Review of QCD critical point fluctuations in beam energy scan theory

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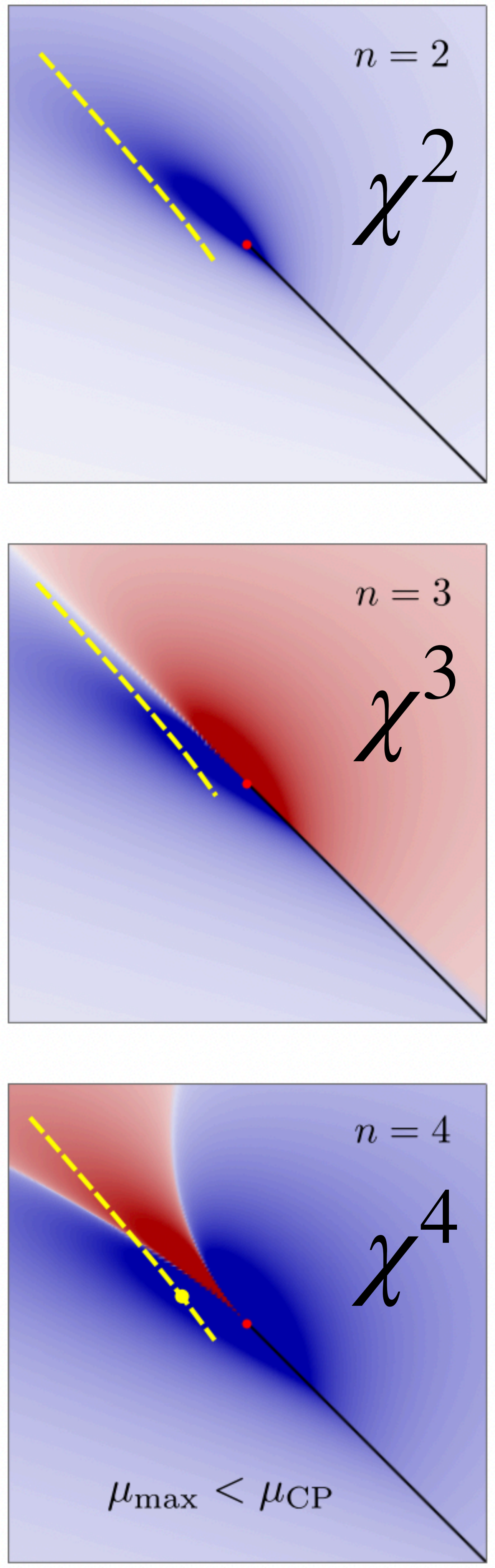
Some topics related to QCD critical point fluctuations in BEST program

1. Universality/Scaling near Critical Point and non-universal map

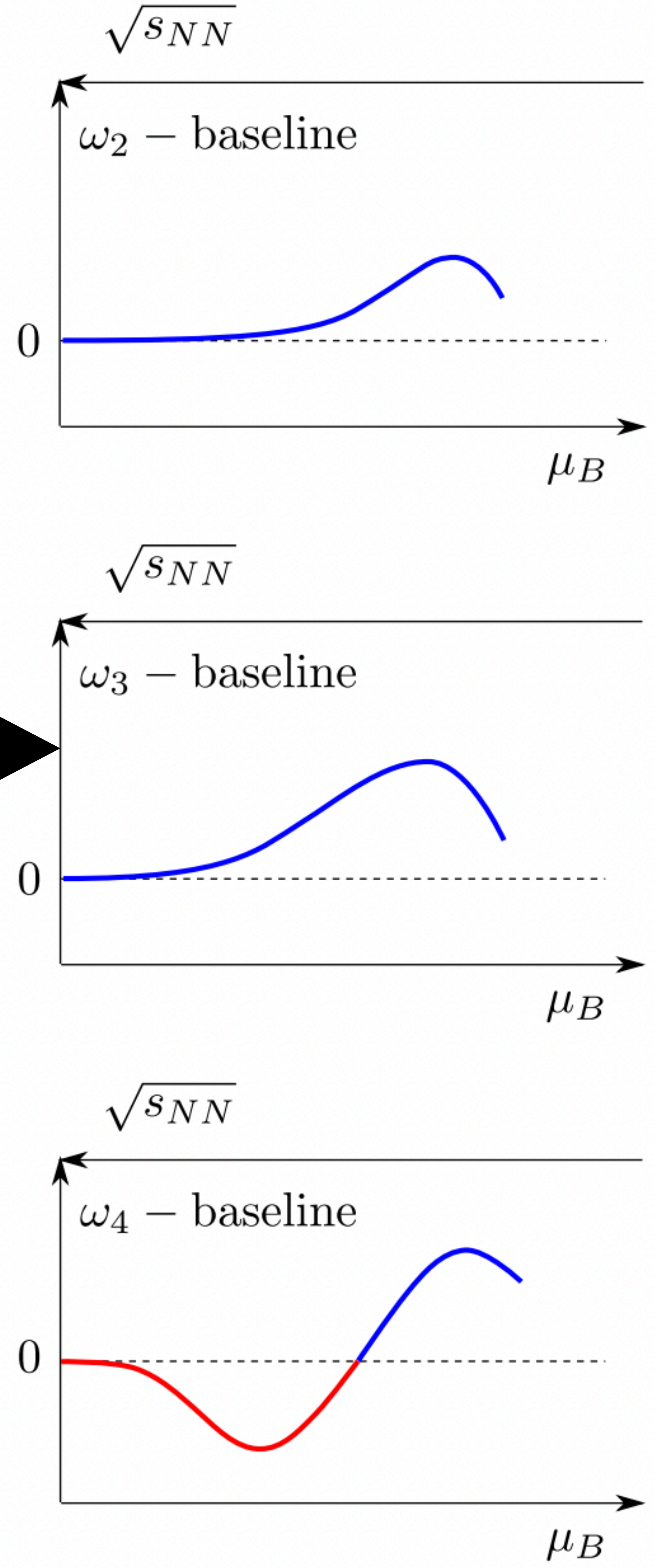
2. A theoretical paradigm to treat universality and charge conserving dynamics together

Critical point and EoS from fluctuation observables

M. Stephanov slides, SQM 24

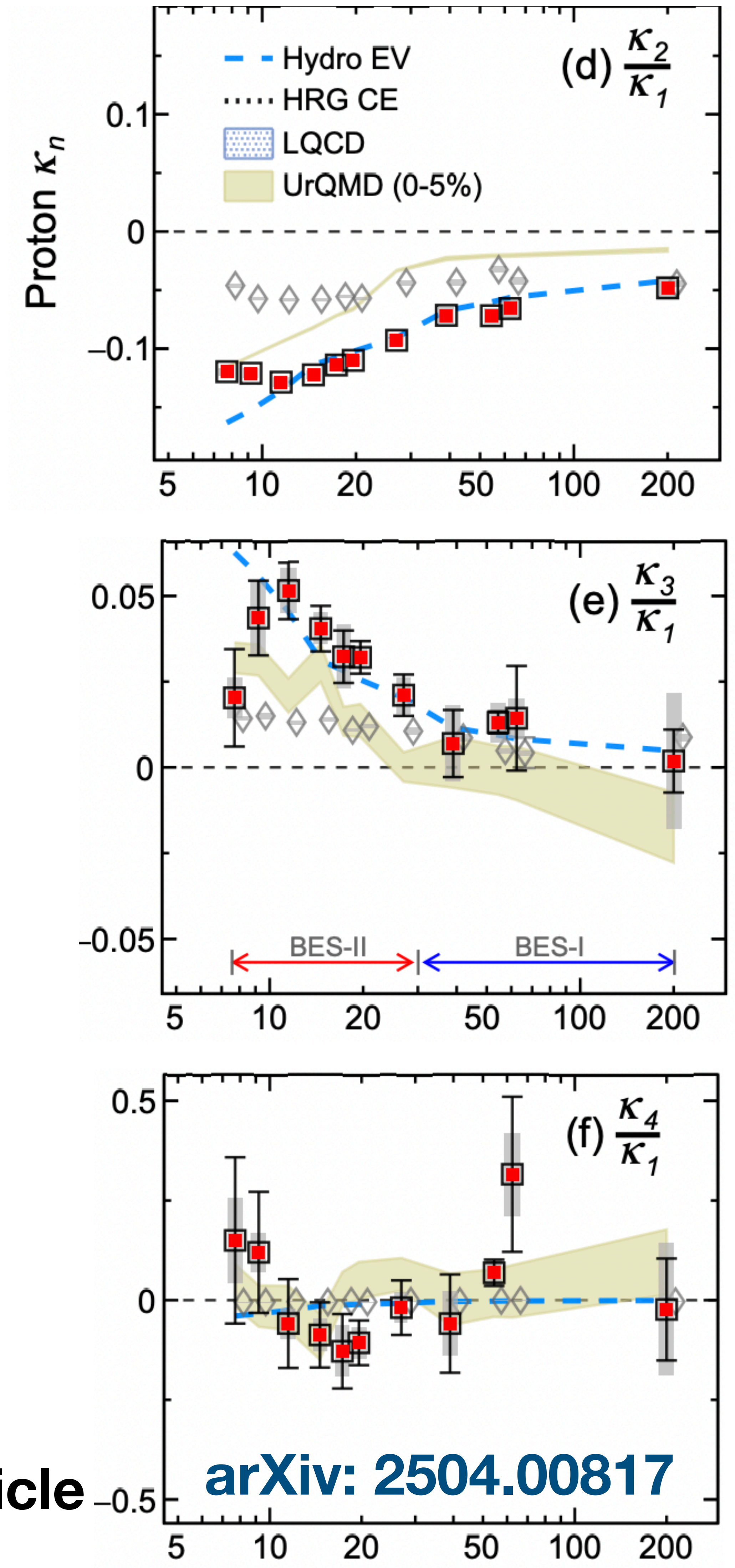


Baryon susceptibilities from 3D Ising-like EoS



Factorial cumulants of particle multiplicity

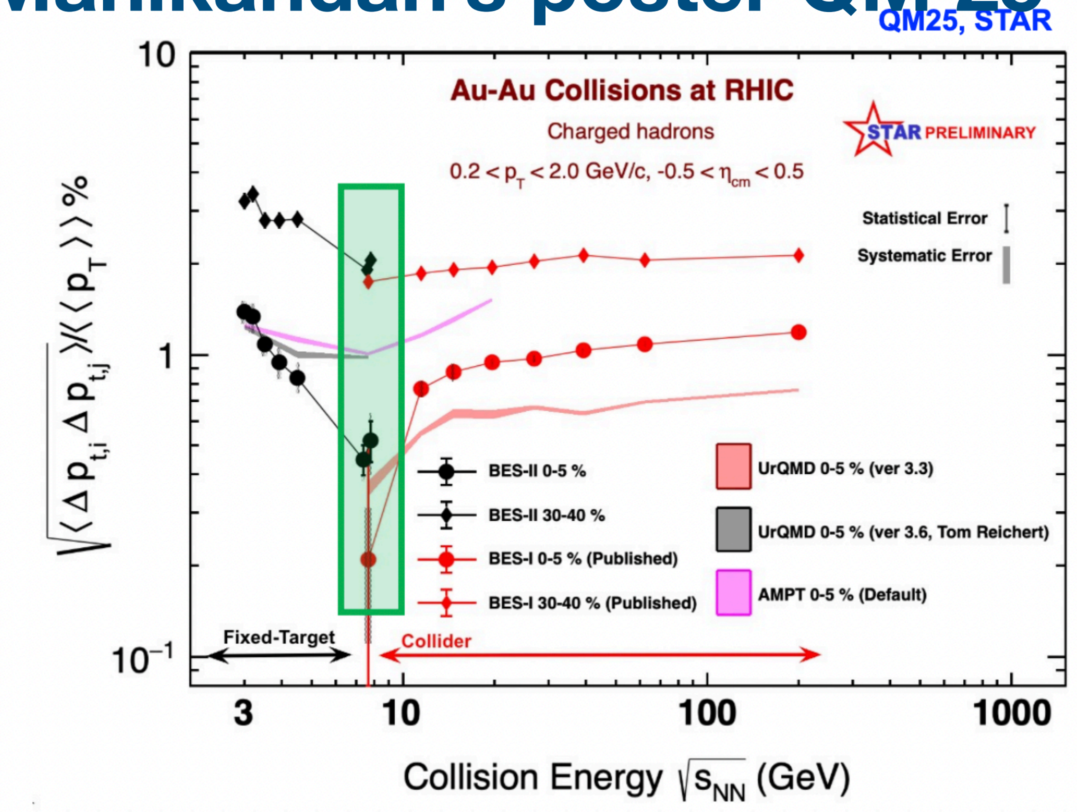
Goal : EoS near CP from data



arXiv: 2504.00817

Collision Energy (GeV)

R. Manikandan's poster QM 25



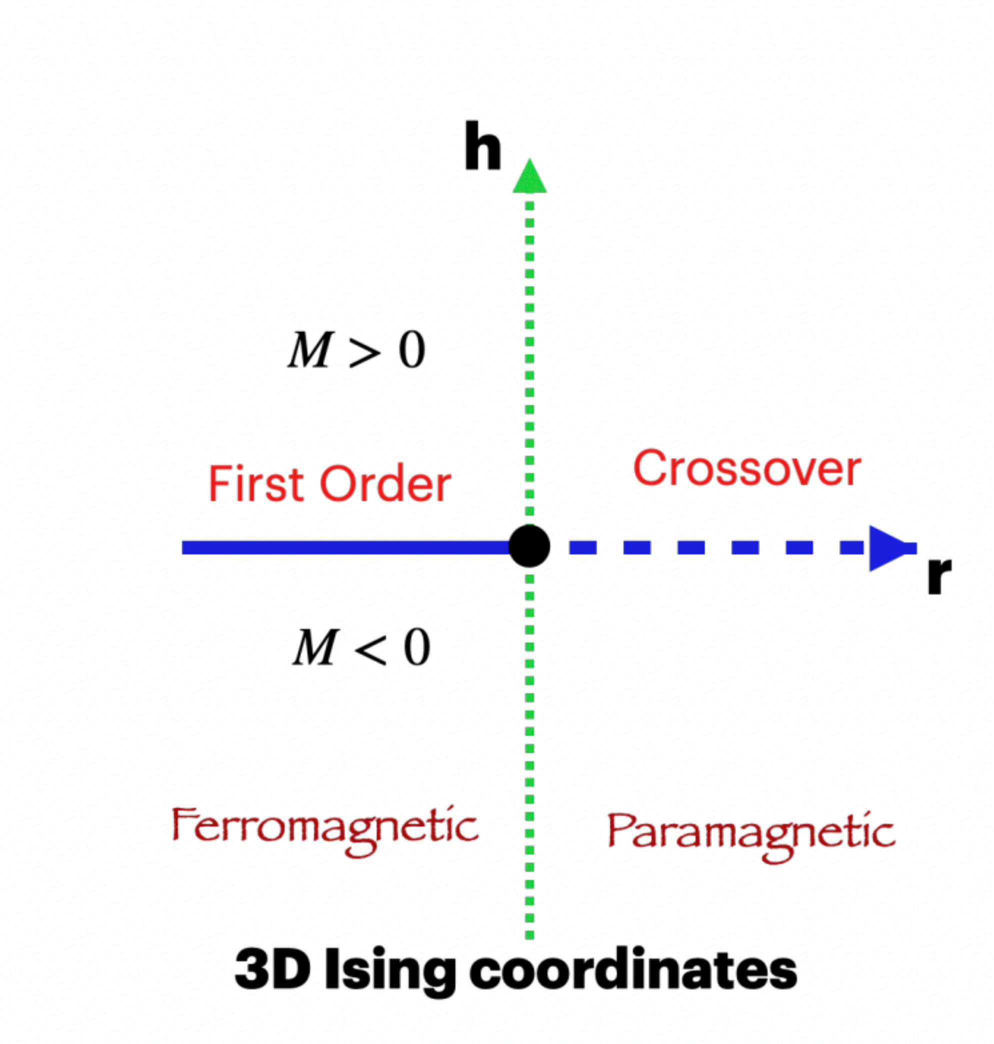
Particle multiplicity factorial cumulants, transverse momentum fluctuations

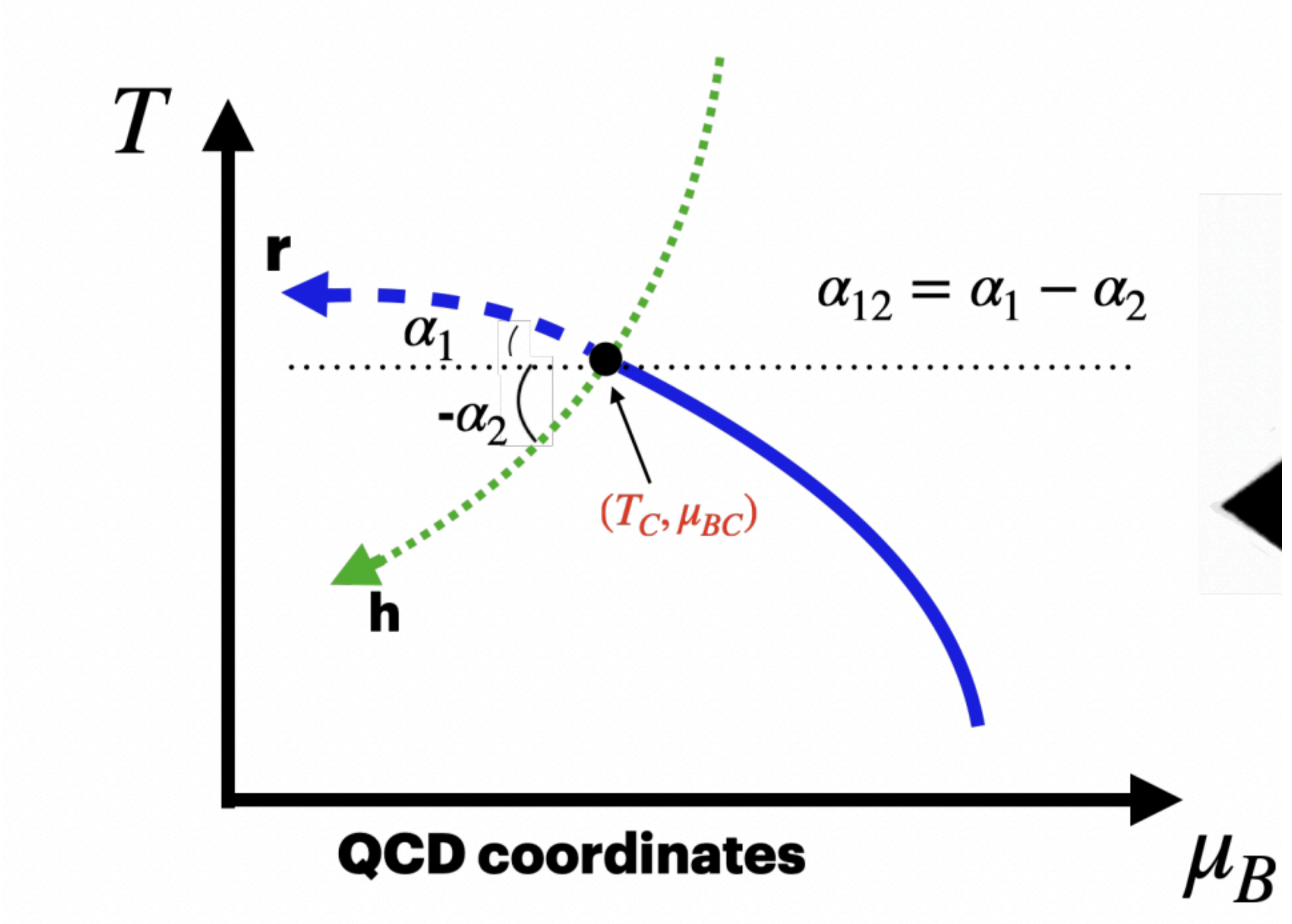
Universal EoS near CP and non-universal map

Minimalistic description of EoS near critical point based on universality

$$P_{\mathrm{QCD}}(\mu,T) \;=\; P_{\mathrm{BG}}(\mu,T) \,+\, A\,G(r(\mu,T),h(\mu,T))$$

