

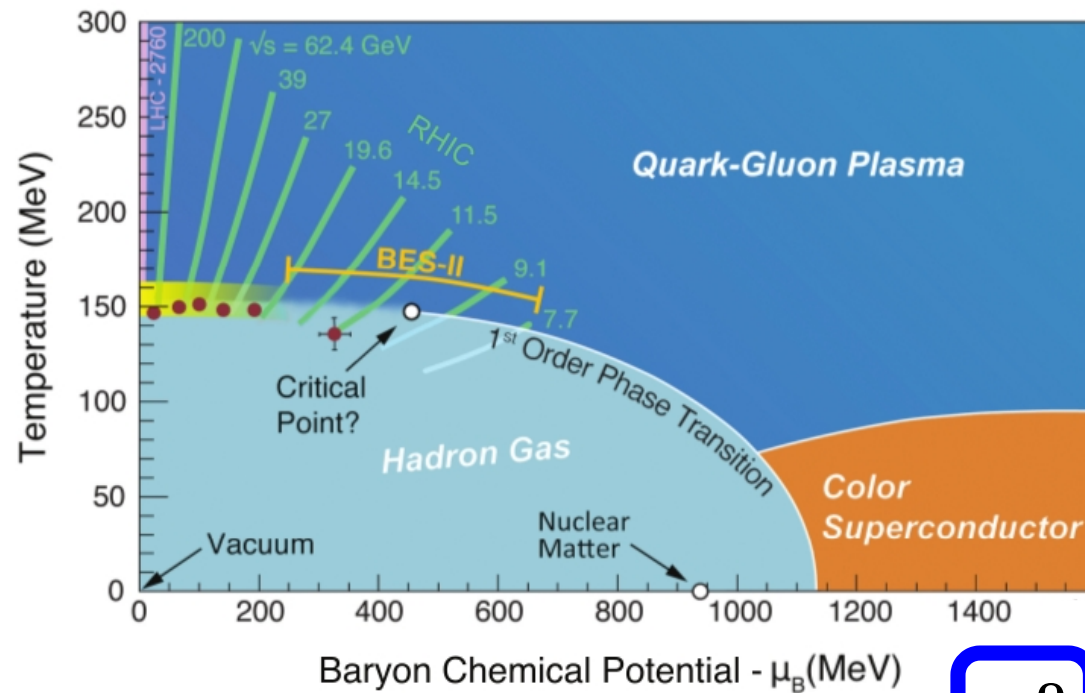
Lattice QCD constraining on the location of a critical point in the QCD phase diagram

Frithjof Karsch
Bielefeld University



- QCD phase diagram and lattice QCD thermodynamics
- Critical behavior in the chiral limit at vanishing and non-chemical potential
 - constraining the location of a critical endpoint
- Universal critical behavior close to the CEP:
 - Z(2) scaling functions and STAR fixed target data

Exploring the phase diagram of strong-interaction matter

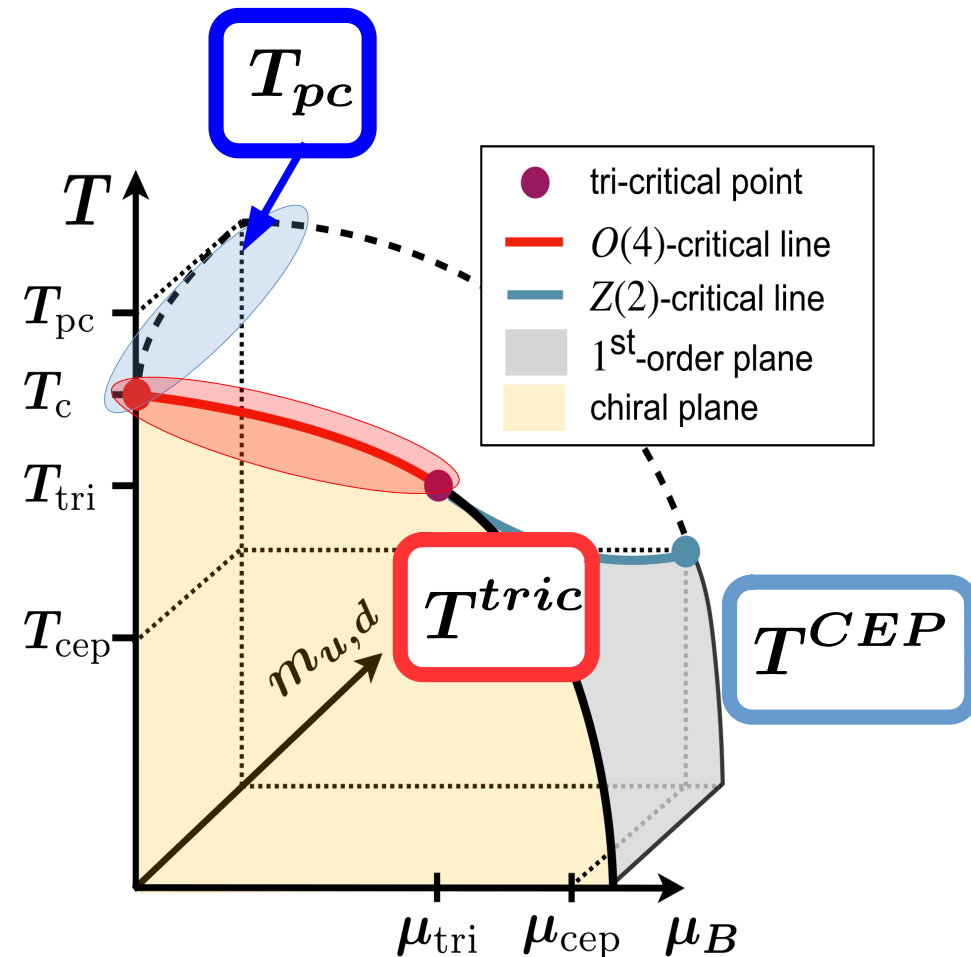


constraining the location of the CEP through information on the chiral phase transition in QCD

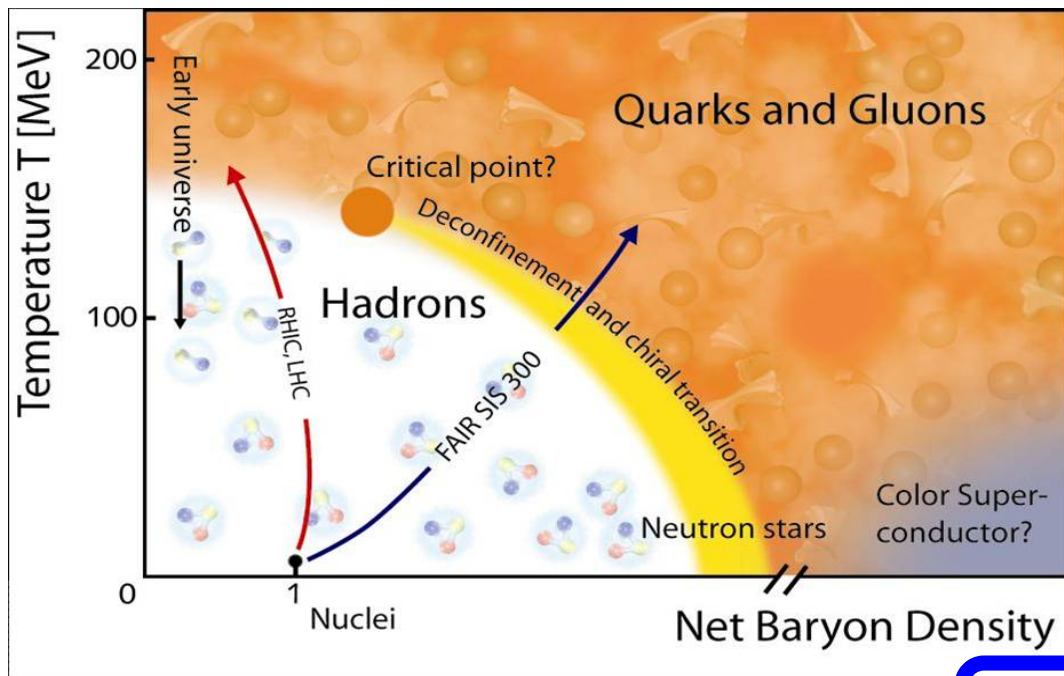
Where is the critical point?



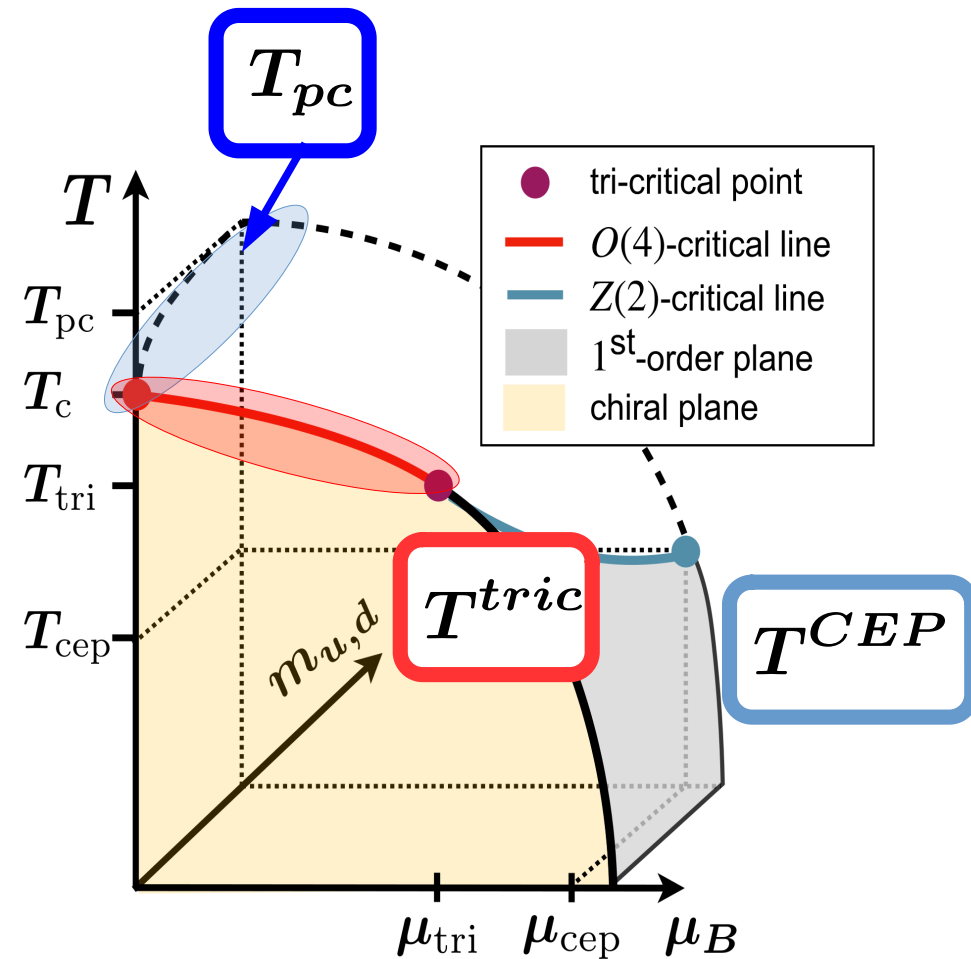
T_c^0



Exploring the phase diagram of strong-interaction matter



constraining the location of the CEP through information on the chiral phase transition in QCD



Where is the critical point?



T_c^0

T_{pc}

T^{tric}

T^{CEP}

Taylor series for finite-density QCD

Radius of Convergence and Pade approximants

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_B, Q, S=0}, \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

constrained Taylor series, demanding strangeness neutrality: $n_S = 0$

and: $n_Q/n_B = 0.4$

$$\hat{\mu}_Q(T, \mu_B) = q_1(T) \hat{\mu}_B + q_3(T) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

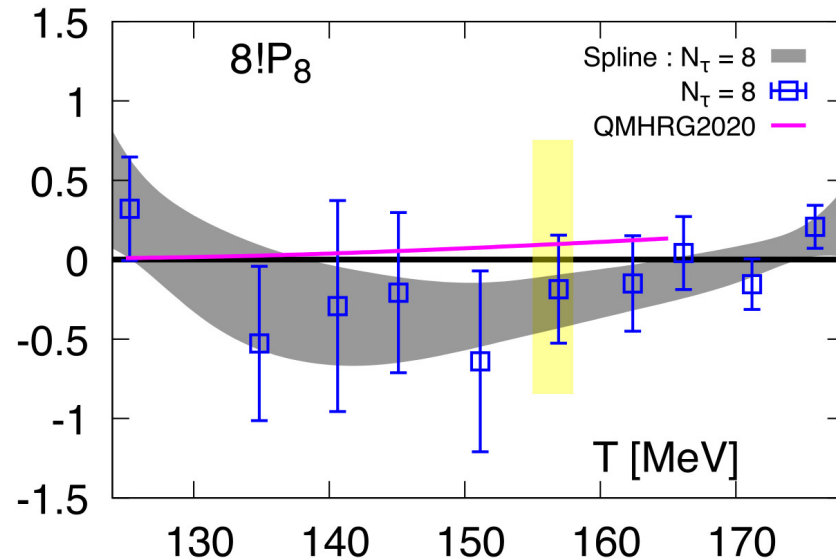
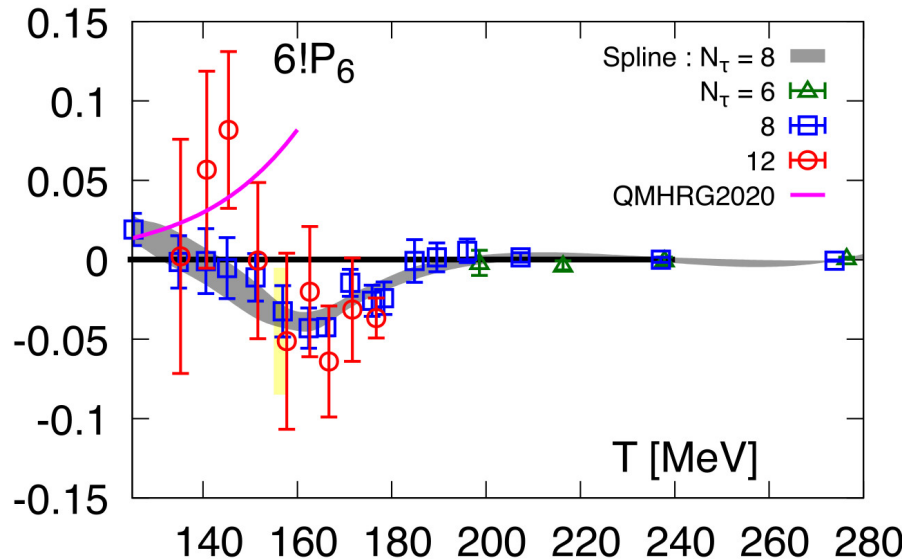
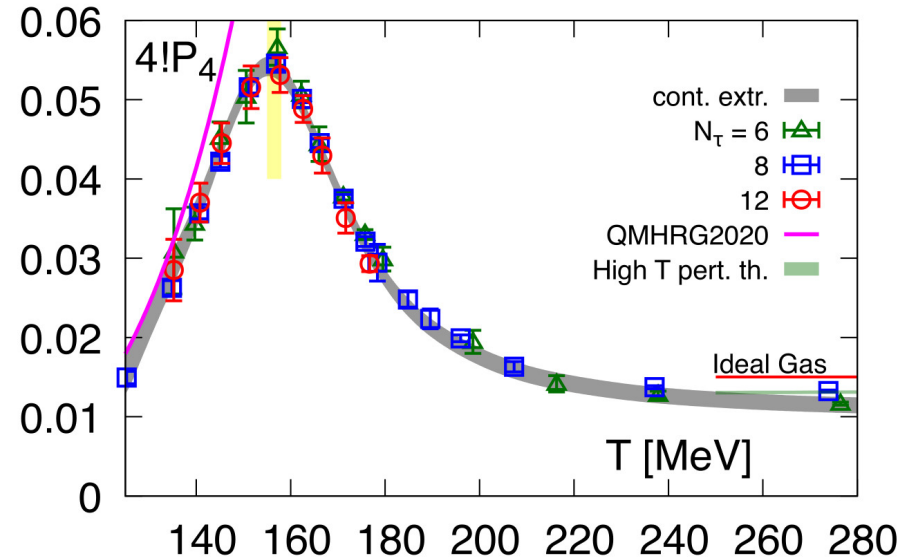
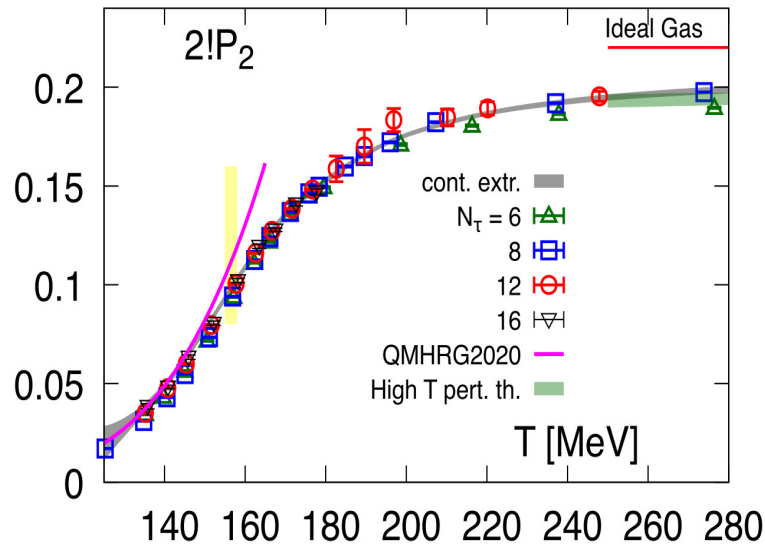
$$\hat{\mu}_S(T, \mu_B) = s_1(T) \hat{\mu}_B + s_3(T) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \frac{1}{(2k)!} \tilde{\chi}_{2k}^B(T) \left(\frac{\mu_B}{T}\right)^{2k}, \quad P_{2k} \equiv \frac{1}{(2k)!} \tilde{\chi}_{2k}^B$$

