Peak of the Speed of Sound in QCD Matter: Violation of the Conformal Limit at Finite Density

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Erice: INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS, 46th COURSE

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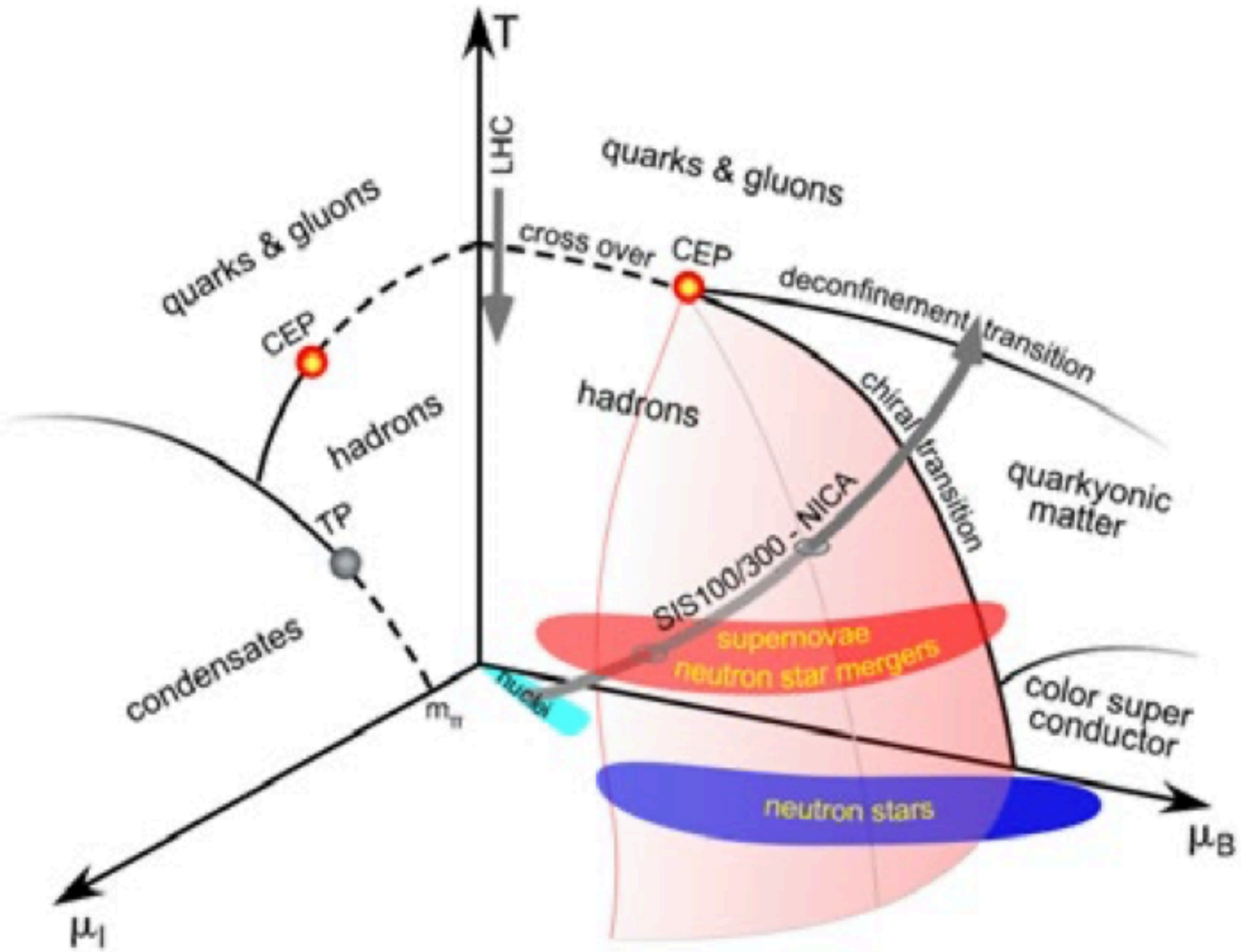
Outline

- Motivation
- Effective models of **QCD** under extreme conditions X LQCD.
- Role of the Medium Separation Scheme (MSS).
- How a isospin asymmetry can influence the phase diagram of **QCD**?
- Conclusions and perspectives

QCD Phase Diagram



QCD Phase Diagram: Open Challenges

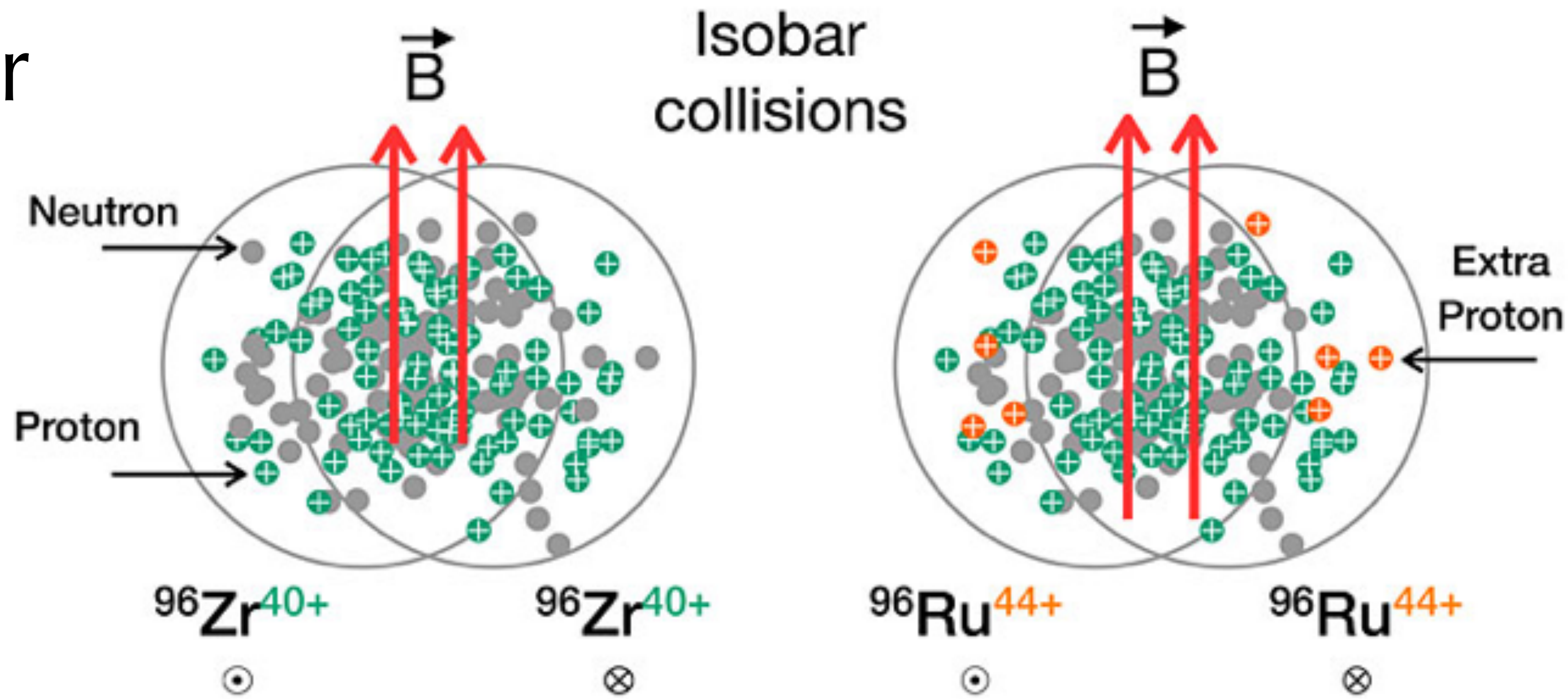
- Fundamental parameters: μB, μI, T, B, …
- EoS → key to strongly interacting matter
- Crucial for:
  - # Early Universe
  - # Neutron stars & supernovae
- Probed by:
  - # Heavy-ion collision
  - # Nuclear structure
  - # Gravitational waves (BH–NS, NS–NS mergers)
- Many aspects still unknown!

Nuclear Physics European Collaboration Committee Long Range Plan 2017

Motivation

Why relevant?

- In RHIC isobar program



- Excess of neutrons over protons

- Neutron star interiors

$$n_I = n_u - n_d$$

Brookhaven Lab, RHIC isobar collisions — "Search for Chiral Magnetic Effect", 31 Aug 2021

Isospin chemical potential

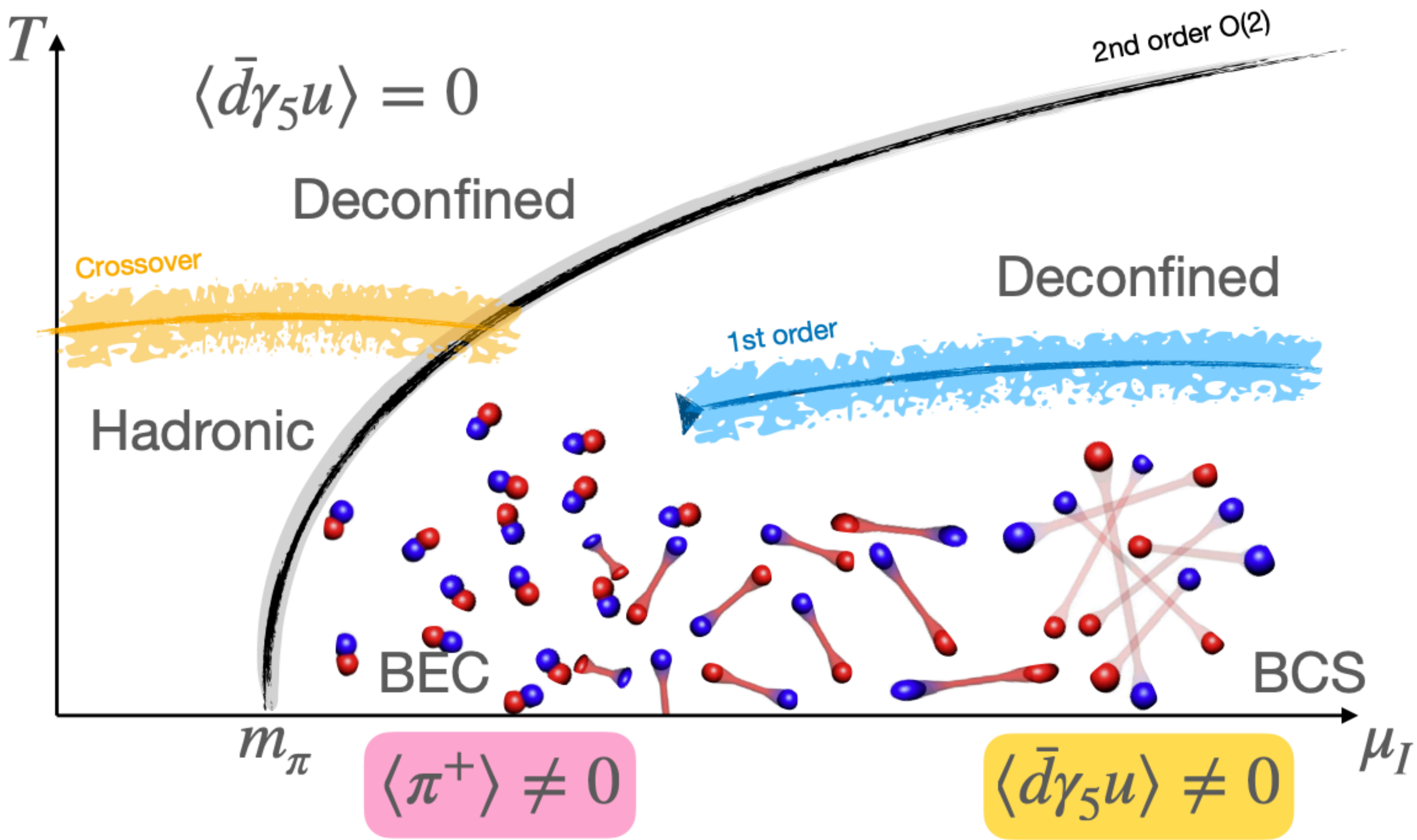
- Grand canonical ensemble
- Chemical potentials $$\boxed{\begin{aligned} \mu_u &= \frac{\mu_B}{3} + \mu_I \\ \mu_d &= \frac{\mu_B}{3} - \mu_I \end{aligned}}$$
- Zero baryon number: $$\boxed{\begin{array}{ccc} \mu_u & = & \mu_I \\ \mu_d & = & -\mu_I \end{array}}$$

