Strongly Correlated Fermi gases

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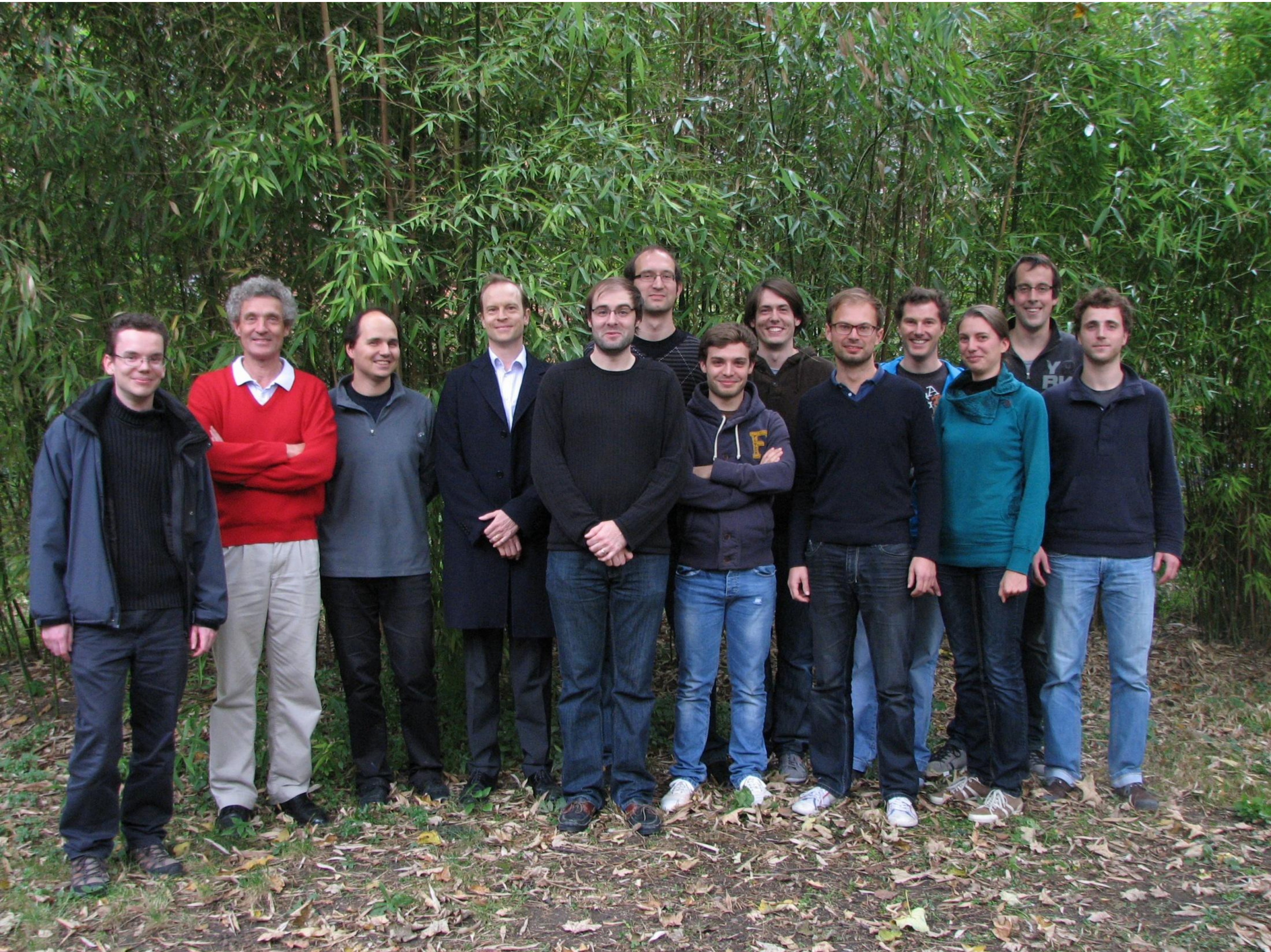
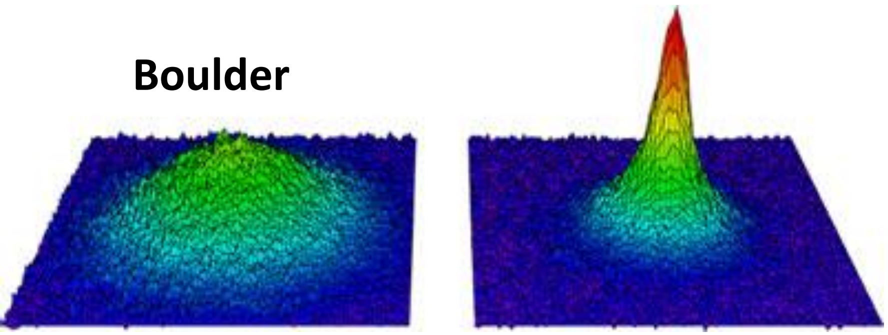
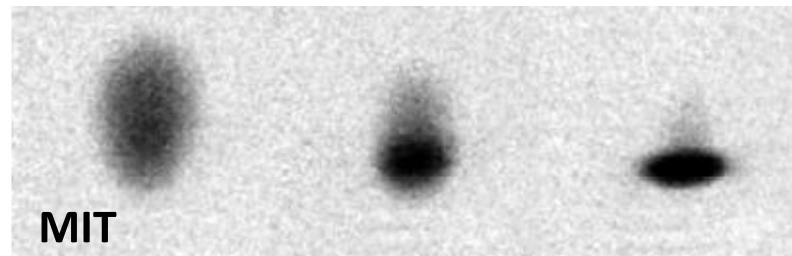


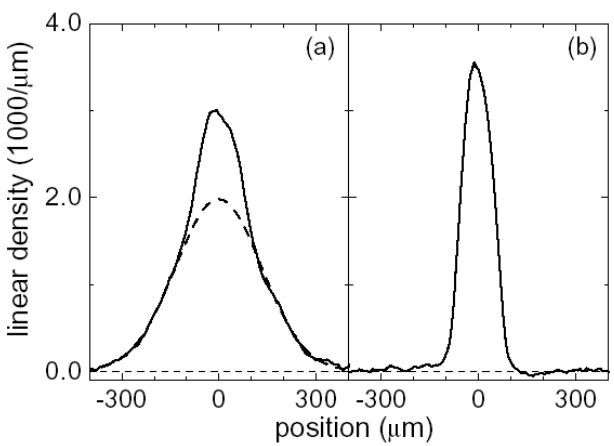
2003-2013: ten years of strongly correlated Fermi gases

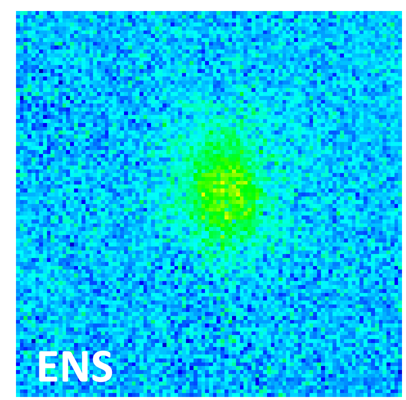
Boulder



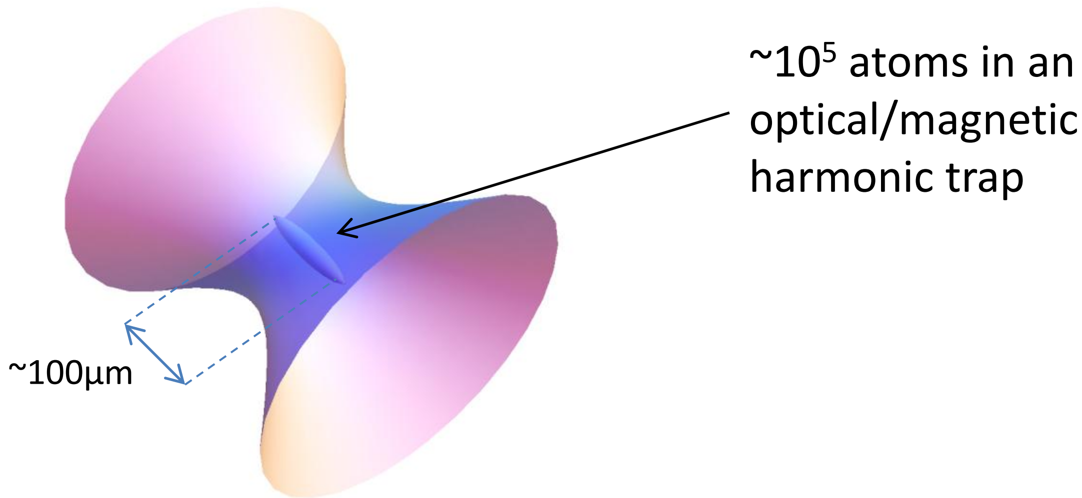


Innsbruck





Some numbers



	Electrons in metal	Ultra cold atoms
Density	$10^{30}/m^3$	$10^{20}/m^3$
Mass	10^{-30}kg	10^{-26}kg
Fermi temperature $\sim n^{2/3}/m$	10^5K	1µK
T/T_F	$<10^{-5}$	$\sim 10^{-2}$