Up to 8th order cumulants are used "frequently"

– Taylor expansion –

control basic features of skewness up to hyper-kurtosis ratios

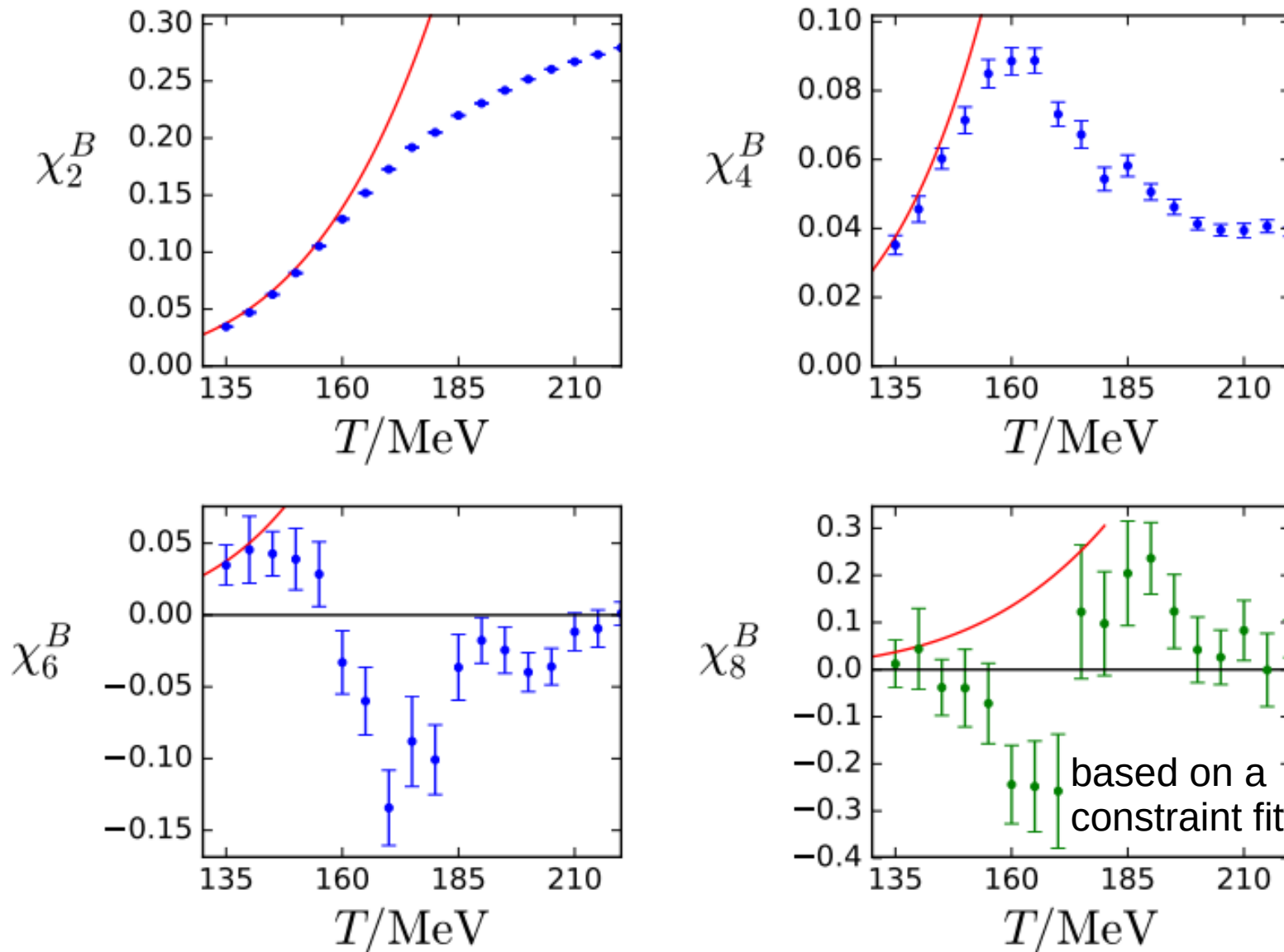


A. Bazavov et al. (HotQCD), Phys. Rev. D 108 (2023) 014510 arXiv:2212.09043

Up to 8th order cumulants are used frequently

- **imag. chem. pot. extrapolations** –

$48^3 \times 12$



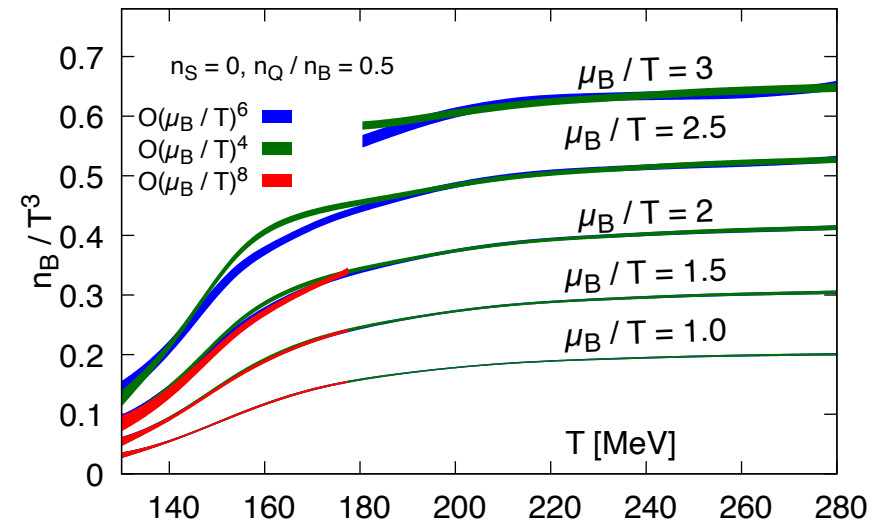
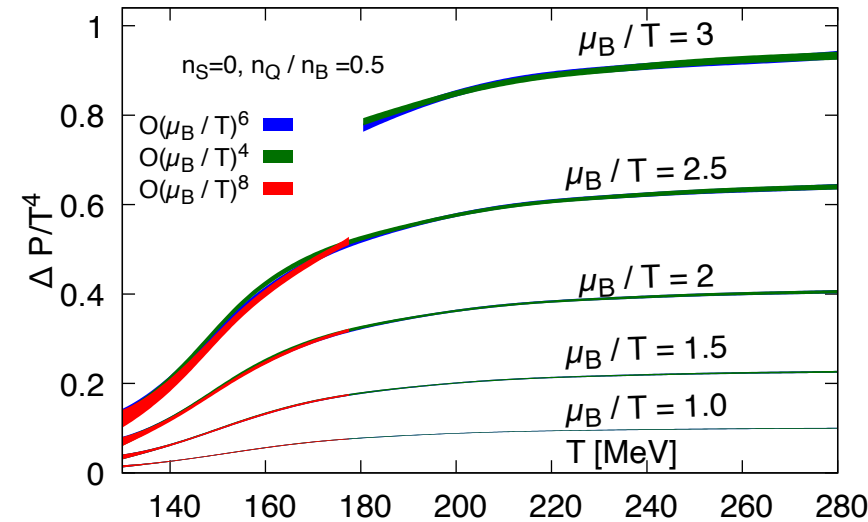
S. Borsanyi et al. , JHEP 10 (2018) 205, arXiv:1805.04445

EoS 2022: Taylor series to 8th order

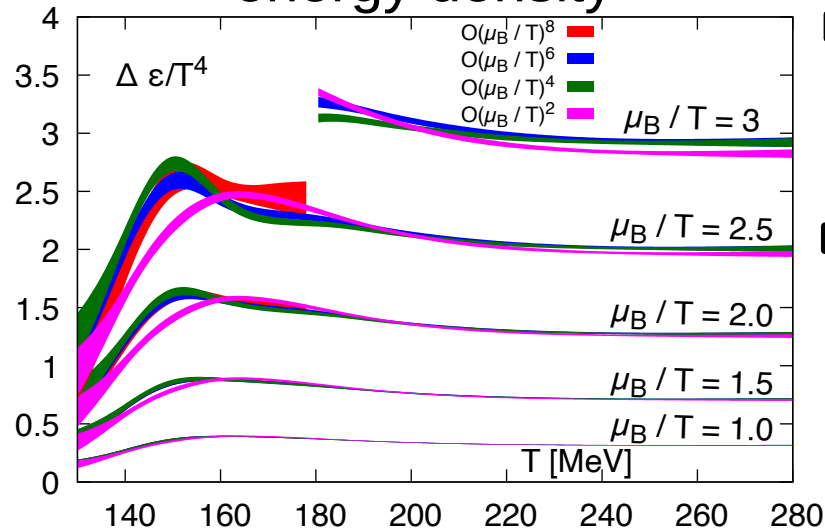
$$\frac{\Delta p(T, \mu_B)}{T^4} = \frac{p(T, \mu_B) - p(T, 0)}{T^4} = P_2(T) \left(\frac{\mu_B}{T} \right)^2 + P_4(T) \left(\frac{\mu_B}{T} \right)^4 + P_6(T) \left(\frac{\mu_B}{T} \right)^6 + \dots$$

pressure

net baryon-number density



energy density



The EoS (pressure & number density) is well controlled for $\mu_B/T \leq 2$ or equivalently $\sqrt{s_{NN}} \geq 11$ GeV



convergence of expansions for higher order derivatives slower, but same radius of convergence as pressure

D. Bollweg et al (HotQCD), arXiv:2212.09043

Comparing Taylor series and Pade resummation

Taylor series

$$\begin{aligned}\frac{\Delta p}{T^4} &\equiv \frac{p(T, \mu_B)}{T^4} - \frac{p(T, 0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T) \left(\frac{\mu_B}{T} \right)^{2k} \\ &= \frac{P_2^2}{P_4} \left(\bar{x}^2 + \bar{x}^4 + c_{6,2} \bar{x}^6 + c_{8,2} \bar{x}^8 + \dots \right) \quad \text{use } P_2 > 0, P_4 > 0 \\ \bar{x} &\equiv \sqrt{P_4/P_2} (\mu_B/T) \\ c_{6,2} &= \frac{P_6 P_2}{P_4^2} \quad , \quad c_{8,2} = \frac{P_8 P_2^2}{P_4^3}\end{aligned}$$

Radius of convergence (Mercer-Roberts estimator):

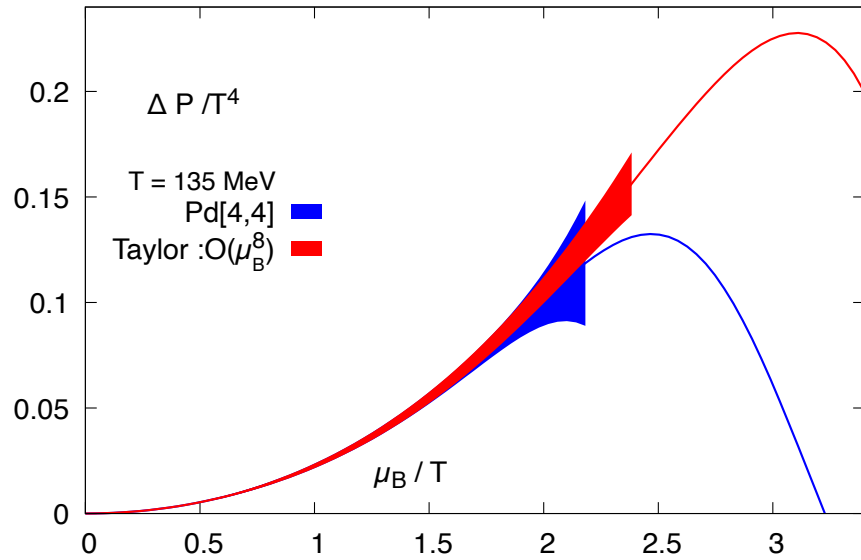
$$r_{c,n} = \left| \frac{P_{n-2} P_{n+2} - P_n^2}{P_{n+4} P_n - P_{n+2}^2} \right|^{1/4}, \quad r_{c,4} = \sqrt{\frac{12 \tilde{\chi}_2^B}{\tilde{\chi}_4^B}} \left| \frac{1 - c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|^{1/4},$$

Pade approximation

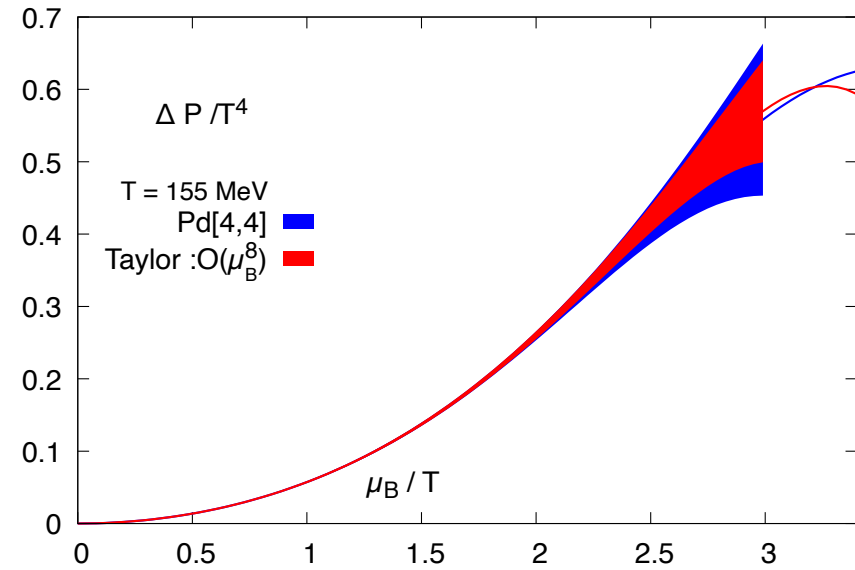
$$P_{[4,4]} = \frac{(1 - c_{6,2}) \bar{x}^2 + (1 - 2c_{6,2} + c_{8,2}) \bar{x}^4}{(1 - c_{6,2}) + (c_{8,2} - c_{6,2}) \bar{x}^2 + (c_{6,2}^2 - c_{8,2}) \bar{x}^4}$$

