Heavy Meson Dynamics in the QCD Conformal Window

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Erice: INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS, 46th Course

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Paper

Paper at 2406.09758 [hep-ph]

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Defect induced heavy meson dynamics in the QCD conformal window

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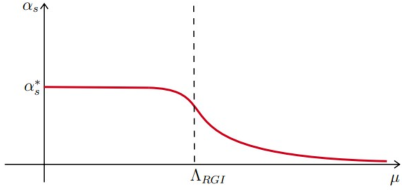
The conformal window of QCD

The Conformal Window (CW) is the region in the N_c *vs* N_f plane where QCD-like theories develop an infrared fixed point (IRFP), while staying asymptotically free in the UV.

The IRFP was first discovered by Banks and Zaks and it is usually called BZ fixed point.



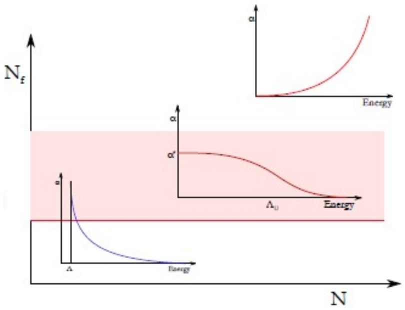


Figure: From F.Sannino arXiv:0911.0931v1 [hep-ph].

The conformal window of QCD

The CW extends from (above):

- N_{AF}, where asymptotic freedom is restored

to (below):

- N_{crit}, where large distance conformality breaking occurs

The coupling α_s grows going from N_{AF} to N_{crit}.

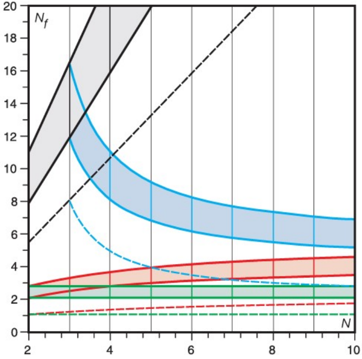


Figure: From F.Sannino and D.Dietrich arxiv:0611341 [hep-ph]. Grey=Fundamental, Blue=A_2, Red=S_2, Green=Adj.

The conformal window of QCD

The CW extends from (above):

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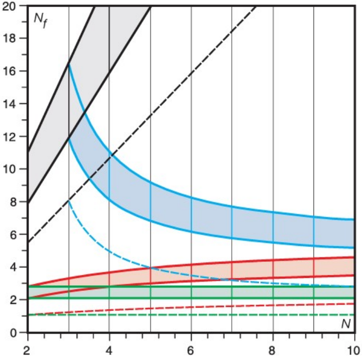
At N_{crit} the theory is strongly coupled. How can we compute N_{crit}?



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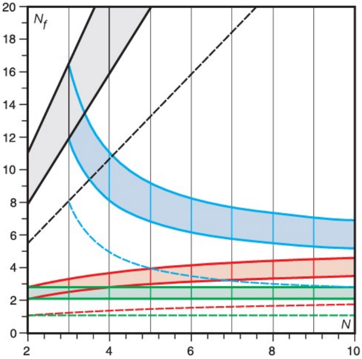

Figure: From F.Sannino and D.Dietrich arxiv:0611341 [hep-ph]. Grey=Fundamental, Blue=A_2, Red=S_2, Green=Adj.

At N_{crit} the theory is strongly coupled. How can we compute N_{crit}? Non-perturbative methods.

The conformal window of QCD

At $N_{AF}(1-\varepsilon)$, that is slightly below the upper boundary of the CW, the theory is weakly coupled and we can trust perturbation theory

$$\alpha_s^* = \frac{44\pi}{21C_A + 33C_r}\varepsilon + \mathcal{O}(\varepsilon^2)$$

C_A and C_r are the quadratic casimirs of the $SU(N)$ adjoint and fermionic matter representations, respectively.