Ultracold collisions

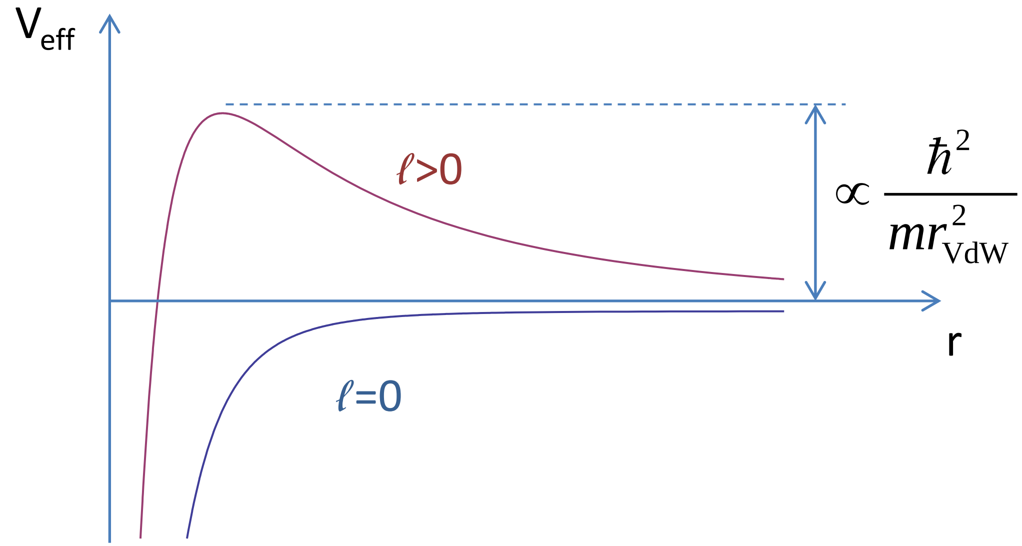
Low temperature: λ_{dB}~0.1µm >>b~a_0 (range of the potential).

For a Van der Waals $1/r^6$ potential, b~r_{VdW} such that

$$V(r) \simeq -\frac{\hbar^2}{m r_{VdW}^2}\left(\frac{r_{VdW}^6}{r^6}\right)$$

Incoming angular momentum ℓ

$$V_{\mathrm{eff}} = V(r) + \frac{\hbar^2 \ell(\ell+1)}{2mr^2}$$

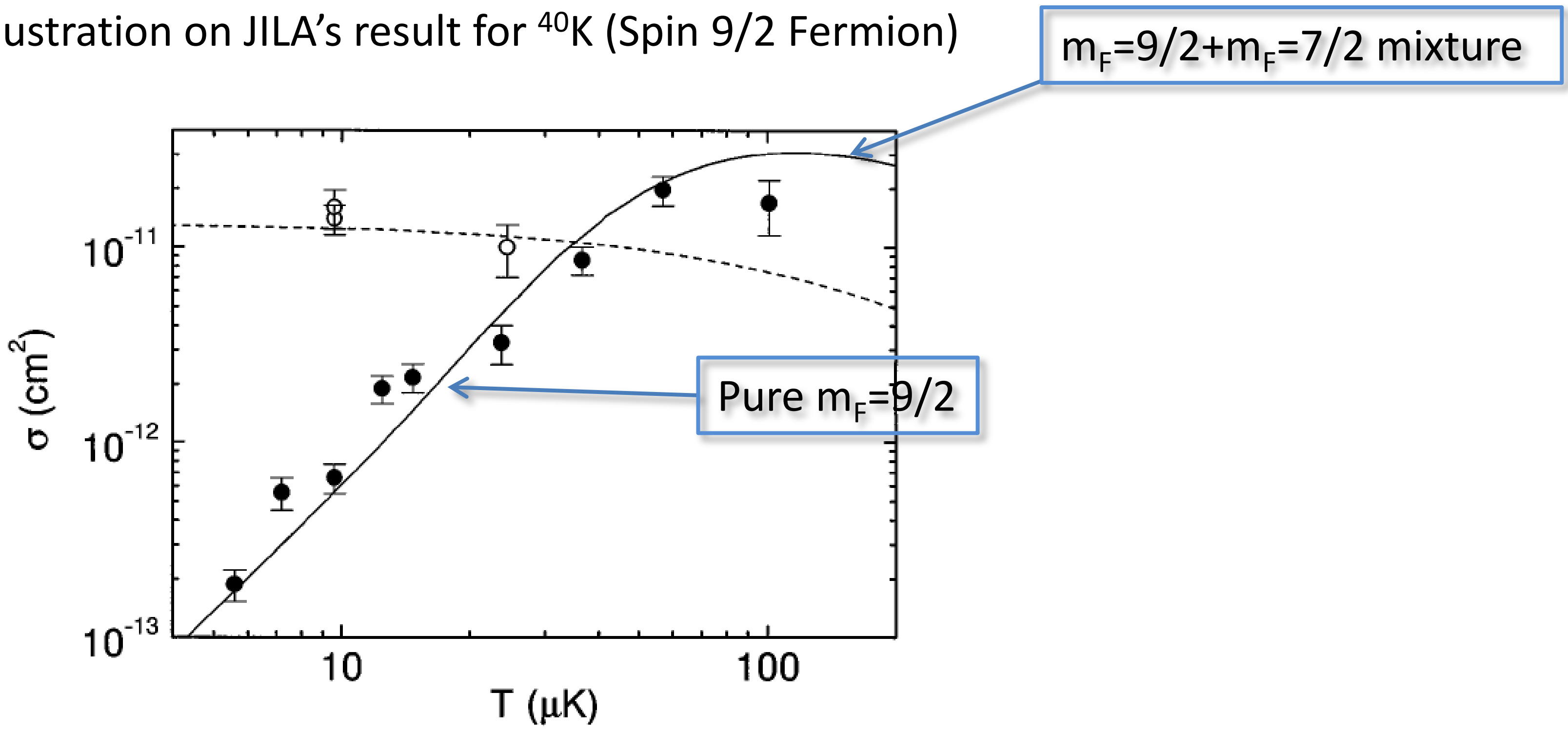


No collision in a ℓ>0 channel at low temperature

Ultracold collisions

No interactions between spin polarized fermions

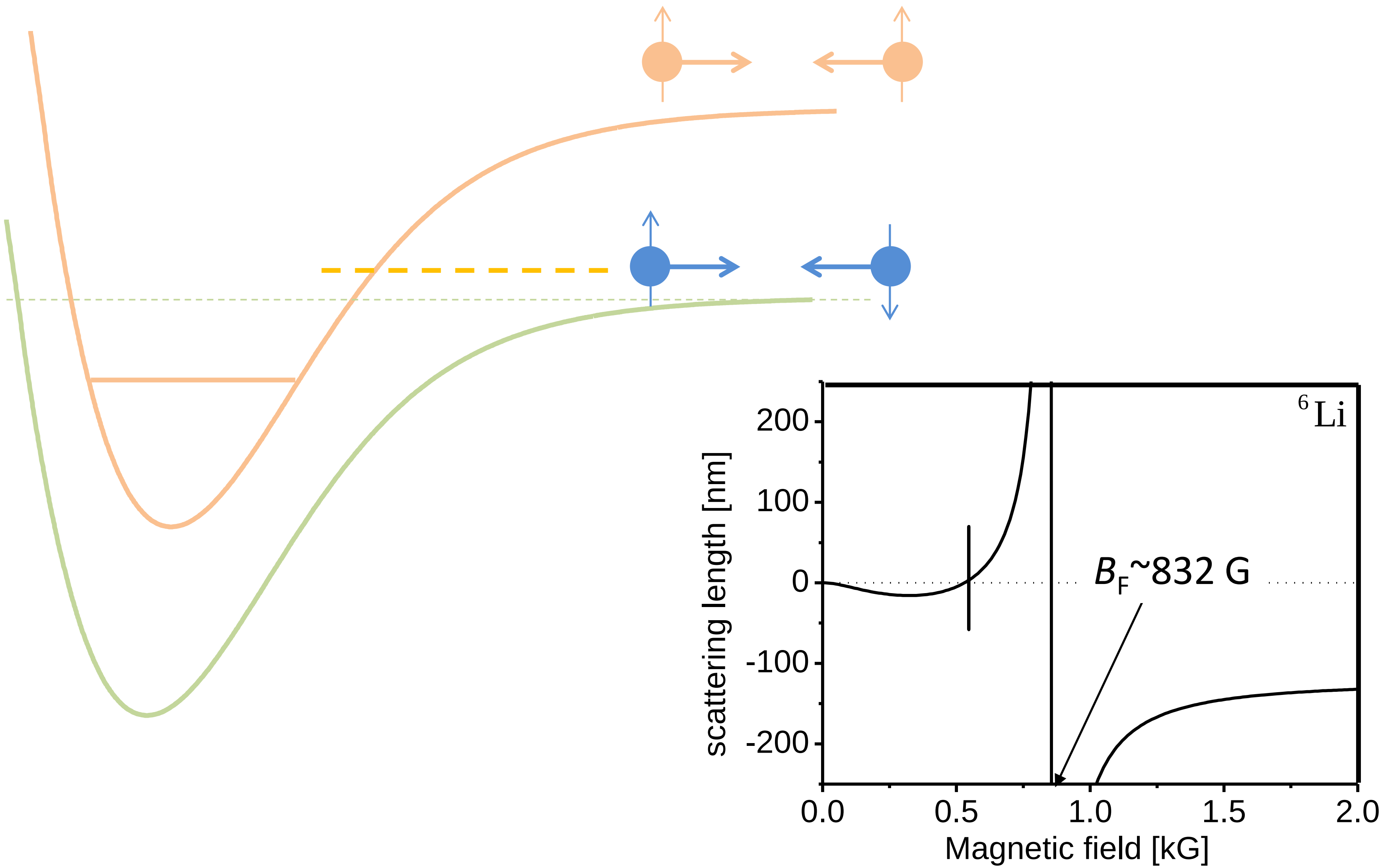
Illustration on JILA's result for ^{40}K (Spin 9/2 Fermion)



