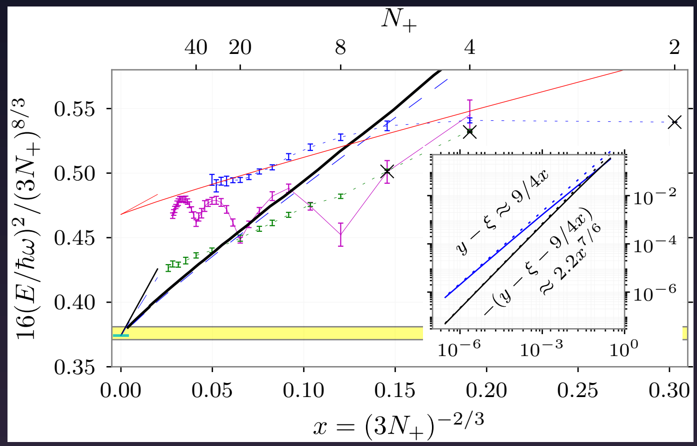
Static and Dynamic Properties near Unitary from DFT

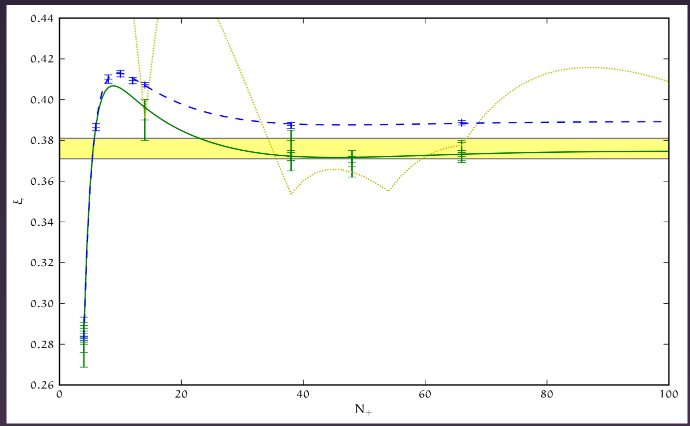
Michael McNeil Forbes
Institute for Nuclear Theory (INT)
University of Washington, Seattle, WA

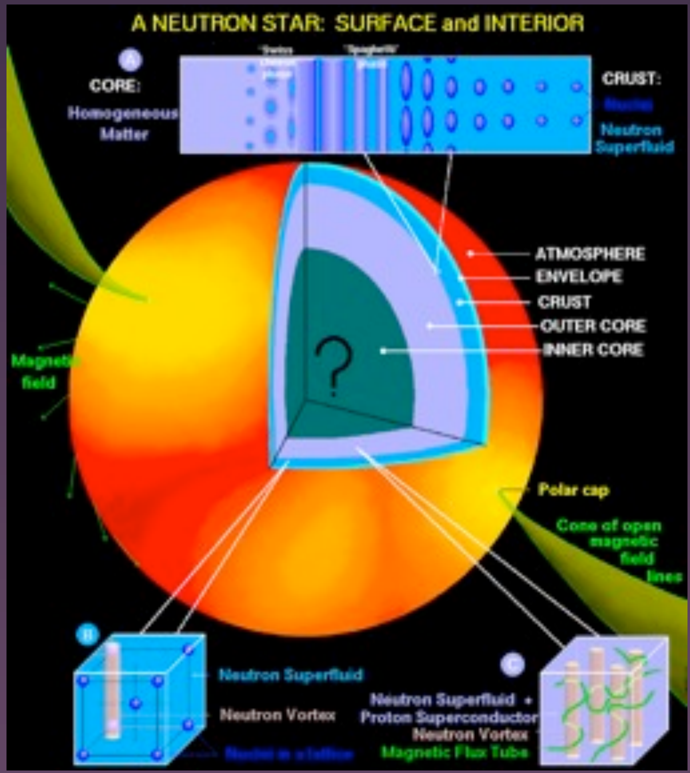
Outline

- DFT for Unitary Fermi Gas
  - Static: boxes and traps
  - Gradient corrections

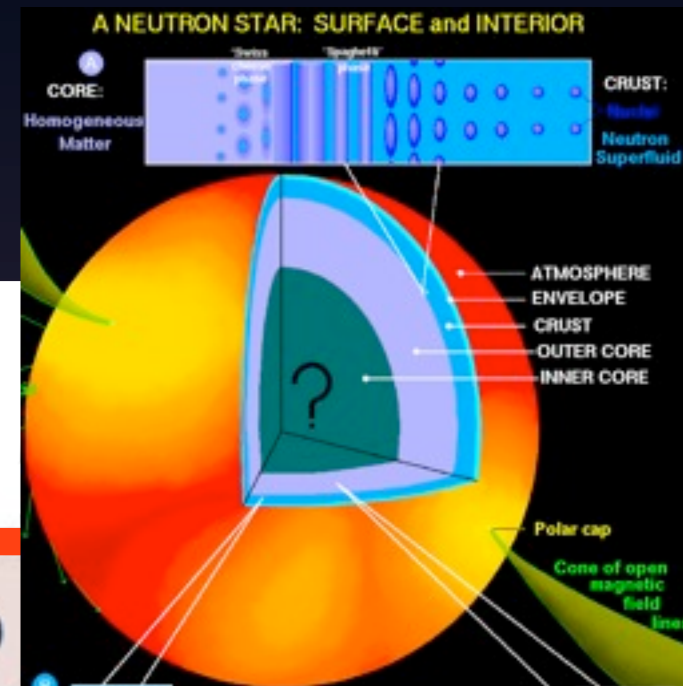
- Gross-Pitaevskii–Equation (GPE)
  - Works for low-energy Unitary Fermi Gas!
  - “Laboratory on your Laptop”
  - scale up to neutron stars (glitching)

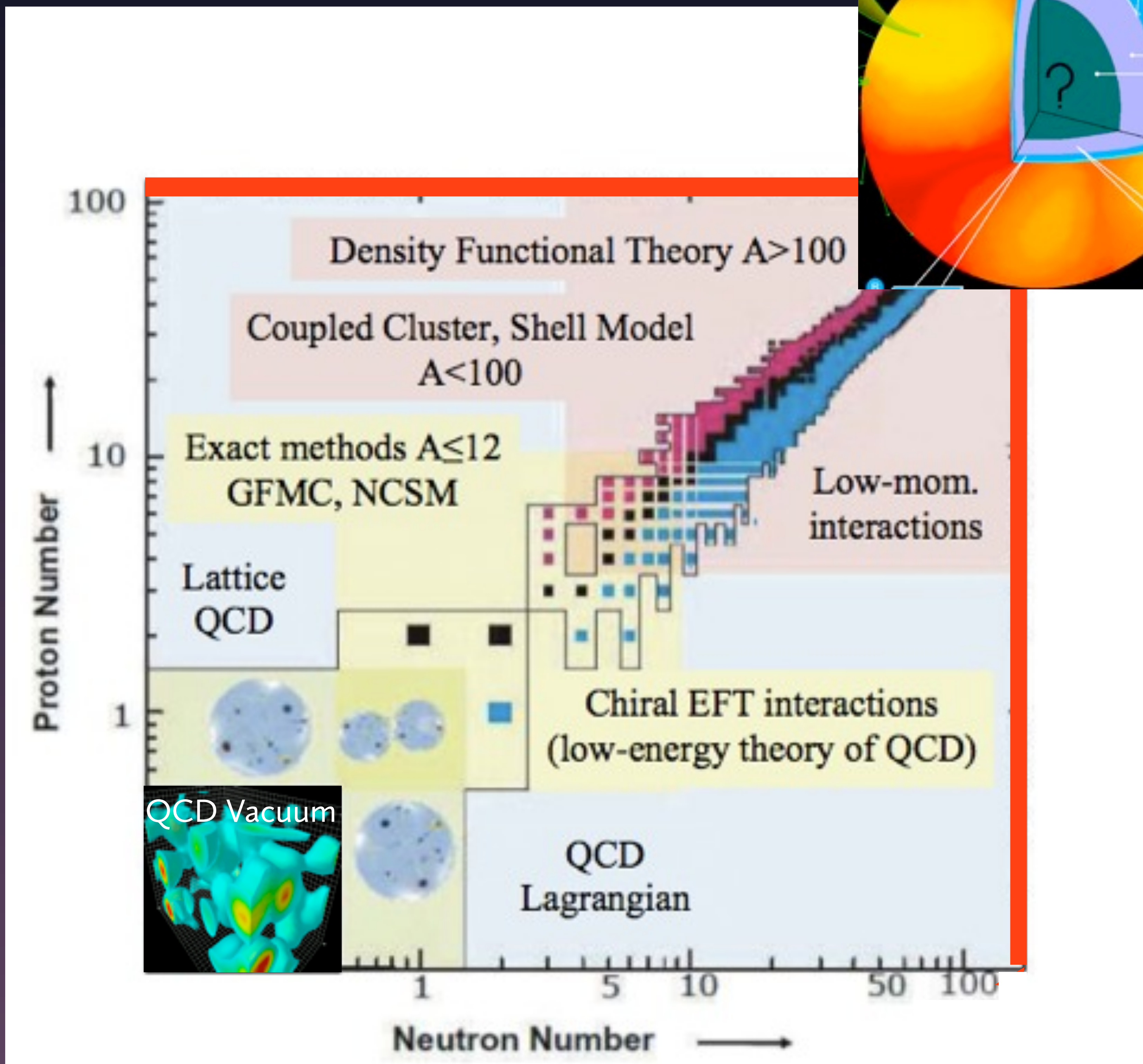




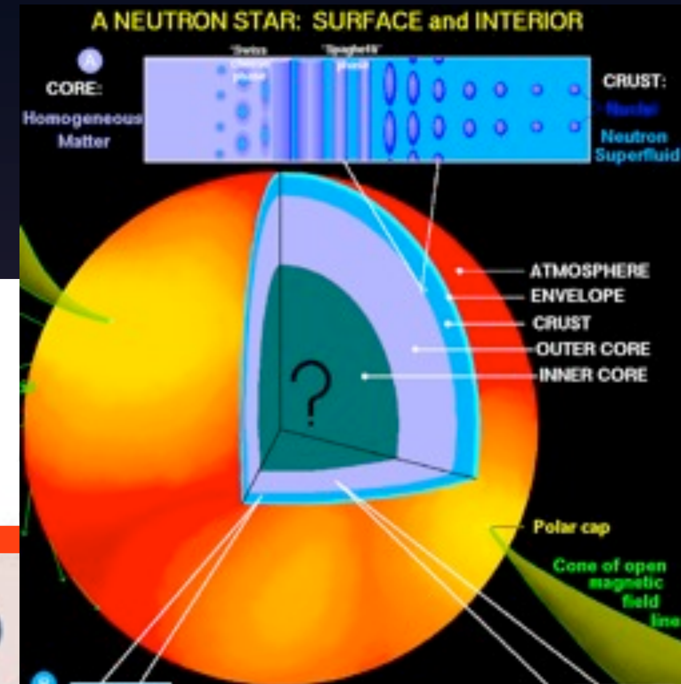


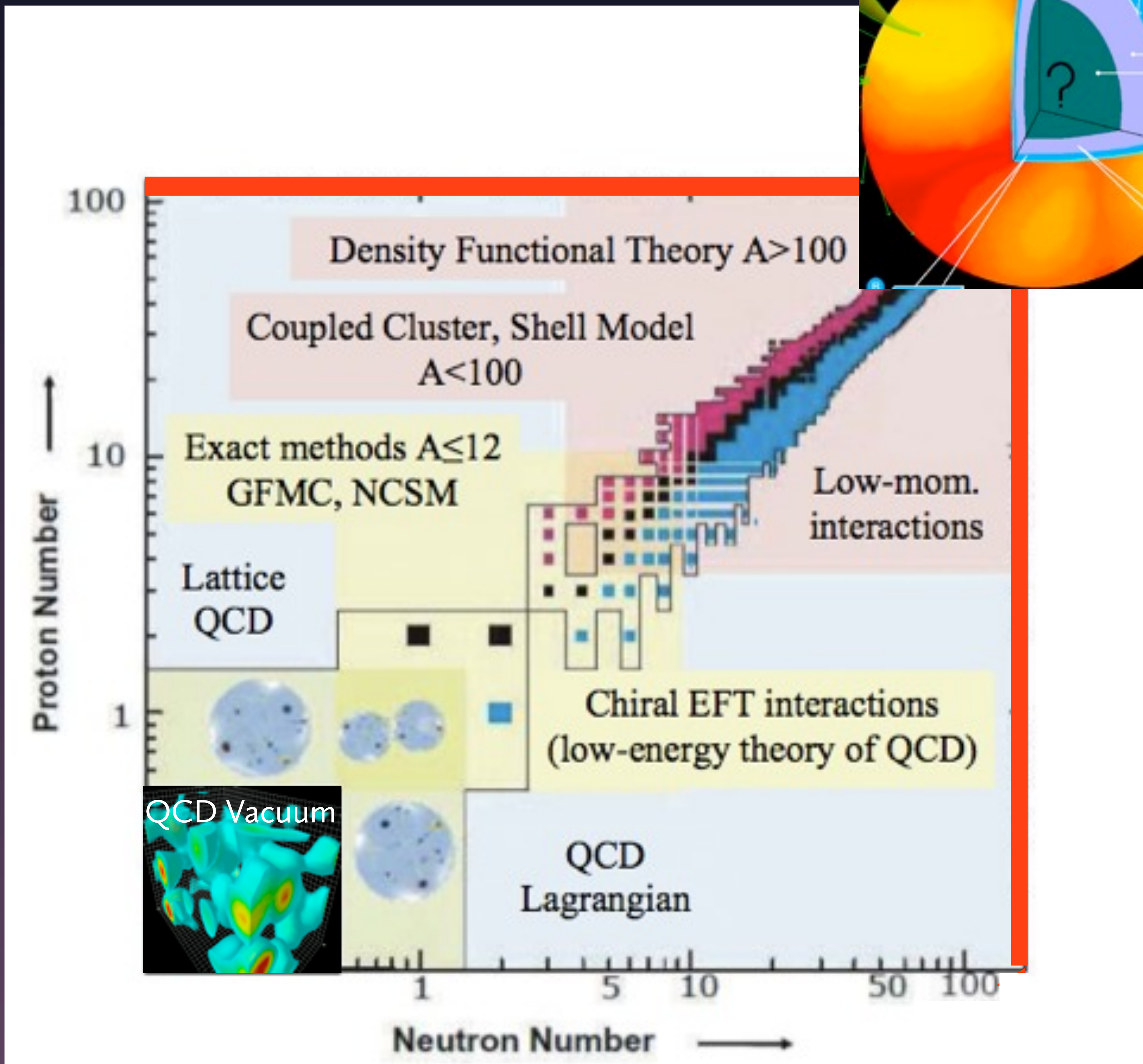
The Nuclear Landscape





QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)





- Lattice QCD,
  nucleons, interactions
- QMC, etc.
  small to medium nuclei
- DFT,
  medium to large nuclei
- Neutron stars?
  Molecular Dynamics
  Hydrodynamics

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

Cold Atoms Benchmarking

- Theoretically clean and simple (universal)
- Well constrained
- Remarkably diverse phase structure
- Convergence of theory, simulation and experiment
- Benchmark for many-body techniques

Unitary Fermi Gas (UFG)

$$\widehat{\mathcal{H}} = \int \left(\widehat{\mathbf{a}}^\dagger \widehat{\mathbf{a}} E_a + \widehat{\mathbf{b}}^\dagger \widehat{\mathbf{b}} E_b\right) - g \int_\Lambda \widehat{\mathbf{a}}^\dagger \widehat{\mathbf{b}}^\dagger \widehat{\mathbf{b}} \widehat{\mathbf{a}}$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \quad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

- Take regulator $\lambda \to \infty$ and coupling $g \to 0$ to fix s-wave scattering length $a^{-1} \propto (\lambda - g^{-1}) = 0$ (unitary limit)
- Universal physics:
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\xi = 0.376(5)$
- Good model of dilute neutron matter in neutron stars

Statics

Density Functional Theory (DFT)

- The (exact) ground state density in any external potential $V(x)$ minimizes a functional (Hohenberg Kohn):

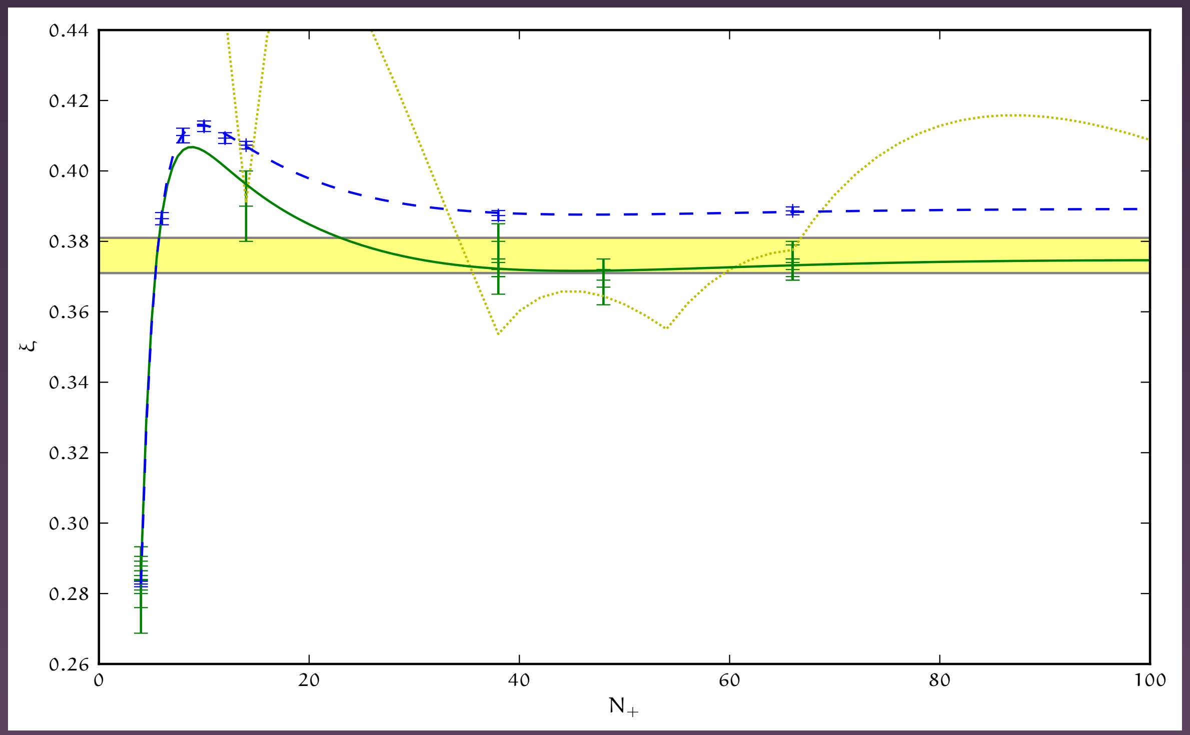
$$\int d^3x \{\mathcal{E}[n(x)] + V(x)n(x)\}$$

- Functional may be complicated (non-local)
  - Need to find physically motivated approximations

SLDA: Superfluid Local Density Approximation

$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Three densities:
  $n \approx \langle a^\dagger a\rangle$, $\tau \approx \langle \nabla a^\dagger \nabla a\rangle$, $\nu \approx \langle ab\rangle$
- Three parameters:
  - Effective mass (m/α)
  - Hartree (β), Pairing (g)



Forbes, Gandolfi, Gezerlis (2012)

BdG: contained in SLDA

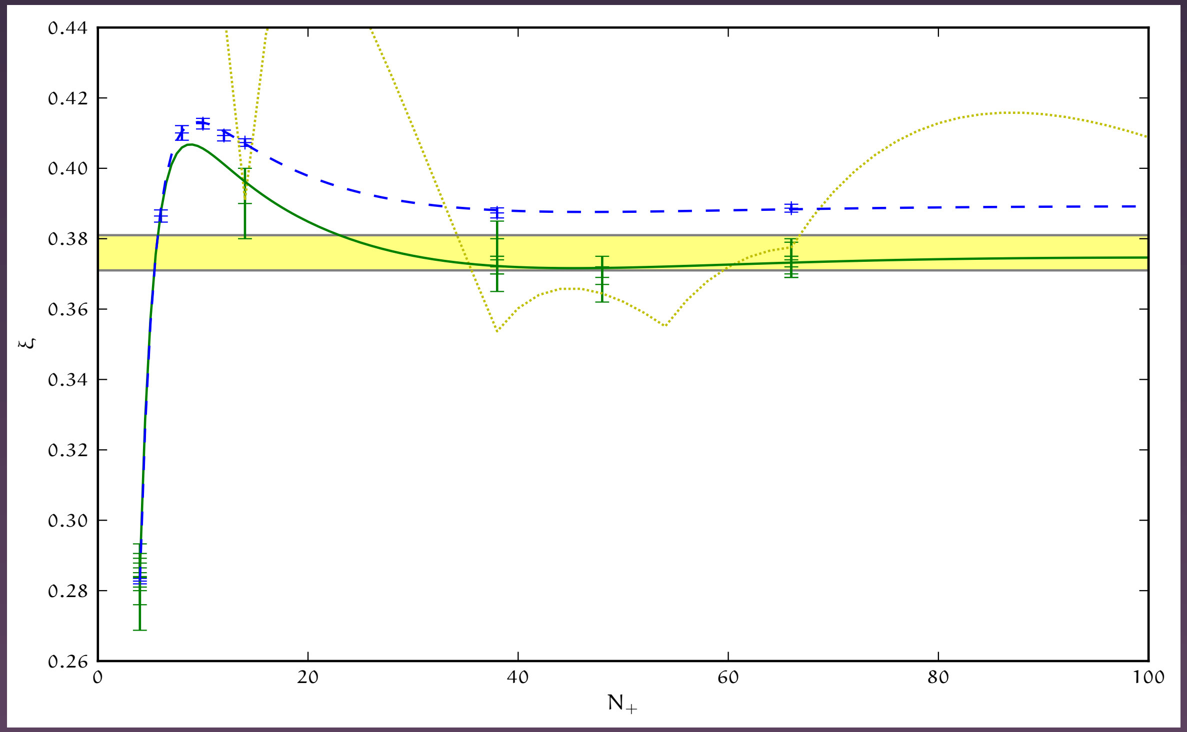
$$\mathcal{E}(n, \tau, \nu) = \alpha \frac{\overset{\langle \nabla \hat{a}^\dagger \nabla \hat{a} \rangle + \langle \nabla \hat{b}^\dagger \nabla \hat{b} \rangle}{\tau}}{m} + \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2} + g_{\text{eff}} \overset{\langle \hat{a}^\dagger \hat{b}^\dagger \rangle \, \langle \hat{b} \hat{a} \rangle}{\nu^\dagger \nu}$$

- Variational: $\mathcal{E} = \langle H \rangle$ (minimize over Gaussian states)
- Bogoliubov-de Gennes (BdG) contained in SLDA
- Unit mass ($\alpha = 1$)
- No Hartree term ($\beta = 0$)
  - (No polaron properties)