3D Ising model





$T_c(\mu_c), \alpha_1(\mu_c)$ Phase boundary

Independent & non-universal parameters

$$\mu_c\,,\ \alpha_{12}\,,\ \rho\,,\ w$$

Weakly constrained in the chiral limit

MP, Stephanov, 19

Parotto et al, 18, Karthein et al, 21, Kahangirwe et al., 24

Examples of recently developed EoSs that have a CP in the Ising universality class but differ in their implementation: Parotto et al, 19, Karthein et al.,21, Grefa et al., 21,Kapusta&Welle,22, Kahangirwe et al.,24

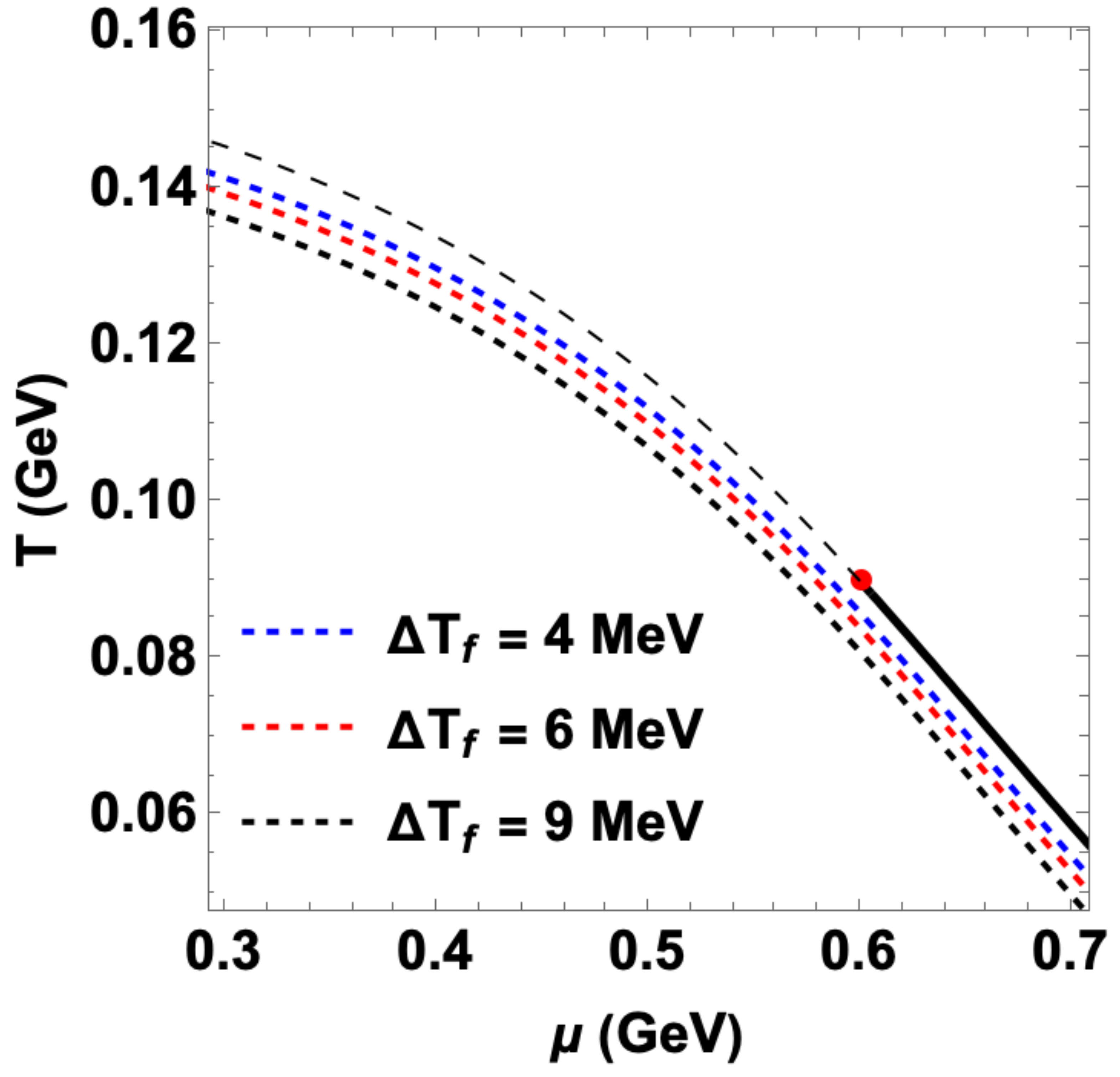
Dependence of factorial cumulants on mapping parameters

Karthein, MP, Rajagopal, Stephanov, Yin

- $\hat{\Delta}\omega_p^k \sim w^{-1.2}\Delta T_f^{1.2-k} \sim \xi^{k(5-\eta)/2-3}$ Magnitude sensitive to w and ΔT_f
- $\bar{\rho} = \rho\, w^{0.36}$ controls the position of peaks/dips along a freeze-out curve
- α_2 affects both the position as well as magnitude of the maxima of the factorial cumulants along the freeze-out curve

M. Pradeep's slides, QM 25, Thurs

Parameters in the model for demonstration



Phase boundary from **Kahangirwe et al., 24 PRD**

$\mu_c = 600$ MeV , $T_c = 89$ MeV , $\alpha_1 \approx 16°$

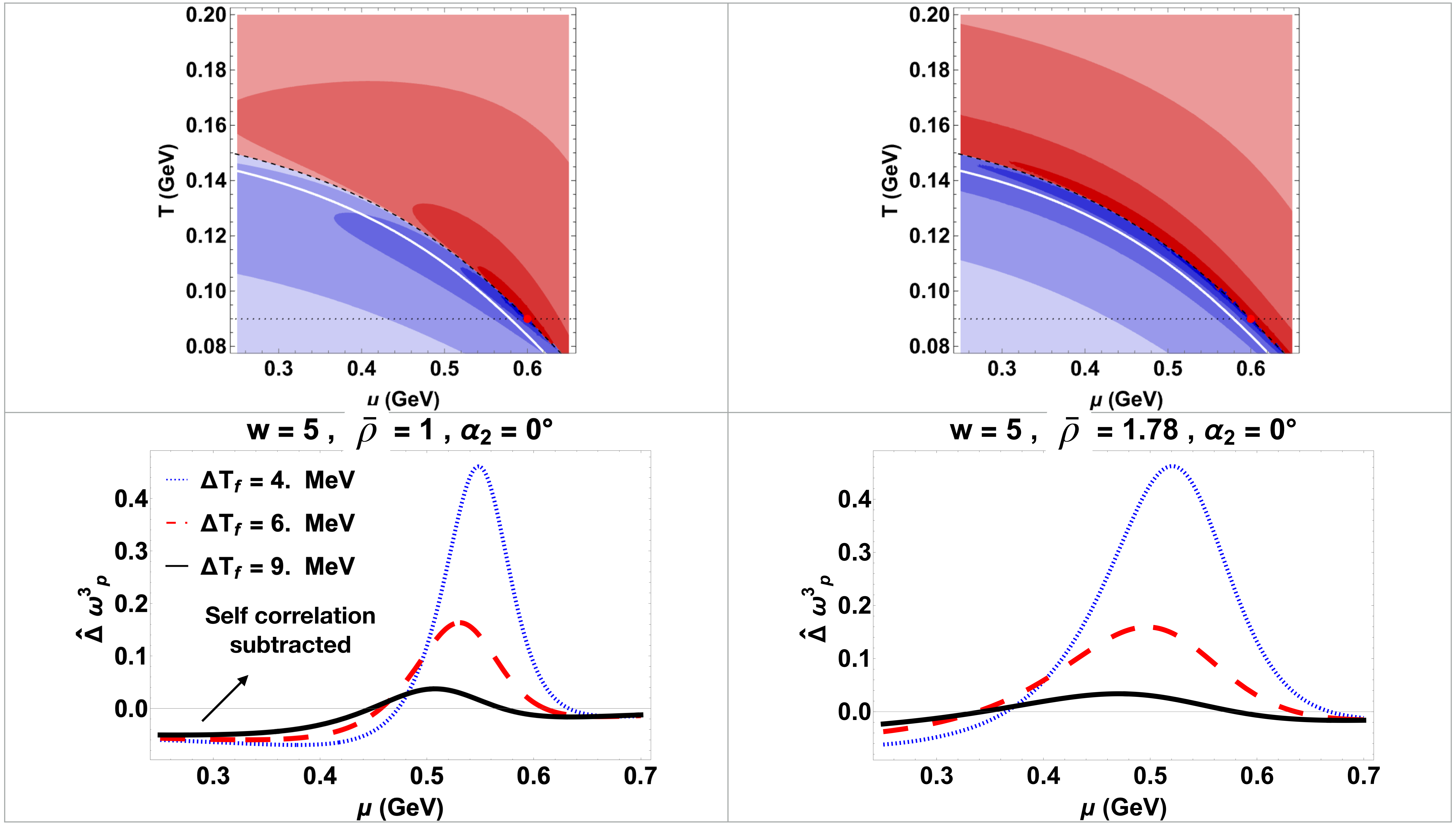
$T_f(\mu) = T_{h=0}(\mu) - \Delta T_f$ Freeze-out curve

$$\bar{\rho} = \rho\, w^{1-\frac{1}{\beta\delta}} = \rho w^{0.36}$$

- We will vary $\bar{\rho}$, w and α_2

$\bar{\rho}$ dependence of third factorial cumulant of proton multiplicity

Karthein, MP, Rajagopal, Stephanov, Yin

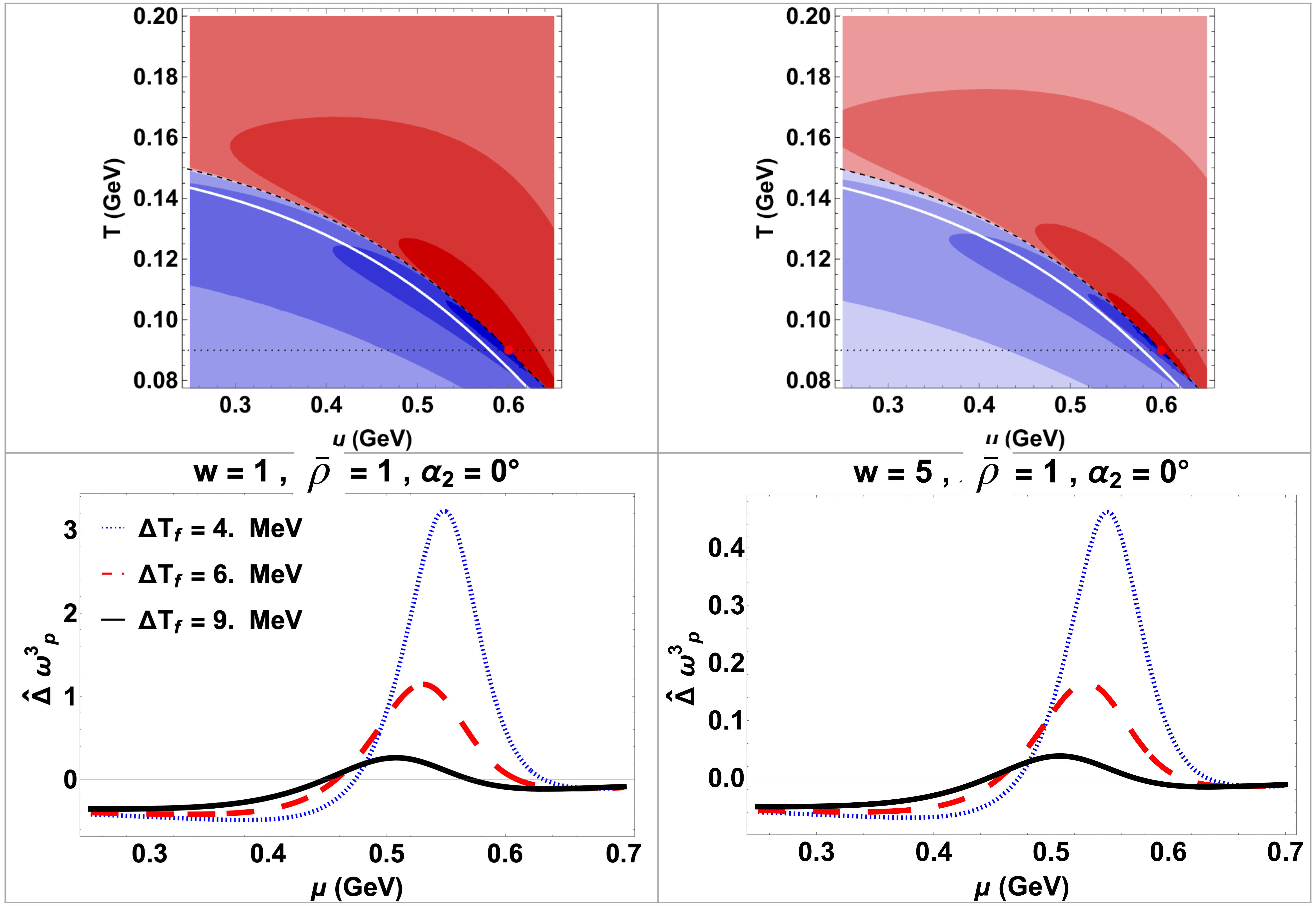


$$\bar{\rho} = \rho\, w^{1-\frac{1}{\beta\delta}} = \rho w^{0.36}$$

- Peak position - sensitive to $\bar{\rho}$
- Peak magnitude - not much change

w dependence of third factorial cumulant of proton multiplicity

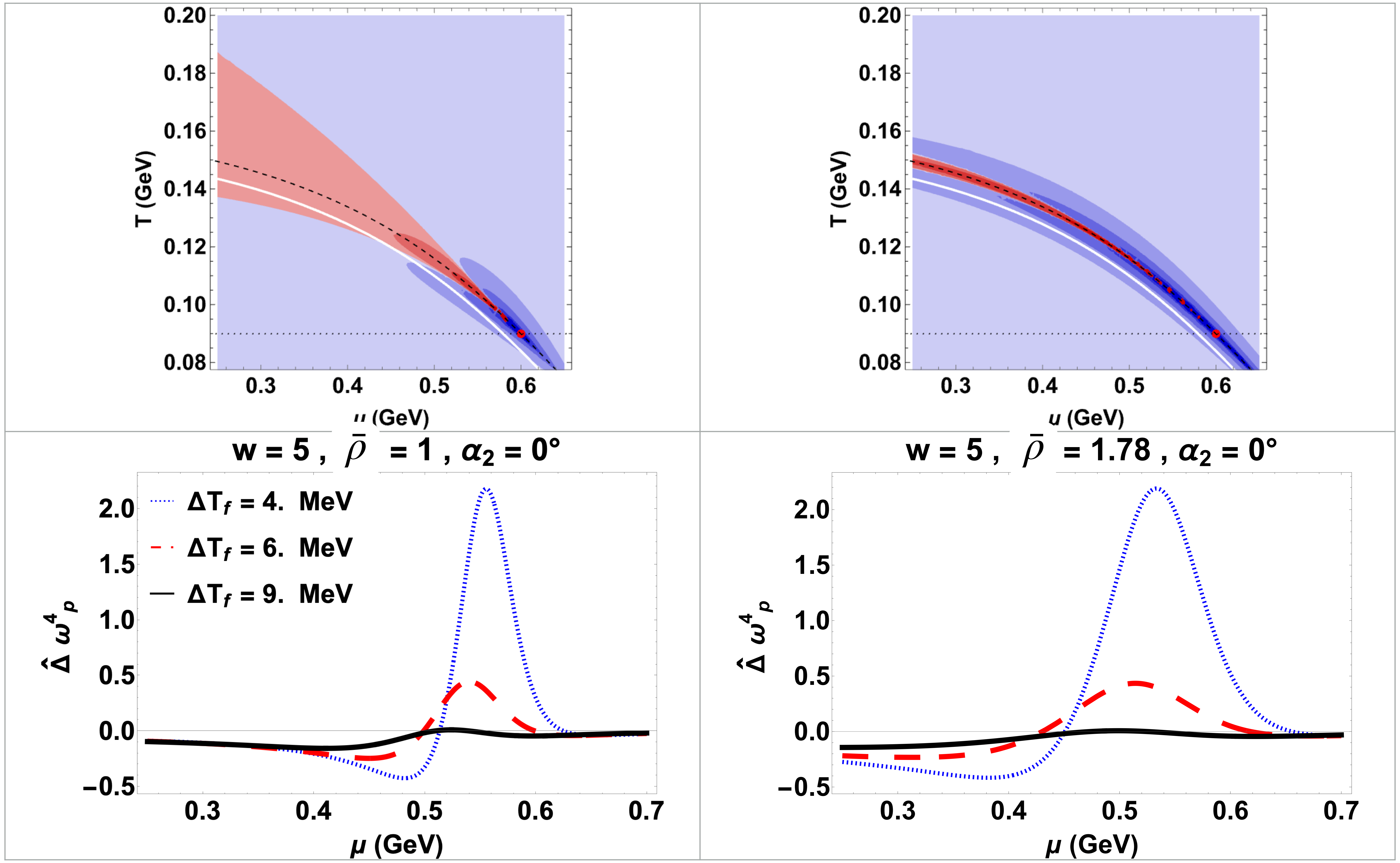
Karthein, MP, Rajagopal, Stephanov, Yin



- Peak position - not changed
- Peak magnitude - sensitive to w

$\bar{\rho}$ dependence of fourth factorial cumulant of proton multiplicity

Karthein, MP, Rajagopal, Stephanov, Yin



$$\bar{\rho} = \rho\, w^{1-\frac{1}{\beta\delta}} = \rho w^{0.36}$$

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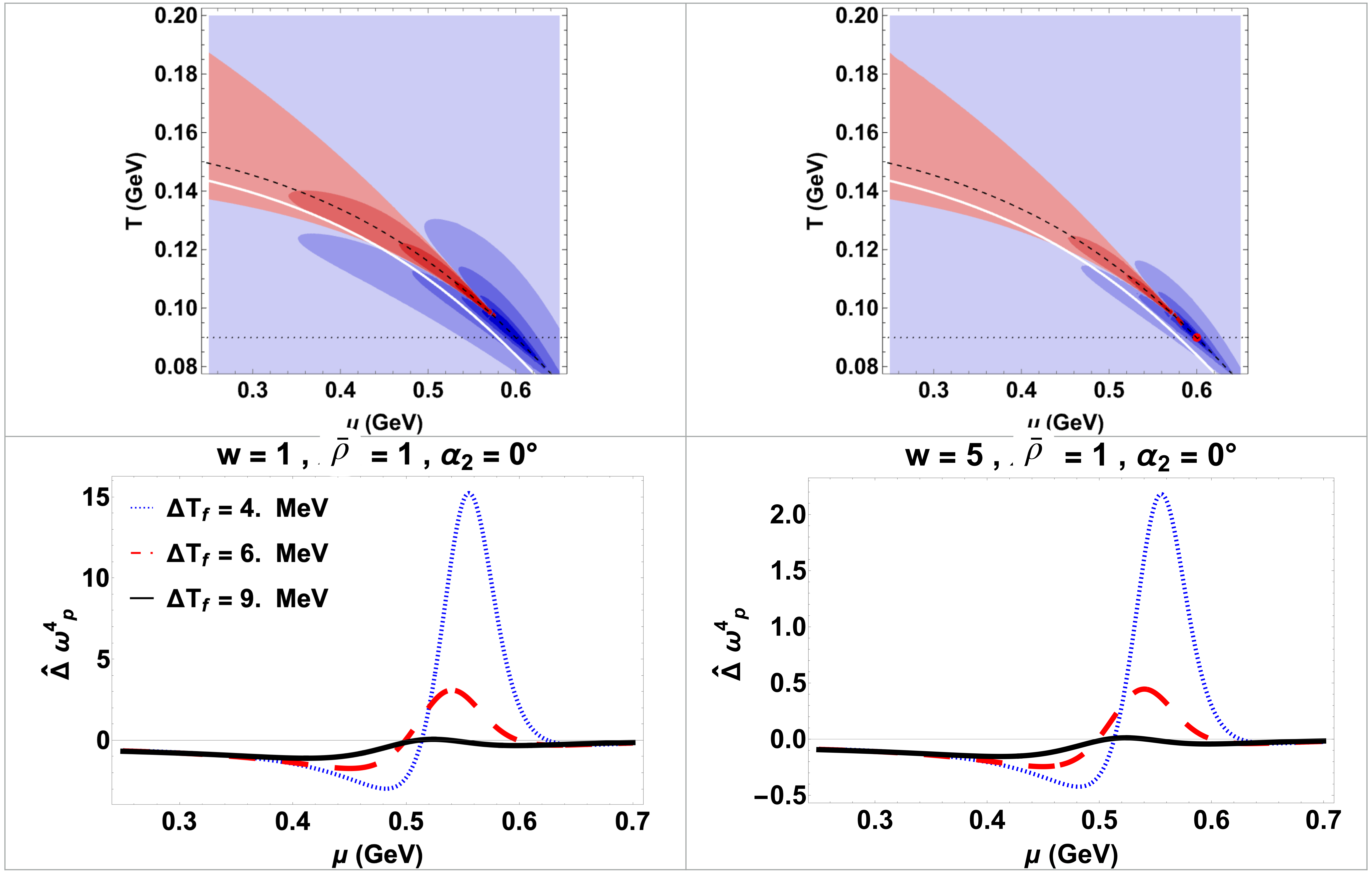
w dependence of fourth factorial cumulant of proton multiplicity