Interactions are insensitive to the details of the potential: the properties of the cloud depend only on the s-wave scattering length (*Universality*).

One can use any model potential as long as $f_k = \dfrac{-a}{1+ika}$

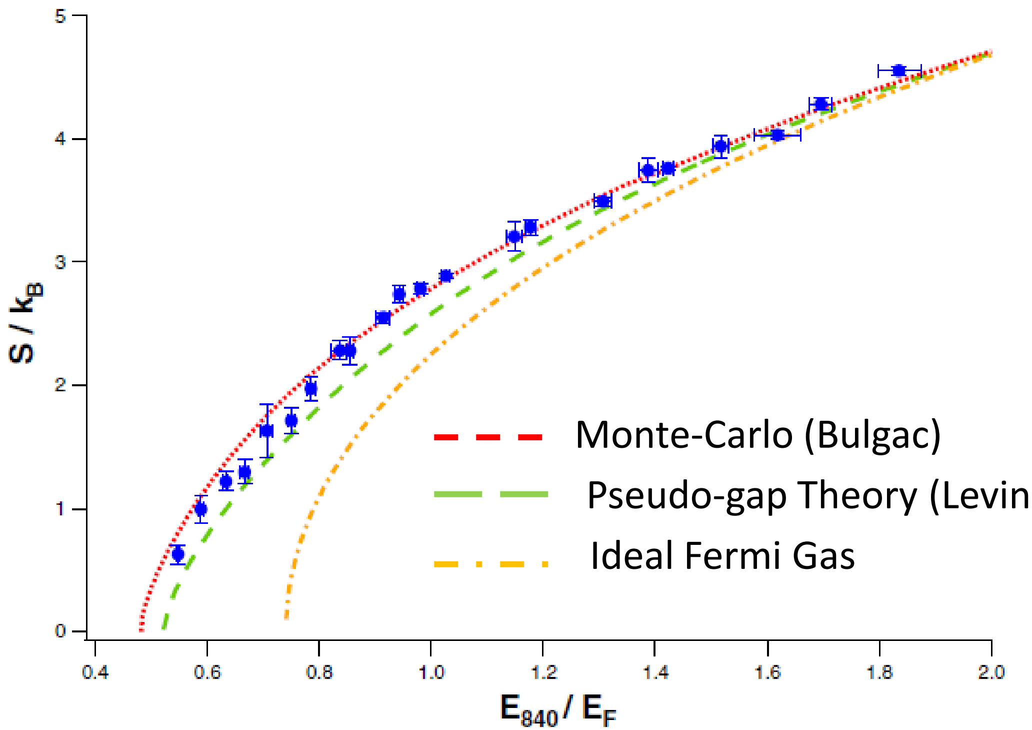
Fano-Feshbach resonances



Global Thermodynamics of Trapped Gases

N, (Atom number) S (Entropy), E (Energy) are well defined quantities for trapped systems.

- Measure Potential energy by *in-situ* density profile
- Measure Interaction+Kinetic Energy by Time of Flight
- Measure Entropy by Adiabatic Sweep to a=0

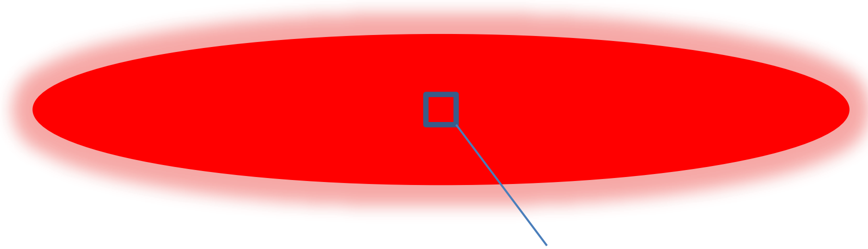


Thermometry:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N$$

a=∞, Luo *et al.* PRL **98** 080402 (2007)

The Local Density Approximation (LDA)

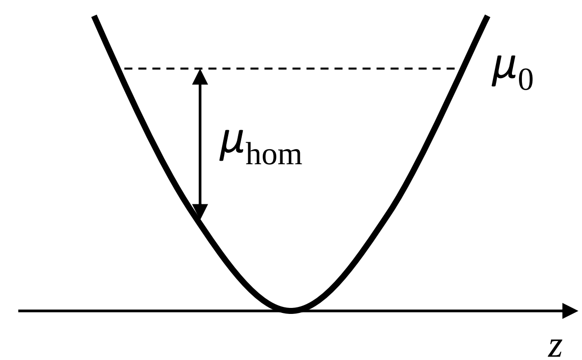


Mesoscopic volume where thermodynamical equilibrium is well defined (T(**r**),μ(**r**),P(**r**))

Equilibrium Condition: T,μ,P uniform.

Local Density Approximation:

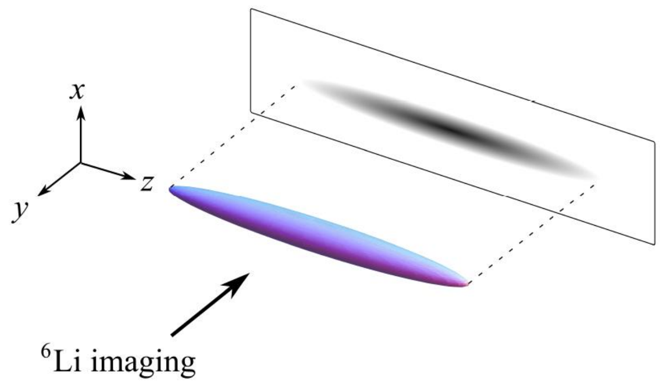
$$\mu(\mathbf{r}) = \mu_{\text{hom}}(n(\mathbf{r}), T(\mathbf{r})) + V(\mathbf{r}) \equiv \mu_0$$



Note: this is the hydrostatic condition $\nabla P + n\nabla V = 0$ with the Gibbs-Duhem relation dP=ndμ at constant T.

