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Baryon-Electric charge Correlations & Chemical Potentials as Probes of Magnetized QCD

Heng-Tong Ding
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HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu,

Phys. Rev. Lett. 132 (2024) 201903, and recent arXiv:2503.18467

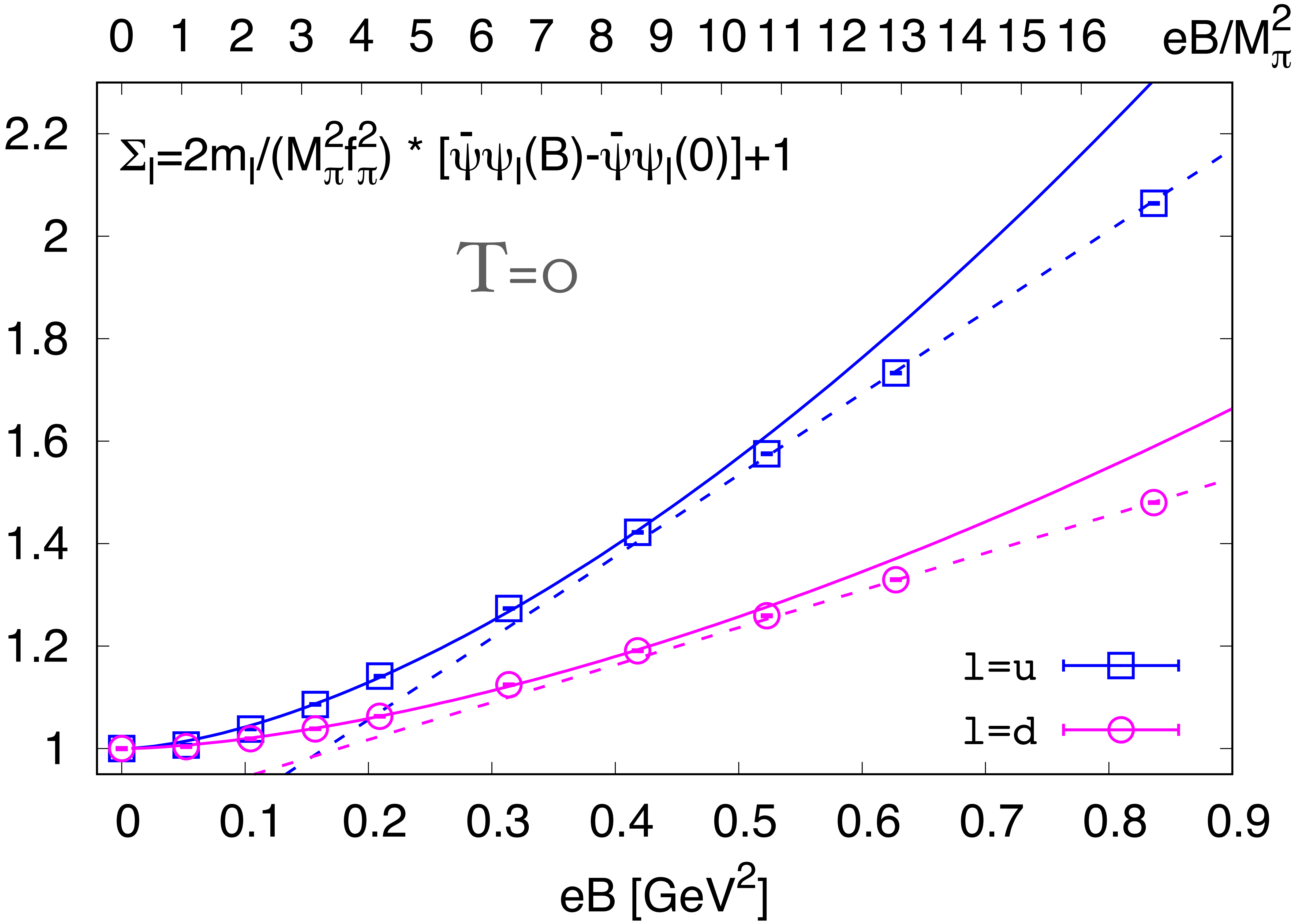
Physics of High Baryon Densities 2025

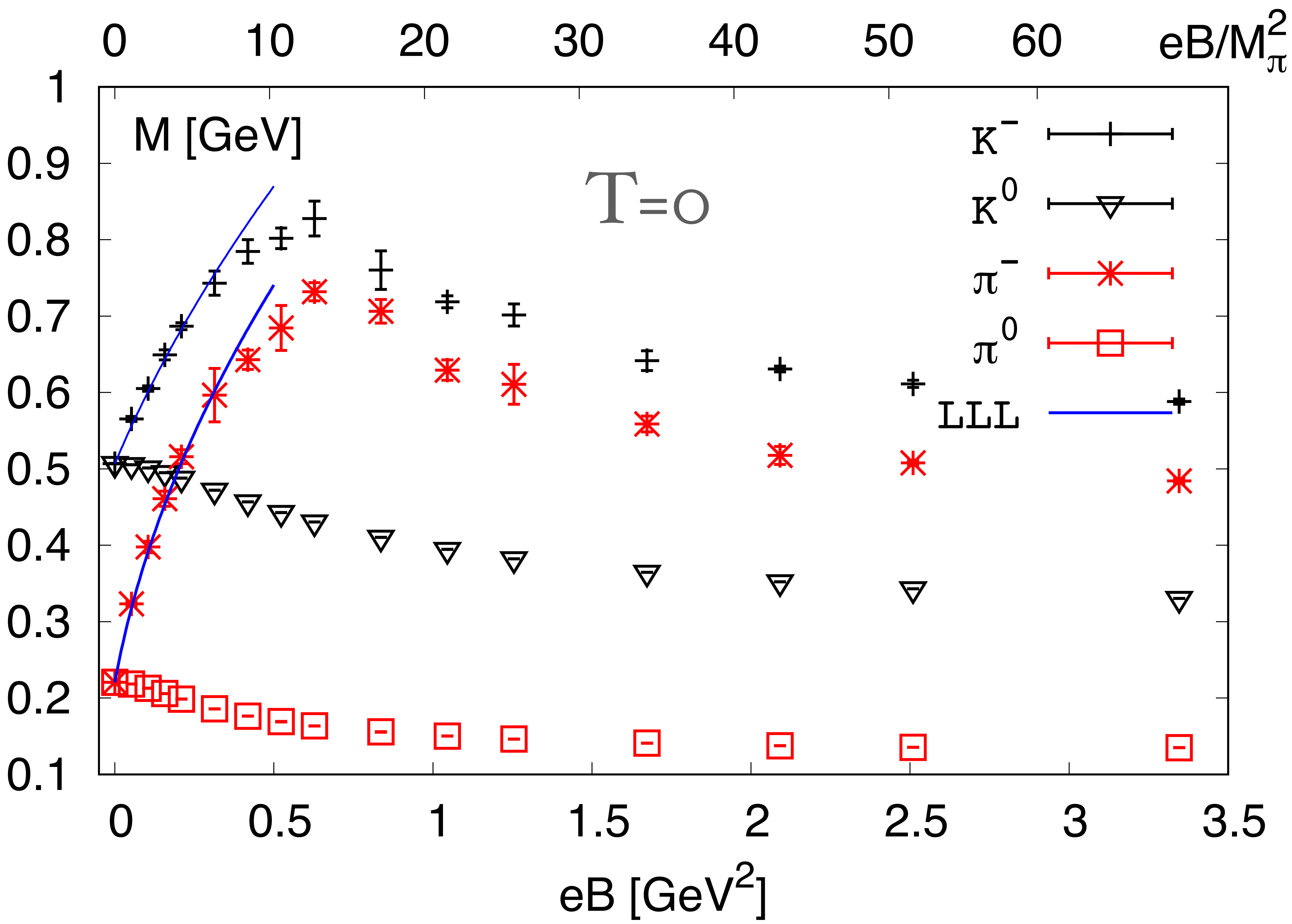
April 13-15, 2025 @ GSI, Darmstadt

Whether imprints of a strong magnetic field exist in the final stage of heavy-ion collisions?

Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates and meson masses

$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV, $32^3 \times 96$ lattice swith $a^{-1} \approx 1.7$ GeV and HISQ action





HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

See also in e.g. Bali et al., Phys.Rev.D86(2012)071502

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505, Luschevskaya et al., NPB 898 (2015) 627

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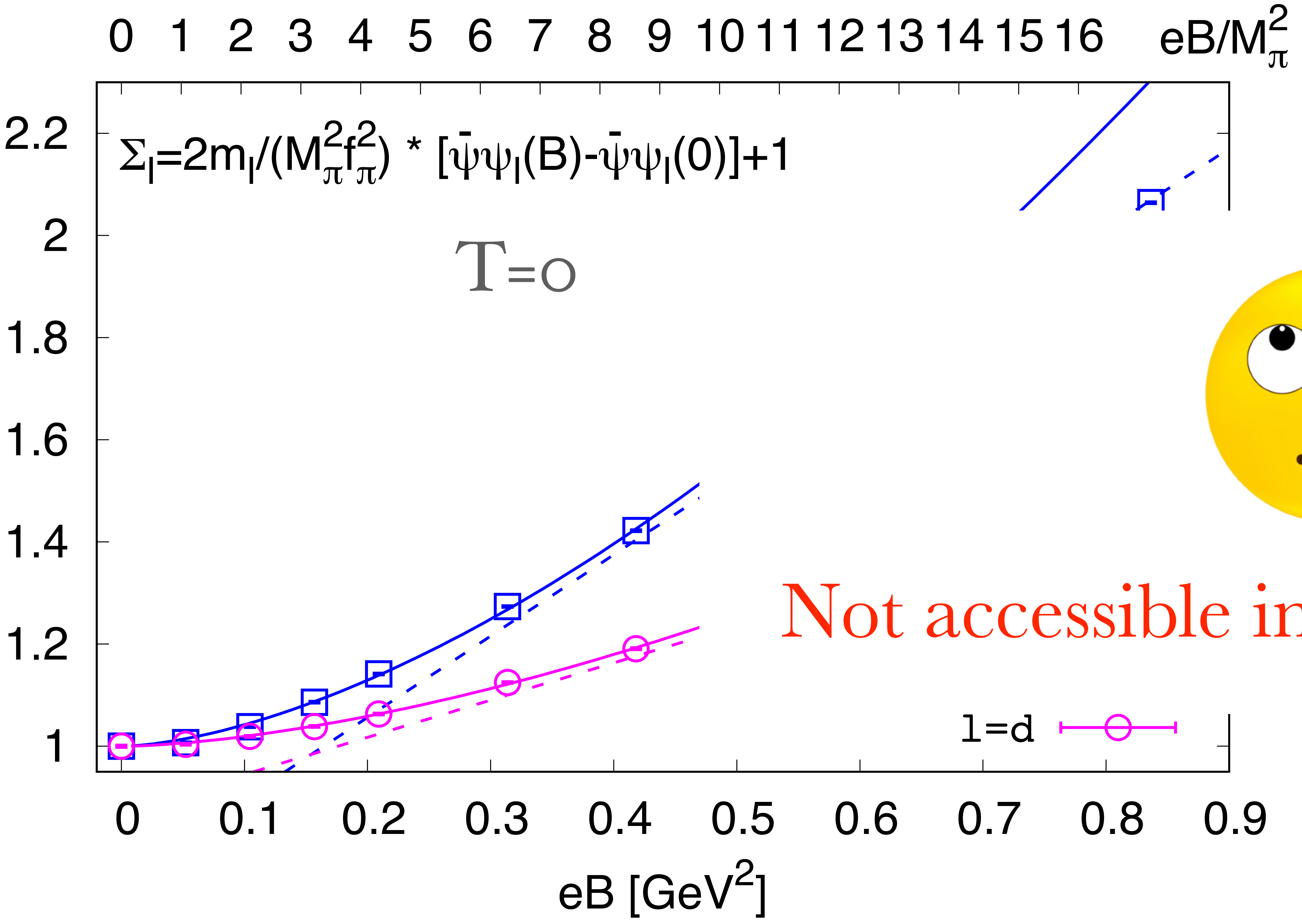

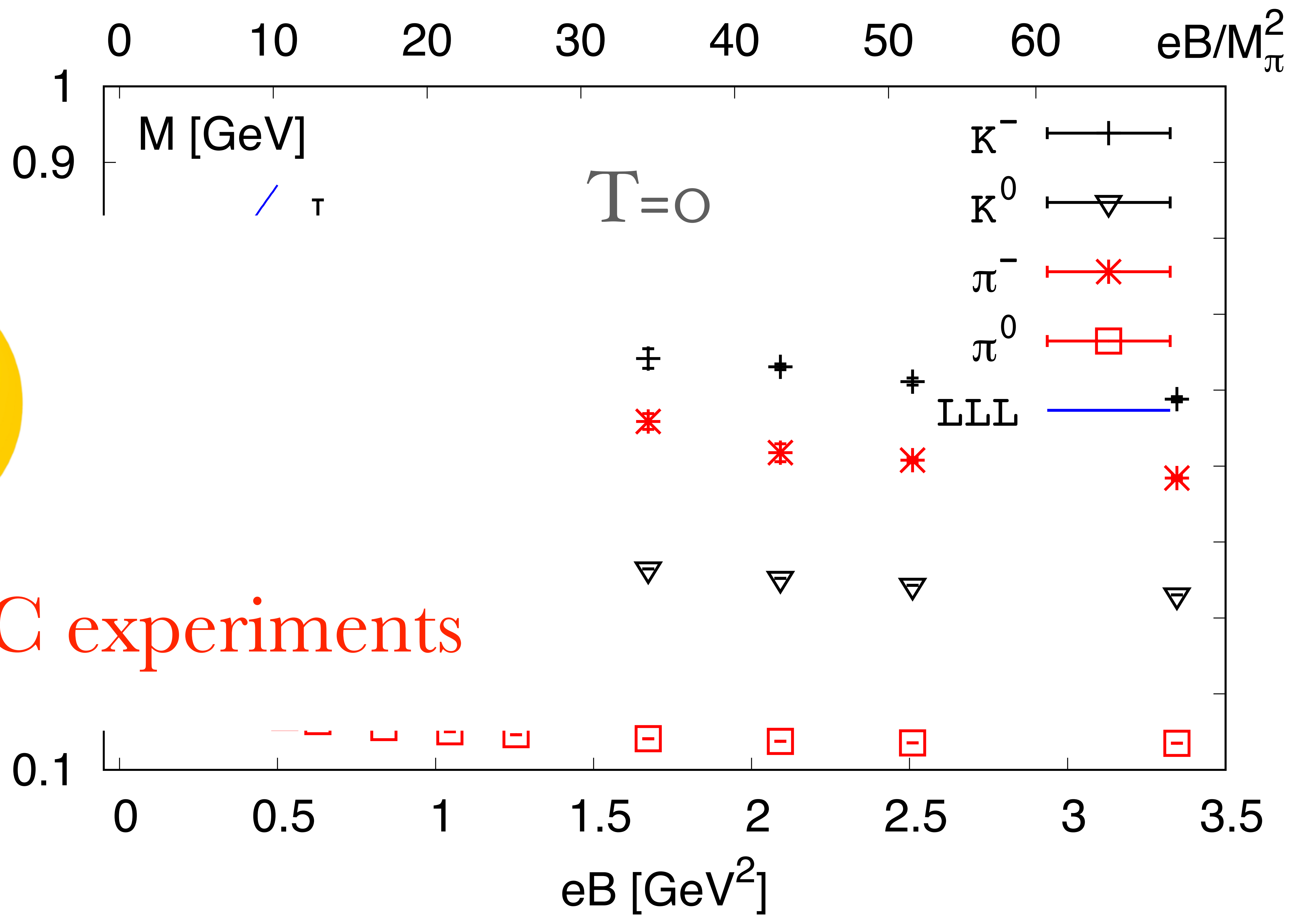

Not accessible in HIC experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

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Fluctuations of net baryon number (B), electric charge (Q) and strangeness (S)

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

See talks of
J. Goswami and A. Pasztor
on Sun

$$\hat{\chi}_{ijk}^{uds} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_u/T)^i \, \partial (\mu_d/T)^j \, \partial (\mu_s/T)^k} \right|_{\mu_{u,d,s}=0}$$

$$\hat{\chi}_{ijk}^{\mathrm{BQS}} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial (\mu_{\mathrm{B}}/T)^i \, \partial (\mu_{\mathrm{Q}}/T)^j \, \partial (\mu_{\mathrm{S}}/T)^k} \right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\boxed{\begin{aligned} \mu_u &= \frac{1}{3}\mu_{\mathrm{B}} + \frac{2}{3}\mu_{\mathrm{Q}} \,, \\ \mu_d &= \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} \,, \\ \mu_s &= \frac{1}{3}\mu_{\mathrm{B}} - \frac{1}{3}\mu_{\mathrm{Q}} - \mu_{\mathrm{S}} \,. \end{aligned}}$$

- At $eB \neq 0$ a lot more could be explored

HRG: Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301 Bhattacharyya et al., EPL115(2016)62003, Marczenko et al., PRC 110(2024)065203, Vovchenko PRC 110(2024)065203

PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009 …

Lattice setup

Arpith Kumar, QM 2025

- HISQ/tree action, line of constant physics adopted from the HotQCD collaboration

- Magnetic field strength: $eB = 6\pi/N_s^2 * N_b * a^{-1}$ with $N_b = 1,2,3,4,6,12,16,24$, i.e. $eB \lesssim 0.8\ \mathrm{GeV}^2 \sim 45\mathrm{m}_\pi^2$

- Temperature ranges from ~145 MeV to ~166 MeV

- Lattice sizes: 32^3x8 and 48^3x12, with additional 64^3x16 at one single value of *eB*

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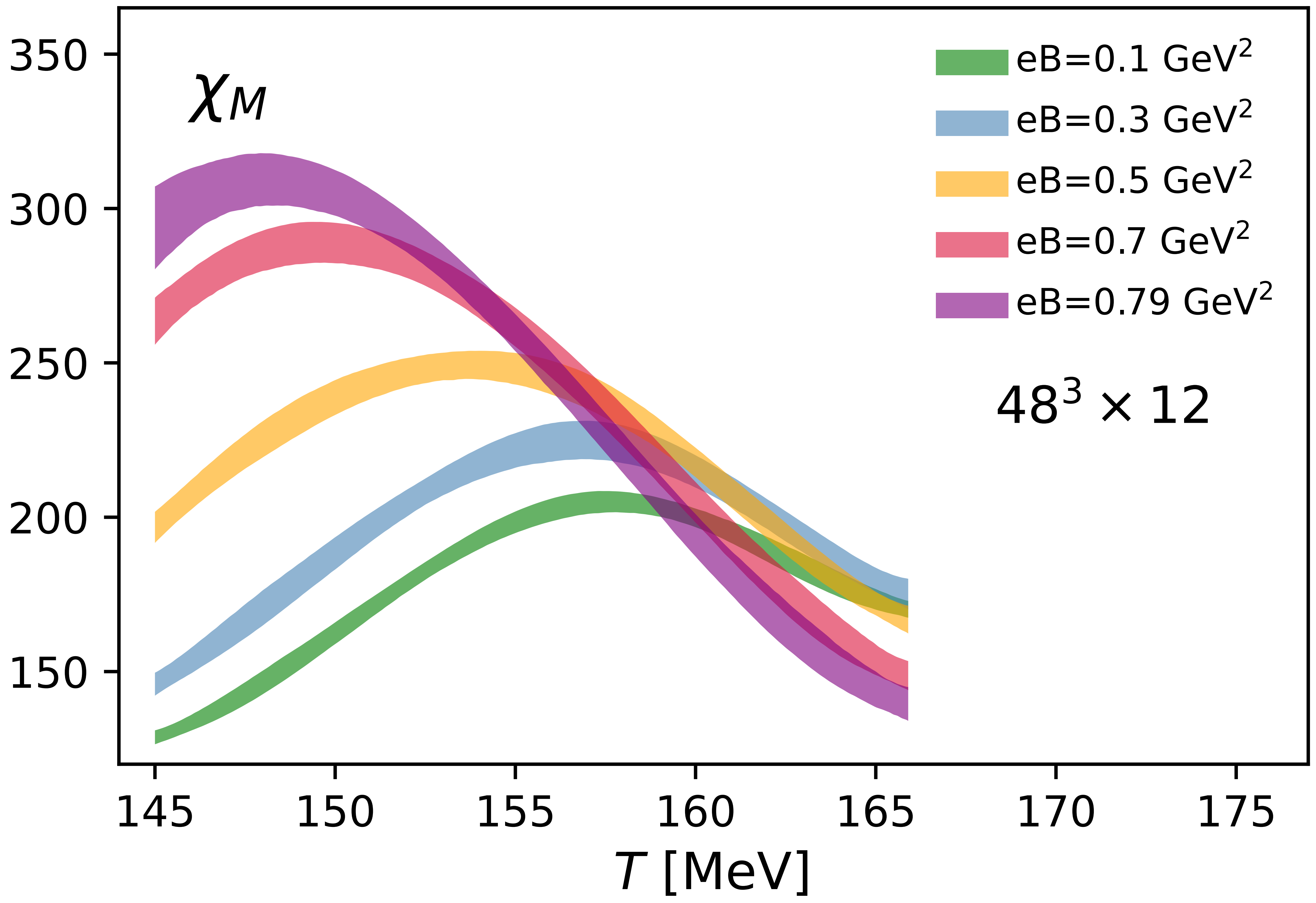
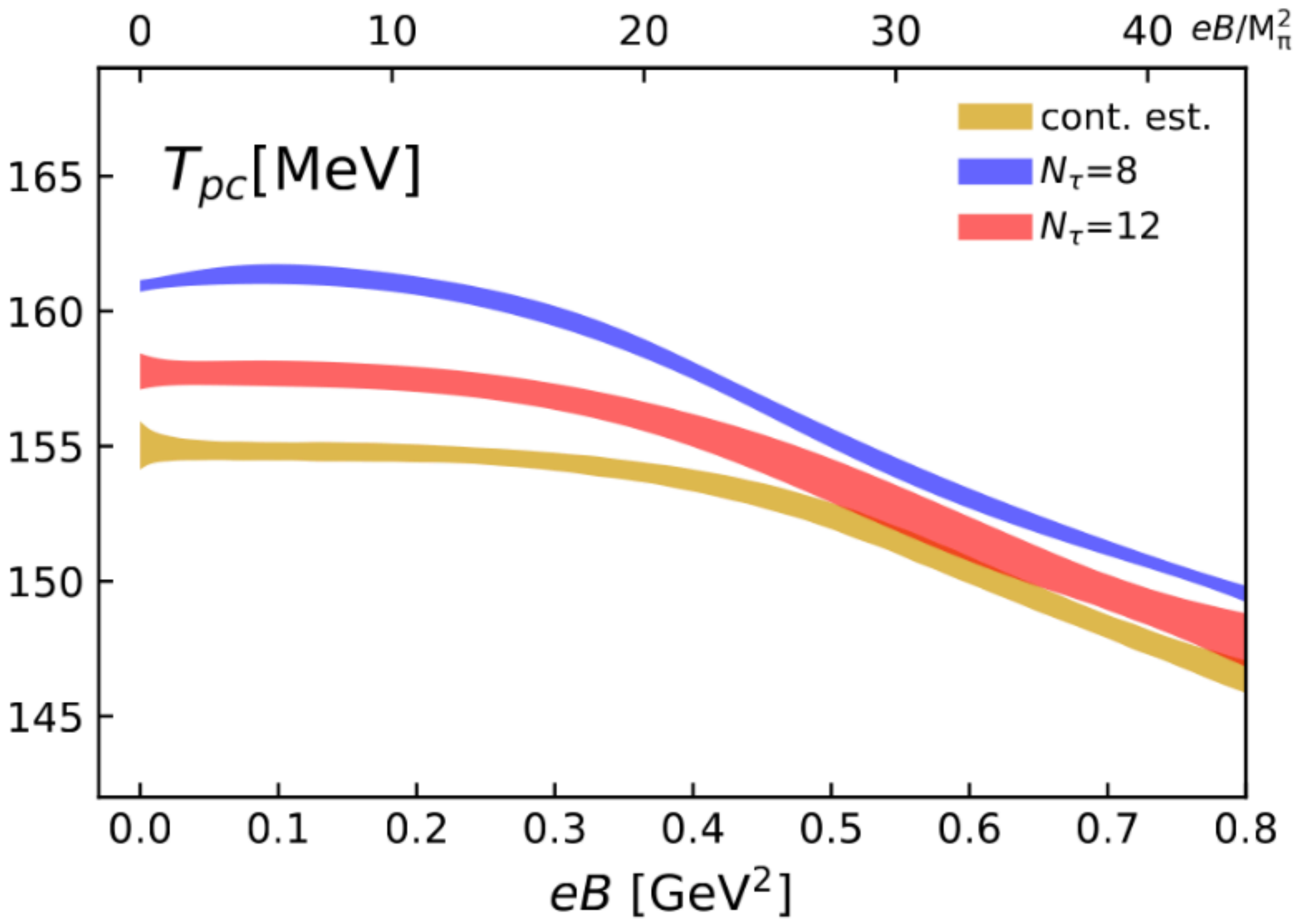
QCD transition temperature in nonzero magnetic fields