Taylor series and Pade approximants

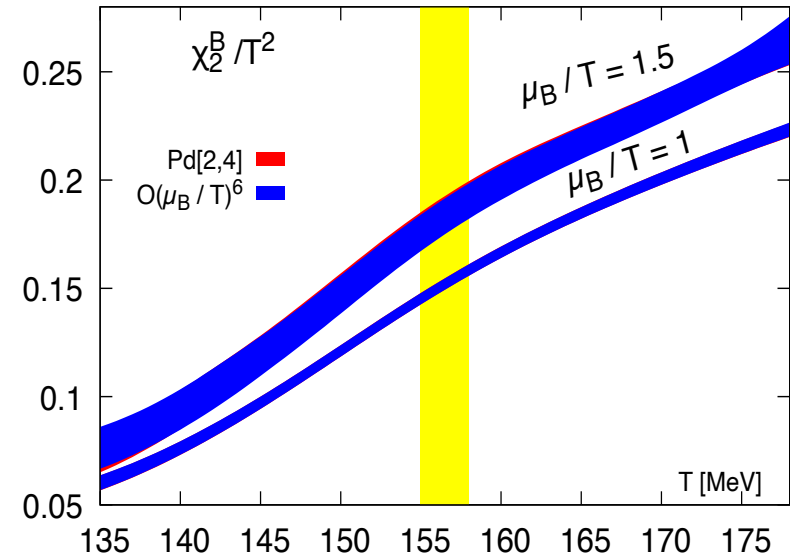
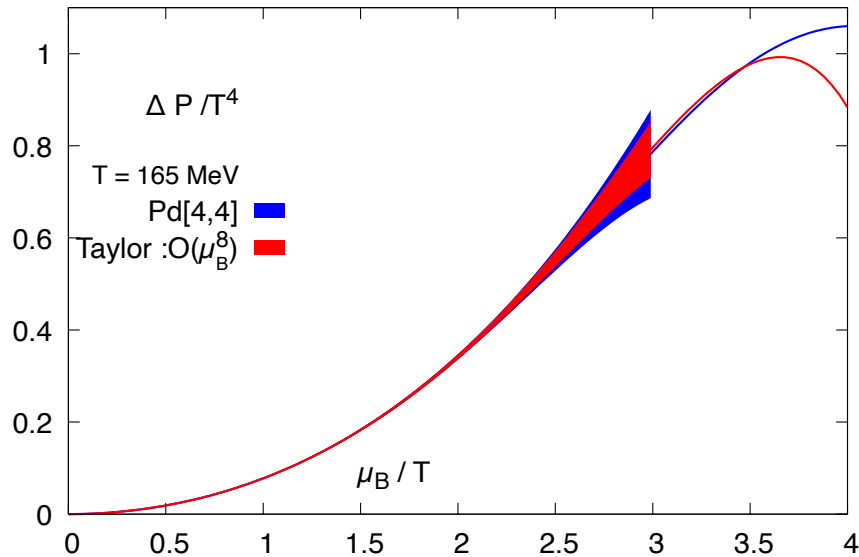
$$\mu_Q = \mu_S = 0$$



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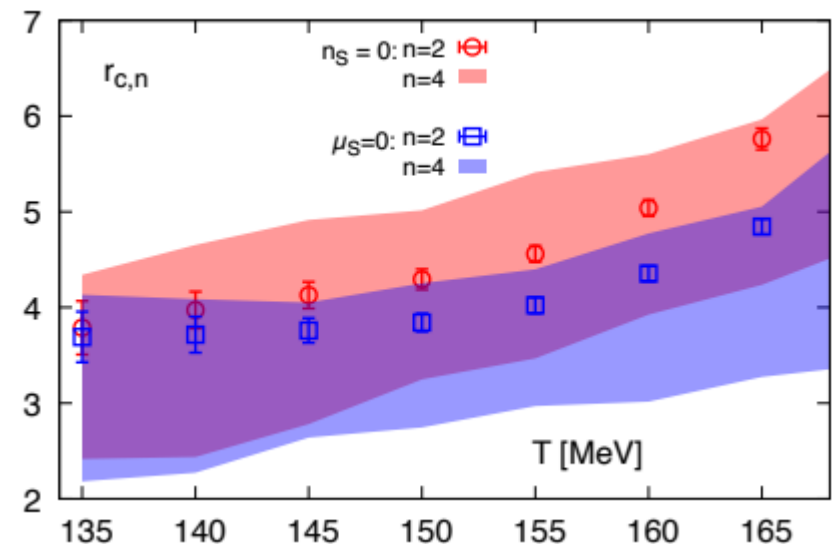
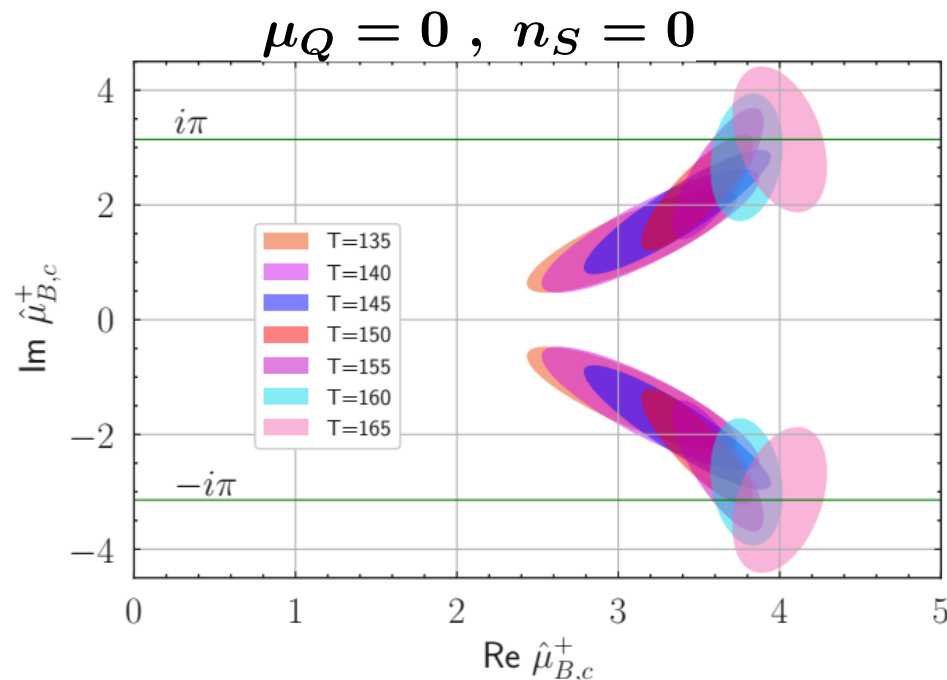
D. Bollweg et al (HotQCD), Phys.Rev.D 105 (2022) 074511

Poles of [n,n] Pade approximants in QCD

$$\hat{\mu}_{B,c}^{\pm} = \pm r_{c,4} e^{\pm i\Theta_{c,4}}, \quad r_{c,4} = \sqrt{\frac{12\tilde{\chi}_2^B}{\tilde{\chi}_4^B} \left| \frac{1 - c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|}^{1/4}, \quad c_{2k,2} = \frac{2\tilde{\chi}_2^B}{(2k)!\tilde{\chi}_{2k}^B} \left(\frac{12\tilde{\chi}_2^B}{\tilde{\chi}_4^B} \right)^{k-1}$$

complex poles move to real axis as temperature decreases

distance of complex pole closest to the origin is identical to the **Mercer-Roberts estimator** for the radius of convergence



HotQCD, arXiv:2202.09184

within current errors poles on the real axis (critical point)
are possible only for

$$T \leq 135 \text{ MeV}, \quad \mu_B/T > 2.5$$

higher statistics will sharpen the constraint

Critical behavior in the chiral limit of QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a **universal scaling function**

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V, T, \vec{\mu})$$

singular

regular

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_l}{T} \right)^2$$

$$h \sim \frac{m_l}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic

$$\frac{\partial^2 \ln Z}{\partial h^2}$$

$$\sim \left(\frac{m_l}{T_c} \right)^{1/\delta-1}$$

↑

~ -0.79

divergence: **strong**

mixed

$$\frac{\partial^2 \ln Z}{\partial h \partial t}$$

$$\sim \left(\frac{m_l}{T_c} \right)^{(\beta-1)/\beta\delta}$$

↑

~ -0.34

moderate

thermal

$$\frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c} \right)^{-\alpha/\beta\delta}$$

↑

~ +0.11

none

O(4) critical
exponents
 $\alpha = -0.21$
 $\beta = 0.38$
 $\delta = 4.82$

Chiral observables in QCD

– chiral condensate: $\langle \bar{\psi}\psi \rangle_l = \frac{\partial P/T}{\partial m_l/T}$, $\langle \bar{\psi}\psi \rangle_l = (\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d)/2$

– chiral order parameter: $M = \frac{2}{f_K^4} [m_s \langle \bar{\psi}\psi \rangle_l - m_l \langle \bar{\psi}\psi \rangle_s]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s \left(\frac{\partial M}{\partial m_u} + \frac{\partial M}{\partial m_d} \right)$ **magnetic**

– mixed chiral susceptibility: $\chi_t = T \frac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_x = T^2 \frac{\partial^2 P/T^4}{\partial \mu_X^2} \Big|_{\mu_X=0}$ **thermal**

$$X = B, S, \dots$$

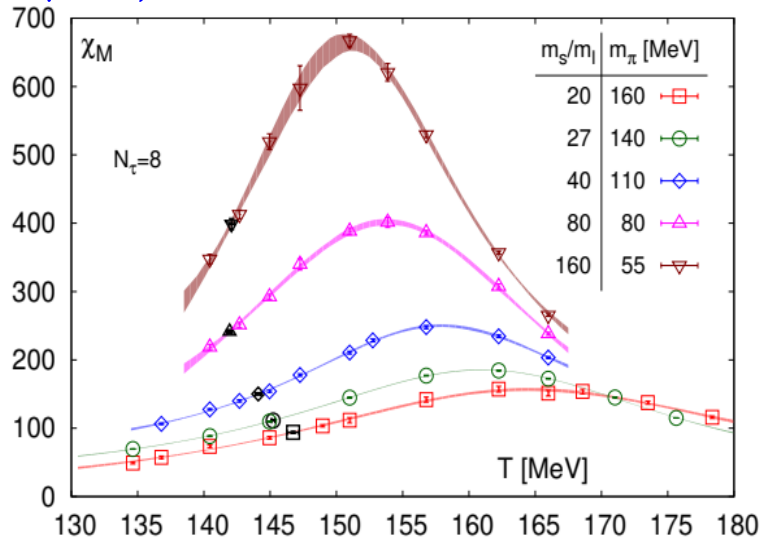
The Chiral **PHASE TRANSITION** in (2+1)-flavor QCD

$$M \sim m_s \partial \ln Z / \partial m_l$$

“magnetic”
susceptibility

$$\chi_M \sim \frac{\partial^2 \ln Z}{\partial m_l^2} \sim (m_s/m_l)^{0.79}$$

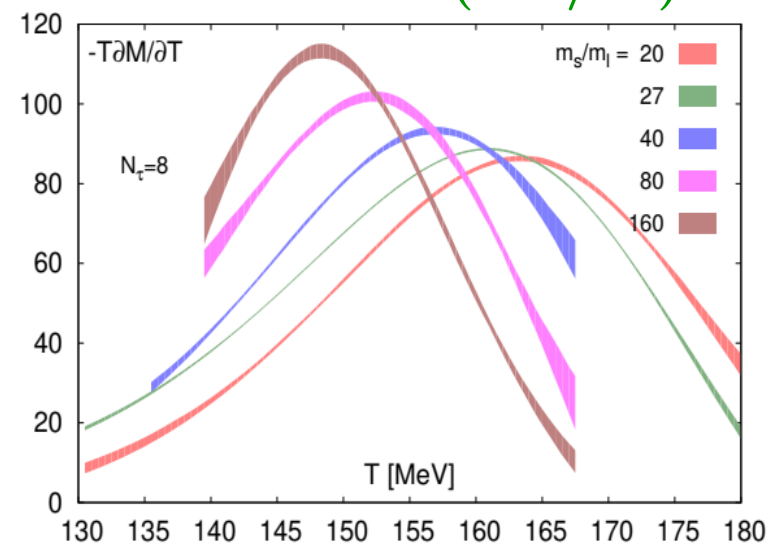
$$(160/27)^{0.79} \sim 4$$



“mixed”
susceptibility

$$\chi_t \sim \frac{\partial^2 \ln Z}{\partial T \partial m_l} \sim (m_s/m_l)^{0.34}$$

$$(160/27)^{0.34} \sim 1.8$$

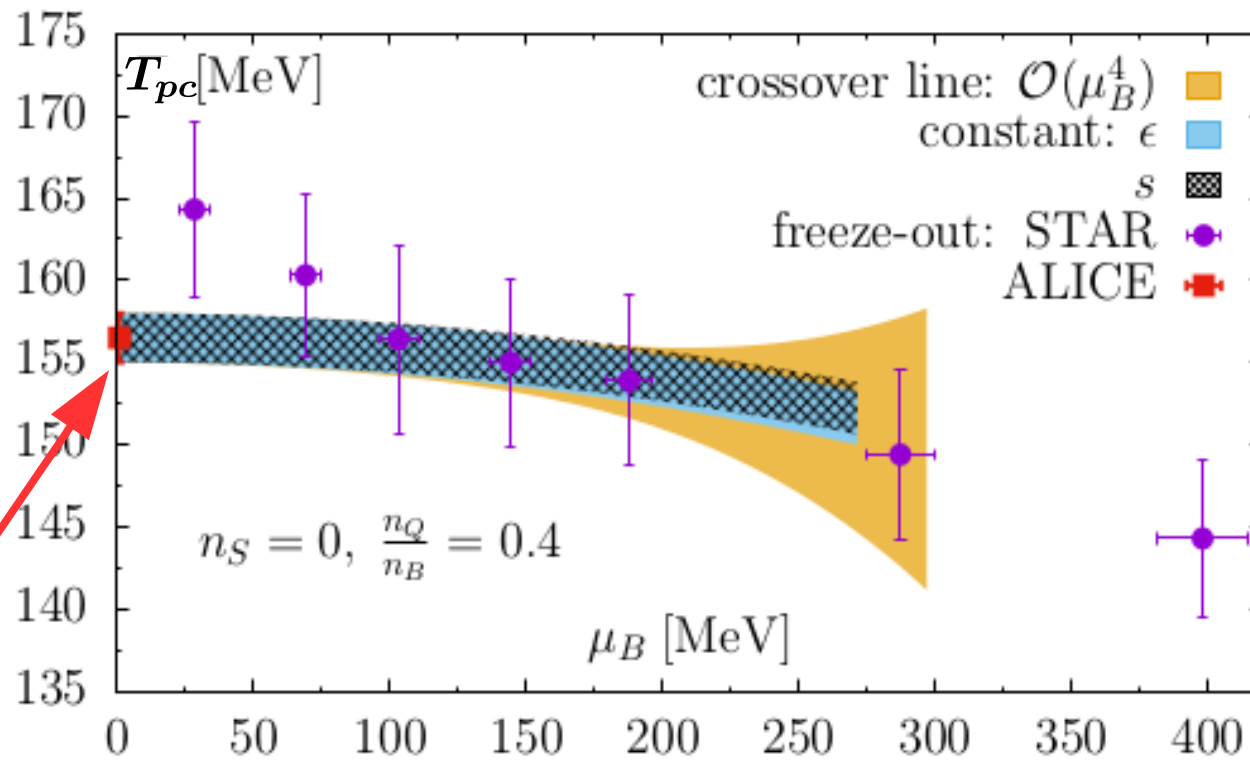


H.T. Ding et al. (HotQCD), PRL 123 (2019) 15, arXiv:1903.04801

Pseudo-critical line for physical quark mass values

$$T_{pc}(\mu_B) = T_{pc}(0) \left(1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(\mu_B)} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T_{pc}(\mu_B)} \right)^4 + \dots \right)$$

