Density functional theory of renormalization group in nuclear matter

Yong-rui Chen

Institut für Theoretische Physik
Justus-Liebig-Universität Gießen

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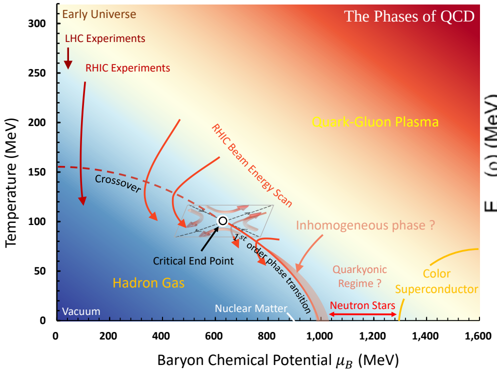
Based on YRC, W.-j. Fu, and Y.-y. Tan, arXiv:2508.02575

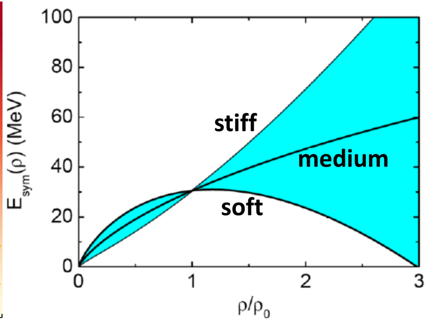
UNTERSTÜTZT VON / SUPPORTED BY Alexander von HUMBOLDT STIFTUNG

Outline

Introduction



QCD phase diagram[1]



symmetry energy of nuclear matter[2]

[1] **Nucl.Tech. 46 (2023) 04, 040002.**

[2] **PHD2024, Wuhan.**

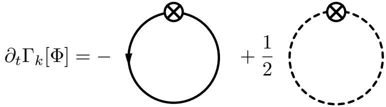
Nucleon-meson effective field theory

The effective action

$$\Gamma_k[\Phi]=\int_x \Big\{\bar{\psi}_N\Big[\gamma_\mu\partial_\mu+m_N-\gamma_0\hat{\mu}_N-h_{\sigma,k}\sigma T^0+\mathrm{i}h_{\omega,k}\gamma_\mu\omega_\mu T^0$$
$$+\mathrm{i}h_{\rho,k}\gamma_\mu\boldsymbol{\rho}_\mu\cdot\boldsymbol{T}\Big]\psi_N+\frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma+\frac{1}{2}m_\sigma^2\sigma^2+\frac{\lambda_3}{3!}\sigma^3+\frac{\lambda_4}{4!}\sigma^4$$
$$+\frac{1}{4}\omega_{\mu\nu}\omega_{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega_\mu+\frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}_{\mu\nu}+\frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}_\mu+V_k(\sigma,\omega,\rho)\Big\}$$

- $\Phi=(\psi_N,\bar{\psi}_N,\sigma,\omega,\rho)$, isospin doublet $\psi_N=(\psi_p,\psi_n)^{\mathsf{T}}$
- The field strength tensors of vector mesons
  $$\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu\,,\quad \boldsymbol{\rho}_{\mu\nu}=\partial_\mu\boldsymbol{\rho}_\nu-\partial_\nu\boldsymbol{\rho}_\mu\,.$$
- nucleon mass m_N and meson mass $(m_\sigma,m_\omega,m_\rho)$ in the vacuum
- Yukawa couplings with the strength $h_{\sigma,k}$, $h_{\omega,k}$ and $h_{\rho,k}$
- $V_k(\sigma,\omega,\rho)$ arises from density fluctuations, $V_{k\to\infty}=0$

Flow of the effective potential



Diagrammatic representation of the flow equation

$$\partial_t \Gamma_k[\Phi] = -\mathrm{Tr}\Big[\big(\partial_t R_{\bar{\psi}\psi,k}\big) G_{\psi\bar{\psi}}\Big] + \frac{1}{2}\mathrm{Tr}\Big[\big(\partial_t R_{\phi\phi,k}\big) G_{\phi\phi}\Big]$$

