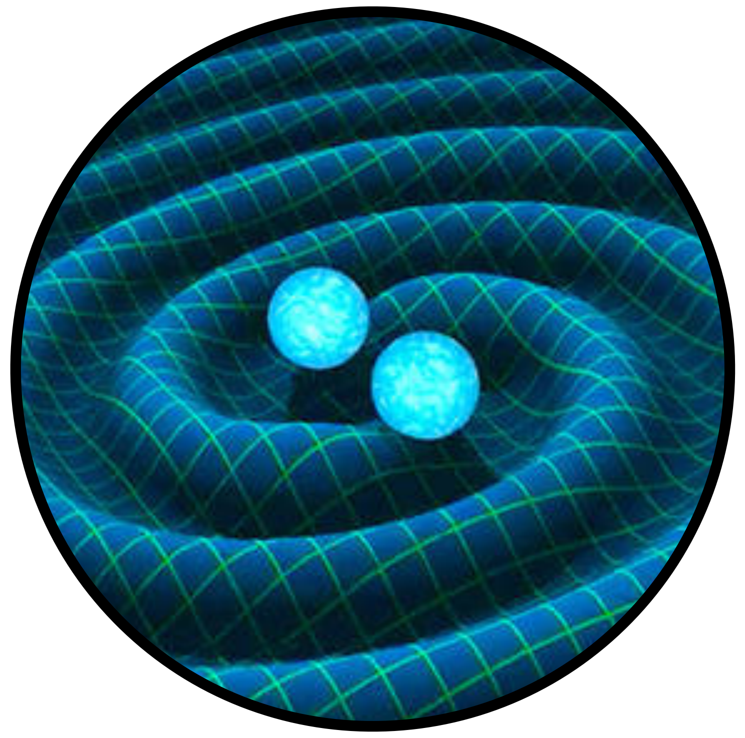


Experiments:

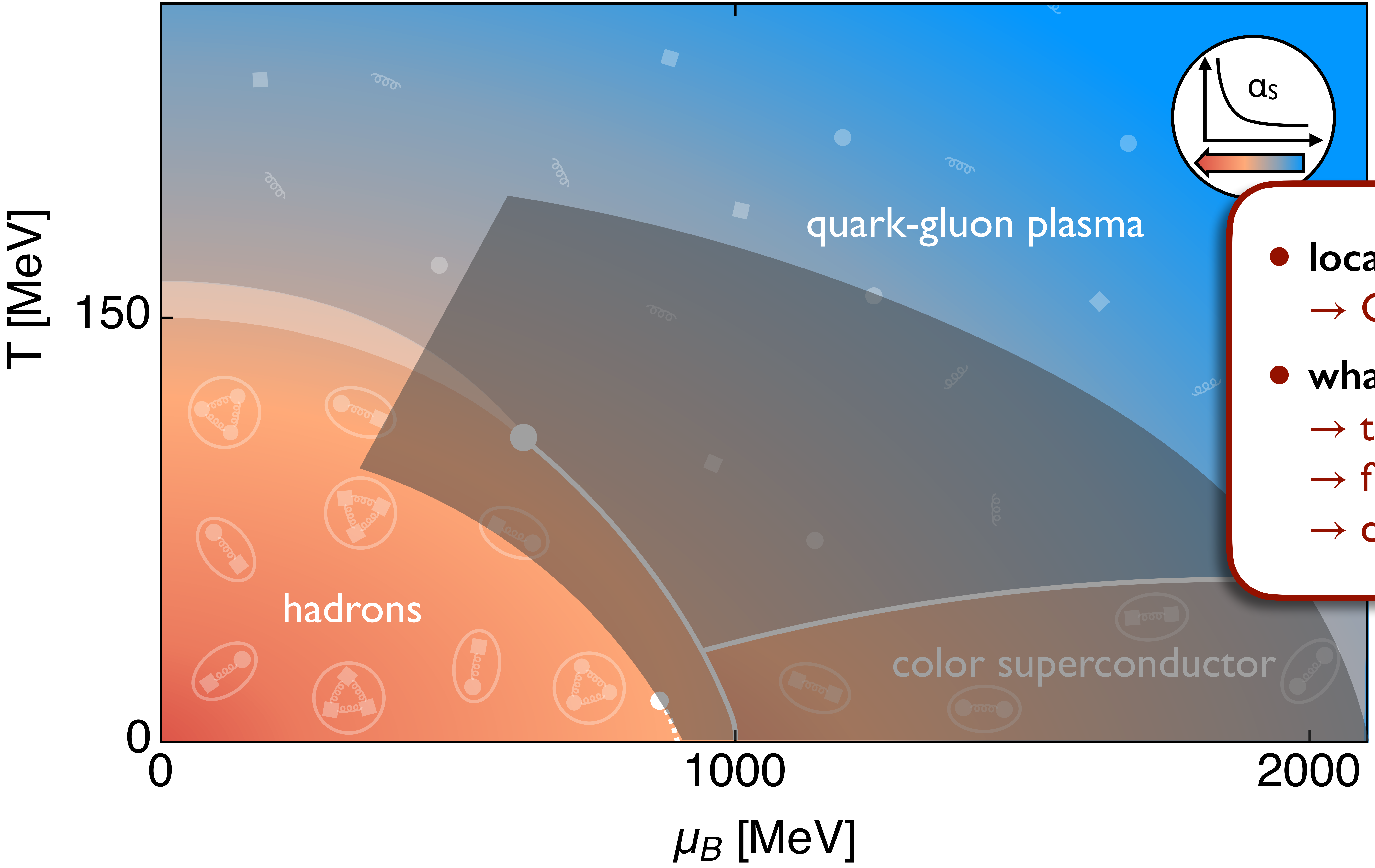
heavy-ion collisions



multi-messenger astronomy



QCD PHASE DIAGRAM IN THIS TALK



- **location of phase transitions?**
  → CEP & chiral transition
- **what happens at large μ_B?**
  → the moat regime + implications
  → finite density EoS
  → color-superconductivity

FUNCTIONAL METHODS

FUNCTIONAL METHODS

The path integral encodes all possible correlation functions of a QFT

$$Z[J] = \int \mathscr{D}\varphi \, e^{iS[\varphi] + i\int_x J(x)\varphi(x)}$$

$$\langle \varphi \cdots \varphi \rangle \sim \frac{\delta}{\delta J} \cdots \frac{\delta}{\delta J} Z[J] \Big|_{J=0}$$

solving a QFT ⇔
knowing all correlation functions

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solving a QFT ⇔
knowing all correlation functions

functional methods provide exact relations for correlation functions

Dyson-Schwinger equations (DSE)

"quantum equations of motion"

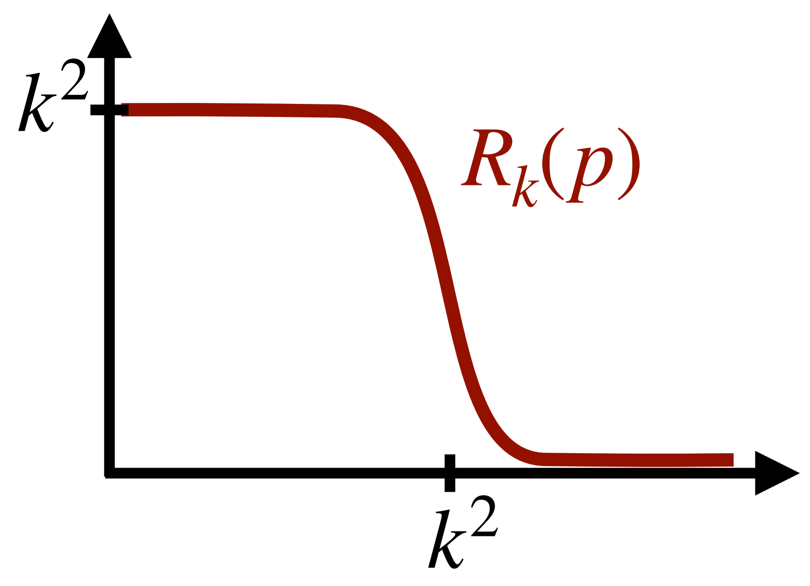
$$\int \mathscr{D}\varphi \left(\frac{\delta S[\varphi]}{\delta \varphi(x)} + J(x) \right) e^{iS[\varphi] + i\int_x J(x)\varphi(x)} = 0$$

functional renormalization group (FRG)

successively integrate out quantum fluctuations

$$Z_k[J] = \int \mathscr{D}\varphi\, e^{i\left(S[\varphi] + \Delta S_k[\phi]\right) + i\int_x J(x)\varphi(x)}$$

$$\Delta S_k[\varphi] = \int_p \frac{1}{2} \varphi(p)\, R_k(p)\, \varphi(-p)$$



specify theory/model by choice of microscopic action $S[\varphi]$

$S[\varphi] = S_{QCD}[q, \bar{q}, A]$: QCD from first principles

FUNCTIONAL METHODS

- define infinite towers of coupled equations for all correlation functions: truncations necessary
- no small parameter in many cases, but apparent hierarchy from low- to high-order correlations
- no sign problem: finite density, real time and complex parameter spaces are all directly accessible
- one/two-loop exact: both intuition and techniques can be leveraged

QCD related reviews:

FRG

[Pawlowski, 0512261]
[Gies, 0611146]
[Rosten, 1003.1366]
[Braun, 1108.4449]
[Dupuis at al., 2006.04853]
[Fu, 2205.00468]

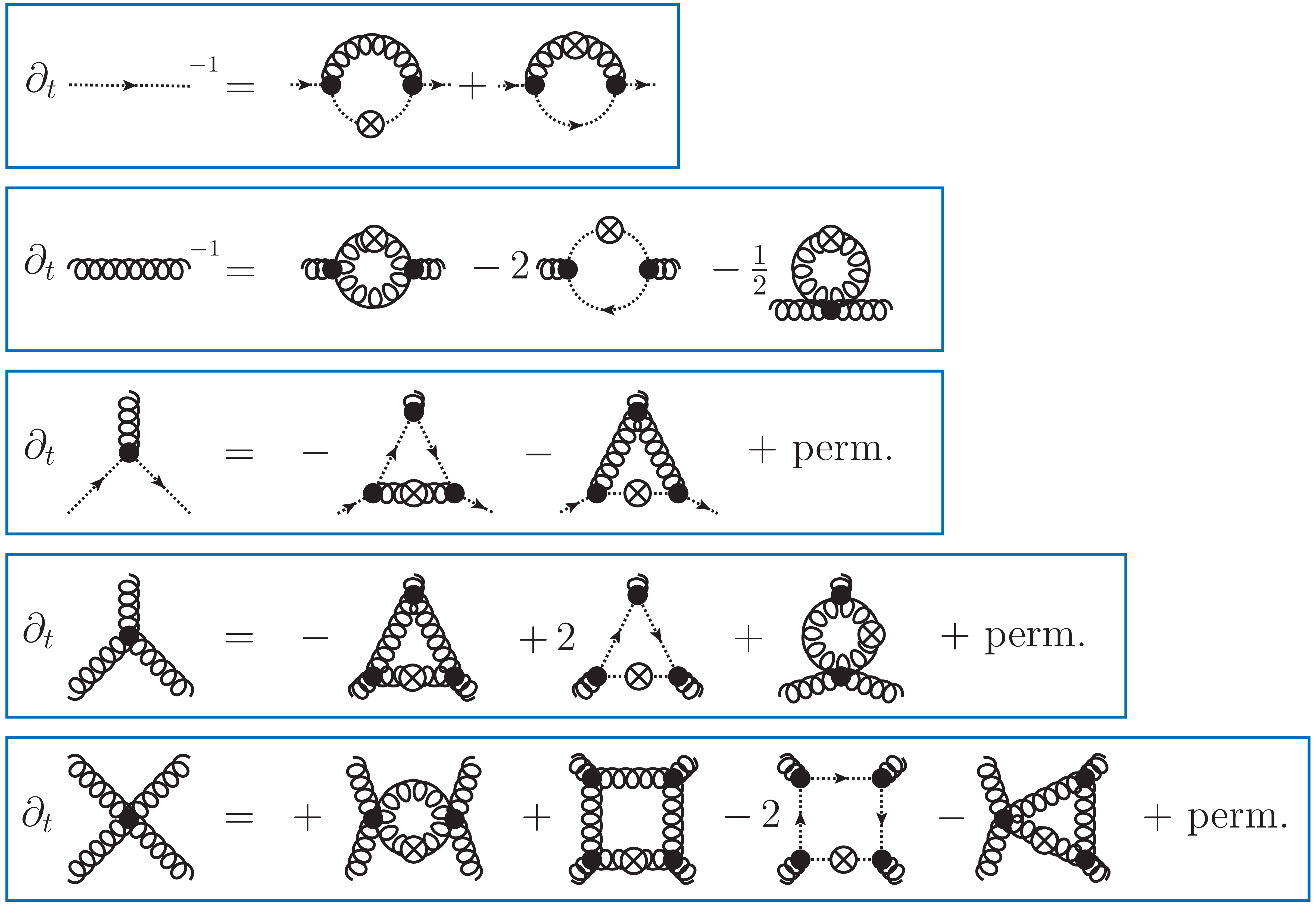
DSE

[Alkofer, von Smekal, 0007355]
[Fischer, 0605173]
[Roberts, Schmidt, 0005064]
[Eichmann at al, 1606.09602]
[Fischer, 1810.12938]
[Huber, 1808.05227]

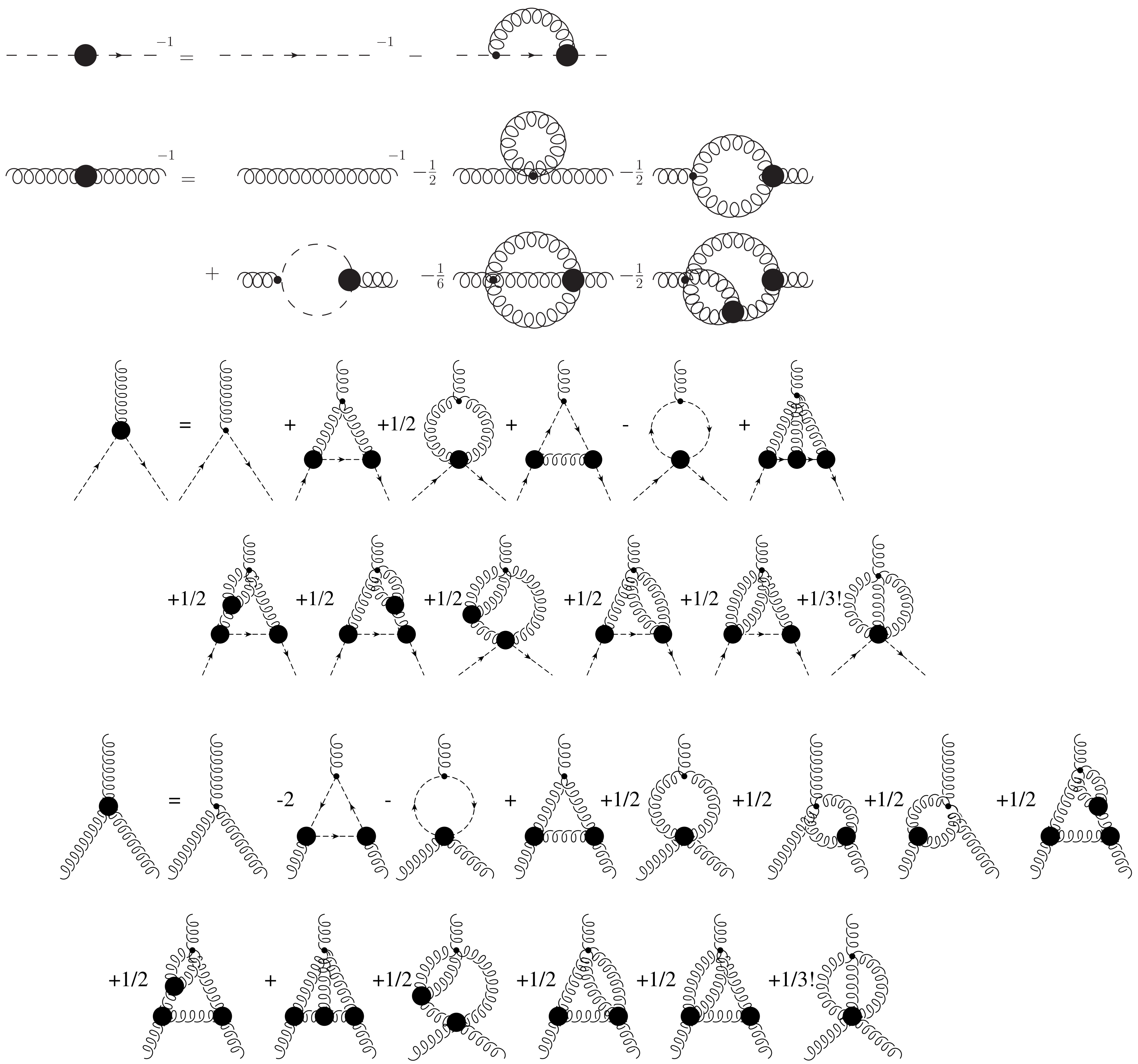
FUNCTIONAL QCD - A GLIMPSE

What we solve - in the gauge sector

FRG [Cyrol et al, 1605.01856]



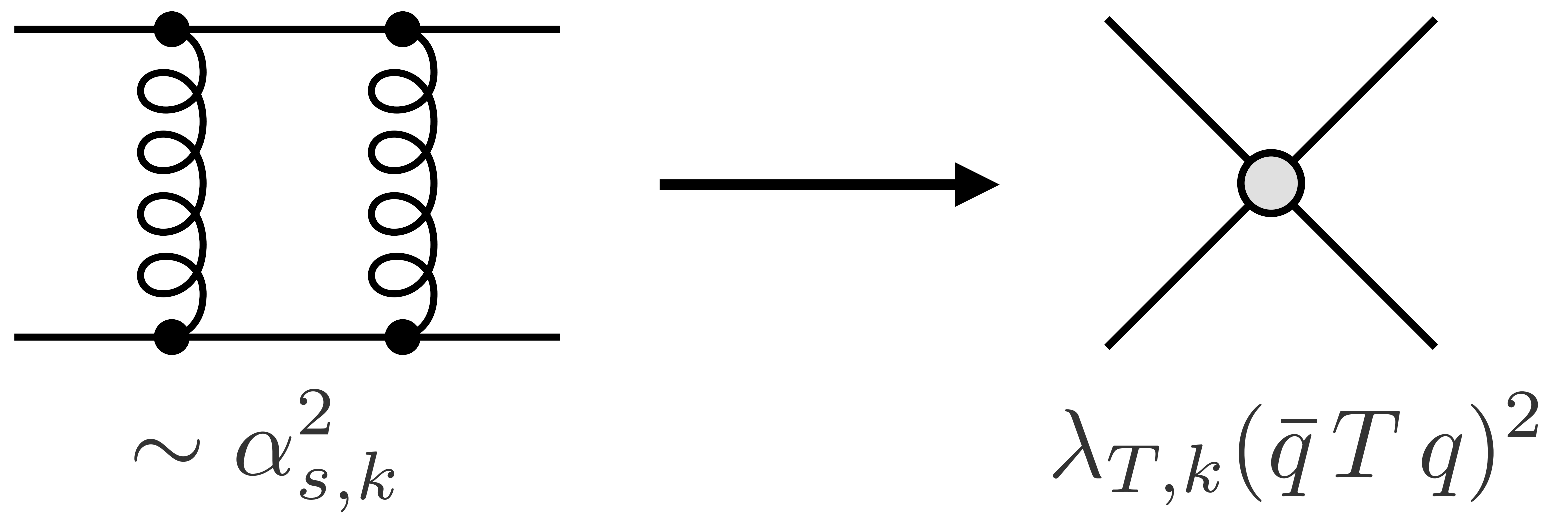
DSE [Huber, Maas, von Smekal, 1207.0222]



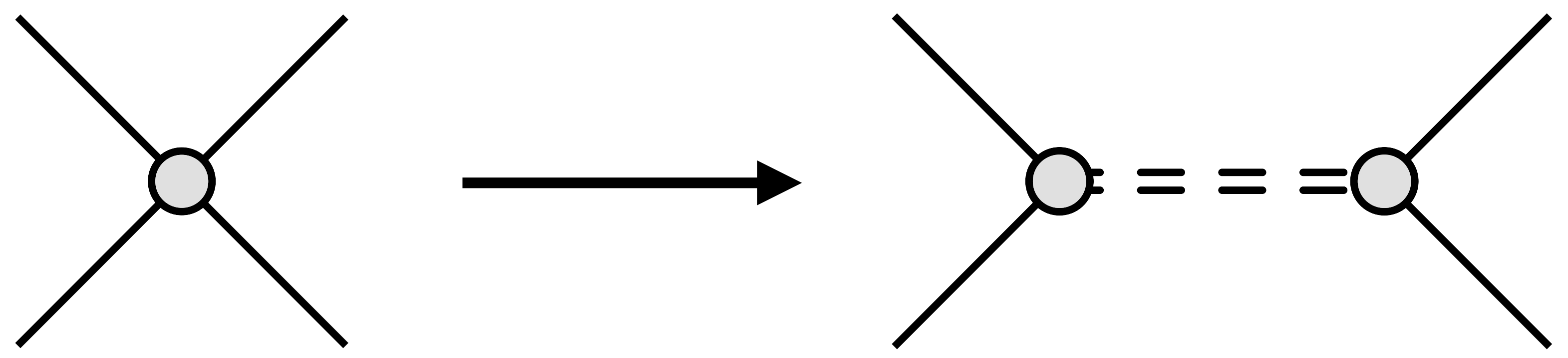
+…+ quark contributions

FUNCTIONAL QCD - EMERGENT ORDER

- $q\bar{q}$-scattering though gluon exchange



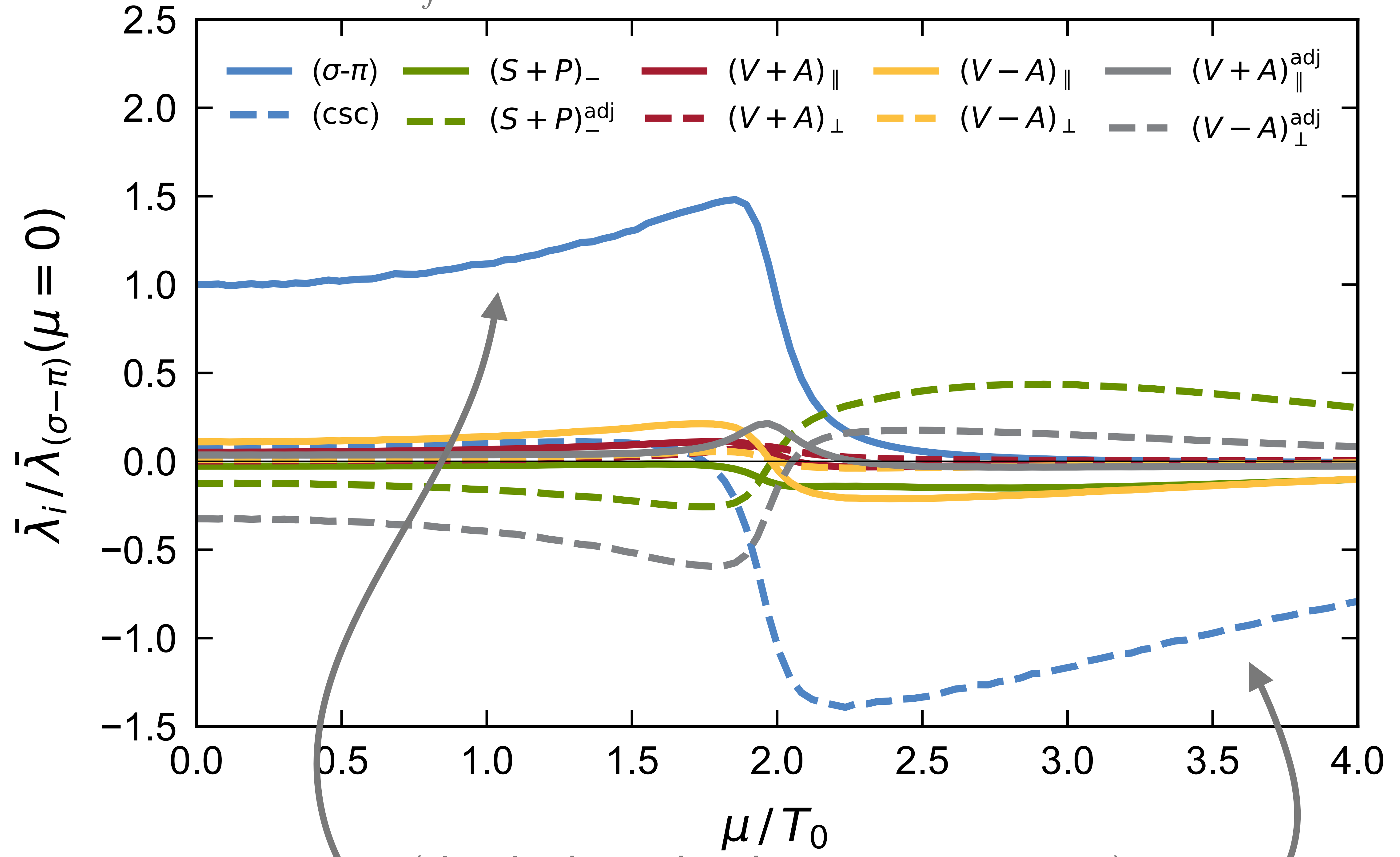
- resonant scattering: bound state/ condensate formation



+ (non-resonant contrib.)

- study **complete** sets of interactions channels

FRG-QCD ($N_f = 2$): [Braun, Leonhardt, Pospiech, 1909.06298]



(closely above the phase transition temp.)

chiral condensate $\sim \langle \bar{q}q \rangle$:
hadronic phase

quark Cooper-pair $\sim \langle qq \rangle$:
color-superconductor

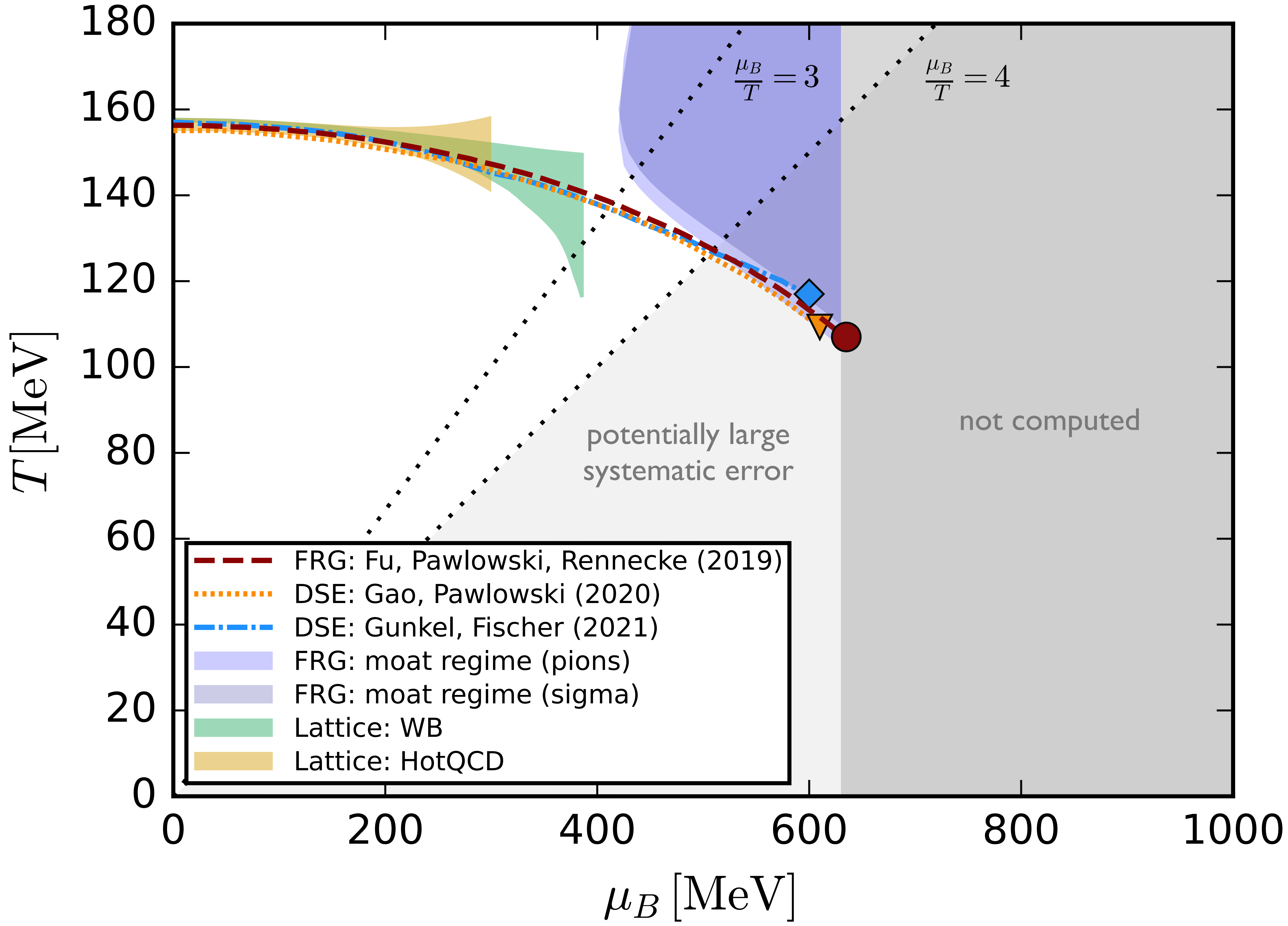
→ **emergent bound states and condensates** from elementary correlations

FRG: "dynamical hadronization" [Gies, Wetterich, 0209183; Pawlowski, 0512261; Fu, Pawlowski, FR, 1909.02991; Fukushima, Pawlowski, Strodthoff, 2103.01129]
DSE/Bethe-Salpeter/Faddeev equations [Roberts, Schmidt, 0005064; Eichmann et al., 1606.09602; Fischer, 1810.12938]

PHASE TRANSITIONS

QCD PHASE DIAGRAM & THE CEP

Results for the chiral transition from direct computations in QCD



- shows only QCD results that agree with lattice data at $\mu_B = 0$
- need to improve/check systematics for $\mu_B/T \gtrsim 4$ (work in progress), but good agreement between different methods and approximations

CEP at $(T, \mu_B) \approx (110, 630)$ MeV

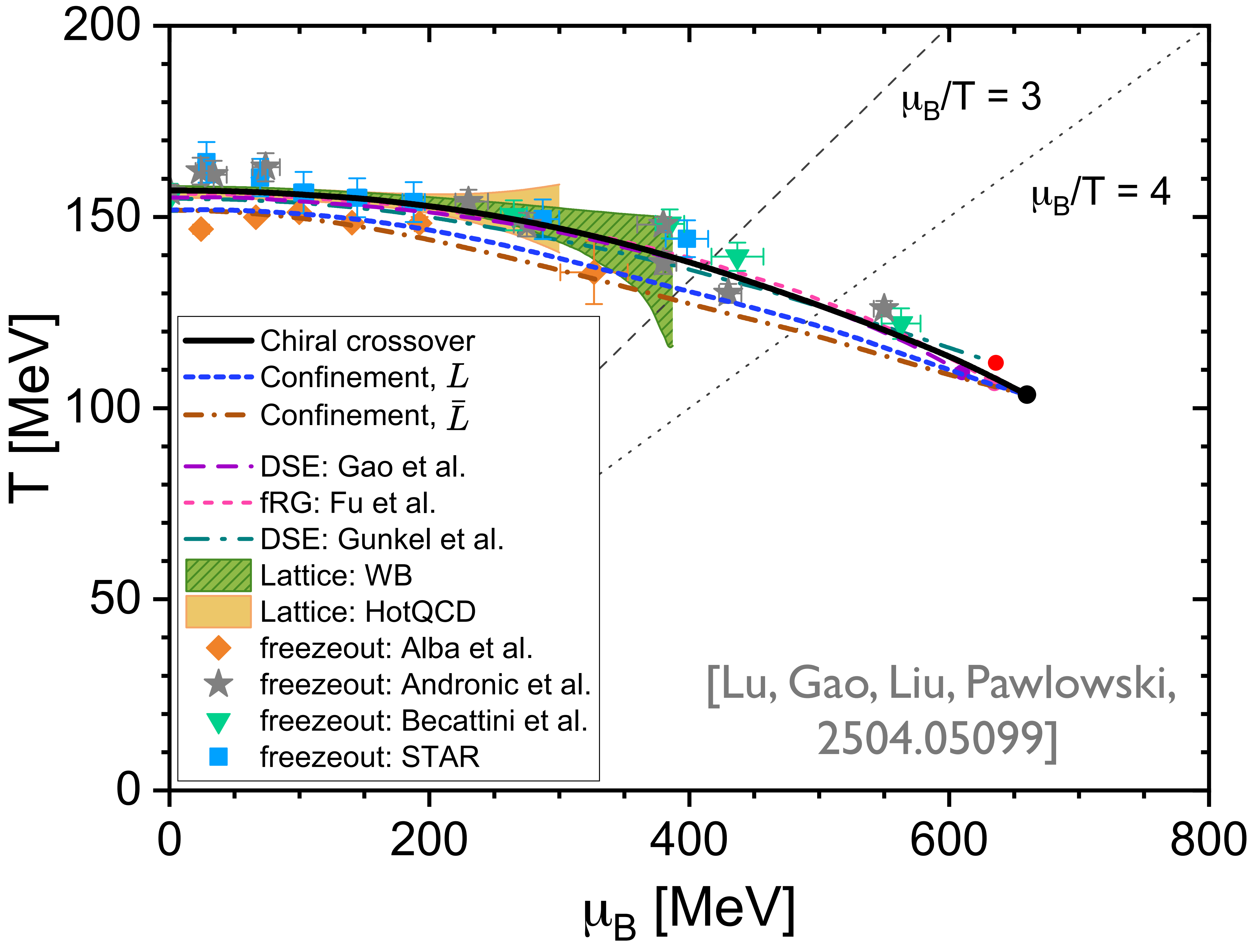
- indications for a new feature: **the moat regime** (more on that later)

[Fu, Pawlowski, FR, 1909.02991]
[Gao, Pawlowski, 2010.13705]
[Gunkel, Fischer, 2106.08356]
[Fu, Pawlowski, Pisarski, FR, Wen, Yin, 2412.15949]

