Lattice QCD and heavy ion collisions

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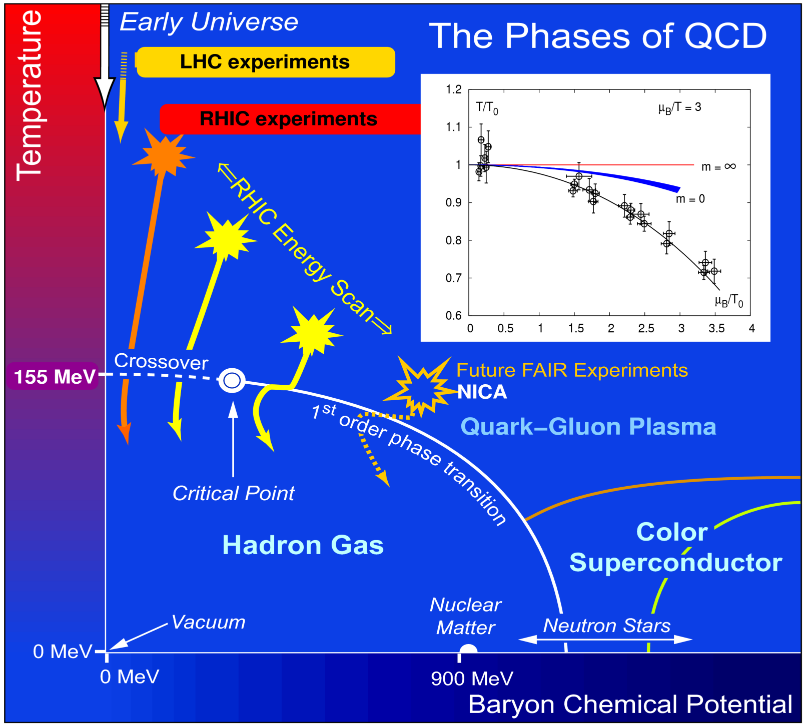


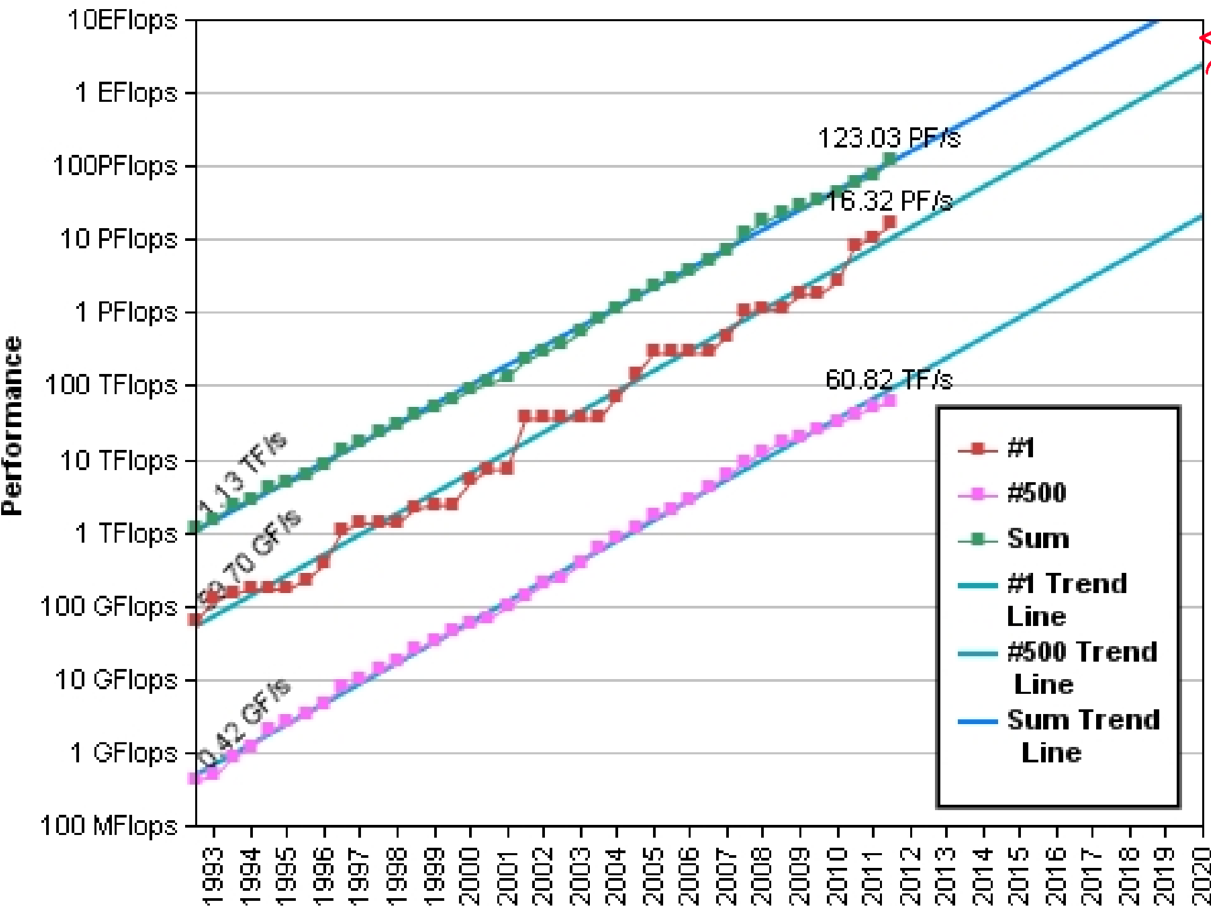
OUTLINE

- The strongly interacting medium
- The crossover transition close to $\mu_B = 0$
- Crossover transition and freeze-out
- The critical point

Future of Computing Hardware (for Lattice QCD)

Computing speed doubles every 1.5 years

⇒ Lattice QCD calculations can reduce cut-off effects by a factor 2-4
or reduce statistical errors by a factor 4 every (5-6) years (action/algo. dep.)



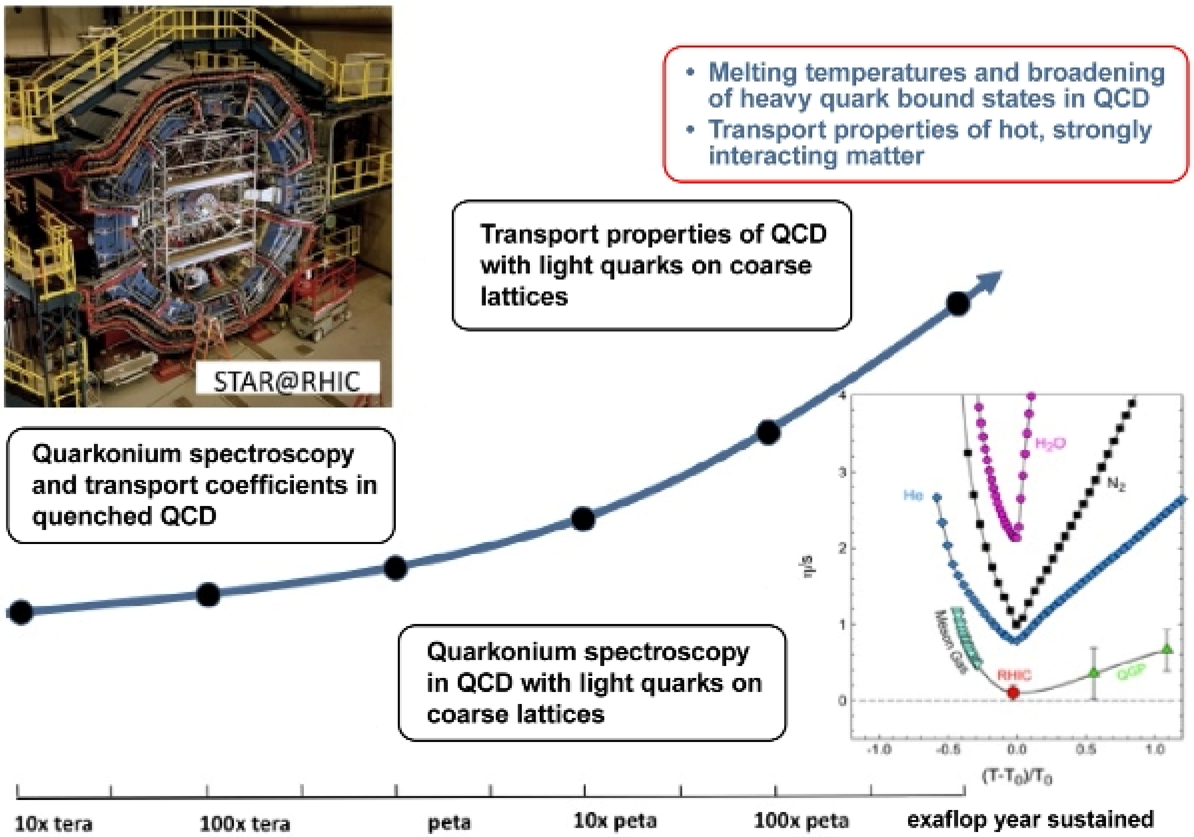
$\lesssim 2020$: 1 Exaflops

peak: 20 Petaflops

BlueGene/Q
Sequoia@LLNL

now also used for QCD thermodynamics with Domain Wall Fermions

The strongly interacting medium: QCD thermodynamics close to T_c



state of the art:
light and heavy quark spectroscopy in quenched QCD (Tokyo-Osaka, Bielefeld-BNL)

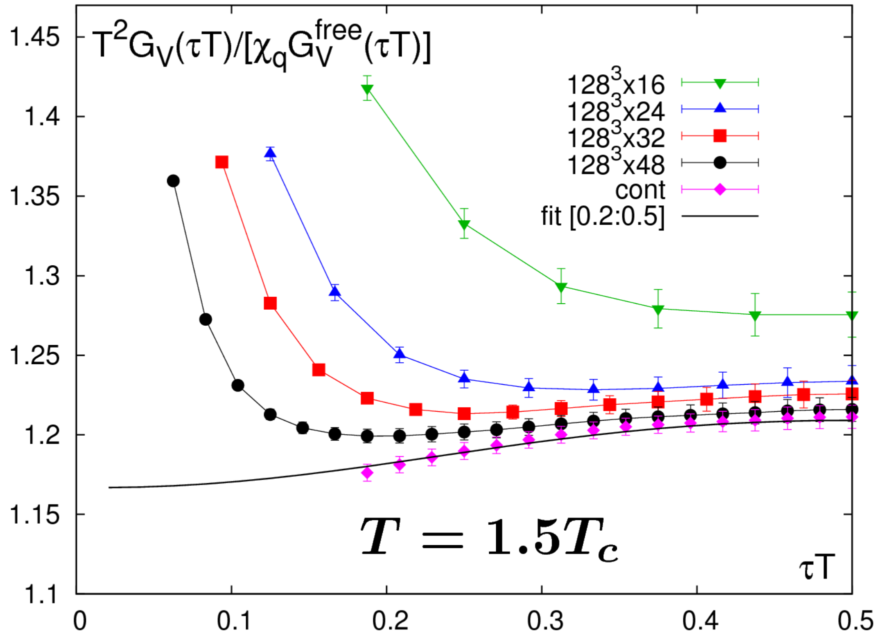
exploratory study of transport properties and first attempts to include effects of dynamical quarks (Dublin-Swansea, Mainz)

goal I: spectral analysis of heavy quark correlation functions ⇨ T-dependence of excited charmonium and bottomonium states

goal II: thermal properties of light quark correlation functions ⇨ dilepton/photon rates at non-vanishing momenta

In-medium properties of hadrons

thermal hadron correlation functions



H.-T. Ding et al, PRD 83 (2011) 034504

ill-posed problem:
need Maximum Entropy Method
(or well-motivated fit ansatz)
to reconstruct spectral functions;
need large lattices to gain enough
information on $G_H(\tau,\vec{r})$

spectral representation of thermal dilepton rates:

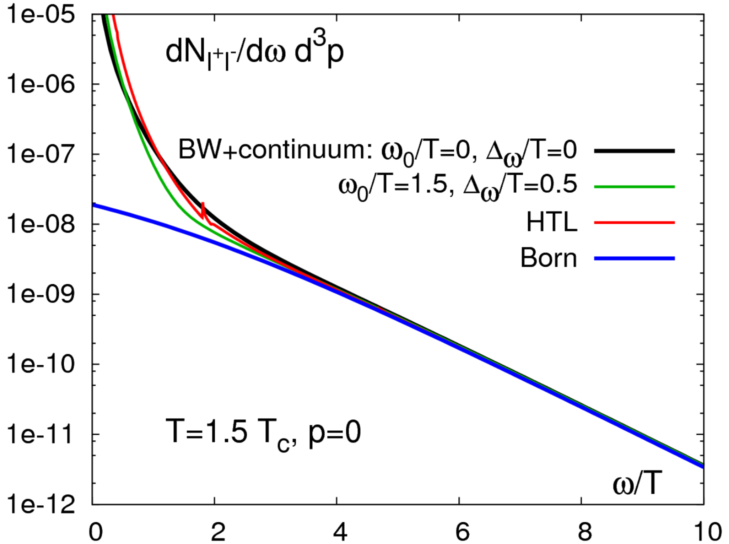
$$\frac{d^4W}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2}\frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(e^{\omega/T}-1)}$$

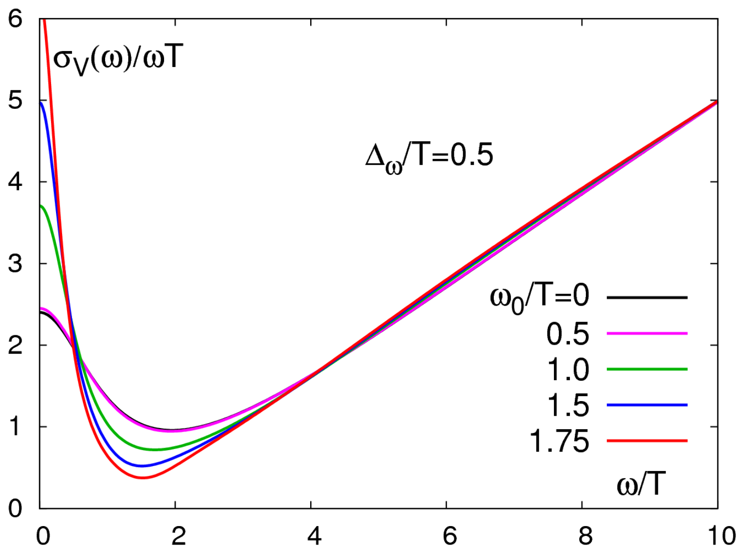
spectral representation of Euclidean correlation functions:

$$G_H^\beta(\tau,\vec{r}) = \int_0^\infty d\omega \int \frac{d^3\vec{p}}{(2\pi)^3}\,\sigma_H(\omega,\vec{p},T)\, e^{i\vec{p}\vec{r}}\,\frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

In-medium properties of hadrons

dilepton rate vs. invariant mass





spectral representation of thermal dilepton rates:

$$\frac{d^4W}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2}\frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(e^{\omega/T}-1)}$$

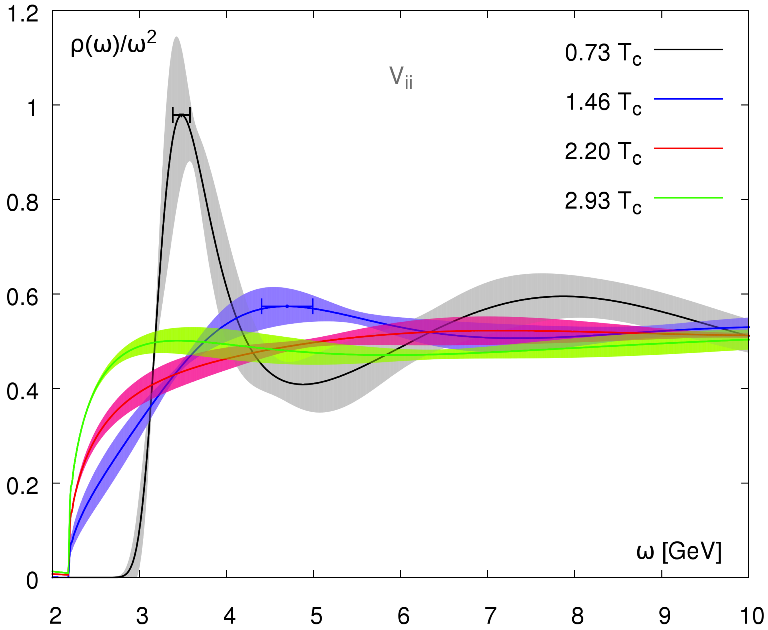
– large thermal lattices $128^3 \times 48$ but no dynamical quarks; low mass enhancement similar to HTL-calculations

– get electrical conductivity from $\sigma_V(0,\vec{0},T)$

$$\frac{1}{3}\sum_f Q_f^2 \le \frac{\sigma}{T} \le \sum_f Q_f^2$$

need to extend calculations to non-zero momenta and need to control influence of light dynamical quarks

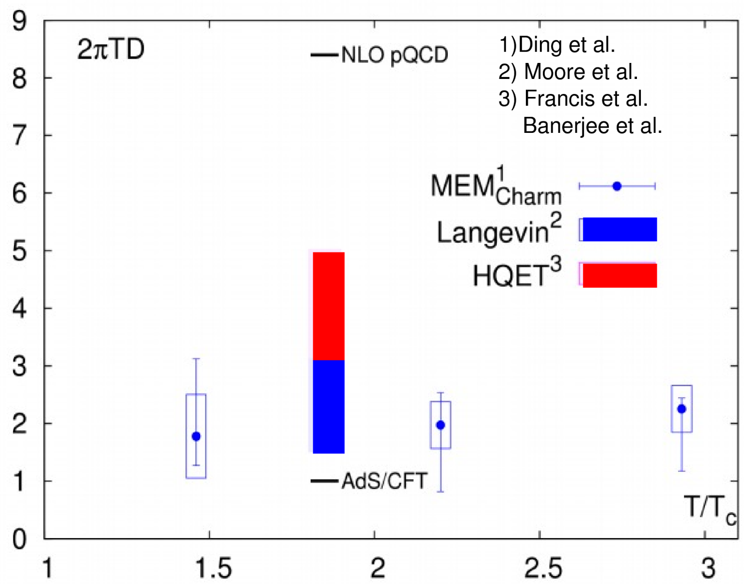
In-medium properties of hadrons



heavy quark spectral functions

– charmonium states are dissolved at $T = 1.5T_c$ (quenched, fine lattices) (H.-T. Ding et al, PRD 86(2012) 014509)

– bottomonium states survive even at $T = 2T_c$ (unquenched, coarse lattice) (G. Aarts et al.,)



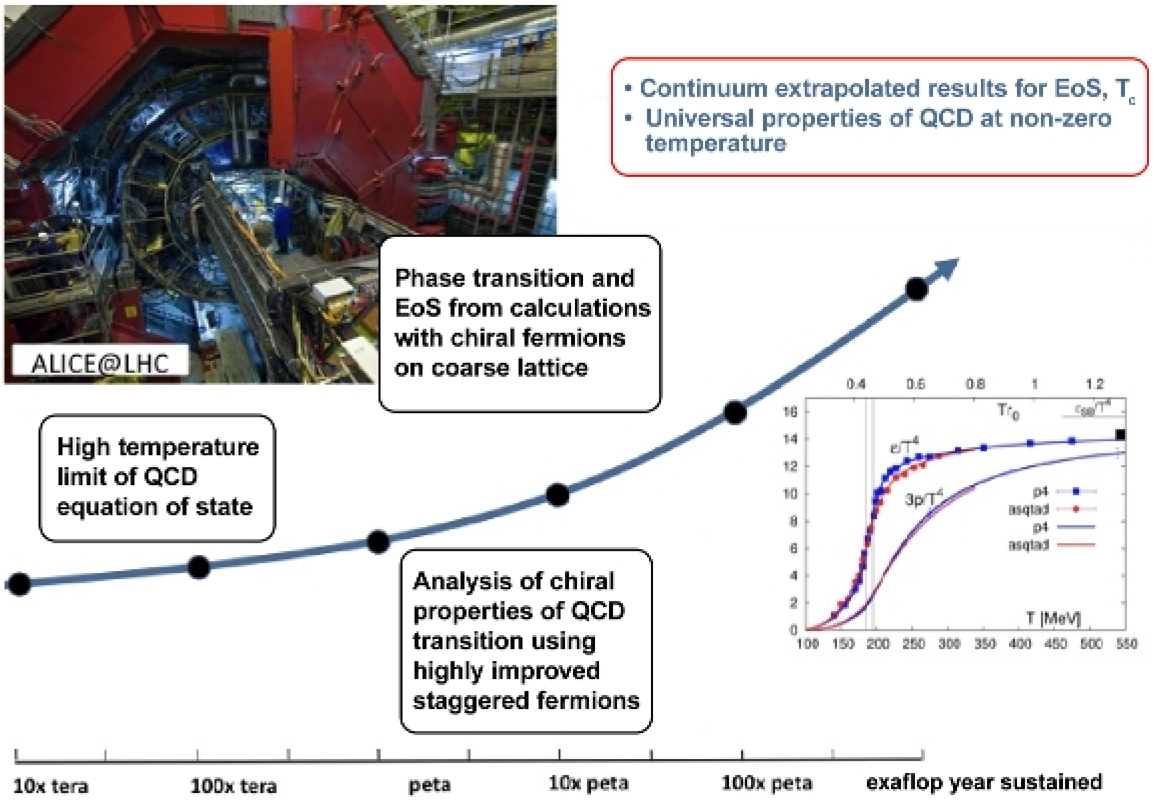
heavy quark diffusion constant

$$D_{charm} \simeq 1/\pi T$$

(H.-T. Ding et al, arXiv:1210.0292)

need to analyze temperatures close to T_c
dynamical quark mass effects will start to become important

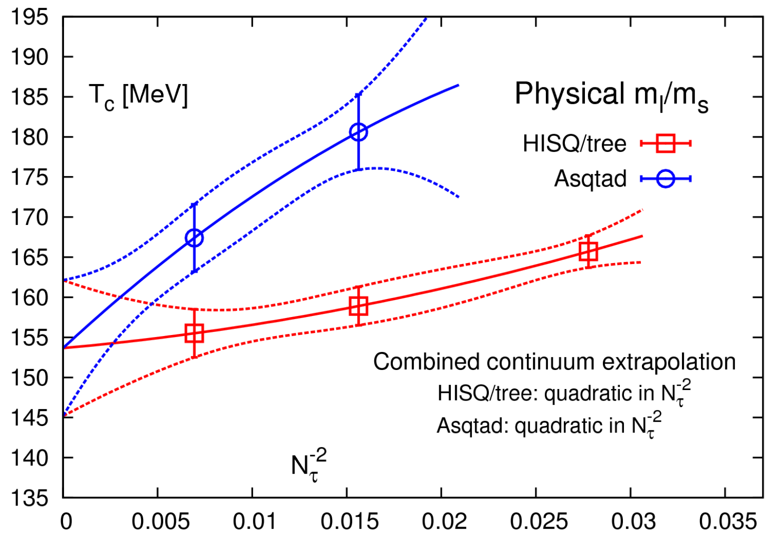
The high energy frontier: Vanishing baryon-chemical potential



explore the crossover transition at vanishing chemical potential and its proximity to the true chiral phase transition through the analysis of freeze-out conditions at LHC: higher order cumulants of charge fluctuations similarly important at LHC as in the Beam Energy Scan at RHIC

Crossover temperature at and close to $\mu_B = 0$

The transition temperature at vanishing chemical potential:



crossover identified by peak in the chiral susceptibility:

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

A. Bazavov et al (HotQCD Collaboration), Phys. Rev. D 85, 054503 (2012)

consistent with transition temperatures determined by the Budapest-Wuppertal collab.

