

New Insights on the Phase Structure with Functional OCD

Jan M. Pawlowski

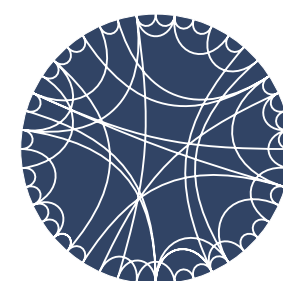
Universität Heidelberg & ExtreMe Matter Institute

GSI Darmstadt, April 13th 2025



fqcd-collaboration.github.io

Plenary talk QM 2025 Rennecke



STRUCTURES
CLUSTER OF
EXCELLENCE



UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

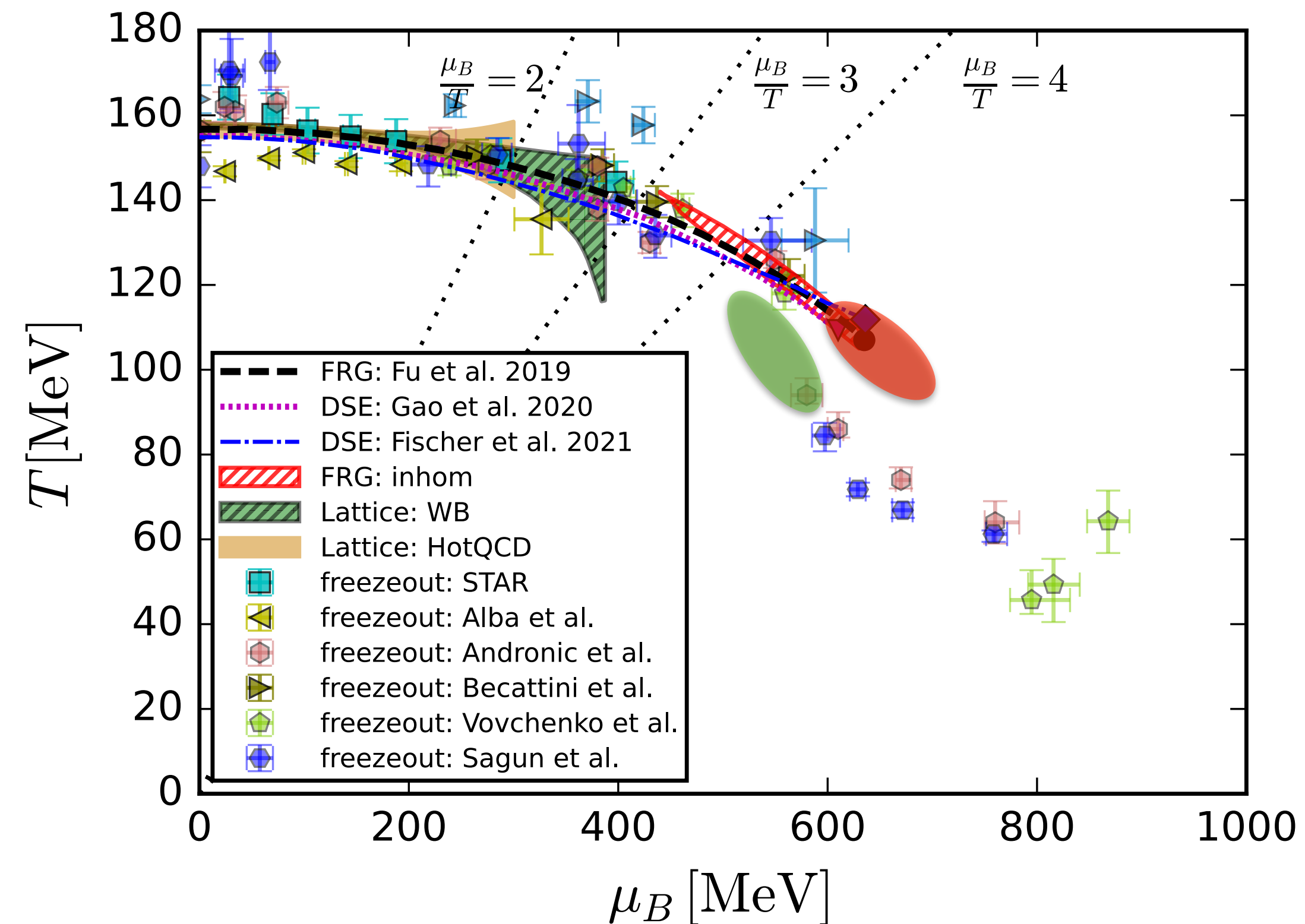
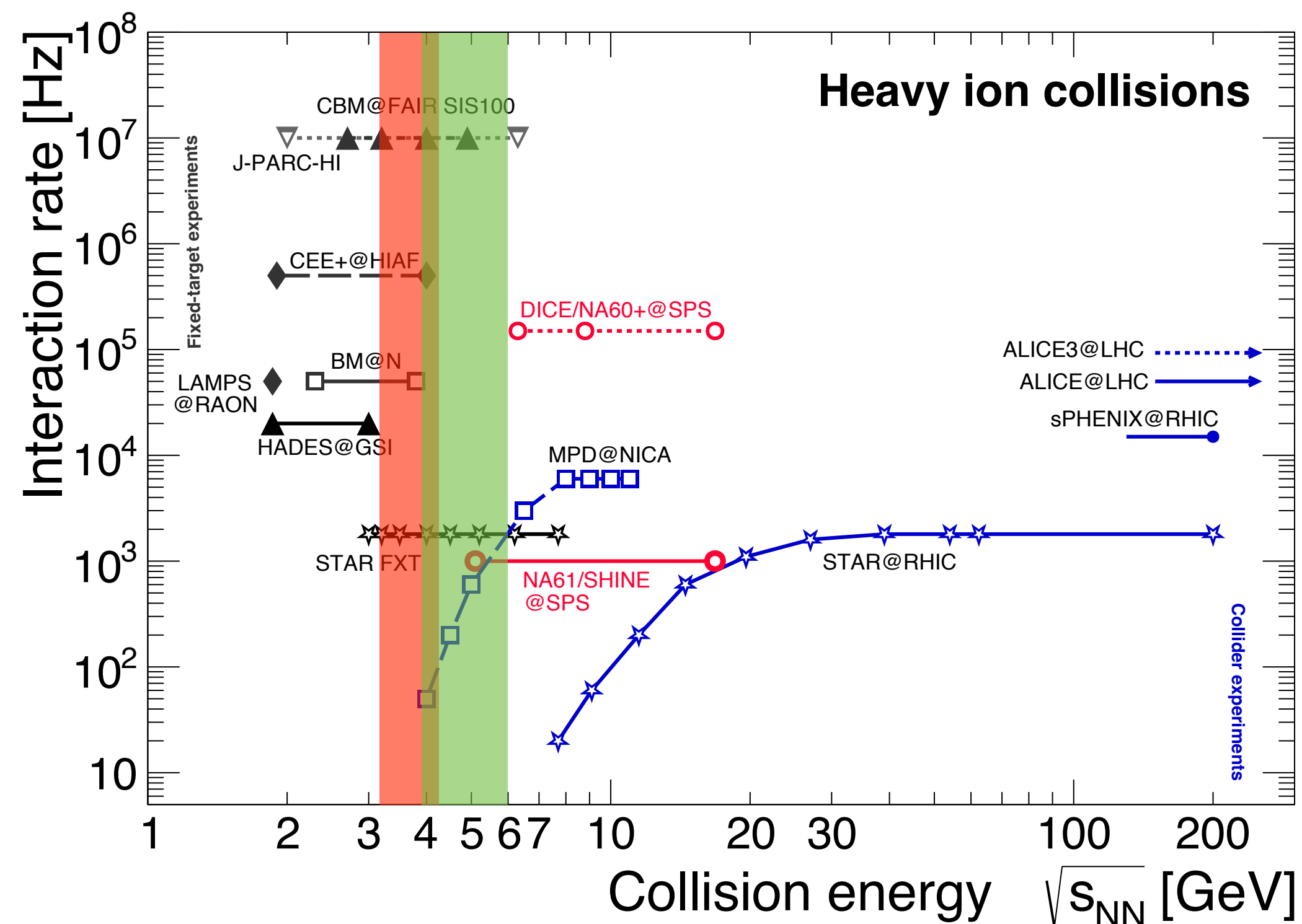


Experimental & Theoretical Landscape

experimental landscape

theoretical landscape

Onset of new physics (CEP)



Peak of kurtosis at the freeze-out line

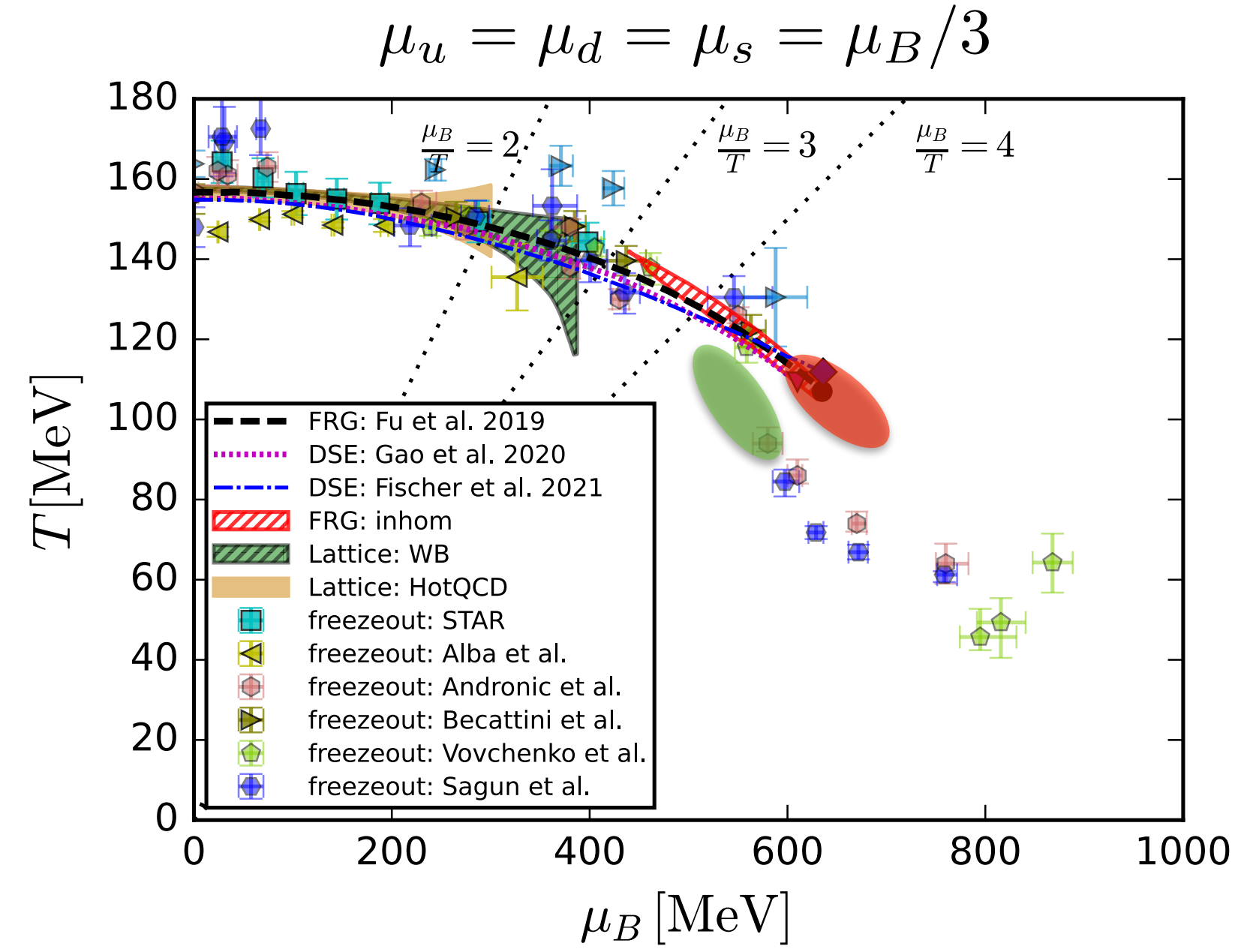
Fu, JMP, Rennecke, PRD 101 (2020) 054032

Galatyuk, A982 (2019) update 2025; CBM, EPJA 53 3 (2017) 60

Gao, JMP, PLB 820 (2021) 136584

Gunkel, Fischer, PRD 104 (2021) 054022

Phase structure of QCD and the CEP



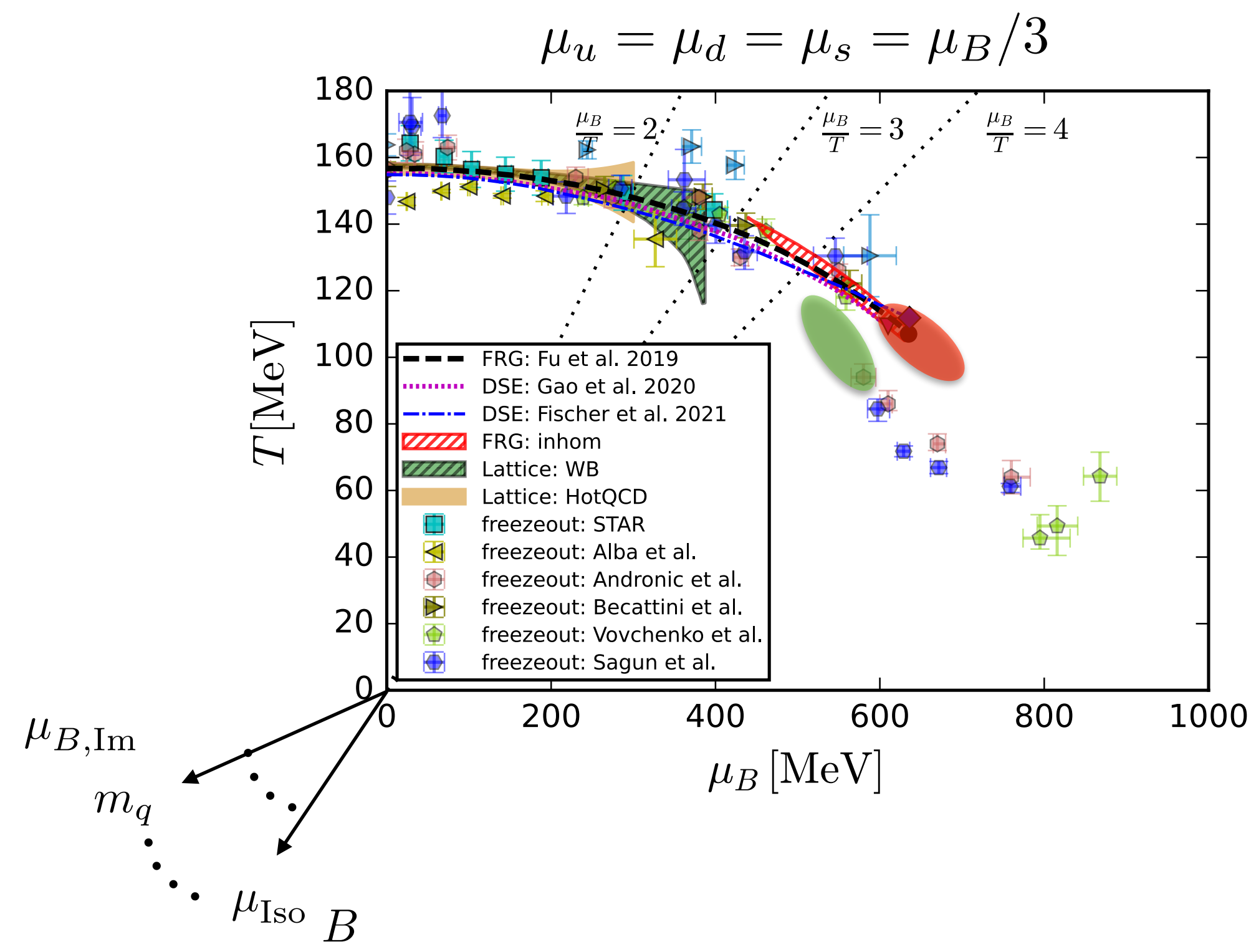
Functional QCD: CEP estimate

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584
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$$(\mu_B, T)_{\text{CEP}} \sim (600 - 650, 105 - 115) \text{ MeV}$$