$$n_I = n_u - n_d$$

QCD at $\mu_I \neq 0$ & $\mu_B = 0$

Conjectured phase diagram

- Son & Stephanov PRL 2001



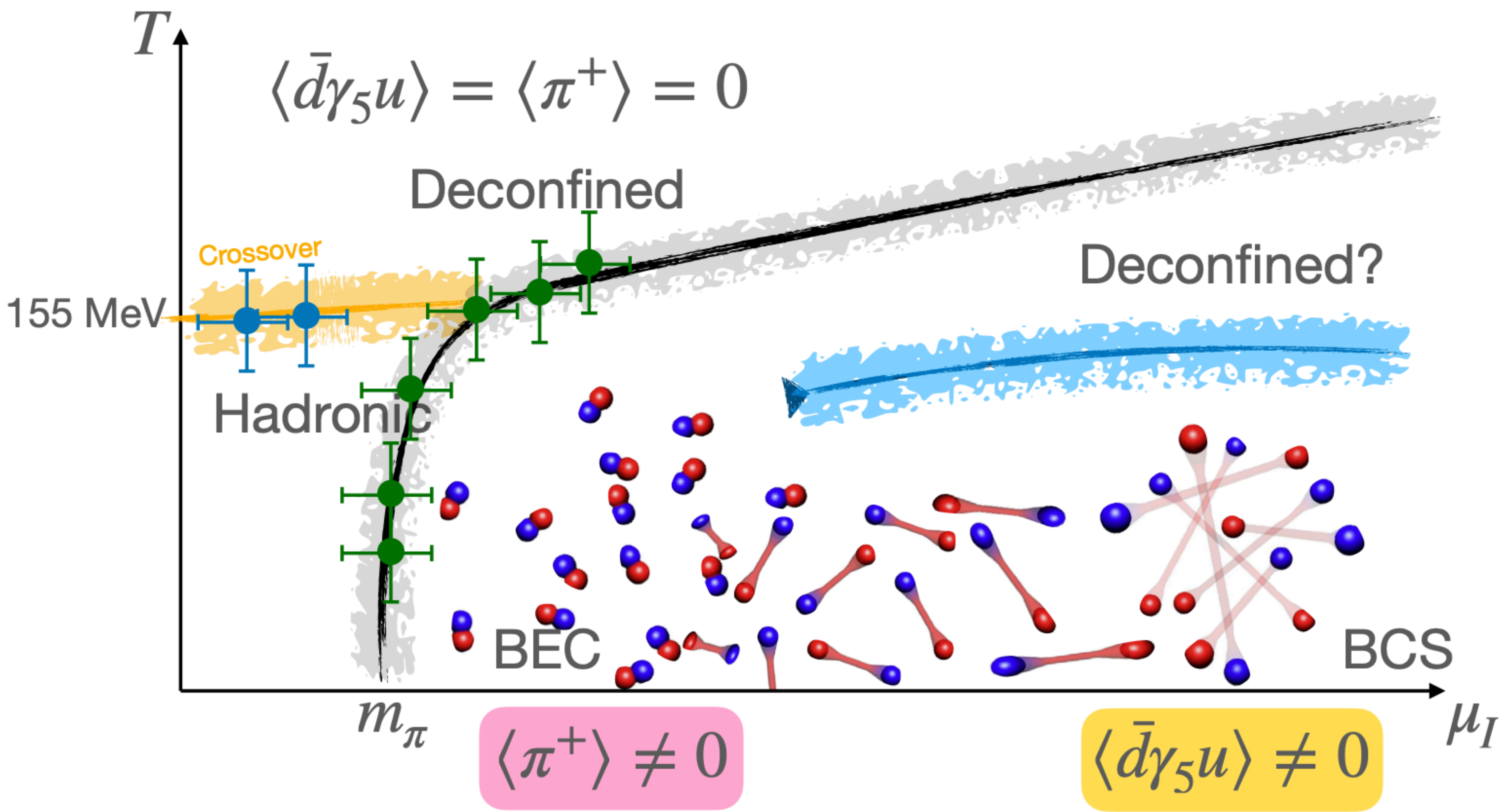
Will Detmold presentation @ Lattice and Continuum Approaches to Strongly Coupled QFT (2025)

QCD at $\mu_I \neq 0$ & $\mu_B = 0$

Conjectured phase diagram

- Status in 2023 - much recent work from Brandt, Cuteri & Endrodi

JHEP 07 (2023) 055 • e-Print: 2212.14016 [hep-lat]



Will Detmold presentation @ Lattice and Continuum Approaches to Strongly Coupled QFT (2025)

Conformal Limit violation: First evidence

PHYSICAL REVIEW LETTERS

Editors' Suggestion

Sound Velocity Bound and Neutron Stars

Paulo Bedaque and Andrew W. Steiner
Phys. Rev. Lett. **114**, 031103 – Published January 2015

If the bound on the speed of sound is actually violated – as it is strongly suggested by our results– the speed of sound, as a function of the energy density, has a peculiar shape. It raises from small values, reaches a maximum with $v_s^2 > 1/3$, lowers to a local minimum with $v_s^2 < 1/3$ and then raises again approaching $v_s^2 = 1/3$ from below at high densities. We find remarkable that such a conclusion can be derived from well established facts.

GW constraints

GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

GW190425

- $R_{M=1.4M_\odot} \leq 15$ km and M_{max} $(2-3)M_\odot$

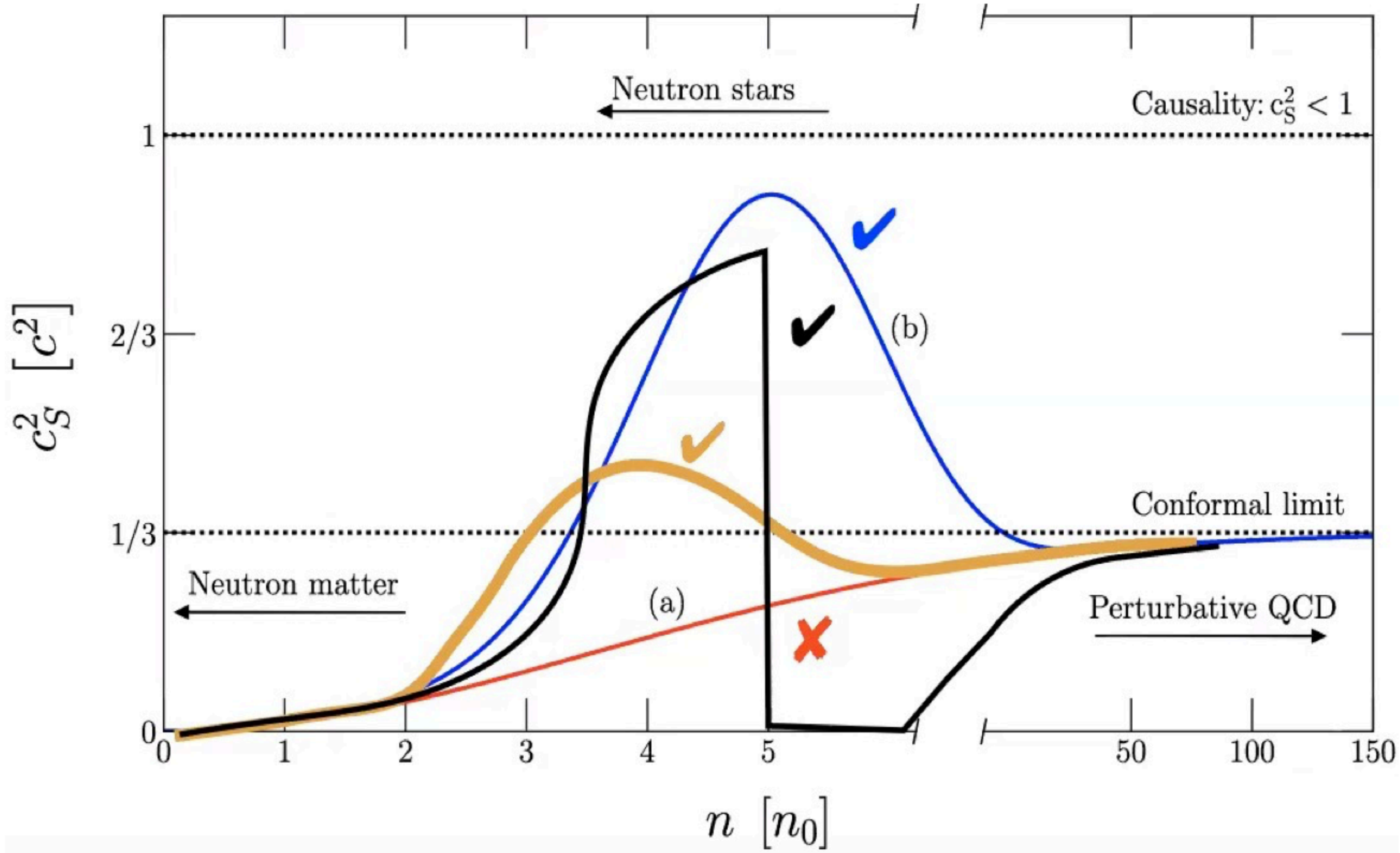


Fig. from S. Reddy presentation @BNL (2021)