Y. Aoki et al., JHEP 0906 (2009) 088

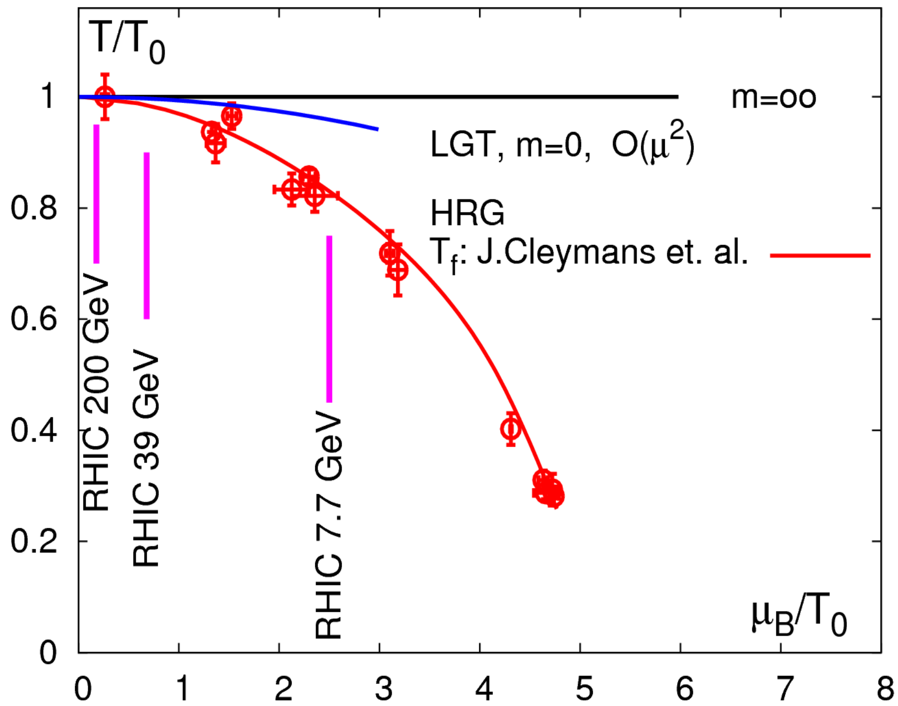
Curvature of the transition line for small μ_B :

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

Bielefeld-BNL, Phys. Rev. D 83, 014504 (2011)

similar: G. Endrodi et al., JHEP 1104, 001 (2011)

Chiral Transition and Freeze-out



(Crossover) transition from lattice QCD:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

freeze-out curve in HIC:
(using HRG as input)

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

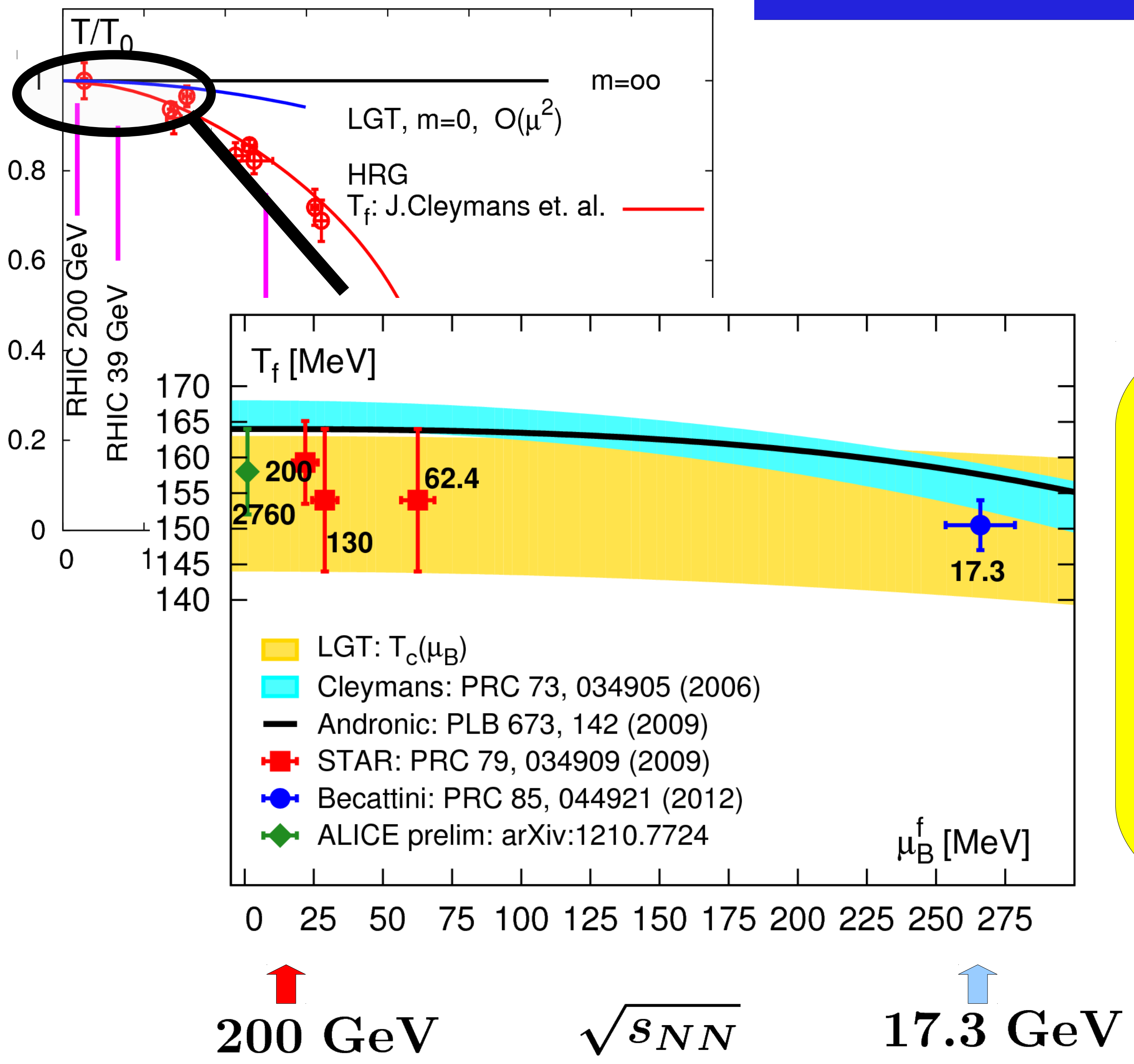
$$\frac{T(\mu_B)}{T_c} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

J. Cleymans et al.,
Phys.Rev. C73, 034905 (2006)

$$T = \frac{164\text{MeV}}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}(\text{GeV})})/0.45)}$$

A. Andronic et al.,
Phys. Lett. B673, 142 (2009)

Chiral Transition and Freeze-out



phenomenological freeze-out curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 2$$

QCD, HRG and Freeze-out in HIC

- in a wide range of beam energies covered by the beam energy scan at RHIC chemical freeze-out seems to happen close to the crossover temperature

Caveat / conceptual problem:

- in the transition region the HRG may not (does not?) provide a good description of the thermodynamics of strongly interacting matter

A. Andronic et al., arXiv:1201.0693

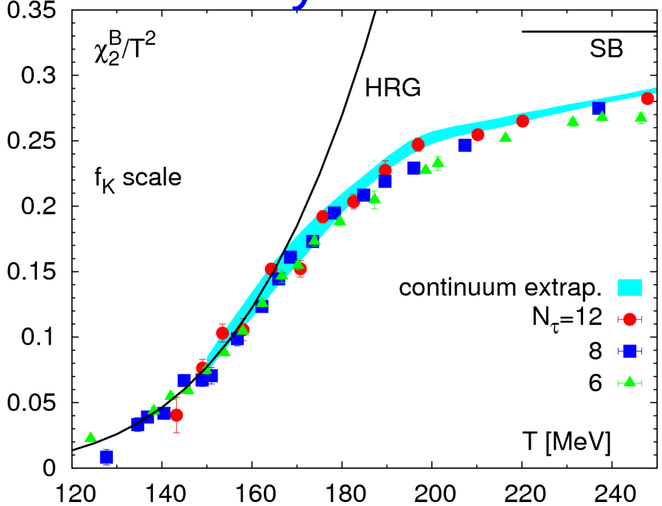
→ The non-interacting HRG works astonishingly well up to

$$T \simeq (160 - 165) MeV$$

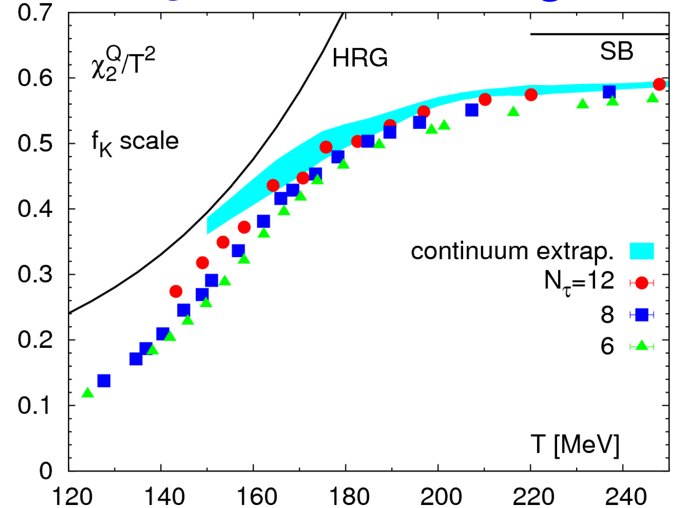
Quadratic charge fluctuations: $\mu_B = 0$

continuum extrapolated results: A. Bazavov et al (hotQCD), arXiv:1203.0784

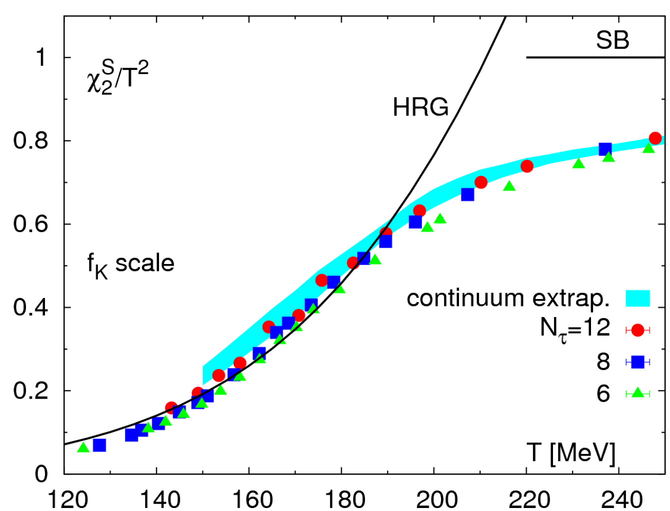
B: baryon number



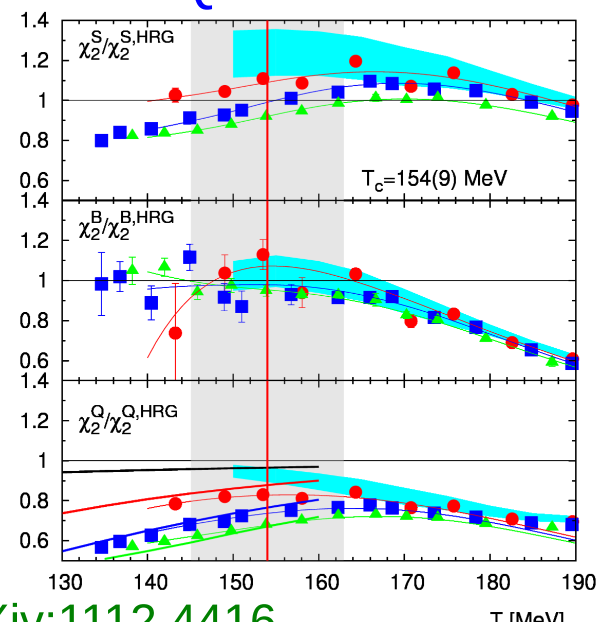
Q: electric charge



S: strangeness



BQS vs. HRG



HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

statistics:
~3000 conf./T
~1500 source vec.