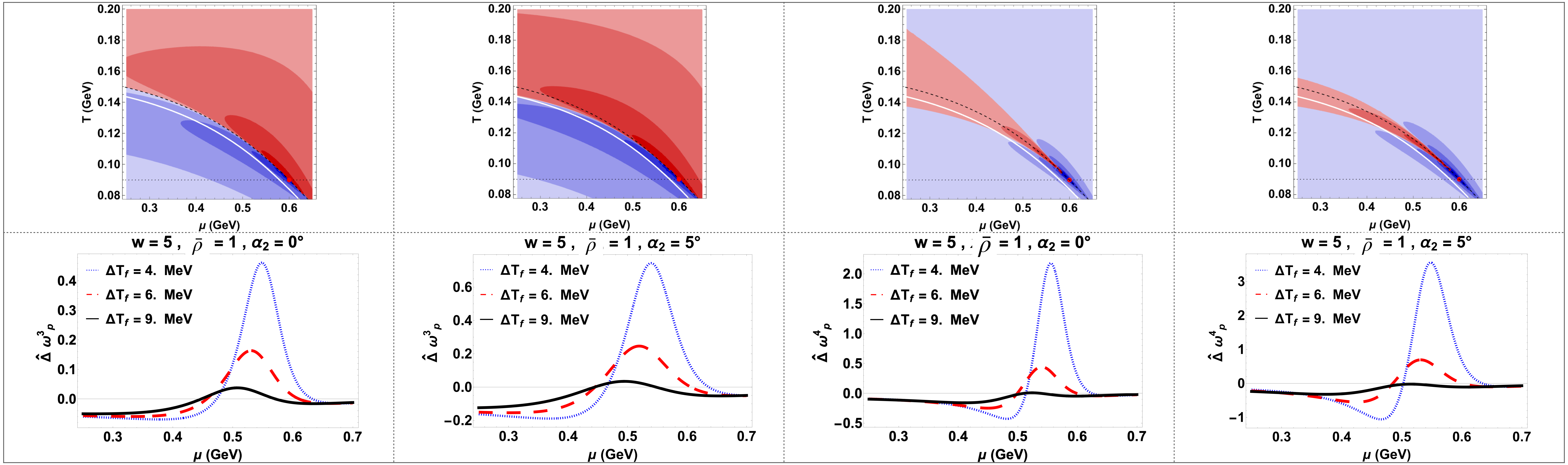
Karthein, MP, Rajagopal, Stephanov, Yin



- Peak position - not changed
- Peak magnitude - sensitive to w

Factorial cumulants can vary by order of magnitude or more within the uncertainties of freeze-out parameters

Slope of h axis (α_2)



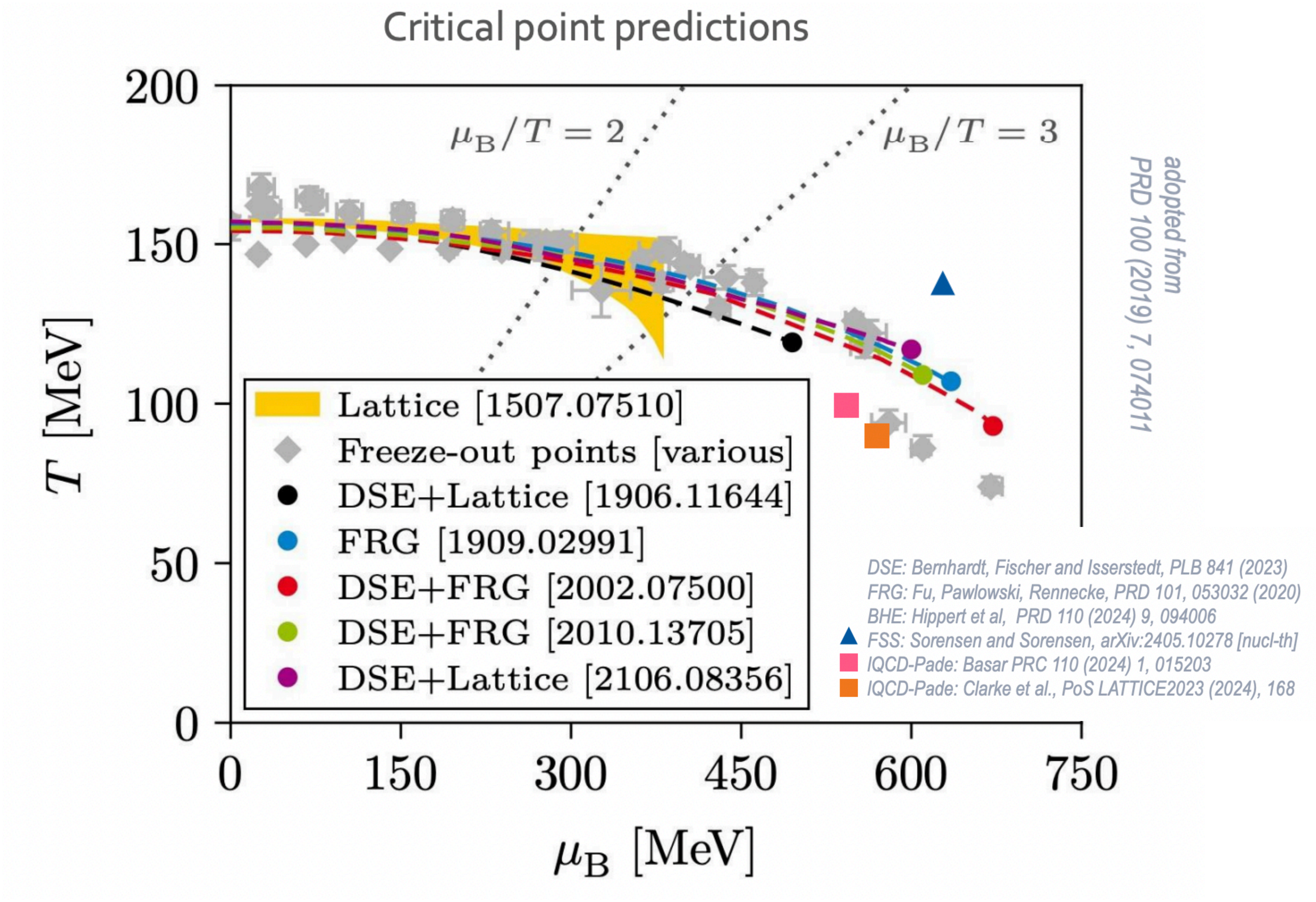
$$\alpha_{12} > 0$$

Universality in the chiral limit

MP, Stephanov, PRD 19

- Magnitude of dips and peaks (and their locations) depend on α_2

Agreement between various theoretical methods on the location of critical point and pseudo-critical curve



Petr Chaloupka's QM 25 Talk, Mon, CBM Collaboration

All CPs are in the Z_2 universality class

How close are these EoSs to one another?

One easy way to answer is computing the non-universal parameters
$\alpha_2\,, \bar{\rho}\,, w$

Inferring non-universal mapping parameters from FRG, Holographic EoS, lattice methods

How well do the different theoretical methods agree with regard to EoS near CP?

Map from van Der Waals gas to Ising Model

MP, Stephanov, 19

$$\tan\alpha_1 = -\left(\frac{3}{2} - \frac{s_c}{n_c}\right)^{-1};$$
$$\tan\alpha_2 = -\left(24 - \frac{s_c}{n_c}\right)^{-1};$$
$$\rho = 5\left(\frac{3P_c}{T_c^4}\right)^{1/4}\sqrt{\frac{4s_c\left(s_c - 3n_c\right) + 13n_c^2}{s_c\left(s_c - 48n_c\right) + 577n_c^2}};$$
$$w = \frac{1}{40}\left(\frac{T_c^4}{3P_c}\right)^{3/4}\sqrt{\frac{s_c}{n_c}\left(\frac{s_c}{n_c} - 48\right) + 577}.$$

- Ratios of derivatives of pressure on the cross-over side **MP, Stephanov, 19**

$$\tan\alpha_1 = \frac{h_\mu}{h_T} = \lim_{r\to 0^+}\frac{P_{\mu\mu}}{P_{\mu T}};$$
$$\tan\alpha_2 = \frac{r_\mu}{r_T} = \lim_{\substack{h=0\\ r\to 0^+}}\frac{P_{\mu\mu\mu}}{P_{TTT}}\frac{P_{TT}}{P_{\mu\mu}}.$$

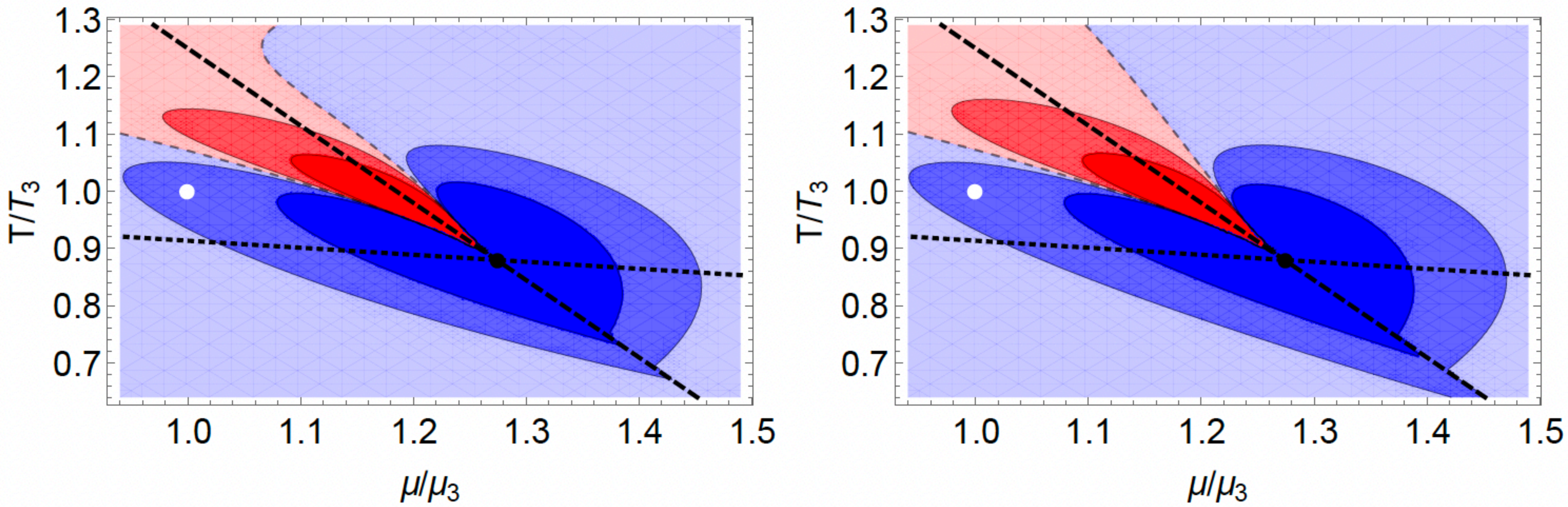
- Discontinuities on the first-order side

Rehr and Mermin, 1973

Mapping Random Matrix Model to Ising model

Halasz et al., PRD 98, MP, Stephanov, PRD 19

$\alpha_1 \approx 13^\circ, \alpha_2 \approx 1^\circ, \rho \approx 0.5, w \approx 1.4$



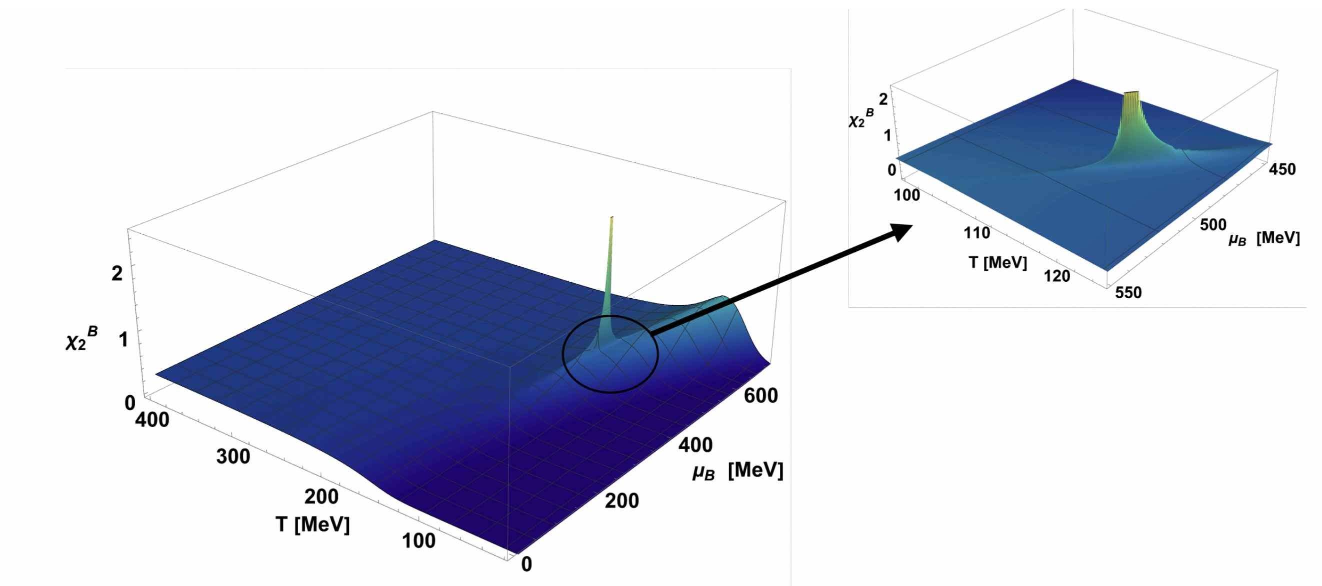
Contours of χ_4 obtained using RMM EoS

Contours of χ_4 obtained using ϕ^4 theory mapped to RMM

Determining the non-universal map will also help in quantifying the scaling region.

