Chiral crossover in the 2 + 1 flavor Nambu-Jona-Lasinio model

International School of Nuclear Physics

46th Course

QCD under extreme conditions – present and future

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QCD at finite temperature and chemical potential

- Lattice QCD has shown that, for zero chemical potential, strongly interacting matter does not undergo through a phase transition, but through a smooth crossover instead[1]
- For nonzero chemical potentials, Lattice QCD is hampered by the notorious sign problem
- It is predicted that, as chemical potential increases, the smooth crossover will extend towards lower temperature values until it reaches a critical end point (CEP), followed by a first order phase transition[2,3,4,5]
- Currently, many experiments are being performed in order to locate evidence for the existence of the CEP and the first order phase transition[6,7,8]
- Effective field theories, like the Nambu-Jona-Lasinio model, are widely used because of their chiral and isospin properties which mimic those of QCD[9,10]

The three flavor NJL model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} + \gamma^{0}\hat{\mu} - \widehat{m}\right)\psi + G\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2] - K\left[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)\right]$$

We set the same chemical potential for each flavor, which approximates the physical conditions during relativistic heavy ion collisions

We work with the partition function, which can be expressed through a path integral of the action[11]

$$Z = \int D\bar{\psi}D\psi \exp\left(\int_0^{\beta} d\tau \int d^3x\, \mathcal{L}_{NJL}\right)$$

The thermodynamic properties of the system are obtained through the grand canonical thermodynamic potential density

$$\Omega = 2G({\sigma_u}^2 + {\sigma_d}^2 + {\sigma_s}^2) - 4K\sigma_u\sigma_d\sigma_s - 2N_c\sum_i \int \frac{d^3p}{(2\pi)^3}E_i - 2TN_c\sum_i \int \frac{d^3p}{(2\pi)^3}\left[ln\left(1+e^{-\frac{E_i+\mu}{T}}\right) + ln\left(1+e^{-\frac{E_i-\mu}{T}}\right)\right]$$

where $E_i = \sqrt{p^2 + {M_i}^2}$ and $M_i = m_i - 4G\sigma_i + 2K\sigma_j\sigma_k$

Gap equations

- The NJL model in non-renormalizable
- A regularization method must be used to get rid of the infinite integrals

We set the following parameters

$\Lambda = 630.9\ \text{MeV}, \qquad m_u = m_d = 5.5\ \text{MeV}, \qquad m_s = 135.9\ \text{MeV}, \qquad G\Lambda^2 = 1.814, \qquad K\Lambda^5 = 9.165$

Which fits the following set of constants[12]

$m_\pi = 138\ \text{MeV}, \qquad f_\pi = 92\ \text{MeV}, \qquad m_\kappa = 495\ \text{MeV}, \qquad m_{\eta'} = 958\ \text{MeV}$

- We solve the following self-consistent gap equation system

$$\frac{\partial\Omega}{\partial\sigma_u} = \frac{\partial\Omega}{\partial\sigma_s} = 0$$

Susceptibilities

- One way to identify if a phase transition is taking place is through the susceptibility of some order parameter
- The constituent mass of the quarks is not an exact order parameter outside of the chiral limit
- Nevertheless, it is interesting to study the behavior of the quark constituent masses in the phase diagram, as a phase transition may still be accurately described from there
- Several different susceptibilities are used in this work. The thermal and vector-scalar susceptibilites can also be used to determine the crossover line[13]
    - Chiral susceptibility (mixed) $\chi_{ij}^{S} = -\frac{\partial \sigma_i}{\partial m_j}$
    - Thermal susceptibility $\chi_i^T = \frac{\partial \sigma_i}{\partial T}$
    - Vector-scalar susceptibility $\chi_i^\mu = \frac{\partial \sigma_i}{\partial \mu}$

The phase diagram

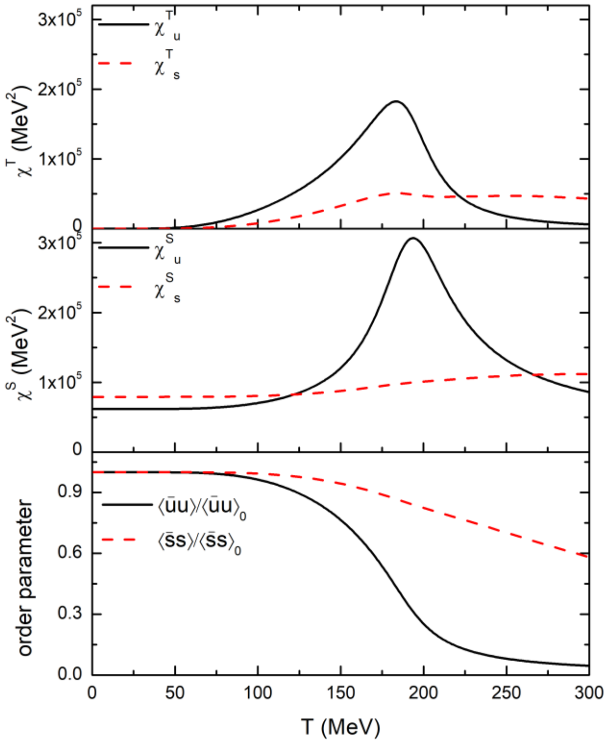
- The location of the first order phase transition is unambiguous, as it is represented by a divergent, sharp peak of the susceptibility at every (μ, T) coordinate where it happens.
- As for the crossover, the most common way to represent it involves the calculation of the projection of the curve of the ridge of the susceptibility, plotted in 3d space, onto the $T-\mu$ plane. In the literature, this is usually depicted in the form of a dashed line, or in some cases, nothing at all. In this work, this criterion is dubbed local.
- The curve of this ridge always reaches the CEP, if there is any. In 3d space, this is seen as the ridge of the susceptibility gradually becoming sharper and sharper (but still continuous) as the chemical potential increases and the temperature decreases. Finally, when the CEP is reached, this sharp but continuous ridge becomes a divergent peak.
- In this work, we extend this approach to define the crossover, not as a line, but as a wide region. We also present an alternative approach to locate the crossover line.

