International School of Nuclear Physics, 46th course.
Erice - Italy.

Centrality mode-by-mode evolution in Pb-Pb collisions

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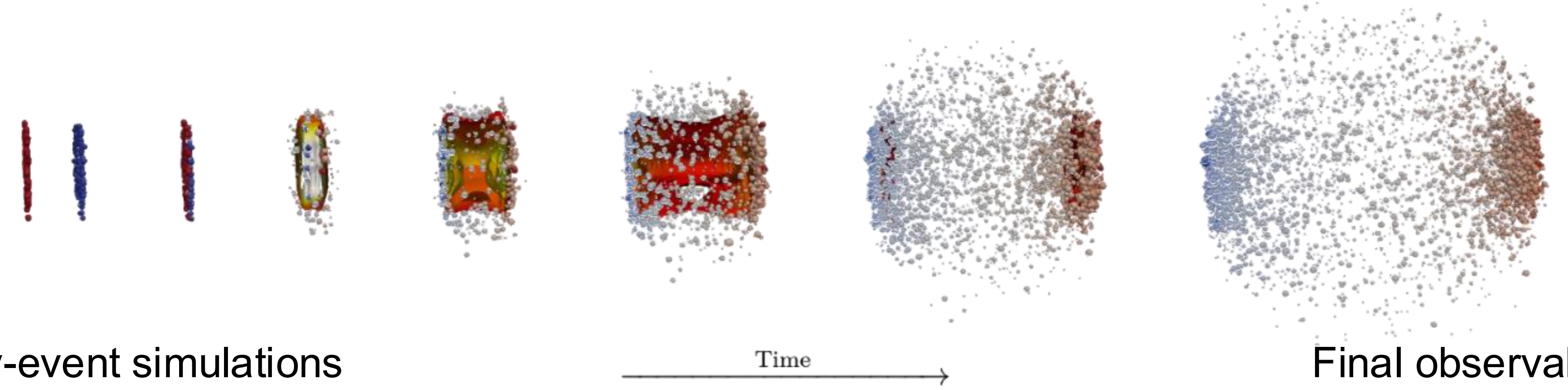
In collaboration with N. Borghini and H. Roch
Presentation based on arXiv: 2508.05336

UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC–TR 211
Strong-interaction matter under extreme conditions

September 2025

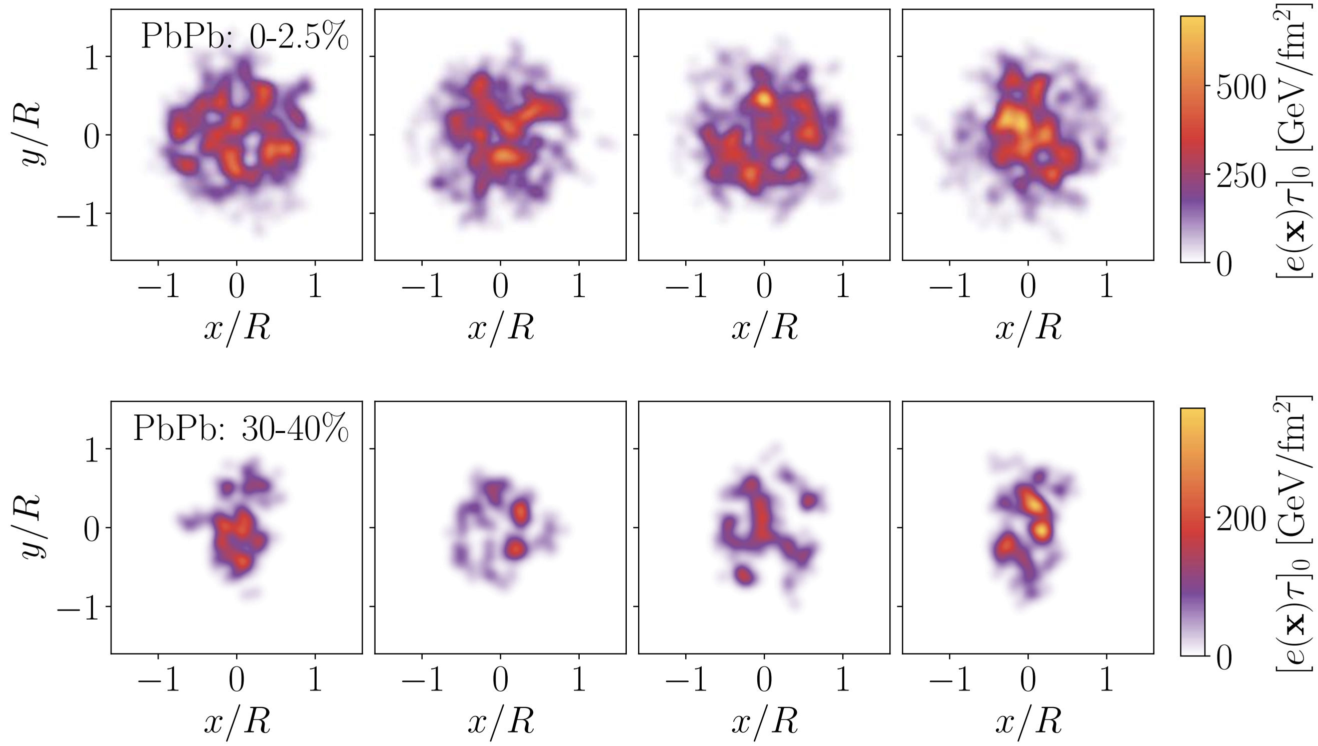
Introduction: Hybrid approach for heavy ion collisions



Event-by-event simulations

Final observables

Each collision deposits energy in a different way, despite a fixed centrality.



Main goal: Study these **fluctuations** in initial states and their evolution.

Theory: Mode-by-mode decomposition

$$\Phi^{(i)}(\boldsymbol{x}) = \bar{\Psi}(\boldsymbol{x}) + \sum_{l} c_l \Psi_l(\boldsymbol{x})$$

Energy profile

Average state

Expansion coefficients

Fluctuation modes

Each initial profile is decomposed into an average state plus a linear combination of fluctuation modes.

Theory: Mode-by-mode decomposition

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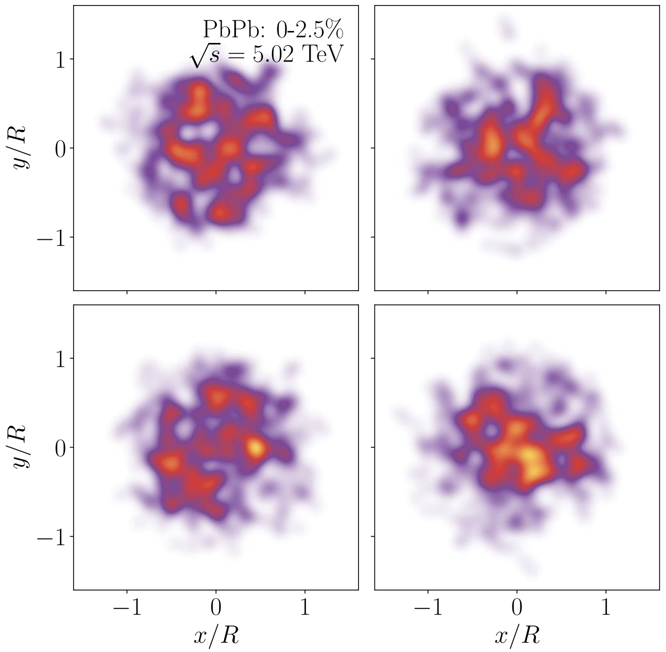
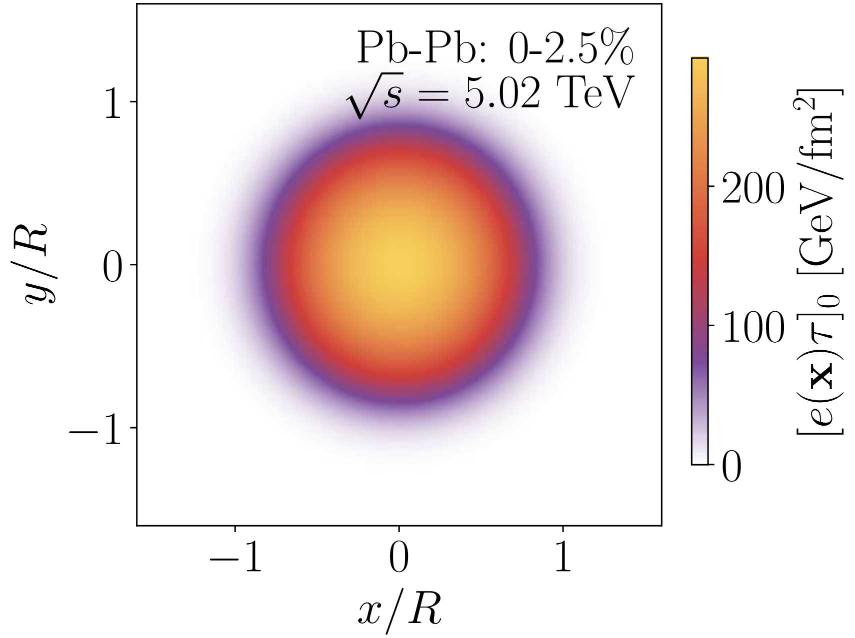
Average state



2^{21} Events

$$\bar{\Psi}(\boldsymbol{x}) \equiv \frac{1}{N_{\mathrm{ev}}} \sum_{i=1}^{N_{\mathrm{ev}}} \Phi^{(i)}(\boldsymbol{x})$$



Theory: Mode-by-mode decomposition

$$\Phi^{(i)}(\boldsymbol{x}) = \bar{\Psi}(\boldsymbol{x}) + \sum_l c_l \Psi_l(\boldsymbol{x})$$

Fluctuation modes

Each initial profile is decomposed into an average state plus a linear combination of fluctuation modes.