(2+1)-flavor QCD physical strange quark sector;

$m_l/m_s = 1/20$

$\Downarrow$

$m_{ps} \simeq 160$ MeV

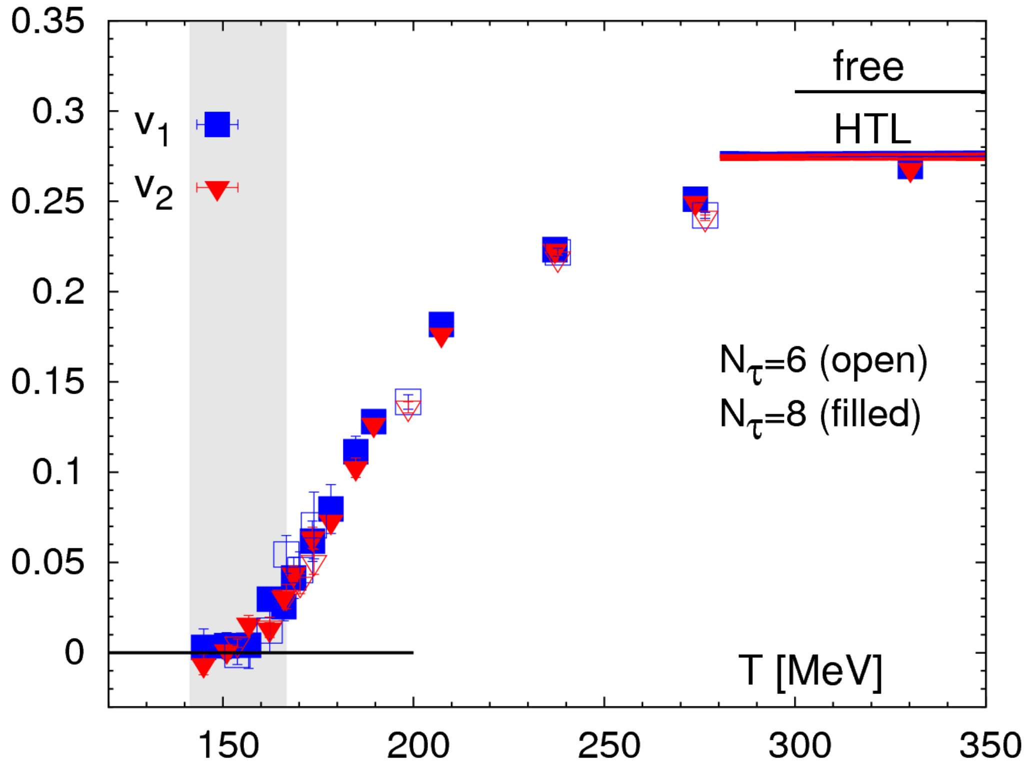
consistent with Wuppertal-Budapest, arXiv:1112.4416

Strangeness-Baryon correlation: Koch-BS at 4th order

$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

} $v_1 = v_2 = 0$ in a HRG



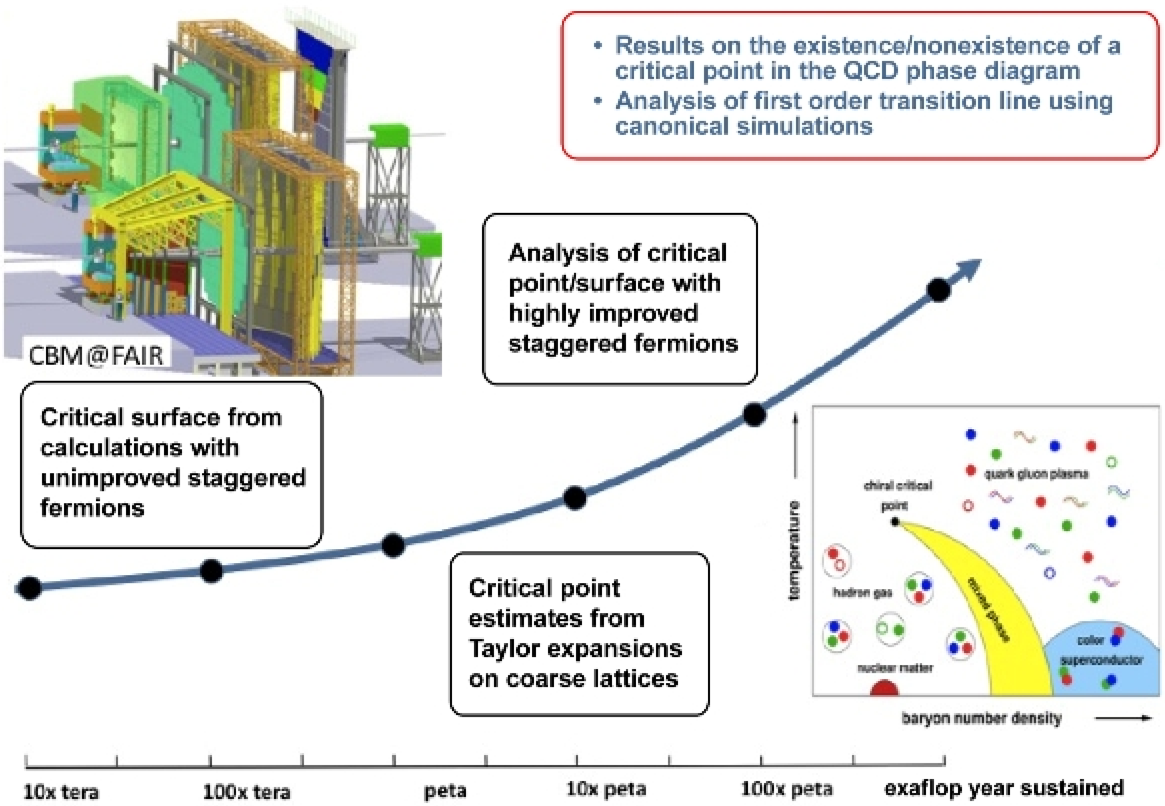
probing net strangeness – net baryon number correlations with 4th order cumulants

uncorrelated HRG only for T<165MeV

(For T< 165MeV HRG is a good approximation even for rather complex observables)

The high density frontier: Non-vanishing baryon-chemical potential

RHIC-BES
FAIR, NICA

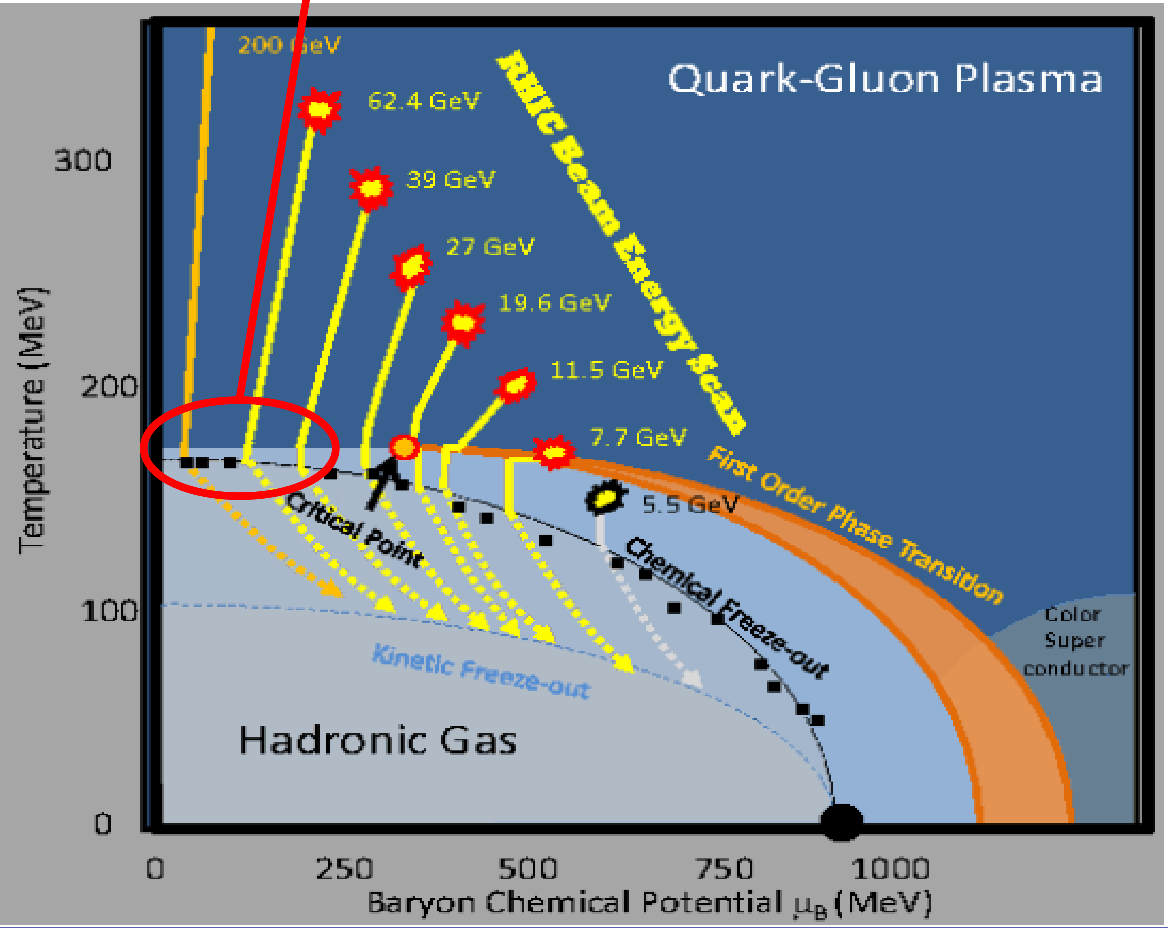


state-of-the-art:
all calculations referring to the existence or non-existence of a critical point are done on coarse lattices with rather simple, non-improved lattice discretisation schemes

all estimates of the location of a critical point are based on calculations that suffer from large cut-off effects (aside from other problems)

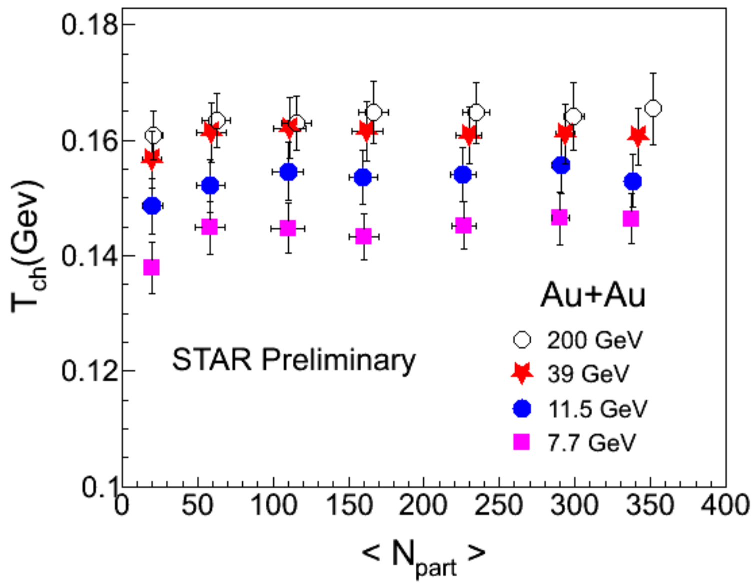
Phase diagram and freeze-out in Heavy Ion Collisions

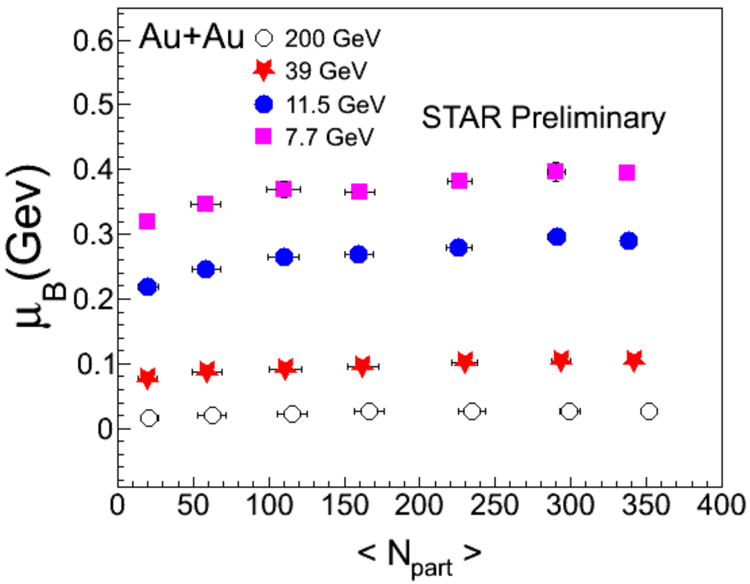
rather than hunting for the critical point right away, let's first establish basic features of cumulants of conserved charges at moderate values of the baryon chemical potential