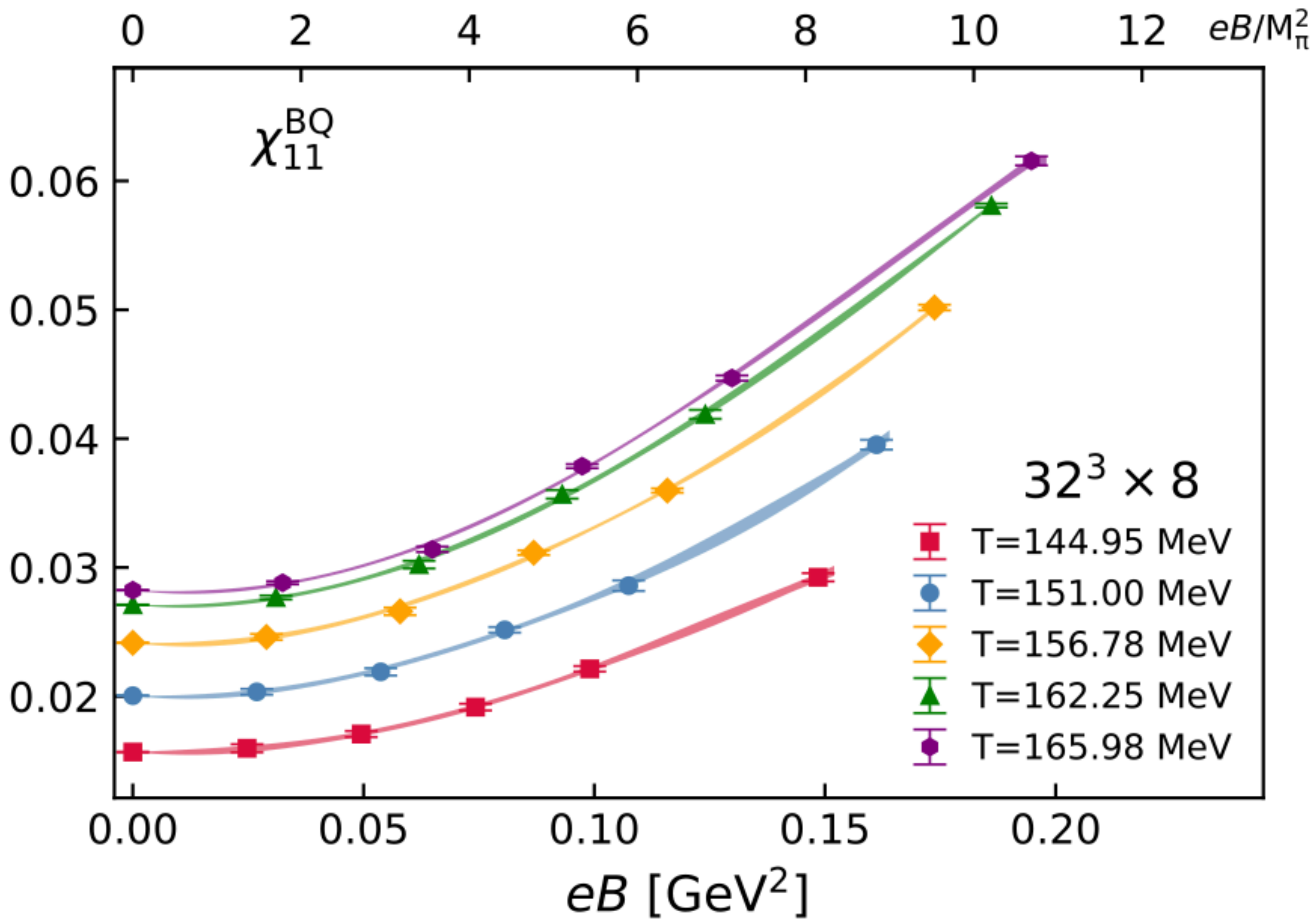


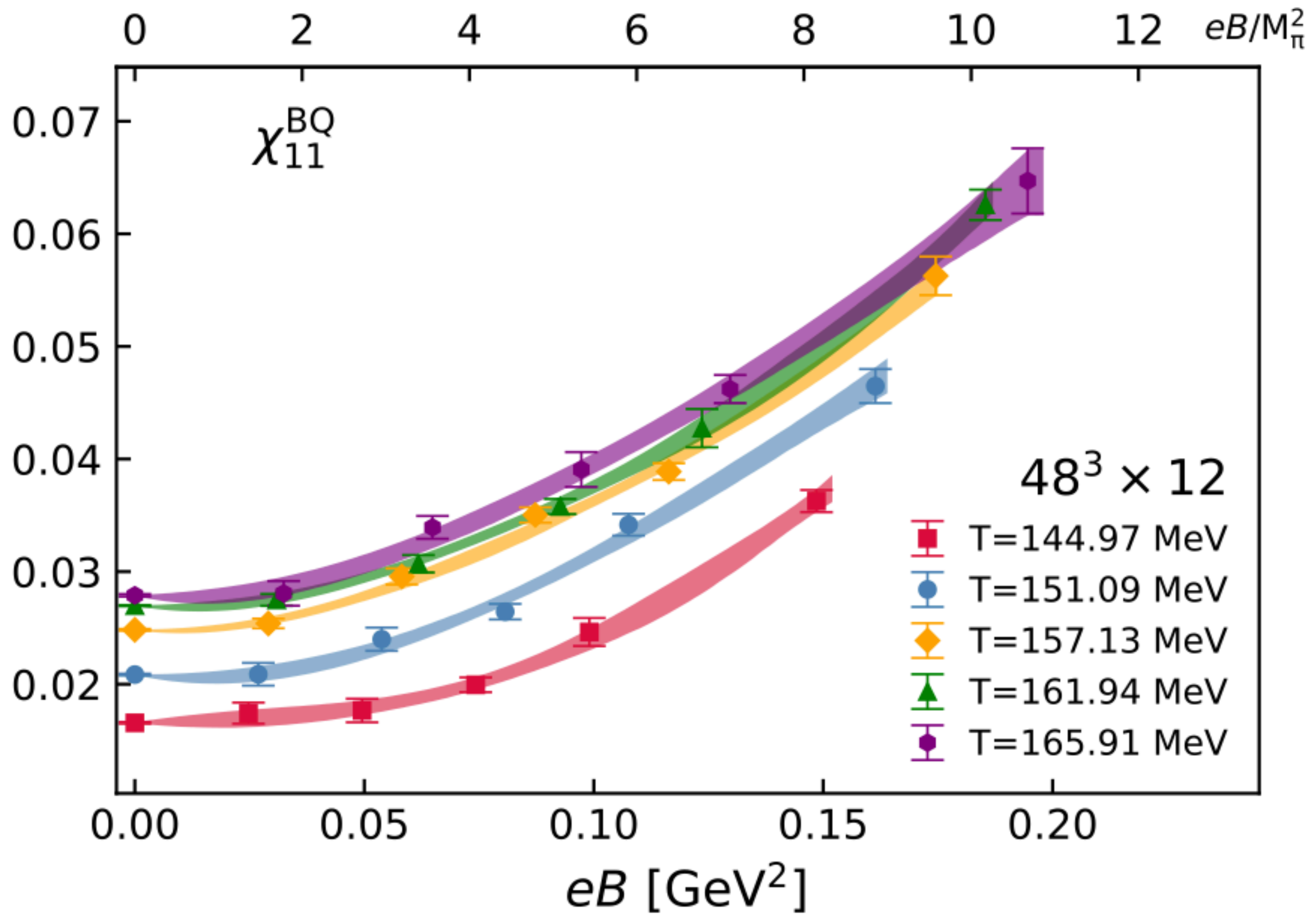


T_{pc} determined as the peak location of chiral susceptibility

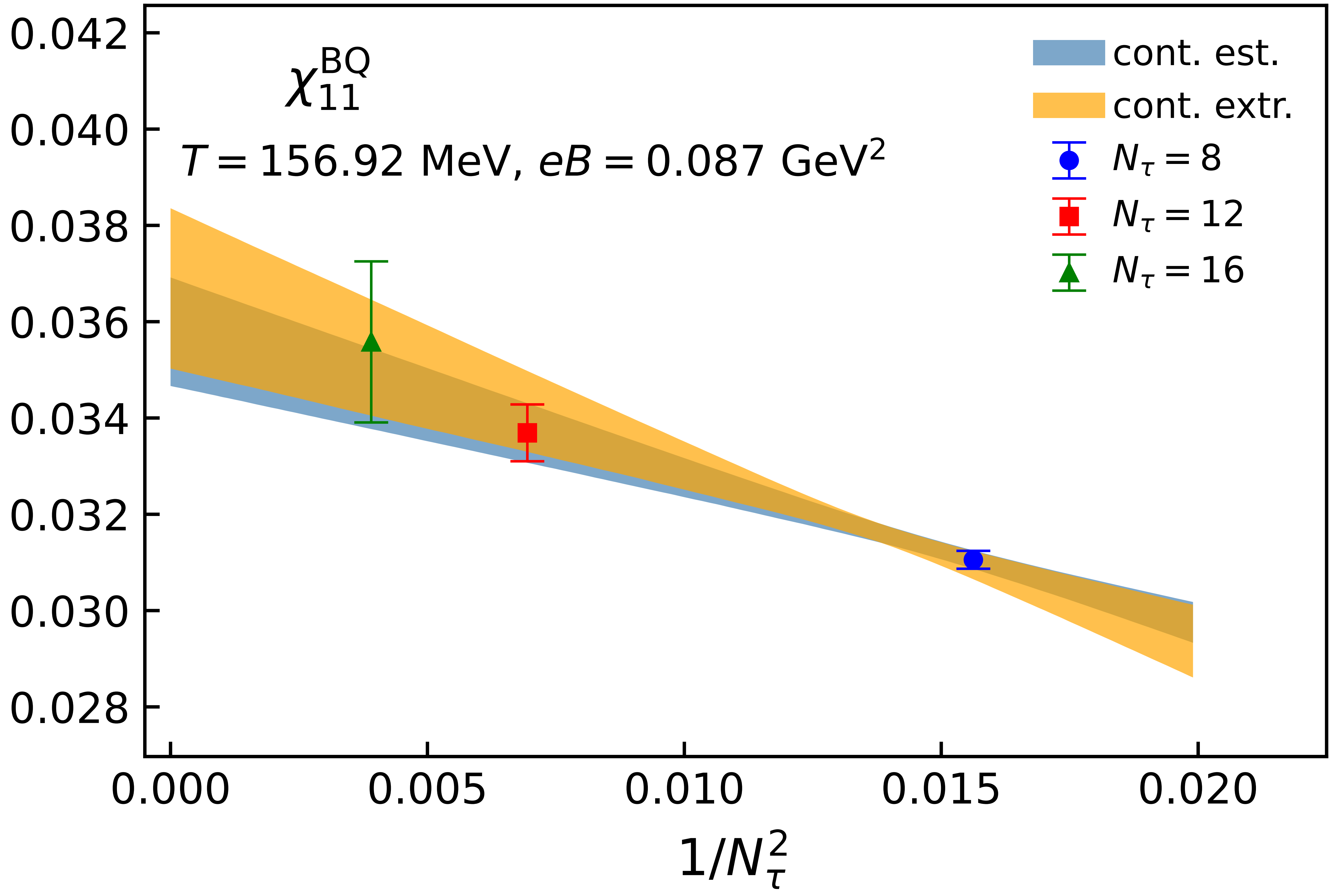
Negligible eB-independence at $eB \lesssim 0.16$ GeV^2