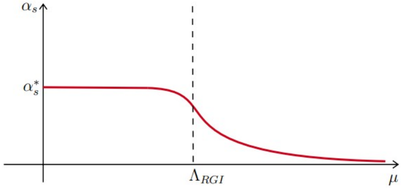

Figure: The running of the coupling constant when a BZ infrared fixed point arises.

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The dynamics of heavy-light states within the Conformal Window

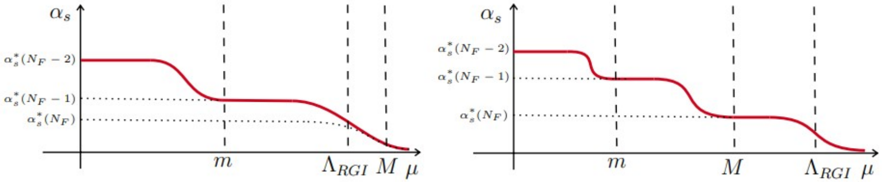


Figure: The IRFP is $\alpha_s^*(N_F-2)$. The dashed lines indicate the running for N_F and N_{F-1} active flavors.

$$\mathcal{L}_{\text{HQET}} = \bar{\psi}(i\gamma_\mu D^\mu - m)\psi + \bar{h}_v iv\cdot Dh_v + \frac{1}{2M}\bar{h}_v(iD_\perp)^2 h_v + \frac{g_s}{4M}\bar{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v + \mathcal{O}(1/4M^2),$$

The dynamics of heavy-light states within the Conformal Window $N_c = 3$

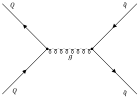
The interaction potential V can be derived by the S-matrix. We can solve the dynamics of the light degree of freedom, the QCD "brown muck", in V. For $N_c = 3$:

- first-order in mass

$$V(\boldsymbol{r}) = -\frac{4}{3}\frac{\alpha_s^*}{r}\left(1 + \boldsymbol{\alpha}_D \cdot \frac{\boldsymbol{P}}{M}\right) + \frac{2}{3M}\frac{\alpha_s^*}{r^2}\boldsymbol{\alpha}_D \cdot \boldsymbol{\sigma}_Q \times \boldsymbol{n} \; ,$$

- infinite mass result

$$V(\boldsymbol{r}) = -\frac{4}{3}\frac{\alpha_s^*}{r}$$



The brown muck wave function and energy is given by the stationary Shröedinger equation

$$(\boldsymbol{\alpha}_D \cdot \boldsymbol{p} + \beta_D m + V(r))\psi = E\psi$$

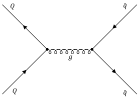
The dynamics of heavy-light states within the Conformal Window $N_c = 3$

The interaction potential V can be derived by the S-matrix. We can solve the dynamics of the light degree of freedom, the QCD "brown muck", in V.

- the S-matrix element has to be projected on the right colour wave-function. If i, j are the IN colours and k, l the OUT colours, then:

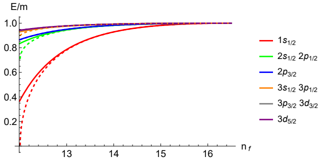
$$\frac{\delta_{ij}}{\sqrt{3}} \frac{\delta_{kl}}{\sqrt{3}}$$

- other colour channels can be safely explored (we have coloured ions!)
- the extension to other colour representations lays in the choice of the proper wave-function.



Color factor for $q\,\bar{q} \to q\,\bar{q}$		
$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$	$C_{\mathbf{1}} \equiv C_F = 4/3$	Attractive
	$C_{\mathbf{8}} = -1/6$	Repulsive
Color factor for $q\,q \to q\,q$		
$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$	$C_{\mathbf{3}} = 2/3$	Attractive
	$C_{\mathbf{6}} = -1/3$	Repulsive