phase diagram at
physical values of
the quark masses



STAR:
arXiv:1701.07065
A. Andronic et al.,
Nature 561 (2018)
321

$$T_{pc} = (156.5 \pm 1.5) \text{ MeV}$$

$$\kappa_2 = 0.012(4)$$

$$\kappa_4 = 0.000(4)$$

A. Bazavov et al. [HotQCD],
Phys. Lett. B795 (2019),
arXiv:1812.08235

$$T_{pc} = (158.0 \pm 0.6) \text{ MeV}$$

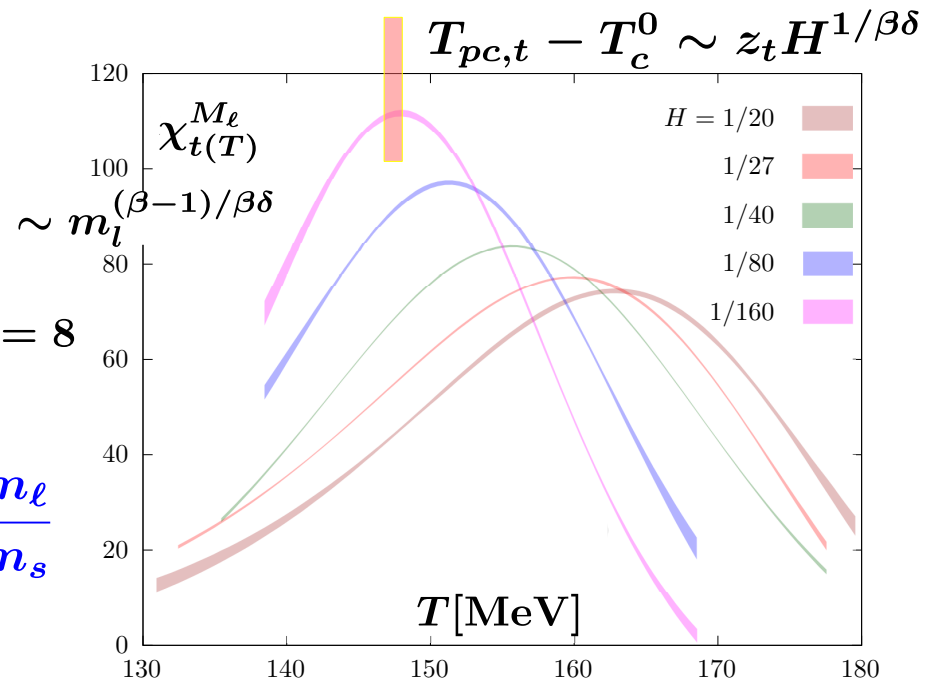
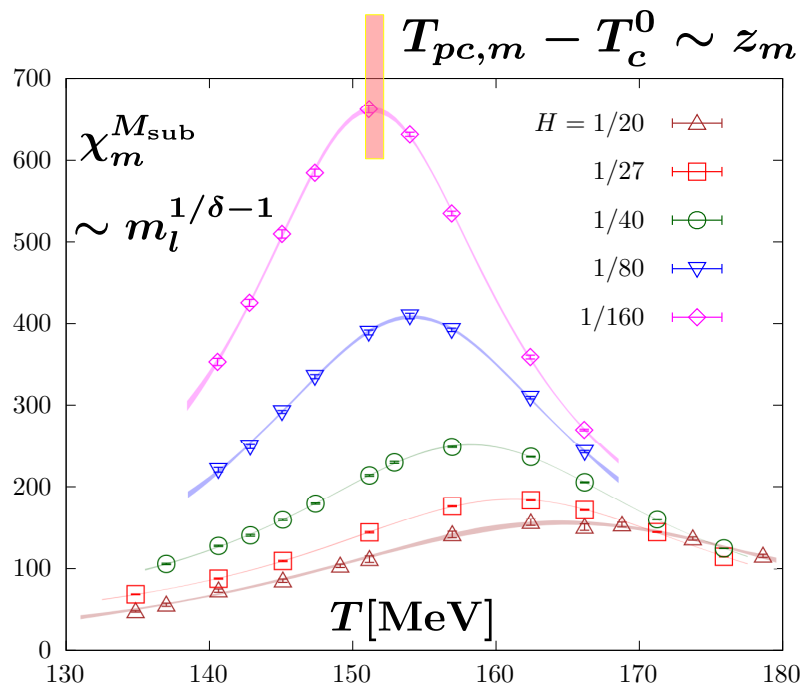
$$\kappa_2 = 0.0153(18)$$

$$\kappa_4 = 0.00032(67)$$

S. Borsanyi, et al,
PRL 125 (2020)
arXiv:2002.02821

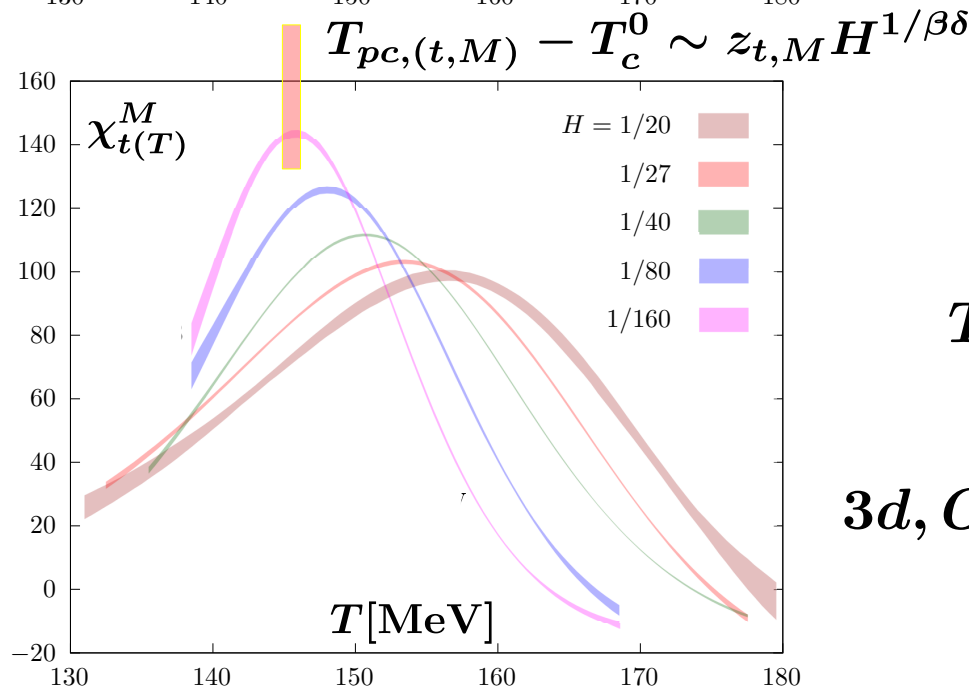
Approaching the chiral limit

Pseudo-critical temperatures from susceptibilities



$$N_\tau = 8$$

$$H = \frac{m_\ell}{m_s}$$



- even in the asymptotic scaling regime T_{pc} is not a unique value
- **depends on observable**

$$T_{pc,x}(H) = T_c^0 \left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} \right)$$

$x = m, t, (t, M)$

$$3d, O(2) : z_m = 1.6675(65)$$

$$z_t = 0.7991(96)$$

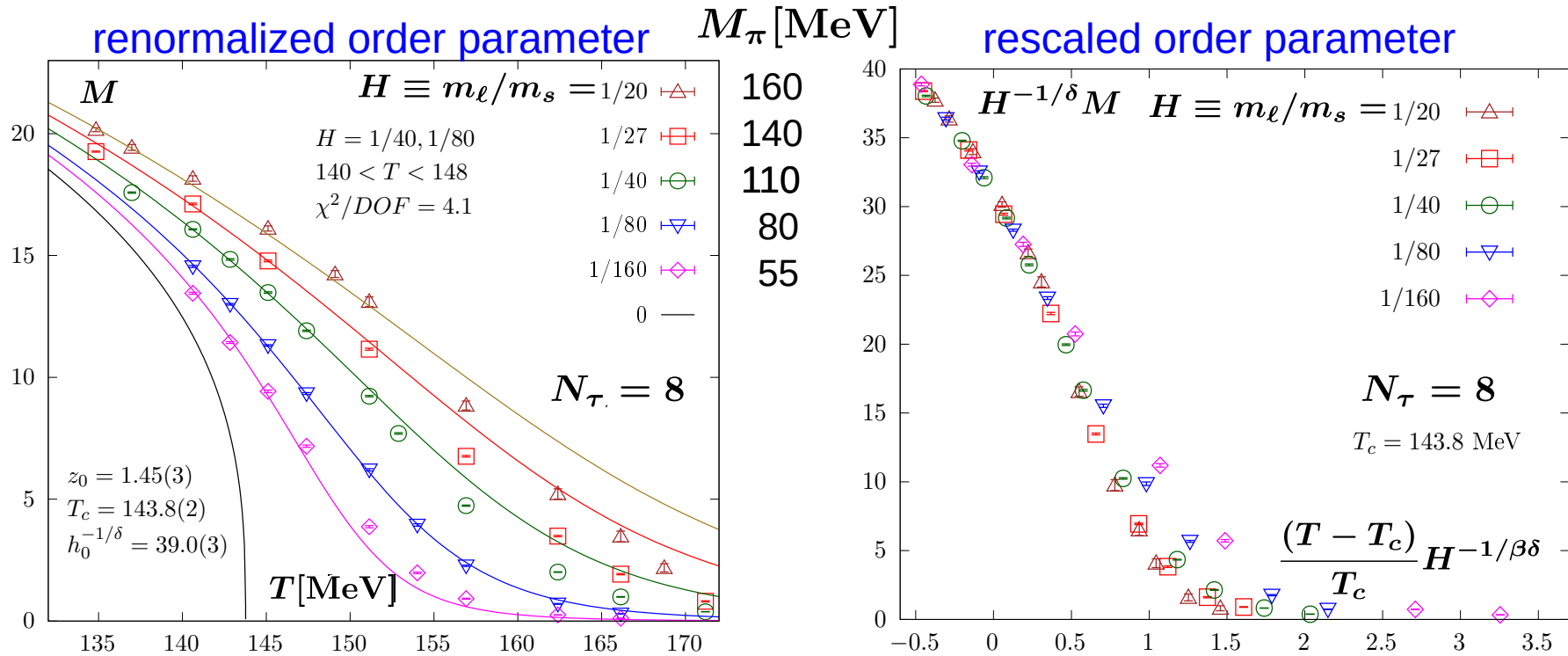
$$z_{t,M} = 0.629(10)$$

Chiral symmetry restoration

$m_{u,d} \rightarrow 0 \quad \longrightarrow \quad SU(2)_L \times SU(2)_R$ unbroken for $T \geq T_c$

improved order parameter with clear connection to universal scaling functions:

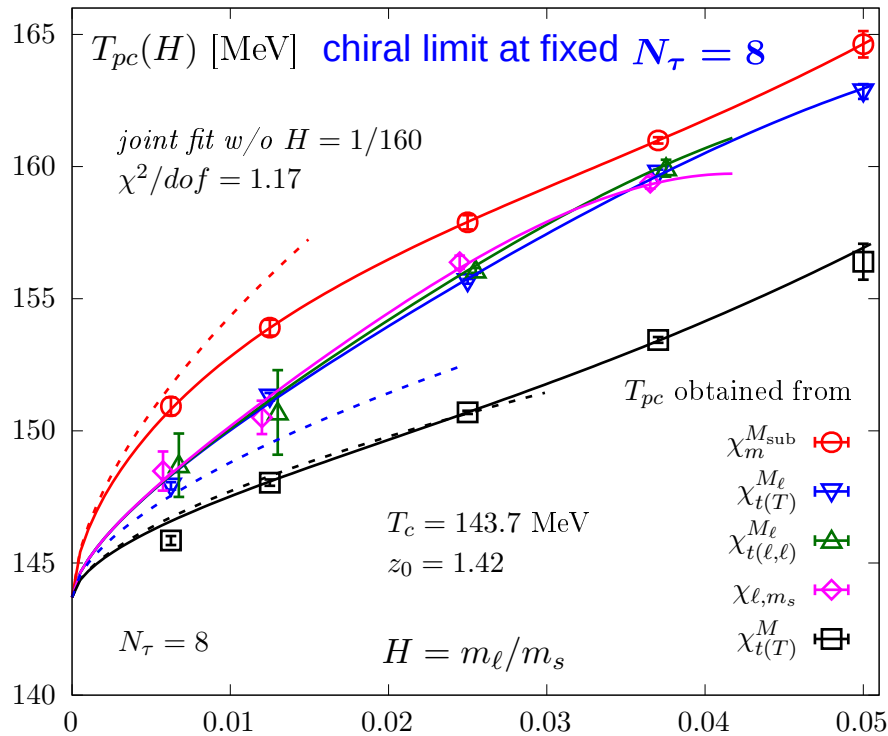
$$M \equiv M_\ell - H\chi_\ell = h^{1/\delta} (f_G(z) - f_\chi(z)) + \tilde{f}_r(T, H)$$



H.T.Ding, O. Kaczmarek, FK, P. Petreczky, Mugdha Sarkar,
C. Schmidt, Sipaz Sharma, arXiv:2403.09390

Pseudo-critical and critical temperatures

$$T_{pc}(H) = T_c + z_x T_c H^{1/\beta\delta}, \quad x = \text{peak}, \delta, \dots$$

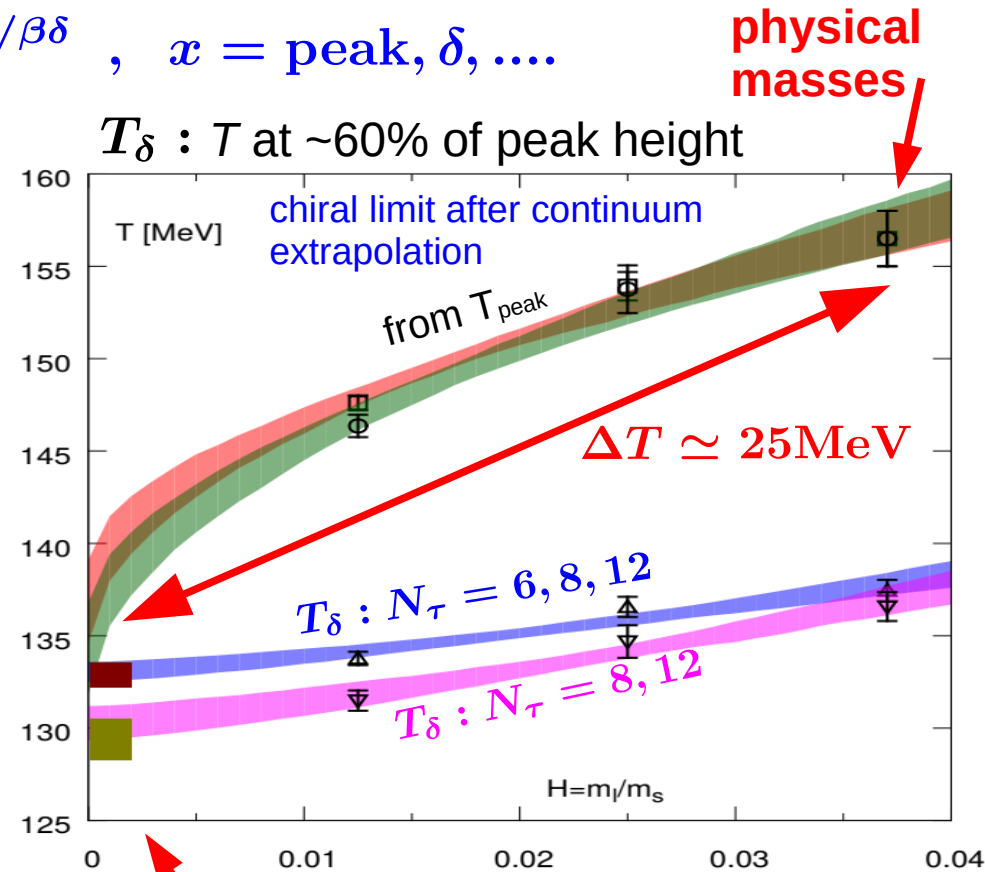


H.T.Ding, O. Kaczmarek, FK, P. Petreczky,
Mugdha Sarkar, C. Schmidt, Sipaz Sharma,
arXiv:2403.09390

physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5) \text{ MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