Phase structure of QCD and the CEP



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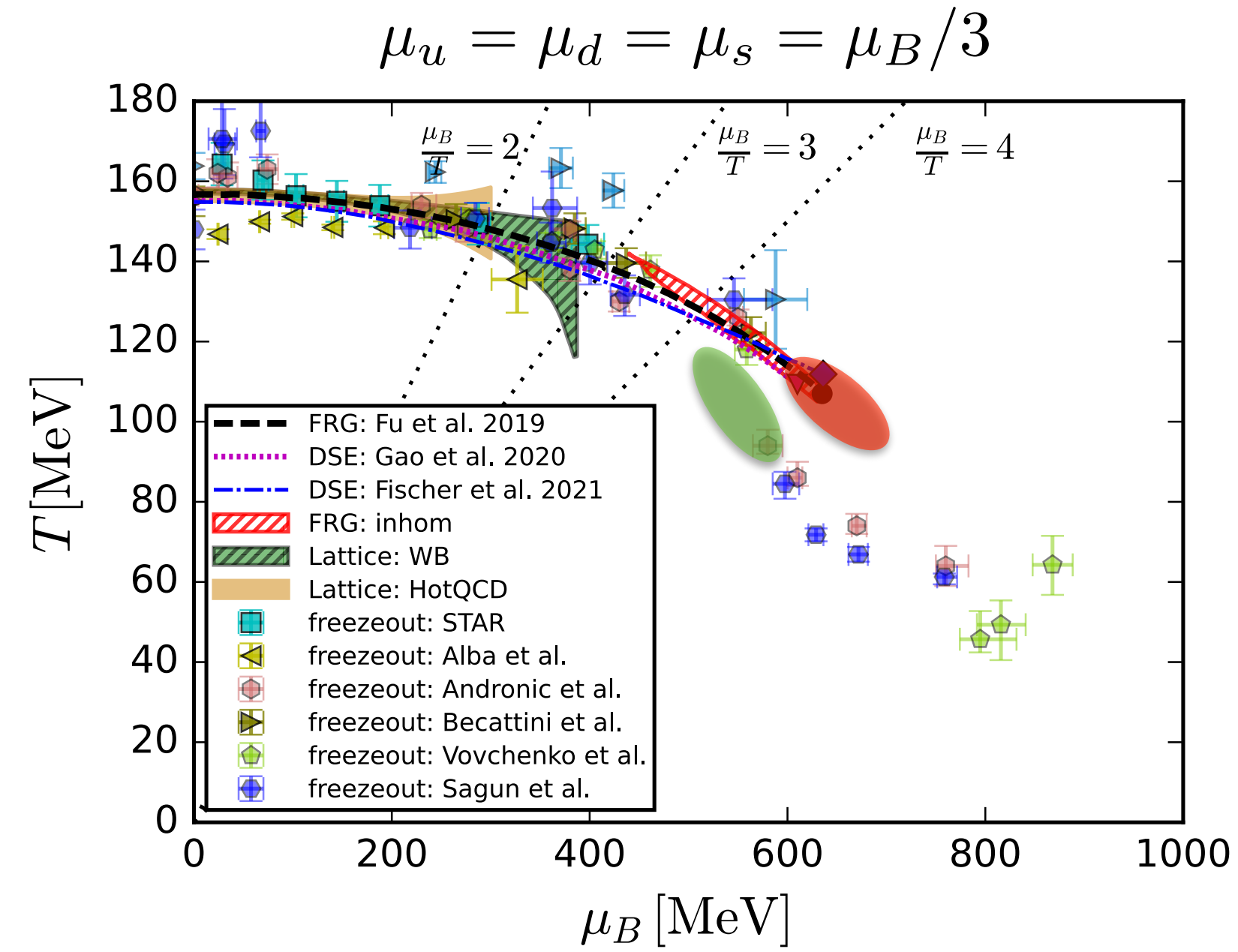
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Collect all possible information/structure
for
physics understanding & extrapolations

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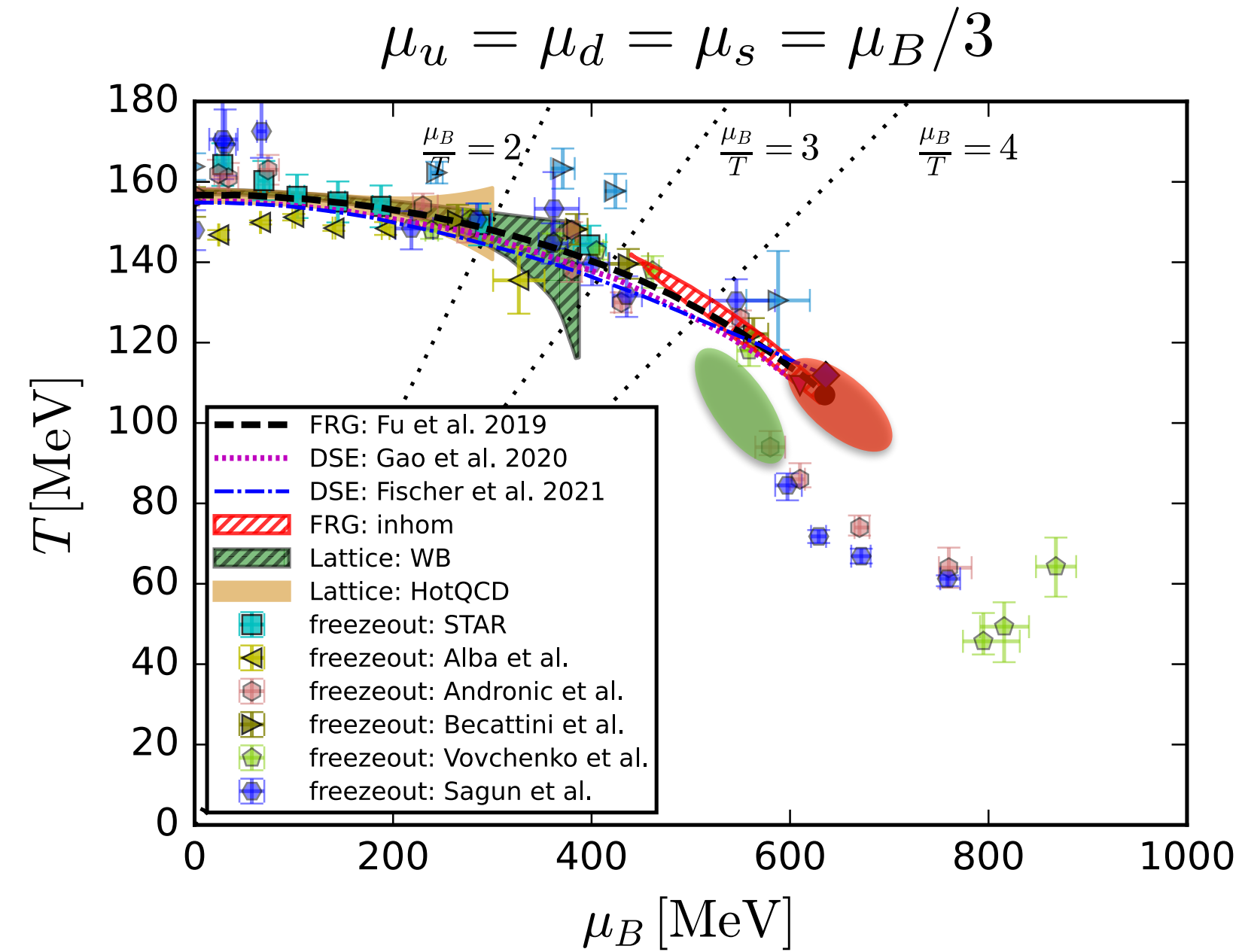
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Estimates & predictions

Requires computations in 1st principles QCD at

$$(\mu_B, T) \sim (\mu_B, T)_{\text{CEP}}$$

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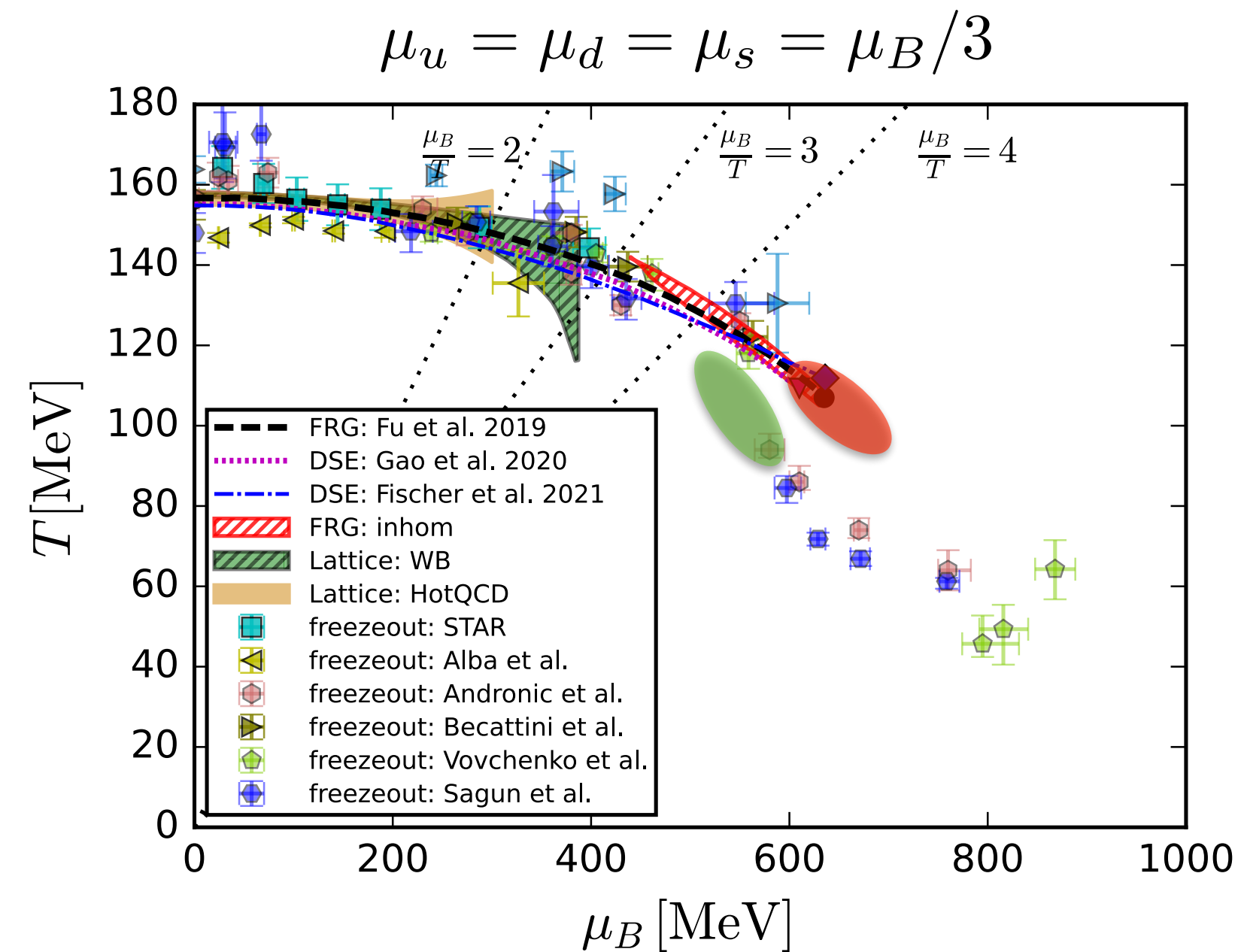
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Extrapolations for Pheno

Requires a discussion of the
explicit & implicit assumptions

Phase structure of QCD and the CEP



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Extrapolations for Pheno

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Lattice extrapolations

low energy effective theories:
QM, NJL, PQM, PNJL, ..., Holography

Functional approaches are the only first principle approaches *to date*
that allow for *direct* computations at finite density with

$$\frac{\mu_B}{T} \gtrsim 3$$

Supplementary Material:

Phase structure from functional QCD: how to

Phase structure from functional QCD: Predictions & estimates

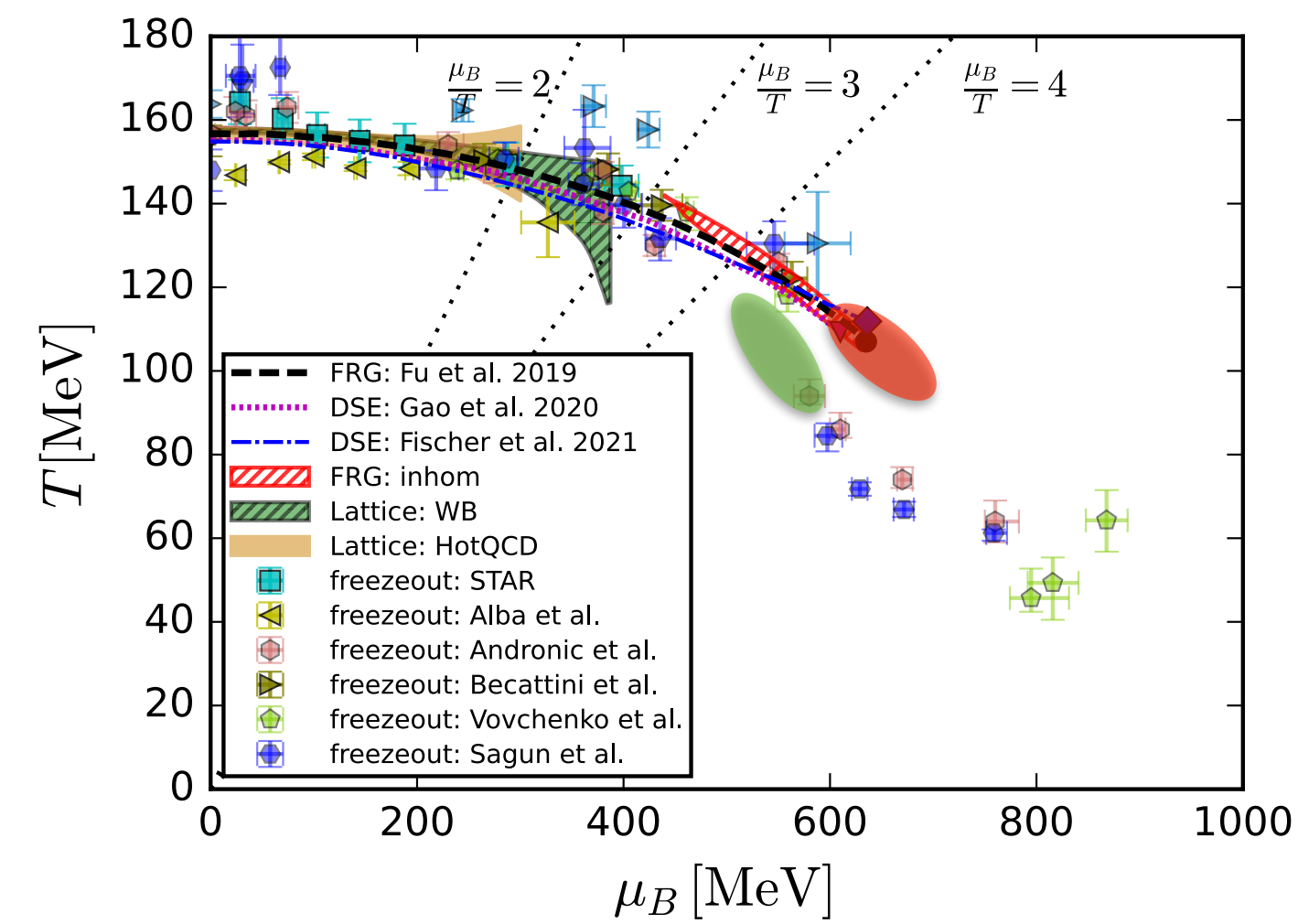
Ripples of the CEP

Fu, Luo, JMP, Rennecke, Wen, Yin, PRD 104 (2021) 9

Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

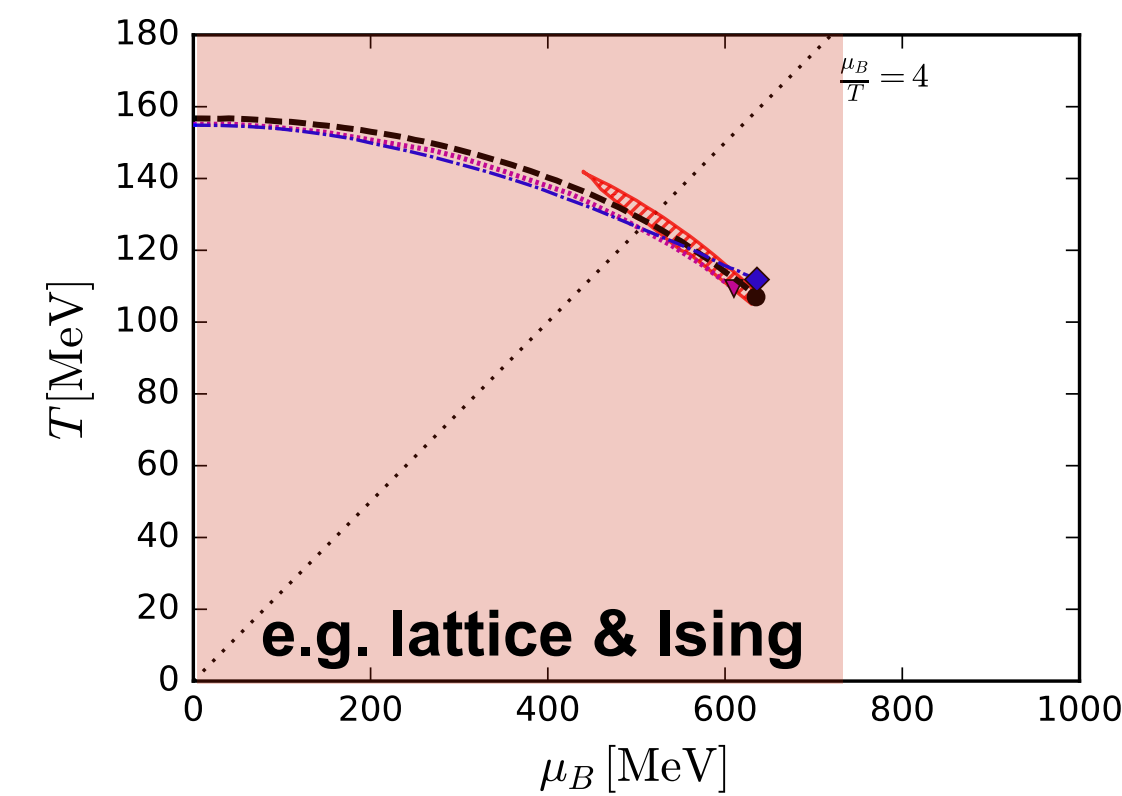
Lu, Gao, Liu, JMP, arXiv:2504.05099

Experimental signatures of the critical end point



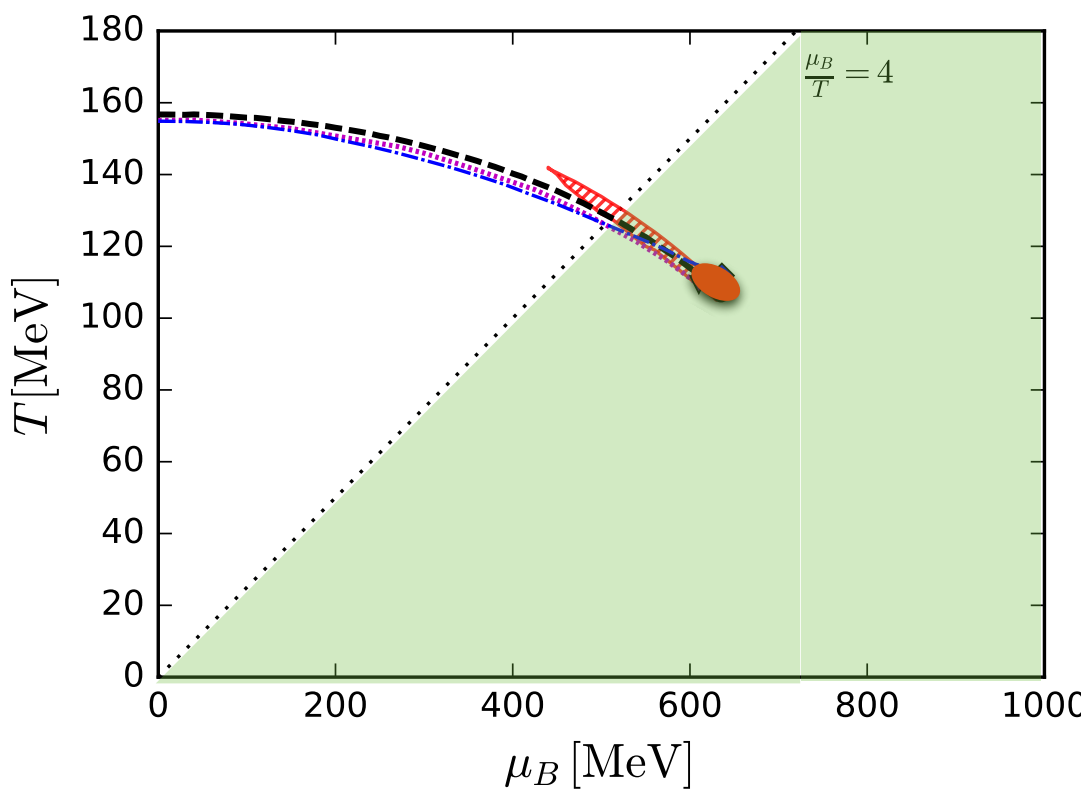
Scenario I

Scaling



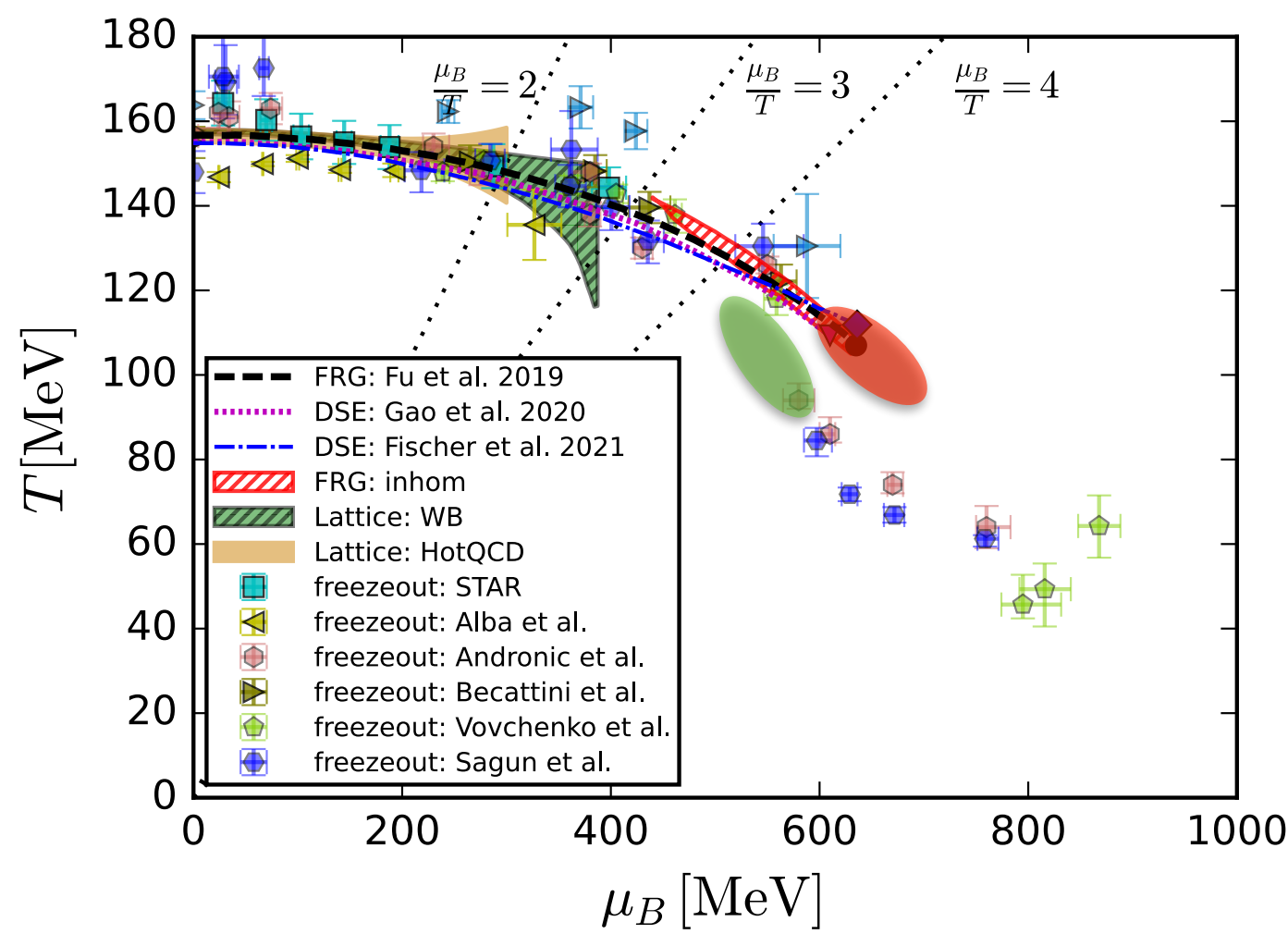
Scenario II

Minimal scaling and new phases

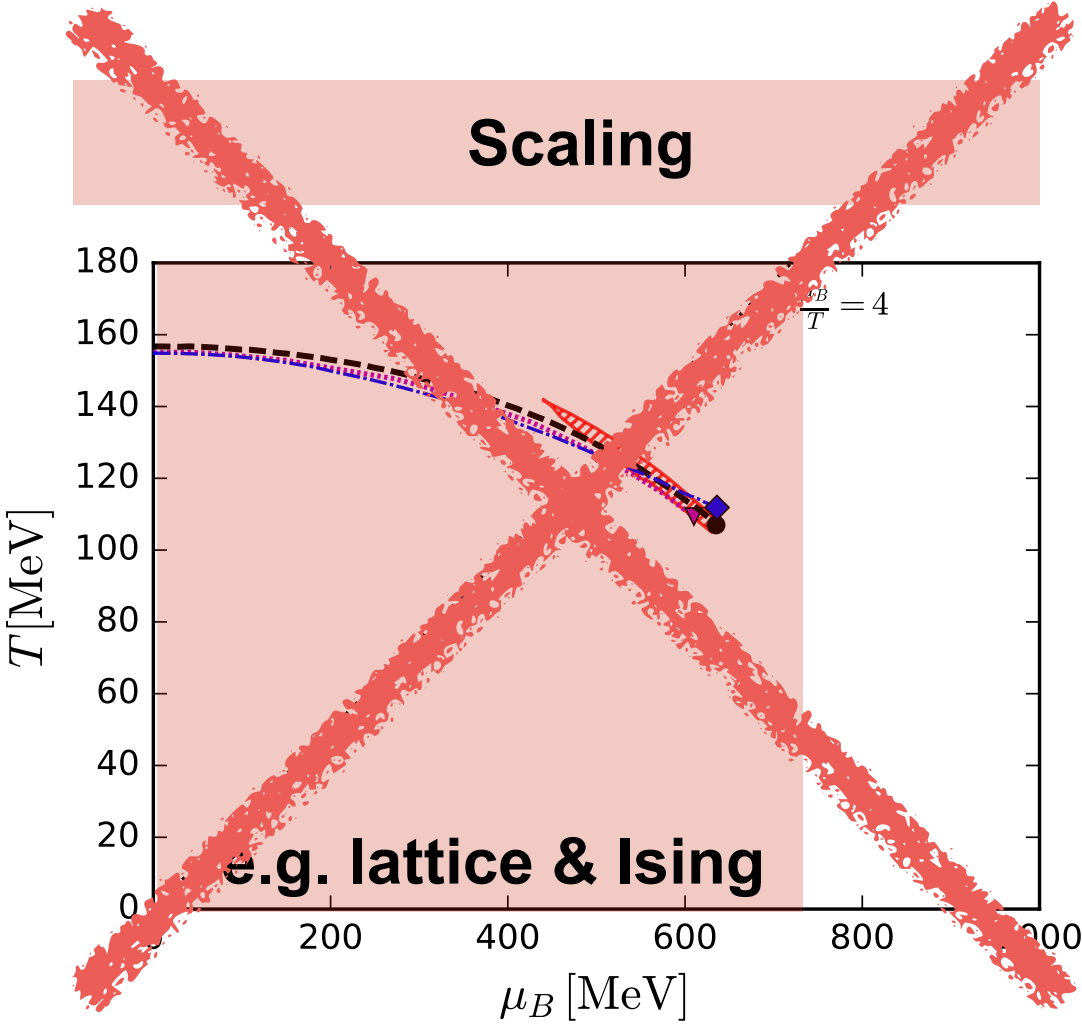


Moat, ...