Crossover criteria

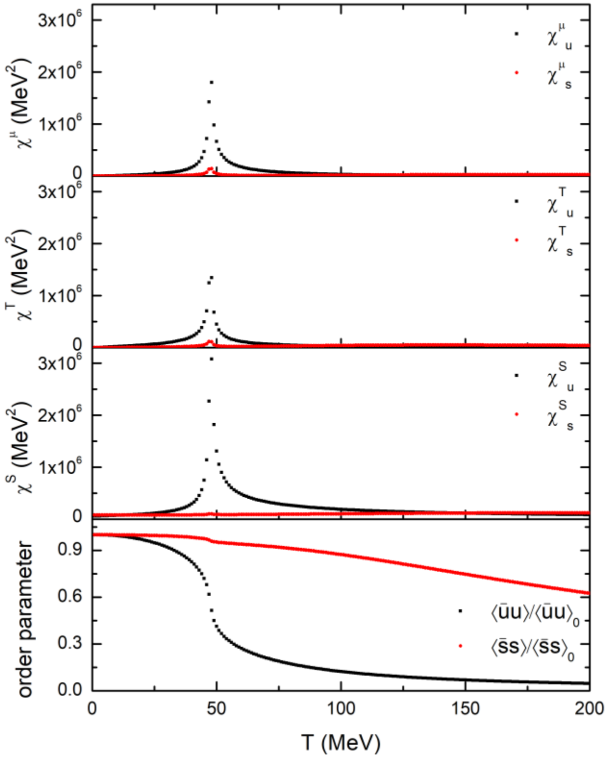
- In this work, we present two different criteria to display the location and the extent of the crossover in the phase diagram.
  - The global criterion. Instead of using the local criterion and the susceptibility to define the crossover line, this alternative criterion uses the absolute value of the order parameter at the CEP to distinguish between phases: higher values belong to one phase while lower values belong to the other one. In case the function is continuous everywhere it was computed, the value of the order parameter at the global maximum of the susceptibility is used instead. This criterion works as long as the jump-type discontinuity becomes wider as the first order transition line goes further, both by increasing the upper limit of the jump and decreasing the lower limit of the jump. This happens in every phase diagram we studied so far.
  - The extended local criterion. An easy way to extend the local criterion commonly used in the literature consists in calculating the determinant of the Hessian matrix at every computed point of the susceptibility in the $T-\mu$ plane. Its sign determines the overall concavity of the surface: positive values imply a definite concavity in all directions while negative values imply the opposite, a concavity depending on the direction it is measured. The susceptibility ridge, characterizing the crossover, extends across every point where the sign of the Hessian matrix is positive, while the curves where it is zero are interpreted as the curves that delimit the crossover itself. This all breaks down, of course, where the Hessian matrix cannot be calculated because the function is non-differentiable (i. e., the first order transition line).

Results

Susceptibilities and order parameters. As expected, the continuous behavior of the susceptibilities, typical of the crossover, appears at $\mu = 0$, only to break down after the CEP is reached (the μ coordinate of the CEP was obtained at $\mu = 316$ MeV).



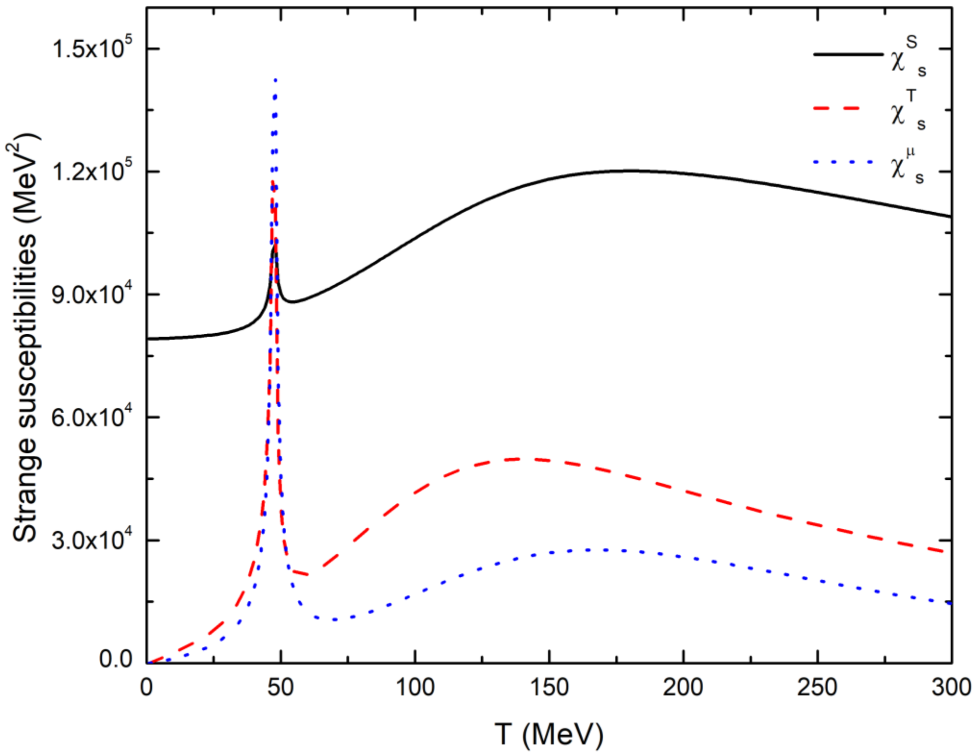
Susceptibilities and order parameters at zero chemical potential