GW constraints

GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

Breakdown of the models

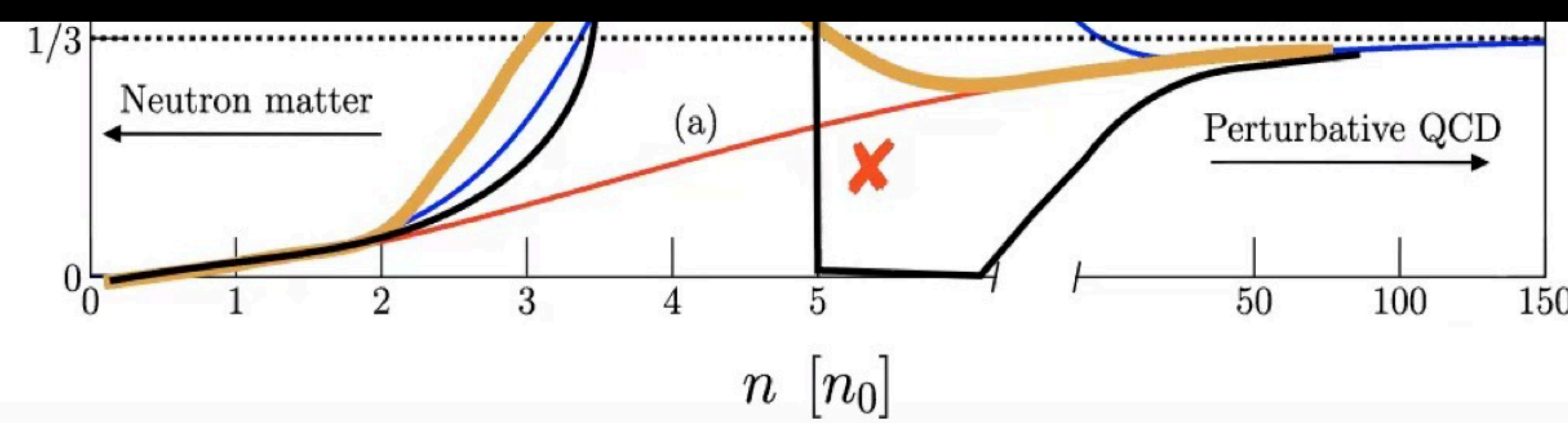


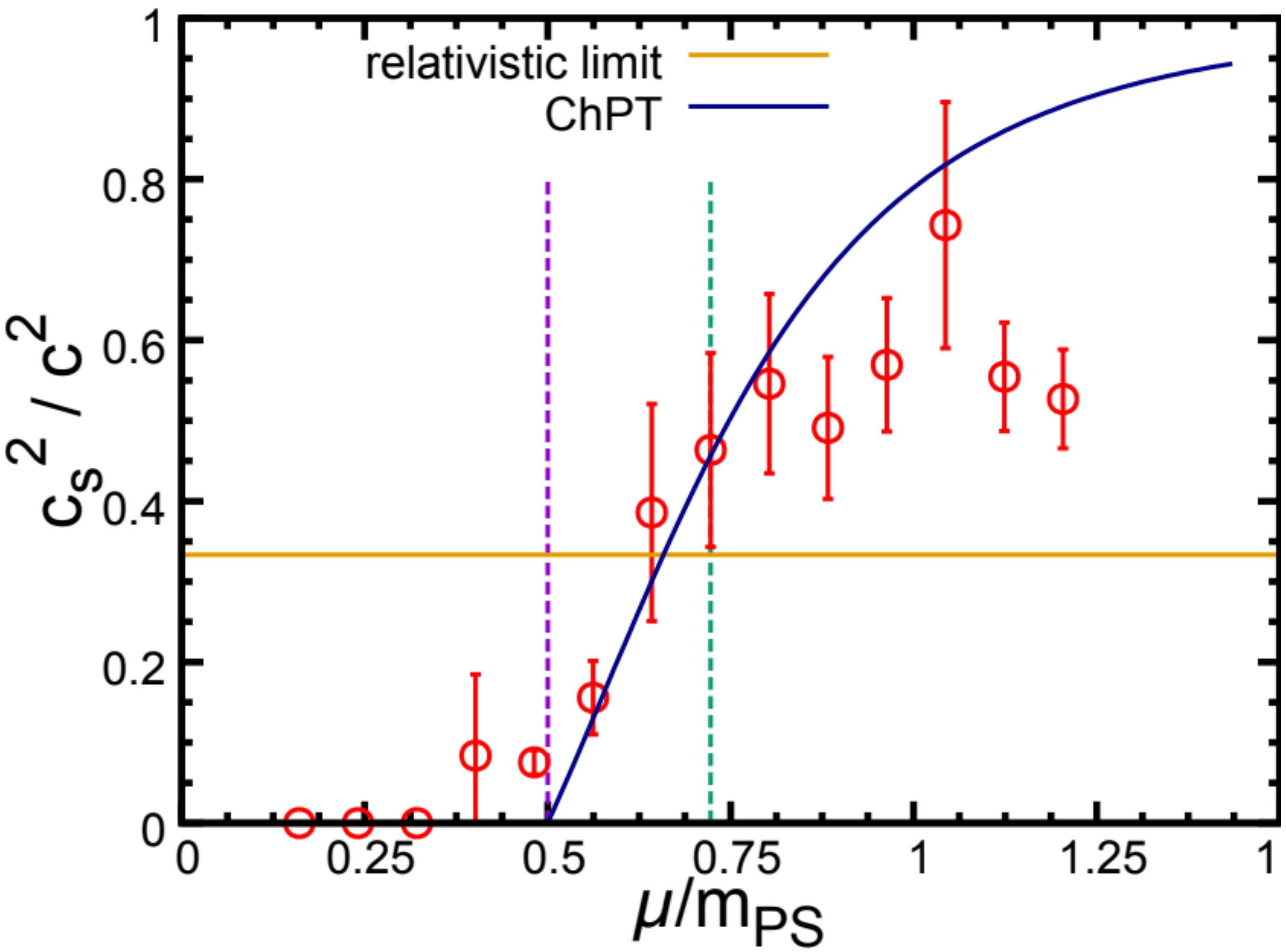
Fig. from S. Reddy presentation @BNL (2021)

