Hu Wei: the tail of the tiger

Sourendu Gupta, ICTS-TIFR

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General Introduction

Forest and tiger

QCD is the theory which gives structure to the universe: baryon masses, baryon acoustic oscillations, galaxies, short range repulsion, stars, planets.

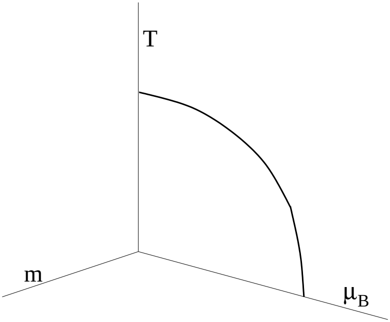
But strong interactions make it difficult to catch this tiger and study it. Only good method is numerical lattice computations.

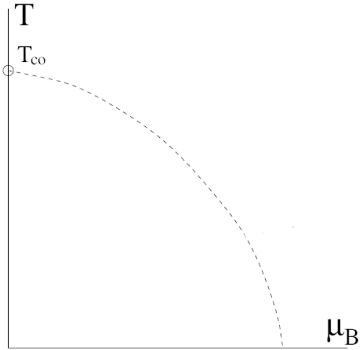
A tiger's reflection

Weinberg: simplify the problem by working only at low-energy. The reflection of a tiger gives its shape and general colour (but don't get distracted by the ripples).

It is possible to understand accurately aspects of QCD by studying effective field theory.

What we know and what we guess





When $T = 0$ line of crossover or 1st order transition has $\mu_B^1 \simeq 1250$ MeV. Normal hadron phase to quark butter phase.

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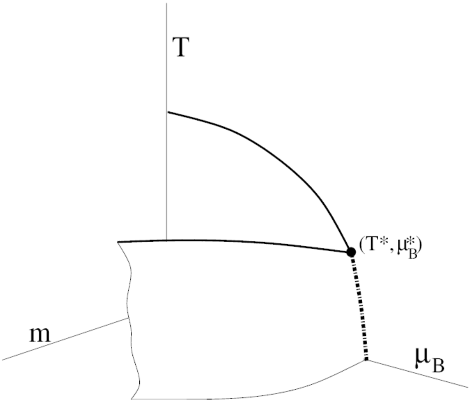

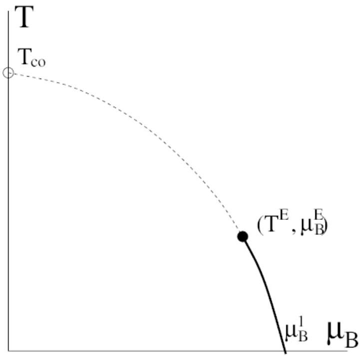

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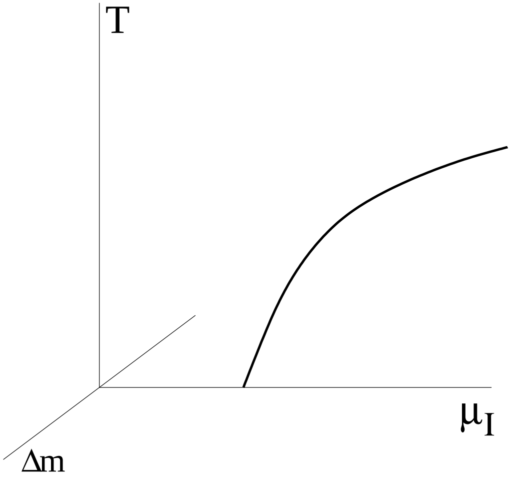
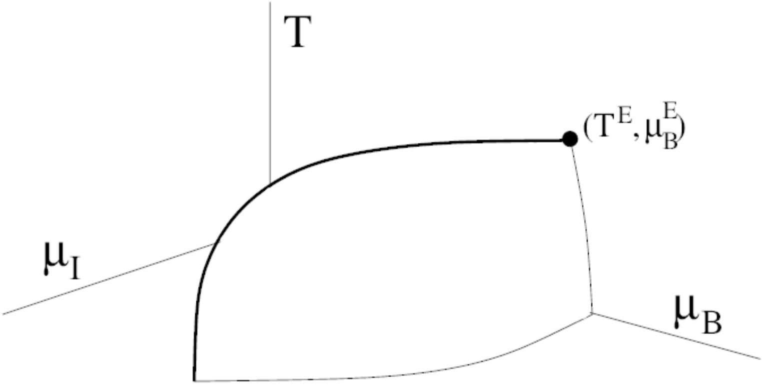
Phase diagram also has isospin



$\Delta m = m_d - m_u$ and $\mu_Q = \mu_I$ for fixed B. Normal hadron phase to pion condensed phase, eventually to colour superconducting phase.

Phase diagram for physical QCD



Physical QCD: $N_f = 1 + 1 + 1$. $m_{\pi^\pm} \neq m_{\pi^0}$ and both about 135 MeV. m_K, m_η physical.