Susceptibilities and order parameters at $\mu = 317$ MeV

Results

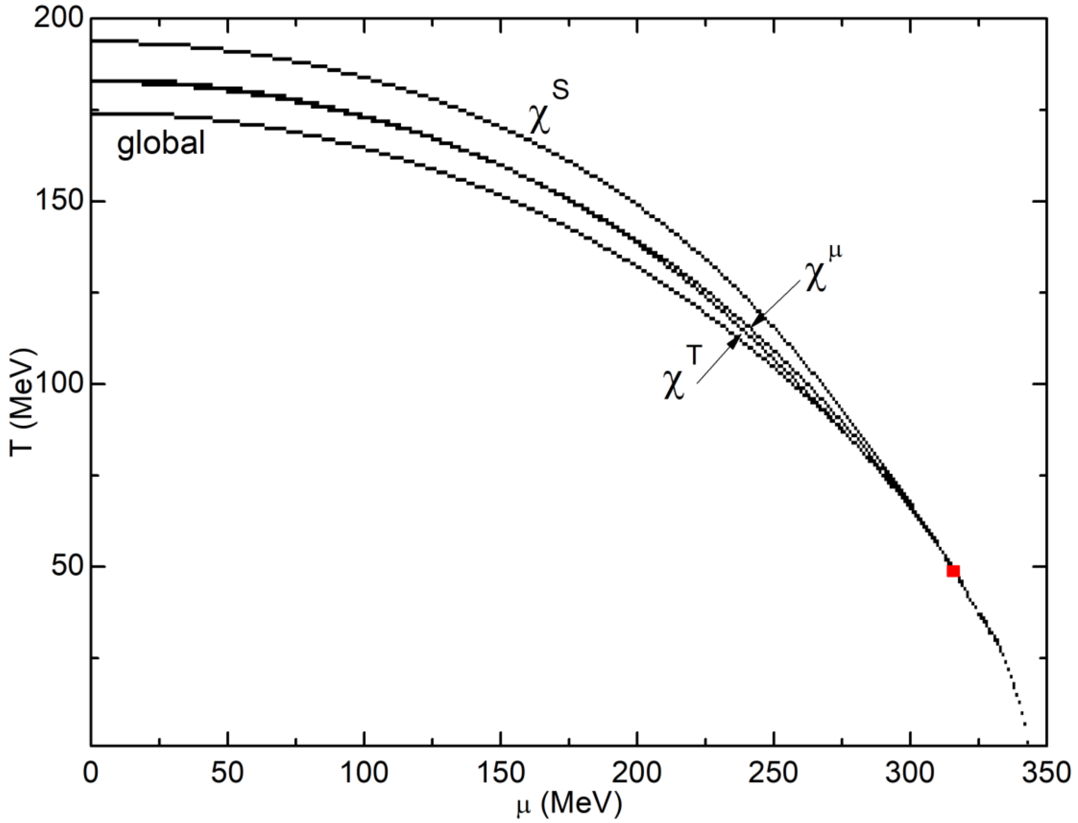
Strange quark susceptibilities. The influence of the chiral phase transition of the light quarks can be seen at the divergencies near the $50\ \mathrm{MeV}$ mark, but the soft bell-like behavior for higher temperature values is interpreted as the strange crossover, still going.



