



Universality in short-distance particle production

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- Universality and the unitary limit
- Schrödinger symmetry
- Nuclear reactions with neutrons
- Neutral charm mesons and the $X(3872)$
- Summary and Outlook

References:

- HWH, **D.T. Son**, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021) [arXiv:2103.12610]
Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022) [arxiv:2107.02831]
Braaten, HWH, Phys. Rev. D **107**, 034017 (2023) [arxiv:2301.04399]

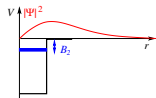
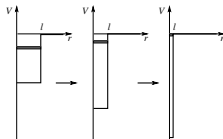


Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

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- Consider short-ranged, resonant S-wave interactions $\Rightarrow$ halo state
- Unitary limit: $a \rightarrow \infty, \ell \sim r_e \rightarrow 0$

$$\mathcal{T}_2(k, k) \propto \begin{bmatrix} \underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \dots} & -ik \end{bmatrix}^{-1} \sim i/k$$

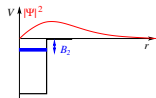
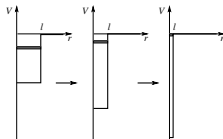


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- Scattering amplitude scale invariant, saturates unitarity bound
- BEC/BCS crossover, neutron matter, ... (cf. Schäfer, Baym, PNAS **118**, e2113775118 (2021))
- System has also (non-relativistic) conformal symmetry/Schrödinger symmetry
Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...

■ Non-relativistic conformal symmetry: Schrödinger symmetry

▣ Galilei symmetry

space + time translations

rotations

Galilei boosts

▣ Scale transformations

$$\mathbf{x} \rightarrow e^{\lambda} \mathbf{x}, \quad t \rightarrow e^{2\lambda} t, \quad \psi \rightarrow e^{-\lambda \Delta} \psi$$

▣ Special conformal transformations

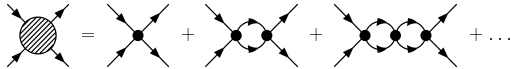
$$\mathbf{x} \rightarrow \frac{\mathbf{x}}{1 + \xi t}, \quad t \rightarrow \frac{t}{1 + \xi t}, \quad \psi \rightarrow \psi' = \dots$$

$\Rightarrow$ preserves angles

■ 12 Parameters

■ Generators: $H, \mathbf{P}, \mathbf{L}, \mathbf{K}, D, C,$ satisfy Schrödinger algebra

- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e \sim \ell$)



- Renormalization group equation: $\Lambda \frac{d}{d\Lambda} \tilde{g}_2 = \tilde{g}_2(1 + \tilde{g}_2)$

- Two fixed points:

– $\tilde{g}_2 = 0 \Leftrightarrow a = 0 \Rightarrow$ no interaction, free particles

– $\tilde{g}_2 = -1 \Leftrightarrow 1/a = 0 \Rightarrow$ unitary limit $\Rightarrow$ conformal/Schrödinger symmetry

- Neutrons: $a \approx -18.6$ fm, $r_e \approx 2.8$ fm $\Rightarrow$ close to unitary limit $\Rightarrow$ approximately conformal
 $\Rightarrow$ multineutron system can be described by conformal field

- Two-point function of (primary) conformal field operator $\mathcal{U}$

$$G_{\mathcal{U}}(t, \mathbf{x}) = -i \langle T \mathcal{U}(t, \mathbf{x}) \mathcal{U}^\dagger(0, \mathbf{0}) \rangle = \mathcal{C} \frac{\theta(t)}{(it)^\Delta} \exp\left(\frac{iM\mathbf{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant $\mathcal{C}$
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega, \mathbf{p}) = -\mathcal{C} \left(\frac{2\pi}{M}\right)^{3/2} \Gamma\left(\frac{5}{2} - \Delta\right) \left(\frac{\mathbf{p}^2}{2M} - \omega\right)^{\Delta - \frac{5}{2}}$$

- pole only for $\Delta = 3/2$ (free field)
- branch cut for $\Delta > 3/2$

- How to calculate scaling dimension Δ ? not given by symmetry $\Rightarrow$ non-perturbative problem

- (1) Δ can be obtained from field theory calculation
- (2) Δ can be obtained from operator state correspondence

$$\Delta \text{ of primary operator} = (\text{Energy of state in HO})/\hbar\omega$$

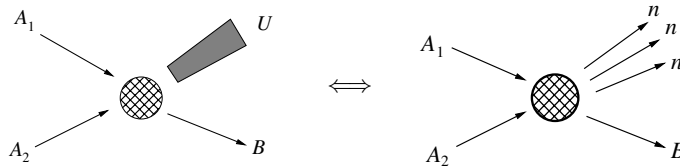
(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