Effective (Quantum) Field Theory

Weinberg's legacy: EFTs for hadrons

Chiral perturbation theory χPT

Very widely used tool for understanding low-energy physics of strong interactions. Roy equations give properties of $\pi\pi$ scattering, extended to πN scattering, captured well in χPT.

Heavy quark effective theory HQET

The systematics of form factors and decay rates of mesons and baryons with one heavy quark: b or c.

Non-relativistic QED and QCD (NRQED, NRQCD)

The Schrödinger's equation for the Hydrogen atom is used as the start of an expansion which exactly reproduces the bound state in QED. Now used in chemistry. Similar approach for QCD bound states with two heavy quarks.

Simple things about the tiger

QCD vacuum breaks chiral symmetry: gives mass to baryons and structure to the universe. Finite temperature buttery melting to chiral symmetric state.

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If this works, then we have caught the tail of the tiger. Can we swing it by its tail?

Thermal EFT setup

Quark EFT for $T > 0$ with chiral symmetry, UV cutoff $\Lambda <$ proton mass

$$
\begin{aligned}
D &= 3 \qquad L_3 = \text{mass terms} \\
D &= 4 \qquad L_4 = \text{thermal kinetic terms}: \quad \overline{\psi}\partial\!\!\!/_0\psi + d_4\overline{\psi}\nabla\!\!\!\!/\,\psi \\
D &= 6 \qquad L_6 = \text{many Nambu JonaLasinio} - \text{like terms} \\
&\qquad\qquad + \text{derivative corrections to propagator} \\
D &= 8 \qquad L_8 = \text{NJL} - \text{like terms with two derivatives} \\
&\qquad\qquad + \text{derivative corrections to propagator} \\
D &= 9 \qquad L_9 = {}'\text{t Hooft} - \text{Schaeffer terms}
\end{aligned}
$$

Use all terms constrained only by symmetry: EFT is not a model but a tool to describe QCD accurately

SG+RS doi:10.1103/PhysRevD.97.036025 and SG+RS+PS in prep

How to do a tractable computation

Introduce a chiral condensate, and do a Hartree-Fock computation. Find the chiral condensate self-consistently: gap equation. This gives the vacuum: spontaneous symmetry breaking.

Examine the effects of fluctuations in the quark fields. The most important fluctuations are the ones that cost least energy: the pseudo-Goldstone bosons. Write the EFT of these fluctuations (thermal χPT). This is a coupled theory of pions, Kaons, η and η' mesons.

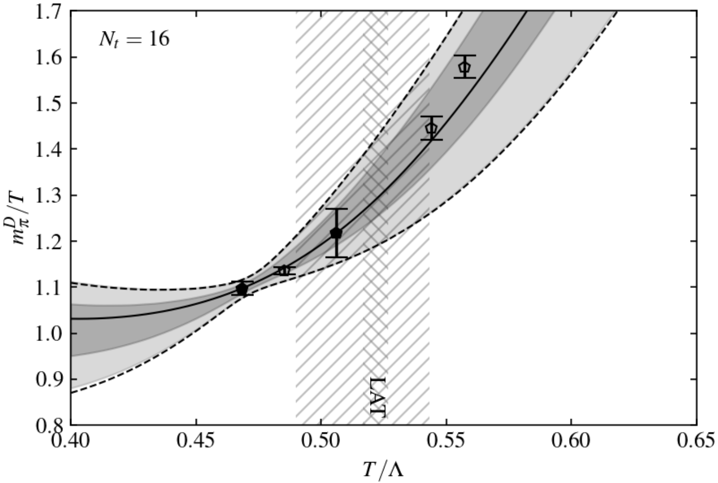
It is straightforward to remove the η' from consideration: requires tuning the 't Hooft-Schaeffer LEC.

We remove the K and η' through a one-loop integration, giving a pion EFT

$$L = \frac{1}{2}m^2\pi^2 + \frac{1}{2}\dot{\pi}^2 + \frac{1}{2}c_4(\nabla\pi)^2 + c_{41}\pi^4 + \cdots$$

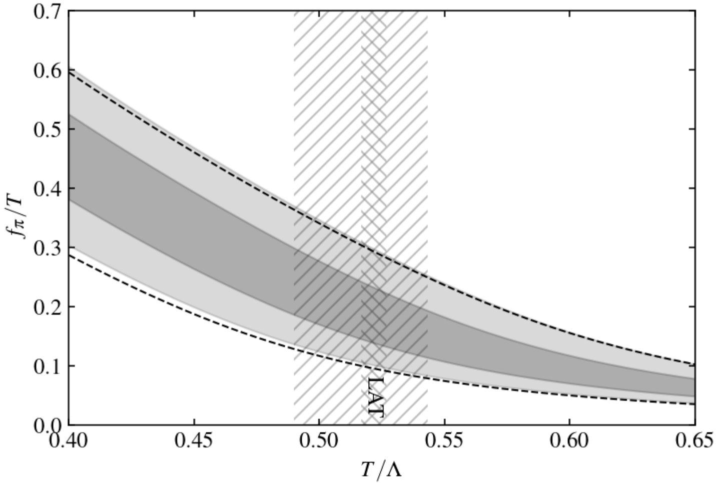
The first 3 terms are LO in chiral power counting, the 4th term is one of several NLO terms.