SLDA: Superfluid Local Density Approximation

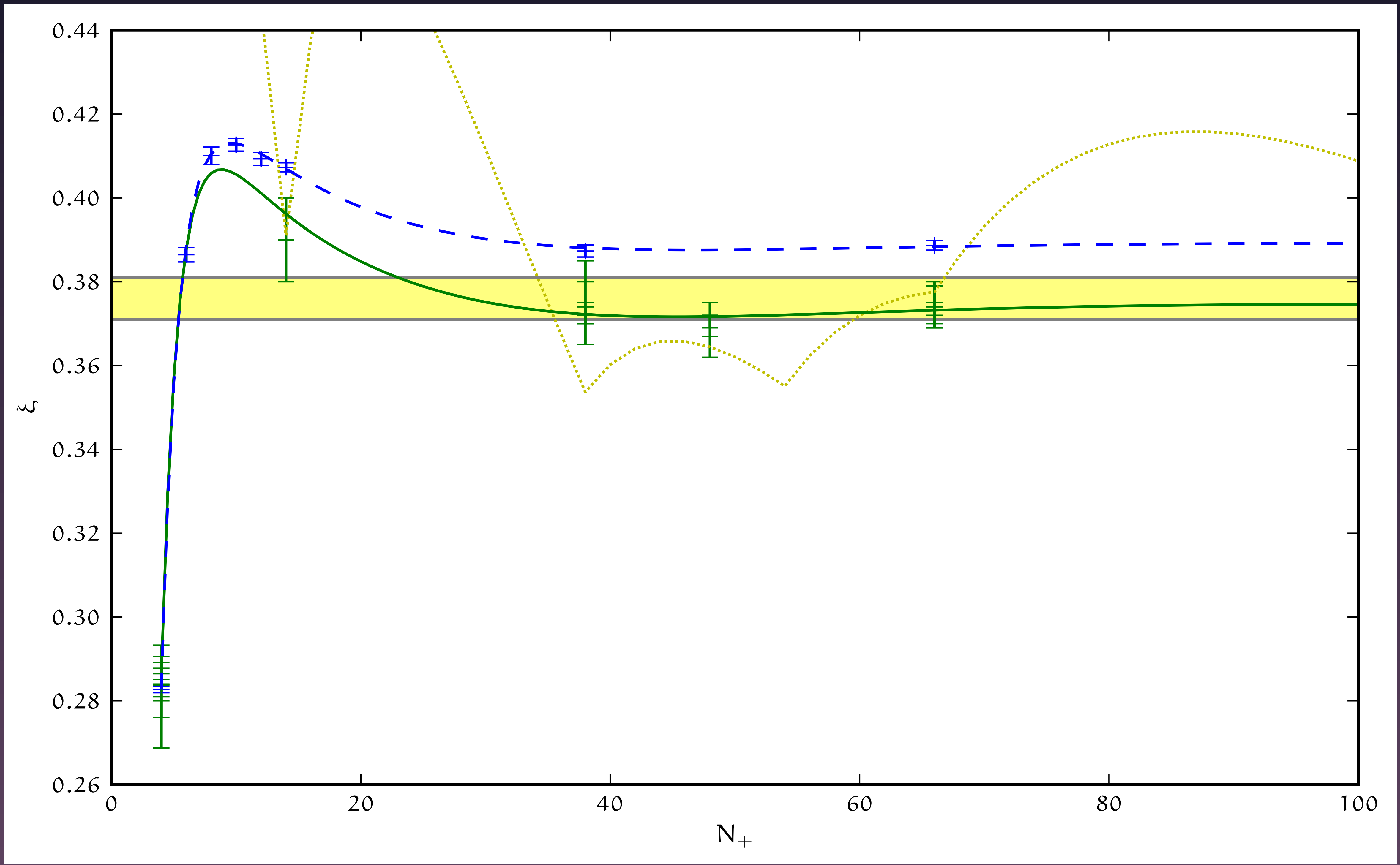
$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Three densities:
  $n \approx \langle a^\dagger a\rangle$, $\tau \approx \langle \nabla a^\dagger \nabla a\rangle$, $\nu \approx \langle ab\rangle$
- Three parameters:
  - Effective mass (m/α)
  - Hartree (β), Pairing (g)



Forbes, Gandolfi, Gezerlis (2012)

SLDA: Superfluid Local

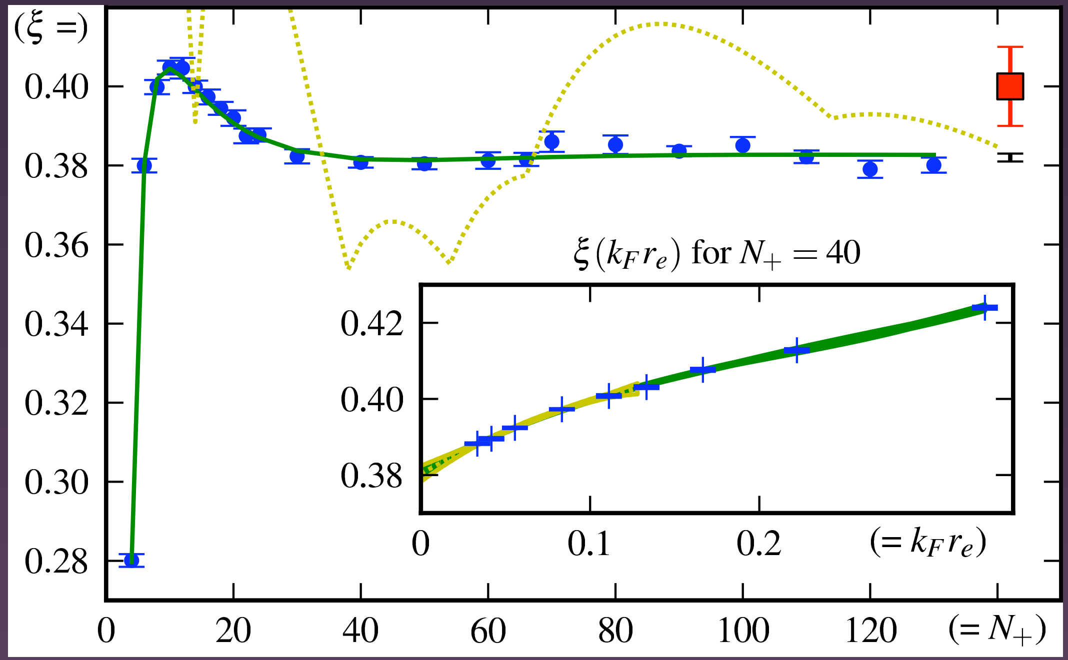


Forbes, Gandolfi, Gezerlis (2012)

SLDA: Fit to QMC using $r_{\text{eff}} = 0$ Extrapolation

$$\mathcal{E}(n, \tau, \nu) = \alpha \frac{\tau}{m} + \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2} + g_{\text{eff}} \nu^\dagger \nu$$

- Three parameters, but
- Independent fits of each N
  - (lots of parameters)
- Can we model range?



Forbes, Gandolfi, Gezerlis (2011, 2012)

Fit directly to QMC

$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\,\frac{(3\pi^2 n)^{5/3}}{10 m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Each parameter becomes a quadratic polynomial:
  - $\alpha(k_F r_e)$, $\beta(k_F r_e)$, $\gamma(k_F r_e)$
- we actually use physical parameters $\xi(k_F r_e)$, $\Delta(k_F r_e)$, $\alpha(k_F r_e)$
- 9 total parameters for all N

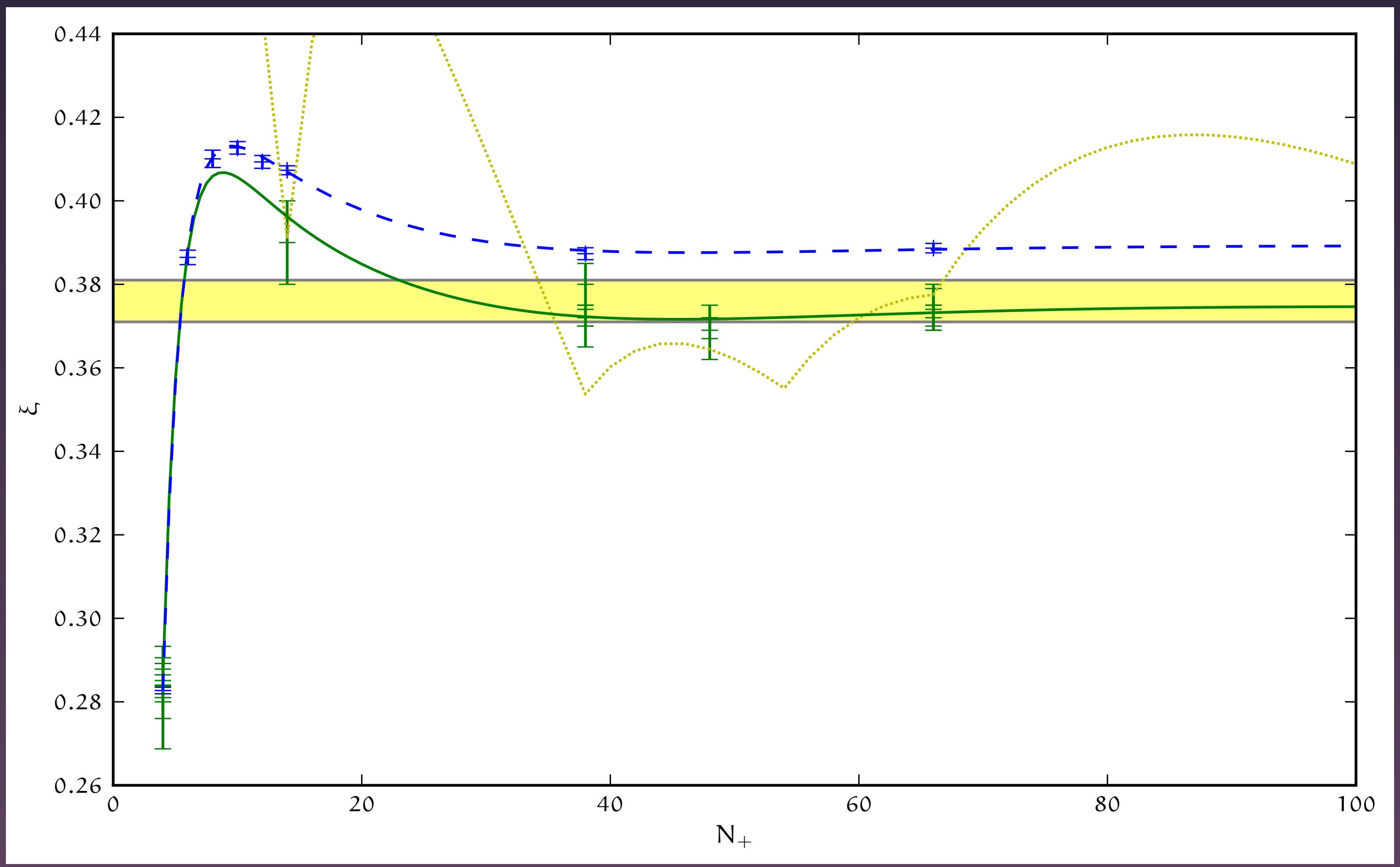
Fit directly to QMC

$$\mathcal{E}(n, \tau, \nu) = \alpha\frac{\tau}{m} + \beta\,\frac{(3\pi^2 n)^{5/3}}{10 m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Not complete story for modeling range:
  - Does not regulate theory
  - No structure for gap (Δ_p)

    probably requires non-local functional

Fit SLDA to box QMC



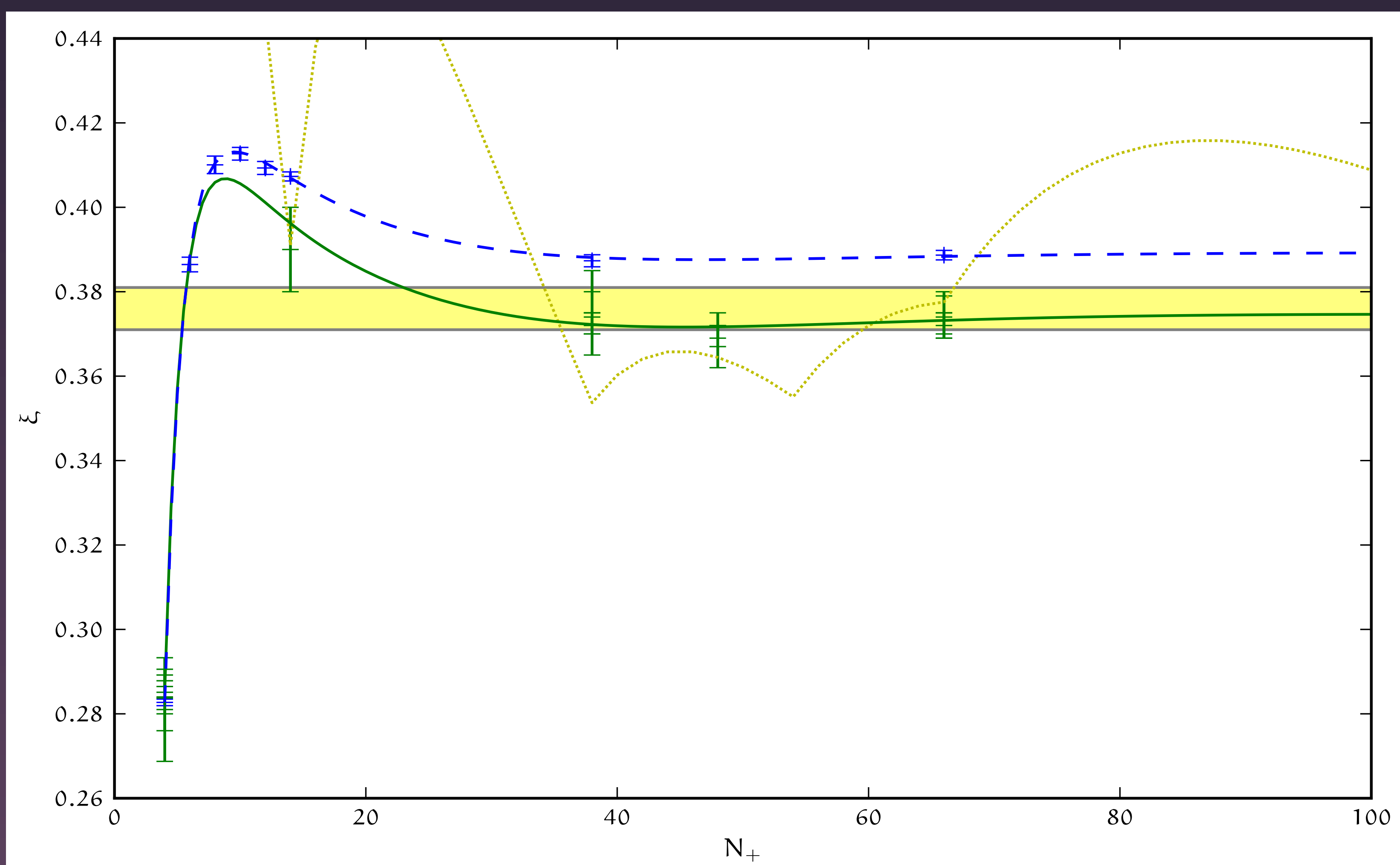
- Fit 60 QMC with 9 parameter model
- Directly use QMC with sub-percent errors
  - reduced $\chi^2 = 6$

Forbes, Gandolfi, Gezerlis PRL (2011)

SLDA parameters

$$\alpha, \xi, \eta = a_0 + a_1 k_F r_e + a_2 (k_F r_e)^2$$

	a_0	a_1	a_2
ξ_{PT}	0.3903(7)	0.121(10)	0.00(3)
	0.3911(4)	0.111(3)	
ξ_{2G}	0.3890(4)	0.128(4)	−0.06(1)
	0.3900(3)	0.111(2)	
η_{PT}	0.99(3)	−2.1(4)	3(1)
	0.90(1)	−0.85(7)	
η_{2G}	0.879(7)	−0.84(3)	0.00(3)
	0.875(8)	−0.82(4)	
α_{PT}	1.34(2)	−1.6(4)	5(2)
	1.303(10)	−0.71(8)	
α_{2G}	1.292(7)	−0.73(6)	0.1(2)
	1.289(7)	−0.69(3)	



Universal slope

$$\xi = \xi + (k_F r_e) S$$

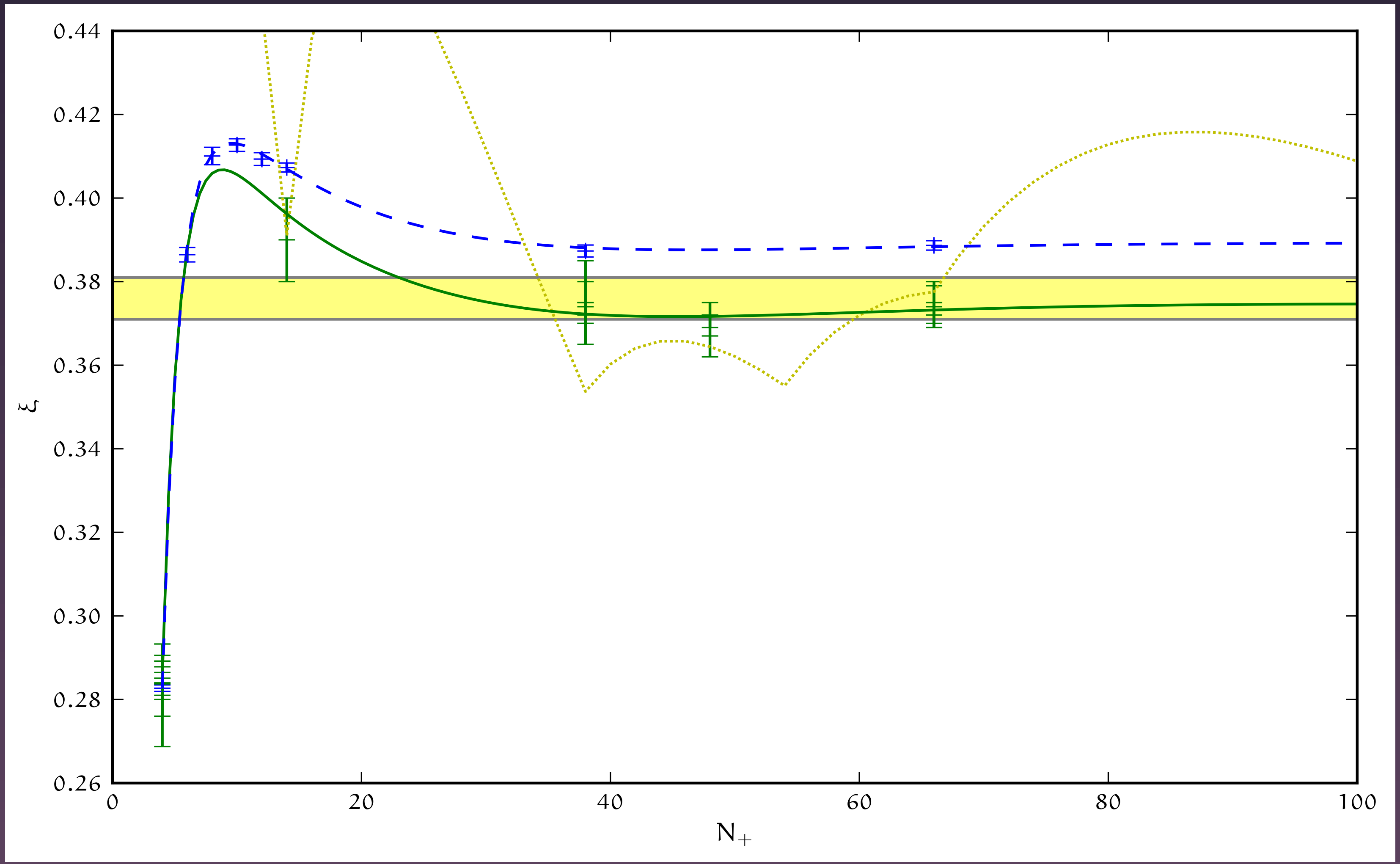
S=0.12(1)

Forbes, Gandolfi, Gezerlis (2012)

SLDA parameters

$$\alpha, \xi, \eta = a_0 + a_1 k_F r_e + a_2 (k_F r_e)^2$$

	a_0	a_1	a_2
ξ_{PT}	0.3903(7)	0.121(10)	0.00(3)
	0.3911(4)	0.111(3)	
ξ_{2G}	0.3890(4)	0.128(4)	−0.06(1)
	0.3900(3)	0.111(2)	
η_{PT}	0.99(3)	−2.1(4)	3(1)
	0.90(1)	−0.85(7)	
η_{2G}	0.879(7)	−0.84(3)	0.00(3)
	0.875(8)	−0.82(4)	
α_{PT}	1.34(2)	−1.6(4)	5(2)
	1.303(10)	−0.71(8)	
α_{2G}	1.292(7)	−0.73(6)	0.1(2)
	1.289(7)	−0.69(3)	



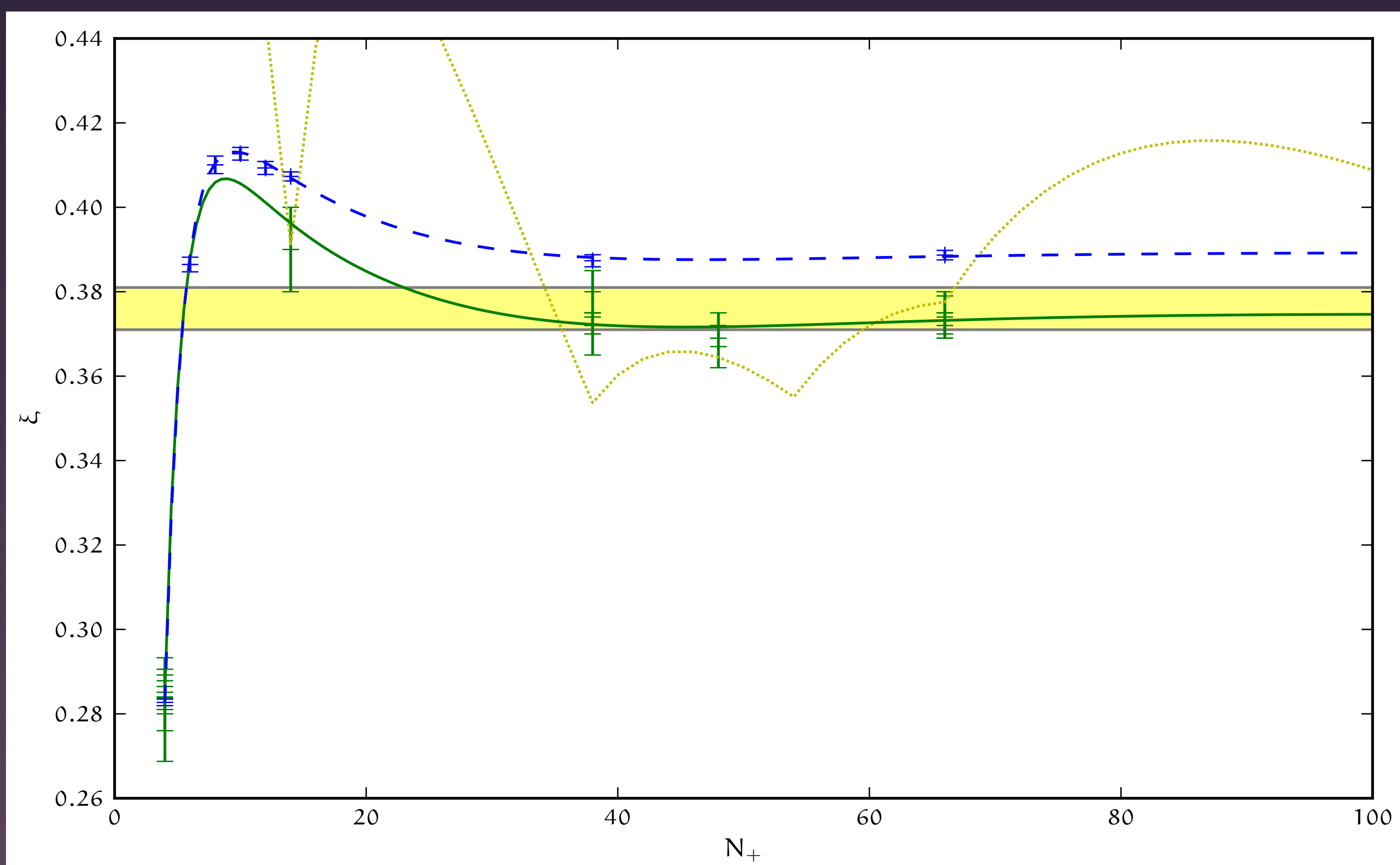
Gap and inverse mass seem too large

Limitation of fixed node approximation?