Local Thermodynamics of Trapped gases

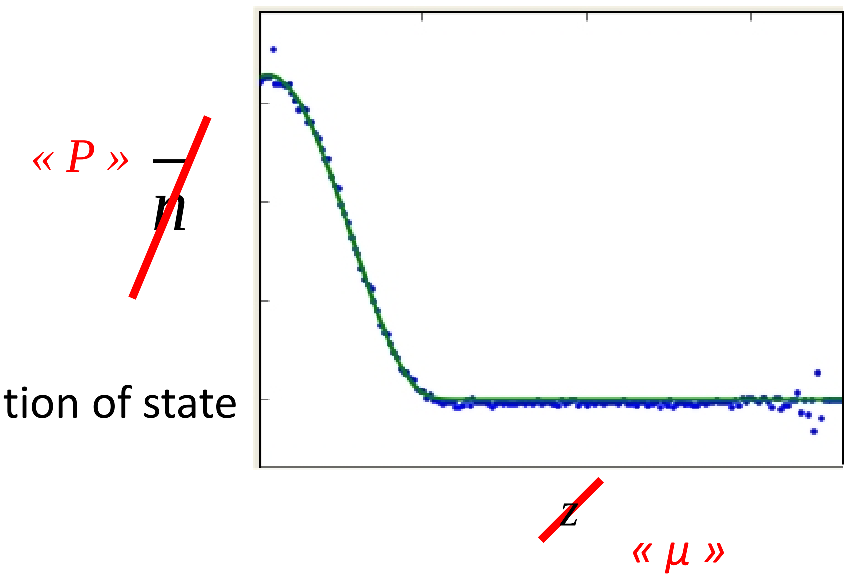
$$\bar{n}(z) = \int dxdy n(x,y,z)$$



•Cylindrical trap and harmonic trapping:
$dxdy = 2\pi r dr = -2\pi d\mu / m\omega^2$

•Gibbs-Duhem $n = dP/d\mu$

$$\bar{n}(z) = -\int \frac{2\pi d\mu}{m\omega^2} \frac{\partial P}{\partial \mu} = P(\mu(z),T)$$

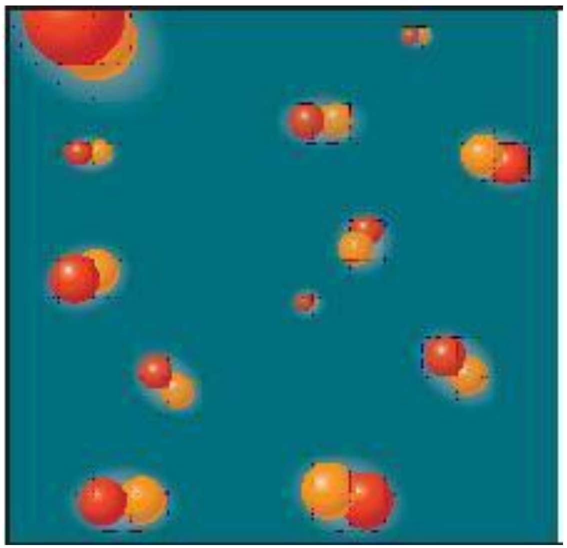


High accuracy (few percent):
•Double integration increases S/N ratio
•One shot yields a whole piece of the Equation of state

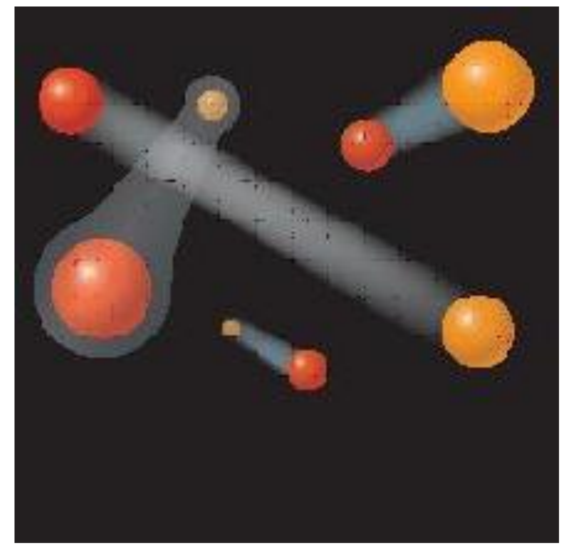
(Similar scheme at Tokyo and MIT)

The zero-temperature Fermi gas: the BEC-BCS crossover

The BEC-BCS crossover



a=∞:
Strongly correlated regime



a>0: strongly attractive regime
BEC of dimers

a<0: weakly attractive regime
BCS state

Equation of State in the BEC-BCS crossover

Goal: calculate the equation of state of the gas as a function of $1/k_F a$

Dimensionally:

$$E = \frac{N}{2} E_b + \frac{3NE_F}{5} \xi(1/k_F a)$$

$$E_F = \frac{\hbar^2 k_F^2}{2m} \qquad k_F = (3\pi^2 n)^{2/3}$$

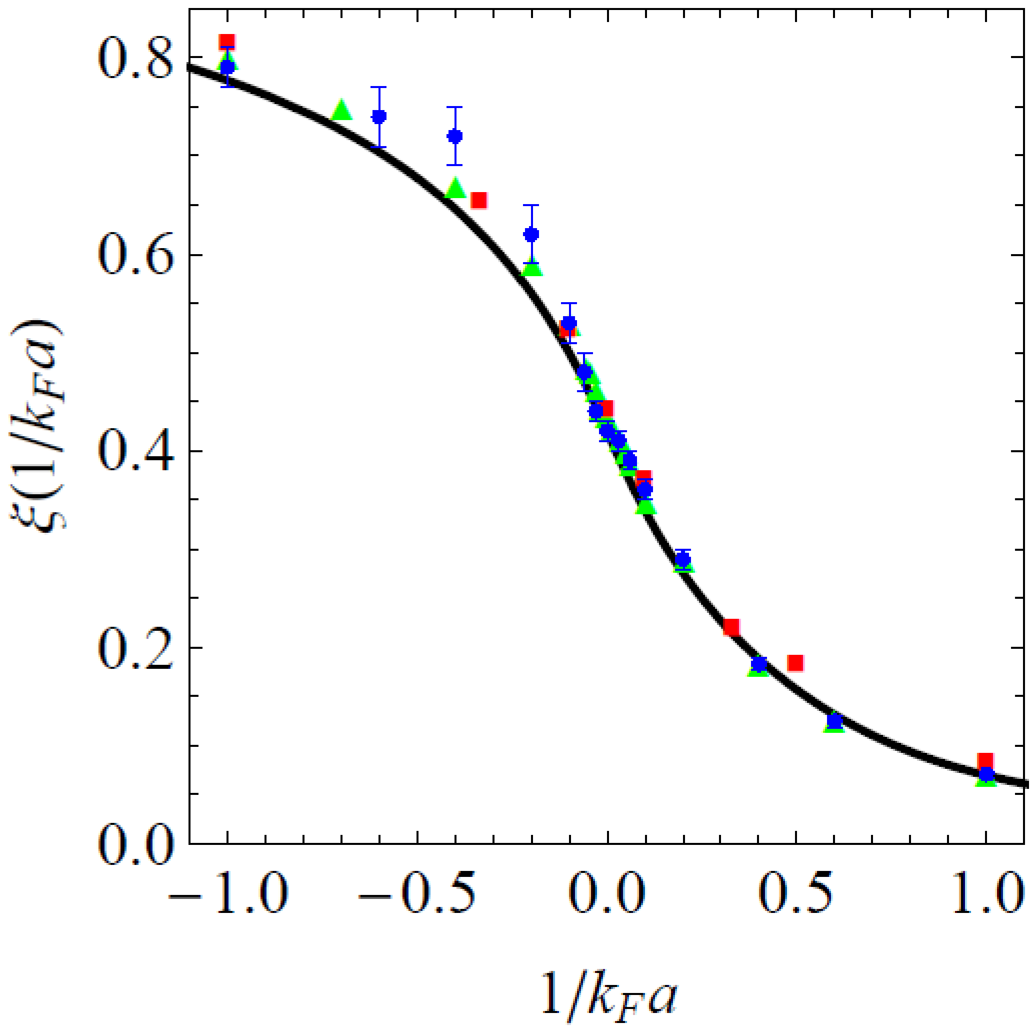
$$E_b = -\frac{\hbar^2}{ma^2} \Theta(a)$$

Asymptotic behavior:

$1/k_F a \longrightarrow 0^-$ *(BCS Limit):* weakly attractive Fermi gas, $\xi \longrightarrow 1$

$1/k_F a \longrightarrow 0^-$ *(BCS Limit): repulsive Bose-Einstein condensate of dimers,* $\xi \longrightarrow 1$

Quantitative Equation of State



Experiment:

Padé interpolation
Navon *et al.* Science **328**, 729 (2010)

