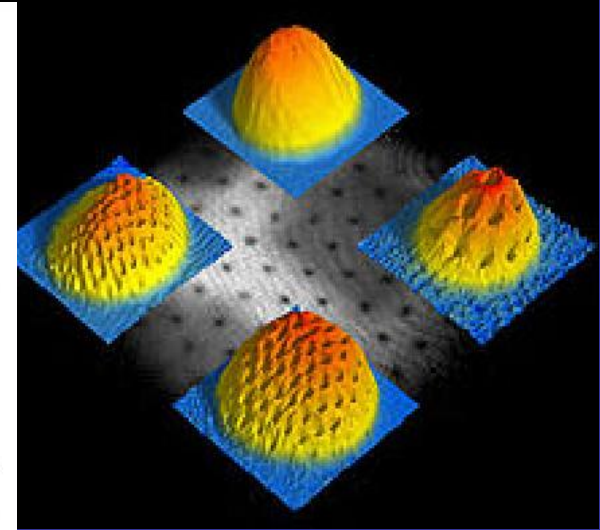
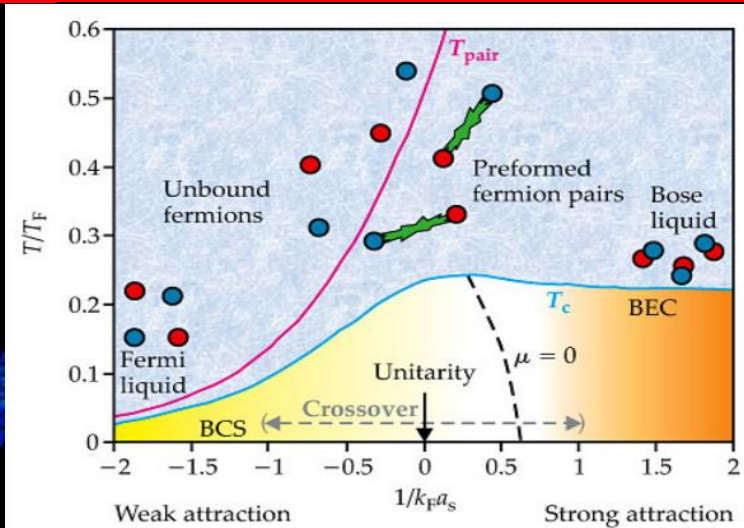
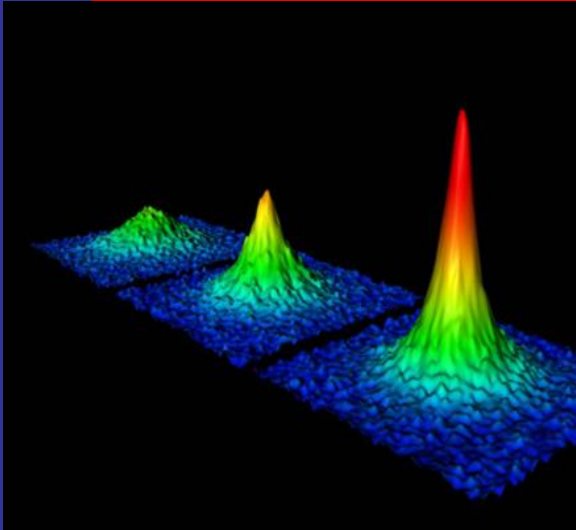


Equilibrium and nonequilibrium properties of unitary Fermi gas from Quantum Monte Carlo



Piotr Magierski

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Collaborators:

- | | |
|---------------|--|
| A. Bulgac | - University of Washington |
| J.E. Drut | - University of North Carolina |
| K.J. Roche | - Pacific Northwest National Lab. |
| G. Wlazłowski | - Univ. of Washington/Warsaw Univ. of Techn. |

Outline

- **BCS-BEC crossover. Unitary regime.**
- **Theoretical approach: Path Integral Monte Carlo (QMC)**
- **Equation of state for the Fermi gas in the unitary regime.**
- **Pairing gap and pseudogap. Spin susceptibility, conductivity and diffusion.**
- **Viscosity.**

What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$n r_0^3 \ll 1 \quad n |a|^3 \gg 1$$

n - particle density
 a - scattering length
 r_0 - effective range

$$\text{i.e. } r_0 \rightarrow 0, a \rightarrow \pm\infty$$

← **NONPERTURBATIVE
REGIME**

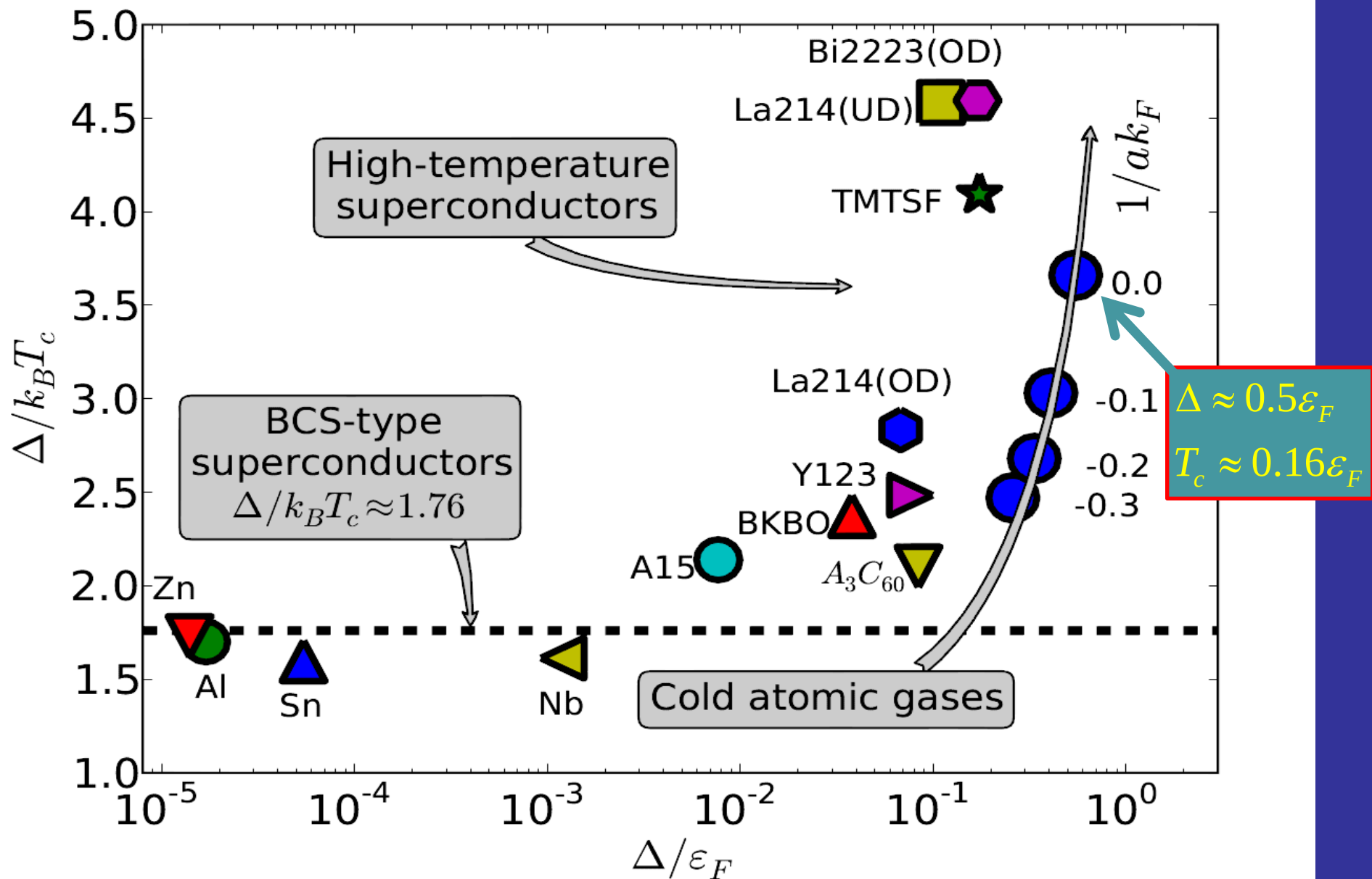
**System is dilute but
strongly interacting!**

Universality: $E(x) = \xi(x) E_{FG} \quad ; \quad x = \frac{T}{\epsilon_F}$

$$\xi(0) = 0.37(1) - \text{Exp. estimate}$$

$$E_{FG} - \text{Energy of noninteracting Fermi gas}$$

Cold atomic gases and high T_c superconductors



Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) - g \int d^3r \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

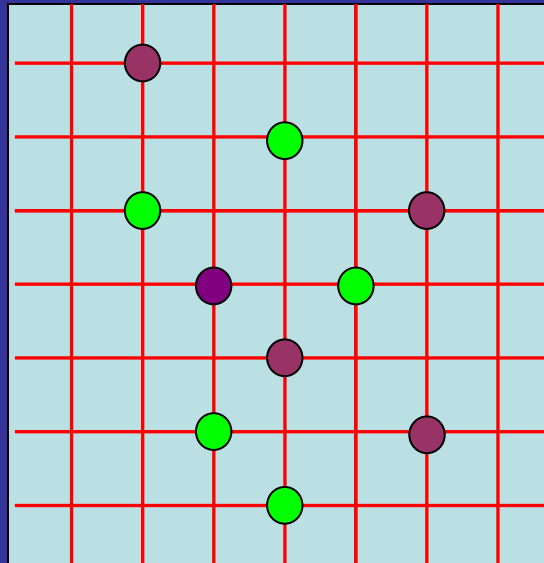
$$\hat{N} = \int d^3r (\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

Path Integral Monte Carlo for fermions on 3D lattice

Coordinate space

L-limit for the spatial correlations in the system

$$k_{\text{cut}} = \frac{\pi}{\Delta x}; \quad \Delta x$$



$$\text{Volume} = L^3$$

$$\text{lattice spacing} = \Delta x$$

● - Spin up fermion: ↑

● - Spin down fermion: ↓

External conditions:

T - temperature

μ - chemical potential

Periodic boundary conditions imposed

Basics of Auxiliary Field Monte Carlo (Path Integral MC)

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) - g \int d^3r \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r (\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{mk_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

$$\frac{1}{g} = \frac{m}{2\pi\hbar^2 \Delta x} \quad \text{- UNITARY LIMIT}$$

$$\hat{U}(\{\sigma\}) = T_\tau \exp \left\{ -\int_0^\beta d\tau [\hat{h}(\{\sigma\}) - \mu] \right\}; \quad \hat{h}(\{\sigma\}) \text{ - one-body operator}$$

$$U(\{\sigma\})_{kl} = \langle \psi_k | \hat{U}(\{\sigma\}) | \psi_l \rangle; \quad |\psi_l\rangle \text{ - single-particle wave function}$$

$$E(T) = \langle \hat{H} \rangle = \int \frac{D[\sigma(\vec{r}, \tau)] e^{-S[\sigma]}}{Z(T)} E[U(\{\sigma\})]$$

$E[U(\{\sigma\})]$ - energy associated with a given sigma field

$$\text{Tr} \hat{U}(\{\sigma\}) = \{\det[1 + \hat{U}_\uparrow(\sigma)]\}^2 = \exp[-S(\{\sigma\})] > 0 \quad \text{- No sign problem for spin symmetric system!}$$

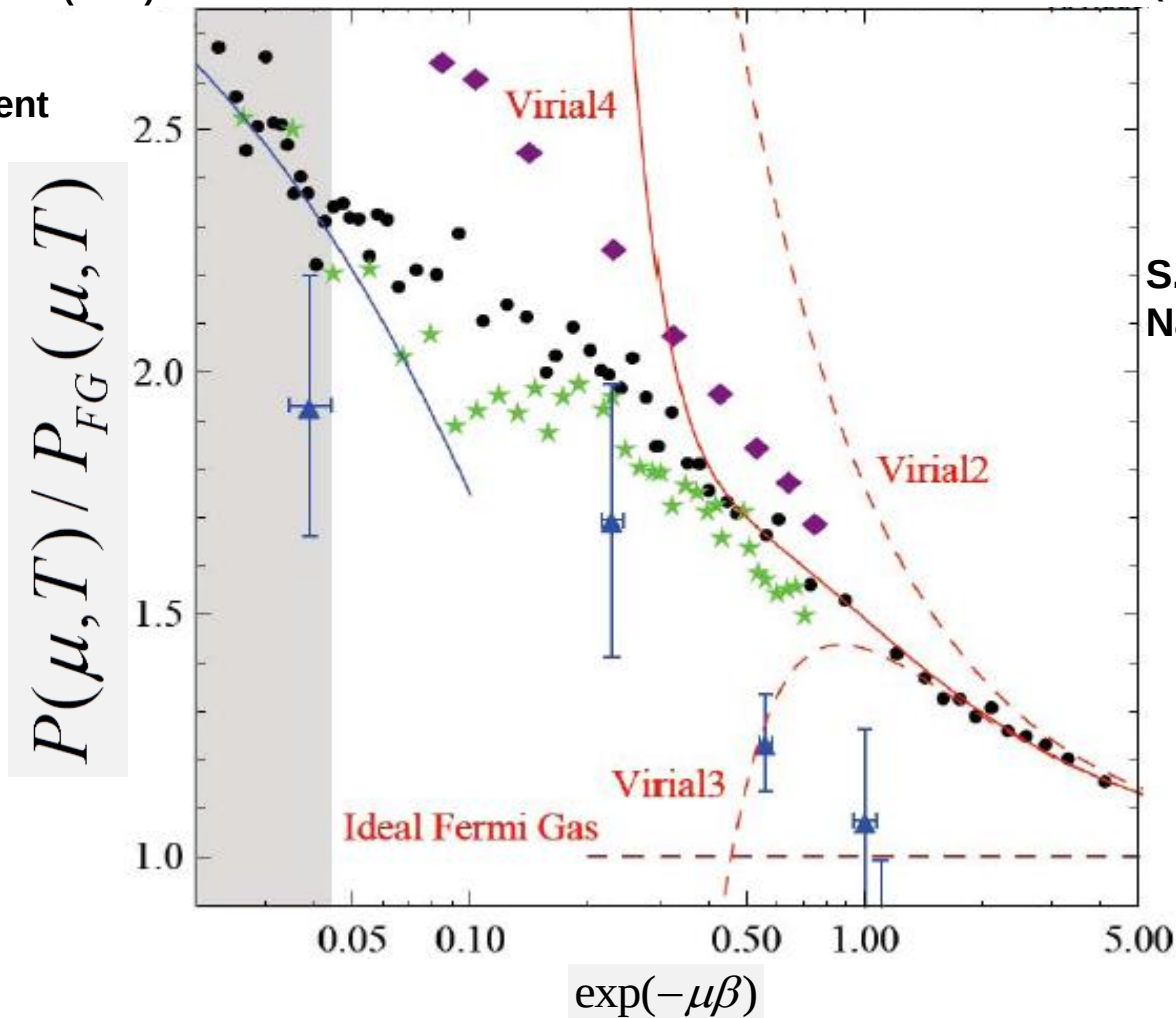
Comparison with Many-Body Theories (1)