A consistent paradigm to describe universality and dynamics together

Kahangirwe et al., PRD 24



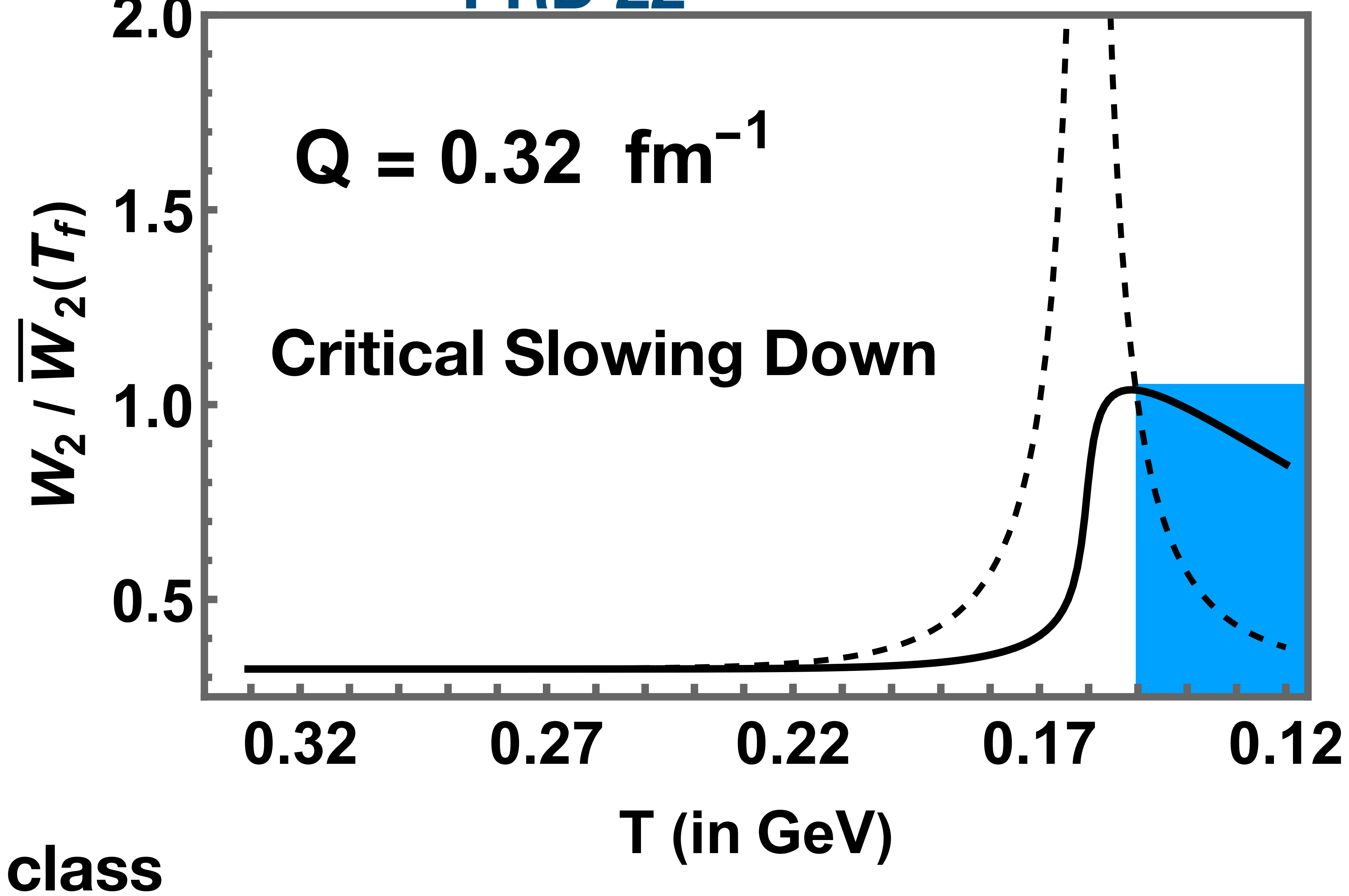
Susceptibilities that diverge at CP

Family of EoS in 3D Ising universality class

$$\{\mu_c, T_c, \alpha_1, \alpha_2, \rho, w\}$$

MP, Rajagopal, Stephanov, Yin , PRD 22

Suppression of peak

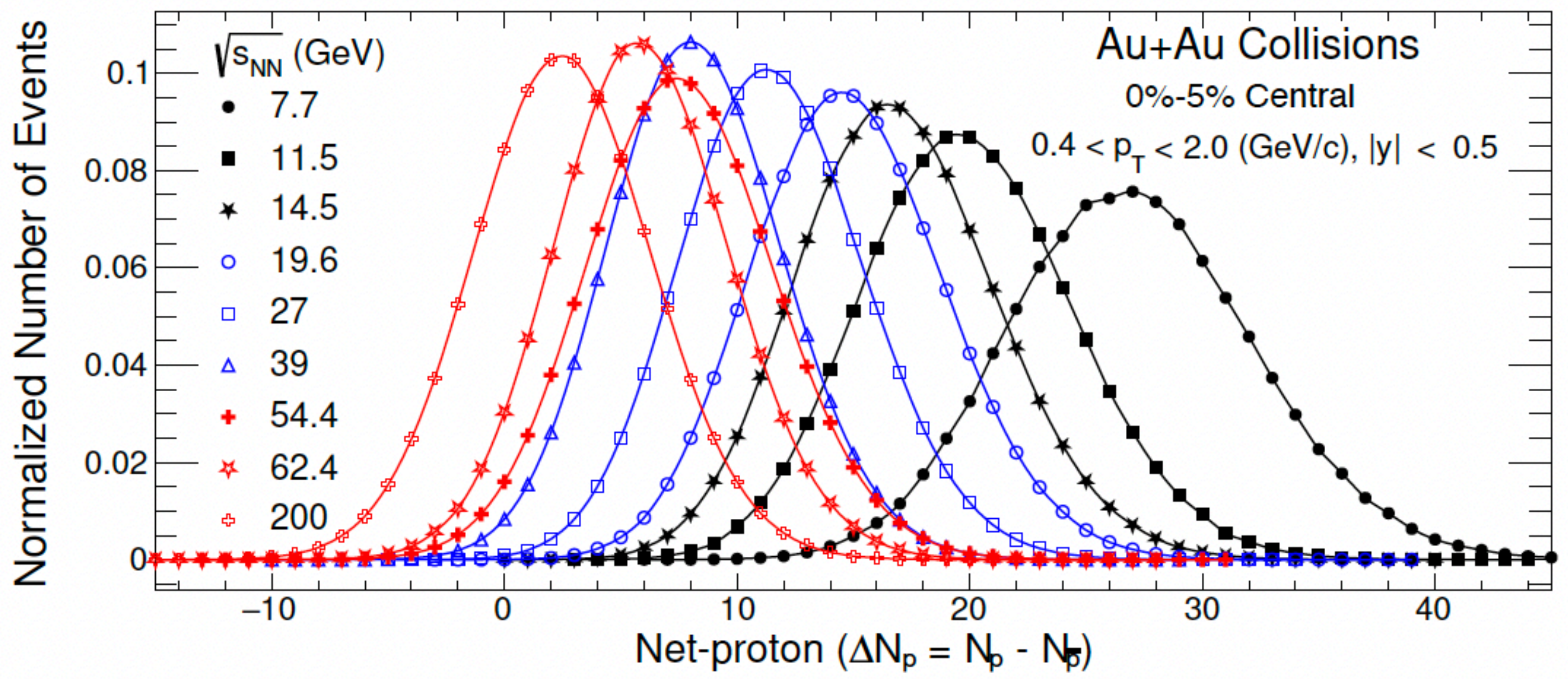


Prolonged memory of CP

Hydro+ $\{f_\xi\}$

Yin, Stephanov, 17, PRD
An et al, 19,20,21,22

STAR Collaboration, PRL 2014, PRL 2021



MP, Stephanov, 23, PRL

Maximum entropy freeze-out

$$\{\mu_f, T_f\}$$

Connecting particle multiplicity fluctuations to hydrodynamic fluctuations

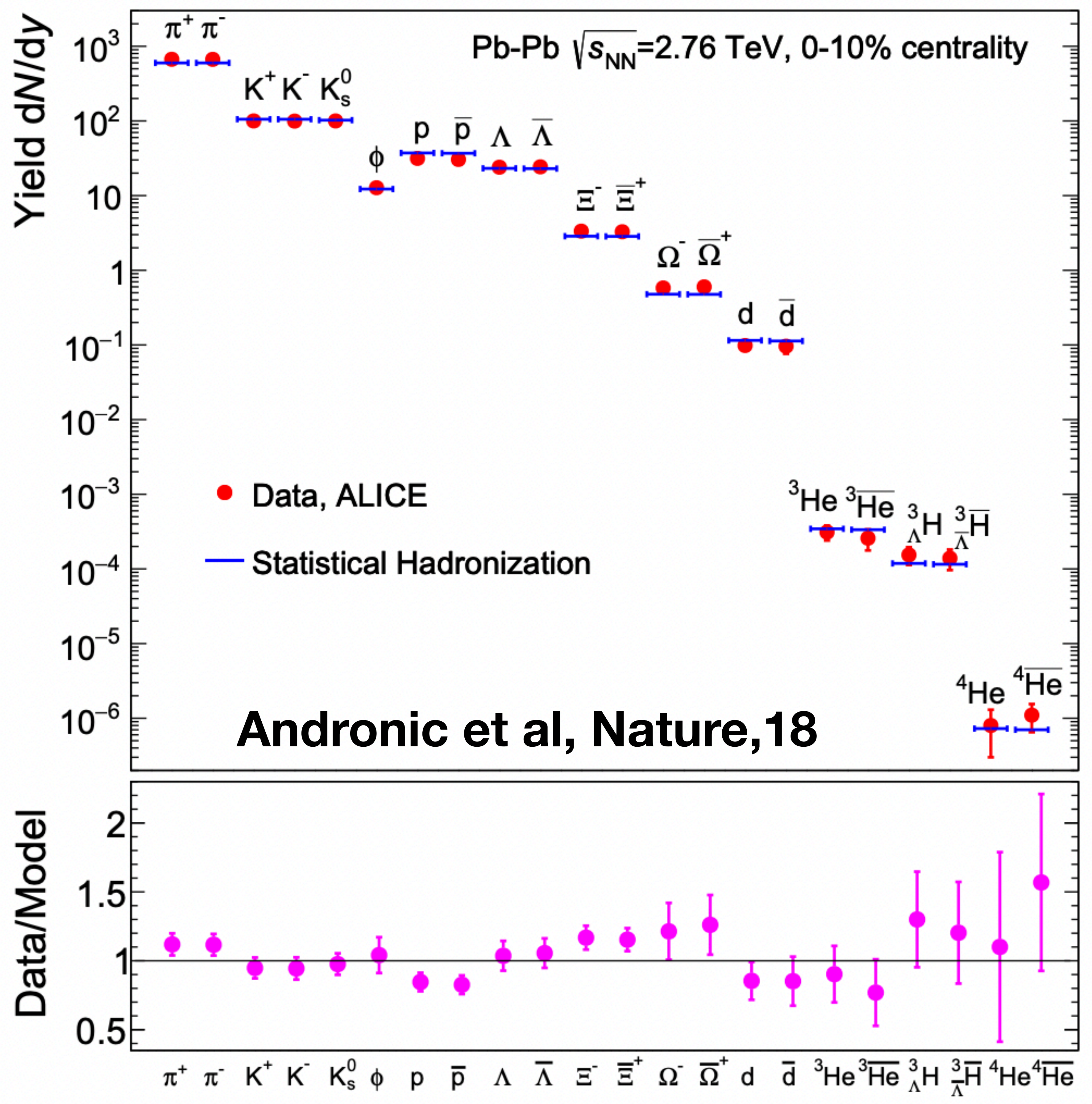
Maximum entropy freeze-out of fluctuations

Maximum entropy freeze-out of mean conserved densities

Maximize entropy of a gas matching energy-momentum densities and charge densities

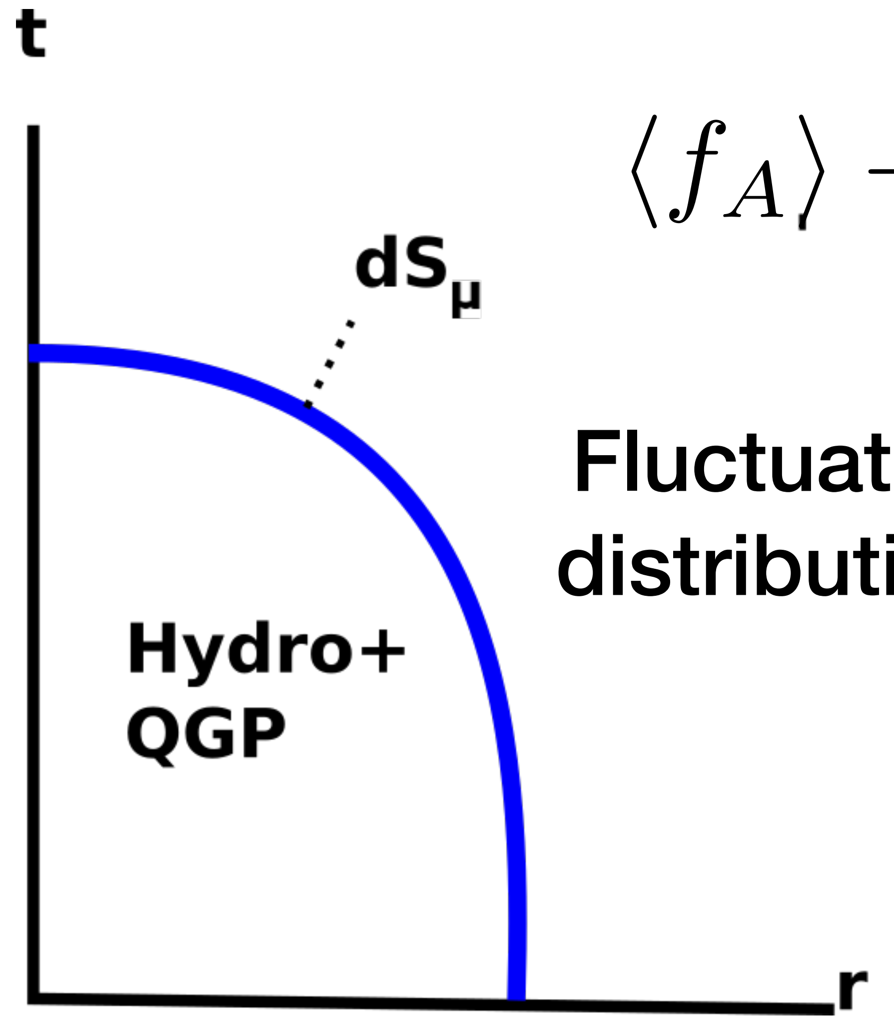
$$\langle \epsilon\, u^{\mu} \rangle = \sum_A \int_{p_A} \bar{f}_A\, p_A^{\mu}\,, \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

Statistical hadron resonance gas model at a temperature and chemical potential corresponding to the average values of conserved densities



See also Chattopadhyay, Everett, Heinz, PRC 21

Maximum entropy freeze-out of event by event hydrodynamic fluctuations



$$\langle f_A \rangle + \delta f_A$$

Fluctuating particle distribution function

MP, Stephanov, PRL 2023

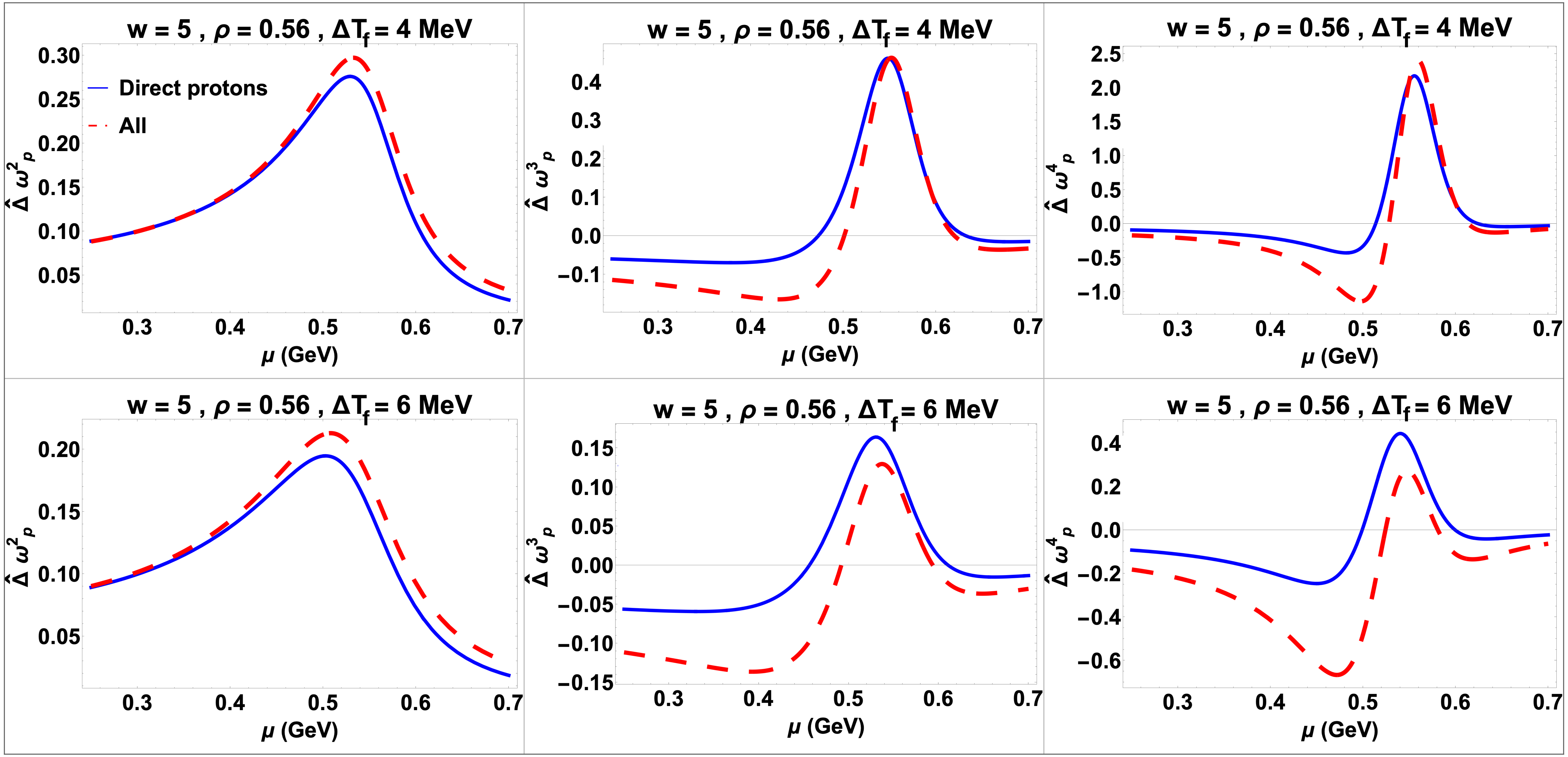
$$\underbrace{\hat{\Delta} G_{ABC}}_{\text{irreducible particle correlations (FC)}} = \underbrace{\hat{\Delta} H_{abc}}_{\text{hydrodynamic correlations}} \underbrace{(\bar{H}^{-1} P \bar{G})^a_A (\bar{H}^{-1} P \bar{G})^b_B (\bar{H}^{-1} P \bar{G})^c_C}_{\text{kinematic factors}}$$

- Mean and n-point correlation functions of energy, momentum and conserved charge densities in both descriptions
- Infinitely many solutions $f_A(x, p)$ that satisfy these matching conditions
- Least biased choice : One which maximizes the entropy of the gas of particles
- Maximum entropy prescription reduces to the Cooper-Frye freeze-out for computing mean multiplicity of particles
- Contribution of decay particles to total cumulants can also be studied

Alternative Particlization Routine : Microcanonical Particlization Koch,Oliinychenko, PRL 2019