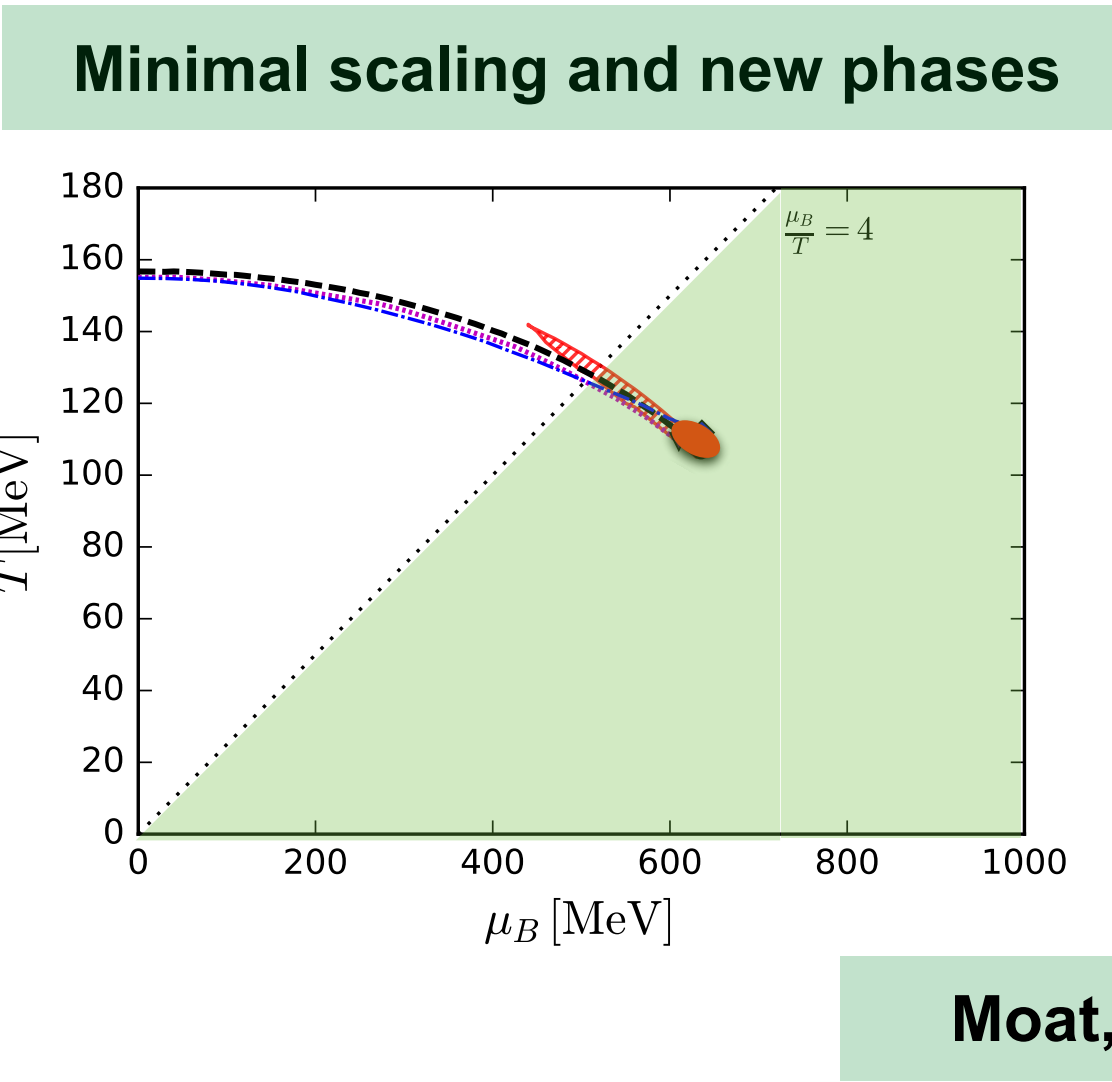
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Scenario II



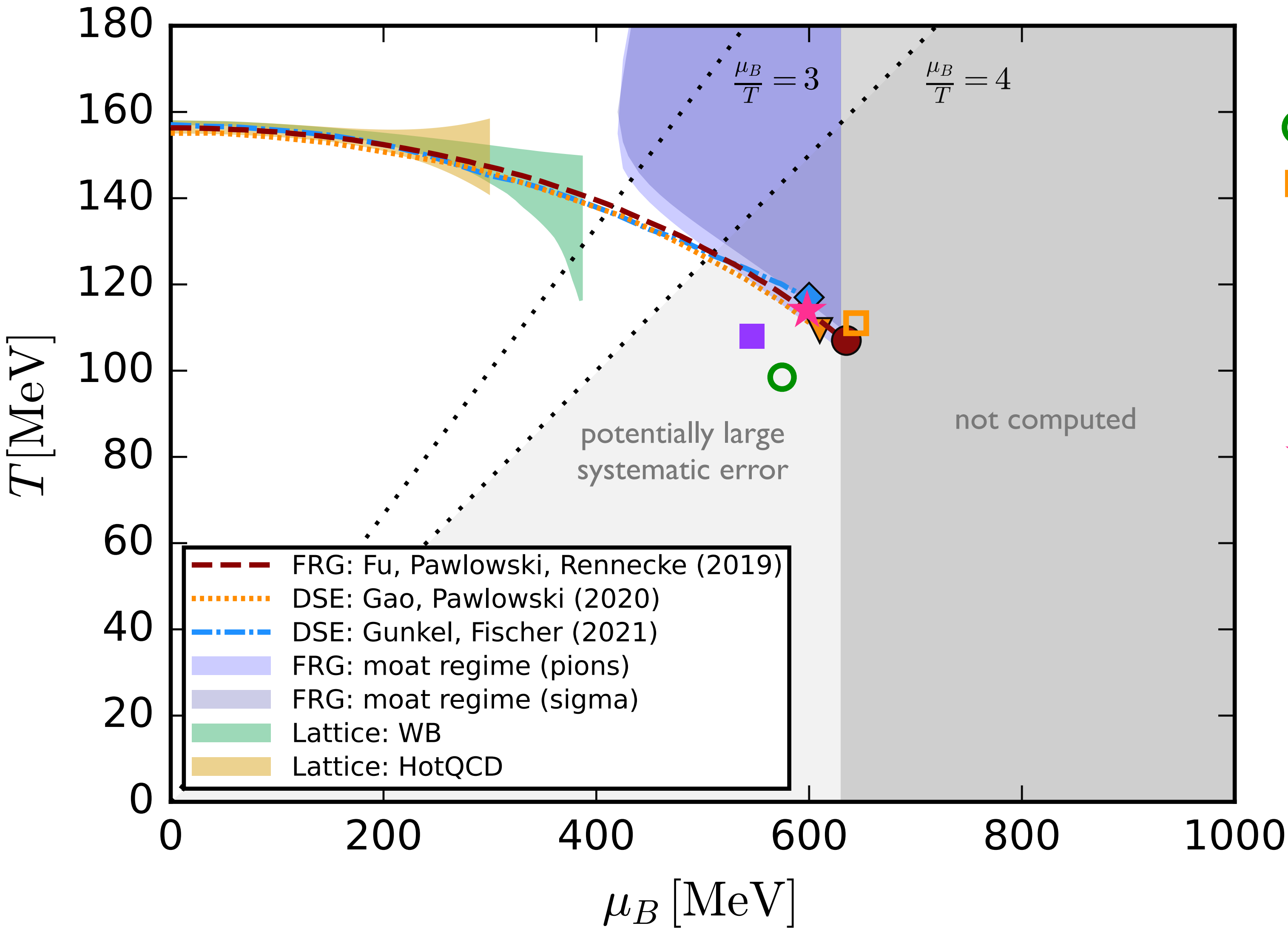
Supplementary Material:

Predictions, estimates & extrapolations and how to use them
Chiral dynamics & soft modes

QCD PHASE DIAGRAM & THE CEP

Modified from Fabian Rennecke's
plenary talk @ QM 2025

FRG & DSE results corroborated by subsequent "extrapolations" of lattice data



Fu, Luo, JMP, Rennecke, Yin, PRD 111, L031502

using Yang-Lee edge singularities:

- conformal Padé [Basar, 2312.06952]
- multi-point Padé [Clarke et al., 2405.10196]
 $N_\tau = 6, 8$ results + continuum estimate
[Schmidt, 2504.00629]

Alexander Adam's talk, Thu 9:40

using thermodynamics:

- constant entropy density [Shah et al. 2410.16206]
Hitansh Shah's talk, Wed 9:40

model-based extrapolation (tiny selection):

- holography [Hippert et al., 2309.00579]
(agrees with [Cai et al., 2201.02004])

CEP location well constrained
by now. And it's in **FAIR** range!

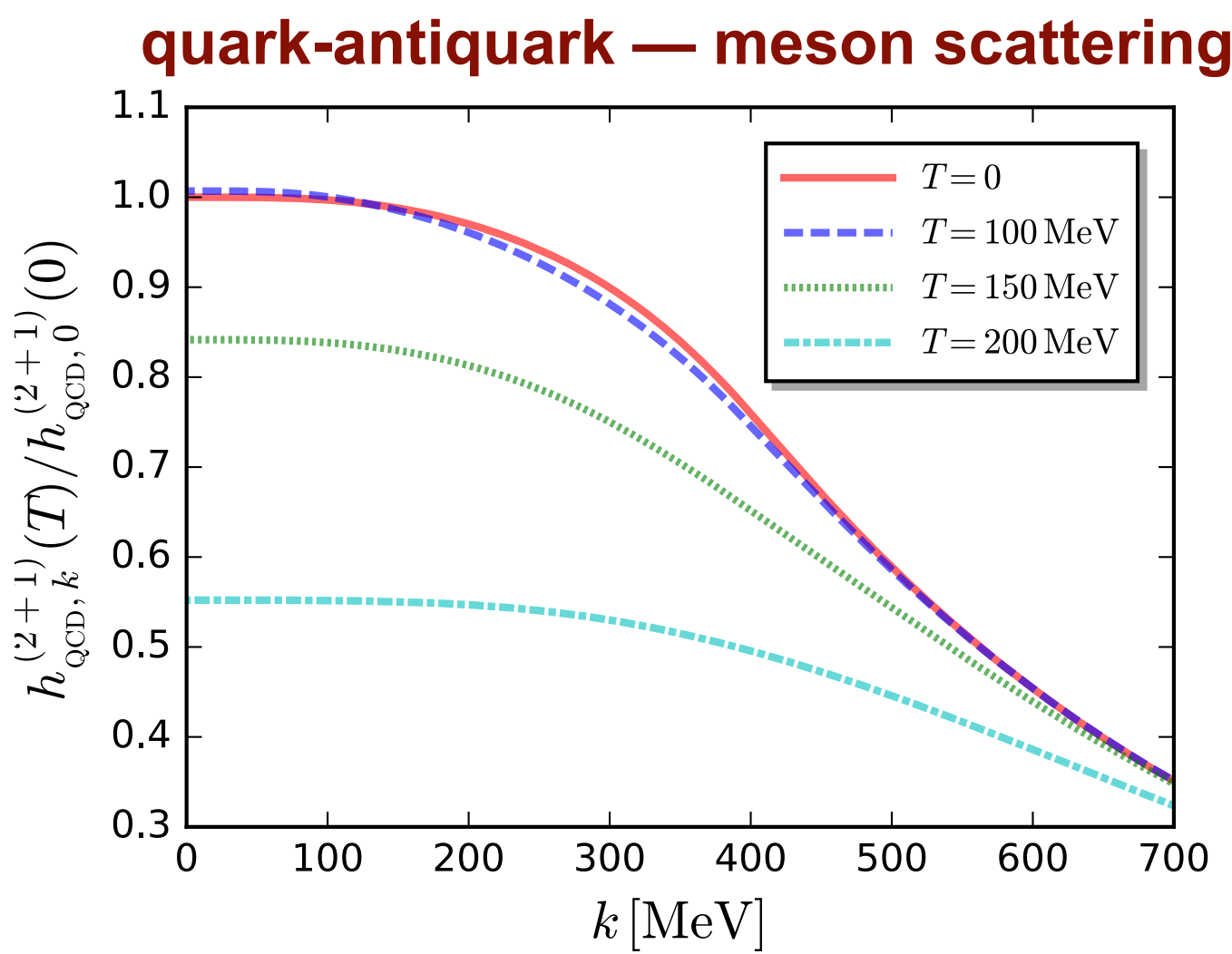
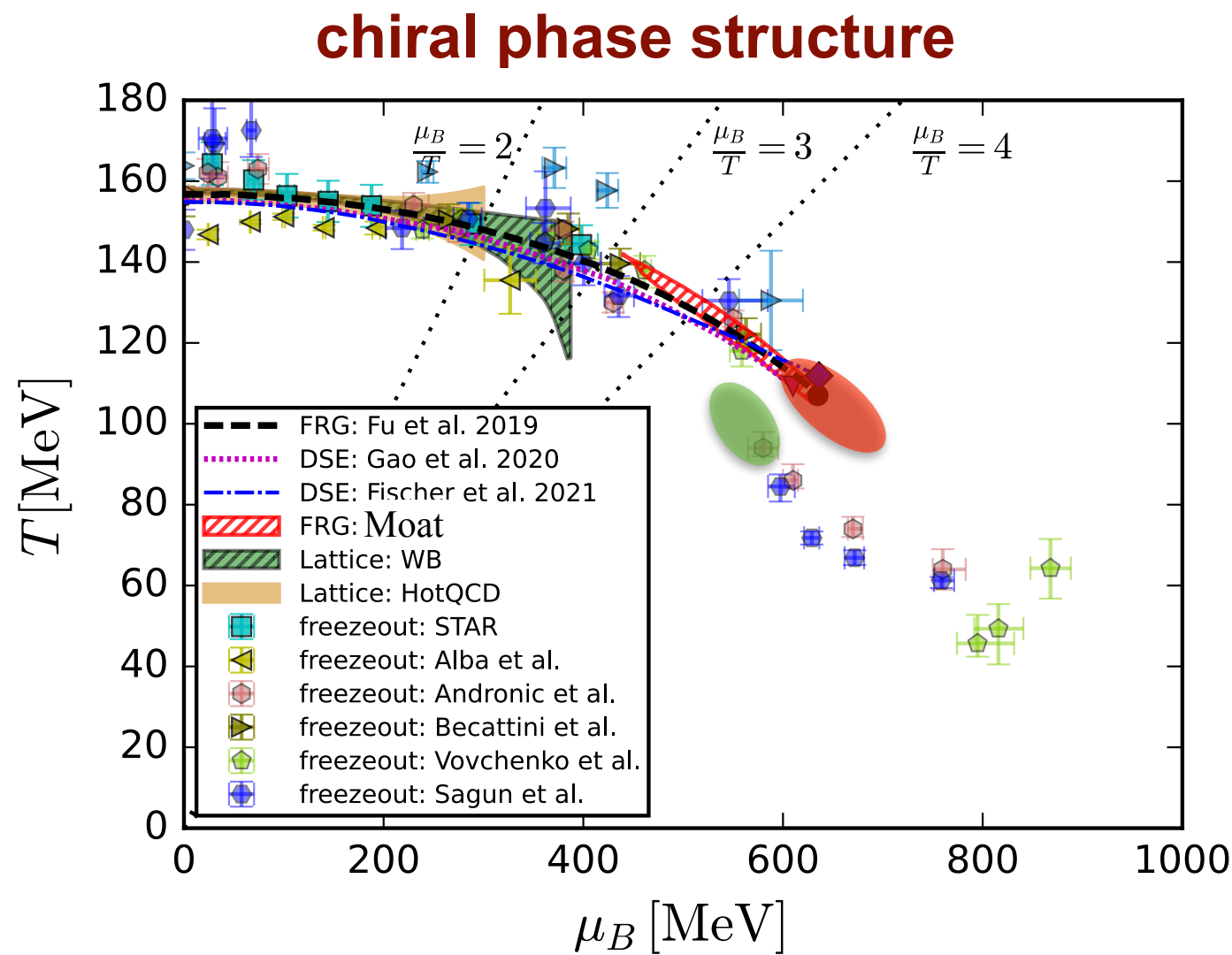
$$\sqrt{s_{NN}} \approx 3.6 - 4.1 \text{ GeV}$$