▲ Diagram. MC
Burovski et al.
PRL96, 160402(2006)

★ QMC
Bulgac, Drut, Magierski,
PRL99, 120401(2006)

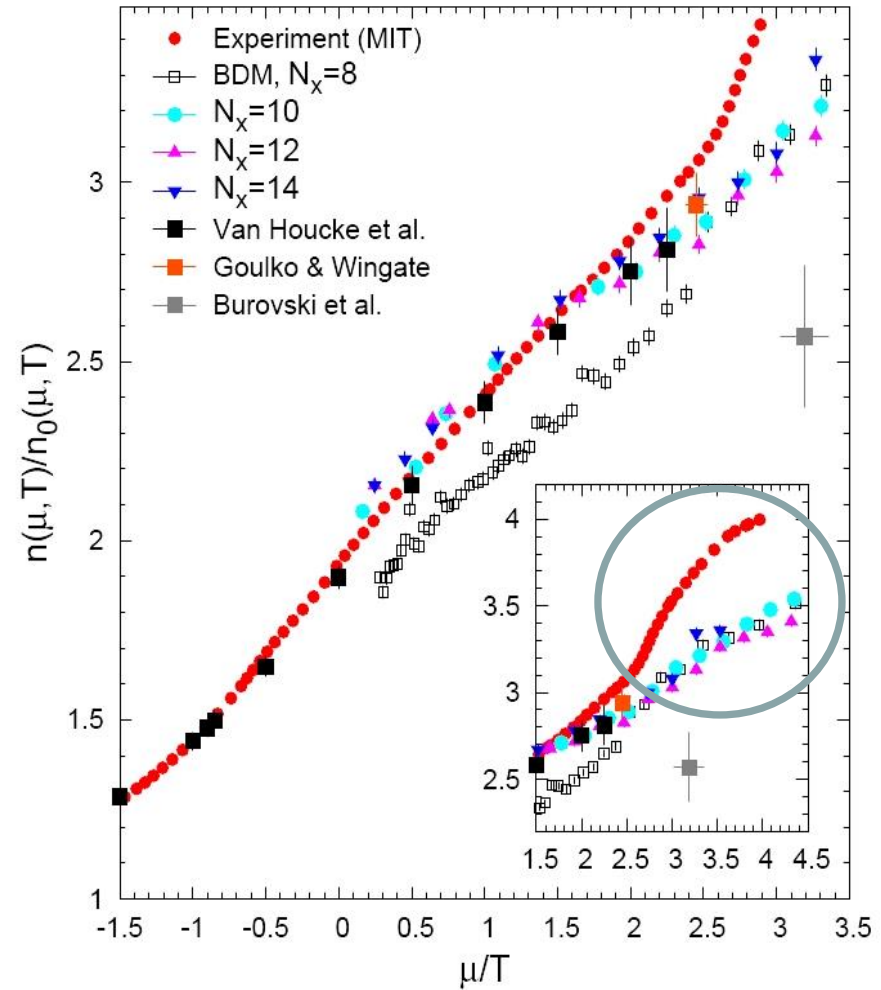
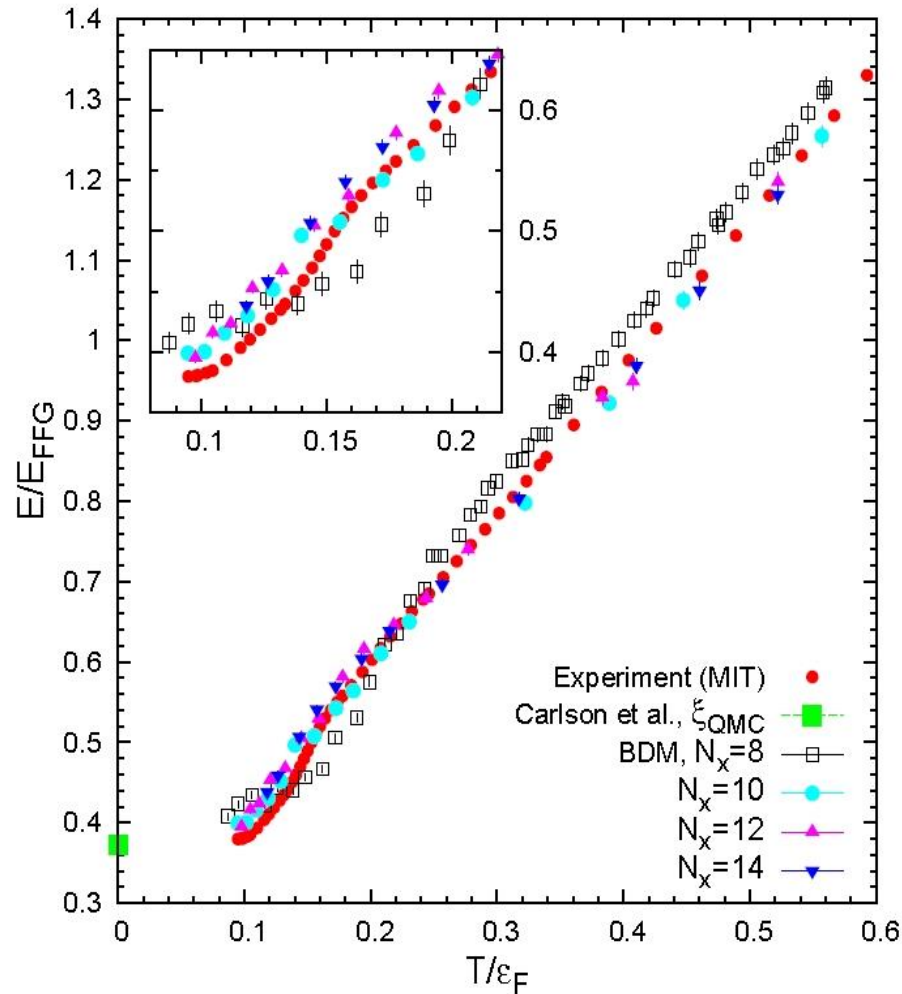
◆ Diagram. + analytic
Haussmann et al.
PRA75, 023610(2007)

● Experiment



S. Nascimbene et al.
Nature 463, 1057 (2010)

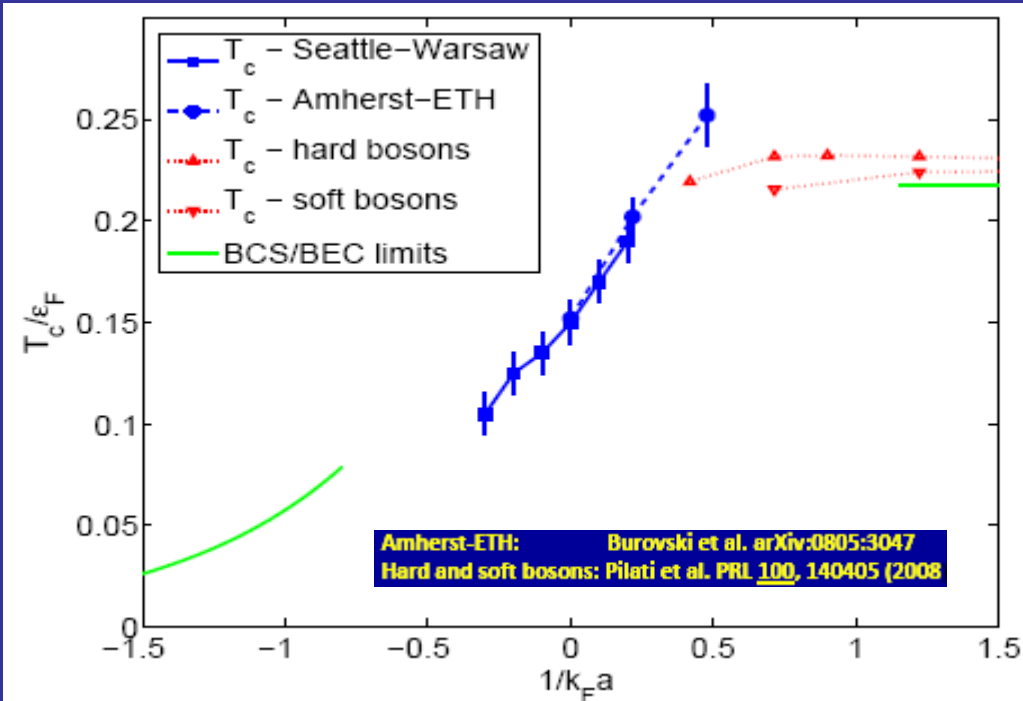
Equation of state of the unitary Fermi gas - current status



Experiment: M.J.H. Ku, A.T. Sommer, L.W. Cheuk, M.W. Zwierlein, Science 335, 563 (2012)

QMC (PIMC + Hybrid Monte Carlo):

J.E.Drut, T.Lähde, G.Włazłowski, P.Magierski, Phys. Rev. A 85, 051601 (2012)



Results in the vicinity of the unitary limit:

- Critical temperature
- Pairing gap

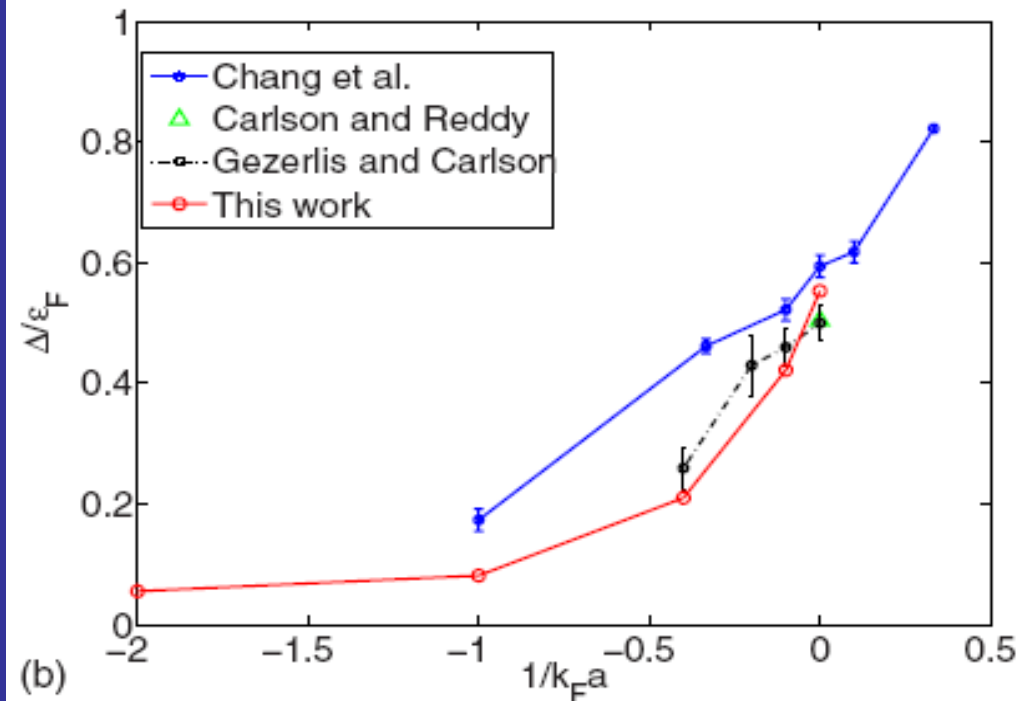
BCS theory predicts:

$$\Delta(T=0)/T_C \approx 1.7$$

At unitarity:

$$\Delta(T=0)/T_C \approx 3.3$$

This is NOT a BCS superfluid!



Local density approximation (LDA) from QMC

Uniform
system

$$\Omega = F - \lambda N = \frac{3}{5} \varphi(x) \varepsilon_F N - \lambda N$$

Nonuniform
system
(gradient
corrections
neglected)

$$\Omega = \int d^3r \left[\frac{3}{5} \varepsilon_F(\vec{r}) \varphi(x(\vec{r})) + U(\vec{r}) - \lambda \right] n(\vec{r})$$
$$x(\vec{r}) = \frac{T}{\varepsilon_F(\vec{r})}; \quad \varepsilon_F(\vec{r}) = \frac{\hbar^2}{2m} \left[3\pi^2 n(\vec{r}) \right]^{2/3}$$

The overall chemical potential λ and the temperature T are constant throughout the system. The density profile will depend on the shape of the trap as dictated by:

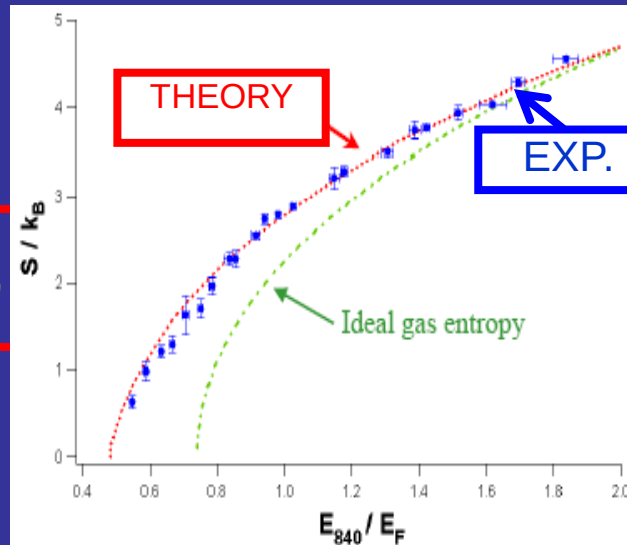
$$\frac{\delta \Omega}{\delta n(\vec{r})} = \frac{\delta (F - \lambda N)}{\delta n(\vec{r})} = \mu(x(\vec{r})) + U(r) - \lambda = 0$$

Using as an input the Monte Carlo results for the uniform system and experimental data (trapping potential, number of particles), we determine the density profiles.