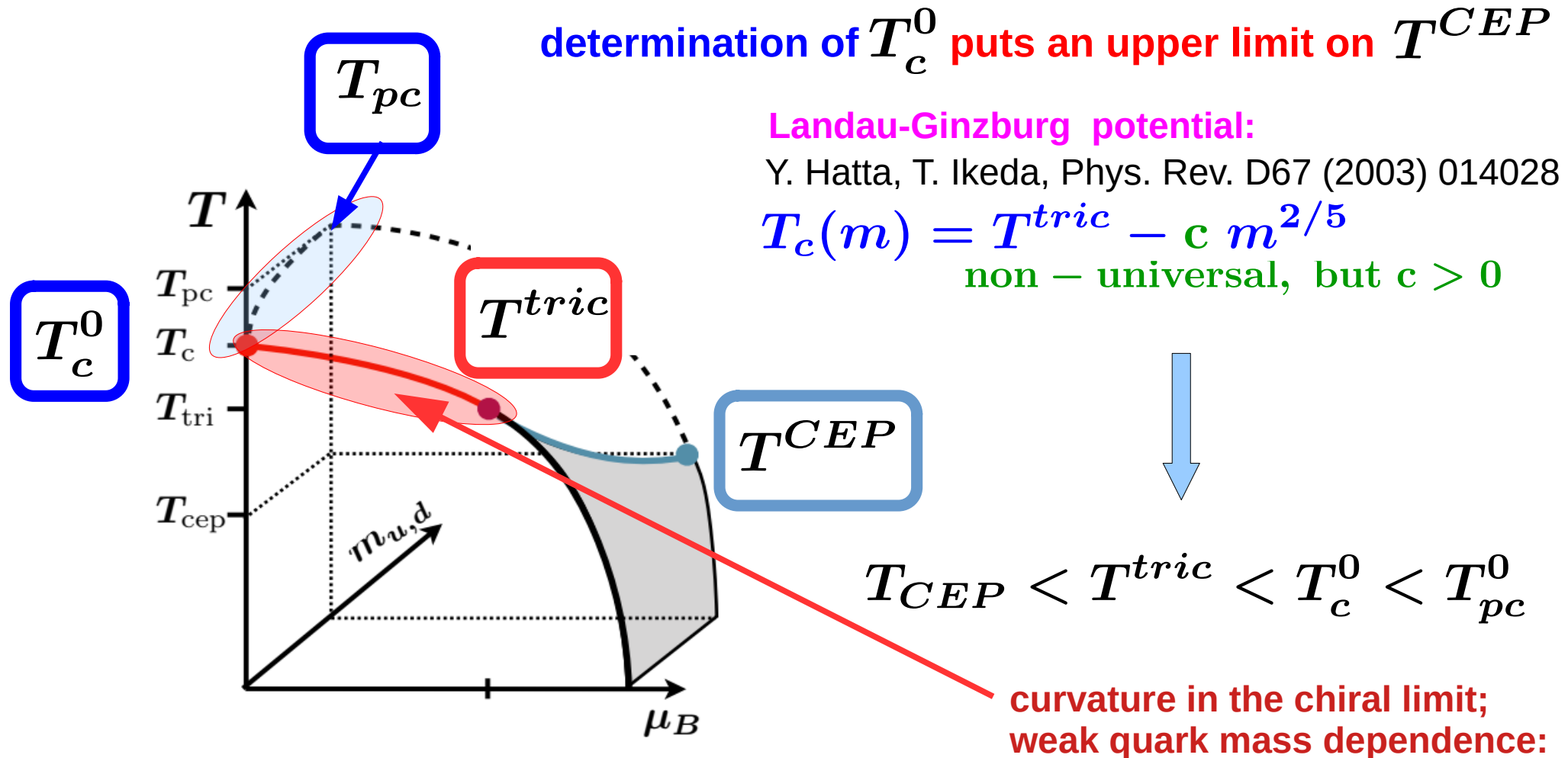


chiral limit extrapolations

$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

The Chiral **PHASE TRANSITION** in (2+1)-flavor QCD at non-vanishing chemical potential



Random Matrix Model A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007

QCD motivated M. Stephanov, Phys. Rev. D73 (2006) 094508

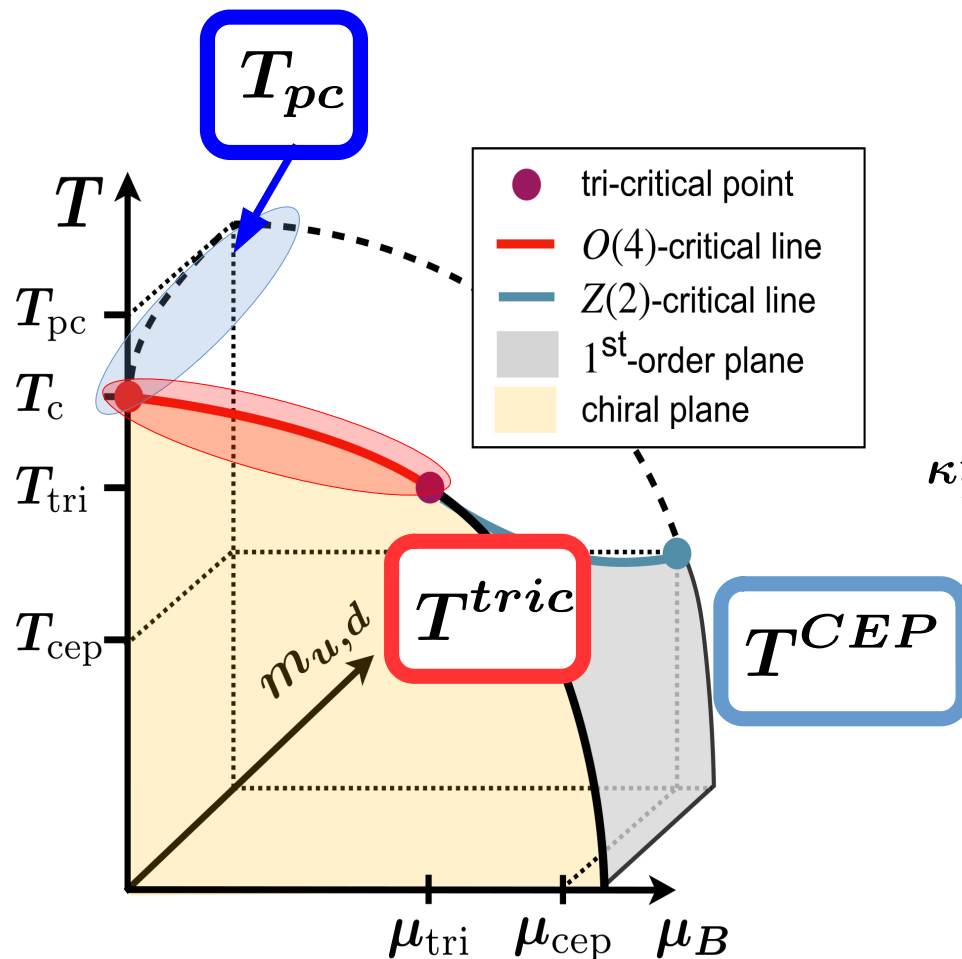
NJL M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

H.-T. Ding et al. PRD 109 (2024) 114516

**Critical temperatures at non-zero chemical potential
– curvature of the critical line in the chiral limit –**

$$T_{pc,x}(H) = T_c^0 \left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} - \kappa_2^B (\mu_B/T)^2 + \dots \right) \quad , \quad H = \frac{m_\ell}{m_s} \quad , \quad z_0 = \frac{h_0^{1/\beta\delta}}{t_0}$$

$$T_c(\mu_B, \dots) = T_c^0 \left(1 - \kappa_2^B \left(\frac{\mu_B}{T} \right)^2 - \kappa_{11}^{BS} \frac{\mu_B}{T} \frac{\mu_S}{T} - \kappa_2^S \left(\frac{\mu_S}{T} \right)^2 + \dots \right)$$



curvature coefficients

$$\kappa_2^f(H) = \frac{1}{2T_c} \left(\frac{\partial^2 M_\ell / \partial \hat{\mu}_f^2}{\partial M_\ell / \partial T} \right)_{(T_c, \vec{\mu}=0)}$$

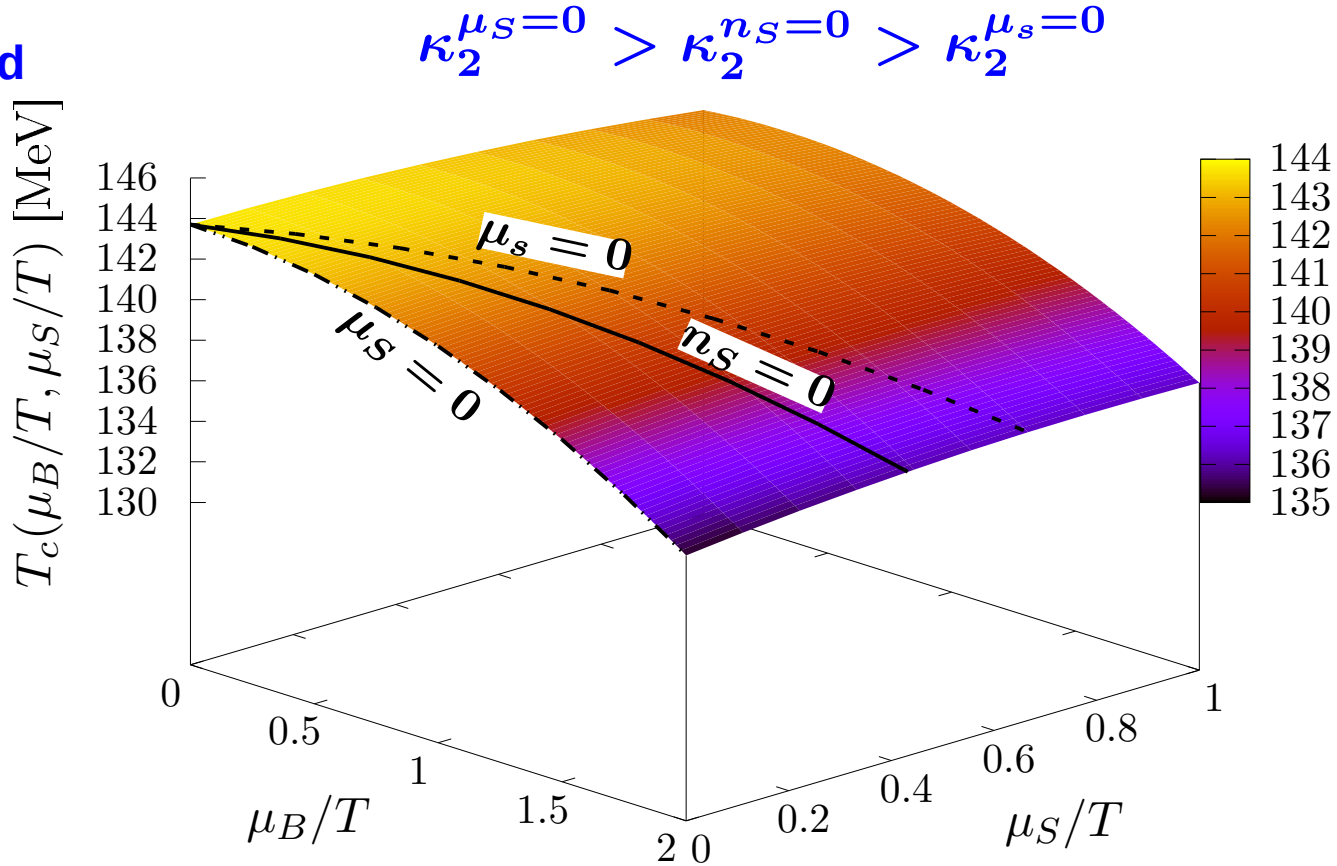
$f = B, S$

$$\kappa_{11}^{BS}(H) = \frac{1}{2T_c} \left(\frac{\partial^2 M_\ell / \partial \hat{\mu}_B \partial \hat{\mu}_S}{\partial M_\ell / \partial T} \right)_{(T_c, \vec{\mu}=0)}$$

H.T.Ding, O. Kaczmarek, FK, P. Petreczky,
Mugdha Sarkar, C. Schmidt, Sipaz Sharma,
arXiv:2403.09390

The critical surface in the $\mu_B - \mu_S$ plane

chiral limit
extrapolated



physical mass
results

$$\kappa_2^B = 0.015(4)$$

$$\kappa_2^{n_S=0} = 0.012(2)$$

HotQCD:
arXiv:1812.08235

$$\mu_S = 0 : \quad \kappa_2^{\mu_S=0} = \kappa_2^B = 0.015(1)$$

$$n_S = 0 : \quad \kappa_2^{n_S=0} = \kappa_2^B \left(1 + s_1^2 \frac{\kappa_2^S}{\kappa_2^B} + 2s_1 \frac{\kappa_{11}^{BS}}{\kappa_2^B} \right) = 0.895(31) \kappa_2^B$$

$$\mu_s = 0 : \quad \kappa_2^{\mu_s=0} = \kappa_2^B \left(1 + \frac{1}{9} \frac{\kappa_2^S}{\kappa_2^B} + \frac{2}{3} \frac{\kappa_{11}^{BS}}{\kappa_2^B} \right) = 0.972(19) \kappa_2^{n_S=0}$$

The Chiral **PHASE TRANSITION** in (2+1)-flavor QCD at non-vanishing chemical potential