Both the nucleon and meson loop are included, $\phi = (\sigma, \omega, \rho)$

Equations of motion(EoM) for the fields

$\omega_\mu\big|_{\Phi_{\mathrm{EoM}}} = \omega_0 \delta_{\mu 0},\ \rho^i_\mu\big|_{\Phi_{\mathrm{EoM}}} = \rho_{03}\delta_{\mu 0}\delta_{i3}$

The effective mass of nucleons

$m_N^* = m_N - \frac{1}{2} h_{\sigma,k}\sigma$

The flow of effective potential (subtract vacuum part)

$$\partial_t V_k(\sigma, \omega_0, \rho_{03}) = -\frac{2}{3\pi^2} k^4 \Big[\Delta\mathcal{F}_{(1)}(\bar{m}_N^{*2}; T, \mu_p^*) + \Delta\mathcal{F}_{(1)}(\bar{m}_N^{*2}; T, \mu_n^*)\Big]$$

The effective chemical potentials for nucleon

$$\mu_p^* = \mu_p - \frac{1}{2}\Big(h_{\omega,k}\mathrm{i}\omega_0 + h_{\rho,k}\mathrm{i}\rho_{03}\Big),\ \mu_n^* = \mu_n - \frac{1}{2}\Big(h_{\omega,k}\mathrm{i}\omega_0 - h_{\rho,k}\mathrm{i}\rho_{03}\Big)$$

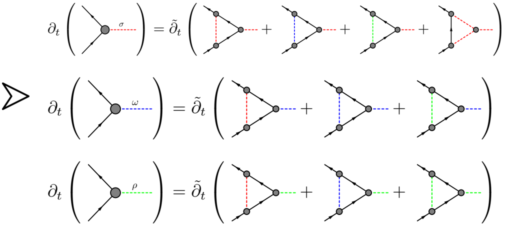
Density dependent Yukawa couplings

Flow equations of the Yukawa couplings:

$$\partial_t h_{\sigma,k} = -\frac{1}{2}\mathrm{Re}\Big[\mathrm{Tr}\Big(T^0 \partial_t \Gamma^{(3)}_{\bar{\psi}\psi\sigma}(p_0)\Big)\Big]$$

$$\partial_t h_{\omega,k} = -\frac{1}{6}\mathrm{Re}\Big[\mathrm{Tr}\Big(T^0 \mathrm{i}\vec{\gamma}_\mu \partial_t (\Gamma^{(3)}_{\bar{\psi}\psi\omega})_\mu(p_0)\Big)\Big]$$

$$\partial_t h_{\rho,k} = -\frac{1}{6}\mathrm{Re}\Big[\mathrm{Tr}\Big(T^3 \mathrm{i}\vec{\gamma}_\mu \partial_t (\Gamma^{(3)}_{\bar{\psi}\psi\rho})^{i=3}_\mu(p_0)\Big)\Big]$$



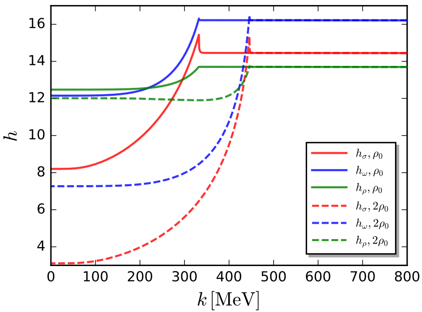
- The fermion momentum $p_0 = (p_0^0 = \pi T,\ \boldsymbol{p}_0 = 0)$
- $\vec{\gamma}_\mu$ is the spatial component of the γ matrix

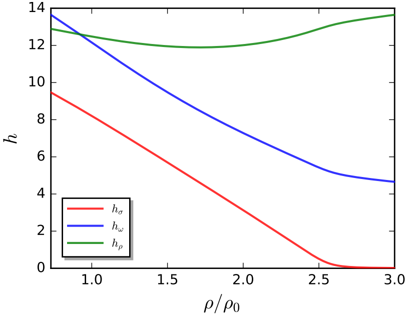
$$\vec{\gamma}_\mu = (1 - \delta_{\mu 0})\gamma_\mu$$