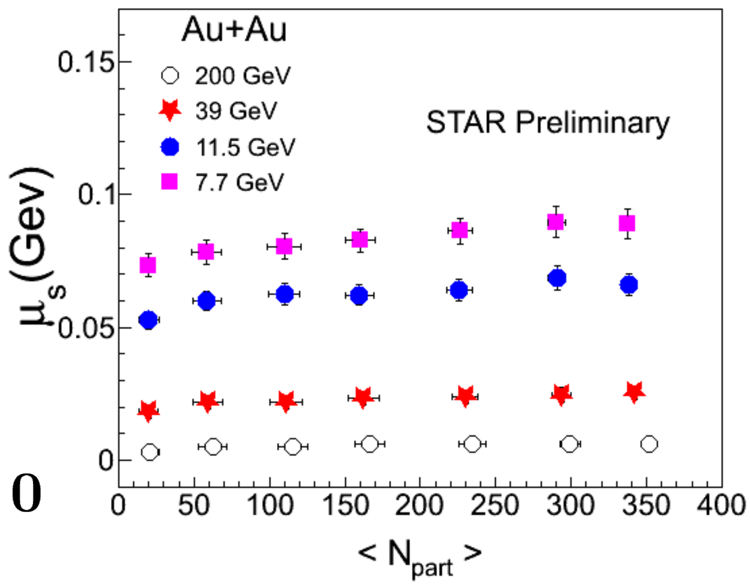
- can cumulants be described by equilibrium thermodynamics with a unique set of temperature (T) and chemical potentials μ_B, μ_S, μ_Q
- do cumulants characterize thermodynamics on the chemical freeze-out curve?
- are thermal (freeze-out) parameter obtained by comparing experimental data with HRG model calculations reproduced by QCD?

Chemical Freeze-out parameter





Sabita Das (STAR Collaboration),
Quark Matter 2012



$$\frac{\mu_S}{\mu_B} \simeq (0.2 - 0.25)$$

only weakly dependent on μ_B

reflects strangeness neutrality: $\langle N_S \rangle = 0$

Cumulants of Net Charge Fluctuations

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$
$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

susceptibilities: $$\chi_{ijk,\mu}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k} \right|_{\mu_B,\mu_Q,\mu_S}$$

moments of
net-charge fluctuation

$$\mathcal{A}_n \sim \langle (\delta N_X)^n \rangle$$

cumulants: $$\chi_2^X \equiv \left. \frac{\partial^2(p/T^4)}{\partial(\mu_X/T)^2} \right|_{\mu=0} = \frac{N_\tau}{N_\sigma^3} \mathcal{A}_2$$

X=B,Q, S

$$\chi_4^X \equiv \left. \frac{\partial^4(p/T^4)}{\partial(\mu_X/T)^4} \right|_{\mu=0} = \frac{1}{N_\sigma^3 N_\tau} (\mathcal{A}_4 - 3\mathcal{A}_2^2)$$

$$\chi_6^X \equiv \left. \frac{\partial^6(p/T^4)}{\partial(\mu_X/T)^6} \right|_{\mu=0} = \frac{1}{N_\sigma^3 N_\tau^3} (\mathcal{A}_6 - 15\mathcal{A}_4\mathcal{A}_2 + 30\mathcal{A}_2^3)$$

Taylor expansions of baryon number susceptibilities

$$\chi_{n,\mu}^B = \sum_{k=0}^{\infty} \frac{1}{k!} \chi_{k+n,0}^B(T) \left(\frac{\mu_B}{T}\right)^k$$

for simplicity:
$\mu_s = \mu_Q = 0$

mean: $M_B = VT^3 \chi_{1,\mu}^B = VT^3 \left(\frac{\mu_B}{T} \chi_2^B + \dots \right)$

variance: $\sigma_B^2 = VT^3 \chi_{2,\mu}^B$

$$= VT^3 \left(\chi_2^B + \frac{1}{2} \left(\frac{\mu_B}{T}\right)^2 \chi_4^B + \dots \right)$$

notation:
$\chi_n^B \equiv \chi_{n,0}^B$

skewness and kurtosis and volume independent ratios of susceptibilities

$$S_B \equiv \frac{\langle (\delta N_B)^3 \rangle}{\sigma_B^3} \;, \quad \kappa_B \equiv \frac{\langle (\delta N_B)^4 \rangle}{\sigma_B^4} - 3$$

$$\frac{\sigma_B^2}{M_B} = \frac{\chi_{2,\mu}^B}{\chi_{1,\mu}^B}, \qquad S_B \sigma_B = \frac{\chi_{3,\mu}^B}{\chi_{2,\mu}^B}, \qquad \kappa_B \sigma_B^2 = \frac{\chi_{4,\mu}^B}{\chi_{2,\mu}^B}$$

Thermal Conditions in Heavy Ion Collisions

expand constraints in Taylor series: $\langle N_S \rangle = 0 \ , \ \langle N_Q \rangle / \langle N_B \rangle = r$

$$\langle N_S \rangle = \left(\chi_{11}^{BS} + q_1 \chi_{11}^{QS} + s_1 \chi_2^S \right) \hat{\mu}_B + \left(m_{S,3} + q_3 \chi_{11}^{QS} + s_3 \chi_2^S \right) \hat{\mu}_B^3$$

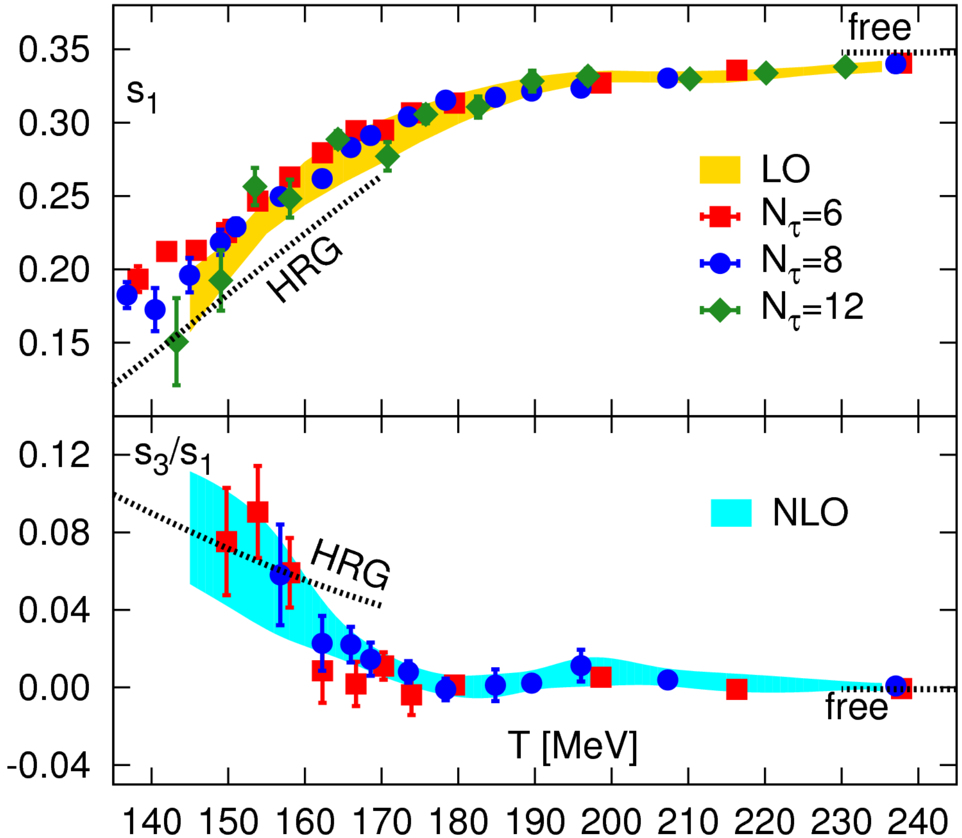
$$\begin{aligned} m_{S,3} \quad = \quad & \frac{1}{6}\chi_{31}^{BS} + \frac{1}{2}\chi_{211}^{BQS} q_1 + \frac{1}{2}\chi_{121}^{BQS} q_1^2 + \frac{1}{6}\chi_{31}^{QS} q_1^3 + \\ & \frac{1}{2}\chi_{22}^{BS} s_1 + \chi_{112}^{BQS} q_1 s_1 + \frac{1}{2}\chi_{22}^{QS} q_1^2 s_1 + \frac{1}{2}\chi_{13}^{BS} s_1^2 + \\ & \frac{1}{2}\chi_{13}^{QS} q_1 s_1^2 + \frac{1}{6}\chi_4^S s_1^3 \end{aligned}$$

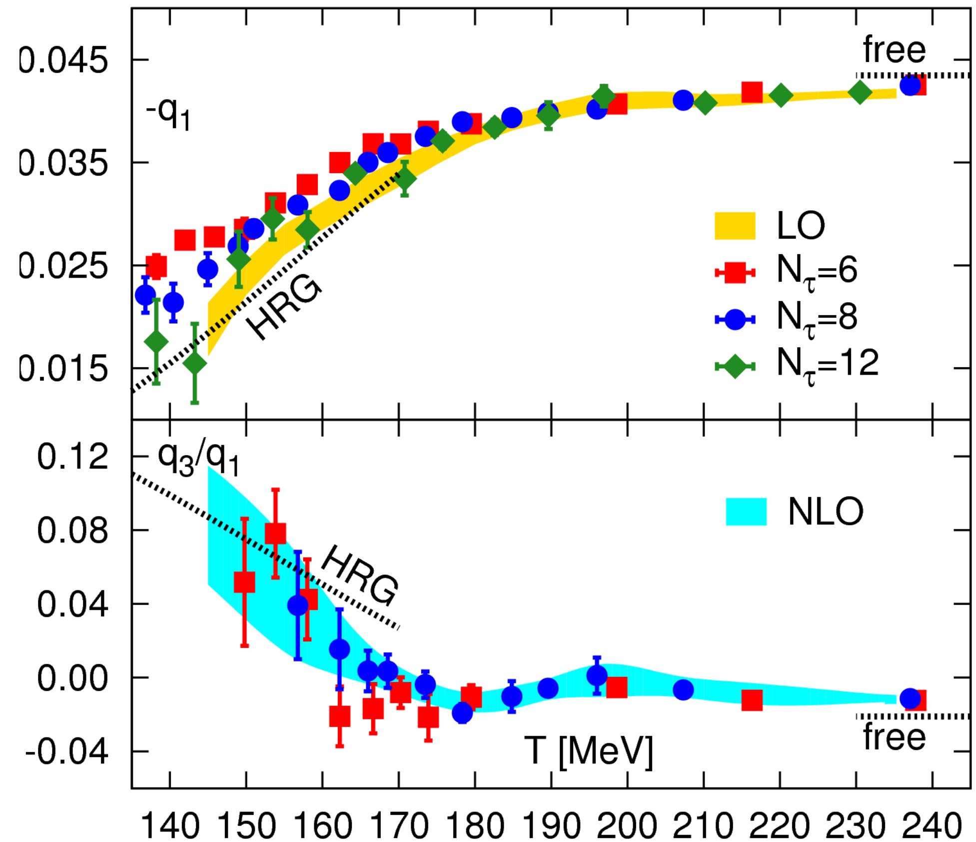
$$\hat{\mu}_B \equiv \mu_B / T$$

similar for: $\langle N_B \rangle \ , \ \langle N_Q \rangle$

solve for: $s_1 , \ q_1 \ , \ s_3 , \ q_3$ } $\mu_S(\mu_B, T), \ \mu_Q(\mu_B, T)$