QCD-assisted low energy effective theory

Direct functional QCD input

Fu, JMP, Rennecke, PRD 101 (2020) 054032

+

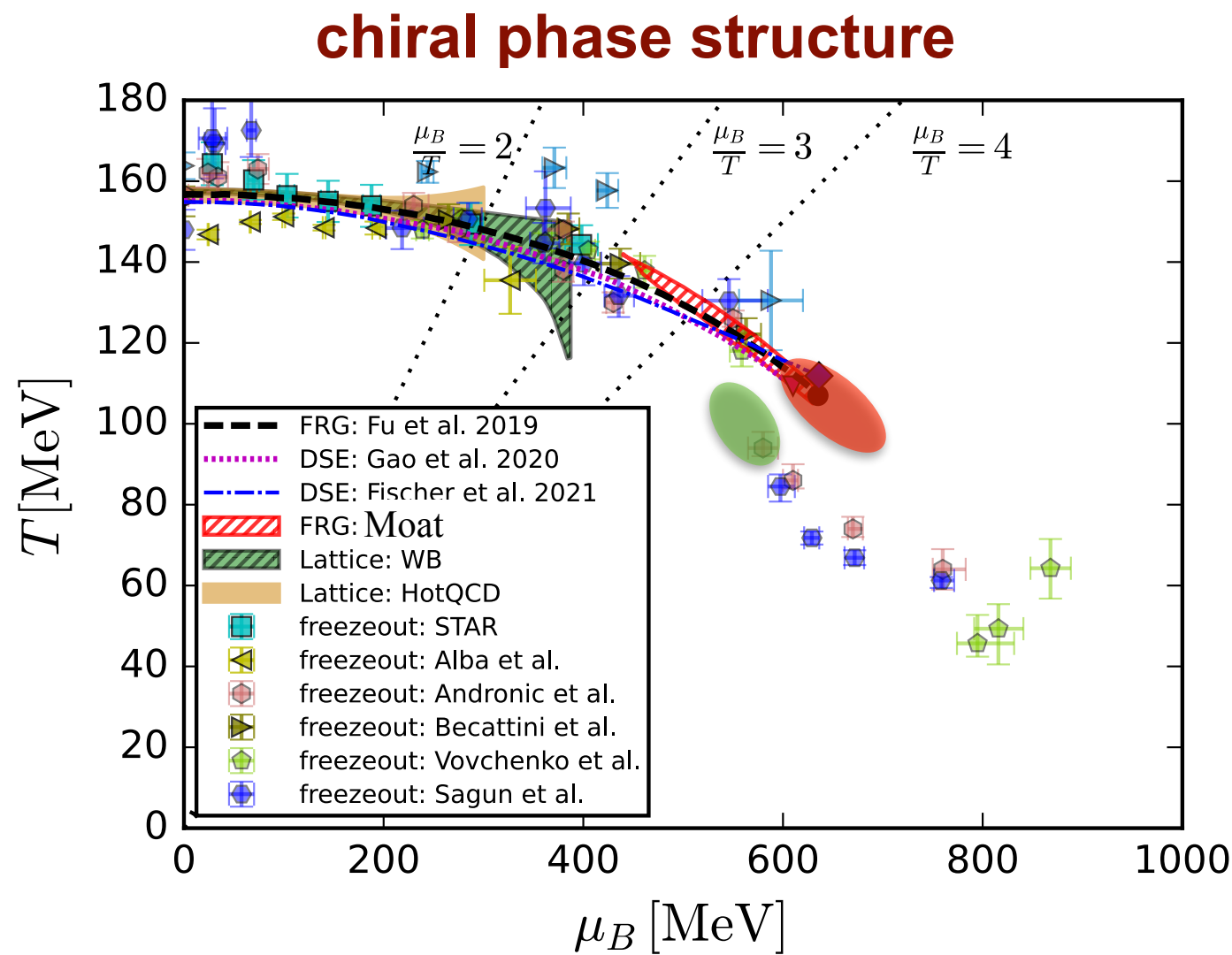


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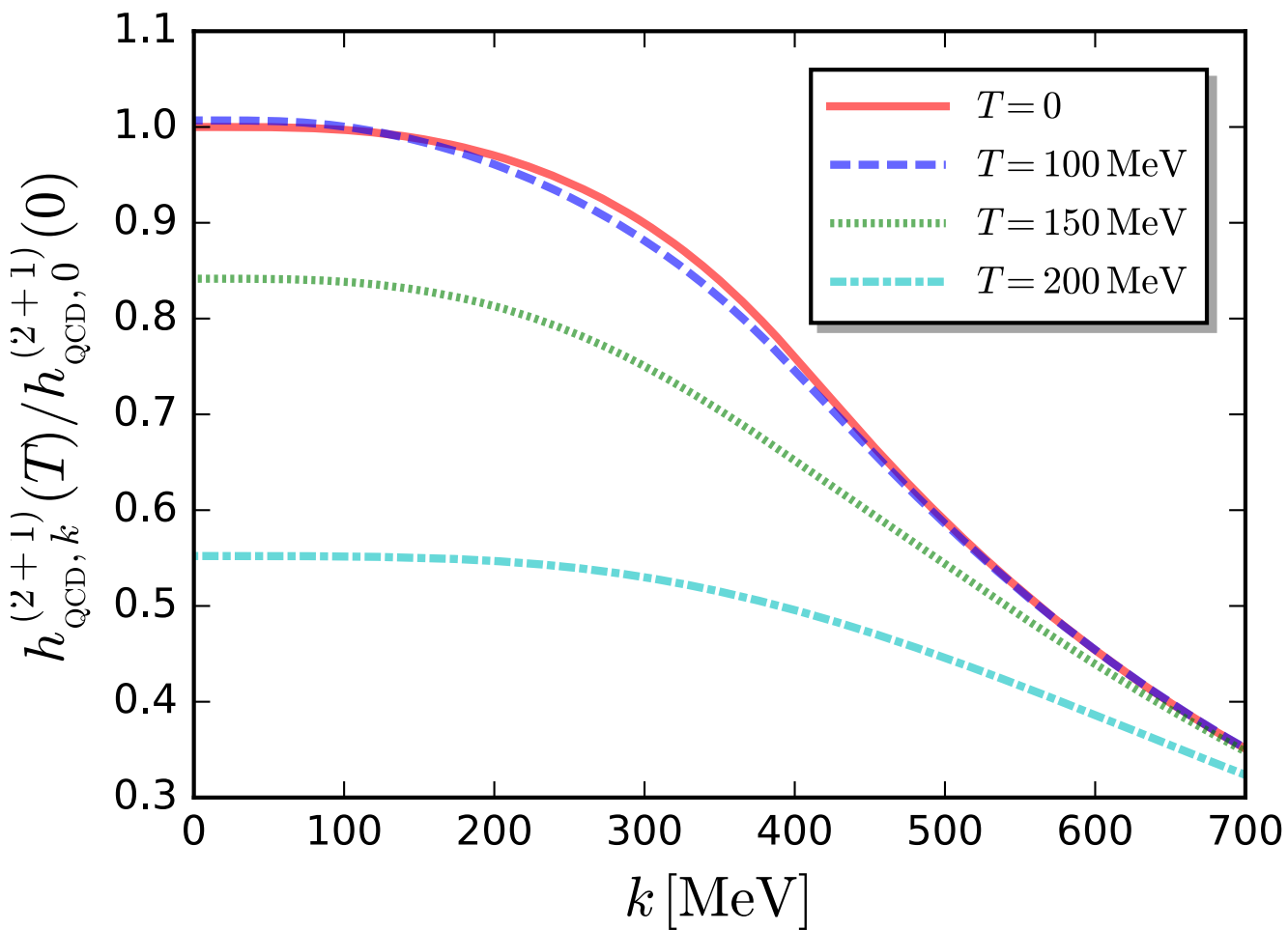
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quark-antiquark — meson scattering