Insights from LQCD

2 color QC2D lattice simulations

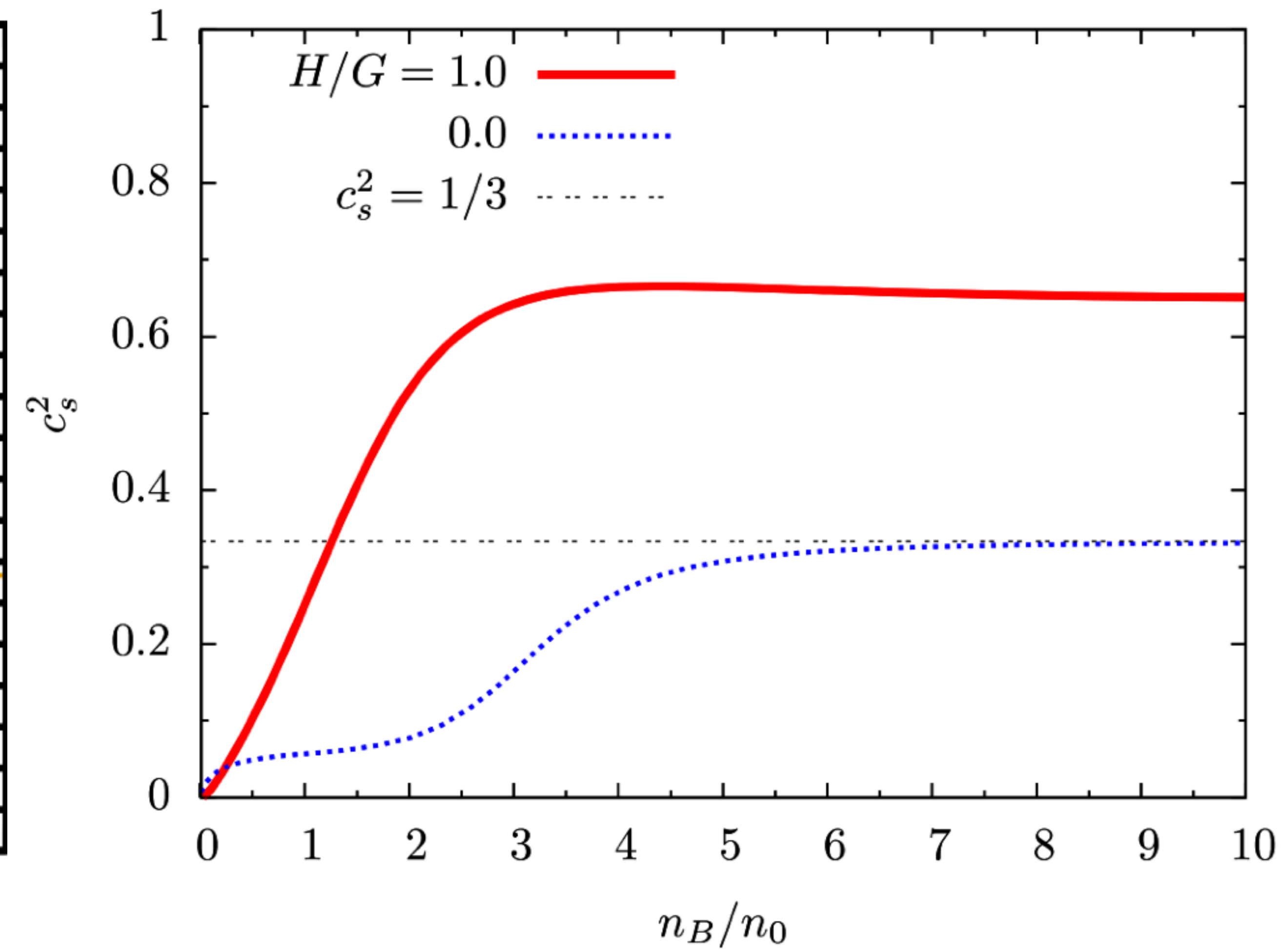
Bump of sound velocity in dense

2-color QCD



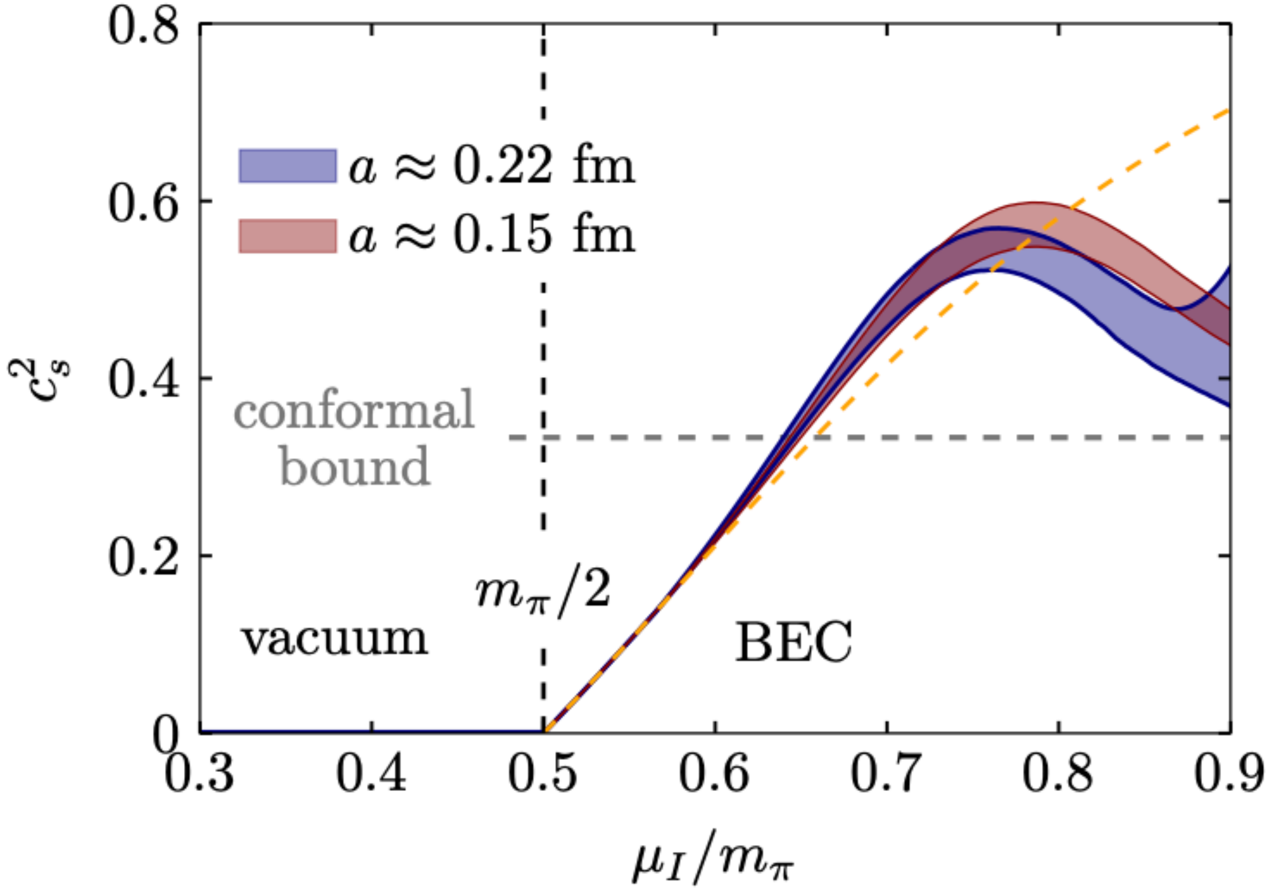
K. Lida and E. Itou, *PTEP* 2022 (2022) 11, 111B01
K. Lida, E. Itou, K. Murakami, D. Suenaga, *JHEP* 10 (2024) 022

2-color NJL model

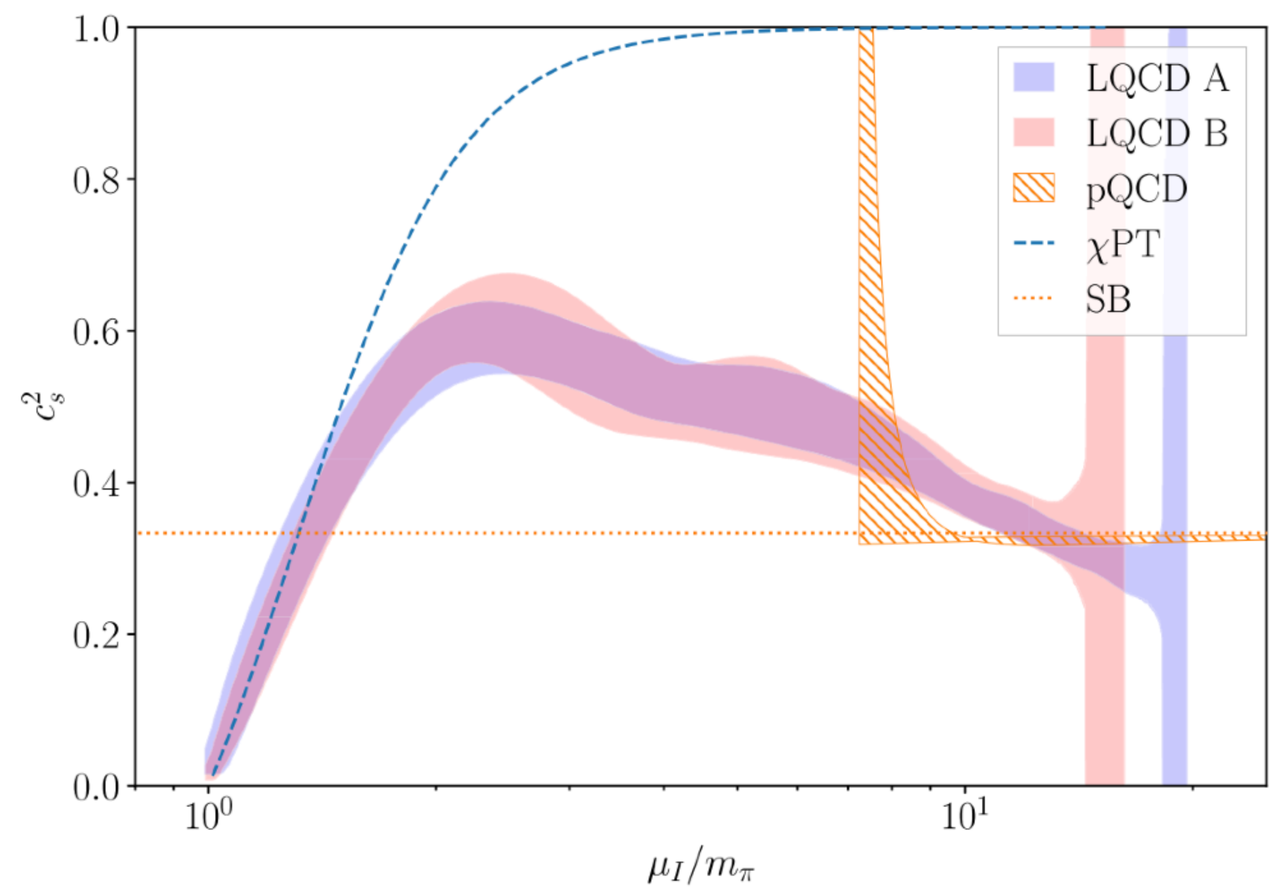


T. Kojo and D. Suenaga, PRD **105**, 076001 (2022)

Recent LQCD results: $\boxed{T = 0,\ \mu_B = 0 \text{ and } \mu_I \neq 0}$



B.B. Brandt, F. Cuteri, G. Endrodi, JHEP 07 (2023) 055

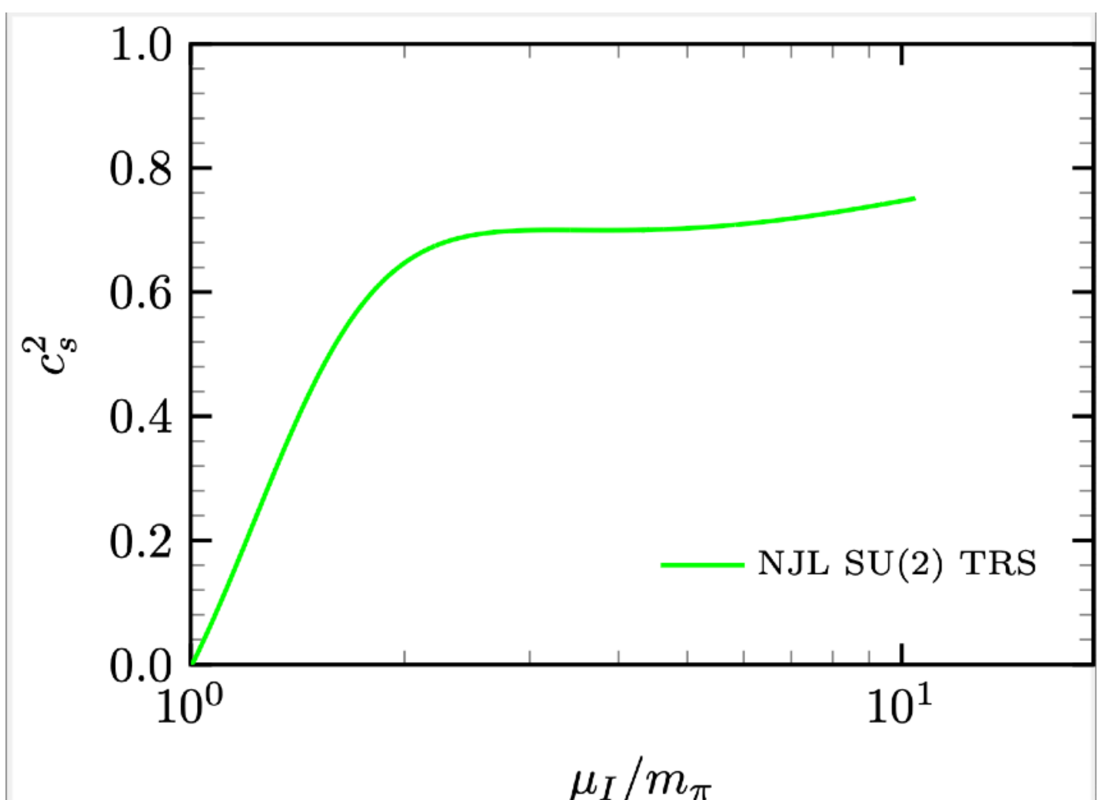


PHYSICAL REVIEW D **108,** 114506 (2023)