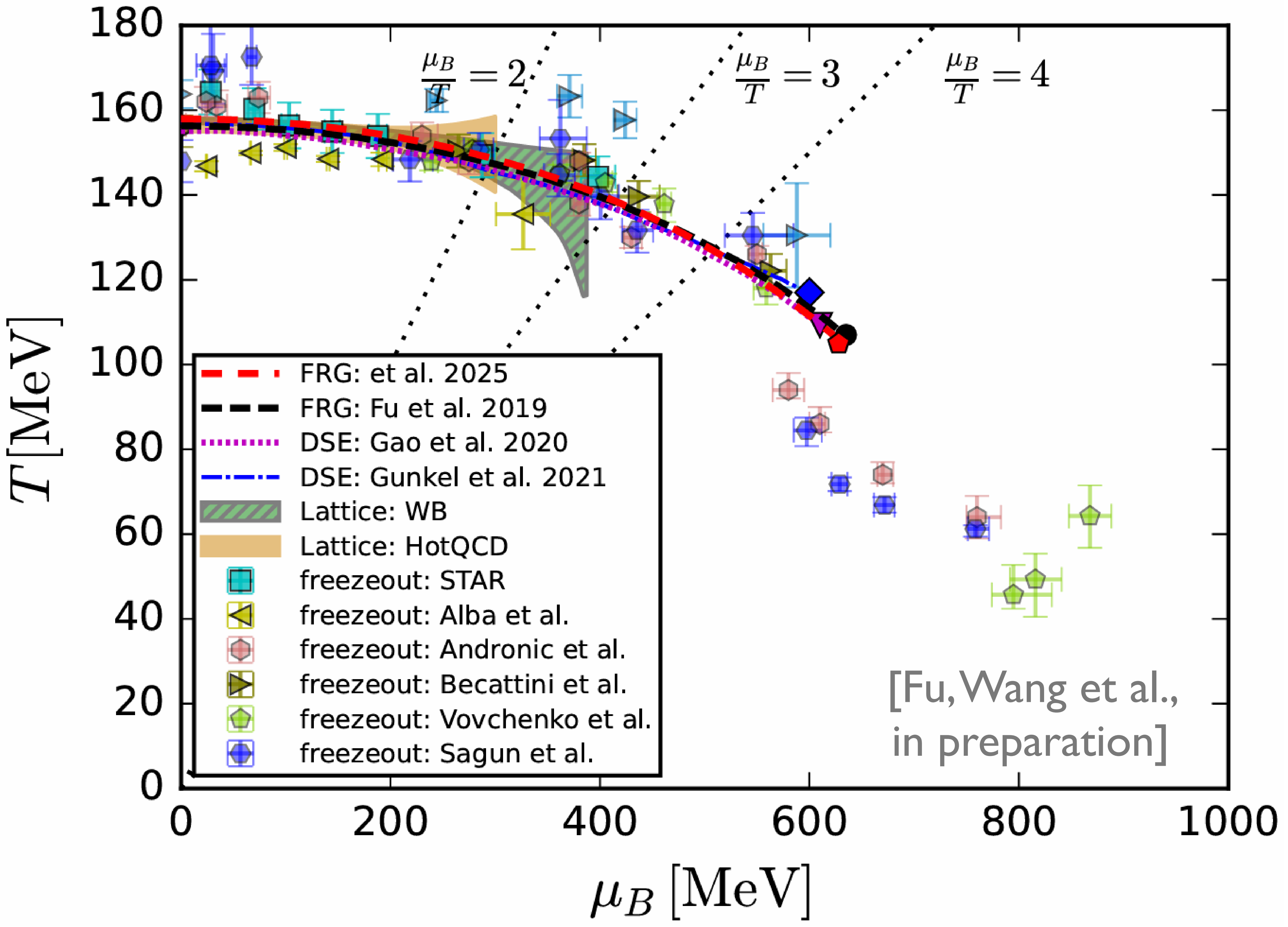
CEP SYSTEMATICS

Examples for tests of systematic errors in CEP location

- effect of nonzero temporal gluon background in DSE-QCD



- Effect of 10 (instead of just 1) four-quark interaction channels in FRG-QCD

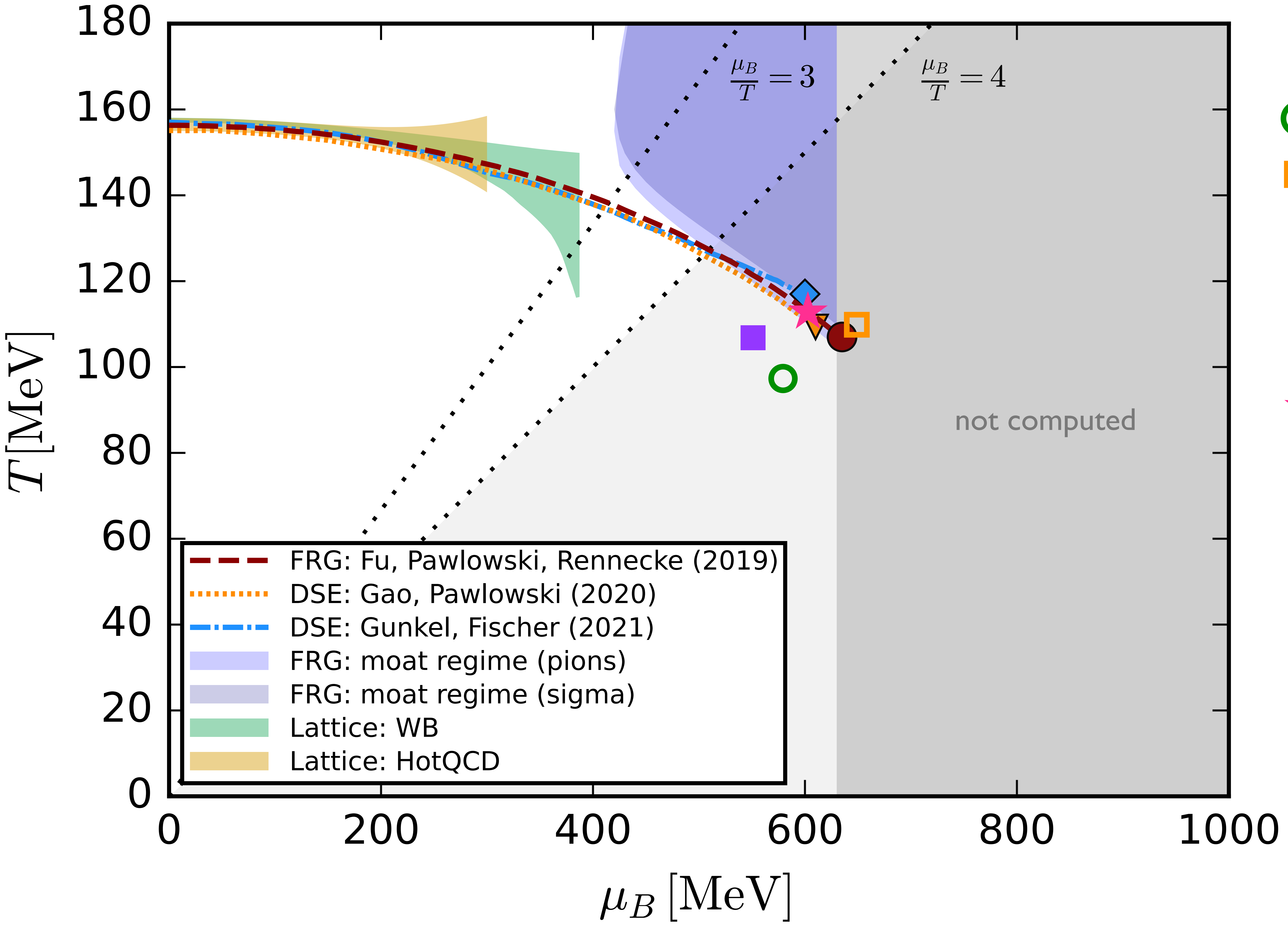


→ strong indication for systematic errors $\lesssim 10\,\%$ on CEP location from functional QCD

Note: not all scenarios where chiral transition/CEP are superseded by other phase (e.g. inhom. phase/Lifshitz point) can be excluded yet, **but something is cooking at $\mu_B \gtrsim 600\,\mathrm{MeV}$**

QCD PHASE DIAGRAM & THE CEP

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952]
- multi-point Padé [Clarke et al., 2405.10196]
  $N_\tau = 6, 8$ results + continuum estimate
  [Schmidt, 2504.00629]

using thermodynamics:

- constant entropy density [Shah et al. 2410.16206]

model-based extrapolation (tiny selection):

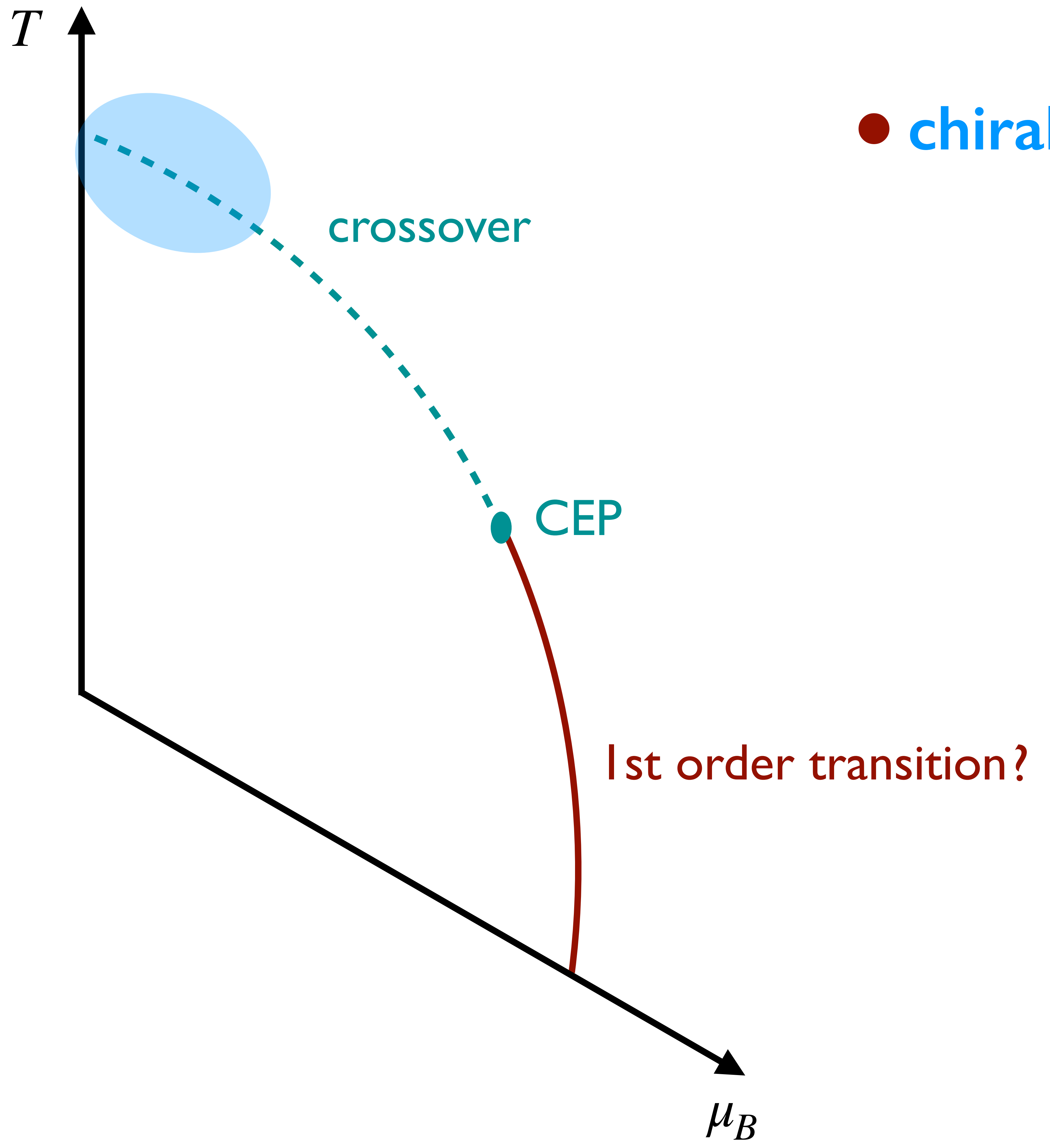
- holography [Hippert et al., 2309.00579]
  (agrees with [Cai et al., 2201.02004])

CEP (/exotic physics) location well constrained. And it's **in FAIR range!**

$\sqrt{s_{NN}} \approx 3.6 - 4.1\,\text{GeV}$

SECOND ORDER TRANSITIONS

Where can we have actual phase transitions in QCD?



- chiral crossover at small μ_B

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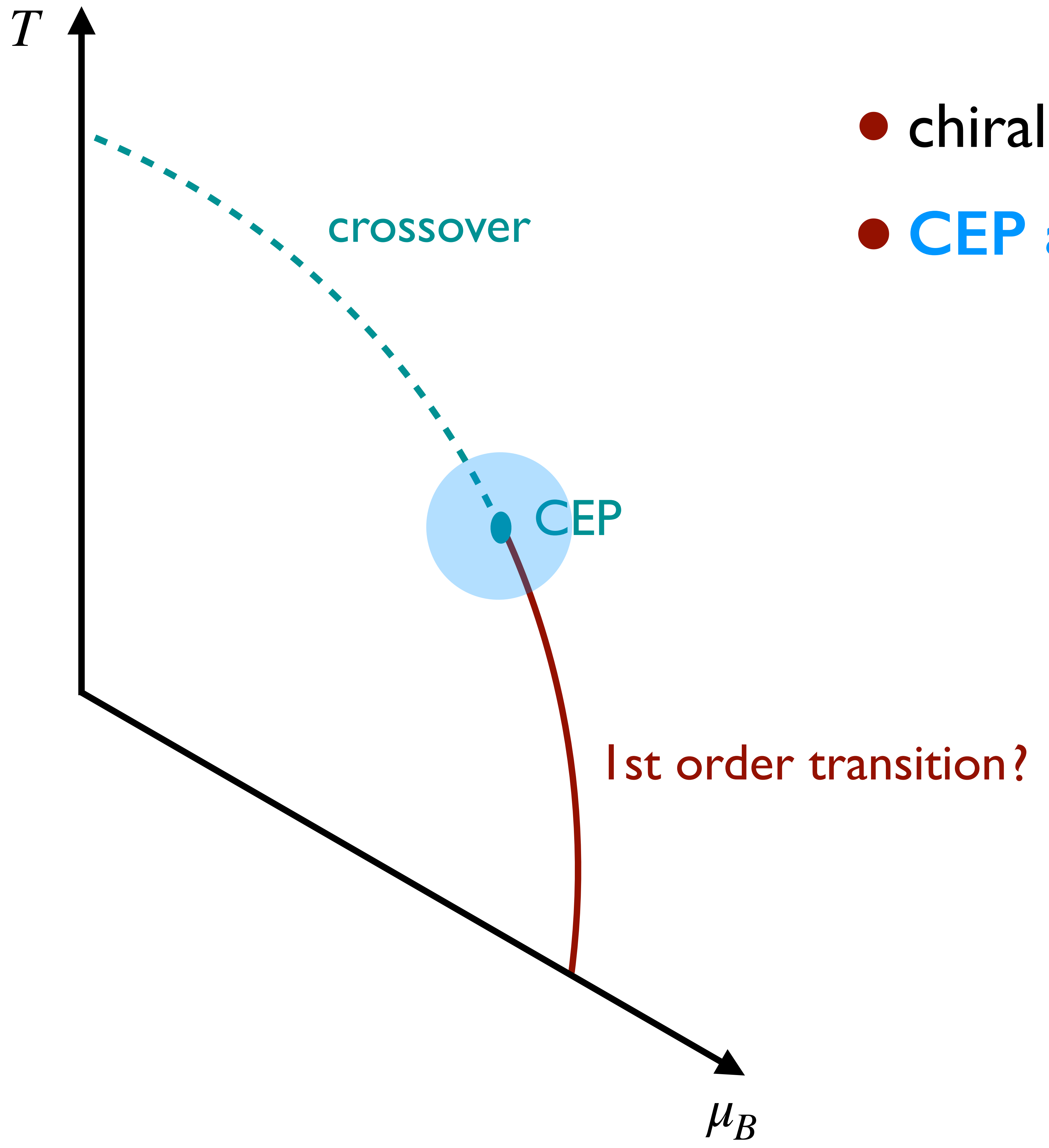

- chiral crossover at small μ
- **CEP at finite μ** (most likely)

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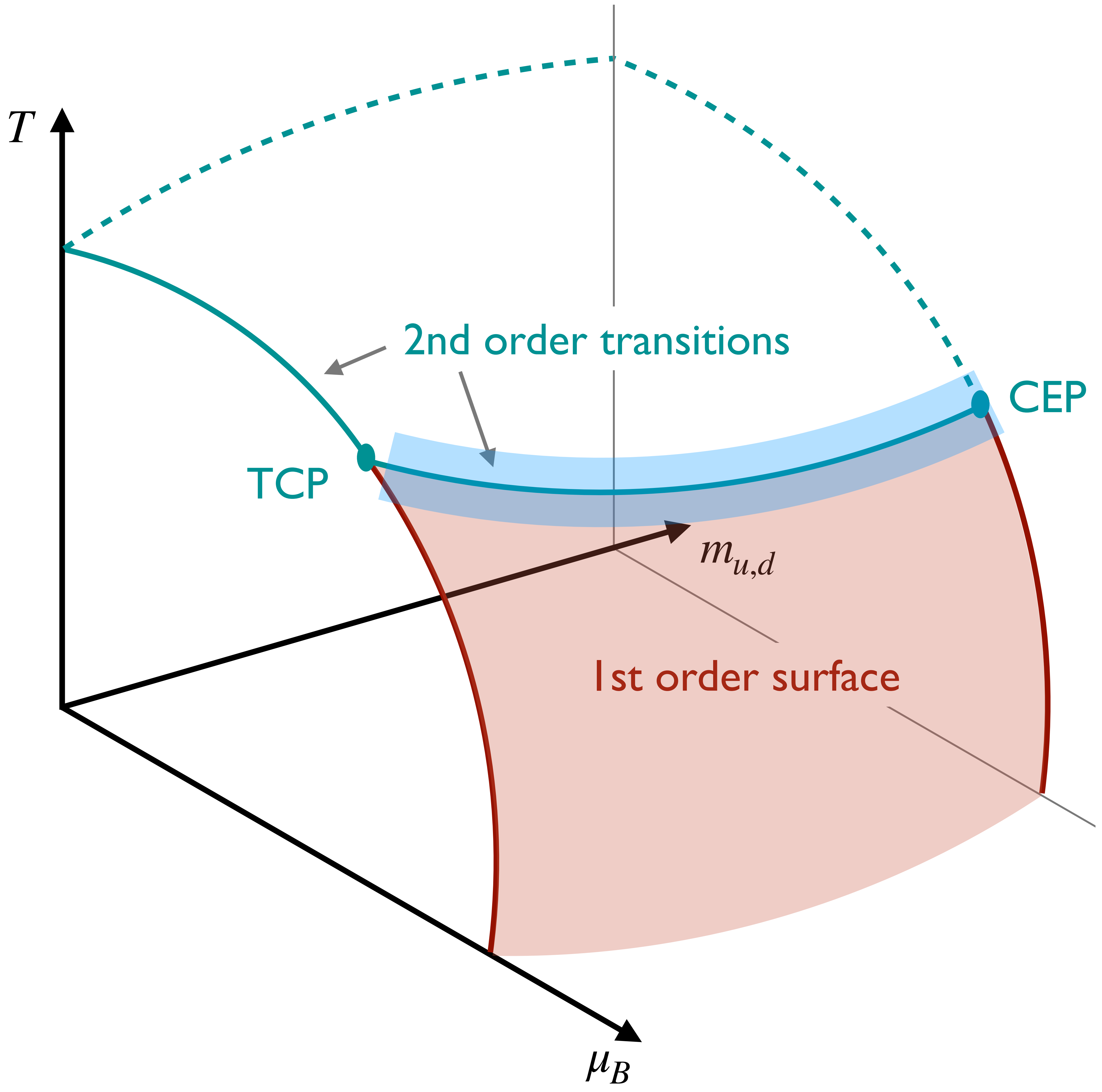

- chiral crossover at small μ
- CEP at finite μ (most likely)
- **extends into light quark mass direction**

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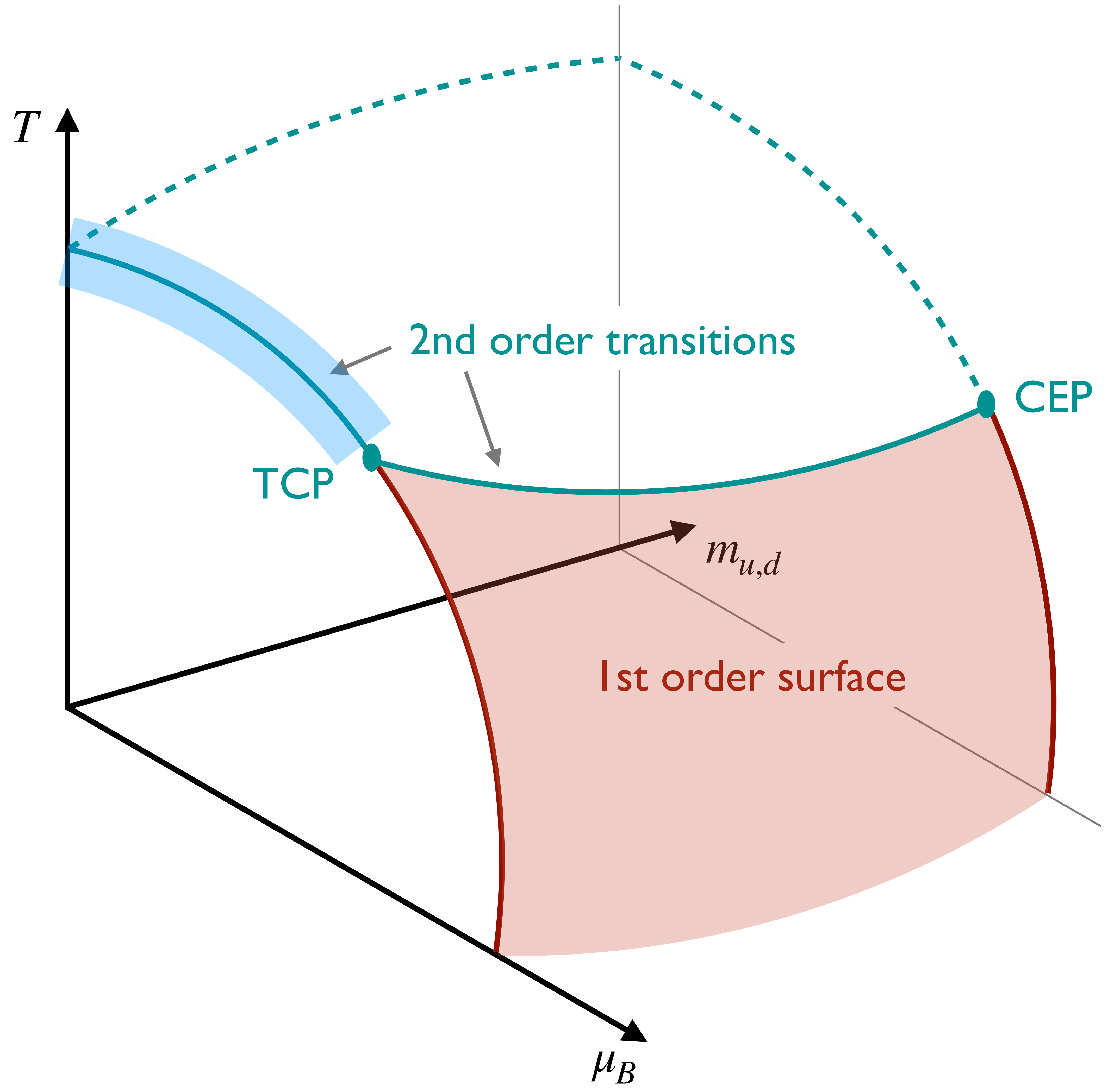

- chiral crossover at small μ
- CEP at finite μ (most likely)
- extends into light quark mass direction
- **second order transition at $m_{u,d} = 0$** (most likely)

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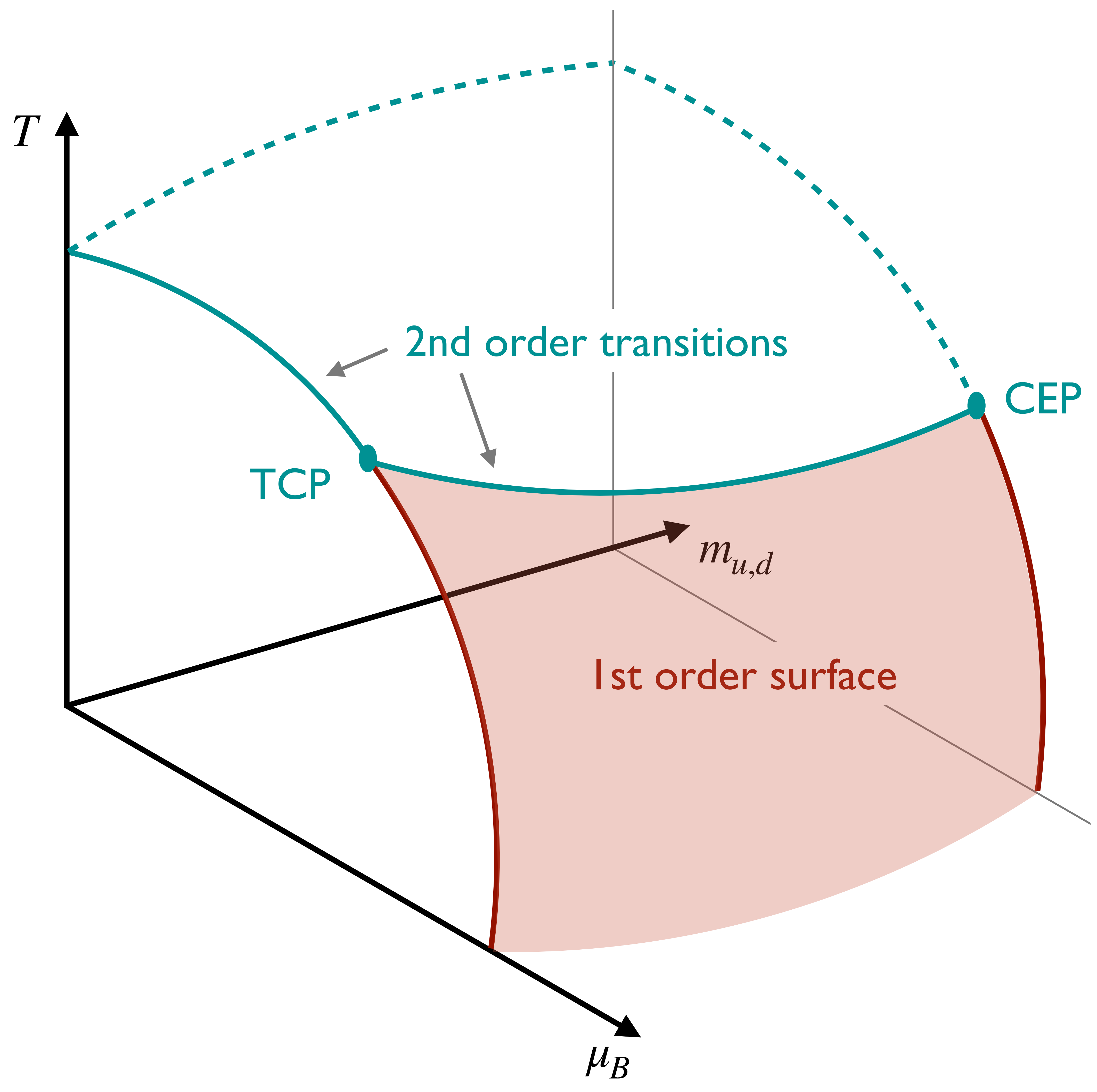


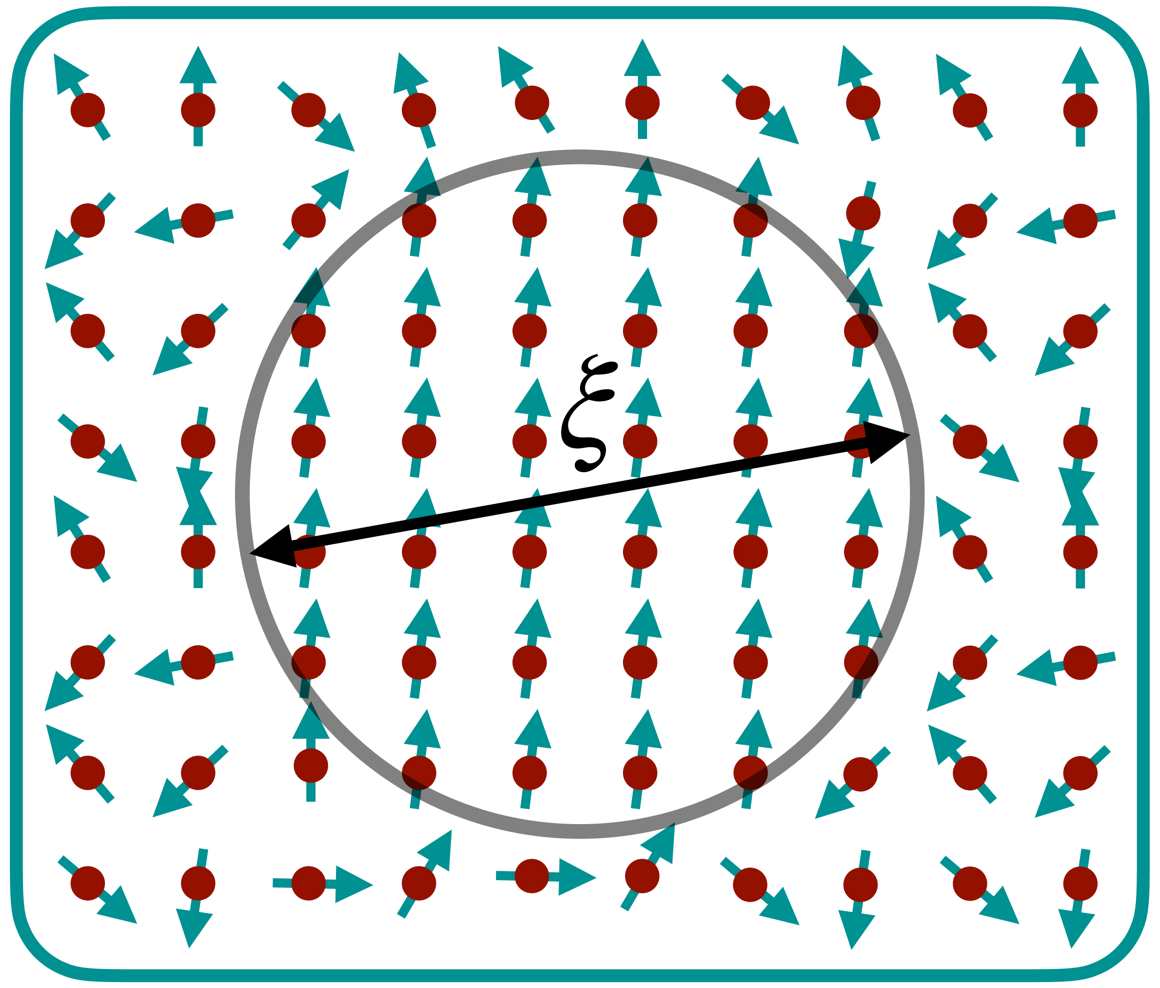
- chiral crossover at small μ
- CEP at finite μ (most likely)
- extends into light quark mass direction
- second order transition at $m_{u,d} = 0$ (most likely)

↓

exploit special features of 2nd order transitions: **critical phenomena**

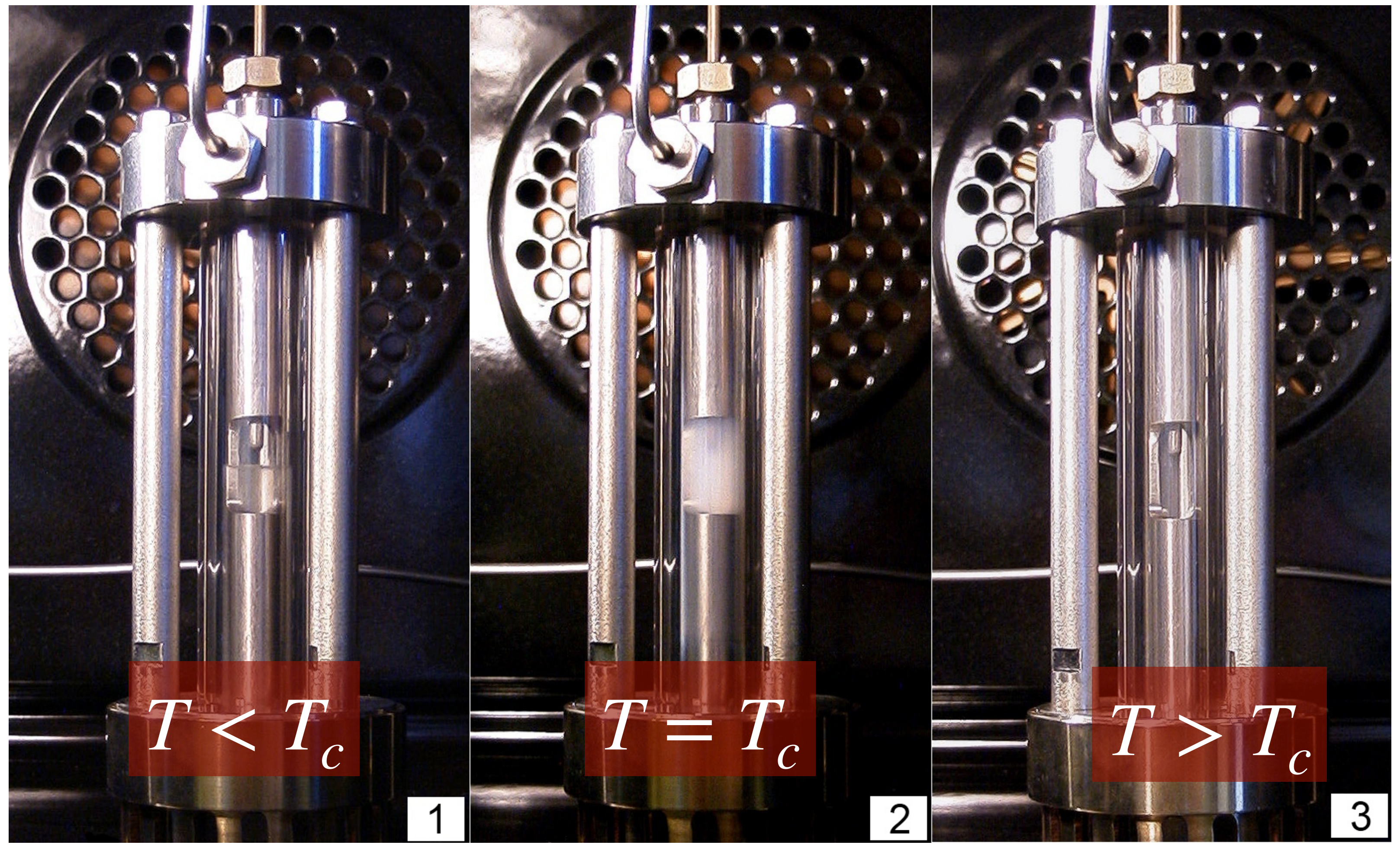
CRITICAL PHENOMENA & UNIVERSALITY

correlation length



2nd order transition:
$\xi \to \infty$

fluctuations on all length scales



critical opalescence of ethane [Wikipedia]

Near the critical point the system is scale invariant and microscopic details are irrelevant