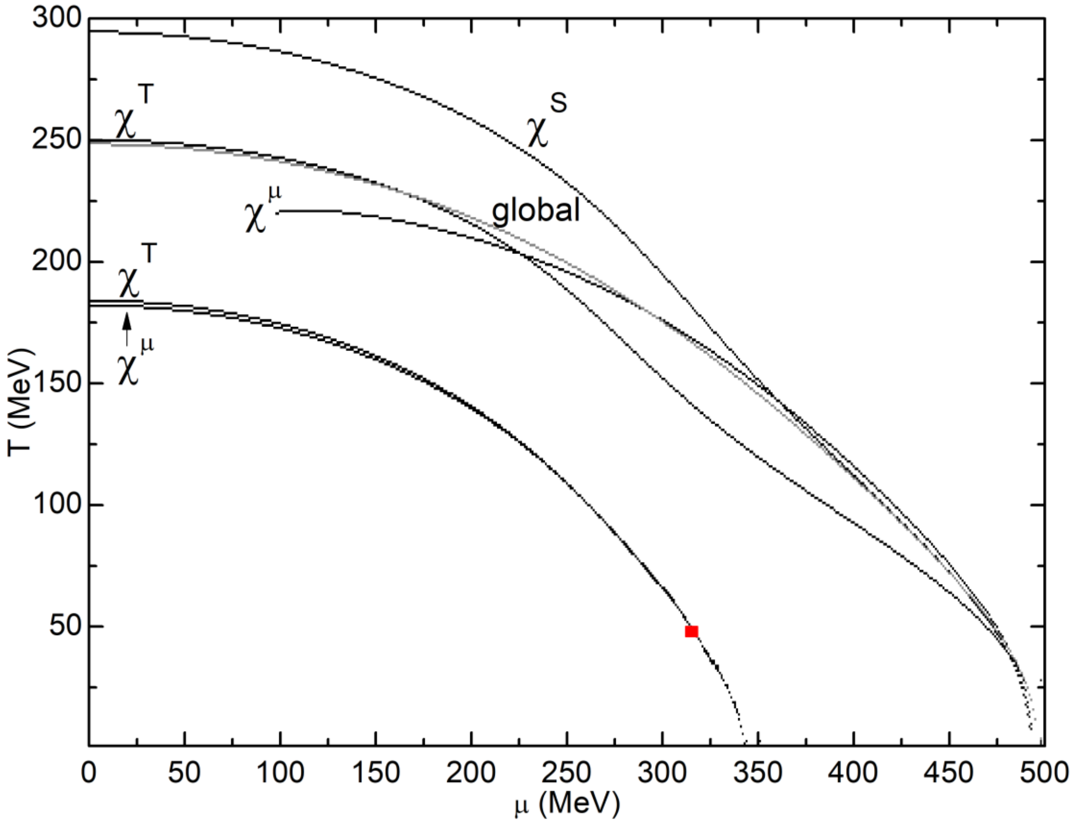
Strange quark susceptibilities at $\mu = 317\ \mathrm{MeV}$

Results

Phase diagrams. We obtained a CEP for the light quark sector at $\mu \cong 316$ MeV, $T \cong 49$ MeV. We obtained no CEP for the strange quark.



Light sector phase diagram



Strange quark phase diagram

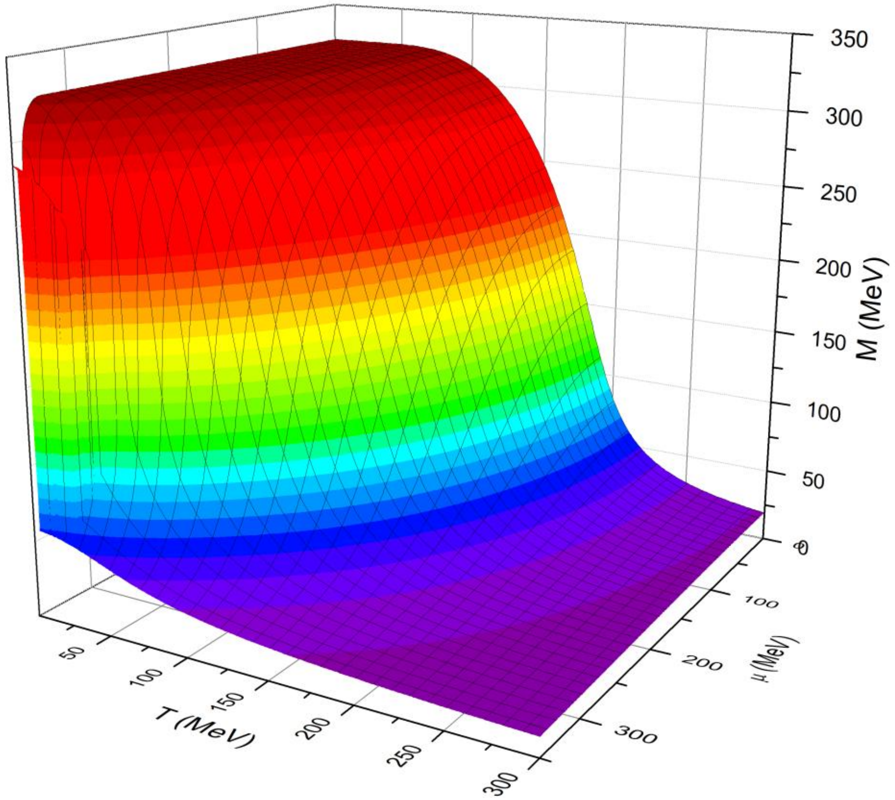
Results

Pseudocritical temperatures. The criterion that fits Lattice QCD data best is the global one. We did not obtain a pseudocritical temperature for the vector-scalar phase diagram for the strange quark.