Lattice quantum chromodynamics at large isospin density

Ryan Abbott,[1,2,*] William Detmold,[1,2] Fernando Romero-López,[1,2] Zohreh Davoudi,[3,4] Marc Illa,[5] Assumpta Parreño,[6] Robert J. Perry,[6] Phiala E. Shanahan,[1,2] and Michael L. Wagman[7]

(NPLQCD Collaboration)



PHYSICAL REVIEW LETTERS **134,** 011903 (2025)

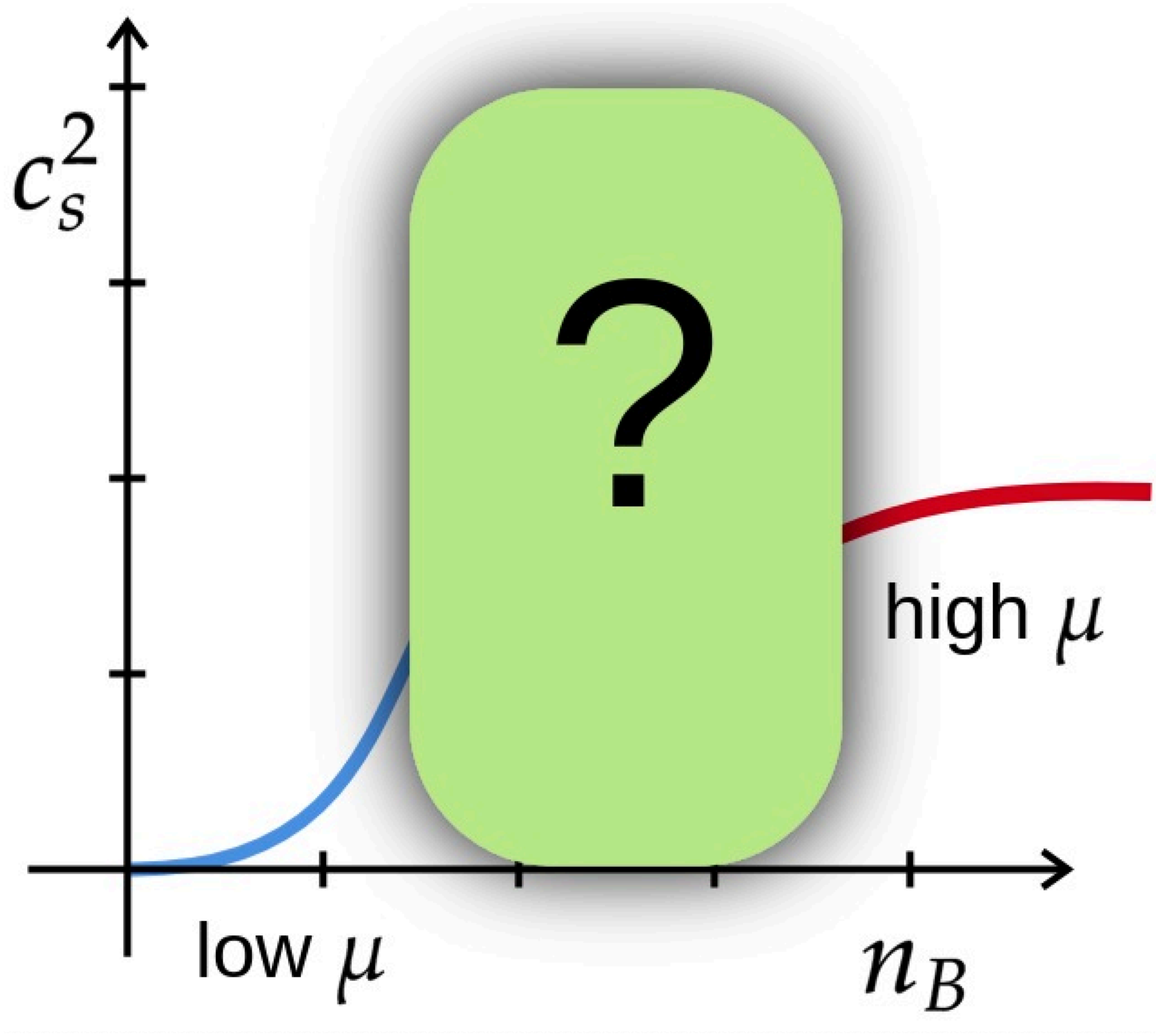
Editors' Suggestion | Featured in Physics

QCD Constraints on Isospin-Dense Matter and the Nuclear Equation of State

Ryan Abbott,[1,2] William Detmold,[1,2] Marc Illa,[3] Assumpta Parreño,[4] Robert J. Perry,[4] Fernando Romero-López,[1,2] Phiala E. Shanahan,[1,2] and Michael L. Wagman[5]

(NPLQCD Collaboration)

To make progress



SU(2) NJL model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

Needs regularization!