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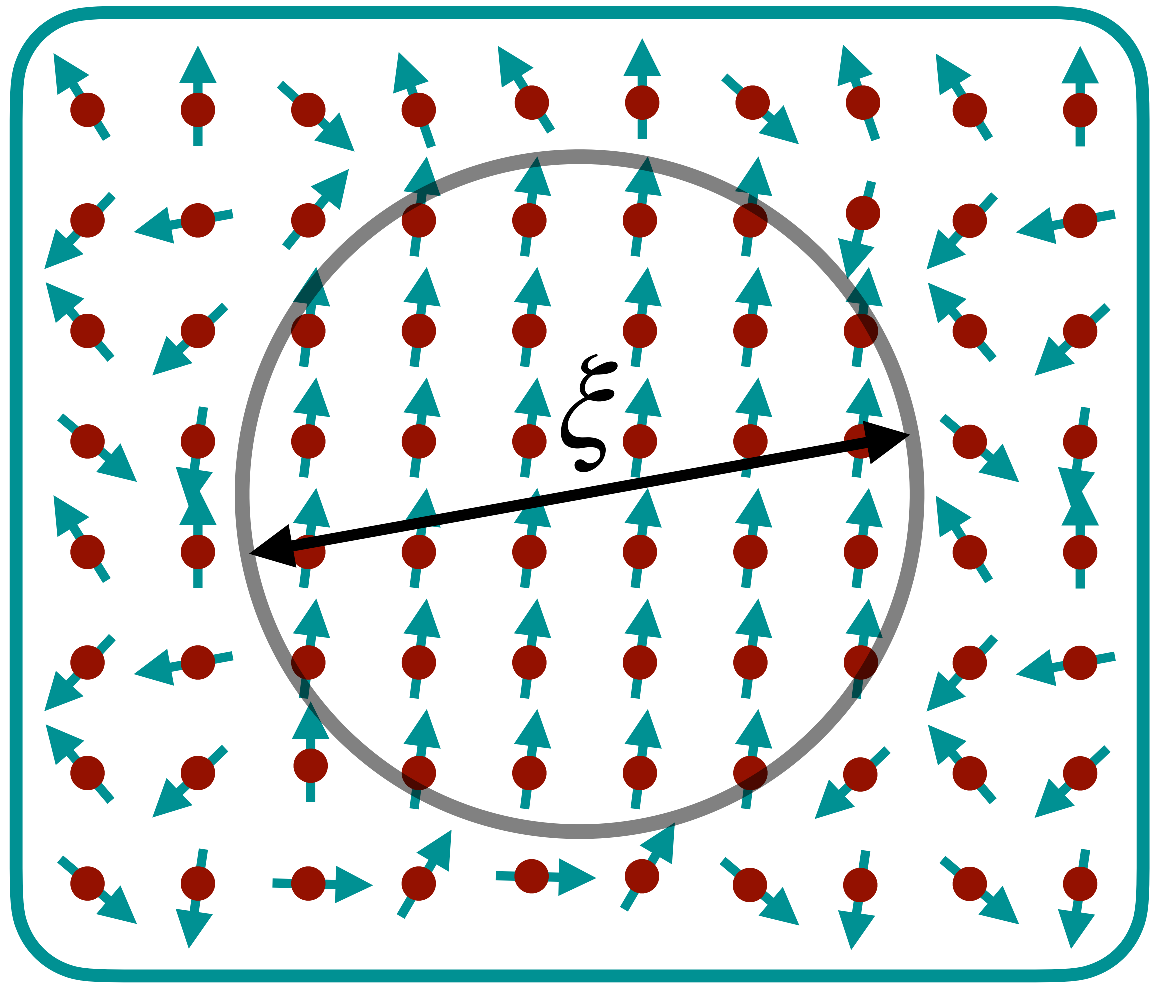

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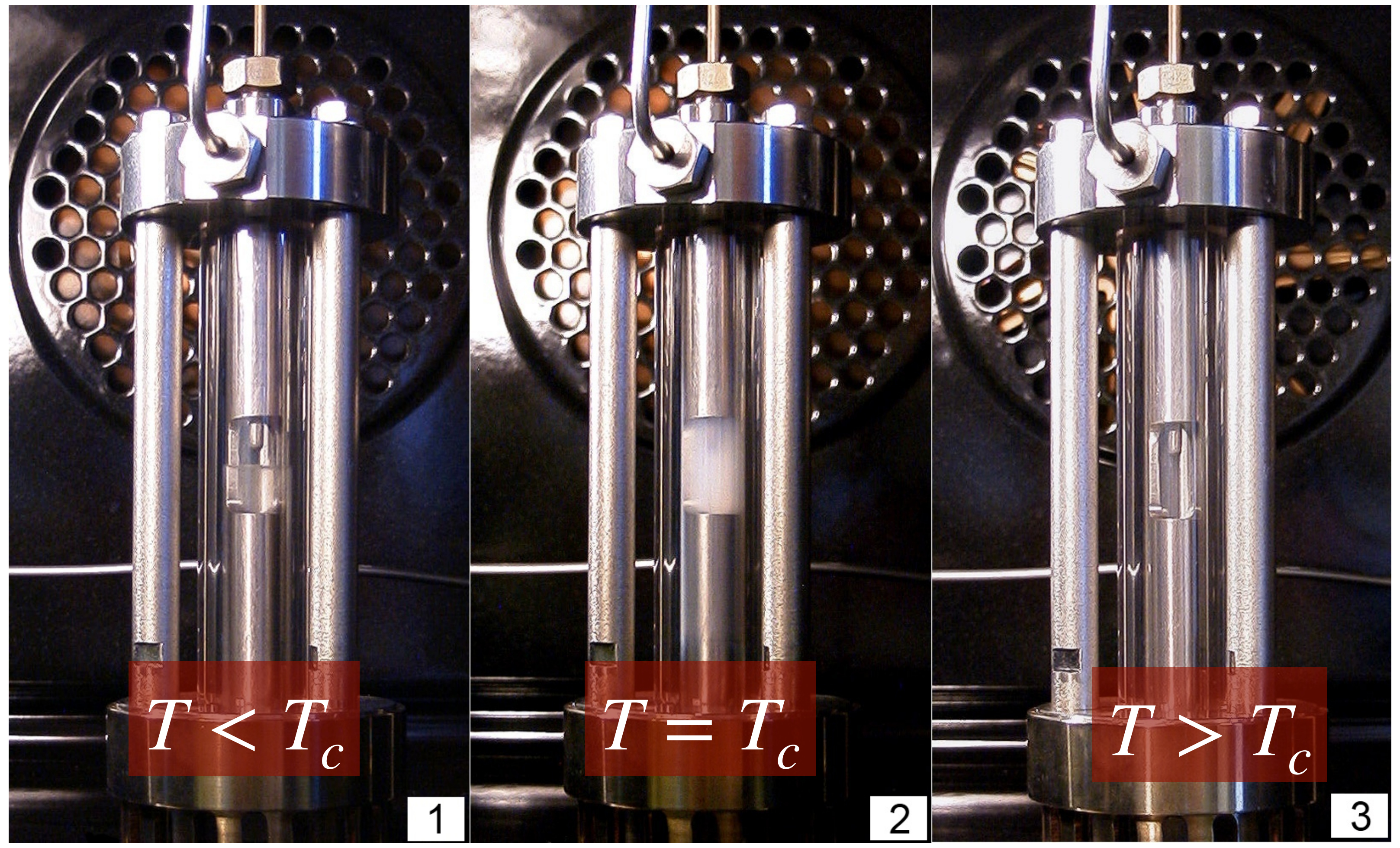
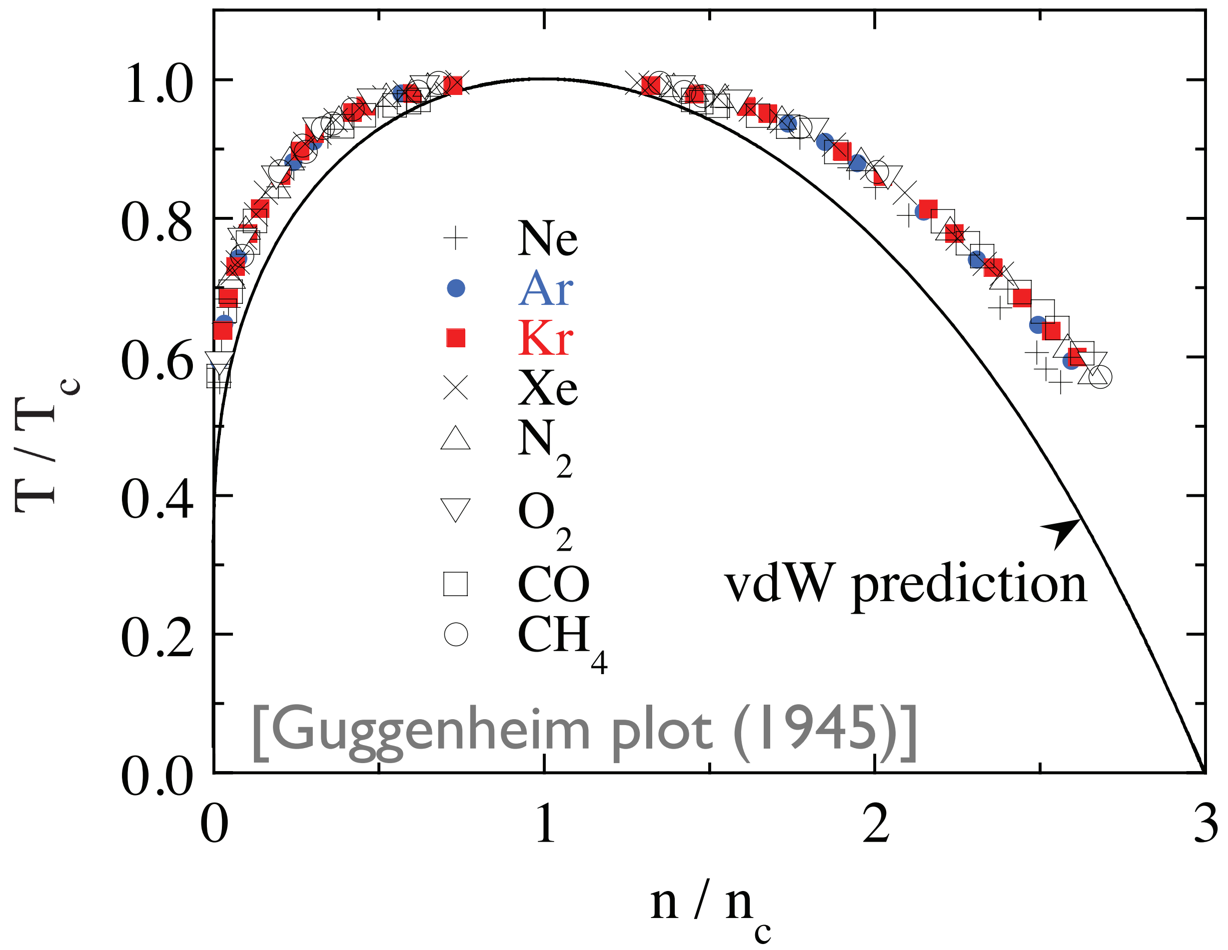


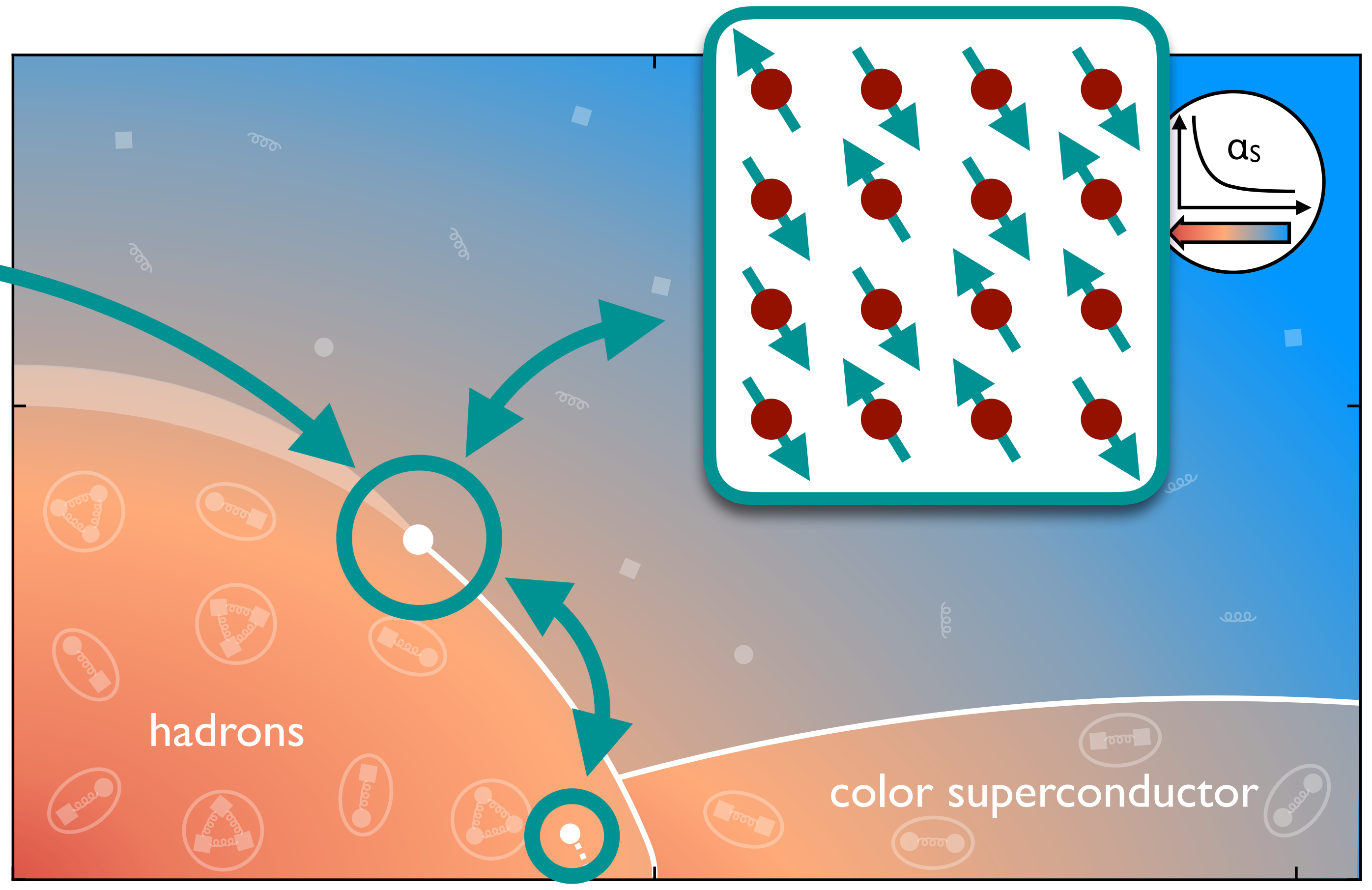
critical opalescence of ethane [Wikipedia]

Near the critical point the system is scale invariant and microscopic details are irrelevant

Universality: main features of the system are described by universal critical exponents, e.g., $\xi \sim (T - T_c)^{-\nu}$

example:
liquid-gas transition
=
3d Ising
=
QCD CEP



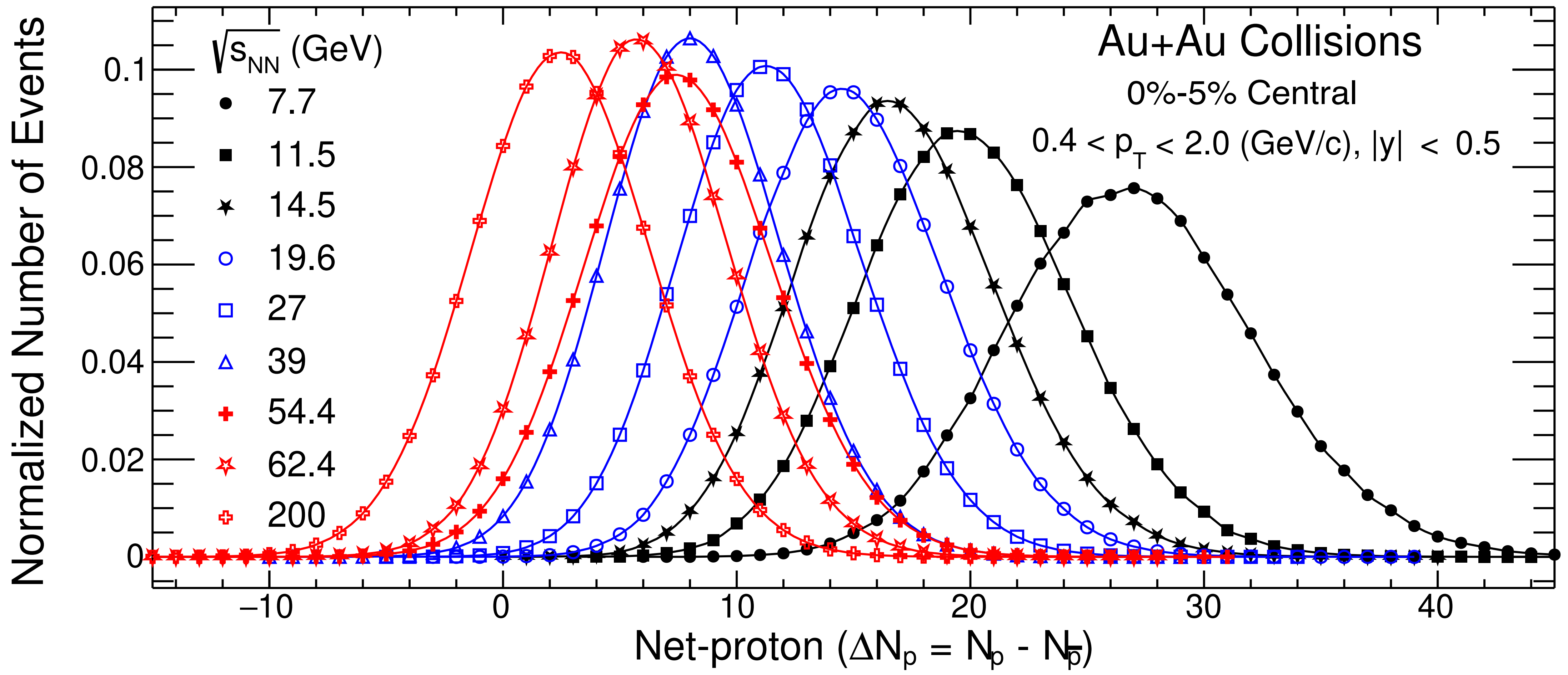


CAN WE EXPLOIT UNIVERSALITY TO FIND THE CEP?

experiment: heavy-ion collisions

- measure net-proton distributions $P(N_P)$

[STAR, 2001.02852]



- net-proton susceptibilities from the distribution

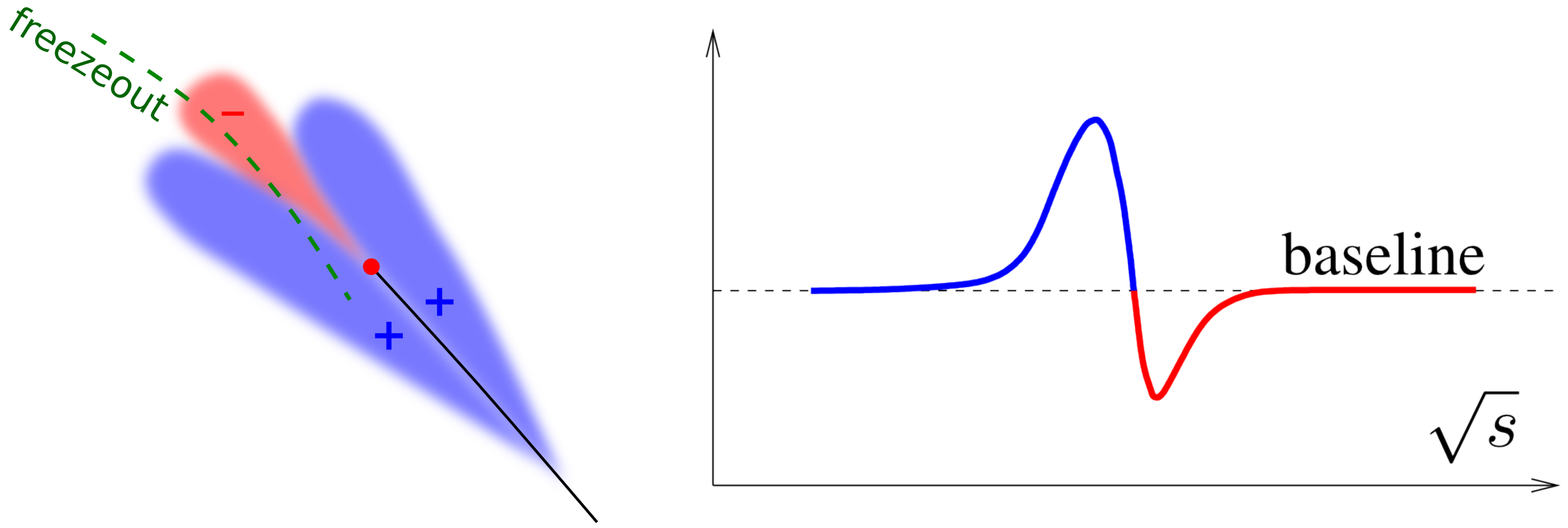
$$\chi_n^P \sim \sum_{N_P} \left[\left(N_P - \langle N_P \rangle \right)^n + \cdots \right] P(N_P)$$

theory

- net-baryon susceptibilities from the pressure

$$\chi_n^B = T^{n-4} \frac{\partial^n p}{\partial \mu_B^n}$$

- χ_n show universal scaling near CEP, e.g., $\chi_4 \sim \xi^7$
- scaling near the CEP: non-monotonic beam-energy dependence of kurtosis $\sim R_{42}^B = \chi_4/\chi_2$

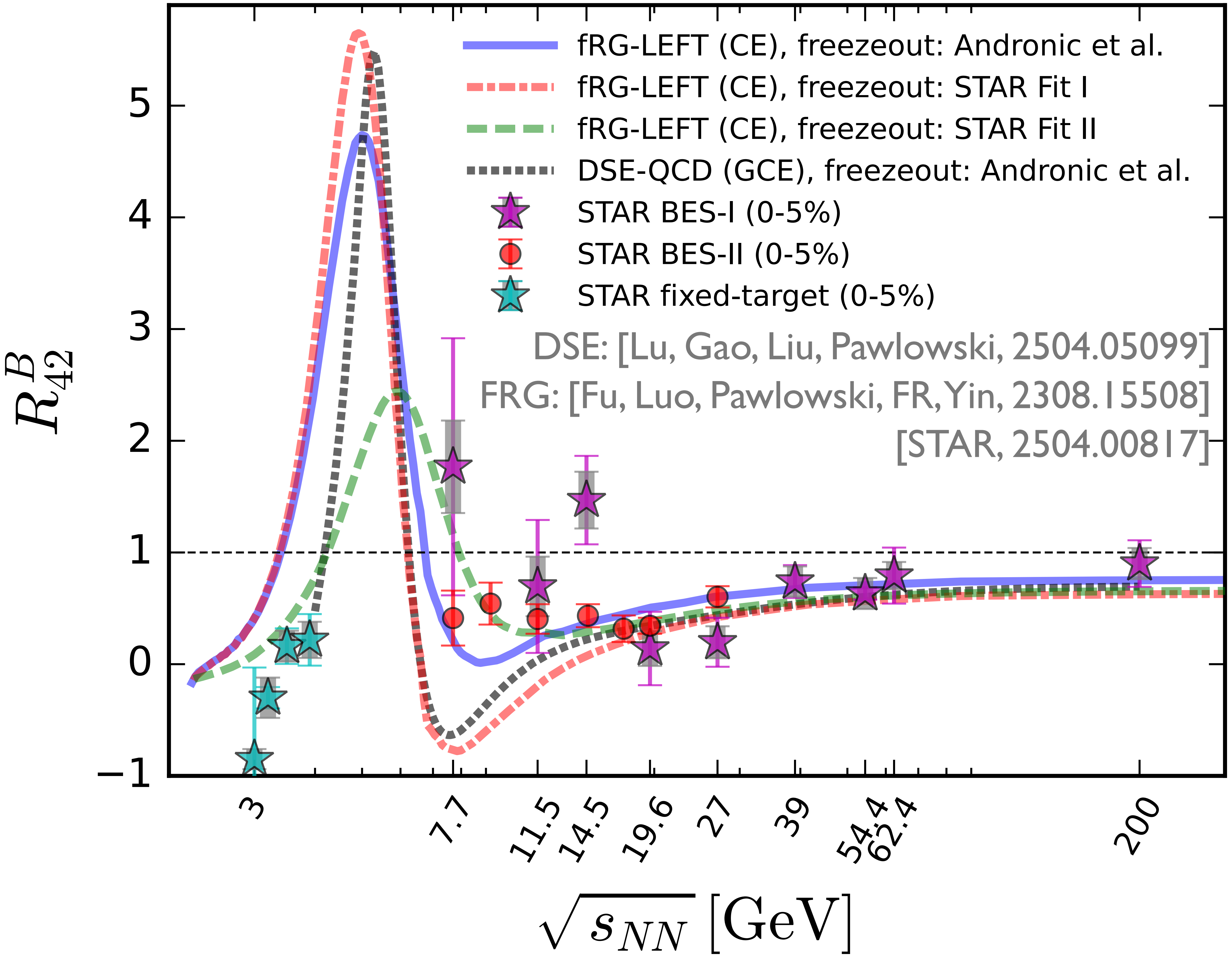


[Stephanov, 0809.3450]

→ measurements can be sensitive to critical fluctuations, but there are **many caveats and subtleties!**

RIPPLES OF THE CEP

net-baryon fluctuations in QCD vs net-protons from STAR



apples to half-apples comparison! [Vovchenko, QM2023]
qualitative features matter here!

- direct calculations: non-monotonicity at low beam-energies
- **no signs of critical scaling** seen along freeze-out

→ **direct signal of narrowed chiral crossover; CEP location encoded in peak height**

[Fu, Luo, Pawlowski, FR, Yin, 2308.15508]

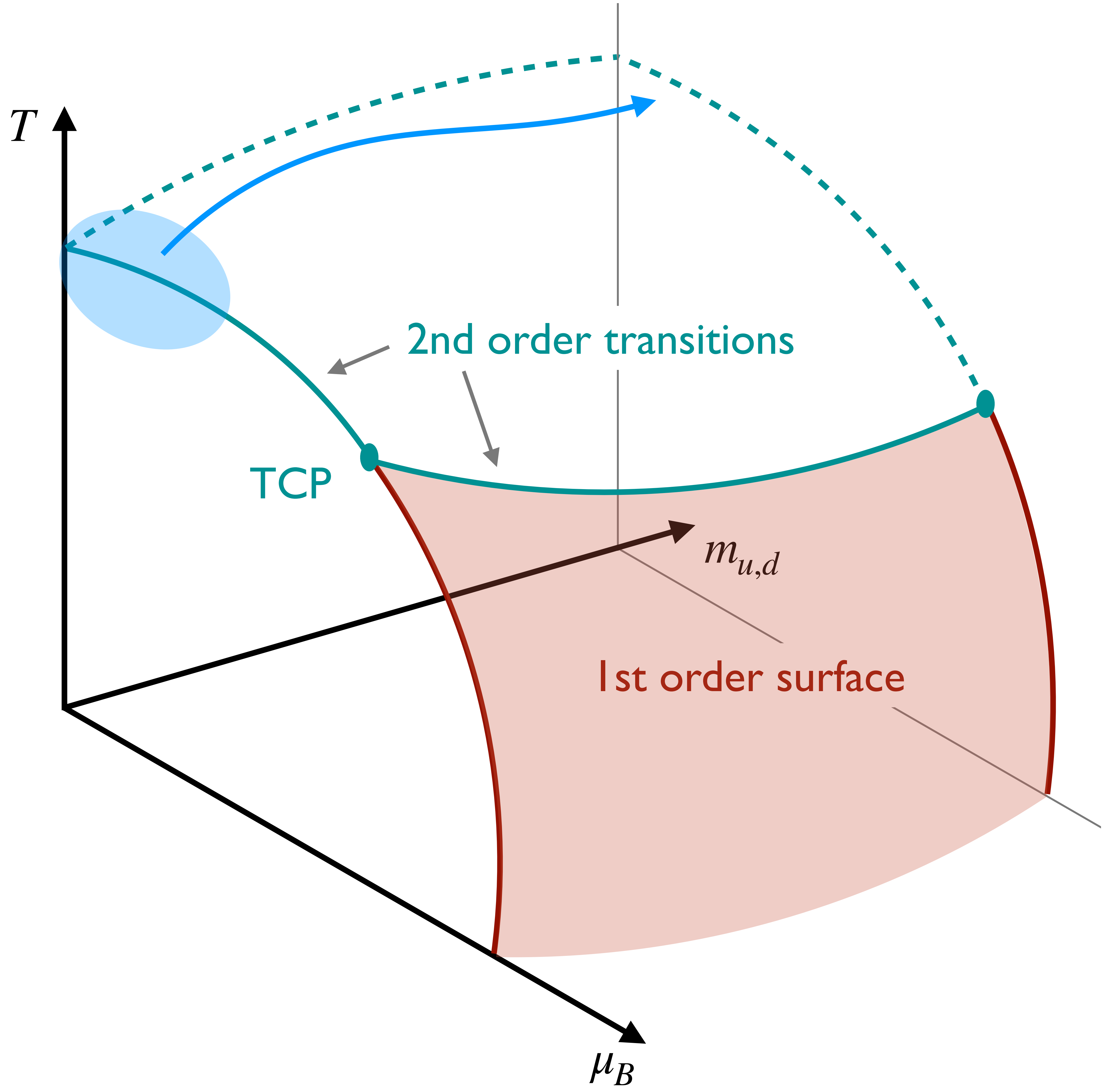
→ **data between $\sqrt{s_{NN}} = 4 - 8\,\mathrm{GeV}$ will be crucial!**

first exploratory DSE-QCD results: [Isserstedt, Buballa, Fischer, Gunkel, 1906.11644]

CHIRAL CROSSOVER VS CHIRAL TRANSITION

Is universality in the chiral limit relevant for the chiral crossover at small μ_B?

- signature in high-order susceptibilities? [Friman et al., 1103.3511; Fu et al., 2101.06035]
- relevant for low-pT pions? [Grossi et al., 2101.10847, 2504.03516, 2504.03514]

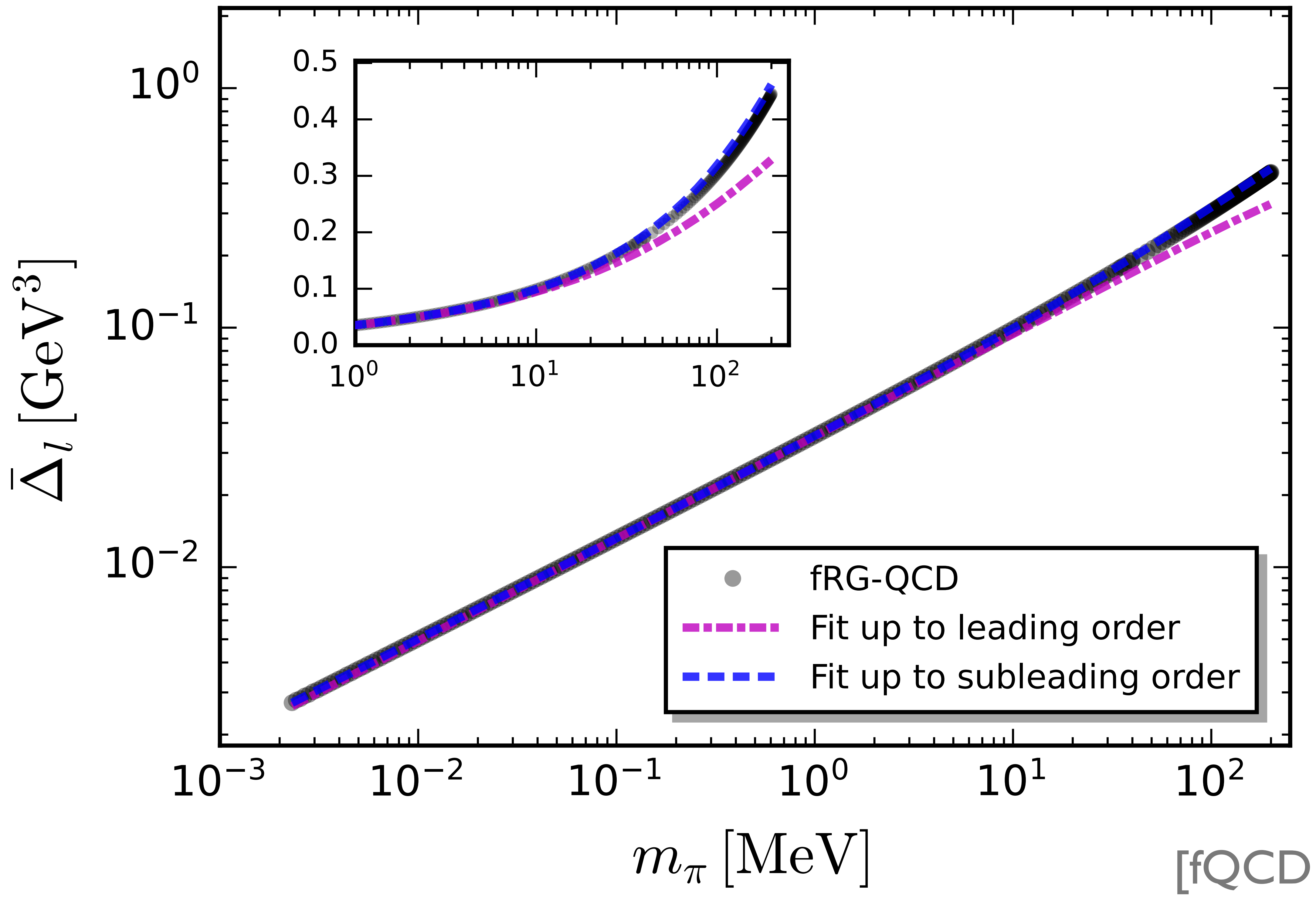


CHIRAL CROSSOVER VS CHIRAL TRANSITION

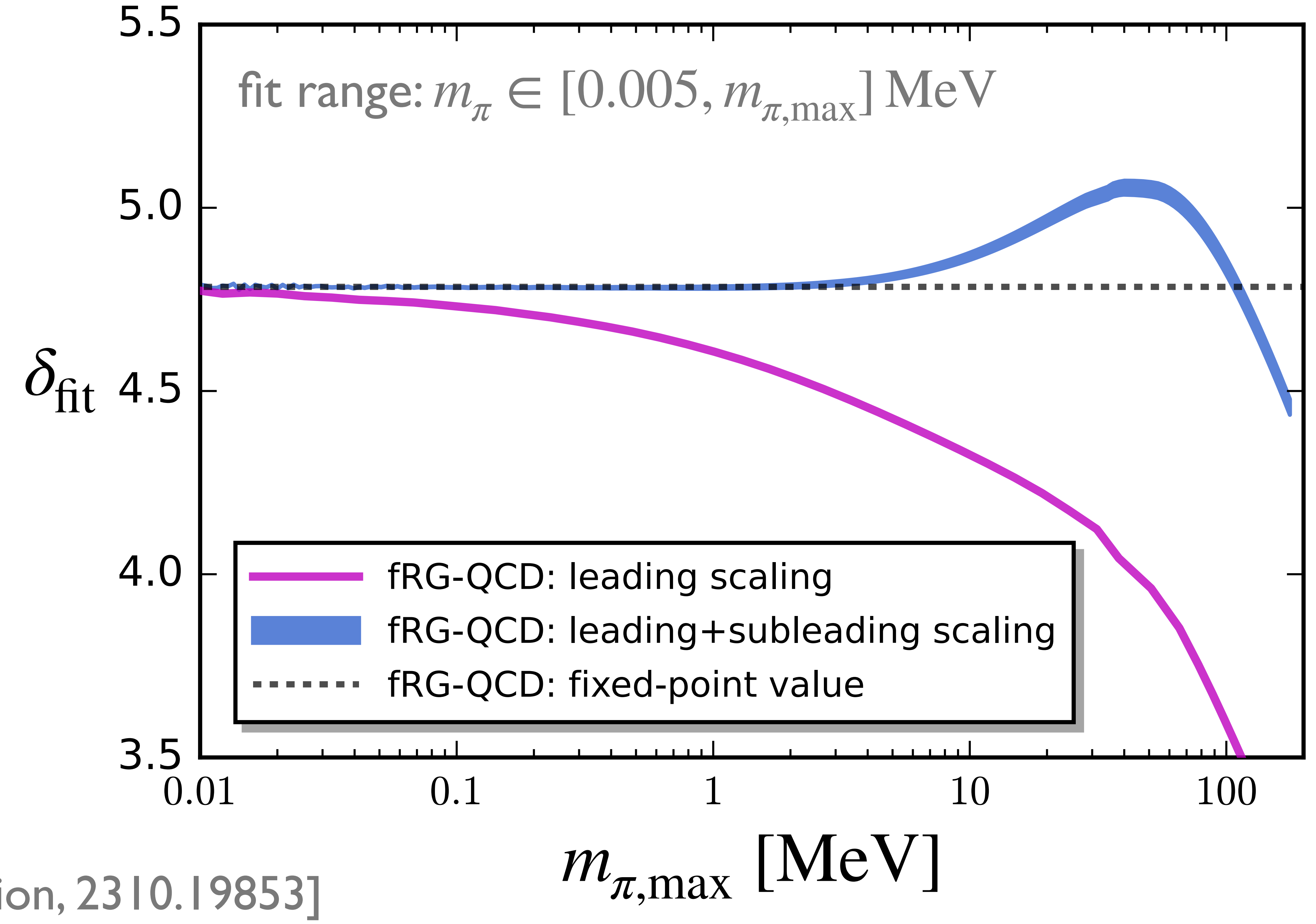
Is universality in the chiral limit relevant for the chiral crossover at small μ_B?