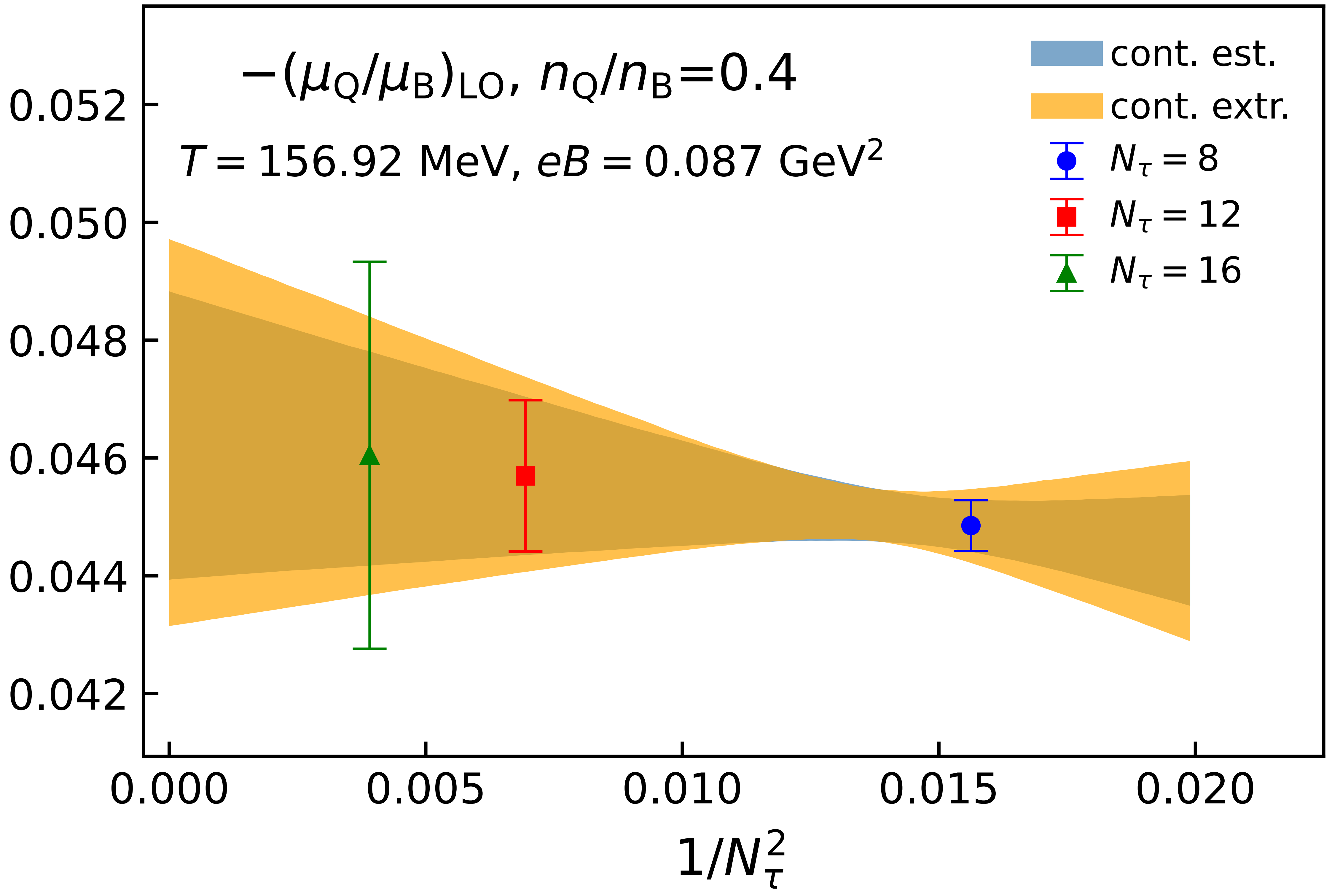
Lattice data on N_τ =8 and 12 lattices



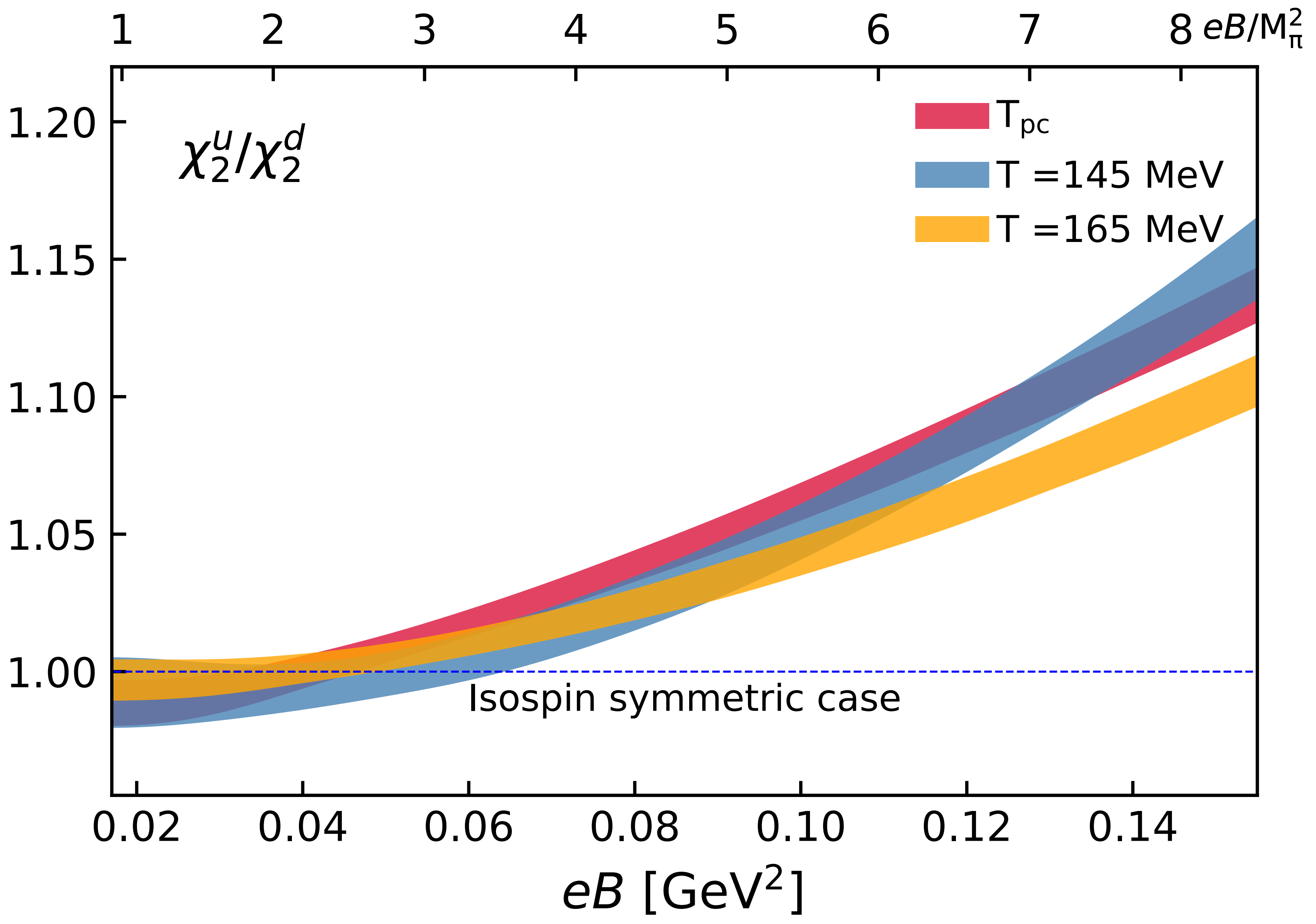


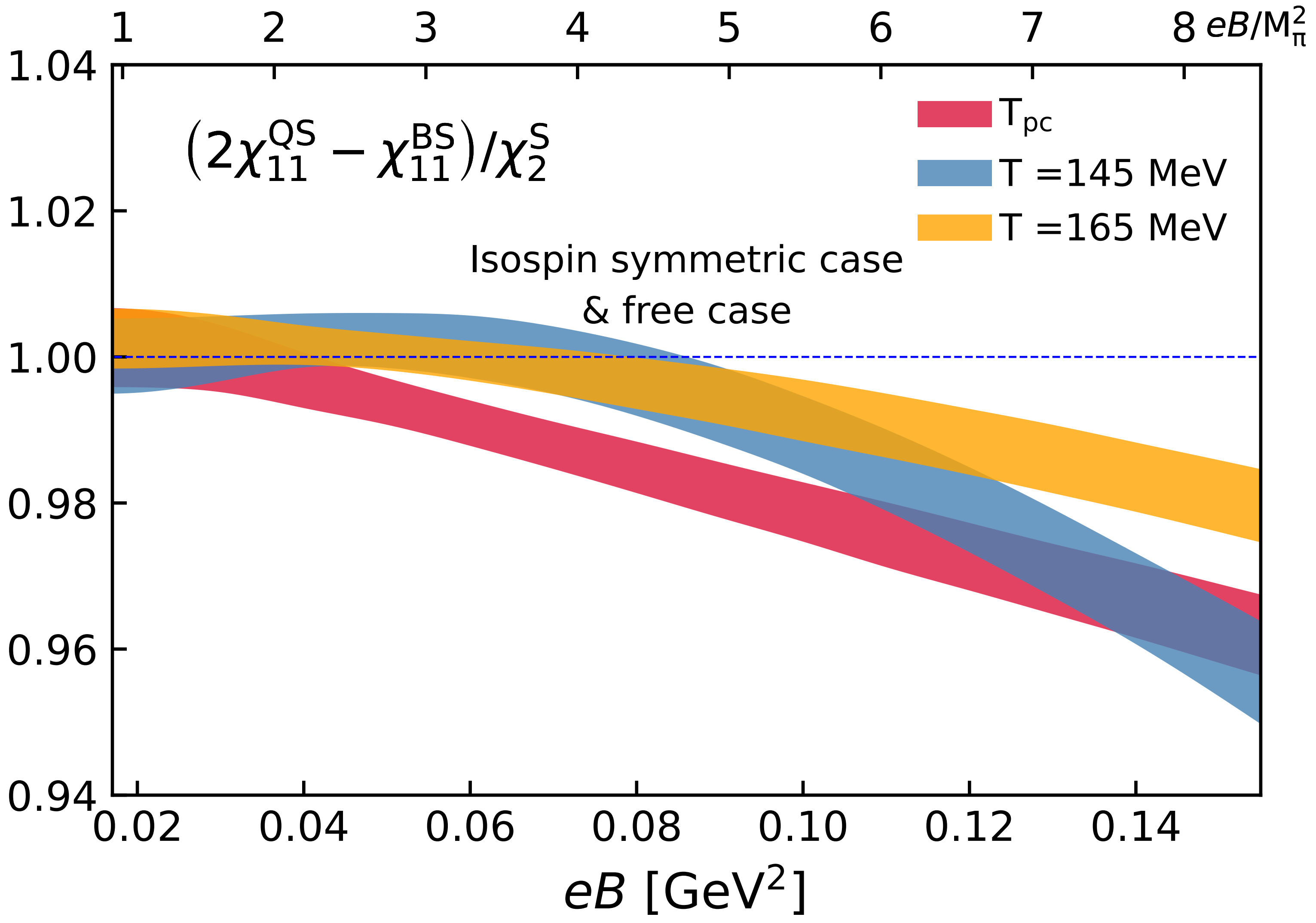
Continuum estimate v.s. continuum limit



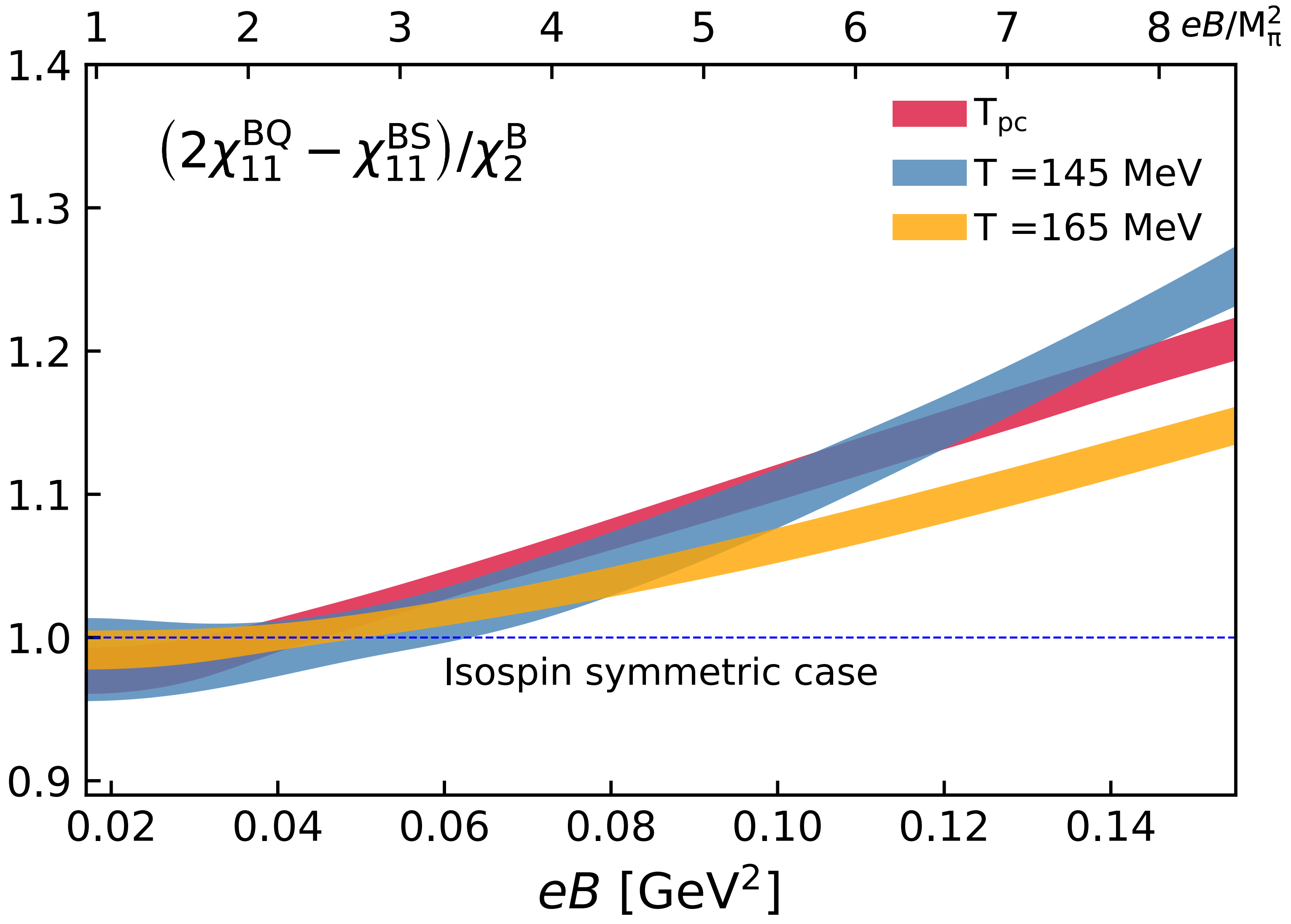


Isospin symmetry breaking at nonzero magnetic field

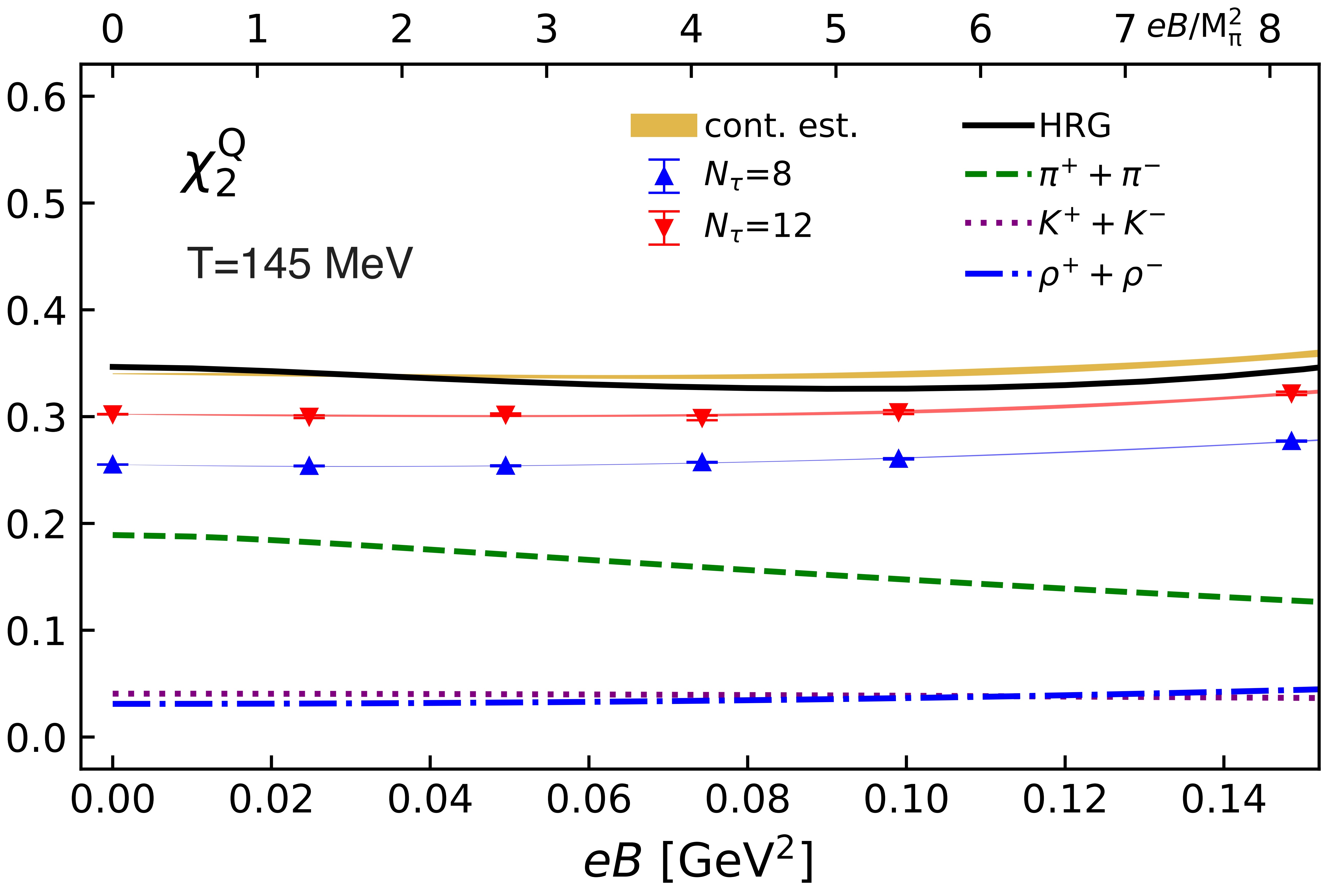




$$\text{At } eB=0,\ \chi_2^u=\chi_2^d,$$
$$2\chi_{11}^{QS}-\chi_{11}^{BS}=\chi_2^S\ ,\ 2\chi_{11}^{BQ}-\chi_{11}^{BS}=\chi_2^B$$



Net electric charge fluctuations at T=145 MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

- In HRG: Thermal pressure arising from charged hadrons in strong magnetic fields

Fukushima & Hidaka, PRL 16'

$$\frac{P_c}{T^4} = \frac{|q_i|B}{2\pi^2 T^3} \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} \frac{e^{n\mu_i/T}}{n} \mathrm{K}_1\left(\frac{n\varepsilon_0}{T}\right)$$

$$\varepsilon_0 = \sqrt{m_i^2 + 2|q_i|B(l + 1/2 - s_z)}$$

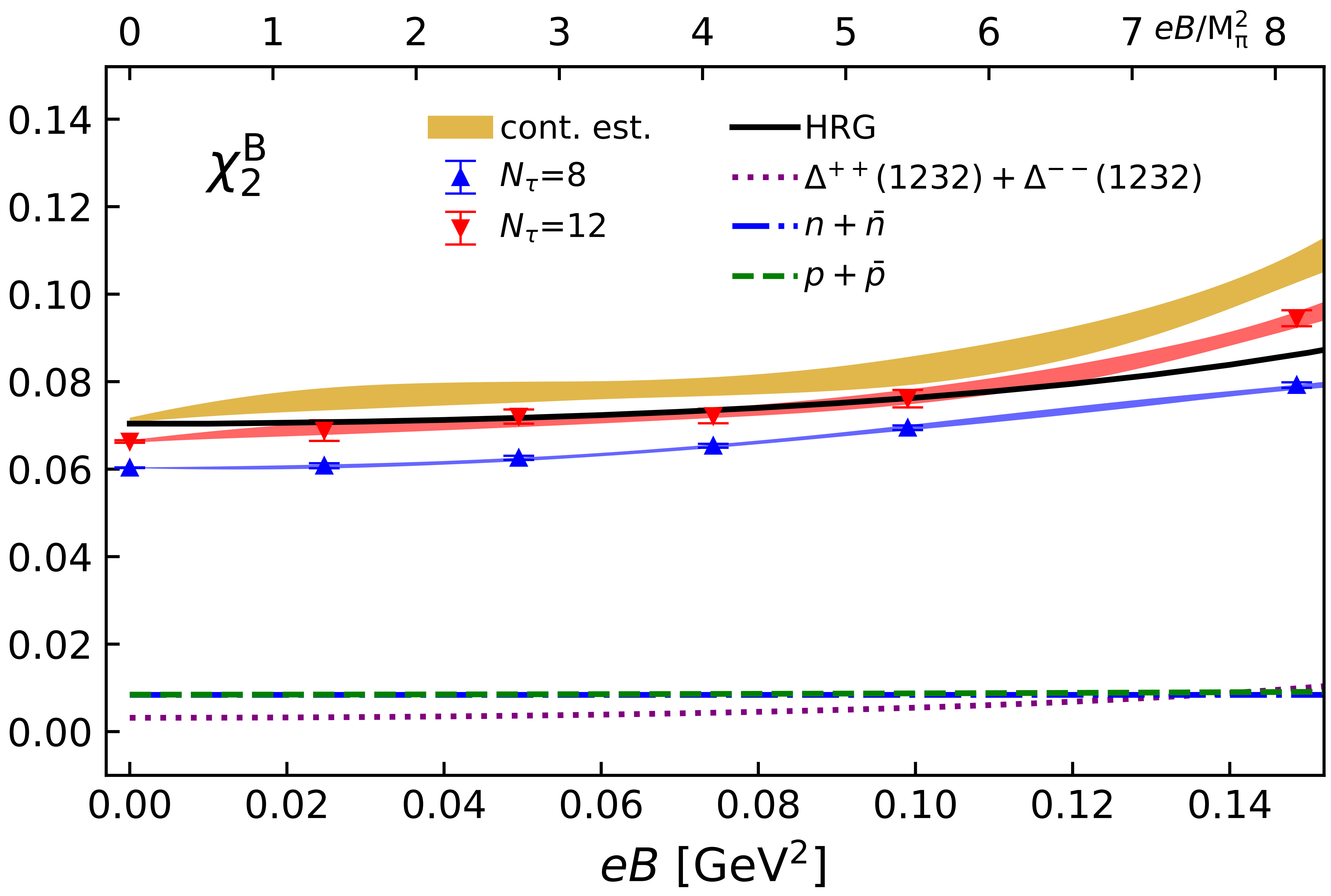
- Diagonal fluctuations from HRG

HTD, Li, Shi & Wang , EPJA 21'

$$\chi_2^X = \frac{B}{2\pi^2 T^3} \sum_i |q_i| X_i^2 \sum_{s_z=-s_i}^{s_i} \sum_{l=0}^{\infty} f(\varepsilon_0)$$

$$f(\varepsilon_0) = \varepsilon_0 \sum_{n=1}^{\infty} (\pm 1)^{n+1} n \, \mathrm{K}_1\left(\frac{n\varepsilon_0}{T}\right)$$

Net baryon number fluctuations at T=145 MeV



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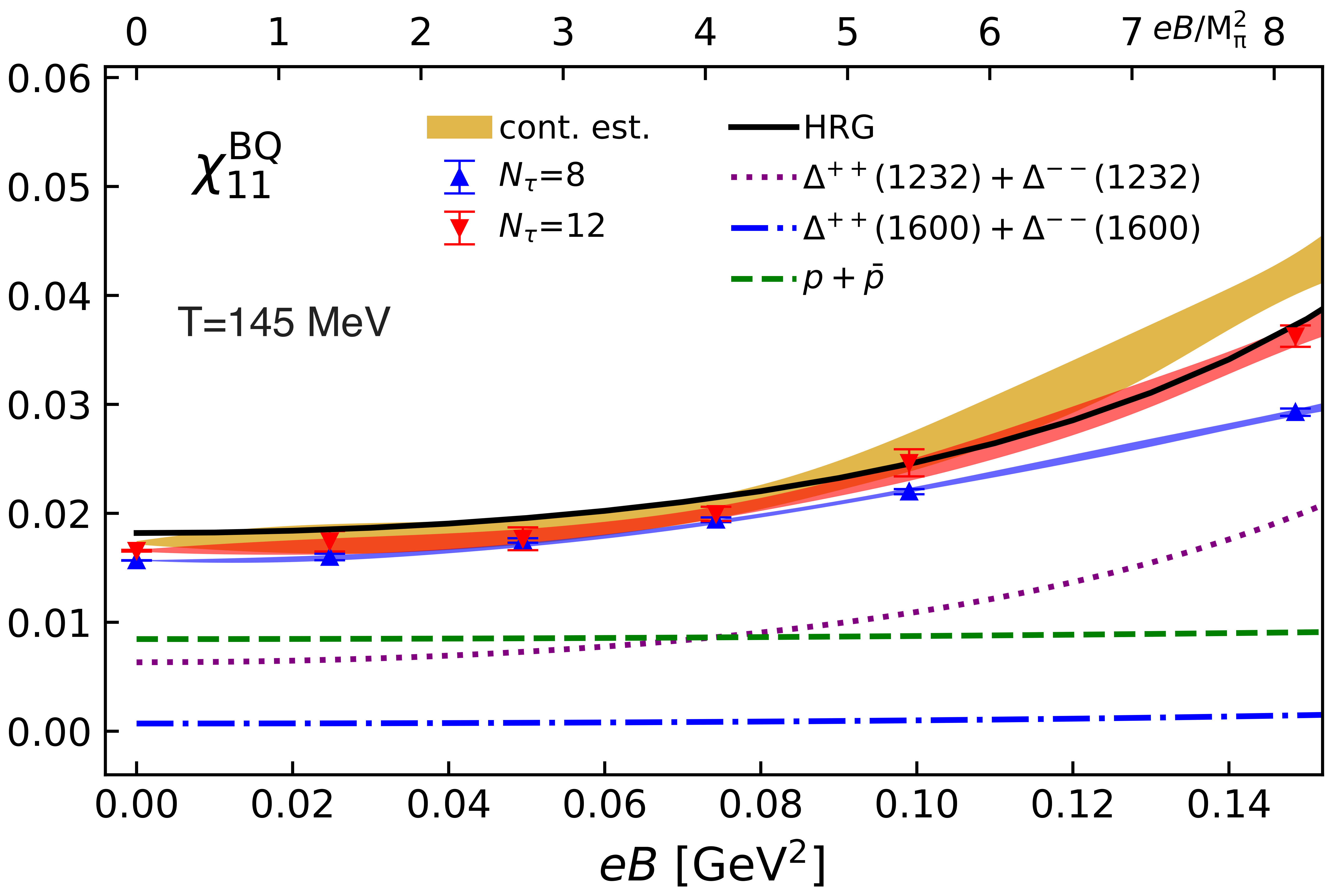
- χ_2^B receives contributions also from neutral baryons

- HRG, where neutral baryons are assumed to be unaffected by eB, undershoots the LQCD results

Baryon electric charge correlation at T=145 MeV



HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

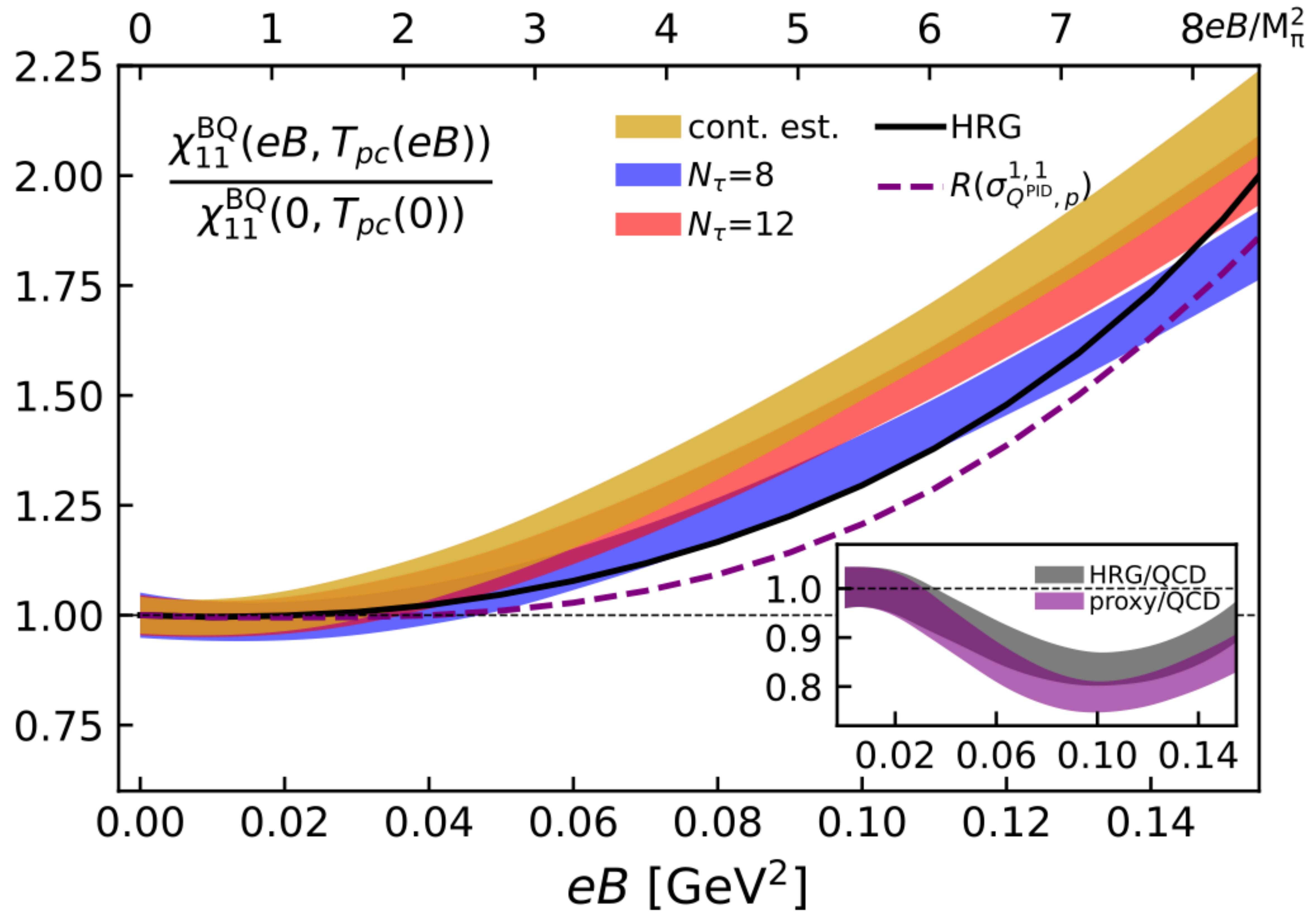
χ_{11}^{BQ}: Magnetometer of QCD

- Most of the eB-dependence comes from doubly charged Delta baryons

$\Delta^{++} \rightarrow p + \pi^+$

- Doubly charged Delta baryons: not-measurable in HIC experiments

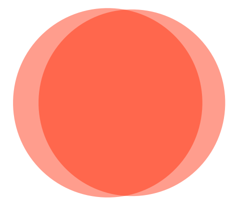
Baryon electric charge correlation along the transition line



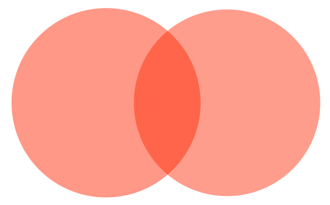
At $eB \lesssim M_\pi^2$: consistent with unity

At $eB \simeq 8M_\pi^2$: ~2 !