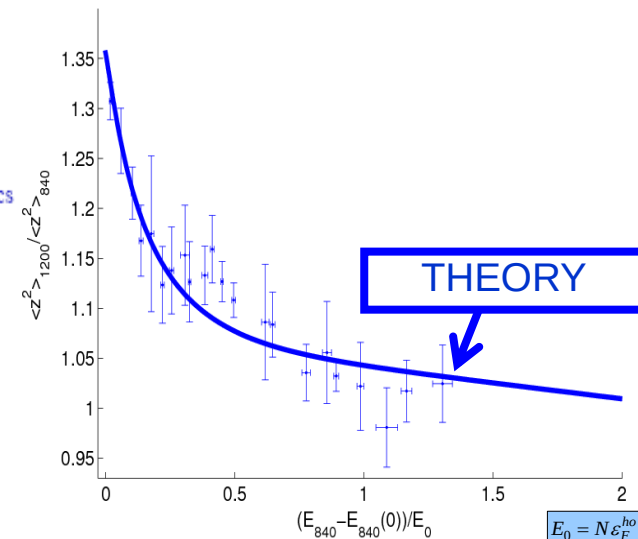
Unitary Fermi gas (^6Li atoms) in a harmonic trap

Experiment:

Luo, Clancy, Joseph, Kinast, Thomas,
Phys. Rev. Lett. 98, 080402, (2007)

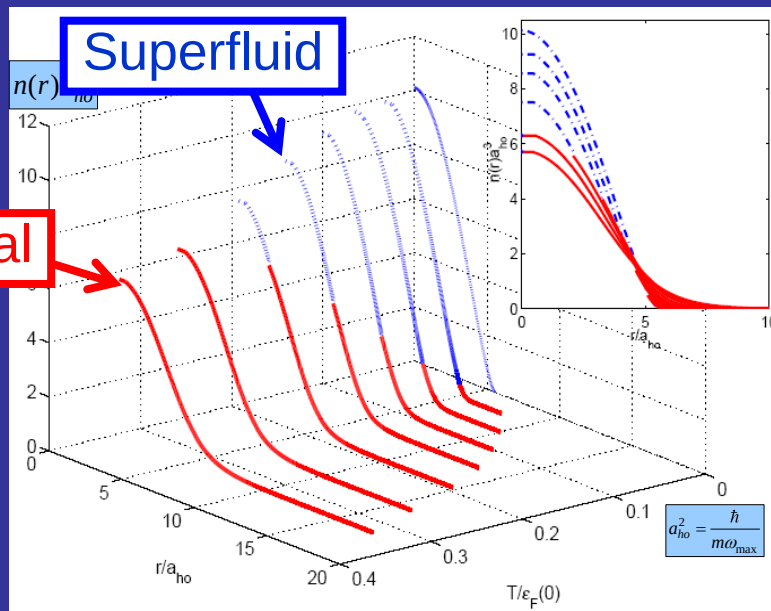


Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.



Ratio of the mean square cloud size at $B=1200\text{G}$ to its value at unitarity ($B=840\text{G}$) as a function of the energy. Experimental data are denoted by point with error bars.

$$B = 1200\text{G} \Rightarrow 1/k_F a \approx -0.75$$



Full *ab initio* theory (no free parameters): LDA + QMC input
Bulgac, Drut, Magierski, Phys. Rev. Lett. 99, 120401 (2007)

$\epsilon_F(0)$ - Fermi energy at the center of the trap

The radial (along shortest axis) density profiles of the atomic cloud at various temperatures.

Pairing gap from spectral function:

Spectral weight function: $A(\vec{p}, \omega)$

$$G^{ret/adv}(\vec{p}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p}, \omega')}{\omega - \omega' \pm i0^+}$$

$$G(\vec{p}, \tau) = -\frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega A(\vec{p}, \omega) \frac{e^{-\omega\tau}}{1 + e^{-\beta\omega}}$$

From Monte Carlo calcs.

$$G(\vec{p}, \tau) = \frac{1}{Z} \text{Tr} \{ e^{-(\beta-\tau)(\hat{H}-\mu\hat{N})} \hat{\psi}_{\uparrow}(\vec{p}) e^{-\tau(\hat{H}-\mu\hat{N})} \hat{\psi}_{\uparrow}^{\dagger}(\vec{p}) \}$$

Constraints

$$\int_{-\infty}^{+\infty} A(\vec{p}, \omega) \frac{d\omega}{2\pi} = 1$$

$$\int_{-\infty}^{+\infty} A(\vec{p}, \omega) (1 + e^{\beta\omega})^{-1} \frac{d\omega}{2\pi} = n(\vec{p})$$

In the limit of independent quasiparticles: $A(\vec{p}, \omega) = 2\pi\delta(\omega - E(p))$

Linear inverse problem

$$G(y) = \int_{-\infty}^{\infty} K(x, y) A(x) dx,$$

G is known from QMC with some error for a number of values of y, usually uniformly distributed within the interval: (0, 1/T)

Maximum entropy method (MEM):

Bayes' theorem:

$$p(\vec{A}|\vec{\tilde{G}}) = \frac{p(\vec{\tilde{G}}|\vec{G})p(\vec{A})}{p(\vec{\tilde{G}})},$$

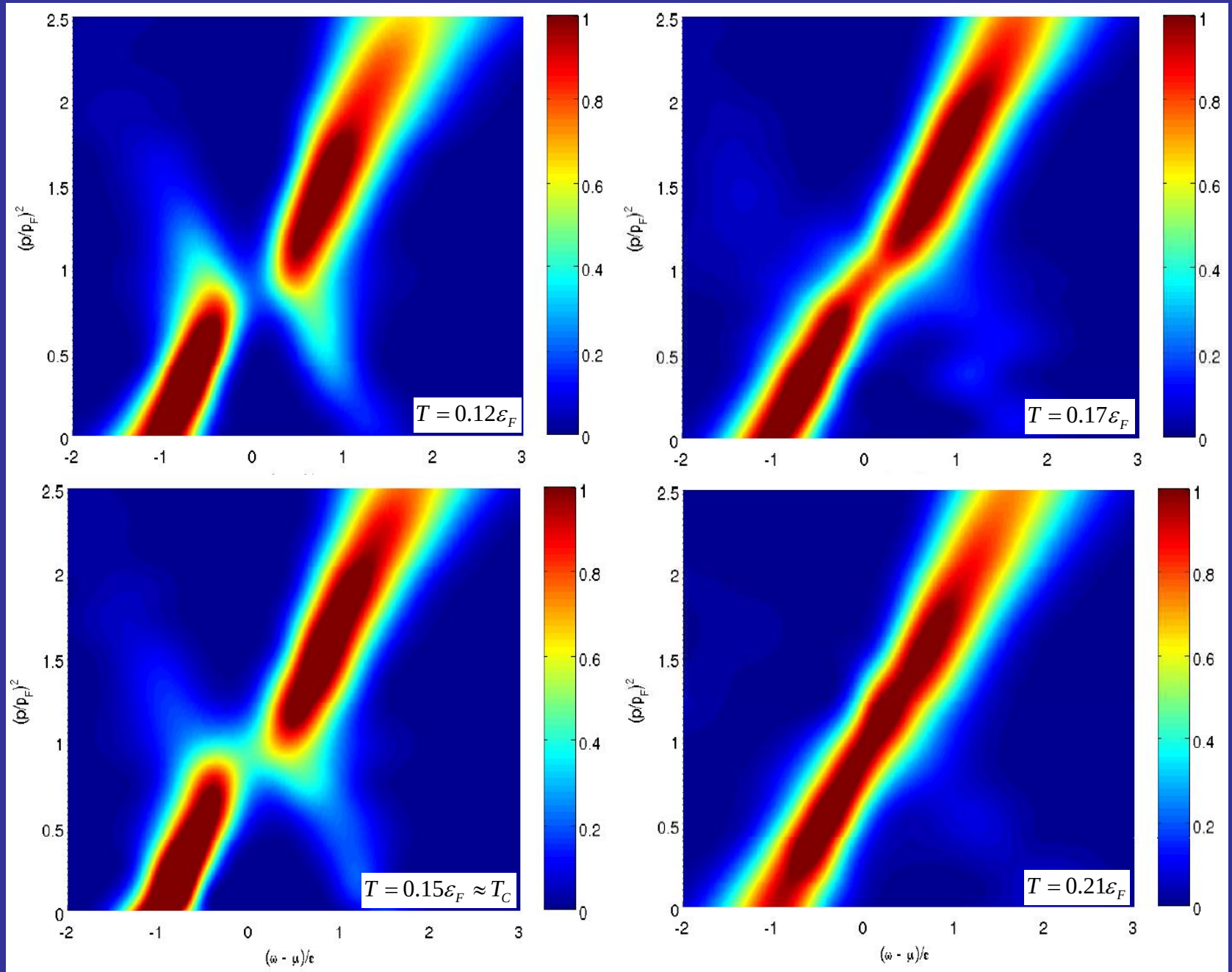
Maximization of conditional probability:

$$\frac{\partial}{\partial A_j} p(\vec{A}|\vec{\tilde{G}}) = 0, \quad j=1, \dots, N,$$

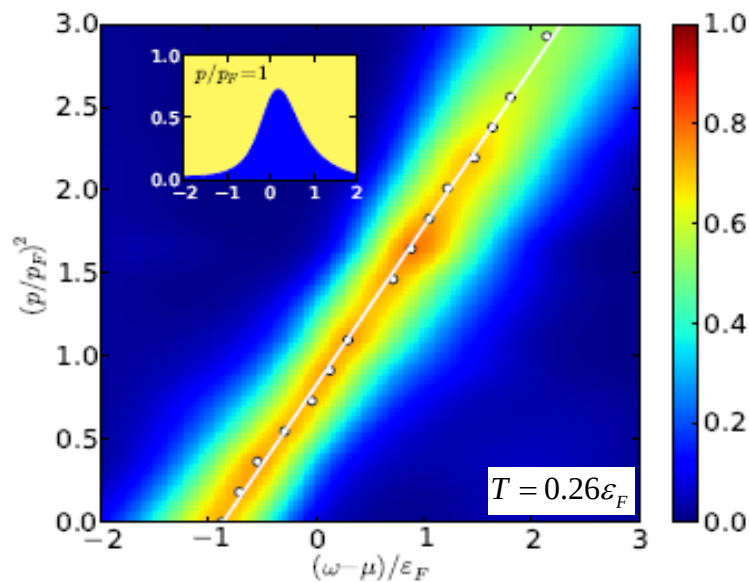
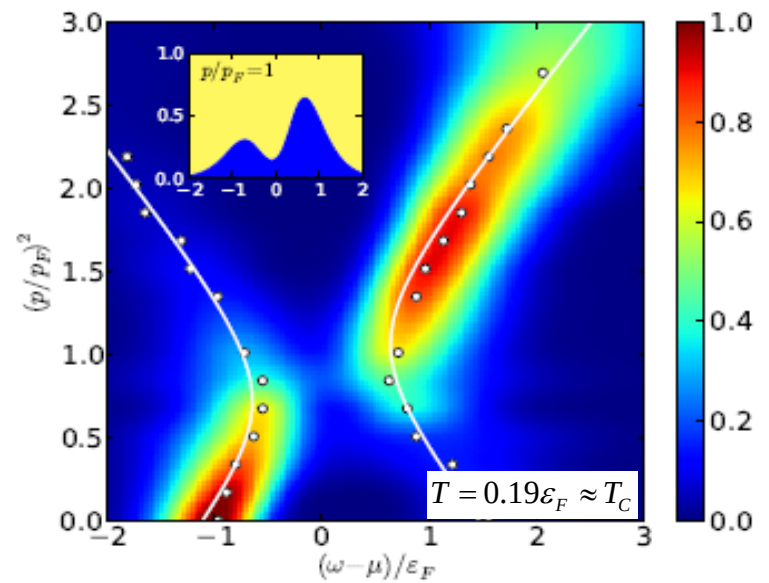
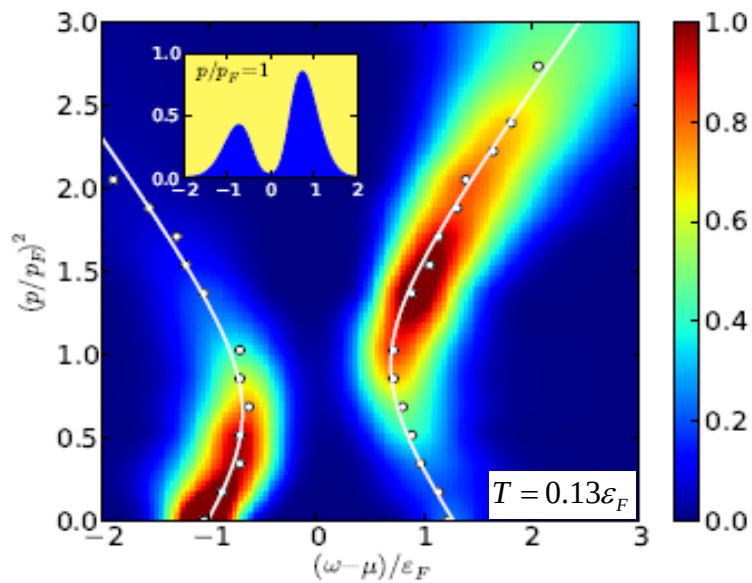
$$p(\vec{A}|\vec{\tilde{G}}) \propto \exp \left(-\frac{1}{2} \sum_{i=1}^{N_\tau} \left(\frac{\tilde{G}_i - G_i}{\sigma_i} \right)^2 - \underbrace{\alpha \sum_{i=1}^N A_i \log \frac{A_i}{M_i}}_{\text{Relative entropy term}} \right),$$