Forbes, Gandolfi, Gezerlis (2012)

Unbiased SLDA fit at $r_{\text{eff}}=0$

N_+	ξ_{N_+}	Method
2	$-0.415332919\cdots$	exact (see section II C)
4	0.288(3), 0.286(3)	exact diagonalization [18]
″	0.28(1)	AFMC [18]
″	0.280(4)	AFMC [12]
14	0.39(1)	AFMC [12]
38	0.370(5), 0.372(2), 0.380(5)	AFMC [12]
48	0.372(3), 0.367(5)	AFMC [12]
66	0.374(5), 0.372(3), 0.375(5)	AFMC [12]
10^6	0.376(5)	experiment [5]



Fit to unbaised results

- $\xi = 0.3742(5)$
- $\Delta = 0.65(1)$
- $\alpha = 1.104(8)$
- $\chi^2 = 0.3$

Forbes, Gandolfi, Gezerlis (2012)

Generalizations?

$$\mathcal{E}(n, \tau, \nu) = \alpha \frac{\tau}{m} + \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2} + g_{\text{eff}} \nu^\dagger \nu$$

- In principle, could have functions of the dimensionless quantity:

$$\frac{\alpha \frac{\tau}{m} + g \nu^\dagger \nu}{n^{5/3}}$$

- No evidence that this complication is needed yet

Gradient Corrections?

$$\mathcal{E}(n, \tau, \nu) = \left(\alpha \frac{\tau}{m} + g\nu^\dagger \nu\right) + \beta \mathcal{E}_{FG}(n) + \boxed{\frac{\delta\lambda}{8m} \frac{(\nabla n)^2}{n}}$$

- Leading order gradient corrections (Weizäcker term)
  - Has no effect (unconstrained by) previous box results

Low Energy Theory

$$\mathcal{L}_{\mathrm{LO+NLO}} = \xi^{-3/2} P_{\mathrm{FG}}(X) + c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + c_2 \frac{(\nabla^2 \phi)^2 - 9m\nabla^2 A_0}{\sqrt{m}} \sqrt{X}$$

$$X = \mu - V(t, \vec{x}) - \partial_t \phi - \frac{(\nabla \phi)^2}{2m} \qquad \langle ab \rangle = |\Delta| e^{2\imath\phi}$$

- Low energy theory of phonons (Son and Wingate 2006)
- Strongly constrained by General Coordinate Covariance
  - generalizes Galilean covariance
  - reduces NLO to 2 new coefficients c_1, c_2
- Three universal coefficients:
  - ξ, c_1, c_2 (I prefer $c_\chi = -6\pi^2(2\xi)^{3/2}(2c_1 - 9c_2)$, $c_\omega = -6\pi^2(2\xi)^{3/2}(2c_1 + 3c_2)$)

Low Energy Theory

$$\chi(q) = \frac{-mk_F}{\hbar^2\pi^2\xi}\left[1 - \frac{c_\chi}{12\xi}\frac{q^2}{k_F^2}\right] \qquad \omega_q = q\underbrace{\frac{\hbar k_F\sqrt{\xi}}{m\sqrt{3}}}_{c_s}\left[1 + \frac{c_\omega}{24\xi}\frac{q^2}{k_F^2}\right]$$

$$c_\chi = -6\pi^2(2\xi)^{3/2}(2c_1 - 9c_2), \quad c_\omega = -6\pi^2(2\xi)^{3/2}\ (2c_1 + 3c_2)$$

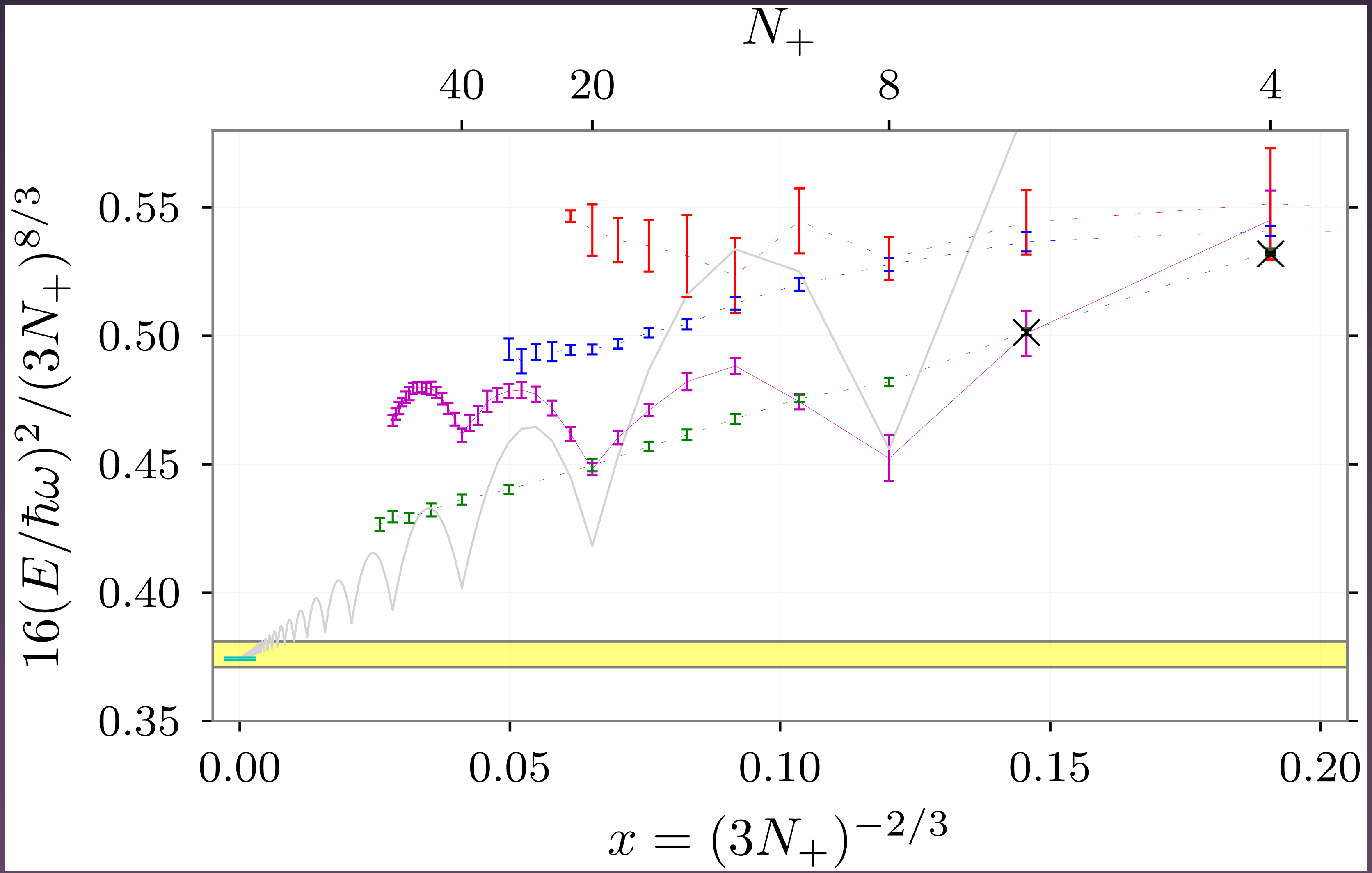
- Leading order: one parameter $\xi = 0.3742(5)$
- Two new universal constants at NLO:
  - Static: $c_\chi = 1.5(3)$ (Forbes 2013: not a proper 1-sigma error)
    - (BdG $c_\chi = 7/3 = 2.33$; ϵ–expansion $c_\chi = 1.6 + O(\epsilon^2)$; GPE $c_\chi = 2.25$)
  - Dynamic: $c_\omega = ?$
    - (BdG $c_\omega = 0.7539$; ϵ–expansion $c_\omega = c_\chi + O(\epsilon^2)$; GPE $c_\omega = c_\chi$)

BdG Juan and Valle 2009; ϵ–expansion Rupak and Schäfer 2007

Harmonic Traps

$$E(N_+) = \frac{\sqrt{\xi}}{4}\hbar\omega\left[\overbrace{(3N_+)^{4/3}}^{\text{LDA}} - \frac{c_\chi}{6\xi}(3N_+)^{2/3} + O(N_+^{5/9})\right],$$

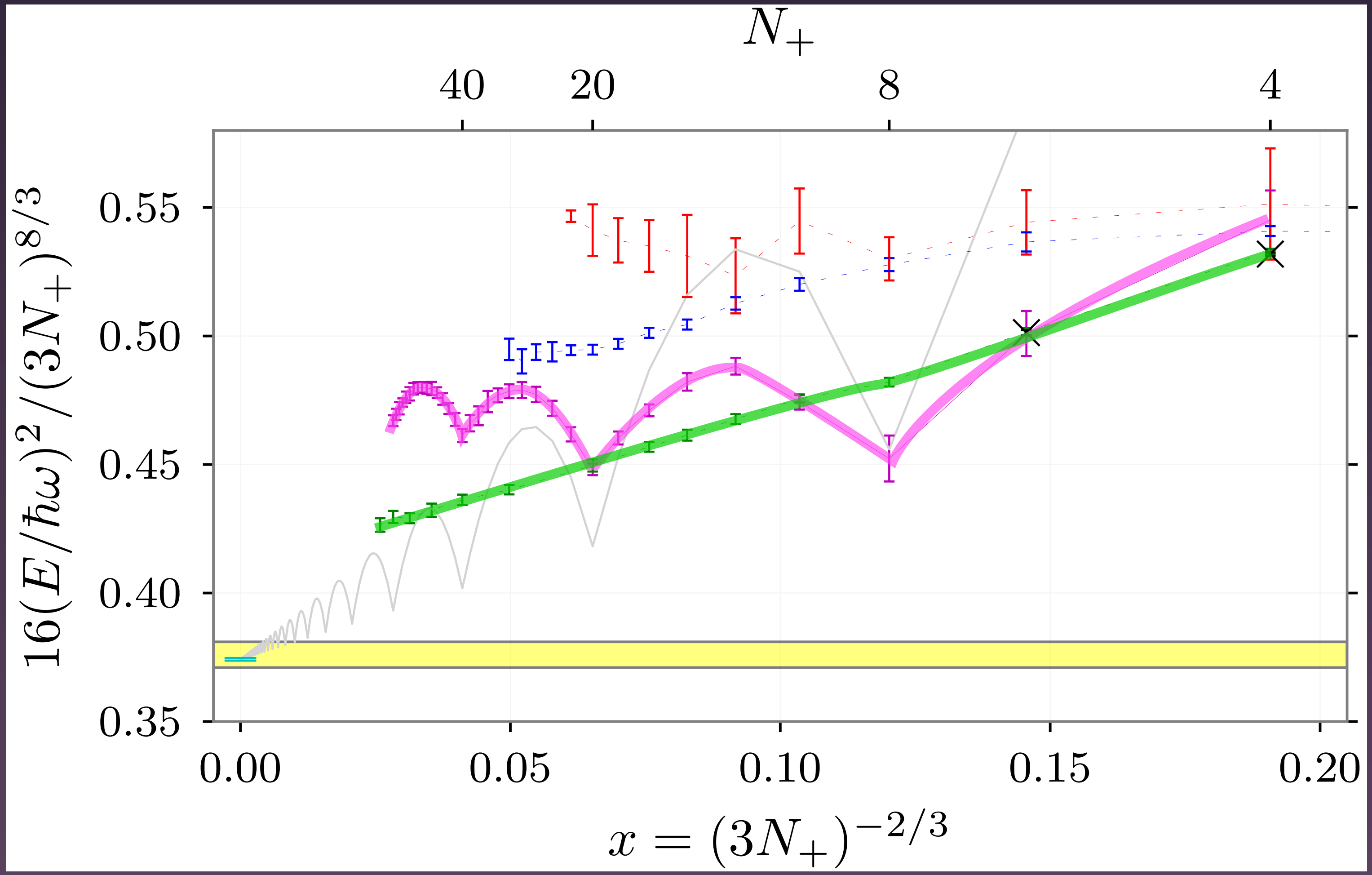
$$\frac{16E^2}{\xi\hbar^2\omega^2(3N_+)^{8/3}} = \boxed{\xi + c_\chi x + \mathcal{O}(x^{7/6}),} \quad x = \frac{1}{(3N_+)^{2/3}}$$



- Static structure factor describes slope

Forbes 2013

QMC Disagreements

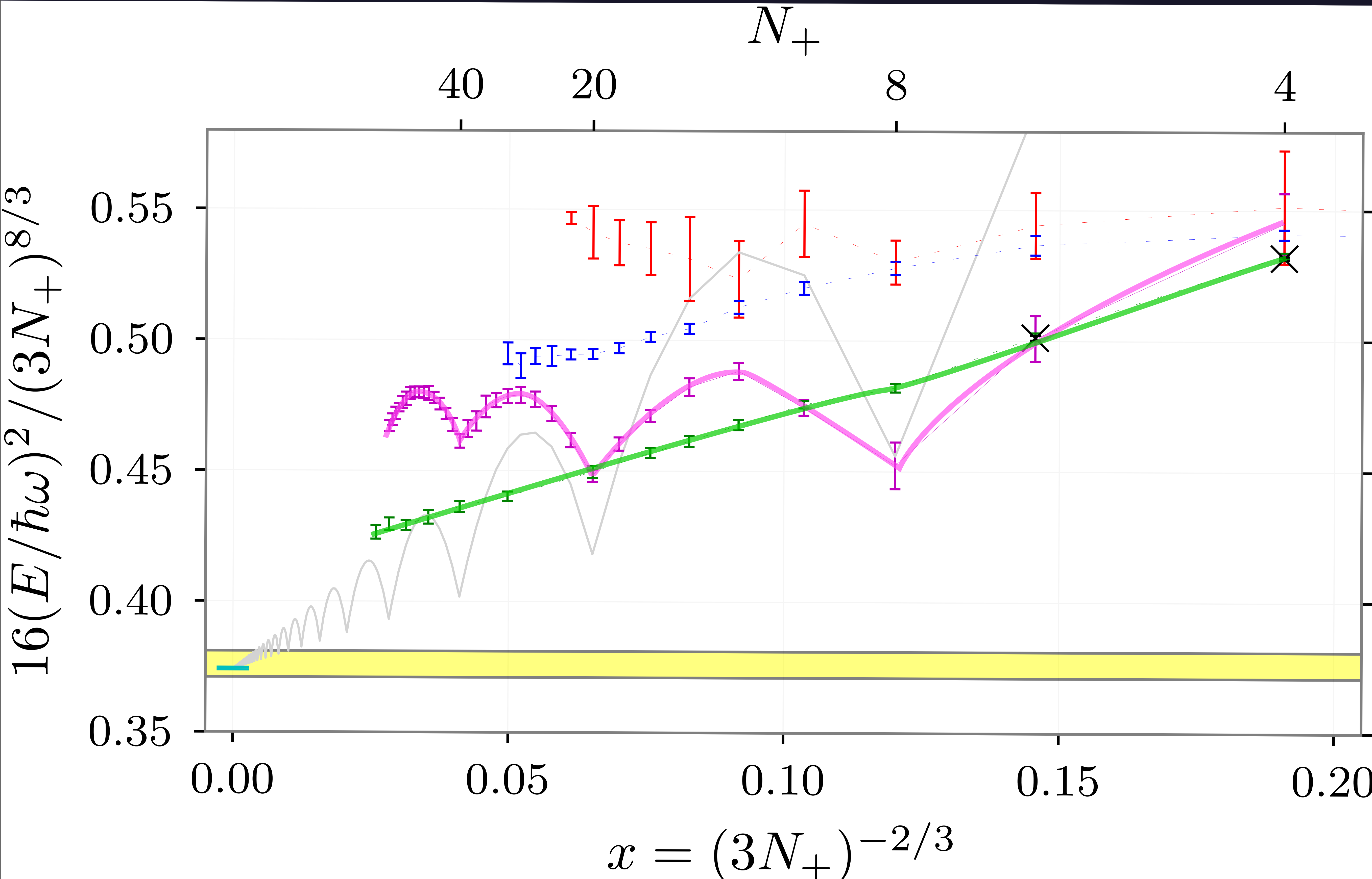


Blume and Daily 2011;
Endres, Kaplan, Lee, Nicholson 2011;
Forbes, Gandalfi, Gezerlis 2012; Forbes 2013

Variational QMC bound (green) not consistent with QMC results

Still need resolution:

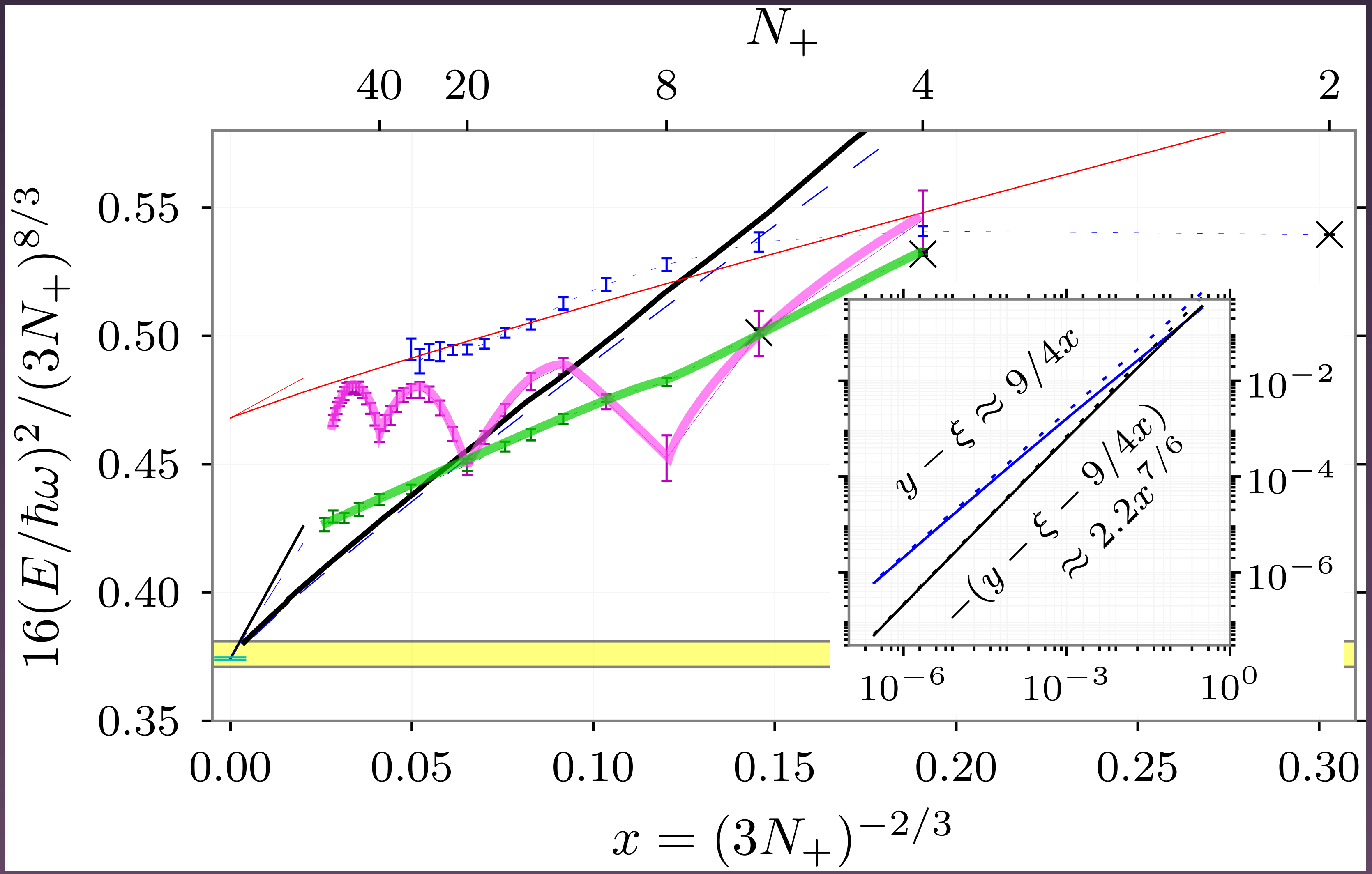
Are there shell effects?