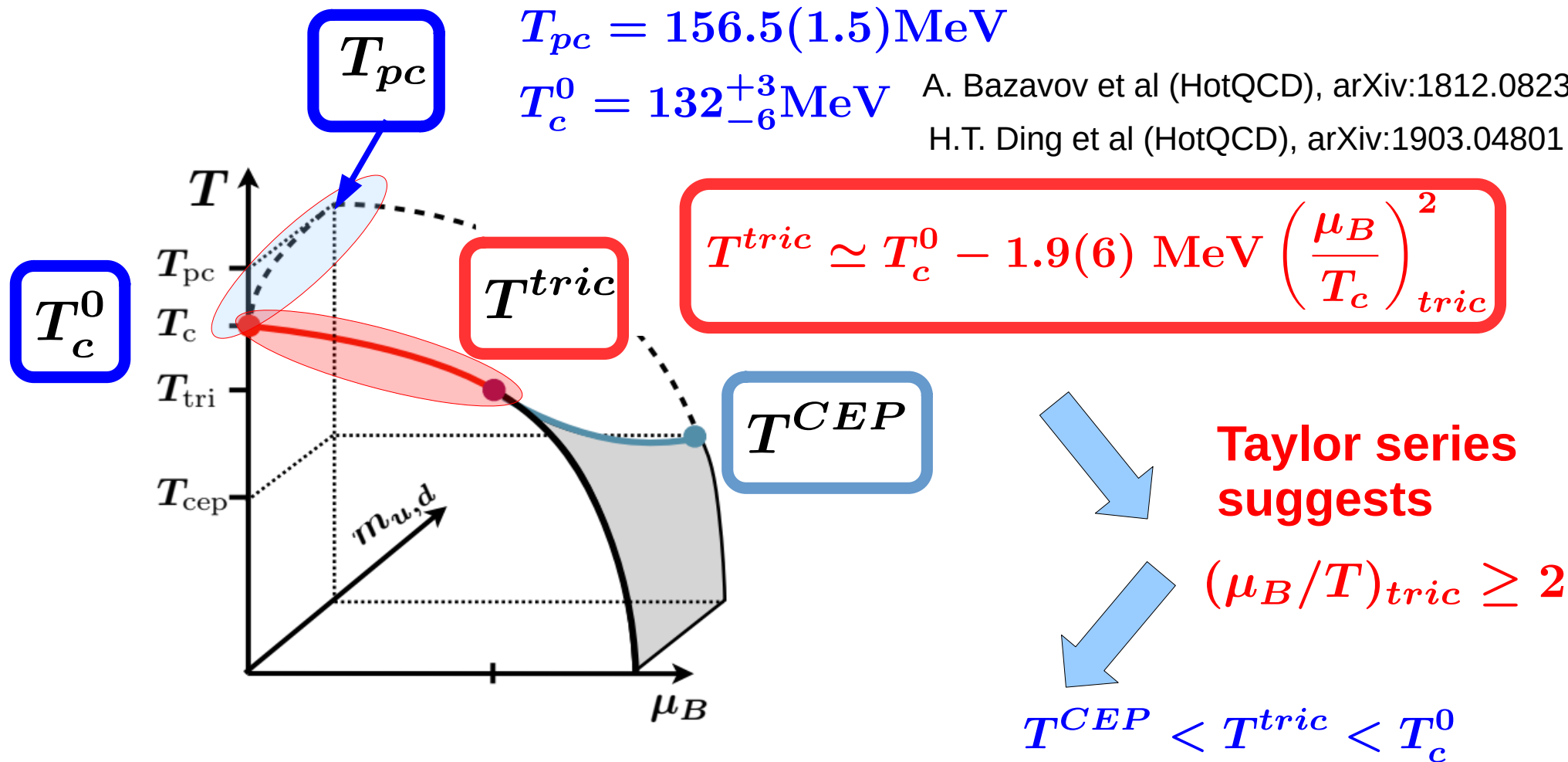
determination of T_c^0 puts an upper limit on T^{CEP}

$$T_{pc} = 156.5(1.5)\text{MeV}$$

$$T_c^0 = 132_{-6}^{+3}\text{MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801



the temperature range below T_c^0 is most important for getting information on a possible CEP

upper limit on T^{CEP} puts constraint on HIC searches for the CEP

– pseudo-critical temperatures at physical quark mass values

$$T_{pc}(\mu_B) = 156.5(1.5)\text{MeV} \left(1 - 0.012(4) \left(\frac{\mu_B}{T} \right)^2 - / + \dots \right)$$

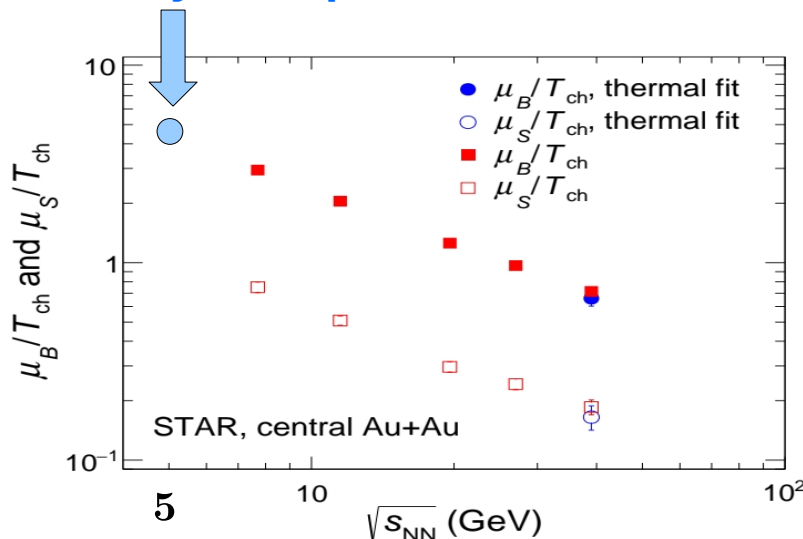
$\rightarrow$ to reach need $T < 132\text{MeV}$ $T < 125\text{MeV}$ $T < 110\text{MeV}$
 $\mu_B/T \simeq 3.6$ $\mu_B/T \simeq 4$ $\mu_B/T \simeq 5$

$$\mu_B \geq 476\text{MeV}$$

$$\mu_B \geq 500\text{MeV}$$

$$\mu_B \gtrsim 550\text{MeV}$$

my extrap.



**Search for the Critical End Point
requires beam energies**

$$\sqrt{s_{NN}} \leq 5 \text{ GeV}$$

STAR freeze-out parameter, arXiv:1906.03732
 also: HADES collaboration, arXiv:1512.07070

upper limit on T^{CEP} puts constraint on HIC searches for the CEP

– pseudo-critical temperatures at physical quark mass values

$$T_{pc}(\mu_B) = 156.5(1.5)\text{MeV} \left(1 - 0.012(4) \left(\frac{\mu_B}{T} \right)^2 - / + \dots \right)$$

to reach
need

$$T < 132\text{MeV}$$

$$\mu_B/T \simeq 3.6$$

$$T < 125\text{MeV}$$

$$\mu_B/T \simeq 4$$

$$T < 110\text{MeV}$$

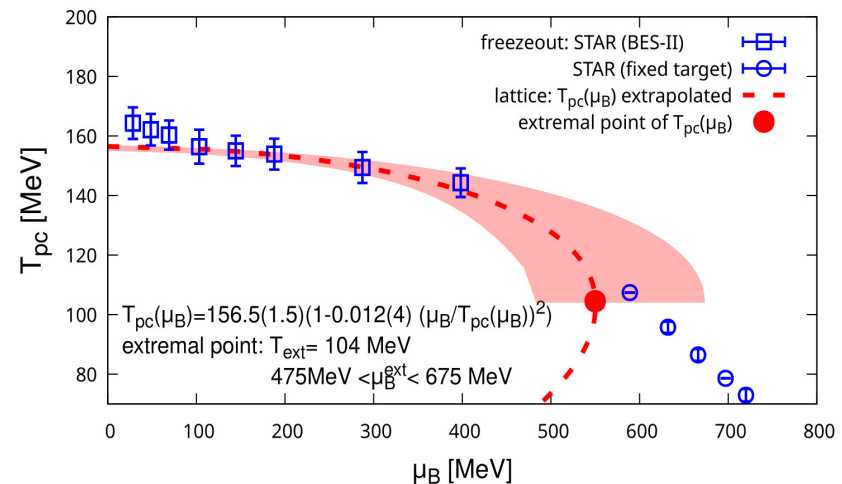
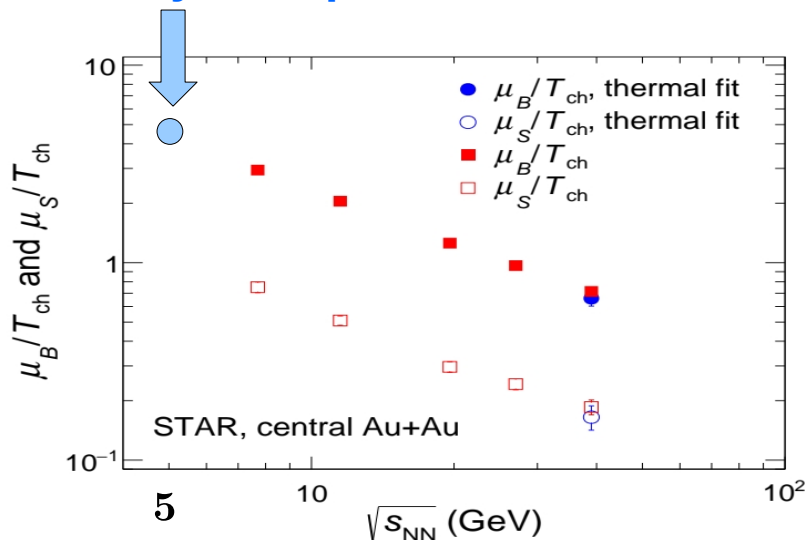
$$\mu_B/T \simeq 5$$

$$\mu_B \geq 476\text{MeV}$$

$$\mu_B \geq 500\text{MeV}$$

$$\mu_B \gtrsim 550\text{MeV}$$

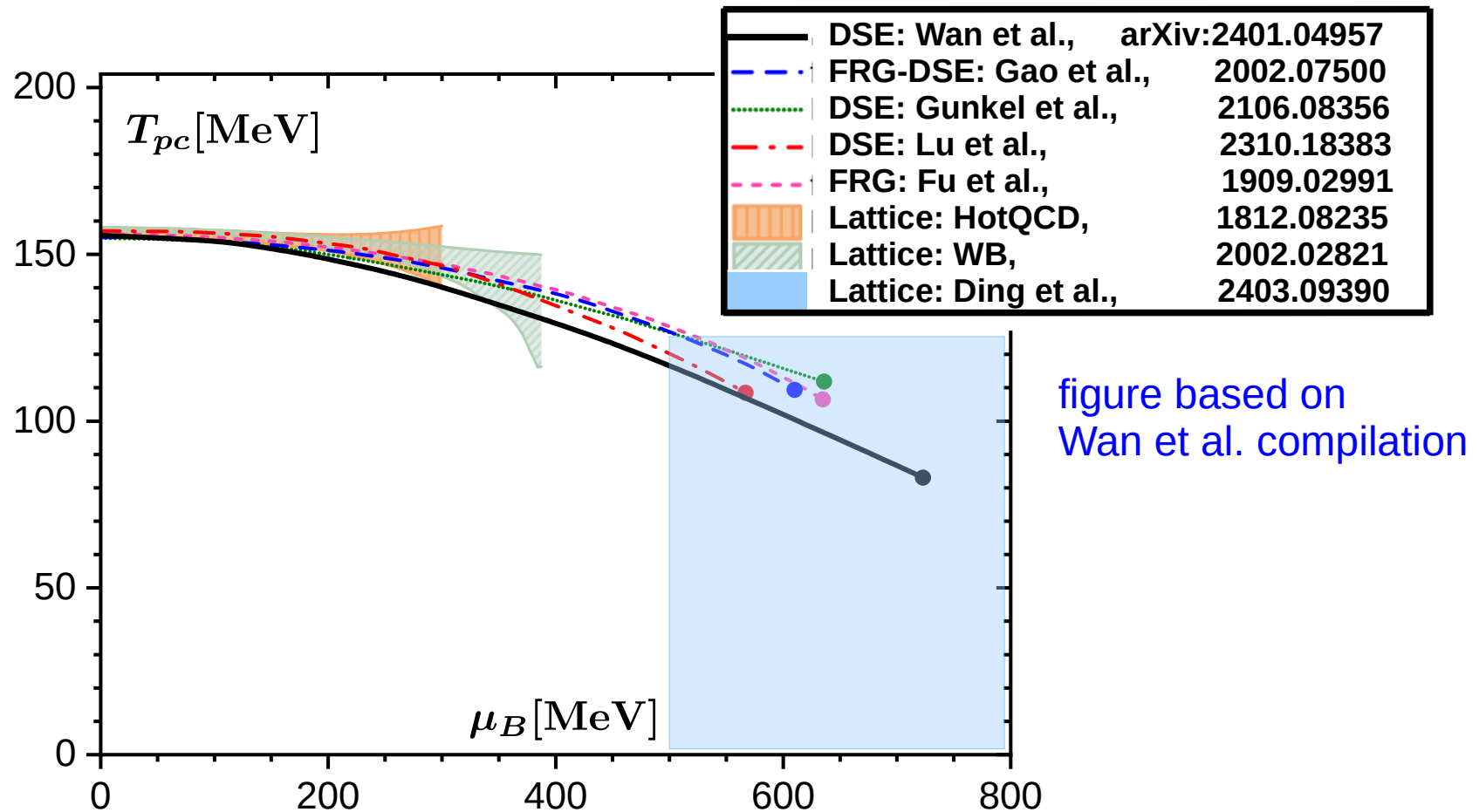
my extrap.



Constraint on allowed region for location of the CEP

Ding,..Mugdha Sarkar,... et al., arXiv:2403.09390

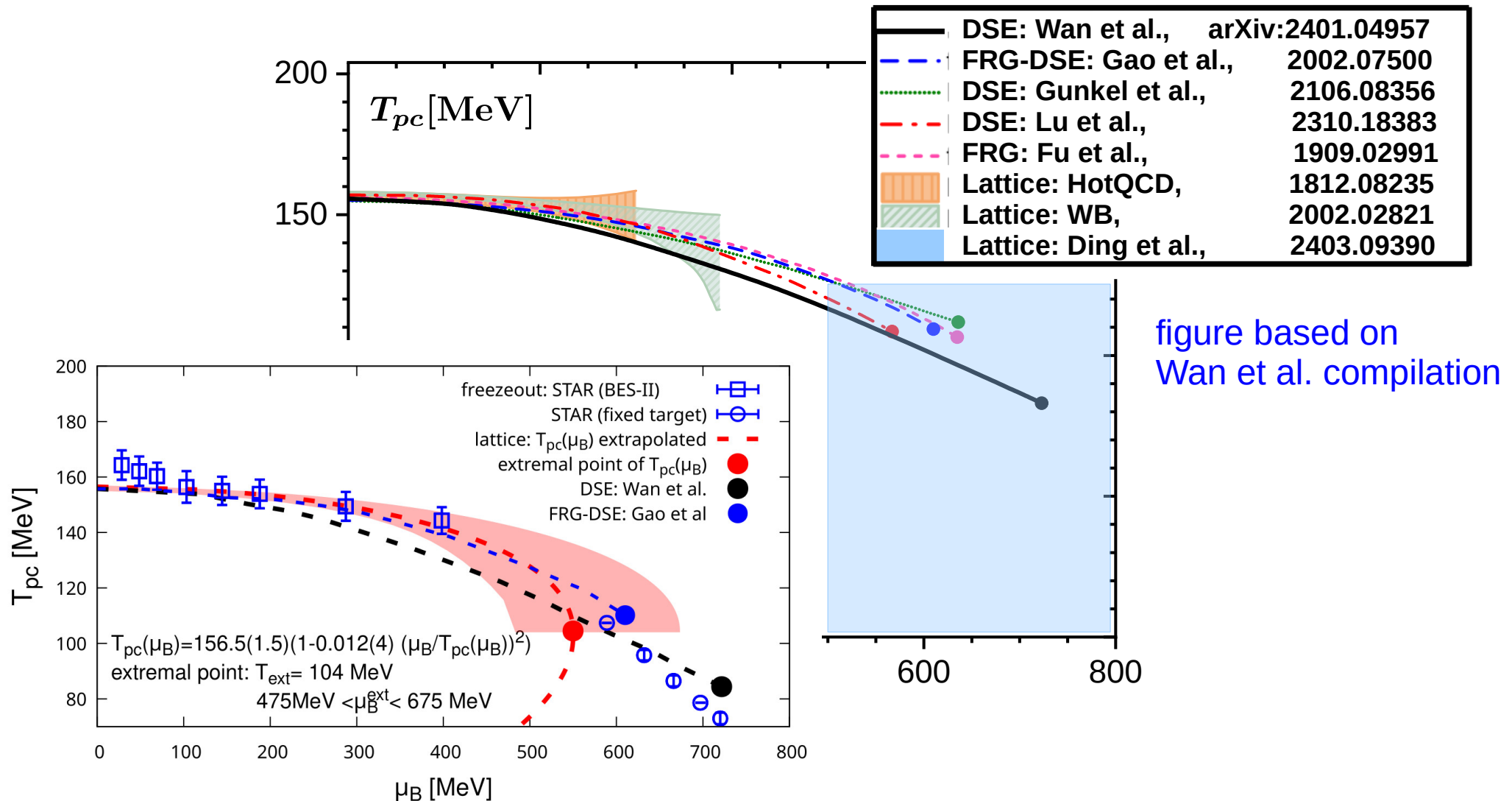
– based on determination of $T_c(\mu_B)$ and apparent convergence of Taylor series for $\mu_B/T \leq 2$