The dynamics of heavy-light states within the Conformal Window $N_c = 3$



The effective coupling is $\alpha = 4/3\alpha_s^*$

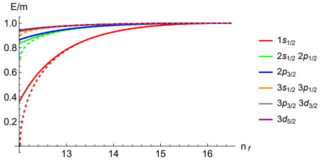
- Non perturbative result

$$E_{nj} = m\left[1 + \left(\frac{\alpha}{n - (j+1/2) + \sqrt{(j+1/2)^2 - \alpha^2}}\right)^2\right]^{-1/2}$$

- Perturbative result

$$E_{nj} = m\left\{1 - \alpha^2\left[\frac{1}{2n^2} + \frac{\alpha^2}{2n^3}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right]\right\} + \mathcal{O}(\alpha^5)$$

The dynamics of heavy-light states within the Conformal Window $N_c = 3$



The "mass" of the brown muck in the ground state is

$$\bar{\Lambda} = m\sqrt{1-\alpha^2}$$

The effective coupling is $\alpha = 4/3\alpha_s^*$

- Non perturbative result

$$E_{nj} = m\left[1 + \left(\frac{\alpha}{n-(j+1/2)+\sqrt{(j+1/2)^2-\alpha^2}}\right)^2\right]^{-1/2}$$

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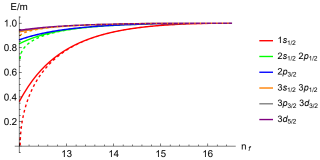
$$E_{nj} = m\left\{1-\alpha^2\left[\frac{1}{2n^2}+\frac{\alpha^2}{2n^3}\left(\frac{1}{j+\frac{1}{2}}-\frac{3}{4n}\right)\right]\right\}+\mathcal{O}(\alpha^5)$$

The dynamics of heavy-light states within the Conformal Window $N_c = 3$



Incidentally, the ground level $(n = 1, j = 1/2)$ only exists if $\alpha < j + 1/2$ corresponding to $N_f > 12$

The effective coupling is $\alpha = 4/3\alpha_s^*$

- Non perturbative result

$$E_{nj} = m\left[1 + \left(\frac{\alpha}{n - (j + 1/2) + \sqrt{(j + 1/2)^2 - \alpha^2}}\right)^2\right]^{-1/2}$$

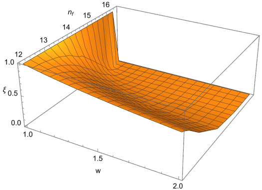
- Perturbative result

$$E_{nj} = m\left\{1 - \alpha^2\left[\frac{1}{2n^2} + \frac{\alpha^2}{2n^3}\left(\frac{1}{j + \frac{1}{2}} - \frac{3}{4n}\right)\right]\right\} + \mathcal{O}(\alpha^5)$$

The dynamics of heavy-light states within the Conformal Window $N_c = 3$

When the heavy quark changes its velocity ($v \to v'$, $w = v \cdot v'$) the transition amplitude is proportional to the superposition of the brown much wave functions before and after the change

$$\xi(w) = \sqrt{\frac{2}{w+1}} \left\langle L, J_L' \middle| L, J_L \right\rangle$$



$$\xi(w) = \frac{1}{w+1} \frac{\left(1+\Phi^2(w)\right)^{-\rho}}{\Phi(w)} \frac{\sin\left(2\rho \arctan\left(\Phi(w)\right)\right)}{\rho}$$

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Confinement

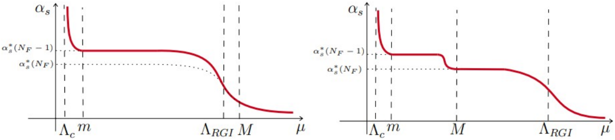
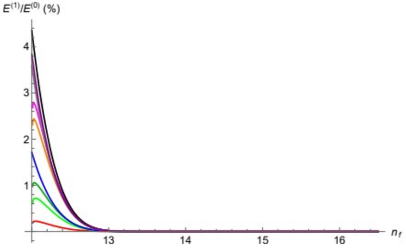


