Electromagnetic Effects on the QCD Coupling

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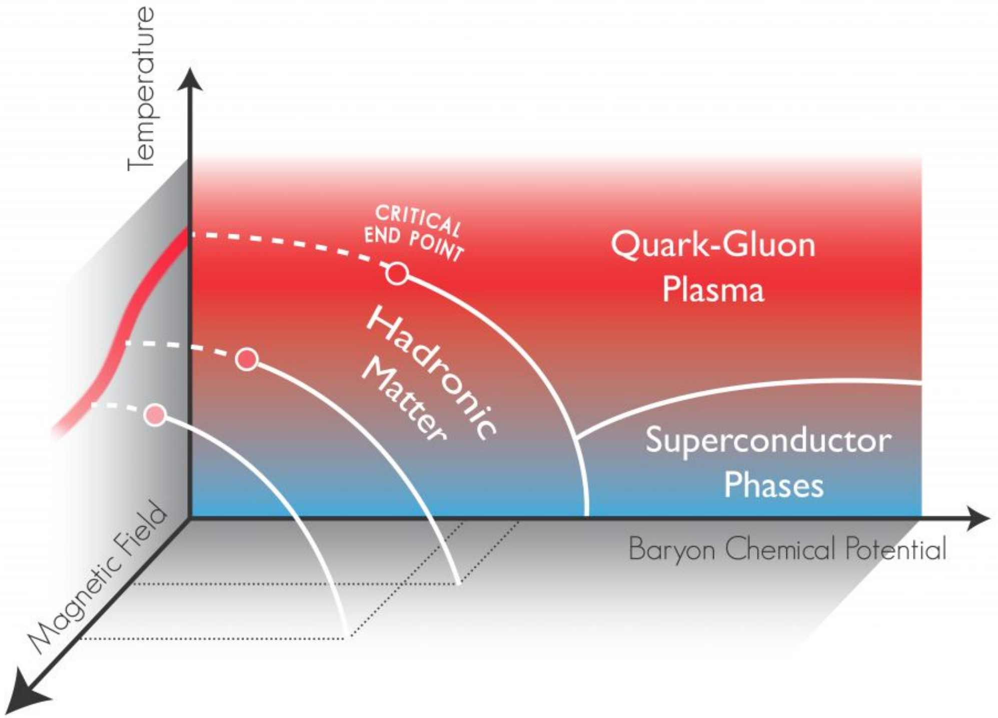
Electromagnetic Effects on the QCD Coupling

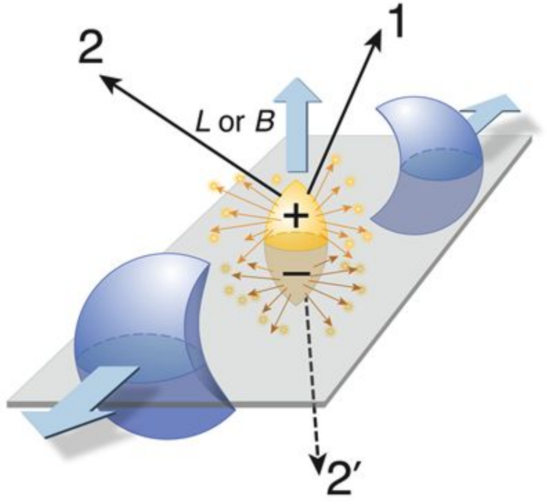
Strong magnetic fields can be found in all systems where we believe the quark-gluon plasma is formed, such as: the early universe, compact stars, and heavy-ion collisions.