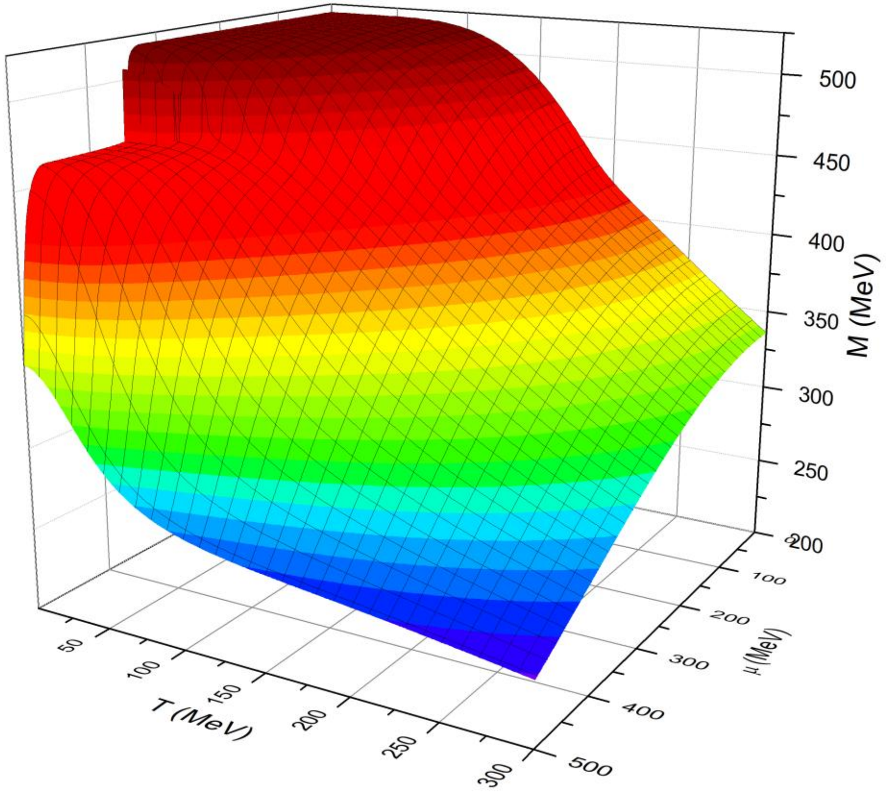
Criterion	Flavor	Phase diagram	Pseudocritical temp.
Local	Light	Chiral	194 MeV
Local	Light	Thermal	183 MeV
Local	Light	Vector-scalar	183 MeV
Local	Strange	Chiral	295 MeV
Local	Strange	Thermal	250 MeV
Local	Strange	Vector-scalar	None
Global	Light	Chiral	174 MeV
Global	Strange	Chiral	249 MeV

Results

Constituent quark mass. The chiral phase transition of the light quarks has a bigger impact on the strange quark mass than its own.



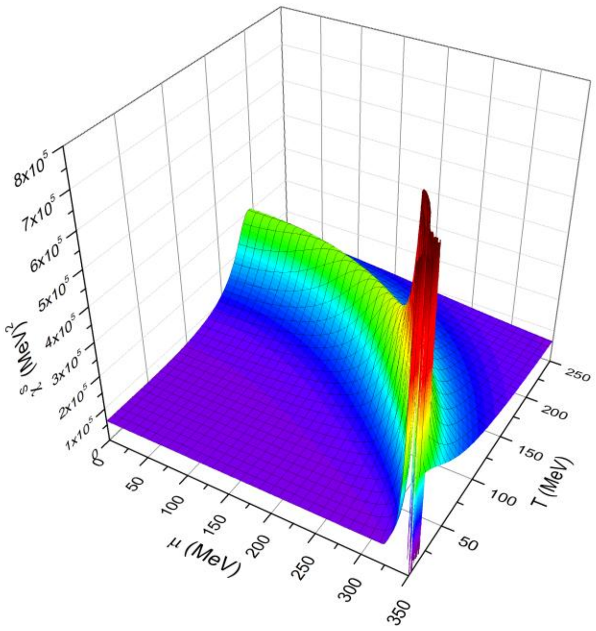
Light quarks constituent mass



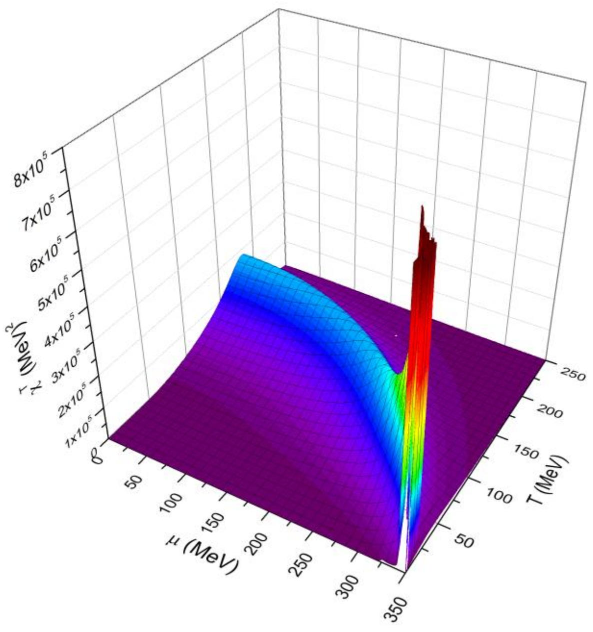
Strange quark constituent mass

Results

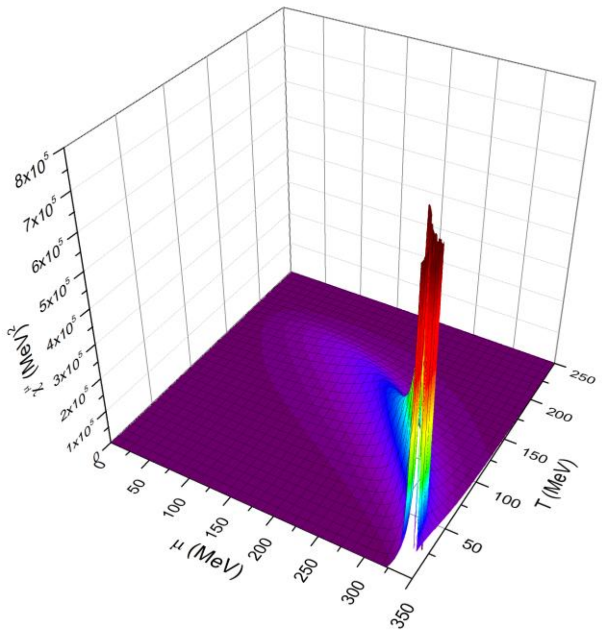
Light quarks susceptibilities. The vector-scalar susceptibility is by far the flattest at the zero chemical potential zone, but we could still recover a pseudocritical temperature from it.