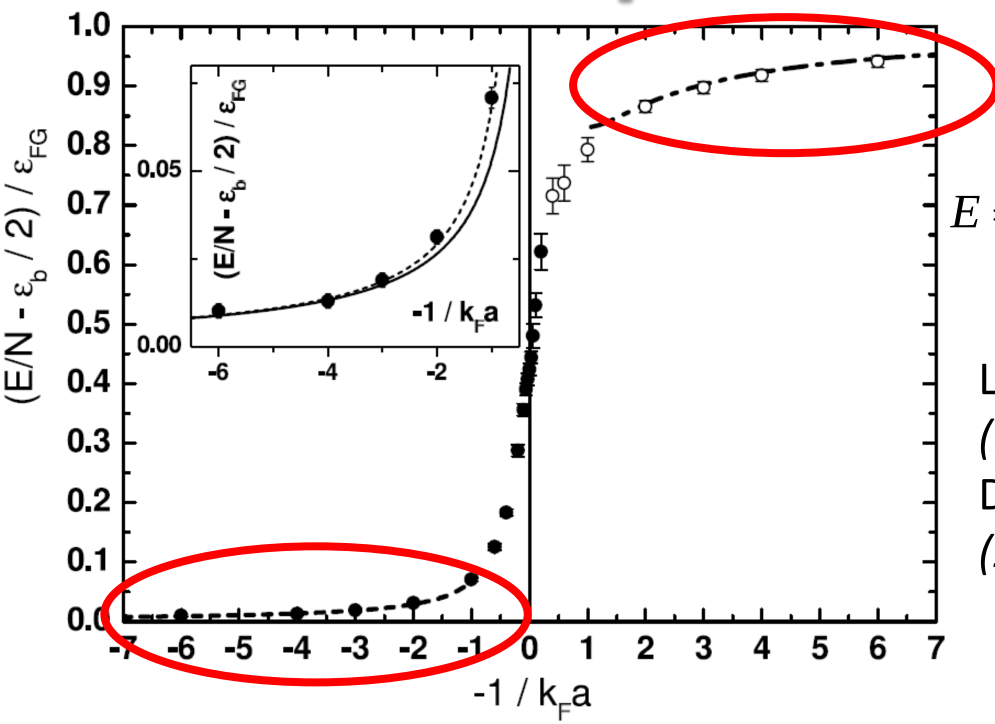
Theory: (Variational Fixed Node Monte-Carlo)

Chang *et al.*, PRA **70**, 043602 (2004)

Astrakharchik *et al.*, PRL **93**, 200404 (2004)

Pilati *et al.*, PRL **100**, 030401 (2008)

Asymptotic expansion of the Equation of State



Lee-Yang expansion:

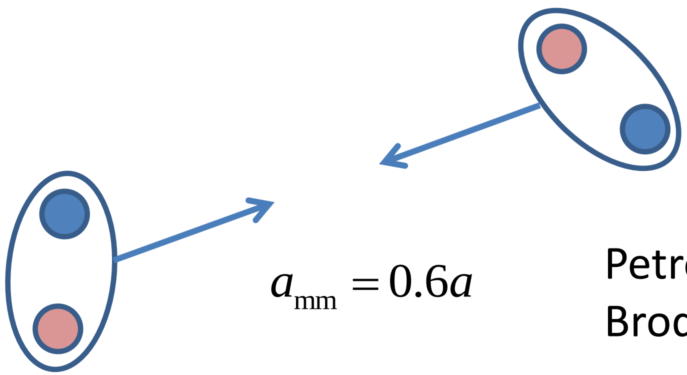
$$E = \frac{3NE_F}{5}\left[1 + \frac{10}{9\pi}k_F a + \frac{4(11-2\ln 2)}{21\pi^2}(k_F a)^2 + \ldots\right]$$

Lee & Yang *Phys. Rev.* ***105****, 1119 (1957) (repulsive fermions)*
Diener *et al. Phys. Rev. A* ***77****, 023626 (2008). (attractive fermions)*

Lee-Huang-Yang expansion: $E = \frac{N}{2}E_b + N\frac{\pi\hbar^2 a_{\mathrm{mm}}}{2m}n\left[1 + \frac{128}{15\sqrt{\pi}}\sqrt{na_{\mathrm{mm}}^3} + \ldots\right]$

Lee, Huang, Yang, Phys. Rev. **106**, 1135 (1957)
X. Leyronas & R. Combescot PRL **99**, 170402 (2007) *(Composite bosons)*

Lee-Huang-Yang corrections and Few body physics



Petrov *et al.* PRL **93**, 090404 (2004) (Schrödinger equation)
Brodsky *et al.* PRA **73**, 032724 (2006) (Diagrammatic)

How far can one describe the dimers as point-like bosons?

Higher order expansion (Wu 1957, Braaten et al. PRL 2002)

$$E = \frac{N}{2} E_b + N \frac{\pi \hbar^2 a_{\mathrm{mm}}}{2m} n \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a_{\mathrm{mm}}^3} + \frac{8}{3}\left(4\pi - 3\sqrt{3}\right) n_{\mathrm{mm}}^3 \ln\left(B n a_{\mathrm{mm}}^3\right) + \ldots \right]$$

Non-universal term:
depends on the internal
structure of the boson.

The Unitary Fermi gas

$$E = \frac{3NE_F}{5}\xi(0)$$ ~ *Ideal Fermi gas*. $\xi(0)$ = « Bertsch parameter »

- BCS mean-field theory: $\xi(0)<0.6$
- DFT (McNeil Forbes *et al.*, PRL 2011): $\xi(0)<0.38(1)$
- Current experimental estimate (ENS-MIT): 0.37(1)

Around unitarity:

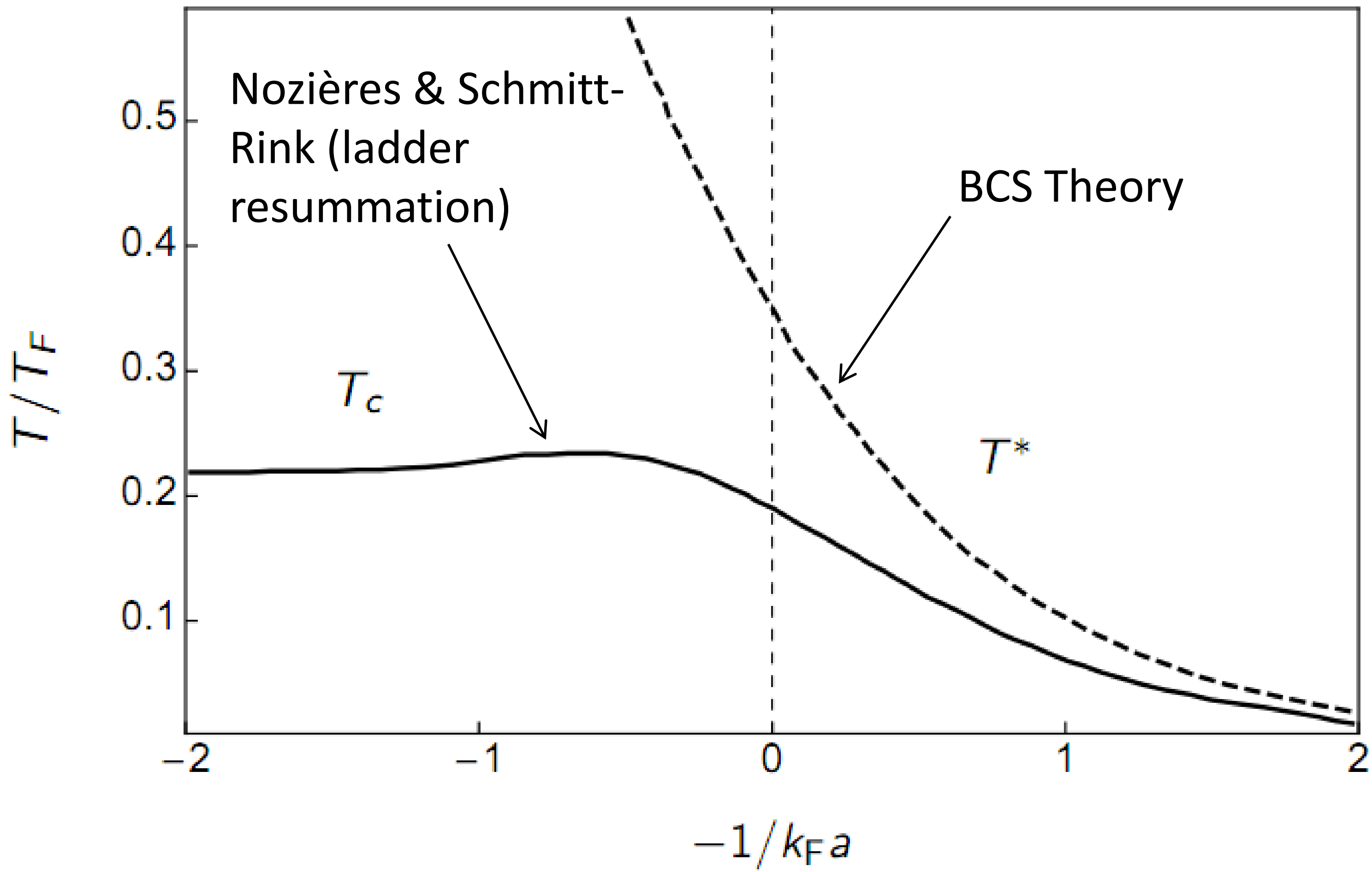
$$\xi(1/k_F a) = \xi(0) + \frac{1}{k_F a}\zeta + \ldots$$

ζ **= Tan's contact**. Related to the tail of the momentum distribution $n_k \simeq -\frac{4\pi m}{\hbar^2 k^4}\frac{\partial E}{\partial(1/a)}$

