Renormalization Invariant Parametrization for Parity Doublet Model

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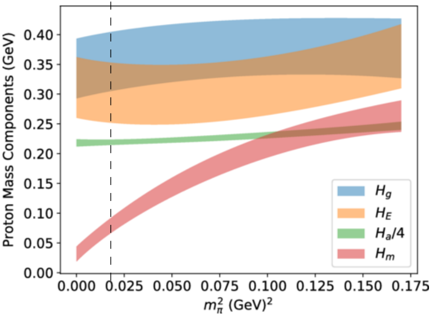
ISNP-2025 September 17, 2025

Parity Doublet Model

$$\mathcal{L} = \bar{N}_1\left[i\not{D} - g_1(\hat{\sigma} + i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}})\right]N_1 + \bar{N}_2\left[i\not{D} - g_2(\hat{\sigma} - i\gamma_5\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}})\right]N_2$$
$$- m_0(\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1) + U_{mes}(\hat{\omega}, \hat{\rho}, \hat{\sigma}, \hat{\boldsymbol{\pi}}, \{g_i\})$$

- Effective model to describe baryons
- Chiral symmetry
  - ⋆ Spontaneous symmetry breaking
  - ⋆ Finite σ expectation value
- Finite mass in the chiral restored phase: $m_0(\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1)$



Yi-Bo Yang. PhysRevLett.121.212001

The Parity Doublet Model (PDM)

$$
\begin{aligned}
\mathcal{L} = &\bar{N}_1\left[i\partial\!\!\!/ - g_\omega \omega\!\!\!/ - g_\rho \tau_3 \rho\!\!\!/ - g_1(\hat{\sigma} + i\gamma_5 \boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}})\right]N_1 \\
&+ \bar{N}_2\left[i\partial\!\!\!/ - g_\omega \omega\!\!\!/ - g_\rho \tau_3 \rho\!\!\!/ - g_2(\hat{\sigma} - i\gamma_5 \boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}})\right]N_2 \\
&- m_0(\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1) + U_{mes}(\hat{\omega}, \hat{\rho}, \hat{\sigma}, \hat{\boldsymbol{\pi}}, \{g_i\})
\end{aligned}
$$

$$\Big\downarrow \text{Diagonalization}$$

$$\mathcal{L} = \bar{N}_+(D\!\!\!\!/ - m_+)N_+ + \bar{N}_-(D\!\!\!\!/ - m_-)N_- + U_{mes}$$

$$m_\pm = \tfrac{1}{2}\sqrt{(g_1+g_2)^2\sigma^2 + 4m_0^2} \pm \tfrac{g_1-g_2}{2}\sigma \qquad \begin{cases} \sigma \to 0 \Rightarrow m_+ = m_- = m_0 \\ \sigma \to f_\pi \Rightarrow m_{+,-} = m_{N,N^*} \end{cases}$$

Thermodynamics (Mean Field)

Grand-Potential

$$\Omega = 2T \sum_{\tau=\pm 1} \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - f\left(\frac{E_\pm - \tilde{\mu}}{T}\right)\right) + \ln\left(1 - f\left(\frac{E_\pm + \tilde{\mu}}{T}\right)\right)$$
$$- 4 \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_\pm^2(\sigma)} + U_{mes}$$

Gap Equations: $\dfrac{\partial\Omega}{\partial\sigma} = 0\,, \quad \dfrac{\partial\Omega}{\partial\omega_0} = 0\,, \quad \dfrac{\partial\Omega}{\partial\rho_0} = 0$

Completely defined thermodynamics of the system!

$$-\Omega = P\,, \quad -\frac{\partial\Omega}{\partial\mu_B} = n_B\,, \quad -\frac{\partial\Omega}{\partial T} = S\,, \quad -\frac{\partial\Omega}{\partial\epsilon} = c_s^2$$

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Divergent contribution

$$\text{Divergent part: } \Omega_{vac} = -4 \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_{\pm}^2(\sigma)}$$

- Regularization:

$$\Omega_{vac} \approx \frac{2}{\pi^2} \sum_{\pm} \left(\frac{m_{\pm}^4}{8\epsilon} - \text{const} \times m_{\pm}^4 + \frac{m_{\pm}^4}{8} \ln \frac{m_{\pm}}{\mu} \right)$$

- Renormalization:
  - ⋆ $m(\sigma) \longleftrightarrow \sigma(m)$
  - ⋆ $U_{mes} = 1/2m^2\sigma^2 + 1/4\lambda\sigma^4 \longrightarrow U_{mes}(m_{\pm}) \equiv a_2(m_+^2 + m_-^2) + a_4(m_+^4 + m_-^4)$

We can renormalize Ω_{vac} using only a_4 as running constant

Renormalization Group (RG) invariance

Renormalization Scale dependent part:

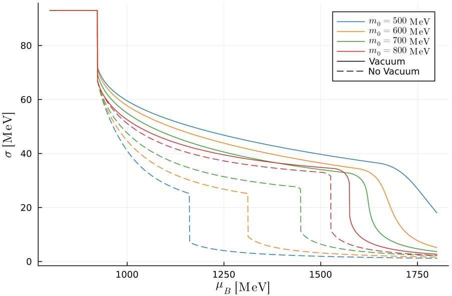
$$\Omega(\ln \mu) = \sum_{\pm} \frac{a_4(\ln \mu)}{4} m_\pm^4 - \frac{m_\pm \ln\left(m^4/\mu\right)}{(2\pi)^2}$$

The Grand Canonical potential is explicitly RG-invariant.