Chiral susceptibility



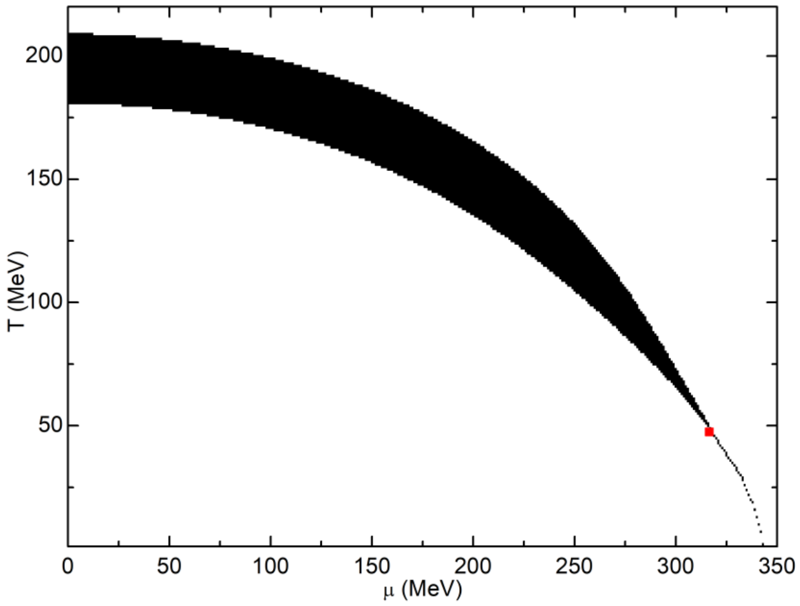
Thermal susceptibility



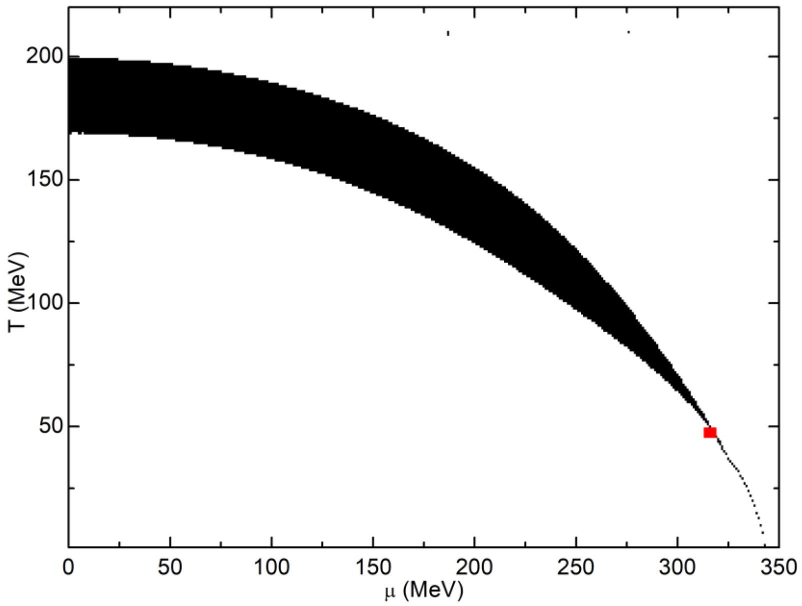
Vector-scalar susceptibility

Results

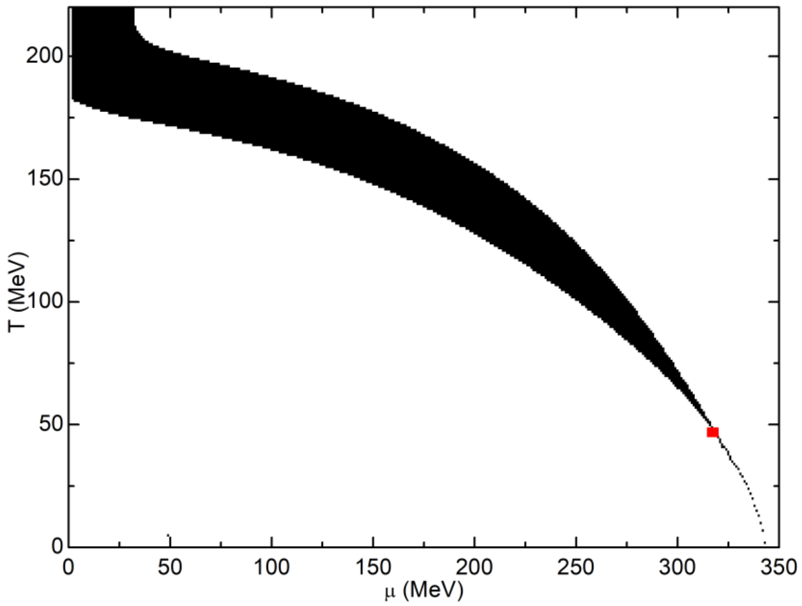
Extended local phase diagrams for the light sector. Despite the different extent of each crossover zone, all three diagrams agree on the location of the CEP and the first order transition line, as expected.