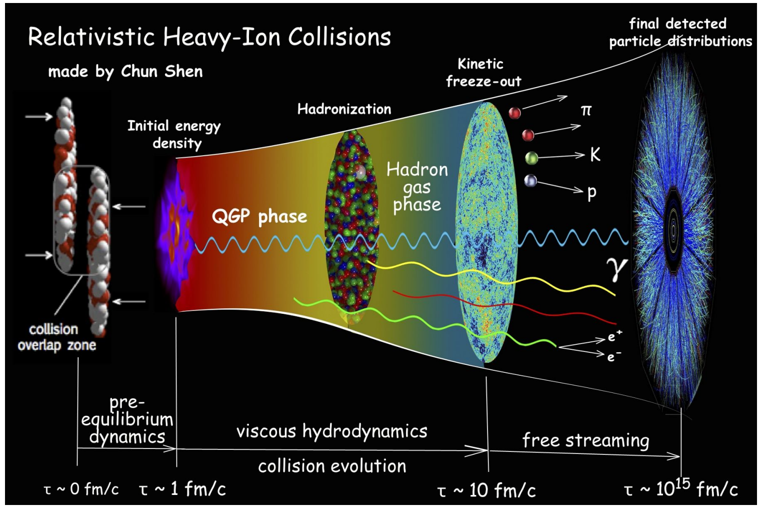


The QGP phase can be found under the conditions of the QCD phase diagram. Under extreme external conditions of temperature and chemical potential, ordinary nuclear matter can overcome a transition to the quark-gluon plasma phase. Recently, it has been found that an external magnetic field can also induce this transition.

Electromagnetic Effects on the QCD Coupling

The strong magnetic field arises after the heavy-ion collision, but decreases logarithmically. The most intense magnetic field estimated in the lab is in non-central heavy-ion collisions. It is expected that magnetic fields reach up to 10^{19} G in this class of collision.





We believe that such a strong magnetic field might be responsible for effects on the QCD coupling constant.

How can we check for effects on the QCD coupling constant?

Beta Function

The beta function can be evaluated through the Callan-Symanzik equation for a n-point Green's function, which, for the massless case, is given by:

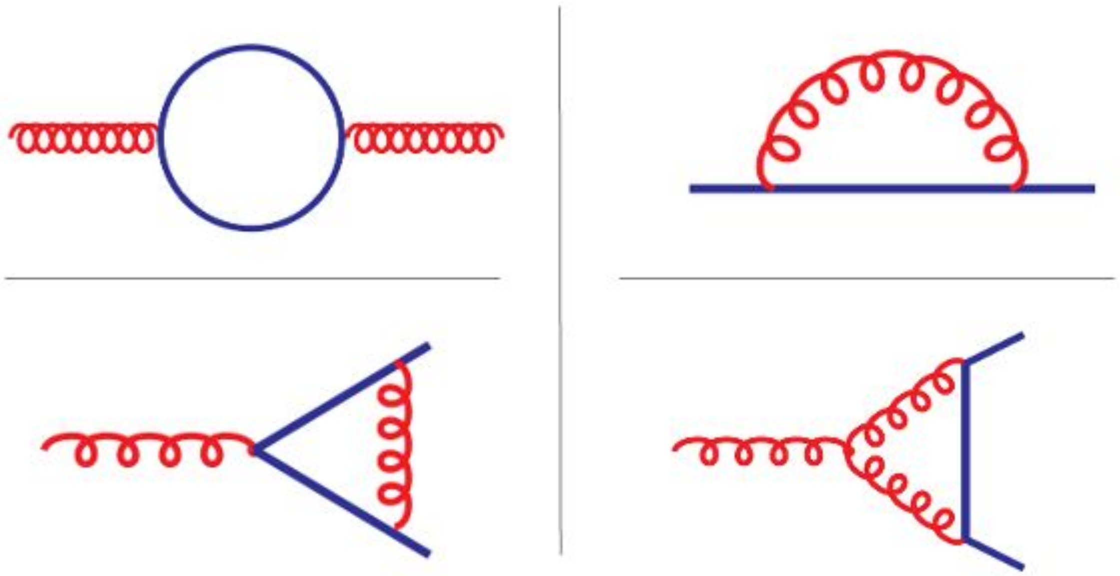
$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+n\,\gamma(g)\right)G^{(n)}(p_1,\ldots,p_n;g(\mu),\mu)=0,$$

where g is the coupling constant, μ, the renormalization scale and the beta function is given by:

$$\beta(g)=\mu\frac{dg}{d\mu}$$

Beta Function

The n-point Green's function of interest (the ones that will be effect by the field), can be represented with Feynman Diagrams.