X(eB)/X(eB=0) : Rcp like observable



Central collision



Peripheral collision

HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

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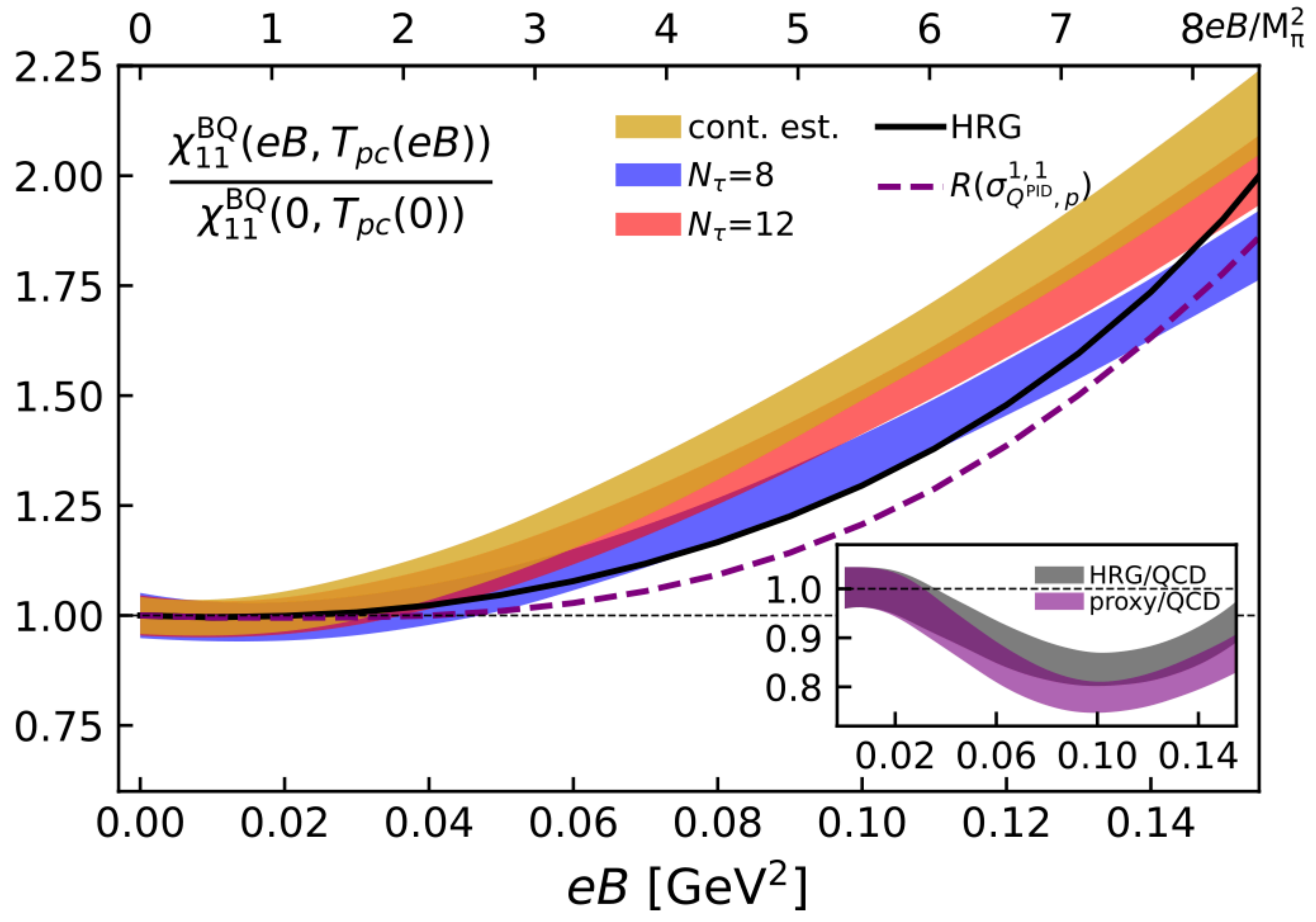

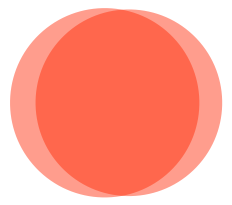

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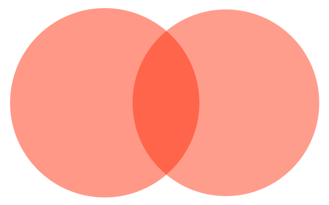


Peripheral collision

HTD, J.-B. Gu, A. Kumar, S.-T. Li, J.-H. Liu, PRL 132 (2024) 201903

Memory carried by the decays of Δ^{++}:

$$\Delta^{++} \rightarrow p + \pi^+$$

- net-B approximated by $\tilde{p}$
- net-Q approximated by Q^{PID}: $\tilde{\pi}^+, \tilde{K}^+, \tilde{p}$

$$\sigma^{i,j,k}_{p,Q^{\mathrm{PID}},K} = \sum_R \left(\omega_{R\to\tilde{p}}\right)^i \left(\omega_{R\to\tilde{Q}^{\mathrm{PID}}}\right)^j \left(\omega_{R\to\tilde{K}}\right)^k$$
$$\times\, I^R_\ell\left(T, \mu_{\mathrm{B}}, \mu_{\mathrm{Q}}, \mu_{\mathrm{S}}, eB\right),$$

$$I^R_\ell\left(T, \mu_{\mathrm{B}}, \mu_{\mathrm{Q}}, \mu_{\mathrm{S}}, eB\right) = \left.\frac{\partial^\ell P_R/T^4}{\partial \hat{\mu}^\ell_R}\right|_{\hat{\mu}_{\mathrm{B,Q,S}}=0}, \; i+j+k=\ell$$

See procedures to construct the proxy using HRG at eB=0, Bellwied et al., Phys. Rev. D 101 (2020) 034506

Proxies with kinematic cuts within HRG

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

$$\frac{P}{T^4} = \frac{1}{T^4}\sum_i P_i = \frac{1}{VT^3}\sum_i \ln \mathcal{Z}_i(V,T,\mu_i,B)\ , \quad \ln \mathcal{Z}_i = \pm V g_i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \ln\left[1 \pm e^{-(E_i-\mu_i)/T}\right]$$

- For collision-generated magnetic fields along the y-axis, i.e. perpendicular to the reaction plane

$$p_{xz}(l,s_y) = \sqrt{2|q|B(l+1/2-s_y)}\ , \qquad E_l(p_y,l,s_y) = \sqrt{m^2+p_y^2+2|q|B(l+1/2-s_y)}$$

- Now with adequate integration variables p_y and ϕ_p:

$$\int d^3\boldsymbol{p} = \int d^2p_{xz}dp_y = \int p_{xz}dp_{xz}dp_y d\phi_p \quad = \int |q_i|\,B dl dp_y d\phi_p$$

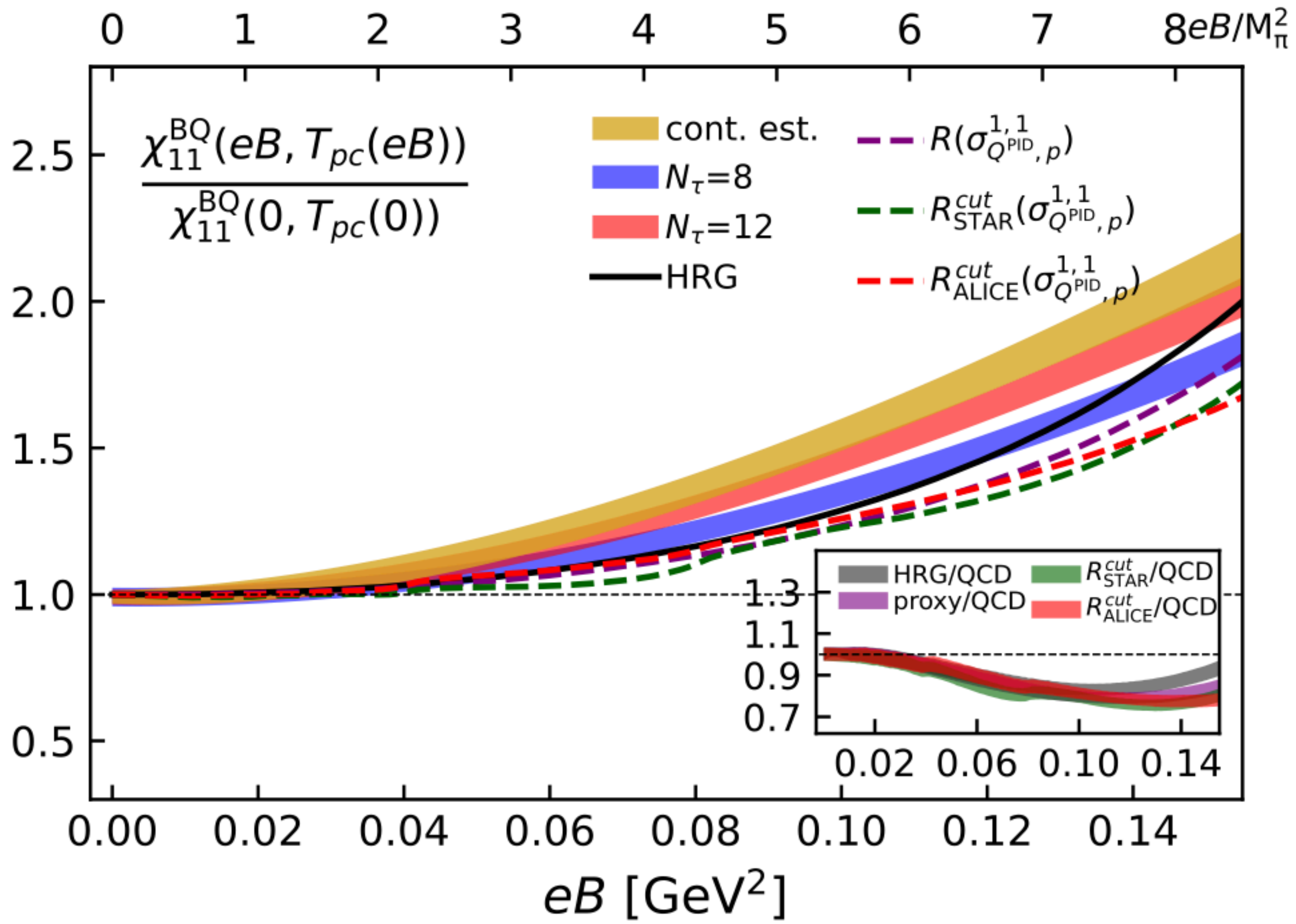
$$\boxed{\begin{aligned}\eta &= \mathrm{arcsinh}(p_{xz}\cos\phi_p/p_T)\\ p_T &= \sqrt{(p_{xz}\sin\phi_p)^2+p_y^2}\end{aligned}}$$

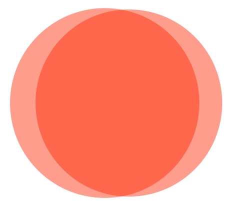
- The pressure arising from charged particles becomes

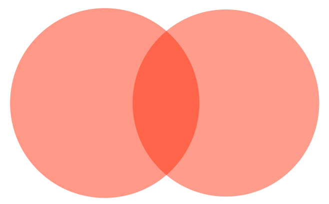
$$\frac{P^{cuts}_{i,charged}}{T^4} = \frac{|q_i|\,B}{(2\pi)^3T^3}\sum_{s_y=-s_i}^{s_i}\sum_{l=0}^{\infty}\int dp_y\int d\phi_p \times \sum_{n=1}^{\infty}\frac{(\pm1)^{n+1}}{n}e^{-n(E_l-\mu_i)/T} \times \Theta(p_{Tmin}, p_{Tmax}, \eta_{min}, \eta_{max})$$

Heaviside step function

Baryon electric charge correlation along the transition line




Central collision


Peripheral collision

- Kinematic cuts for the ALICE detector

  $|\eta| < 0.8$, $0.2 < p_T < 2$ GeV/c for π and K & $0.4 < p_T < 2$ GeV/c for proton

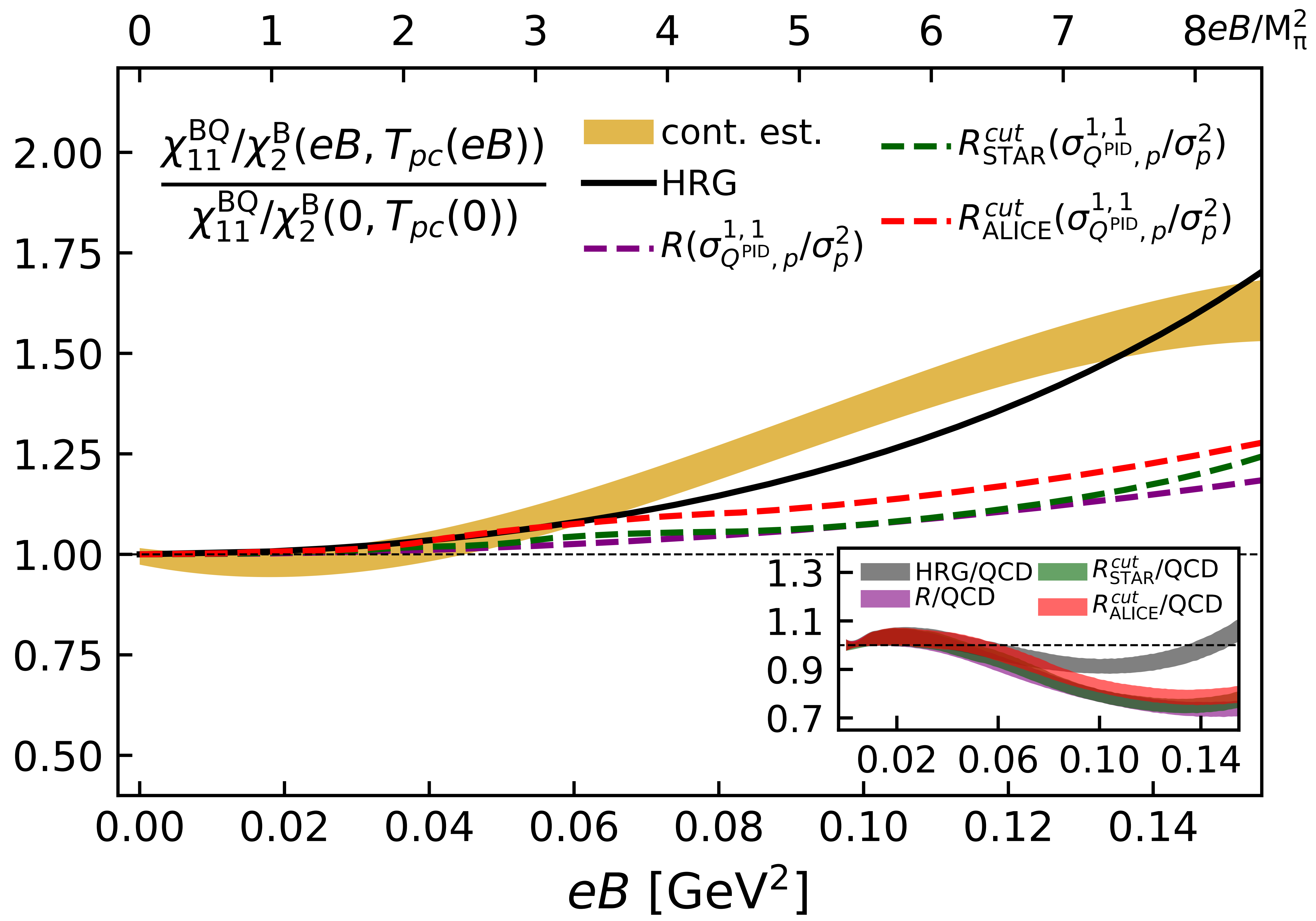
- Kinematic cuts for the STAR detector

  for π and K and proton: $|\eta| < 0.5$, $0.4 < p_T < 1.6$ GeV/c

- Both proxies with kinematic cuts applied to ALICE and STAR catches up at least 70% of the eB-dependence of BQ correlation

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

Double ratios $R(\chi_{11}^{\rm BQ}/\mathcal{O}_1) \equiv \dfrac{\chi_{11}^{\rm BQ}/\mathcal{O}_1(eB, T_{pc}(eB))}{\chi_{11}^{\rm BQ}/\mathcal{O}_1(eB=0, T_{pc}(eB=0))}$, $\mathcal{O}_1 \equiv \{\boxed{\chi_2^{\rm B},}\ \chi_2^{\rm Q}, \chi_{11}^{\rm QS}\}$



Central collision

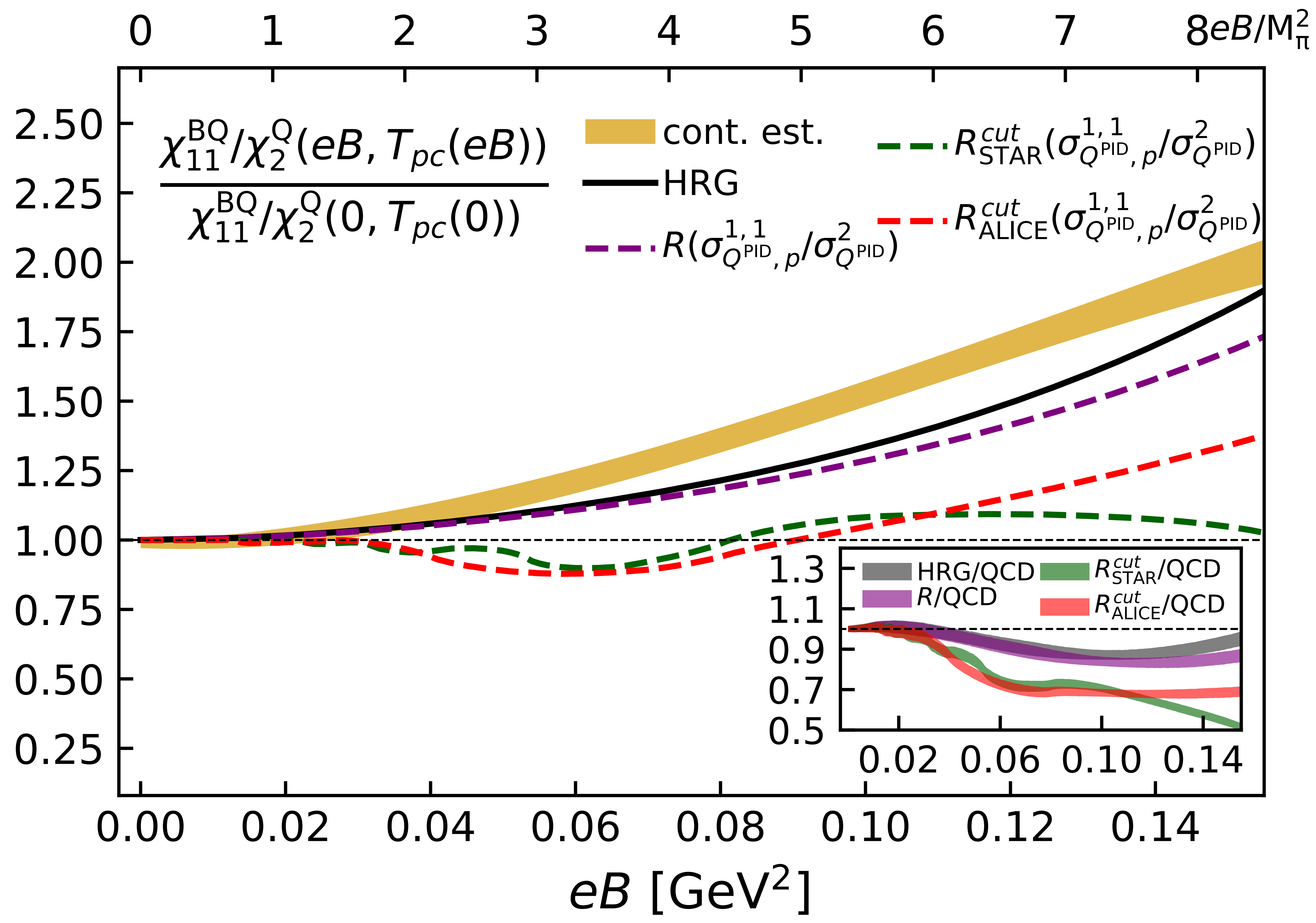
Peripheral collision

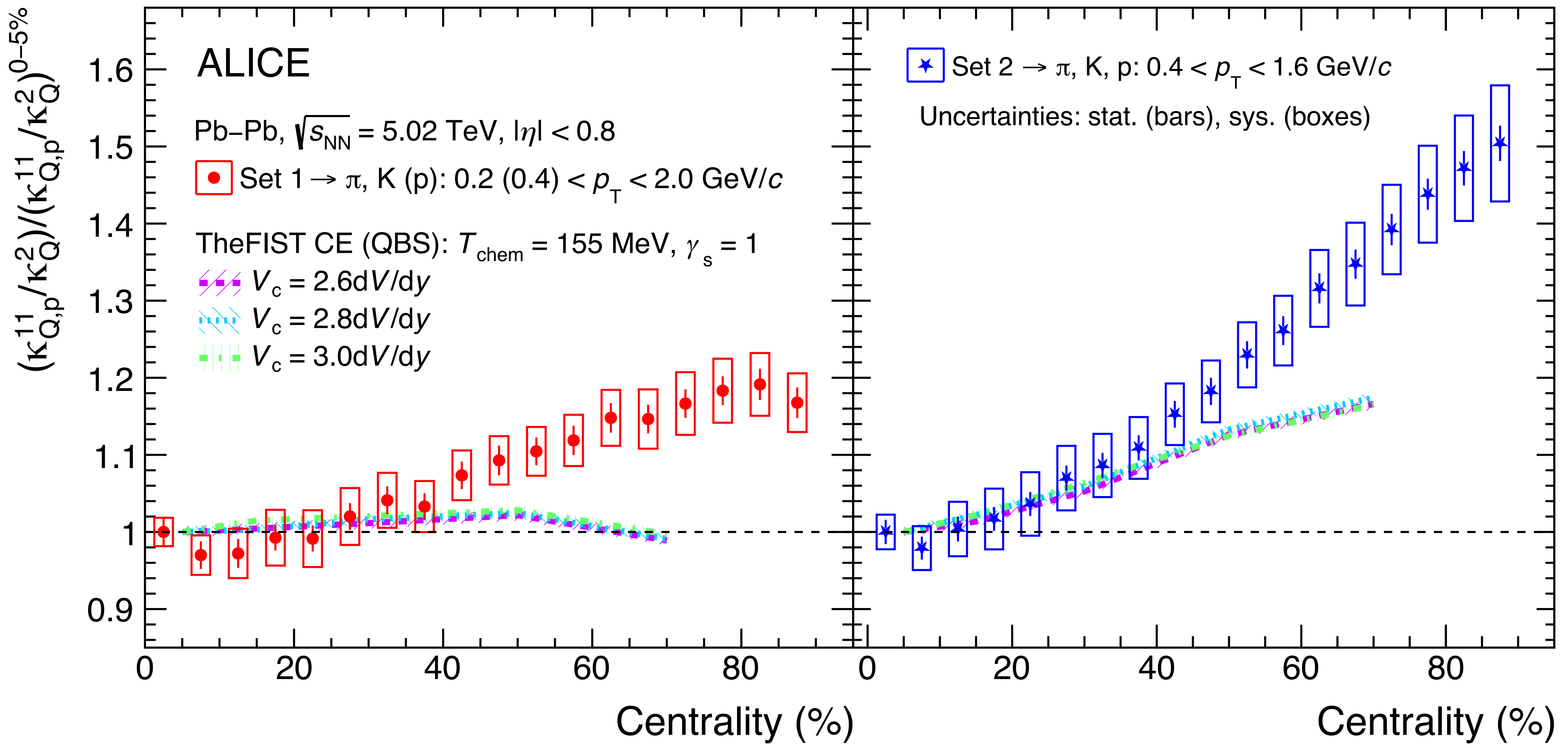
- ☑ At $eB \simeq 8M_\pi^2$, LQCD ~1.6
- ☑ expected weaker response $\chi_2^{\rm B}$ most sensitive after $\chi_{11}^{\rm BQ}$
- ☑ Proxies with cuts reaches to about 1.25

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

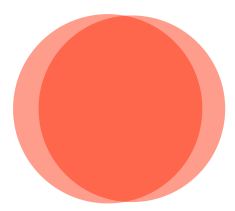
Double ratios

$$R(\chi_{11}^{BQ}/\mathcal{O}_1) \equiv \frac{\chi_{11}^{BQ}/\mathcal{O}_1(eB, T_{pc}(eB))}{\chi_{11}^{BQ}/\mathcal{O}_1(eB=0, T_{pc}(eB=0))}, \quad \mathcal{O}_1 \equiv \{\chi_2^B, \boxed{\chi_2^Q}, \chi_{11}^{QS}\}$$





ALICE Collaboration, arXiv:2503.18743 [nucl-ex]



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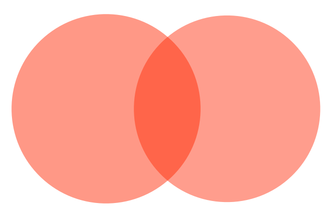