Relative entropy term

Spectral weight function at unitarity: $(k_F a)^{-1} = 0$



Spectral weight function at the BEC side: $(k_F a)^{-1} = 0.2$

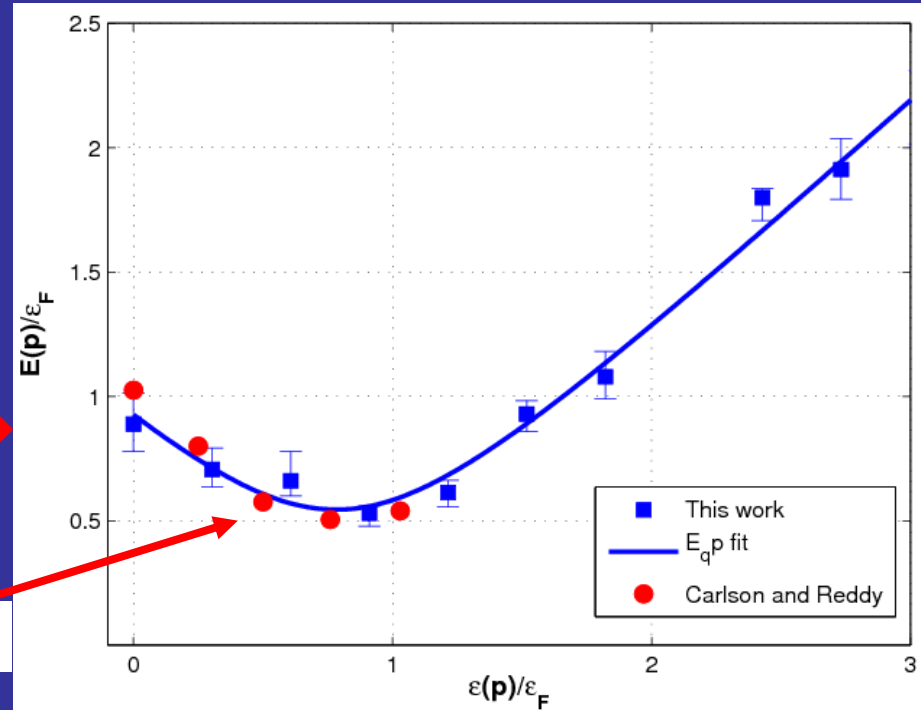


Single-particle properties

$$E(p) = \sqrt{\left(\frac{p^2}{2m^*} + U - \mu\right)^2 + \Delta^2}$$

Quasiparticle spectrum
extracted from spectral weight
function at $T = 0.1\varepsilon_F$

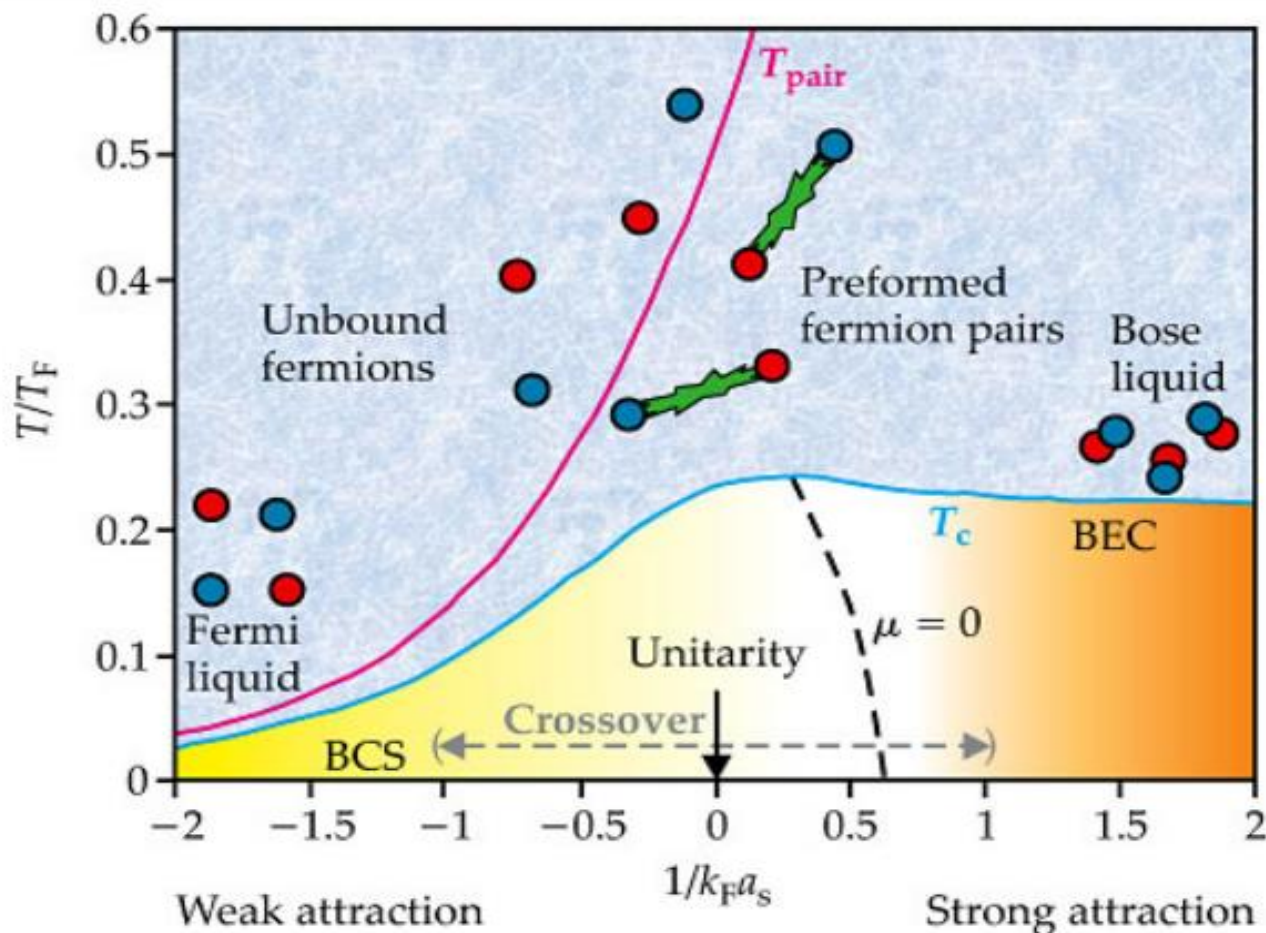
Fixed node MC calcs. at $T=0$



Effective mass: $m^* = (1.0 \pm 0.2)m$

Mean-field potential: $U = (-0.5 \pm 0.2)\varepsilon_F$

Weak temperature dependence!

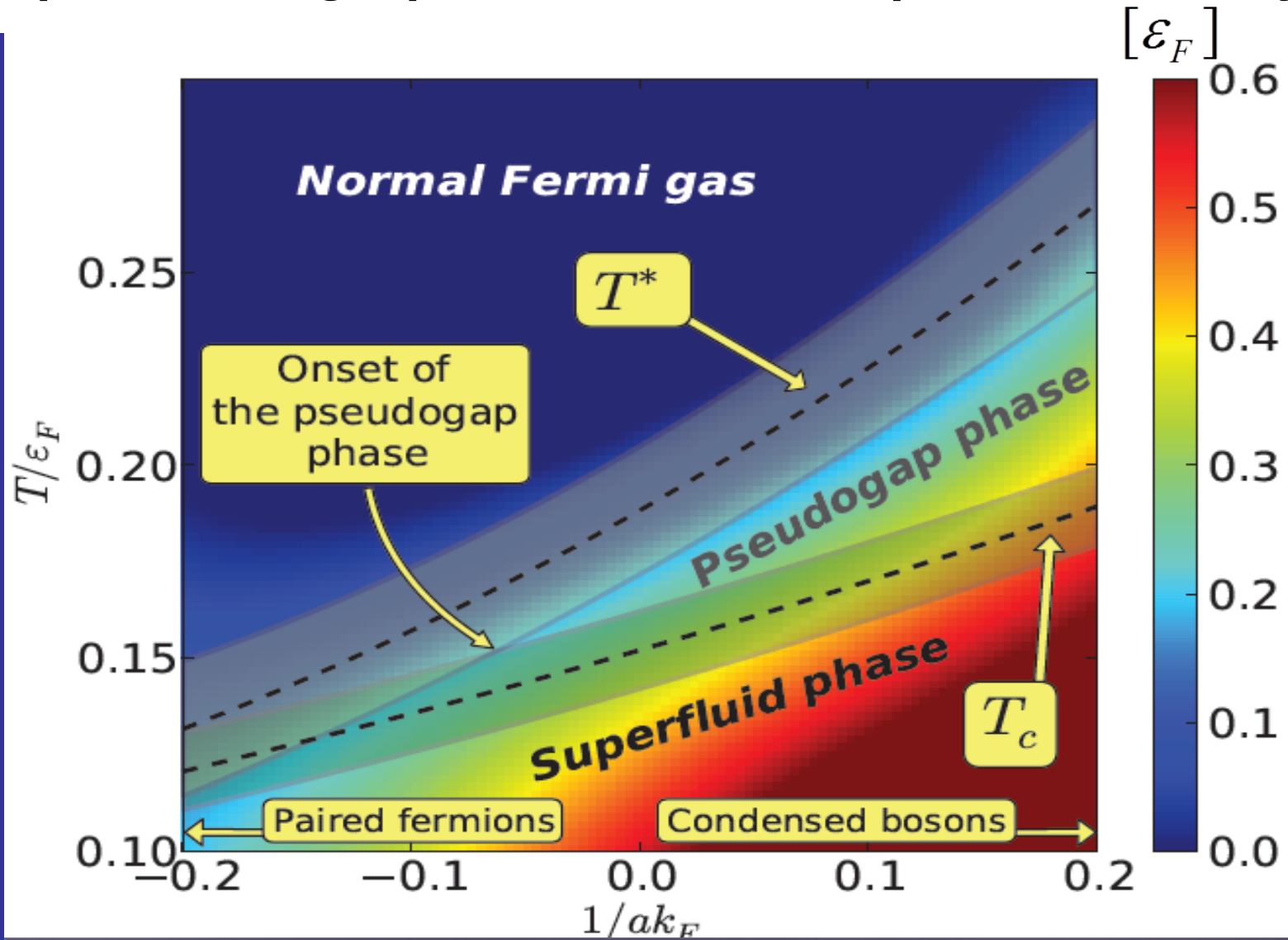


Pairing pseudogap: suppression of low-energy spectral weight function due to incoherent pairing in the normal state ($T > T_c$)

Important issue related to pairing pseudogap:

- Are there sharp gapless quasiparticles in a normal Fermi liquid
YES: Landau's Fermi liquid theory;
NO: breakdown of Fermi liquid paradigm

Gap in the single particle fermionic spectrum - theory



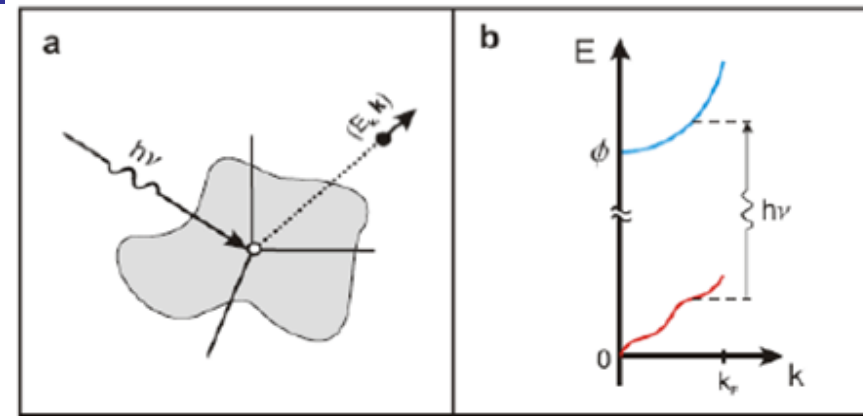
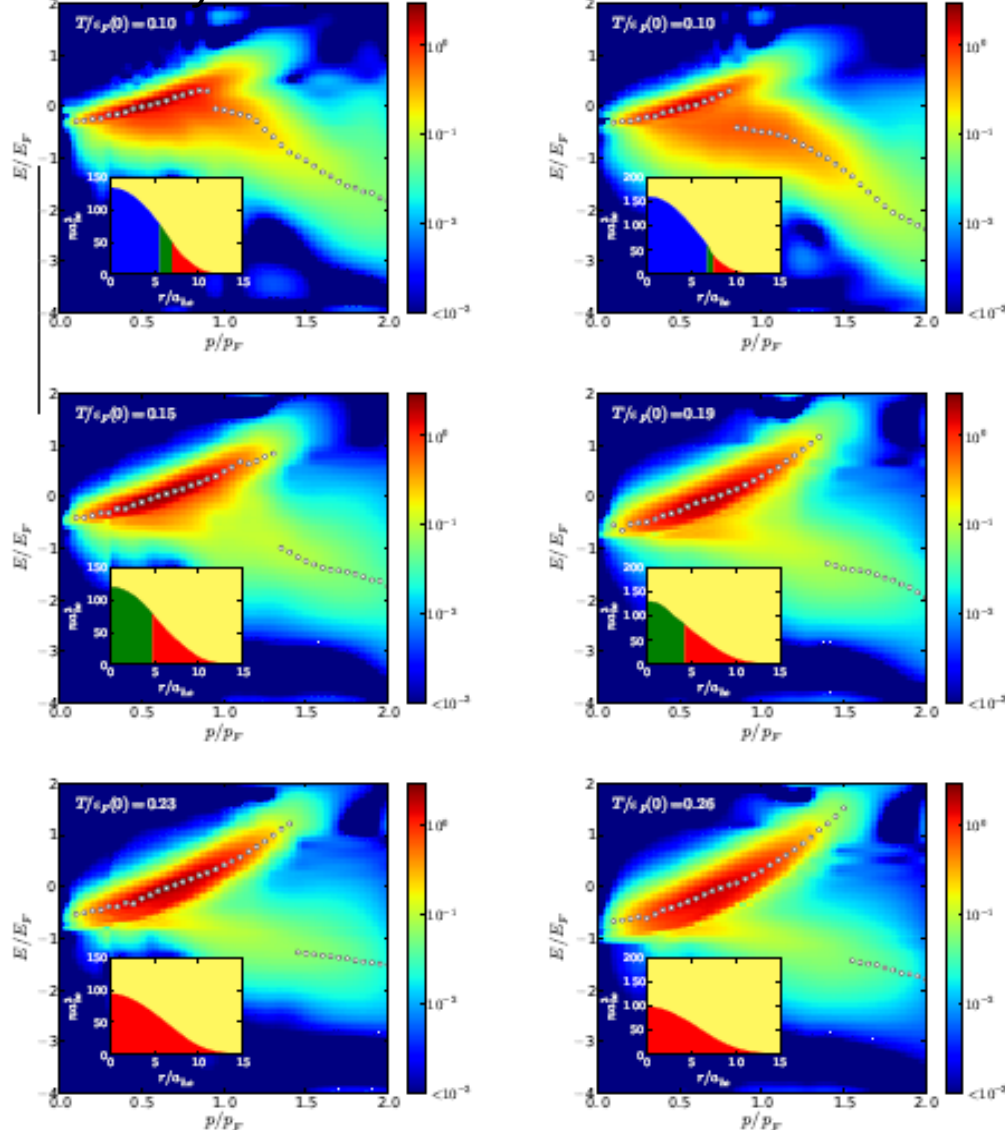
Ab initio result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.