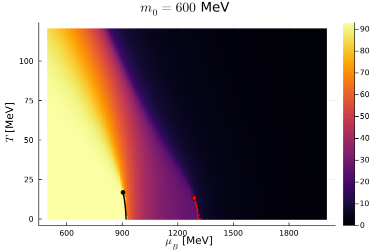
$$\frac{\mathrm{d}\Omega}{\mathrm{d}\ln \mu} = \left(\frac{\partial}{\partial \ln \mu} + \beta_{a_4} \frac{\partial}{\partial a_4}\right)\Omega(\ln \mu) = 0$$

μ can be fixed arbitrarily ($\mu = m_0$)

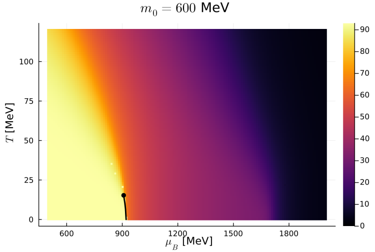
Results (phase diagrams)



Without Vacuum Contribution

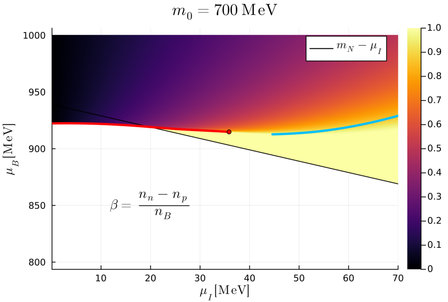


With Vacuum Contribution



- The Vacuum term push the chiral transition to high μ_B (and high n_B)
- 1^{st} order transition $\rightarrow$ crossover
- Effect bigger with low m_0

Results (phase diagrams)



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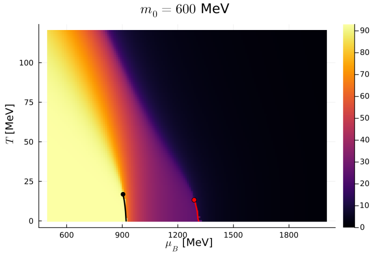
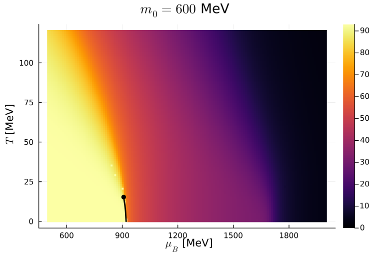
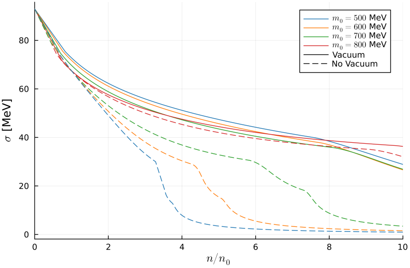
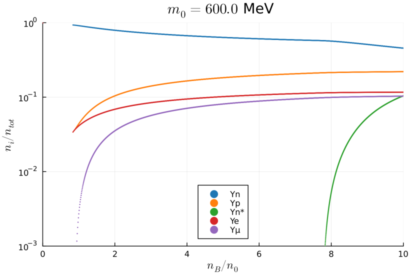


With Vacuum Contribution



Results (β-Eq)





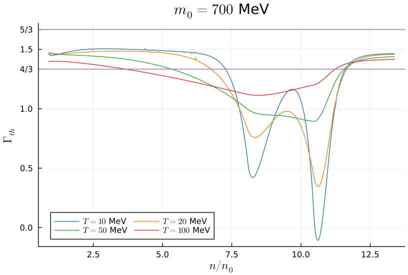
Vacuum contribution + β-Eq
=
χ-transition out of densities
of astrophysics interest

m_0	M_{max}	n_{max}
500 MeV	$1.84\,M_\odot$	$7.0\,n_0$
600 MeV	$1.77\,M_\odot$	$7.7\,n_0$
700 MeV	$1.69\,M_\odot$	$7.6\,n_0$
800 MeV	$1.52\,M_\odot$	$8.4\,n_0$

Results ($T \neq 0$ - A New Hope)

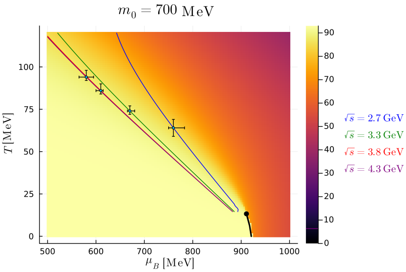
$$P_{th} = P(\mu, \beta, T) - P(\mu, \beta, 0)$$
$$\varepsilon_{th} = \varepsilon(\mu, \beta, T) - \varepsilon(\mu, \beta, 0)$$

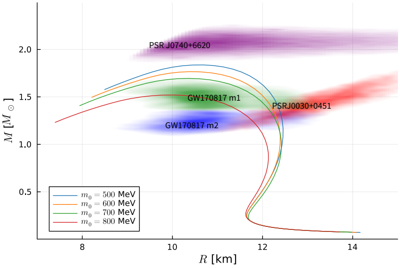
$$\Gamma_{th} = 1 + \frac{P_{th}}{\varepsilon_{th}}$$



- The adiabatic index encode the finite T effects on EoS
- Resonances strongly modify Γ_{th}
- Effect in NS mergers ($n \sim 5 - 6\, n_0,\ T \sim 60\,\mathrm{MeV}$)

Outlook





- $SU(3)$ extension of the model to include hyperons
- Use the RG-invariant framework on one-derivative extension of PDM (kinetic mixing)
- Include pion condensation (lattice calculation as benchmark[1,2])
- Apply the developed models to neutron star mergers, supernovae, low-energy heavy-ion collisions

[1] JHEP07(2023)055
[2] PhysRevD.97.054514

Thank you for your attention!