Study the size of the scaling region using the chiral condensate, $\langle \bar{q}q \rangle(T, m_\pi) \sim m_\pi^{2/\delta} f_G(z) + f_{\rm reg}(T, m_\pi)$

- chiral condensate for different $m_{u,d}$ at T_c



- infer breakdown of scaling from critical fit to data:



[fQCD Collaboration, 2310.19853]

→ **no universality for $m_\pi \gtrsim 5\,\mathrm{MeV}$: critical physics irrelevant* for physical quark masses at small μ_B**

fits of the form $\bar{\Delta}_l(m_\pi) = B_c\, m_\pi^{2/\delta}\left(1 + a_m m_\pi^{2\theta_H}\right) + c_1\, m_\pi^2 + c_2\, m_\pi^4$ break down for $m_\pi \gtrsim 25\,\mathrm{MeV}$

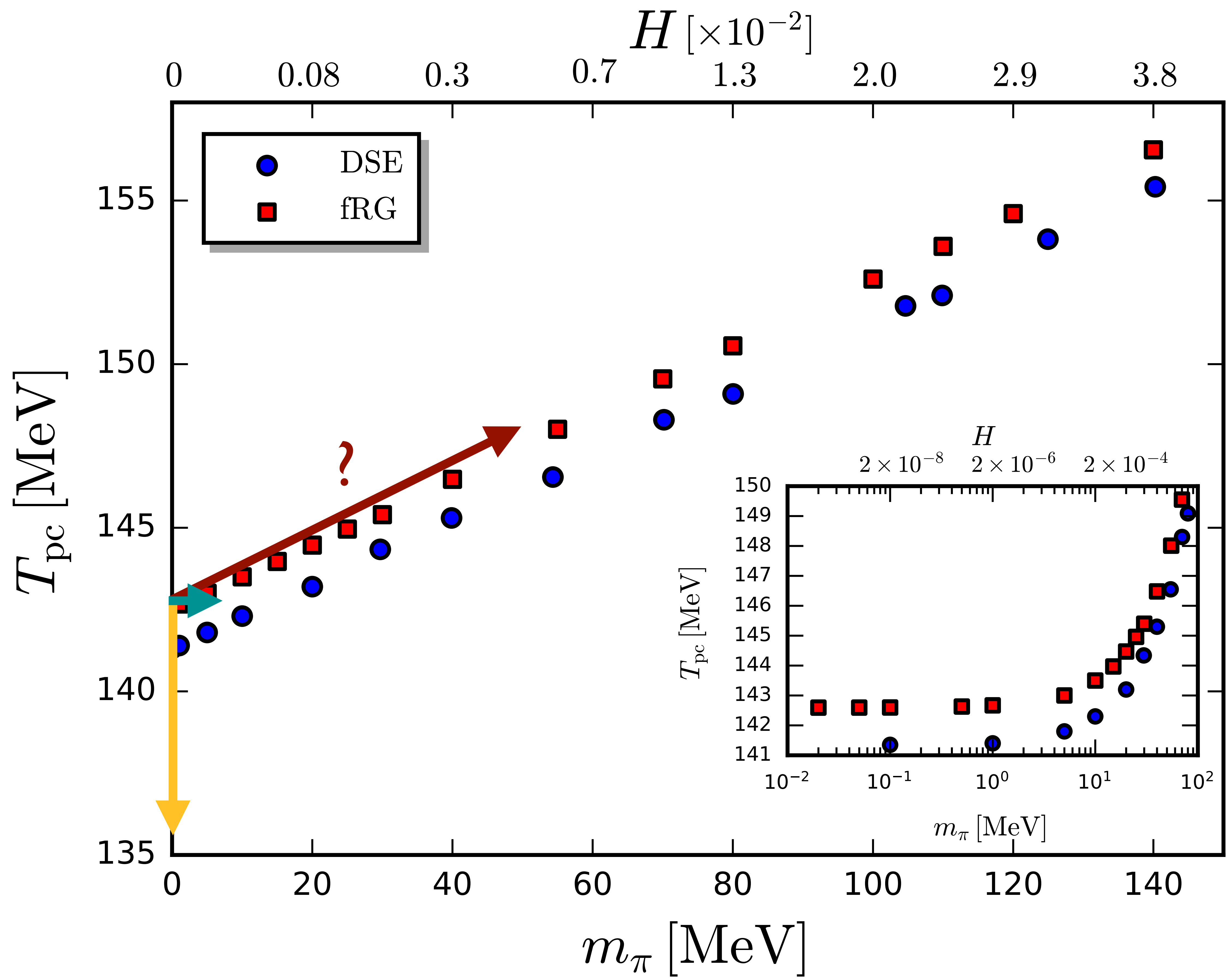
CHIRAL CROSSOVER VS CHIRAL TRANSITION

Is critical scaling in the chiral limit relevant for the chiral crossover at small μ_B?

Study the siz…

- chiral c… to data:

*size of scaling regime not yet known along $T_{\rm pc}$



- $m_\pi \lesssim 5\,\mathrm{MeV}$ at $T = T_c$ ✓
- $T_c - T \lesssim 7\,\mathrm{MeV}$ at $m_\pi = 0$ –
- $T_{\rm pc}(m_\pi) \lesssim ???$ ✗

FRG: [Braun et al., 2003.13112]
DSE: [Gao, Pawlowski, 2112.01395]
DSE: [Bernhardt, Fischer, 2309.06737]
FRG: [fQCD Collaboration, 2310.19853]

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THE MOAT REGIME

EXCITATIONS AT ZERO DENSITY

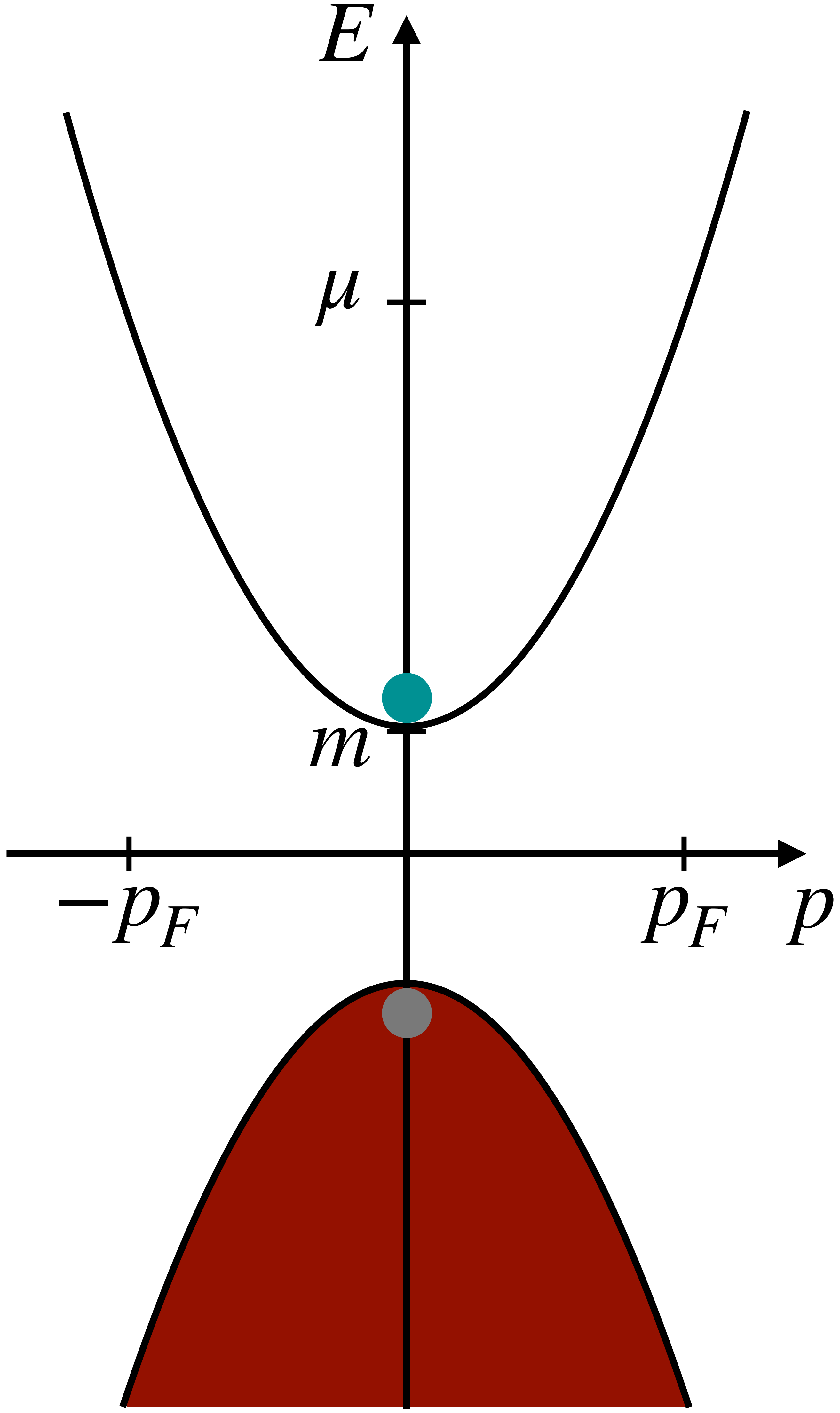
What kind of excitation do we expect? Look at Dirac cone at $\mu = 0$

- energy of a relativistic particle

$$E(p) = \pm\sqrt{\mathbf{p}^2 + m^2}$$

- Fermi momentum $E(p_F) = \mu$

$$p_F = \sqrt{\mu^2 - m^2}$$



small-μ excitations

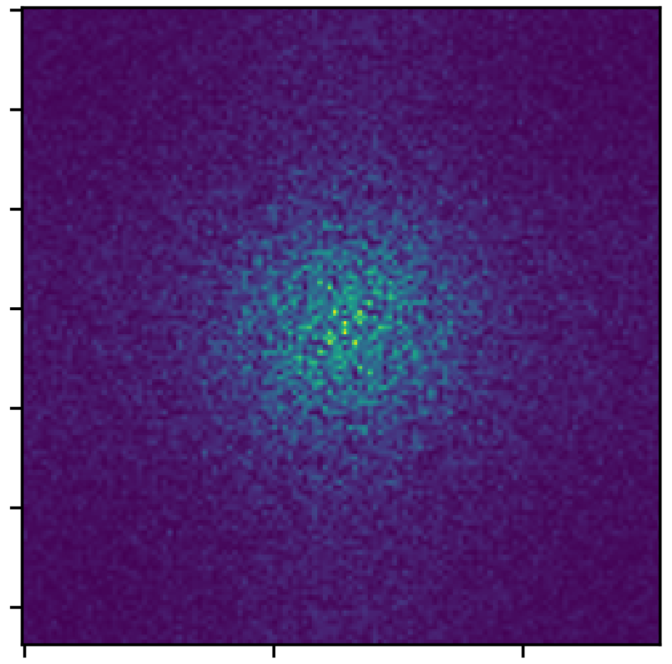
particle-antiparticle

$$\bar{\psi}_+(0)\,\psi_-(0)$$

zero net-momentum

homogeneous excitation

($\rightarrow$ chiral condensate)



EXCITATIONS AT NONZERO DENSITY

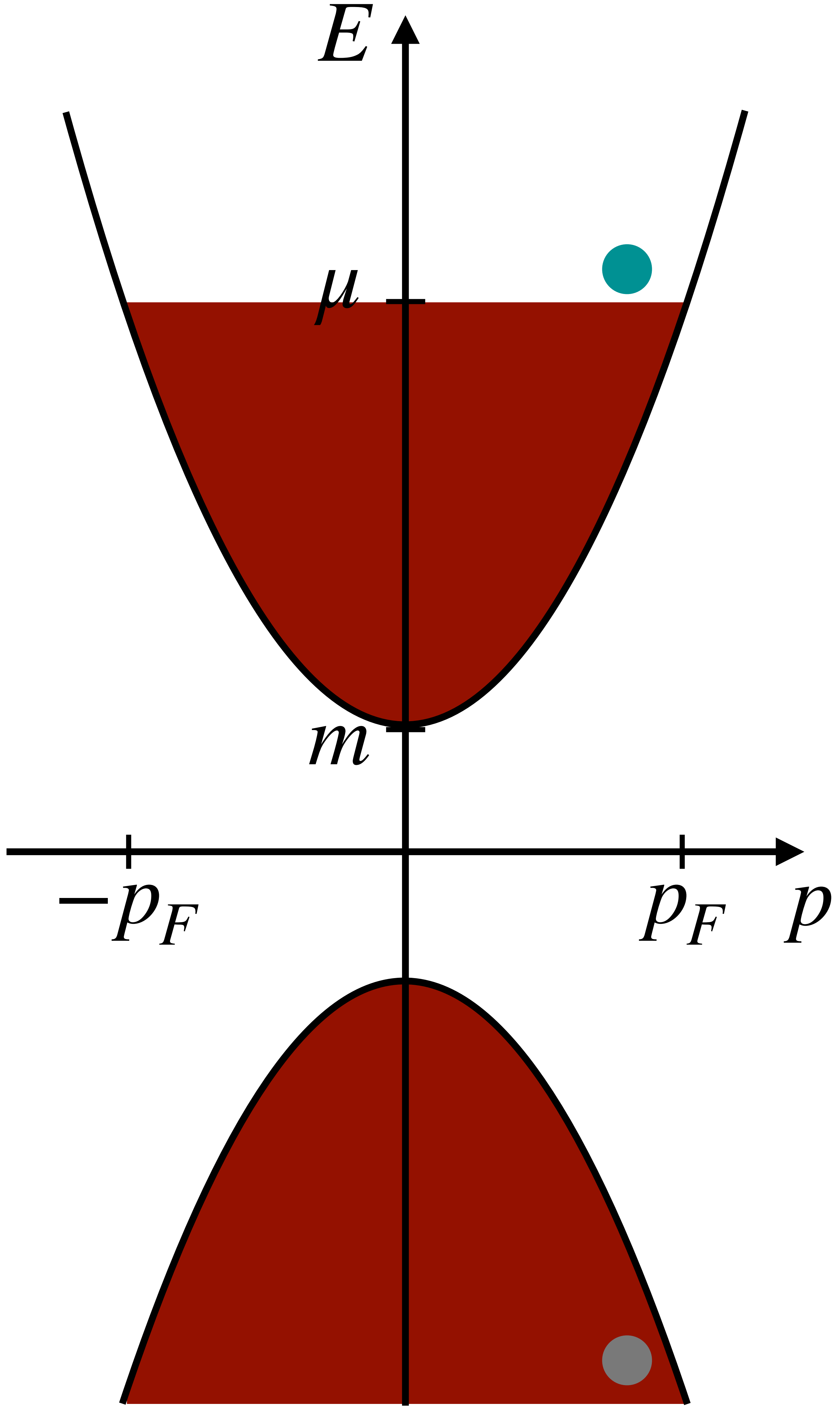
What kind of excitation do we expect? Look at Dirac cone at $\mu > 0$

- energy of a relativistic particle

$$E(p) = \pm\sqrt{\mathbf{p}^2 + m^2}$$

- Fermi momentum $E(p_F) = \mu$

$$p_F = \sqrt{\mu^2 - m^2}$$

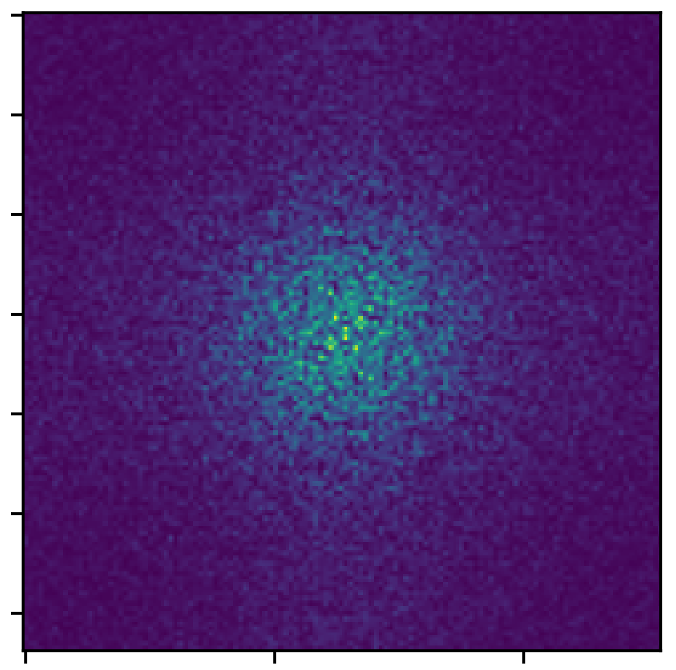


large-μ excitations

particle-antiparticle

$$\bar{\psi}_+(p_F)\,\psi_-(p_F)$$

zero net-momentum, but energy penalty $\Delta E \approx 2\mu$:
homogeneous excitation
disfavored at large μ

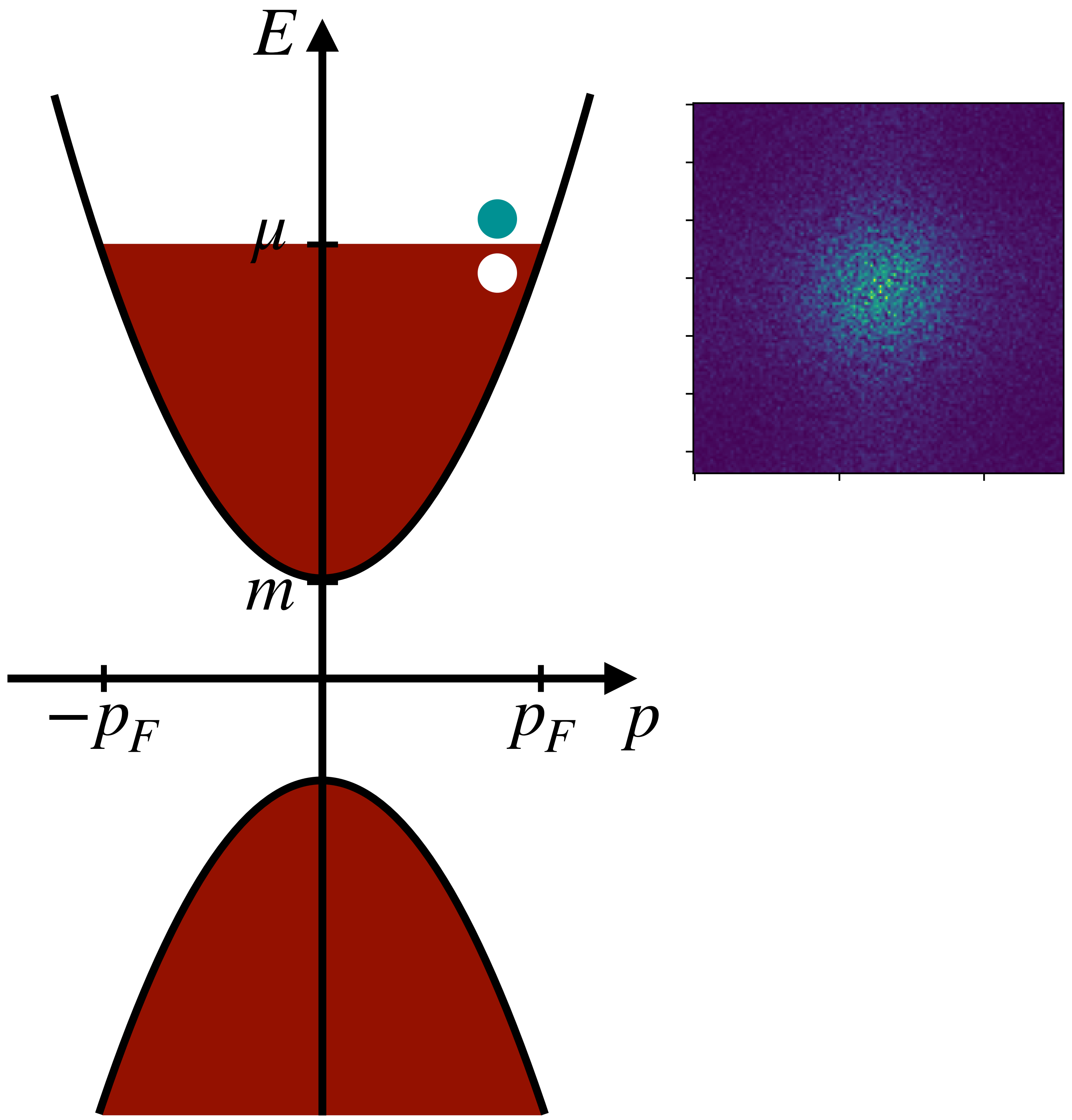


EXCITATIONS AT NONZERO DENSITY

What kind of excitation do we expect? Look at Dirac cone at $\mu > 0$

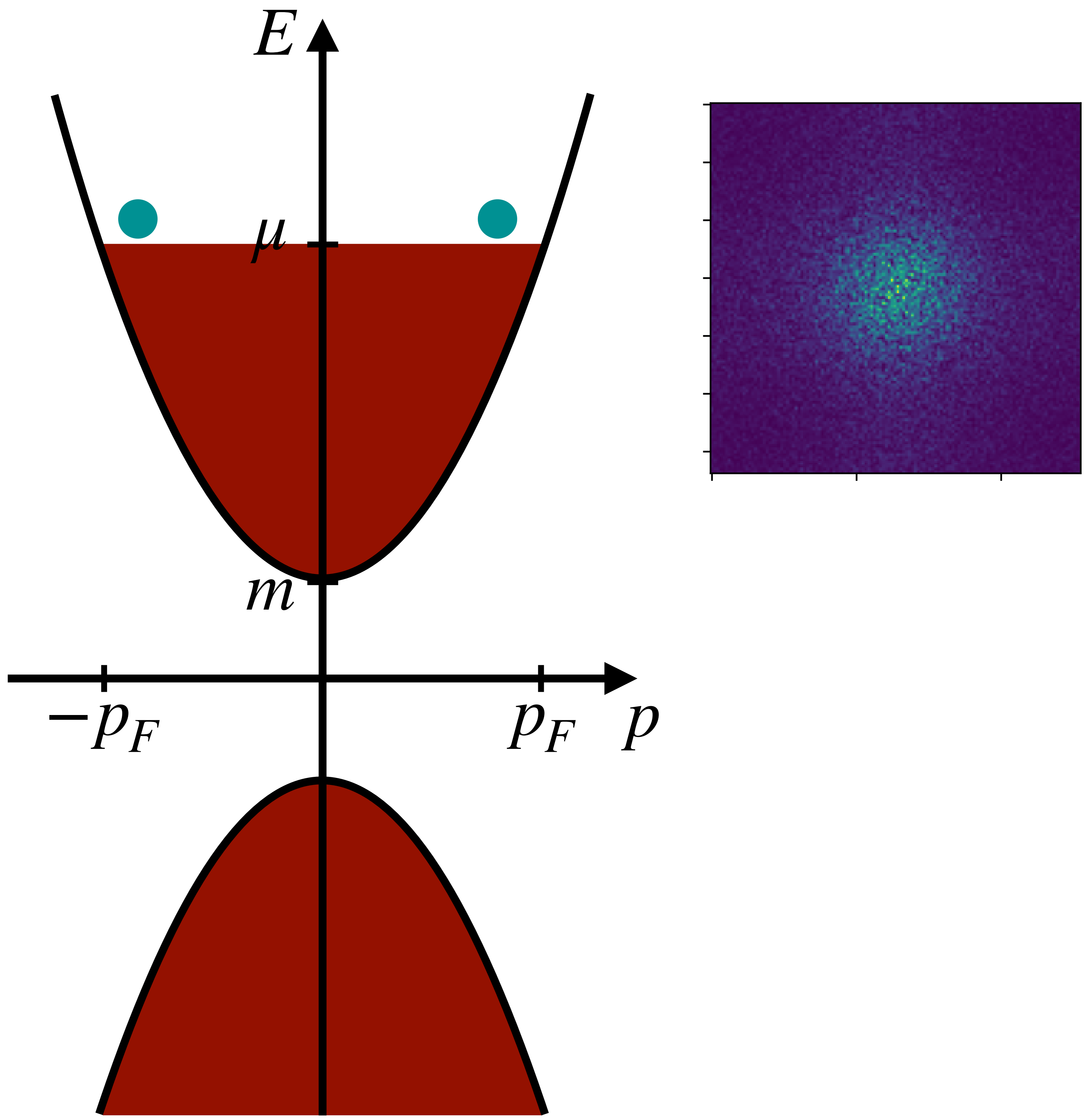
particle-hole excitation $\bar{\psi}_+(p_F)\,\psi_+(p_F)$
with net-momentum $= 0$

$\rightsquigarrow\ \supset$ chiral condensate



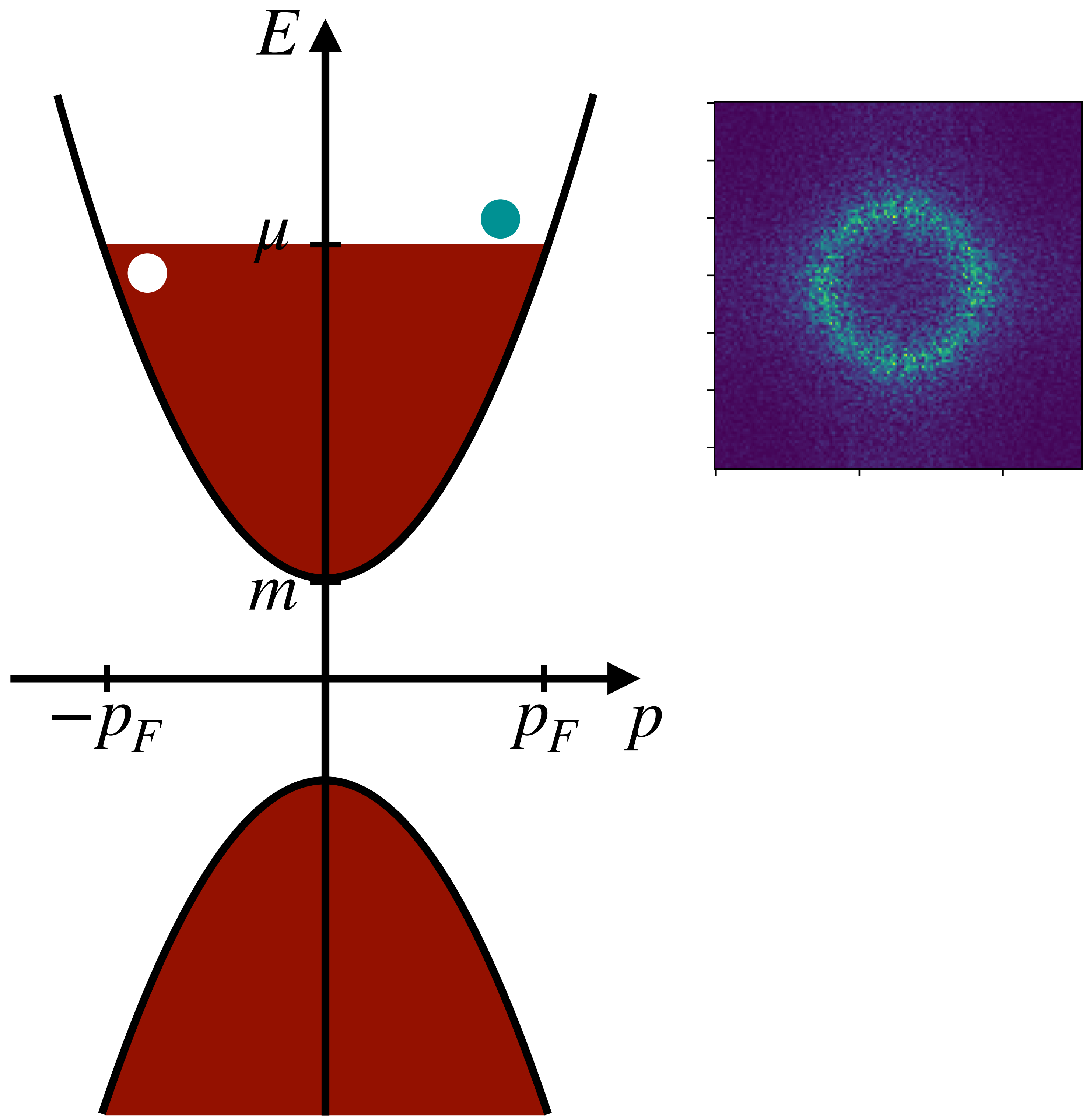
diquark excitation $\psi_+(-p_F)\,\psi_+(p_F)$
with net-momentum $= 0$

$\rightsquigarrow$ color-superconductor



particle-hole excitation $\bar{\psi}_+(-p_F)\,\psi_+(p_F)$
with net-momentum $= 2p_F$

$\rightsquigarrow$ spatial modulations/
inhomogeneous phase



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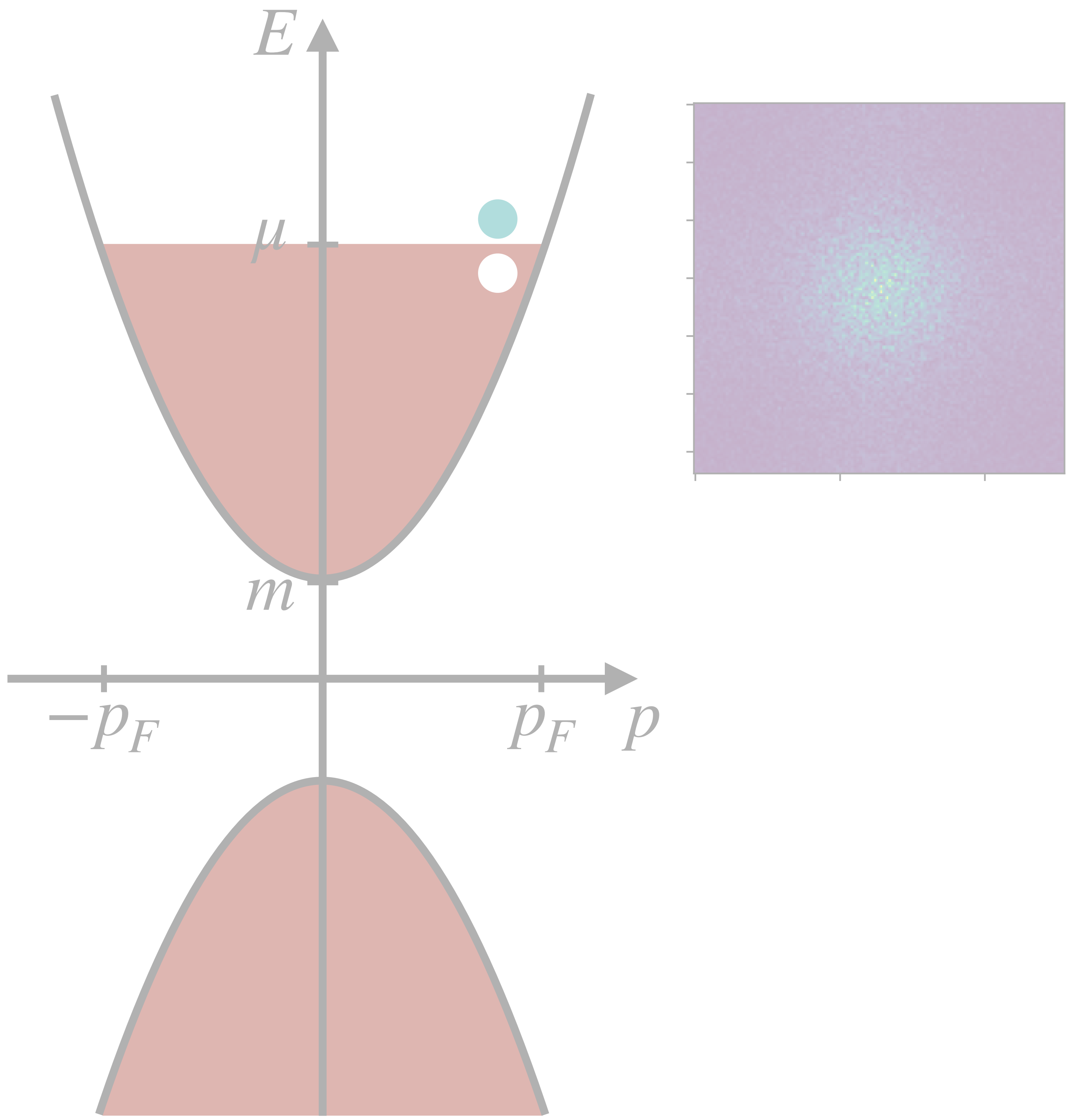
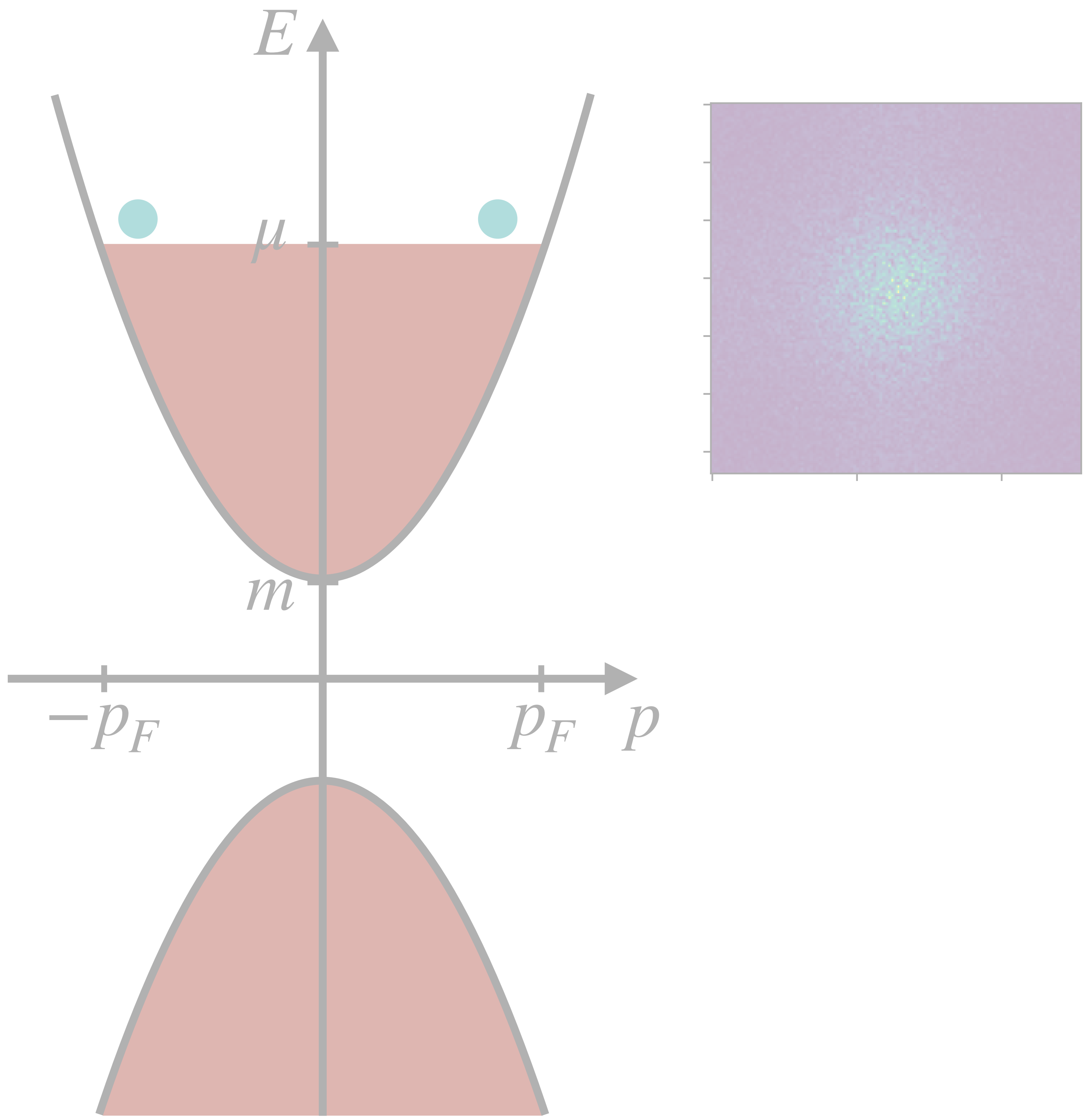
⇝ $\supset$ chiral condensate