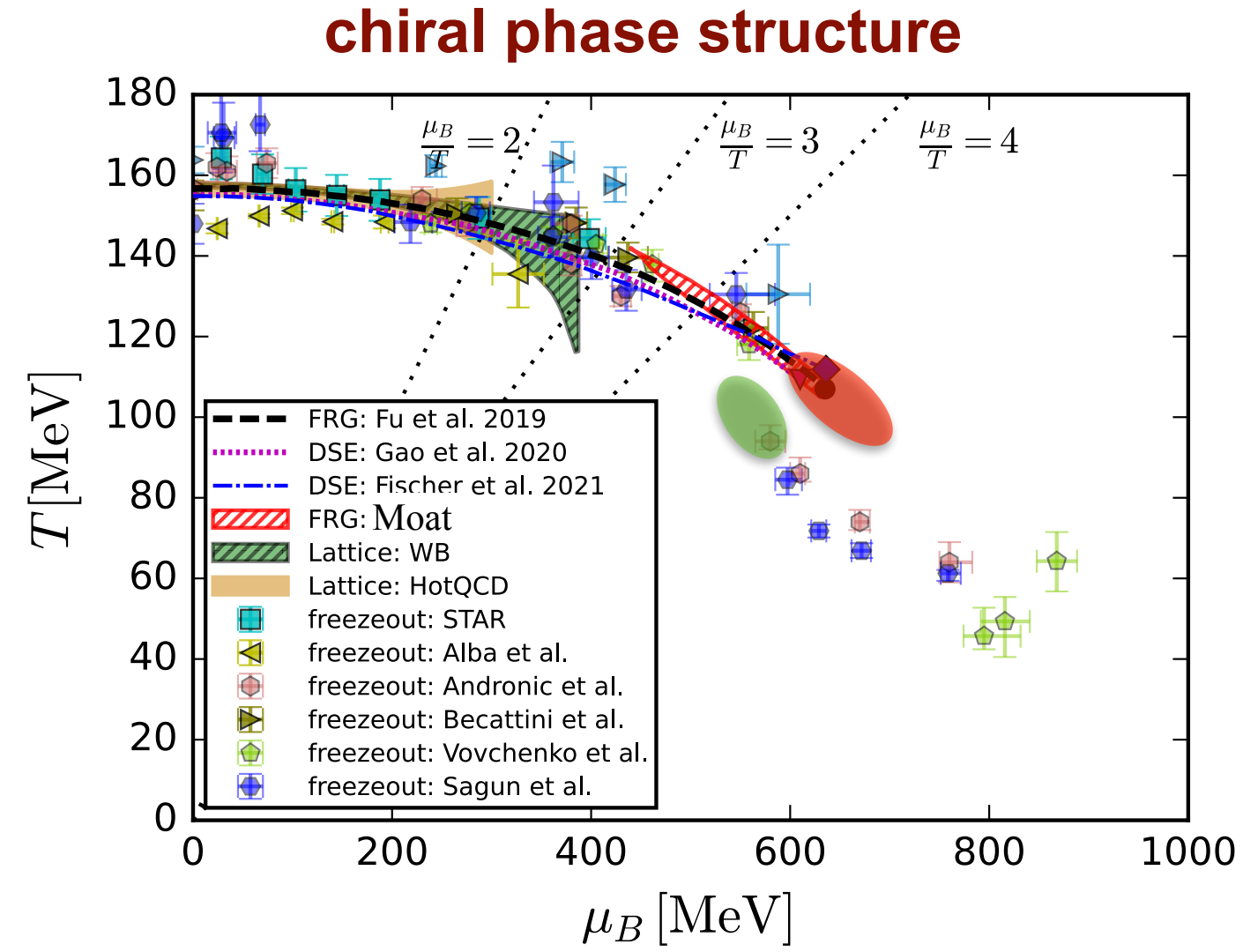


low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

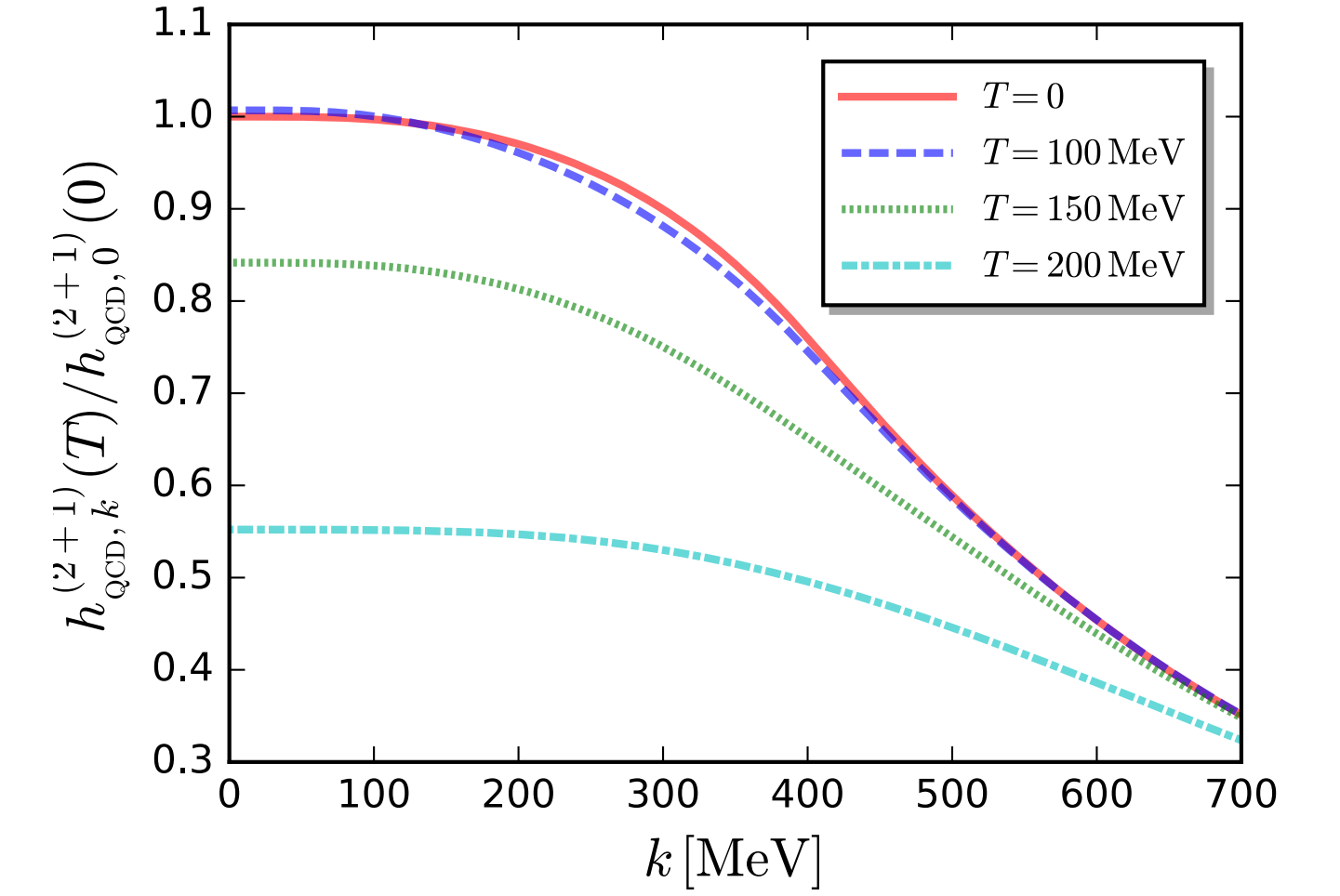
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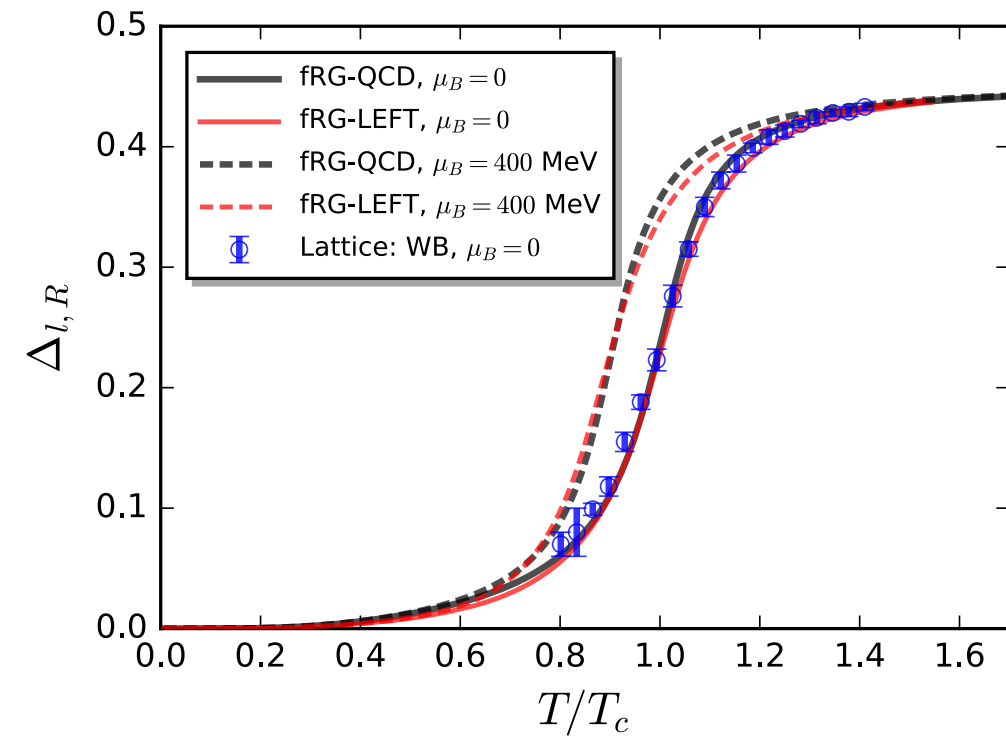


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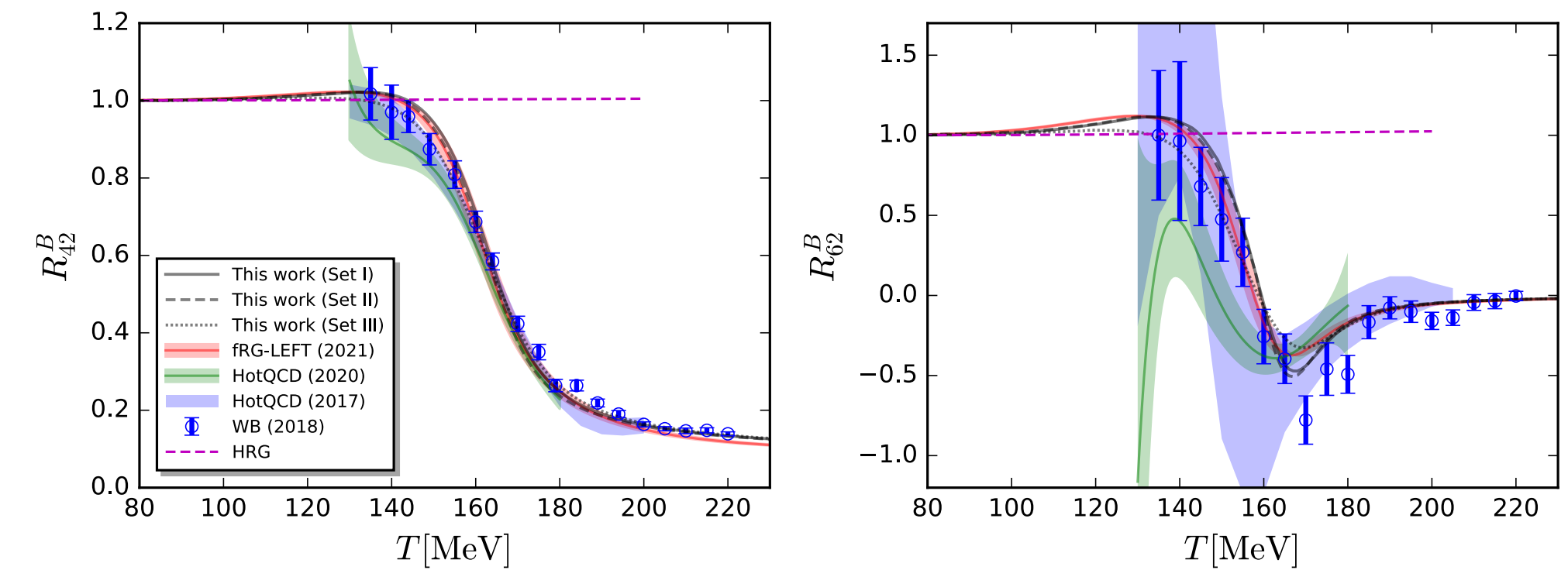
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renormalised chiral condensate



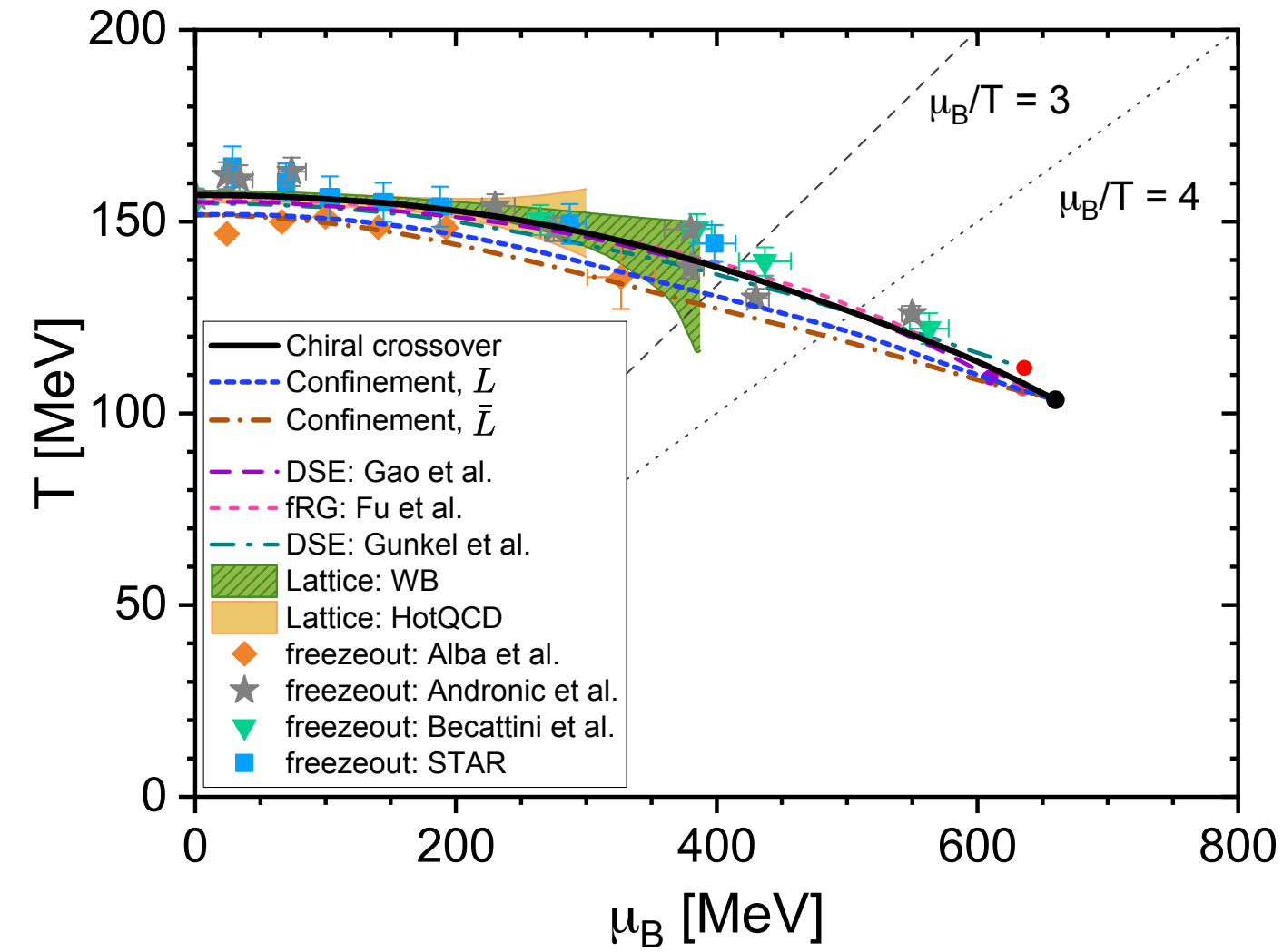
Benchmarks with lattice and fQCD
at
vanishing density and fQCD at finite density

baryon number fluctuations



Functional QCD

chiral & 'confinement' phase structure



$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \left(\text{Loop 1} - \text{Loop 2} - \text{Loop 3} - \frac{1}{6} \text{Loop 4} + \text{Loop 5} \right)$$

The diagram illustrates the calculation of the derivative of the difference between the grand potential Γ and the entropy S with respect to the baryon chemical potential A_0 . It shows five Feynman-like diagrams representing different contributions to the baryon number fluctuations. The first three diagrams are subtracted, and the fourth is subtracted with a weight of 1/6, while the fifth is added.

DSE

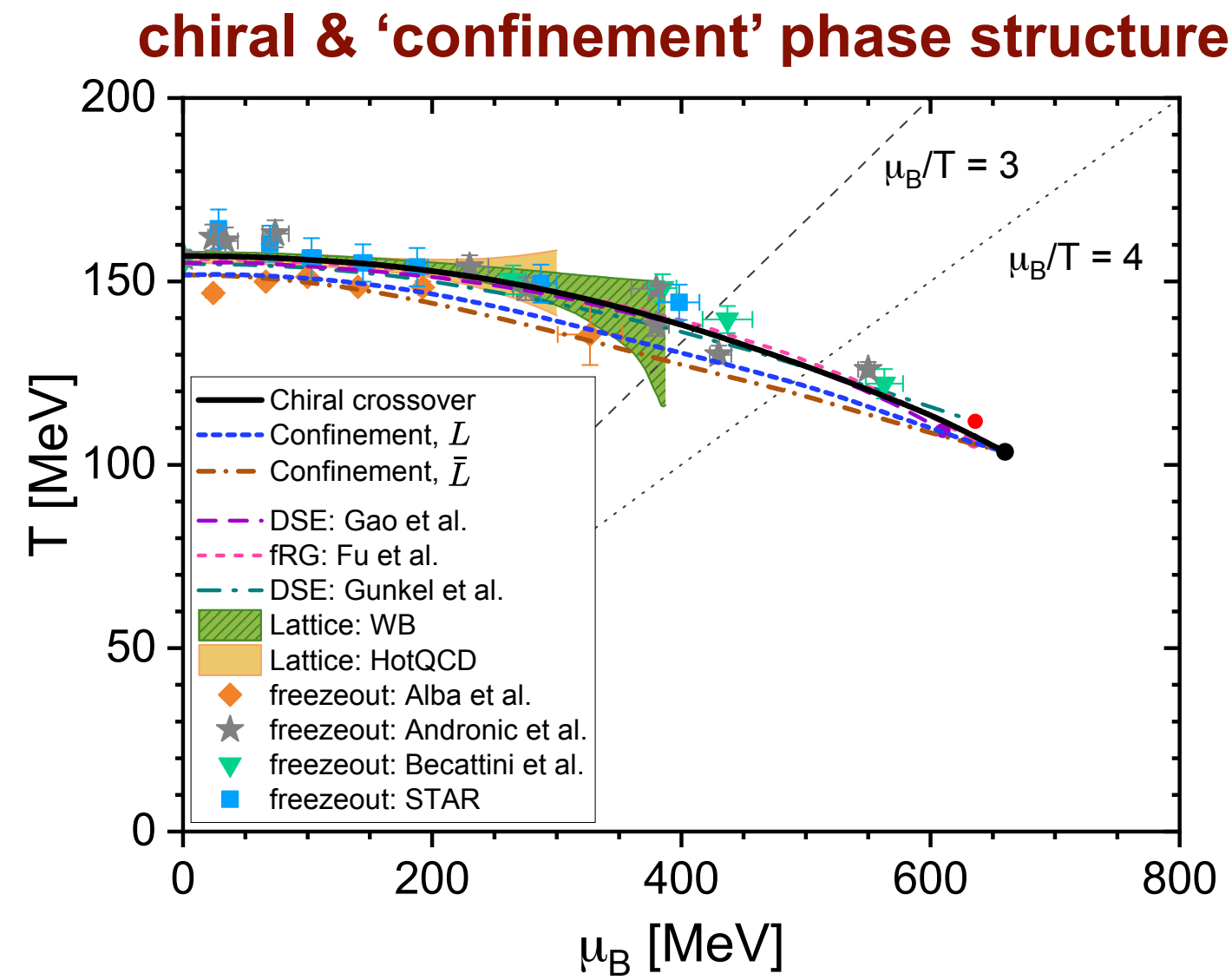
L: 'cousin' of the Polyakov loop

Braun, Gies, JMP, PLB 684 (2010) 262
 Fister, JMP, PRD 88 (2013) 045010
 Herbst, Lücker, JMP, arXiv:1510.03830