But how can we compute the Feynman Diagrams?

Feynman Diagrams

One way to evaluate the Feynman Diagrams is using the Path Integral Method.

In quantum mechanics, there is a superposition principle, when a process can take place in more than one way. Its total amplitude is the coherent sum of the amplitudes for each way. Therefore, the path integral is defined as:

$$\sum_{\text{all paths}} e^{iS[x(t)]}/\hbar = \int Dx(t)\; e^{iS[x(t)]}/\hbar$$

Action of the theory

Feynman Diagrams

The n-point correlation function tells how quantum fields at different spacetime points are connected through interactions. Using the path integral method, the n-point correlation function reads:

$$\langle 0|\, T[\phi_1(x_1)\ldots\phi_n(x_n)]\,|0\rangle = \frac{\int D\phi\ \phi(x_1)\ldots\phi(x_n)\ e^{i\int d^4x\,\mathcal{L}(\phi)}}{\int D\phi\ e^{i\int d^4x\mathcal{L}(\phi)}};$$

For the QCD case, the Lagrangian is given by:

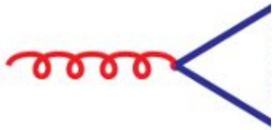
$$\begin{aligned}\mathcal{L}_{QCD} &= \bar{\psi}_{i\alpha}(i\not{D}_\mu - m)^{\alpha\beta}\delta_{ij}\psi_{j\beta} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \\ &= \bar{\psi}_{i\alpha}\,(i\not{\partial}_\mu - m)^{\alpha\beta}\delta_{ij}\,\psi_{j\beta} + \bar{\psi}_{i\alpha}\,g_s\,(\gamma^\mu)^{\alpha\beta}\,A^a_\mu t^a_{ij}\,\psi_{j\beta} - \frac{1}{4}\,F^a_{\mu\nu}F^{\mu\nu}_a \\ &- g_s\,f^{abc}\,(\partial_\mu A^a_\nu)\,A^{\mu\,b}\,A^{\nu\,c} - \frac{1}{4}\,g_s^2\,f^{abc}\,f^{ade}\,A^b_\mu\,A^c_\nu\,A^{\mu\,d}\,A^{\nu\,e}.\end{aligned}$$

Feynman Diagrams

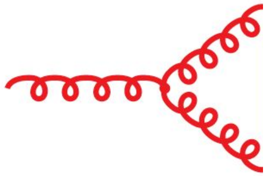
$$\bar{\psi}_{i\alpha}\,(i\partial\!\!\!/_\mu - m)^{\alpha\beta}\delta_{ij}\,\psi_{j\beta}$$

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

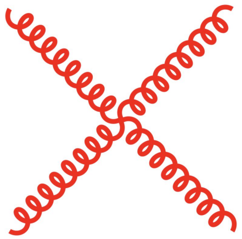
$$\bar{\psi}_{i\alpha}\,g_s\,(\gamma^\mu)^{\alpha\beta}\,A^a_\mu t^a_{ij}\,\psi_{j\beta}$$



$$-\,g_s\,f^{abc}\,(\partial_\mu A^a_\nu)\,A^{\mu\,b}\,A^{\nu\,c}$$



$$-\frac{1}{4}\,g_s^2\,f^{abc}\,f^{ade}\,A^b_\mu\,A^c_\nu\,A^{\mu\,d}\,A^{\nu\,e}$$



How can we describe the background magnetic field?