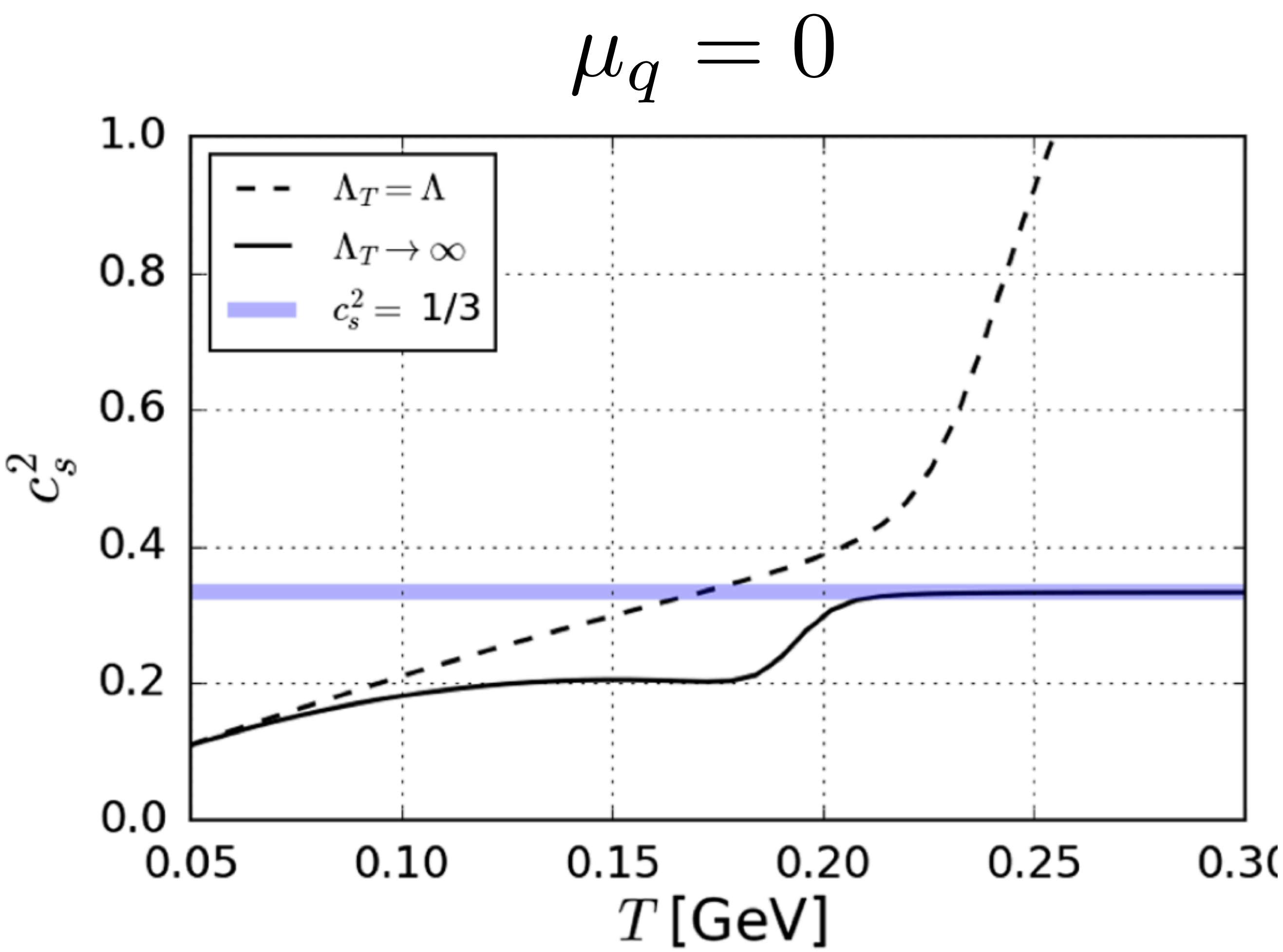
Regularize vacuum contributions

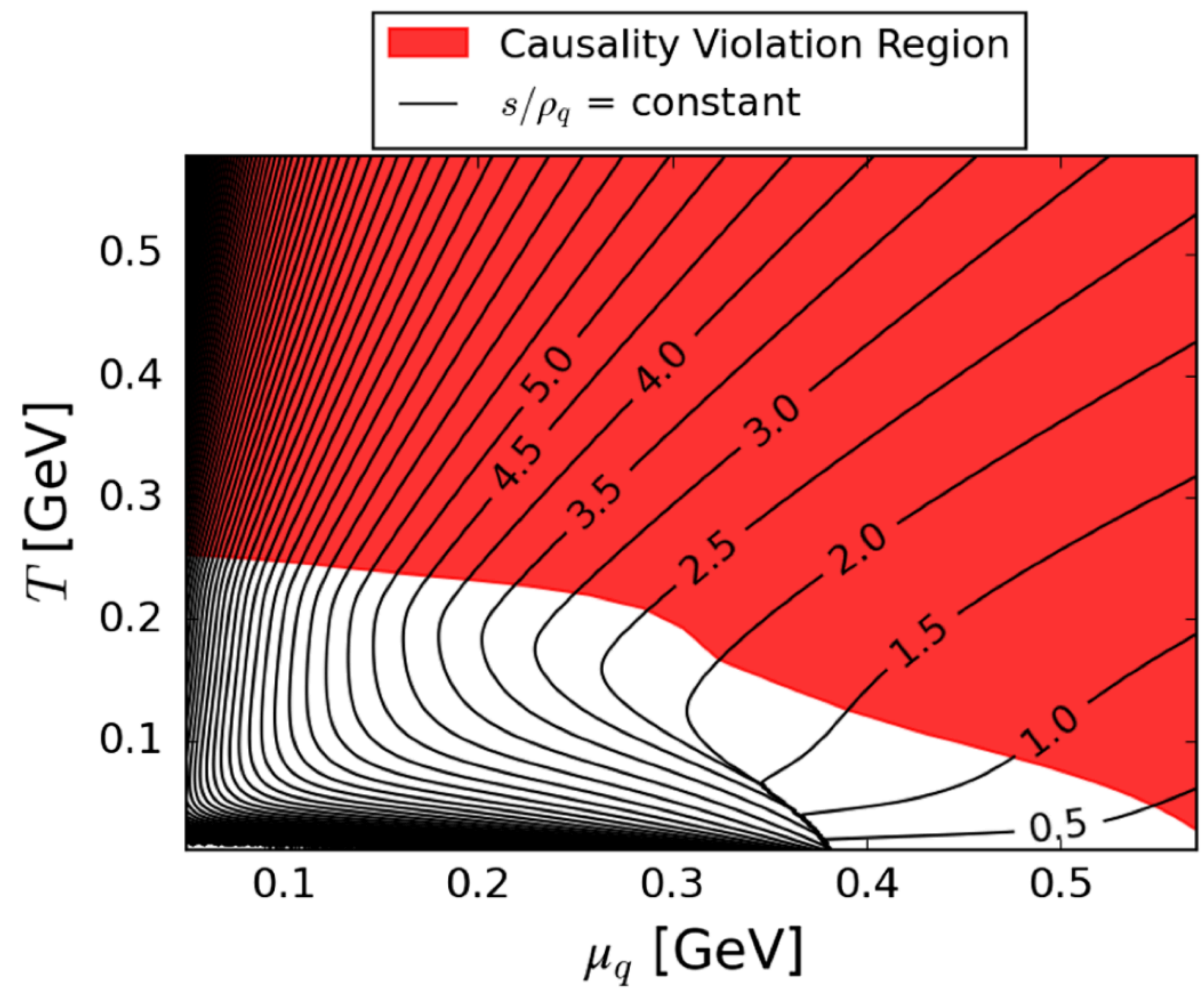
DO NOT regularize medium contributions

$$\begin{aligned}\Omega(T,\mu_q) \quad &= \frac{(M-m_0)}{4G} - 2N_cN_f\int_\Lambda \frac{d^3k}{(2\pi)^3}\,\omega(k) \\ &-4N_cN_fT\int_{\boxed{\Lambda_T}} \frac{d^3k}{(2\pi)^3}\left[\log\left(1+e^{-(\omega(k)+\mu_q)/T}\right)\right. \\ &\left.+\log\left(1+e^{-(\omega(k)-\mu_q)/T}\right)\right]\end{aligned}$$

where $\omega(k) = (k^2+M^2)^{1/2}$

Causality violation





RLSF, A.E.B.Pasqualotto, W.R. Tavares, S. Avancini and G. Krein, Phys. Rev. D 107, 096017 (2023)

Medium Separation Scheme

- Let's suppose an arbitrary energy relation ω including some chemical potential μ and function(s) of a condensate(s) ⟨...⟩:

$$\omega(k) = \sqrt{[f(\vec{k}) \pm \mu]^2 + \langle \cdots \rangle}$$

<u>Chiral chemical potential:</u>

$$\mu \to \mu_5$$
$$\langle \cdots \rangle \to M_q(\sim \langle \bar{q}q \rangle)$$
$$\omega(k) = \sqrt{(|\vec{k}| \pm \mu_5)^2 + M_q^2}$$

<u>Isospin chemical potential:</u>

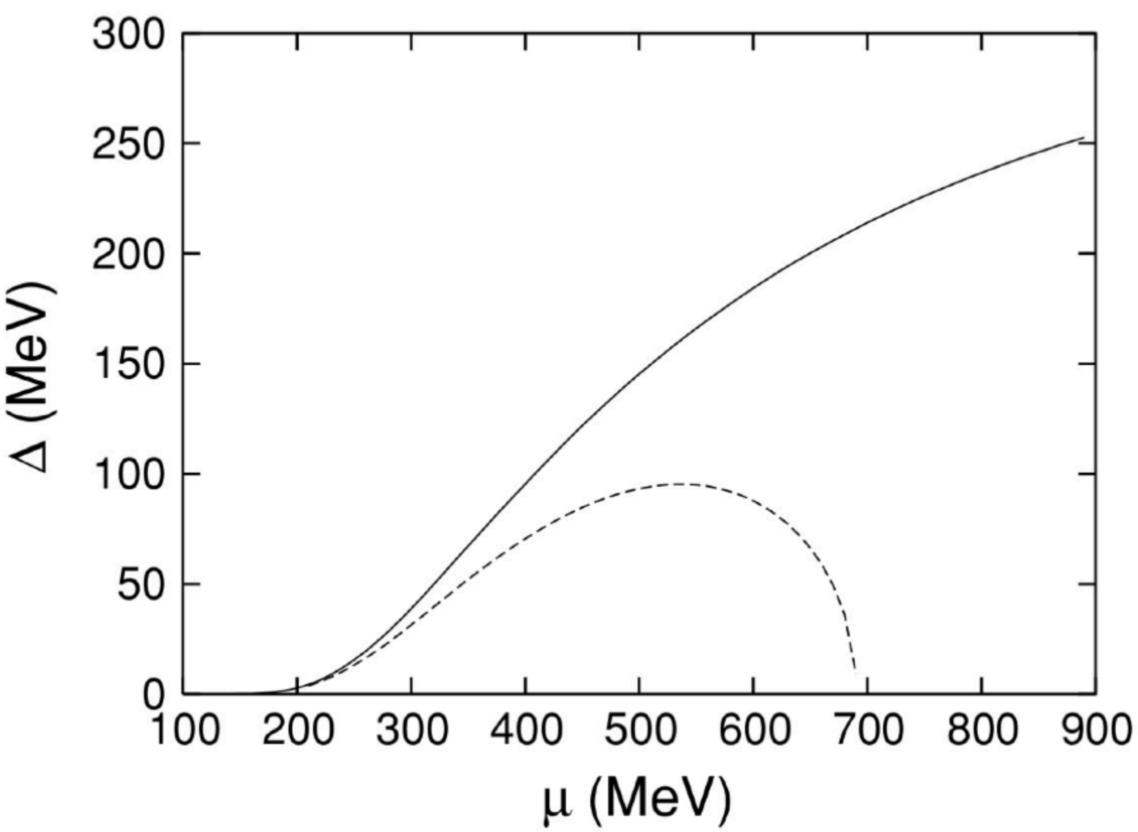
$$\mu \to \mu_I$$
$$\langle \cdots \rangle \to \Delta_\pi(\sim \langle \bar{q} i \gamma_5 \tau_\pm q \rangle)$$
$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_I\right)^2 + \Delta_\pi^2}$$

<u>Quark chemical potential:</u>

$$\mu \to \mu_q$$
$$\langle \cdots \rangle \to \Delta_c(\sim \langle qq \rangle)$$
$$\omega(k) = \sqrt{\left(\sqrt{k^2 + M_q^2} \pm \mu_q\right)^2 + \Delta_c^2}$$

2SC CSC

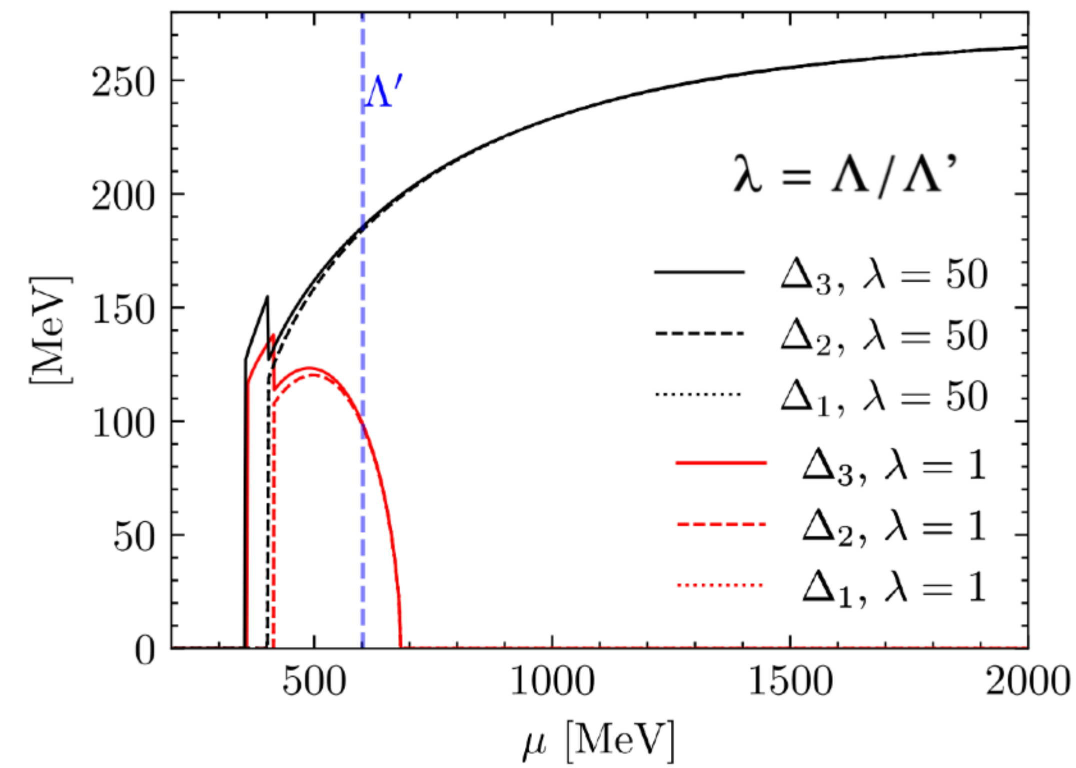
NJL+MSS X NJL + RG X QMM



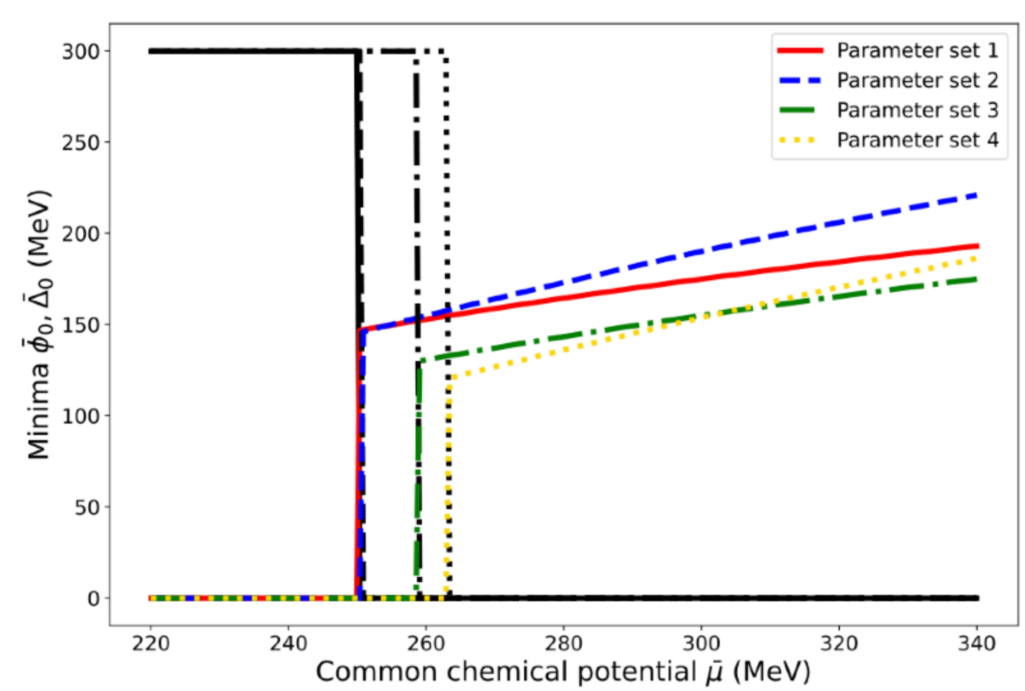
RLSF, G. Dallabona, O.A. Battistel and G. Krein, PRC 73, 018201 (2006)