N_+
40
20
8
4
$16(E/\hbar\omega)^2/(3N_+)^{8/3}$
0.55
0.50
0.45
0.40
0.35
0.00
0.05
0.10
0.15
0.20
$x = (3N_+)^{-2/3}$

Thermodynamic Limit

$$E(N_+) = \frac{\sqrt{\xi}}{4}\hbar\omega\left[\overbrace{(3N_+)^{4/3}}^{\text{LDA}} - \frac{c_\chi}{6\xi}(3N_+)^{2/3} + O(N_+^{5/9})\right],$$

$$\frac{16E^2}{\xi\hbar^2\omega^2(3N_+)^{8/3}} = \boxed{\xi + c_\chi x + \mathcal{O}(x^{7/6}),} \quad x = \frac{1}{(3N_+)^{2/3}}$$

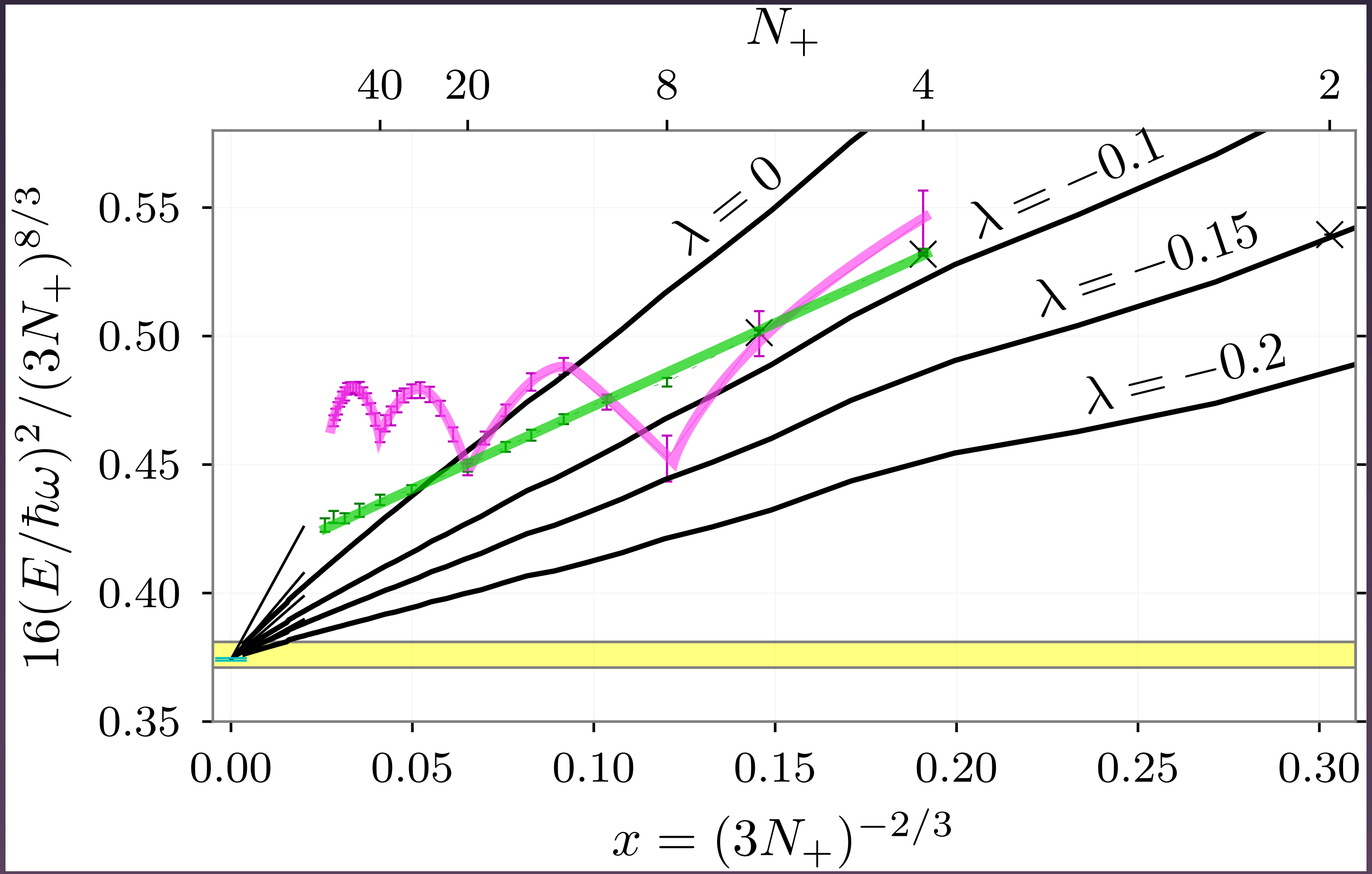


- Thermodynamic limit is far away!
- Hard to see asymptotic behaviour (N~10^8)
- Validity of LDA?

Forbes 2013

Gradient Corrections?

$$\mathcal{E}(n,\tau,\nu) = \left(\alpha\frac{\tau}{m} + g\nu^\dagger\nu\right) + \beta\mathcal{E}_{FG}(n) + \frac{\lambda}{8m}\frac{(\nabla n)^2}{n}$$

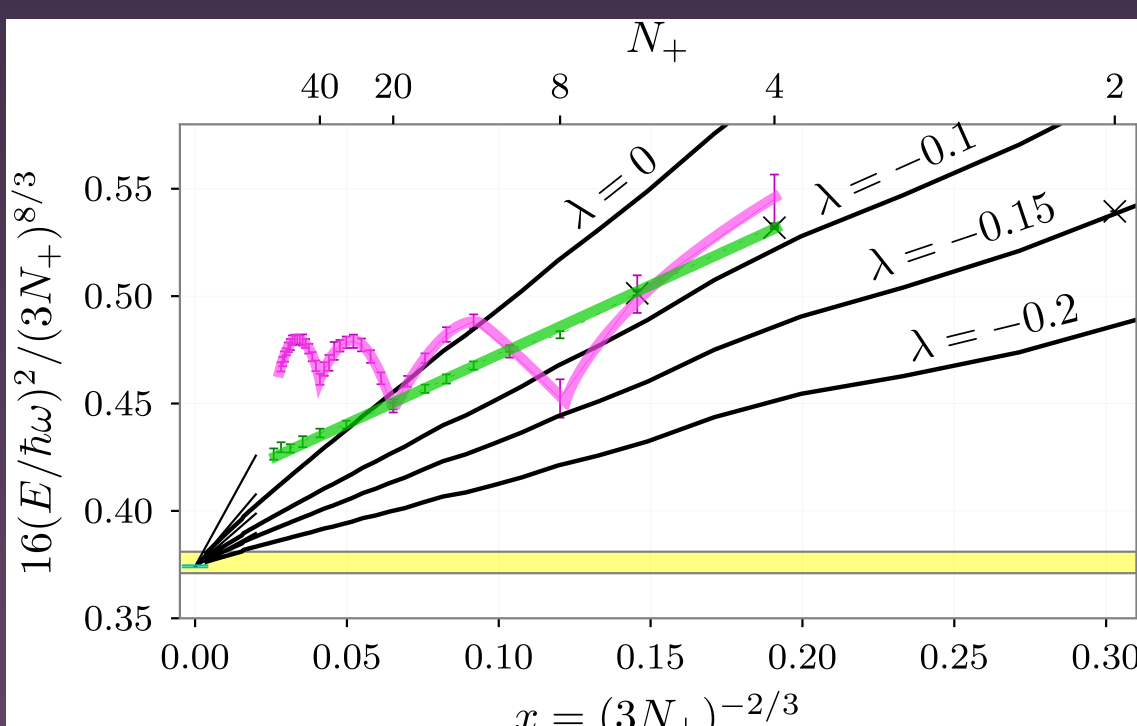


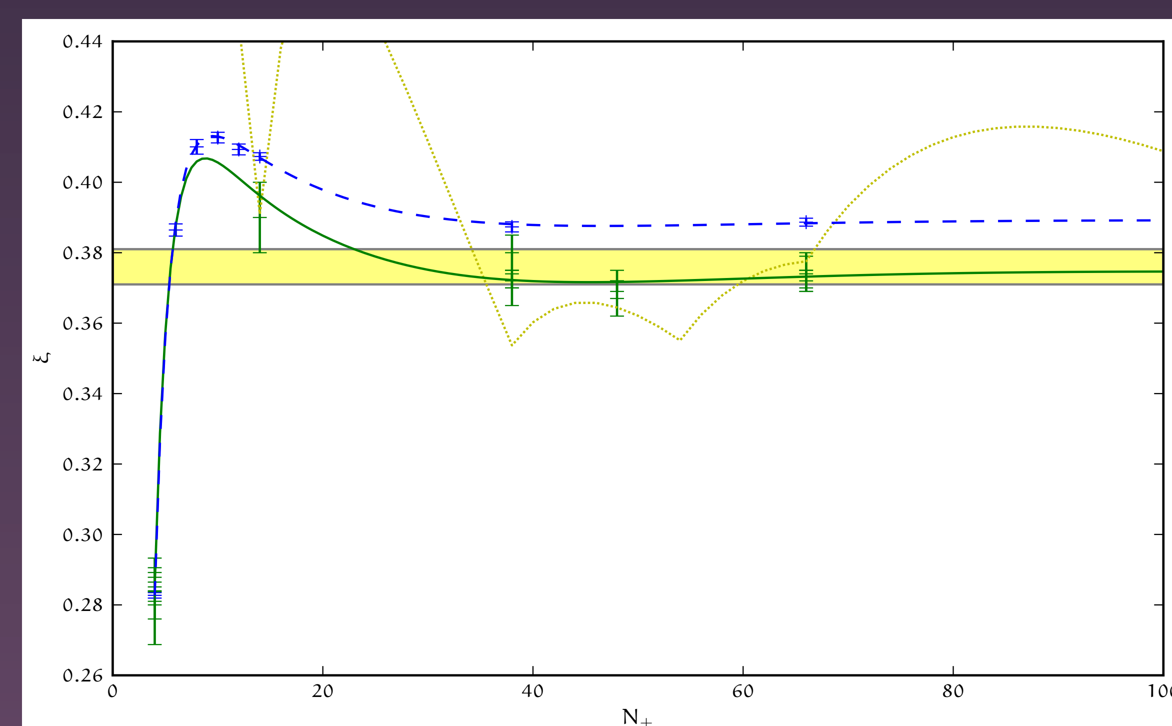
- Negative!
- c_χ=1.5(3)
- Only LO gradient affects c_χ
- Still a “small” effect (~10%)

SLDA Summary

$$\mathcal{E}(n,\tau,\nu) = \left(\alpha\frac{\tau}{m} + g\nu^\dagger\nu\right) + \beta\mathcal{E}_{FG}(n) + \frac{\lambda}{8m}\frac{(\nabla n)^2}{n}$$

- Works remarkably well

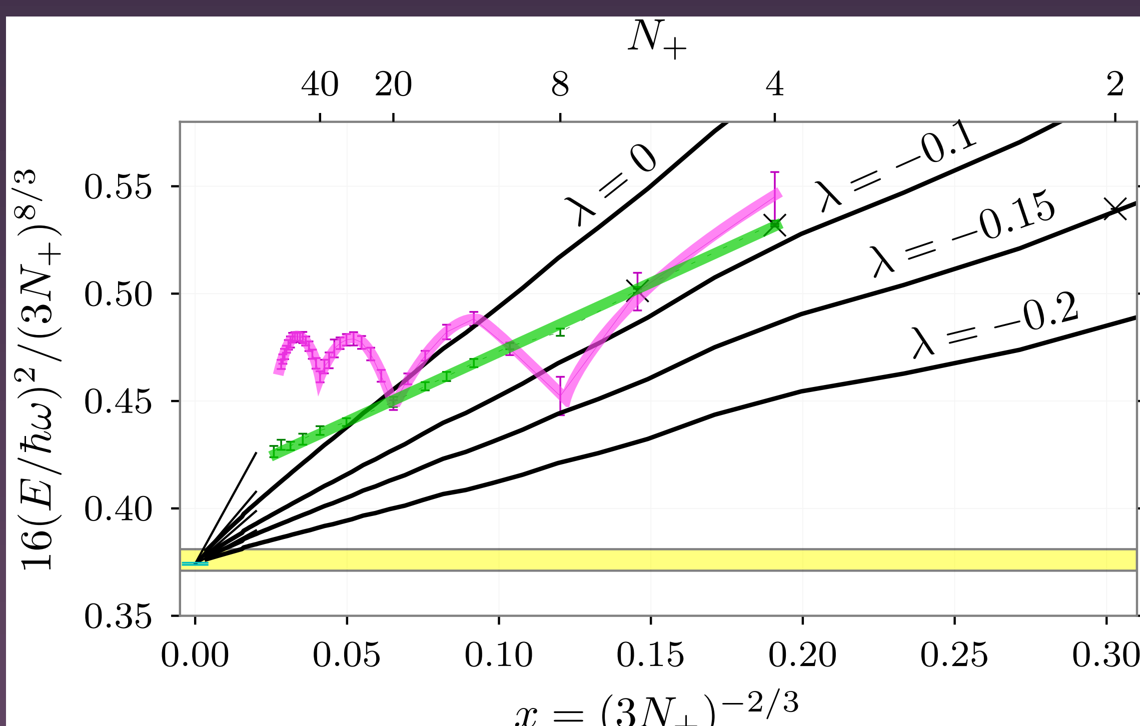


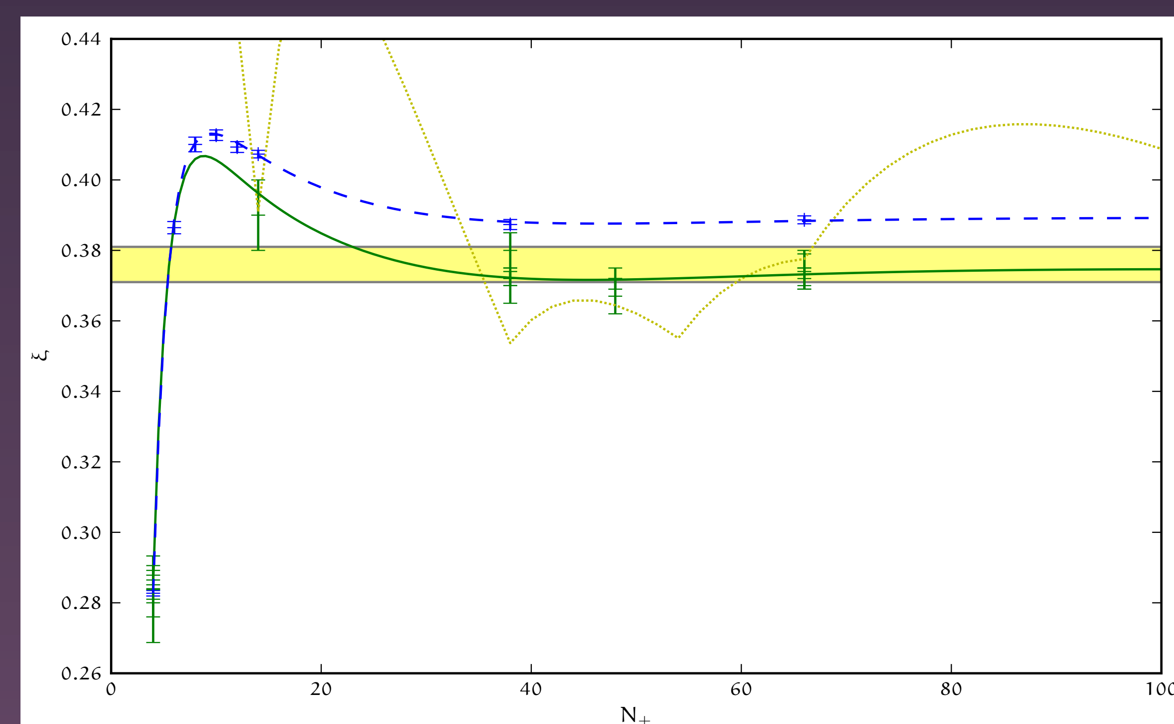


SLDA Summary

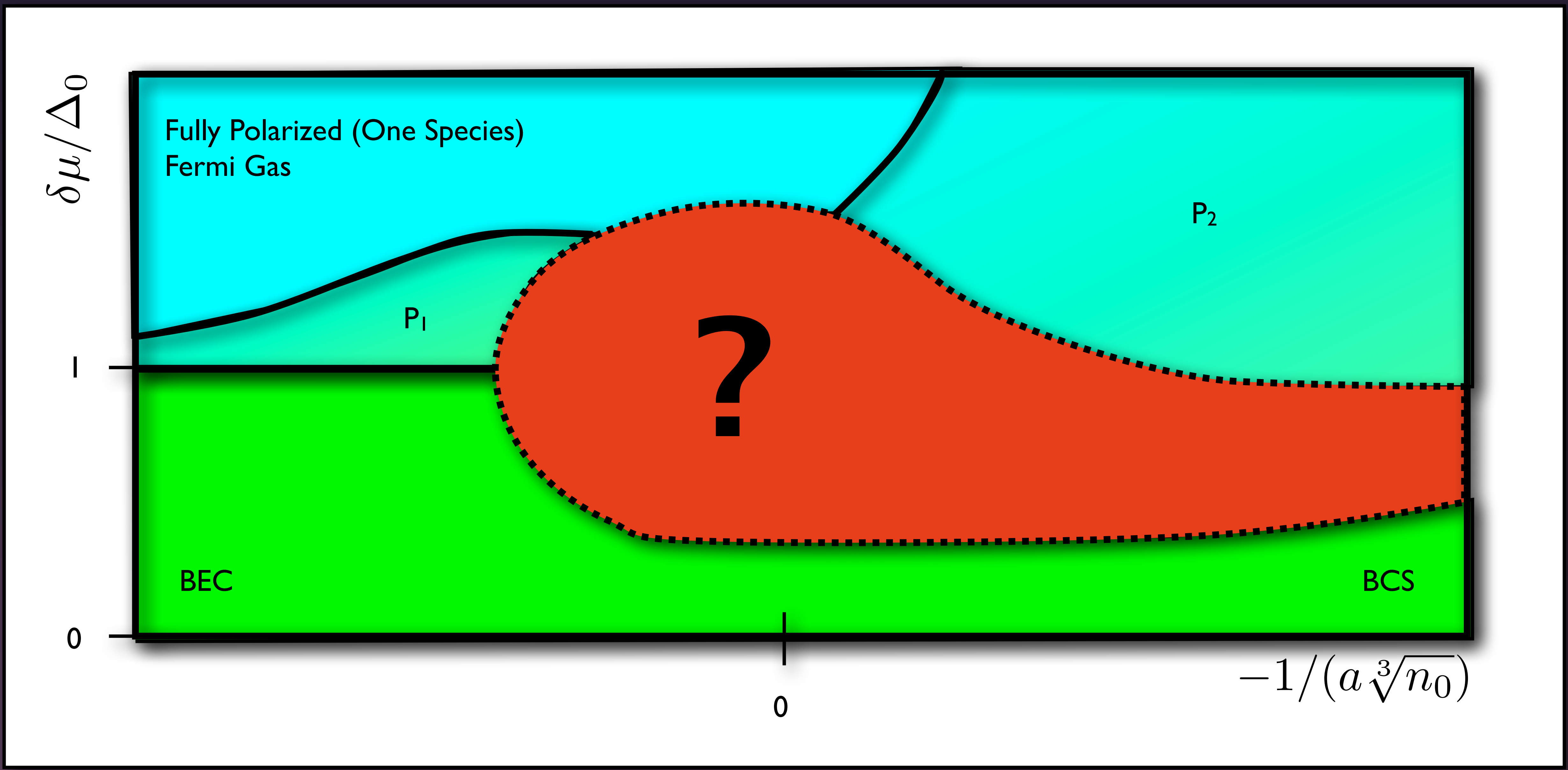
$$\mathcal{E}(n,\tau,\nu) = \left(\alpha\frac{\tau}{m} + g\nu^\dagger\nu\right) + \beta\mathcal{E}_{FG}(n) + \frac{\lambda}{8m}\frac{(\nabla n)^2}{n}$$

- Extrapolation: small to large systems (IR convergence)



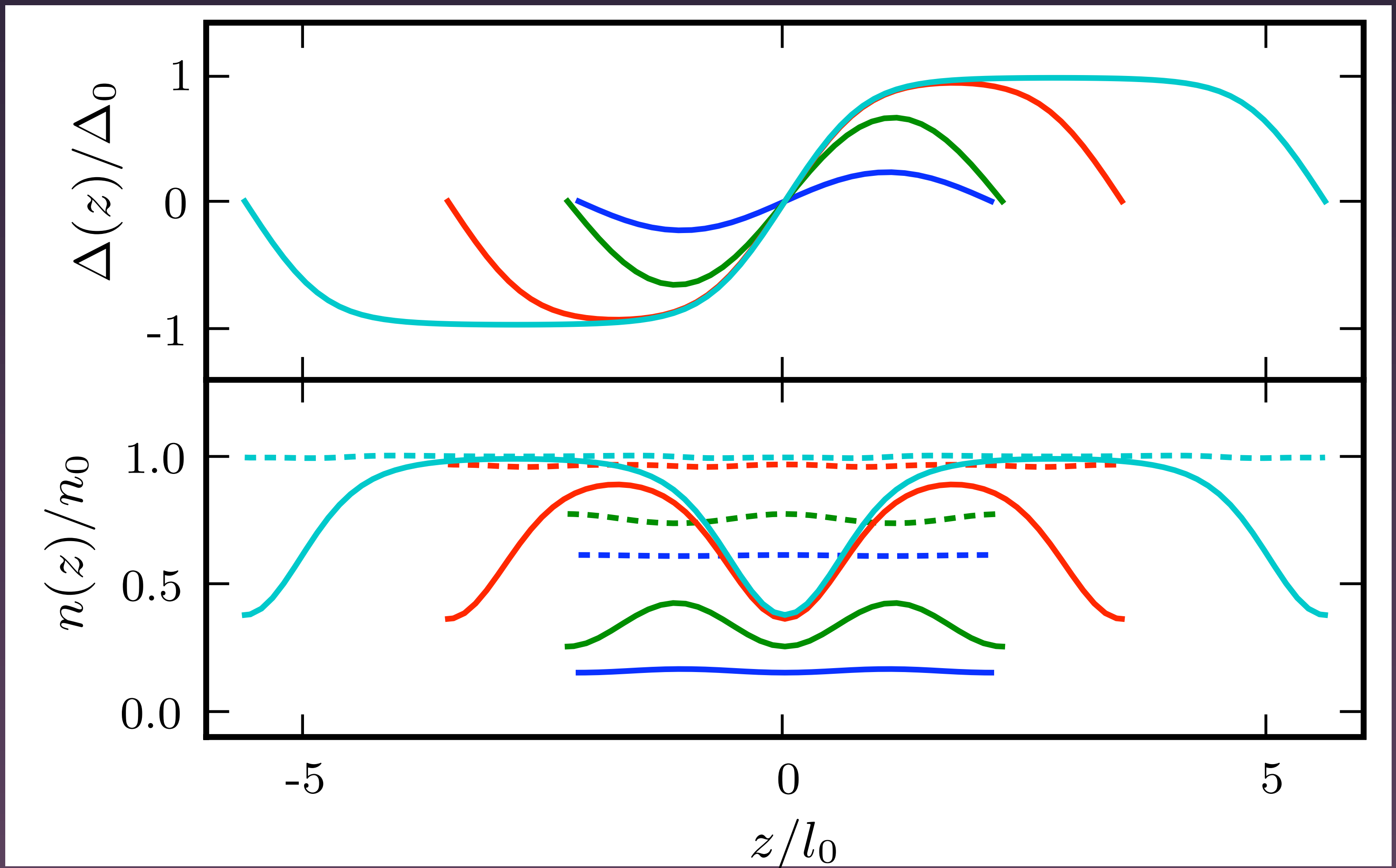


Phase Structure



Based on D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk PRL 97 020402 (2006)

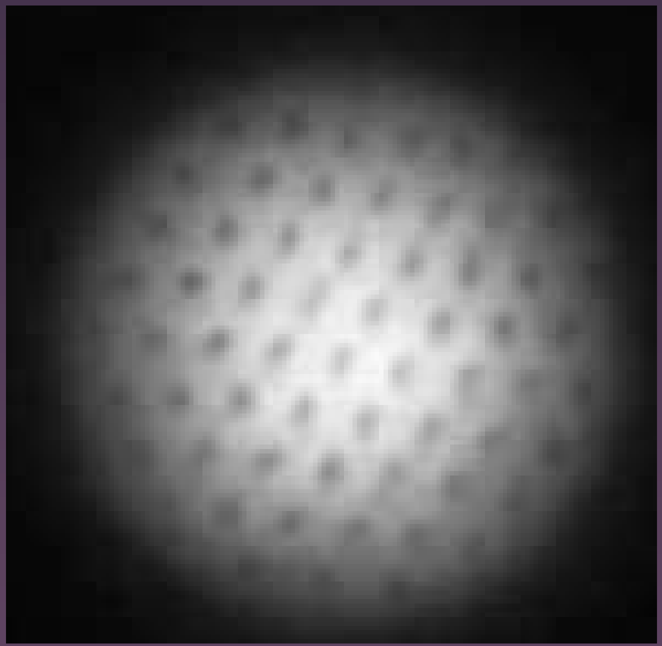
ASLDA DFT predicts (FF)LO at Unitarity: Supersolid!