Autocorrelation between the fluctuation part $\rho \equiv \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} \Phi^{(i)} \Phi^{(i)\mathsf{T}} - \bar{\Psi}\bar{\Psi}^{\mathsf{T}}$

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This density matrix can be diagonalized $\rho \tilde{\Psi}_l = \lambda_l \tilde{\Psi}_l$

Importance of the mode (λ_l)

Orthonormal basis ($\tilde{\Psi}_l$)

Theory: Mode-by-mode decomposition

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Assuming that the expansion coefficients are:

- Centered $\langle c_l \rangle = 0$
- Uncorrelated $\langle c_l c_{l'} \rangle = \delta_{ll'}$
- Normalized to 1

What are called modes are an unnormalized, orthogonal basis

$$\Psi_l \equiv \sqrt{\lambda_l} \tilde{\Psi}_l$$

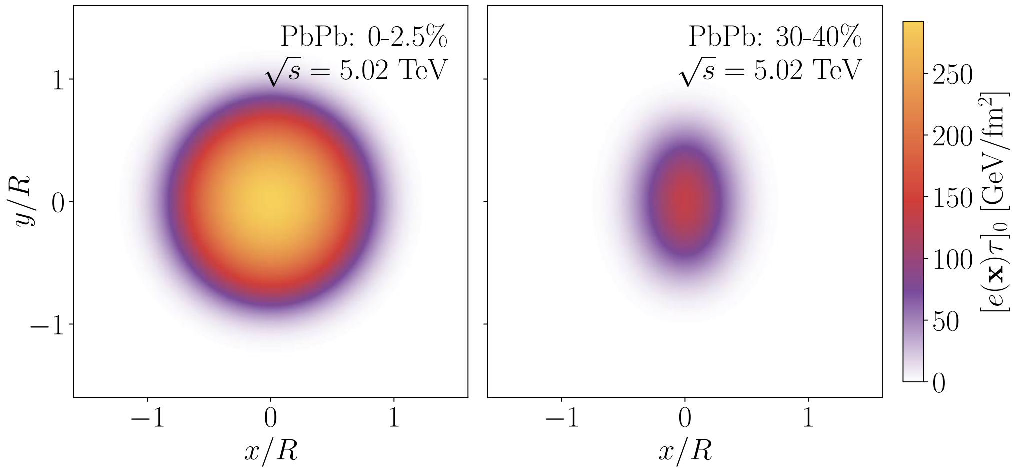
Notice that the fluctuation modes should have units of energy

Theory: Mode-by-mode decomposition

$$\Phi^{(i)}(\boldsymbol{x}) = \bar{\Psi}(\boldsymbol{x}) + \sum_l c_l \Psi_l(\boldsymbol{x})$$

Fluctuation modes

Average state

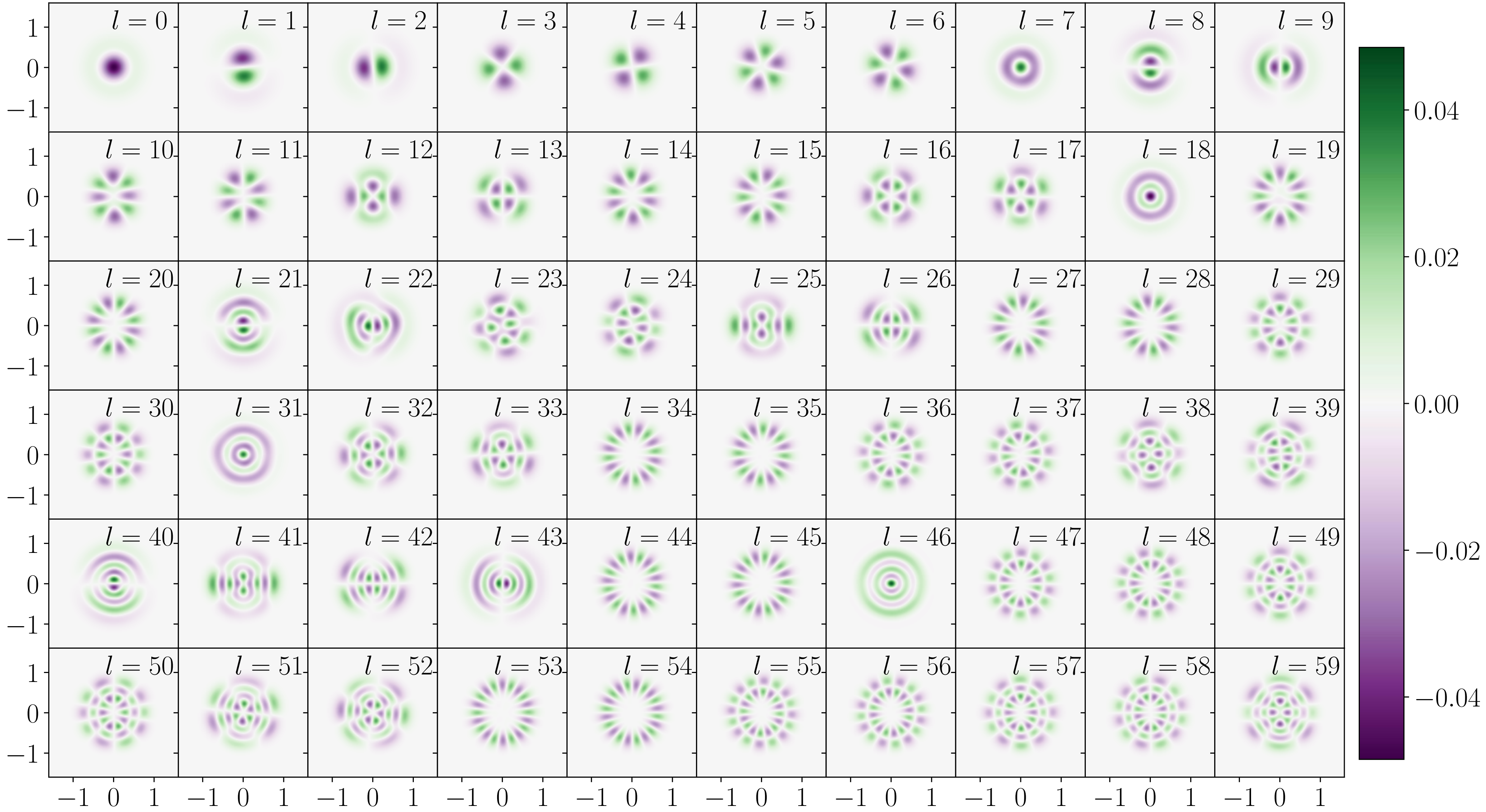


Each initial profile is decomposed into an average state plus a linear combination of fluctuation modes.

Results: Fluctuation modes*

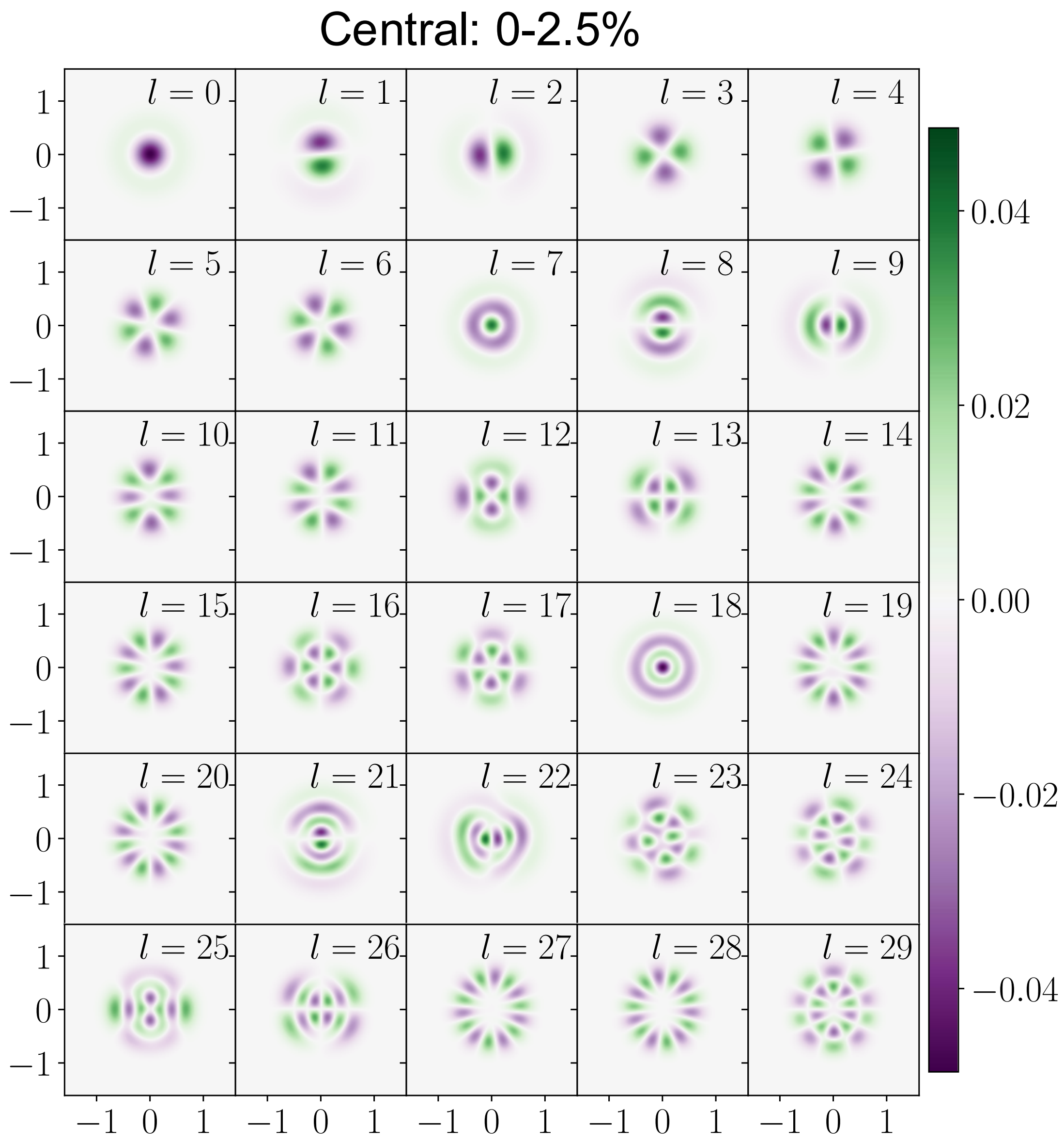
Central: 0-2.5%

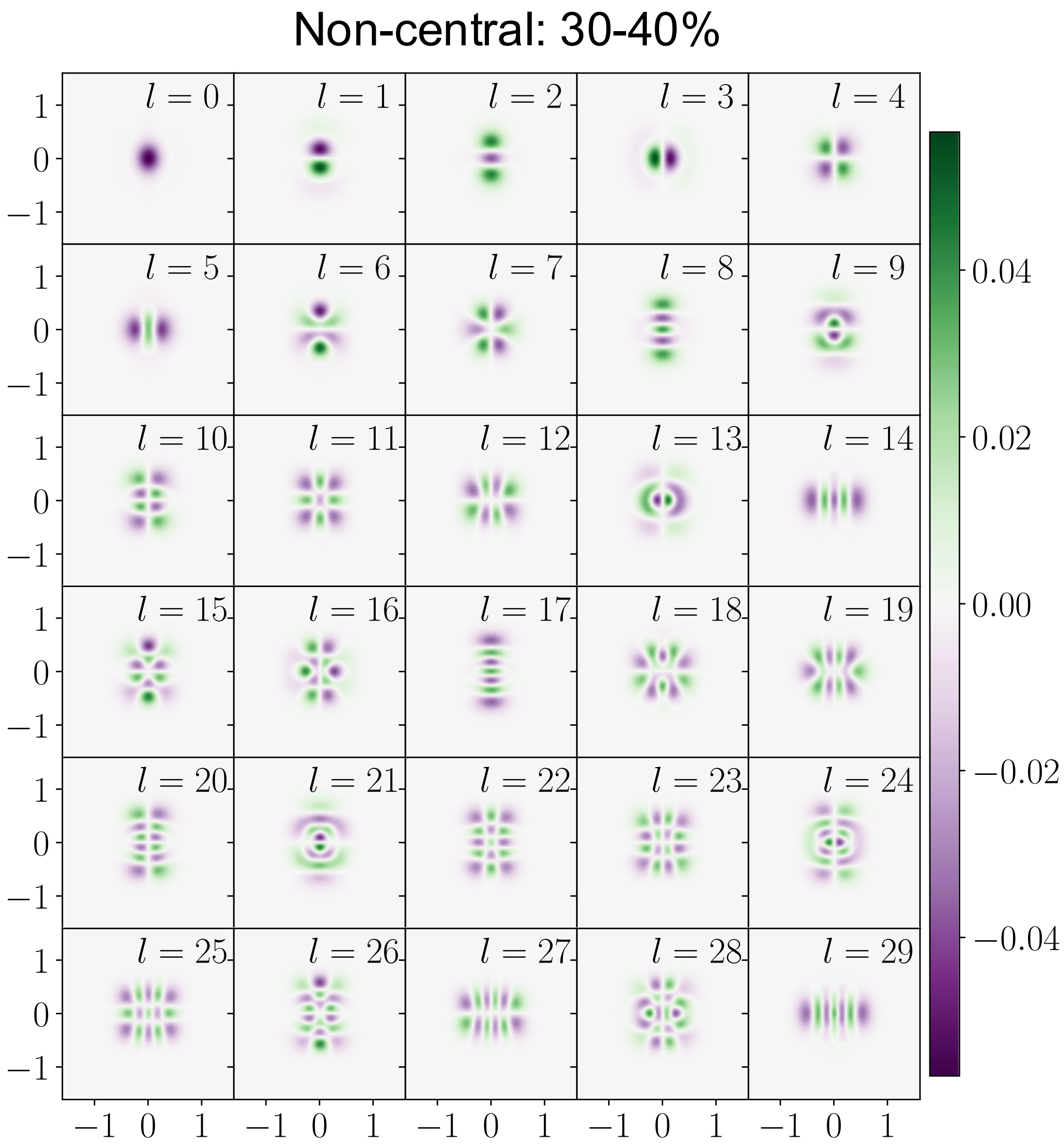
Modes are ordered by importance (eigenvalues)
Modes have symmetries: circular, dipole, quadrupole, sextupole...



*The plot shows the normalized modes, for a better comparison.

Results: Comparison of modes in different centralities





Theory: Computation of observables

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$$O_\alpha(\Phi^{(i)}) = O_\alpha(\bar{\Psi}) + \sum_l L_{\alpha,l} c_l^{(i)} + \frac{1}{2} \sum_{l,l'} Q_{\alpha,ll'} c_l^{(i)} c_{l'}^{(i)} + \mathcal{O}(c_l^3)$$

Taylor expansion around the average state in orders of the expansion coefficients

Linear response

$$L_{\alpha,l} = \left. \frac{\partial O_\alpha}{\partial c_l} \right|_{\bar{\Psi}}$$

Quadratic response

$$Q_{\alpha,ll'} = \left. \frac{\partial^2 O_\alpha}{\partial c_l \partial c_{l'}} \right|_{\bar{\Psi}}$$

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But for a numerical implementation, the modes are plugged into the average state

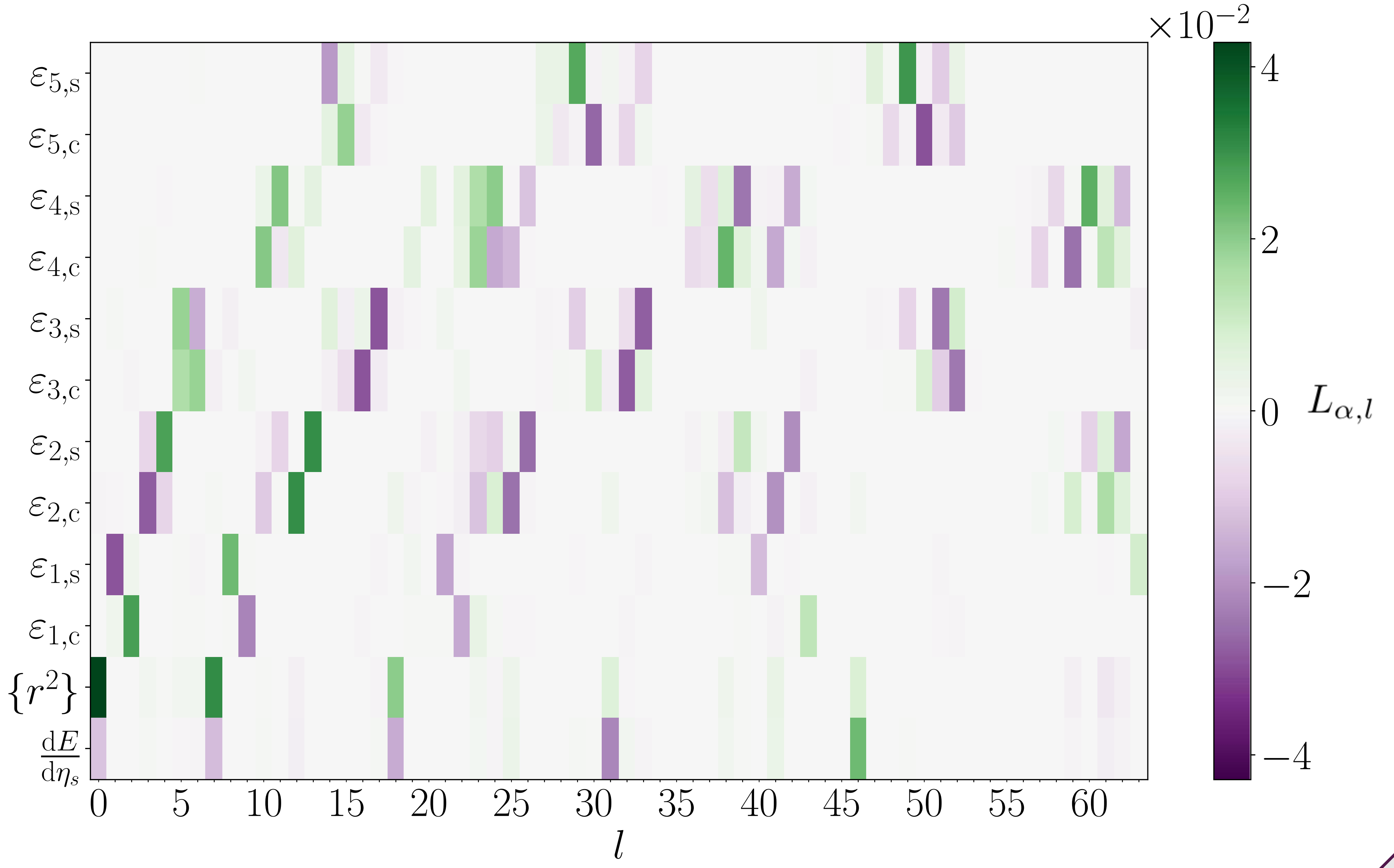
$$\Psi_l^\pm = \bar{\Psi} \pm \xi \Psi_l$$

Small coefficient. Here: 0.5

$$L_{\alpha,l} = \frac{O_\alpha(\Psi_l^+) - O_\alpha(\Psi_l^-)}{2\xi} \qquad Q_{\alpha,ll} = \frac{O_\alpha(\Psi_l^+) + O_\alpha(\Psi_l^-) - 2O_\alpha(\bar{\Psi})}{\xi^2}$$

Results: Linear response coefficients in initial states

Pb-Pb
Central 0-2.5%



Results: Linear response coefficients in initial states

Pb-Pb
Central 0-2.5%

Eccentricities:

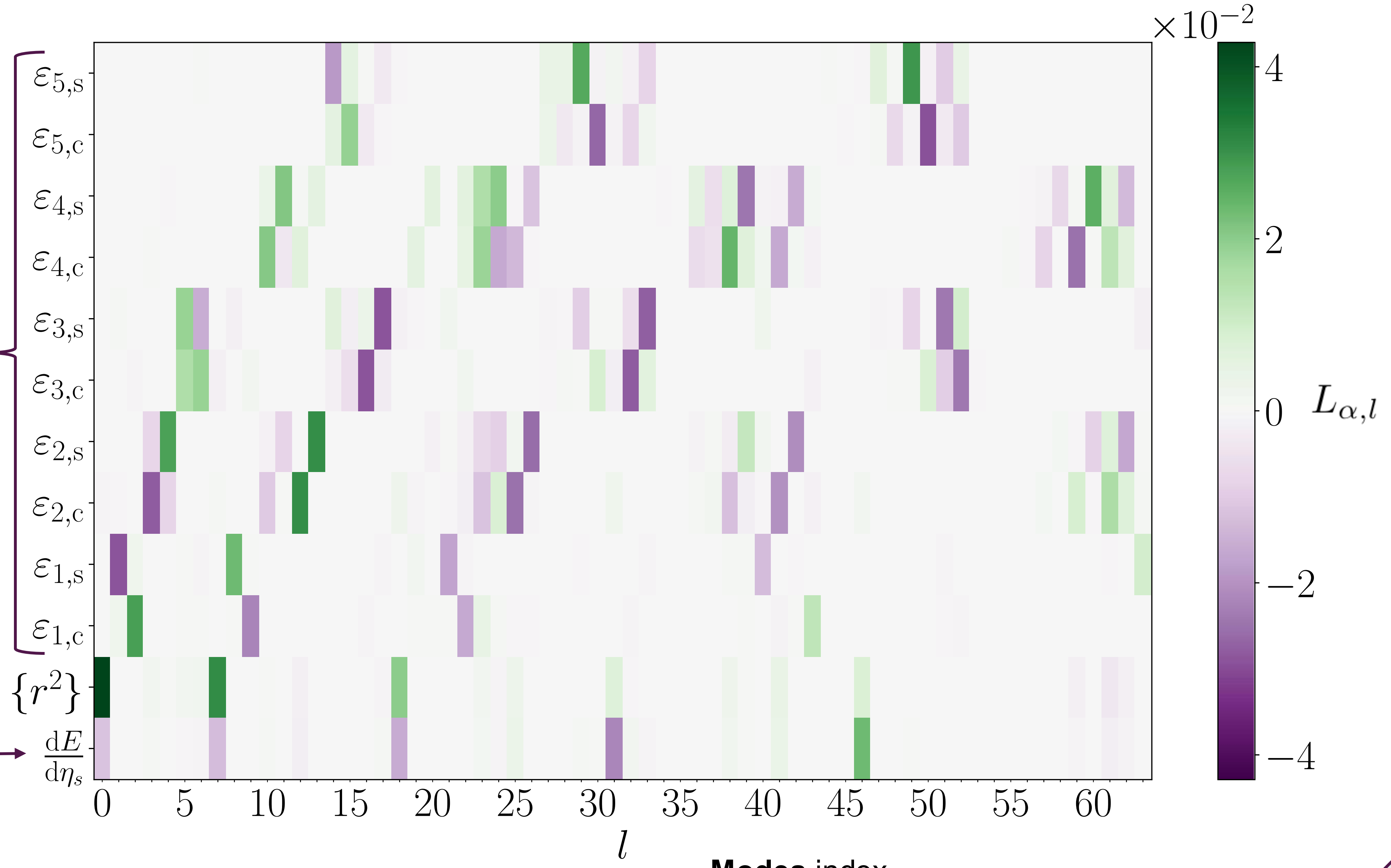
$$\varepsilon_1 e^{i\Phi_1} \equiv -\frac{\int r^3 e^{i\theta} e(r,\theta) r dr d\theta}{\int r^3 e(r,\theta) r dr d\theta}$$

$$\varepsilon_n e^{i\Phi_n} \equiv -\frac{\int r^n e^{in\theta} e(r,\theta) r dr d\theta}{\int r^n e(r,\theta) r dr d\theta}$$

Eccentricities are split into **cosine** and **sine** components

Average square **radius**

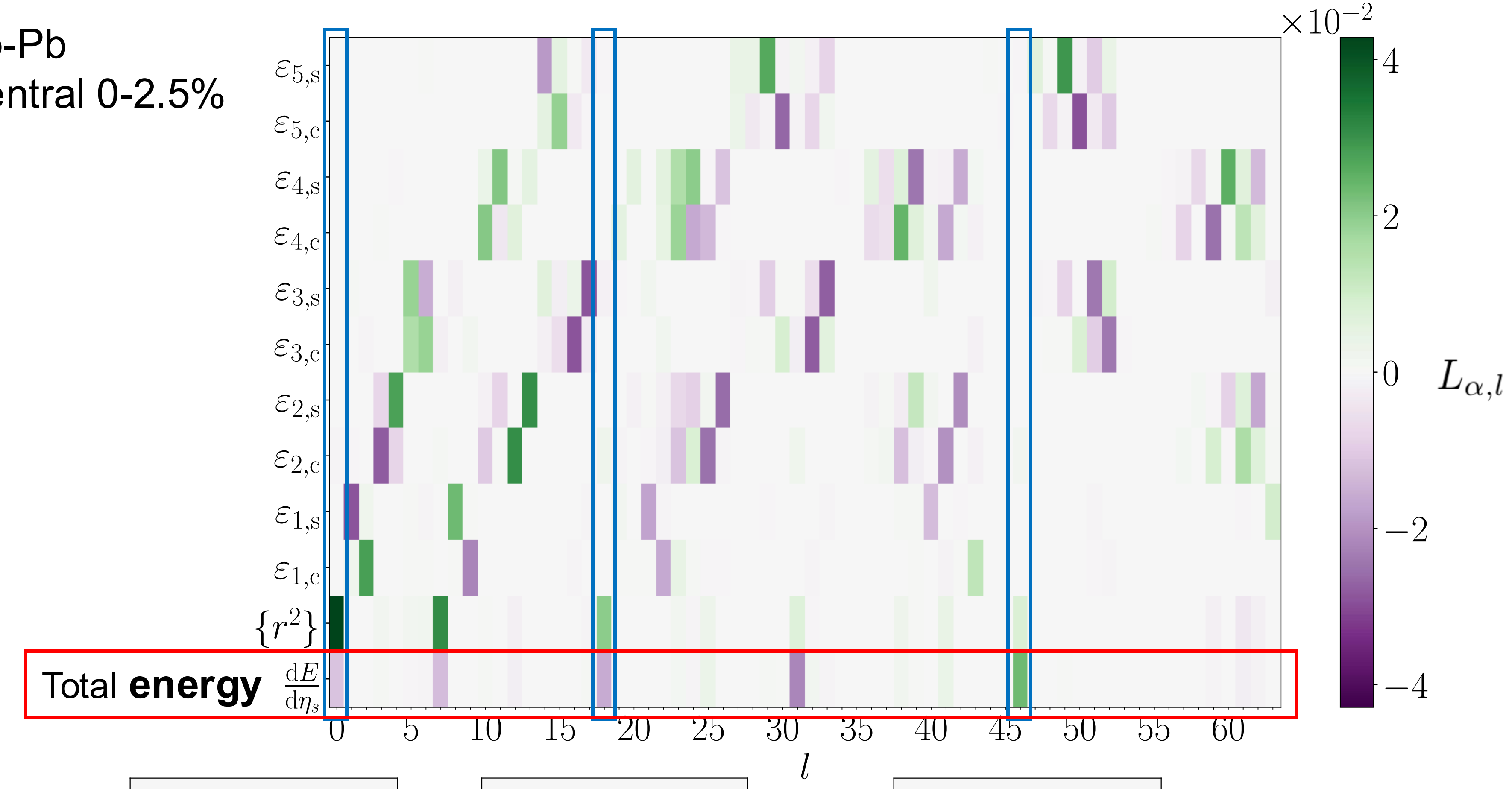
Energy per unit space-time rapidity

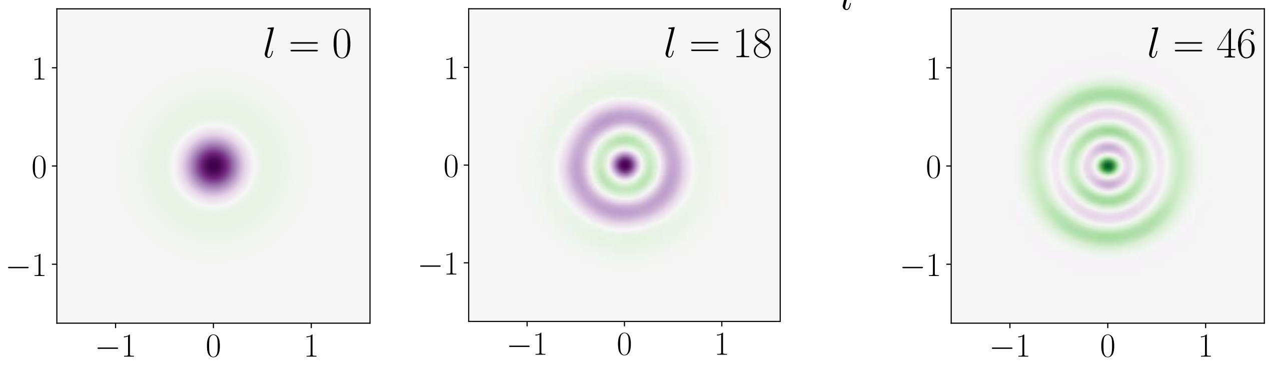


Modes index

Results: Connection between observables and modes

Pb-Pb
Central 0-2.5%

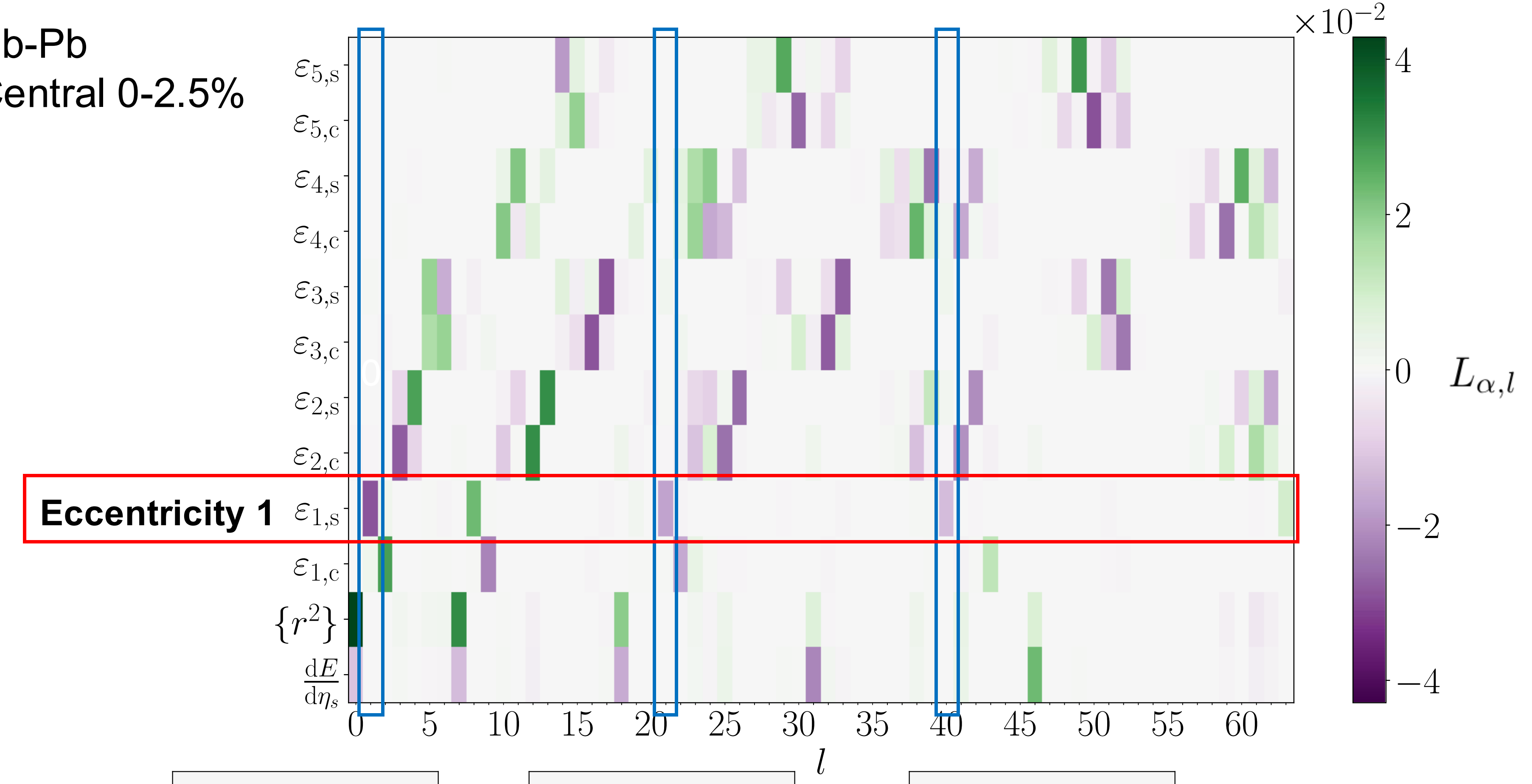


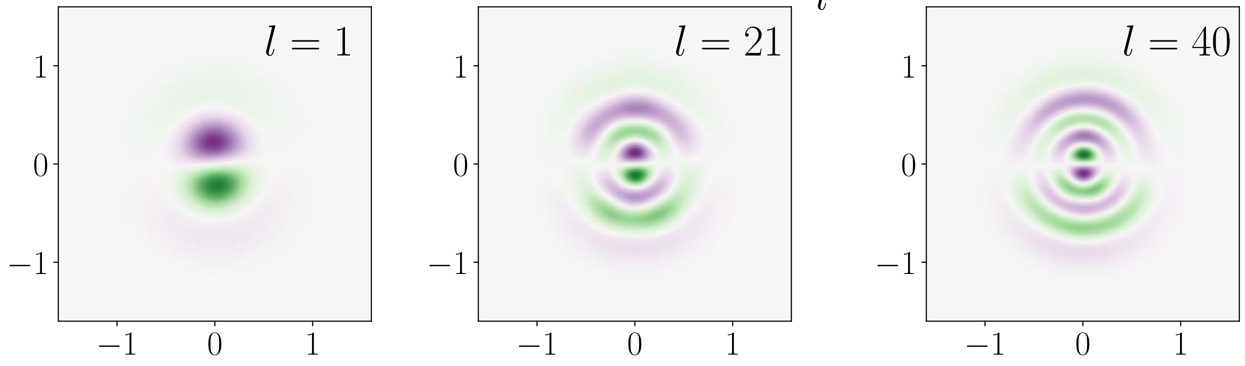


Modes that carry energy have the same symmetry as the average state.

Results: Connection between observables and modes

Pb-Pb
Central 0-2.5%

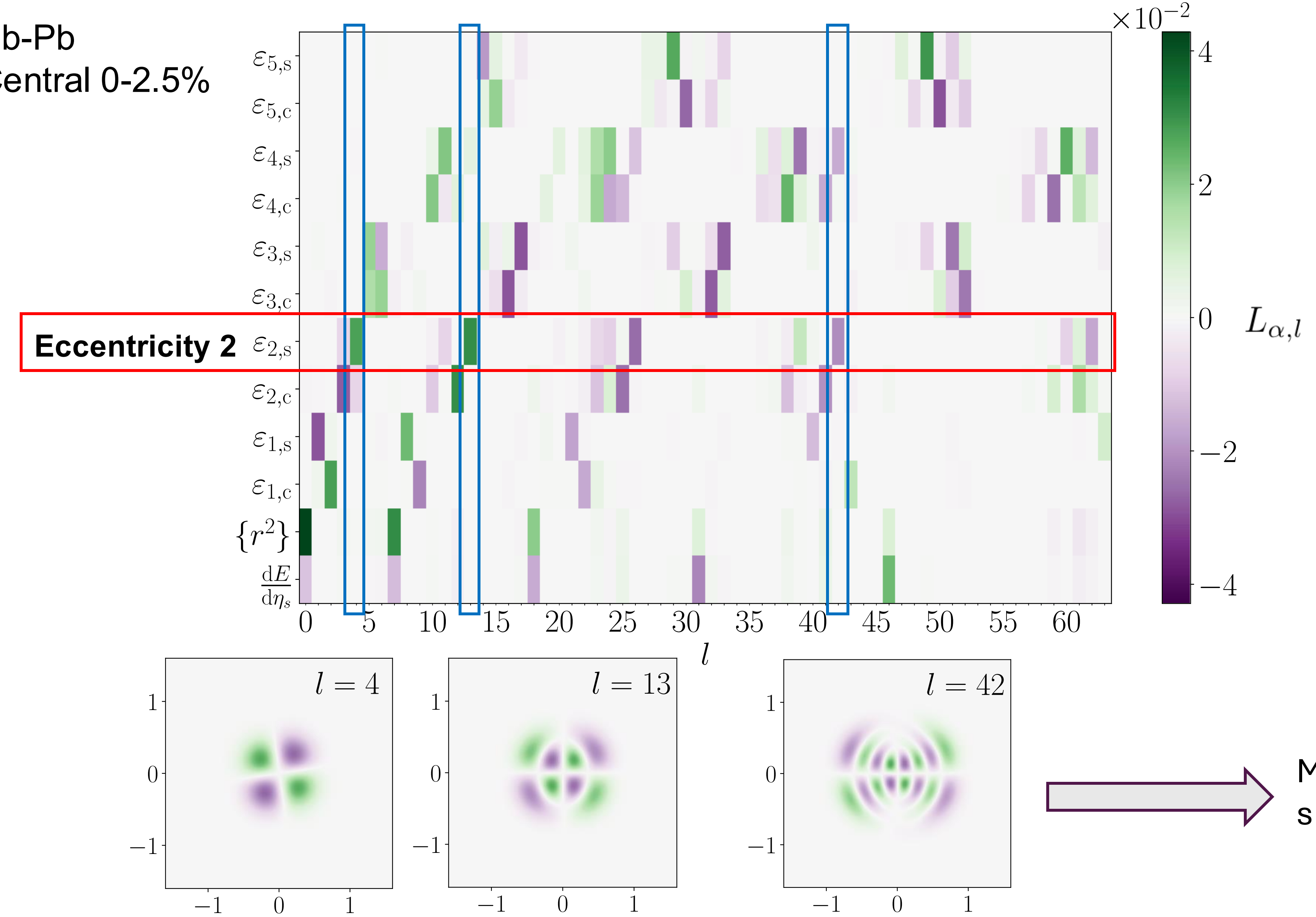




Modes with dipole shape, carry eccentricity 1

Results: Connection between observables and modes

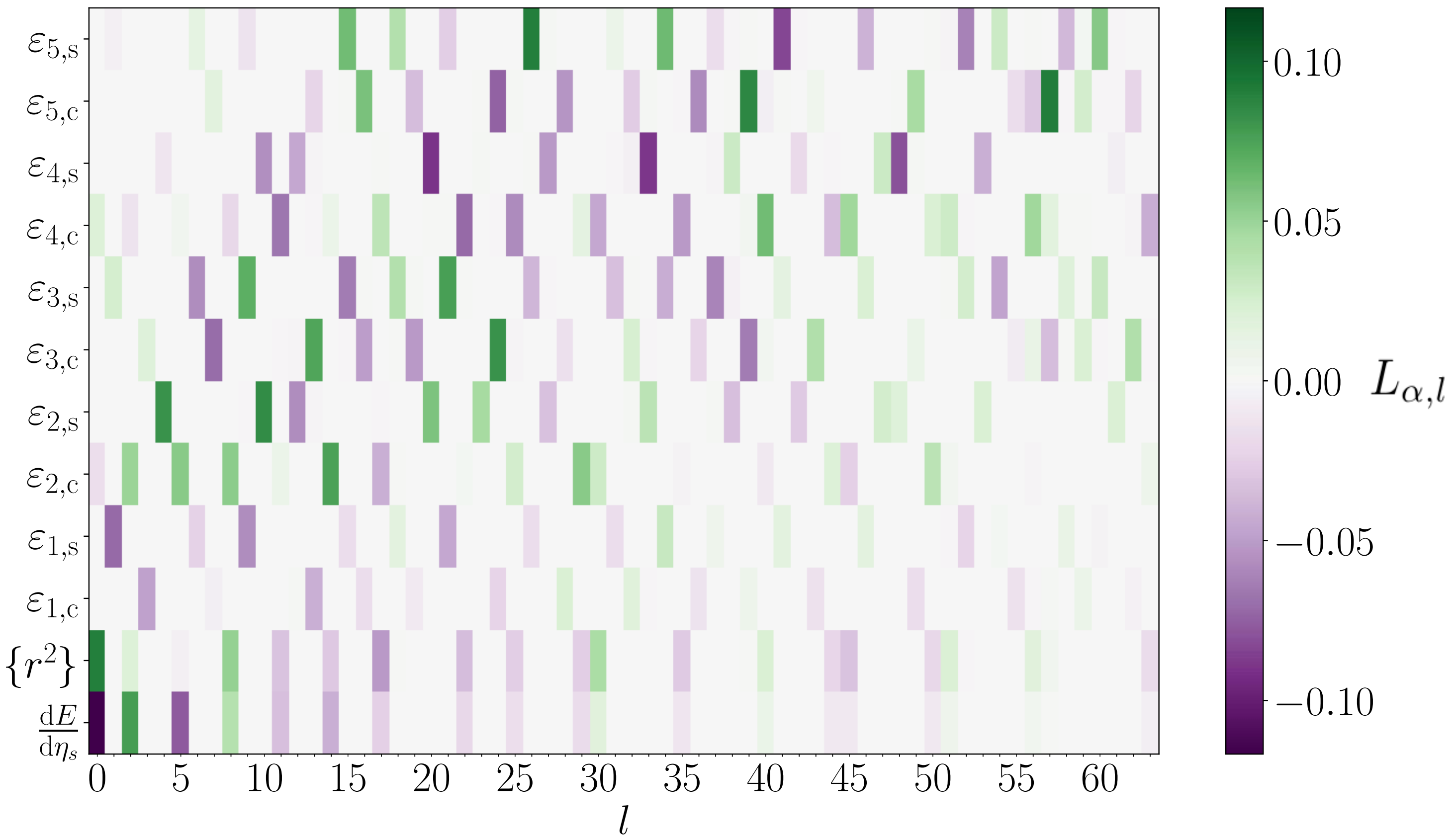
Pb-Pb
Central 0-2.5%



Modes with a quadrupole shape, carry eccentricity 2

Results: Connection between observables and modes

Pb-Pb
Non-central 30-40%



Results: Connection between observables and modes

Pb-Pb
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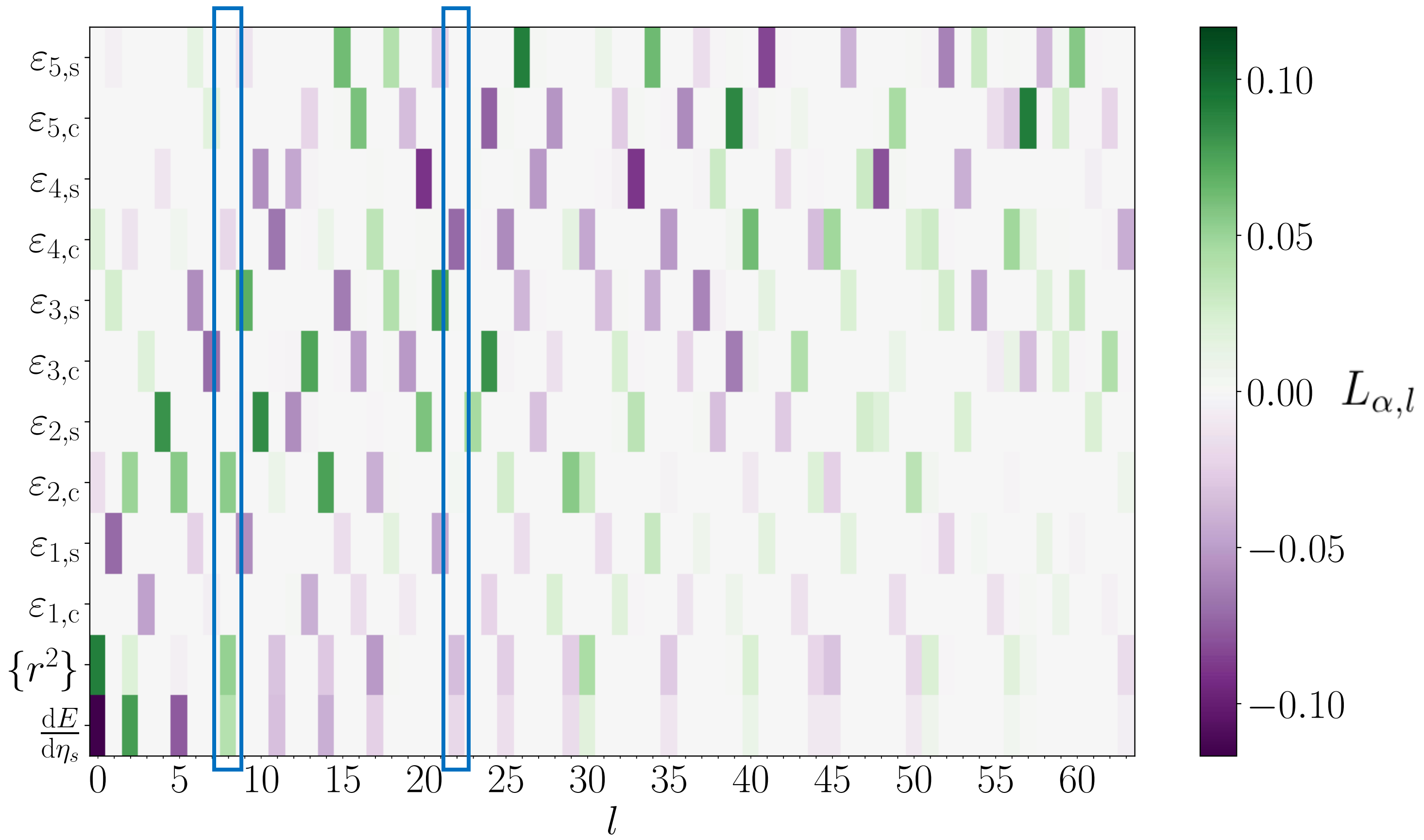
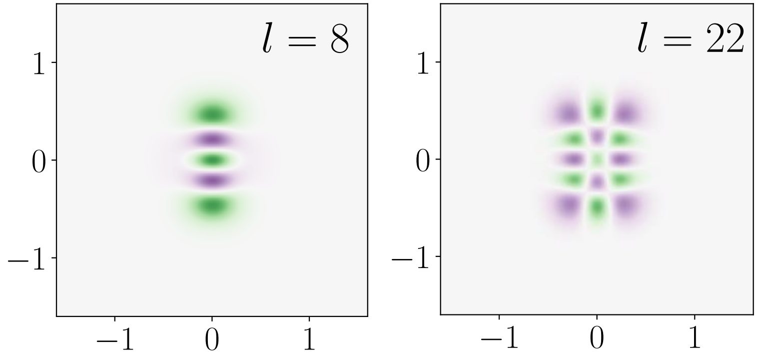
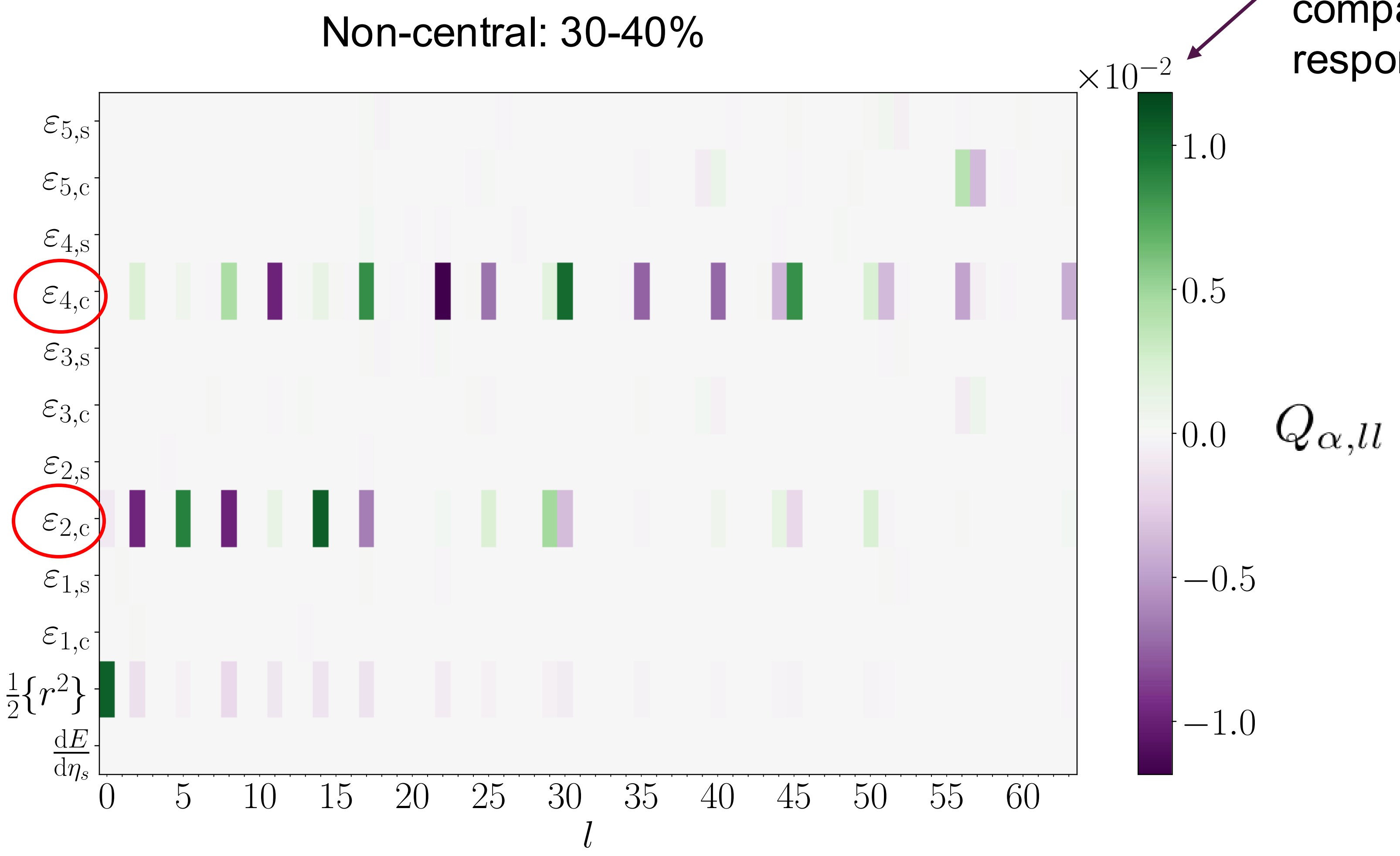




Modes in non-central events are more busy and carry more than one observable

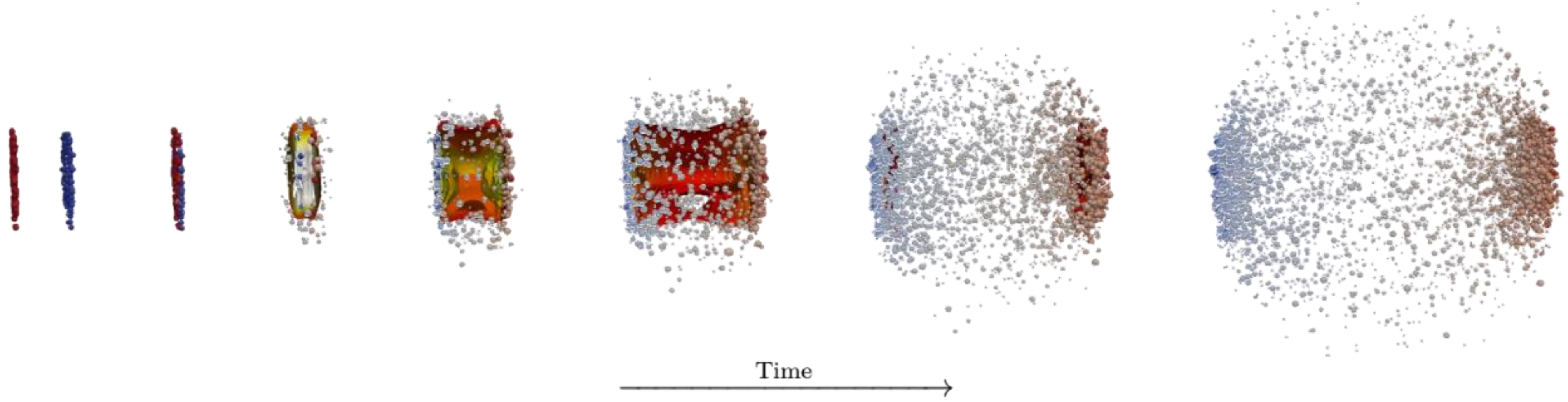
Results: Quadratic response coefficients

Quadratic dependencies in even eccentricities



Smaller contribution compared to the linear response 10^{-1}

Evolution of the modes



Initial state → Pre-equilibrium → Hydrodynamics → Hadronic interactions

(MCGlauber) (KøMPøST) (MUSIC) (SMASH)

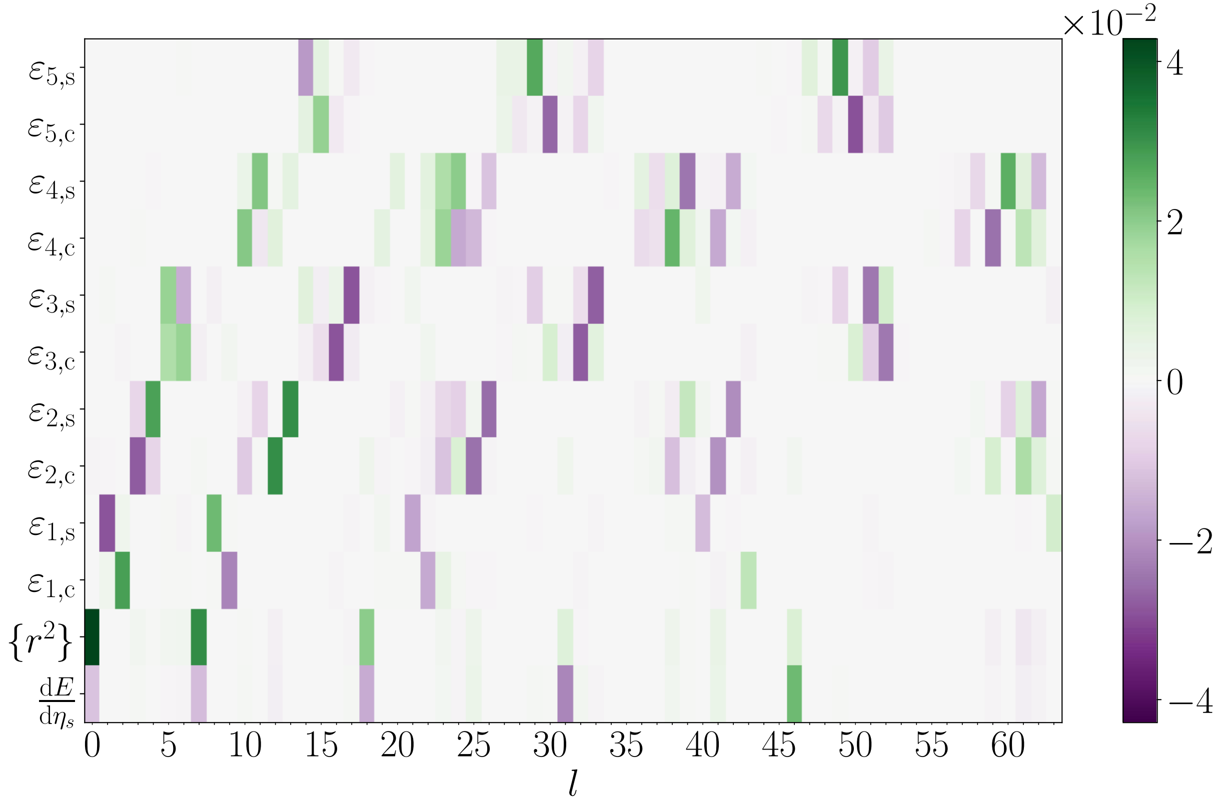
Average State
+
Modes

Evolution

Final observables

Results: Linear response coefficients for modes evolution

Central: 0-2.5%

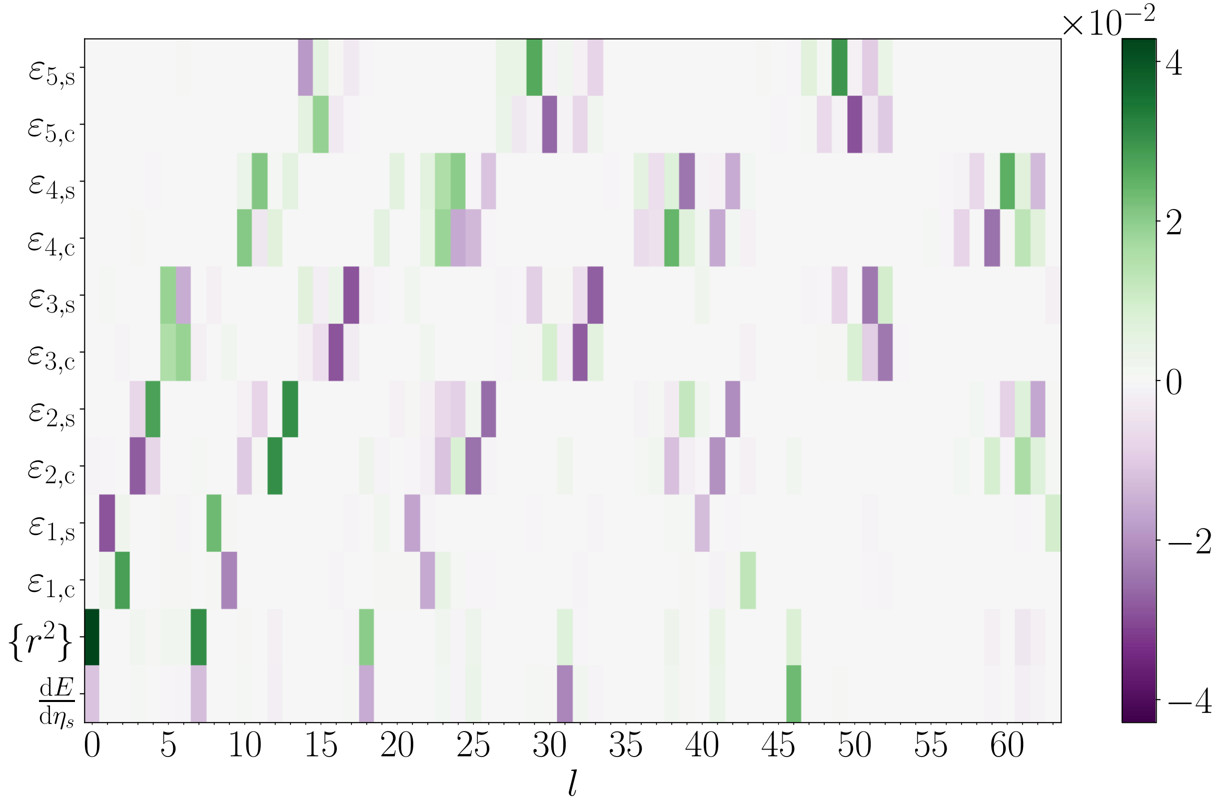


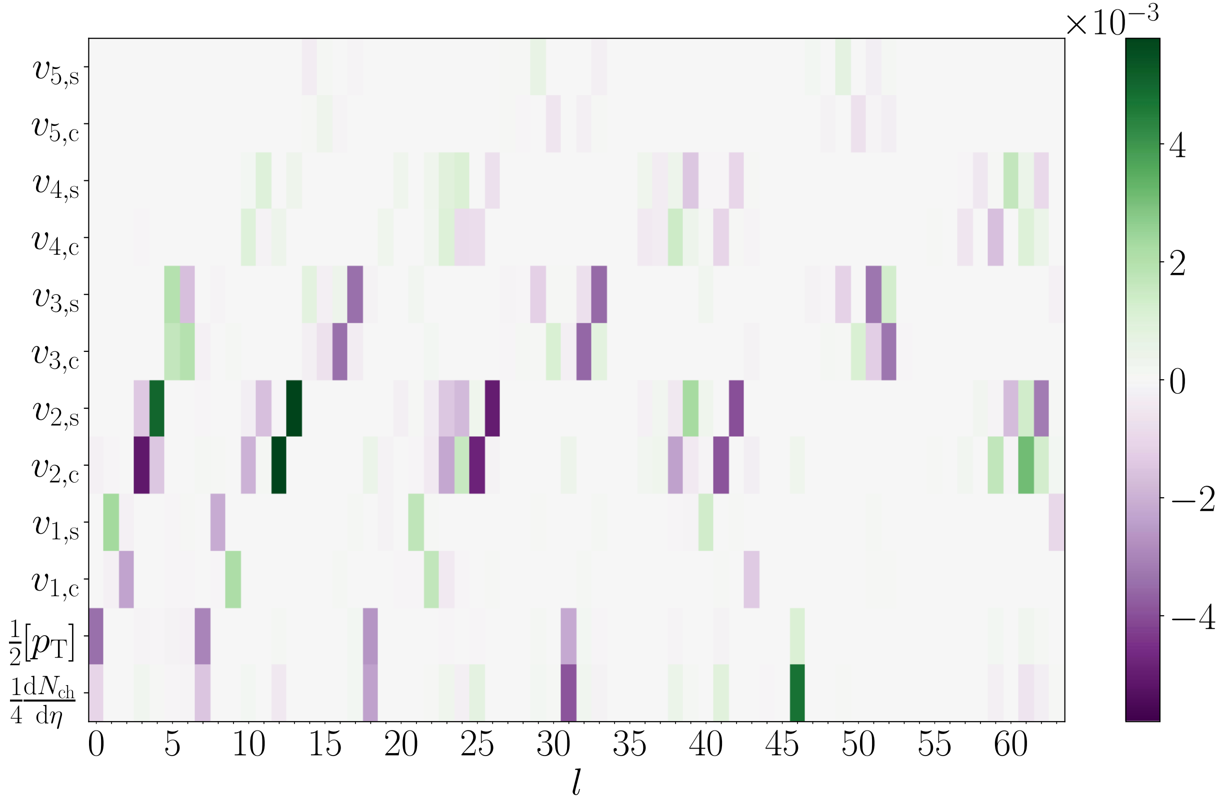
Initial state → KøMPøST + MUSIC

Energy and eccentricities

Results: Linear response coefficients for modes evolution

Central: 0-2.5%





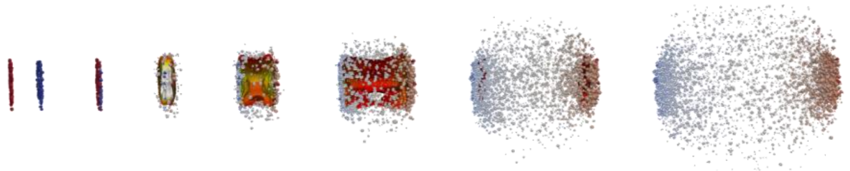
Initial state → Final state

Energy and eccentricities

KøMPøST + MUSIC

Multiplicity, transverse momentum and flow coefficients (anisotropy in momentum space)

Conclusions

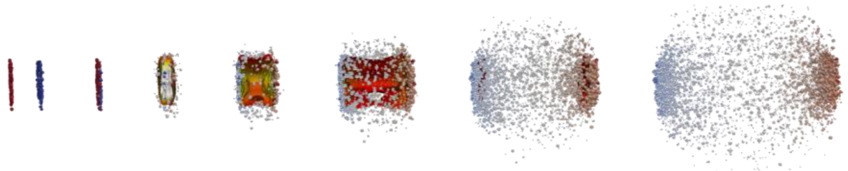


We determine the average state and the uncorrelated modes for 2 centralities in Pb-Pb collisions at 5.02 TeV.

Modes in non-central bins are affected by variations in the impact parameter.

Whole hydrodynamic and hadronic evolution is expensive, but possible.

Conclusions



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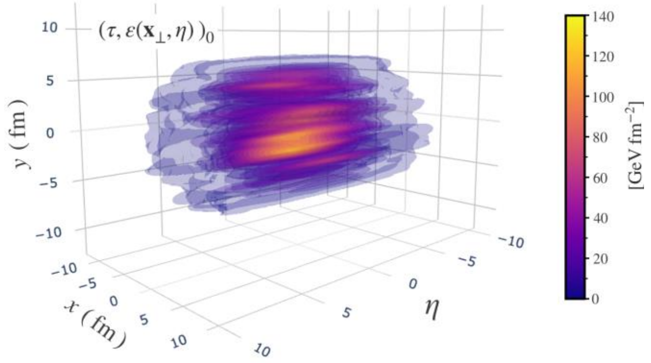
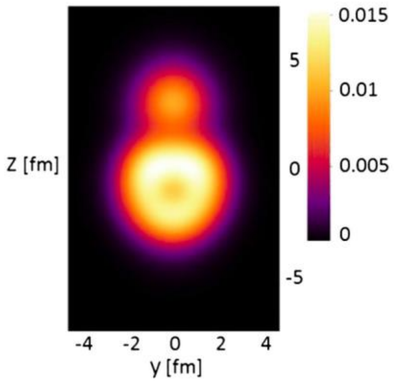
Whole hydrodynamic and hadronic evolution is expensive, but possible.

Next steps in the extension of the model

- Deformed nuclei
- Apply in more realistic initial condition codes with hotspot and using first principles nuclear structure configurations
- 3D production and evolution
- ...





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Thank you!

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September, 2025.

Theory: Mode-by-mode decomposition

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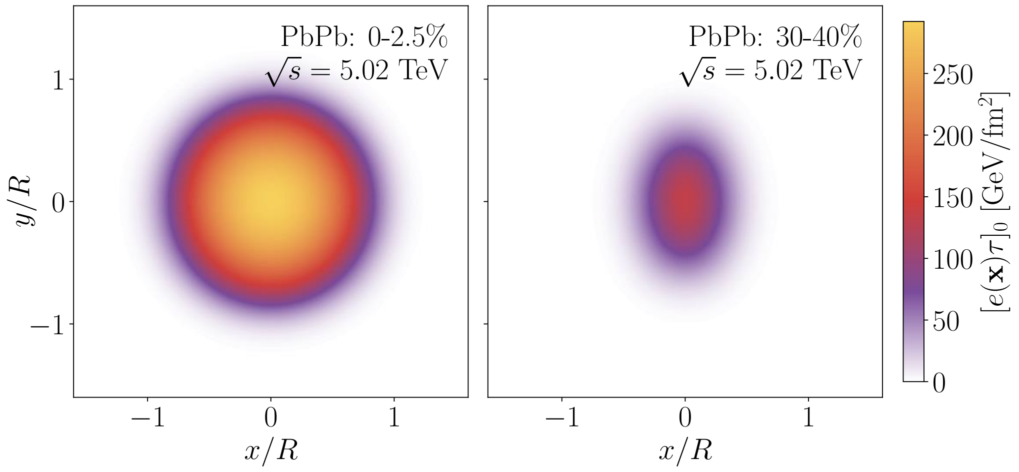
Average state

Fluctuation modes

Each initial profile is decomposed into an average state plus a linear combination of fluctuation modes.



Numerical details to apply the theory:

- Millions of events (2^{21} events)
- Each profile is a vector of size N_{pts} (192^2)
- The density matrix is a matrix of N_{pts} X N_{pts} (192^2 X 192^2)
- The procedure yields N_{pts} modes, but the first few ones already yield a good description of bulk observables

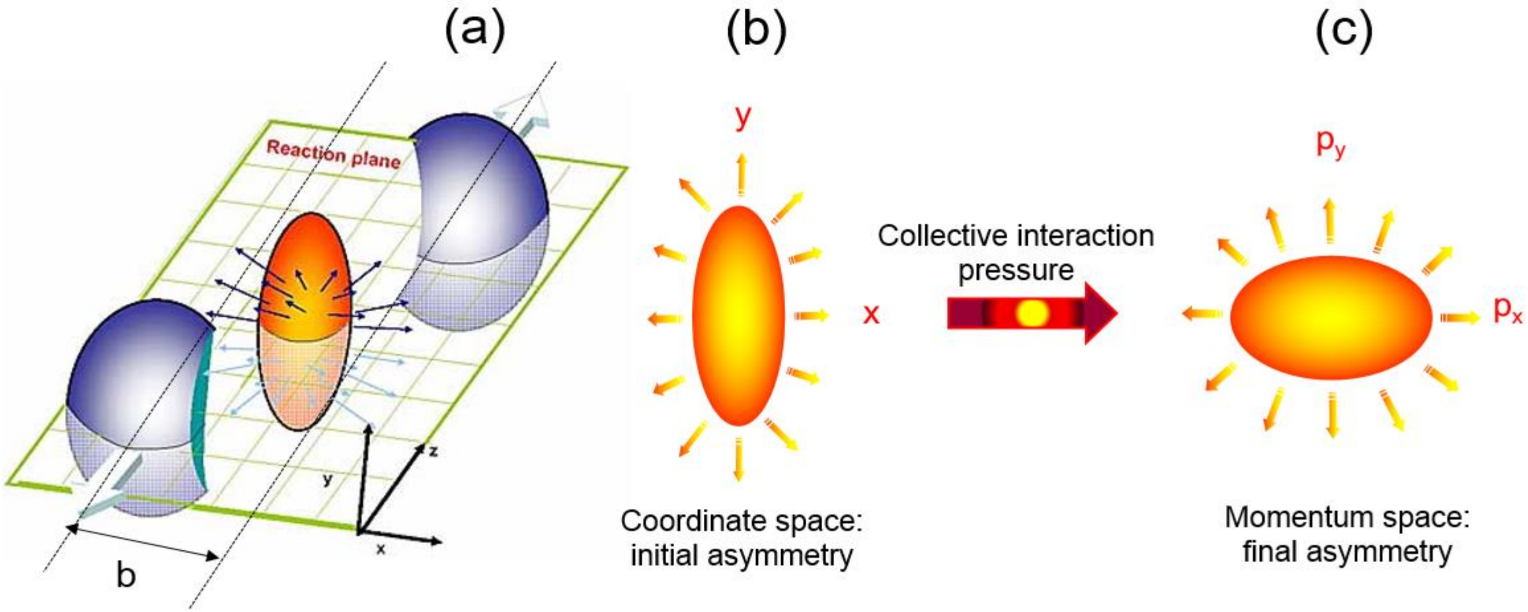
N_{pts} is the number of points in the grid. The box size for Pb-Pb simulations is 21.184 fm and the spacing size used is 0.11 fm, which gives a split of 192 X 192.

Introduction: Eccentricities and flow coefficients

$$\varepsilon_1 e^{i\Phi_1} \equiv -\frac{\int r^3 e^{i\theta} e(r,\theta) r dr d\theta}{\int r^3 e(r,\theta) r dr d\theta}$$

$$\varepsilon_n e^{i\Phi_n} \equiv -\frac{\int r^n e^{in\theta} e(r,\theta) r dr d\theta}{\int r^n e(r,\theta) r dr d\theta}$$

$$v_n e^{in\Psi_n} \equiv \frac{\int e^{in\phi_p} \frac{dN_{ch}}{p_T dp_T d\phi_p d\eta} p_T dp_T d\phi_p}{\int \frac{dN_{ch}}{p_T dp_T d\phi_p d\eta} p_T dp_T d\phi_p}$$



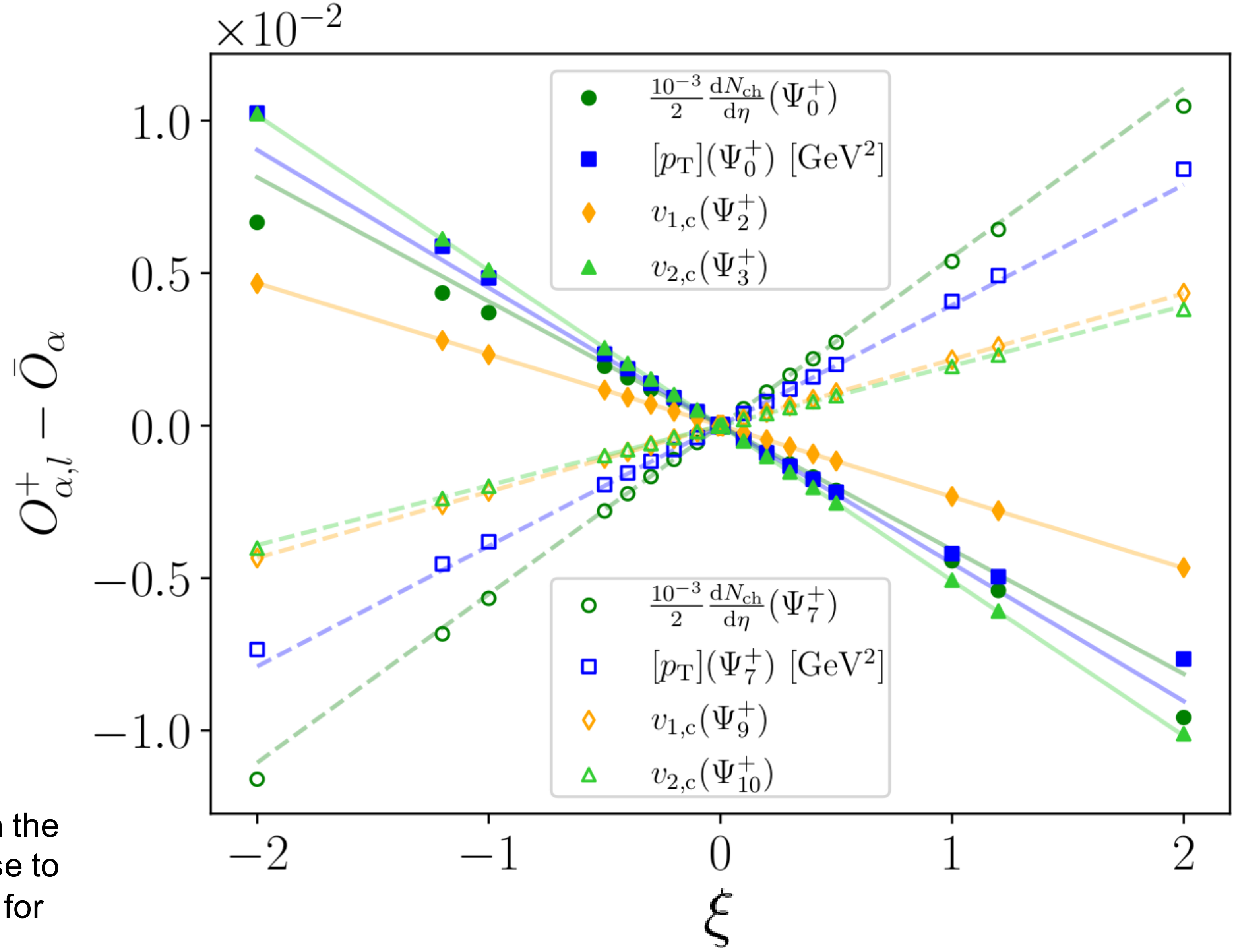
Theory: Linearity

$$\Psi_l^{\pm} = \bar{\Psi} \pm \xi \Psi_l$$

Here: 0.5

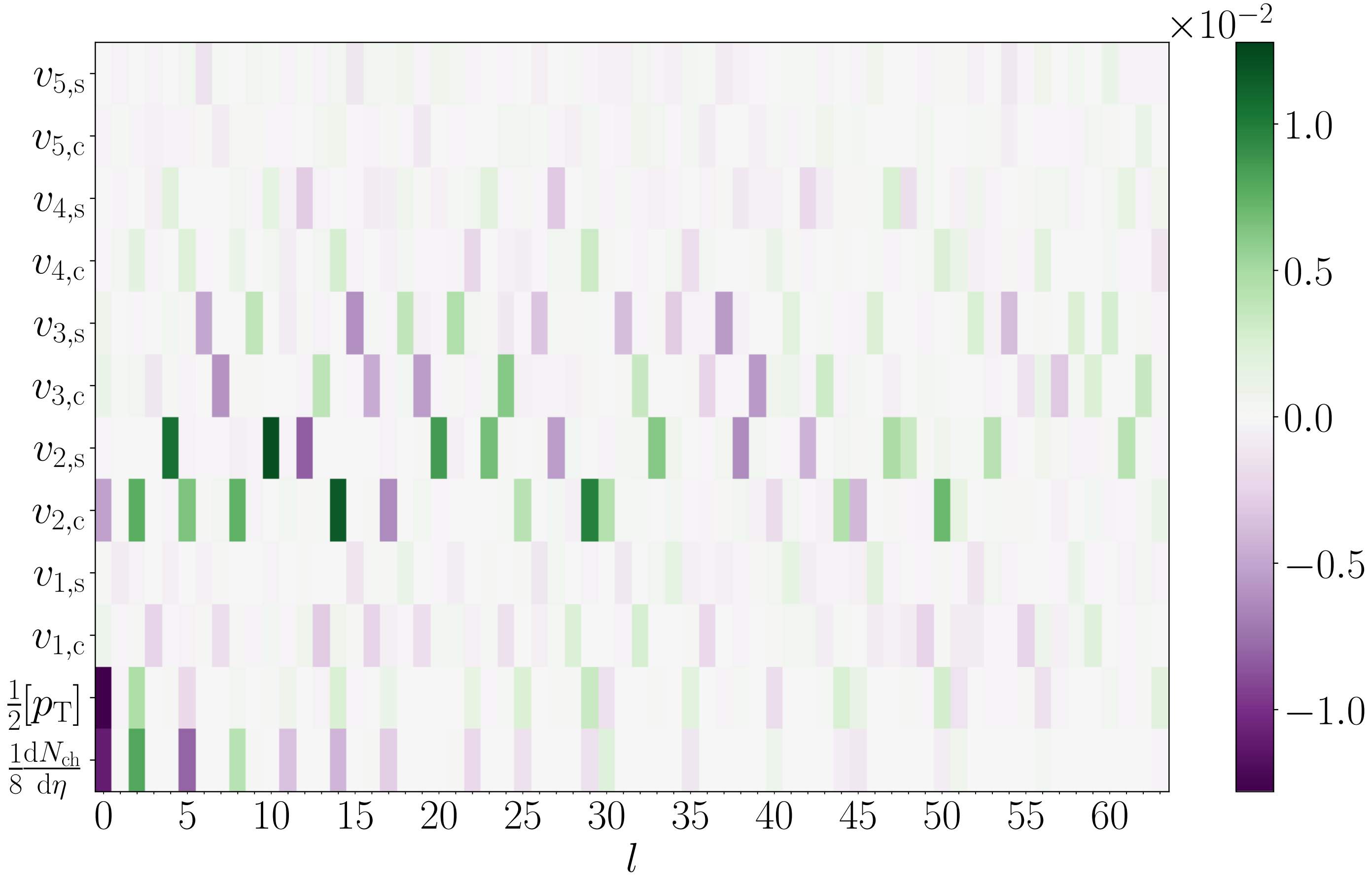
$$L_{\alpha,l} = \frac{O_\alpha(\Psi_l^+) - O_\alpha(\Psi_l^-)}{2\xi}$$

Higher values increase the signal in the observables, but also allow the noise to grow — an important consideration for hadronic interactions.



Results: Final linear response coefficients with hadronic interactions

Non-central: 30-40%



Similar to the results from only decays in the spectra, but it is more expensive to run to get a noise smaller than the signal.

For the signal to be visible in Pb-Pb in 30-40% at 5.02 TeV, with multiplicity around 500 particles, it is necessary to run 20000 oversamplings in the Cooper-Frye step.

The statistical noise is proportional to the number of particles produced $1/\sqrt{M}$