Strangeness and Electric Charge Chemical Potentials





HRG at LO and NLO, respectively

Bielefeld-BNL, PRL 109 (2012) 192302

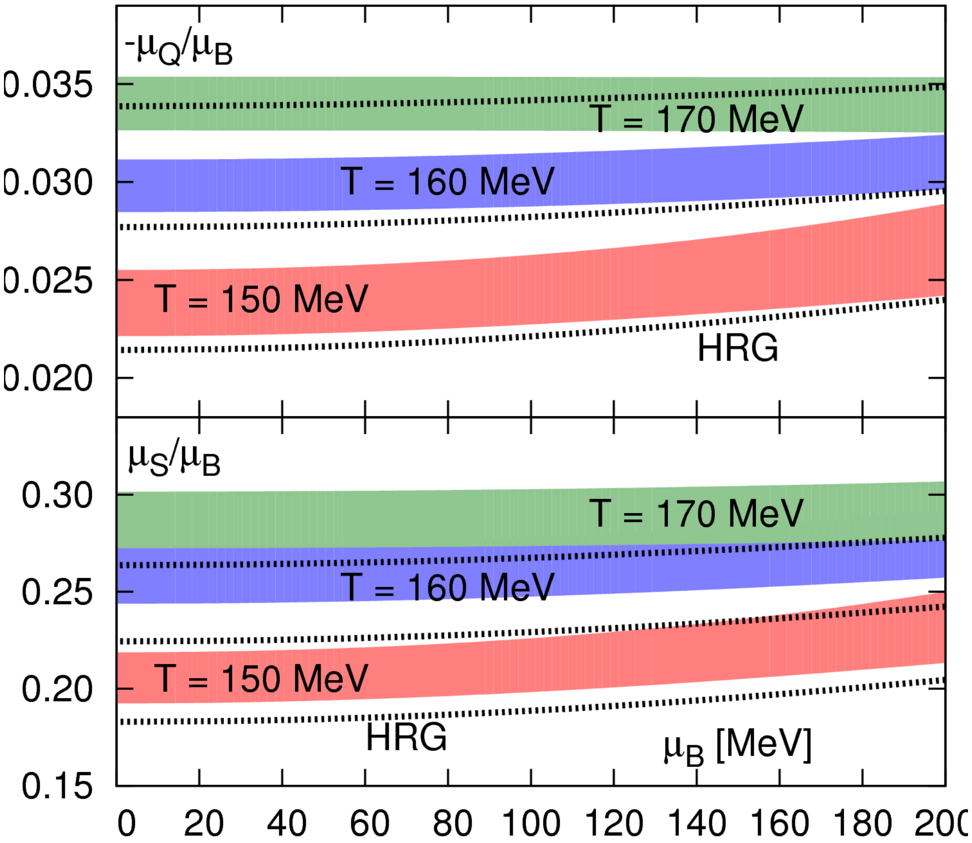
LO: continuum extrapolation

NLO: $N_\tau = 8$ interpolation

NLO correction is below 10% for all T > 140 MeV and $\mu_B/T \leq 1$

Next to Leading Order (NLO) results at fixed T

for 150MeV < T < 170MeV QCD and HRG agree within ~10% on μ_S/μ_B , μ_Q/μ_B



NLO Taylor expansions for electric charge and strangeness chemical potentials are well behaved for

$$\mu_B/T \lesssim 1.3$$

tempting to compare with STAR result (QM'12),

$\frac{\mu_S}{\mu_B} \simeq (0.2 - 0.25)$ However,..

for orientation: $\mu_B = 1.3T \Leftrightarrow$

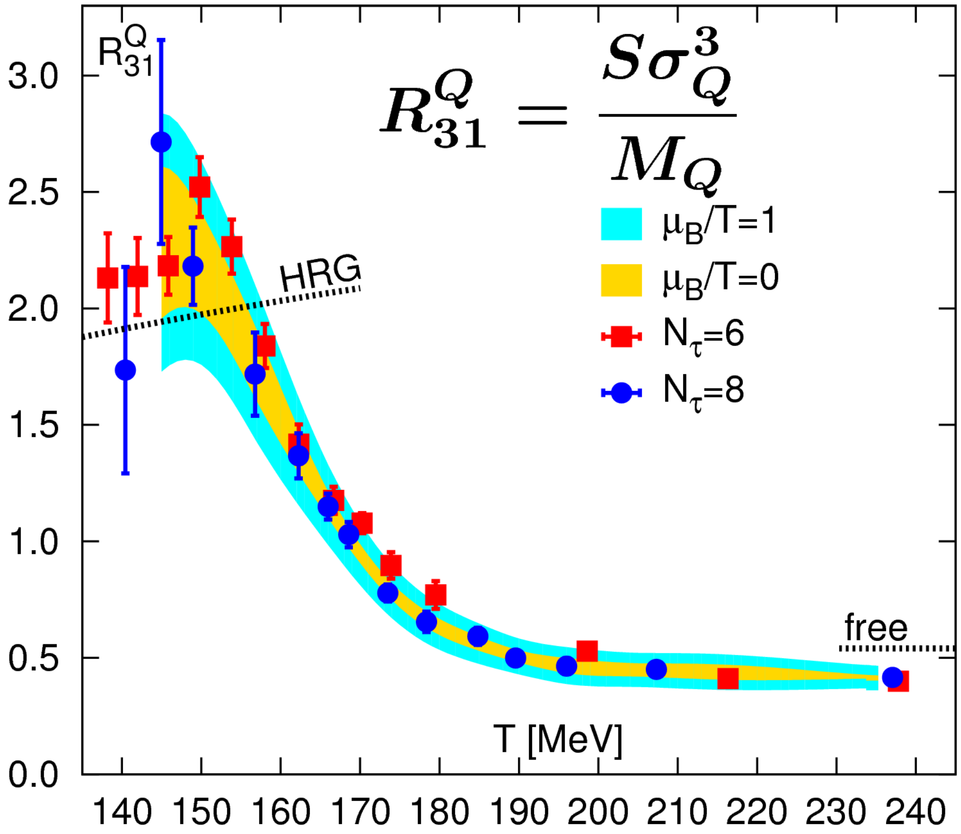
$\mu_B = 200\ \mathrm{MeV\ at}\ T = 160\ \mathrm{MeV}$