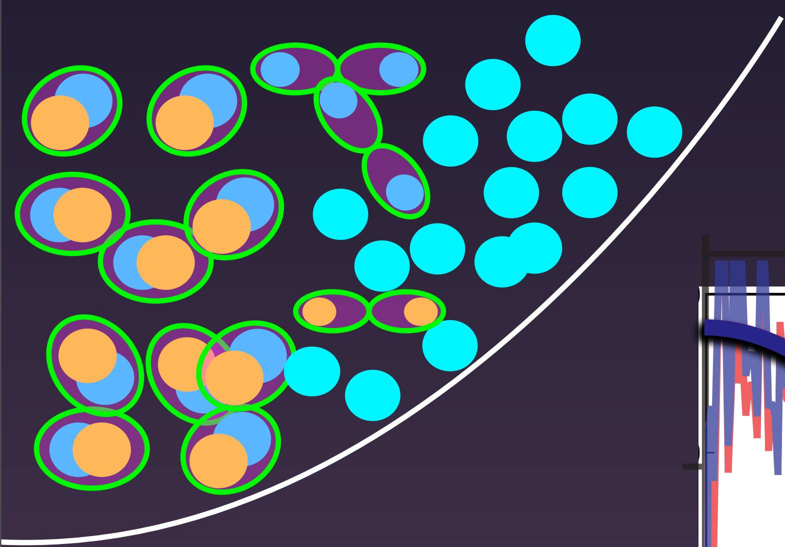
Bulgac and Forbes PRL 101 (2008) 215301

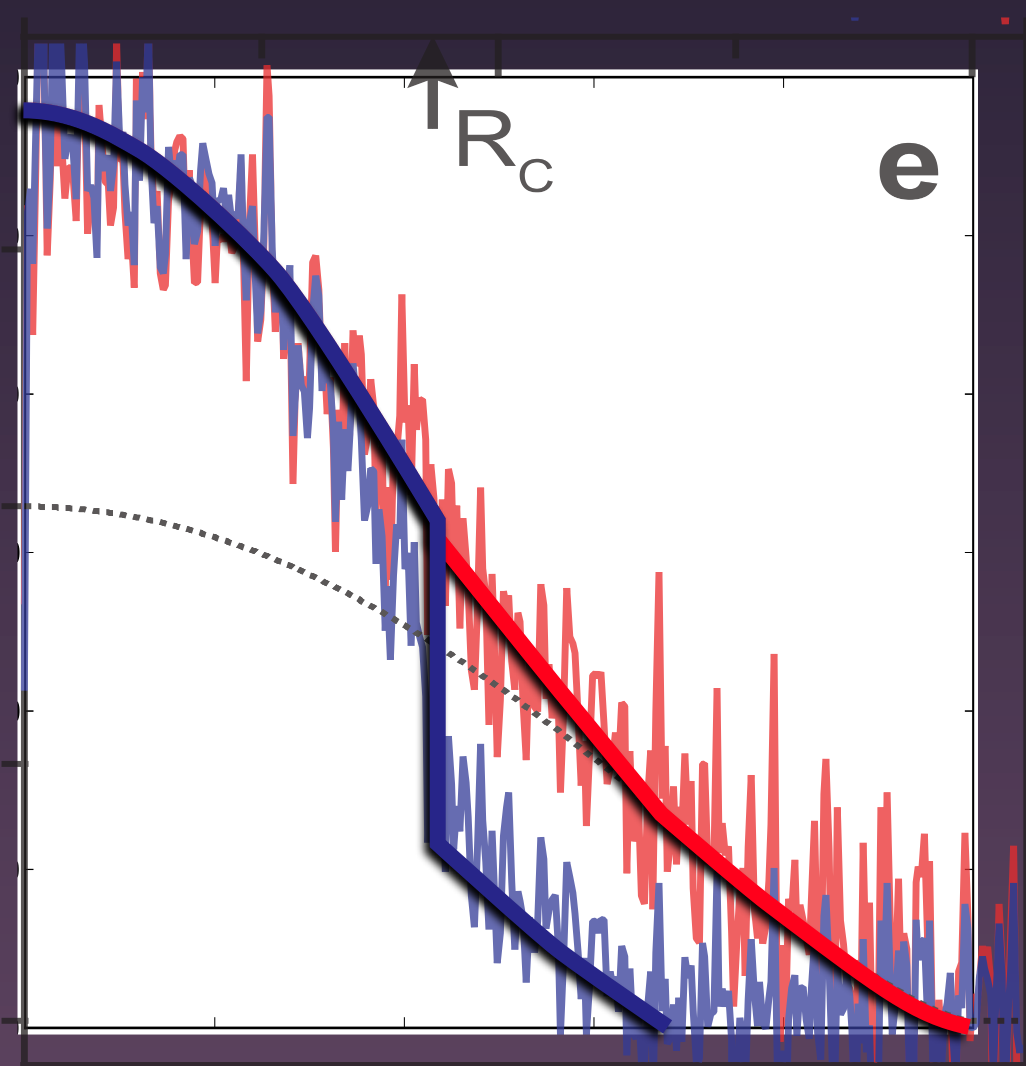
Large density contrast (factor of 2)

Similar to contrast of vortex core



Observations: Nothing?





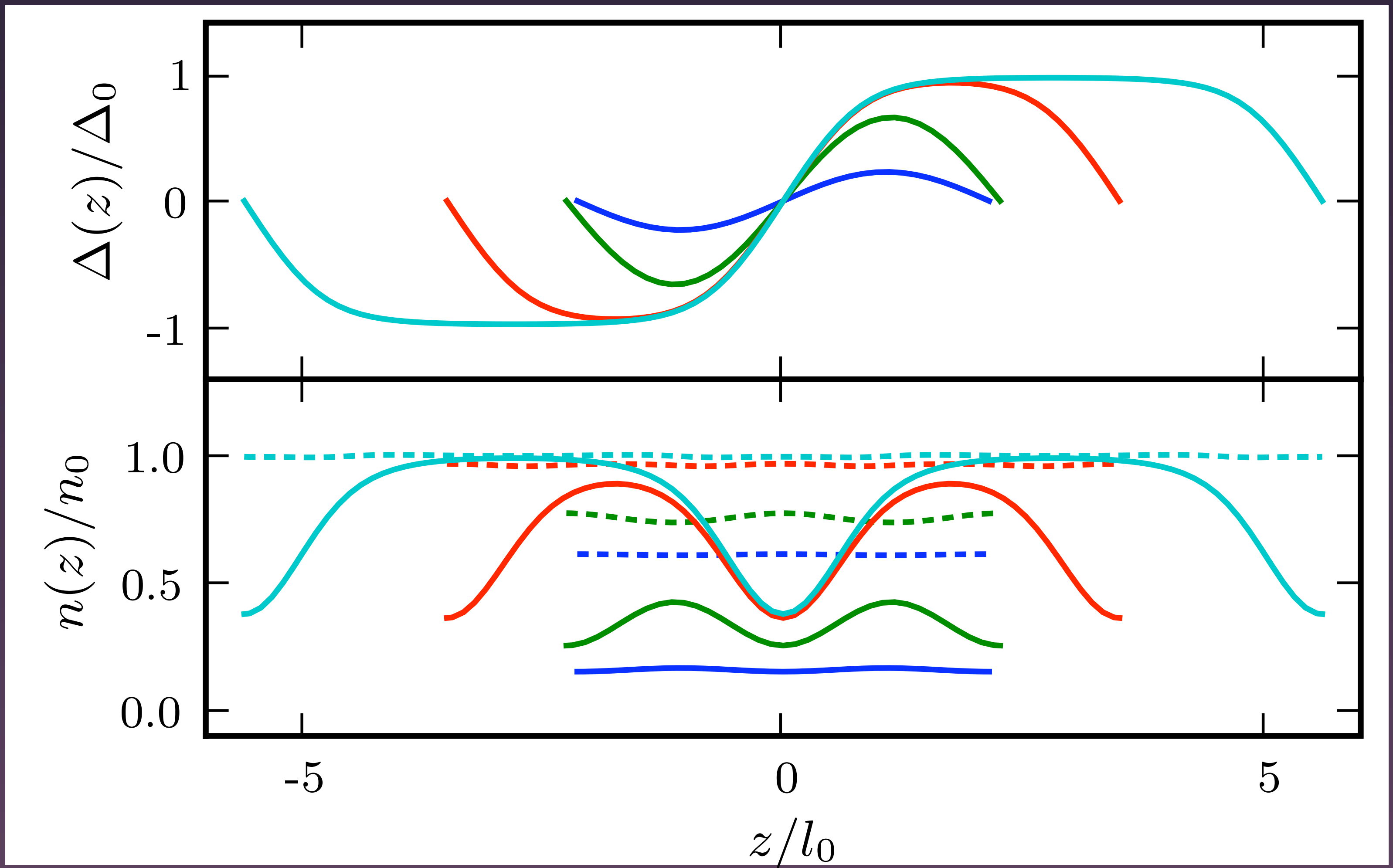
MIT Experimental data from Shin et. al (2008)

Paired core

Polarized wings

Maybe there are no interesting polarized superfluid phases?

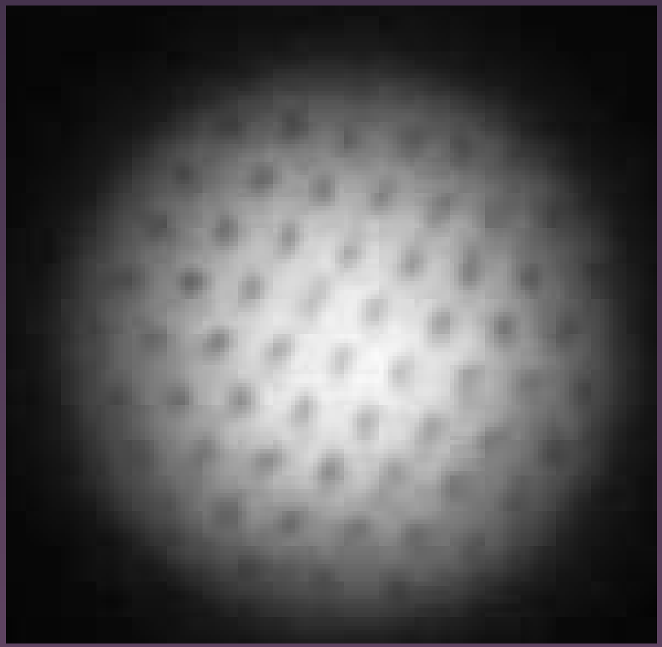
DFT predicts (FF)LO at Unitarity: Supersolid!



Bulgac and Forbes PRL 101 (2008) 215301

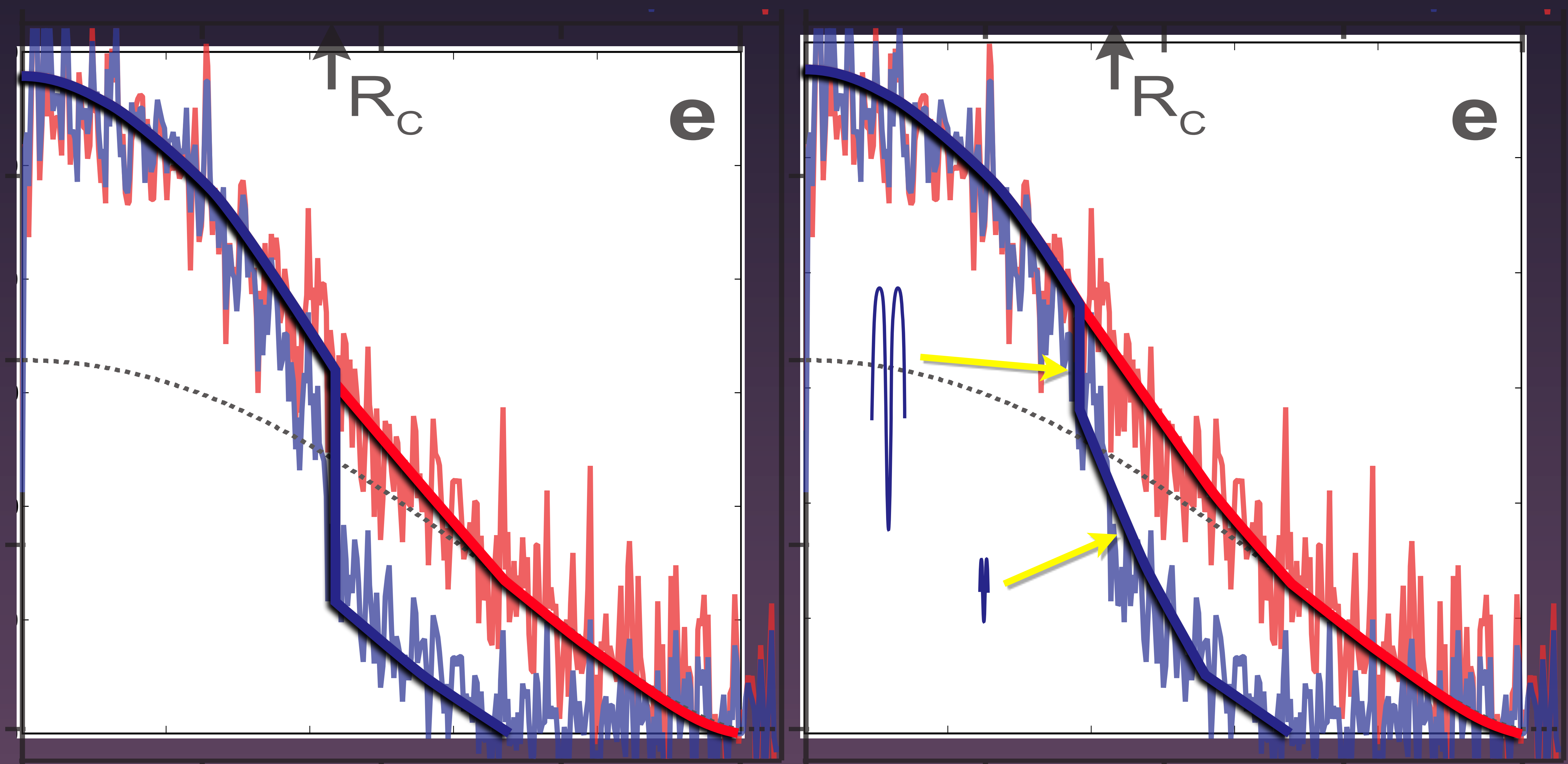
Large density contrast (factor of 2)

Similar to contrast of vortex core



Observations: Inconclusive

- Need detailed structure or novel signature



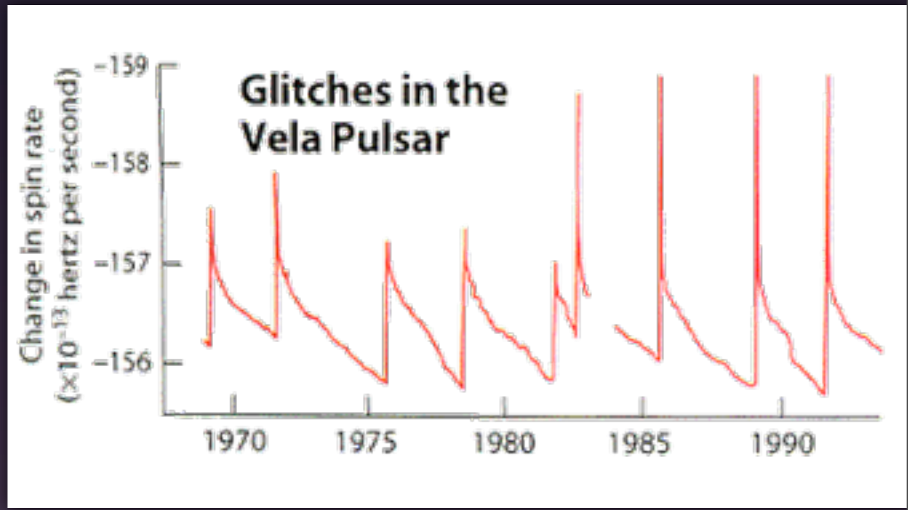
MIT Experimental data from Shin et. al (2008)

Why FFLO not seen?

- It is not there:
  - Other homogenous phases might be better.
  - T might be too high (fluctuations kill 1D FFLO).
  - Trap frustrates formation (traps are not flat enough).
- It is not seen:
  - Noise washes out signature.
  - Small physical volume for FFLO.
- Need a nice flat trap: Large physical volume of FFLO

Dynamics

Application: Vortex Pinning



Pulsar Astronomy by Andrew G. Lyne and Francis

- Pulsar glitching (neutron stars)
  - Massive vortex unpinning events?

    Anderson and Itoh (1975)

- Large scale events (thousands of vortices)
  - Too big for DFT – use GPE

- Need Vortex-Defect interactions (force)
  - Use DFT to calculate and then fit GPE

TDDFT (TDSLDA)

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Need to evolve each hundreds of thousands of wavefunctions
- Possible for moderate systems (nuclei) using supercomputers, resonances, induced fission etc.
- Probably not for glitching dynamics

GPE model for UFG

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}(\rho_F) \right)$$

$$i\partial_t\Psi = \left(-\frac{\nabla^2}{4m_F} + 2[V_F + \xi\epsilon(\rho_F)] \right) \Psi$$

- Think:
  - Boson = Fermion pair (dimer)
- Galilean Covariant (fixes mass)
- Match Unitary Equation of State

$$\rho_F = 2|\Psi|^2$$

$$\mathcal{E}_{FG} \propto \rho_F^{5/2}$$

$$\epsilon_F = \mathcal{E}'_{FG}(\rho_F) \propto \rho_F^{3/2}$$

GPE model = Extended Thomas Fermi (ETF)

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\sqrt{\rho_F}|^2}{8m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}_{FG}(\rho_F) + \frac{4\lambda - 1}{8m_F}(\nabla\sqrt{\rho_F})^2 \right)$$

- In the absence of currents (i.e. no vortices), kinetic and Weizsäcker terms behave the same

  Vortices etc. appear as kinks in $\sqrt{\rho_F}$

- See Salasnich for a discussion

GPE model = Extended Thomas Fermi (ETF)

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\sqrt{\rho_F}|^2}{8m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}_{FG}(\rho_F) + \cancel{\frac{4\lambda - 1}{8m_F}}(\nabla\sqrt{\rho_F})^2 \right)$$

- Weizsäcker term “breaks” vortices
- (Also does not match experiment)

GPE model for UFG

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}(\rho_F) \right)$$

$$i\partial_t\Psi = \left(-\frac{\nabla^2}{4m_F} + 2[V_F + \xi\epsilon(\rho_F)] \right) \Psi$$

- Dynamics are much easier than SLDA
  - Only one wavefunction to evolve
- Contains superfluid hydrodynamic equations
- Match to low-energy physics

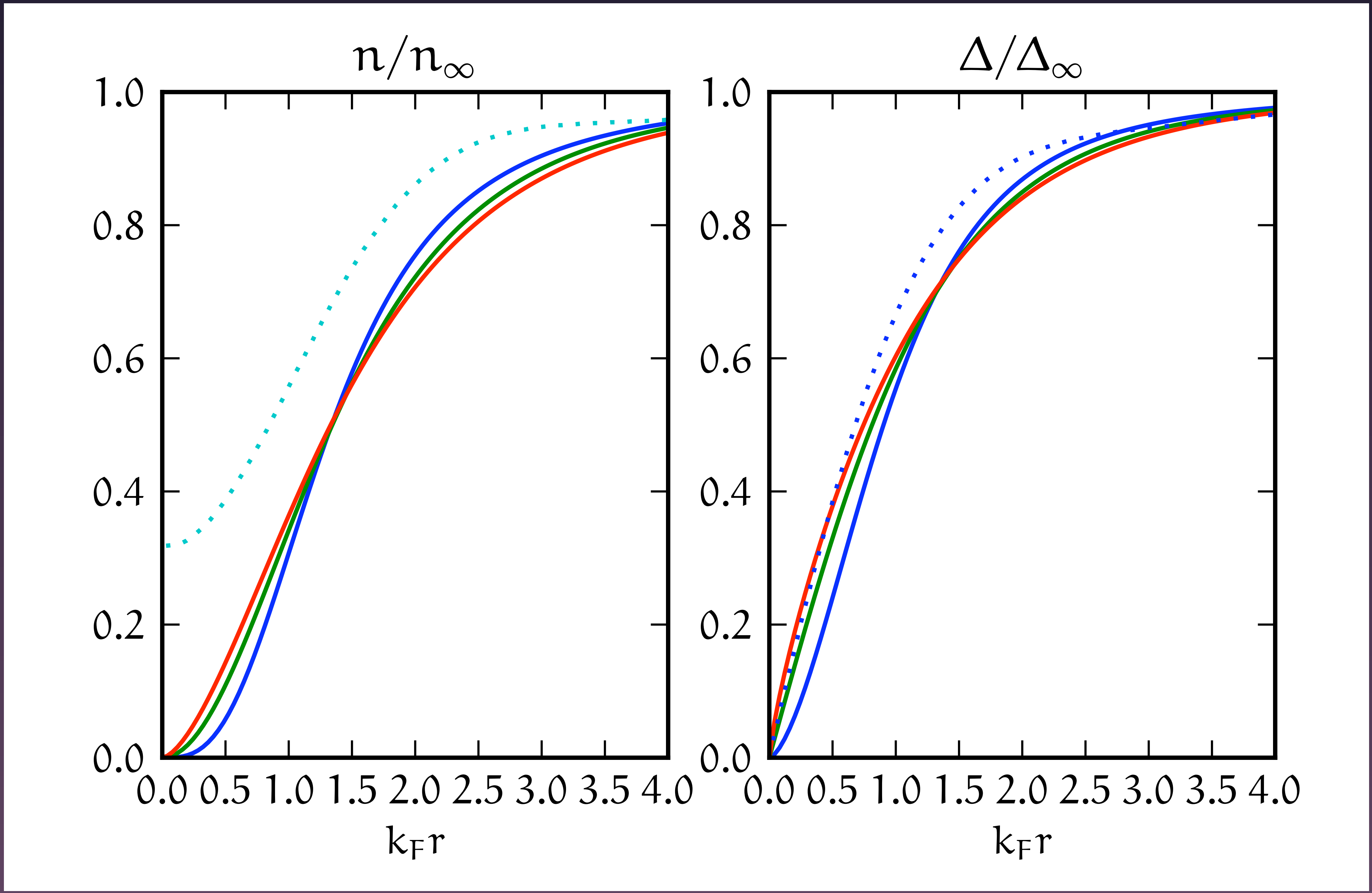
Matching Theories:
The Good

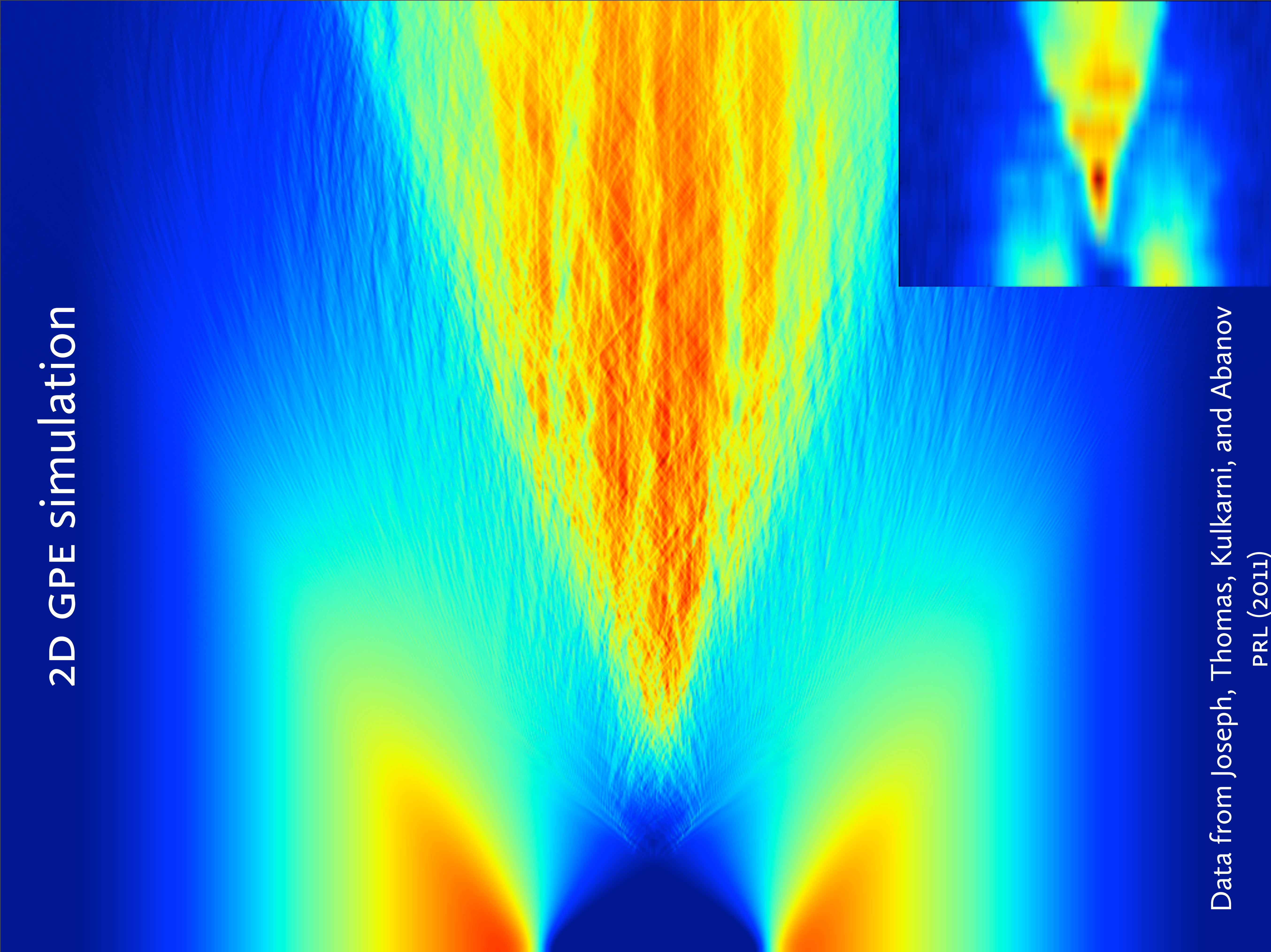
- Galilean Covariance (fixes mass/density relationship)
- Equation of State
- Hydrodynamics
  - speed of sound (exact)
  - phonon dispersion (to order q^3)
  - static response (to order q^2)