N	S	L	$\mathcal{O}$	Δ	Δ_{free}
2	0	0	$\psi_1\psi_2$	2	3
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272	5.5
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622	6.5
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)	8
5	1/2	1	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2\nabla_i\psi_1$	7.6(1)	10.5

- Connection between Δ and energy of particles in a trap
- Predictions for $N \leq 70$ available (M.G. Endres et al., PRA **84**, 043644 (2011))

High-energy nuclear reaction with multi-neutron final state

(HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))



$$\Delta Mc^2 + E_{A_1} + E_{A_2} = E_B + E_U$$

■ **Assumption:** energy scale of primary reaction $\gg E_U - \frac{\mathbf{p}^2}{2M_U} = E_n^{cms}$

■ **Factorization:** $\frac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \text{Im } G_U(E_U, \mathbf{p})$

■ **Reproduces Watson-Migdal treatment of FSI for $2n$**



- Two ways to do experiments

- (a) detect recoil particle B

$$\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta-5/2}, \quad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$$

- (b) detect all final state particles **including neutrons**

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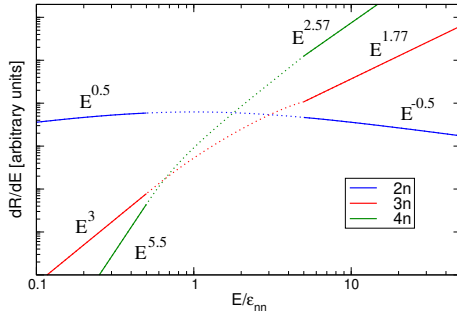
- $2n$ case can be understood from dimer propagator ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \Rightarrow \text{Im } G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

- See also (M.Göbel et al., Phys. Rev. C **104**, 024001 (2021)) for ${}^6\text{He}(p, p_\alpha)2n$

(Göbel, HWH, Phillips, Phys. Rev. C **110**, 024003 (2021)) for hard core knockout from ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, ${}^{17}\text{B}$, ${}^{19}\text{B}$, ${}^{22}\text{C}$

■ Power law behavior at low energies



- $N = 5$: $E^8 \rightarrow E^{5.1}$
- $N = 6$: $E^{10.5} \rightarrow E^{6.2}$
- ...

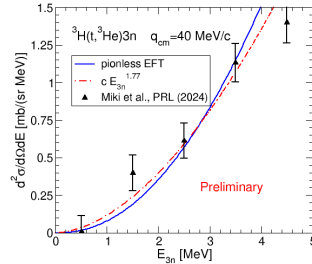
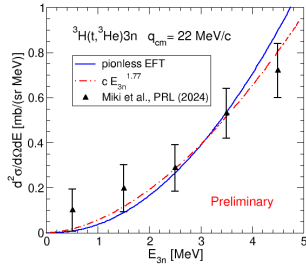
■ Predictions for $N \leq 70$ available
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■ No peak except for $2n$

Braaten, HWH, Phys. Rev. D **107**, 034017 (2023)

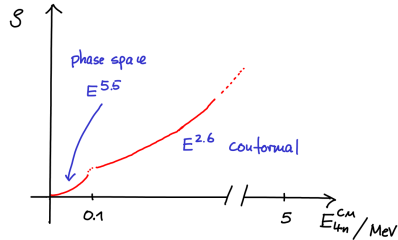
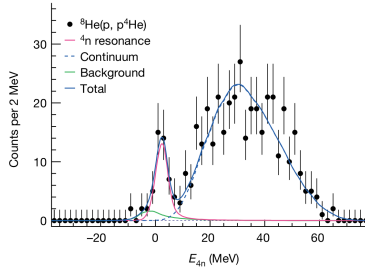
$3n$ distributions from experiment

- $3n$ case consistent with previous experiments for $^3\text{H}(\pi^-, \gamma)3n$ (Miller et al., Nucl. Phys. A **343**, 347 (1980))
- $3n$ distributions from $^3\text{H}(^3\text{H}, ^3\text{He})3n$ (Miki et al., Phys. Rev. Lett. **133**, 012501 (2024))



- Small q_{cm} data consistent

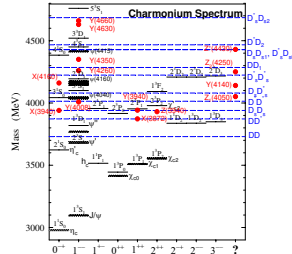
- Search for tetraneutron resonances in $^8\text{He}(p, p\alpha)4n$ (M. Duer et al., Nature **606**,678 (2022))



- Structure from initial state?
- Dineutron correlations can produce peak (Lazauskas, Hiyama, Carbonell, Phys. Rev. Lett. **130**, 102501 (2023))