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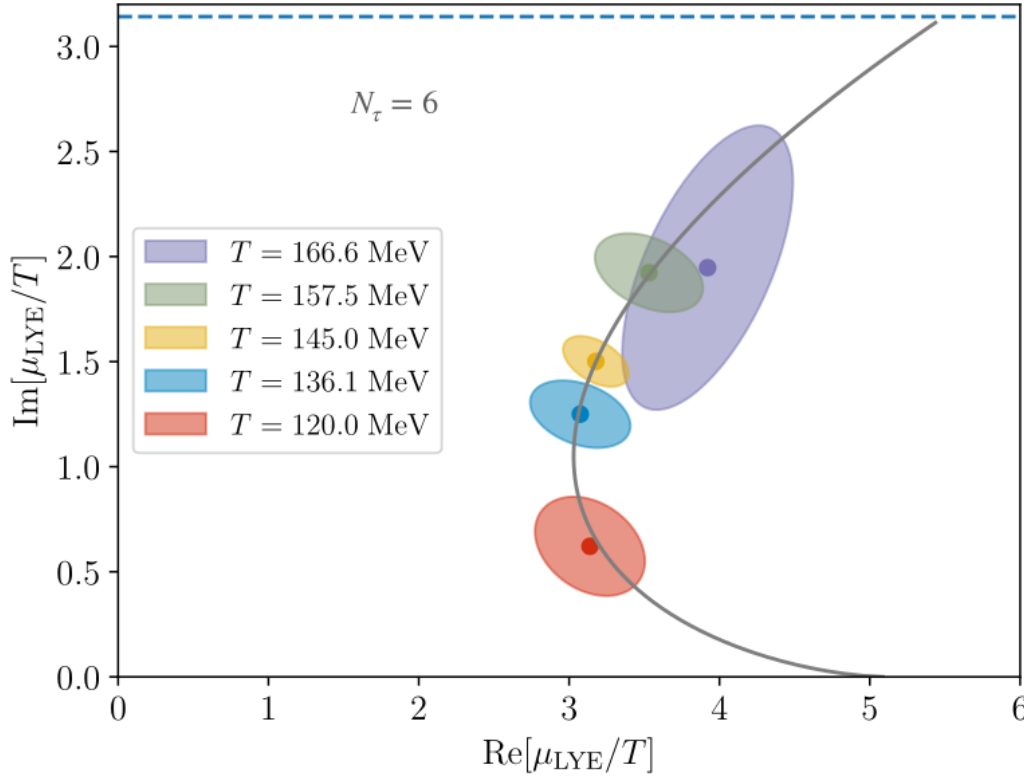
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Lee-Yang edge singularities from imaginary chemical potential calculations

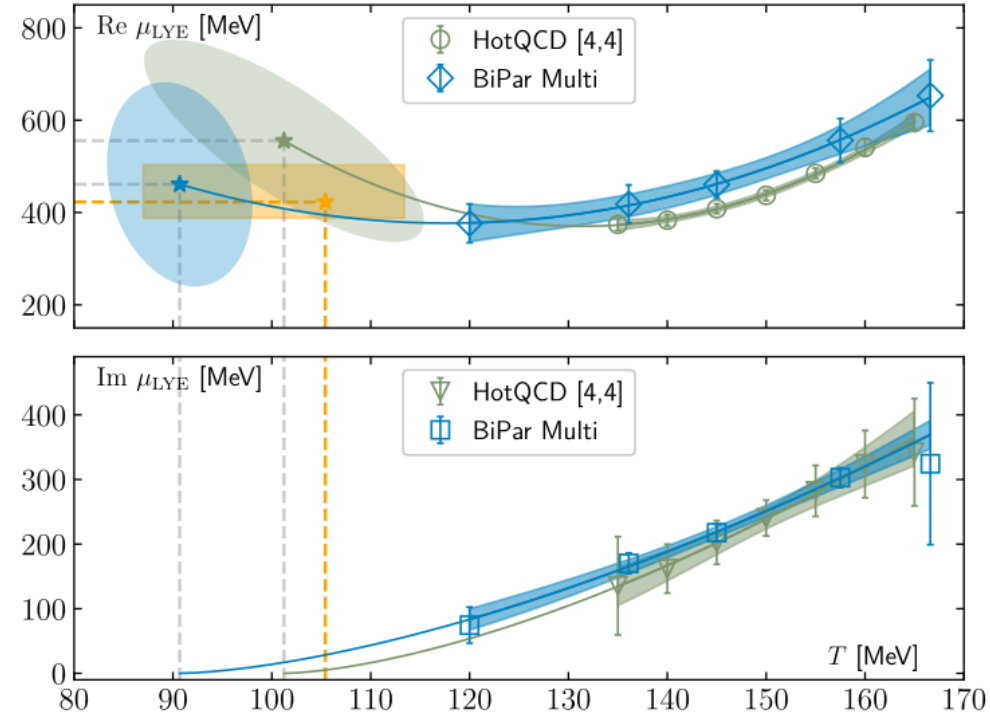
slide: C. Schmidt, ECT*, Trento, Italy, Sep 8-12, 2025

❖ Perform one fit for $N_\tau = 8$ (LT=4) and $\mathcal{O}(10^5)$ fits for $N_\tau = 6$ (LT=6)



❖ Ellipses show 1σ confidence region, using the Pearson correlation coefficient

❖ $N_\tau = 6$ singularities shown here are chosen on the basis of the χ^2 of the scaling fit (“best fit”)



$$\text{Re}[\mu_{\text{LYE}}] = \mu_B^{\text{CEP}} + c_1 \Delta T + c_2 \Delta T^2$$

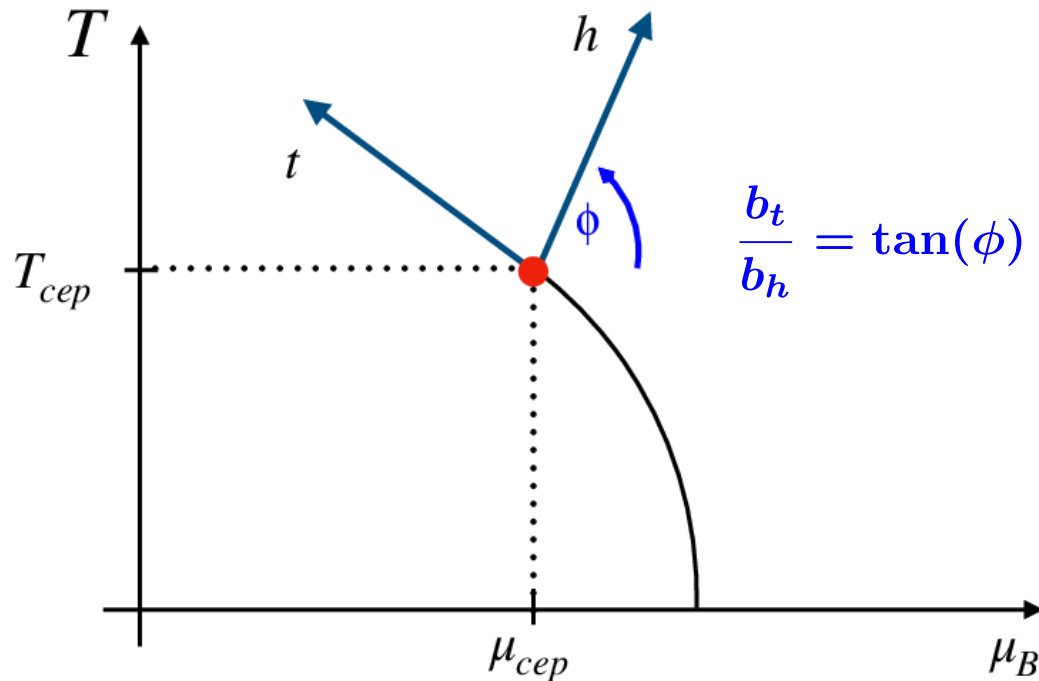
$$\text{Im}[\mu_{\text{LYE}}] = c_3 \Delta T^{\beta\delta},$$

❖ Orange box shows the AIC weighted result for $N_\tau = 6$, based on $\mathcal{O}(10^5)$ scaling fits

D.A. Clarke (Bielefeld-Parma),
arXiv:2405.10196

$$(T^{\text{cep}}, \mu_B^{\text{cep}}) = (105_{-18}^{+8} \text{ MeV}, 422_{-35}^{+80} \text{ MeV})$$

Universal critical behavior of QCD in the vicinity of the CEP



$$t \sim \frac{T - T_{cep}}{T_{cep}} + b_t \frac{\mu^2 - \mu_{cep}^2}{\mu_{cep}^2}$$

$$h \sim \frac{T - T_{cep}}{T_{cep}} + b_h \frac{\mu^2 - \mu_{cep}^2}{\mu_{cep}^2}$$

Note: symmetry restored region, $z > 0$, corresponds to $t > 0$

conserved charge fluctuations: $\chi_n^B = - \left. \frac{\partial^n f / T^4}{\partial \hat{\mu}_B^n} \right|, \quad \hat{\mu}_B \equiv \frac{\mu_B}{T}$

singular contribution in Z(2) universality class: $\chi_n^B = - \frac{\partial^n f_{sing} / T^4}{\partial \hat{\mu}_B^n}$

$$f_{sing} = h^{2-\alpha} f_f(z), \quad z = z_0 \frac{t(T, \mu)}{|h(T, \mu)|^{1/\beta\delta}}$$

Critical behavior of QCD in the vicinity of the CEP

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a **universal scaling function**

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V, T, \vec{\mu})$$

singular

regular

$$t \sim (T - T_c)/T_c + b_t(\mu^2 - \mu_c^2)/\mu_c^2$$

$$h \sim (T - T_c)/T_c + b_h(\mu^2 - \mu_c^2)/\mu_c^2$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic

$$\frac{\partial^2 \ln Z}{\partial h^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1}$$

↑

~ -0.79

divergence: strong

mixed

$$\frac{\partial^2 \ln Z}{\partial h \partial t}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta}$$

↑

~ -0.43

moderate

thermal

$$\frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$$

↑

~ -0.07

very moderate

Z(2) critical exponents

$$\alpha = +0.108$$

$$\beta = 0.326$$

$$\delta = 4.805$$

Critical behavior of QCD in the vicinity of the CEP

- universal critical behavior in cumulants in the vicinity of CEP is quite different from that in the chiral limit
- already second order cumulants diverge, dominant singular behavior of cumulants is "magnetization like" at CEP whereas it is "energy like" in the chiral limit
- HOWEVER: sub-leading corrections to the magnetization-like divergence are energy-like. Their calculation requires knowledge of the universal free energy scaling function (C. Nonaka, M Asakawa, Phys.Rev.C71(2005)044904), e.g.

$$\chi_1(T, \mu) = Ab_h h^{1/\delta} \left(f_G(z) + \frac{b_t}{b_h} f'_f(z) h^{1-1/\beta\delta} \right)$$

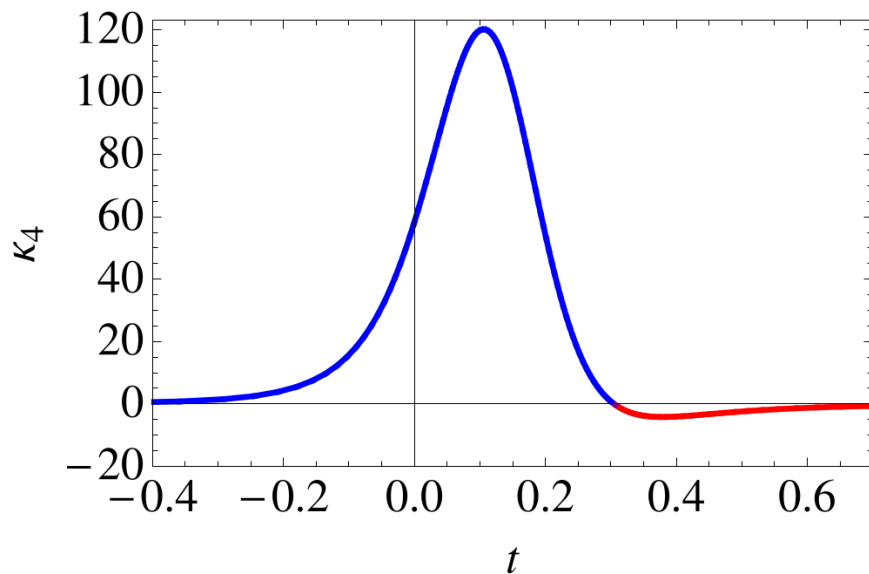
although parametrically suppressed, it can give a sizable modification of the scaling function, in particular when

$\phi \simeq \pi/2$ cumulants almost energy-like
i.e. divergence in $\chi_2(T, \mu_B)$
will show up only very close to CEP