Matching Theories: The Bad

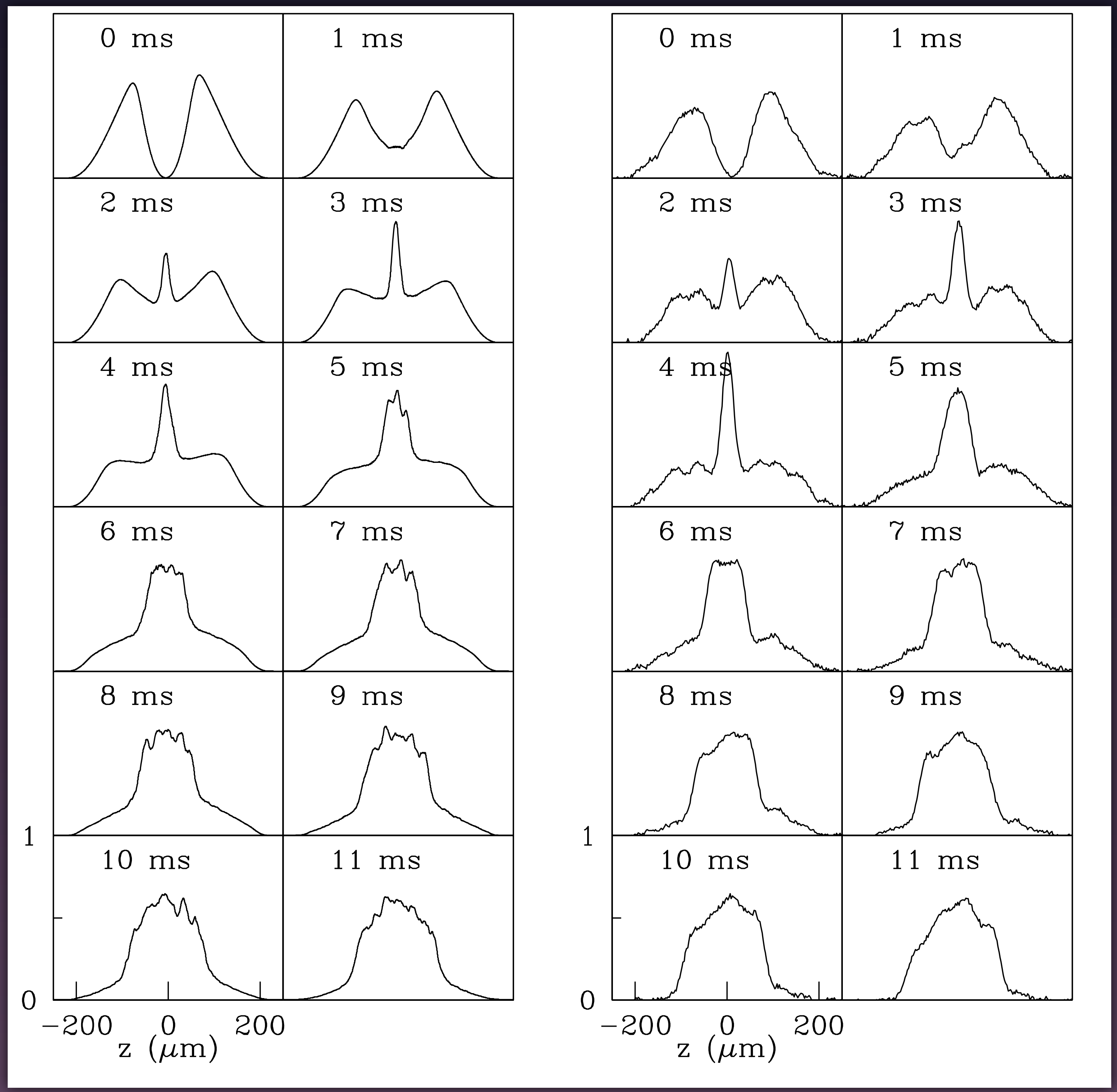
- GPE has $\rho=2|\Psi|^2$
  - Density vanishes in core of vortex
  - Implies $\int|\Psi|^2$ conserved
    - (Approximate conservation $\int|\Psi|^2$ in Fermi simulations provides measure of applicability)
- No “normal state”
  - Two fluid model needed?
  - Coarse graining (transfer to “normal” component)

Vortex Structure



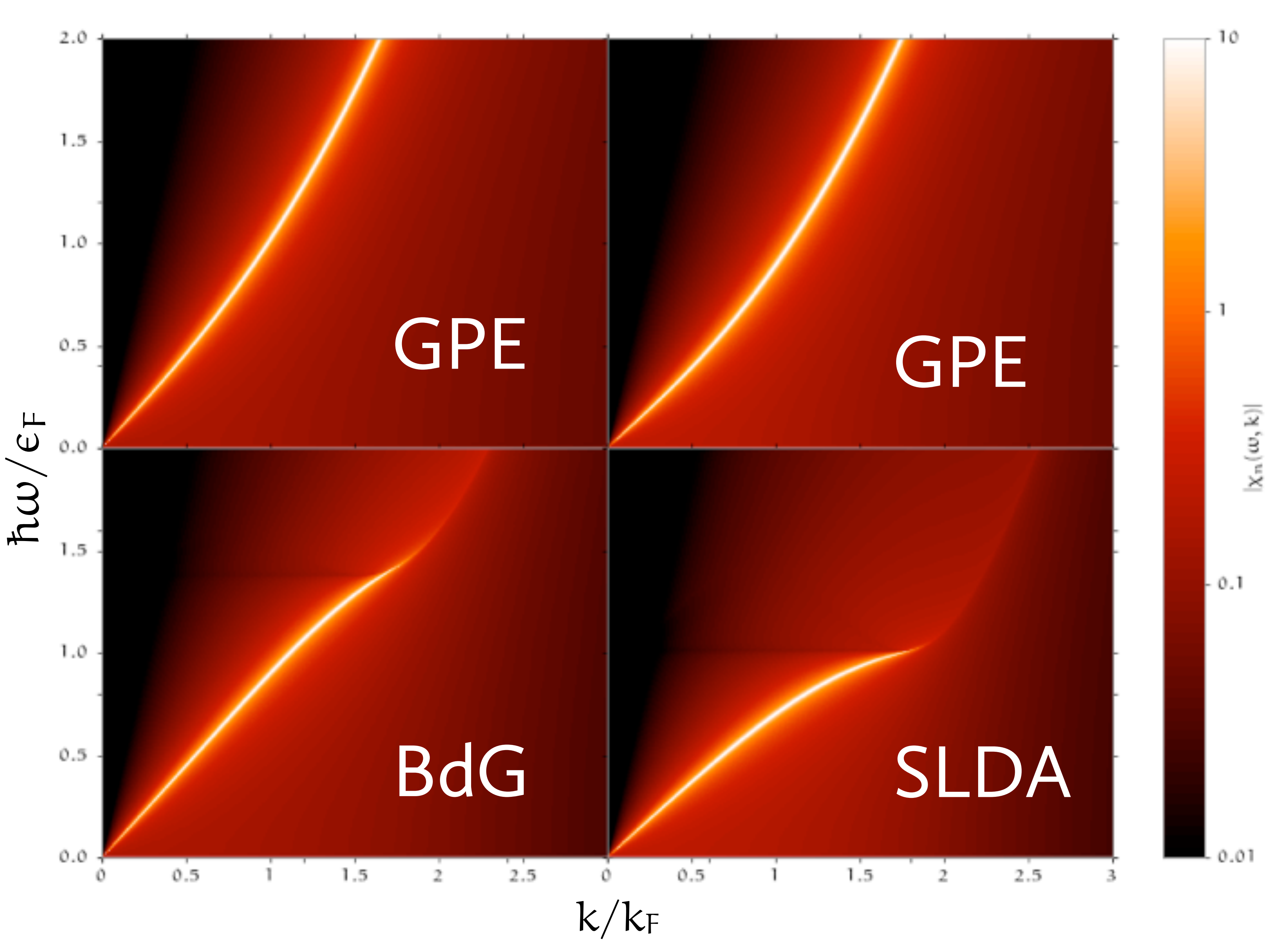

2D GPE simulation
Data from Joseph, Thomas, Kulkarni, and Abanov
PRL (2011)

GPE vs. Experiment



Ancilotto, L. Salasnich, and F. Toigo (2012)

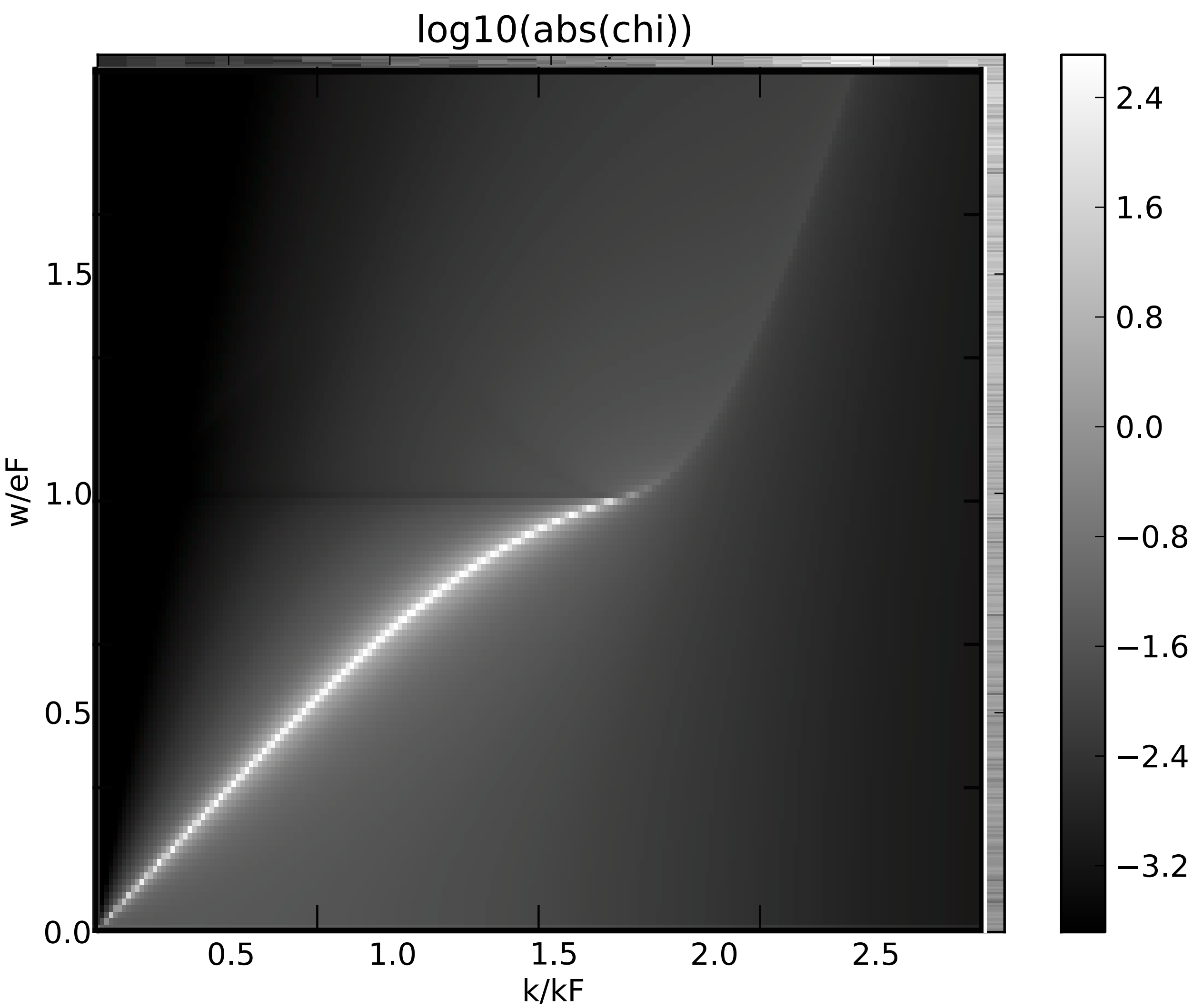
Linear Response



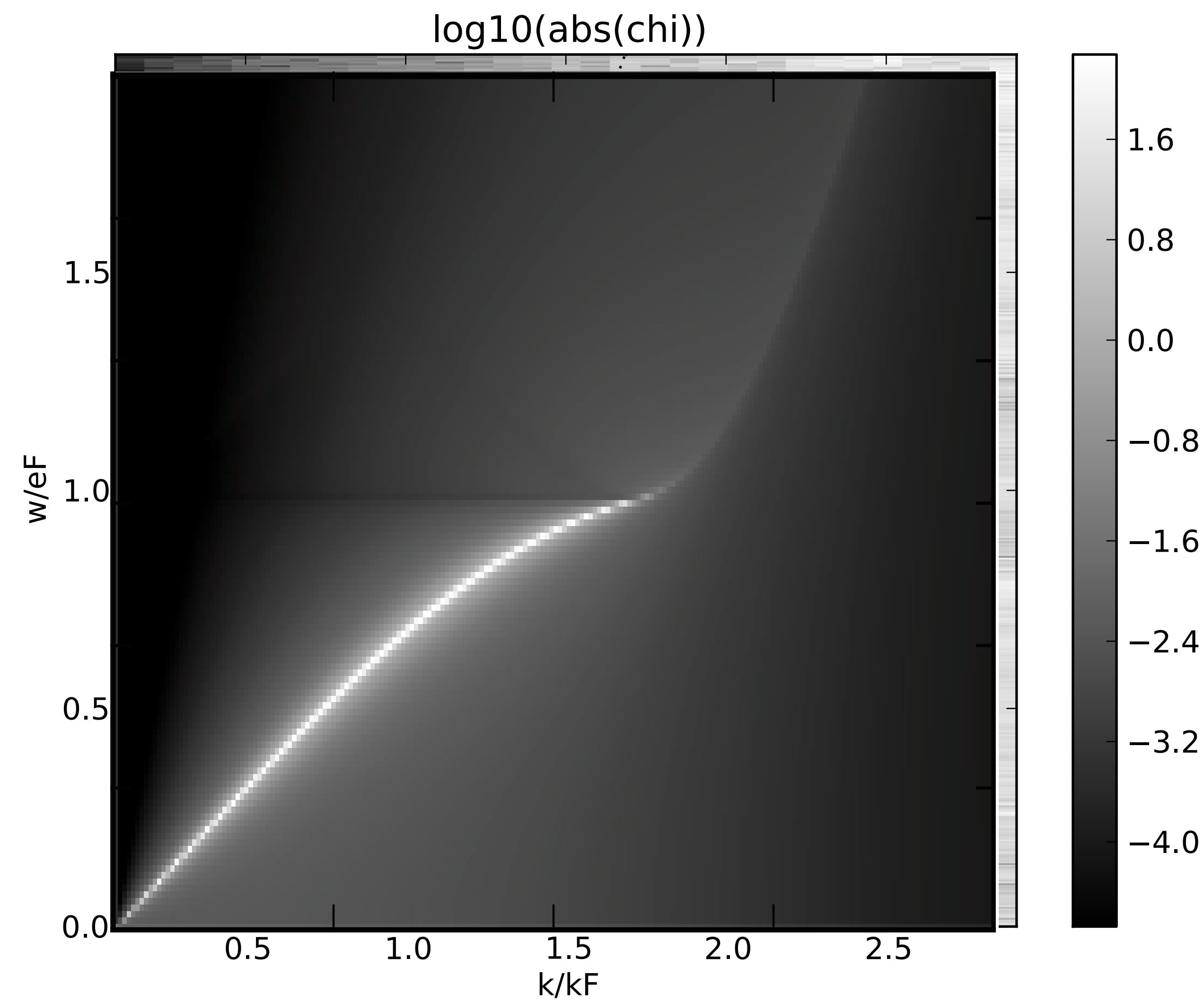
Work with Rishi Sharma (TRIUMF)

Non-Linear Response?

V=0.05



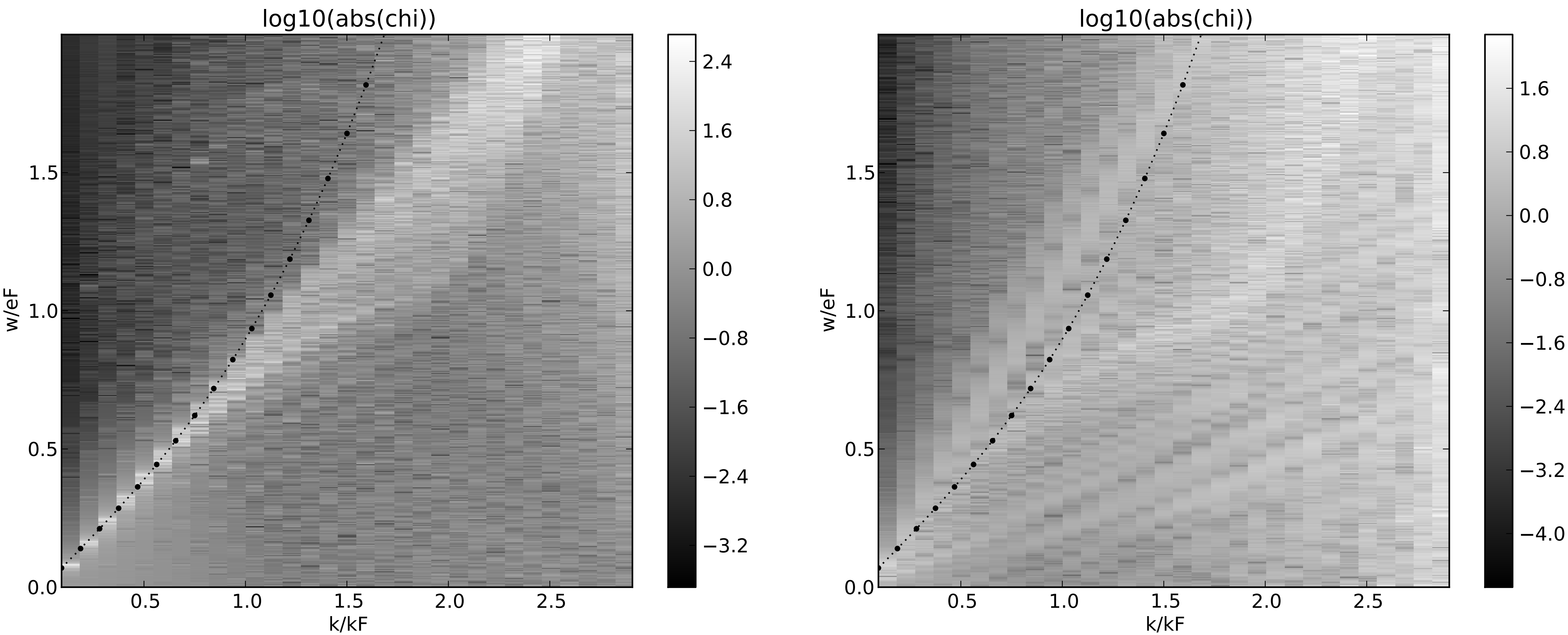
V=0.5



Work with Alan Luo and Rishi Sharma

Non-Linear Response?