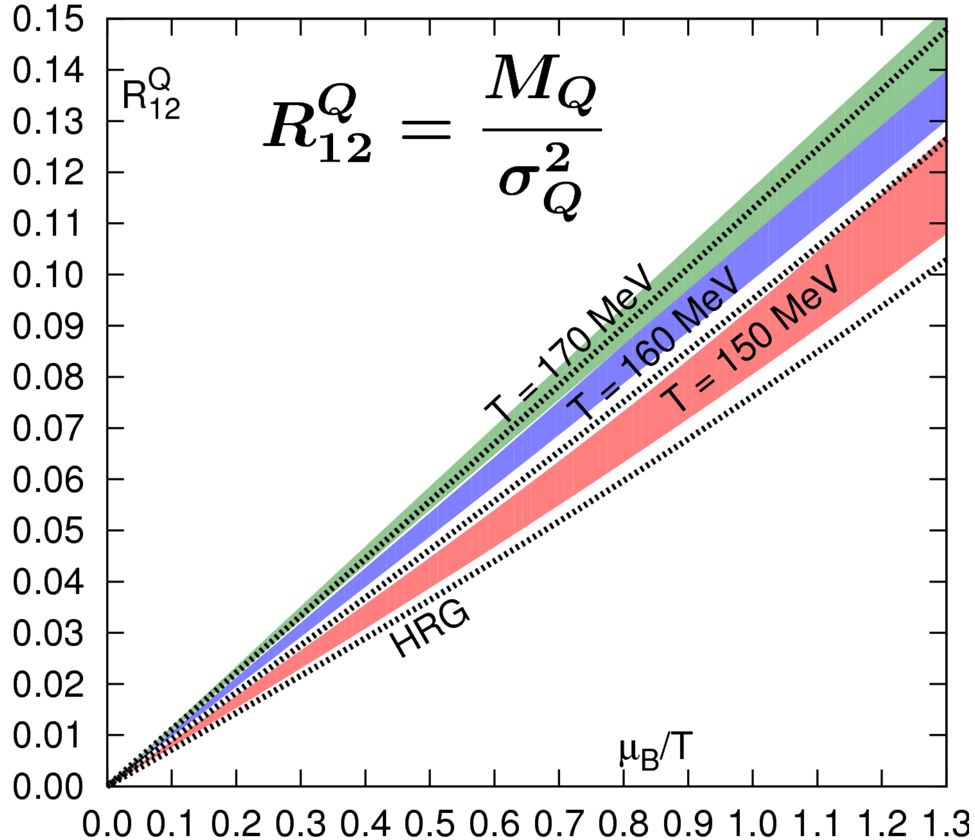
this covers RHIC experiments down to

$$\sqrt{s_{NN}} \simeq 20\ \mathrm{GeV}$$

Bielefeld-BNL, PRL 109 (2012) 192302

Determination of T and μ_B





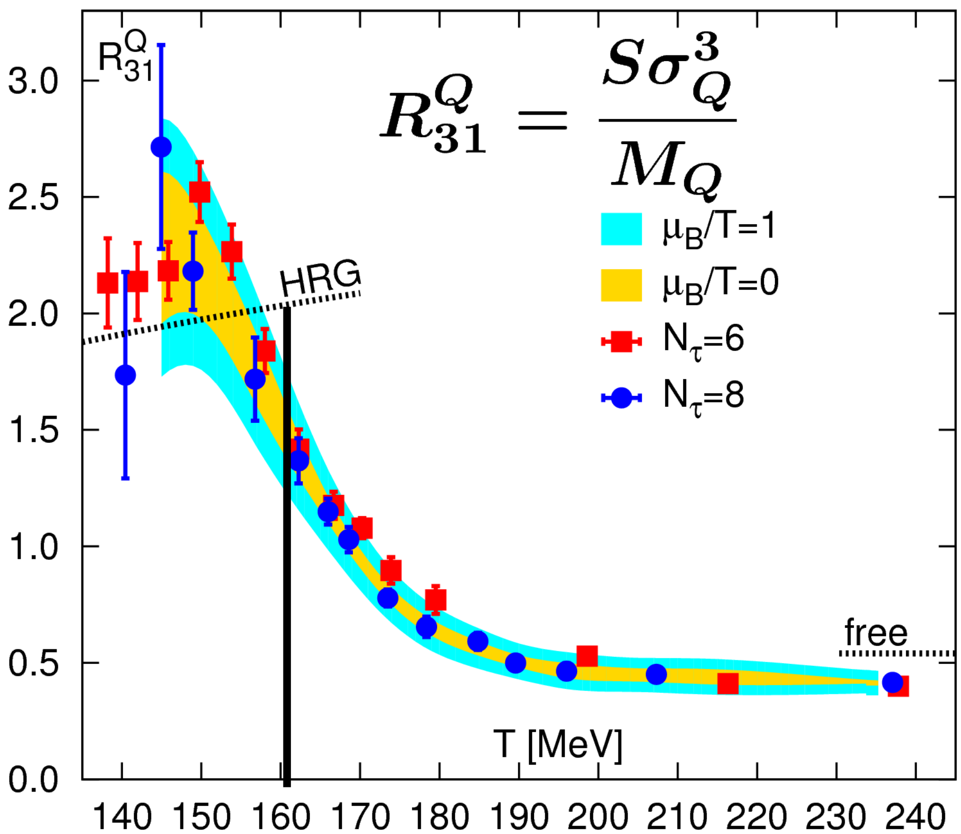
Bielefeld-BNL, PRL 109 (2012) 192302

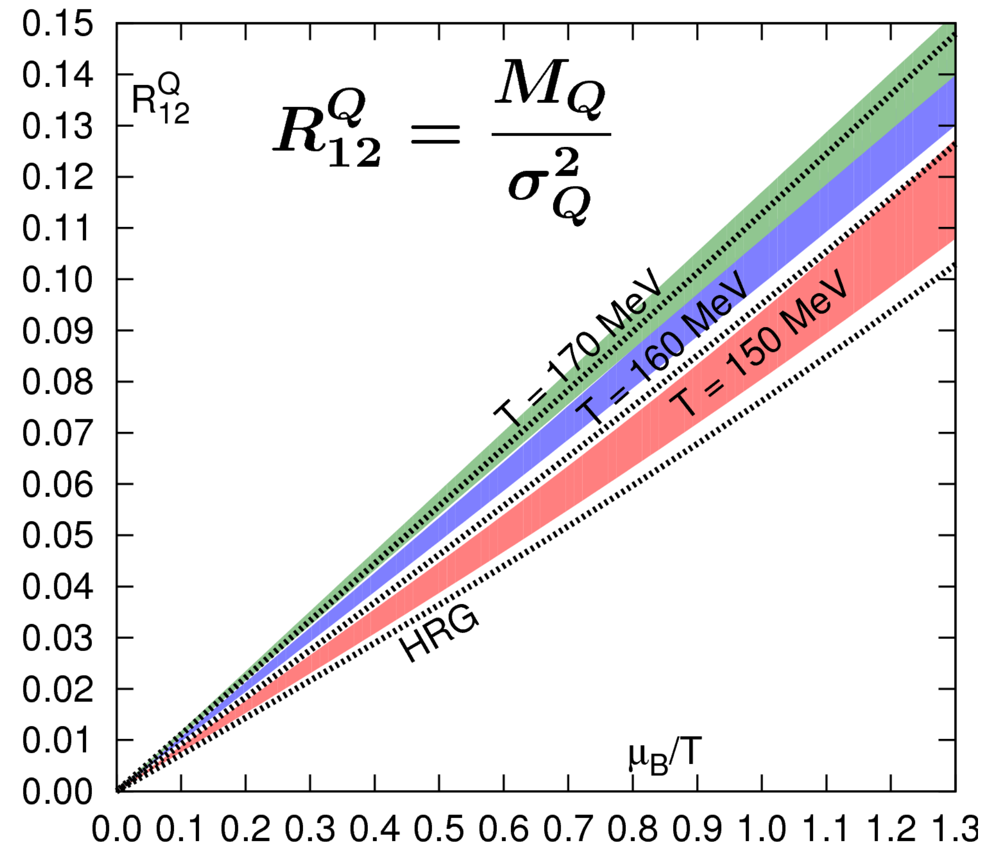
need data for these two observables to determine

$$T,\ \mu_B$$

from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

Determination of T and μ_B





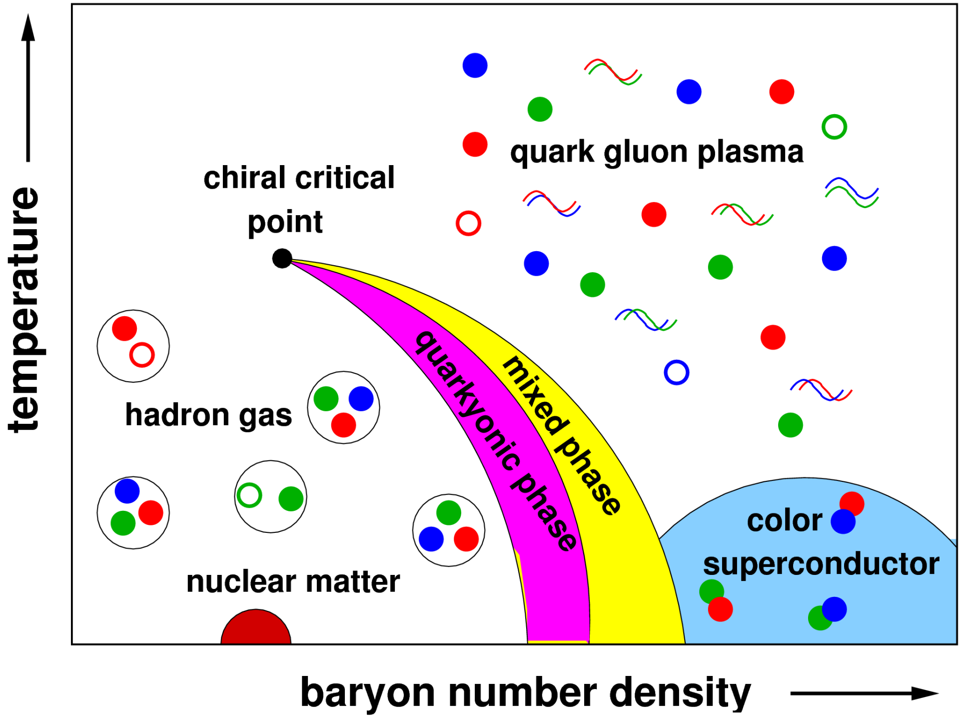
Bielefeld-BNL, PRL 109 (2012) 192302

expect significant deviations for R^Q_{31} from HRG, if

$$T_{freeze} \geq 160\ \mathrm{MeV}$$

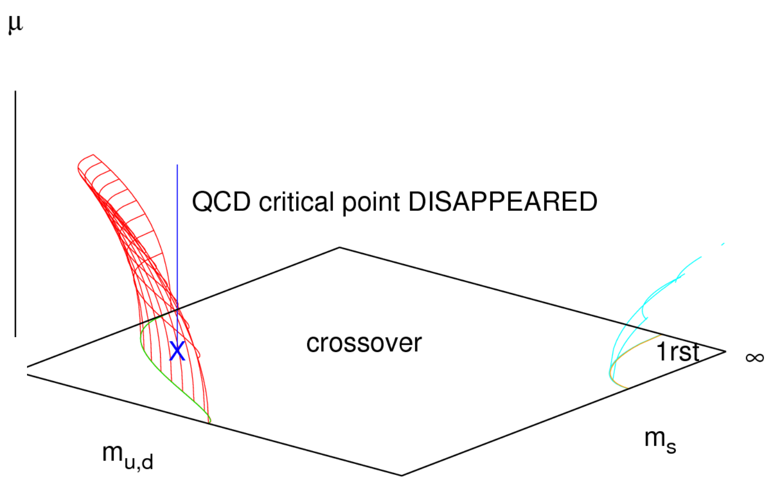
from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

A critical point ???



Is there a critical point
in the QCD phase diagram?

no unique answer so far

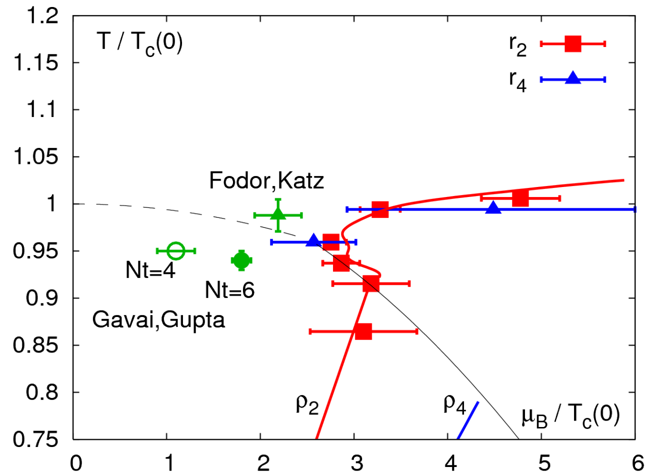


deForcrand, Philipsen

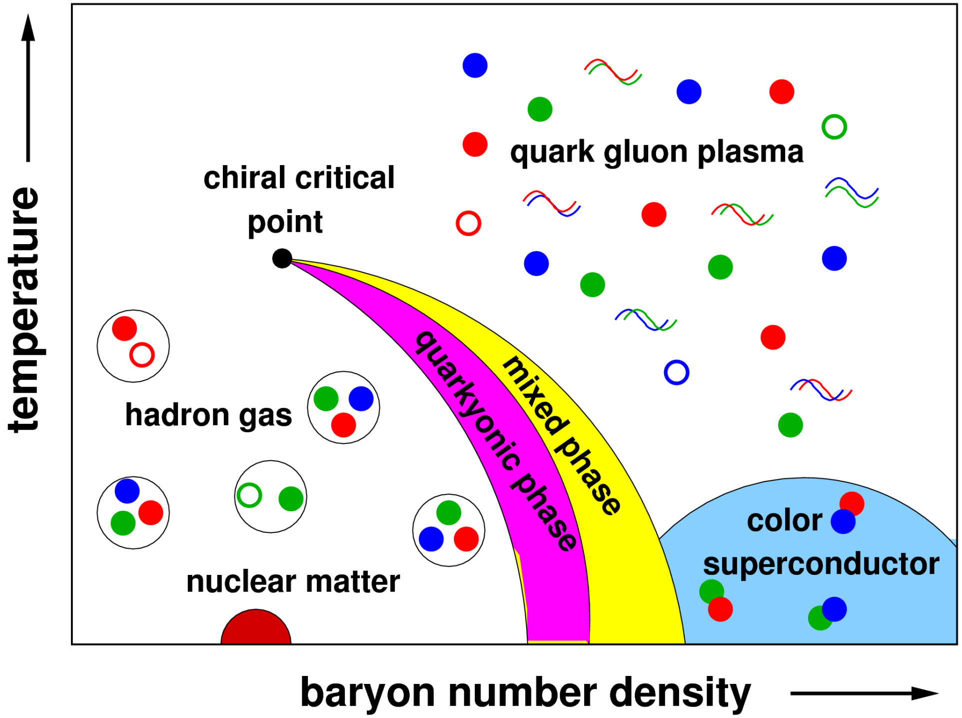
critical point estimate
from Taylor expansion

$$\rho = \lim_{n\to\infty} \rho_n$$

$$\rho_n = \sqrt{\frac{(n+2)!\chi_n^B}{n!\chi_{n+2}^B}}$$



A critical point ???



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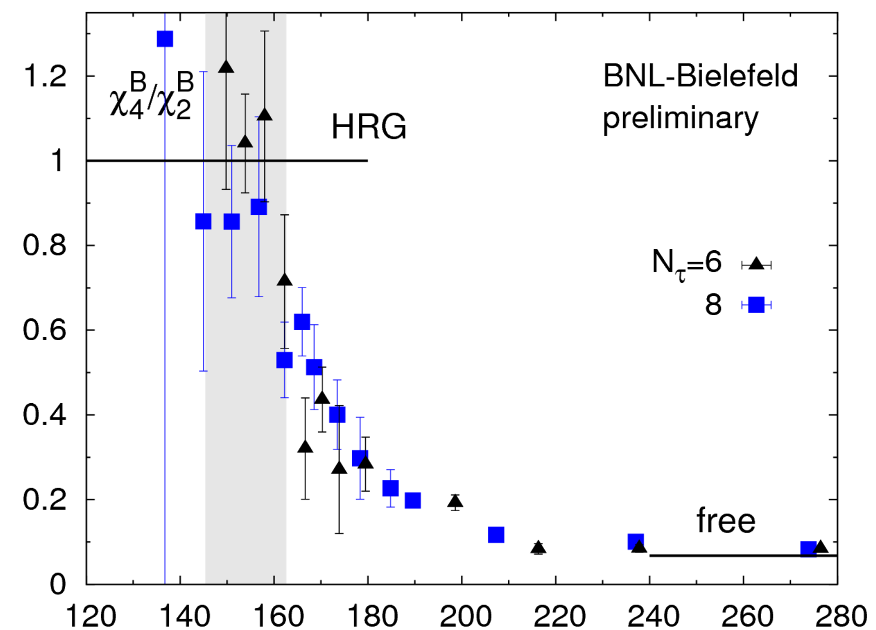


critical point estimate
from Taylor expansion

$$\rho = \lim_{n\to\infty} \rho_n$$

$$\rho_n = \sqrt{\frac{(n+2)!\chi_n^B}{n!\chi_{n+2}^B}}$$

basic input: cumulant ratios

ratios consistent with HRG
⟺ estimator consistent with infinite radius of convergence

Conclusions

– the transition temperature and the freeze-out temperature agree within current statistical accuracy at zero and non-zero baryon chemical potential at least up to $\mu_B = 200\ \mathrm{MeV}$ which covers beam energies in heavy ion experiments down to about 20 GeV.