Peripheral collision

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

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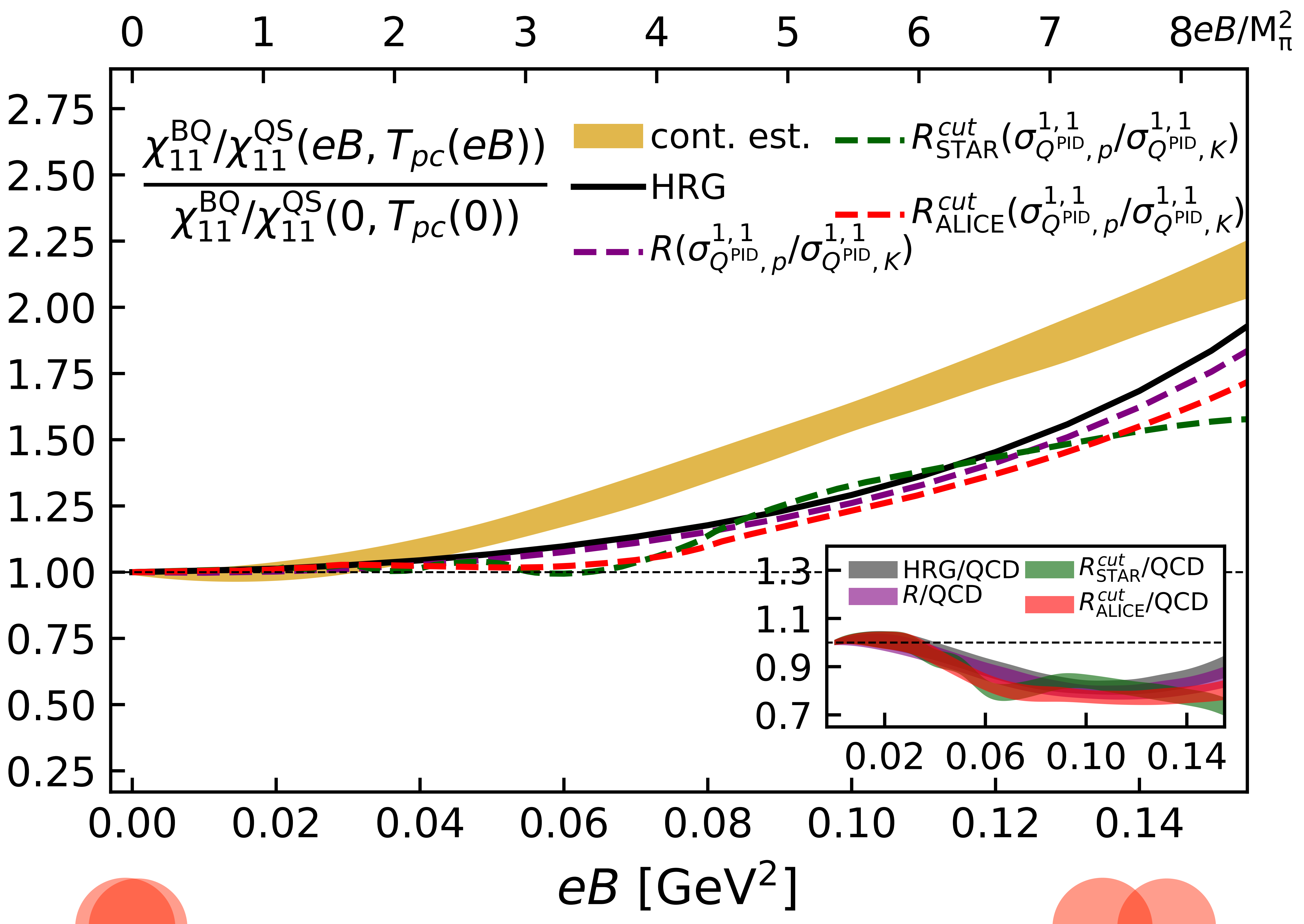
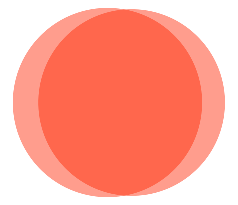
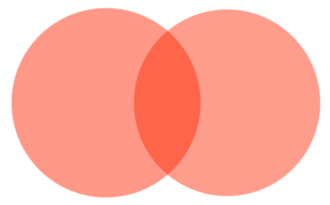




Central collision



Peripheral collision

~ 2.2 Lattice QCD

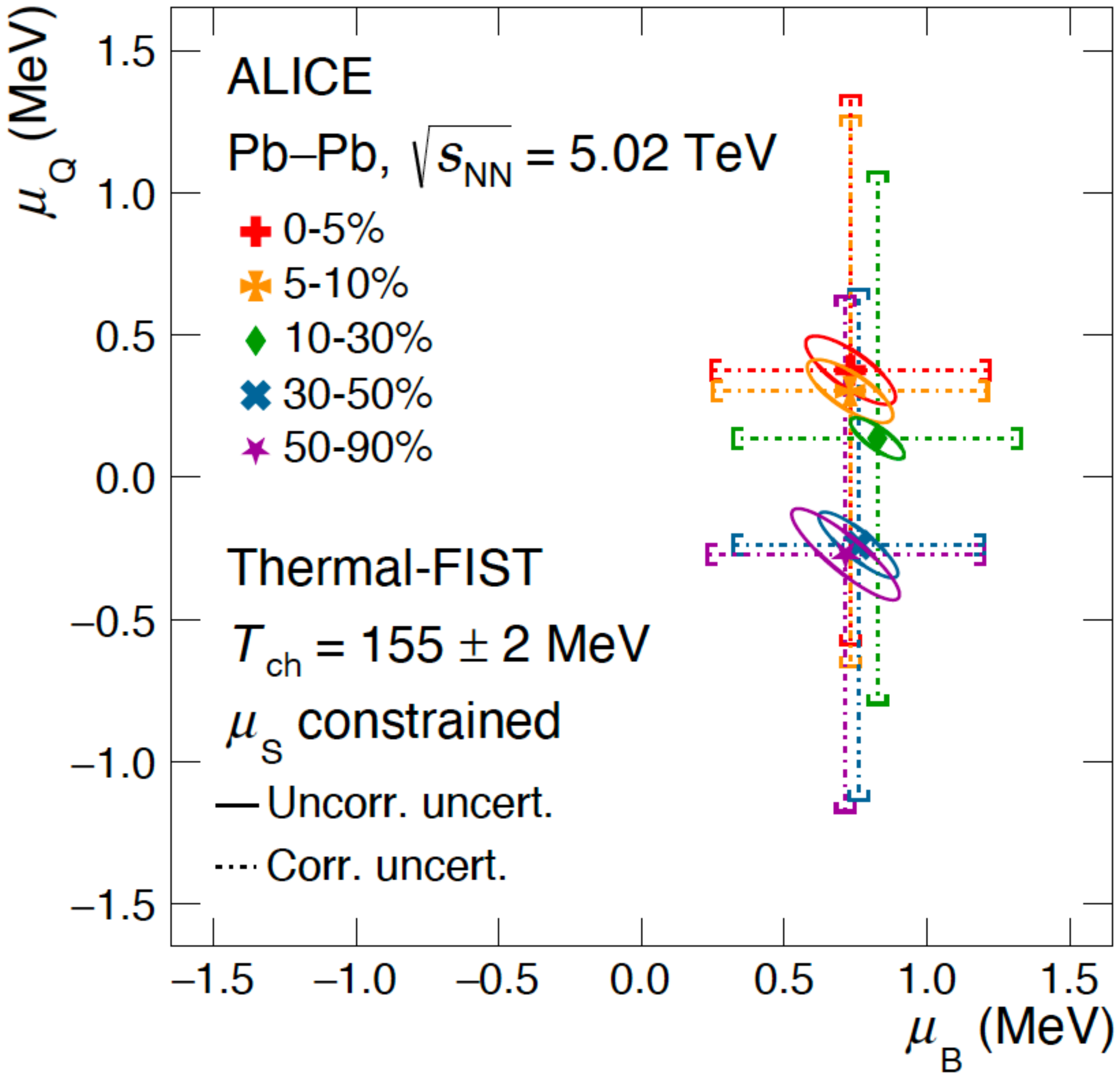
$\sim 1.6 - 1.75$ Proxies

$\sim 75 - 84\,\%$ Proxies/LQCD

$R(\chi_{11}^{BQ}/\chi_{11}^{QS})$
Even more effective probe in both ALICE & STAR!

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

Electric charge chemical potential over baryon chemical potential



ALICE, arXiv: 2311.13332

- μ_Q/μ_B can be extracted from thermal fits to particle yields, which typically utilize hadron spectra without considering magnetic field effects

- μ_Q/μ_B is also connected to fluctuations of B,Q,S

$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

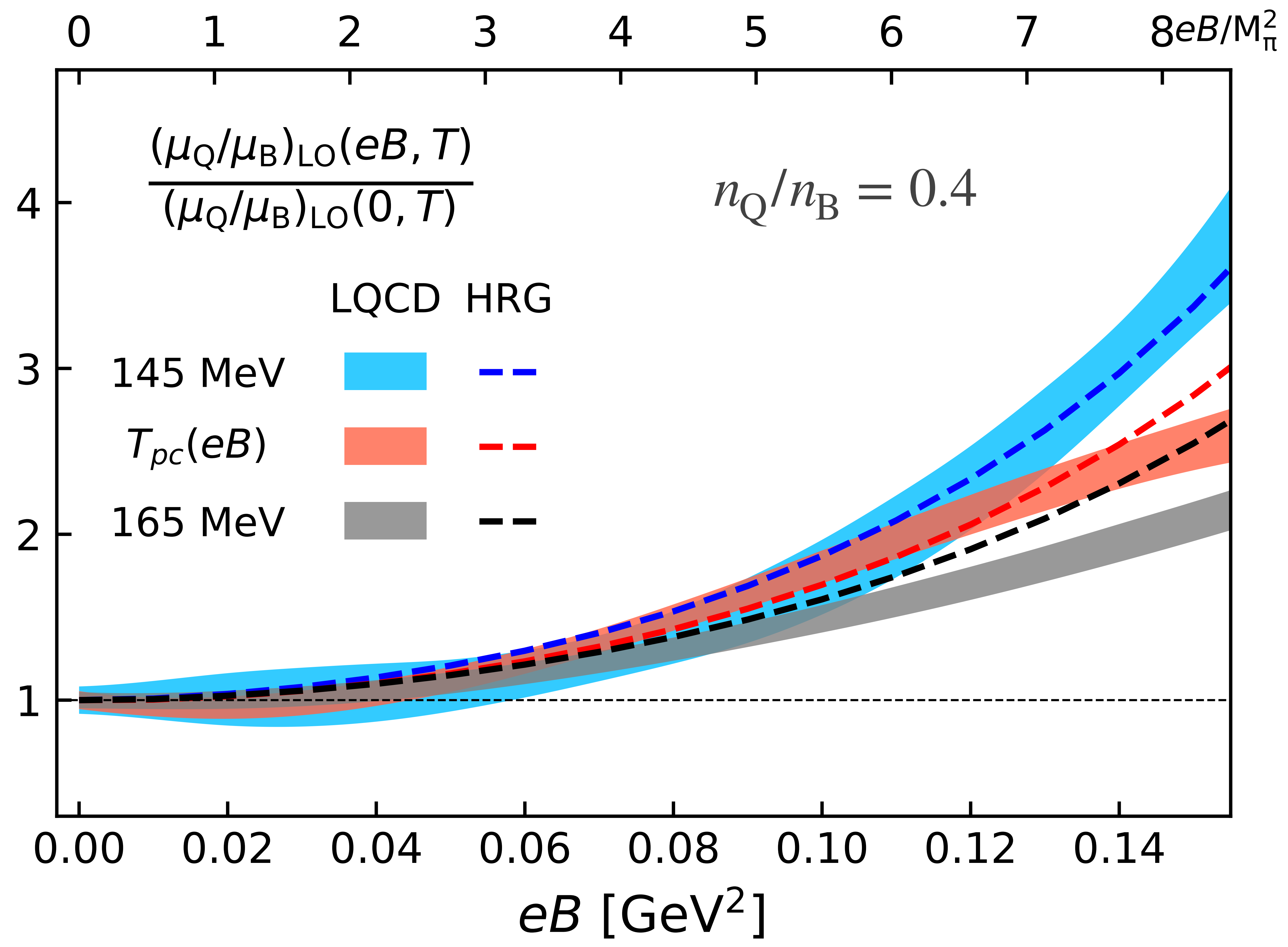
$$q_1 = \frac{r(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}) - (\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}{(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}) - r(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}$$

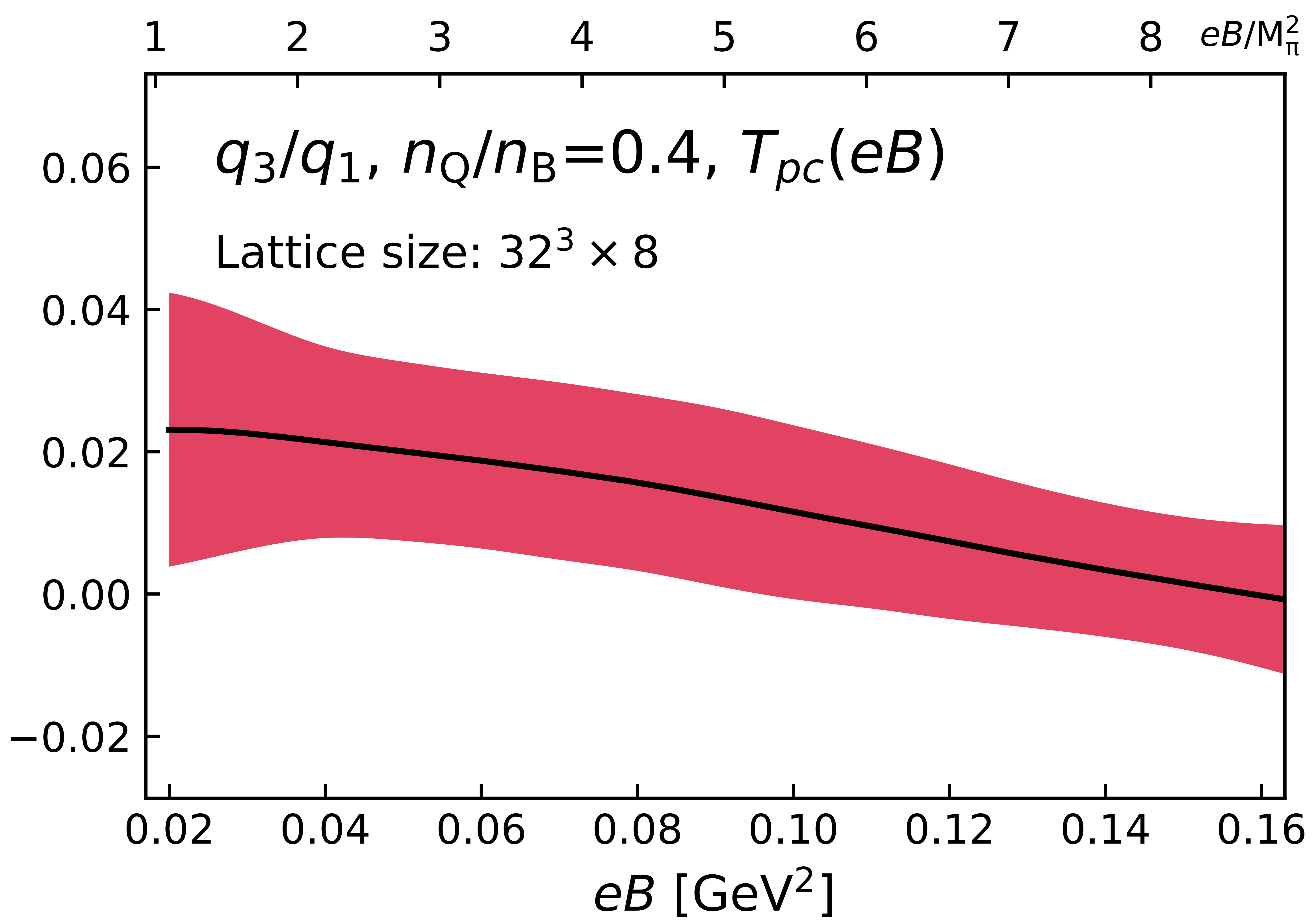
$$r = n_Q/n_B$$

HotQCD, PRL 109 (2012) 192302

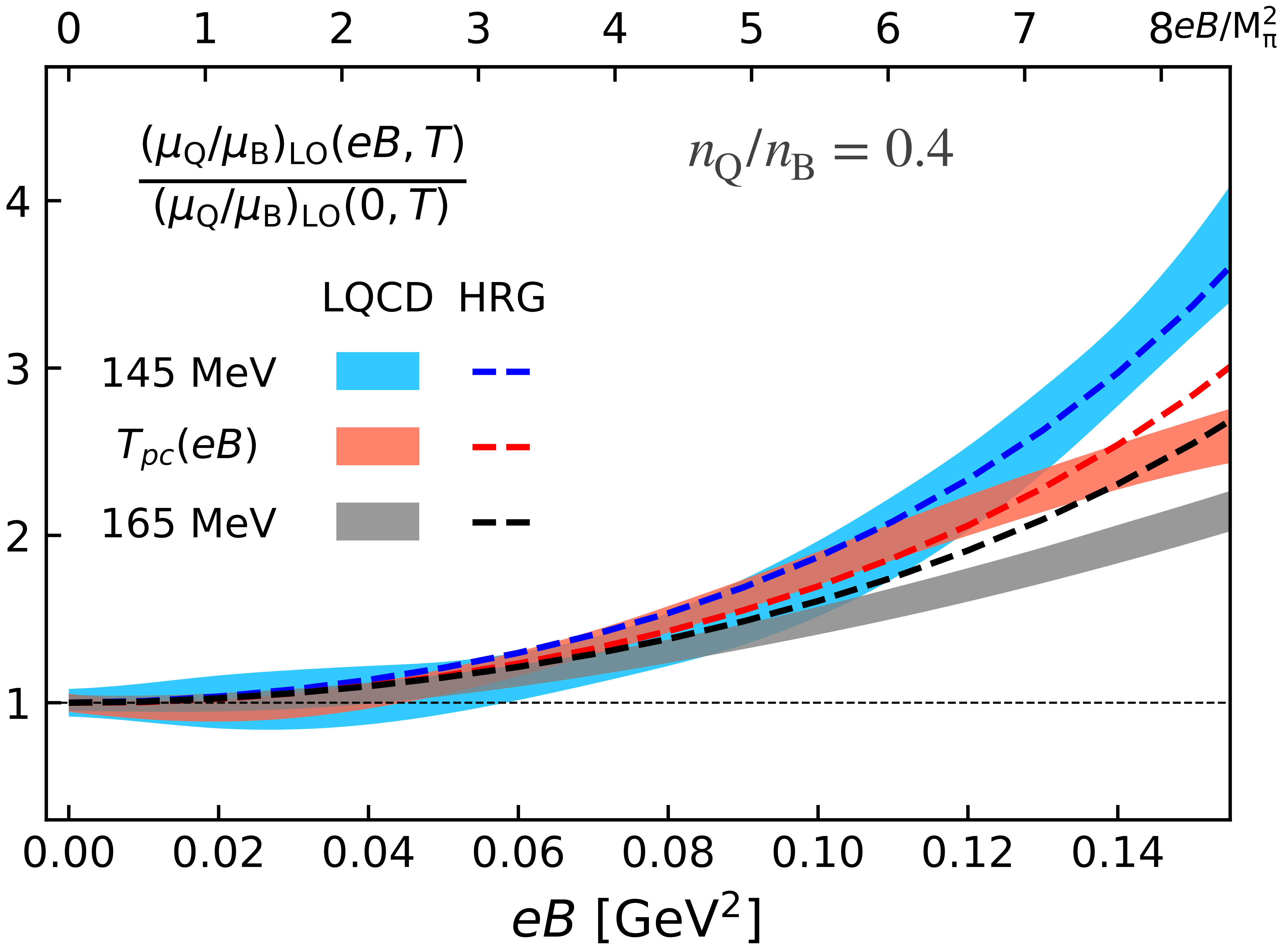
μ_Q/μ_B in nonzero magnetic fields

$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

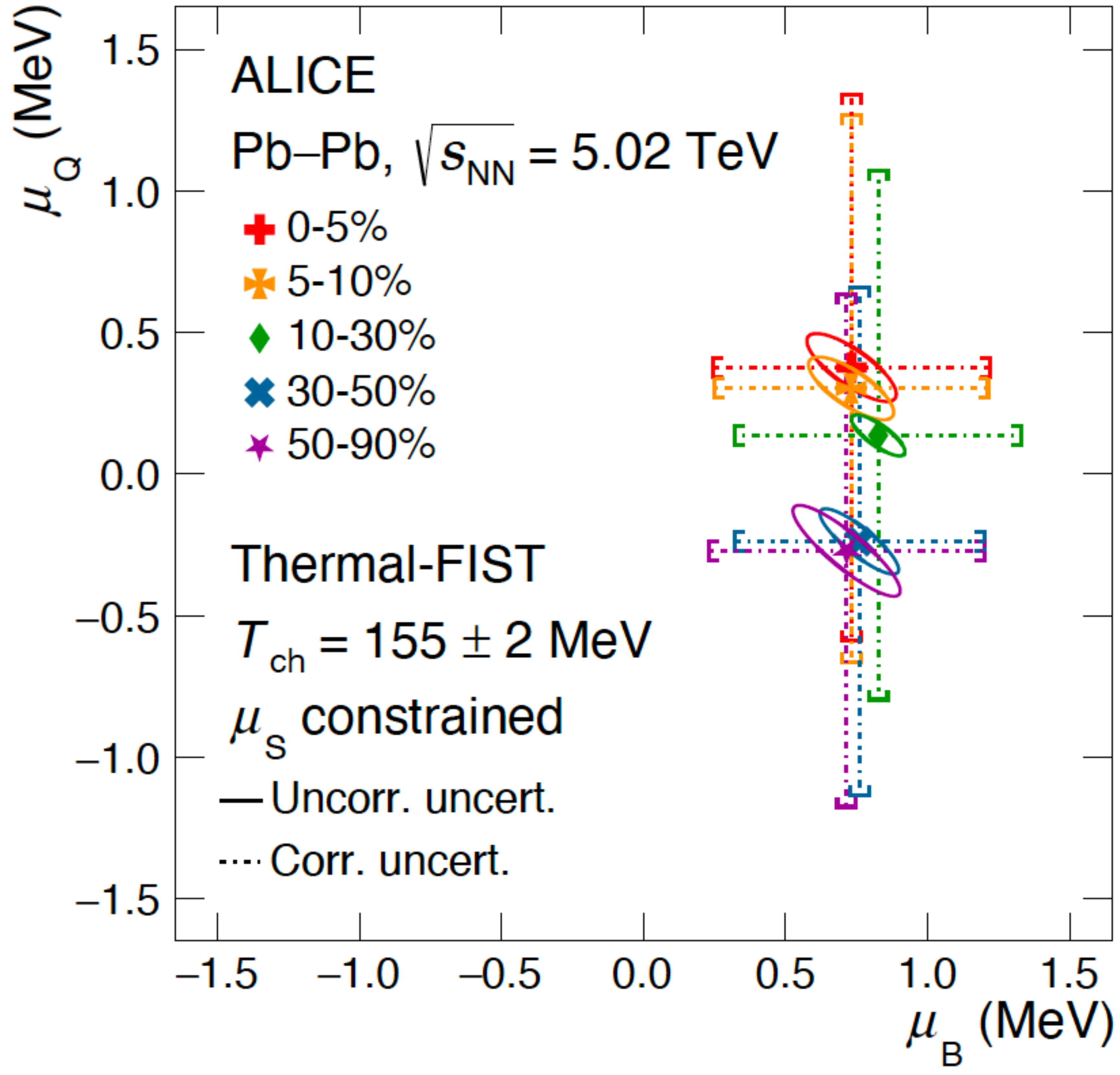




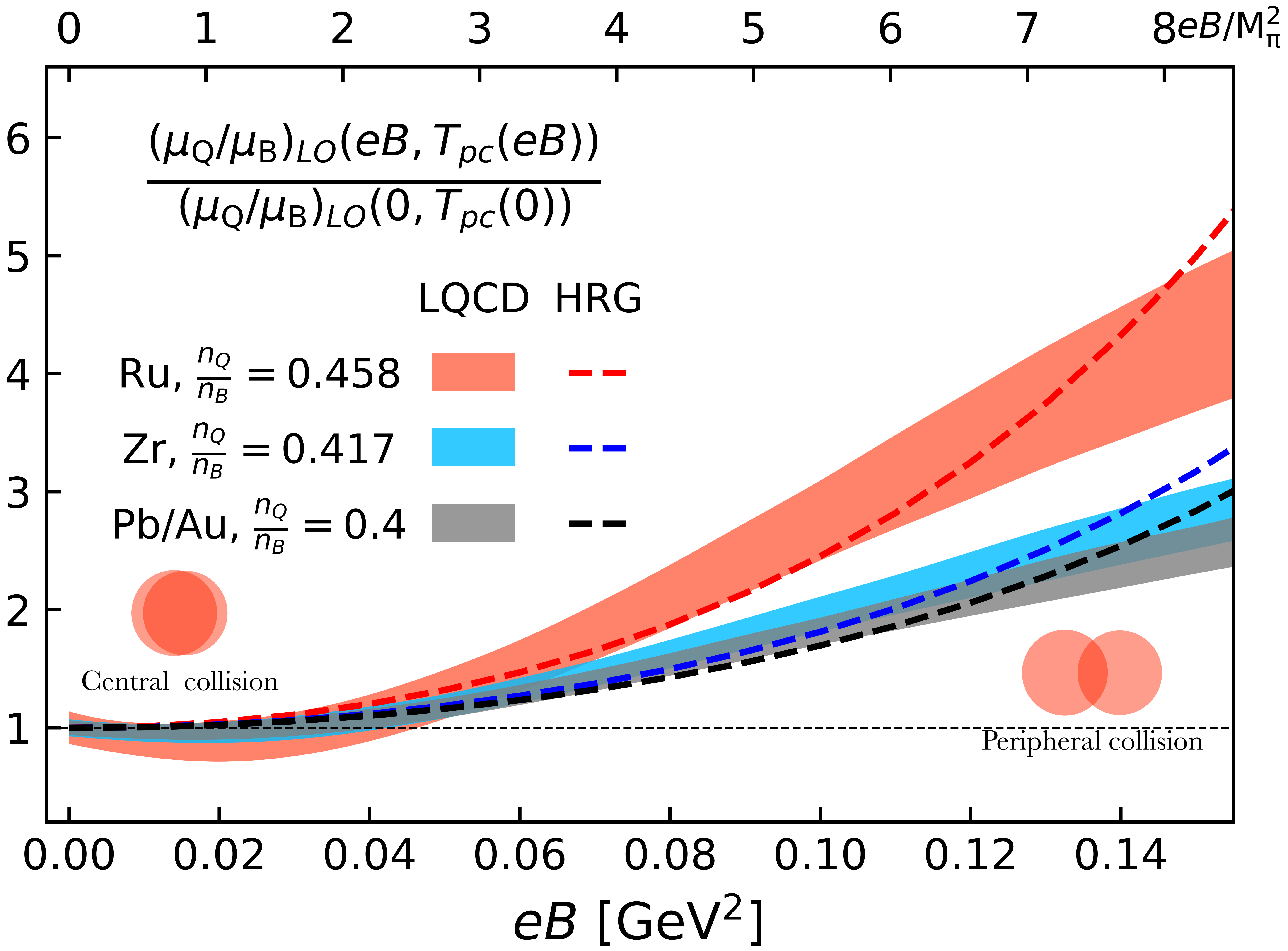
μ_Q/μ_B in nonzero magnetic fields



Suggest performing Thermal-FIST analyses with eB effects incorporated into the hadron spectrum