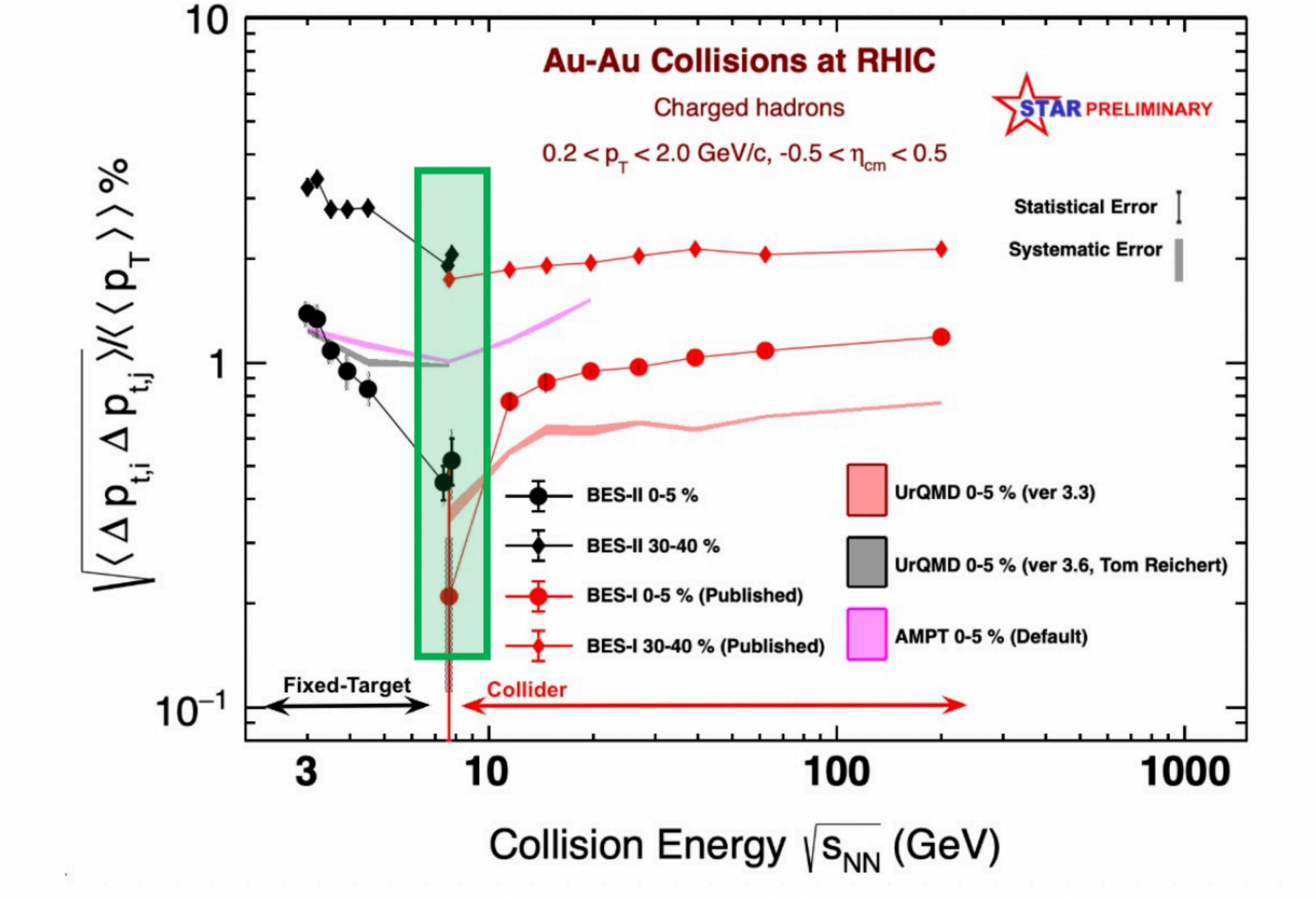
Contributions of resonances

Karthein, MP, Rajagopal, Stephanov, Yin



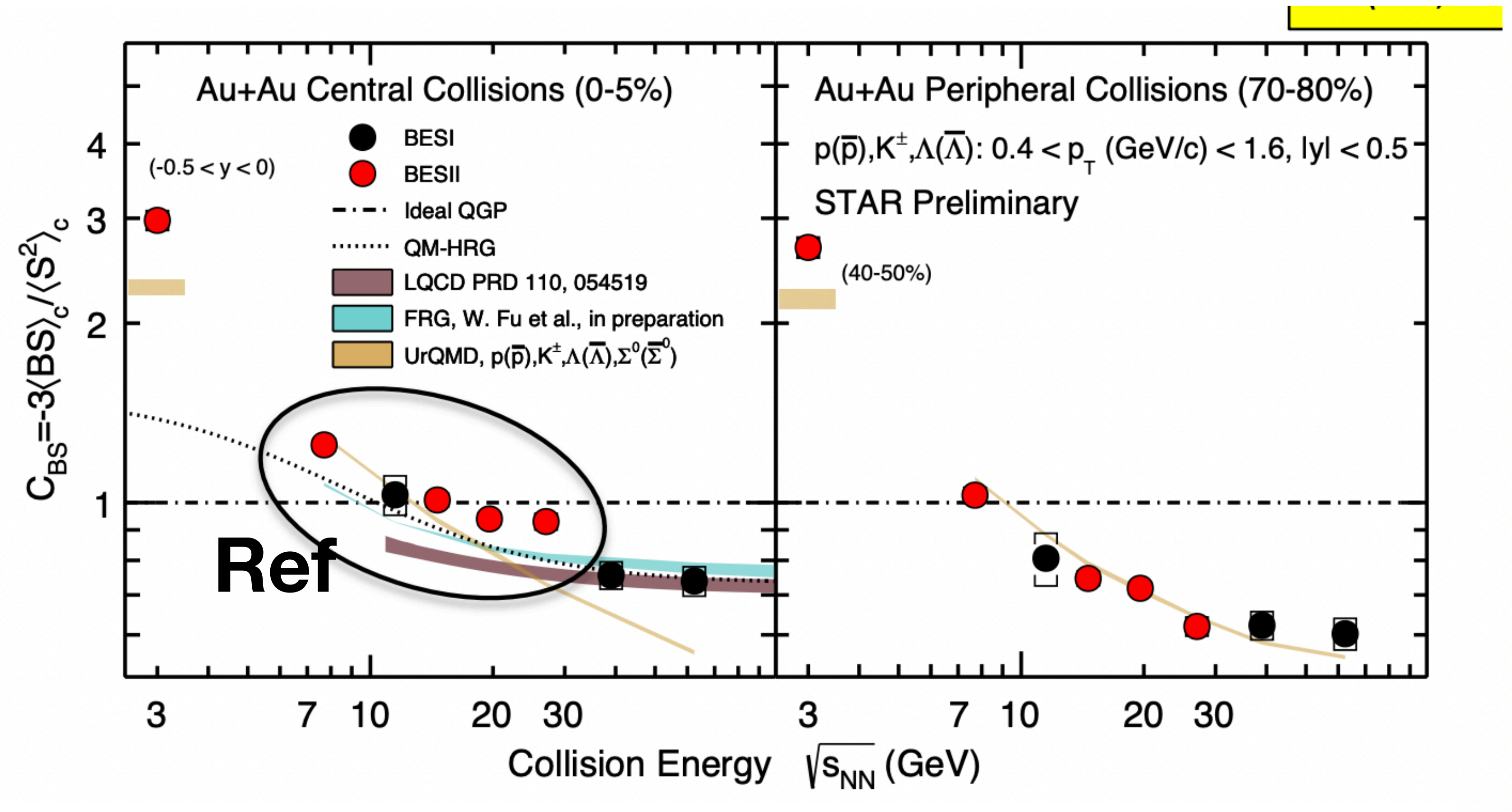
Other fluctuation observables

R. Manikandan's poster, QM 25

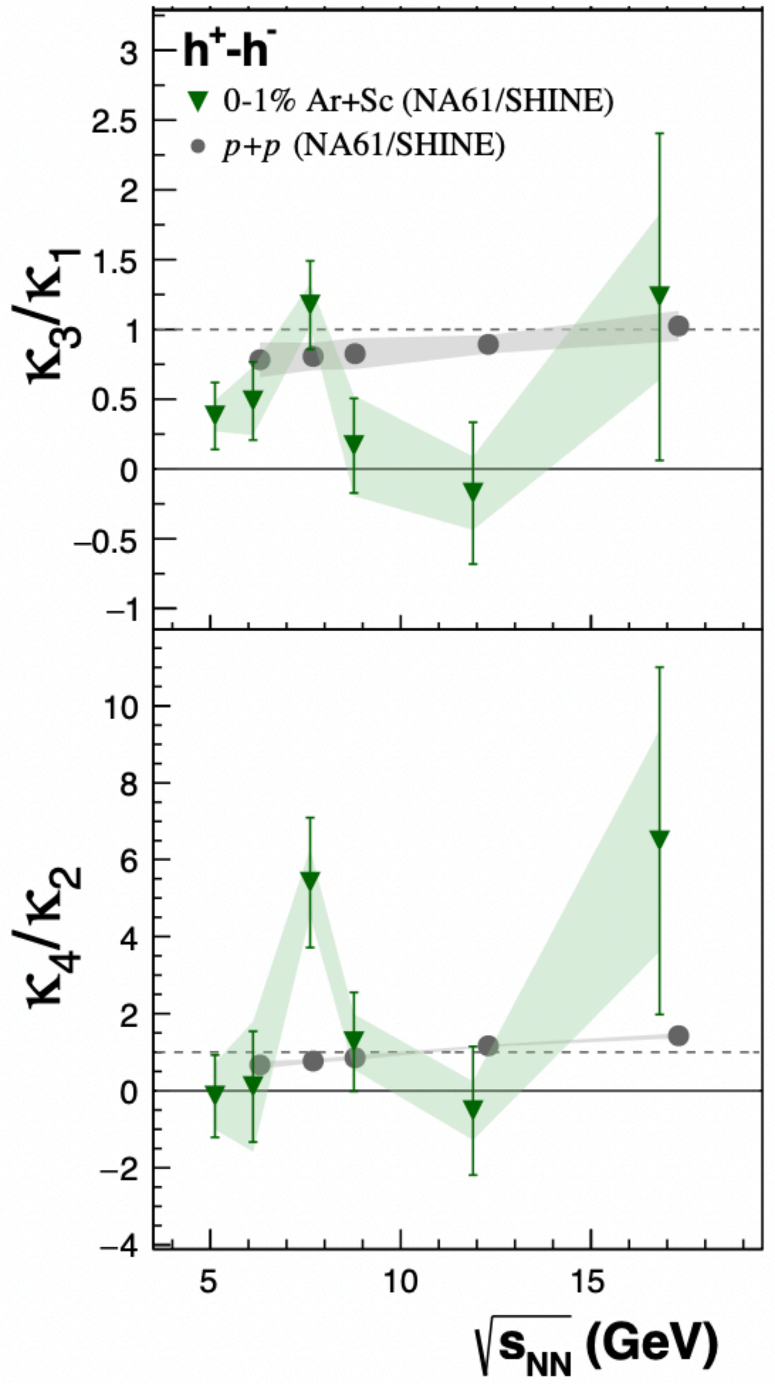


- Transverse momentum fluctuations

- Pion multiplicity cumulants, mixed particle multiplicity cumulants
- Balance functions



Hanwen Feng , STAR, QM 25 slides



Maja Mackowiak-Pawlowska, QM 25 slides, NA61/SHINE

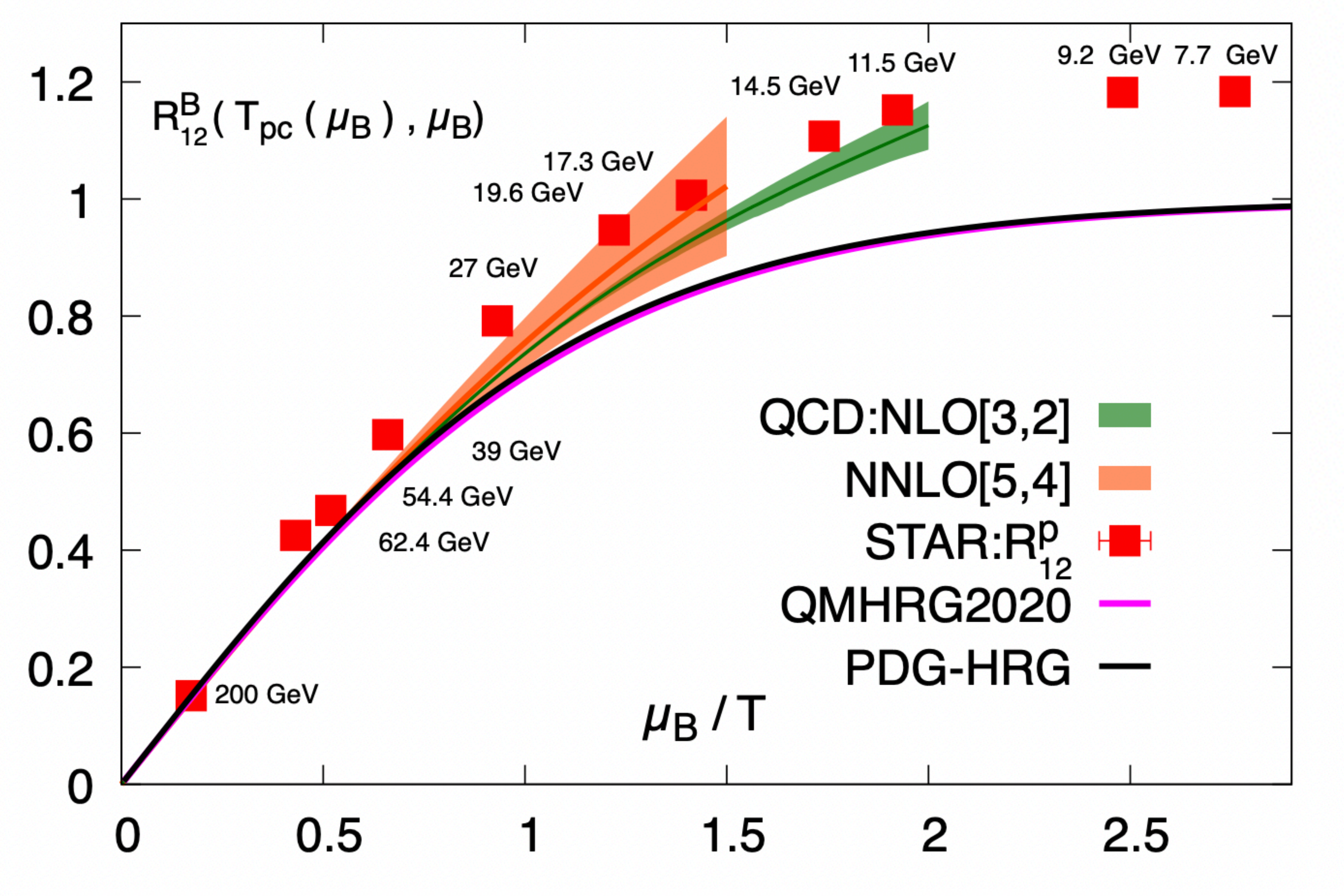
Summary and looking forward

- Data shows deviations from non-critical baselines from Hydro, baryon susceptibility ratios from lattice extrapolations and FRG calculations - Explanations?
- Remarkable agreement between theoretical methods with regard to location of critical point - What about the other EoS parameters?
- Equilibrium estimates of factorial cumulants of proton multiplicities computed using ME method - Finite time and size effects?
- Equations for deterministic evolution of hydro correlations derived and implemented for two-point correlations in semi-realistic settings - Two-point and three point correlation functions dynamics with universal EoS?
- Stochastic hydrodynamic simulations using Metropolis approach, tested in simple scenarios - Towards realistic simulations?
- Molecular dynamic simulations with critical point - Implementing long-range correlations due to critical point
- Spinodal dynamics in first-order phase transition in longitudinally invariant systems studied - Towards realistic simulations and correlations?
- Freeze-out - Where does the freeze-out curve lie relative to phase boundary?
- Data-theory comparisons to constrain $\{\mu_c, T_c, \alpha_1, \alpha_2, \rho, w, f_\xi\}$
- Tremendous progress in initial state correlations and background flow descriptions - Integrating with fluctuation dynamics

Back up

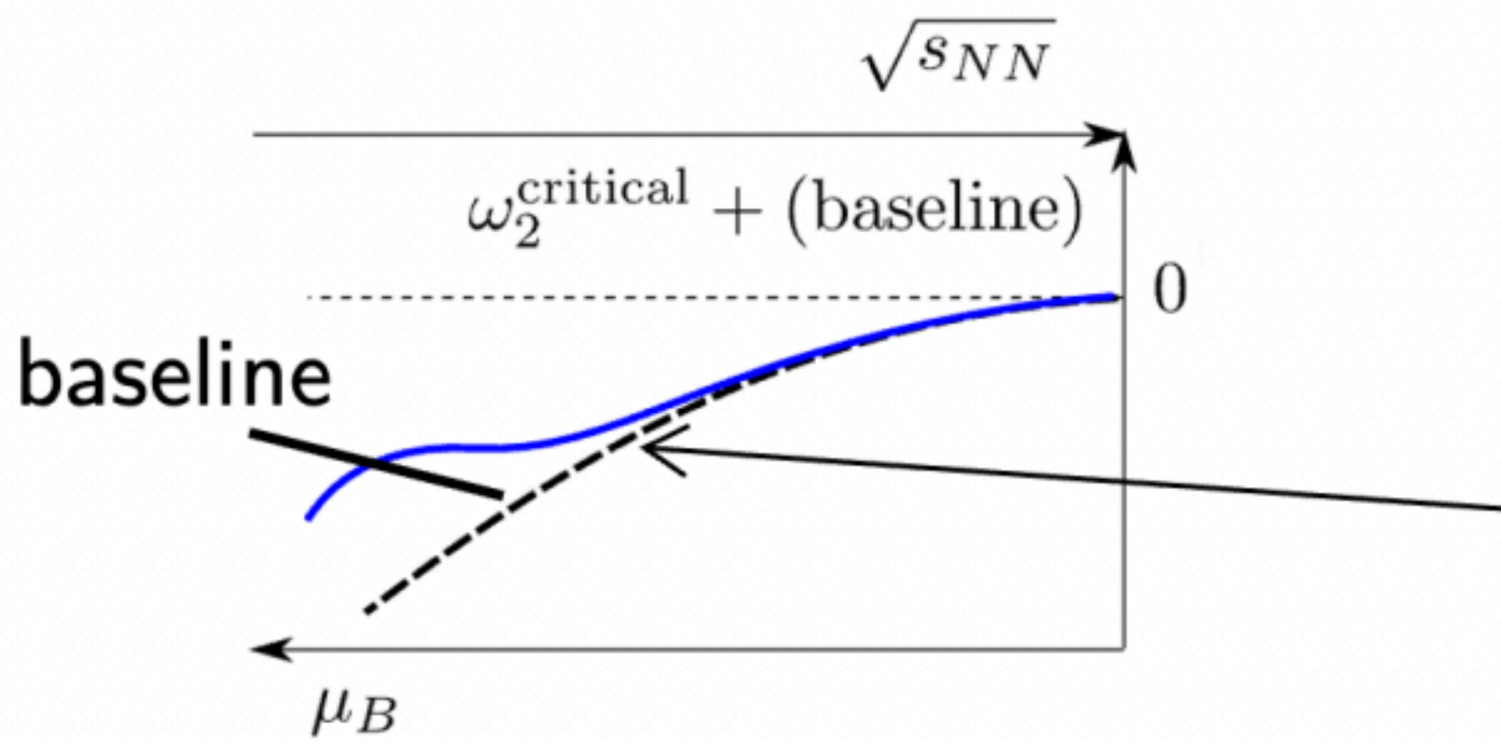
κ_n : theory and experiment

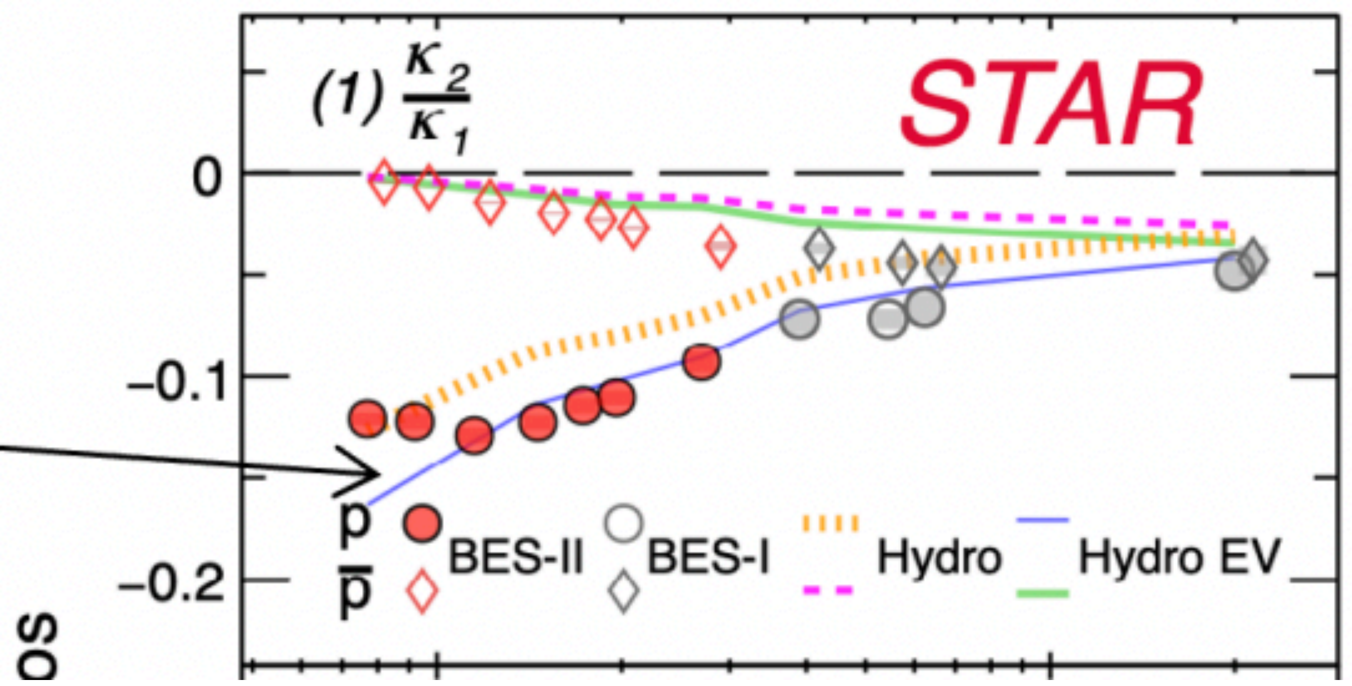
κ_2/κ_1 from theory and experiment



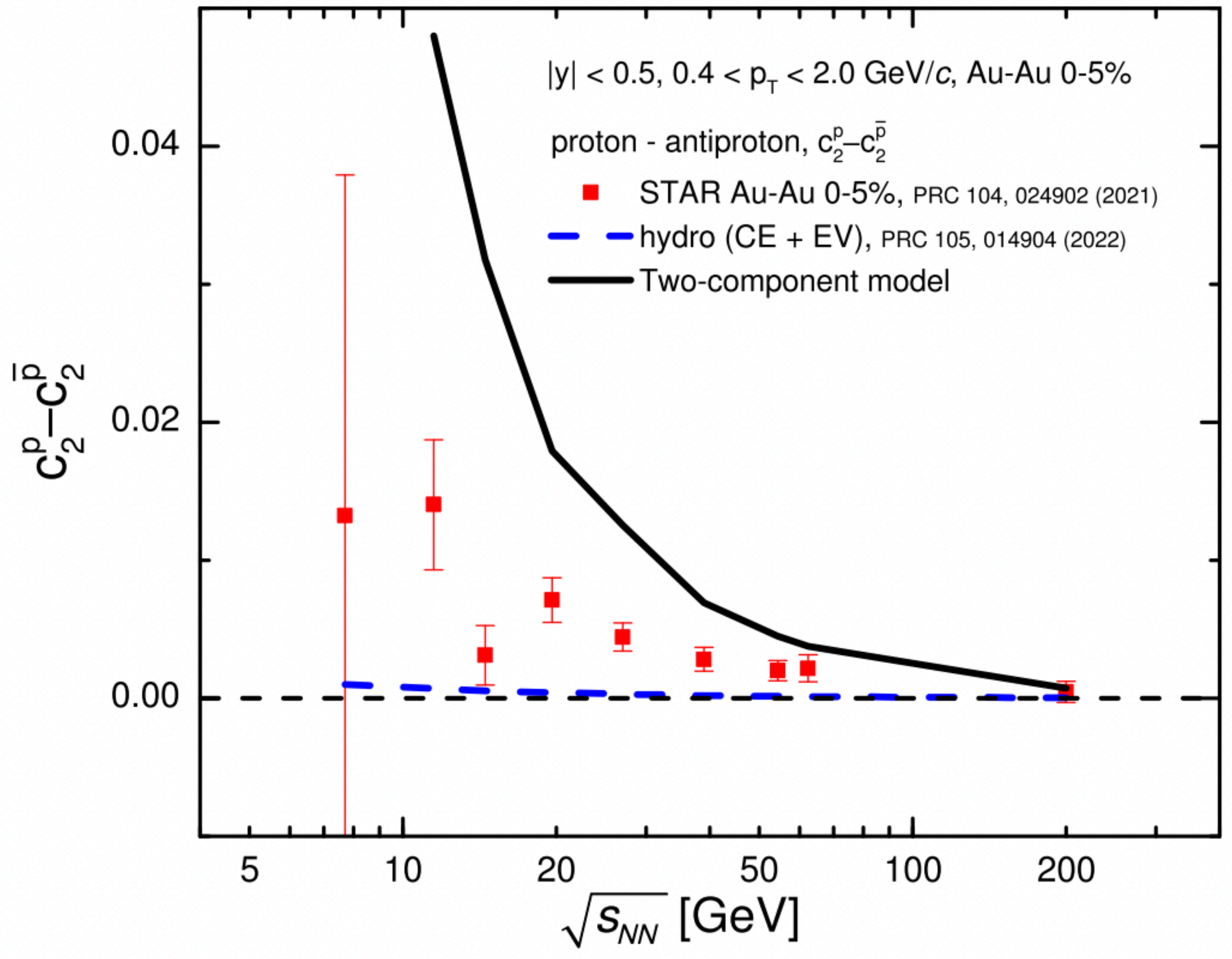
HotQCD 2024 : Phys.Rev.D 110 (2024); STAR results : CPOD2024