diquark excitation $\psi_+(-p_F)\,\psi_+(p_F)$
with net-momentum $= 0$

⇝ color-superconductor



particle-hole excitation $\bar{\psi}_+(-p_F)\,\psi_+(p_F)$
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⇝ spatial modulations/
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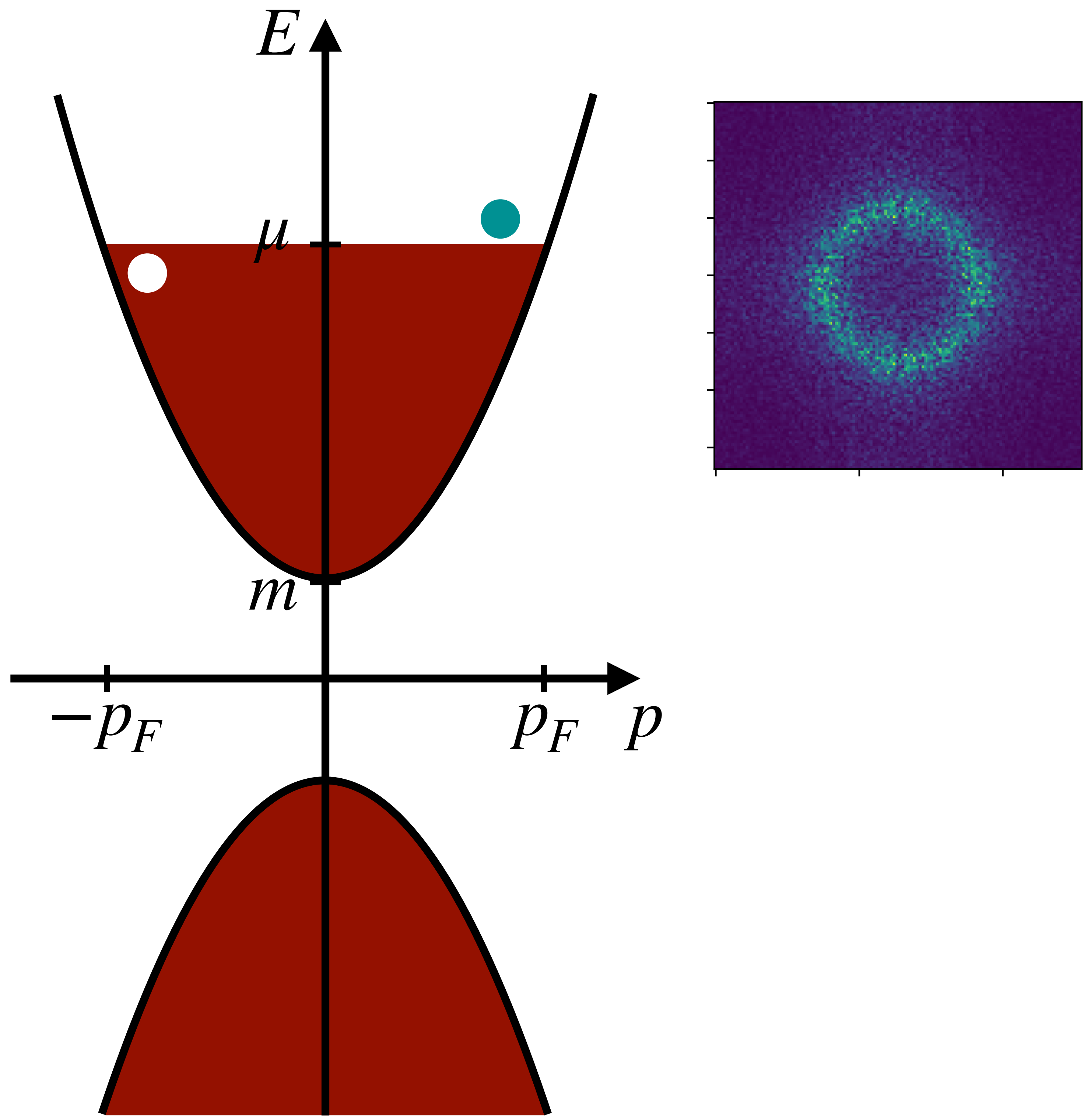


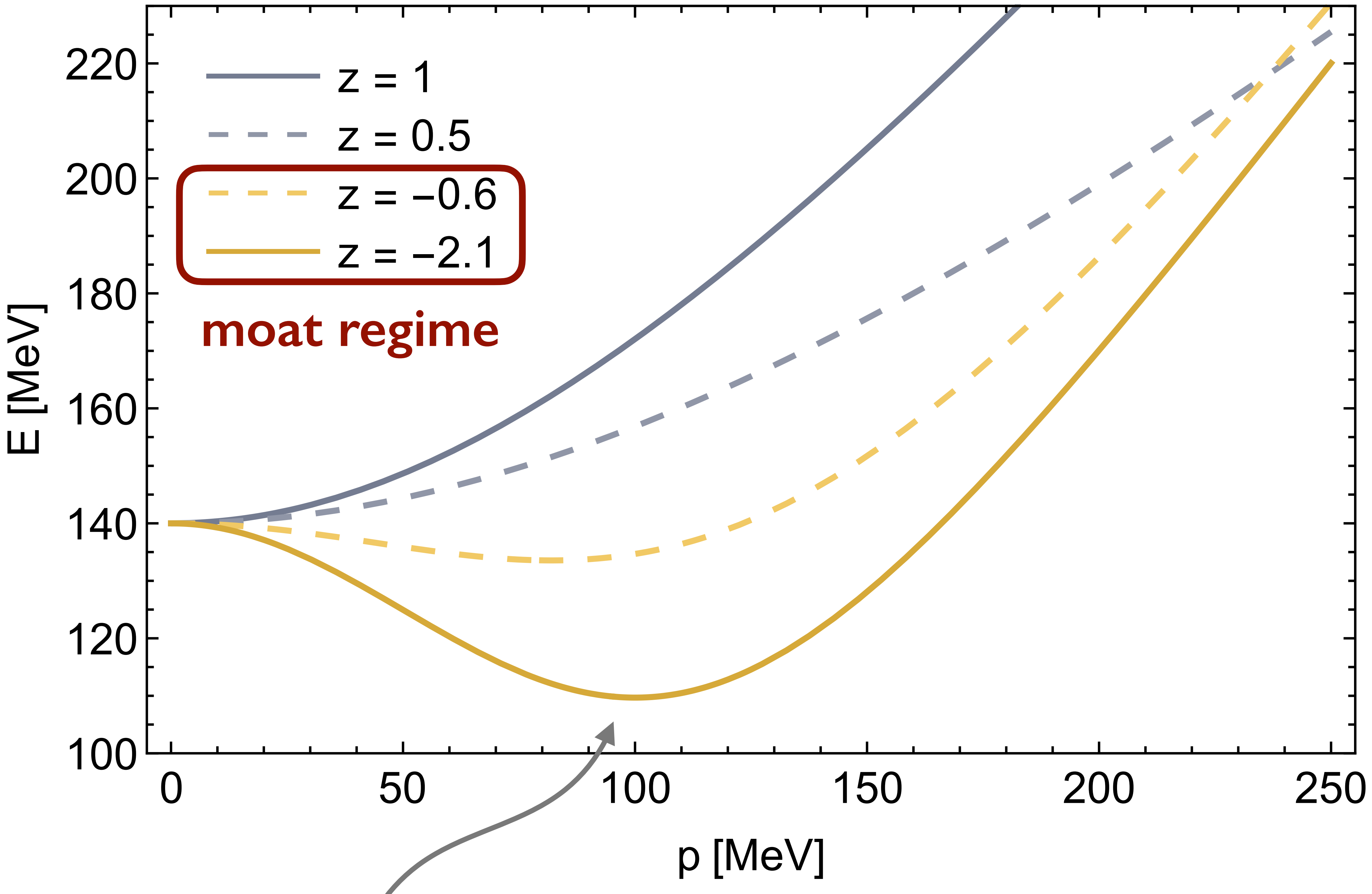
THE MOAT REGIME

[Pisarski, FR, 2103.06890]

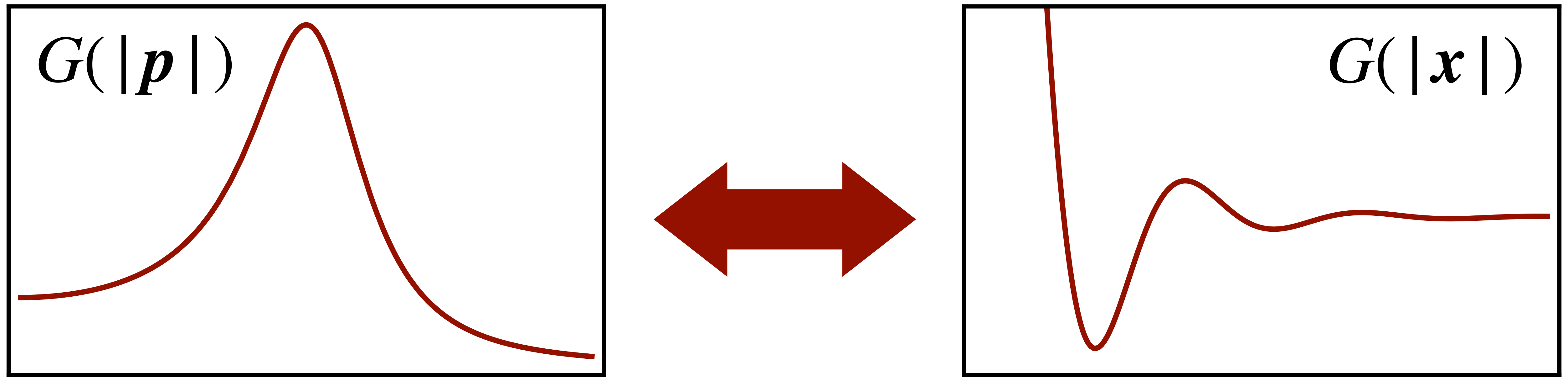
[source: Simon Ledingham]

moat: (static) boson dispersion is minimal at nonzero momentum

$$\sqrt{1/G(p_0=0,\boldsymbol{p}^2)} = E(\mathbf{p}^2) = \sqrt{Z(\mathbf{p^2})\,\mathbf{p^2}+m^2} \approx \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p^6}) + \bar{m}^2}$$



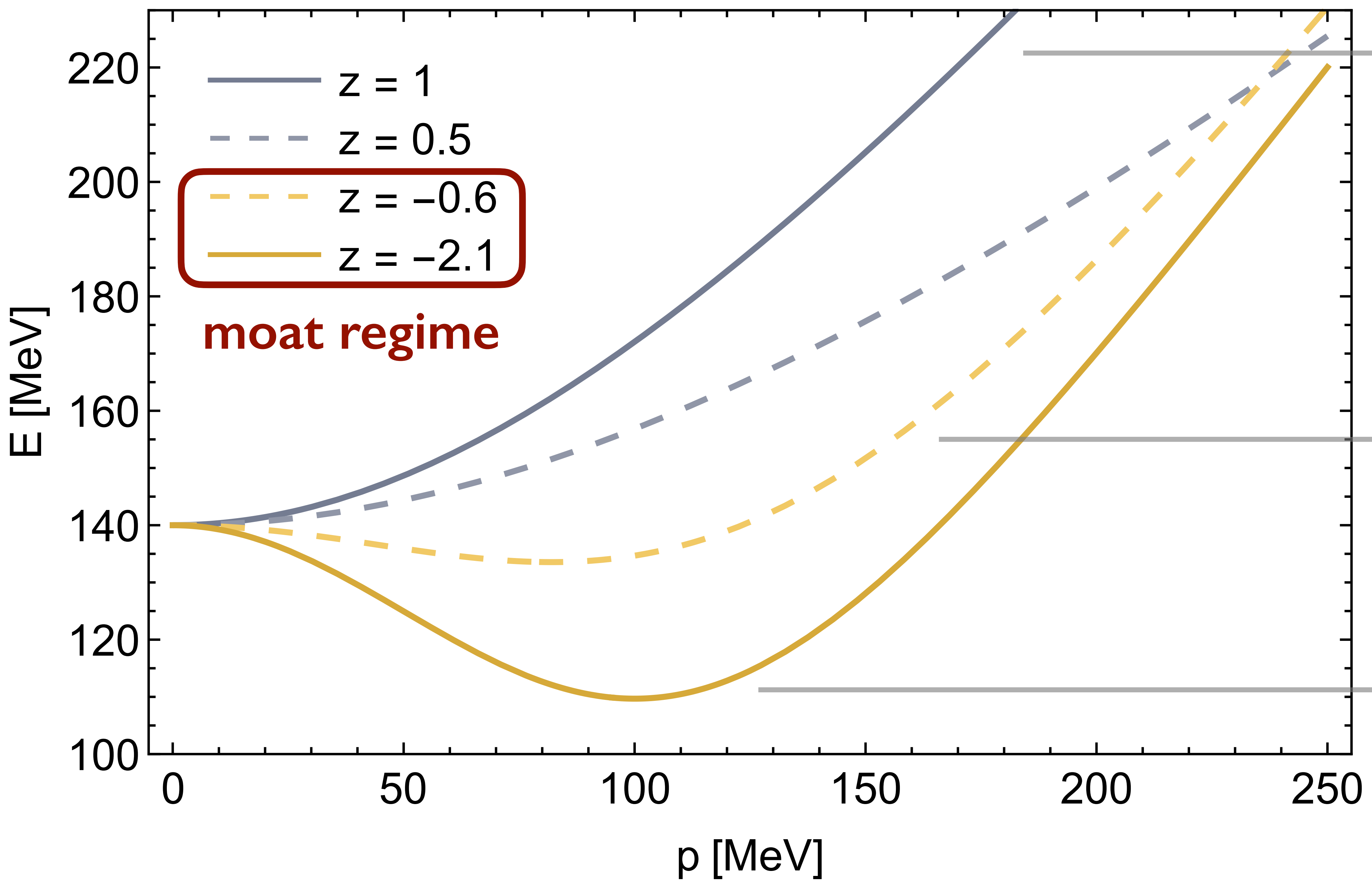
"gain energy by going faster"



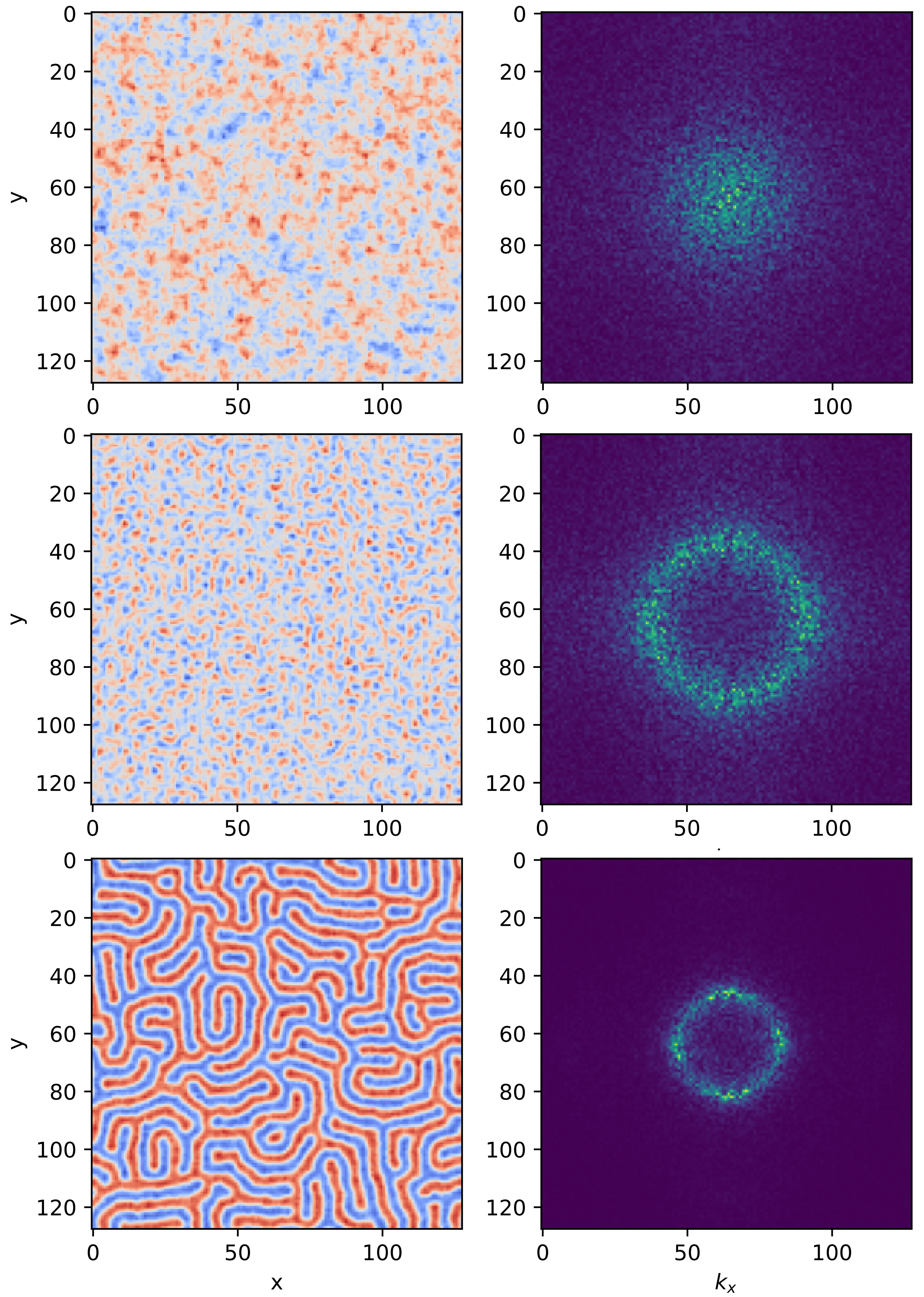
favored momentum **spatial modulations**

MOAT REGIME & PATTERNS

$$E(\mathbf{p}^2) = \sqrt{Z(\mathbf{p}^2)\,\mathbf{p}^2 + m^2} \approx \sqrt{z\,\mathbf{p}^2 + w\mathbf{p}^4 + \mathcal{O}(\mathbf{p}^6) + \bar{m}^2}$$



2d Ising model with different bare z



moat regime ⟷ pattern formation

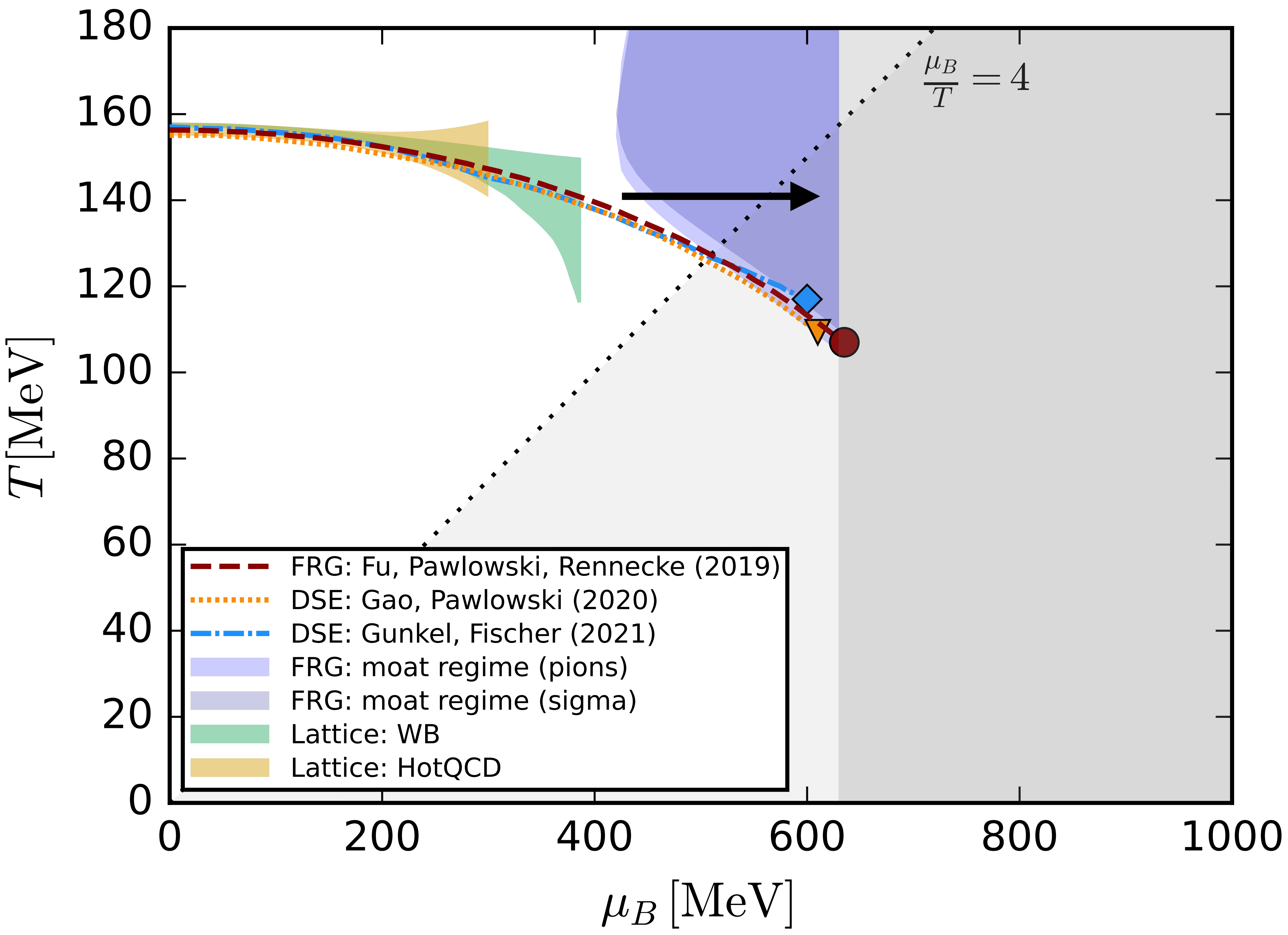
[Harhoff, FR, Riedel, Schlichting (in preparation)]
see also: [Schindler et al., 1906.07288; Valgushev, Winstel, 2403.18640]

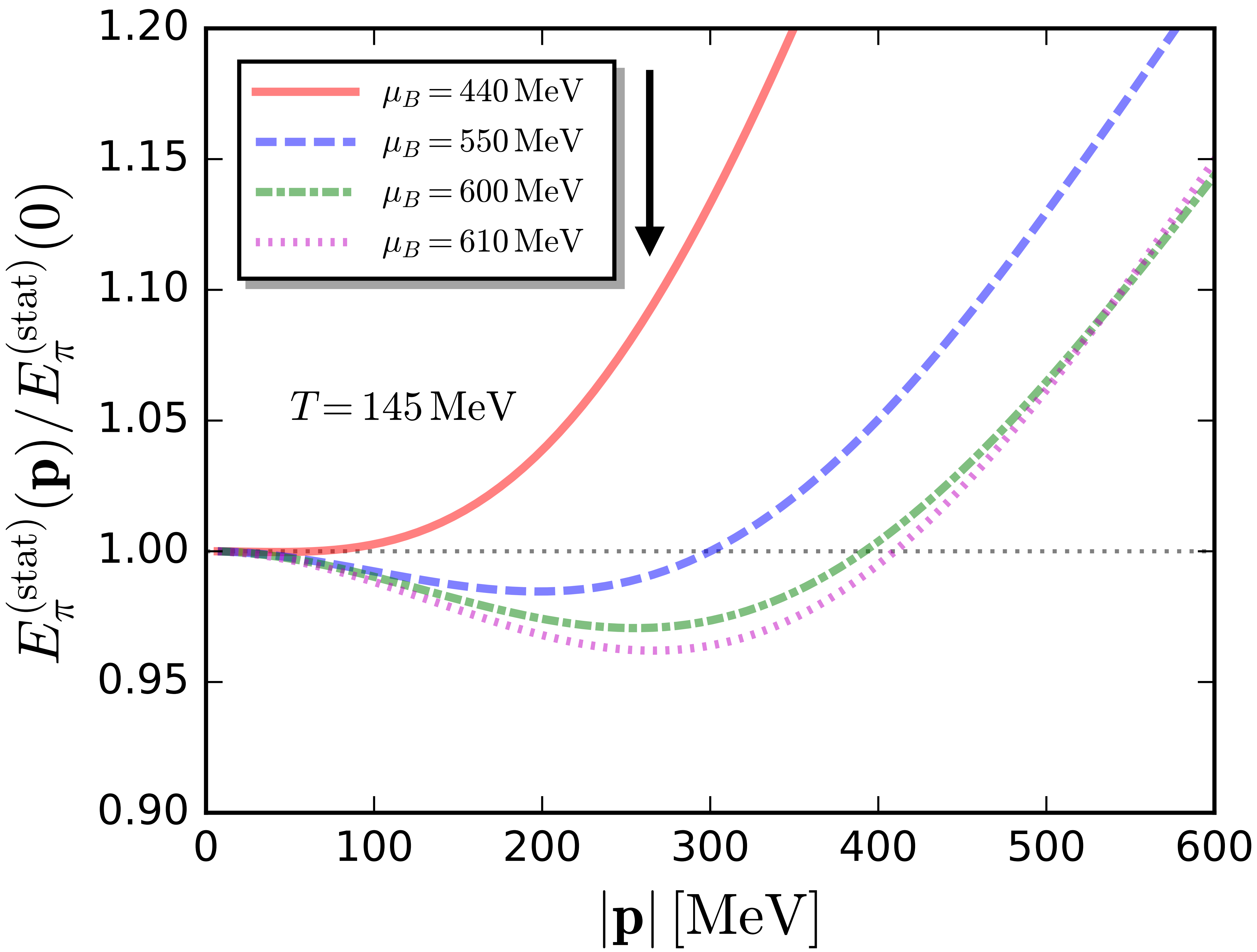
THE MOAT REGIMES IN QCD

[Fu, Pawlowski, Pisarski, FR, Wen, Yin, 2412.15949]

- moat regime appears to depend on the species
- the lighter the meson, the stronger the signal → pions are good probes

[Töpfel, Pawlowski, Braun, 2412.16059; Cao, 2504.18874; FR, Yin (in preparation)]

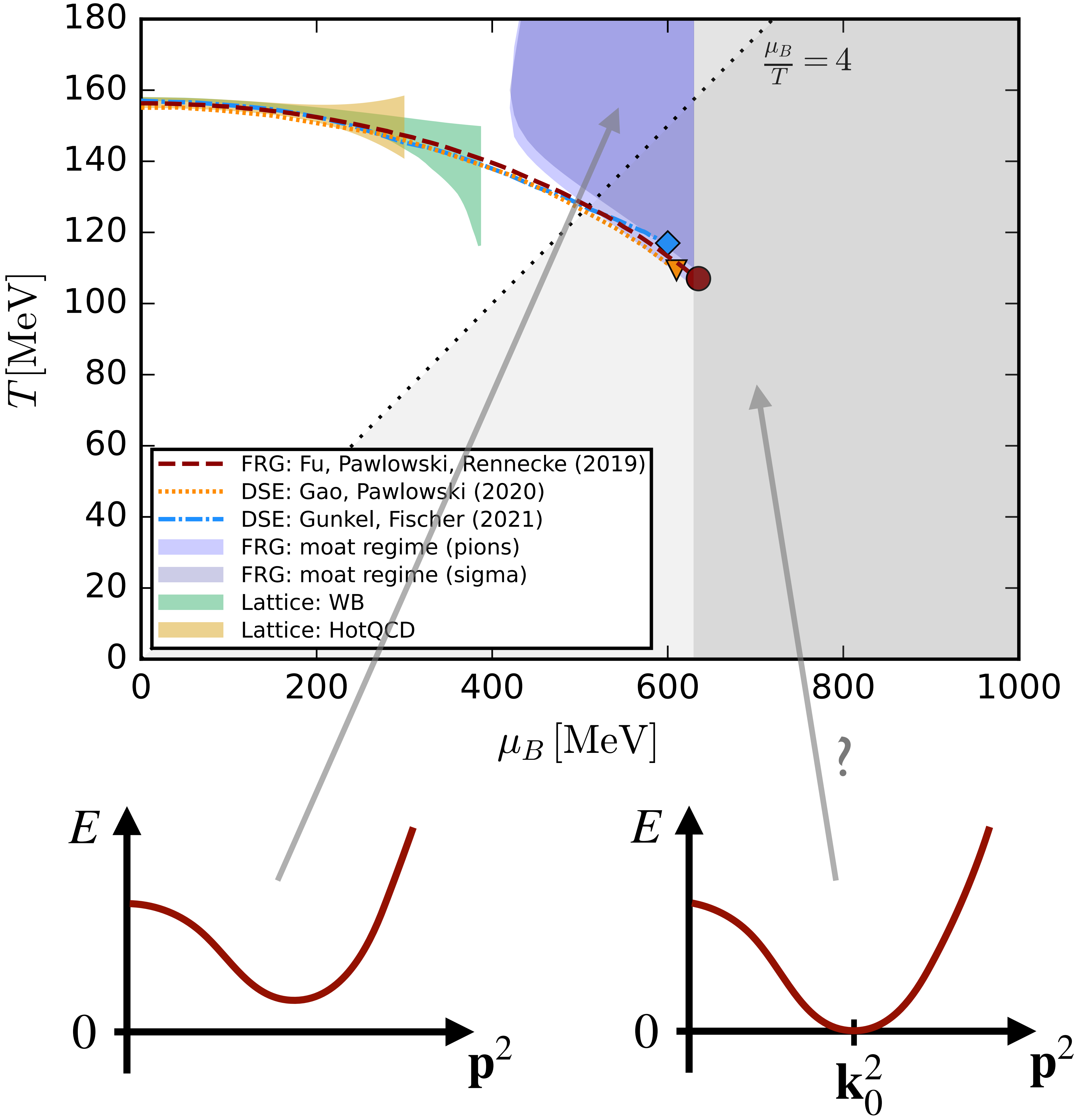




→ evidence for spatial modulations & pattern formation in the phase diagram

IMPLICATIONS OF THE MOAT

The energy gap might close:



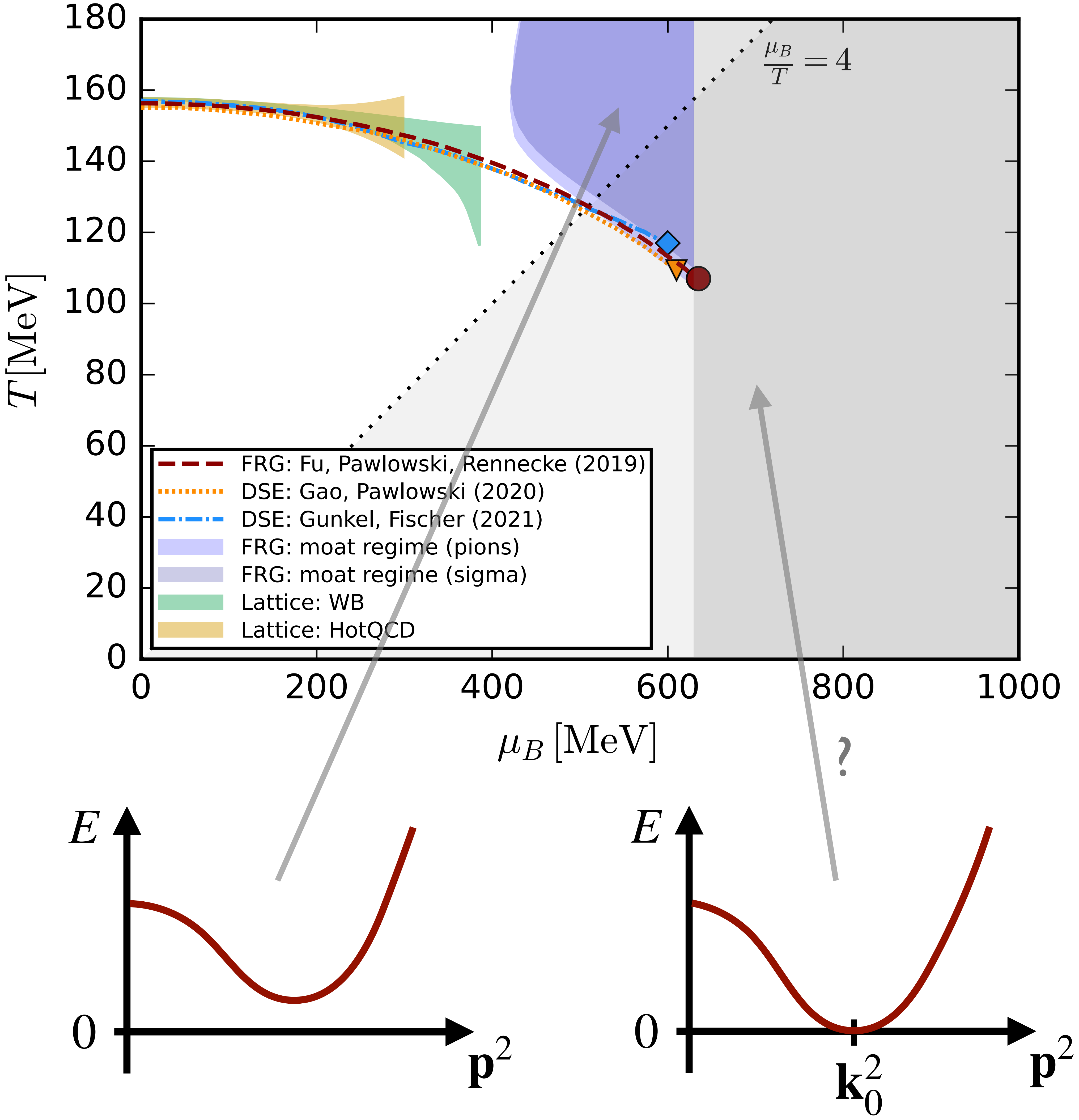
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

IMPLICATIONS OF THE MOAT

The energy gap might close:

DSE: [Motta, Buballa, Fischer; *PRD* (2023-25)]
FRG: [Fu, Pawlowski, Pisarski, FR, Wen, Yin; PRD (2024)]



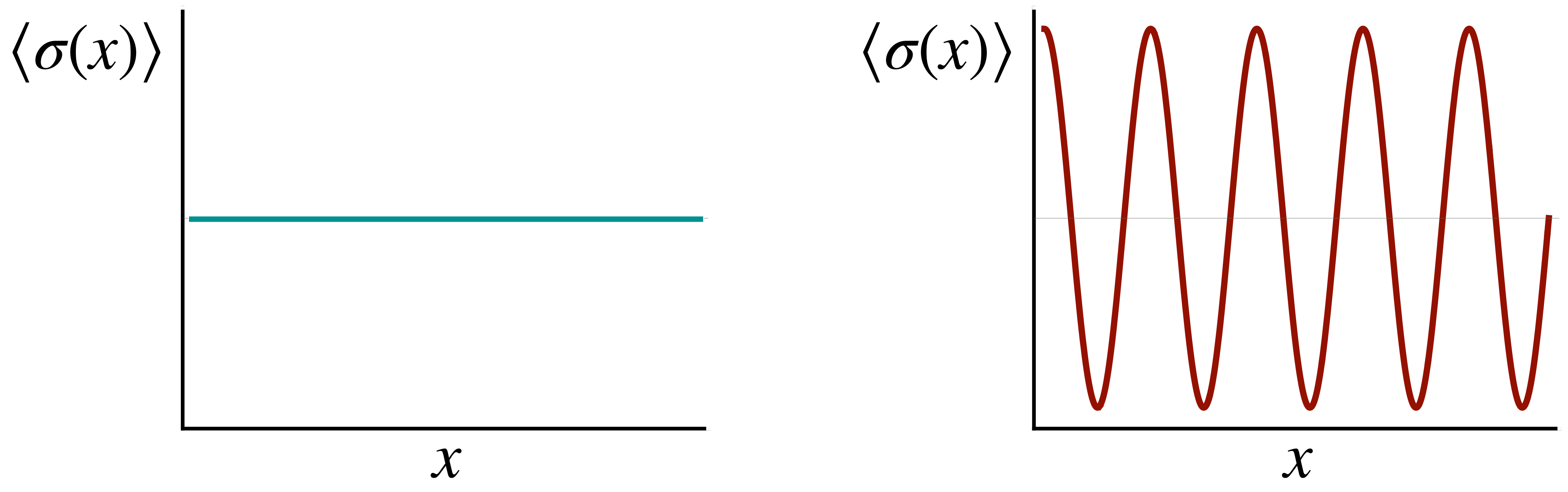
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an **inhomogeneous condensate**

homogeneous

inhomogeneous



IMPLICATIONS OF THE MOAT

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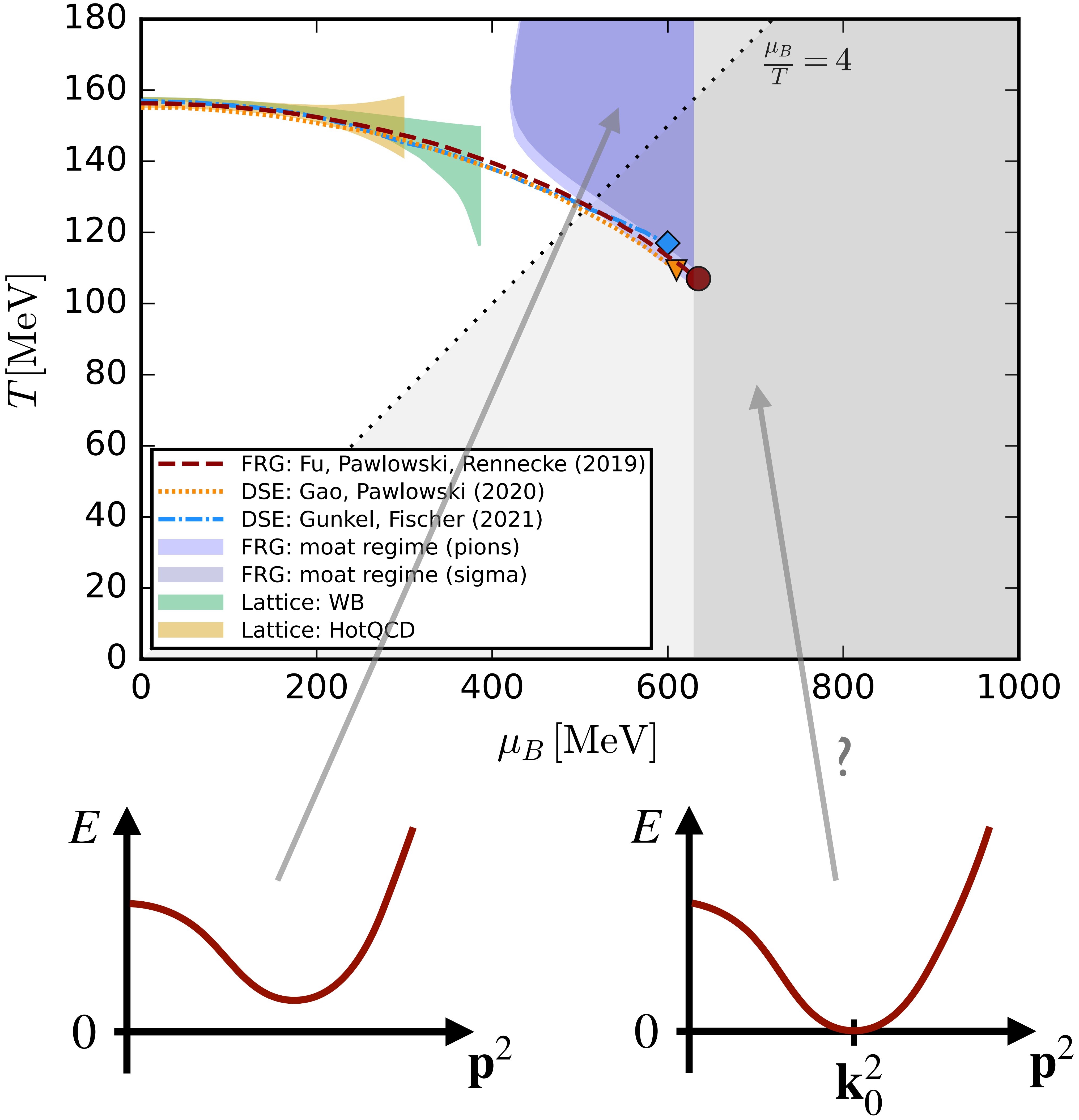


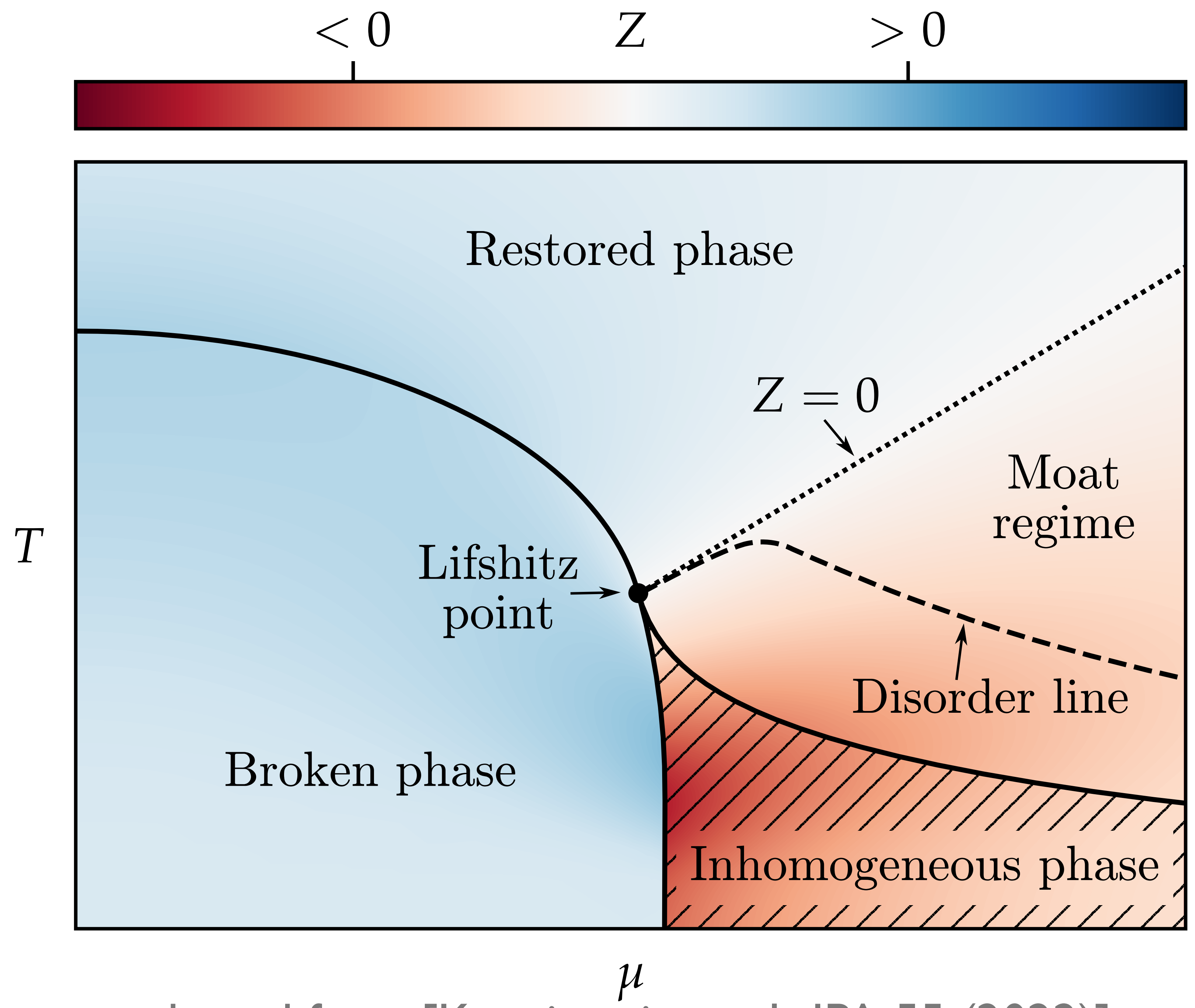
$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:

→ instability towards formation of an inhomogeneous condensate

common feature of low-energy models,

[Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel, 2410.22418]

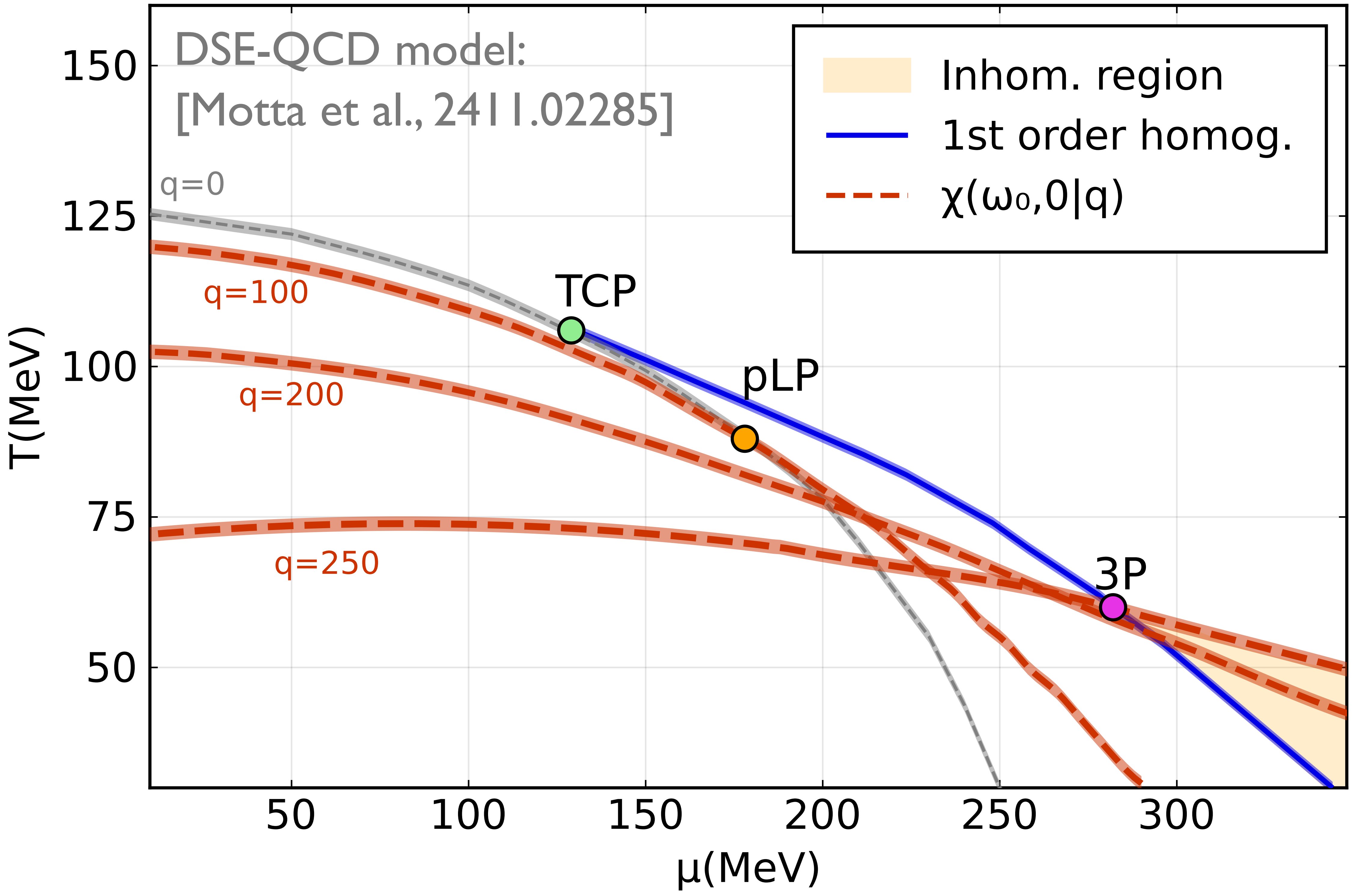


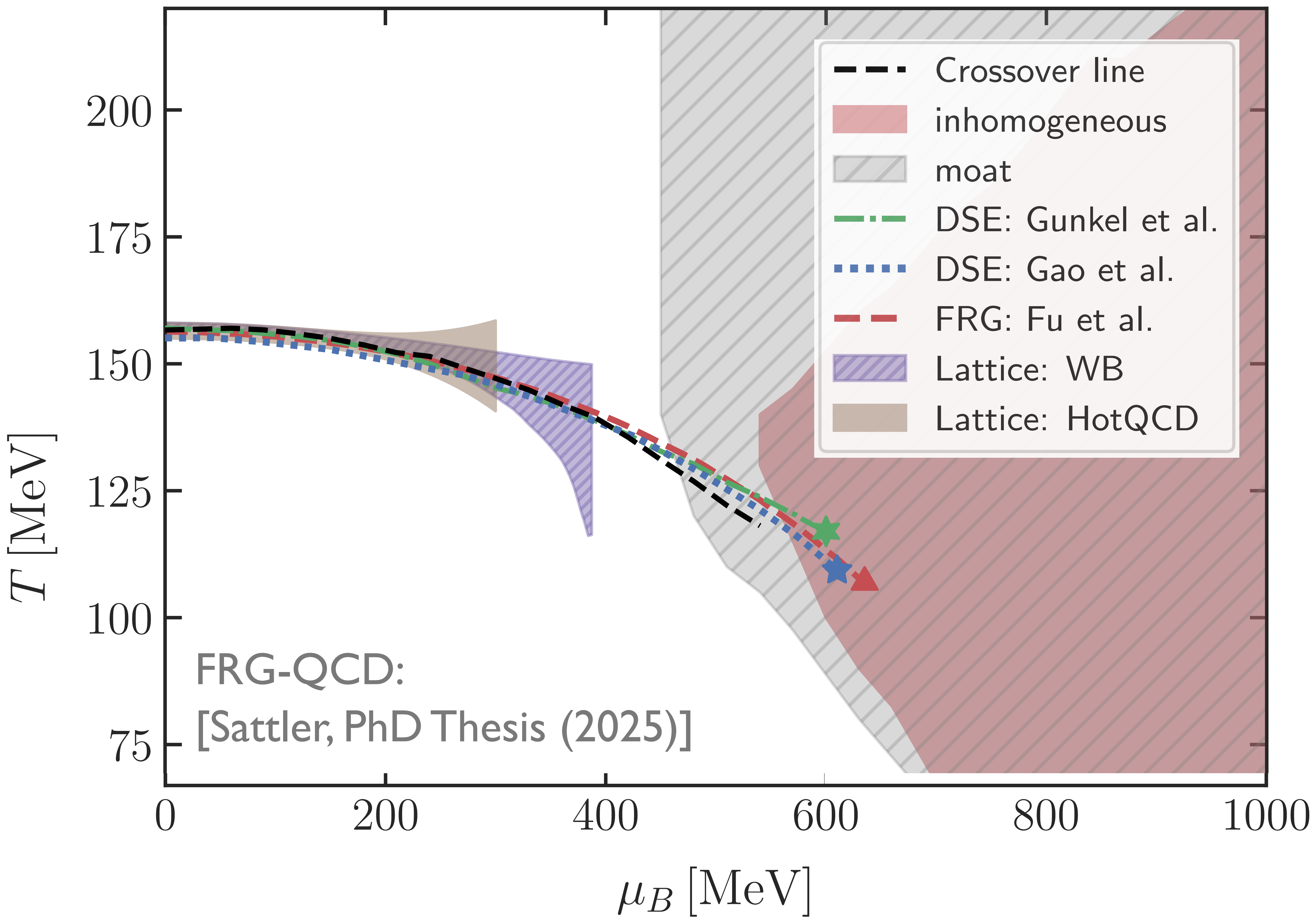
adapted from [Koenigstein et al., JPA 55 (2022)]

- many types of inhomogeneous phases possible (crystals, liquid crystals, ... depends on which spatial symmetries are broken)
- in any case, they are always accompanied by a moat regime

IMPLICATIONS OF THE MOAT

First hints for inhomogeneous phases from exploratory/preliminary QCD studies:





FRG: instability at finite length scale?

→ **liquid crystal?**

[Lee et al., 1504.03185]
[Hidaka et al., 1505.00848]

SEARCH FOR MOATS IN HICS

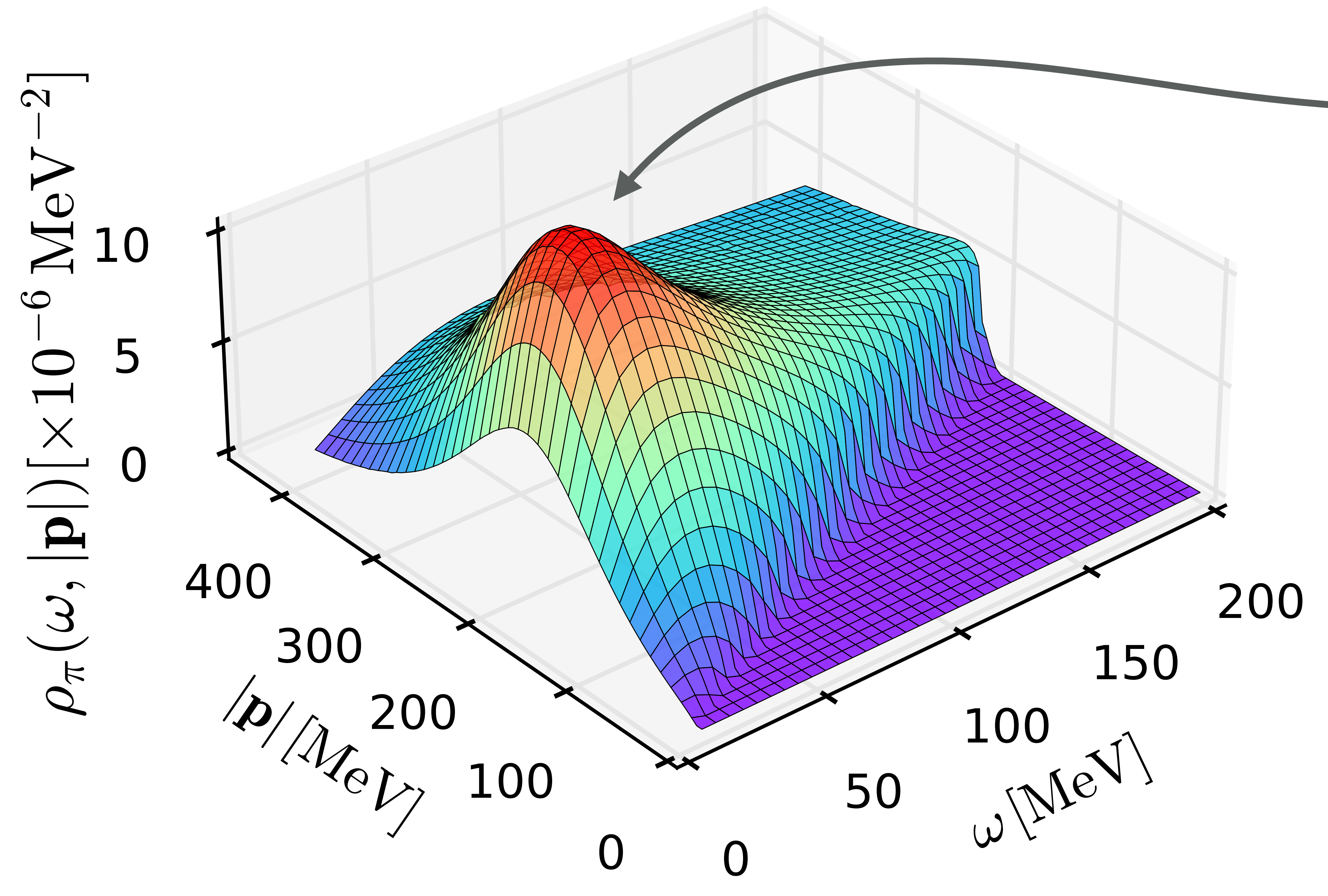
[Pisarski, FR, 2103.06890]

intuitive idea

Characteristic feature of moats: particles with minimal energy at nonzero momentum

$\Longrightarrow$ modified particle production at nonzero momentum

- description of particle production requires knowledge of real-time correlation functions
- directly accessible with the FRG [Floerchinger; JHEP (2012), Kamikado, Strodthoff, von Smekal, Wambach; EPJ C (2014), ...MANY more...]



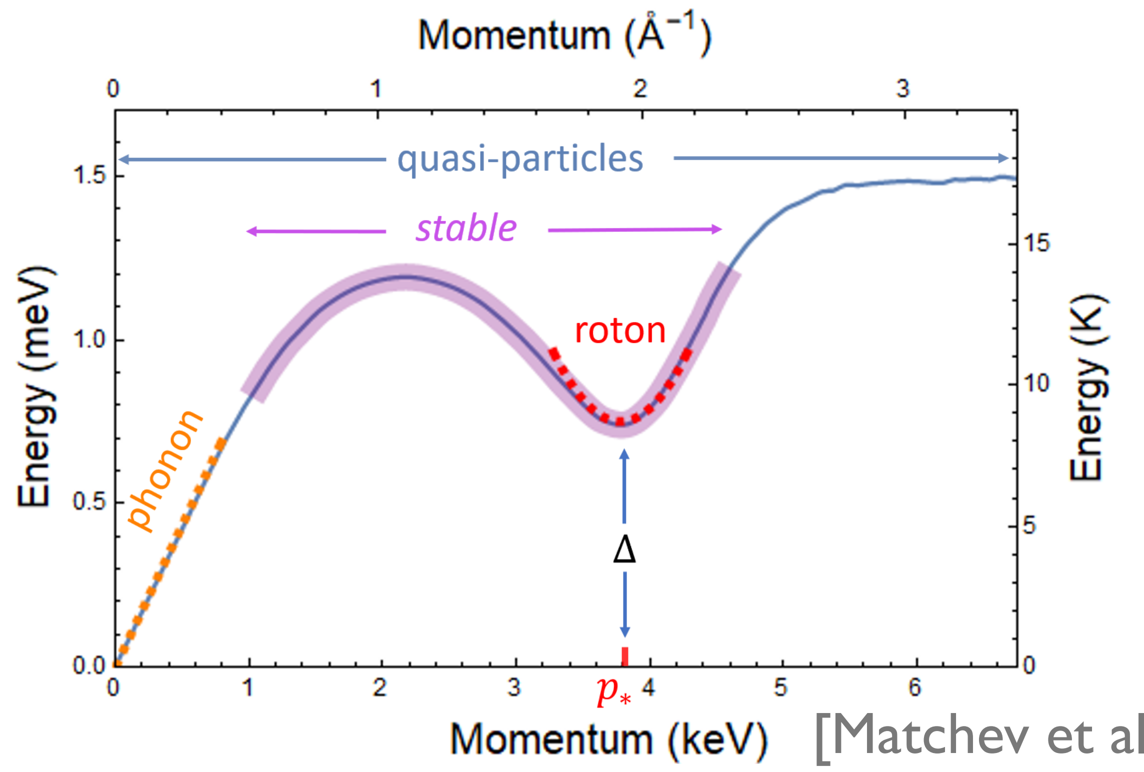
pion spectral function in the moat regime

quasi-particle like peak in the spacelike region:

the "moaton"

[Fu, Pawlowski, Pisarski, FR, Wen, Yin; PRD (2025)]

inspired by the roton in superfluid Helium II



[Matchev et al.; JHEP (2022)]

SEARCH FOR MOATS IN HICS

[Pisarski, FR, 2103.06890]

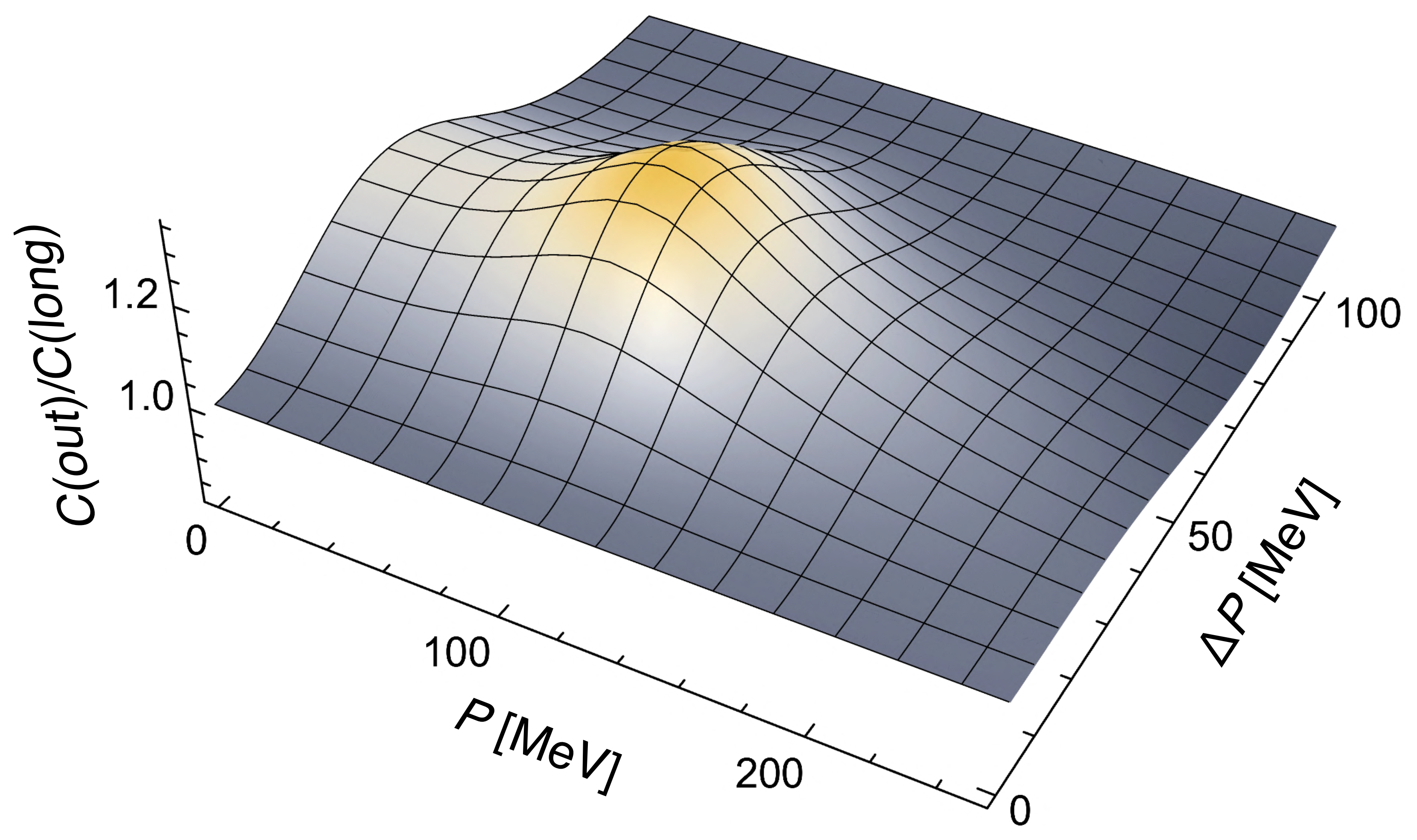
intuitive idea

Characteristic feature of moats: particles with minimal energy at nonzero momentum

$\Longrightarrow$ **modified particle production at nonzero momentum**

- HBT correlations

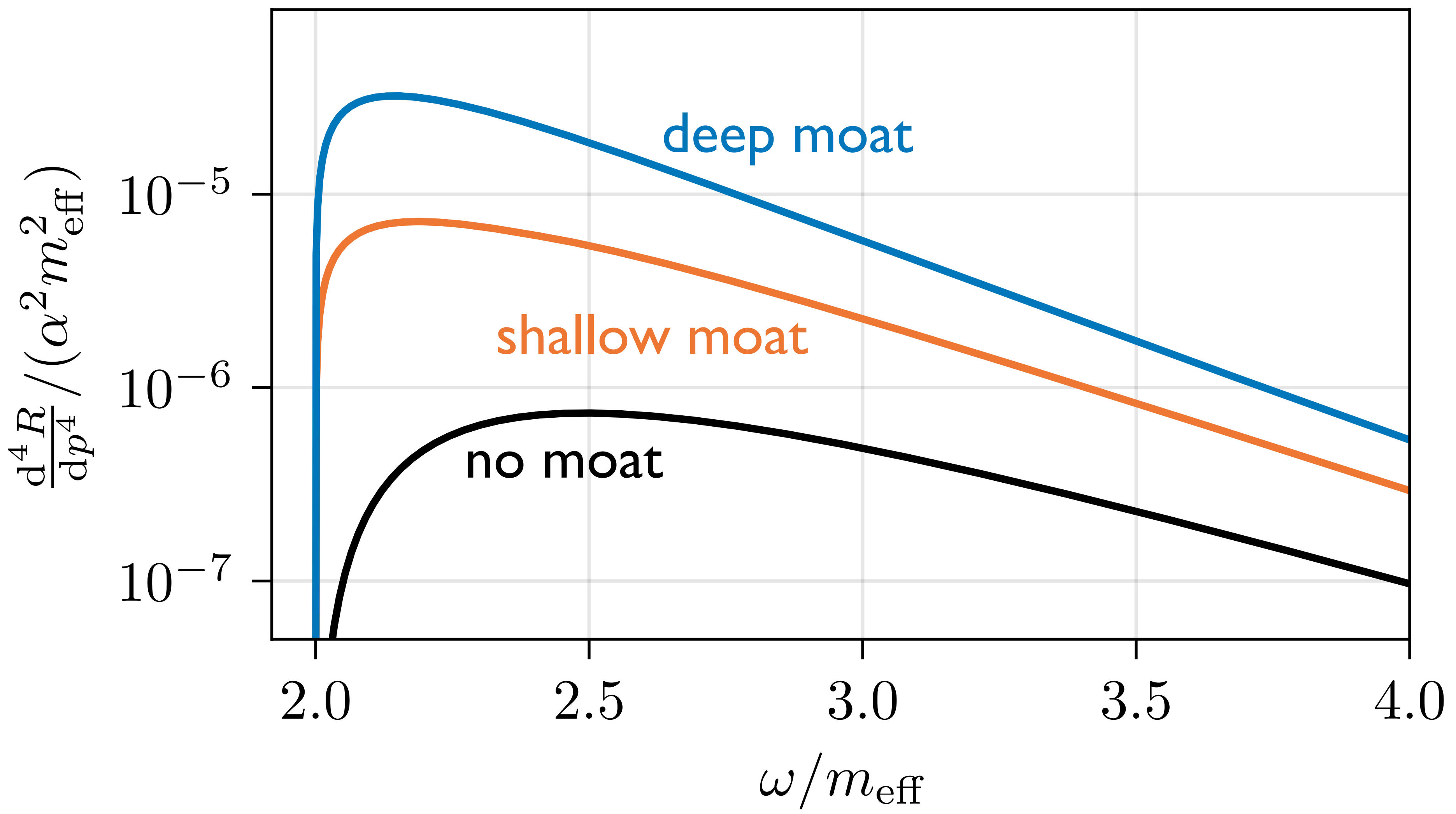
[FR, Pisarski, Rischke, 2301.11484]



→ characteristic peak at nonzero average pair momentum

- dilepton production

[Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel, 2410.22418]



→ enhanced dilepton rate from "moaton threshold"

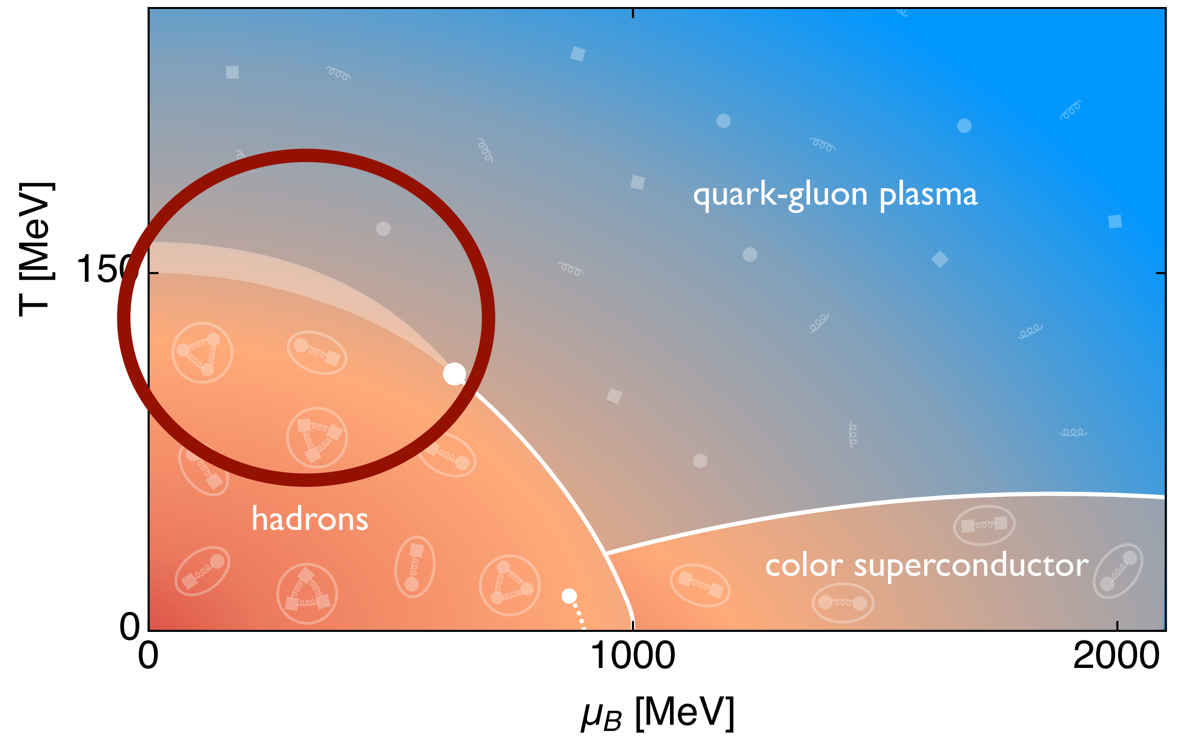
signals of inhomogeneous phases: [Fukushima et al., 2306.17619; Hayashi, Tsue, 2407.08523]

so far, predictions are based on very simplistic models, not QCD!