J. Goswami, FK, S. Mitra, M. Sarkar, Sipaz Sharma, C.Schmidt, work in progress

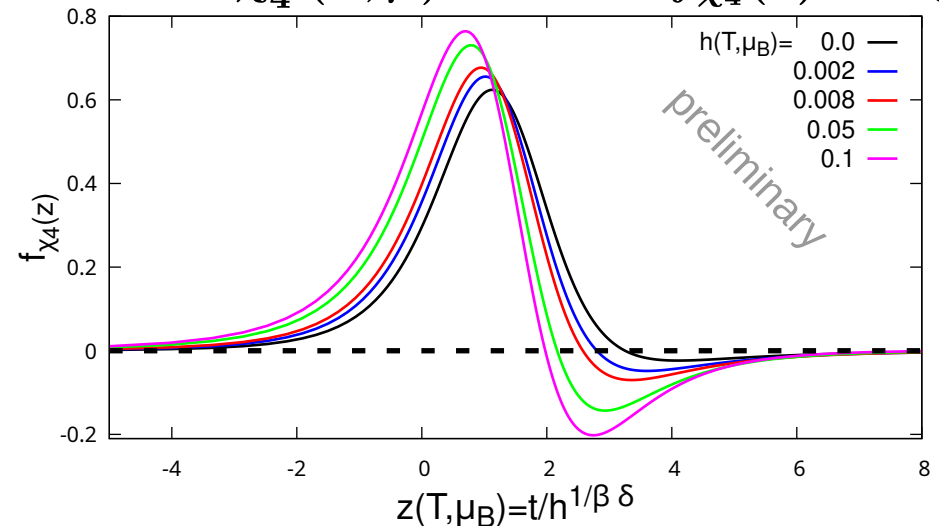
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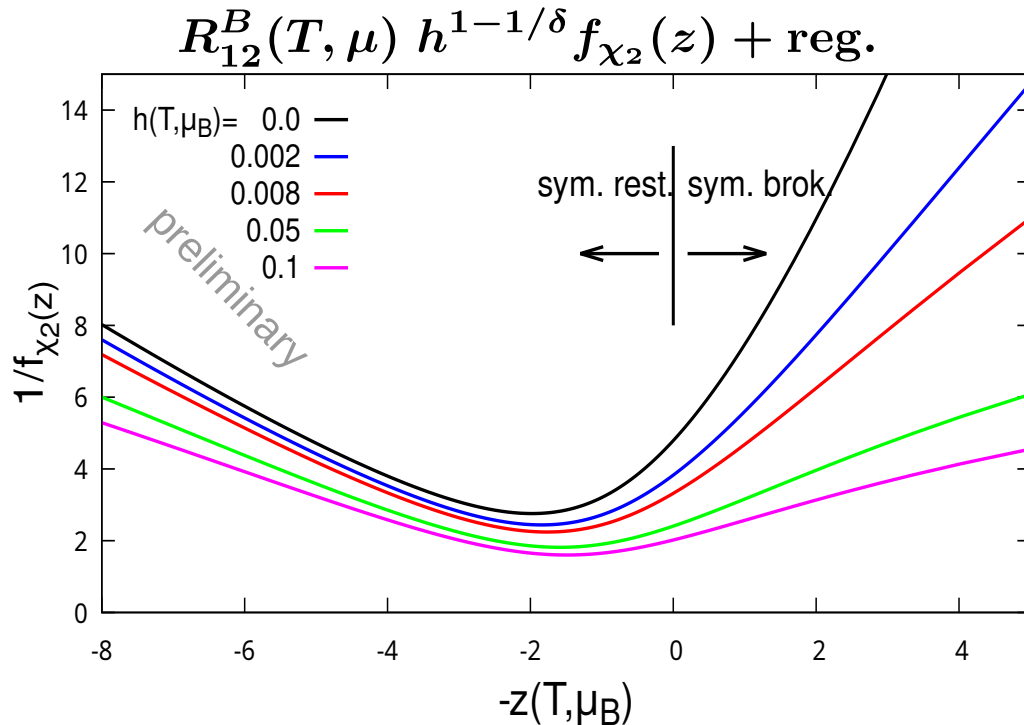
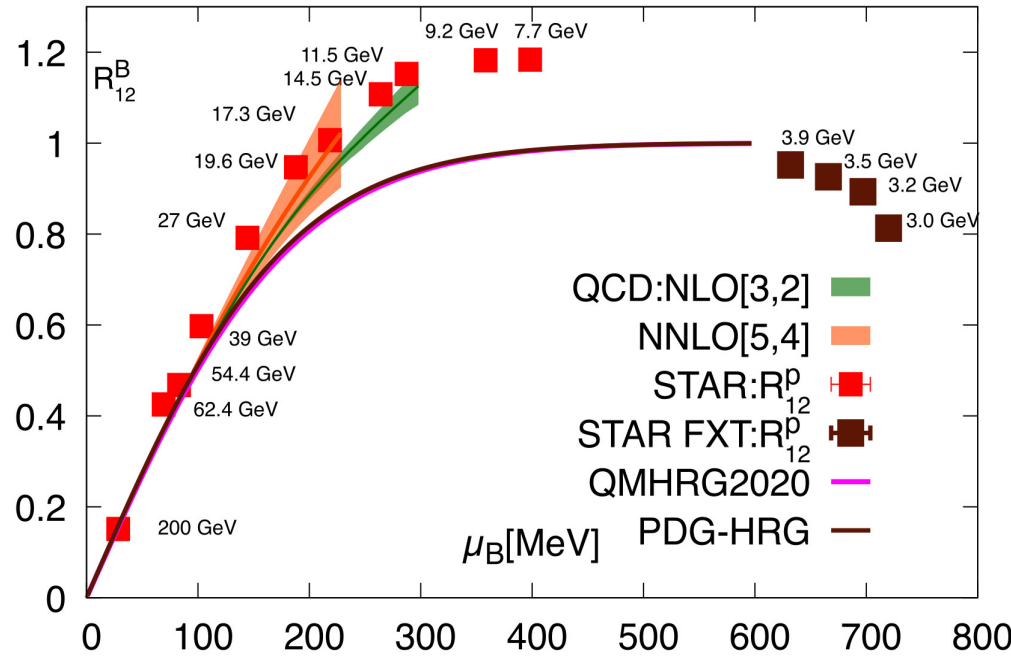
M.A. Stephanov, PRL, 107, (2011) 052301

taking into account sub-leading corrections $\chi_4^B(T, \mu) = h^{1/\delta-3} f_{\chi_4}(z) + \text{reg.}$

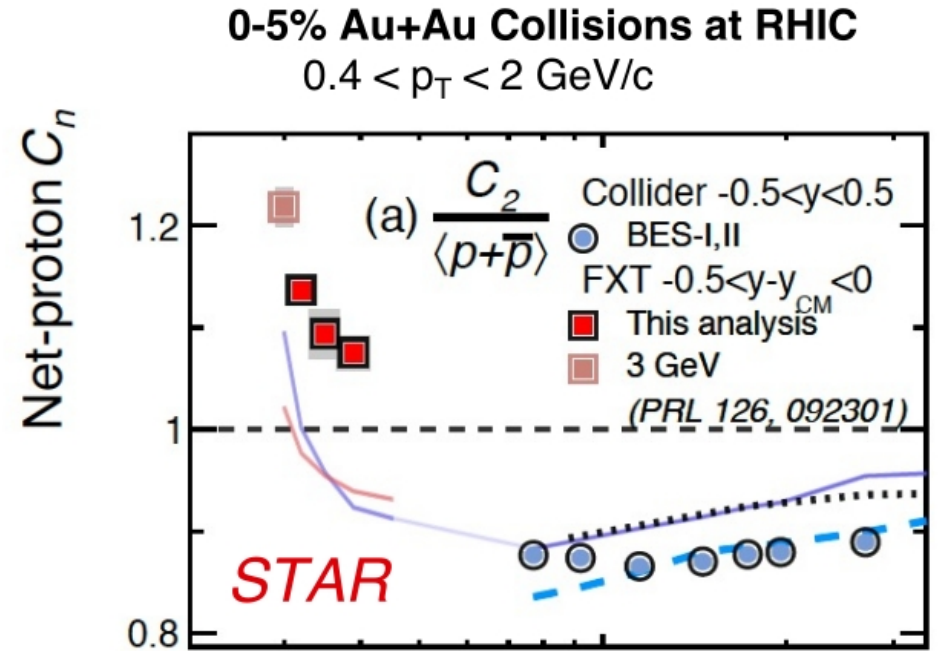


J. Goswami, FK, S. Mitra, M. Sarkar, Sipaz Sharma, C.Schmidt, work in progress

Inverse net-baryon number fluctuation ratio



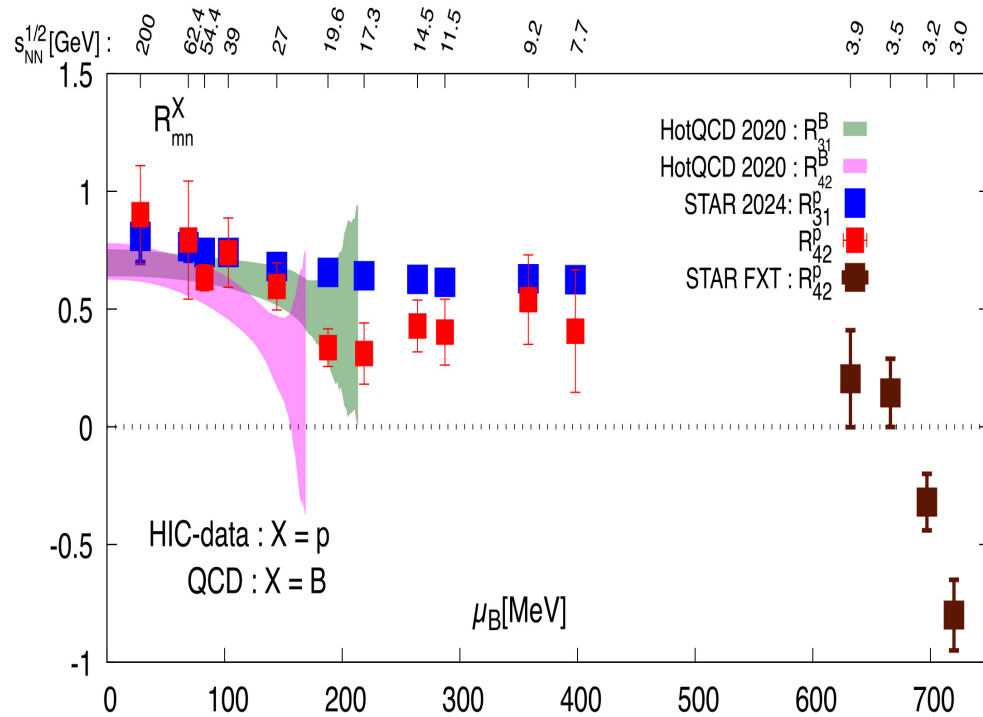
QCD on the pseudo-critical line vs. STAR data at freeze-out



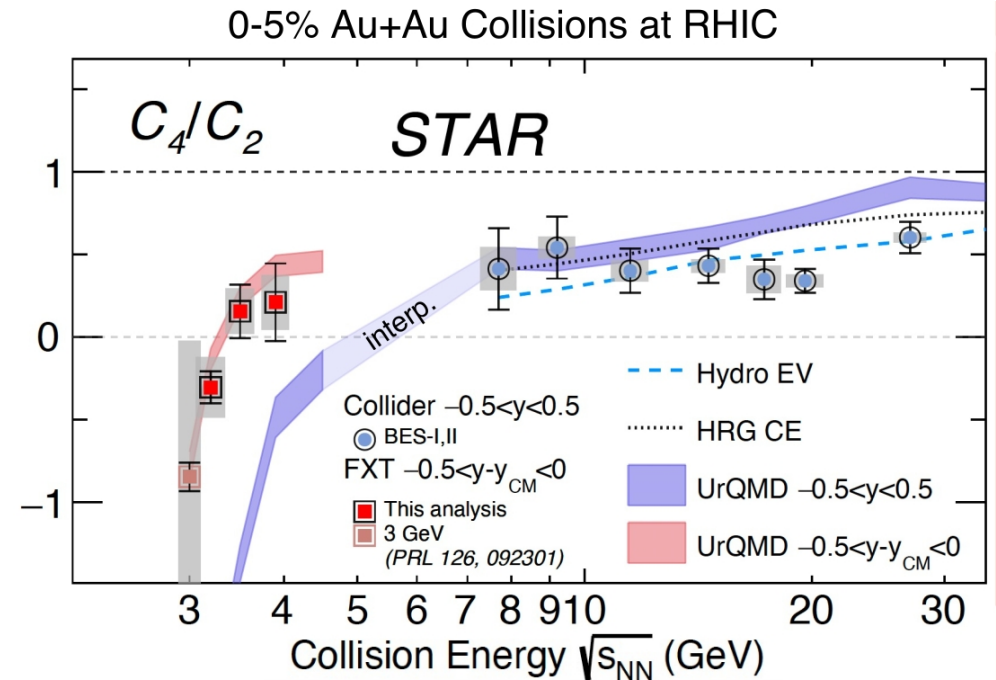
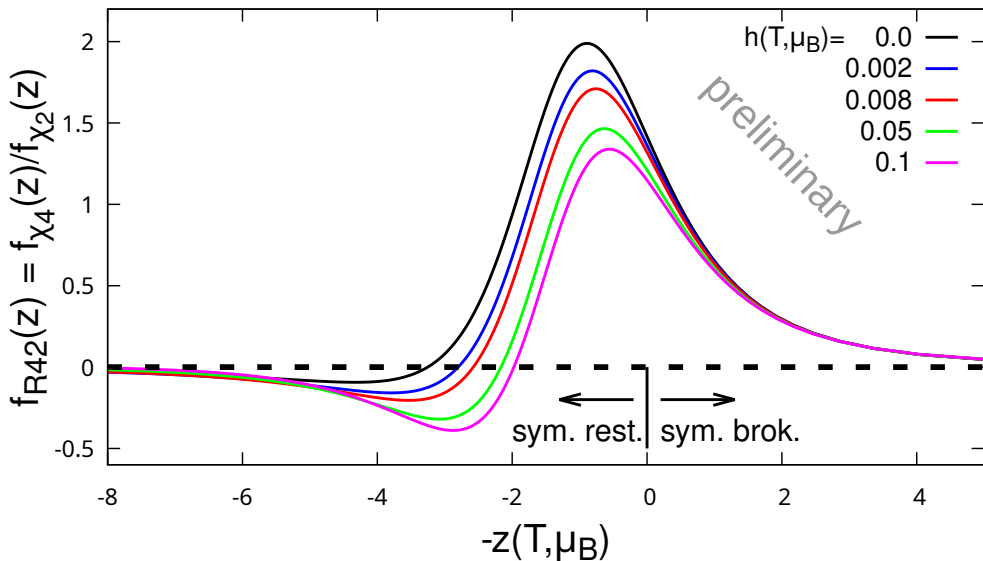
Z. Sweger (STAR Coll.),
Quark Matter 2025

Note: Already at 7.7 GeV the total to net proton number ratio is close to unity, STAR, PRL. 135 (2025) 142301

Kurtosis ratio: QCD on the pseudo-critical line vs. STAR data at freeze-out

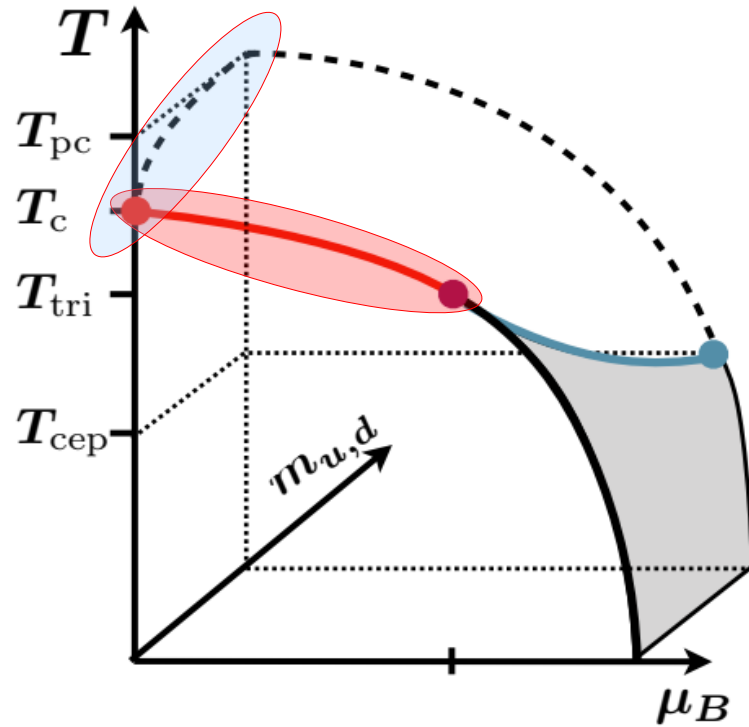


$$R_{42}^B(T, \mu) = h^{-2} f_{R42}(z) + \text{reg.}$$



Z. Sweger (STAR Coll.),
Quark Matter 2025

Conclusions



What we learned so far about the CEP in QCD from lattice QCD calculations:

I) the critical temperature is below $T_c = 132_{-6}^{+3}$ MeV

II) the corresponding critical chemical potential is likely to be above 500 MeV

→ Taylor expansions need to be resummed in order to reach higher μ_B/T without relying on extrapolations

- radius of convergence obtained from Taylor series agrees well with pole closest to the origin found from Padé approximants

- no CEP for $\mu_B/T \leq 2.5$

- CEP not in the BES-II range (in collider mode)

- EoS (pressure & number density) well controlled for

$\mu_B/T \leq 2.0 \forall T > 135$ MeV
(larger range for higher T)

$\mu_B \geq 500$ MeV must be explored