J. Goswami, QM 25 slides





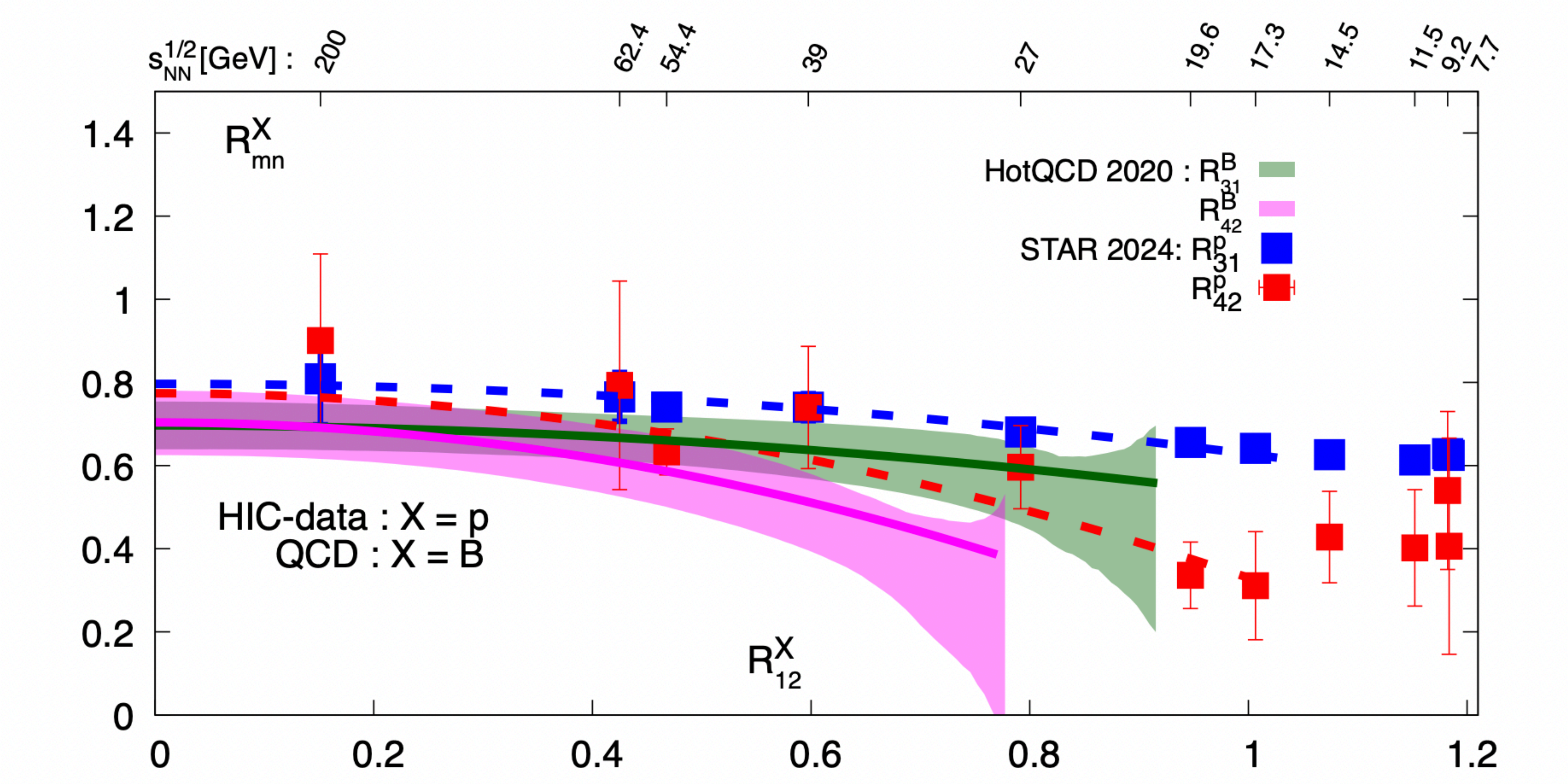
Vovchenko et al, arXiv:2504.01368v1,arXiv:2503.16405v1



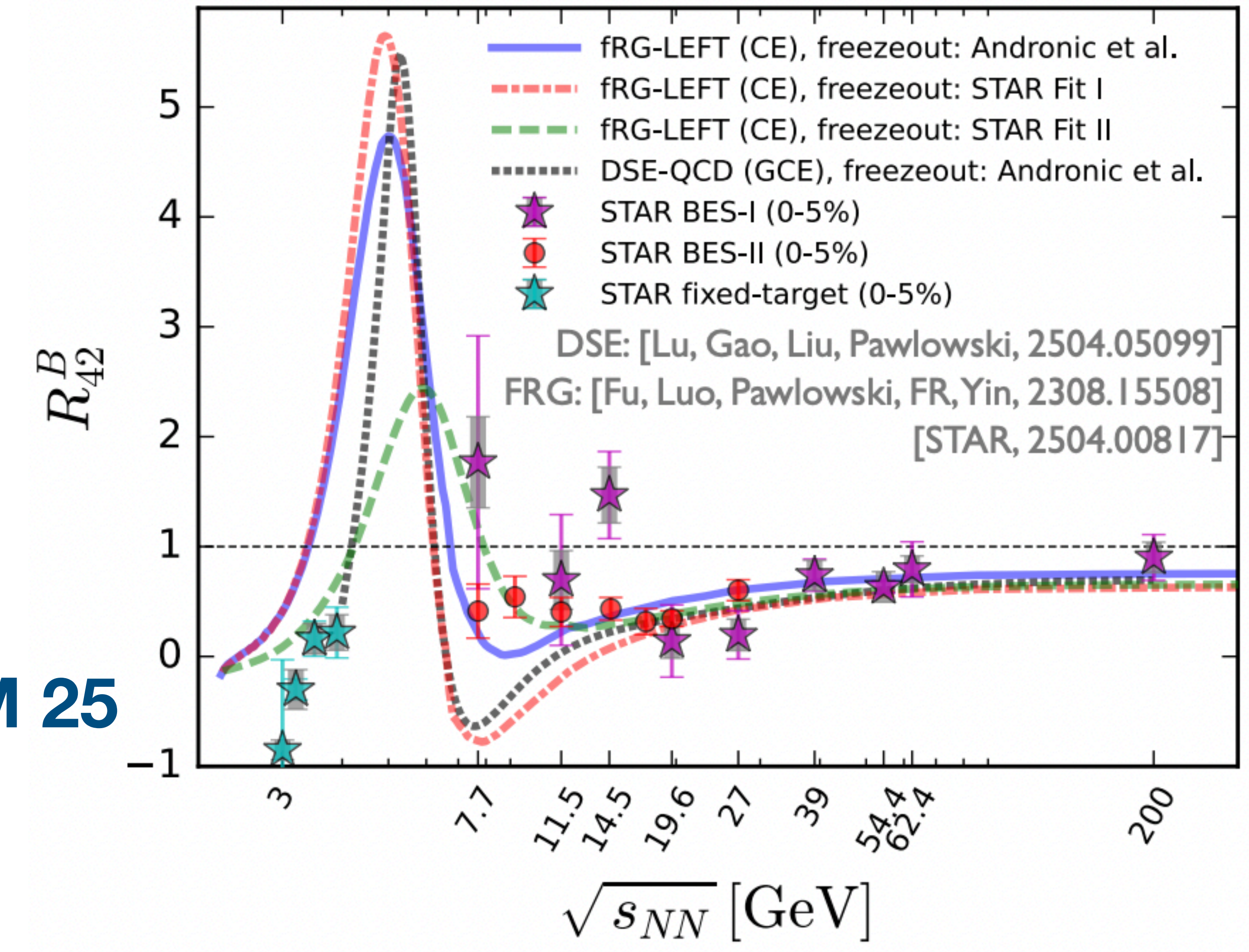
- Deviations from HRG, non-critical hydro baselines
- Agreement with lattice QCD NNLO till $\sqrt{s_{NN}} = 17.3$ GeV
- Global conservation and volume fluctuations alone can't explain data

$$-\hat{C}_2^{\bar{p}} / \langle N_{\bar{p}} \rangle^2 > -\hat{C}_2^{p} / \langle N_p \rangle^2$$

κ_3/κ_1 and κ_4/κ_2 from theory and experiment

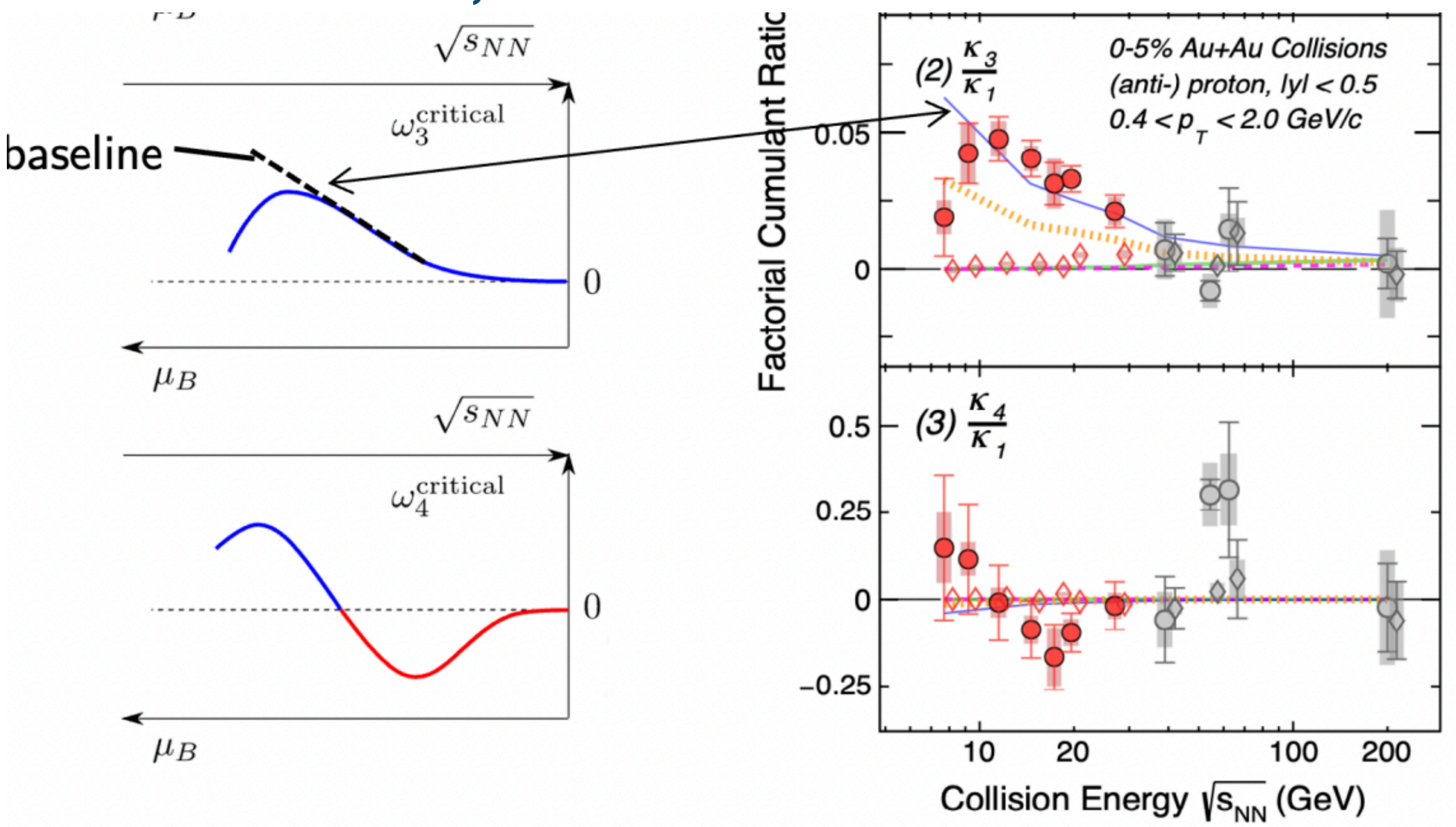


J. Goswami, QM 25 slides



F. Rennecke's slides , QM 25

Vovchenko et al, arXiv:2504.01368v1



- Good agreement between STAR ,Lattice QCD and FRG/DSE for $\sqrt{s_{NN}} \leq 19.6$ GeV
- No consensus between FRG and data at lower $\sqrt{s_{NN}}$
- Deviation from non-critical hydro-baseline at lower $\sqrt{s_{NN}}$

Dependence of factorial cumulants of proton multiplicity on non-universal parameters

Universal dynamics near the QCD critical point

Dynamic universality near the critical point

- Stochastic hydrodynamics with critical diffusion for entropy per baryon (Model H) **Son & Stephanov, 04, PRD**

- Deterministic approach to fluctuations : Deterministic equations for evolving n-point functions

$$\partial_t \psi = -\nabla \cdot \mathsf{Flux}[\psi, H, H_3, H_4, \ldots];$$
$$\partial_t H = \mathsf{F}[\psi, H, H_3, H_4, \ldots];$$
$$\partial_t H_3 = \mathsf{F}_3[\psi, H, H_3, H_4, \ldots];$$
$$\vdots$$

An et al 2009.10742, PRL

Hydro+ : Stephanov, Yin,17

- Stochastic approach **Kapusta, Mueller, Stephanov, 2011, PRC**

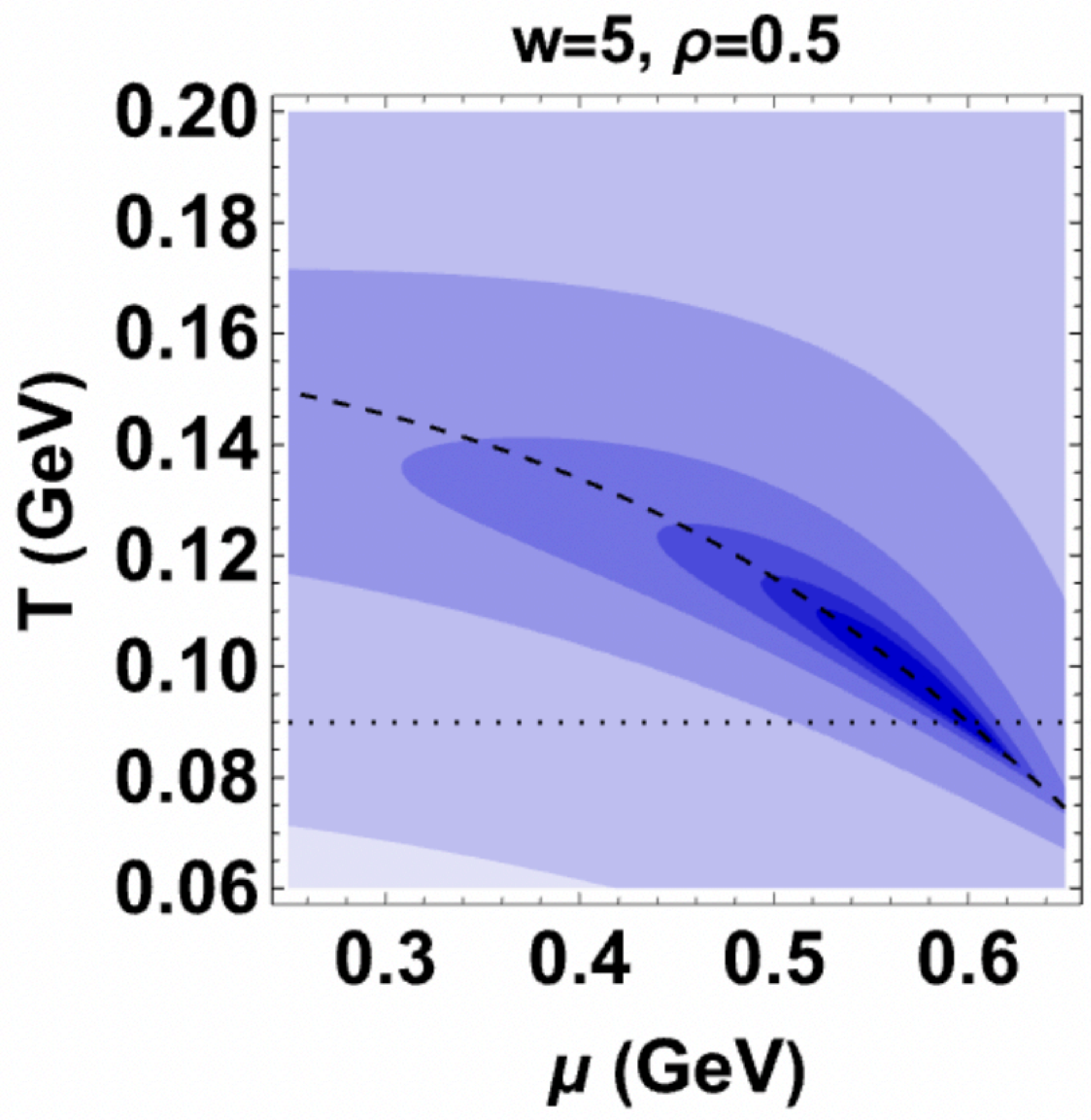
$$\partial_t \tilde{\psi} = -\nabla (F[\tilde{\psi}] + \zeta)$$