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

D.T. Son, Phys. Rev. D 59, 094019 (1999)



H. Gholami, M. Hofmann and M. Buballa, PRD 111, 014006 (2025)



J.O. Andersen and M. P. Nødtvedt, PRD 111, 034031 (2025)

SU(2) NJL model

$\boxed{T = 0, \mu_B = 0 \text{ and } \mu_I \neq 0}$

In the mean-field approximation, the thermodynamic potential is given by

$$\Omega_{\mathrm{NJL}} = \frac{\sigma^2 + \Delta^2}{4G} - 2N_c \int_\Lambda \frac{d^3k}{(2\pi)^3} \left(E_k^+ + E_k^-\right)$$

where $E_k^\pm = \sqrt{\left(E_k \pm \frac{\mu_I}{2}\right)^2 + \Delta^2}$, $E_k = \sqrt{k^2 + M^2}$

From these equations we obtain

$$\sigma = 4GN_c M \, I_\sigma$$
$$\Delta = 4GN_c \Delta \, I_\Delta$$

with the definitions

$$I_\sigma = \sum_{s=\pm 1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} \frac{E_k + s\mu_I}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$

$$I_\Delta = \sum_{s=\pm 1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$

TRS scheme

Medium Separation Scheme

$$I_\Delta = \frac{1}{\pi}\sum_{j=\pm 1}\int_{-\infty}^{+\infty} dx \int_\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{x^2+(E_k+j\mu_I)^2+\Delta^2}$$

Using the identity

$$\frac{1}{x^2+(E_k+j\mu_I)^2+\Delta^2} = \frac{1}{x^2+k^2+M_0^2} + \frac{M_0^2-\Delta^2-\mu_I^2-M^2-2j\mu_I E_k}{(x^2+k^2+M_0^2)\left[x^2+(E_k+j\mu_I)^2+\Delta^2\right]}$$

M_0 is the vacuum quark mass!

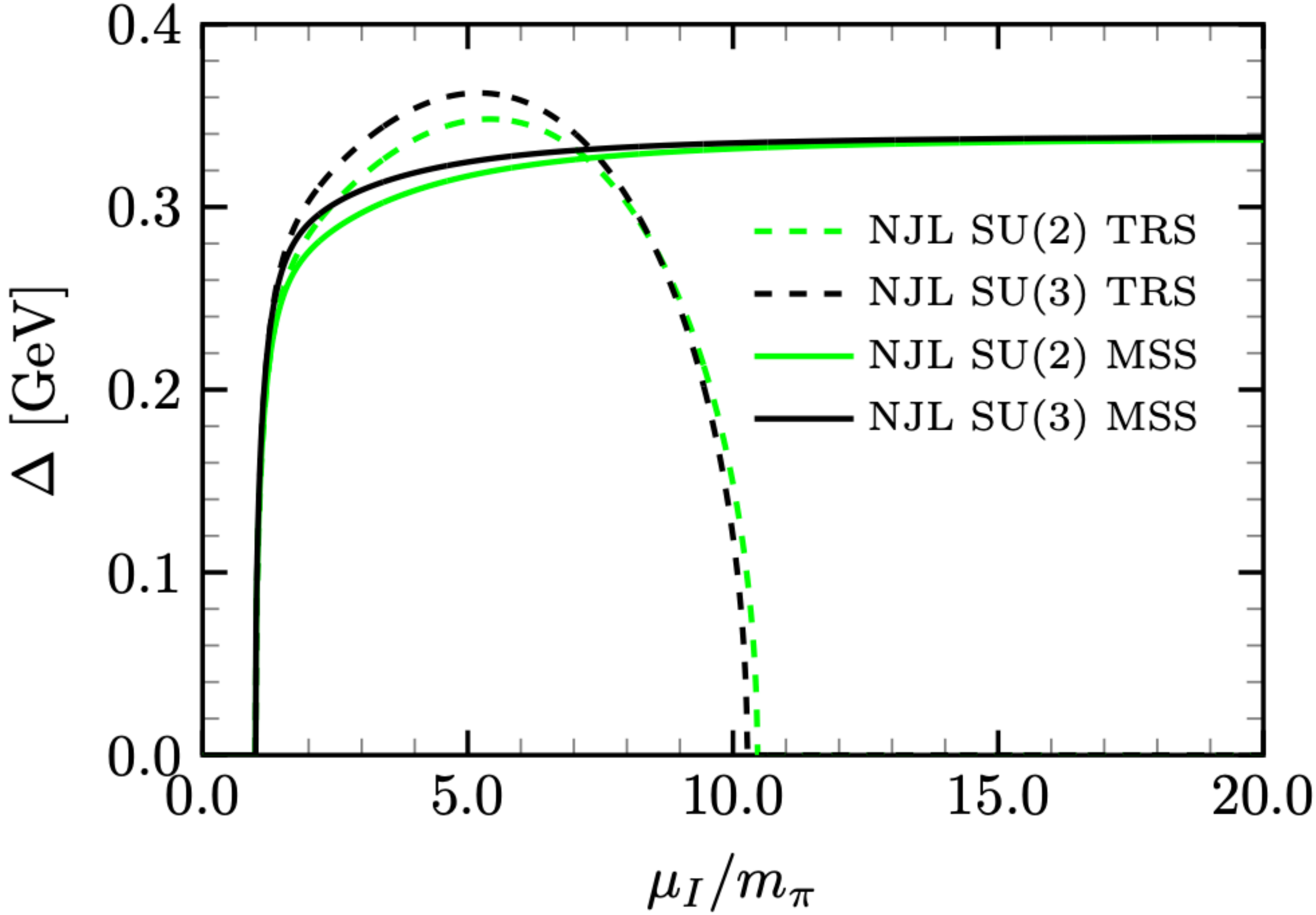
$$I_\Delta = \frac{1}{2}\underbrace{I_{\mathrm{quad}}(M_0)}_{\text{Divergent}} - (\Delta^2 - M_0^2 - 2\mu_I^2 + M^2)\underbrace{I_{\log}(M_0)}_{\text{Divergent}} + \underbrace{I_{\mathrm{finite}}}_{\text{No Regulator}}$$

$$I_{\mathrm{quad}}(M_0) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{p^2+M_0^2}}$$

$$I_{\log}(M_0) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+M_0^2)^{\frac{3}{2}}}$$

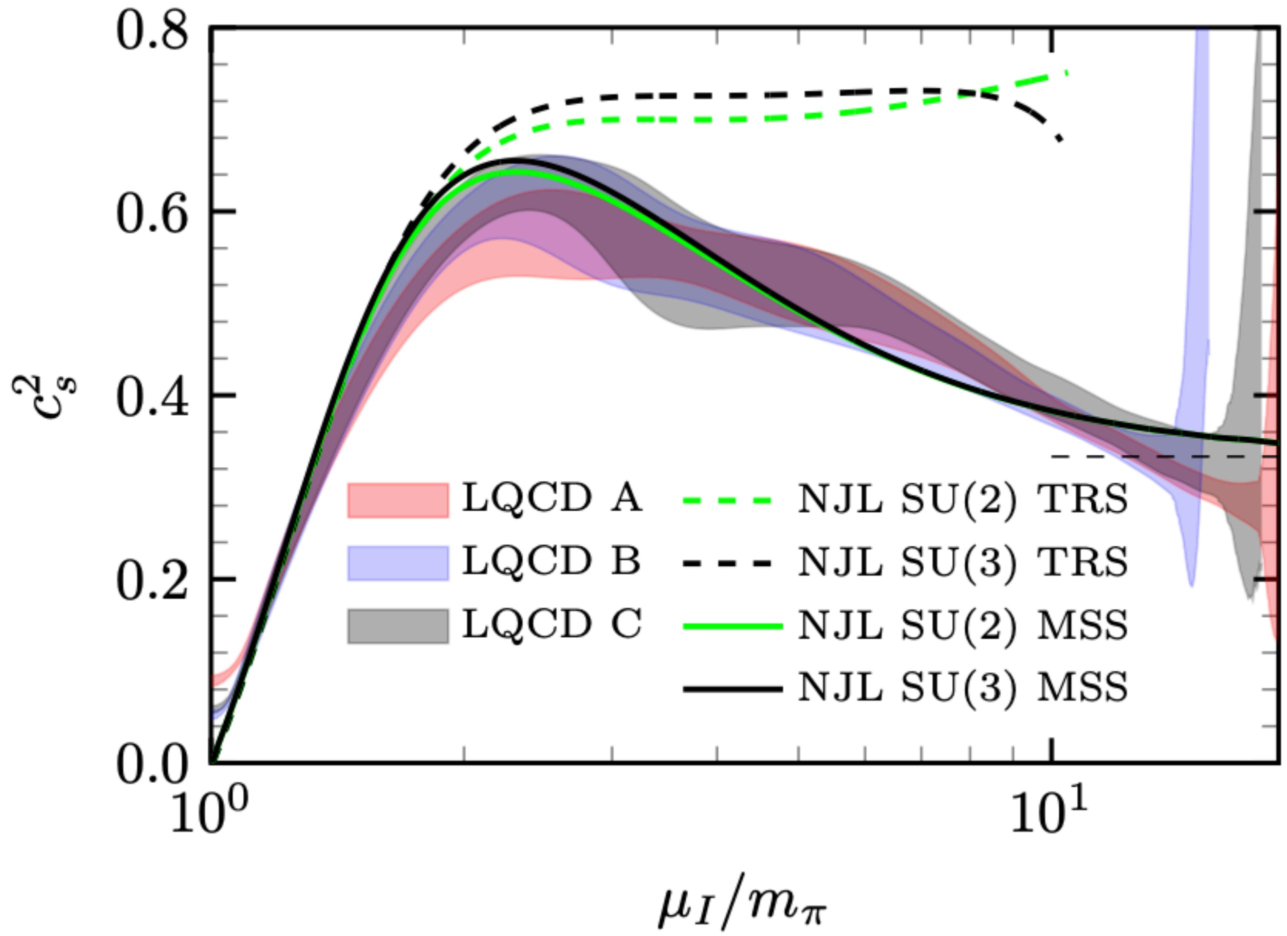
T=0 NJL + isospin asymmetry

Pion condensate



RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, e-Print: 2507.14343 [hep-ph]