Energy distribution curves (EDC) from the spectral weight function

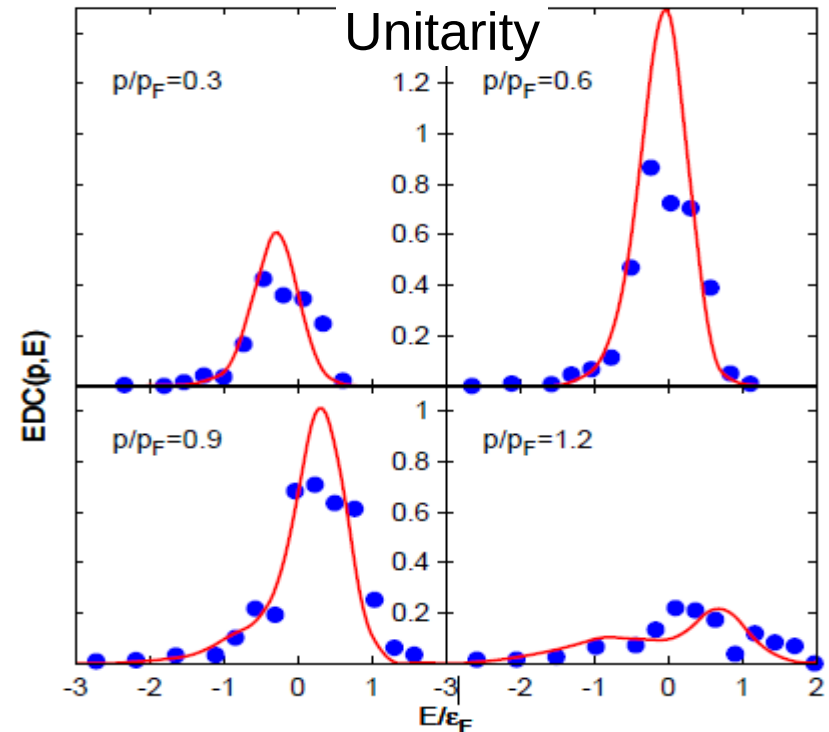
$$\text{EDC}(p, E, T) = C p^2 \int_0^\infty dr r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E - \mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E - \mu(r)),$$

Unitarity

BEC side



Unitarity



Experiment (blue dots): Gaebler et al. *Nature Physics* 6, 569(2010)

QMC (red line): Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.* 107, 145304 (2011)

Spin susceptibility and spin drag rate

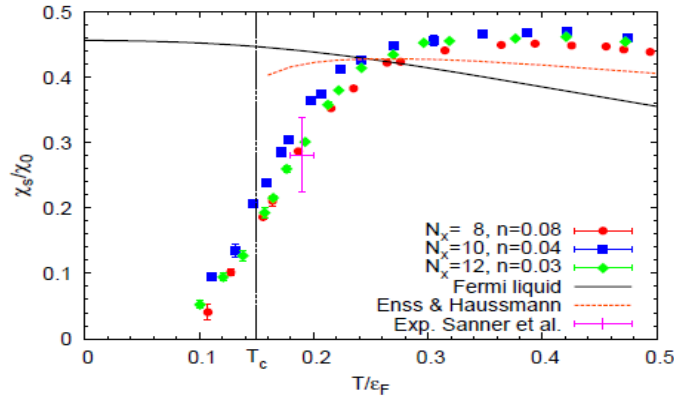


FIG. 2: (Color online) The static spin susceptibility as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line indicates the critical temperature of superfluid to normal phase transition $T_c = 0.15 \varepsilon_F$. For comparison Fermi liquid theory prediction and recent results of the T -matrix theory produced by Enss and Haussmann [25] are plotted with solid and dashed (brown) lines, respectively. The experimental data point from Ref. [15] is also shown.

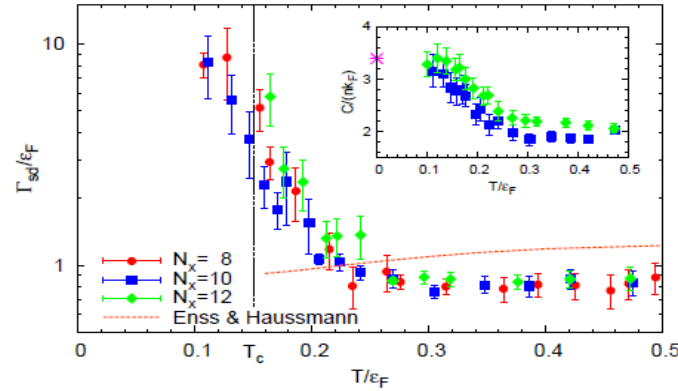


FIG. 3: (Color online) The spin drag rate $\Gamma_{sd} = n/\sigma_s$ in units of Fermi energy as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line locates the critical temperature of superfluid to normal phase transition. Results of the T -matrix theory are plotted by dashed (brown) line [25]. The inset shows extracted value of the contact density as function of the temperature. The (purple) asterisk shows the contact density from the QMC calculations of Ref. [29] at $T = 0$.

$$\Gamma = \frac{n}{\sigma_s} \quad - \text{spin drag rate}$$

$$\sigma_s(\omega) = \pi \rho_s(q=0, \omega) / \omega \quad - \text{spin conductivity}$$

$$G_s(q, \tau) = \frac{1}{V} \left\langle \left(\hat{j}_{q\uparrow}^z(\tau) - \hat{j}_{q\downarrow}^z(\tau) \right) \left(\hat{j}_{-q\uparrow}^z(0) - \hat{j}_{-q\downarrow}^z(0) \right) \right\rangle$$

$$G_s(q, \tau) = \int_0^\infty \rho_s(q, \omega) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh[\omega\beta/2]} d\omega$$

$$D_s = \frac{\sigma_s}{\chi_s}$$

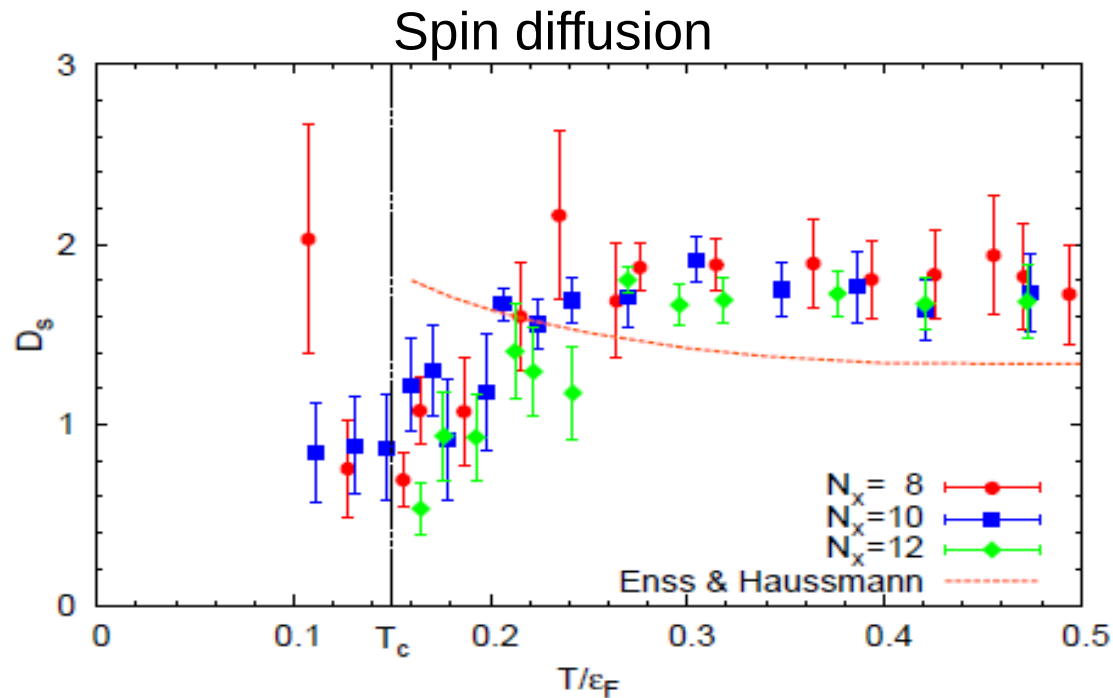


FIG. 4: (Color online) The spin diffusion coefficient obtained by the Einstein relation $D_s = \sigma_s/\chi_s$ as function of temperature. The notation is identical to Fig. 3.

No minimum is seen in QMC down to 0.1 of Fermi energy

Estimate from
kinetic theory at low T:

$$D_s \sim p_F l \sim n^{1/3} n^{-1/3} \sim 1$$

Wlazłowski, Magierski, Bulgac, Drut, Roche,
Phys. Rev. Lett. 110, 090401,(2013)

Pseudogap at unitarity – theoretical predictions

Path Integral Monte Carlo	- YES
Dynamic Mean Field	- YES
Selfconsistent T-matrix	- NO
Nonselfconsistent T-matrix	- YES

Hydrodynamics at unitarity

No intrinsic length scale $\longrightarrow$ Uniform expansion keeps the unitary gas in equilibrium

Consequence:

uniform expansion does not produce entropy = bulk viscosity is zero!

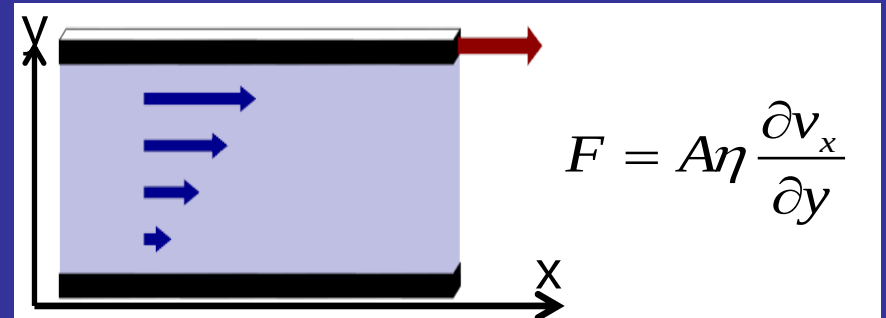
Shear viscosity:

For any physical fluid:

$$\frac{\eta}{S} \geq \frac{\hbar}{4\pi k_B}$$

KSS conjecture

Kovtun, Son, Starinets, Phys.Rev.Lett. 94, 111601, (2005)
from AdS/CFT correspondence



Maxwell classical estimate: $\eta \sim$ mean free path

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: unitary Fermi gas, quark-gluon plasma

Shear viscosity

$$\eta(\omega) = \pi \rho_{xyxy}(q=0, \omega) / \omega$$

$$G_{xyxy}(q, \tau) = \int d^3r \left\langle \hat{\Pi}_{xy}(r, \tau) \hat{\Pi}_{xy}(0, 0) \right\rangle e^{iqr}$$

$$G_{xyxy}(q, \tau) = \int_0^\infty \rho_{xyxy}(q, \omega) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh[\omega\beta/2]} d\omega$$

$$i \left[\hat{j}_k(r), \hat{H} \right] = \partial_l \hat{\Pi}_{kl}(r)$$

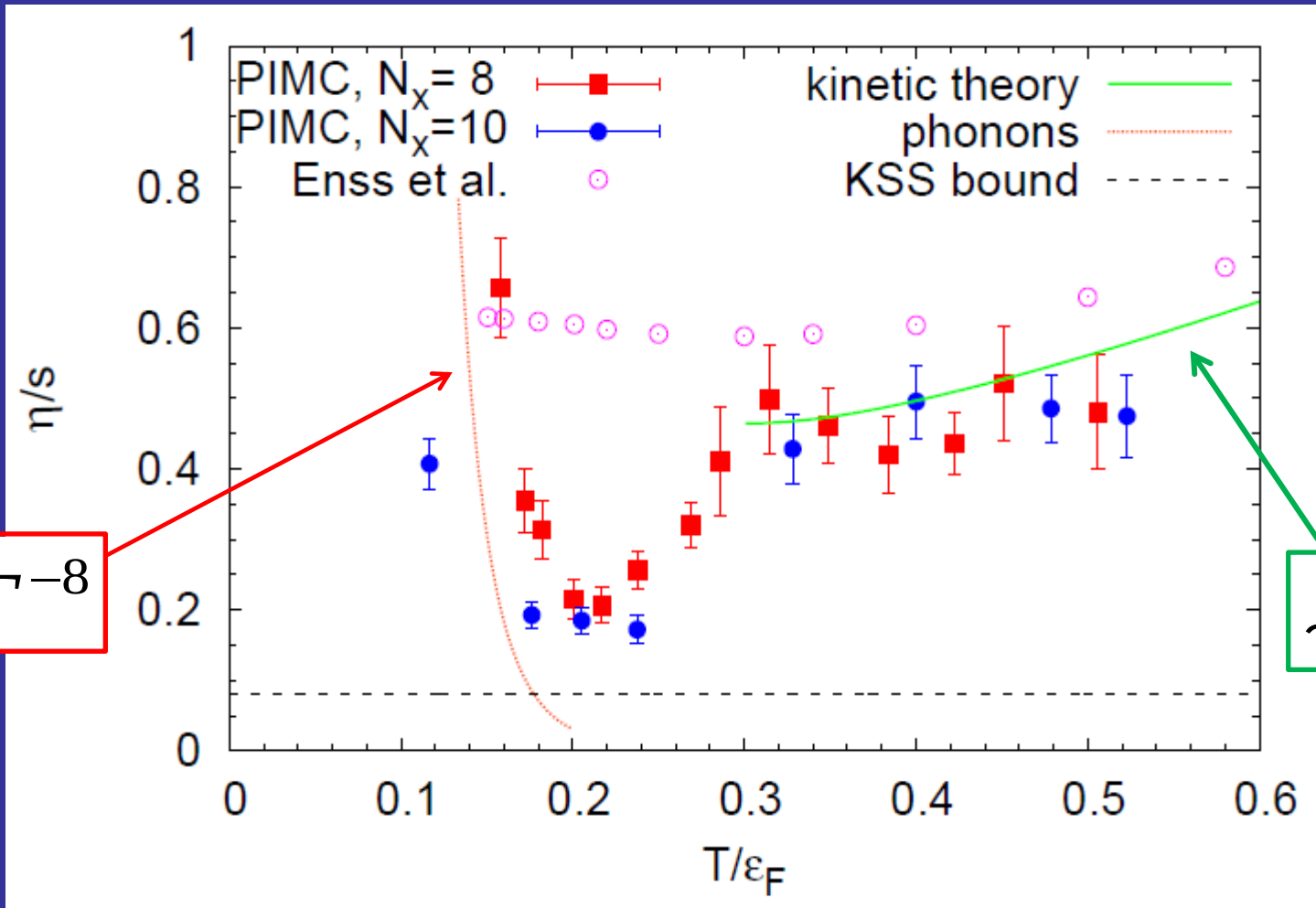
Additional symmetries and sum rules:

$$G(\tau) = G(\beta - \tau)$$

$$\frac{1}{\pi} \int_0^\infty d\omega \left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}} \right] = \frac{\varepsilon}{3} \varepsilon - \text{energy density}$$