Correlation length

Karthein, MP, Rajagopal, Stephanov, Yin

$$\Gamma(Q) = \frac{T}{6\pi\eta_R\xi^{-3}} K(Q\xi)$$

Relaxation rate for the slow mode **Kawasaki, 70**

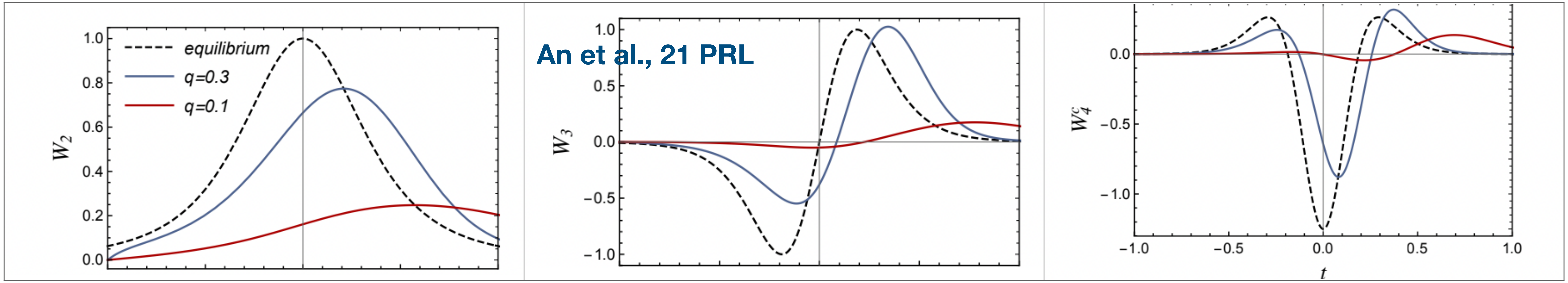


$$\xi_{QCD} = f_\xi \, \xi_{Ising}$$

Dynamics adds a parameter , f_ξ **to the list of parameters to constrain.**

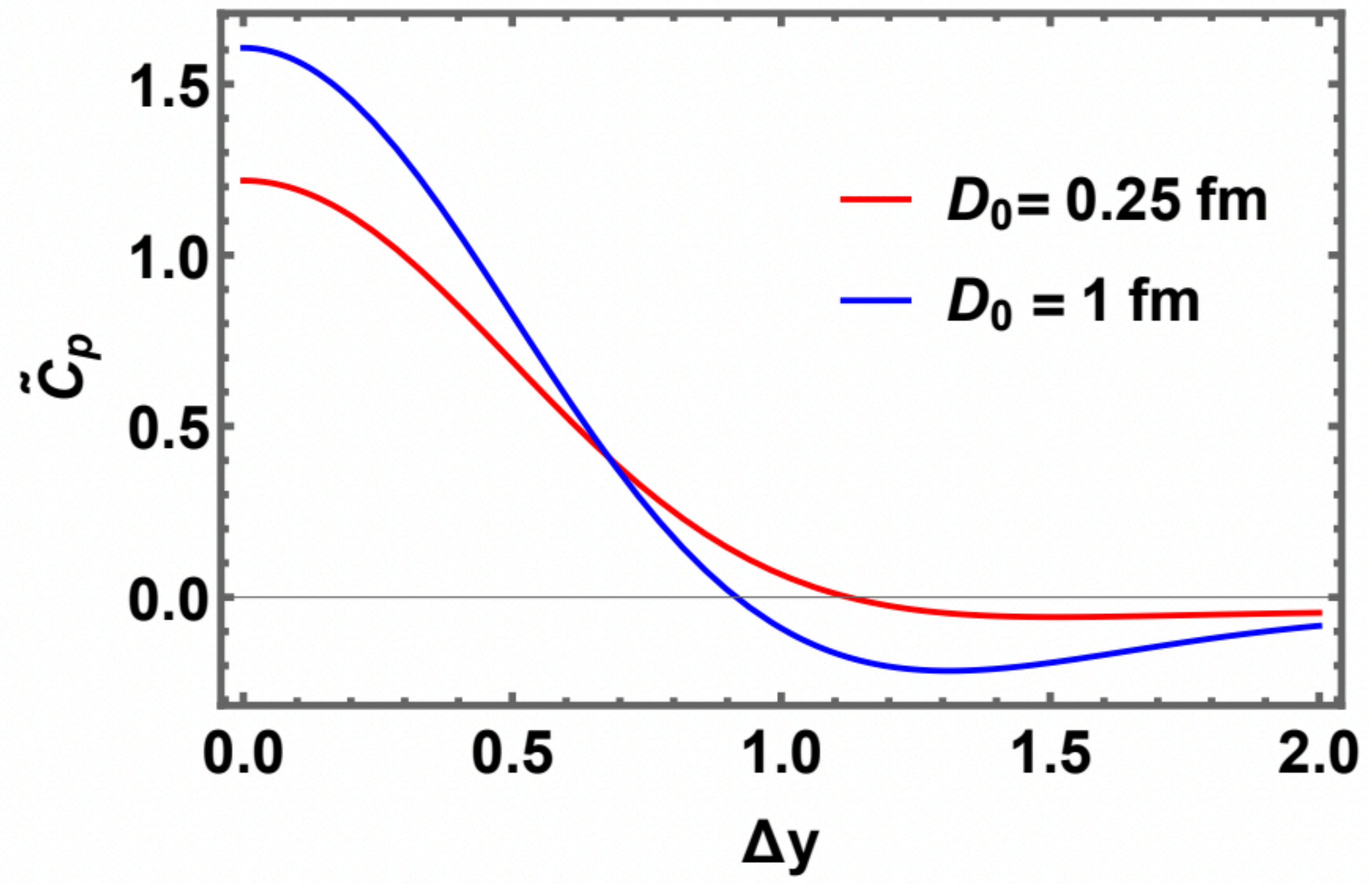
Phenomenological consequences

- Memory effects **Mukherjee et al,15, Rajagopal et al, 19, Du et al, 20, MP et al, 22**



- Lesser correlations in momentum space to HRG at large momenta relative due to conservations

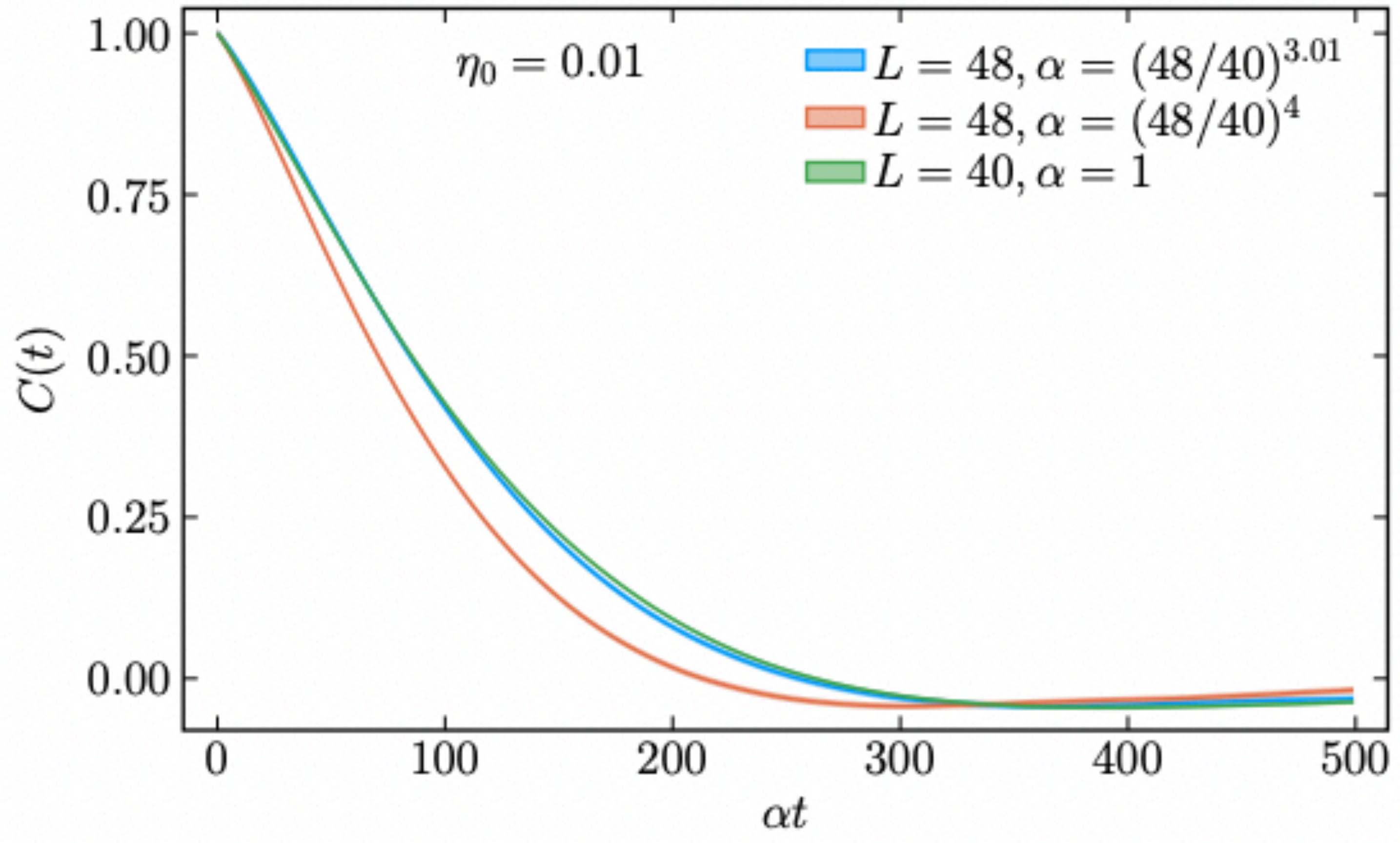
MP, Rajagopal, Stephanov, Yin, 22, PRD



Stochastic relativistic hydrodynamics

- Metropolis approach for simulating hydrodynamic fluctuations and dissipation simultaneously (Chattopadhyay et al., 24,25) in the "density frame" in a relativistically covariant way (Bhambure et al,24)

$$C(t) = \langle \phi_k(0)\phi_{-k}(t) \rangle$$



Dynamic Scaling:

$$z(\eta{=}0.01) = 3.07$$

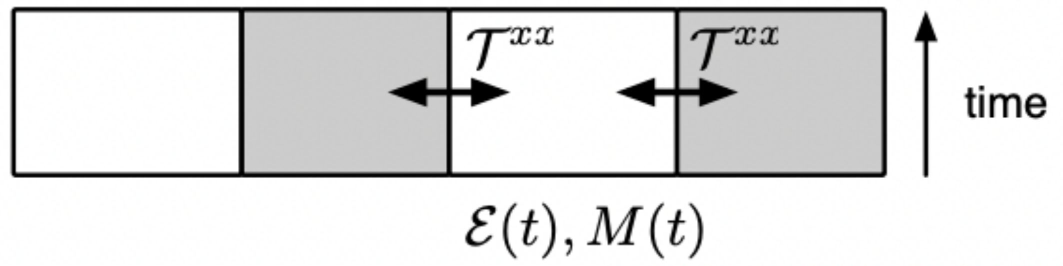
T. Schaefer's slides, QM 25, Tue

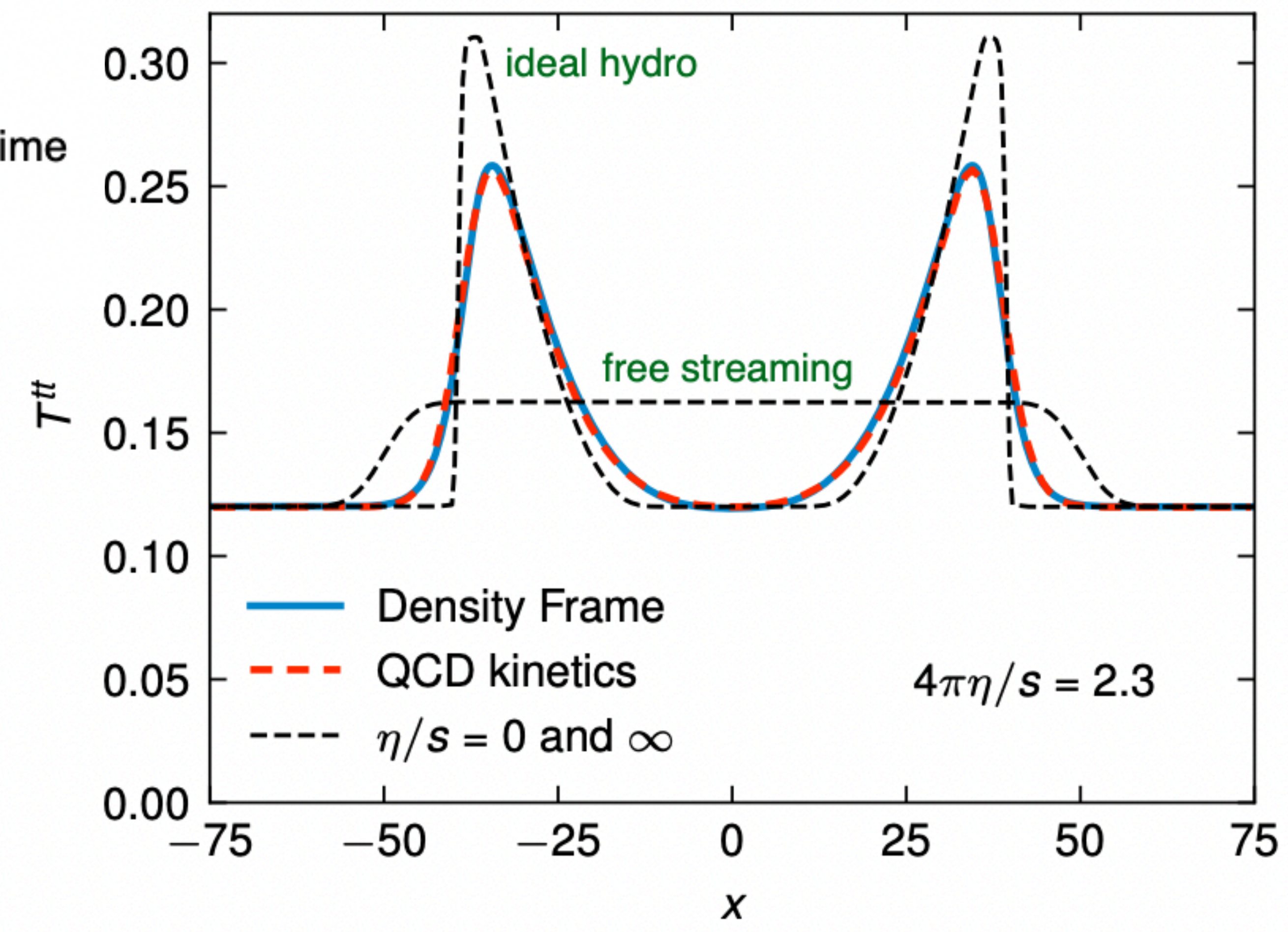
Comparison of QCD kinetics with DF hydro and MUSIC

Covariant QCD kinetics in 1+1D by Fabian Zhou, Aleksas Mazeliauskas

D. Teaney's slides, QM 25 Tue

$$\frac{\ell_{\rm mfp}}{L} = 0.12$$





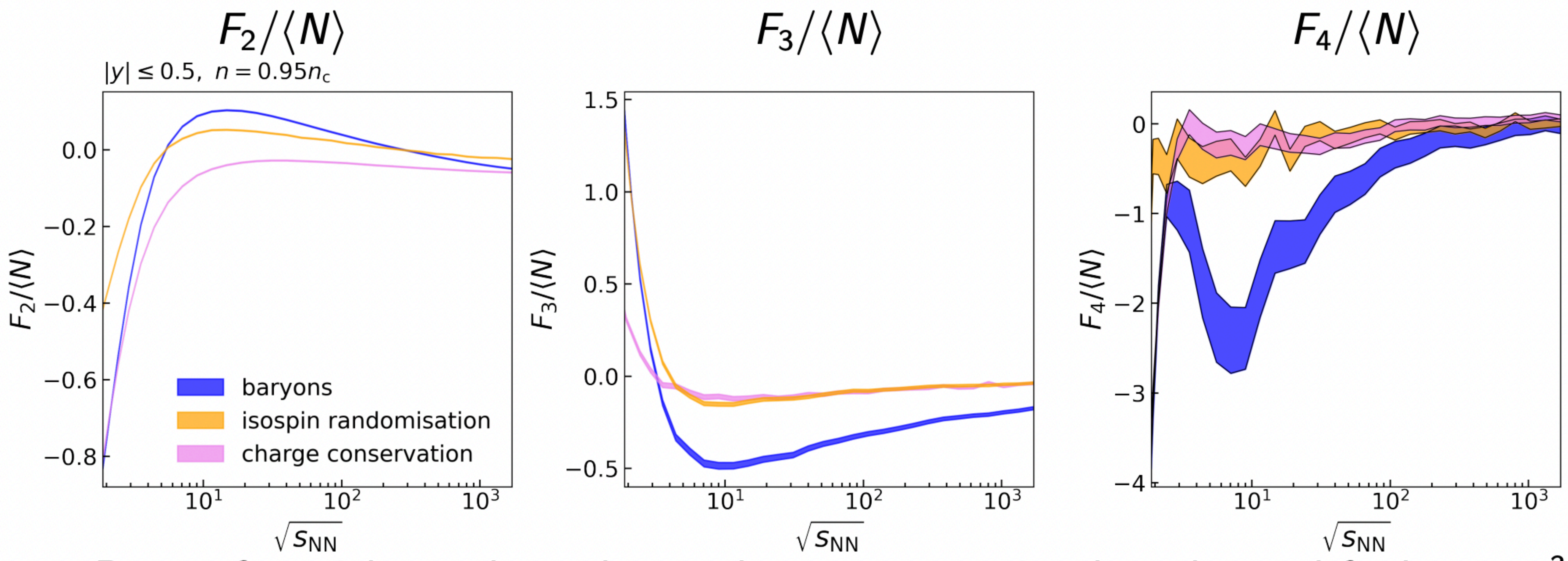
Hydro is good and density-frame works well. Better than MUSIC?

Getting ready for realistic hydrodynamical simulations!

Higher order cumulants from molecular dynamic simulations

V. Kuznietsov's slides, QM 25,Tue

Factorial cumulants at fixed $|y| < 0.5$ acceptance ($n = 0.95n_c$)



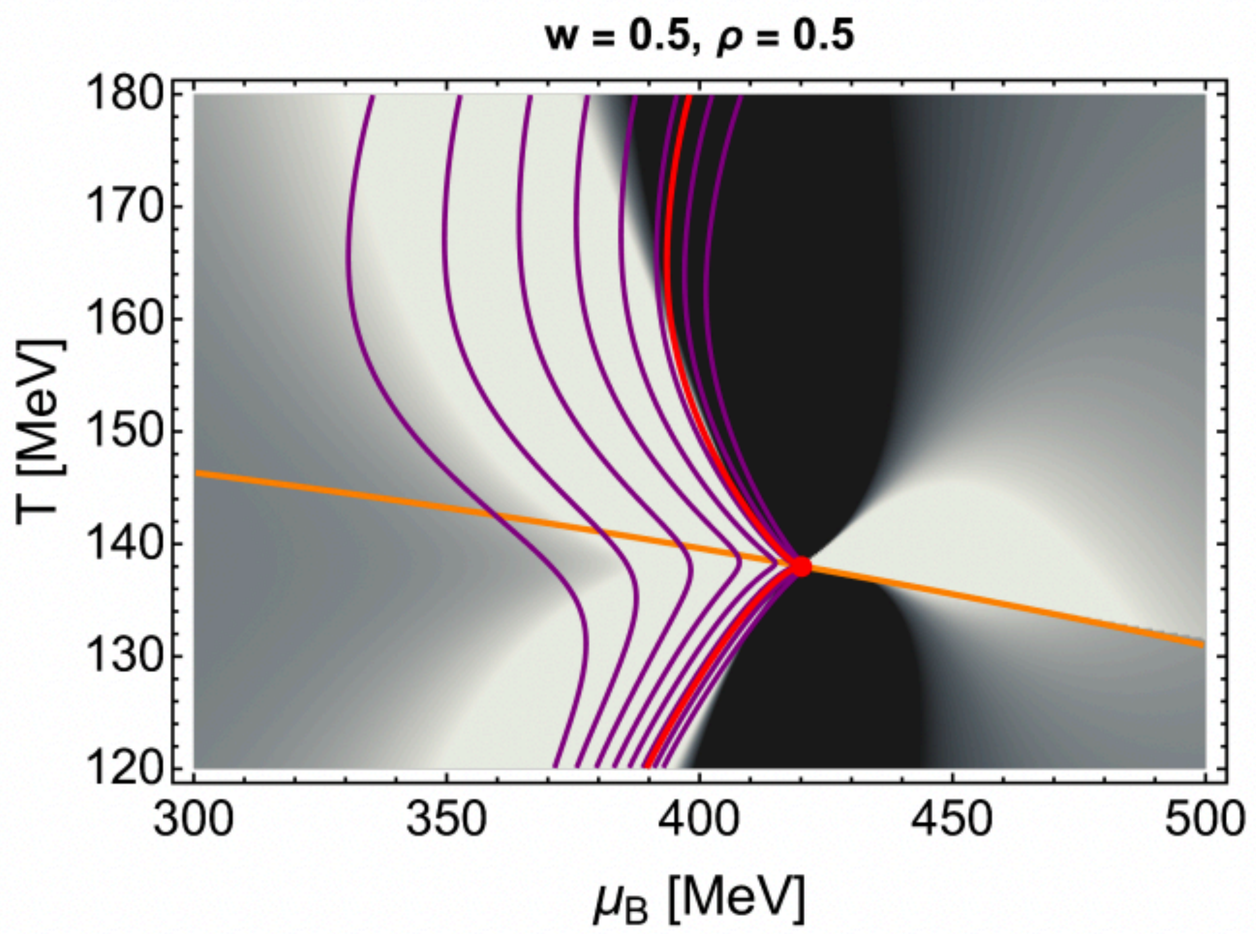
Proton factorial cumulants do not show as strong signal as observed for baryons.[3]

[3]NB: Once again, this is not a prediction of actual $\sqrt{s_{NN}}$ dependence but an estimation of the signal for a freeze-out at critical density occurring at a given $\sqrt{s_{NN}}$.