T=0 Sound velocity



$$\epsilon = -P + \mu_I n_I$$
$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

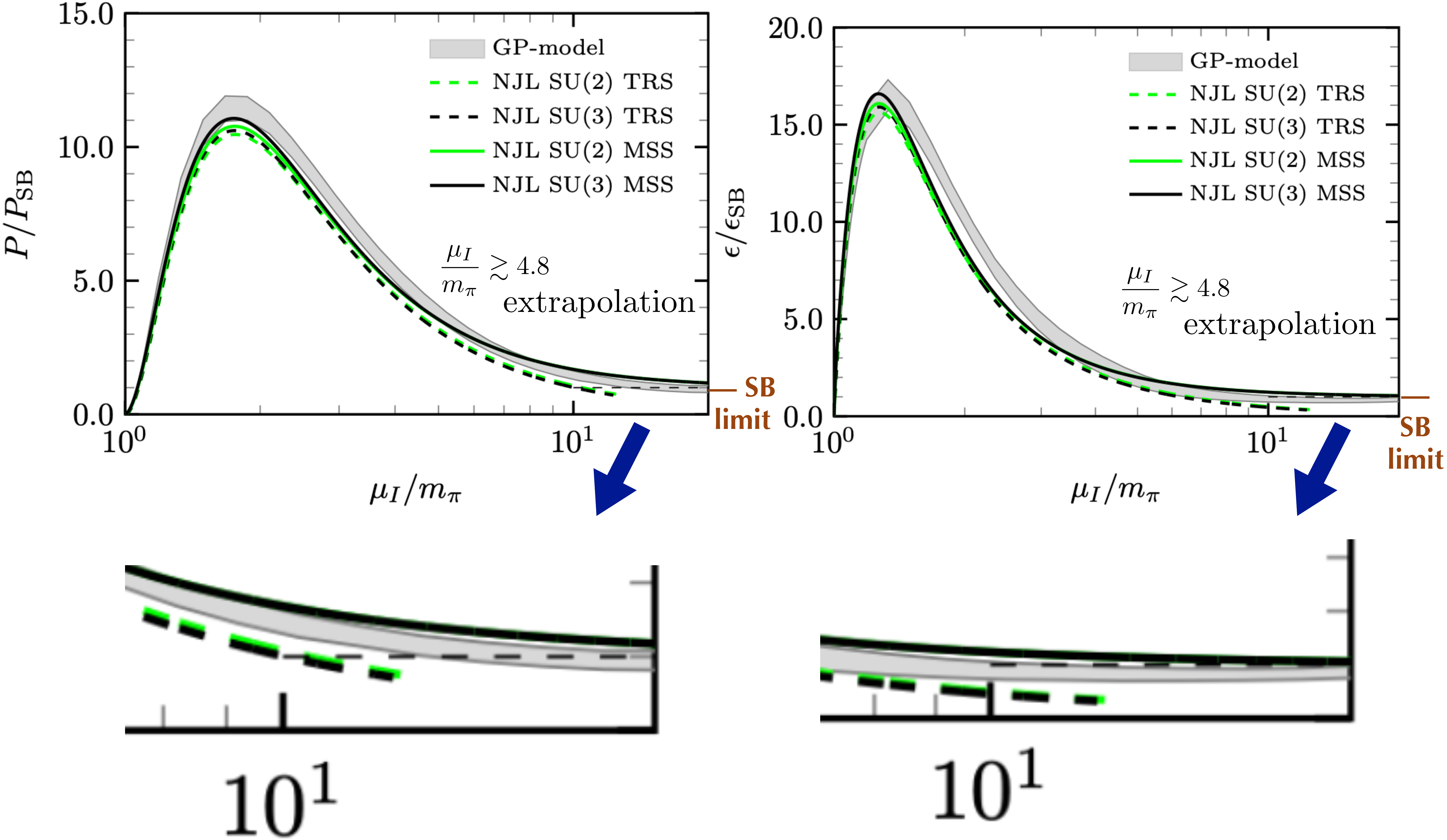
This data does not control for finite lattice spacing effects, and there are also uncontrolled systematics at large and small μ_I

Updated bounds based on the continuum limit lattice QCD results at the physical quark masses, χPT, and perturbative QCD through the GP-model.

RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, e-Print: 2507.14343 [hep-ph]

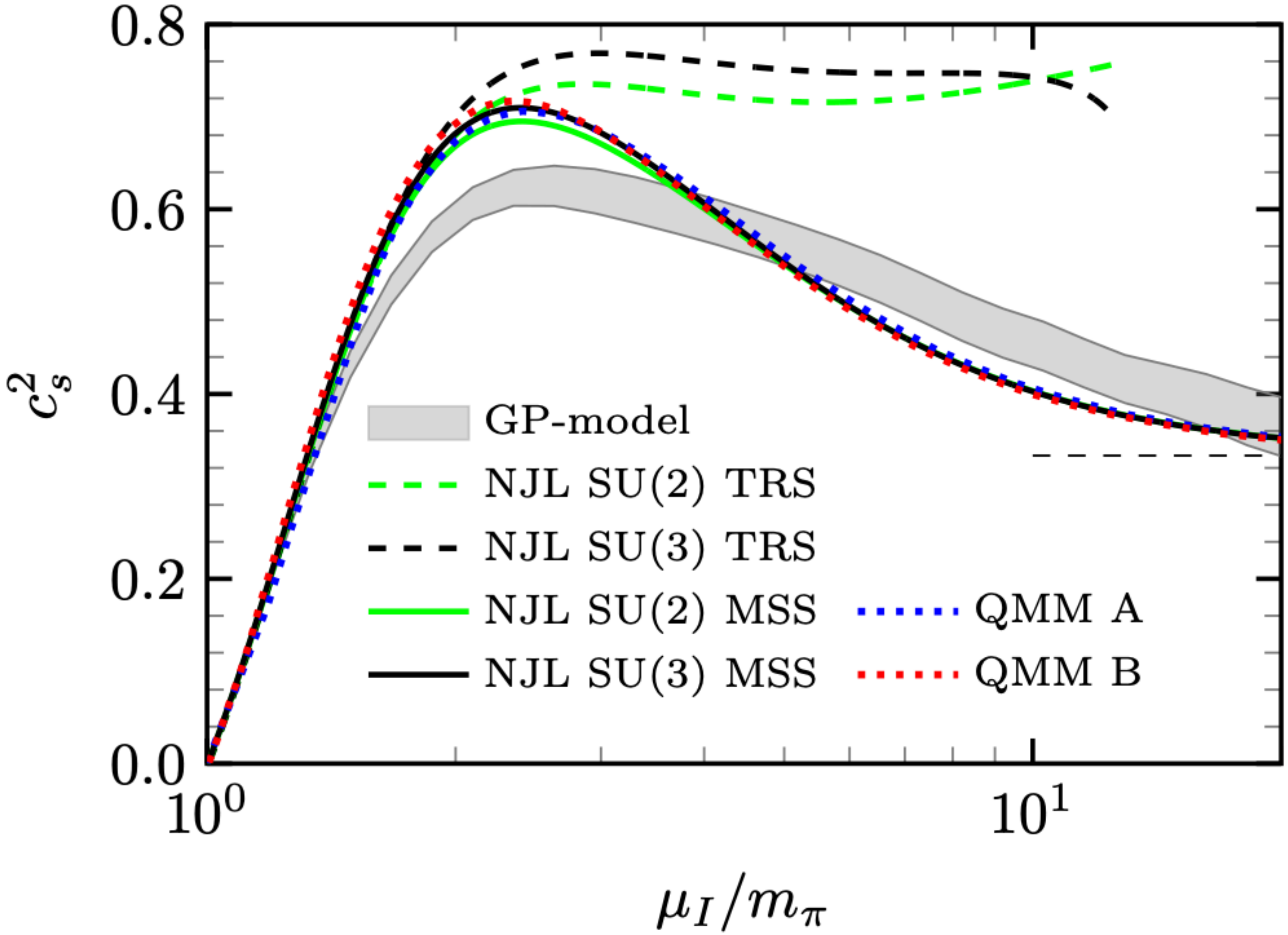
R. Abbott et al. [NPLQCD], Phys. Rev. Lett. 134, no.1, 1 (2025)

Thermodynamics



RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, e-Print: 2507.14343 [hep-ph]

Speed of sound peak in isospin QCD: a natural prediction of the Medium Separation Scheme



No adjust of NJL parameters, Just disentangle medium contributions from the vacuum!

RLSF, B. Lopes, D.C. Duarte, R.O. Ramos, e-Print: 2507.14343 [hep-ph]

R. Abbott et al. [NPLQCD], Phys. Rev. Lett. 134, no.1, 1 (2025)

(A) J. O. Andersen and M. P. Nødtvedt, [arXiv:2502.10229 [hep-ph]].
(B) B. B. Brandt, V. Chelnokov, G. Endrodi, G. Marko, D. Scheid and L. von Smekal,[arXiv:2502.04025 [hep-ph]]

Final Remark: QCD under extreme conditions

- Finite temperature T
- Finite magnetic field eB
- Chiral chemical potential μ_5
- Two color QC2D
- Finite isospin chemical potential μ_I

NO
Sign Problem!

LQCD can be used as a benchmark platform for comparing different effective models used in the literature.

Conclusion:

- MSS results are consistent with LQCD across different contexts and with QMM and FRG methods (CSC).

Perspectives:

- Comparison with RG approach
- Regulator dependence of inhomogeneous phases
- Finite densities + confinement effects

CNPq
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FAPERGS

Thank you for your attention!

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