Explorative DSE studies EoS & fluctuations of the net-baryon charge

Isserstedt, Buballa, Fischer, Gunkel, PRD 100 (2019) 074011
 Isserstedt, Fischer, Steinert, PRD 103 (2021) 054012

Functional QCD



On-shell

$$\frac{\delta(\Gamma - S)}{\delta A_0} = \left[\frac{1}{2} \text{ (loop 1)} - \text{ (loop 2)} - \text{ (loop 3)} - \frac{1}{6} \text{ (loop 4)} + \text{ (loop 5)} \right]_{L, \bar{L}} = 0$$

DSE

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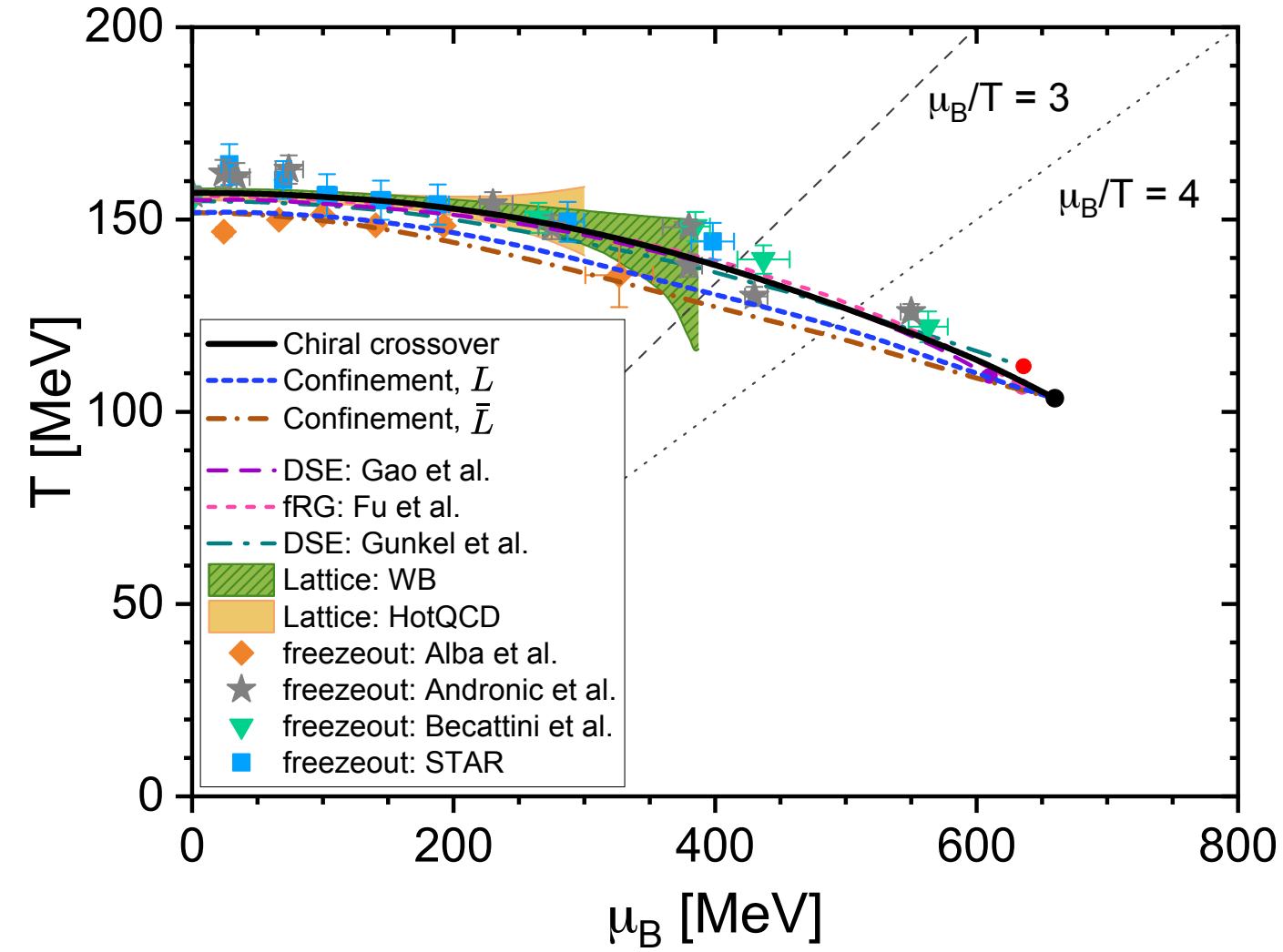
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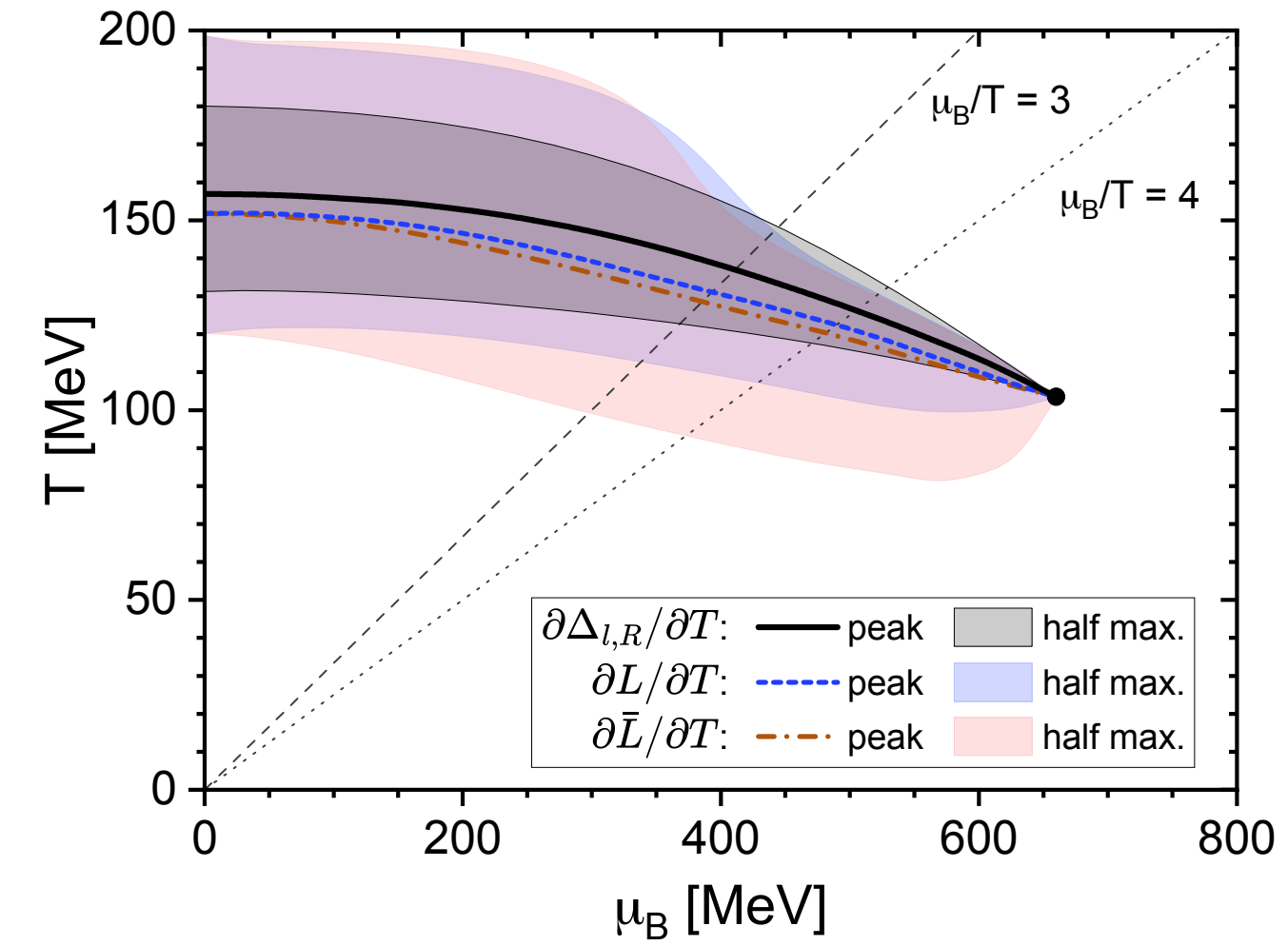
Off-shell $L = \bar{L} = 1$

Functional QCD

chiral & 'confinement' phase structure

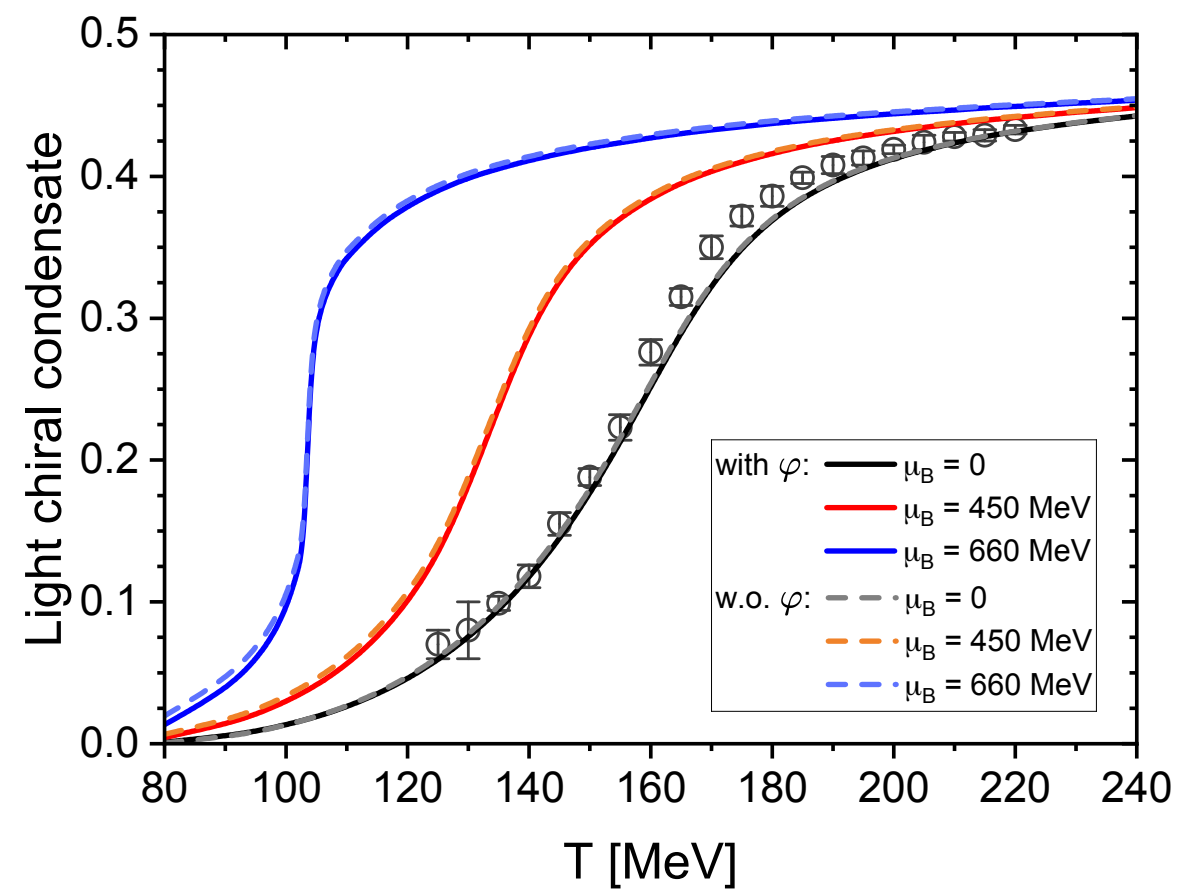


Crossovers with width



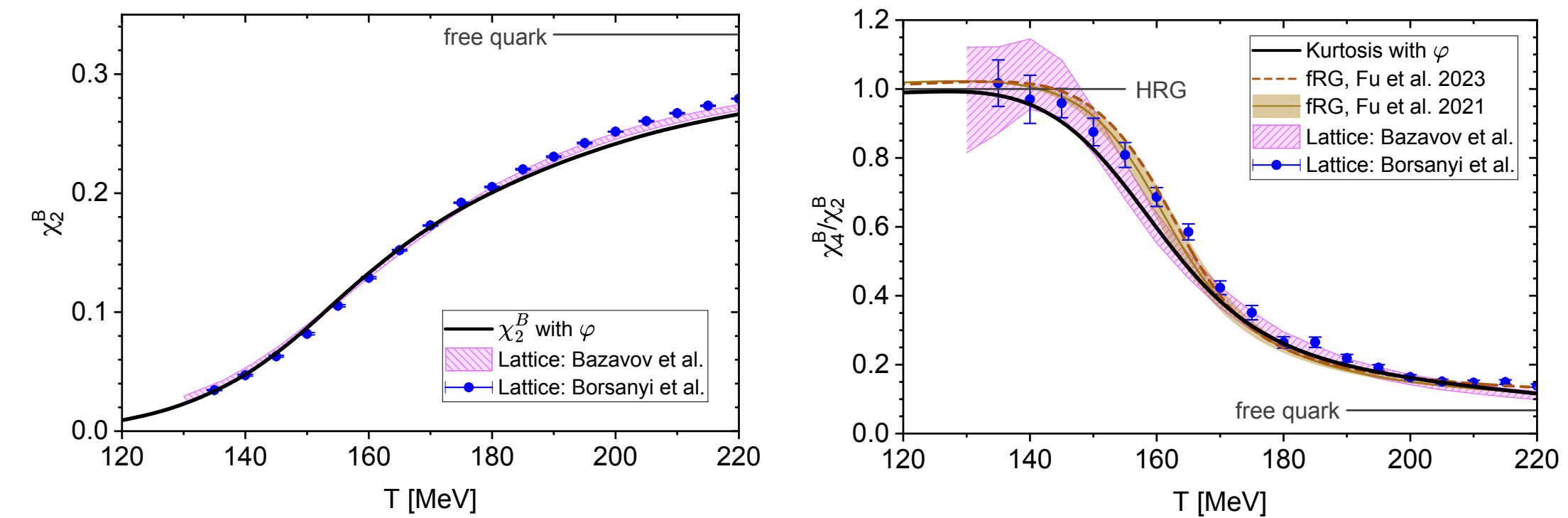
DSE

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Benchmarks with lattice and fQCD
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baryon number fluctuations