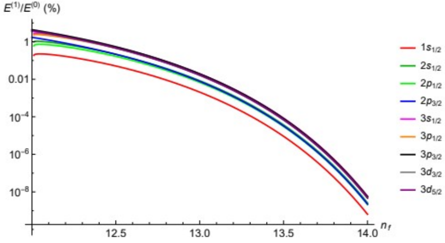
Figure: Below the mass of the light quarks, the pure Yang-Mills dynamics generates a confining scale Λ_c. The strong coupling does not reach the IRFP and diverges at Λ_c.

$$V_c = \Lambda_c^2 r \quad \Lambda_c = e^{-\frac{2\pi}{\beta_0 \alpha_s(m)}}$$

$$(\boldsymbol{\alpha}_D \cdot \boldsymbol{p} + \beta_D m - \frac{\alpha}{r} + \beta_D \Lambda_c^2 r)\psi = E\psi$$

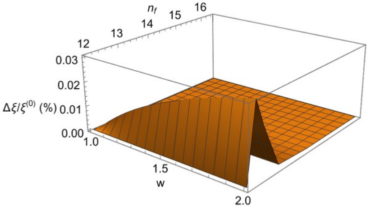
Confinement





$$E_{nj}^{(1)} = \left\langle \psi_{njm}^{(0)} \right| \beta_D \Lambda_c^2\, r \left| \psi_{njm}^{(0)} \right\rangle = \Lambda_c^2 \int dr\, r^3 (|g_n(r)|^2 - |f_n(r)|^2)$$

Confinement



$$\psi_{1,1/2,\pm 1/2} = \psi^{(0)}_{1,1/2,\pm 1/2} + \sum_{n,j,m \neq 1,1/2,\pm 1/2} \frac{\left\langle \psi^{(0)}_{njm} \middle| \beta_D \Lambda_c^2 r \middle| \psi^{(0)}_{1,1/2,\pm 1/2} \right\rangle}{E_1^{(0)} - E_{nj}^{(0)}} \psi^{(0)}_{njm}$$

Retaining the most relevant order in α and the contribution from the 2s orbital

$$\frac{\left\langle \psi^{(0)}_{njm} \middle| \beta_D \Lambda_c^2 r \middle| \psi^{(0)}_{1,1/2,\pm 1/2} \right\rangle}{E_1^{(0)} - E_n^{(0)}} \approx \frac{16\,\Lambda_c^2}{m^2 \alpha^3} \frac{n^{9/2}(n-1)^{n-3}}{(n+1)^{n+3}}$$

The dynamics of heavy-light states within the Conformal Window $N_c = 2$

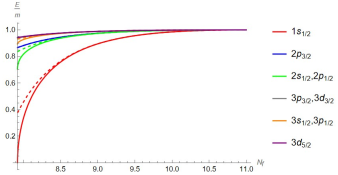
For $N_c = 2$ and fundamental fermions:

- the colour singlet wave-function is

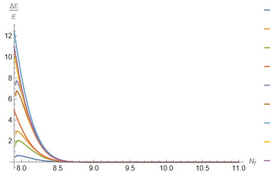
$$\frac{\epsilon_{ij}}{\sqrt{2}} \frac{\epsilon_{kl}}{\sqrt{2}}$$

- infinite mass result

$$V(\boldsymbol{r}) = -\frac{3}{4} \frac{\alpha_s^*}{r}$$



- the CW starts at $N_f^{AF} = 11$,
- α_c is reached at $N_f = 8$,
- β_1 changes sign at $N_f = 5.5$.



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Conclusions

- I have solved the heavy meson dynamics within the conformal window
- I have computed two fundamental dynamical quantities for the mesons: 1) the brown muck mass 2) the Isgur-Wise function
- I have studied the effect of a confinement dynamics in a safe a controllable manner
- the framework can be tested by first principle lattice computations, offering a new way to gain information about gauge theories and conformal dynamics

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Outlook

- Extend the framework to other matter representations
- Probe the lower boundary of the conformal window
- Build dark-sector of the Standard Model
- Going beyond the defect approximation in conformal field theory