μ_Q/μ_B in different collision systems



$$\mu_Q/\mu_B = q_1 + q_3\mu_B^2 + \mathcal{O}(\mu_B^4)$$

$$q_1 = \frac{r(\chi_2^B\chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}) - (\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}{(\chi_2^Q\chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}) - r(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS})}$$

$$r = n_Q/n_B$$

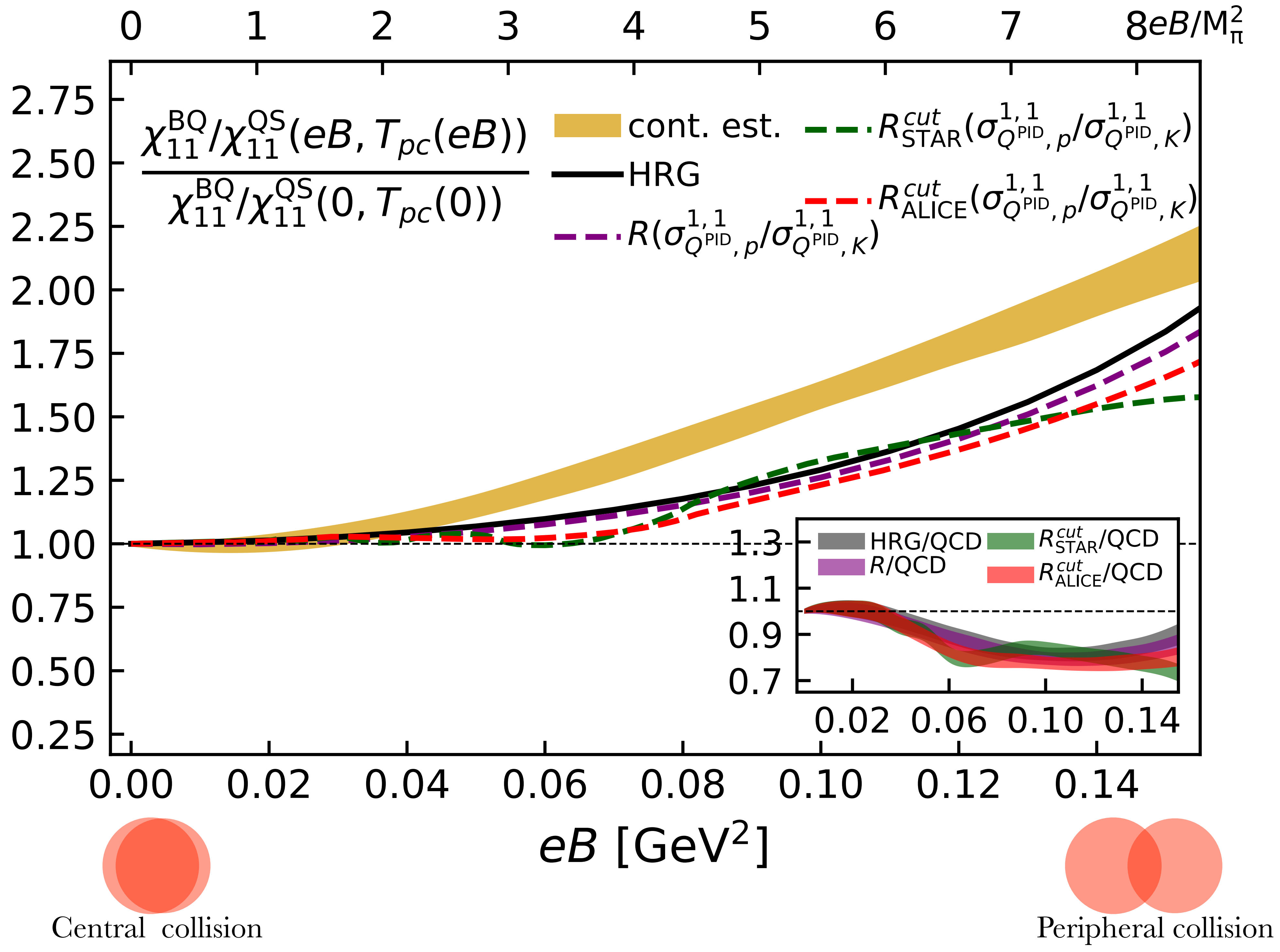
${}^{96}_{44}$Ru $+{}^{96}_{44}$ Ru: r=0.458

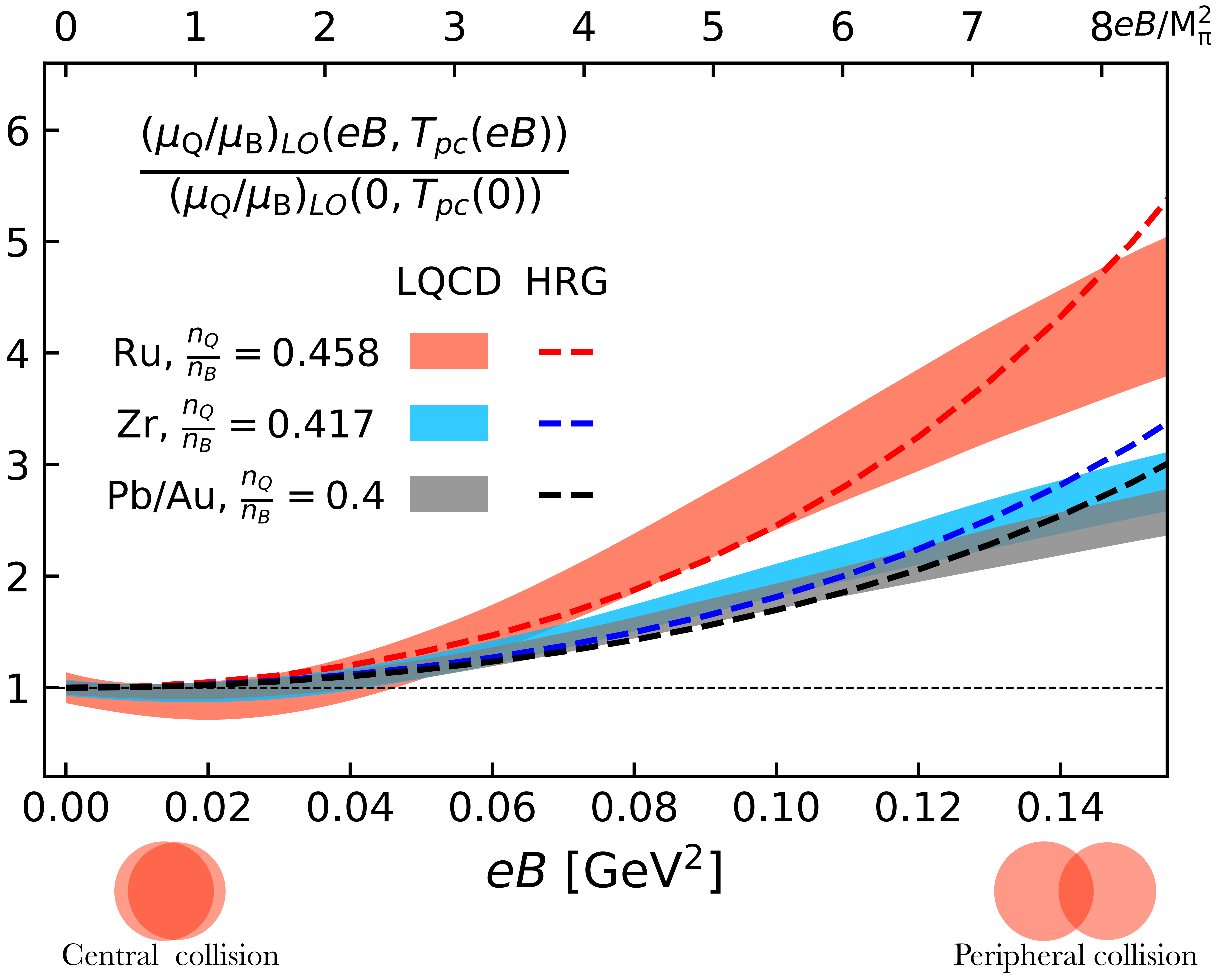
${}^{96}_{40}$Zr $+{}^{96}_{40}$ Zr: r=0.417

${}^{208}_{82}$Pb $+{}^{208}_{82}$ Pb: r=0.4

Summary

- A first lattice QCD computation of 2nd fluctuations of B, Q, S in nonzero magnetic fields
- QCD baselines for effective theories and model studies
- Probes to detect imprints of magnetic fields in HIC: χ_{11}^{BQ} measured from proxy and μ_Q/μ_B obtained from thermal fits to particle yields in HIC



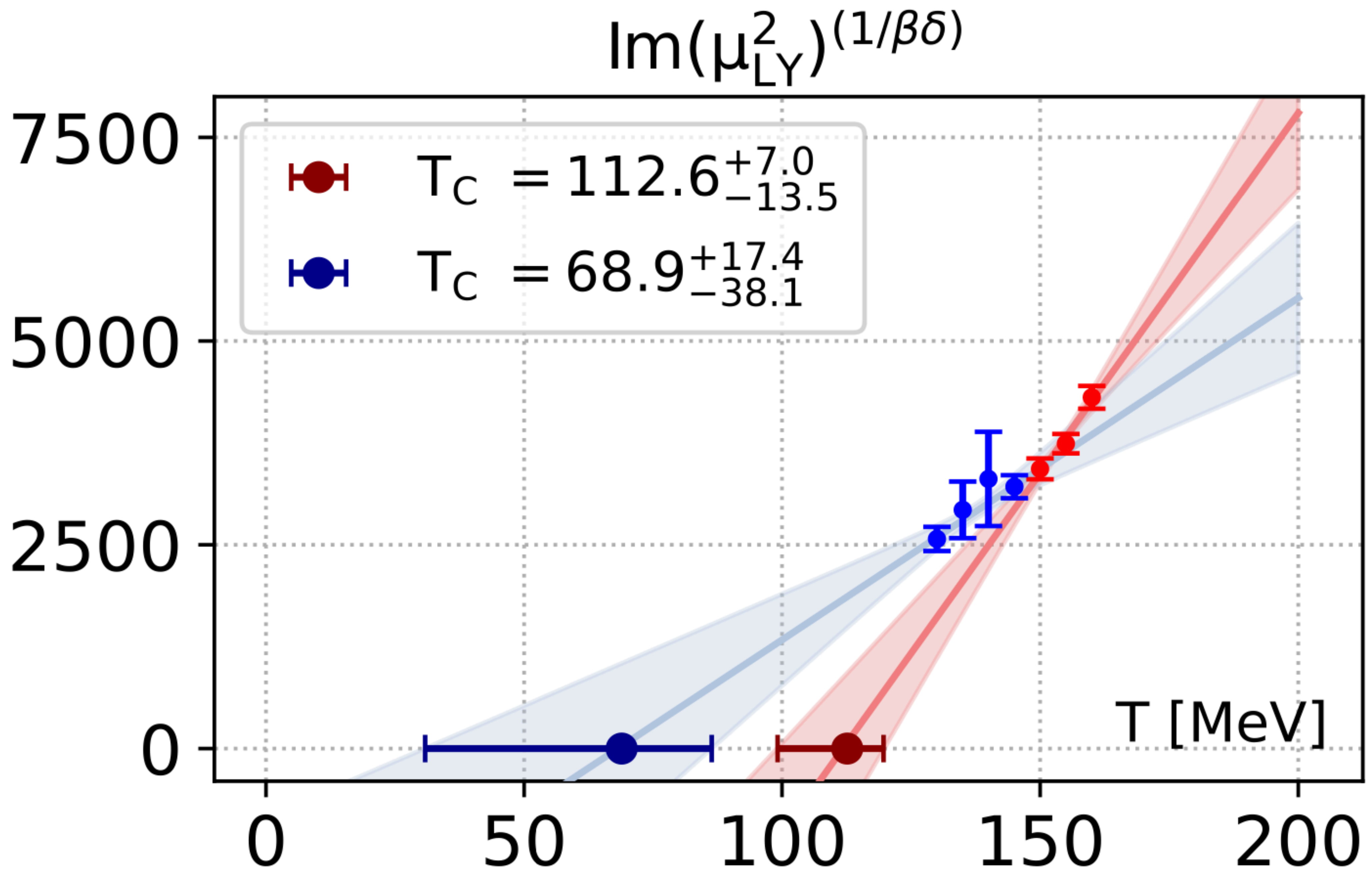


Searching for the CEP from LQCD Simulations at a Low Temperature

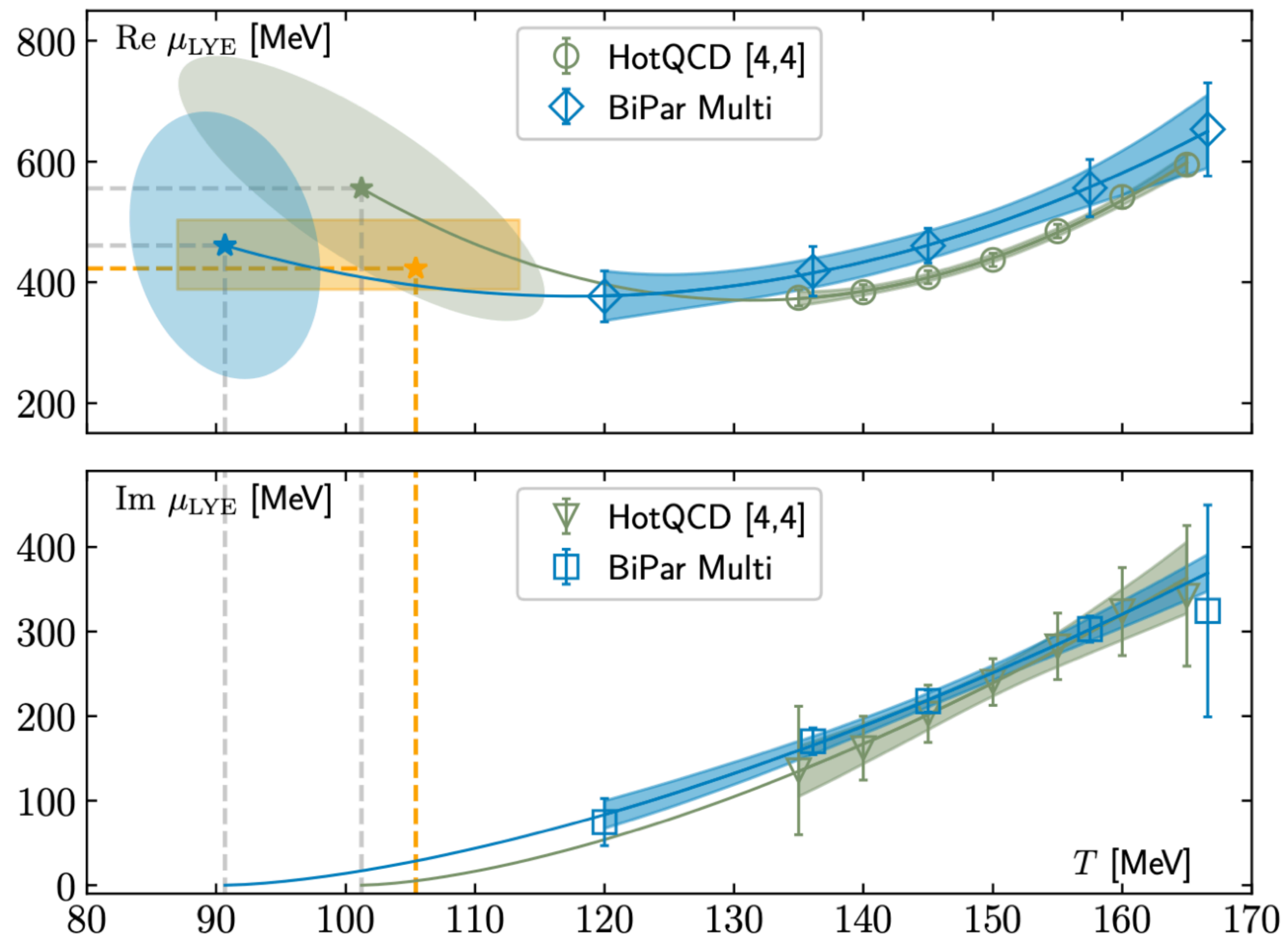
Poster by Jin-Biao Gu
in QM2025

- Current results from LQCD and effective theory suggest: $T_c^{CEP} \simeq 110(?)$ MeV

See Jamie Karthein's talk on Sun.



Alexander Adam, QM 2025



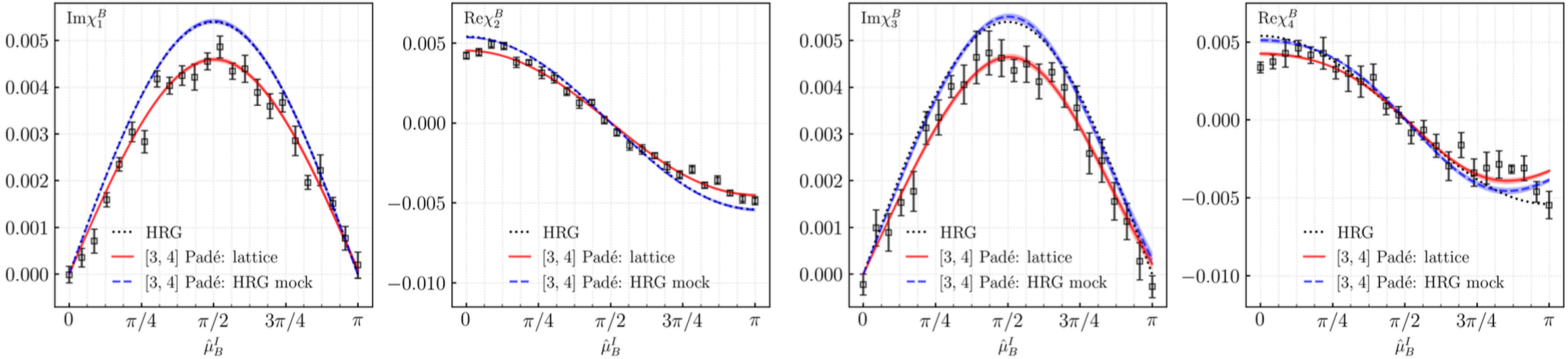
Bielefeld-Parma 2405.10196

- Results are based on the extrapolations from LQCD simulations at $T \gtrsim 120$ MeV

Searching for the CEP from LQCD Simulations at a Low Temperature

Poster by Jin-Biao Gu in QM2025

- We performed LQCD simulations directly at T=107 MeV with imaginary μ_B on $20^3 \times 10$ lattices, with ~200k configurations for each $i\mu_B$

