Cold and dense QCD matter and neutron stars:

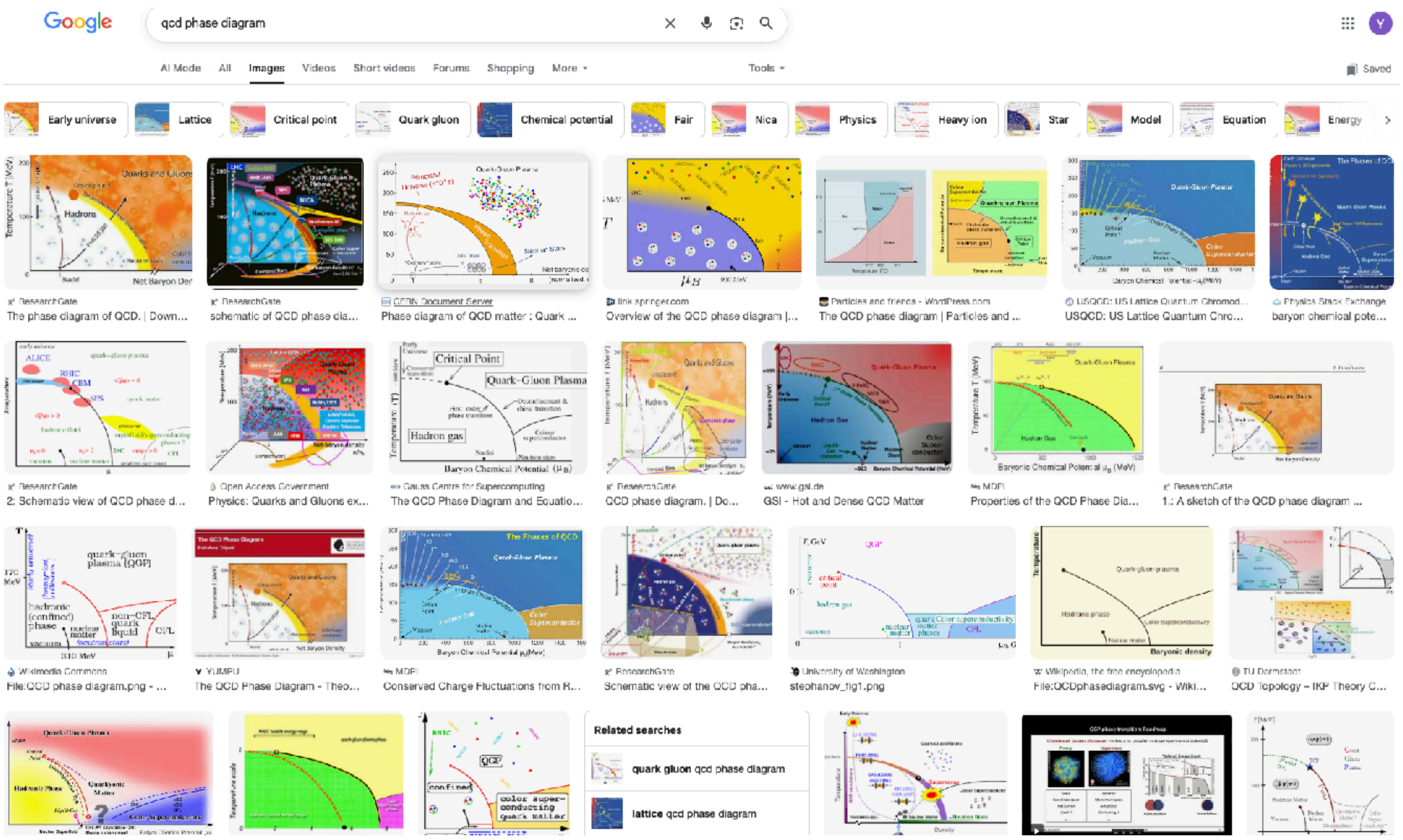
quark deconfinement at high T and μ_B

Yuki Fujimoto

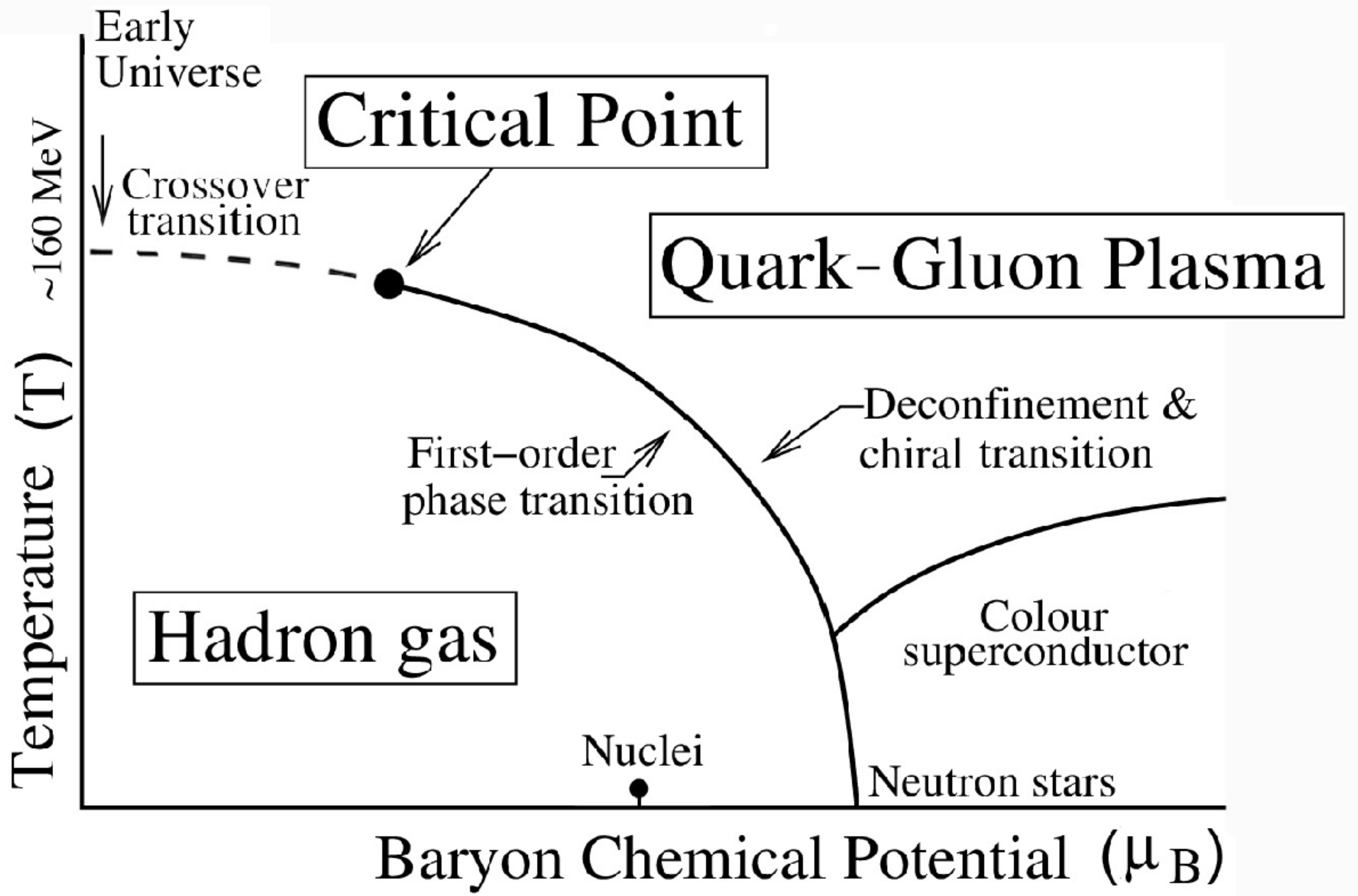
(RIKEN-Berkeley Center, UC Berkeley → Niigata U)

September 17, 2025 - Erice: International School of Nuclear Physics, 46th Course

Many versions of QCD phase diagrams...



Conventional QCD phase diagram

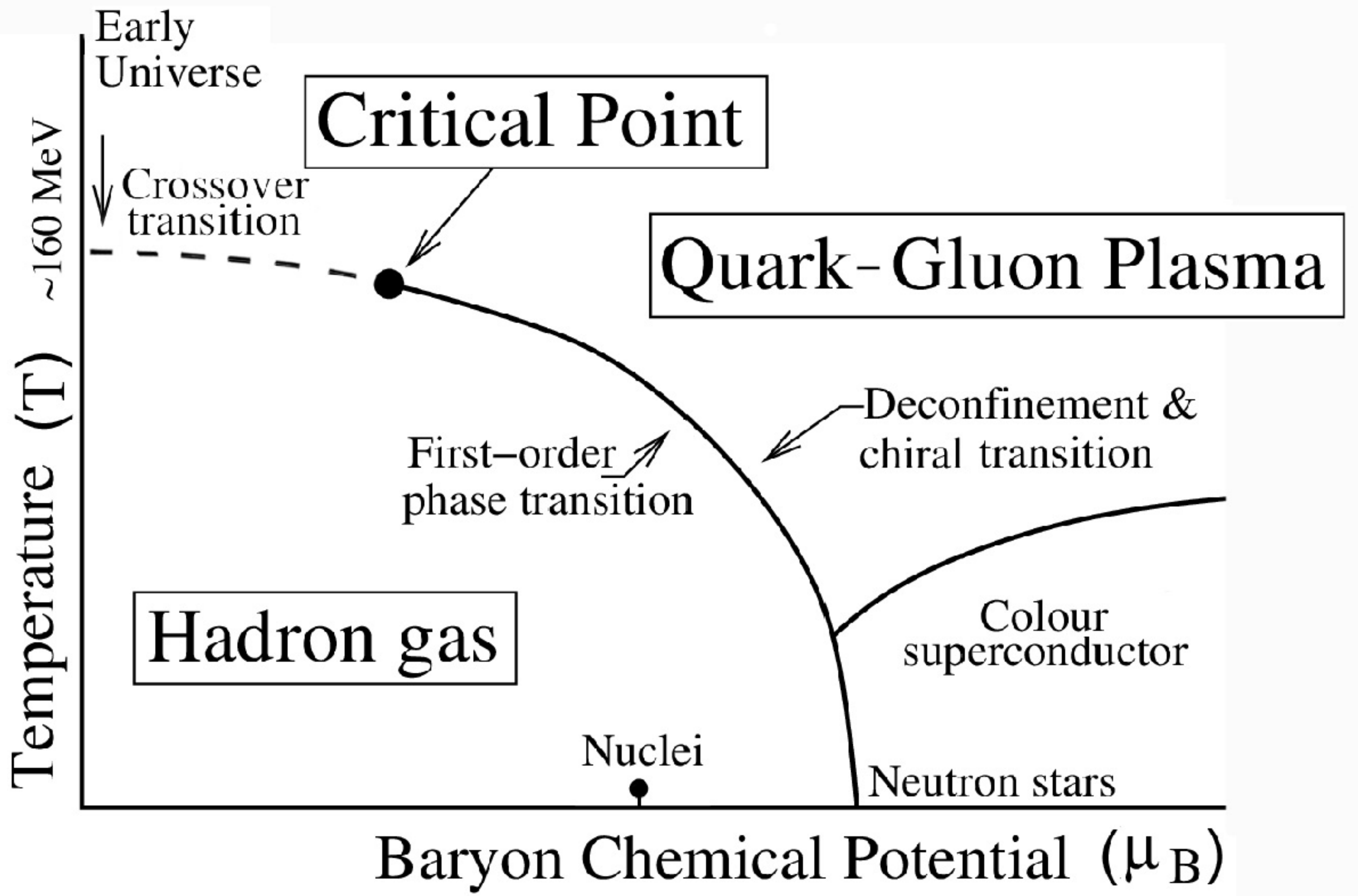


taken from: https://www.gauss-centre.eu/results/elementaryparticlephysics/the-qcd-phase-diagram-and-equation-of-state

Conventional QCD phase diagram

Two main transitions:

1. Confinement → Deconfinement … **No order parameters**
2. Chiral phase transition



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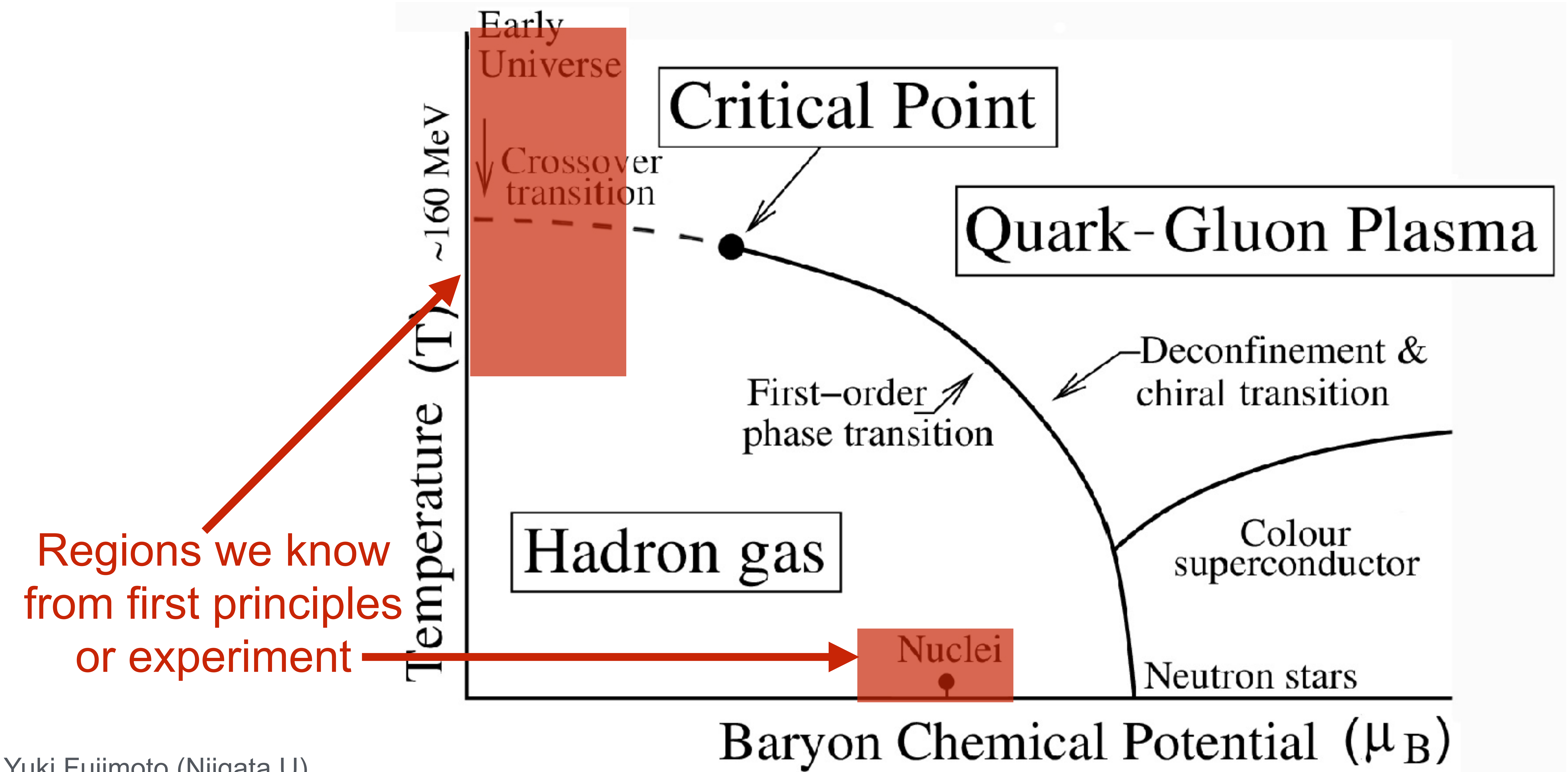

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In this talk

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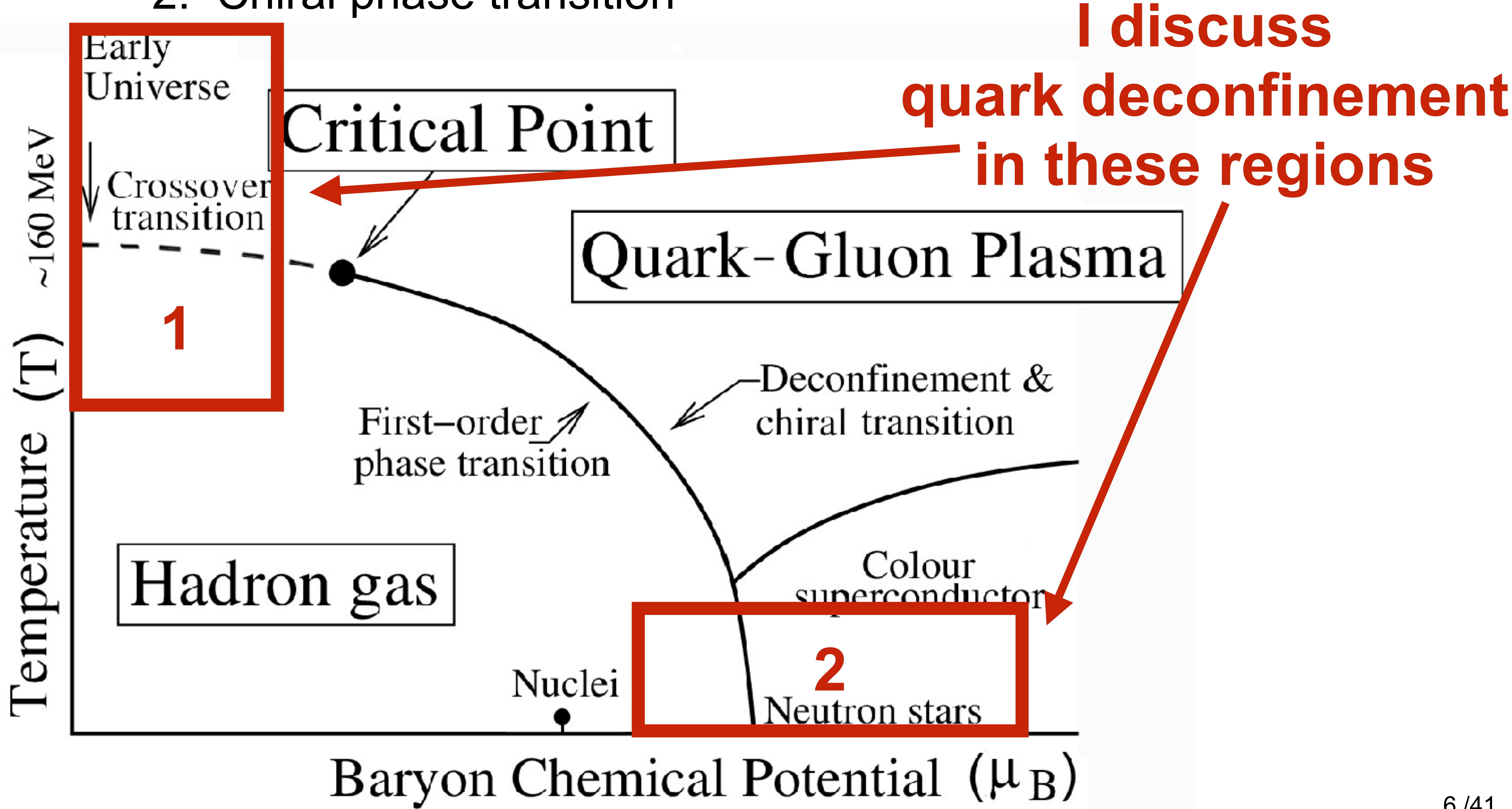

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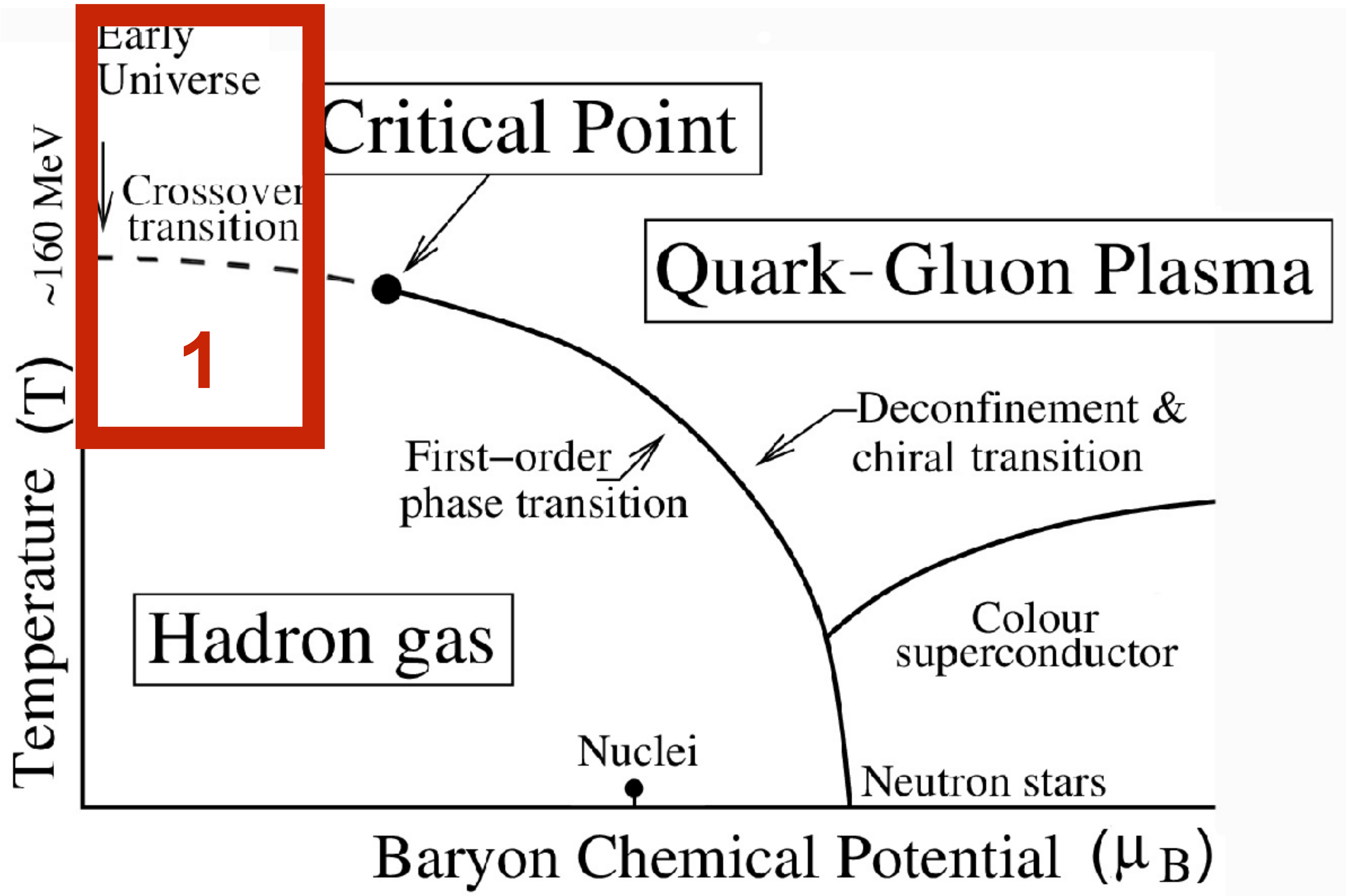
1. Interpretation of quark deconfinement in crossover at high *T*



Quark deconfinement is hard to capture

- Order parameter for deconfinement — **Polyakov loop**:

$$L(\boldsymbol{x}) = \mathscr{P} \exp \left[-ig \int_0^{1/T} dx_4 \, A_4(\boldsymbol{x}, x_4) \right]$$

and its thermal expectation value: $\Phi = \langle \ell(\boldsymbol{x}) \rangle = e^{-f_q/T}$ $\quad \left(\ell = \frac{1}{N_c} \mathrm{tr}\, L \right)$

- In the **confined (disordered) phase:** $\Phi = 0$

Deconfined (ordered) phase: $\Phi \neq 0$ $\quad \to T_d \simeq 285\,\mathrm{MeV}$ for pure SU(3)

Borsanyi et al., PRD 105 (2022)

… related to breaking of the center symmetry Z_3 of $\mathrm{SU}(3)$

- With dynamical quarks, the center symmetry is explicitly broken

→ No well-defined order parameter Φ when there are dynamical quarks

Deconfinement in EoS of hadrons & resonances

Schematic behavior of high-temperature equation of state:

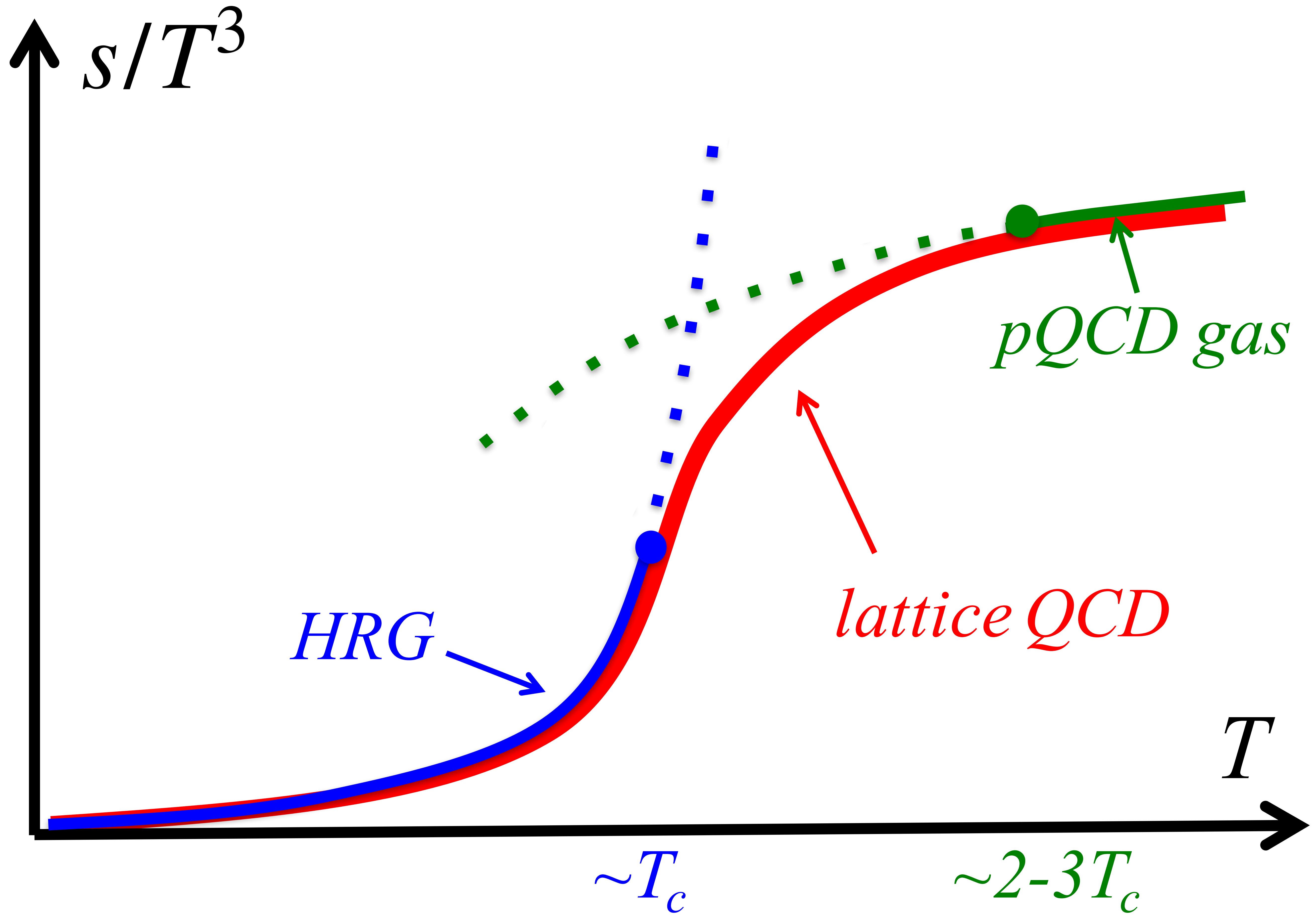


Figure adapted from: Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

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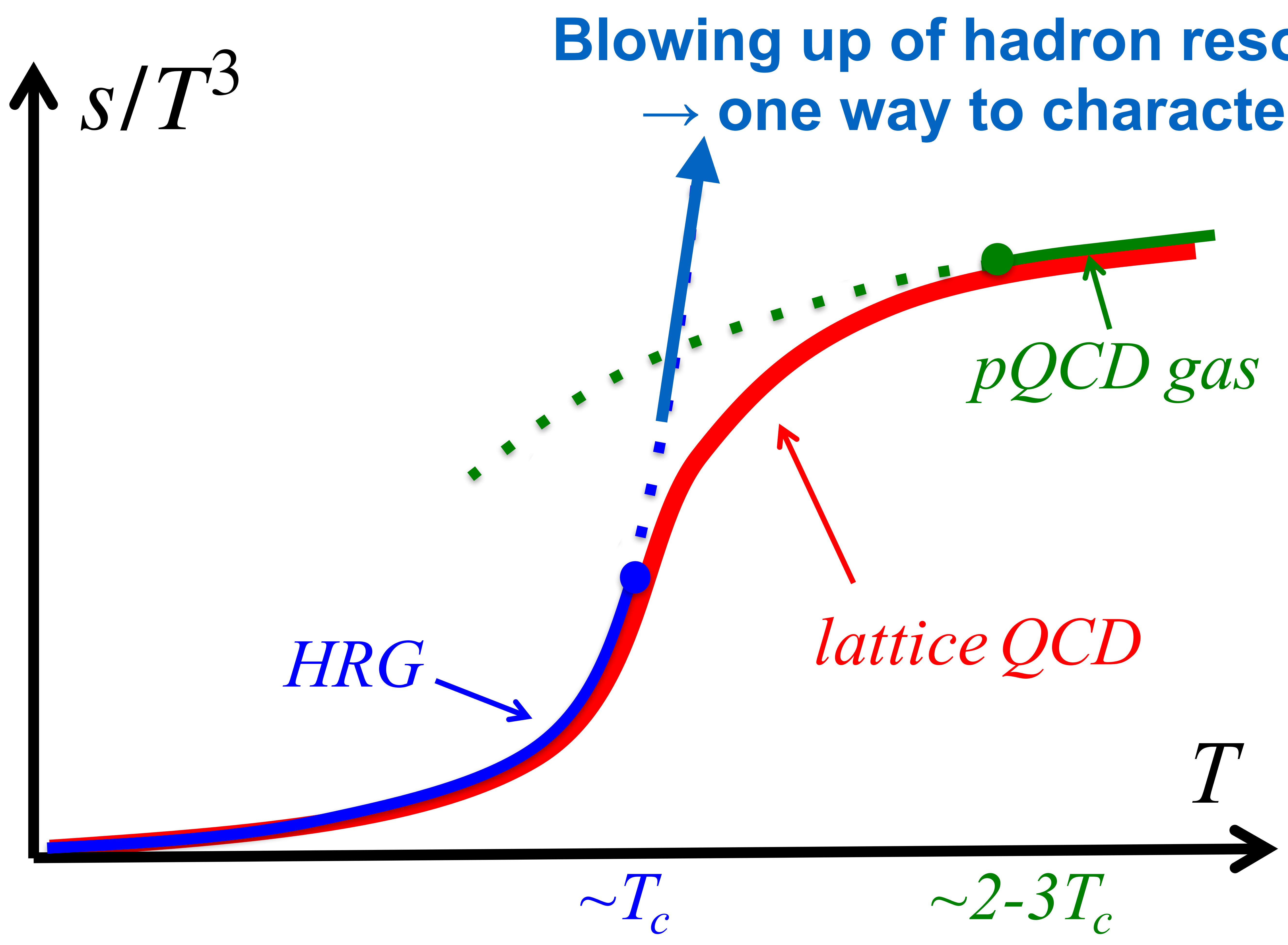

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Hagedorn temperature

Partition function: $Z \propto \int_{m_0}^{\infty} dm\, \rho(m) e^{-m/T}$,

$$\rho(m) \propto m^a \exp(m/T_{\mathrm{H}})$$

T_H: Hagedorn's limiting temperature

Hagedorn (1965)

T_H … Later reinterpreted as temperature of quark-gluon liberation

Cabibbo & Parisi (1975)

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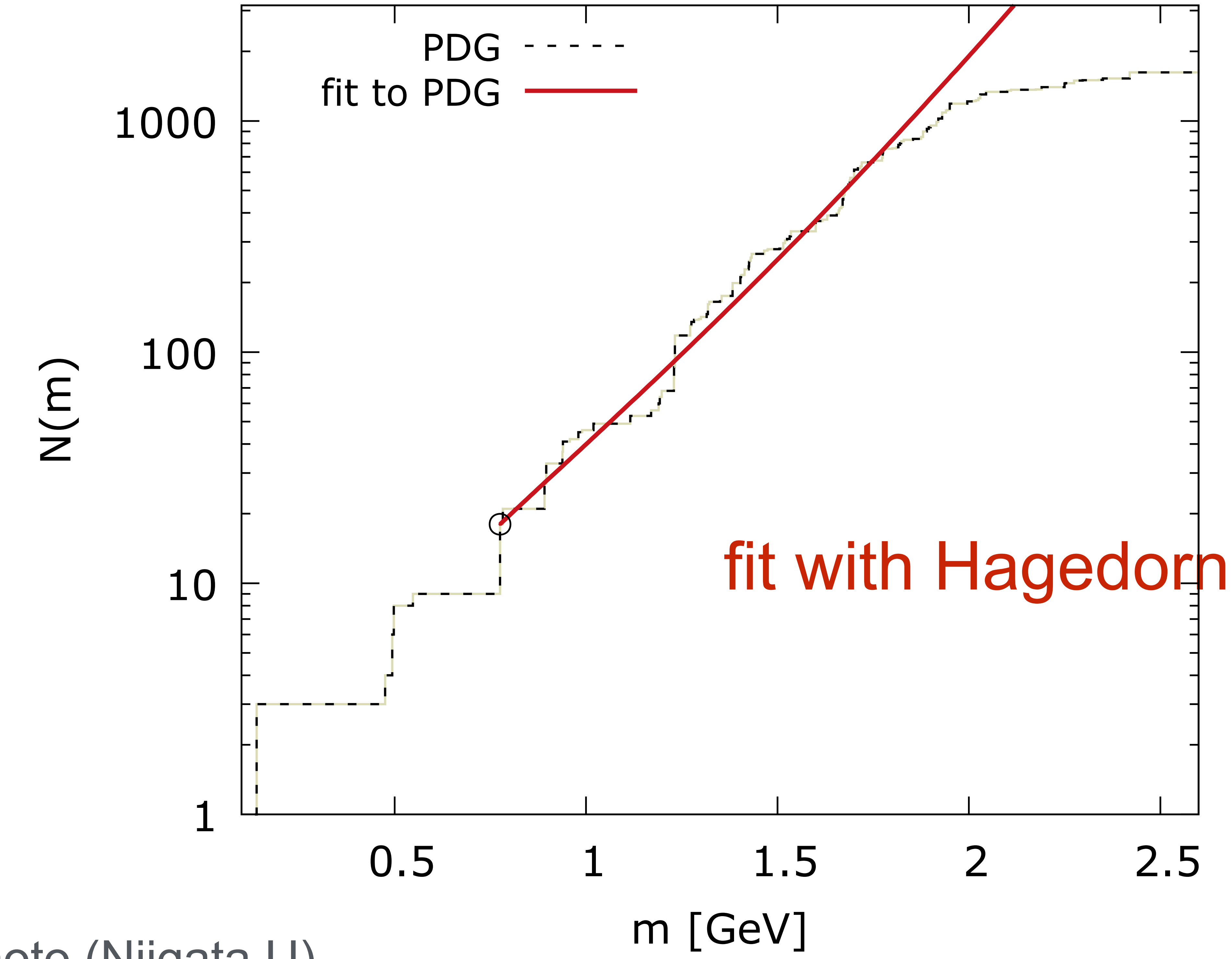
T_H: Hagedorn's limiting temperature

Hagedorn (1965)

Integrated mass spectrum
of hadrons & resonances in PDG:



fit with Hagedorn spectrum

T_H … Later reinterpreted as
temperature of quark-gluon liberation

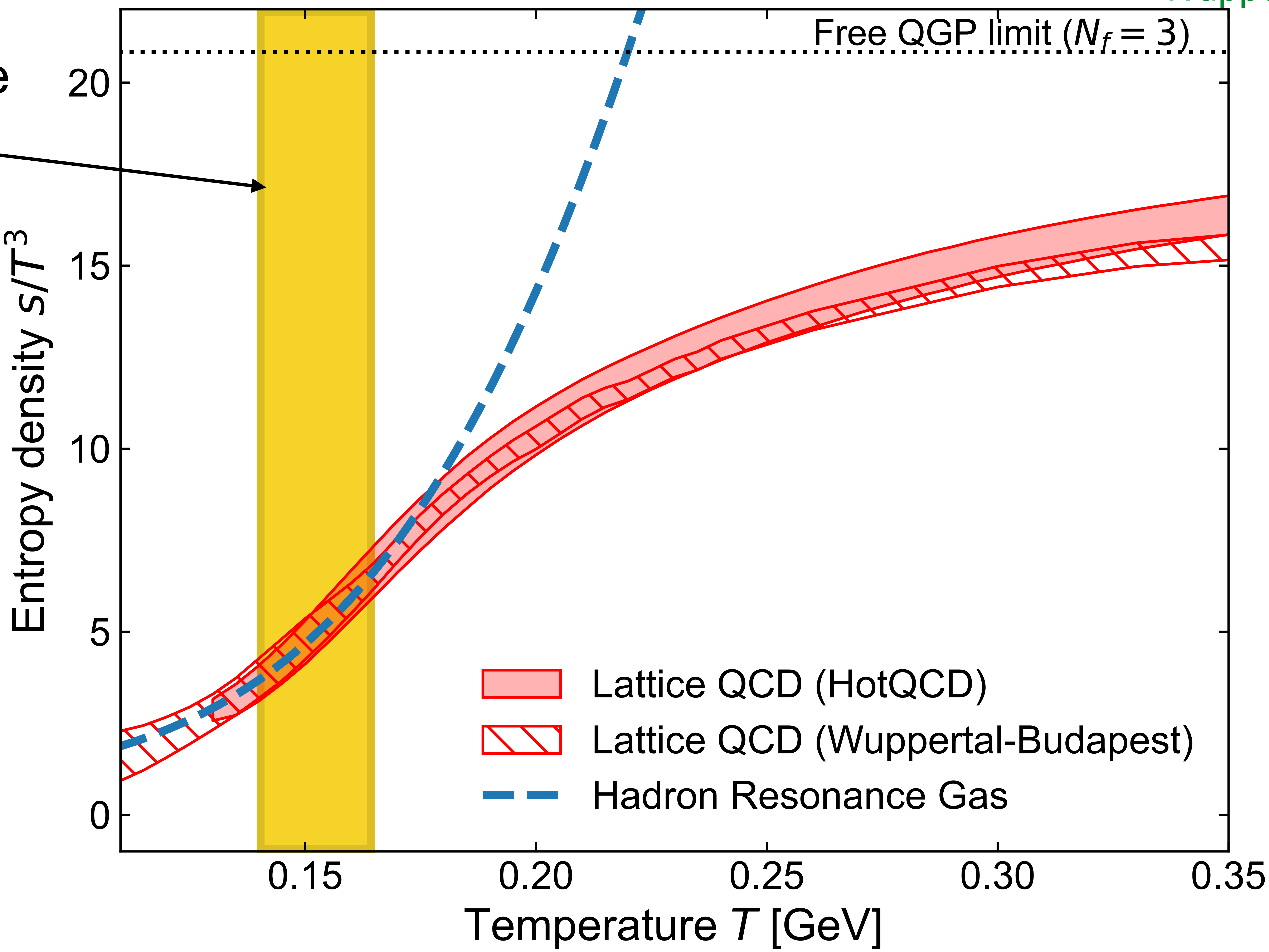
Cabibbo & Parisi (1975)

Figure adapted from:
Lo,Marczenko,Redlich,Sasaki (2015)

Lattice QCD equation of state

Critical temperature
for chiral transition
& deconfinement

Wuppertal-Budapest (2014);
HotQCD (2014)



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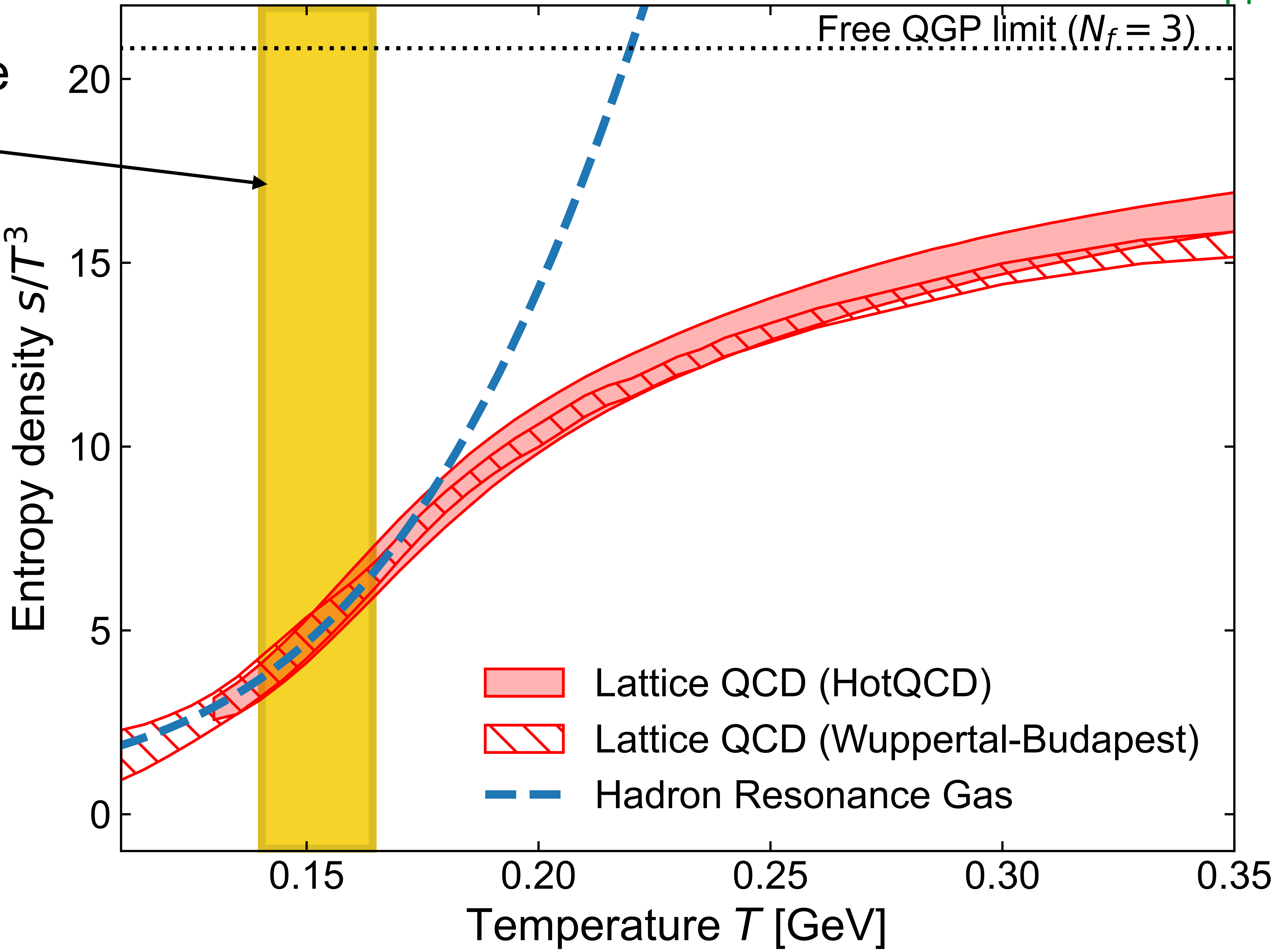


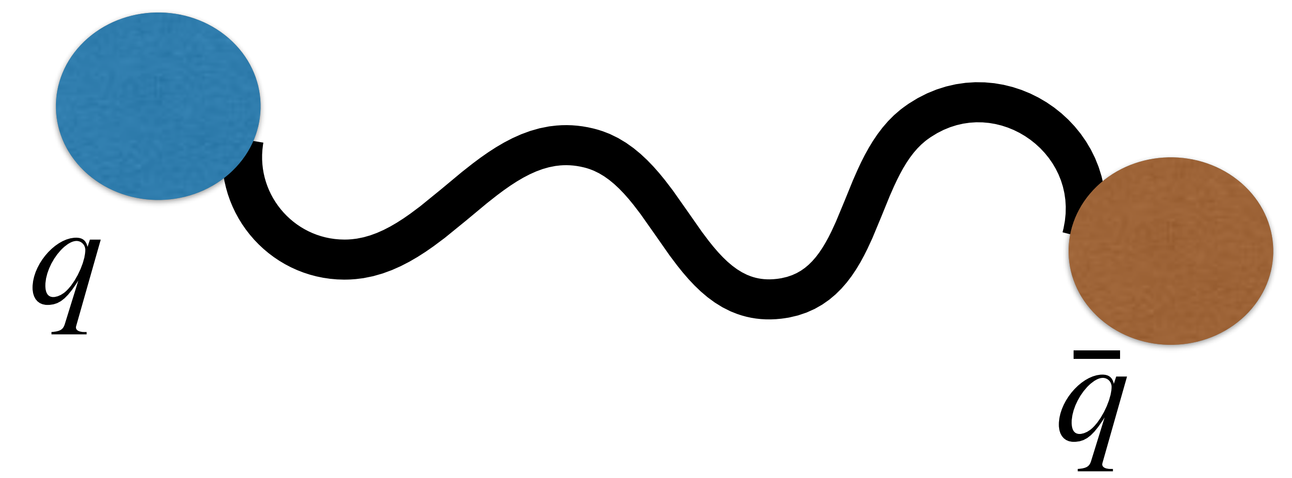
Hagedorn spectrum from string theory

Hagedorn spectrum of open strings:

$$\rho(m) = \frac{\sqrt{2\pi}}{6T_H}\left(\frac{T_H}{m}\right)^{3/2} e^{m/T_H}$$

See, e.g., Green,Schwarz,Witten

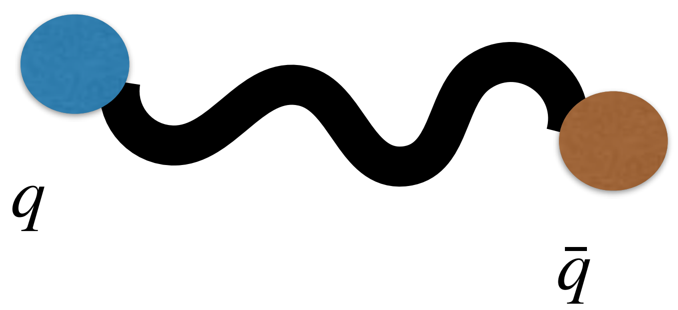
… Mesons can be approximately regarded as an **open string** (w/ quarks attached at the end)



This stringy picture is implied from the Regge trajectory

Fitting EoS with open string formula

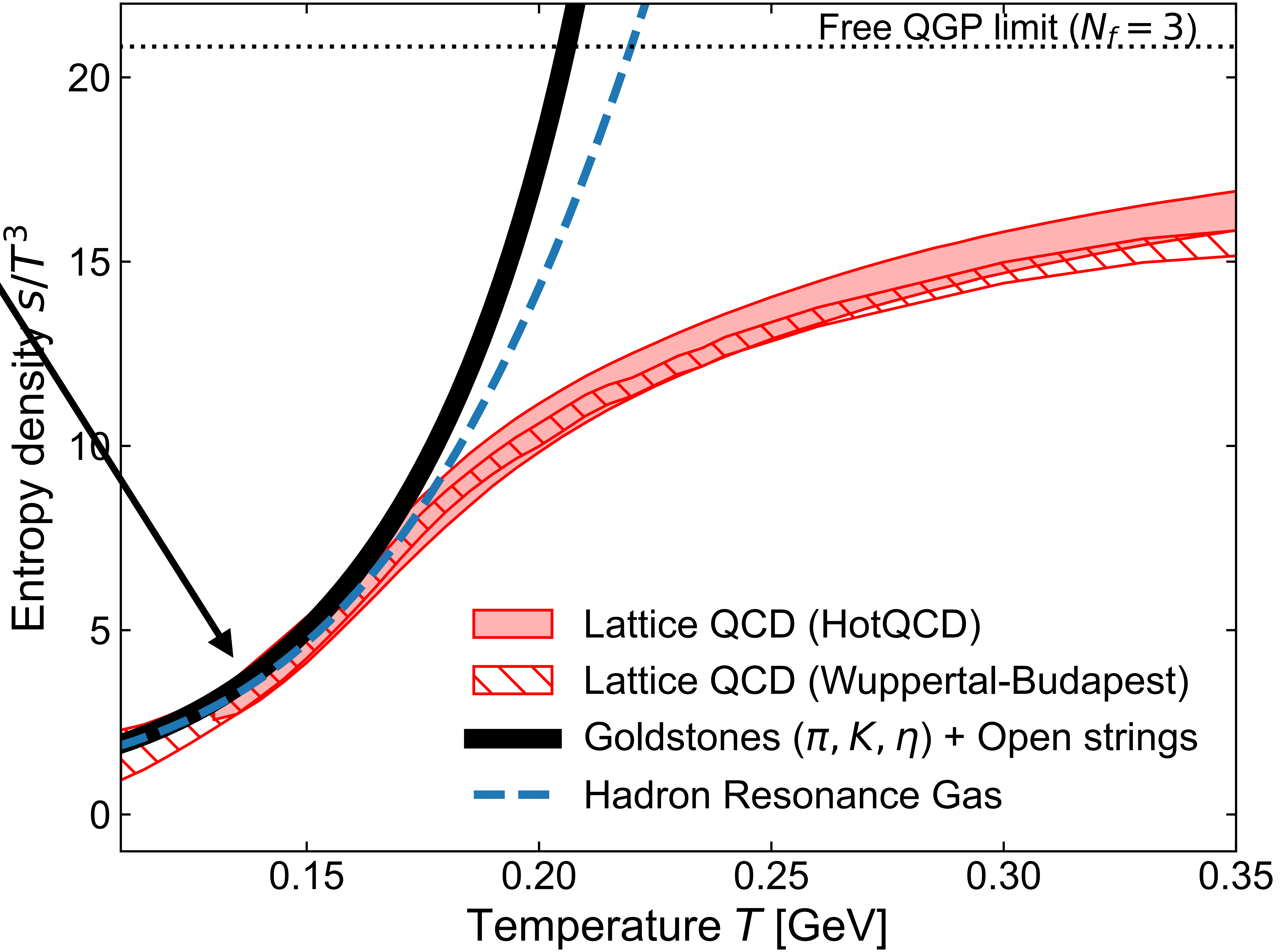
Fujimoto, Fukushima, Hidaka, McLerran (2025)



Fit with open string

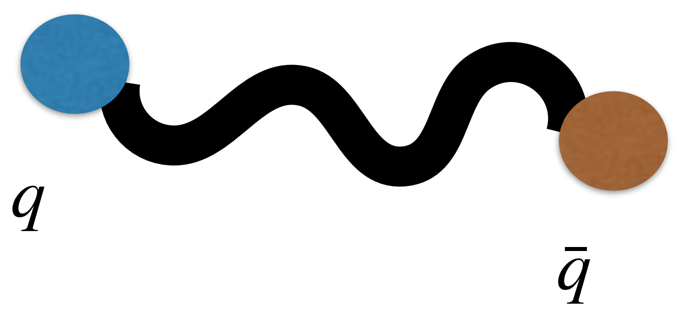
w/ $T_H \simeq 300$ MeV

… the value is common for quarkless theory (pure glue theory)



Deconfinement may take place at higher T

Fujimoto,Fukushima,Hidaka,McLerran (2025)



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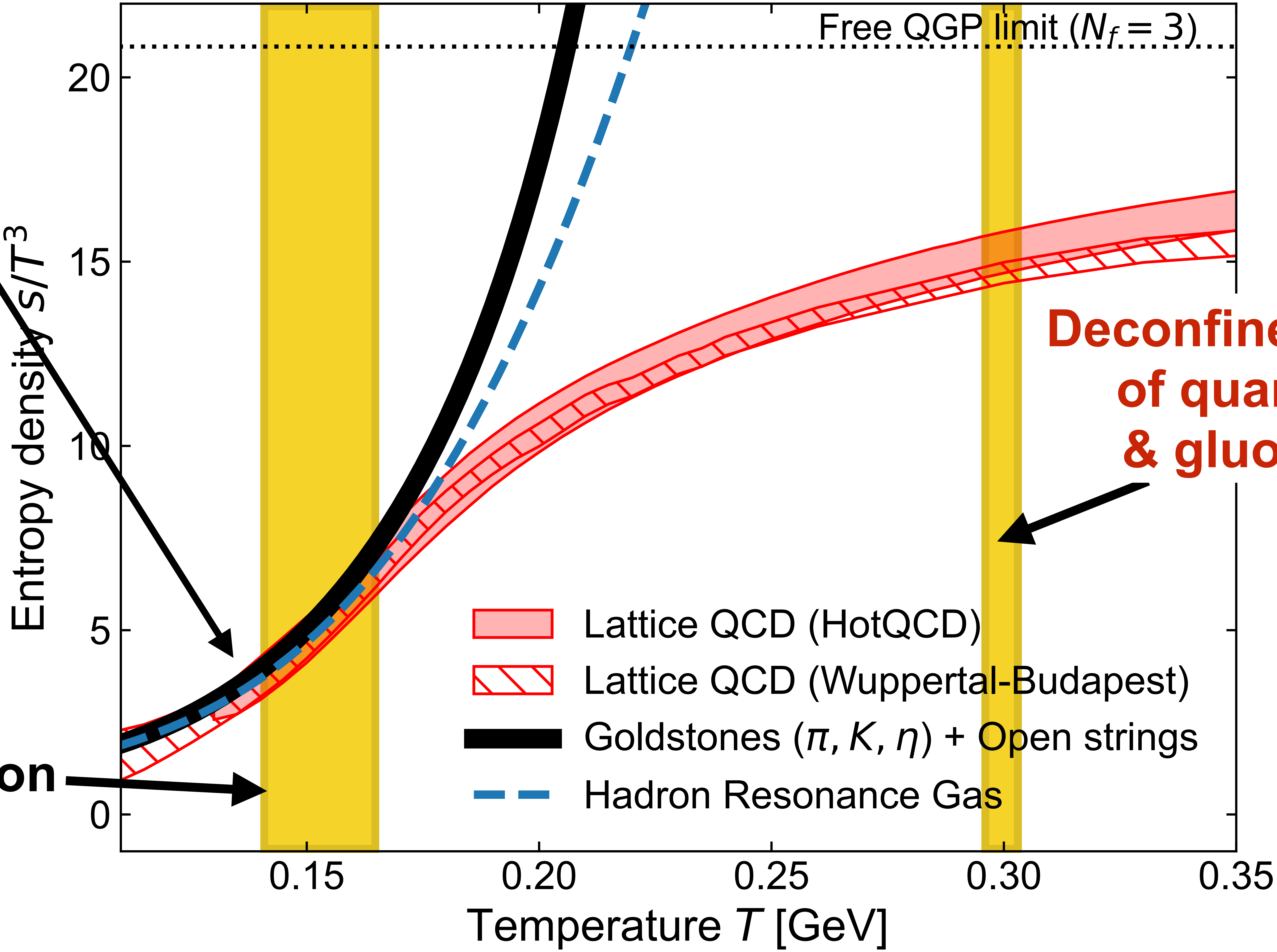

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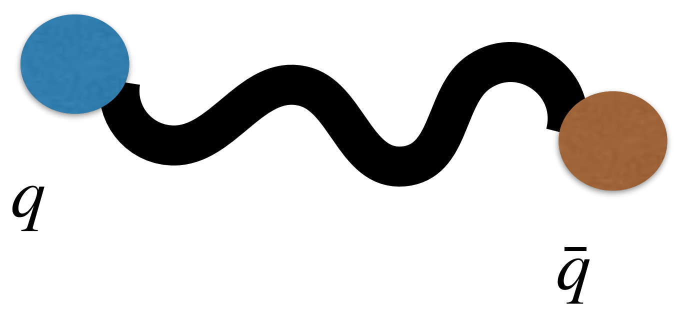

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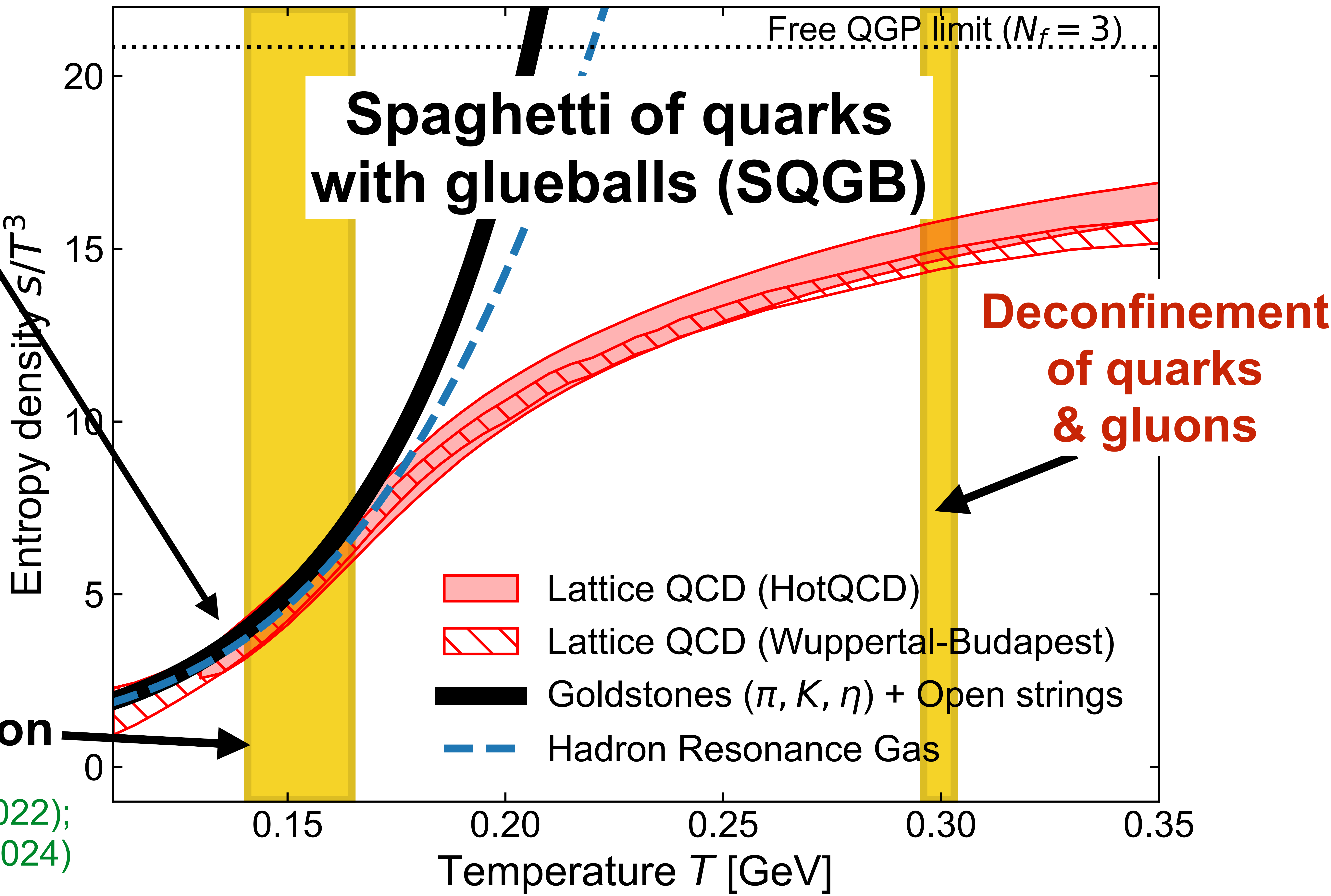
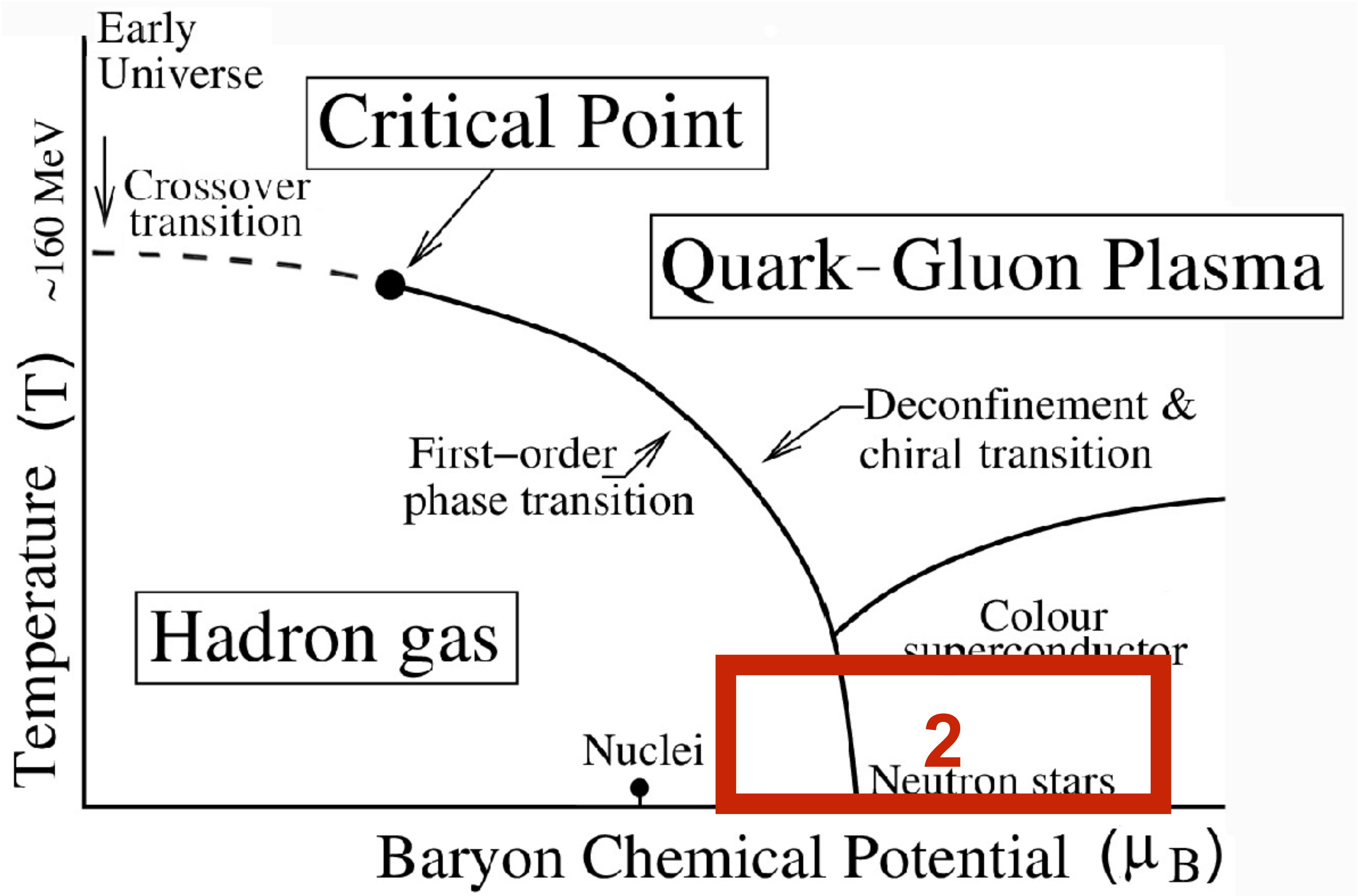
Chiral transition



See also: Glozman,Philipsen,Pisarski (2022); Cohen,Glozman (2024)

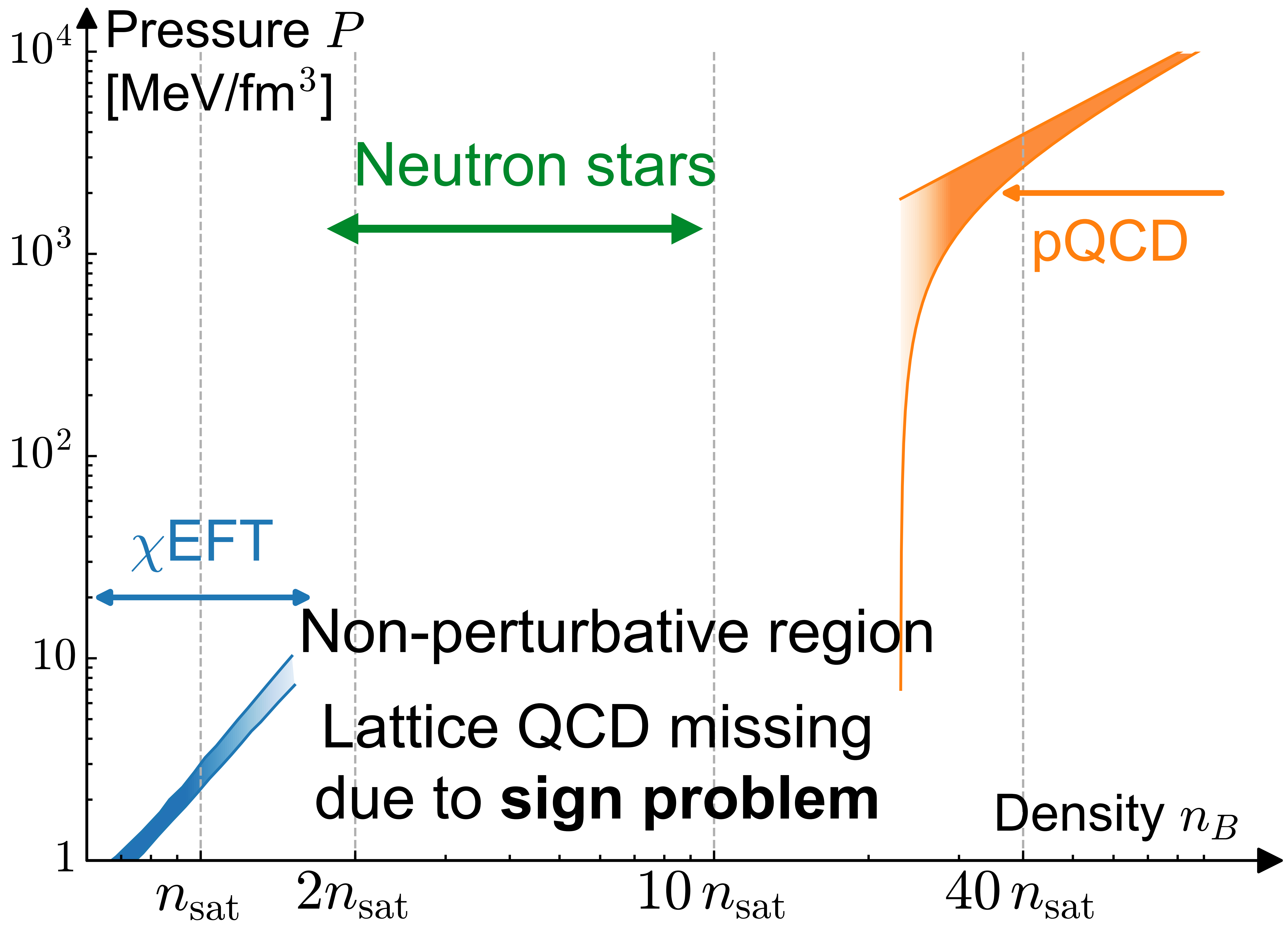
2. Quark (de)confinement at large μ_B



Challenges in finite-μ_B QCD

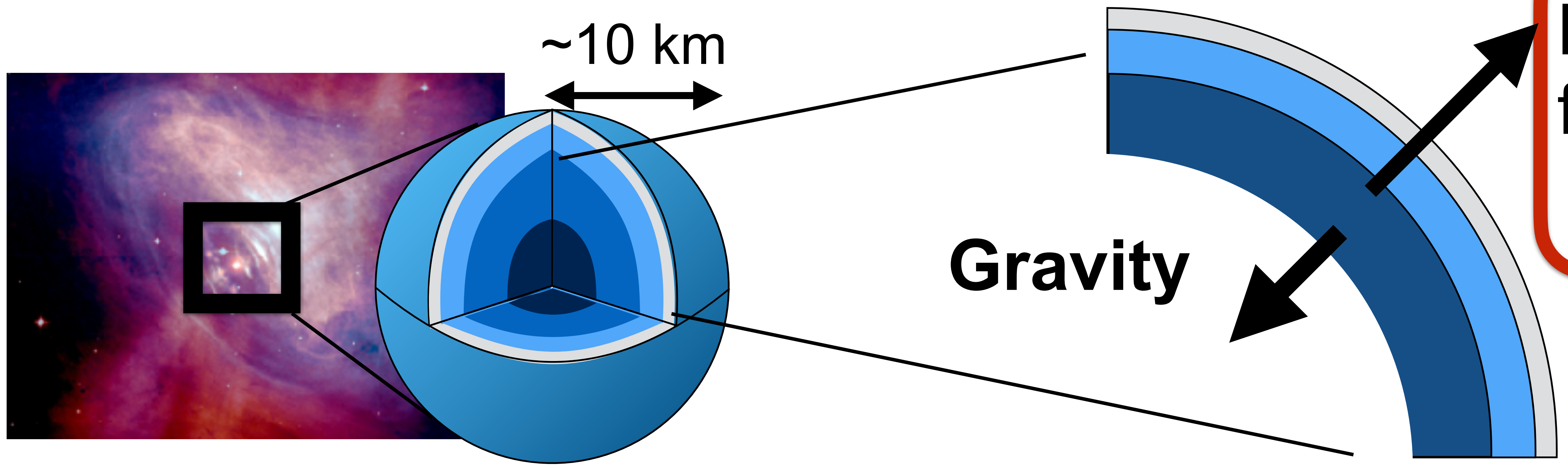
First-principles calculation only available by perturbation theory

Freedman,McLerran(1978);
Baluni(1979);
Kurkela,Romatschke,Vuorinen,
Gorda,Säppi,
Paatelainen,Seppänen+(2009-)



Tews,Krüger,Hebeler,Schwenk(2013);
Drischler,Furnstahl,
Melendez,Philips(2020);
Keller,Hebeler,Schwenk(2022);
& many others

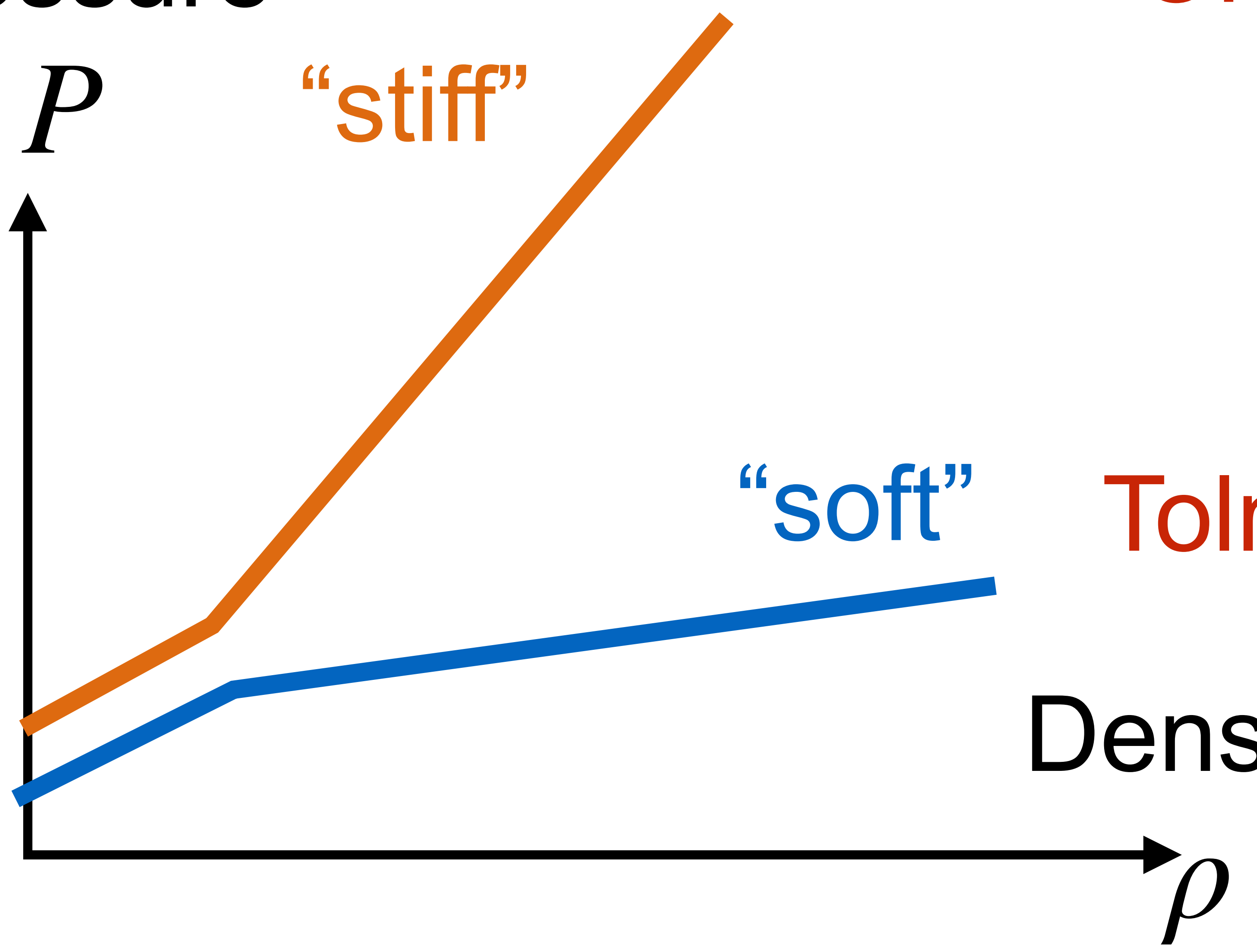
Neutron star structure and equation of state



Pressure
from nuclear force
(= Strong interaction)

Equation of state (EoS)

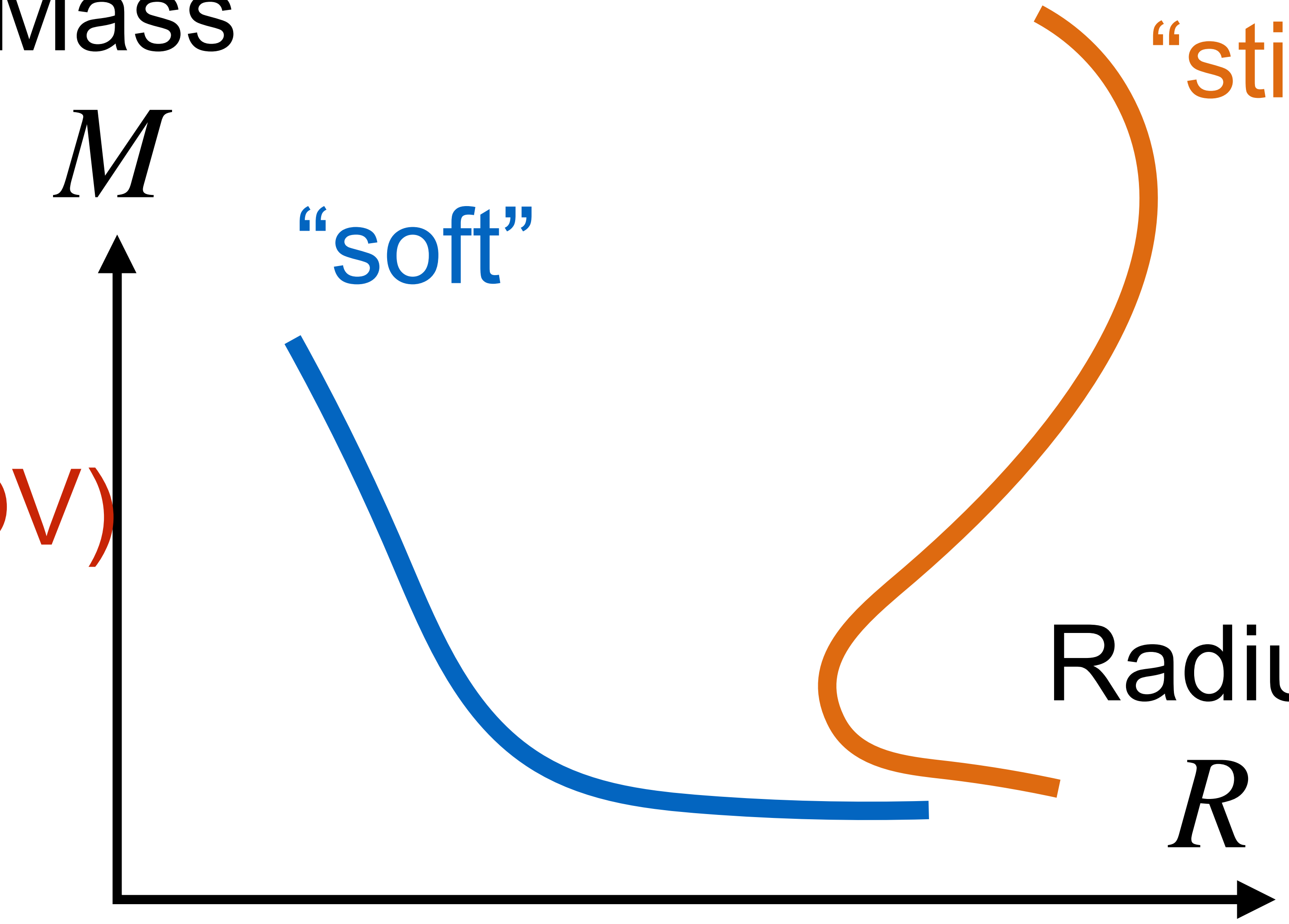
Equation of state



One-to-one correspondence via general relativity

Tolman-Oppenheimer-Volkoff (TOV) equation

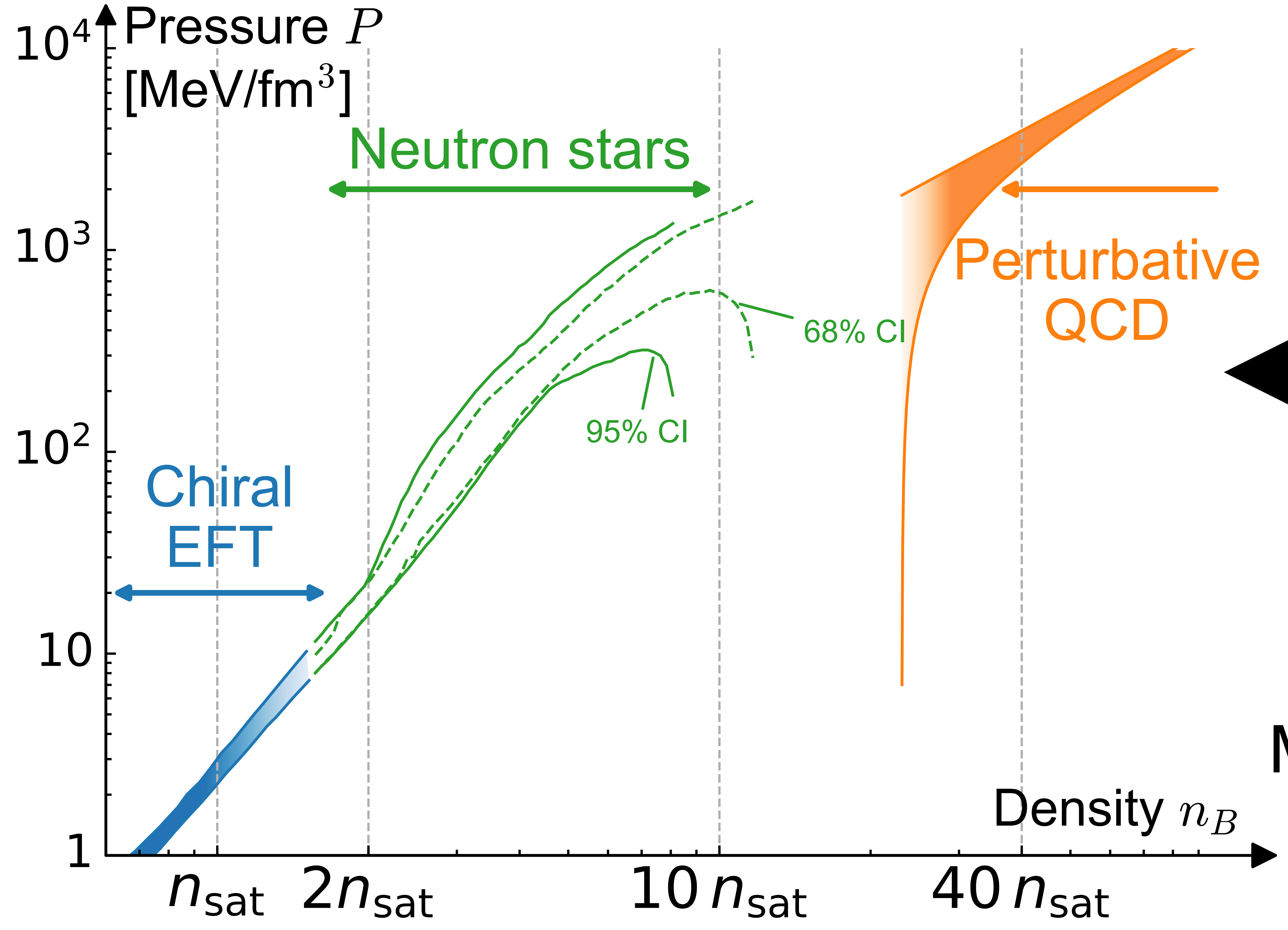
Neutron star structure (Mass-radius relation)



Neutron star structure and equation of state

One-to-one correspondence
through TOV eqn

Equation of state



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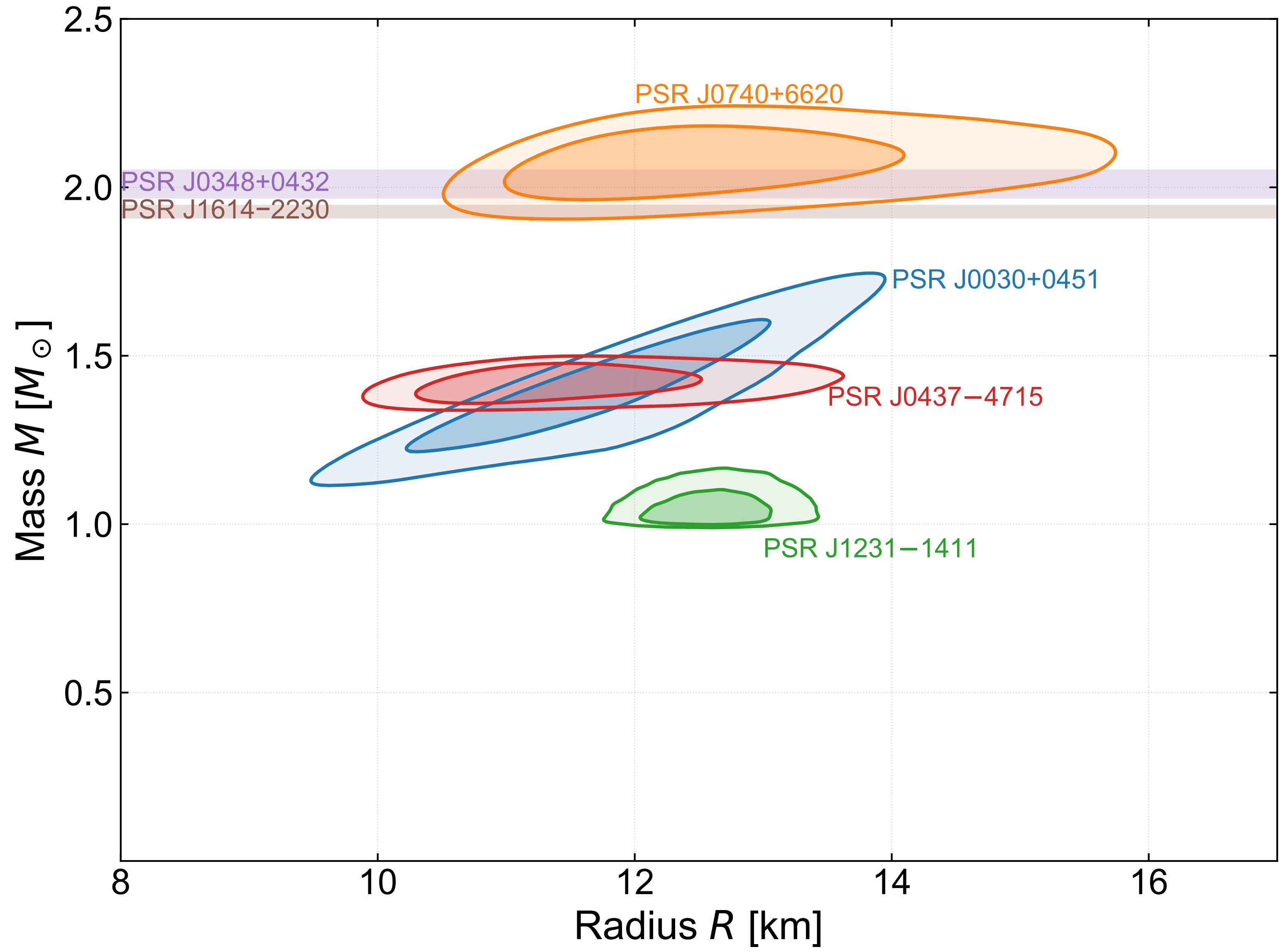
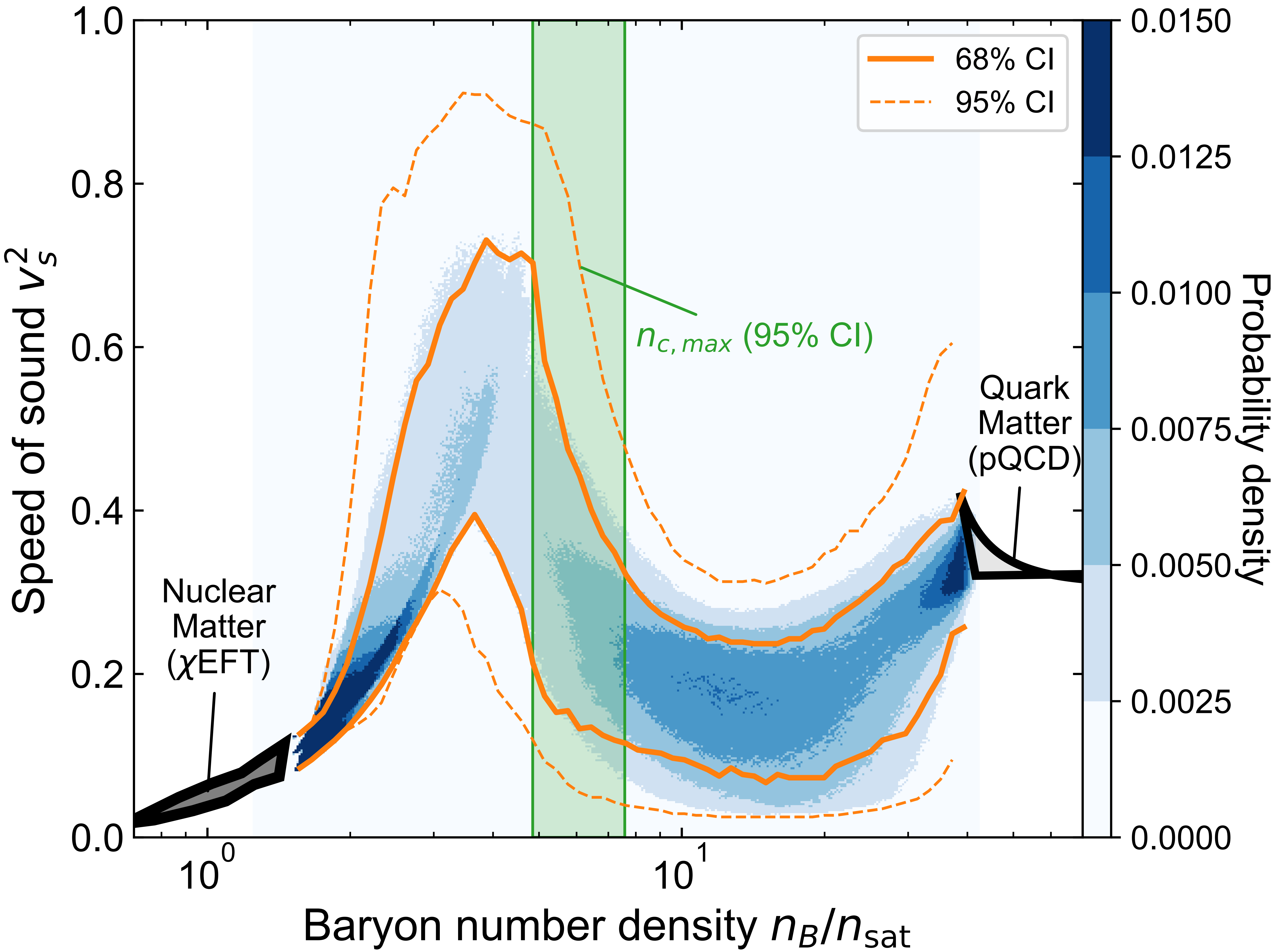


Inverse problem

Map neutron-star mass-radius relation
to the EoS using statistical methods
(Bayesian, Neural networks,
Gaussian process, etc.)

Emerging picture of neutron star EoS

Figure based on method by Altiparmak,Ecker,Rezzolla (2022)



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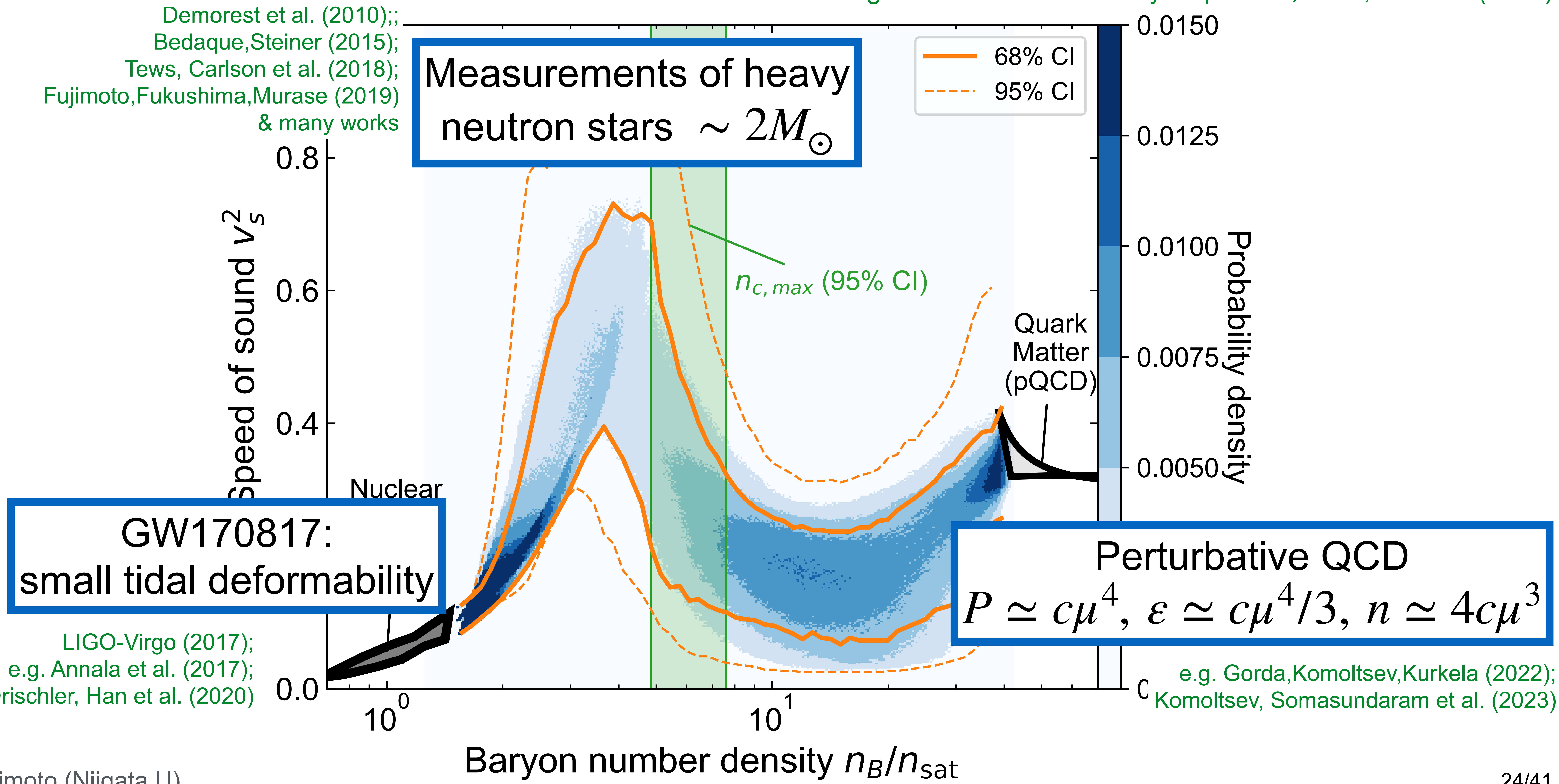
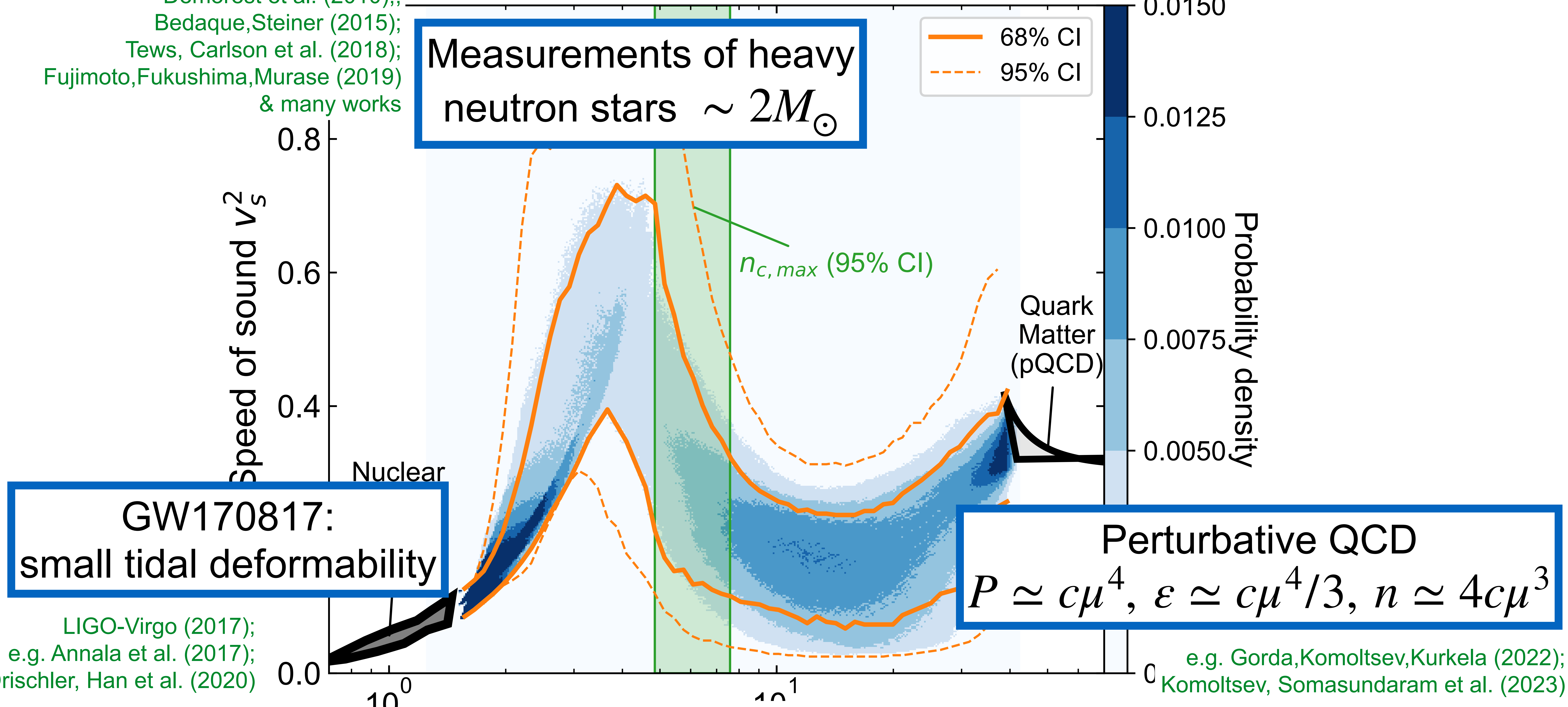


Emerging picture of neutron star EoS

Figure based on method by Altiparmak,Ecker,Rezzolla (2022)

Demorest et al. (2010);;
Bedaque,Steiner (2015);
Tews, Carlson et al. (2018);
Fujimoto,Fukushima,Murase (2019)
& many works



LIGO-Virgo (2017);
e.g. Annala et al. (2017);
Drischler, Han et al. (2020)

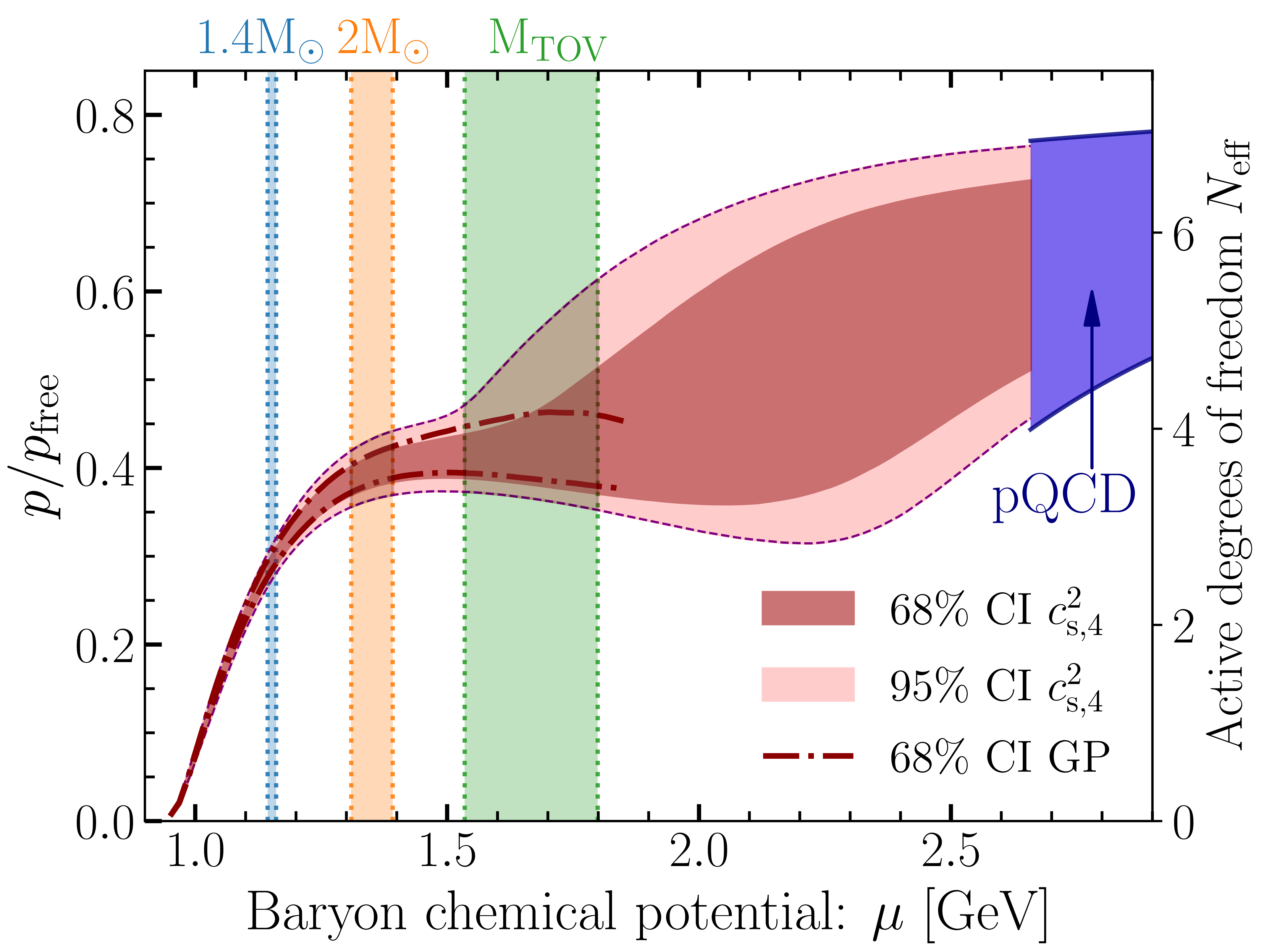
e.g. Gorda,Komoltsev,Kurkela (2022);
Komoltsev, Somasundaram et al. (2023)

What can be learned from this?

Increasing degrees of freedom in EoS → Color?

Recent Bayesian inference of neutron star EoS:

Effective d.o.f. in EoS ≈



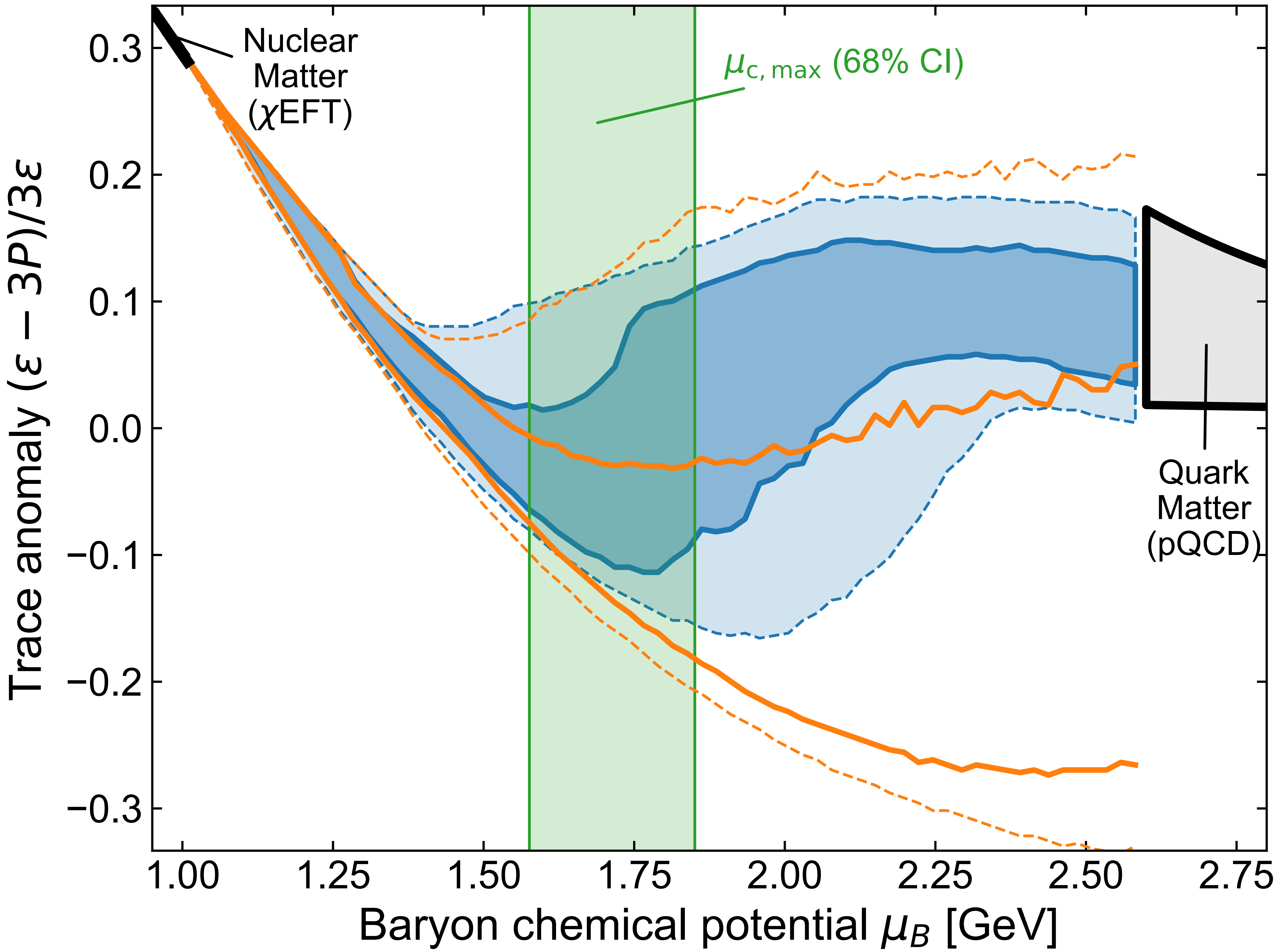
Effective d.o.f keeps increasing → may indicate appearance of color

Annala,Gorda,Hirvonen,Komoltsev,Kurkela,Nättilä,Vuorinen (2023)

Behavior of the trace anomaly

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Trace anomaly $\Delta = \dfrac{\varepsilon - 3P}{3\varepsilon}$



Behavior of the trace anomaly

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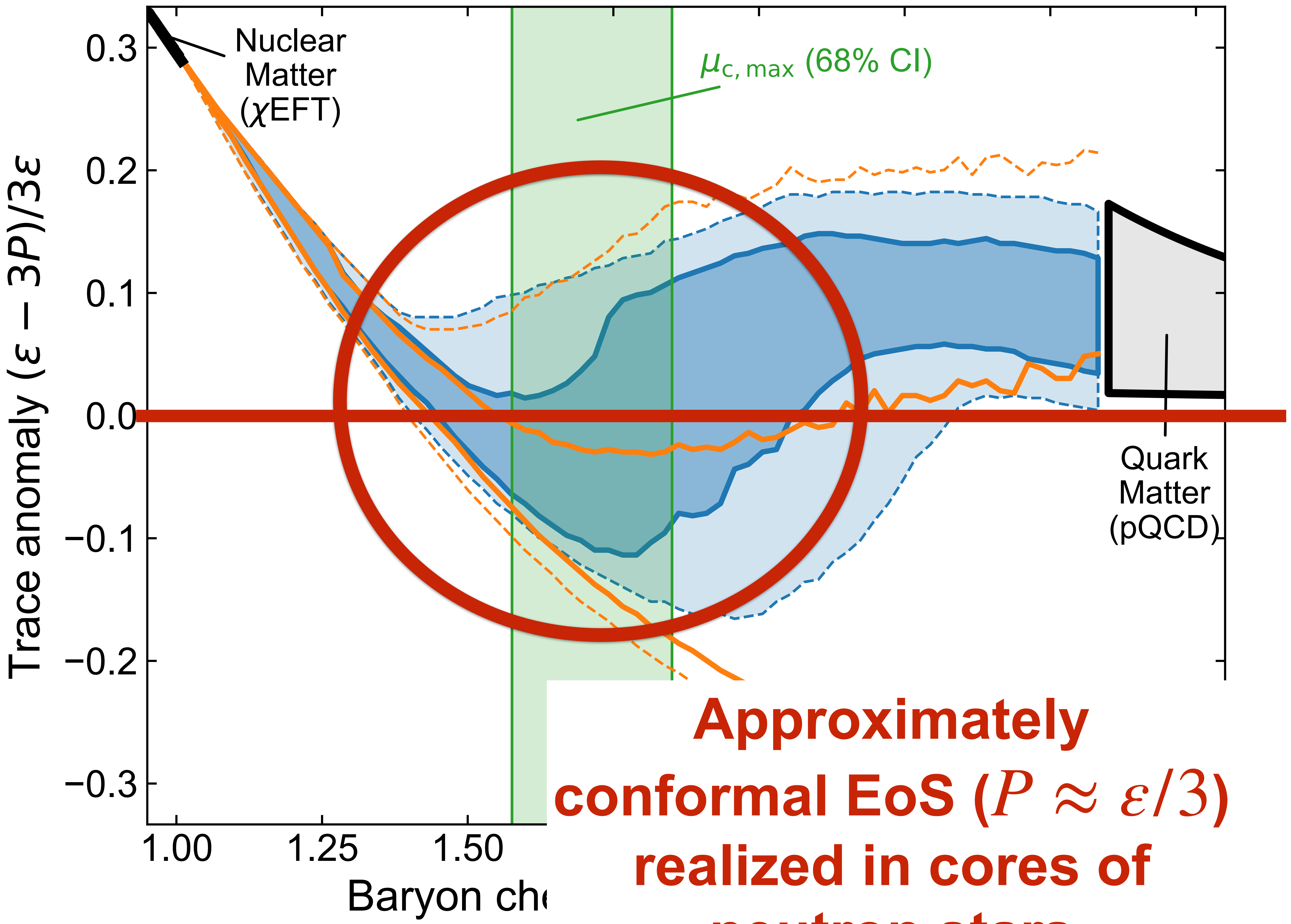
Trace anomaly $\Delta = \dfrac{\varepsilon - 3P}{3\varepsilon}$

Approximately conformal EoS in NS

QCD favors conformal EoS around the NS core density
→ onset of quark matter?

See also: Marczenko et al. (2022); Annala et al. (2023)



Approximately conformal EoS ($P \approx \varepsilon/3$) realized in cores of neutron stars

Trace anomaly and speed of sound

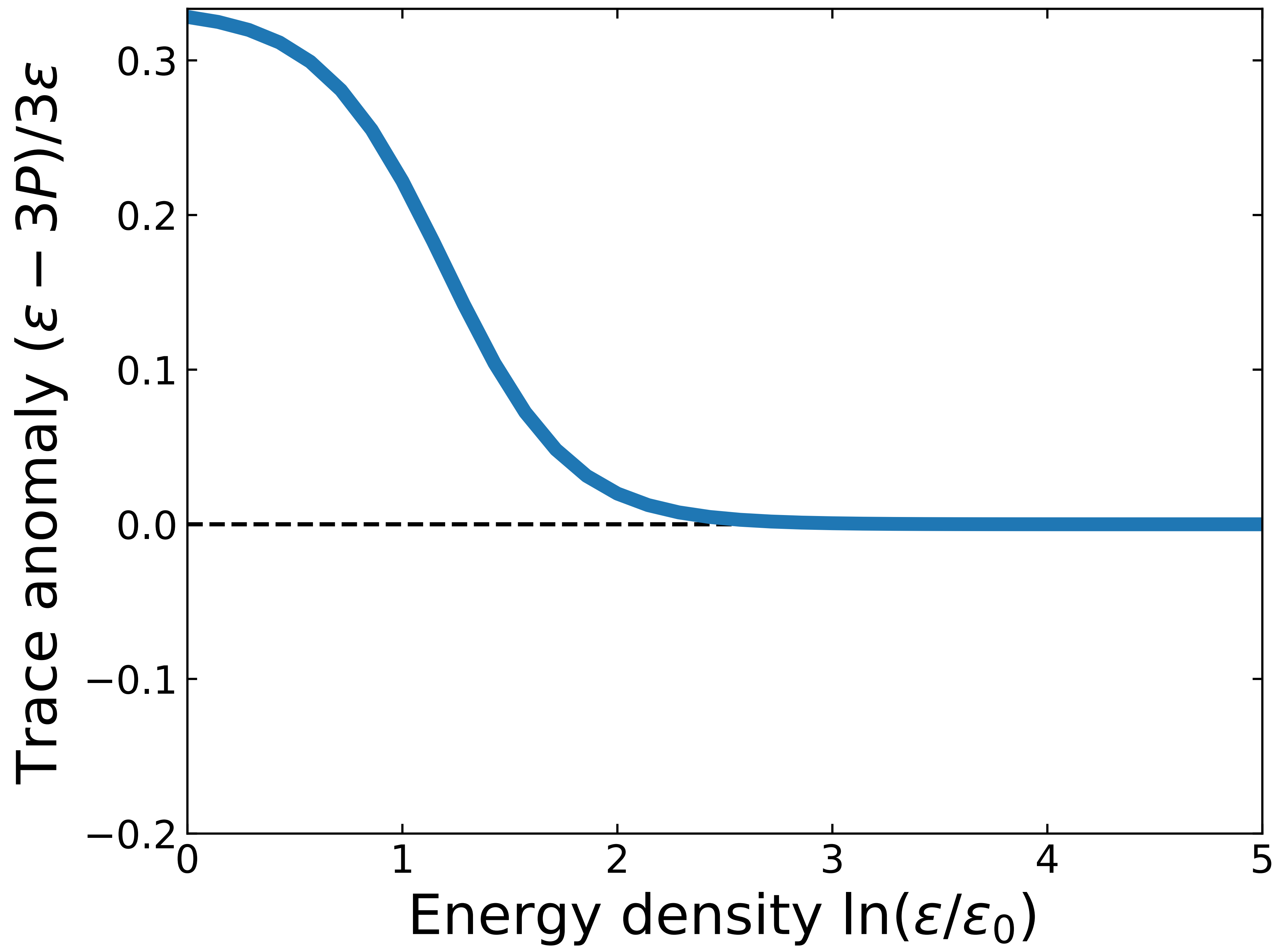
Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

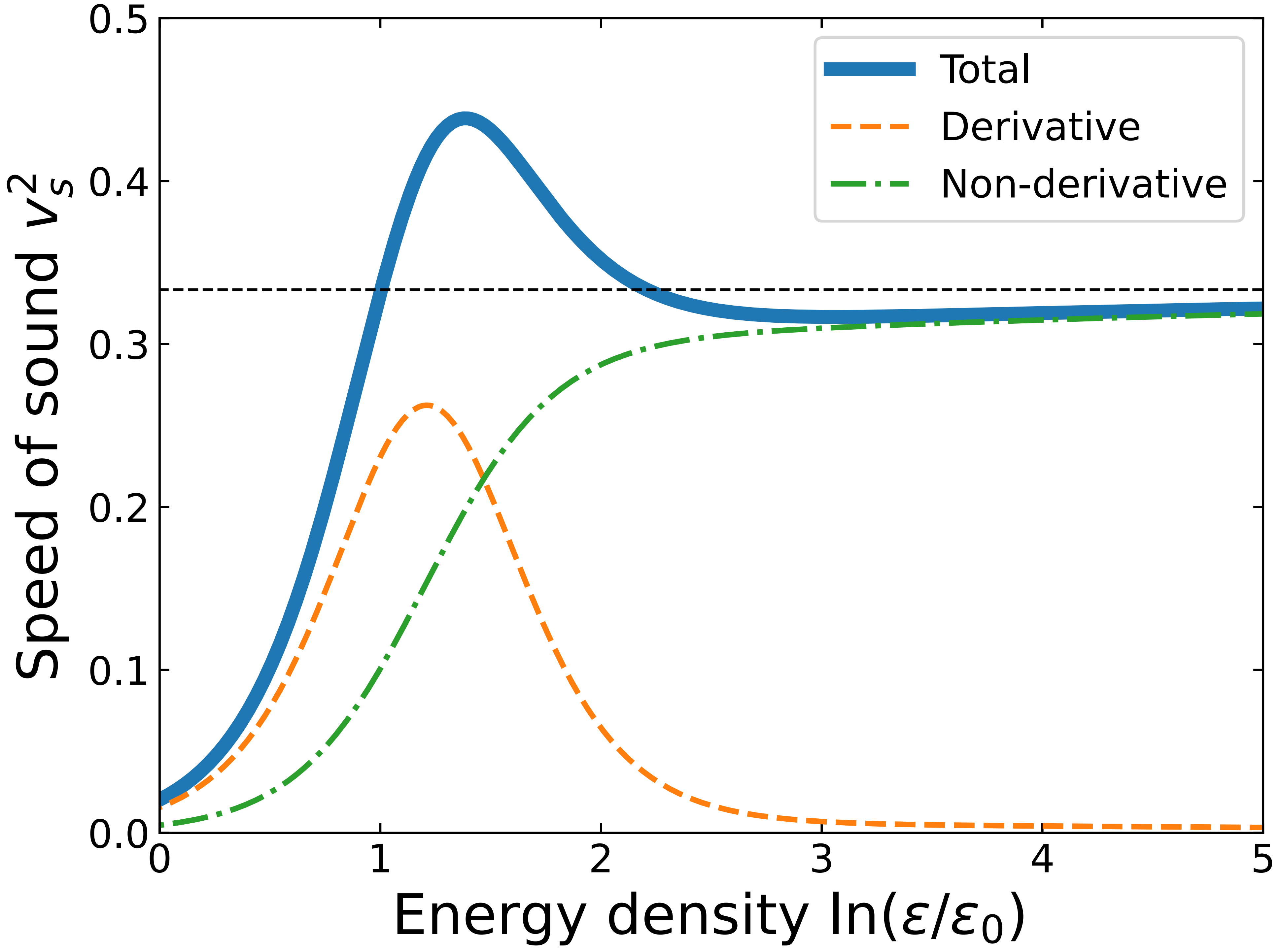
Rapid approach to $\Delta \to 0$ naturally spikes v_s^2

Trace anomaly $\Delta = \dfrac{\varepsilon - 3P}{3\varepsilon}$

Sound velocity $v_s^2 = \varepsilon \dfrac{d\Delta}{d\varepsilon} + \left(\dfrac{1}{3} - \Delta\right)$

Derivative Non-derivative



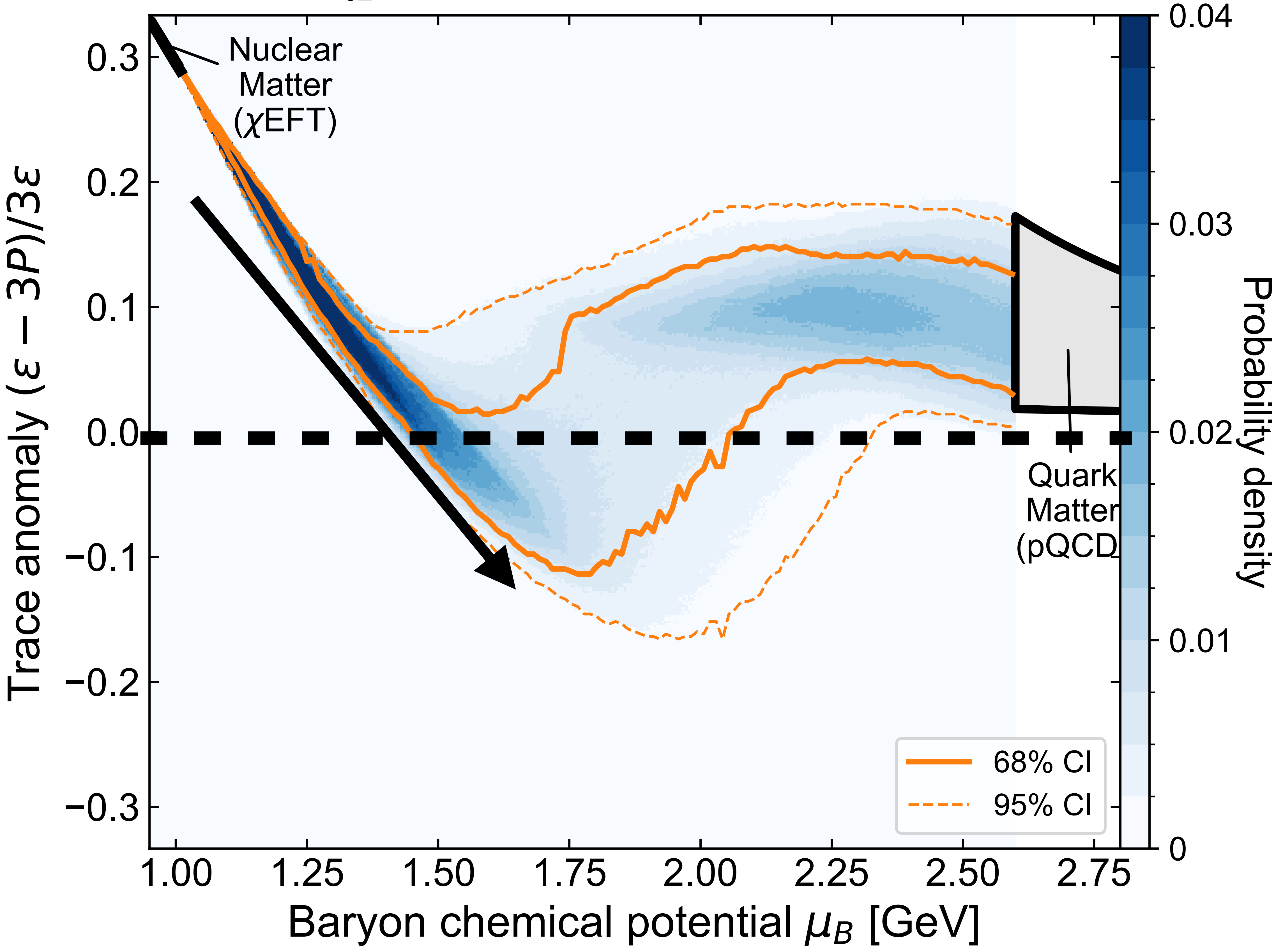


Trace anomaly and speed of sound

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

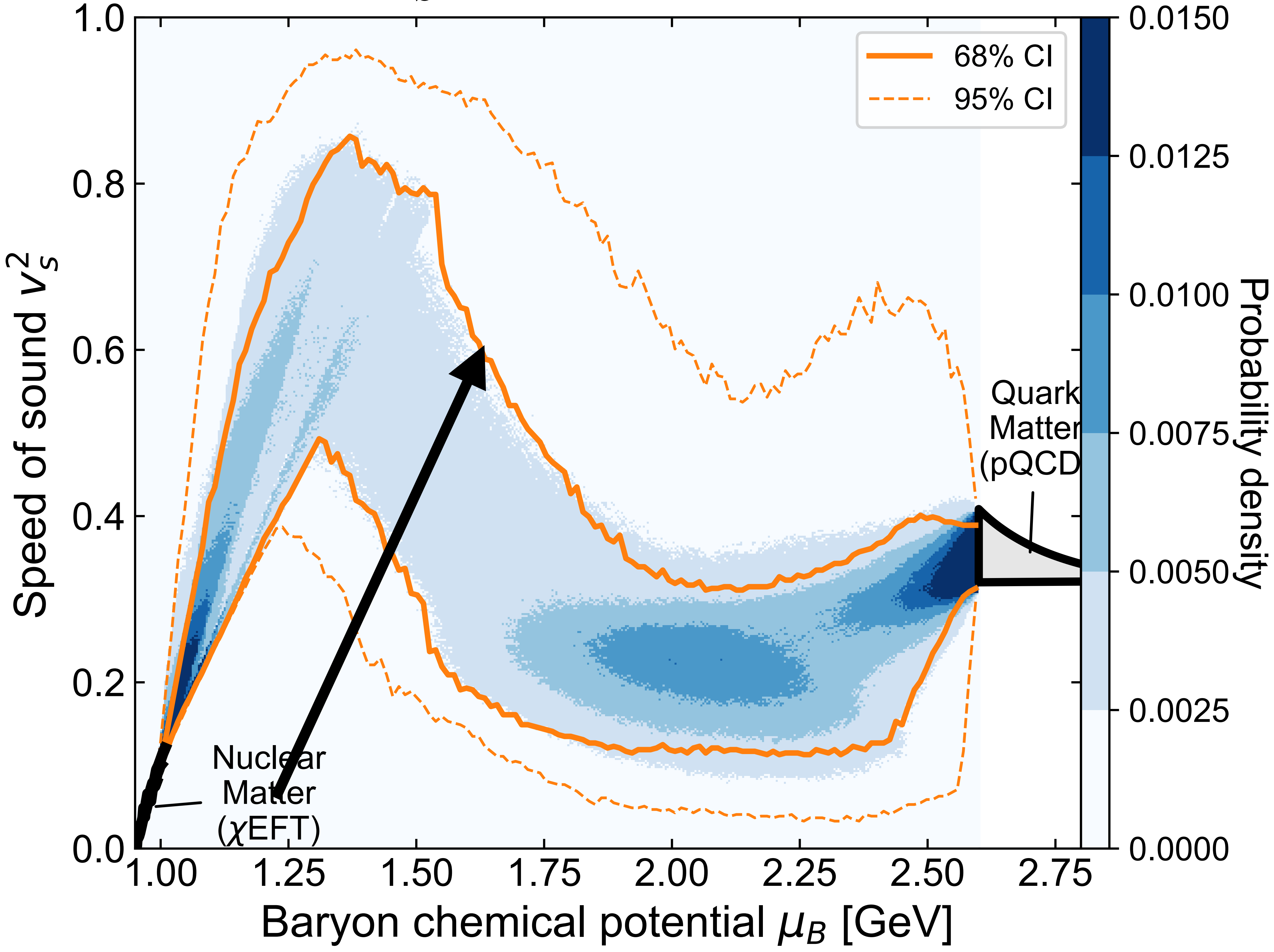
Normalized trace anomaly:

$$\Delta_{\rm tr} = (\varepsilon - 3P)/3\varepsilon$$



Sound speed:

$$v_s^2 = dP/d\varepsilon$$



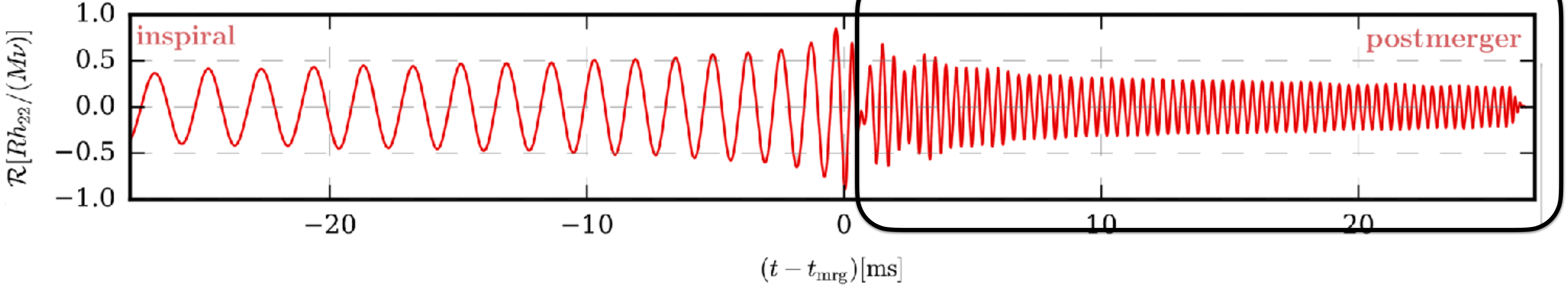
Rapid approach to $\varepsilon - 3P \to 0$ drives the peak in v_s^2 in actual data

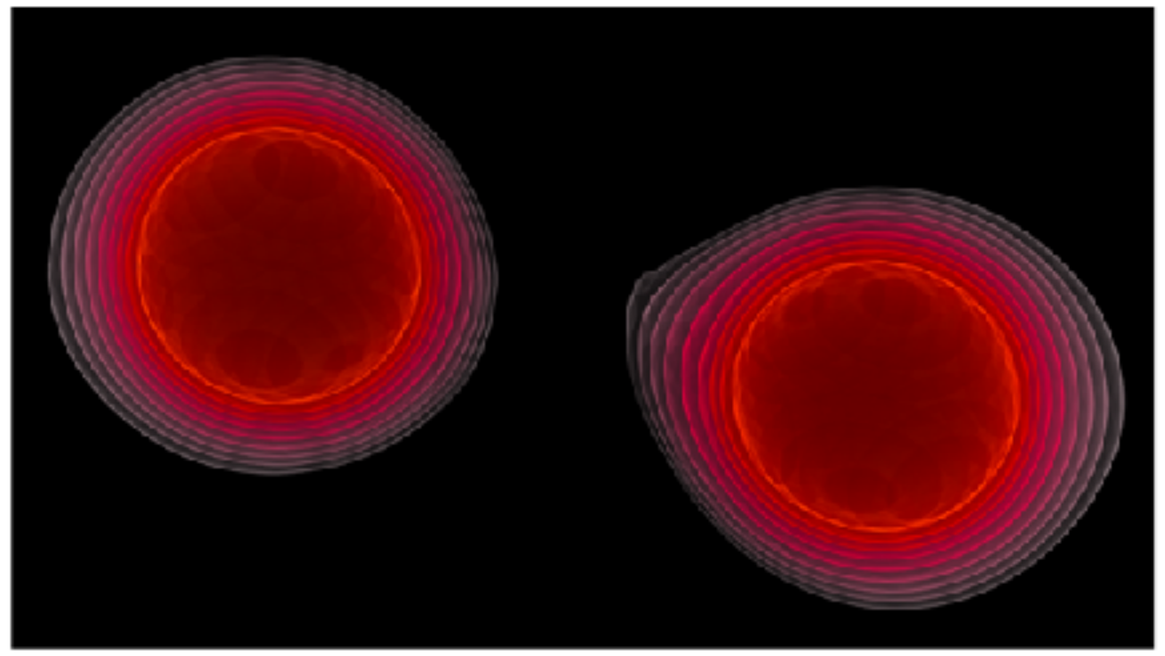
Binary neutron star mergers

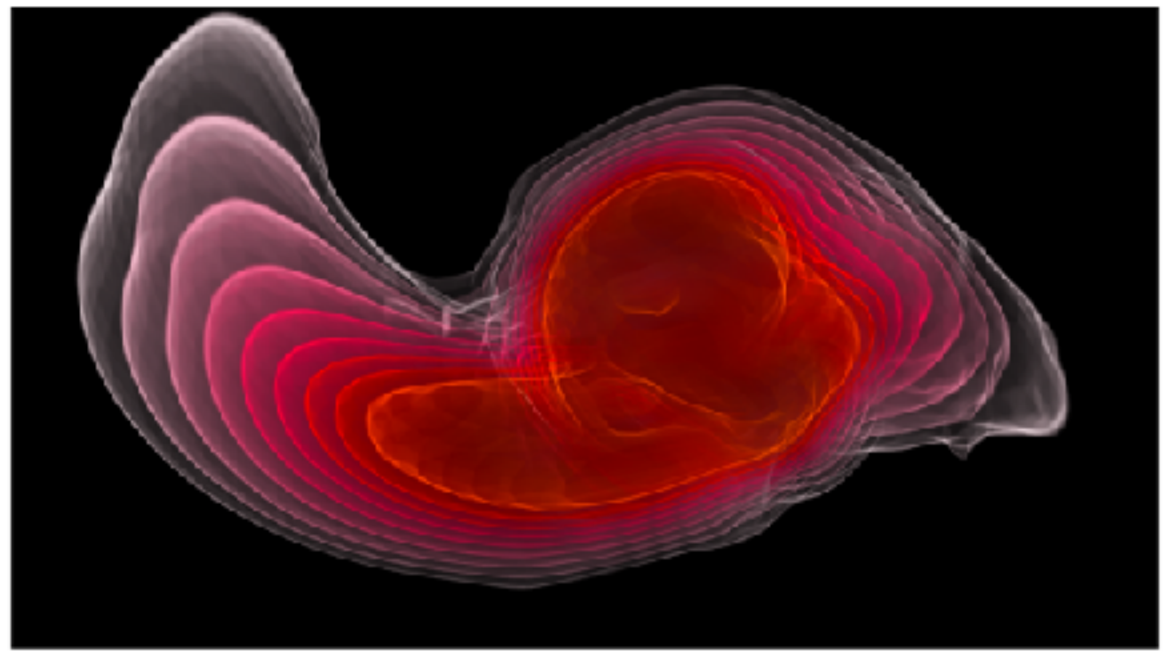
What can we learn from the future gravitaional wave observation of binary neutron star mergers?

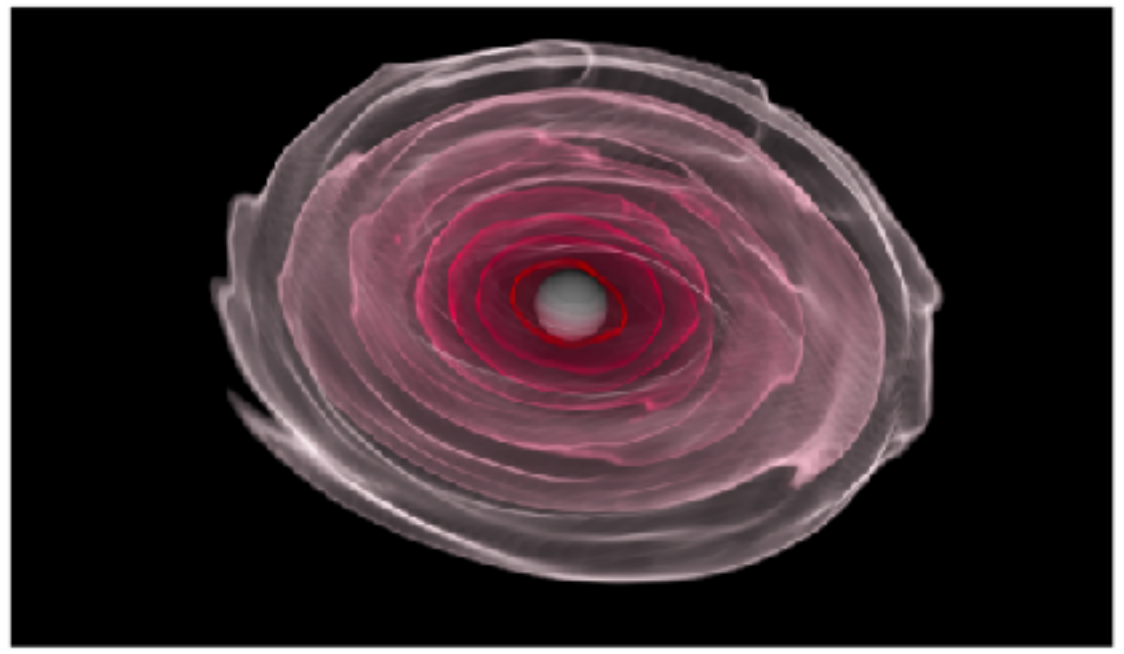
Postmerger phase contains more information on the EoS

Inspiral phase is what we currently observed







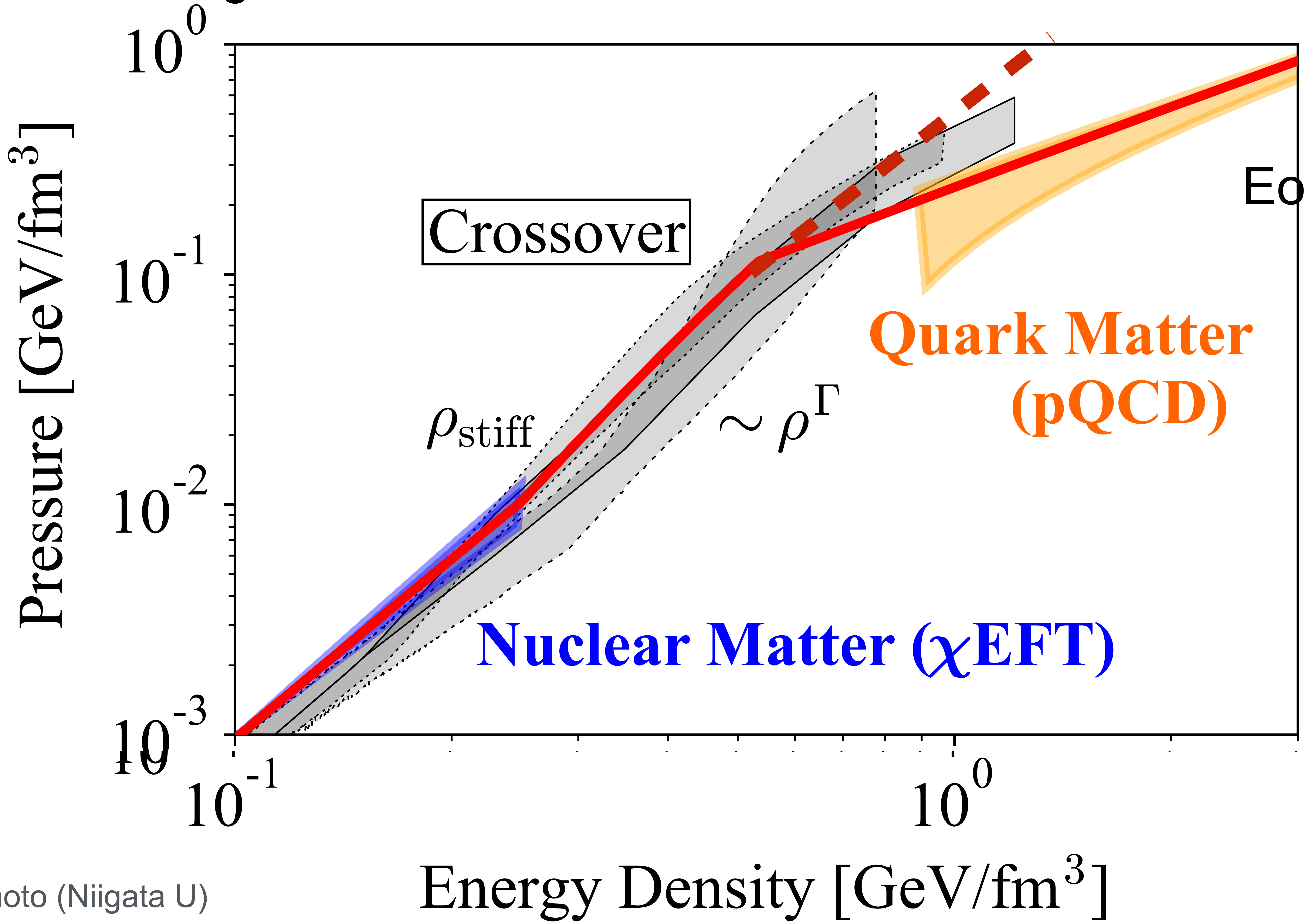


From: Dietrich,Hinderer,Samajdar (2020)

Crossover EoS & possible gravitational wave signal

Fujimoto,Fukushima,Hotokezaka,Kyutoku, PRL130 (2022); PRD111 (2024)

Modeling of EoS:



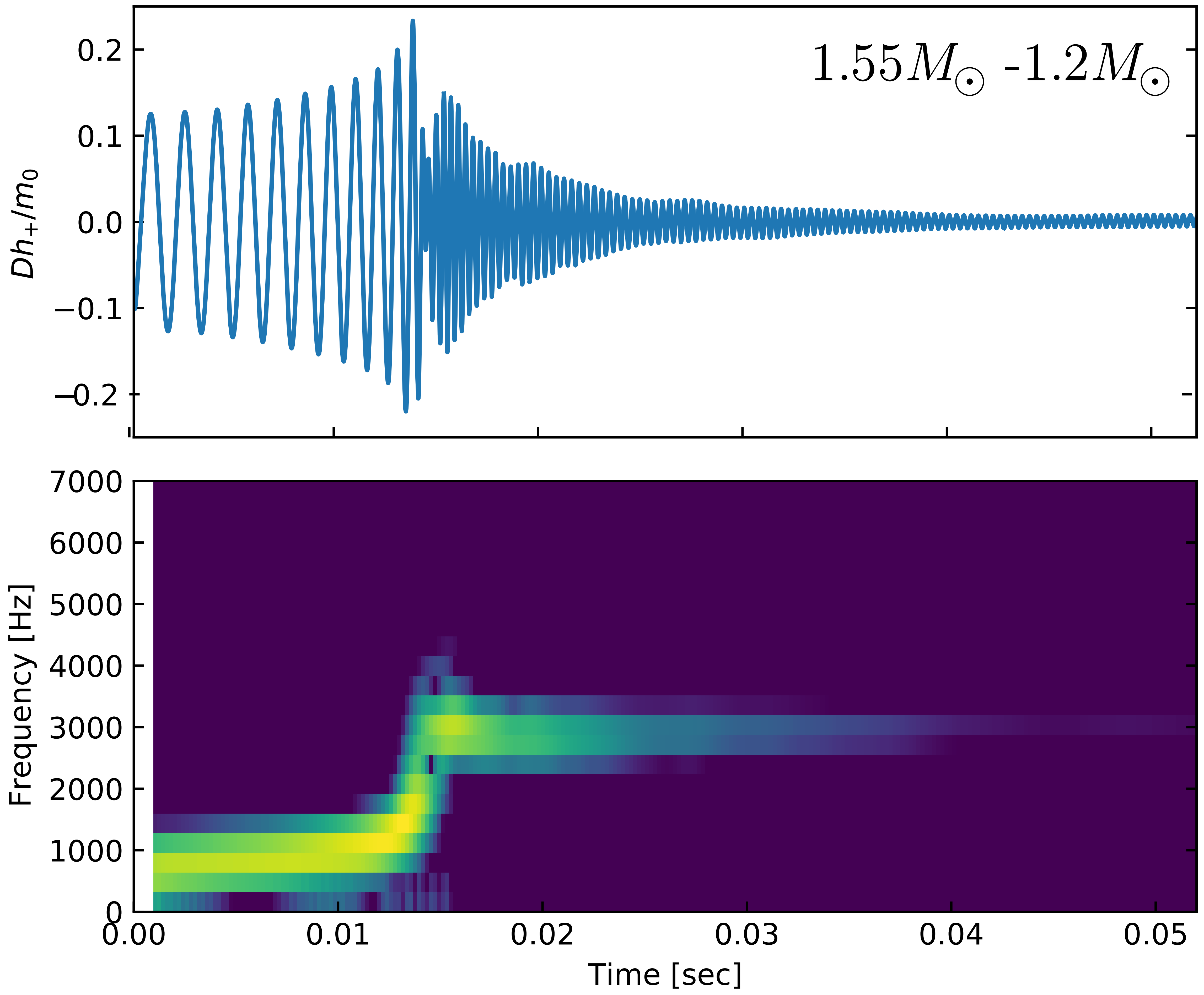
EoS constructed by extrapolating two ab initio calculations

We contrast two cases:
1) w/ softening
2) w/o softening

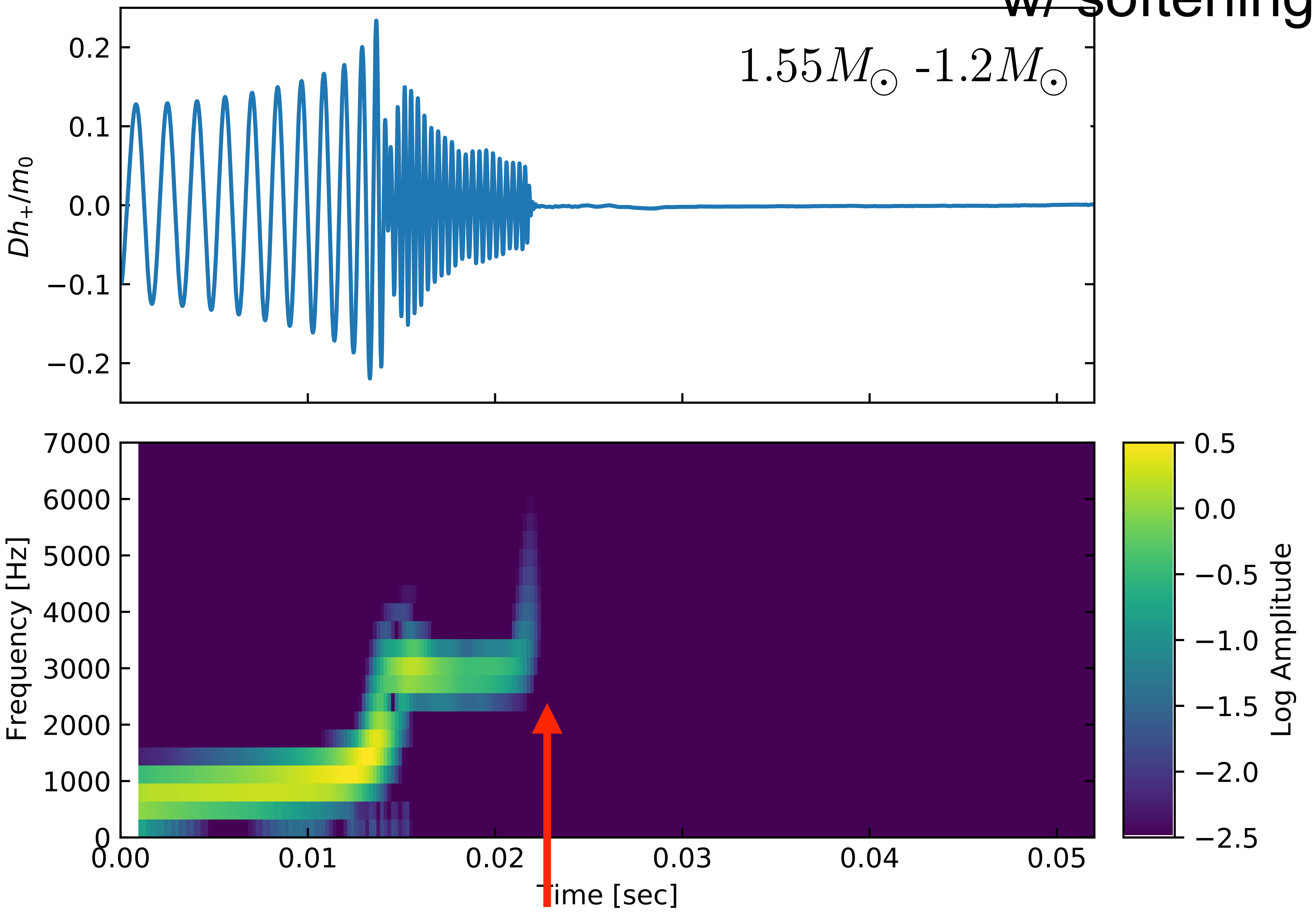
Crossover EoS & possible gravitational wave signal

Fujimoto,Fukushima,Hotokezaka,Kyutoku, PRL130 (2022); PRD111 (2024)

w/o softening



w/ softening

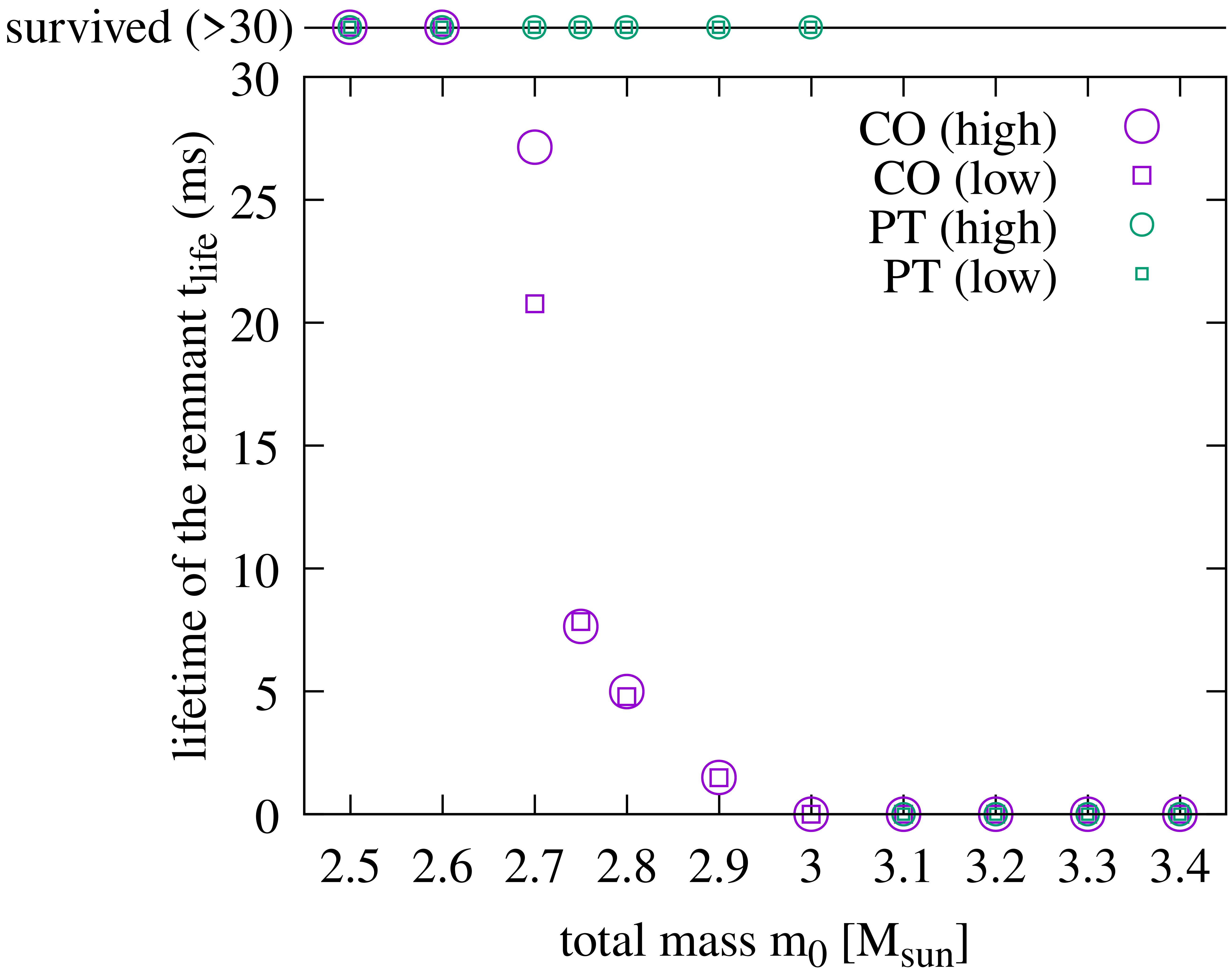


Early collapse to black hole
in the postmerger signal

Lifetime of the merger remnant

Fujimoto,Fukushima,Hotokezaka,Kyutoku, 2408.10298

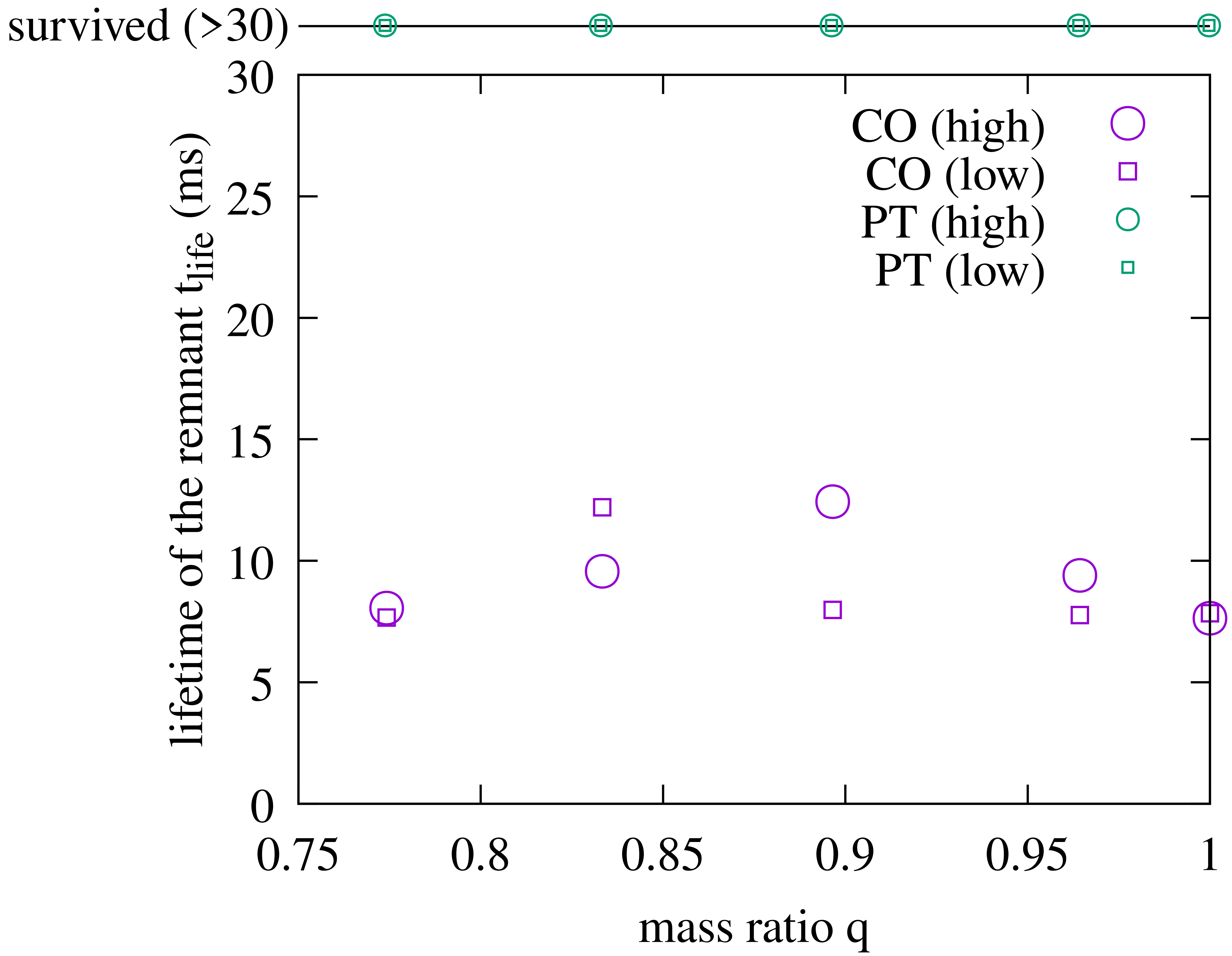
Lifetime is determined primarily by the total mass of binary



Weak dependence on mass ratio

Fujimoto,Fukushima,Hotokezaka,Kyutoku, 2408.10298

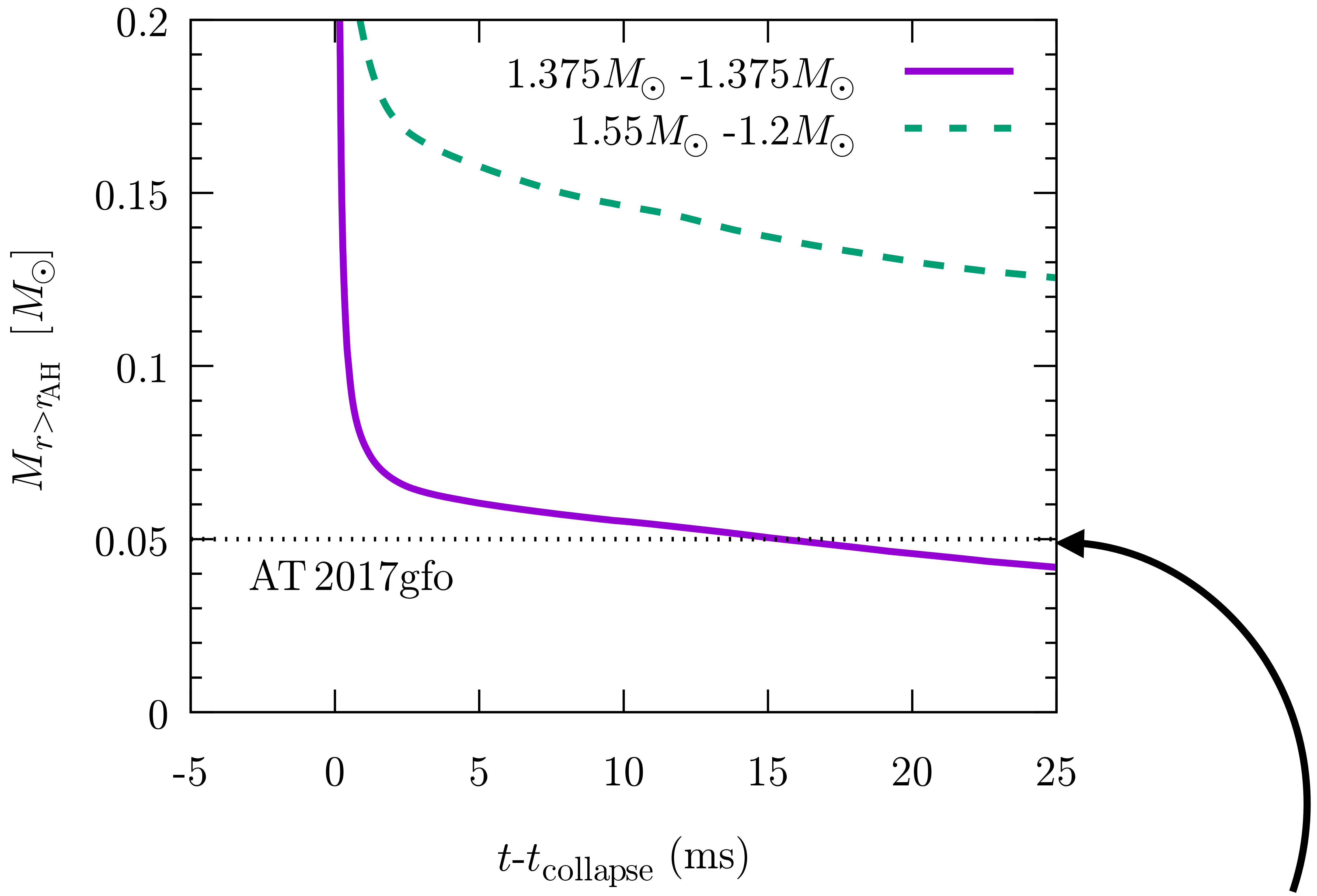
For total mass $m_0 = 2.75\,M_\odot$, mass ratio dependence is weak



Consistency with kilonova AT2017gfo

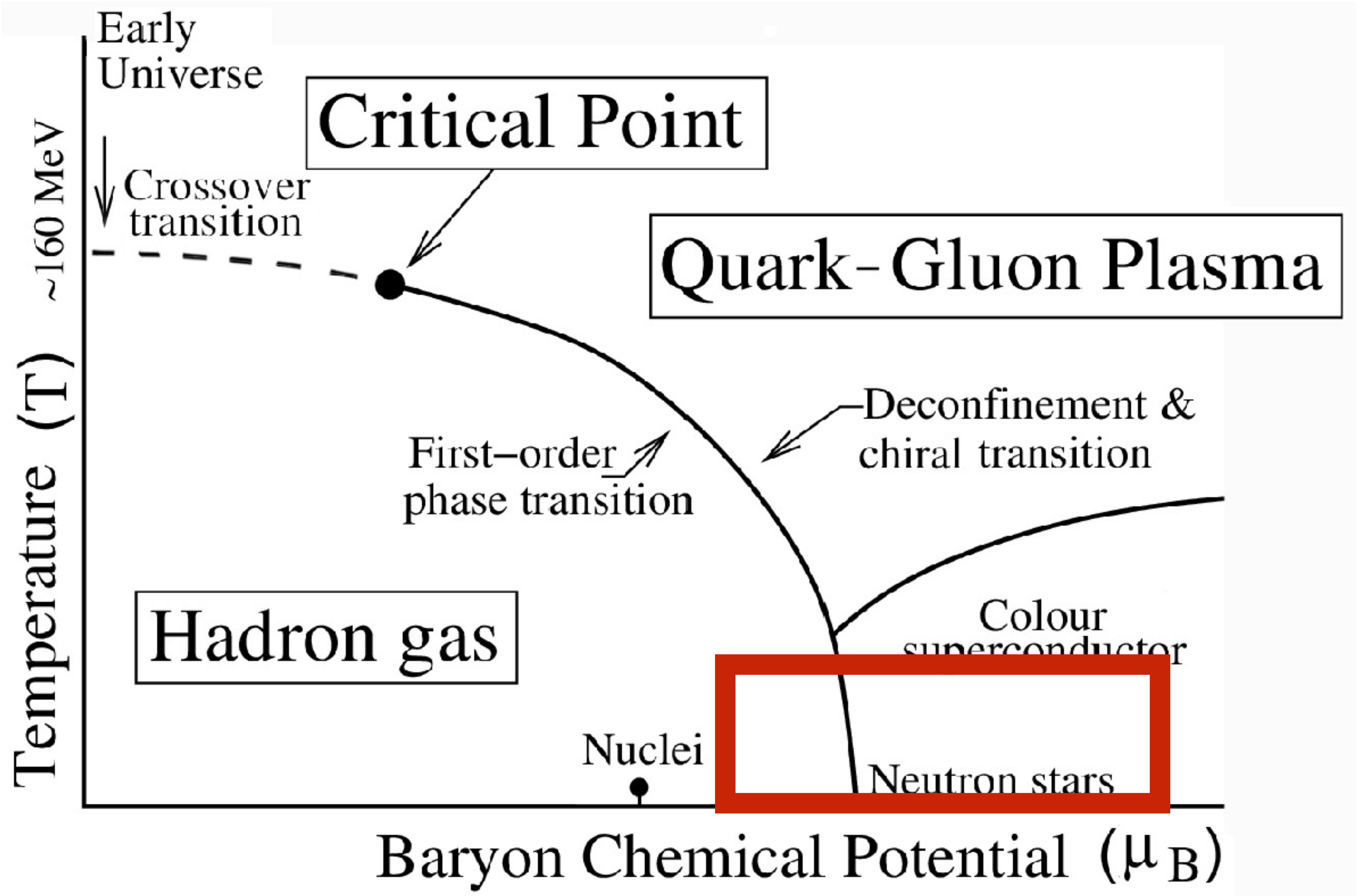
Fujimoto,Fukushima,Hotokezaka,Kyutoku, PRL130 (2023): 2408.10298

Remnant mass outside the apparent horizon of the BH



AT2017gfo, electromagnetic counterpart of GW170817, requires ejection of $\approx 0.05 M_\odot$ for its observed luminosity

3. Complications in quark deconfinement & possible directions to tackle finite-μ QCD



Quark (de)confinement may be complicated

- Several specific QCD cases & QCD-like theories indicate the possibility of absence of deconfinement & confinement persisting even at high density…

- **Large-N_c limit of QCD (QCD is $N_c = 3$):** no screening of confining potential
  McLerran,Pisarski (2007)

- **$N_c = 2$ QCD:** may remain confined at high density implied from lattice QCD
  Hands,Kim,Skullerud (2010); Iida, Itou, et al. (2024)

- **QCD at finite *isospin* density ($\mu_u = \mu,\ \mu_d = -\mu$):**
  weak-coupling calculation suggests confinement remains Son,Stephanov (2000)

- **Two-flavor color superconductor in QCD:**
  effective theory after Higgs phenomenon suggests confinement
  Rischke,Son,Stephanov (2001)

Quarkyonic matter
— Duality between confinement and deconfinement

Theoretical observations:
- Quark confinement persists at high baryon density
- Color is liberated, and N_c dependence appears in the equation of state

Quark confinement may persist in the regime quarks are natural d.o.f.
→ duality (continuity) between the confined nucleons and deconfined quarks

So, no sharp phase boundary between confinement and deconfinement
… crossover EoS

See: McLerran,Reddy (2019); Fujimoto,Kojo,McLerran (2023, 24);
Bluhm,Fujimoto,McLerran,Nahrgang (2024) for actual model calculations

Possible directions on the lattice

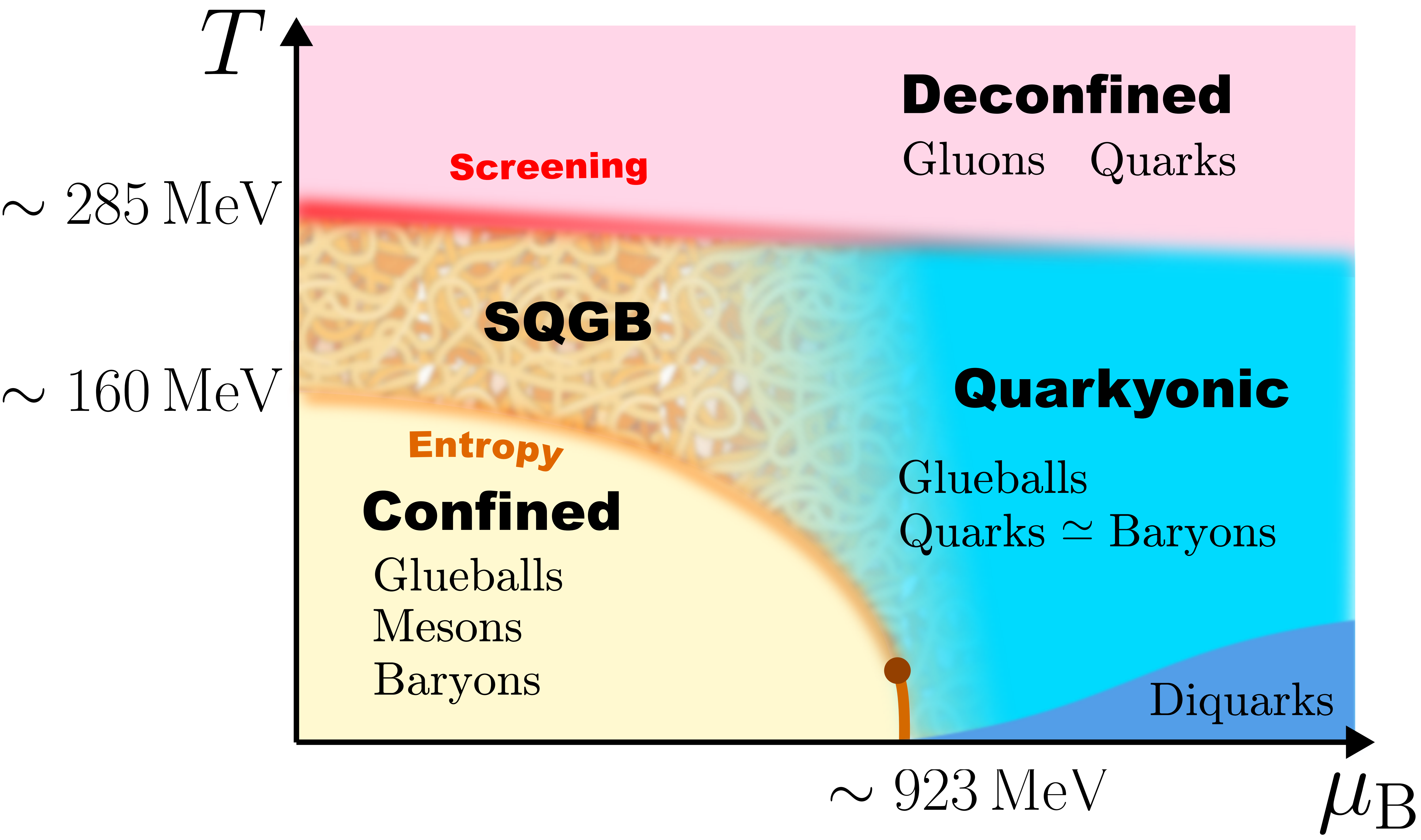
There are a few theories similar to finite-μ_B QCD **without sign problem:**

- **QCD at finite isospin chemical potential** ($\mu_u = \mu_I/2,\ \mu_d = -\mu_I/2$)
  One can constrain the symmetric nuclear matter EoS through the QCD inequality
  Fujimoto, Reddy (2023); Abbott et al. (NPLQCD) (2023, 24)

- **QCD at finite μ_B and $N_c = 2$**

- **QCD at finite μ_B in 1+1 dimensions**

Summary



- Deconfinement is difficult to capture
- Hagedorn temperature (≈ deconfinement) may be $T_H \sim 300$ MeV
  … common for quarks & gluons
- Neutron star may favor quark-like EoS
  … may be tested by future binary mergers
- Confinement may persist at large density
  … Quarkyonic regime.

Bonus materials

Comment 1: Difference in Hagedorn spectrum

Conventionally used form (from bootstrap condition):

$$\rho(m) = \frac{a}{(m^2 + m_0^2)^{5/4}} e^{m/T_H}$$

… ambiguity in the power-law in the prefactor

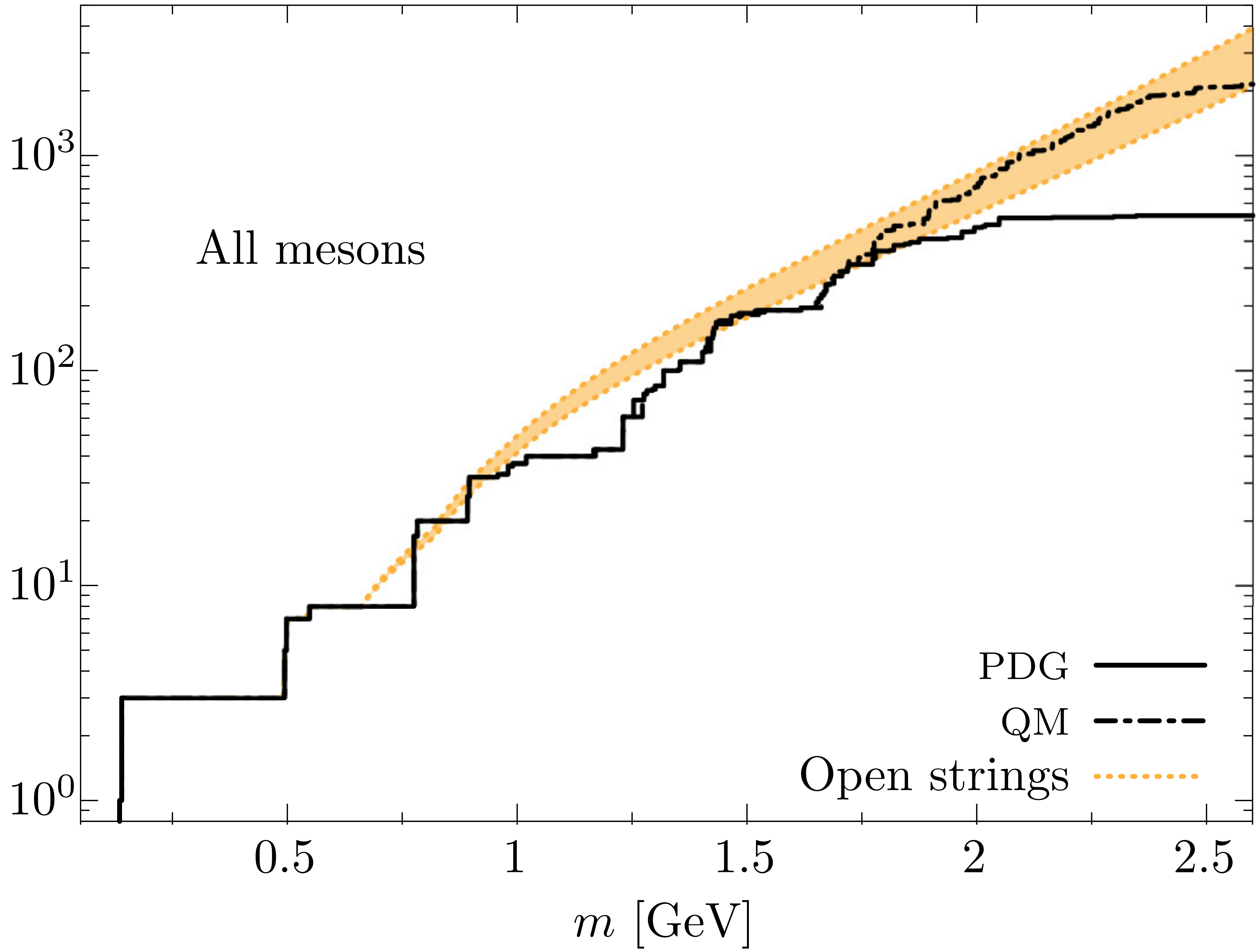
Hagedorn spectrum from string theory:

$$\rho(m) = \frac{\sqrt{2\pi}}{6T_H} \left(\frac{T_H}{m}\right)^{3/2} e^{m/T_H}$$

… no ambiguity in the prefactor

Fitting PDG with open string Hagedorn spectrum

Marczenko,Kovacs,McLerran,Redlich (2025)

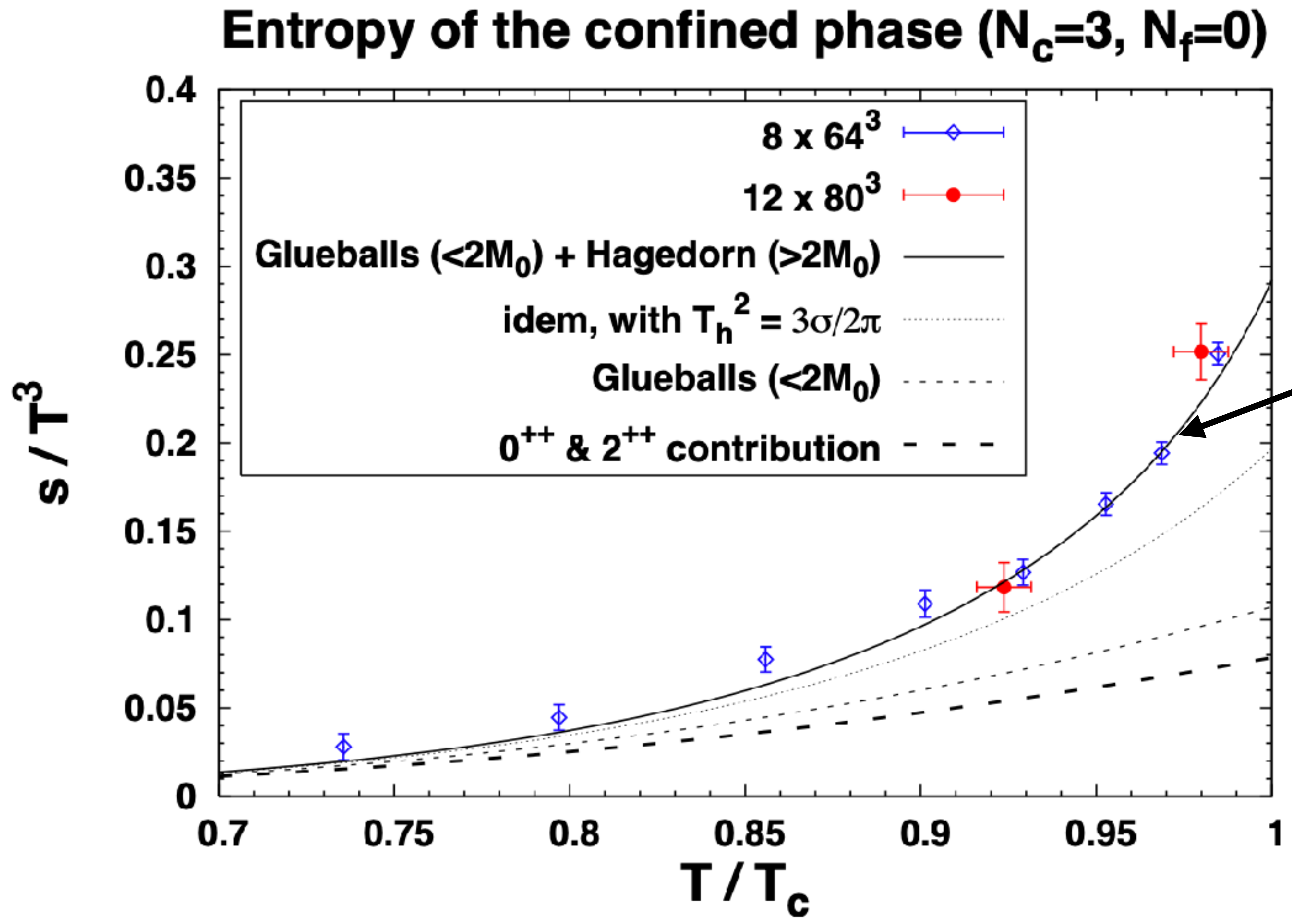


Comment 2: The same works in pure glue QCD

Pure SU(3) Yang-Mills → Deconfinement takes place at $T_d \simeq 285\,\mathrm{MeV}$

→ Below T_d is explained by glueballs

Borsanyi et al., PRD 105 (2022)



Solid line is fit w/ Hagedorn spectrum

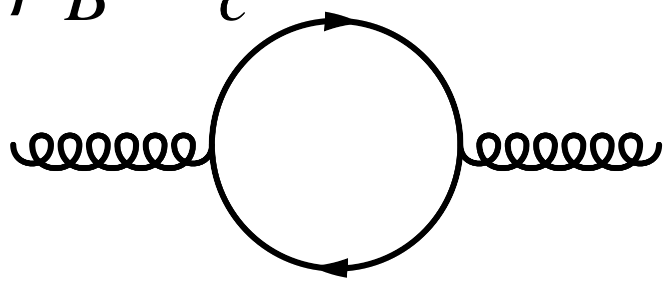
Meyer, PRD 80 (2009)

Duality/continuity in dense QCD: Quarkyonic matter

Quark deconfinement at high density may not be that simple…

McLerran & Pisarski (2007): **Quarkyonic matter**

$\mu = \mu_B / N_c$



$$m_D^2 \sim \frac{\lambda_{\text{'t Hooft}}\, \mu^2}{N_c} \to 0$$

… deconfinement is never affected by quark medium in large-N_c QCD

Quark confinement persists in the regime quarks are natural degrees of freedom

→ duality/continuity between the baryonic and quark d.o.f

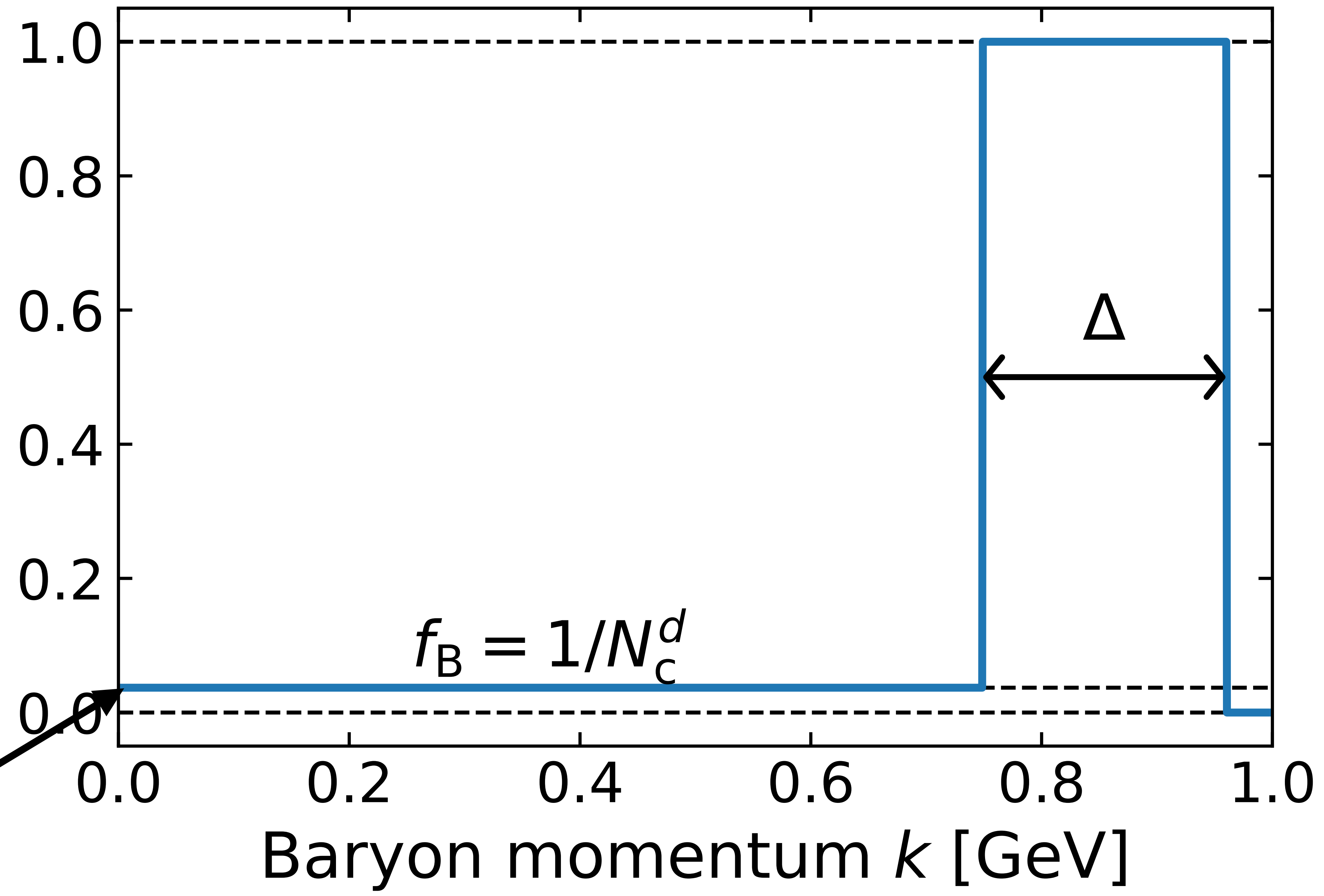
cf. similar situation can be found in
- QCD at finite isospin density — Son,Stephanov (2000)
- Two-flavor color superconductor — Rischke,Son,Stephanov (2001)
- Two-color QCD — Hands,Kim,Skullerud (2010); Iida et al. (2024)

Upshot of the duality

Fujimoto,Kojo,McLerran, PRL 132 (2023)

Non-trivial modification in baryon distribution:

f_{B}



Kojo (2021)

This low-momentum part becomes highly suppressed due to the **quark Pauli principle**

$k_{F,\mathrm{baryon}} = N_c k_{F,\mathrm{quark}} \rightarrow$ Phase space for baryons = $k_{F,\mathrm{baryon}}^3 = N_c^3 k_{F,\mathrm{quark}}^3$

for quarks $= k_{F,\mathrm{quark}}^3$

N_c^3 times larger

QCD inequality

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA] \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA] \left|\det \mathscr{D}(\frac{\mu_I}{2})\right|^2 e^{-S_G}$

u quark   d quark

charge conjugation symmetry $\mu_B \to -\mu_B$

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA] \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c}) e^{-S_G} = \int [dA] \,\mathrm{Re}\left[\det \mathscr{D}(\frac{\mu_B}{N_c})\right]^2 e^{-S_G}$

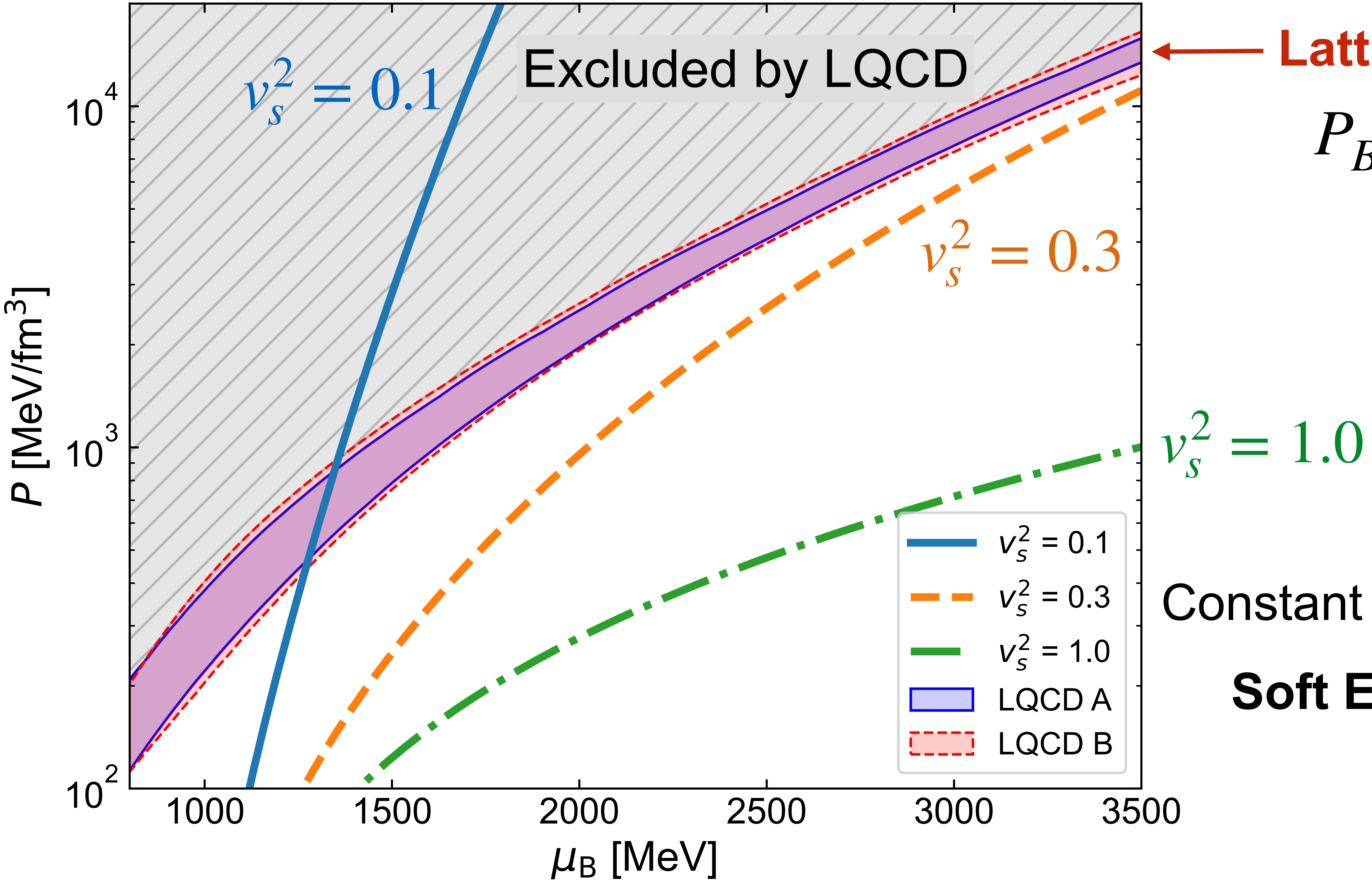
Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\mathrm{Re}\, z^2 \leq |z^2| = |z|^2$:

$$Z_B(\mu_B) \leq \int [dA] \left|\det \mathscr{D}(\frac{\mu_B}{N_c})\right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound

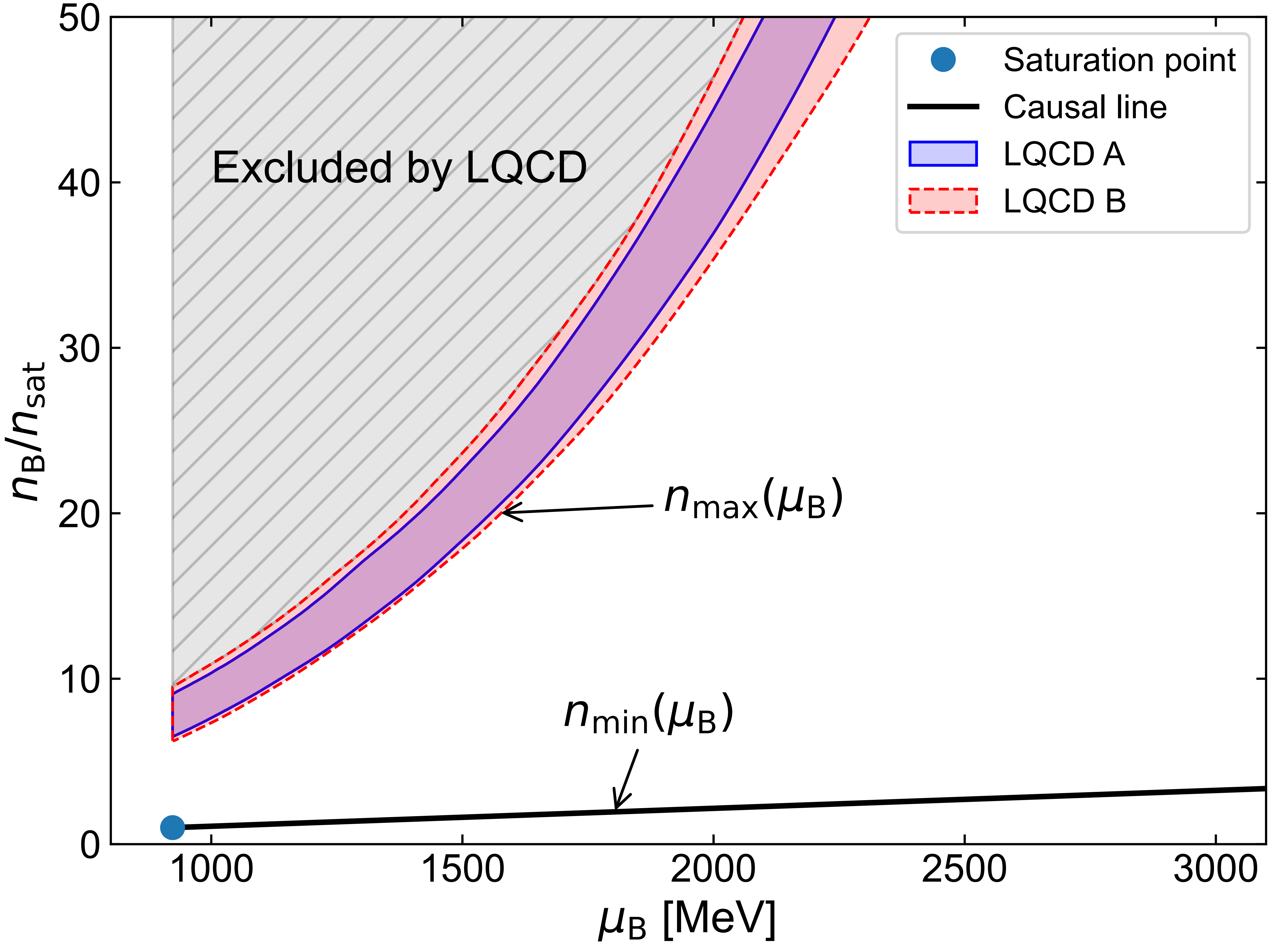
$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller P at a given ε) is excluded

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\mu_{\rm sat}}^{\mu_B} d\mu'\, n_B(\mu') \;\leq P_I\!\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

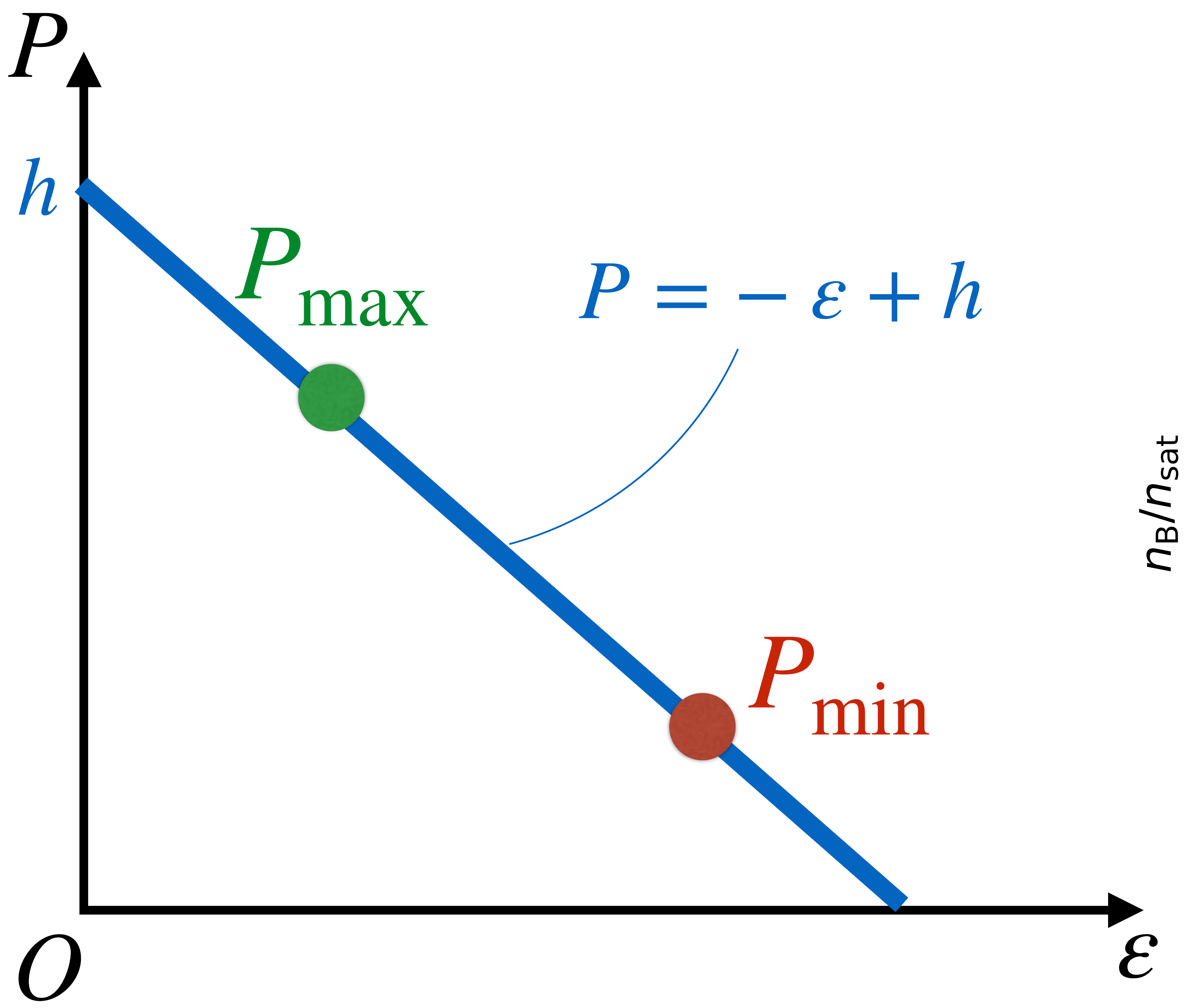
Lower bound of the integral must be specified

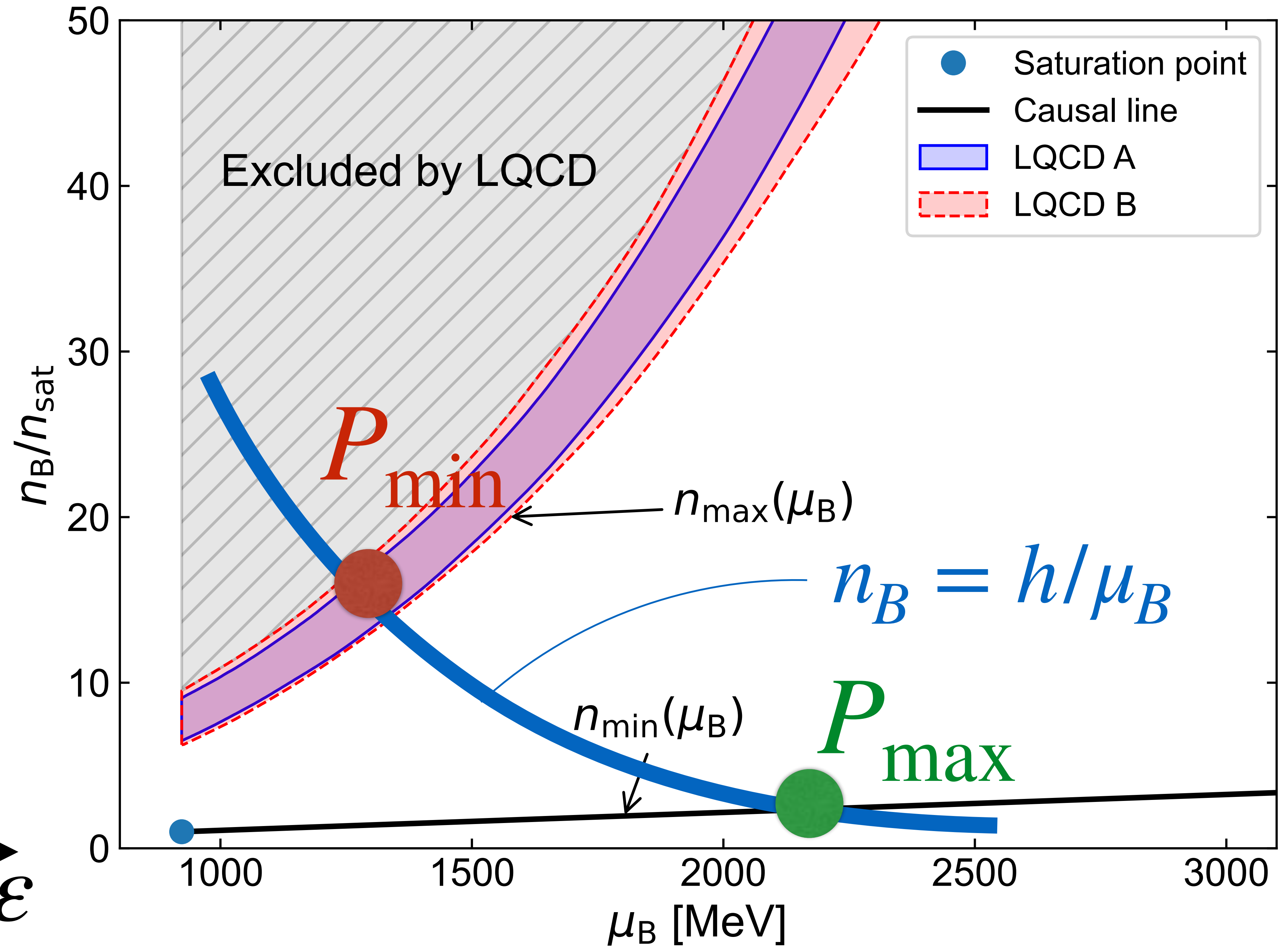
fix it to the **empirical saturation property**

Bounds on $P(\varepsilon)$

Isenthalpic line: $h = \mu_B n_B = \varepsilon + P = \mathrm{const}$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)





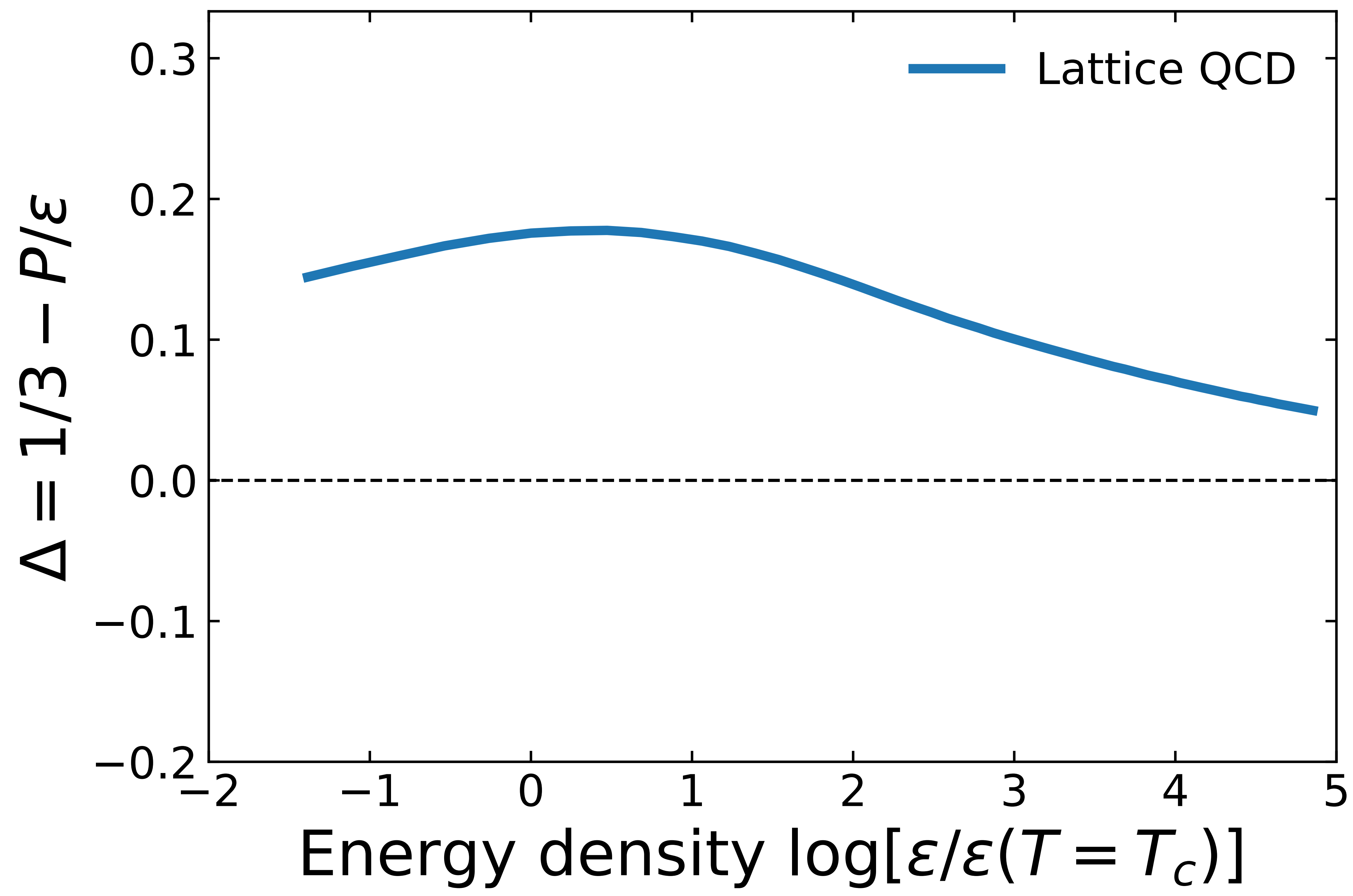
by changing value of h, the trajectories of P_{min} (P_{max}) gives the lower (upper) bound for $P(\varepsilon)$

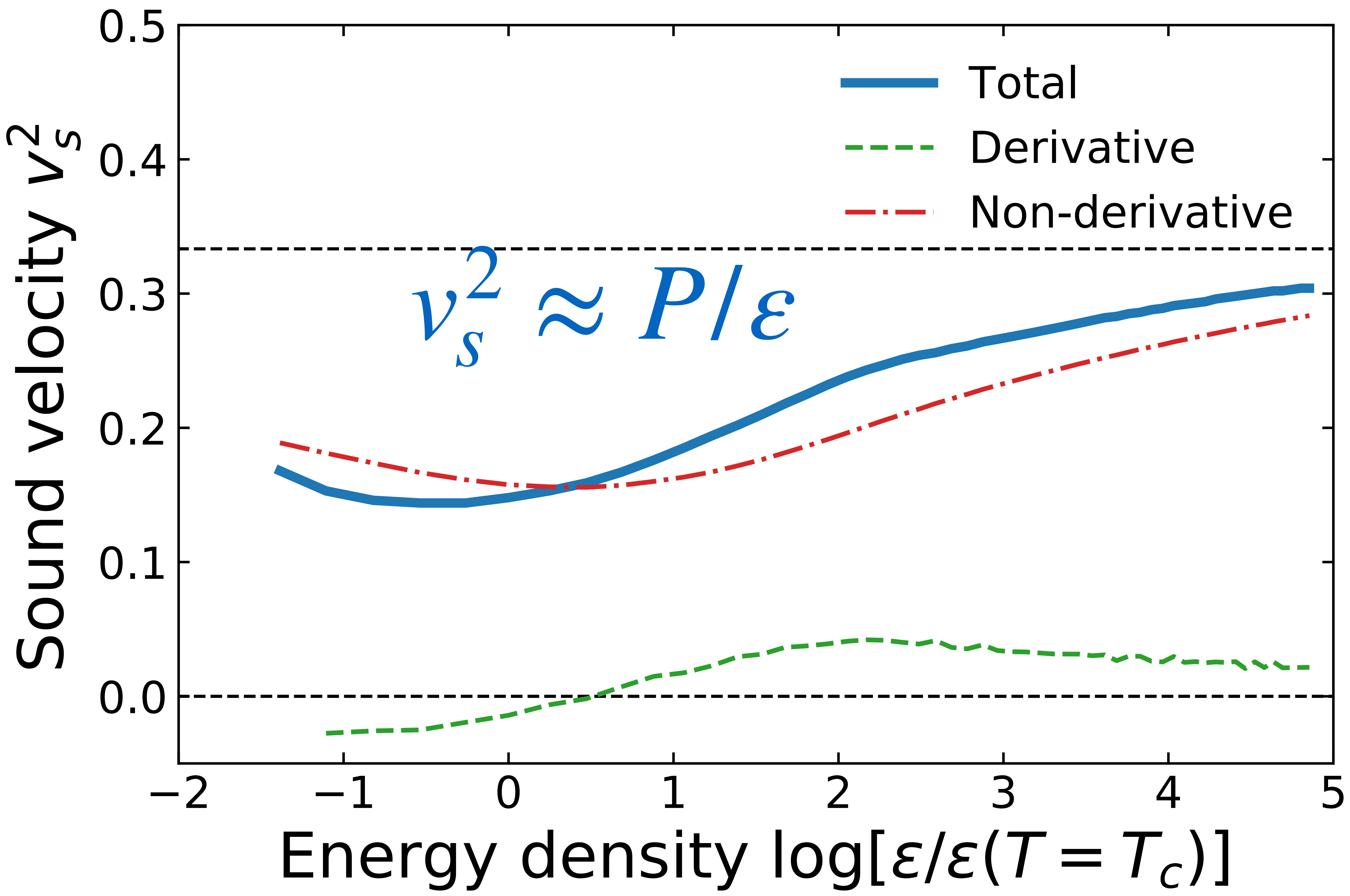
At finite temperature

Trace anomaly $\Delta_{\rm tr} = \dfrac{\varepsilon - 3P}{3\varepsilon}$

Sound velocity $v_s^2 = \varepsilon \dfrac{d\Delta_{\rm tr}}{d\varepsilon} + \left(\dfrac{1}{3} - \Delta_{\rm tr}\right)$

Derivative | Non-derivative





HotQCD (2014)

$\Delta_{\rm tr}$ is slowly varying → Derivative component is irrelevant

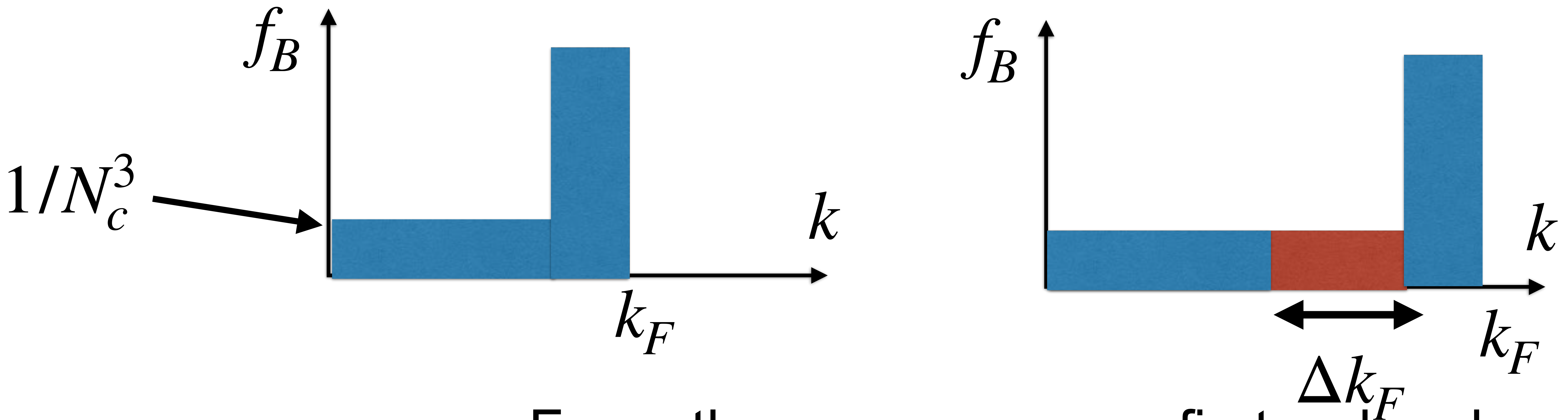
Quarkyonic matter favors large sound speed

Fujimoto,Kojo,McLerran, PRL 132 (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_\mathrm{B}}{\mu_\mathrm{B} dn_\mathrm{B}/d\mu_\mathrm{B}} \to \frac{\delta\mu_\mathrm{B}}{\mu_\mathrm{B}} \sim v_s^2 \frac{\delta n_\mathrm{B}}{n_\mathrm{B}}$$

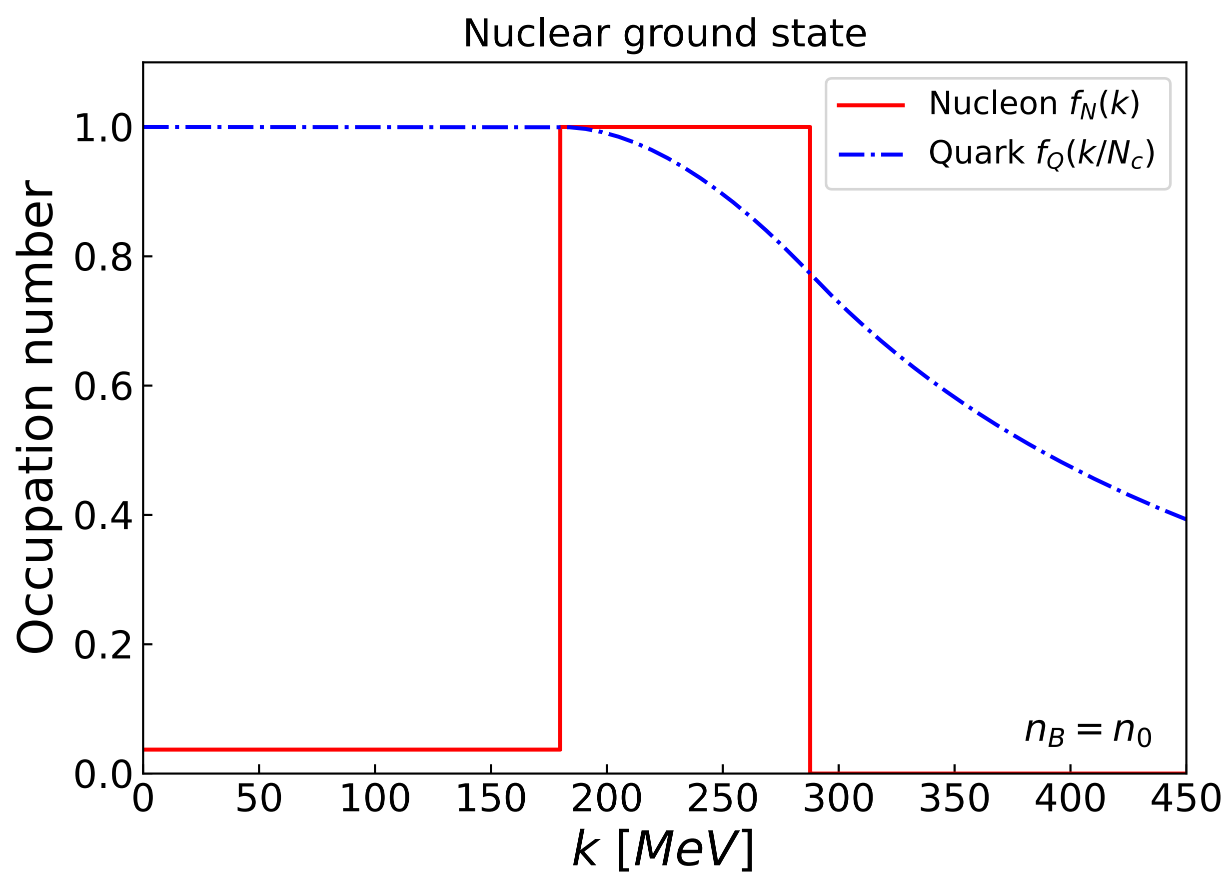
If baryons have underoccupied state, the change in density is small while the change in chemical potential is large

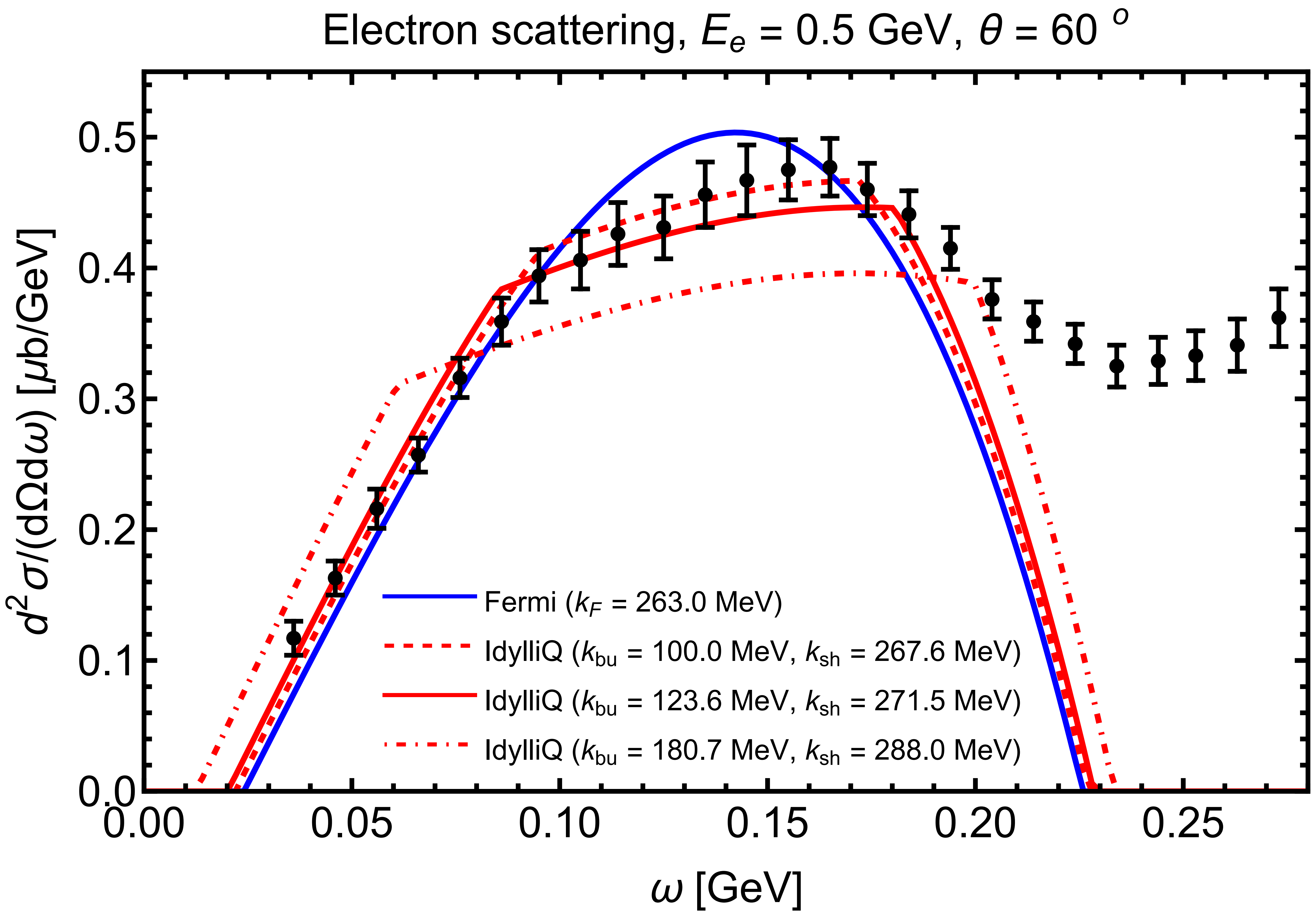


→ Favor the crossover over first-order phase transition ($v_s^2 = 0$)

Possible signature of Quarkyonic matter in experiment

Koch,McLerran,Miller,Vovchenko, PRC 110 (2024)





Suppression in low-momentum part of baryon distribution explains the data well

Possible relation of Quarkyonic matter to the phase diagram

Bluhm, Fujimoto, McLerran, Nahrgang (2024)