- The medium part of threshold function

$$\Delta\mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2, \bar{m}_b^2; T, \mu, p_0) = \Big[\mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2, \bar{m}_b^2; T, \mu, p_0) - \mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2, \bar{m}_b^2; 0, 0, 0)\Big] \times \Big[n_F(\bar{m}_f^2; T, \mu) + n_F(\bar{m}_f^2; T, -\mu)\Big]$$

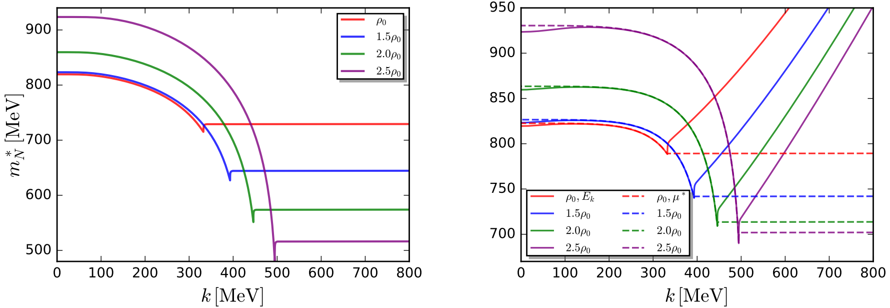
Numerical results





- Yukawa couplings as functions of k(left panel) and as functions of the baryon density(right panel)
- Yukawa couplings decrease when RG scale lower down
- h_σ, h_ω decrease with the increase of the baryon density (**medium-induced screening**)

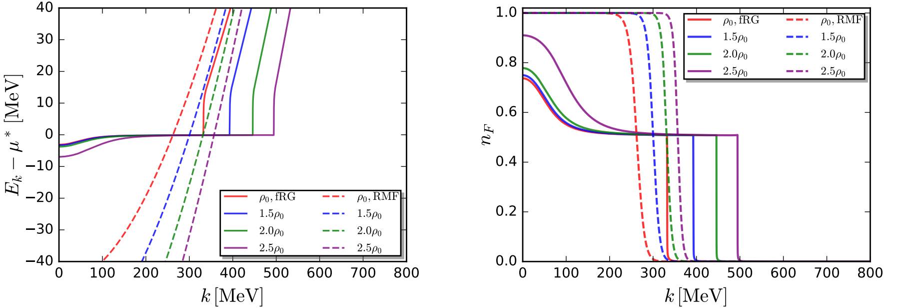
Numerical results



m_N^* and μ^* as functions of the RG scale k at several different baryon densities

- effective nucleon mass develops a little dip and increases subsequently
- effective mass is smaller in the UV but larger in the IR
- effective energy of quasi-nucleons stays close to the effective chemical potential (**locking of Fermi surface**)

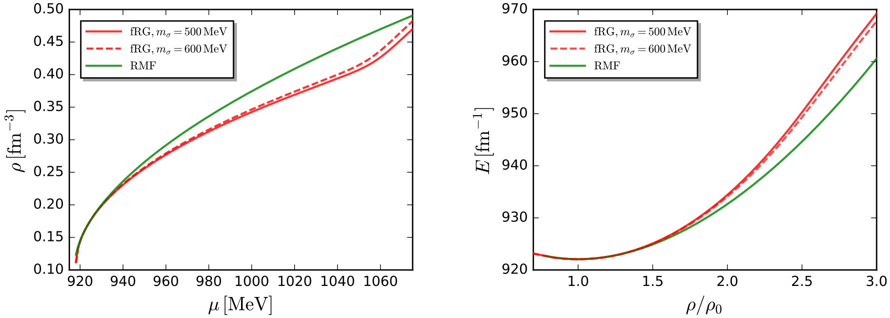
Numerical results



$E_k - \mu^*$ and fermionic distribution function as a function of the RG scale k

- $E_k - \mu^*$ stays around 0 in a sizable region of k
- distribution function is not far away from 1/2 in a sizable region of k
- The novel phenomenon has never been observed in RMF