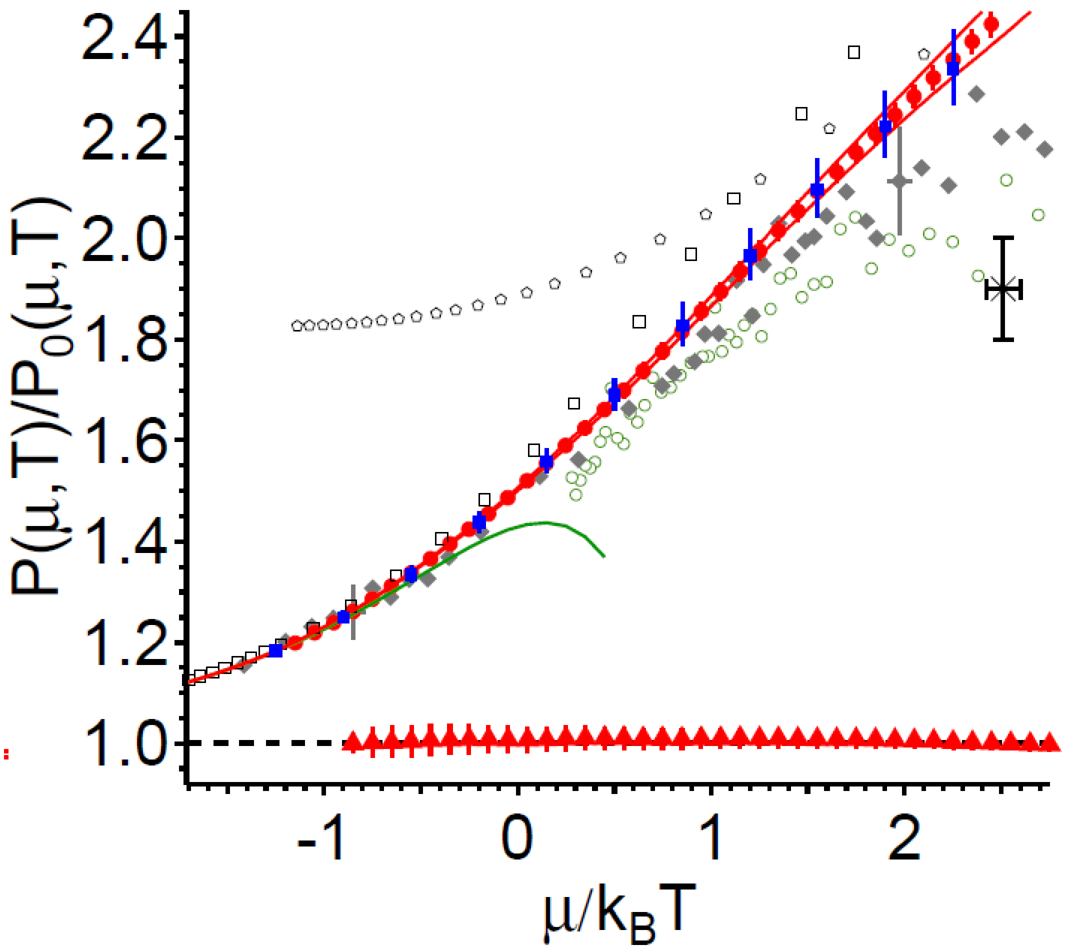
ζ=0.93 (5) from Equation of State Measurement (ENS). Compatible with *Static Structure Factor Measurement* (Melbourne), *Direct Momentum Distribution* and *Photoemission Spectroscopy* (JILA)

The Unitary Fermi Gas at Finite Temperature

Critical temperature of the Fermi gas



Finite temperature equation of state of the Unitary Fermi gas.



$P_0(\mu,T)$: equation of state of the ideal Fermi gas.

Experiments:

MIT

ENS

Tokyo

Theory

Bold Diagrammatic MC

Auxilliary Field QMC

First order bold diagram

3rd order virial expansion

High temperature behavior: Virial Expansion

Grand canonical Partition Function $$\Xi = \sum_{N=0}^{\infty} Z_N e^{N\mu/k_B T}$$

In the dilute limit, fugacity $z = e^{\mu/k_BT} \longrightarrow 0$

The Nth term of the expansion (« Virial » expansion) is given by the solution of the N-body problem.

$$P(\mu,T) = -\frac{k_B T}{V}\ln(\Xi) = 2\frac{k_B T}{\lambda_{\text{th}}^3}\sum_{N\geq 1} b_N z^N$$

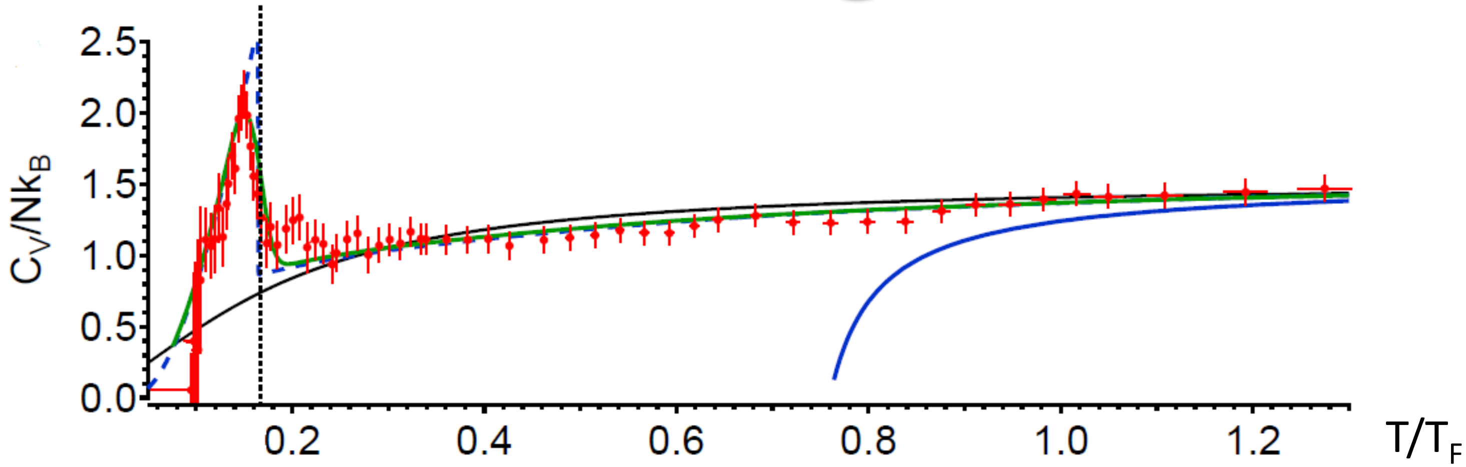
Ideal Fermi gas

$$b_N = \frac{(-1)^{N+1}}{N^{5/2}} + b'_N$$

N	b'_N (ENS exp.)	b'_N (Theory)
2	NA	$1/2^{1/2}$
3	-0.35(2)	-0.355*
4	0.096(15)	-0.016**

*Wu, Liu, Drummond PRL 2009, Kaplan & Sun PRL 2011, Leyronas PRA 2011
** Rakshit, Daily & Blume, PRA 2012