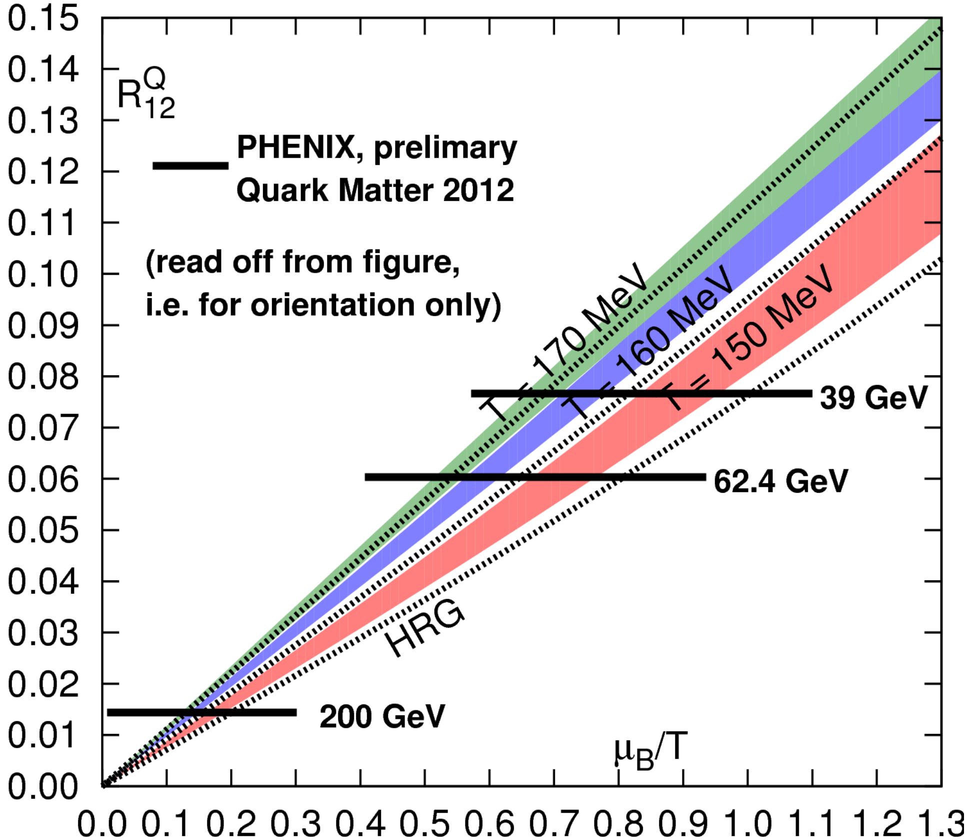
– higher order cumulants of net charge fluctuations are very promising observables to search for critical behavior and to make contact between (lattice) QCD and HIC experiments.

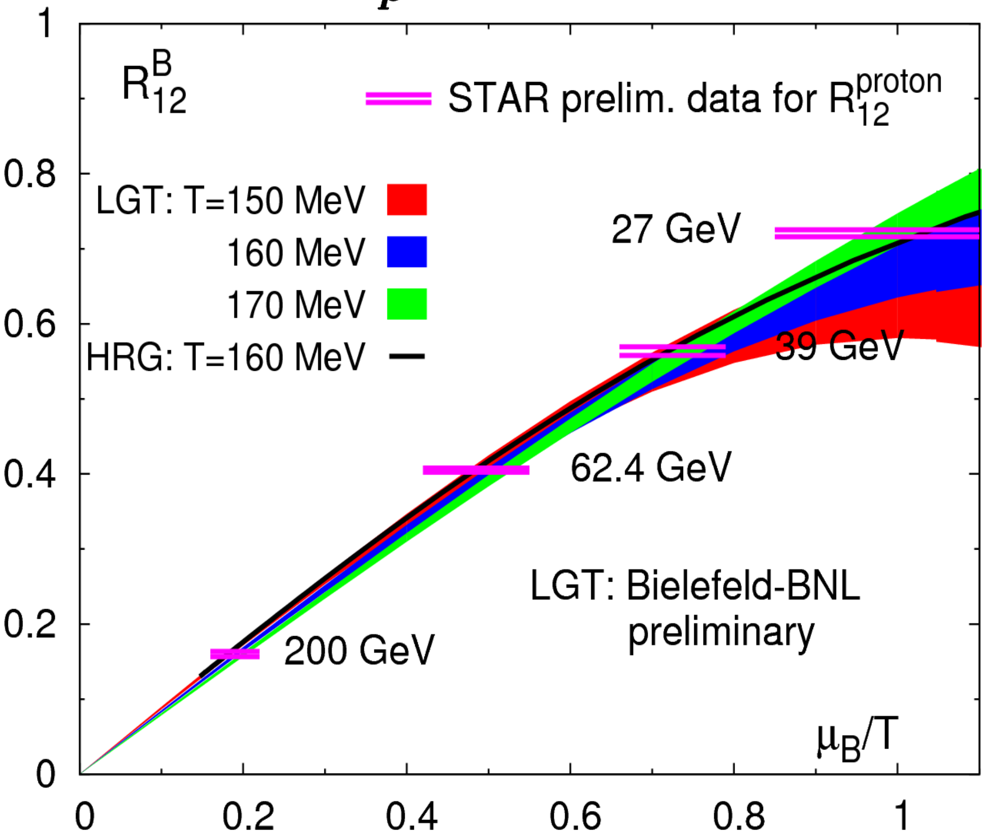
– through a comparison between equilibrium QCD calculations and HIC data on cumulants up to 6^{th} order (?) it will become possible to test whether fluctuations of conserved charges can consistently be described by equilibrium thermodynamics with a unique set of freeze-out parameters.

– HRG and QCD calculations of freeze-out parameters seem to agree on the (10-20)% level – needs to be checked in more detail

Mean over variance: HIC vs. LGT

$$R_{12}^{Q} = \frac{M_Q}{\sigma_Q^2} \qquad R_{12}^{p} = \frac{M_p}{\sigma_p^2} \qquad R_{12}^{p} = R_{12}^{B} \;??$$





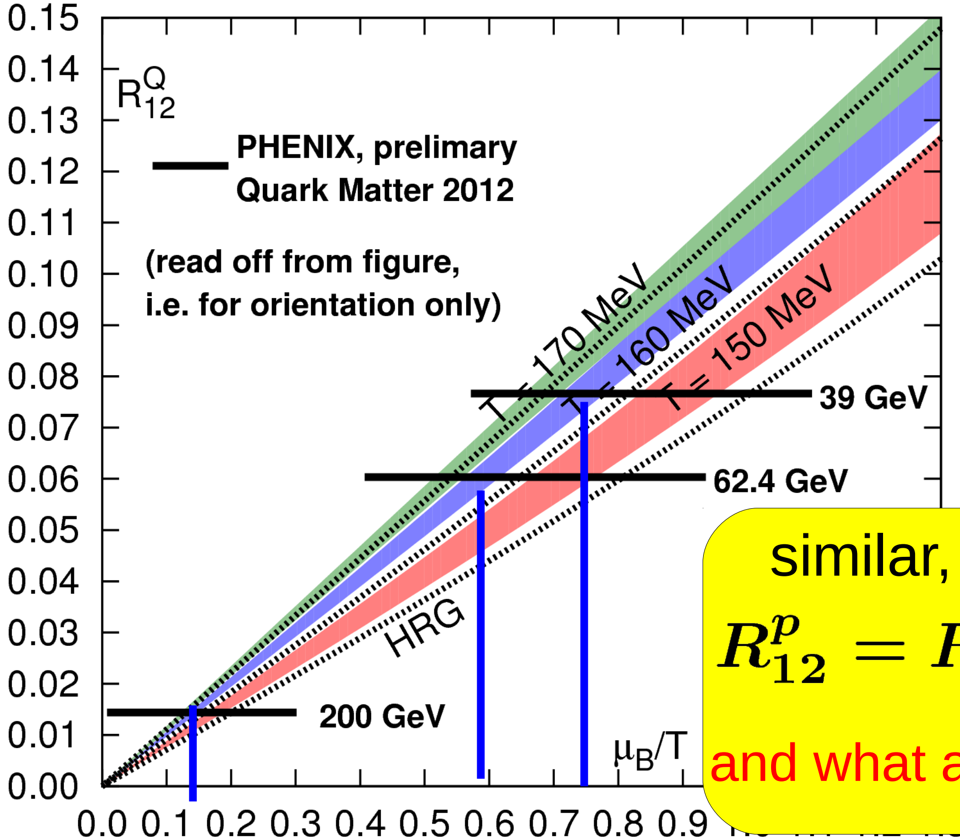
PHENIX data: J. Mitchell, Quark Matter 2012

STAR data: X. Luo, Quark Matter 2012

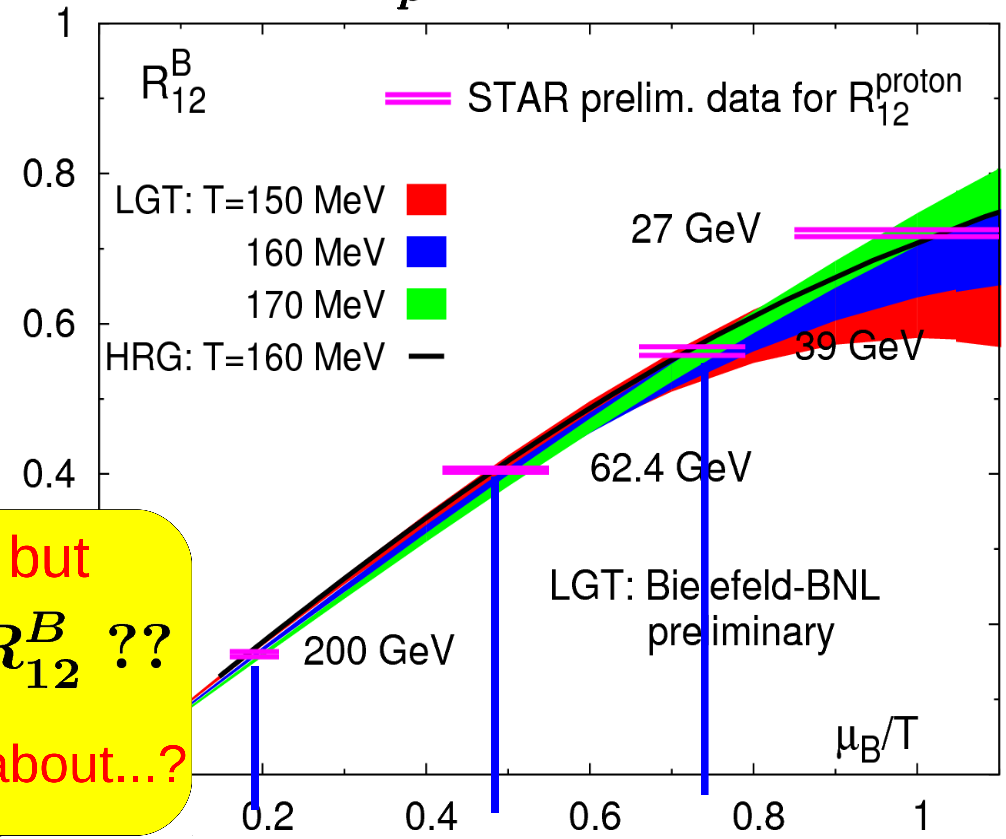
– need data for $R_{31}^{Q} = S_Q \sigma_Q^3 / M_Q$ to extract $T_{freeze}^{cumulants}$

Mean over variance: HIC vs. LGT

$$R_{12}^{Q} = \frac{M_Q}{\sigma_Q^2} \qquad R_{12}^{p} = \frac{M_p}{\sigma_p^2}$$



PHENIX data: J. Mitchell, Quark Matter 2012



STAR data: X. Luo, Quark Matter 2012

similar, but

$$R_{12}^{p} = R_{12}^{B} \;??$$

and what about...?

– need data for $R_{31}^{Q} = S_Q \sigma_Q^3 / M_Q$ to extract $T_{freeze}^{cumulants}$