Hydrodynamics with first-order phase transitions

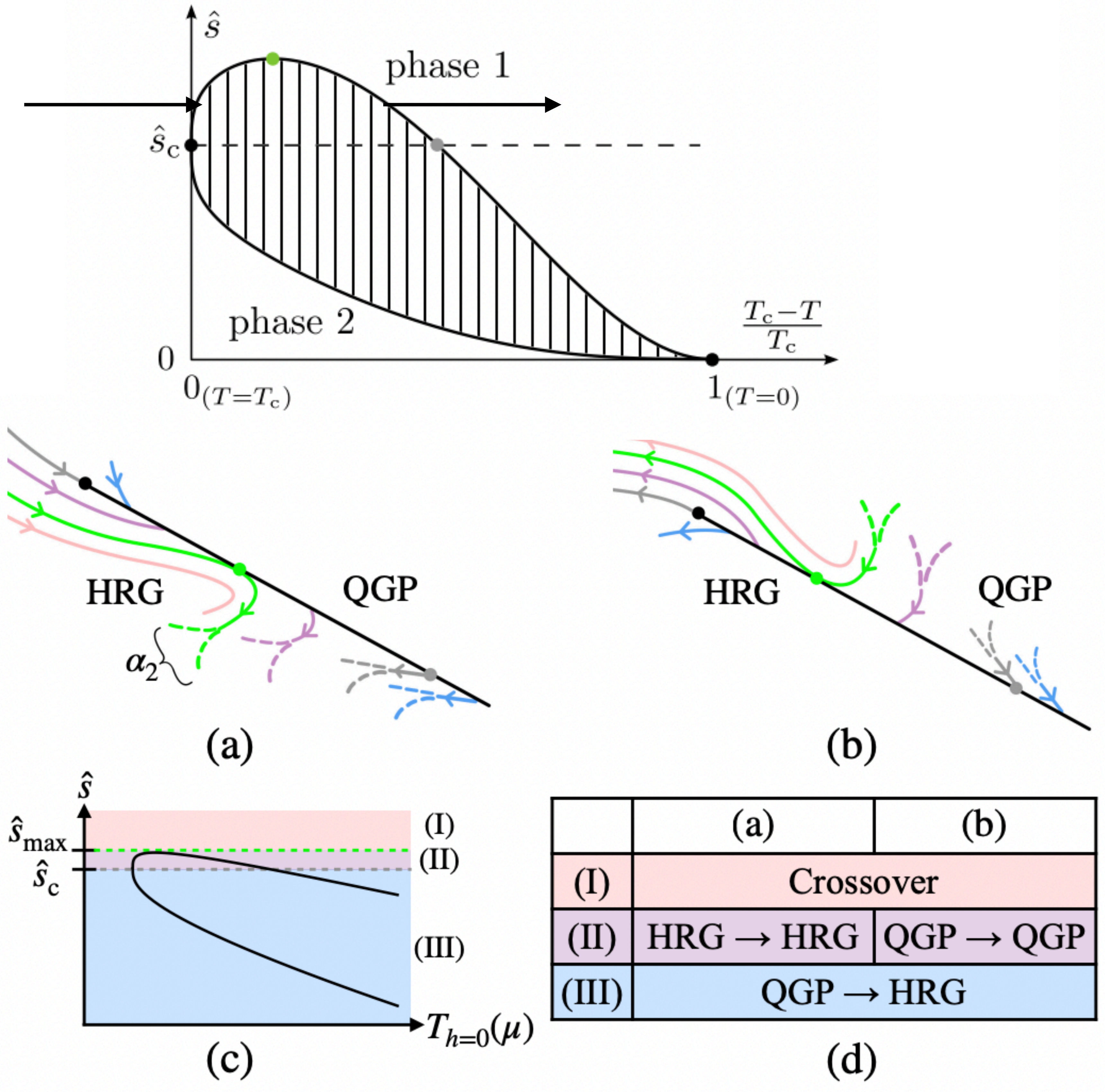
Hydrodynamics near first order phase transition

Critical lensing



Dore et al., 22, PRD
Nonaka & Asakawa,05, PRC

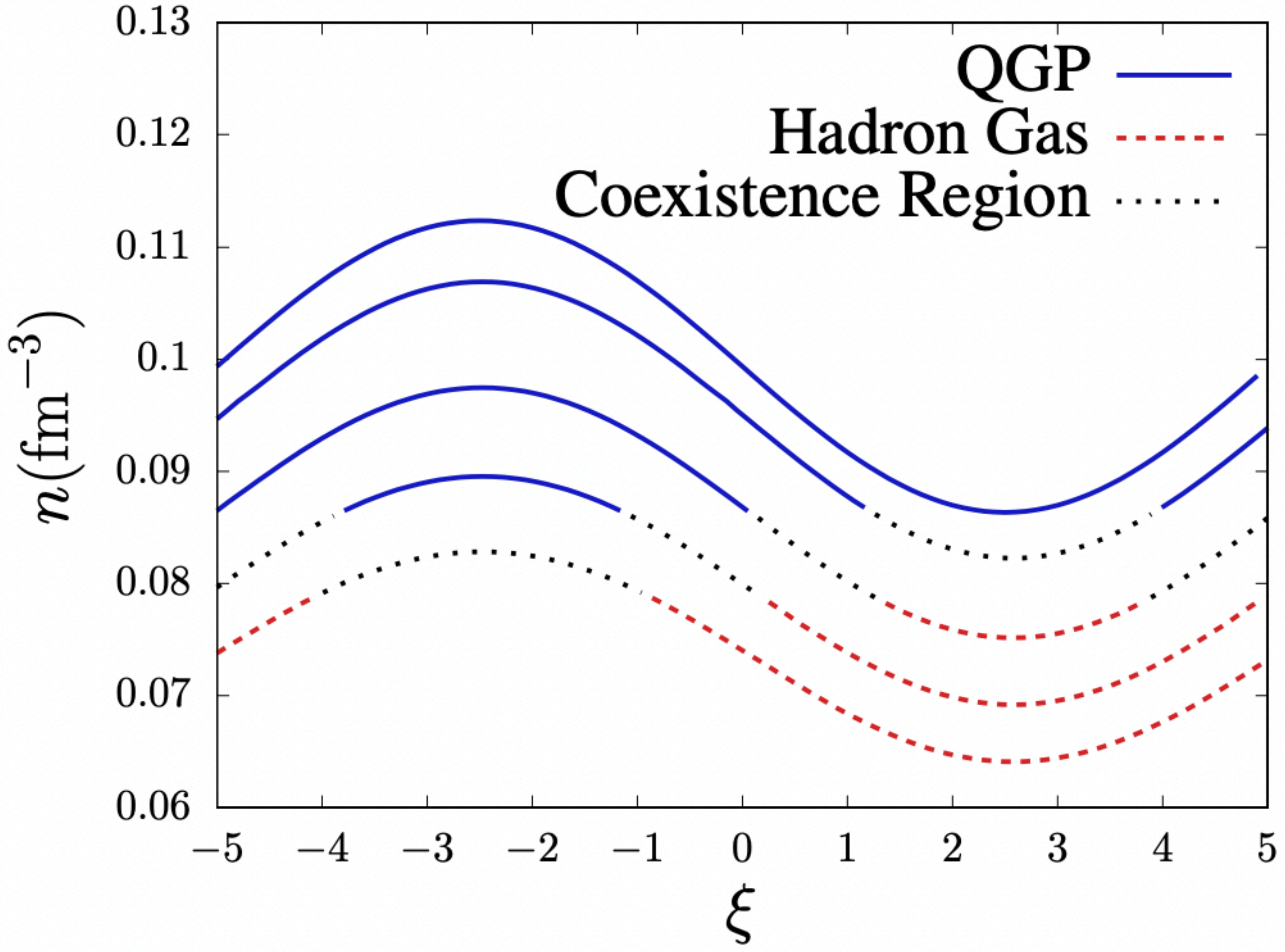
Universal features of isentropes near CP



	(a)	(b)
(I)	Crossover	
(II)	HRG → HRG	QGP → QGP
(III)	QGP → HRG	

(d)

MP, Sogabe, Stephanov,Yee, 24, PRC



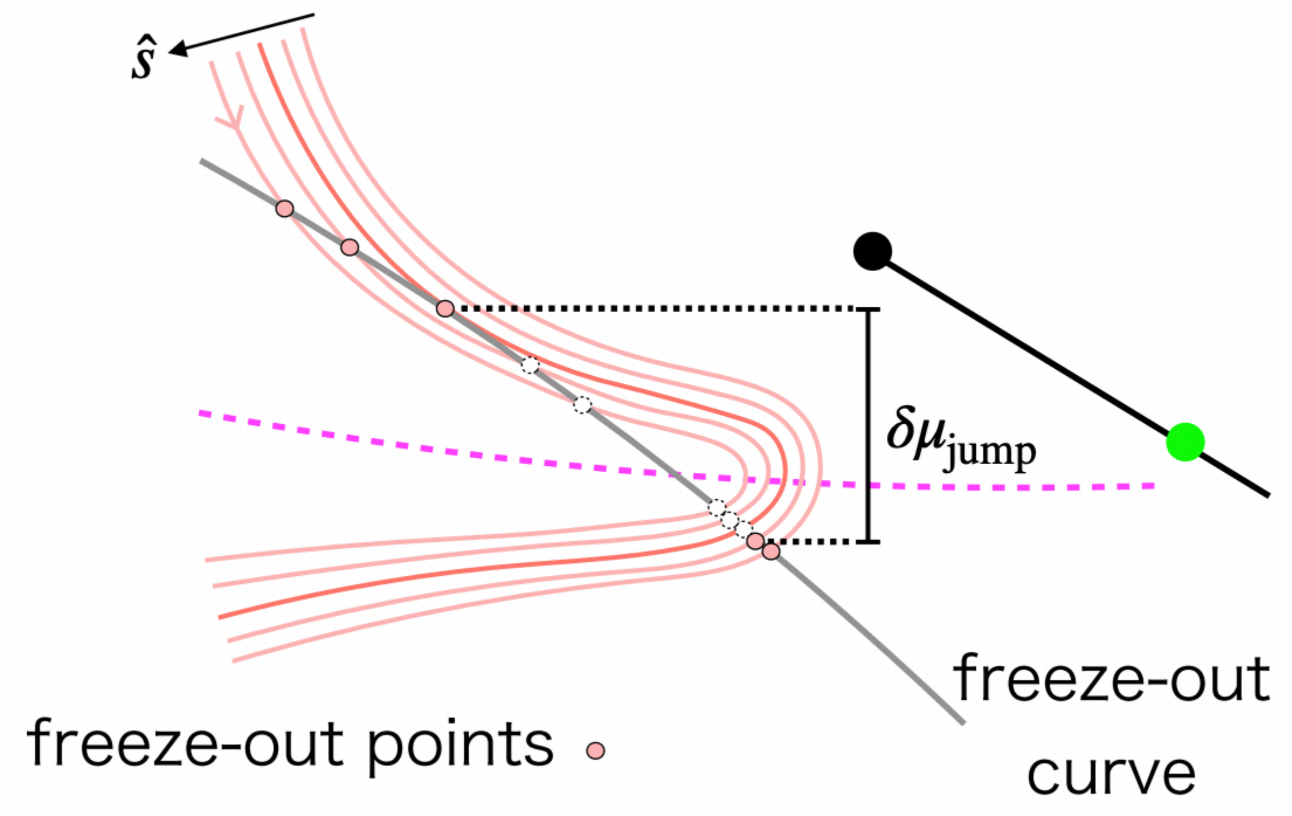
Kapusta, Singh, Welle, 24, PRC

Spinodal decomposition in relativistic fluids

See Karthein, Koch, Ratti, PRD 25 also

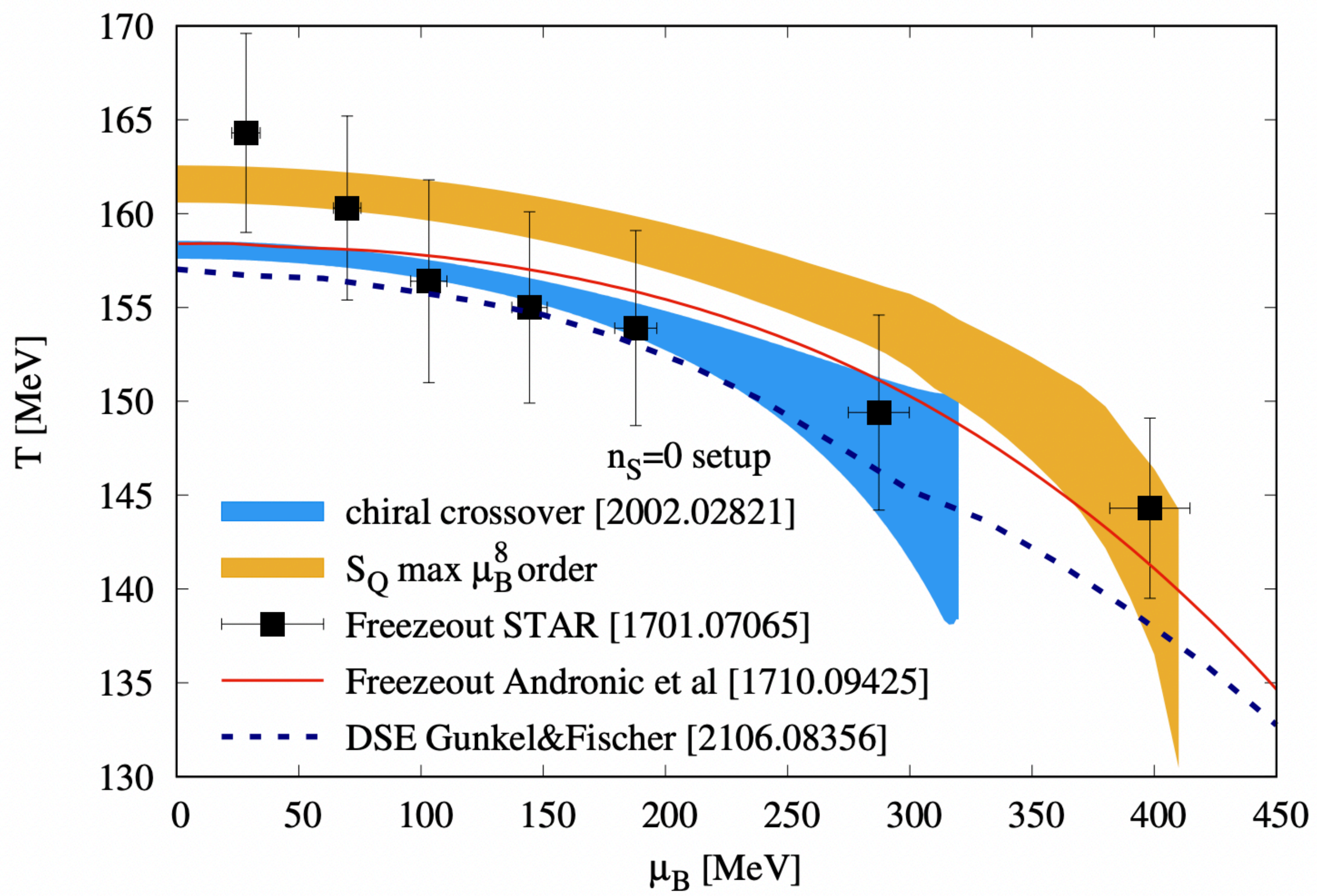
- How do spinodal dynamics affect these newly identified class of hydro trajectories on the first-order side?
- Correlations?.

Freeze-out curve



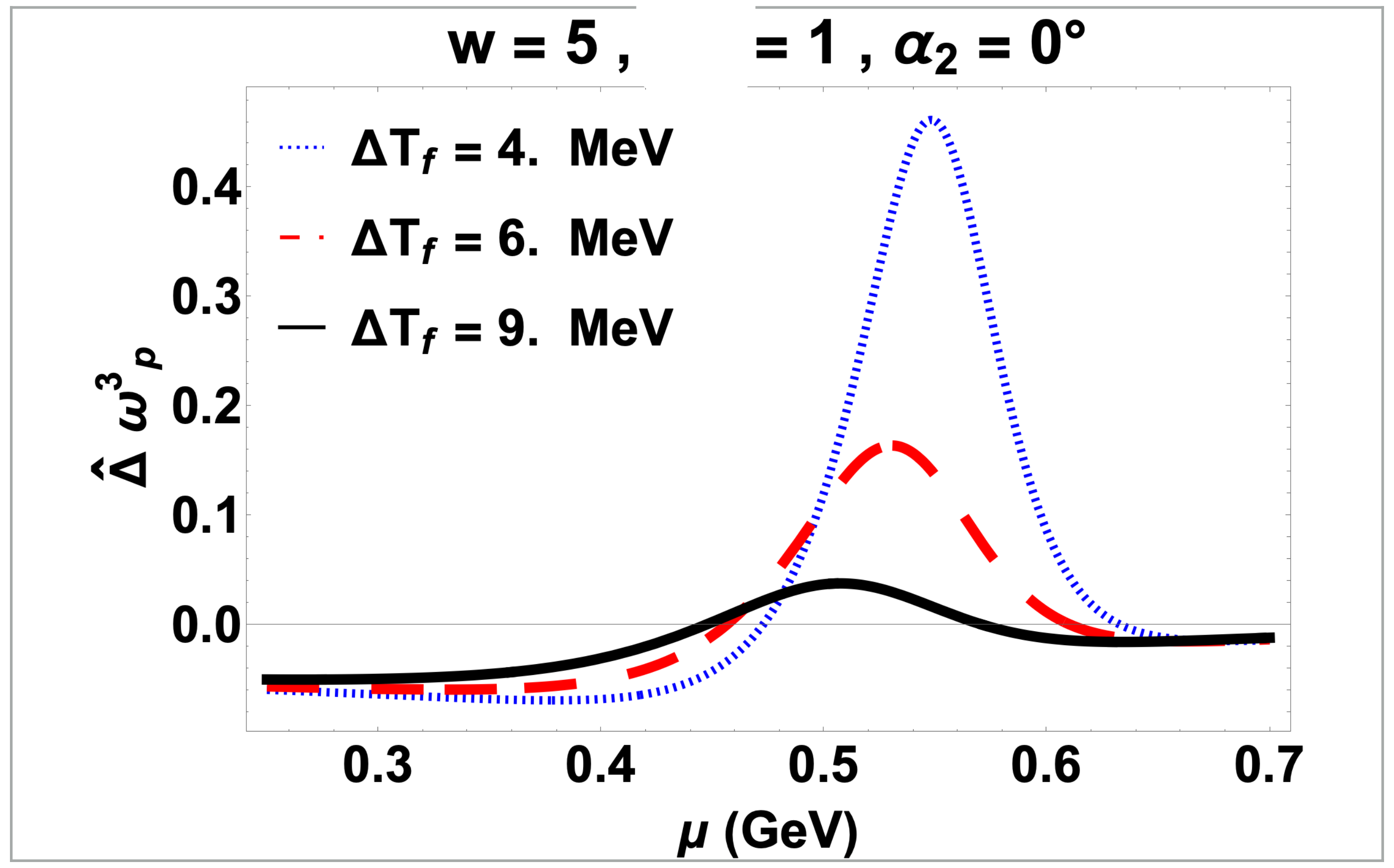
See N. Sogabe's talk on Wed, QM 2025

Universal topography of hydro trajectories near CP : Discontinuity in freeze-out curve



Borsanyi et al.,*Phys.Rev.D* 111 (2025) 1, 014506

$$T_f(\mu) = T_{h=0}(\mu) - \Delta T_f$$



$$\hat{\Delta}\omega_p^k \sim \Delta T_f^{1.2-k}$$

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Within the uncertainties of freeze-out parameters, higher order cumulants can vary by order of magnitudes

Understanding the dynamical mechanism of freeze-out is important!