Critical regime

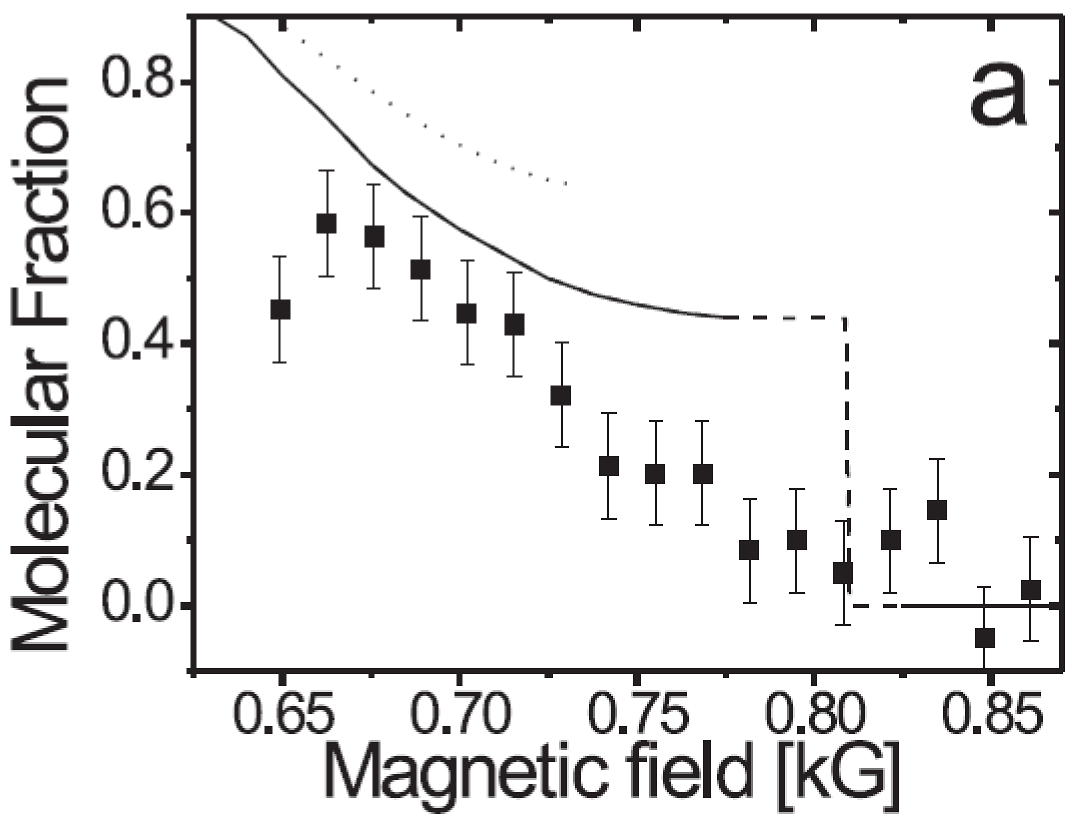


		T_c/T_F
Exp	ENS (Nascimbène *et al.* Nature 2010)	0.157(15)
	MIT (Ku *et al.* Science 2012)	0.167(13)
The.	Self-consistent T-Matrix[1]	0.16
	Det. Monte-Carlo[2]	0.173(6)
	Det. Monte-Carlo[3]	0.152(7)
	Aux. Field Monte-Carlo[4]	0.15(1)

[1] R. Haussmann, W. Rantner, S. Cerrito, and W. Zwerger, Phys. Rev. A **75**, 023610 (2007)
[2] O. Goulko and M. Wingate, Phys. Rev. A **82**, 053621 (2010).
[3] E. Burovski, N. Prokof'ev, B. Svistunov, and M. Troyer, Phys. Rev. Lett. **96**, 160402 (2006).
[4] A. Bulgac, J. E. Drut, and P. Magierski, Phys. Rev. A **78**, 023625 (2008).

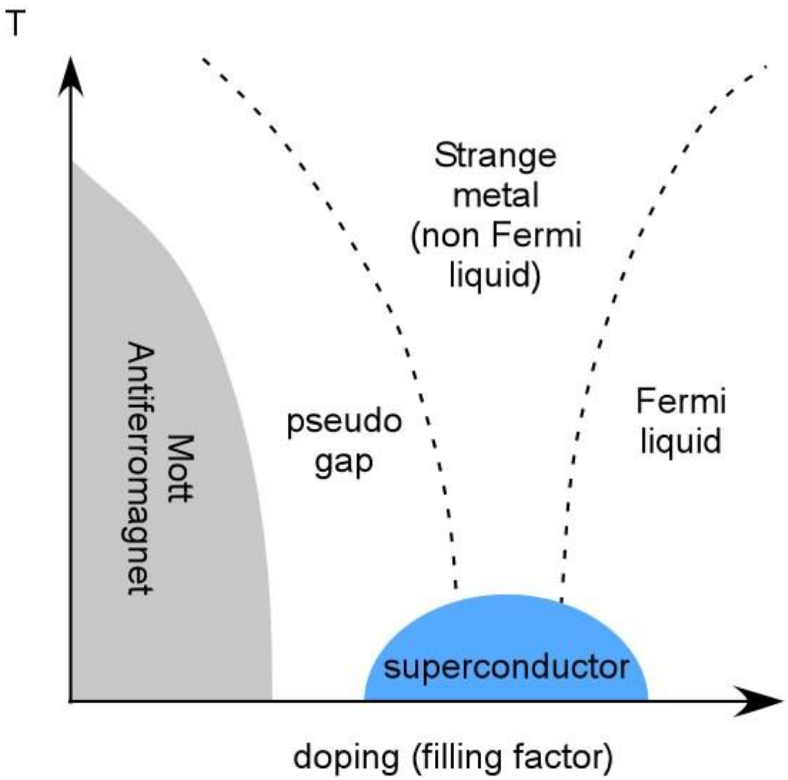
And T*?
(Are there preformed pairs above Tc?)

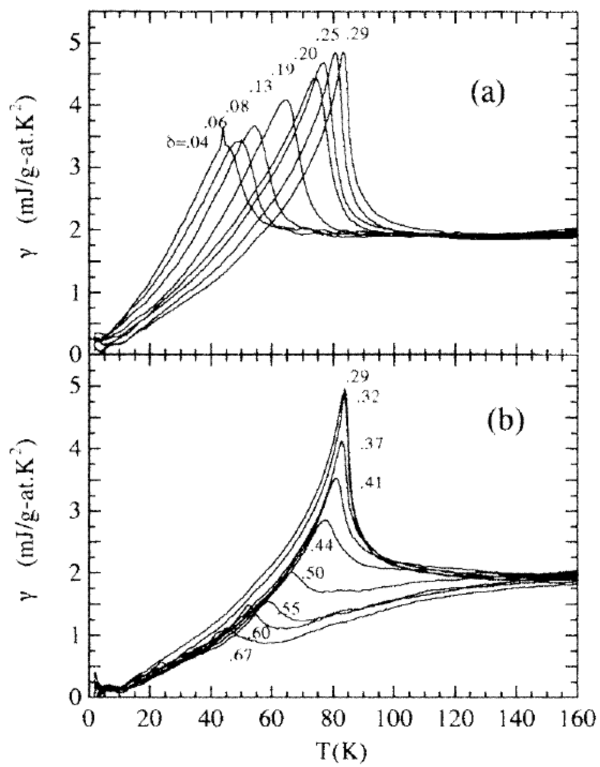
For $k_F a \ll 1$: well defined molecules, T* given by Law of Mass Action.



And T* at Unitarity?

Motivation: is there a relationship between preformed pairs and the pseudogap region in high T_c superconductors.



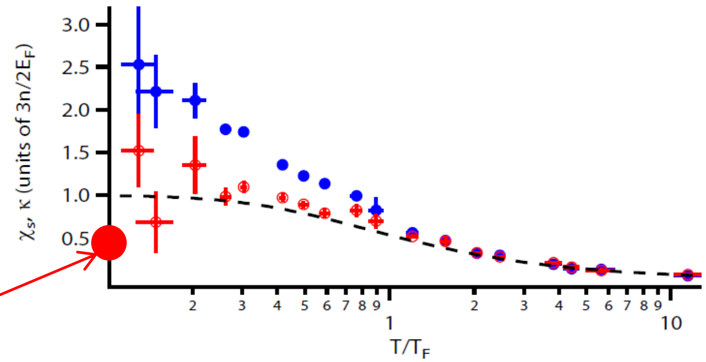


$$\gamma = C_V / T$$

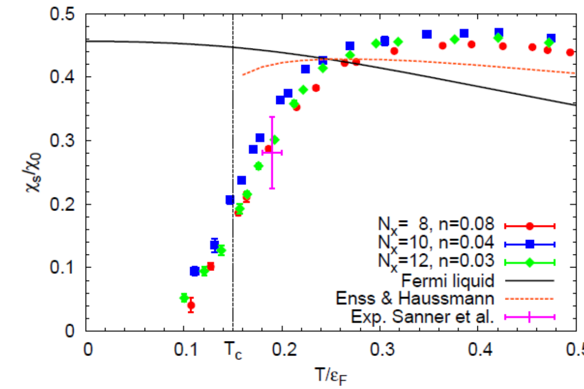
And T*?
(Are there preformed pairs above Tc?)

Spin susceptibility:

Sommer et al. Nature **472**, 201-204 (2011)

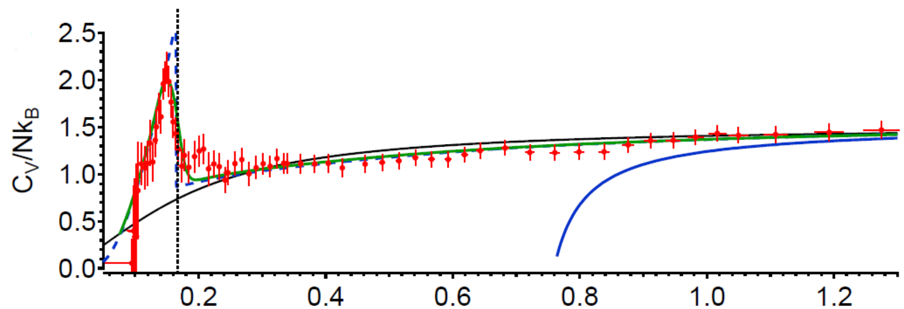


ENS, T=0 Value

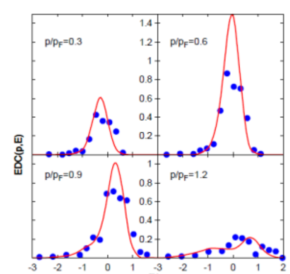


Wlazlowski et al. Phys. Rev. Lett. **110**, 090401 (2013)

Specific Heat:



Spectral function



Gaebler et al. Nat. Phys. (2010)
Magierski et al. PRL (2010)

Conclusion

•Apart from qualitative issues, outstanding quantitative theoretical and experimental understanding of the **thermodynamic properties** of strongly correlated Fermi gases (also true for the phase diagram of spin imbalanced systems).