Landau Levels

In quantum mechanics, the Landau levels describe the discrete energy states of electrons confined to a plane due to the influence of a perpendicular magnetic field. The external field modifies the dispersion of fermions, leading to this quantization. This is done by considering the Dirac equation in the presence of a Maxwell field,

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0 \qquad \partial_{\mu} \to \partial_{\mu} + iqA_{\mu}.$$

using the Landau Gauge:

$$A^{\mu} = (0, -B\,x_2, 0, 0).$$

Landau Levels

with scalar dispersion relation:

$$p_0^2 = p_3^2 + m^2 + (2n+1)|q|B,$$

and fermionic dispersion relation

$$p_{0ns}^2 = p_3^2 + m^2 - q\,B\,s + (2n+1)|q|B$$

spin projection

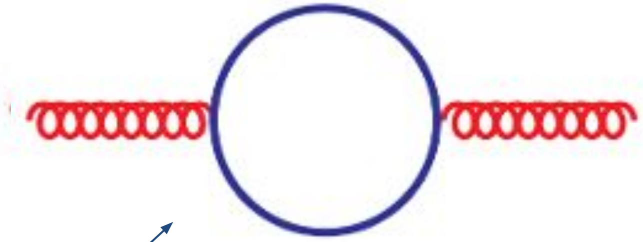
What about in Quantum Field Theory?

Gluon Polarization Tensor

Using the Landau Gauge:

$$A^0 = A^1 = A^3 = 0 \quad and \quad A^2 = Bx$$

we can write the Schwinger 2-point function in the (LLL):



Kenji Fukushima. Physical Review D, 83(11):111501, 2011.

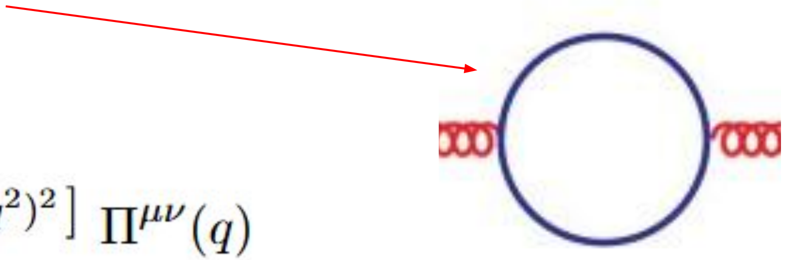
$$\langle\psi(x)\bar{\psi}(y)\rangle = \int \frac{dp^2 dp^3}{(2\pi)^2} \sum_k \frac{e^{-i\omega_p(x^0-y^0)+ip^2(x^2-y^2)+ip^3(x^3-y^3)}}{2\omega_p}$$
$$\times P_k(x)(\omega_p\gamma^0 - \tilde{\mathbf{p}} \cdot \gamma + m)P_k(y),$$

Projection matrix with Landau wave functions

where: $\tilde{\mathbf{p}} = (0, \sqrt{2eBk}, p^3)$ and $\omega_p = \sqrt{\tilde{\mathbf{p}}^2 + m^2}$

Gluon Polarization Tensor

At the lowest Landau Level (LLL) approximation,
we have the following self-energy:



$$\Pi^{\mu\nu}(k,q) = (2\pi)^4\,\delta^4(k+q)\; e^{-\frac{1}{2eB}\left[(q^1)^2+(q^2)^2\right]}\;\Pi^{\mu\nu}(q)$$

with:

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\,\frac{e^2\,|eB|}{2\pi^2}\;I(q/m)$$

where $\Pi^{1\nu} = \Pi^{2\nu} = \Pi^{\mu 1} = \Pi^{\mu 2} = 0$ and:

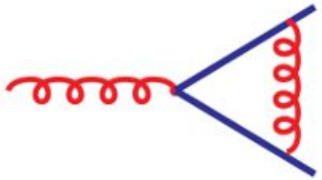
$$I(q/m = x) = \int_0^1 dy\;\frac{y\,(1-y)}{y\,(1-y) - x^{-2}} = 1 - \frac{4\,sin^{-1}(x/2)}{x\,\sqrt{4-x^2}}.$$

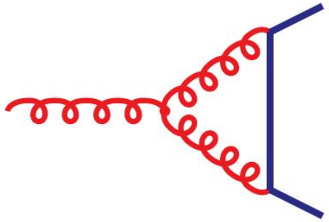
Kenji Fukushima. Physical Review D, 83(11):111501, 2011.

Next Steps

The next step will involve reproducing the one-loop correction to the fermion propagator with an electromagnetic background:

and then evaluate the vertex:





To finally evaluate the Beta Function.

Grazie!

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