Neutral charm mesons and $X(3872)$

- **New $c\bar{c}$ states at B factories: X, Y, Z**
(cf. Godfrey, arXiv:0910.3409)
 - ▣ Challenge for understanding of QCD
 - ▣ Unitary limit relevant?
- $X(3872)$ (Belle, CDF, BaBar, D0, LHCb)
- **Nature of $X(3872)$?**
 - ▣ $\bar{D}^0 D^{0*}$ -molecule, tetraquark, charmonium hybrid, ...



$$m_X = (3871.65 \pm 0.06) \text{ MeV}, \quad \Gamma = (1.19 \pm 0.21) \text{ MeV}, \quad J^{PC} = 1^{++} \quad (\text{PDG 2023})$$

- **Assumption:** $X(3872)$ is weakly-bound $D^0\bar{D}^{0*}$ -molecule
 - $\Rightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}$, $B_X = (0.07 \pm 0.12) \text{ MeV} \approx 1/(2\mu_{D\bar{D}^*} a^2)$
 - $\Rightarrow$ **universal properties** (Braaten et al., 2003-2008; ...)

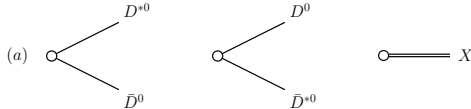
Neutral charm mesons and $X(3872)$

- Approximate unparticles of three D^0/D^{0*} mesons
- Interaction of $X(3872)$ with $D^0, \bar{D}^0, D^{0*}, \bar{D}^{0*}$ determined by large a
(Canham, HWH, Springer, PRD **80**, 014009 (2009))

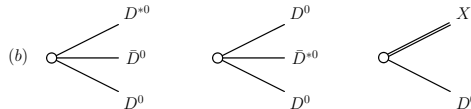
$$a_{D^0 X} = -9.7a \quad a_{D^{*0} X} = -16.6a$$

- Richer structure because of $X(3872)$ (bound state)

two charm mesons



three charm mesons



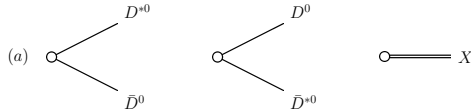
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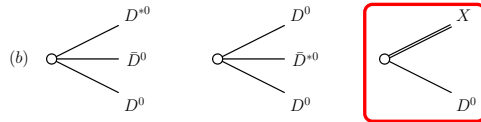
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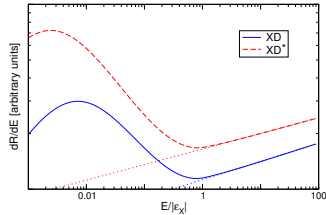
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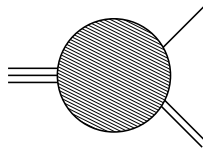
three charm mesons



- Universal scaling for unparticles of three neutral charm mesons
(Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022))



XD point production

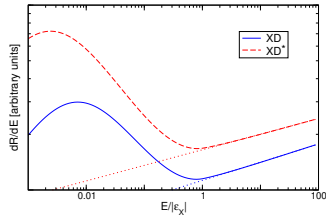


$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2 \sqrt{E} \approx E^{0.1} \quad \text{for } E/|\epsilon_X| \gg 1 \quad \text{from conformal 3-pt. function}$$

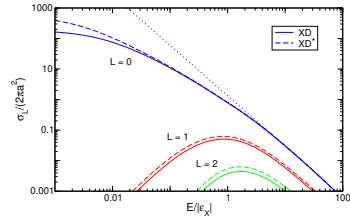
$$\Delta_1=3/2, \quad \Delta_2=2, \quad \Delta_3 \approx 3.10119/3.08697 \quad (\text{Braaten, HWH, Phys. Rev. D } \mathbf{107}, 034017 (2023))$$

■ Universal scaling for unparticles of three neutral charm mesons

(Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022))



XD point production



XD elastic scattering

$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2 \sqrt{E} \approx E^{0.1}, \quad \sigma \sim E^{-1.6} \quad \text{for } E/|\epsilon_X| \gg 1$$

$$\Delta_1=3/2, \quad \Delta_2=2, \quad \Delta_3 \approx 3.10119/3.08697$$



- Universality in strongly interacting quantum systems
- High-energy nuclear reactions with final state neutrons
 - ⇒ (approximate) **conformal symmetry**
 - ⇒ **power law behavior of observables** determined by scaling dimension Δ
- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles



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- Other applications & extensions
 - ▣ Neutral charm mesons
 - ▣ Two-component fermions in ultracold atom physics
 - ▣ Systems with the Efimov effect, EM charge?
 - ⇒ bosonic atoms, nucleons, α particles
 - ⇒ **complex scaling dimensions**
 - ⇒ scale symmetry broken

Additional transparencies



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■ (Relativistic) unparticle (Georgi, Phys. Rev. Lett. **98**, 221601 (2007))