Numerical results



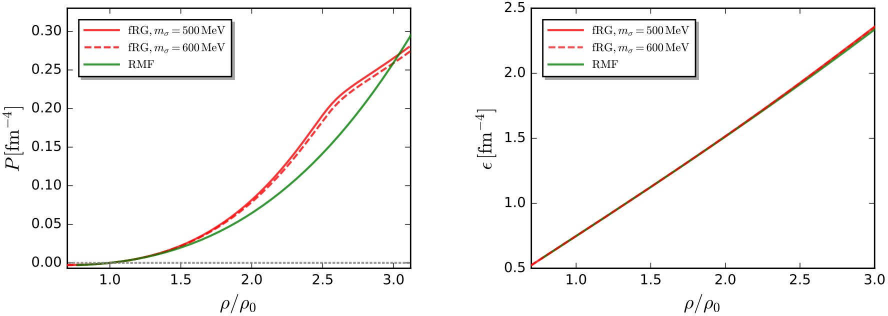
Nucleon density (left panel). Energy per nucleon (right panel)

The proton and neutron densities

$$\rho_p = -\frac{\partial\Omega(T,\mu_p,\mu_n)}{\partial\mu_p}, \qquad \rho_n = -\frac{\partial\Omega(T,\mu_p,\mu_n)}{\partial\mu_n}$$

The nucleon density $\rho = \rho_p + \rho_n$

Numerical results

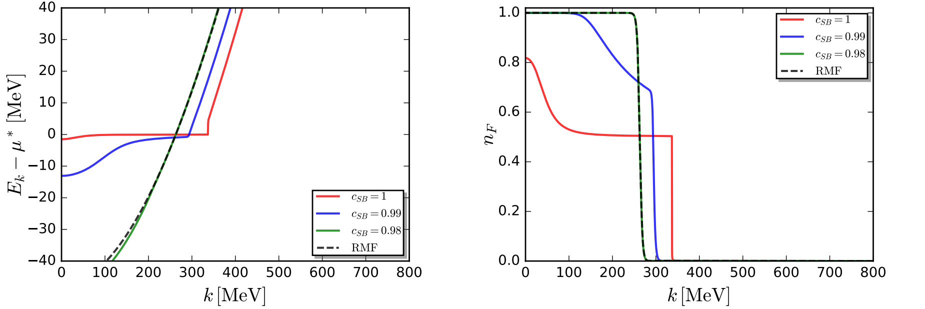


The pressure reads $p = -\Omega(T, \mu_p, \mu_n)$, entropy density $s = -\partial\Omega/\partial T$
The energy density

$$\varepsilon = Ts + \mu_p \rho_p + \mu_n \rho_n - p$$

- EoS of nuclear matter: stiffer $\rho_0 \lesssim \rho \lesssim 2.5\rho_0$, softer $\rho \gtrsim 2.5\rho_0$

Interpolation between the fRG and RMF



$$\Delta\mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2,\bar{m}_b^2;T,\mu,p_0) = \Big[\mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2,\bar{m}_b^2;T,\mu,p_0) - \mathcal{FB}_{(n_f,n_b)}(\bar{m}_f^2,\bar{m}_b^2;0,0,0)\Big] \times \Big[n_F(\bar{m}_f^2;T,c_{SB}\mu) + n_F(\bar{m}_f^2;T,-c_{SB}\mu)\Big]$$

- flows of Yukawa couplings are turned down continuously with the decrease of c_{SB}
- results of fRG are very close to the RMF when c_{SB} reduce to 0.98

Summary

- **Medium-induced screening** of couplings, leading to a stiffer equation of state (EoS) at moderate densities ($\rho_0 \lesssim \rho \lesssim 2.5\rho_0$) and a softer EoS at higher densities $\rho_0 \gtrsim 2.5\rho_0$
- **Locking of Fermi surface**, a novel RG-scale dependent phenomenon in which the effective energy of quasi-nucleons stays close to the running Fermi surface
- A **self-consistent density dependence of Yukawa couplings** that plays a substantial role in the behavior of high density nuclear matter

Thanks for your attentions!