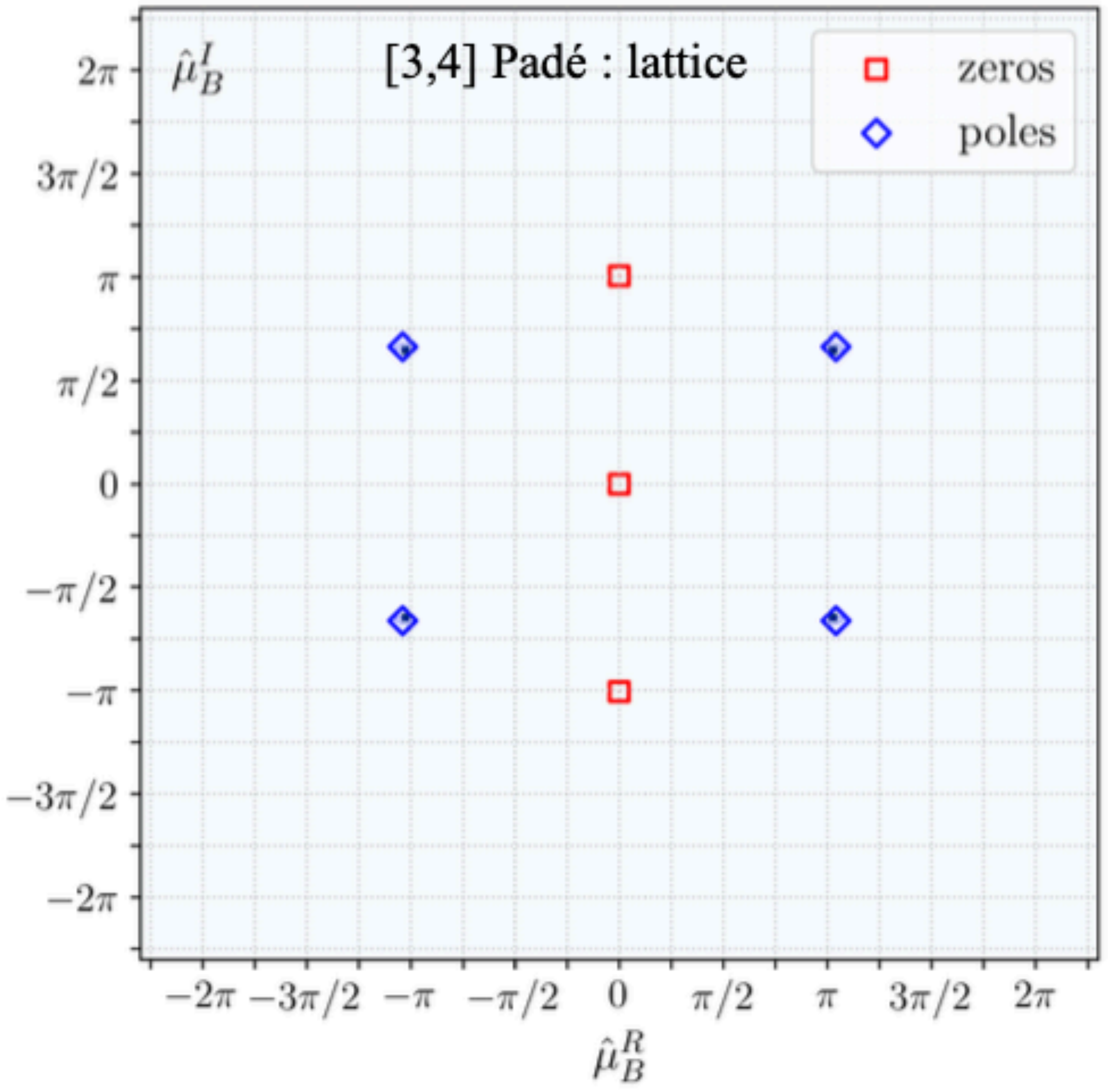


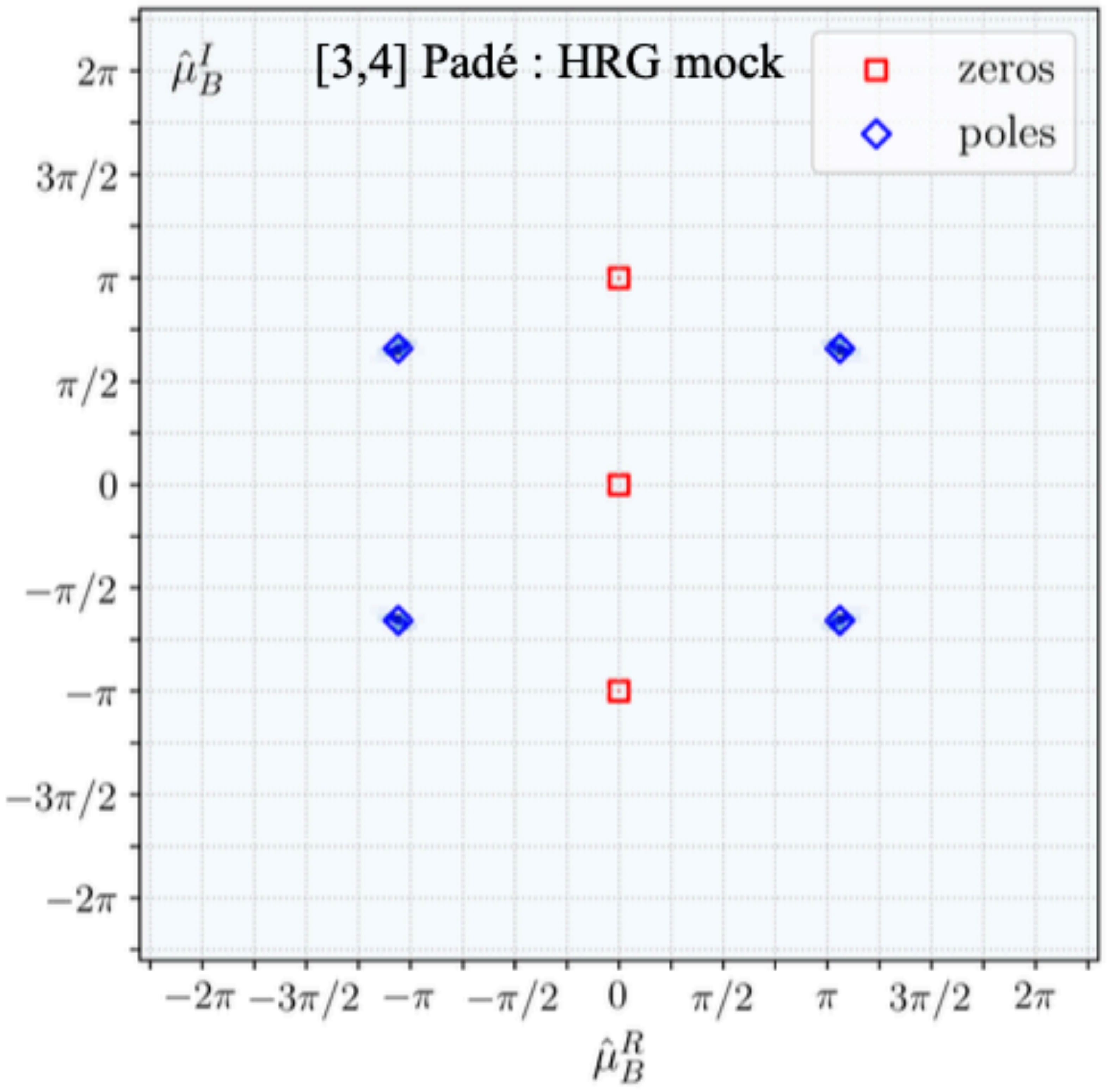
Padé fit: $$R_n^m(\hat{\mu}_B) \equiv \frac{P_m(\hat{\mu}_B)}{1+Q_n(\hat{\mu}_B)} = \frac{\sum_{i=0}^{(m-1)/2} a_{2i+1}\hat{\mu}_B^{2i+1}}{1+\sum_{j=1}^{n/2} b_{2j}\hat{\mu}_B^{2j}}$$

Searching for the CEP from LQCD Simulations at a Low Temperature

Poster by Jin-Biao Gu in QM2025

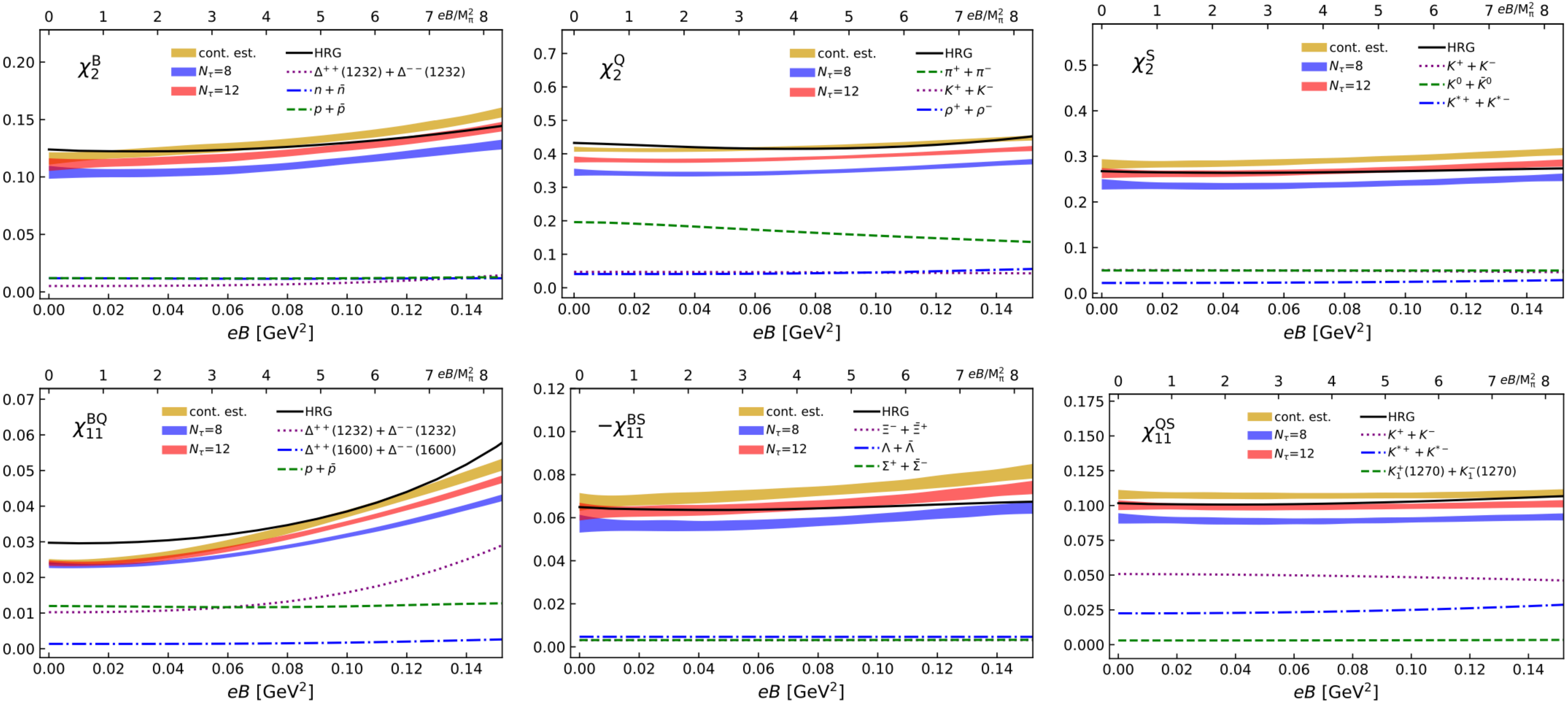
- We performed LQCD simulations directly at T=107 MeV with imaginary μ_B on $20^3 \times 10$ lattices, with ~200k configurations for each $i\mu_B$





Backup

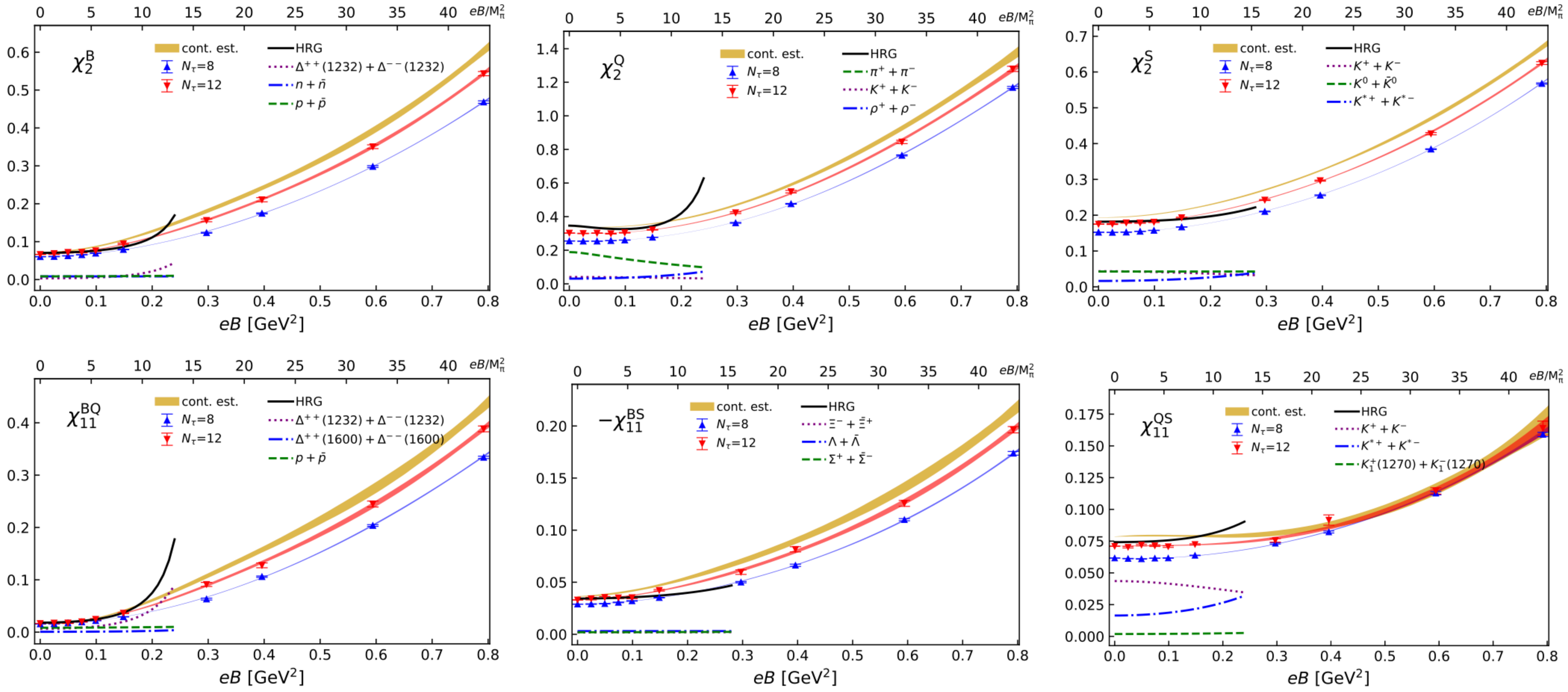
2nd order fluctuations along the transition temperature and their comparison to HRG



HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

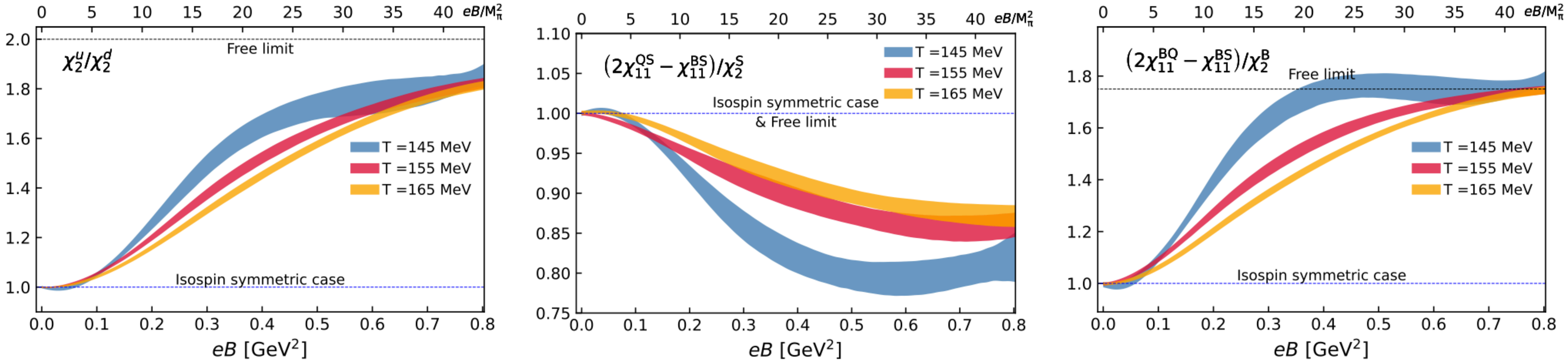
2nd order fluctuations and their comparisons to HRG at T=145 MeV

HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467



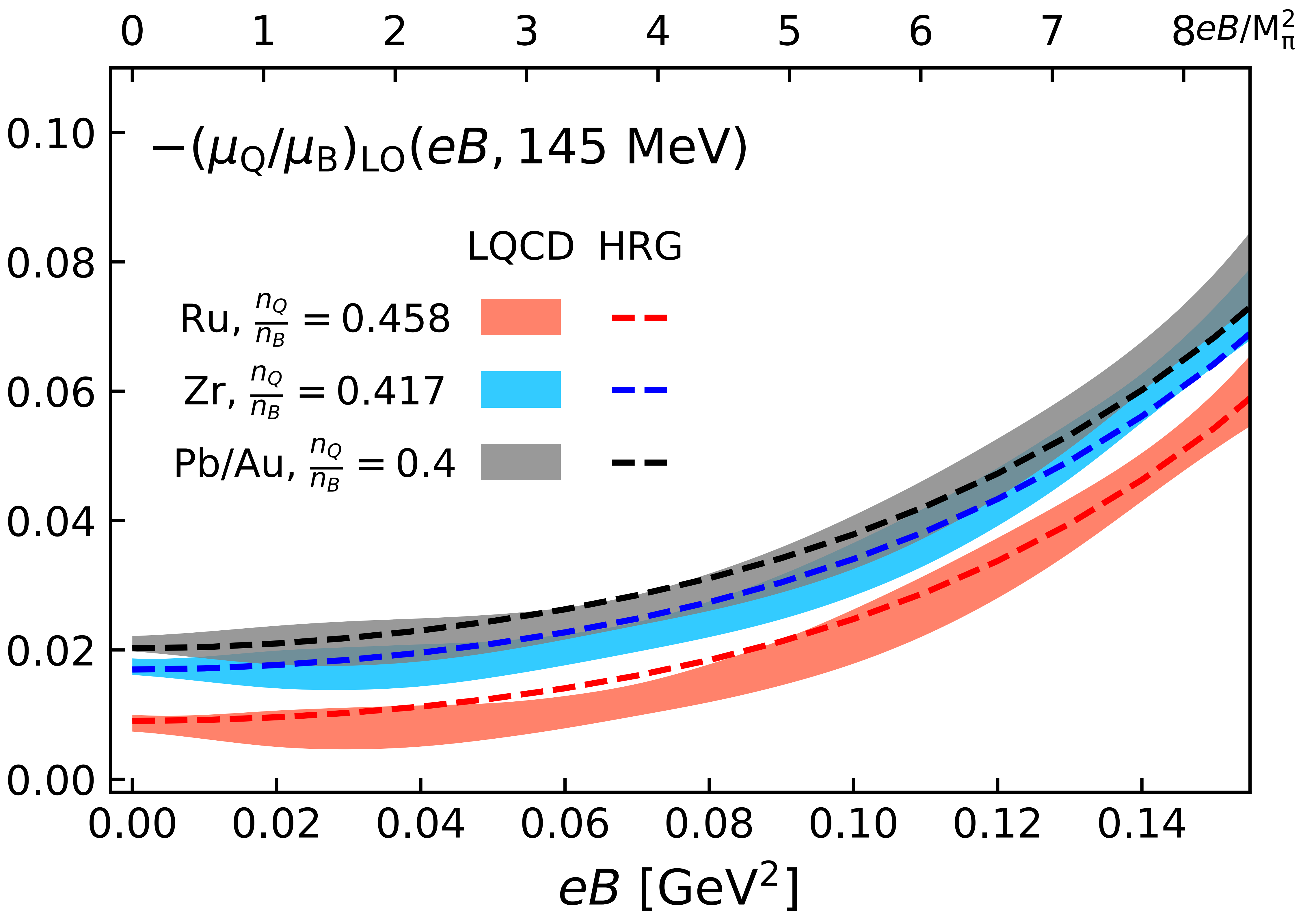
Hierarchy in eB-dependence: $\chi_{11}^{BQ} > \chi_2^{B} > \chi_{11}^{BS} > \chi_2^{Q} > \chi_2^{S} > \chi_{11}^{QS}$

Isospin symmetry breaking in strong magnetic fields

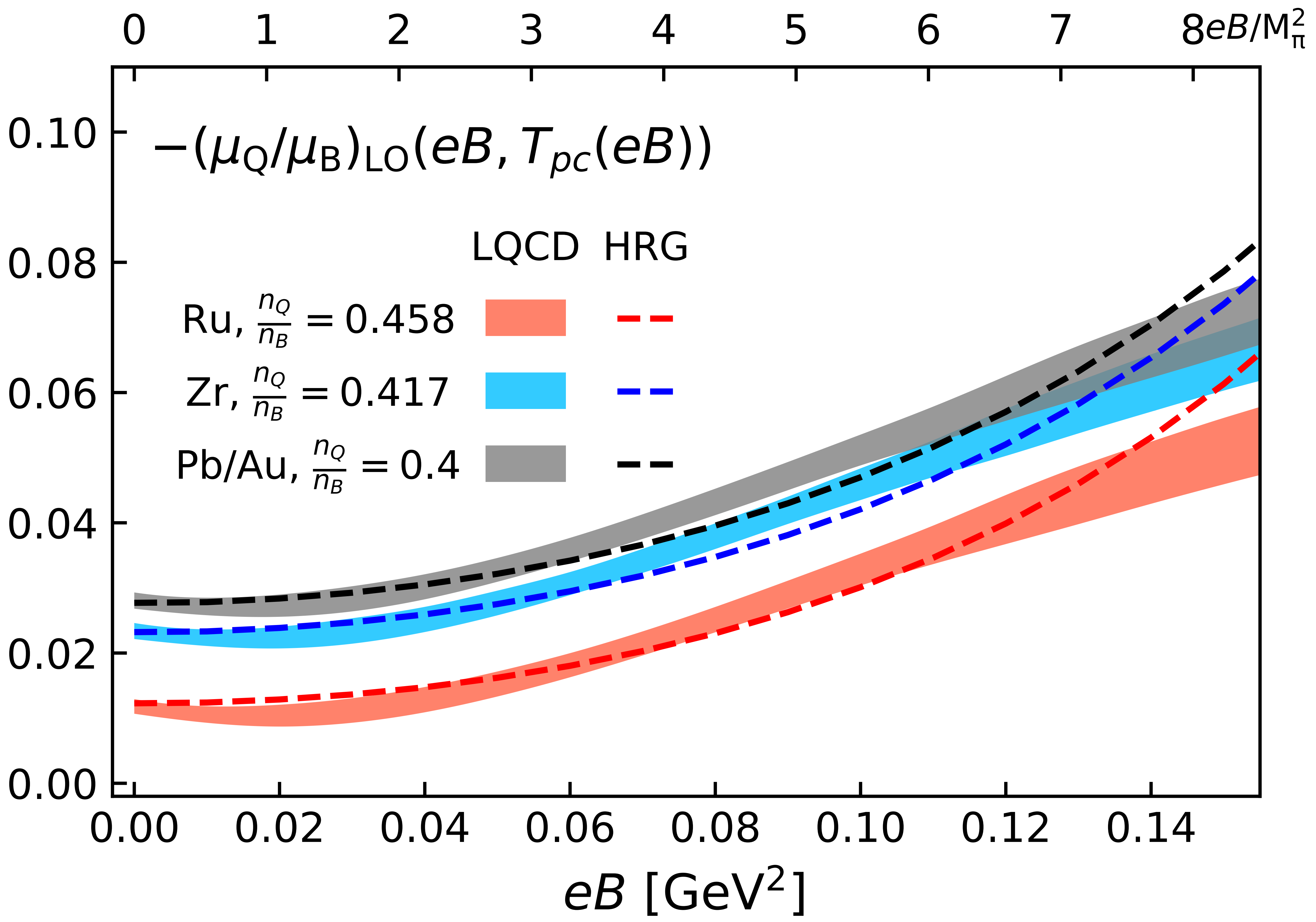


HTD, J.-B. Gu, A. Kumar, S.-T. Li, arXiv:2503.18467

μ_Q/μ_B



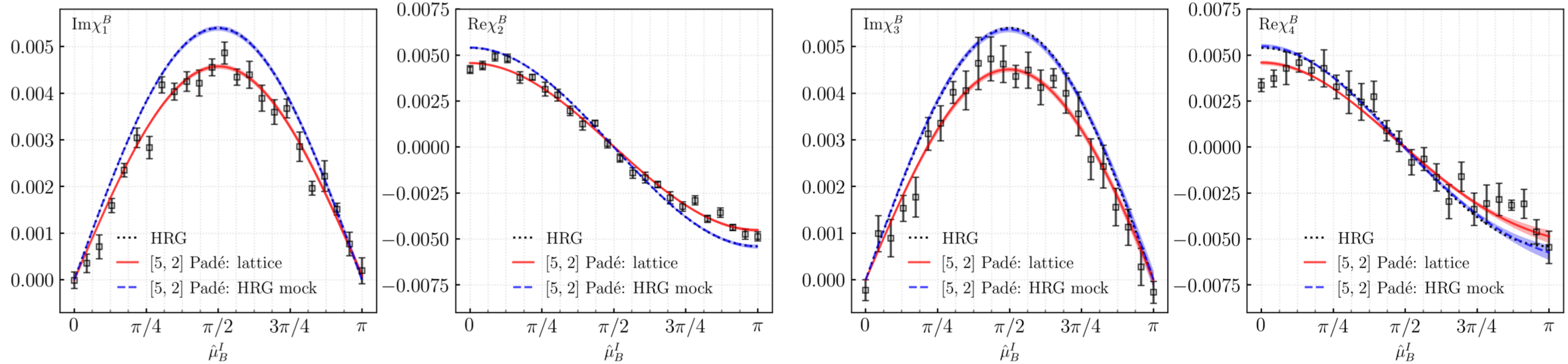
$-(\mu_Q/\mu_B)_{LO}(eB, 145\ \mathrm{MeV})$
LQCD HRG
Ru, $\frac{n_Q}{n_B} = 0.458$
Zr, $\frac{n_Q}{n_B} = 0.417$
Pb/Au, $\frac{n_Q}{n_B} = 0.4$
eB [GeV^2]
eB/M_π^2