Chiral



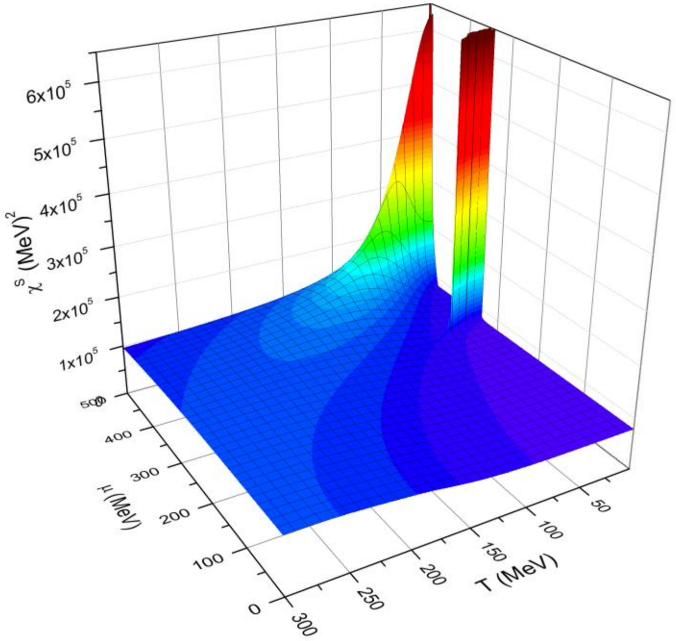
Thermal



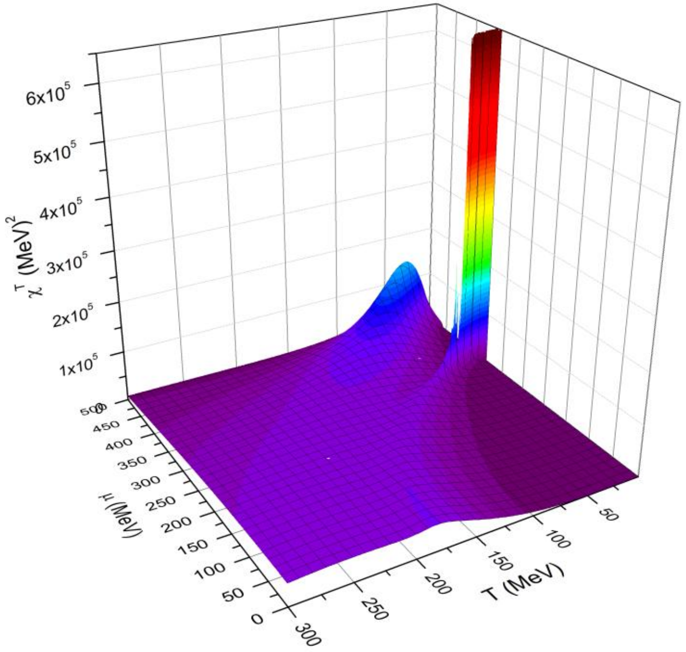
Vector-scalar

Results

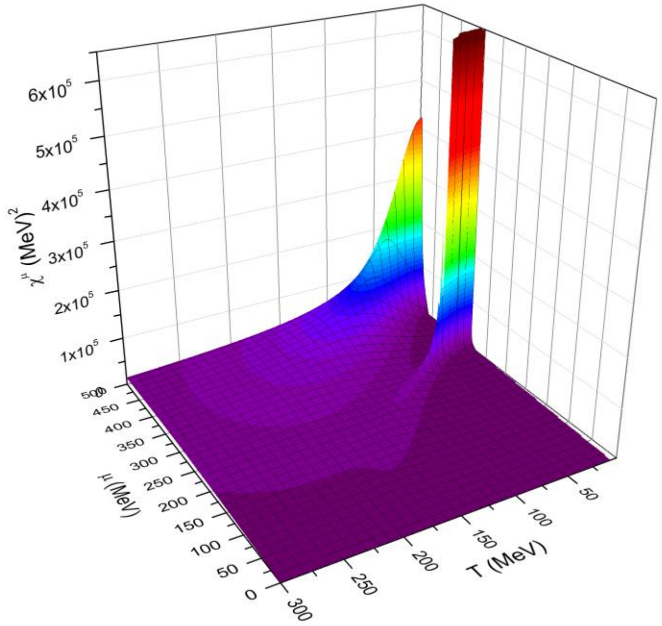
Strange quarks susceptibilities. The vector-scalar susceptibility is by far the flattest at the zero chemical potential zone, but we could still recover a pseudocritical temperature from it.