Input pion properties to fix LECs



SG+RS+PS in prep, using HotQCD doi:10.1103/PhysRevD.100.094510

EFT predictions for thermal pions



Looking forward to measurements from HotQCD, others

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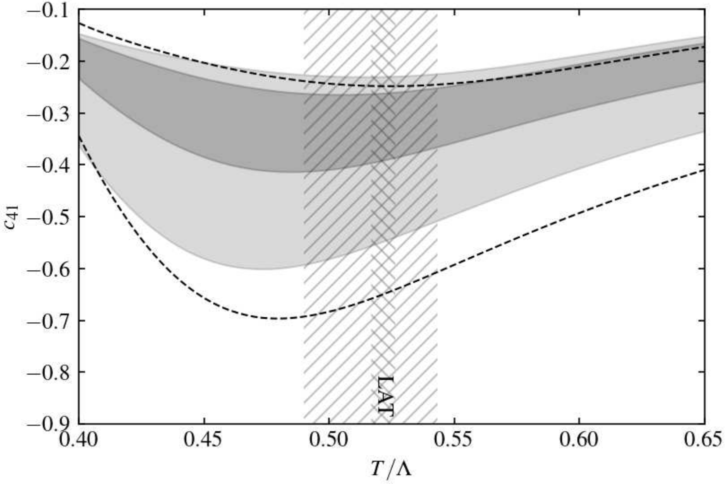

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EFT predictions for the phase diagram

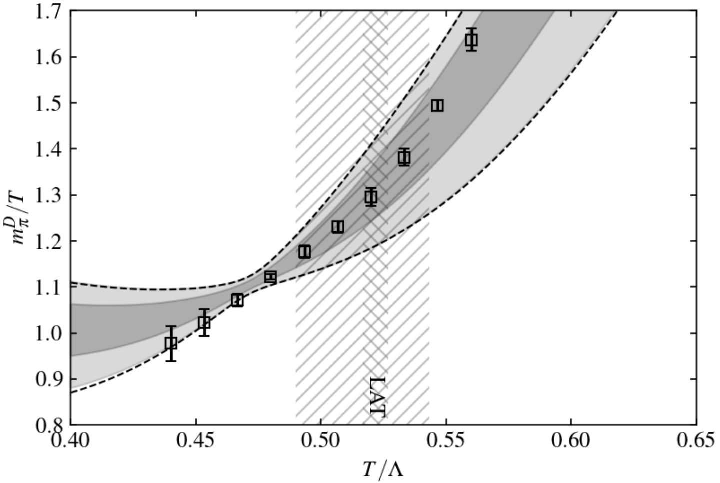
	T_{co} (MeV)	κ_2	κ_4
HotQCD 2017	156.5 (1.5)	0.015 (4)	-0.001 (3)
BHJW 2020	158.0 (0.6)	0.0153 (18)	0.00032 (67)
EFT	**157 (7)**	**0.0169 (20)**	**0.00014 (2)**

EFT is used to compute the phase diagram in the HF approximation.

Errors in T_c for the EFT are induced from the pion Debye mass. Improved lattice measurement of pion screening mass will reduce the errors in the EFT prediction. Needed to understand how closely pion properties are related to the phase diagram.

Quark butter

Pion properties continuous across phases



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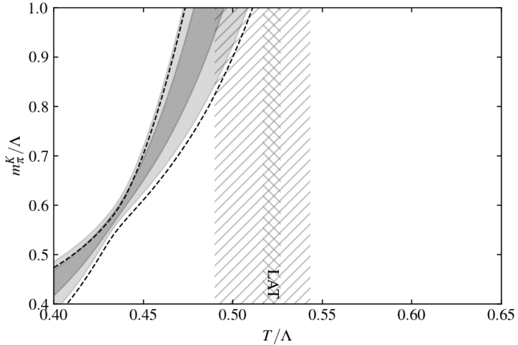
Analytic continuation: kinetic mass

In Minkowski space

$$E = \sqrt{m^2 + c_4 p^2} \simeq m + \frac{p^2}{(2m/c_4)} + \cdots$$

Pole mass (rest mass) is m, but kinetic energy needs a different mass.
Kinetic mass $m^K = m/c_4$. **SG+RS doi:10.1142/S0217751X20300215**



Increasing m^K imply that scattering channels which are open at low T close rapidly as T increases.

Increasing m_π^K reason why the pion decouples from the dynamics above T_c

Tiger becomes quark butter

1. Chiral crossover properties are closely tied to the behaviour of pions at finite temperature: implied by chiral symmetry
2. The pion mass and other properties are continuous across the chiral crossover. Implies that pions remain strongly interacting matter. Can be excited by appropriate probe.
3. The kinetic mass increases very rapidly, and reaches the UV cutoff of the EFT just below T_c. Implies that pions stop participating in the dynamics at around T_c.
4. Hadron matter to quark matter transition is quite complicated near the crossover; some aspects of hadrons remain above T_{co}. This is one aspect of quark butter.