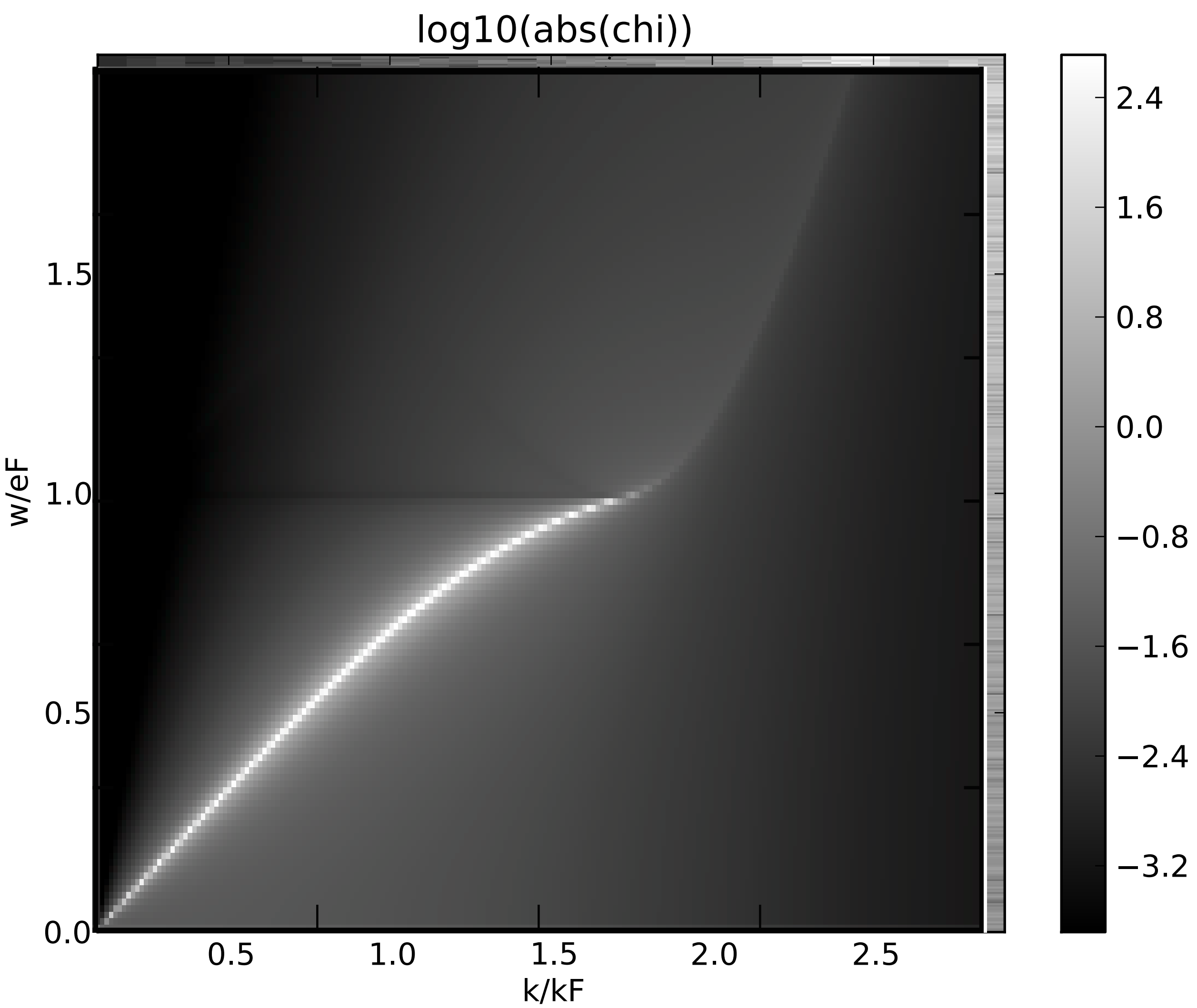
V=0.05 V=0.5



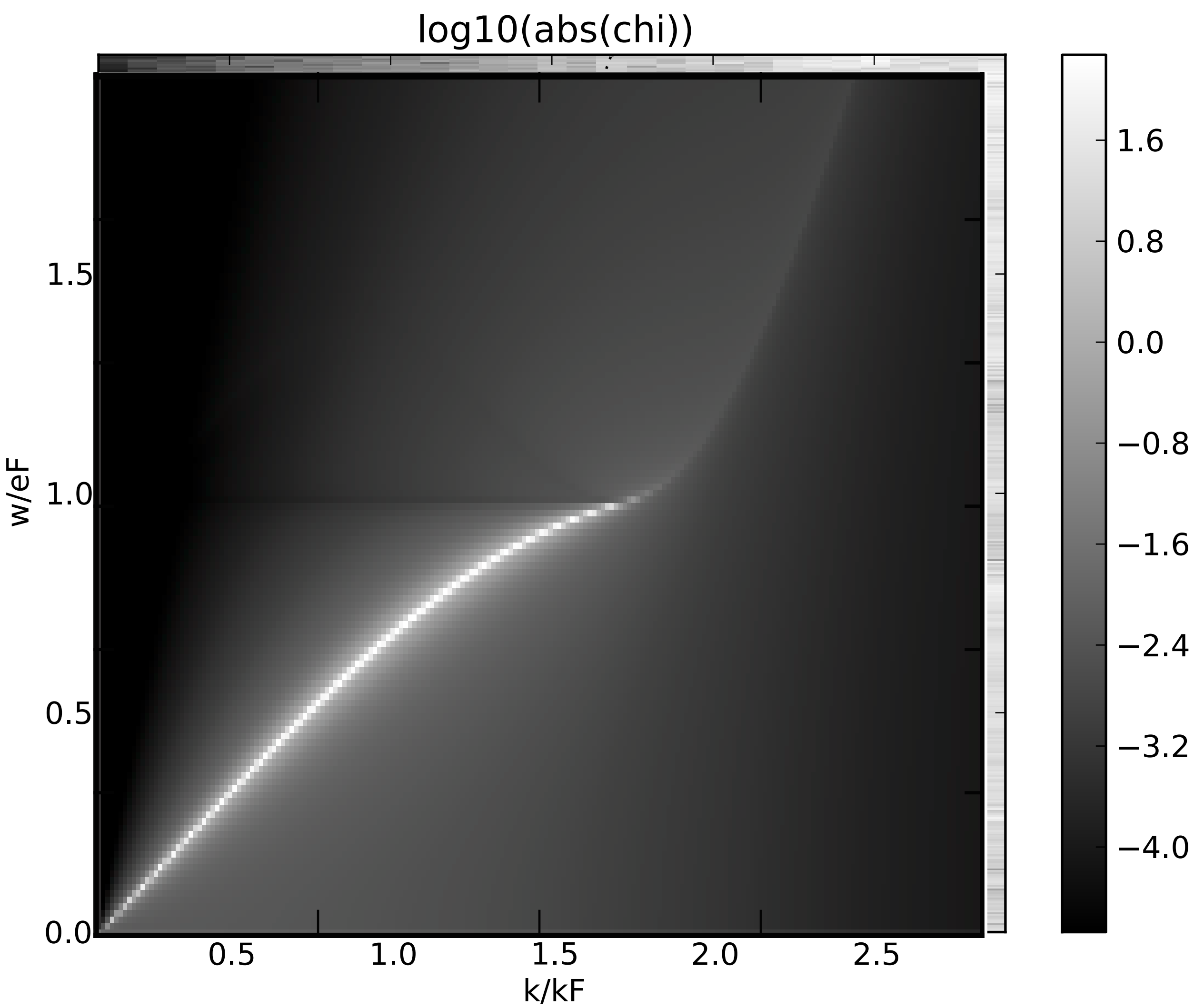
Work with Alan Luo and Rishi Sharma

Non-Linear Response?

V=0.05



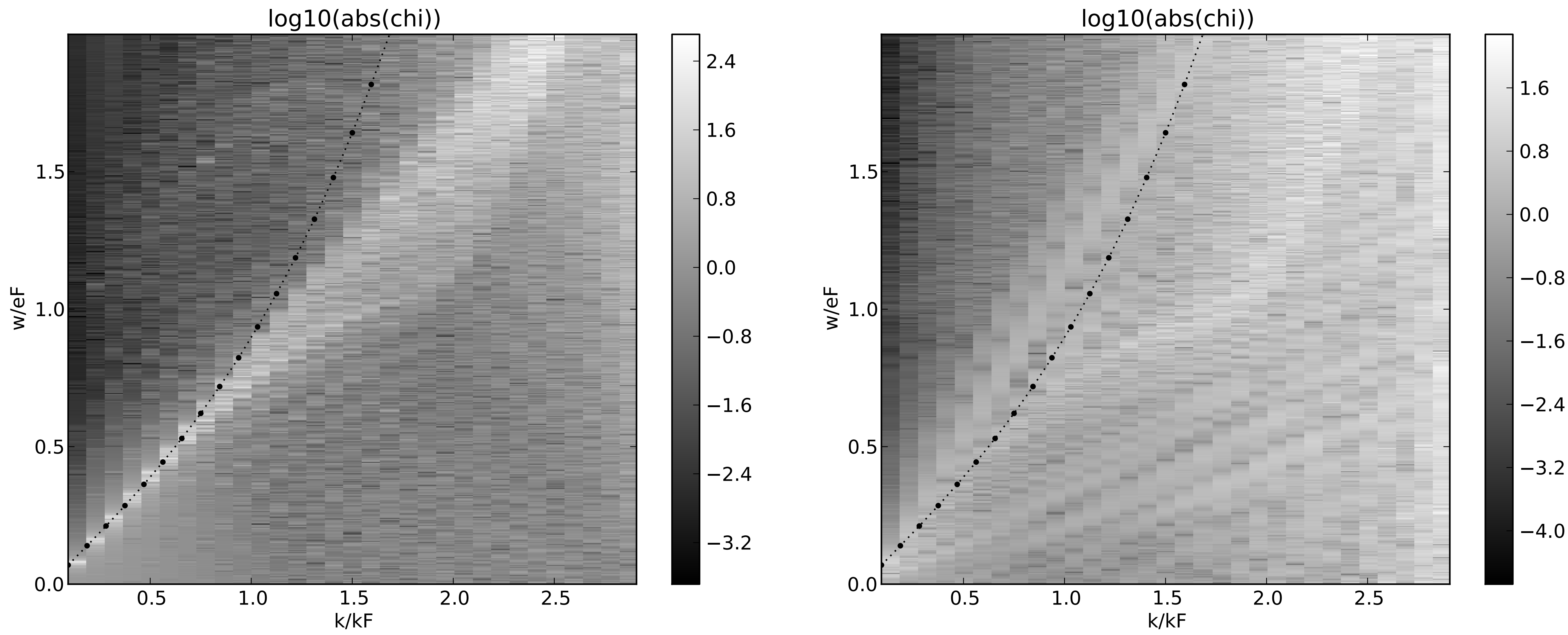
V=0.5



Work with Alan Luo and Rishi Sharma

Non-Linear Response?

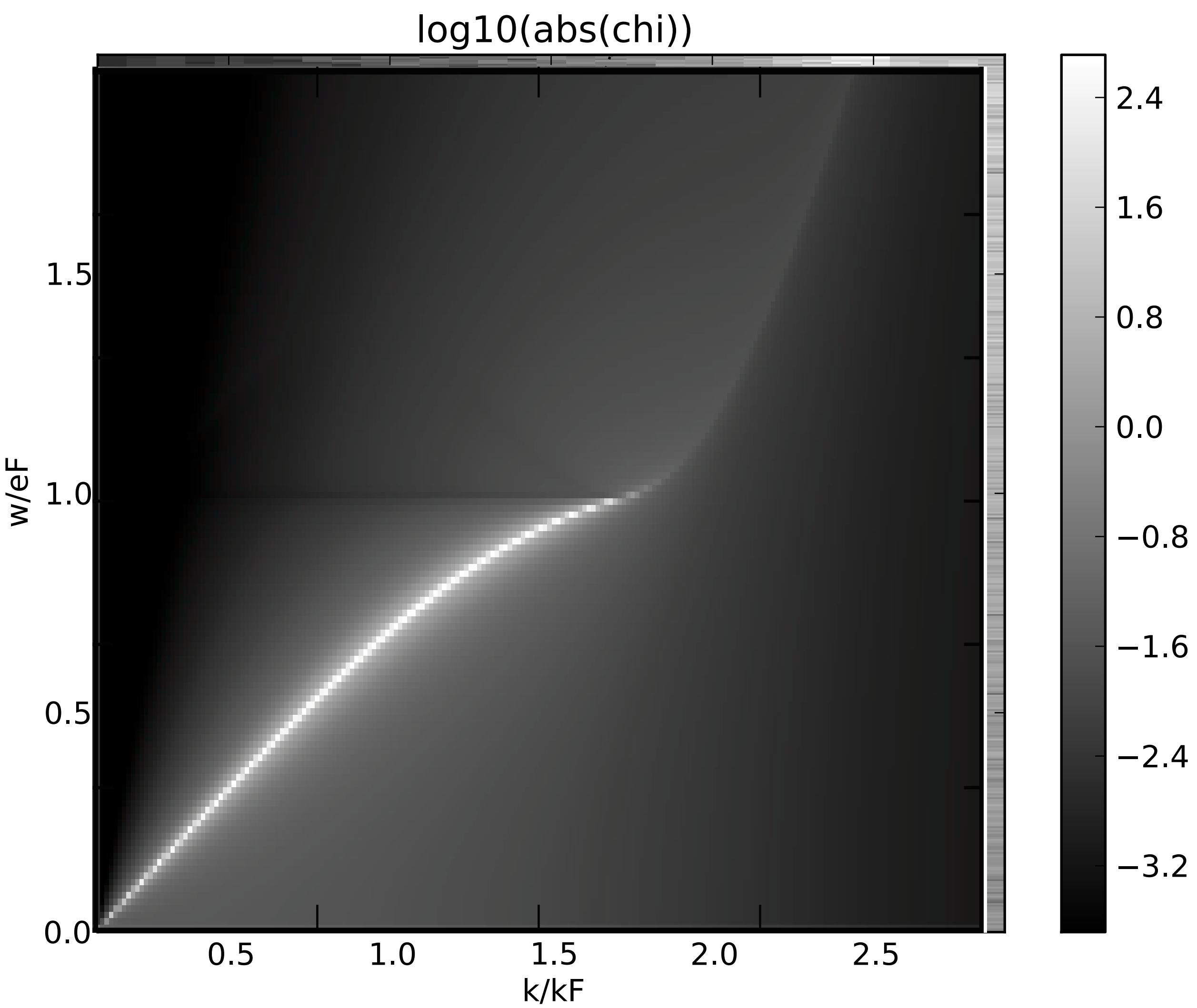
V=0.05 V=0.5



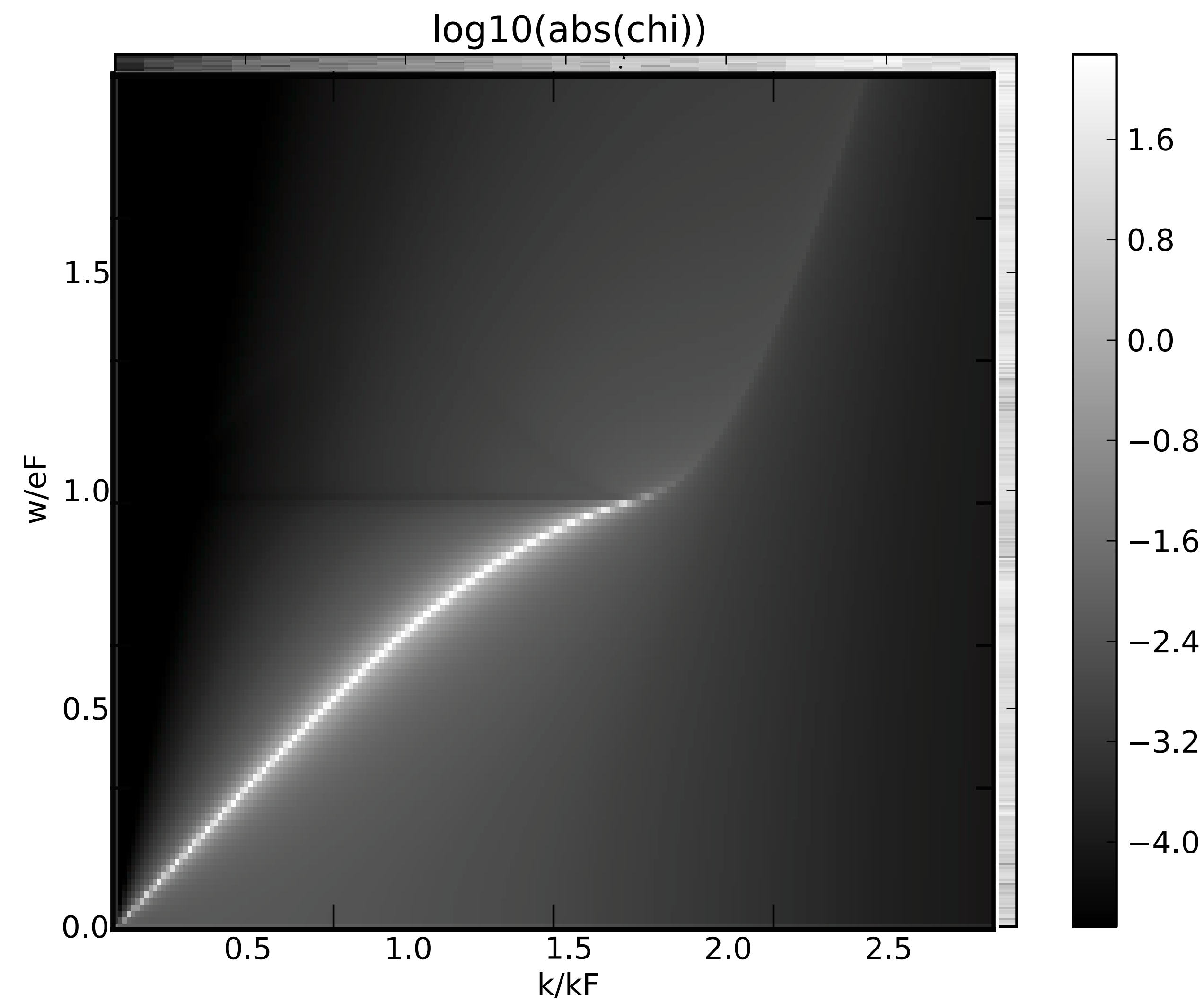
Work with Alan Luo and Rishi Sharma

Non-Linear Response?

V=0.05



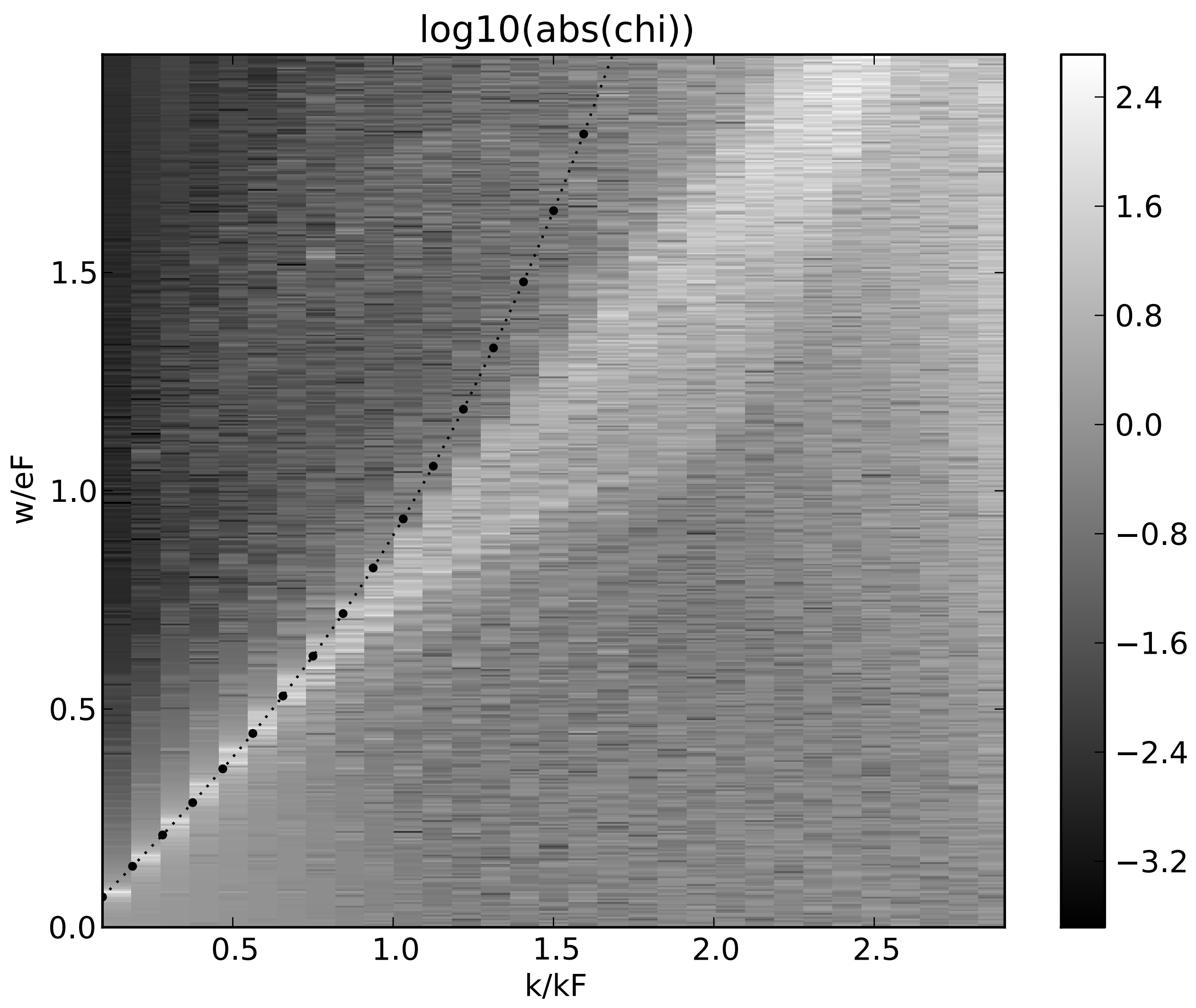
V=0.5



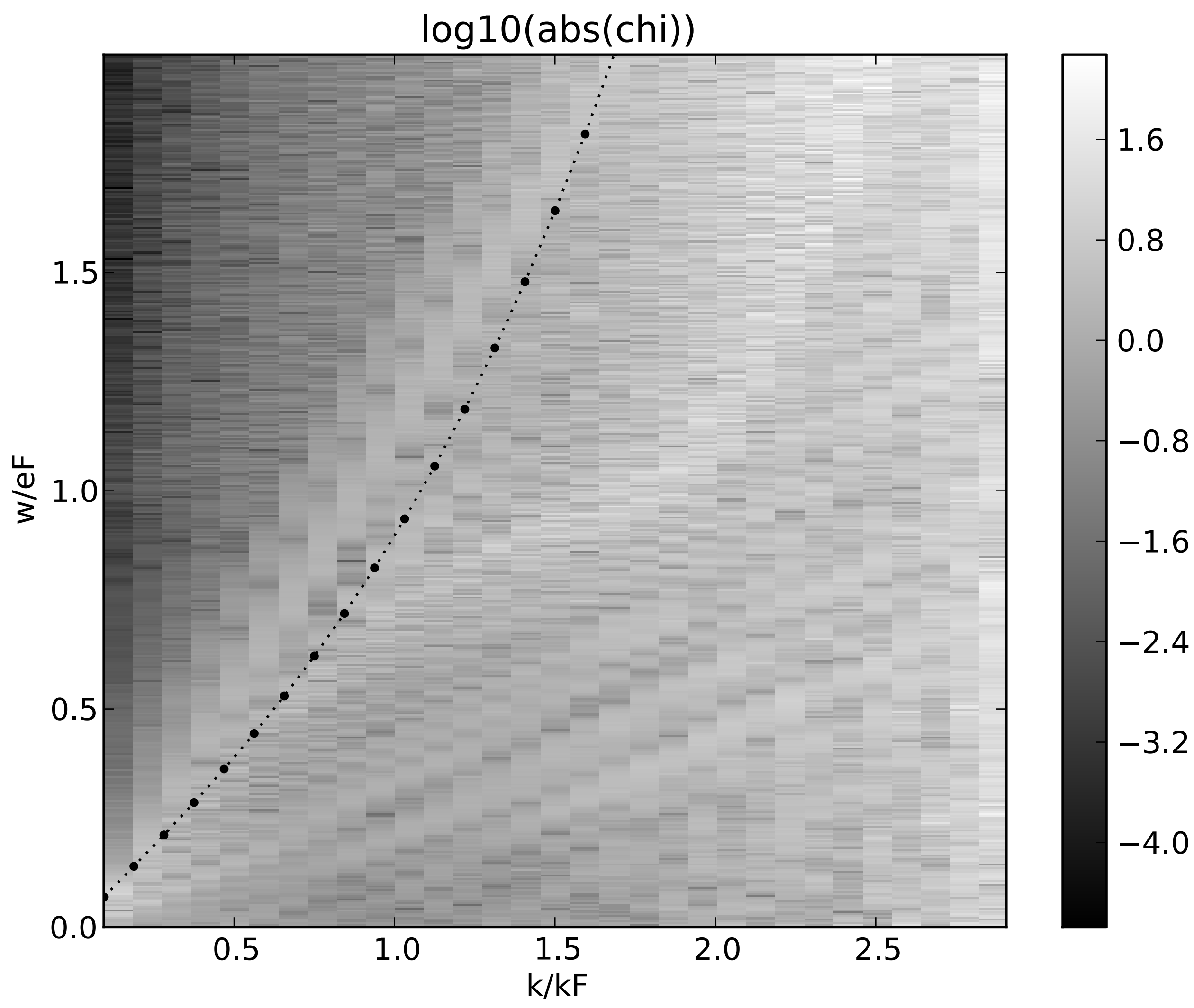
Work with Alan Luo and Rishi Sharma

Non-Linear Response?