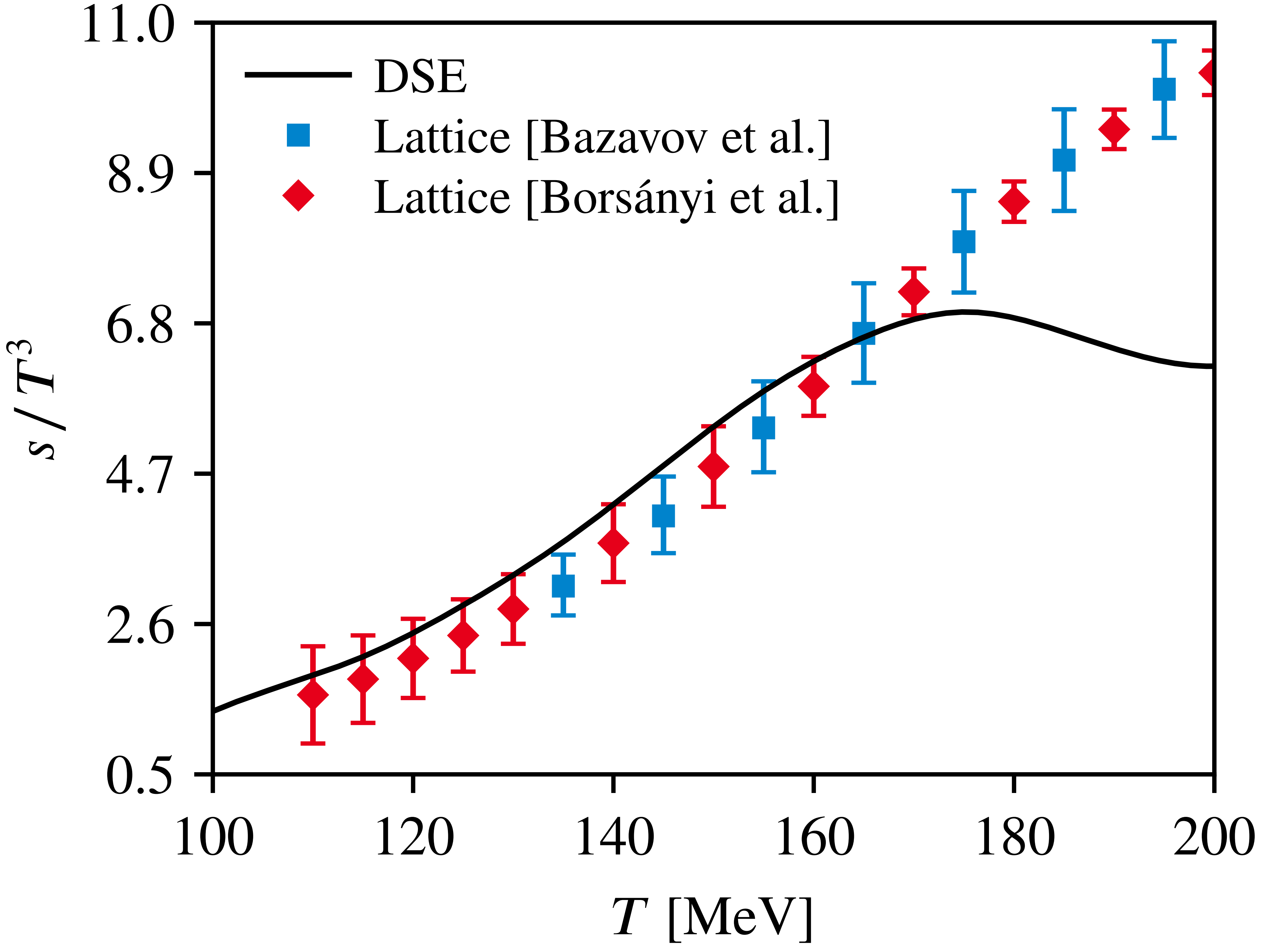
EOS OF DENSE QCD MATTER

HOT AND SOMEWHAT DENSE EOS

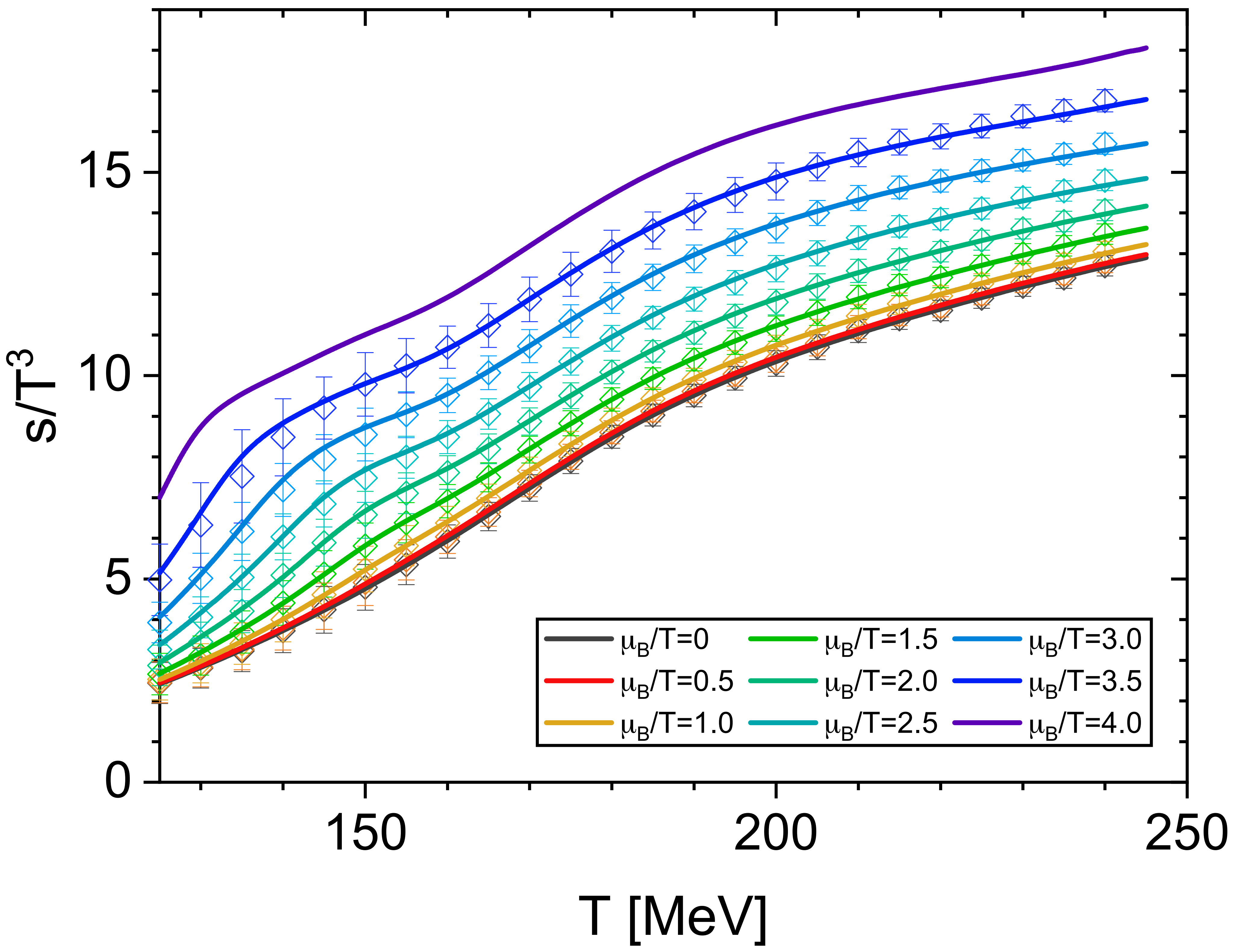
Rapid progress in the past years. Example: entropy density



- **2020** [Isserstedt, Fischer, Steinert, 2012.04991]



- **2025** [Lu et al., 2504.05099]



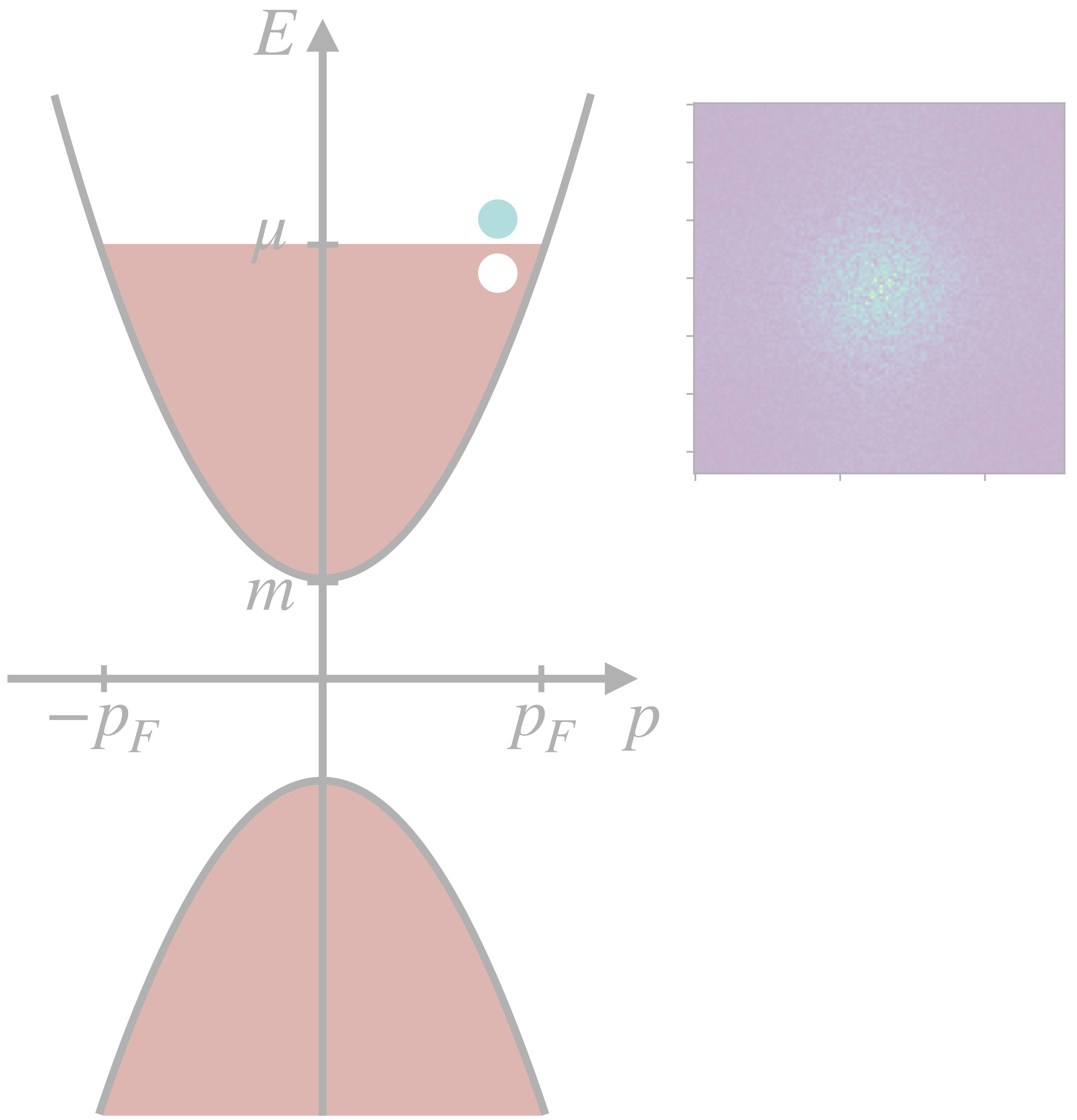
more on QCD-EoS: [Isserstedt et al. 1906.11644; Bernhardt, Fischer, Isserstedt, 2208.01981; Lu et al., 2310.18383]

EXCITATIONS AT NONZERO DENSITY

What kind of excitation do we expect? Look at Dirac cone at $\mu > 0$

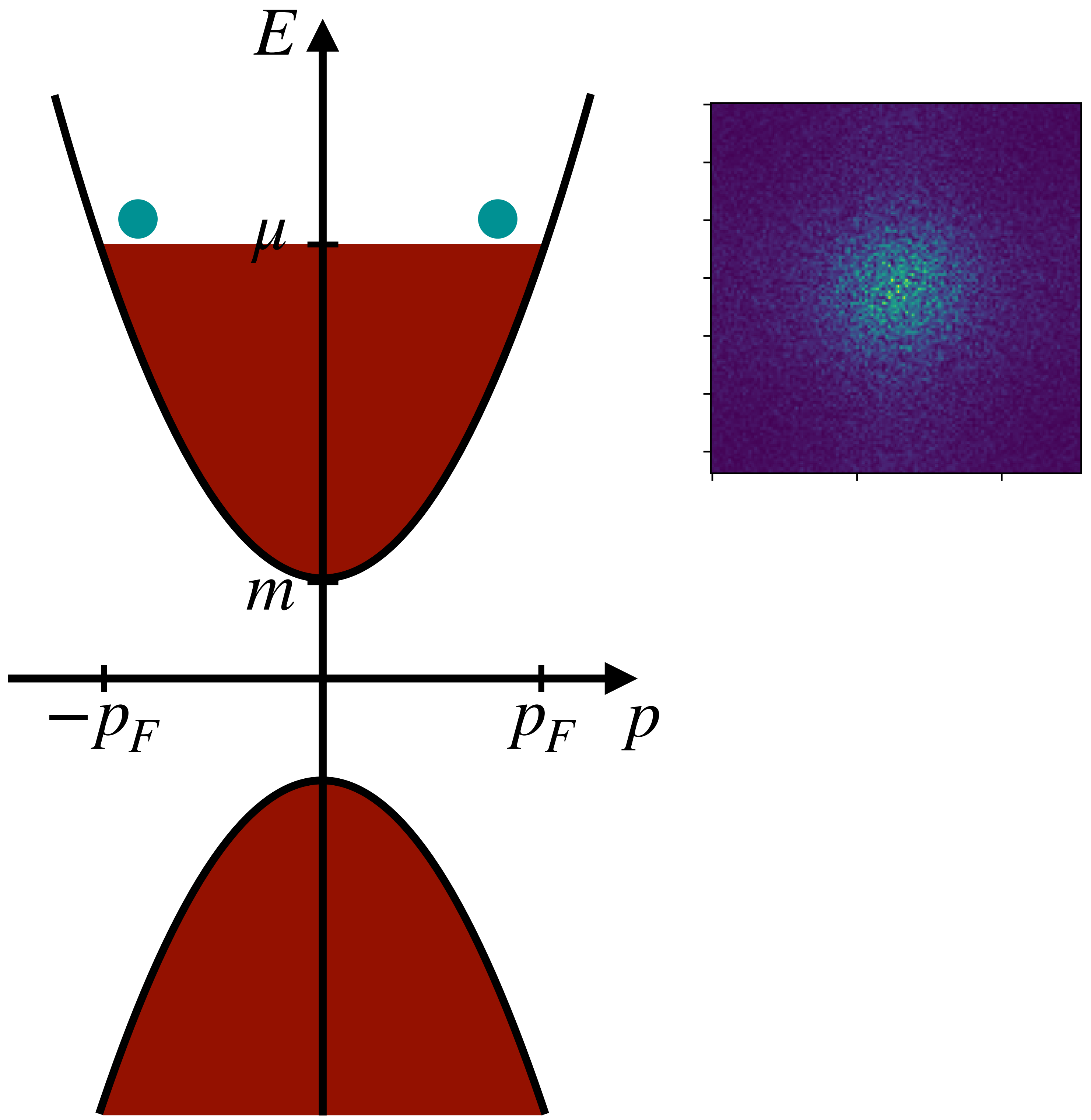
particle-hole excitation $\bar{\psi}_+(p_F)\,\psi_+(p_F)$
with net-momentum $= 0$

⤳ $\supset$ chiral condensate



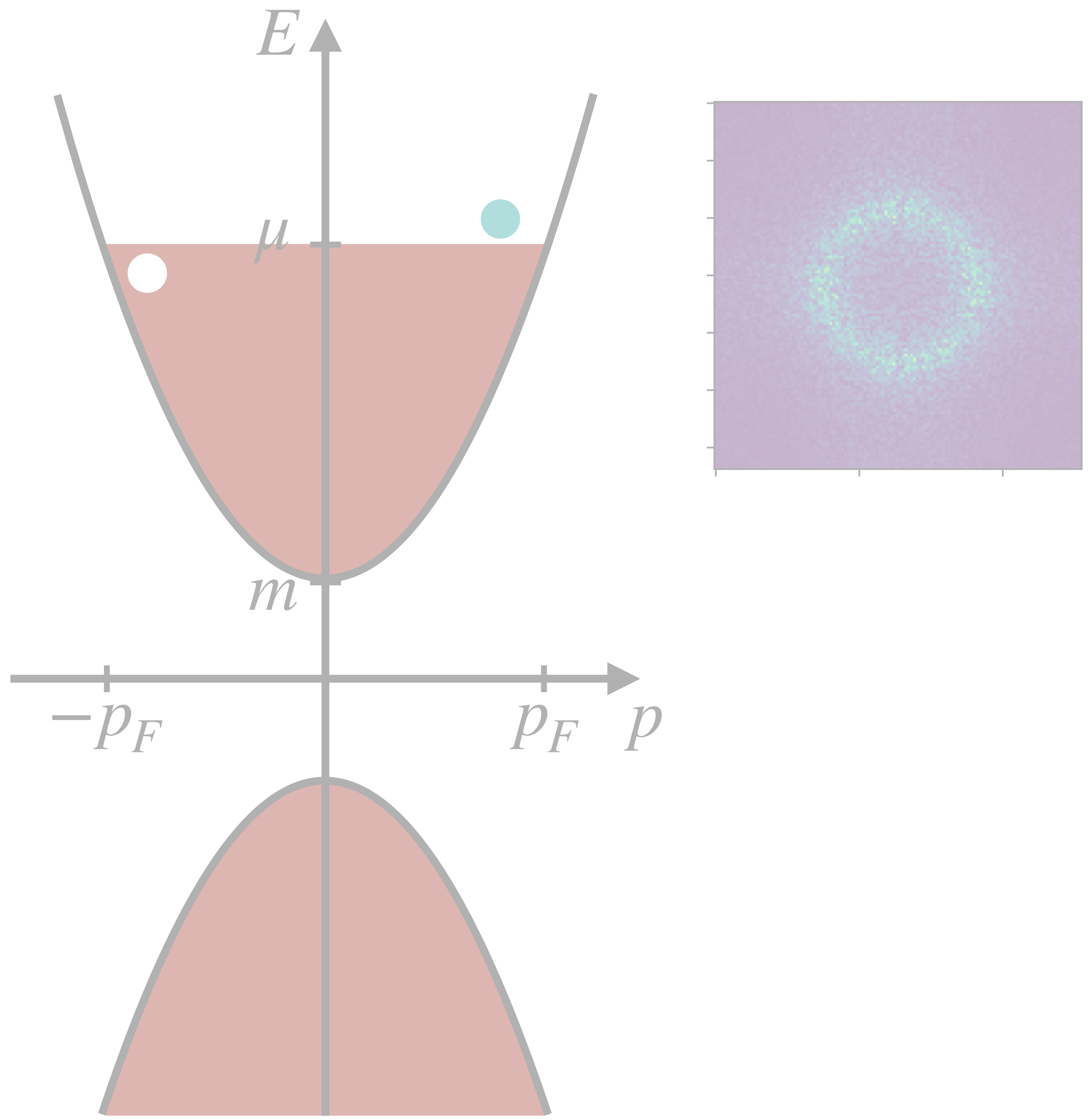
diquark excitation $\psi_+(-p_F)\,\psi_+(p_F)$
with net-momentum $= 0$

⤳ color-superconductor



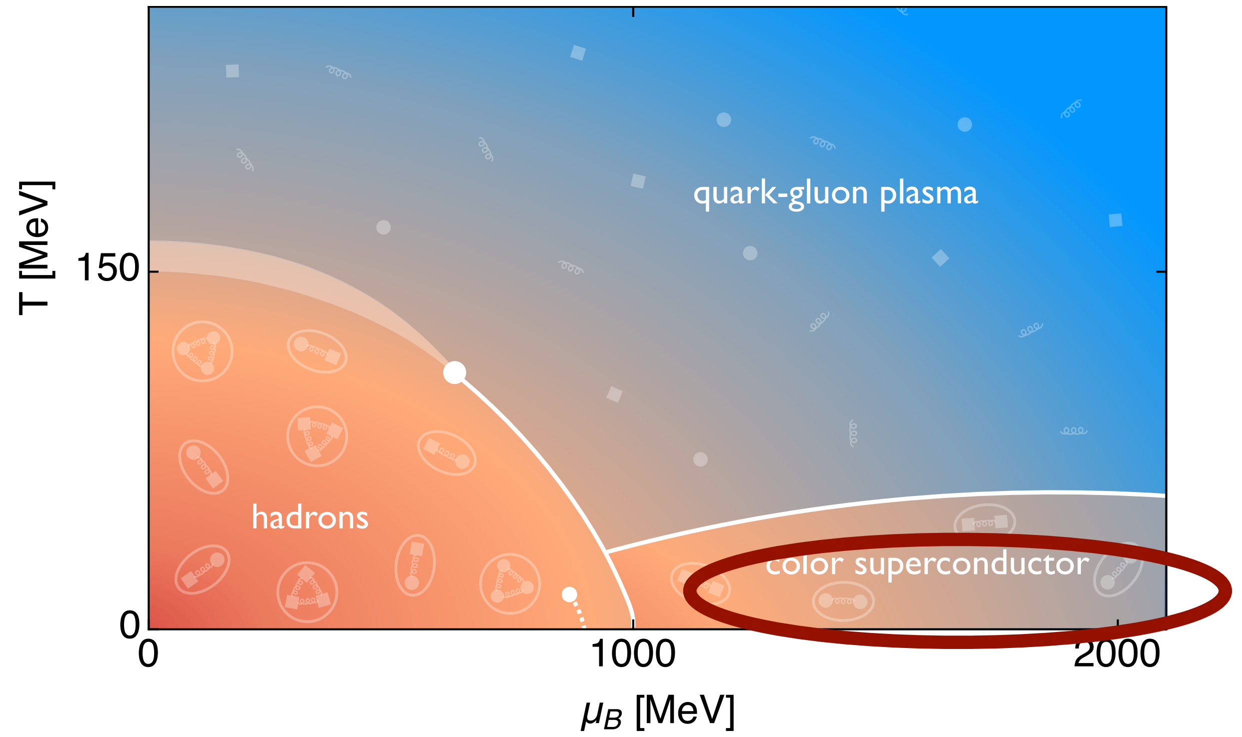
particle-hole excitation $\bar{\psi}_+(-p_F)\,\psi_+(p_F)$
with net-momentum $= 2p_F$

⤳ spatial modulations/ inhomogeneous phase

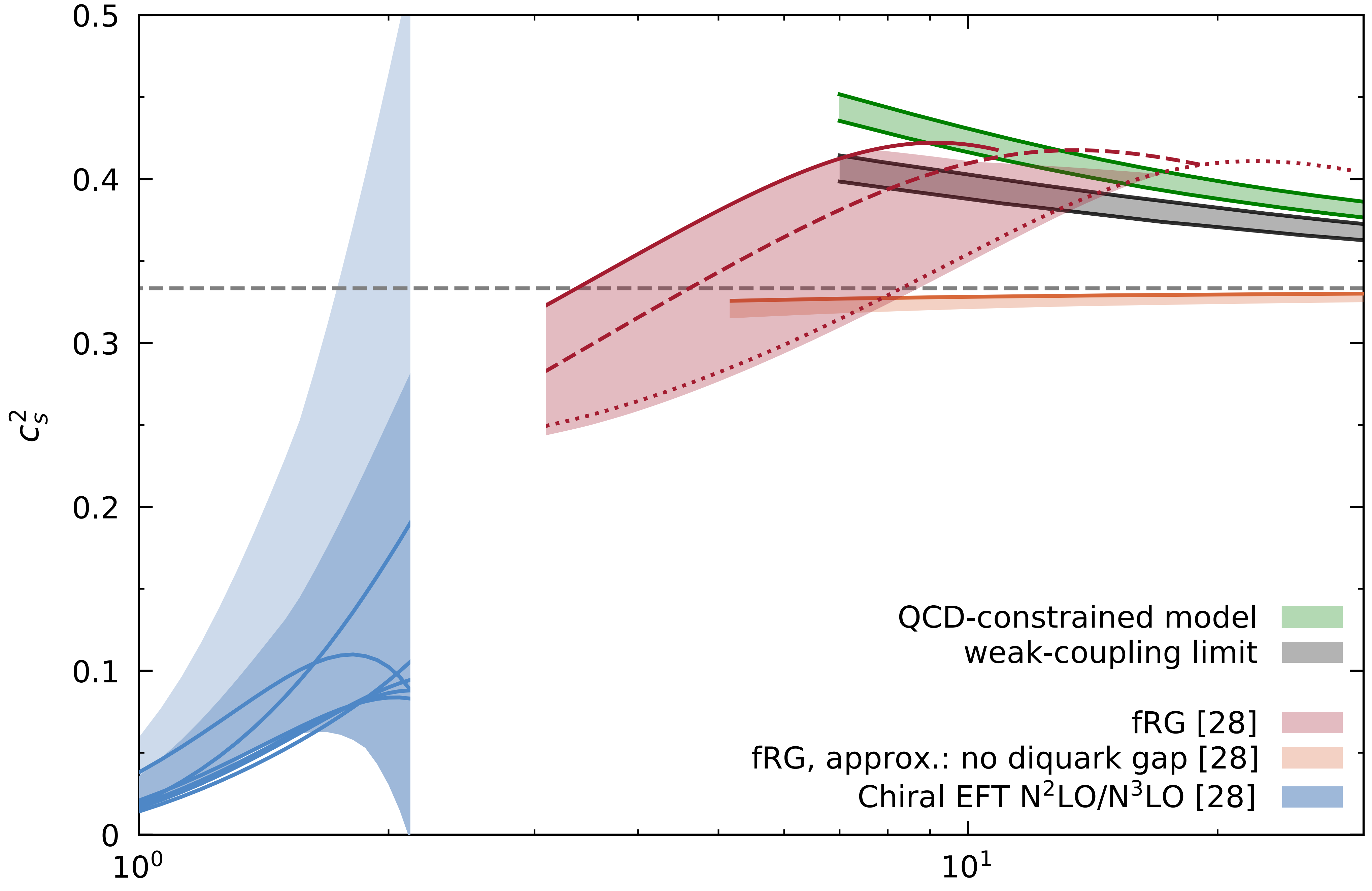


COLD AND VERY DENSE EOS

EoS and color-superconducting gap in the non-perturbative high-density regime

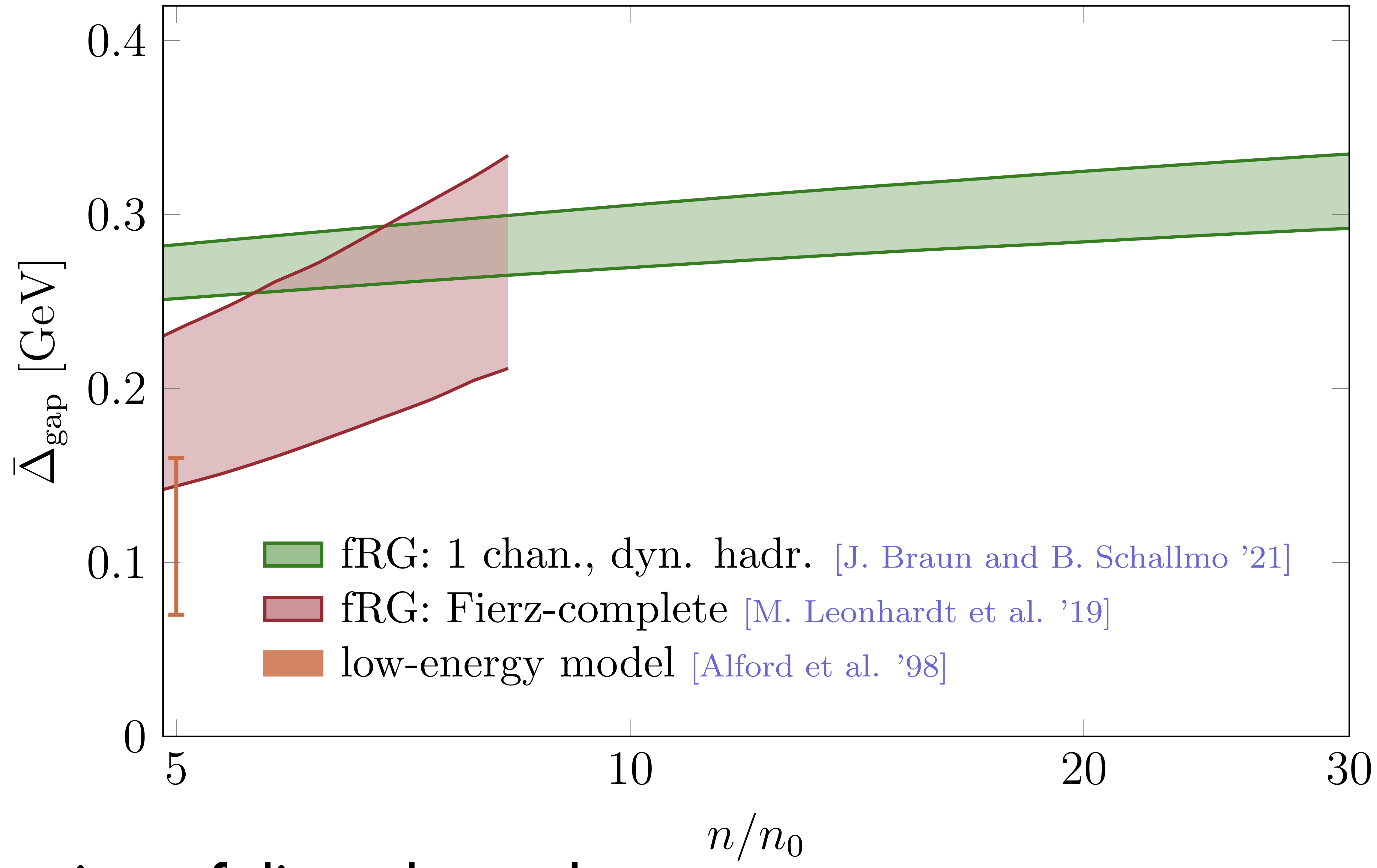


- speed of sound^2



[Braun, Schallmo, 2106.04198]
[28]=[Leonhardt et al., 1907.05814]

- diquark condensate (2SC)



- $c_s^2 > 1/3$ tied to formation of diquark condensate
- $c_s^2 \to 1/3$ from below for very large density [Braun, Geissel, Schallmo, 2206.06328]

DSE: [Müller, Buballa, Wambach, 1303.2693; Müller, Buballa, Wambach, 1603.02865]
more FRG: [Braun, Schallmo, 2204.00358; Braun, Geissel, Gorda, 2403.18010]