- ▣ field ψ in relativistic conformal field theory
- ▣ ψ characterized by scaling dimension Δ , massless
- ▣ hidden conformal symmetry sector beyond Standard model (weakly coupled)
- ▣ no evidence at LHC so far
(CMS Coll., EPJC **75**, 235 (2015), PRD **93**, 052011, JHEP **03**, 061 (2017))

■ (Non-relativistic) unparticle/unnucleus

- ▣ non-relativistic analog of Georgi's unparticle
- ▣ field ψ in non-relativistic conformal field theory
(cf. Nishida, Son, Phys. Rev. D **76**, 086004 (2007))
- ▣ ψ characterized by scaling dimension Δ and mass M
- ▣ free field has $\Delta = 3/2 \iff$ mass dimension
 $\Rightarrow$ lowest possible value (unitarity)
- ▣ N neutrons are (approximate) unparticle with mass Nm_N and scaling dimension $\Delta = ?$

■ Imaginary part of propagator

$$\mathrm{Im} G_{\mathcal{U}}(\omega, \mathbf{p}) \sim \begin{cases} \delta\left(\omega - \frac{\mathbf{p}^2}{2M}\right), & \Delta = \frac{3}{2}, \\ \left(\omega - \frac{\mathbf{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\mathbf{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

■ General $\mathcal{U}$ does not behave like a particle (“unparticle/unucleus”) $\Rightarrow$ continuous energy spectrum

■ Examples of unnuclei

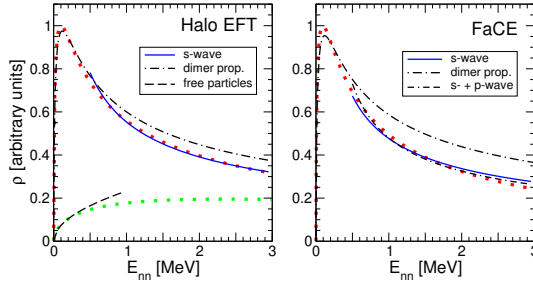
■ N free fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = Nm_\psi$, $\Delta = 3N/2$

■ N interacting fields: $\mathcal{U} = \psi_1 \dots \psi_N$, $M = Nm_\psi$, $\Delta > 3/2$

■ Corrections from finite a and r_0 (S. Dutta, R. Mishra, D.T. Son, Phys. Rev. D **109**, 016001 (2024))

$$\mathrm{Im} G_{\mathcal{U}}(\omega, 0) \sim \omega^{\Delta - \frac{5}{2}} \theta(\omega) \left(1 + \frac{c_1}{a\sqrt{m\omega}} + c_2 r_0 \sqrt{m\omega} \right), \quad c_2 = 0$$

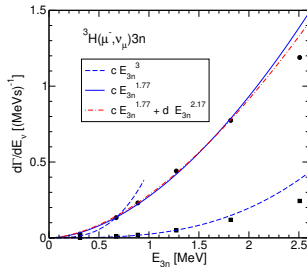
- Two-neutron spectrum for ${}^6\text{He}(p, p\alpha)2n$ (Göbel et al., Phys. Rev. C **104**, 024001 (2021))



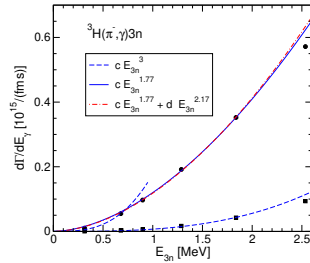
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■ Radiative muon/pion capture on the triton (AV18 + UIX)



Golak et al., PRC **98**, 054001 (2018)



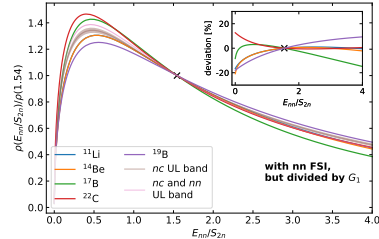
Golak et al., PRC **94**, 054001 (2016)

■ Conformal prediction: $\frac{d\Gamma}{dE} \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}$, $0.1 \text{ MeV} \ll E_{3n} \ll 5 \text{ MeV}$

Universality in hard knock-out reactions

- Universality of $2n$ distributions in hard knock-out of the core
- Calculate $2n$ distributions of different halo nuclei in halo EFT

	A	S_{2n} [keV]	E_{nc}^* [keV]	E_{nn}^* [keV]
^{11}Li	9	369	26	
^{14}Be	12	1266	510	
^{17}B	15	1384	83	118
^{19}B	17	90	5	
^{22}C	20	100	68	



- $S_{2n} \gg E_{nc}^*, E_{nn}^* \implies$ nuclei close to unitary limit $\implies$ shape accurate to 20%

(Göbel, HWH, Phillips, Phys. Rev. C **110**, 024003 (2024))