V=0.05



V=0.5

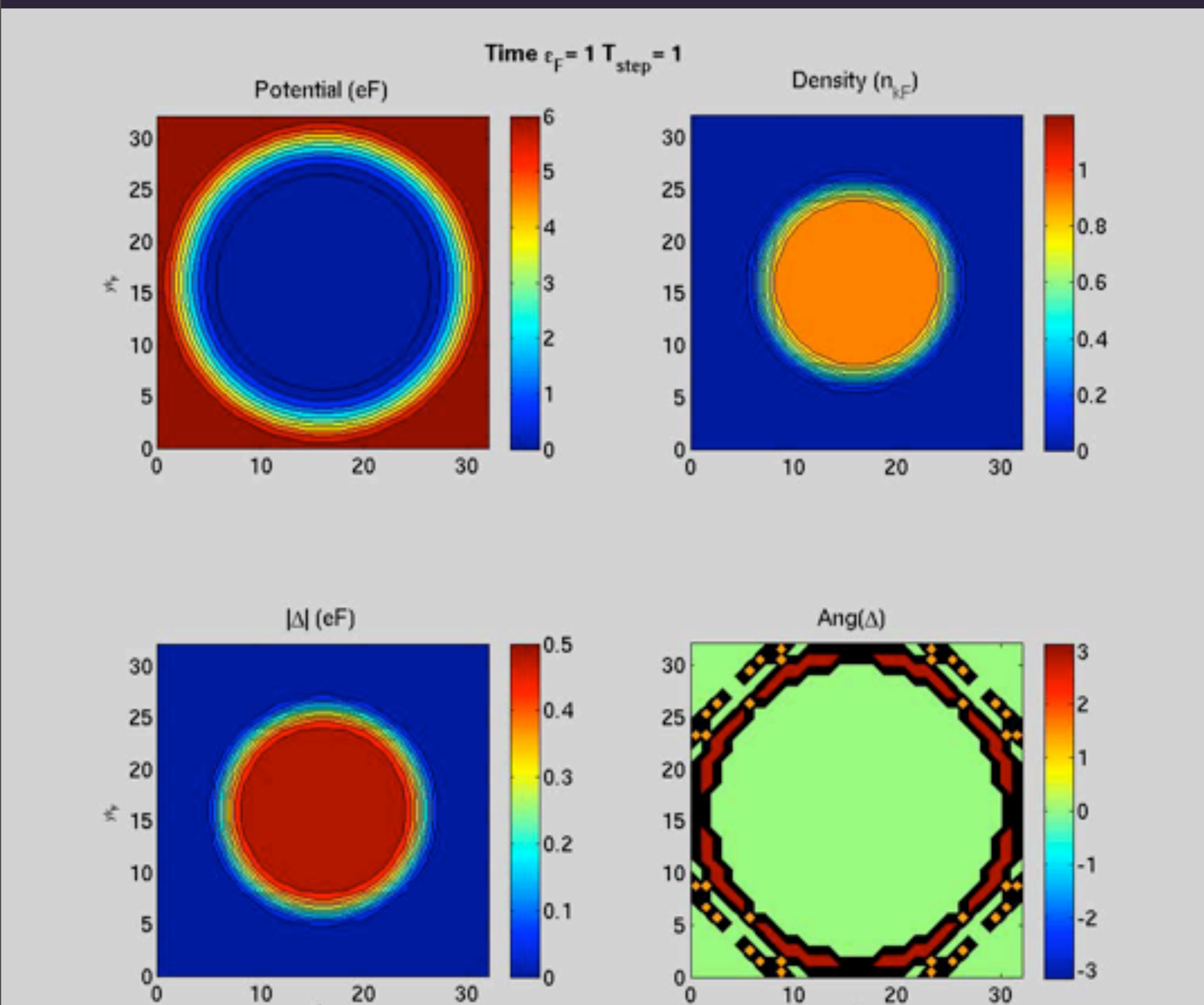


Work with Alan Luo and Rishi Sharma

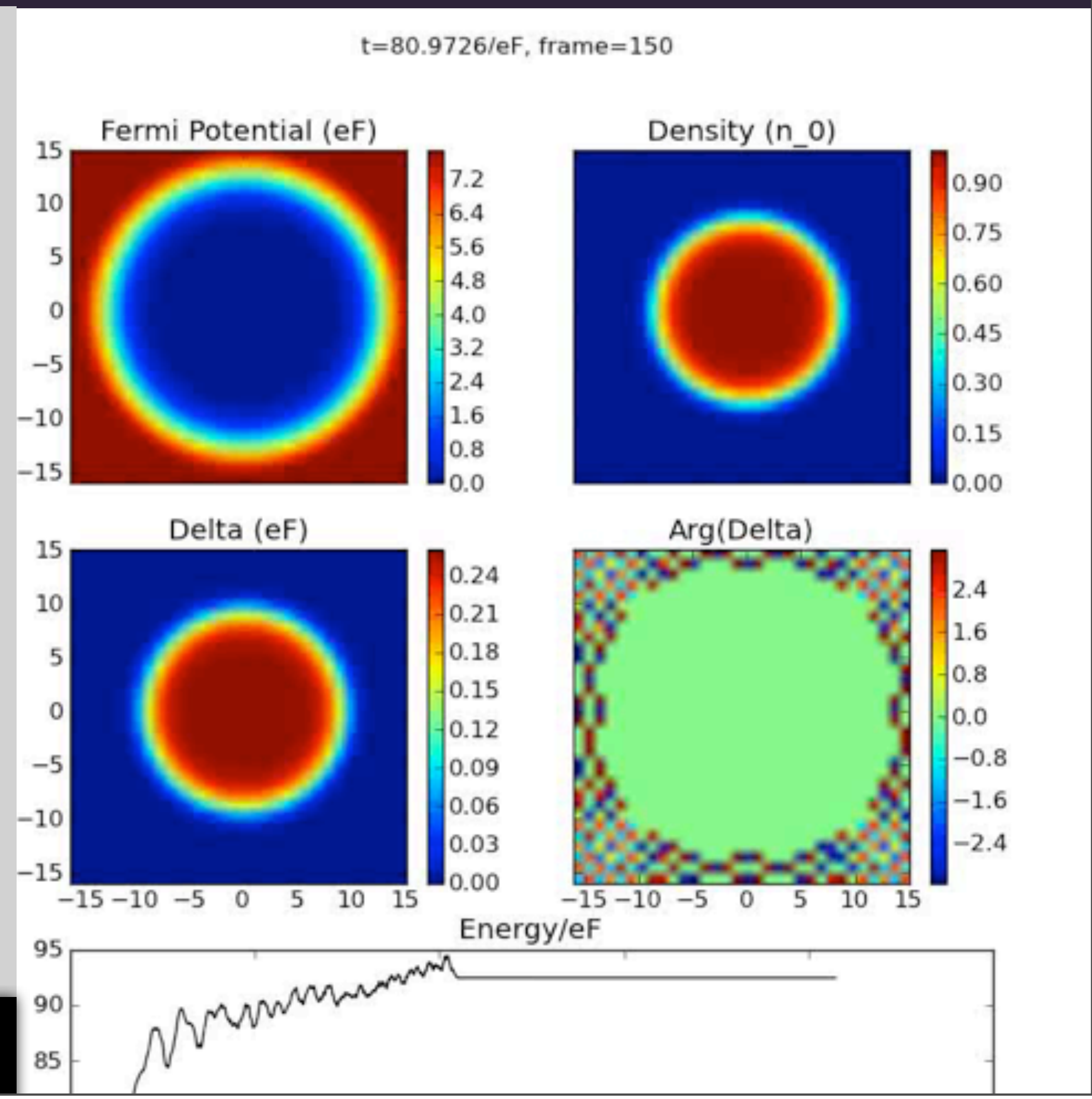
Comparison

Fermions
SLDA TDDFT

Gross Pitaevskii
model

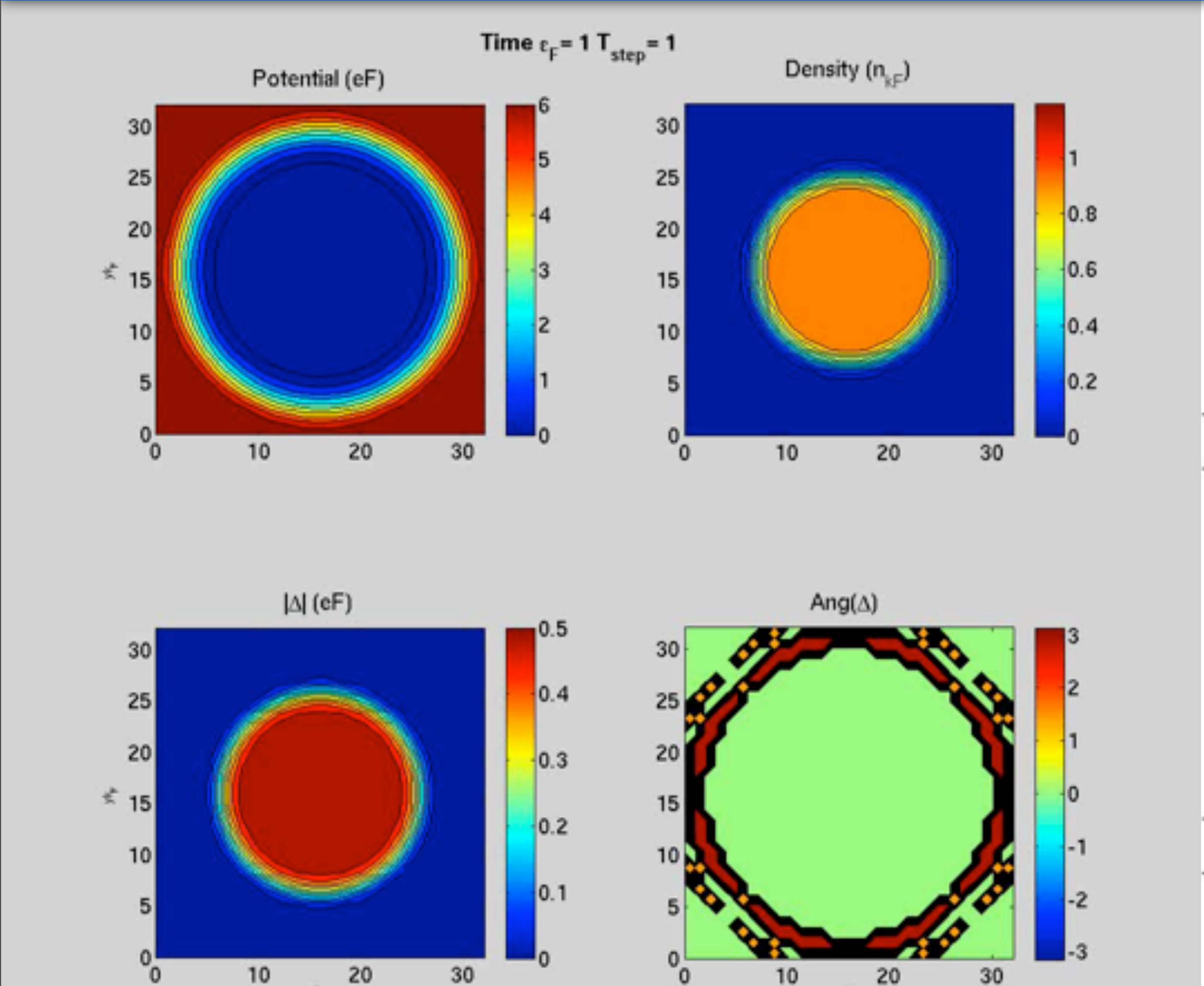


Bulgac et al. (Science 2011)

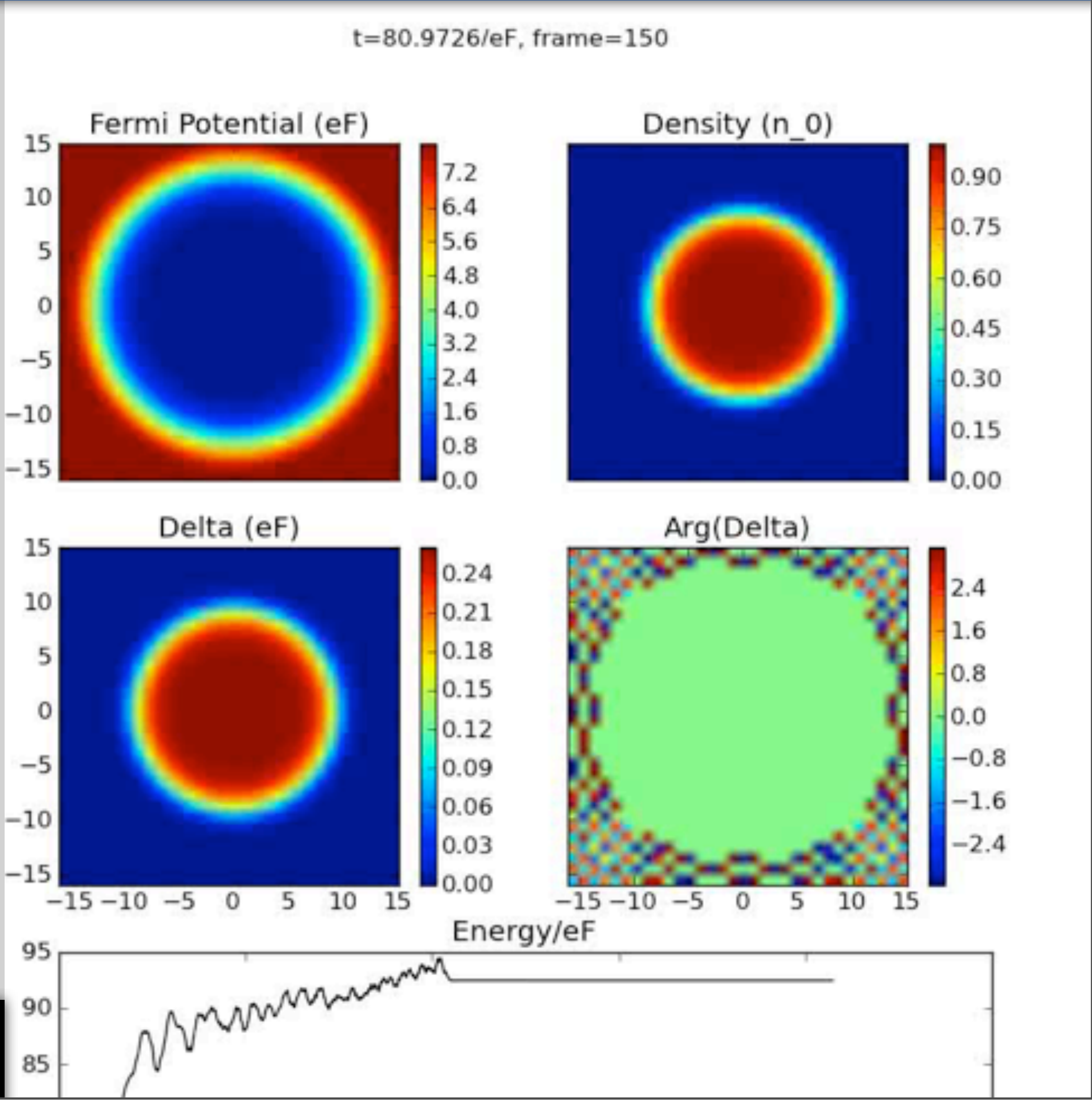


- Fermions:
- Simulation hard!
- Evolve 10^4–10^6 wavefunctions
- Requires supercomputers

- GPE:
- Simulation much easier!
- Evolve 1 wavefunction
- Use supercomputers to study large volumes



Bulgac et al. (Science 2011)



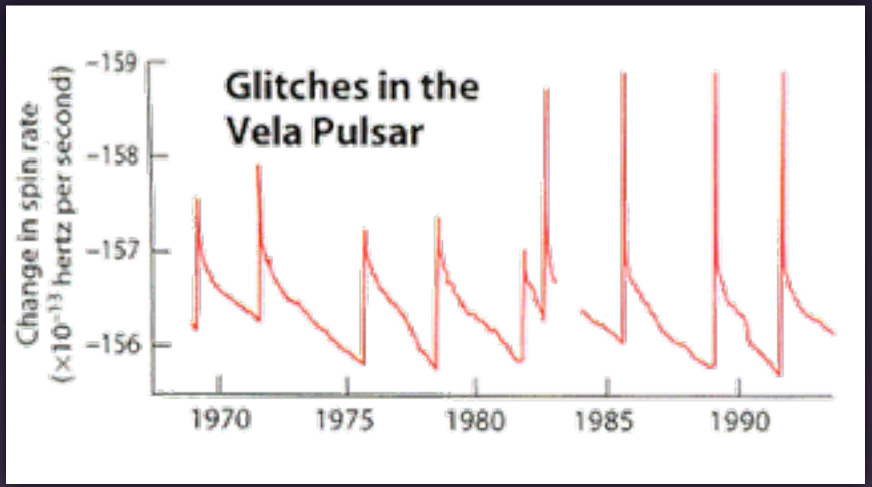
GPE Simulations of Unitary Fermi Gas

“A Laboratory on your Laptop”

Application: Vortex Pinning

- Pulsar glitching (neutron stars)
  - Massive vortex unpinning events?

    Anderson and Itoh (1975)



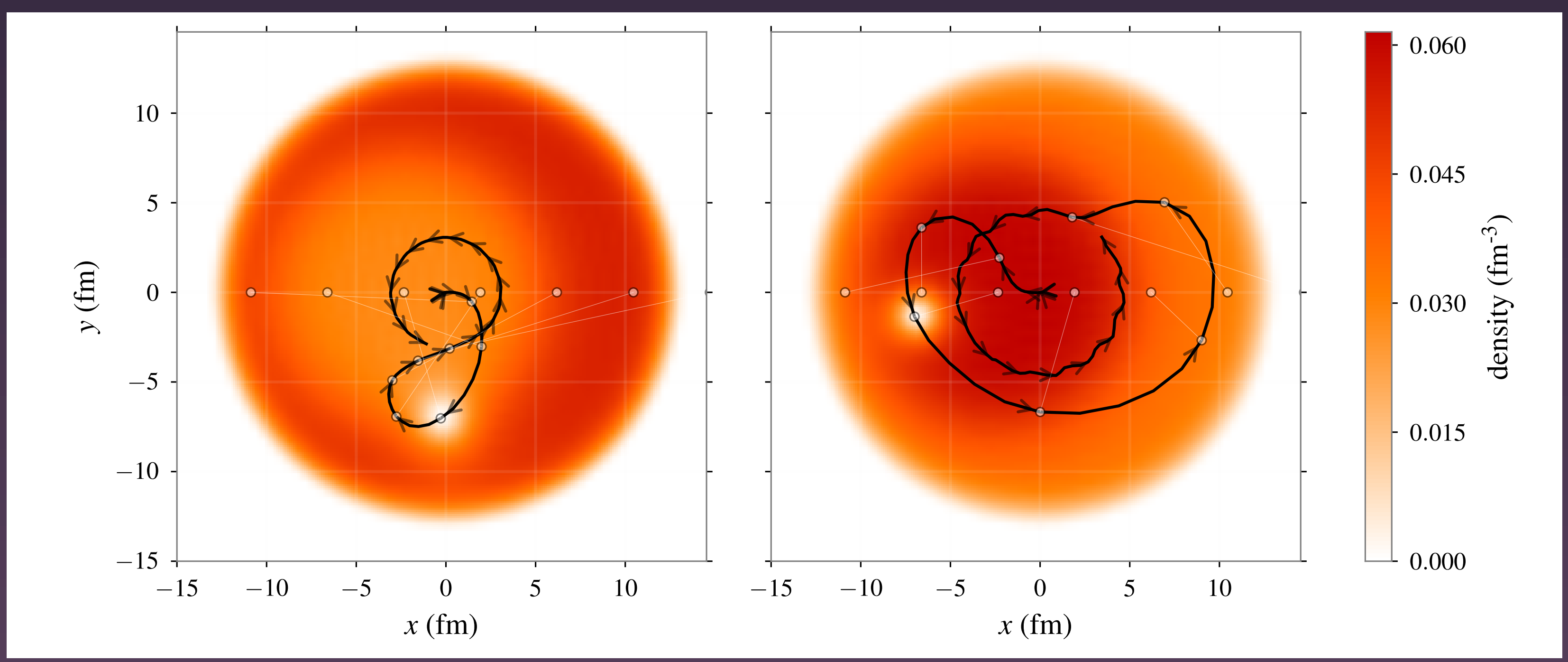
Pulsar Astronomy by Andrew G. Lyne and Francis

- Large scale events (thousands of vortices)
  - Too big for DFT – use GPE

- Need Vortex-Defect interactions (force)
  - Use DFT to calculate and then fit GPE

Pinning Force

$$\frac{dE}{dt} = -\vec{v} \cdot \vec{F}$$



Bulgac, Forbes, Sharma 2013

Thermodynamics

- Well defined: (unlike vortex mass)
- Accessible from dynamic simulations
- Extract from stirring simulations

Applications

- Fast qualitatively accurate simulation:
  - Design initial conditions and $V(t)$ for experiment and expensive fermion DFT calculations
  - Develop intuition for quantum hydrodynamics
- Framework to attack large-scale simulations
  - Neutron star glitches (vortex depinning?)
  - Multi-scale simulations

Future Work

- Deal with pair-breaking
  - Two fluid model: transfer energy and mass to a normal component
  - Stochastic extensions?
- More flexible model
  - How to get past Galilean invariance?
- Multiscale model - matching
  - Is GPE enough?
    - database of vortex/vortex interactions?
    - spawn small fermionic solvers to deal with collisions?

Conclusion

- Slda dft: Excellent agreement with expt. and qmc
  - Tool for extrapolating from small to large

    (Generalize to aid with IR convergence?)

- Gpe models works quite well for low energy dynamics
  - Quantitative agreement at unitarity

    (Can their domain of validity be expanded?)

- "A Laboratory on your Laptop"
  - A feasible solution for bulk superfluid dynamics?
  - Neutron star glitches?