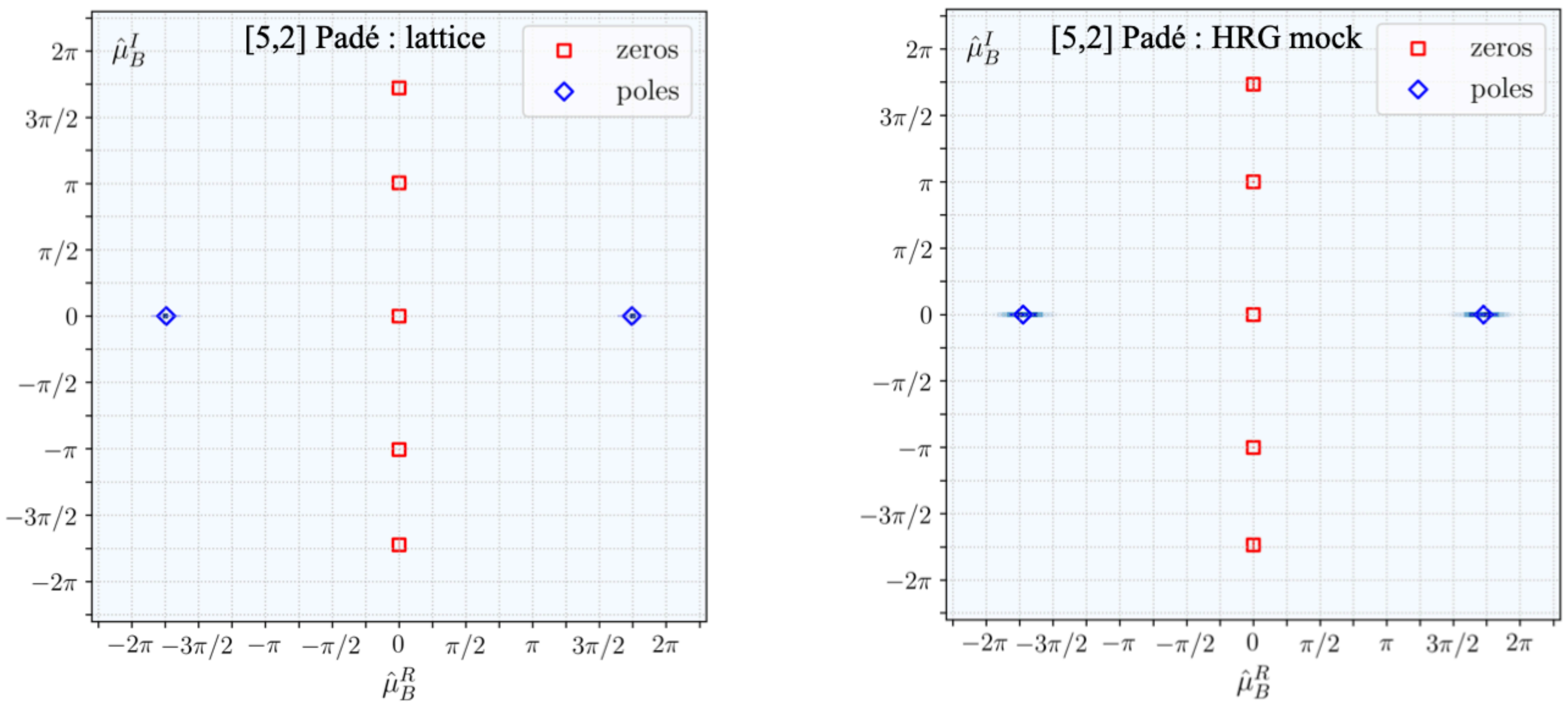


$-(\mu_Q/\mu_B)_{LO}(eB, T_{pc}(eB))$
LQCD HRG
Ru, $\frac{n_Q}{n_B} = 0.458$
Zr, $\frac{n_Q}{n_B} = 0.417$
Pb/Au, $\frac{n_Q}{n_B} = 0.4$
eB [GeV^2]
eB/M_π^2

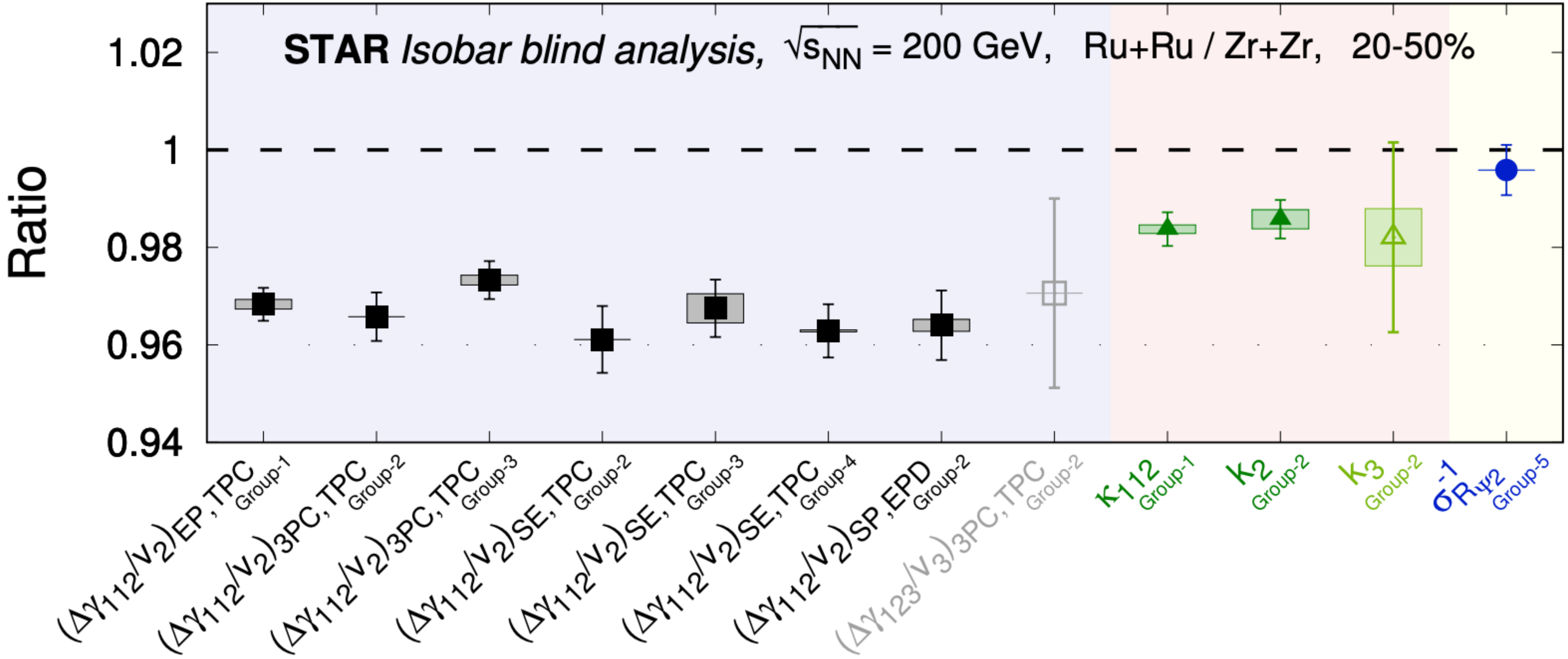
Searching for the CEP from LQCD Simulations at a Low Temperature

Poster by Jin-Biao Gu
in QM2025



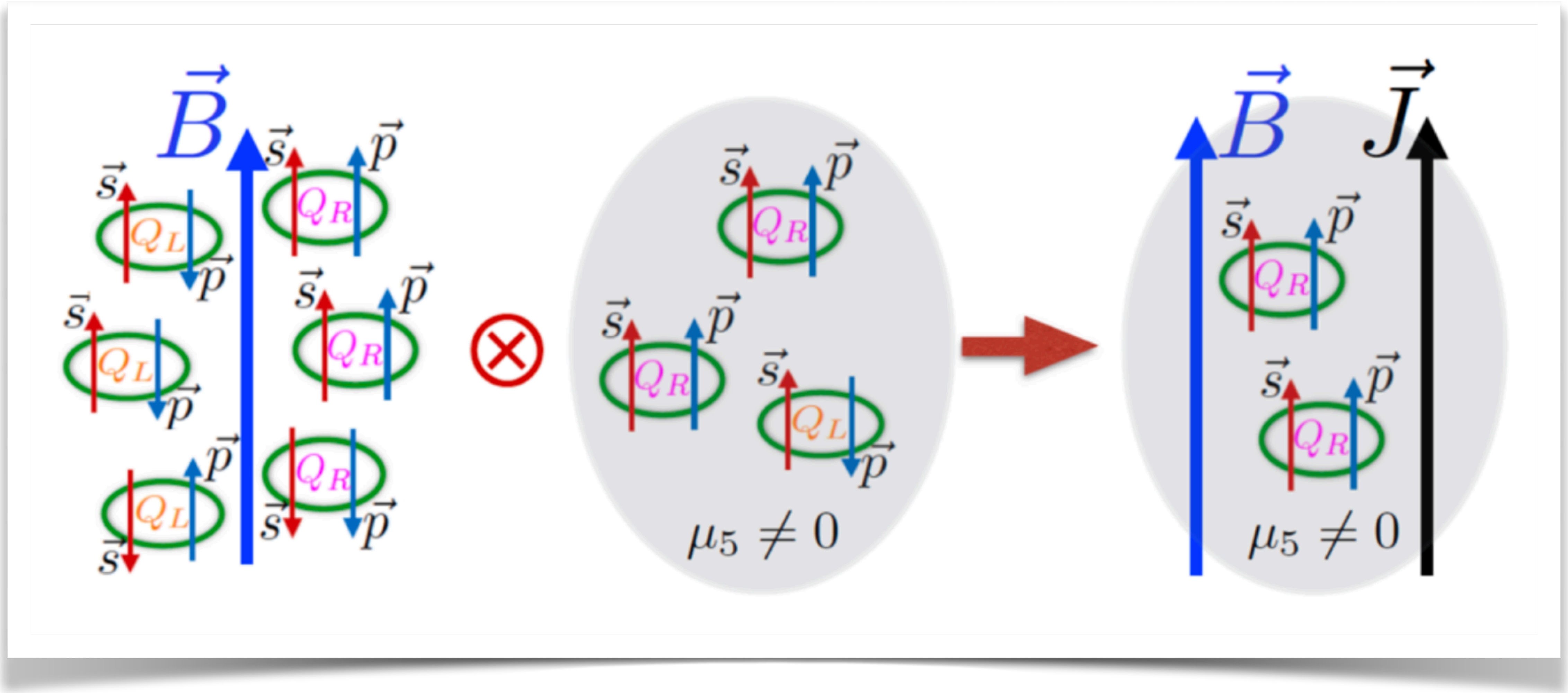


Search for the Chiral Magnetic Effect with Isobar Collisions



STAR collaboration, *Phys.Rev.C* 105 (2022) 1, 014901

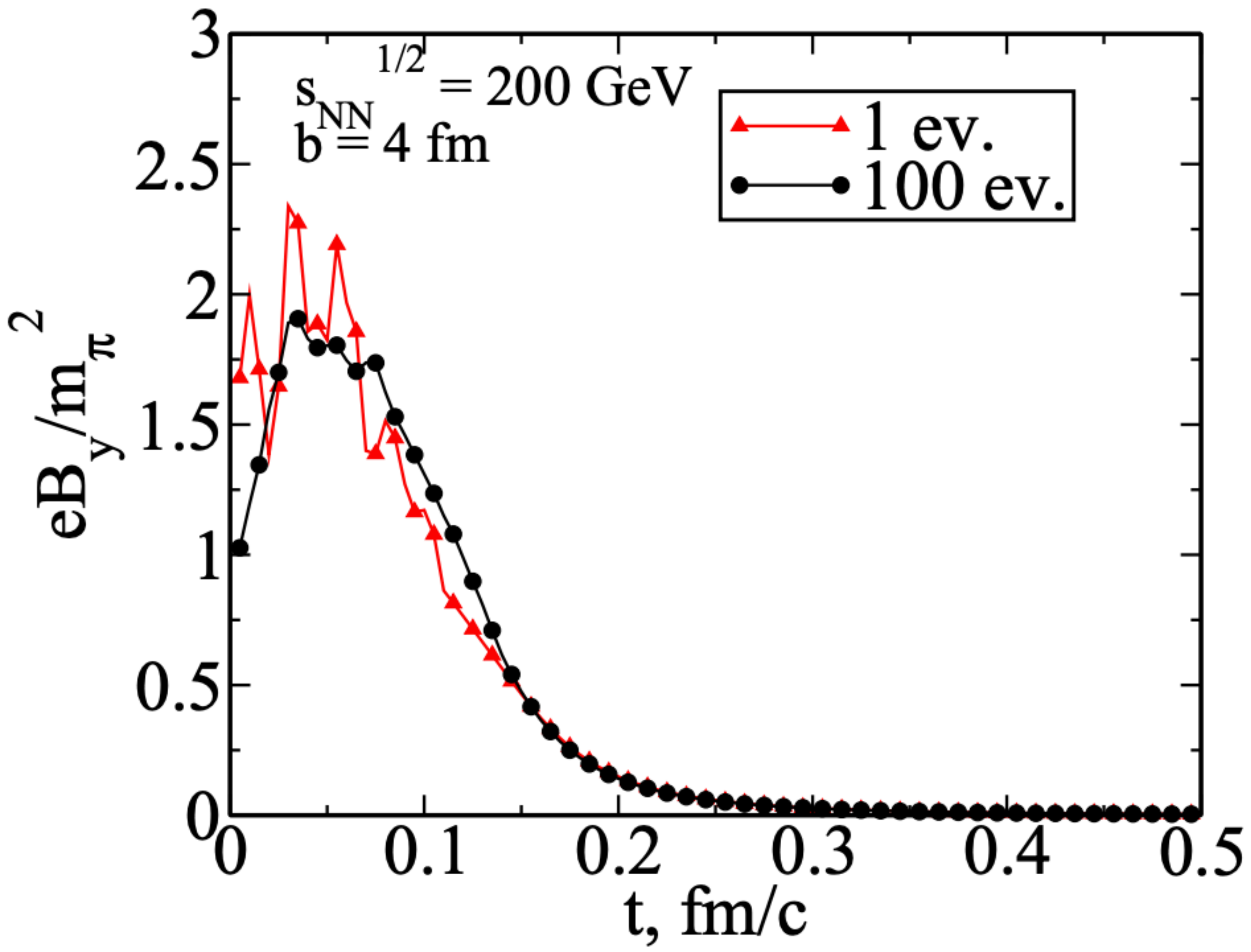
Search for chiral magnetic effect



See recent reviews e.g.
D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

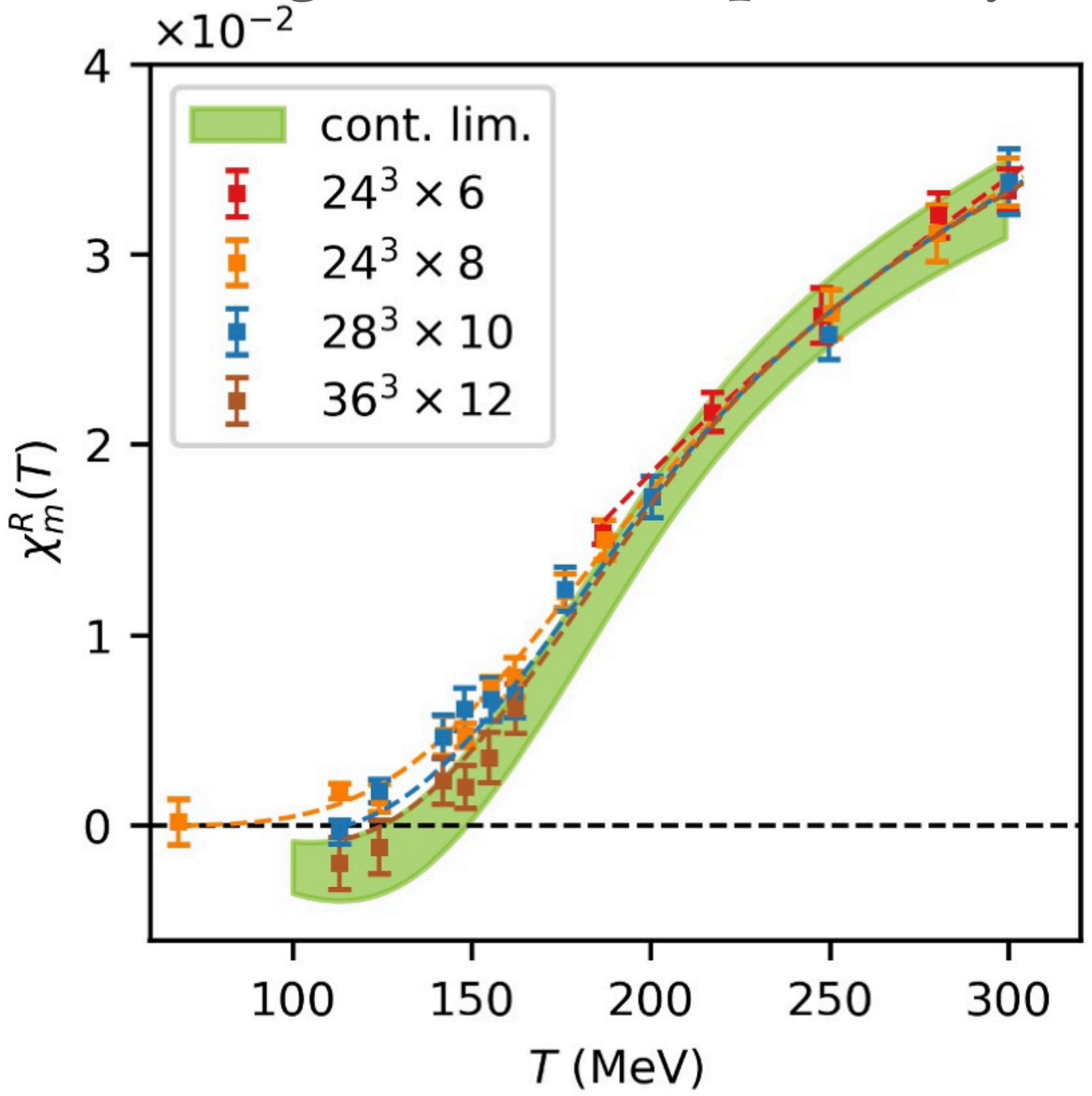
- ☑ Axial U(1) anomaly
  - HTD, Li, Mukherjee, Tomiya, Wang & Zhang, PRL 126 (2021) 082001
  - HTD, Huang, Mukherjee & Petreczky, PRL 131 (2023) 161903
  - Kaczmarek, Shanker & Sharma, PRD108 (2023) 094501
  - Alexandru, Bonano, D'Elia & Horvath, arXiv:2404.12298
  - … …
- ▶ Strong magnetic field

Magnetic field created in the early stage of HIC



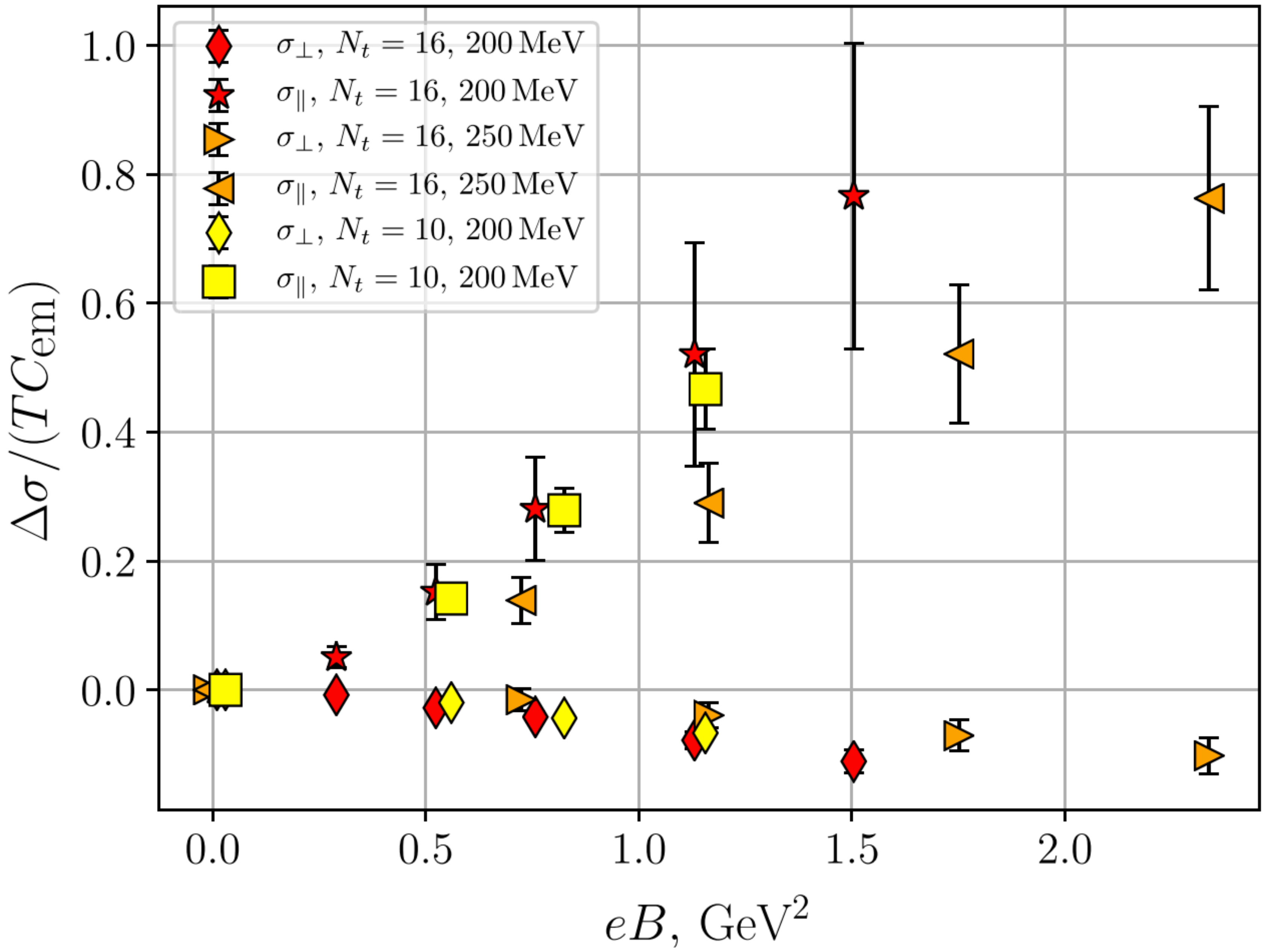
Skokov, Illarionov and Toneev, IJMPA 24 (2009) 5925

LQCD results on
Magnetic susceptibility



Brandt et al, JHEP 07 (2024) 207

LQCD results on
difference to electromagnetic
conductivity at eB=0



Astrakhantsev et al., PRD 102 (2020) 054516

t=0: RHIC: $eB \sim 5M_\pi^2$
LHC: $eB \sim 70M_\pi^2$

$T \gtrsim 150$ MeV: Paramagnetic
$T \lesssim 150$ MeV: Diamagnetic

$eB \uparrow$ Parallel: $\uparrow$
Transverse: $\downarrow$