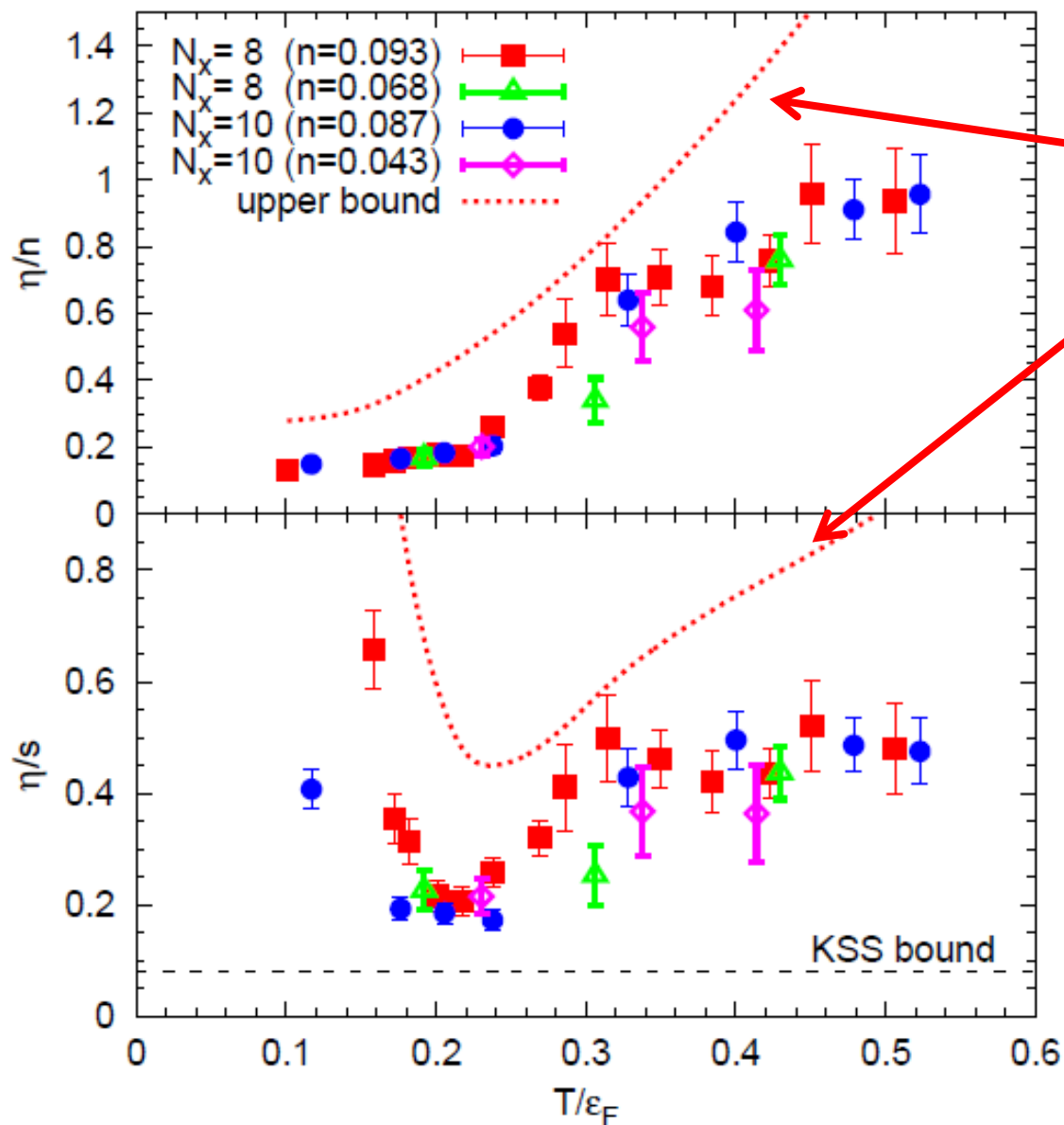
$$\eta(\omega \rightarrow \infty) \simeq \frac{C}{15\pi\sqrt{m\omega}}.$$

Shear viscosity to entropy density ratio



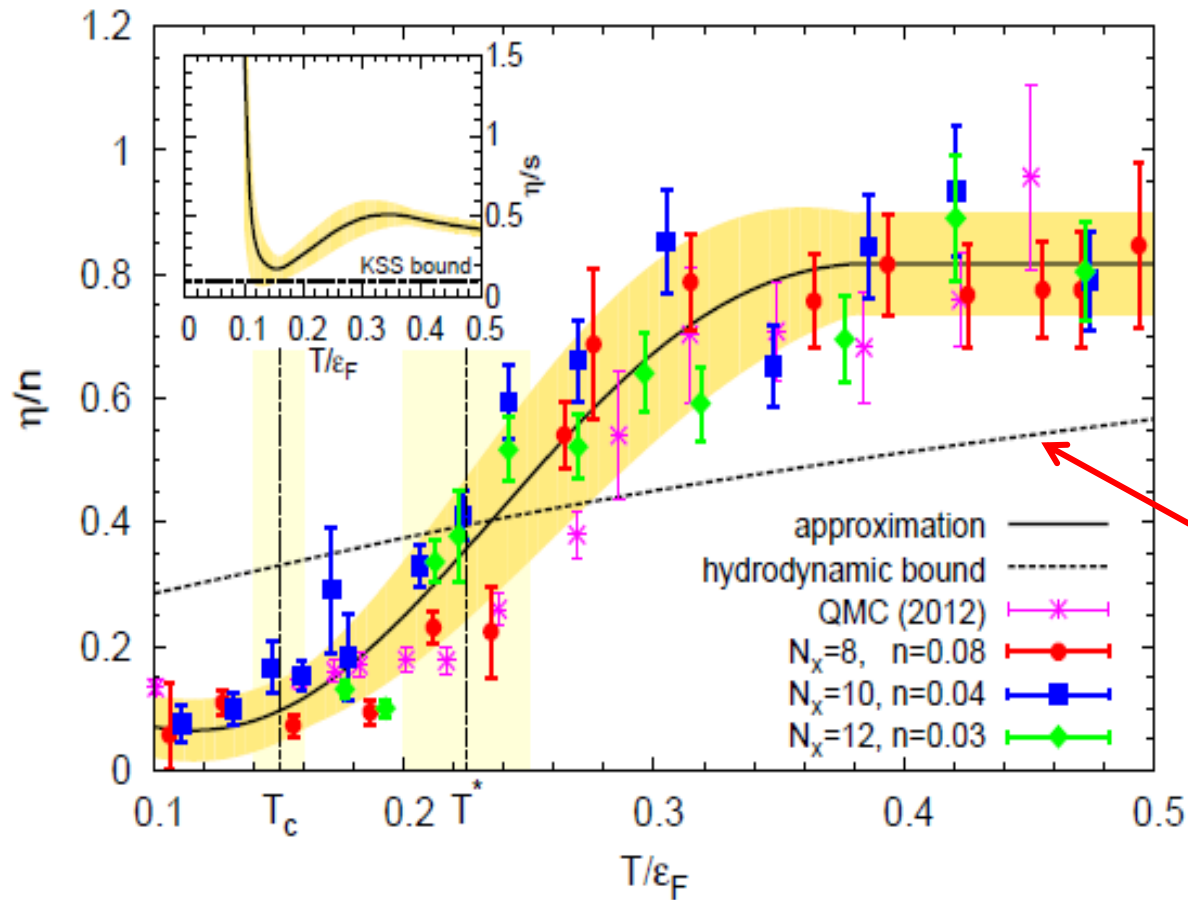
$$\sim T^{-8}$$

$$\sim T^{3/2}$$



Uncertainties related to numerical analytic continuation

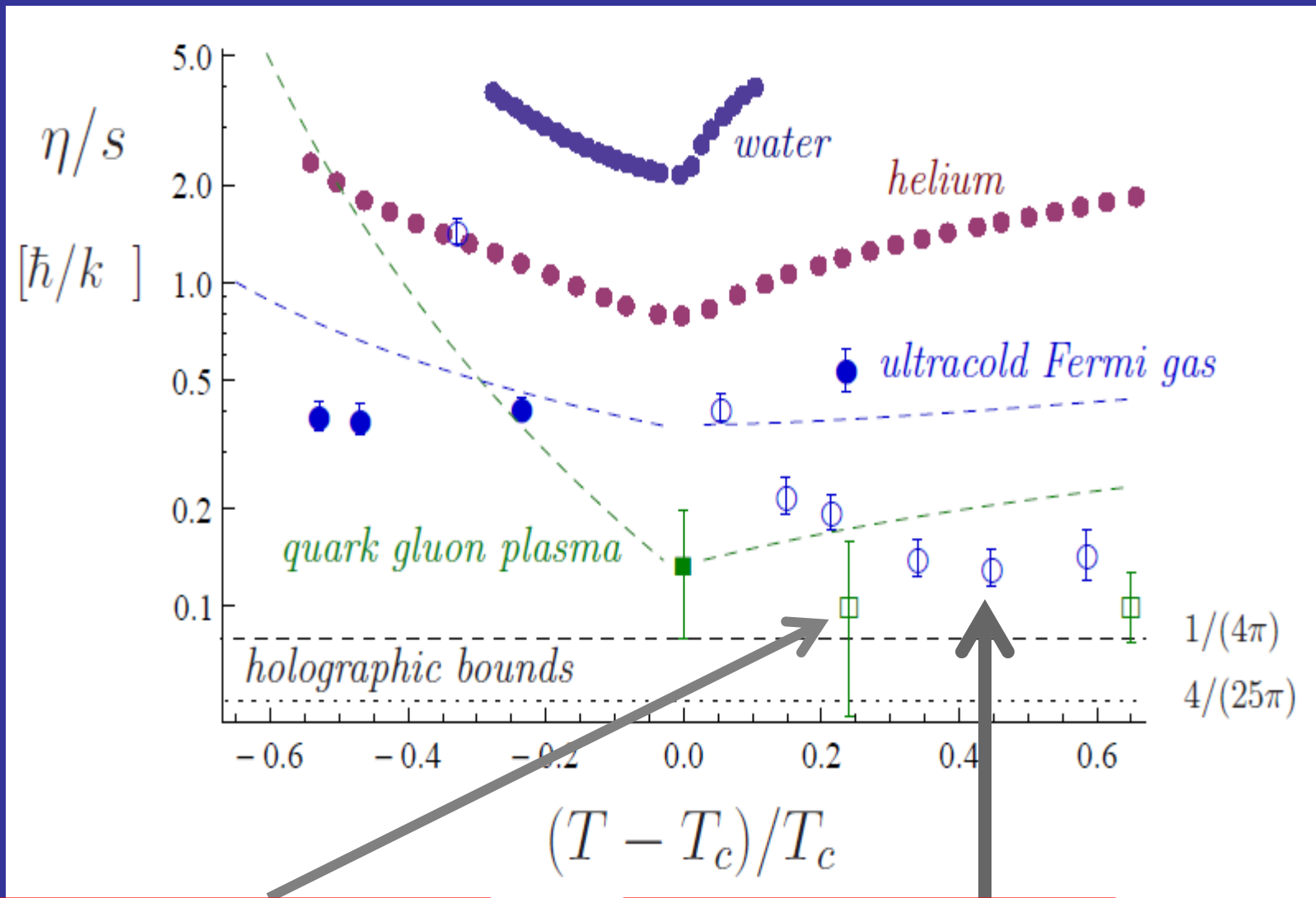
Preliminary results: Wlazłowski et al.



C. Chafin, T. Schafer,
PRA87,023629(2013)
P.Romatschke, R.E. Young,
arXiv:1209.1604

Shear viscosity to entropy ratio – experiment vs. theory

(from A. Adams et al.1205.5180)



Lattice QCD (SU(3) gluodynamics):
H.B. Meyer, Phys. Rev. D 76, 101701 (2007)

QMC calculations for UFG:
G. Wlazłowski, P. Magierski, J.E. Drut,
Phys. Rev. Lett. 109, 020406 (2012)

Conclusions for DFT for unitary Fermi gas at finite temperature:

- 1) Effective mass is approximately 1 and has a weak temperature dependence
- 2) Gradient corrections are relatively weak (see also Michael Forbes talk)

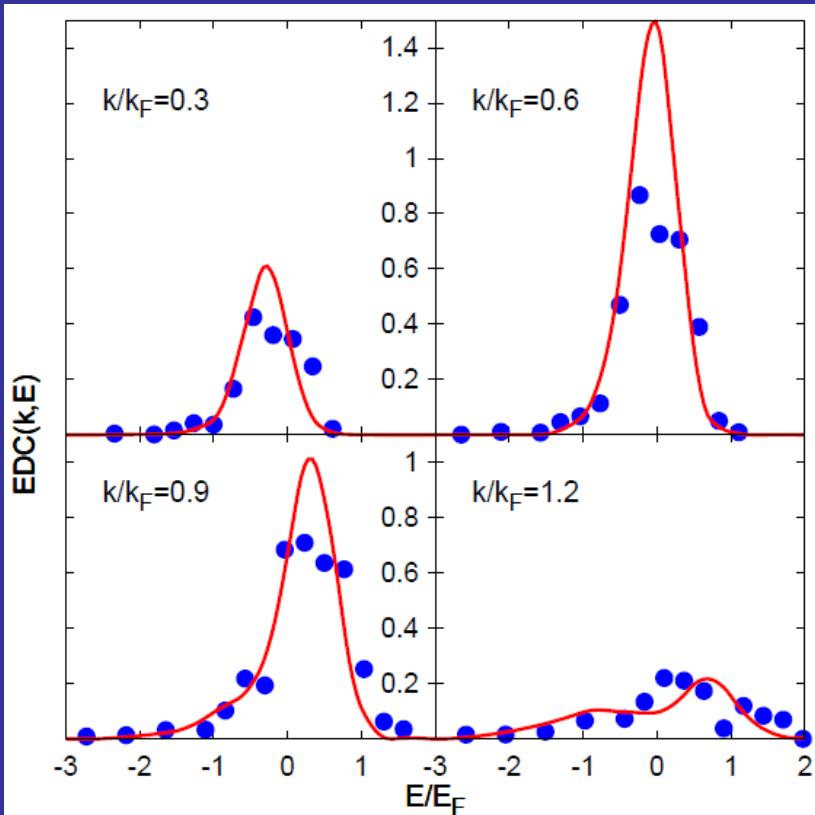
Problems:

- 1) Proper description of pseudogap within DFT: new type of a density?
- 2) Proper value of critical temperature within DFT.