Chiral susceptibility



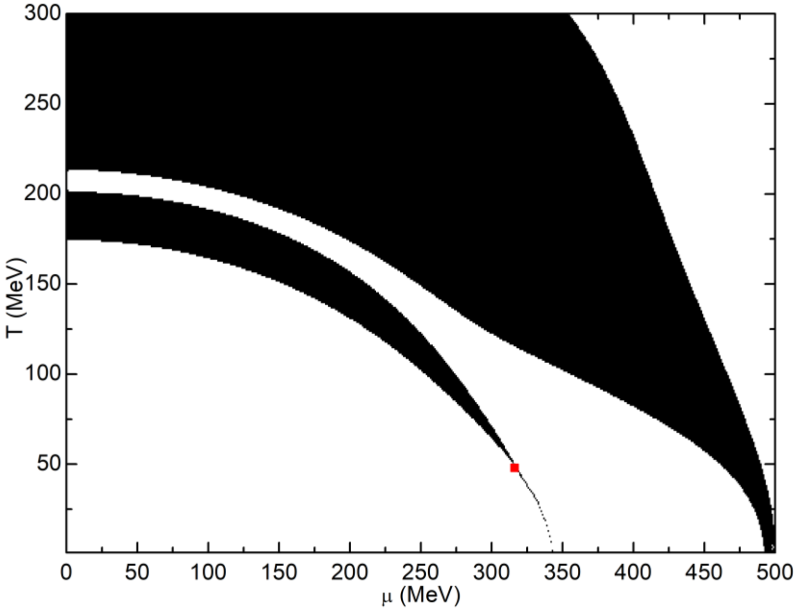
Thermal susceptibility



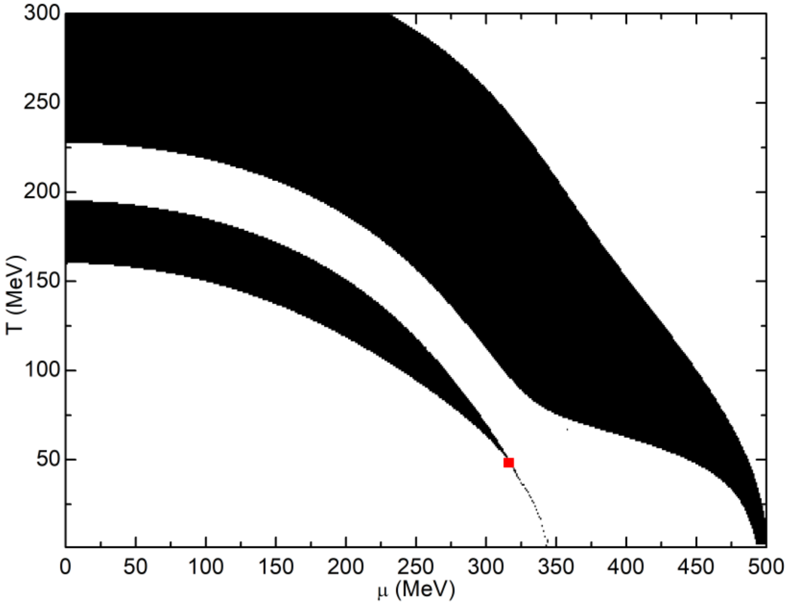
Vector-scalar susceptibility

Results

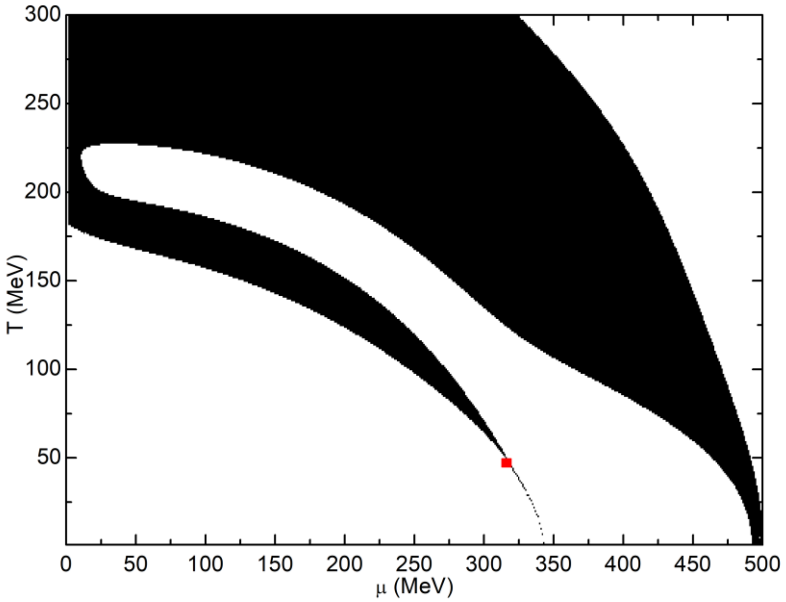
Extended local phase diagrams for the strange quark. Even if there is no critical end point for the strange quark, the dramatic thinning of the region at the $T = 0$ limit suggests that, with a similar parameter set, a CEP may be obtained for the strange quark.



Chiral



Thermal



Vector-scalar

Conclusions and future work

- We conclude that the chiral phase diagram for the light quarks has a strong effect on the strange quark, but not vice-versa. We think it is because of the 't Hooft mixing term in combination with the very high ratio of m_s/m_u.
- Even if the local criterion is defined in terms of the CEP, which is completely outside of the scope of Lattice QCD, it is still the criterion that fits best, at least with the employed parameter set.
- Given the behavior of the strange susceptibilities at very high chemical potentials, it is possible that, with a slightly different parameter set, a strange CEP could be found alongside the light quark CEP, at different (μ, T) coordinates.
- For future work, we could study the thermodynamics of the model beyond the leading order, instead of settling with the basic mean-field approach.

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Thank you for your attention