•If there is indeed a pseudogap phenomenon, is it related to High-T_c?

• **Dynamical properties** are not so thoroughly explored (transport phenomena, excitation spectra, quench experiments – eg Higgs Mode).

• Can we go **beyond the 3D, zero range model**? Finite range effect, lower dimensional problems.

Thermodynamic Properties of the Fermi Liquid

FERMI LIQUID PARADIGM: the low temperature thermodynamics of a normal Fermi gas is dictated by the behaviour of long lived quasi-particles with renormalized parameters (*effective* mass...)

THERMODYNAMIC CONSEQUENCES OF THE FERMI LIQUID HYPOTHESIS

- Finite compressibility at T=0
- Finite magnetic susceptibility at T=0
- Specific heat ~T

GRAND CANONICAL EQUATION OF STATE

Effective mass

$$\frac{P(\mu,T)}{P_0(\mu,0)} = \xi_n^{-3/2}\left(1+\frac{5\pi^2}{8}\xi_n\frac{m^*}{m}\left(\frac{k_B T}{\mu}\right)^2+\frac{15}{8}\chi b^2+\ldots\right)$$

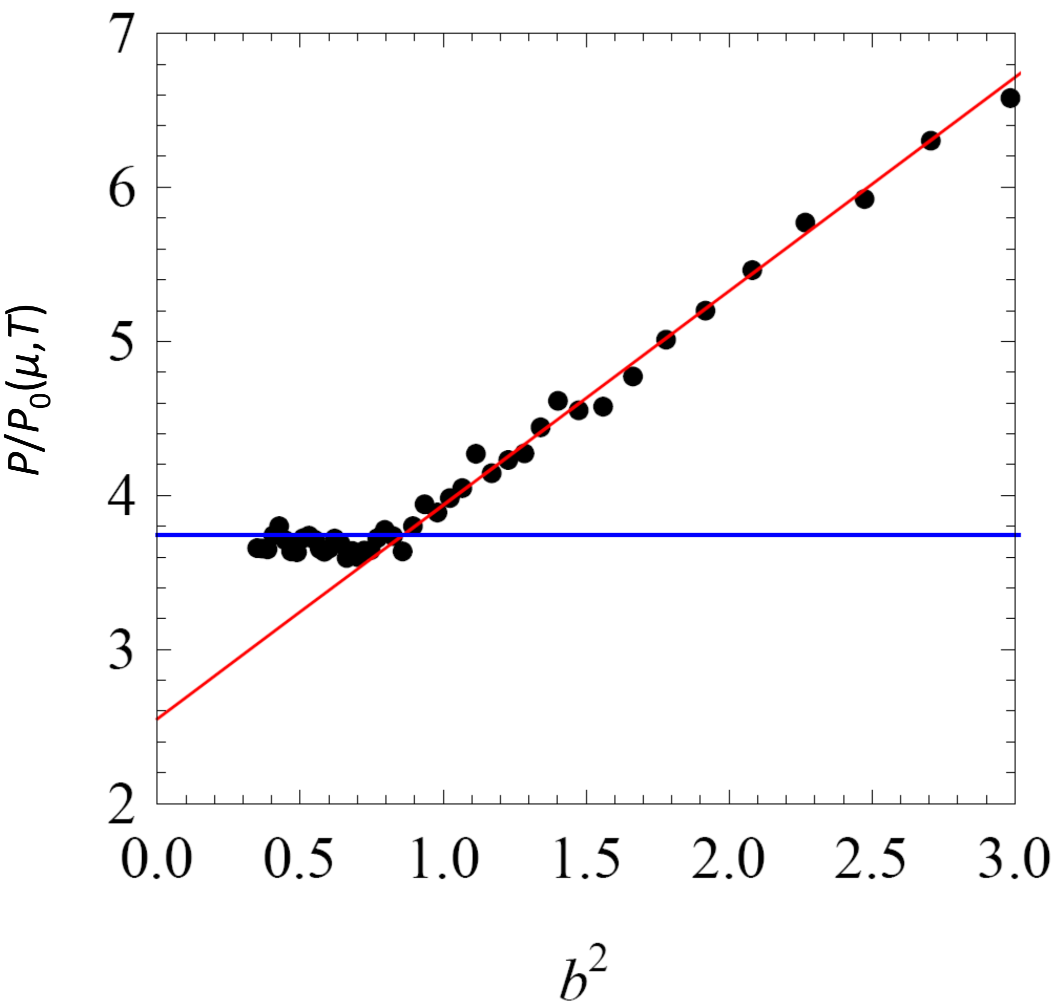
Susceptibility

Compressibility

$$\mu = \frac{\mu_\uparrow + \mu_\downarrow}{2} \qquad b = \frac{\mu_\uparrow - \mu_\downarrow}{\mu_\uparrow + \mu_\downarrow}$$ (effective magnetic field)

Spin susceptibility of the Normal Phase
(S. Nascimbène et al. PRL 106, 215303 (2011))

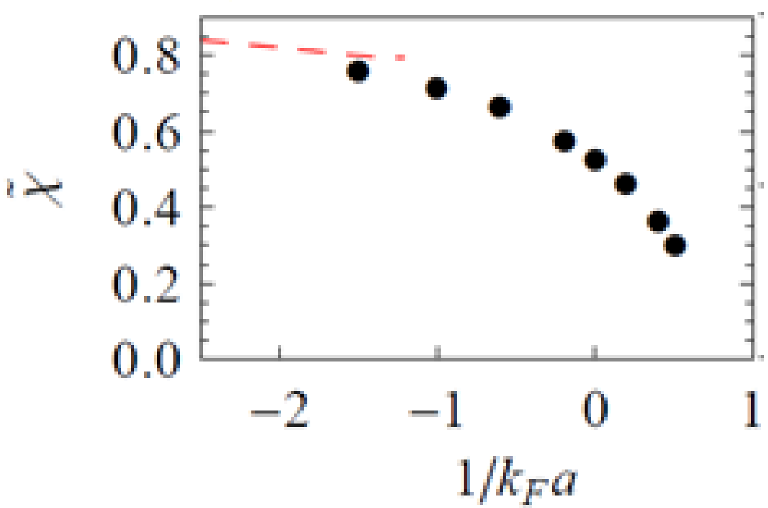
Apply a « magnetic field » by creating a spin imbalanced Fermi gas (see also Rice and MIT)



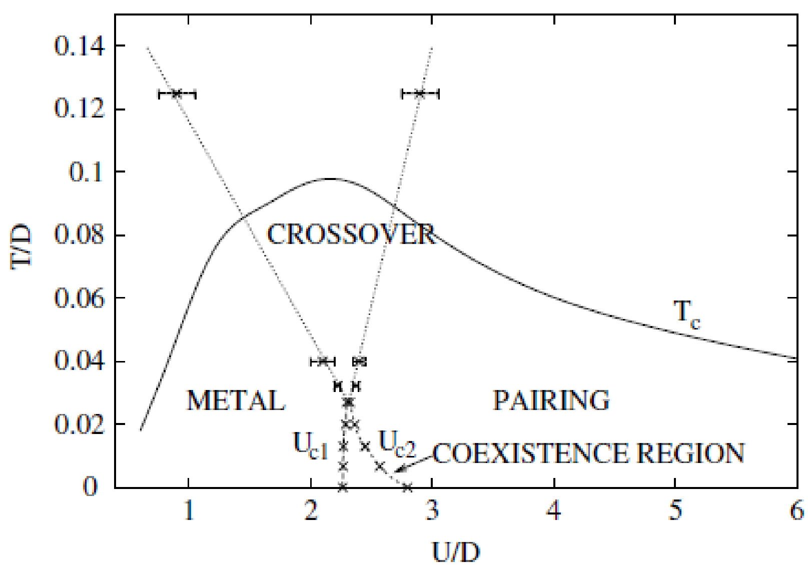
	Exp.	Theory.
m^*/m	1.13	???
ξ_n	0.51(2)	0.56
χ	0.27(1)	0.27

* Fixed Node Monte Carlo (S. Girogini, Trento)

Is there an underlying quantum phase transition in the normal phase?



Fixed Node Monte-Carlo



Fermi-Hubbard Model