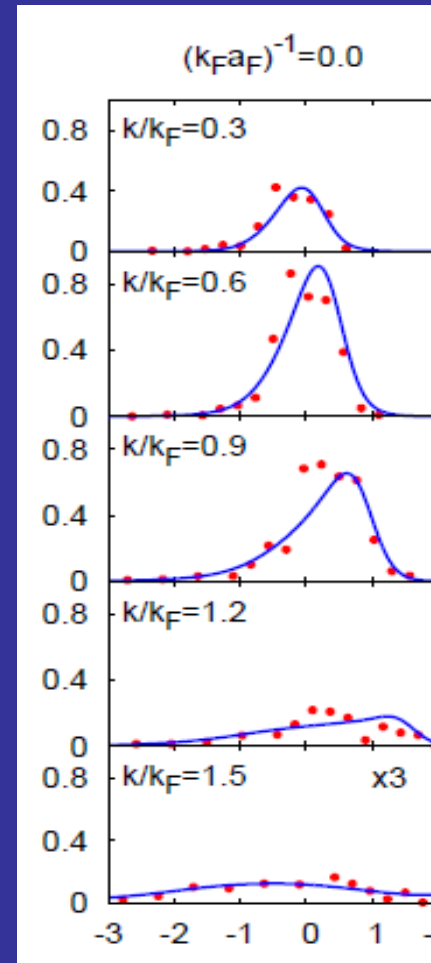
Theory vs Experiment (photoemission spectr.)

$$EDC(k, E) \sim A(k, \omega) f(\omega)$$

PIMC



Non selfconsistent t-matrix approx.



Single-particle properties

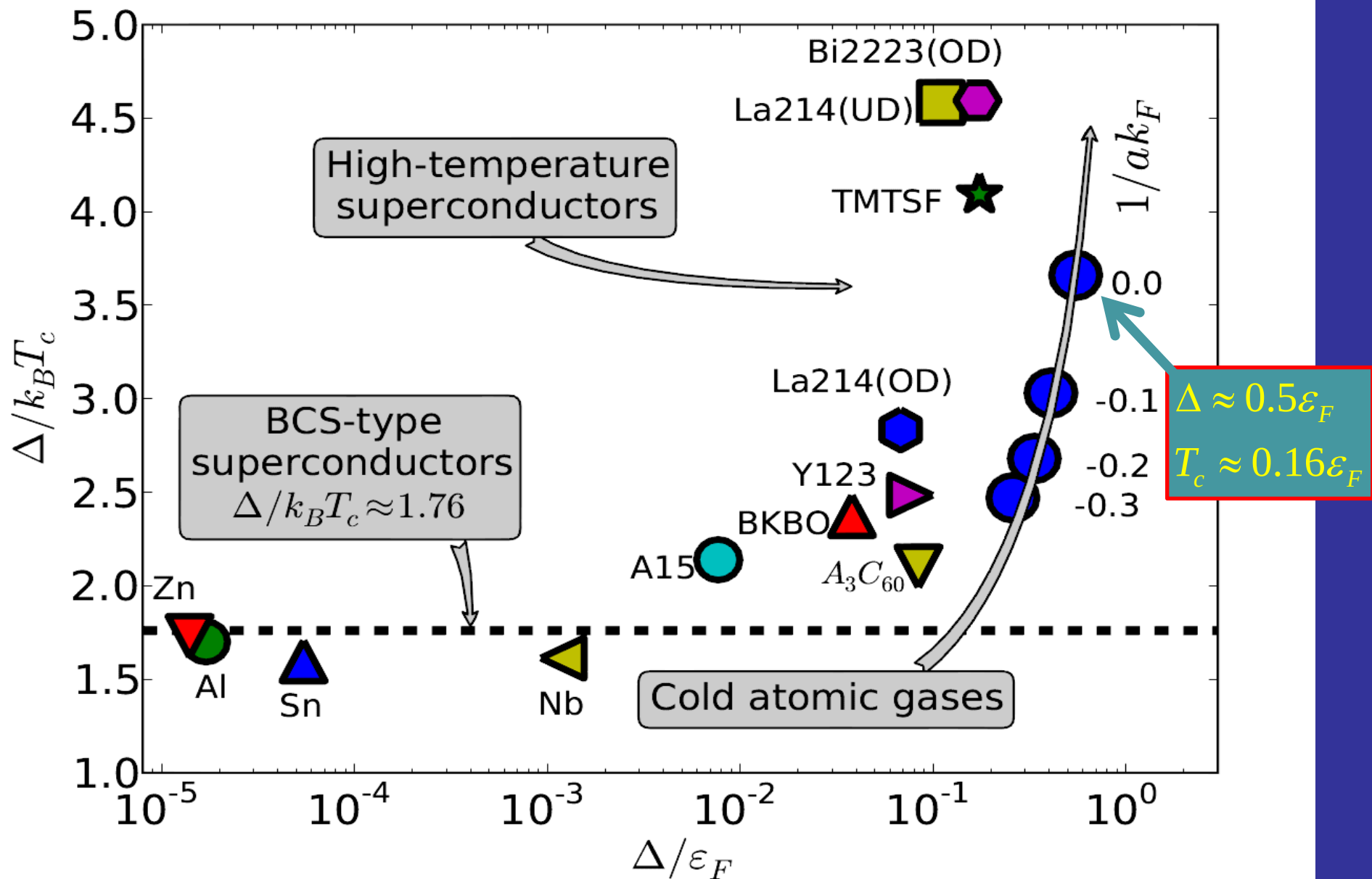
$$E(p) = \sqrt{\left(\frac{p^2}{2m^*} + U - \mu\right)^2 + \Delta^2}$$

Effective mass: $m^* = (1.0 \pm 0.2)m$

Self energy: $U = (0.5 \pm 0.2)m$

Weak temperature dependence!

Cold atomic gases and high T_c superconductors



Finite size scaling:

$$|T_c - T_{ij}| = \kappa g(L_i, L_j),$$

$$g(L_i, L_j) = L_j^{-(\omega+1/\nu)} \left[\frac{\left(\frac{L_j}{L_i}\right)^\omega - 1}{1 - \left(\frac{L_i}{L_j}\right)^{1/\nu}} \right]$$

$$\nu = 0.671$$

$$\omega = 0.8$$

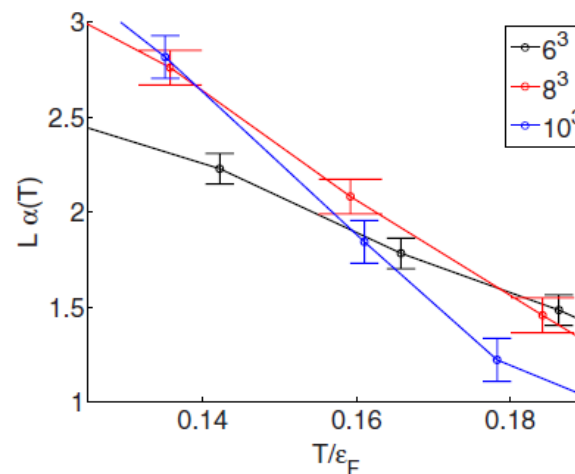


FIG. 11. (Color online) Condensate fraction $\alpha(T)$ scaled with the lattice size L , black squares for 6^3 , red stars for 8^3 , and blue circles for 10^3 (highest on the left) lattices, respectively, for the case $1/k_F a = -0.1$. The error bars correspond to the statistical errors.

Effective range corrections:

$$k_F r_{\text{eff}} \approx 0.4 - 0.5$$

CM corrections:

$$\begin{aligned} -\frac{1}{g} &= \frac{m}{4\pi\hbar^2 a} - \frac{m\Lambda}{2\pi^2\hbar^2} + \frac{m|K|}{8\pi^2\hbar^2} \\ &= \frac{m}{4\pi\hbar^2 a} - \frac{m\Lambda}{2\pi^2\hbar^2} \left(1 - \frac{|K|}{4\Lambda}\right). \end{aligned}$$

CM momentum
of a fermion pair

Total systematic error does not exceed 10-15%

Static and dynamic responses from QMC

Static:

$$\hat{H}(t) = \hat{H} - \hat{A}X$$

$$\delta B = \Phi X$$

$$\Phi = \int_0^{1/T} d\tau \left\langle \Delta \hat{A}(\tau) \Delta \hat{B} \right\rangle_0; \Delta \hat{A} = \hat{A} - \left\langle \hat{A} \right\rangle_0$$

Dynamic:

$$\hat{H}(t) = \hat{H} - \hat{A}X(t)$$

$$\delta B(t) = i \int_{-\infty}^{+\infty} dt \Phi(t-t') X(t')$$

$$\Phi(t) = i\theta(t) \left\langle [\hat{A}(t), \hat{B}] \right\rangle_0$$

$$\Phi(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{A^{(+)}(\omega')}{\omega - \omega' + i0^+} \left(1 - e^{-\omega'/T}\right) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{A^{(-)}(\omega')}{\omega - \omega' + i0^+} \left(e^{\omega'/T} - 1\right)$$

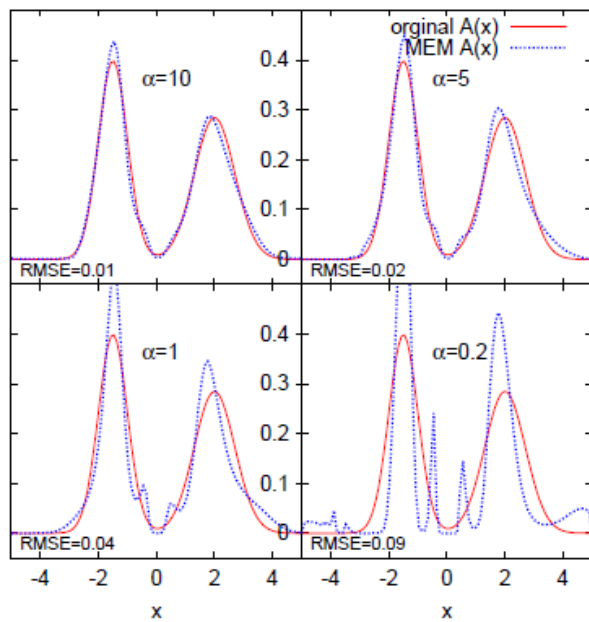


Figure 5: (Color online) Reconstruction of the artificial object function $A(x)$ by self-consistent MEM for different values of parameter α .

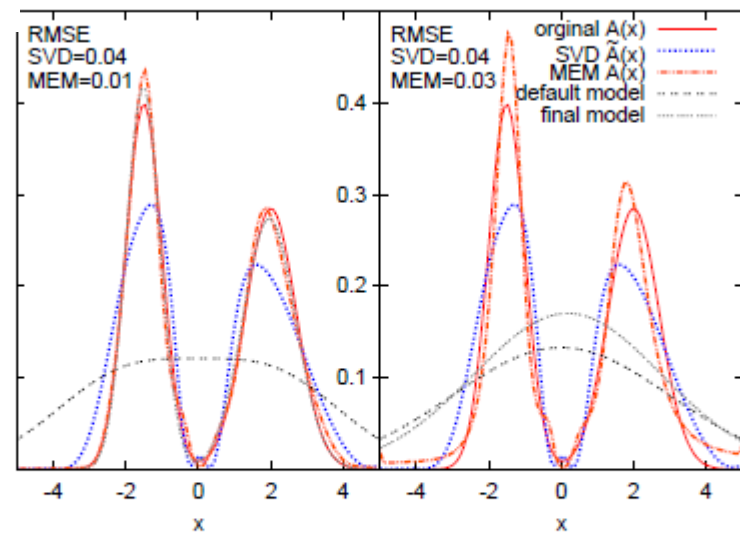
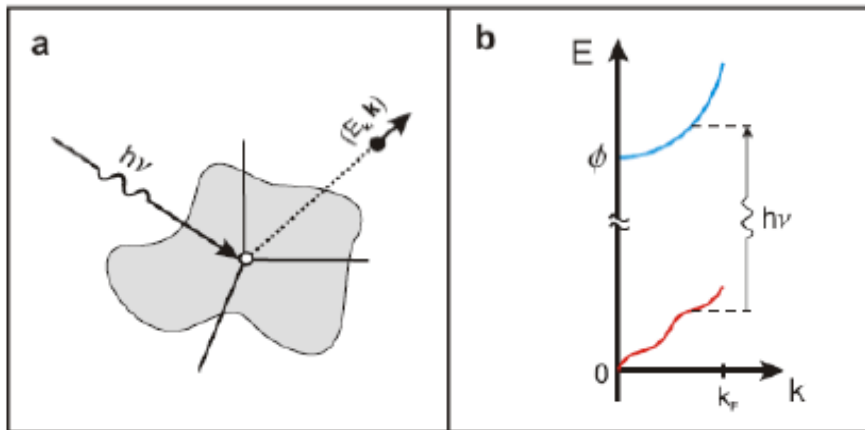


Figure 6: (Color online) The reconstruction ability of the spectral function for the full problem (data with noise + external constraints) of the SVD and MEM methods. The left panel shows the solution of the self-consistent MEM with a combination of two Gaussian functions as a default model class. The right panel shows the solution of the self-consistent MEM with Gaussian functions as a default model class.

RF spectroscopy in ultracold atomic gases

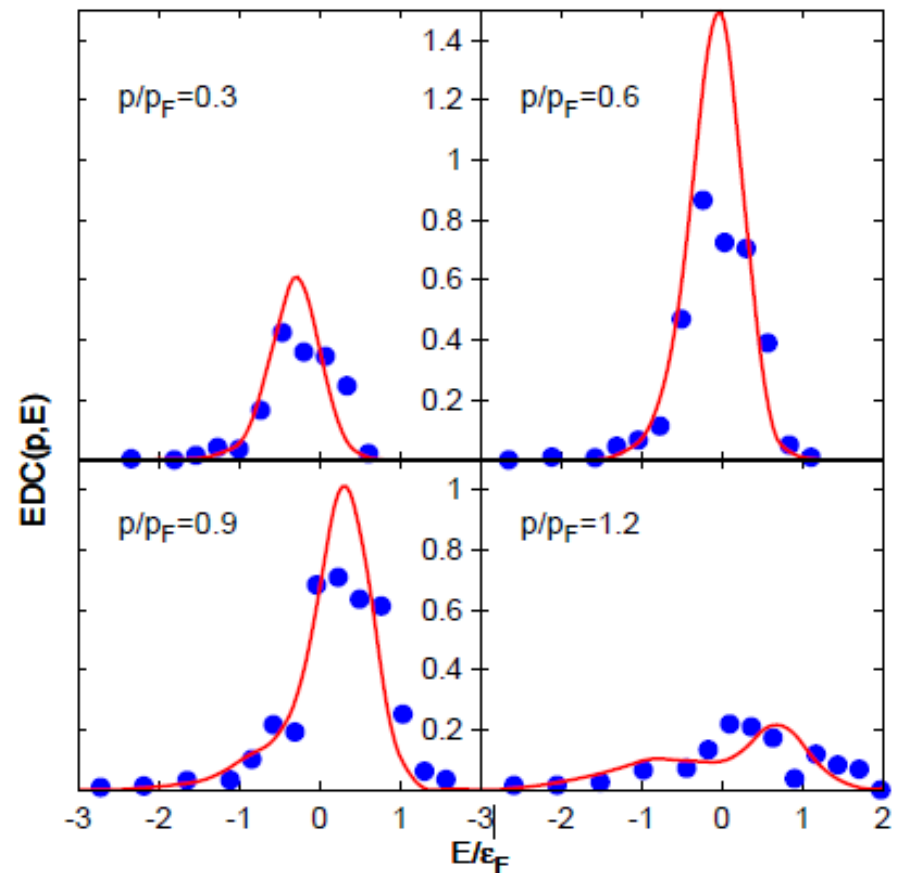


$$-E_s + h\nu = \frac{\hbar^2 k^2}{2m} + \phi$$

$$E(N) = E(N-1) + E_s$$

Stewart, Gaebler, Jin, Using
photoemission spectroscopy
to probe a strongly
interacting Fermi gas,
Nature, 454, 744 (2008)

$$\text{EDC}(p, E, T) = C p^2 \int_0^\infty dr r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E - \mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E - \mu(r)),$$



Experiment (blue dots): D. Jin's group
Gaebler et al. *Nature Physics* 6, 569(2010)

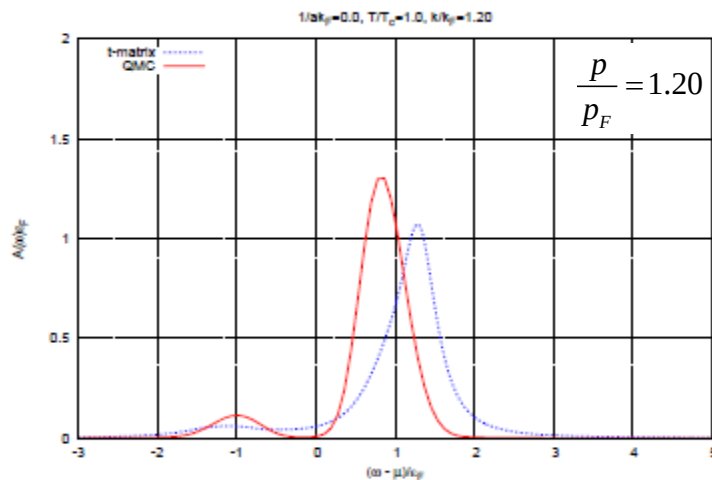
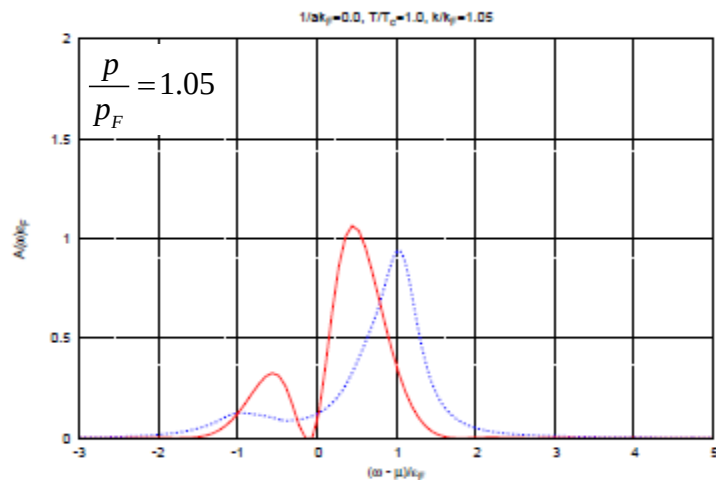
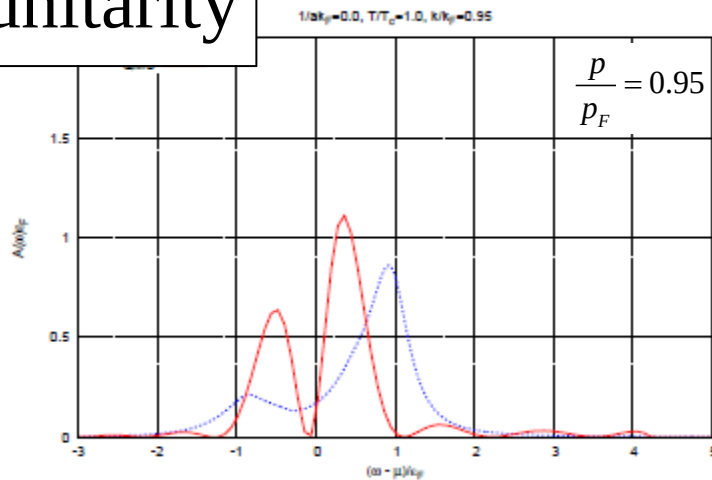
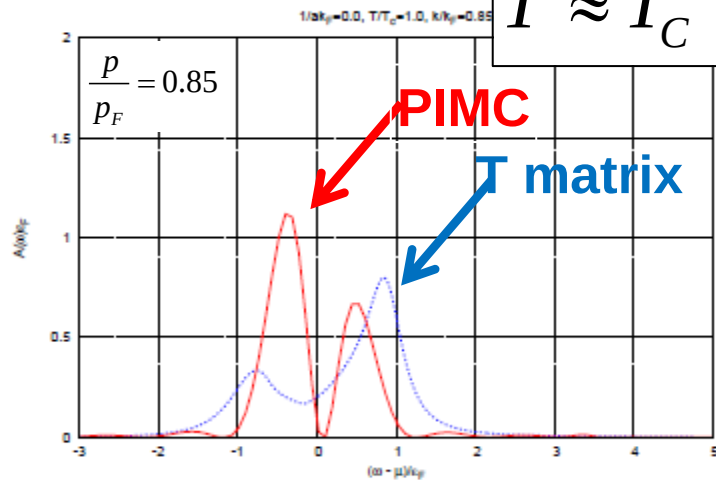
Theory (red line):

Magierski, Wlazłowski, Bulgac,
*Phys.Rev.Lett.*107,145304(2011)

PIMC vs T-matrix approach (preliminary results)

$T \approx T_C$ at unitarity

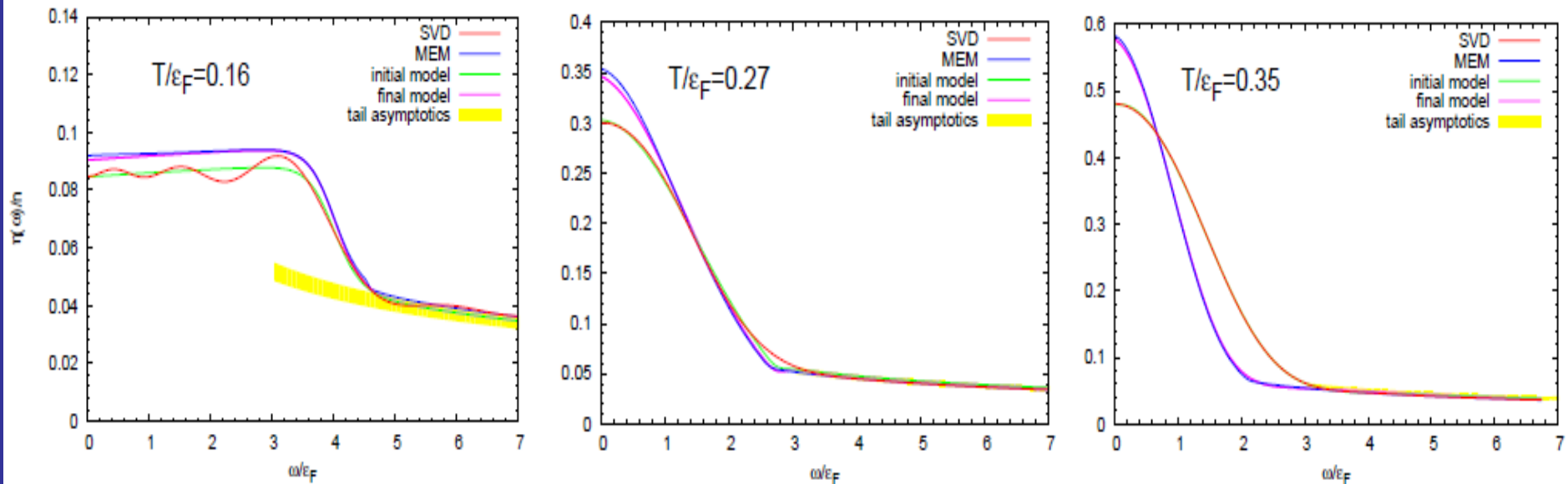
$A(p, \omega)$



$\frac{\omega - \mu}{\epsilon_F}$

T matrix gap at T_c is about twice larger than the *ab initio* gap

Combined Maximum Entropy and SVD methods were applied



Additional symmetries and sum rules:

$$G(\tau) = G(\beta - \tau)$$

$$\frac{1}{\pi} \int_0^\infty d\omega \left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}} \right] = \frac{\varepsilon}{3} - \text{energy density}$$

$$\eta(\omega \rightarrow \infty) \simeq \frac{C}{15\pi\sqrt{m\omega}}.$$

Details of calculations, improvements and problems

- Currently we can reach 14^3 lattice and perform calcs. down to $x = 0.06$ (x – temperature in Fermi energy units) at the densities of the order of 0.03.
- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.
- Update field configurations using the Metropolis importance sampling algorithm. QMC calculations can be split into two independent processes:
 - 1) sample generation (generation of sigma fields),
 - 2) calculations of observables.
- Change randomly at a fraction of all space and time sites the signs the auxiliary fields $\sigma(r,\tau)$ so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .
- At low temperatures use Singular Value Decomposition of the evolution operator $U(\{\sigma\})$ to stabilize the numerics.
- MC correlation “time” ≈ 200 time steps at $T \approx T_c$ for lattices 10^3 . Unfortunately when increasing the lattice size the correlation time also increases. One needs few thousands uncorrelated samples (we usually take about 10 000) to decrease the statistical error to the level of 1%.

1. Create an SVD solution and apply it to construct the class of default models $\mathcal{M}(x; \vec{f})$;
2. Use the “self-consistent” MEM with constructed class of models $\mathcal{M}(x; \vec{f})$ to produce final solution.

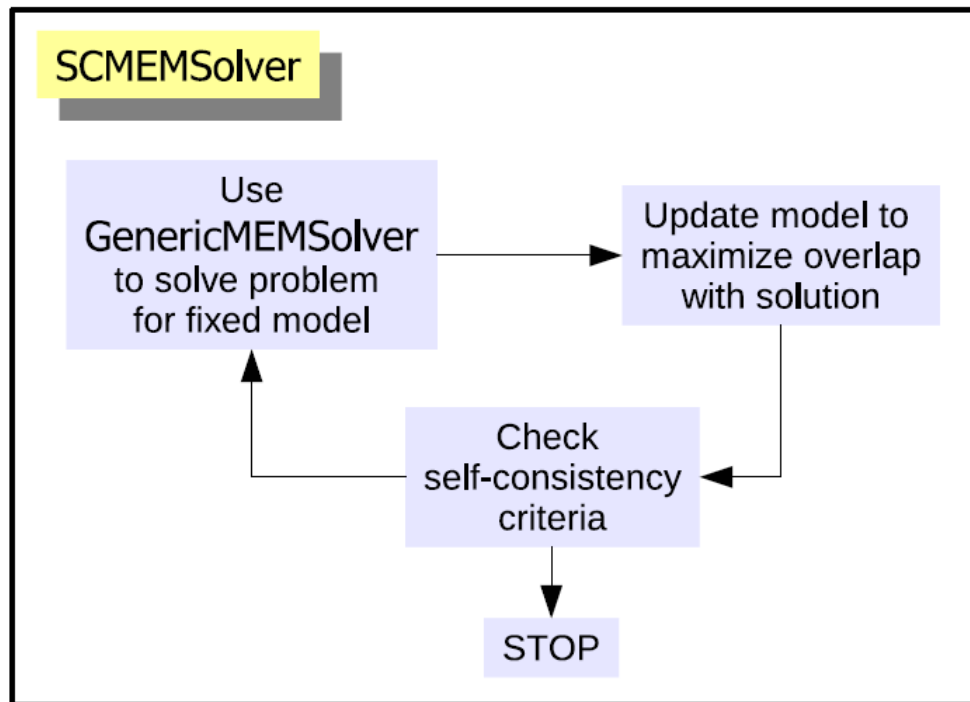


Figure 2: (Color online) The algorithm scheme for SCMEMSolver.