CONCLUSION

functional methods can be used to study the QCD from first principles

→ solid predictions, rapid progress & new features at finite density

QCD at small μ_B

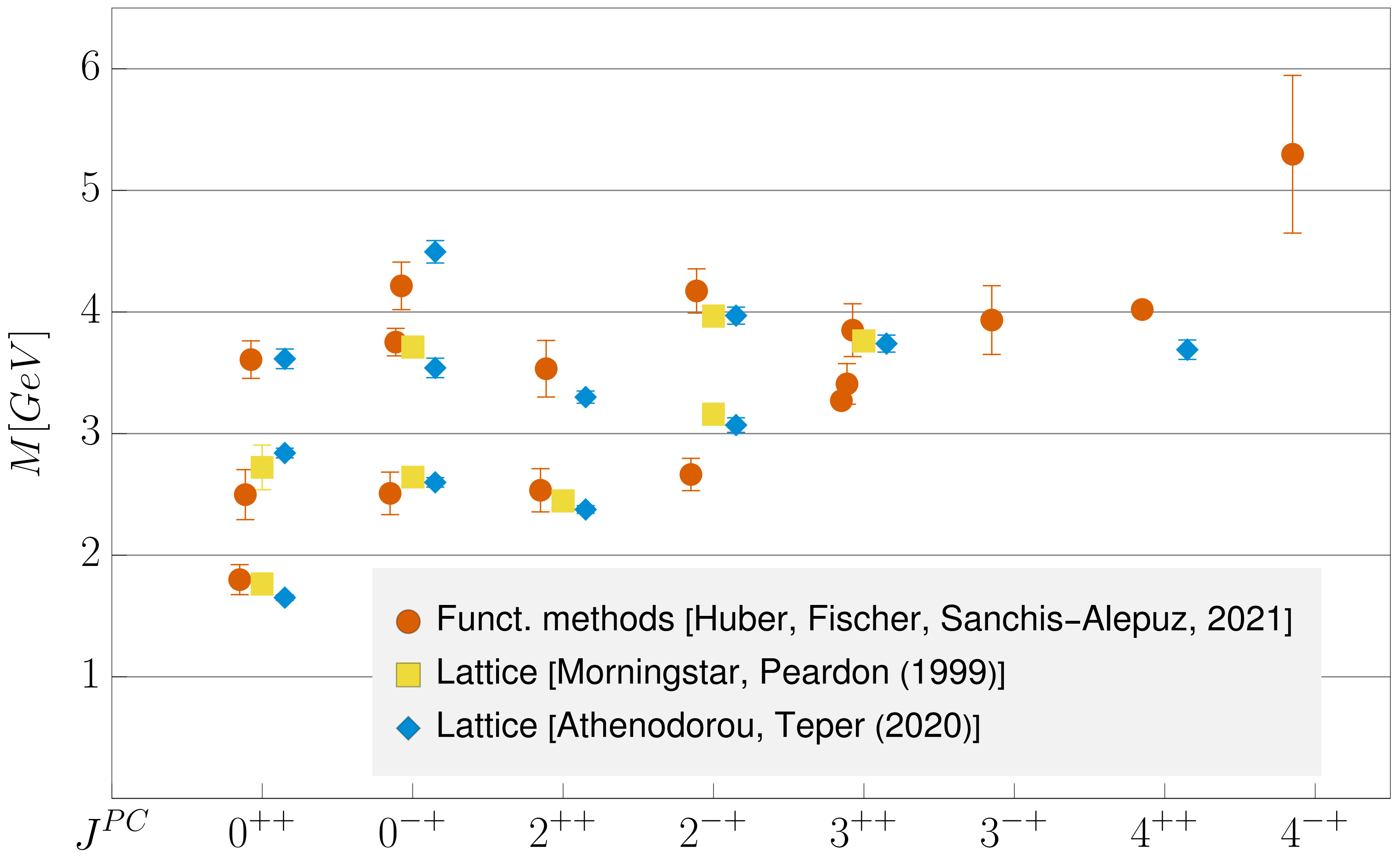
QCD at large μ_B

BACKUP

FUNCTIONAL QCD - SOME BENCHMARKS

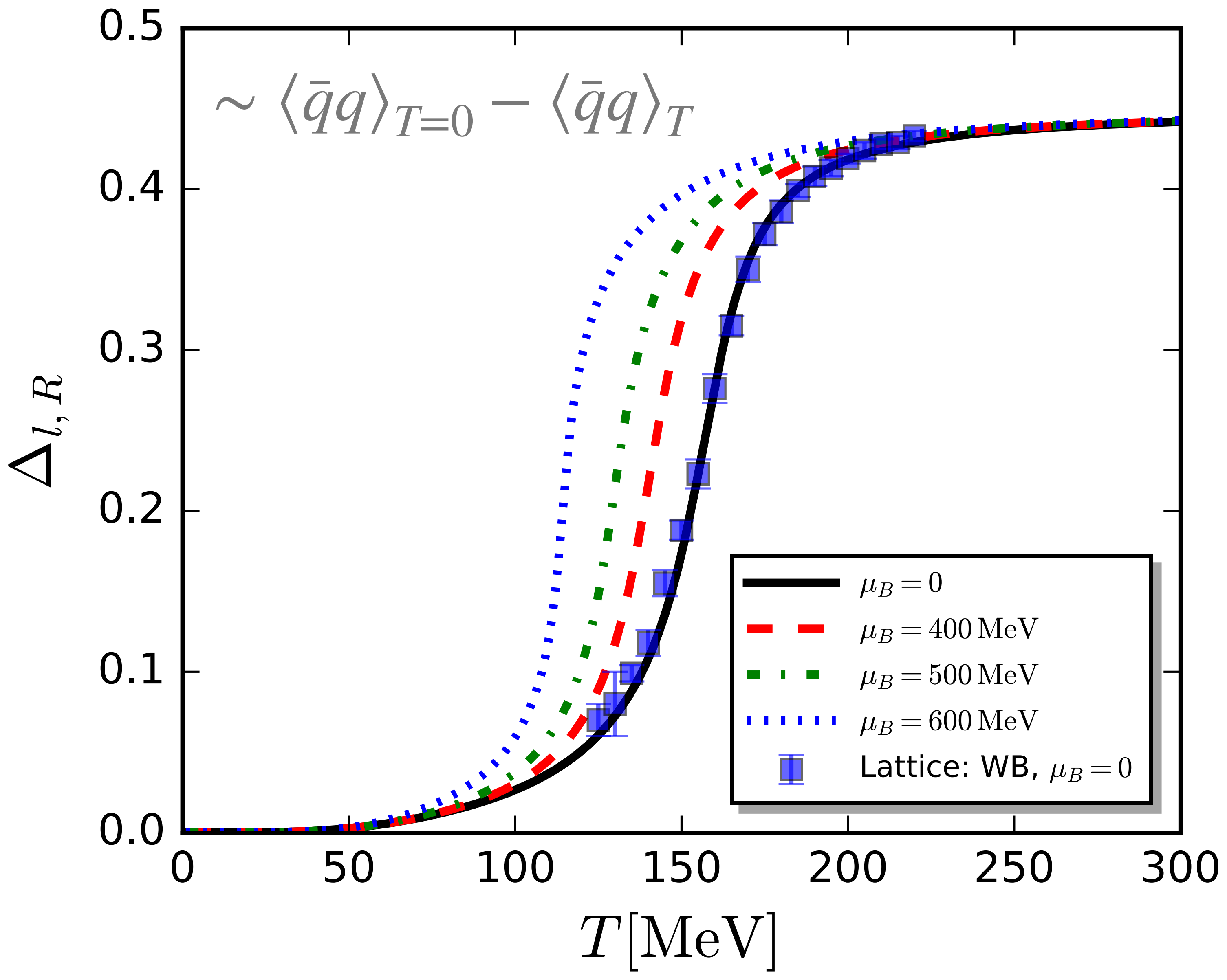
Comparison with lattice results:

glueball masses in quenched QCD



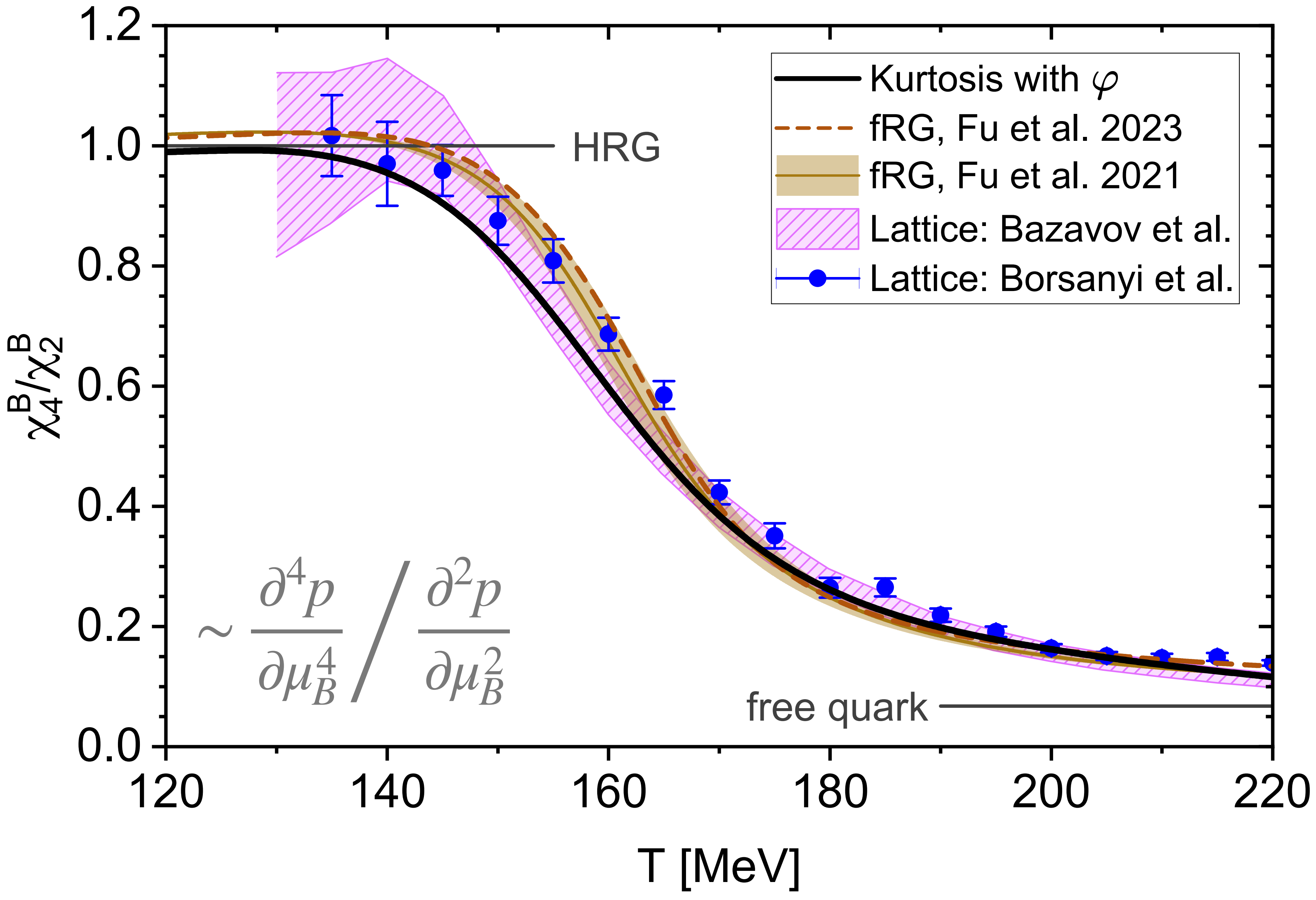
[Huber, Fischer, Sanchis-Alepuz, 2110.09180]

(renormalized) chiral condensate in QCD



[Fu, Pawlowski, FR, 1909.02991]

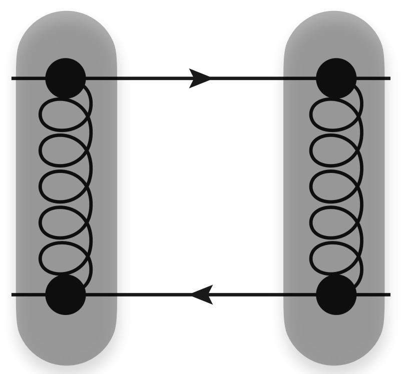
kurtosis of net-baryon distribution (EoS)

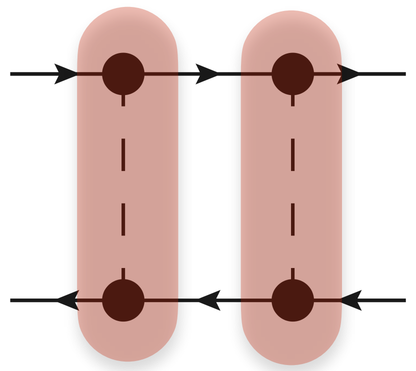


DSE-QCD:[Lu, Gao, Liu, Pawlowski, 2504.05099]
FRG-LEFT: [Fu, Luo, Pawlowski, FR, Wen, 2101.06035]

EMERGENT LOW-ENERGY PHYSICS

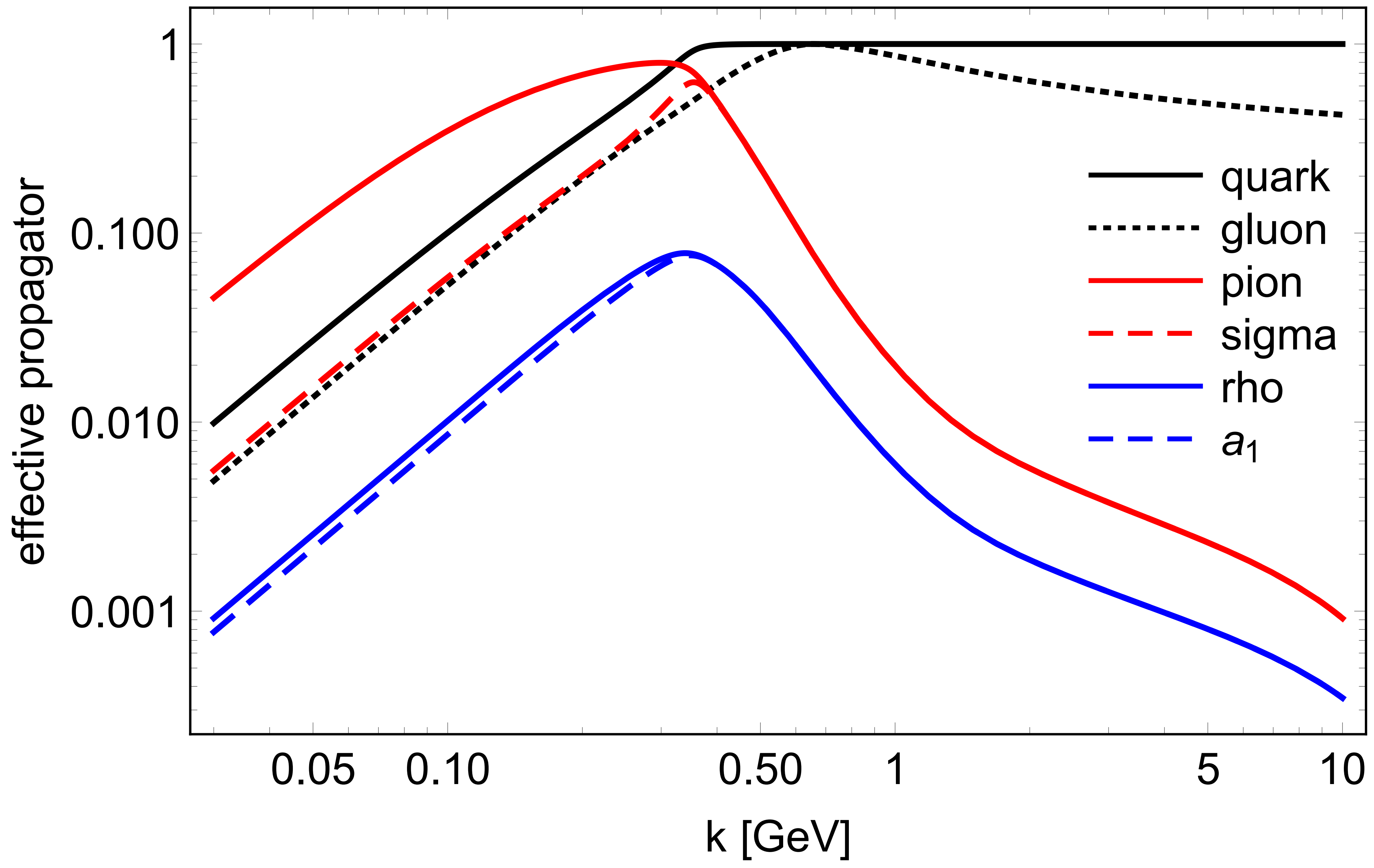
Different dominant degrees of freedom at different energy scales





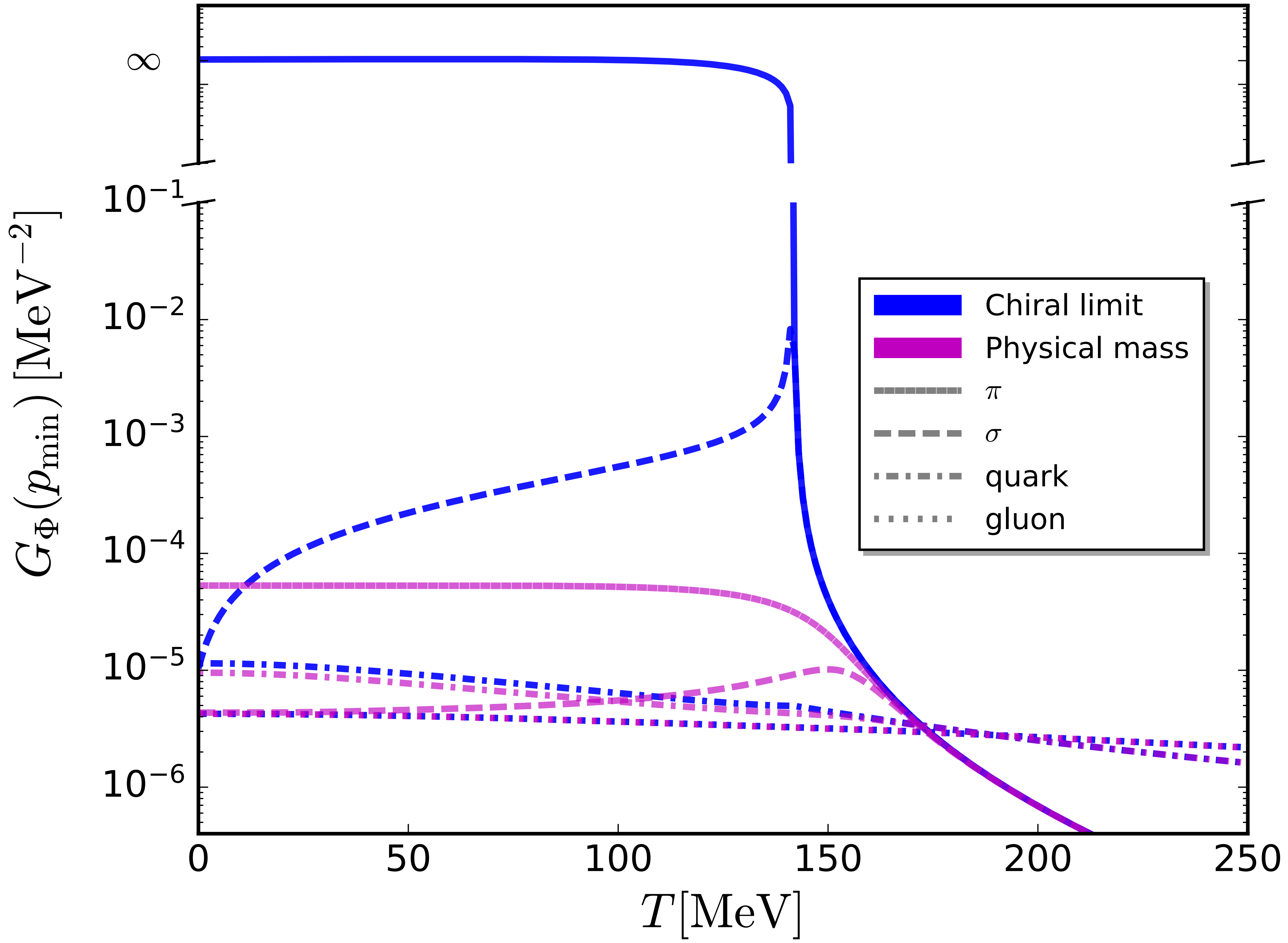
- vacuum

[FR, 1504.03585]



- finite T; physical point & chiral limit

[fQCD Collaboration, 2310.19853]



→ soft modes at low energies: **emergent EFTs** (e.g. χPT)

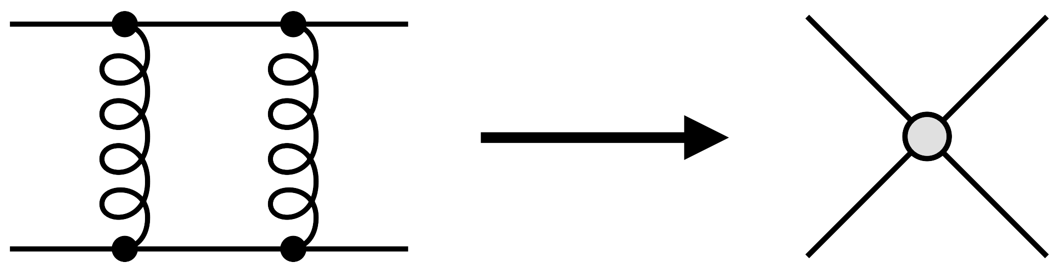
cf. [Divotgey, Eser, Mitter, 1901.02472]

TOWARDS NUCLEAR MATTER FROM FIRST PRINCIPLES

Important benchmark: nuclear liquid-gas transition from quark-gluon correlations.

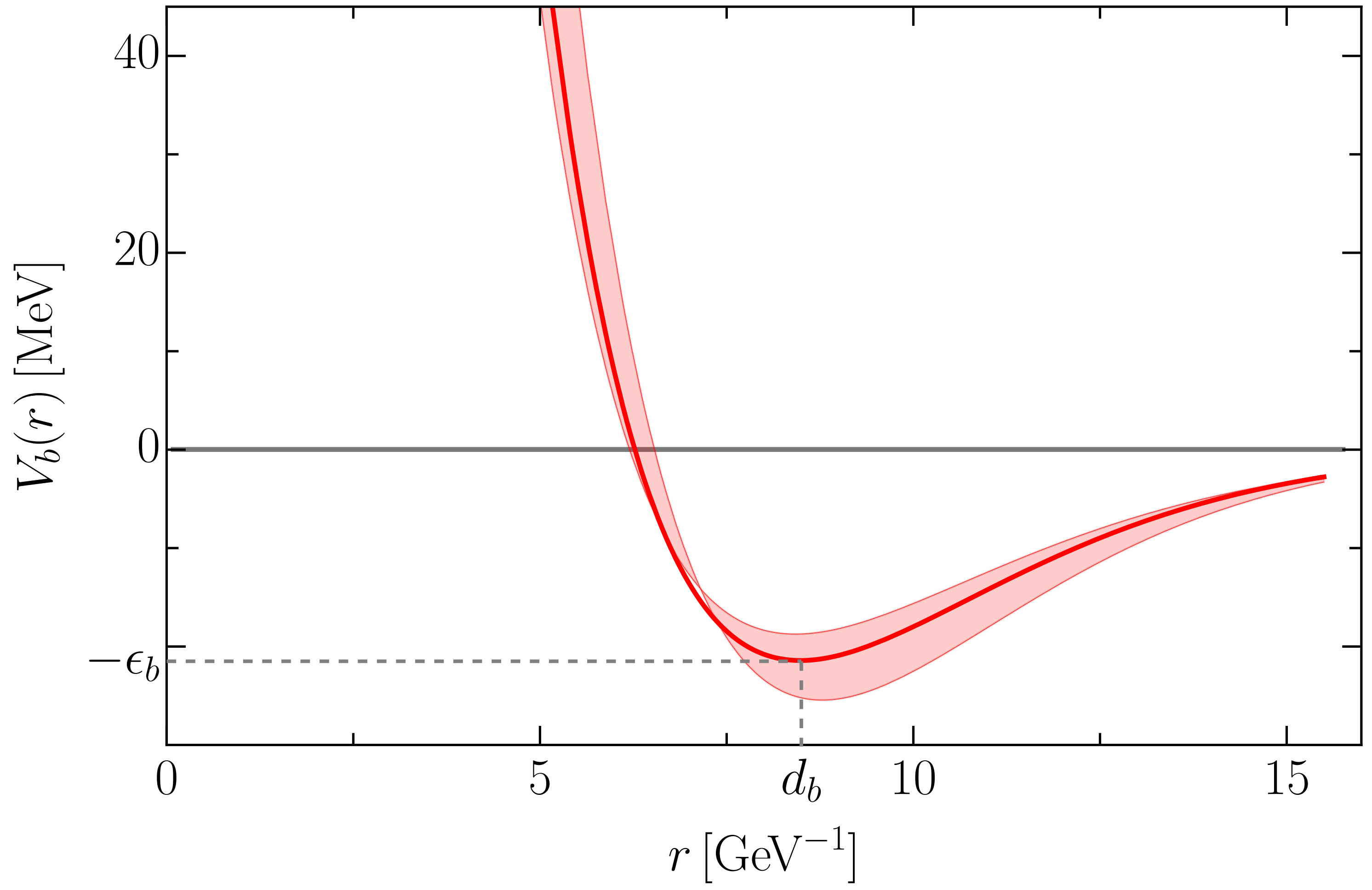
First exploratory studies available (involving some modeling)

FRG: density-channel interactions



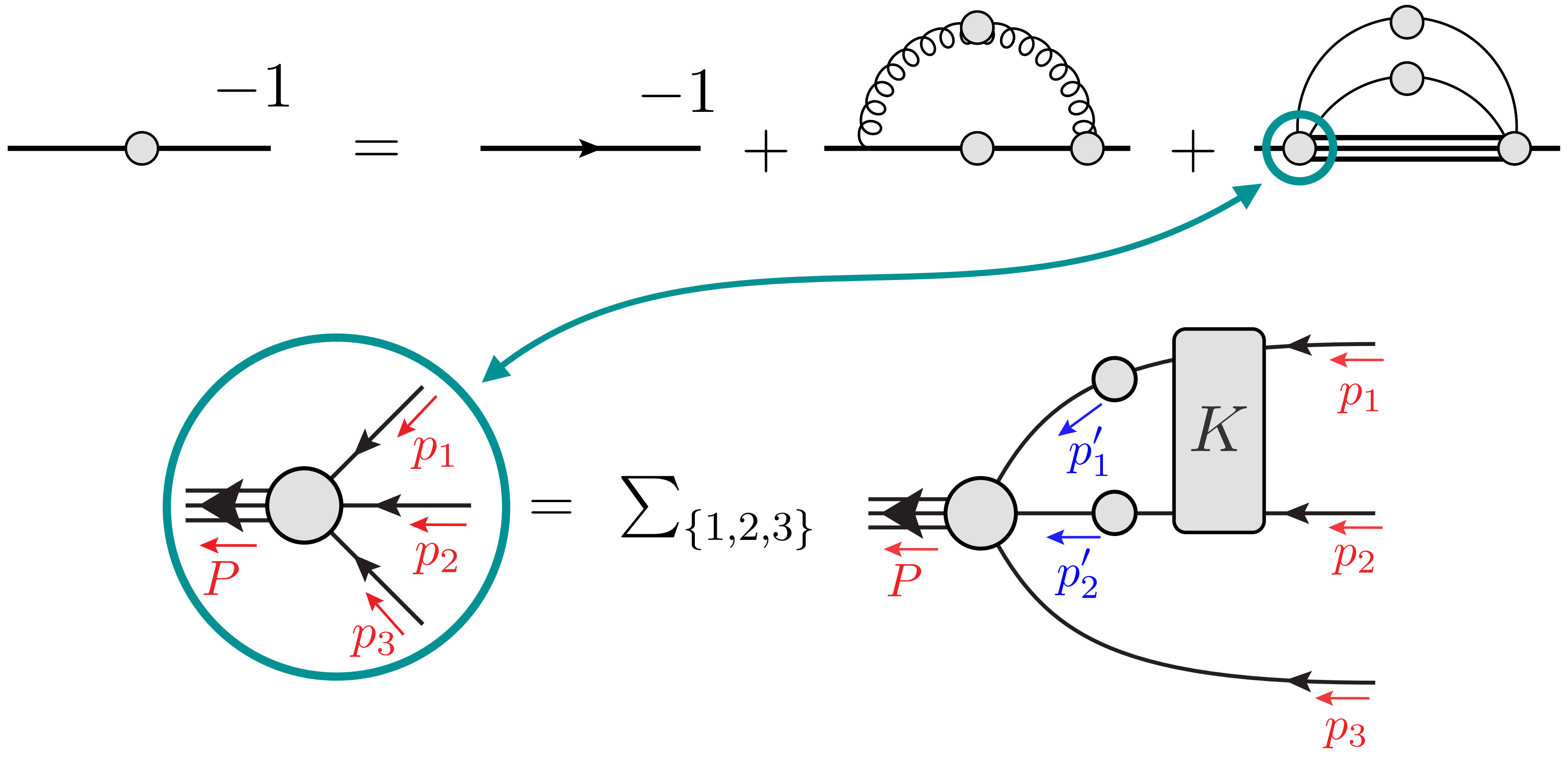
$\sim \lambda_V (\bar{q}\gamma^0 q)^2$

- short-distance repulsion from 1st principles
- first estimates for nuclear matter properties encouraging $(n_0 \approx 0.21(16)\,\mathrm{fm}^{-3}, \epsilon_b \lesssim 21(5)\,\mathrm{MeV})$



[Fukushima et al., 2308.16594]

DSE: nucleon contribution



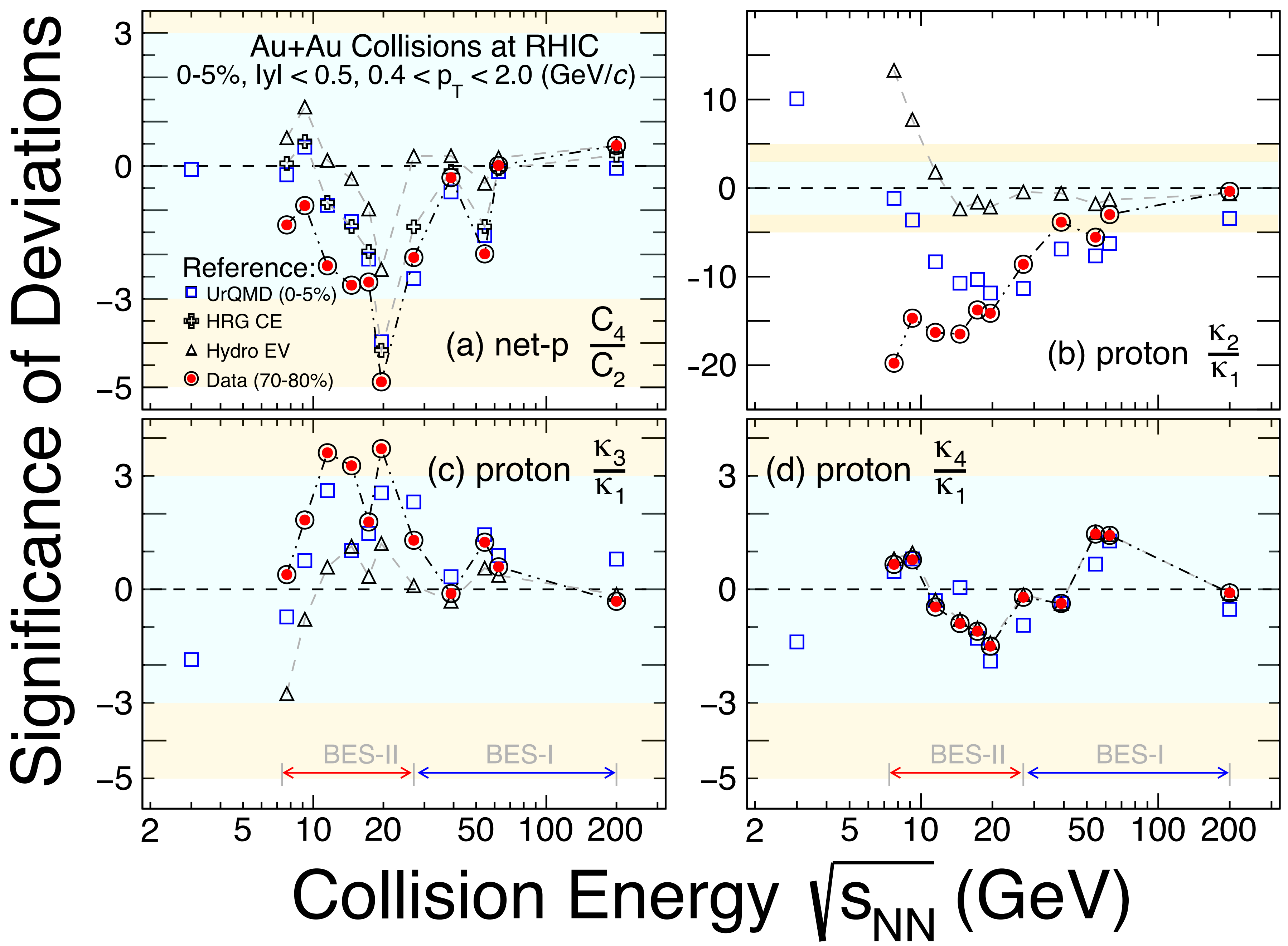
- saturation density: $n_0 \approx 0.15\,\mathrm{fm}^{-3}$
- binding energy: $\epsilon_b \approx 15.9\,\mathrm{MeV}$

[Gao et al., 2504.00539]

WHAT'S GOING ON AT 19.6 GEV?

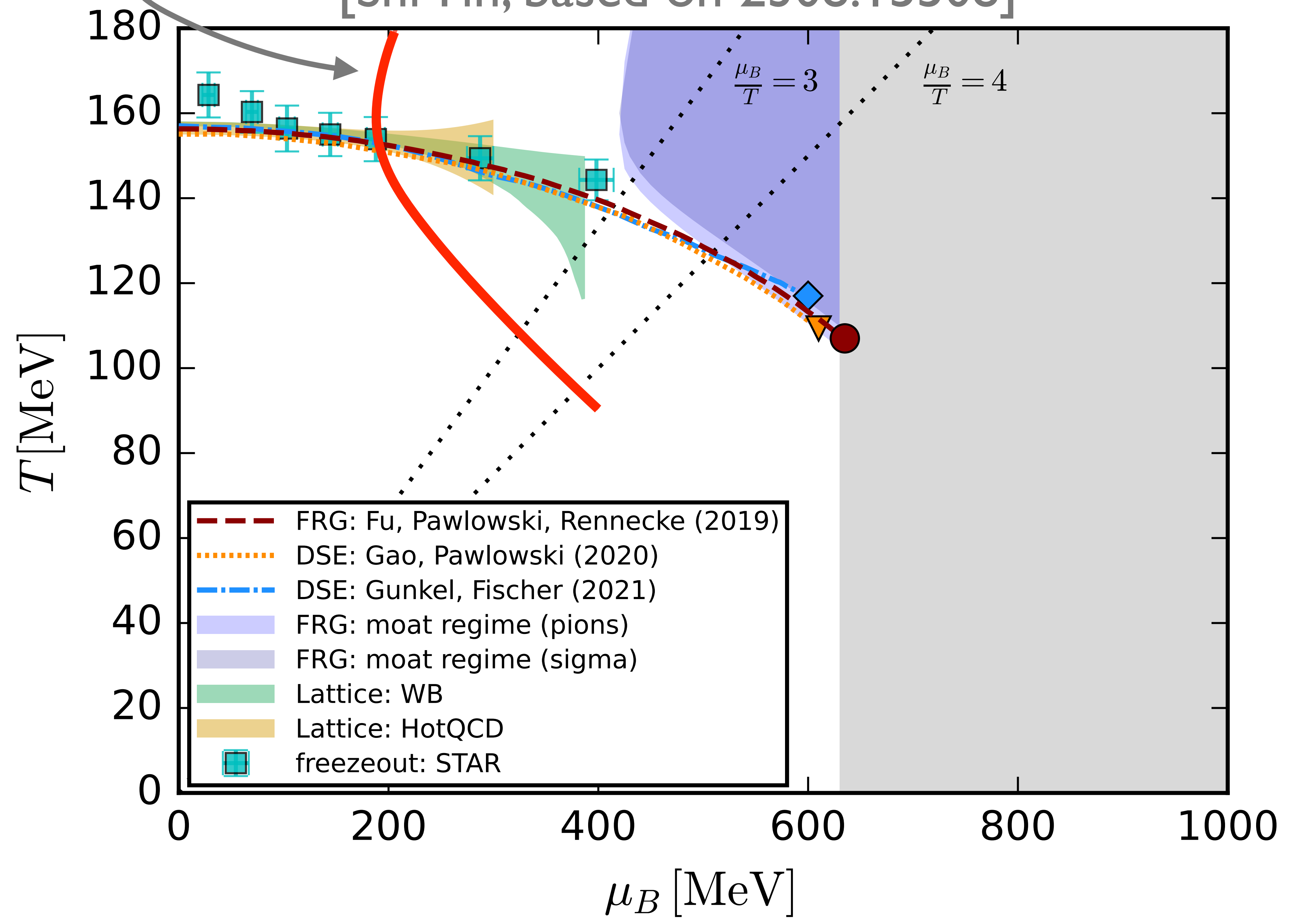
2-5σ deviation at $\sqrt{s} = 19.6\,\mathrm{GeV}$ from STAR BES-II

[STAR, 2504.00817]

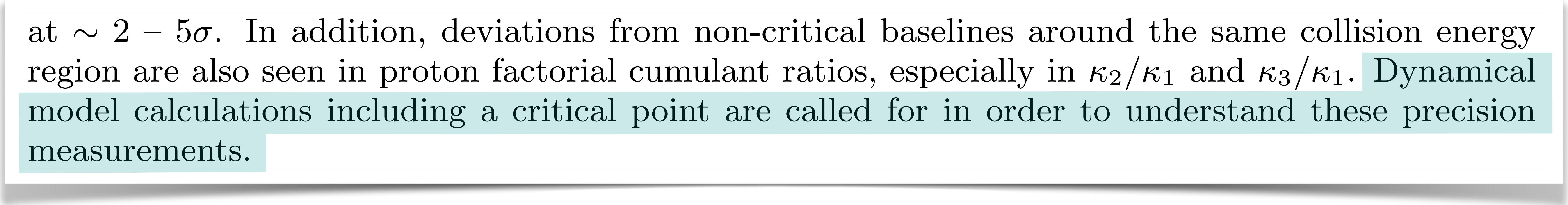


nothing remarkable in the phase diagram there

isentrope through $19.6\,\mathrm{GeV}$ freeze-out point
[Shi Yin, based on 2308.15508]



from the abstract of [STAR, 2504.00817]


at $\sim 2-5\sigma$. In addition, deviations from non-critical baselines around the same collision energy region are also seen in proton factorial cumulant ratios, especially in κ_2/κ_1 and κ_3/κ_1. Dynamical model calculations including a critical point are called for in order to understand these precision measurements.

- phase diagram well understood in this region from lattice, FRG & DSE
- too far away from CEP for dynamical critical phenomena to be relevant

→ ?

latest lattice estimate: $\mu_{B,\mathrm{CEP}} \gtrsim 450\,\mathrm{MeV}$ [Borsanyi et al., 2502.10267]: