Application range of perfect spin hydrodynamics

Valeriya Mykhaylova
Jagiellonian University

In collaboration with Z. Drogosz, W. Florkowski, S. Bhadury, S. Kar

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JAGIELLONIAN UNIVERSITY IN KRAKÓW

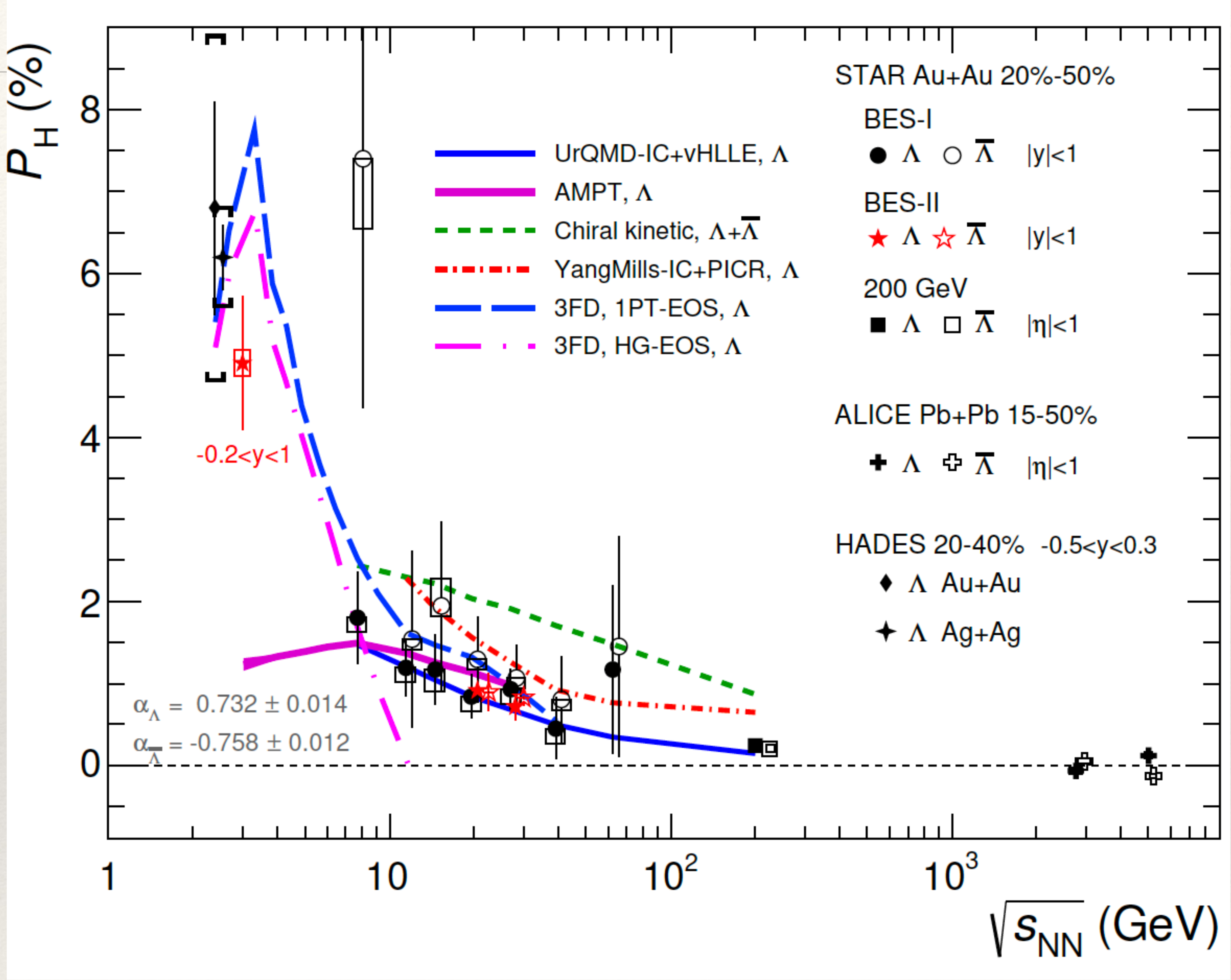
NATIONAL SCIENCE CENTRE POLAND

Spin hydrodynamics

- Relativistic hydro → data interpretation
- ‘Spin’ measurements → hydrodynamics + spin
- <u>Various theoretical methods:</u> density operators, entropy principle, grad of hydrodynamic fields, Lagrangian effective theories …

  (see works of V. Ambrus, F. Becattini, W. Florkowski, D. Rischke, R. Ryblewski, E. Speranza, …)

- Our formalisms: Kinetic theory & Wigner function
- One of the issues: <u>applicability range.</u>



Global spin polarization o Λ hyperons

[Niida, Voloshin, Int. J. Mod. Phys. E 33 (2024)]

Kinetic theory — classical spin description

Local equilibrium distribution in spin-extended phase space

$$f^{\pm}(x,p,s) = \exp\left(-p_\mu \beta^\mu \pm \frac{\mu}{T} + \frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right), \quad \beta^\mu = \frac{u^\mu}{T}, \qquad u'^\mu = (1,\vec{0})$$

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spin chemical potential

$$\omega_{\mu\nu} = \frac{\Omega_{\mu\nu}}{T}$$

Spin polarization tensor

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Electric- and magneticlike vectors:

$$\vec{e} = (e^1, e^2, e^3), \quad \vec{b} = (b^1, b^2, b^3)$$

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spin vector

$s \perp p$

spin chemical potential

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Spin polarization tensor

$$s^{\mu\nu} = \frac{1}{m}\epsilon^{\mu\nu\alpha\beta} p_\alpha s_\beta$$

internal agular momentum

[Mathisson, Acta Phys. Polon. 6, 163 (1937)]

$$s_\beta = (0,\vec{\mathfrak{s}})$$

$$\mathfrak{s}^2 = 3/4$$

eigenvalue of the Casimir operator in SU(2)

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Physical quantities obtained from

$$\int dP \int dS\, f^{\pm}(x,p,s) \;\; \cdots$$

Perfect spin hydro: spin tensor conserved — no spin-orbit coupling, can be added by dissipation

[Florkowski, Hontarenko, PRL 134 '25; Drogosz, Florkowski, Hontarenko PRD 110 '25]

Scalar density in kinetic theory

classical spin

Tensors can be obtained by taking derivatives w.r.t. β^μ and $\omega_{\mu\nu}$

$$n = \int dP \int dS \exp\left(-p_\mu \beta^\mu + \frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right) = \int dP \exp\left(-p_\mu\beta^\mu\right) \int dS \exp\left(\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right)$$

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$$\int dS \exp\left(\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right) = \frac{2\sinh(\mathfrak{s}\,|\vec{b}_*|)}{\mathfrak{s}\,|\vec{b}_*|} \qquad \text{with} \qquad \vec{b}_* = \frac{1}{m}\left(E_p\vec{b} - \vec{p}\times\vec{e} - \frac{\vec{p}\cdot\vec{b}}{E_p+m}\vec{p}\right)$$

Spin measure

$*$ particle rest frame

$$\mathfrak{s}^2 = 3/4$$

eigenvalue of the Casimir operator in SU(2)

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Spin measure

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eigenvalue of the Casimir operator in SU(2)

$$\boxed{n = 2\int dP' \exp\left(-\frac{E_{p'}}{T}\right)\frac{\sinh(\mathfrak{s}\,|\vec{b}'_*|)}{\mathfrak{s}\,|\vec{b}'_*|}}$$

local rest frame

$u'^\mu = (1,\vec{0})$

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divergence possible!

Criterion of convergence

$$\boxed{n = 2\int dP' \exp\left(-\frac{E_{p'}}{T}\right)\frac{\sinh(\mathfrak{s}\,|\,\overrightarrow{b'_*}\,|)}{\mathfrak{s}\,|\,\overrightarrow{b'_*}\,|}}$$

$*$ particle rest frame

$'$ local (fluid's) rest frame

$$\vec{b}' = \gamma\,(\vec{b} - \vec{v}\times\vec{e}) - \frac{\gamma^2}{\gamma+1}\vec{v}(\vec{v}\cdot\vec{b})$$

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$$\tau \equiv -\frac{E_{p'}}{T} + \mathfrak{s}\,|\,\overrightarrow{b'_*}\,|, \qquad \max(\tau) < 0$$

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For large $|\overrightarrow{p'}|$: $$\tau = -\frac{|\vec{p}'|}{T} + \mathfrak{s}\frac{|\vec{p}'|}{m}|\vec{b}' - \hat{p}'\times\vec{e}' - (\hat{p}'\cdot\vec{b})p|$$

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$$\mathfrak{s}^2 = 3/4$$

$$\boxed{\mathfrak{s}\sqrt{\overrightarrow{b'}^2 + \overrightarrow{e'}^2 + 2|\overrightarrow{e'}\times\overrightarrow{b'}|} < \frac{m}{T}}$$

$\vec{b}, \vec{e}$ — components of $\omega^{\mu\nu}$

spin polarization tensor

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see also the result of Abboud, Gavassino, Singh, Speranza, arXiv:2506.19786

Local equilibrium Wigner function

NEW

$$W^{\pm}(x,k) = \pm\frac{1}{2}\int \mathrm{d}P\,\delta^{(4)}(k \mp p)(p_\mu\gamma^\mu \pm m)X^{\pm}(x,p)$$

$$X^{\pm}(x,p) = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu + \gamma_5 a_\mu\gamma^\mu\right], \qquad a_\mu(x,p) = -\frac{1}{4m}\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}(x)p^\nu$$

Differs from widely used version by Becattini, Chandra, Del Zanna, Grossi, Annals Phys. 338 '13

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- Accurate definition of the mean polarization vector.
- Spin density matrix consistent with Landau-Lifshitz.

[Drogosz, arXiv:2509.0601]

- Conserved currents agree with kinetic theory at any order in ω up to numerical factor.
- Perfect spin hydro is non-linearly causal and its equations are symmetric hyperbolic.

[S.Bhadury,Z.Drogosz, W.Florkowski, S.Kar,V.M. 2505.02657]

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traces over the spinor space

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$$-\omega_{\max} \leq e^i,\, b^i \leq \omega_{\max}$$

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$$\boxed{\omega_{\max}\sqrt{\frac{1+v}{1-v}} < \frac{2}{2+\sqrt{2}}\,\frac{m}{T}}$$

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Applicability range?

longitudinal relativistic Doppler effect

$$\boxed{\omega_{\max}\sqrt{\frac{1+v}{1-v}} < \frac{2}{2+\sqrt{2}}\frac{m}{T}}$$

$$-\omega_{\max} \leq e^i,\ b^i \leq \omega_{\max}$$

Local equilibrium Wigner function

Criteria of convergence

$$\boxed{\frac{1}{2}\sqrt{\overrightarrow{b'}^2 + \overrightarrow{e'}^2 + 2|\overrightarrow{e'} \times \overrightarrow{b'}|} < \frac{m}{T}}$$

$$\boxed{\omega_{\max}\sqrt{\frac{1+v}{1-v}} < \frac{2}{2+\sqrt{2}}\frac{m}{T}}$$

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More restrictive: $(\vec{b}, \vec{v}, \vec{e}) \perp$ to each other,

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$$|\vec{b}| = |\vec{e}|, \ (\vec{v} \times \vec{e}) \parallel \vec{b}$$

but easier to apply: need $\omega_{\max}$, v only

vs. $\vec{b} = (b_1, b_2, b_3)$, $\vec{e} = (e_1, e_2, e_3)$, $\vec{v} = (v_1, v_2, v_3)$

Variety of the application ranges

Variables	Kinetic theory – classical spin	Wigner function — quantum spin
$\omega_{\max},\ v$	$\mathfrak{s}\,\omega_{\max}\left(2+\sqrt{2}\right)\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$	$\frac{1}{2}\,\omega_{\max}\left(2+\sqrt{2}\right)\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$
$\vec{e},\ \vec{b},\ v$	$\mathfrak{s}\sqrt{\vec{b}^2+\vec{e}^2+2\,\lvert\vec{e}\times\vec{b}\rvert}\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$	$\frac{1}{2}\sqrt{\vec{b}^2+\vec{e}^2+2\,\lvert\vec{e}\times\vec{b}\rvert}\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$
$\vec{e},\ \vec{b},\ \vec{v}$	$\mathfrak{s}\sqrt{\overrightarrow{b'}^2+\overrightarrow{e'}^2+2\,\lvert\overrightarrow{e'}\times\overrightarrow{b'}\rvert} < \frac{m}{T}$	$\frac{1}{2}\sqrt{\overrightarrow{b'}^2+\overrightarrow{e'}^2+2\,\lvert\overrightarrow{e'}\times\overrightarrow{b'}\rvert} < \frac{m}{T}$

in local rest frame

The bound depends on the „level" of given variables

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$\omega_{\max}$, v	$\mathfrak{s}\,\omega_{\max}\left(2+\sqrt{2}\right)\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$	$\frac{1}{2}\omega_{\max}\left(2+\sqrt{2}\right)\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$
$\vec{e}$, $\vec{b}$, v	$\mathfrak{s}\sqrt{\vec{b}^2+\vec{e}^2+2\lvert\vec{e}\times\vec{b}\rvert}\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$	$\frac{1}{2}\sqrt{\vec{b}^2+\vec{e}^2+2\lvert\vec{e}\times\vec{b}\rvert}\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$
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in local rest frame

The bound depends on the „level" of given variables

$\mathfrak{s}^2 = 3/4$

Practical applicability

Perfect spin hydro with conserved spin tensor $\omega_{\mu\nu}$ $\rightarrow$ last stages of HIC / HIC at moderate energies.

$$\frac{1}{2}\omega_{\max}\left(2+\sqrt{2}\right)\sqrt{\frac{1+v}{1-v}} < \frac{m}{T}$$

$$\omega_{\max} < 0.2, \quad m = 0.4 - 1.1 \ \text{GeV}, \quad T = 0.15 \ \text{GeV}, \quad v = 0.5$$

$$m/T \approx 3\text{-}10, \quad \sqrt{(1+v)/(1-v)} \approx 1.7$$

Values of ω cannot be arbitrary — like $\mu < m$ for bosons

Abboud, Gavassino, Singh, Speranza, arXiv:2506.19786

Summary

- Proper formulation of the local equilibrium Wigner function for particles with spin 1/2 — long-standing problem solved — principal tool in spin hydrodynamics.
- Criteria constraining the application of perfect spin hydro have been defined.
- Our perfect spin hydro is non-linearly causal and its equations are symmetric hyperbolic.