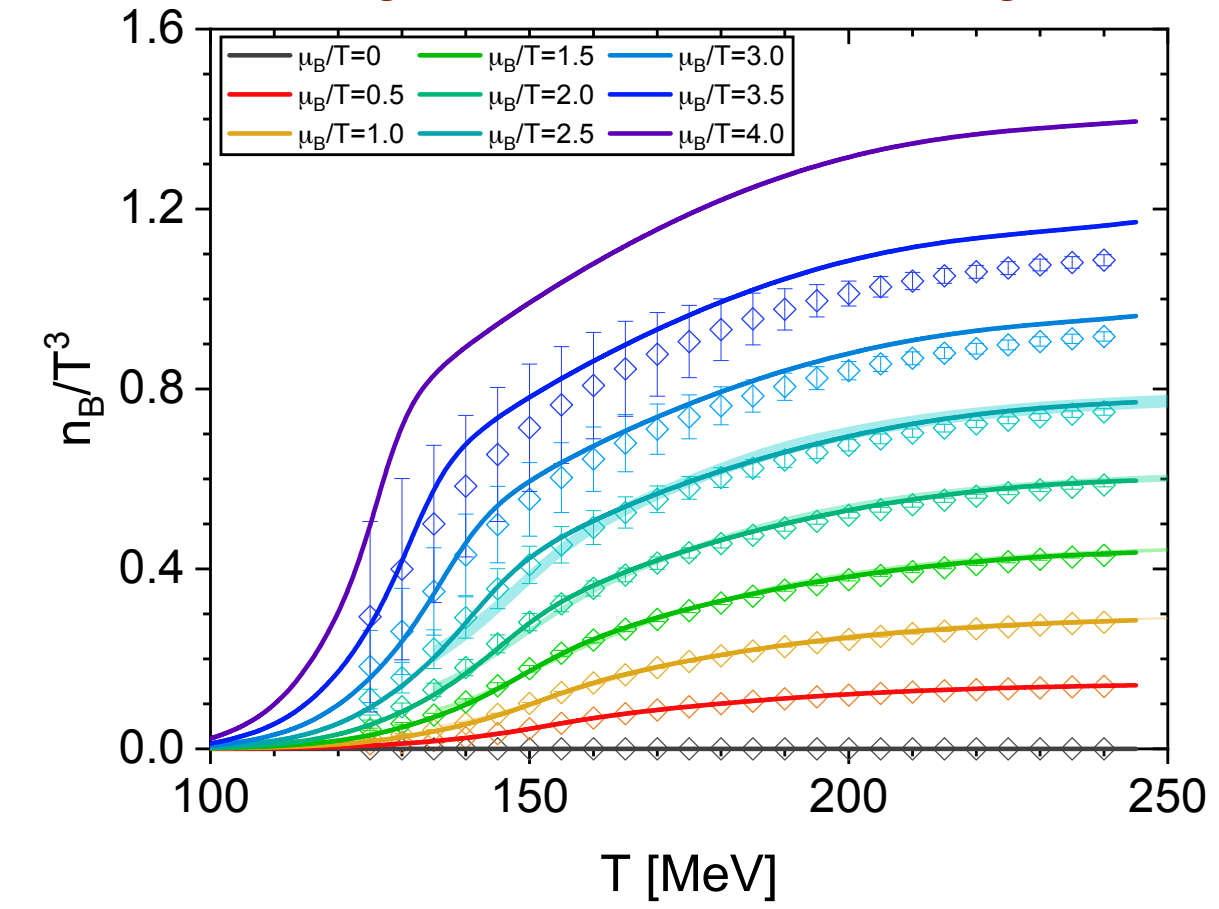


Functional QCD

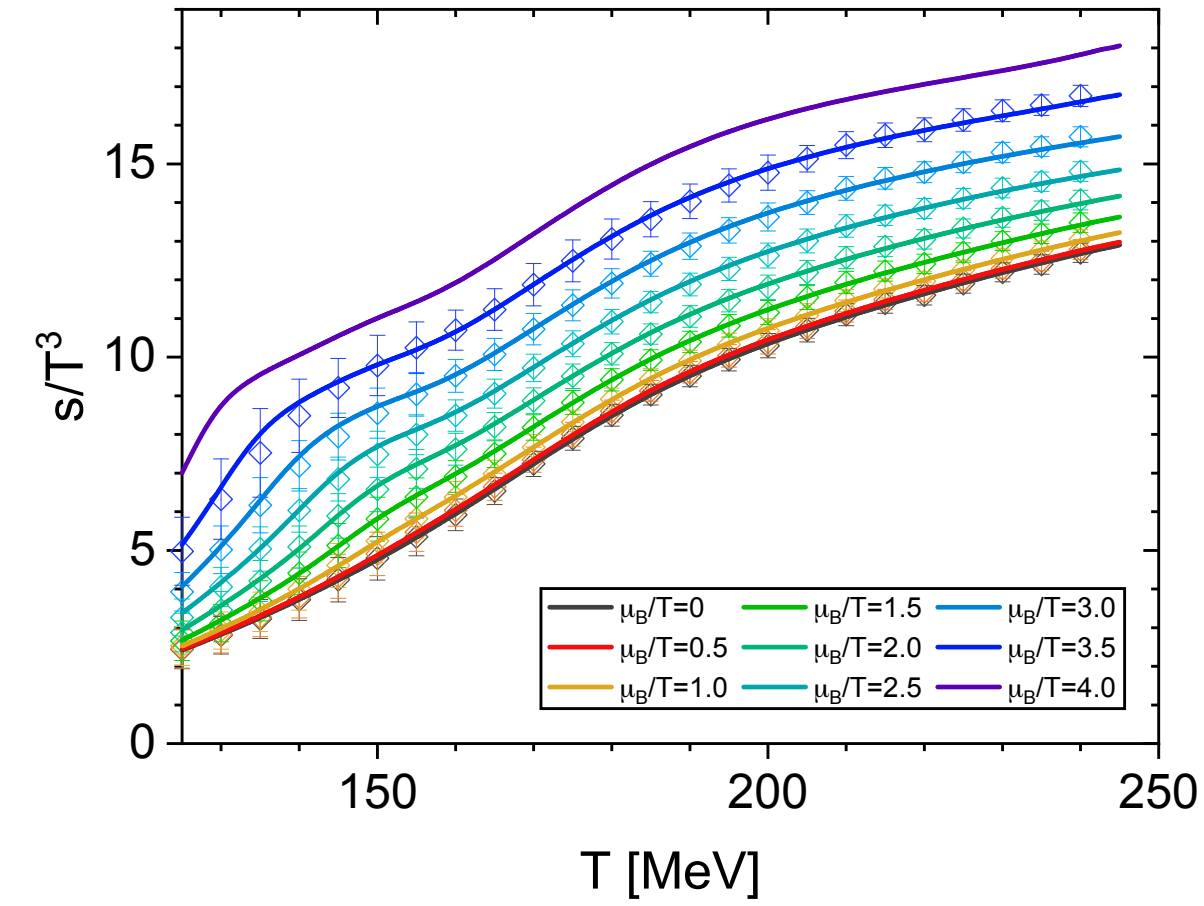
Observables

Thermodynamics

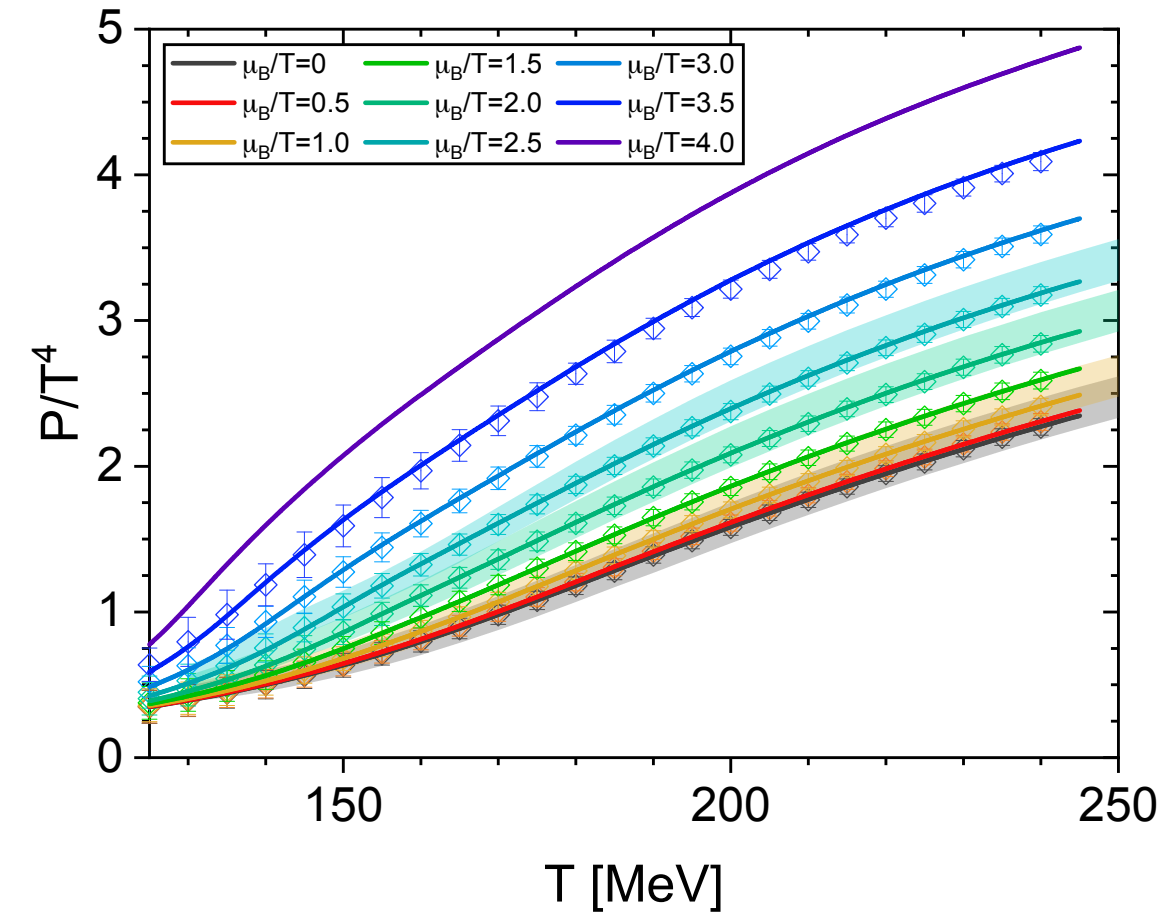
baryon number density



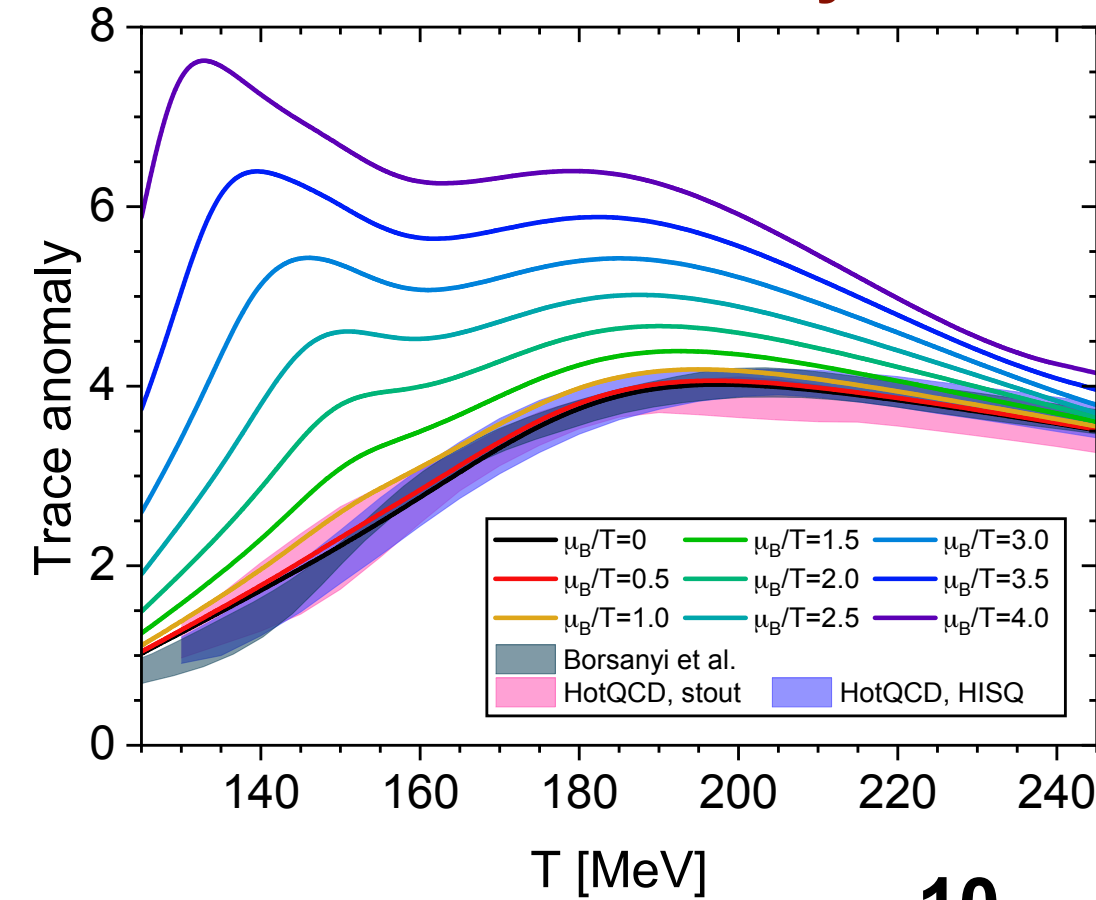
entropy



Pressure



trace anomaly

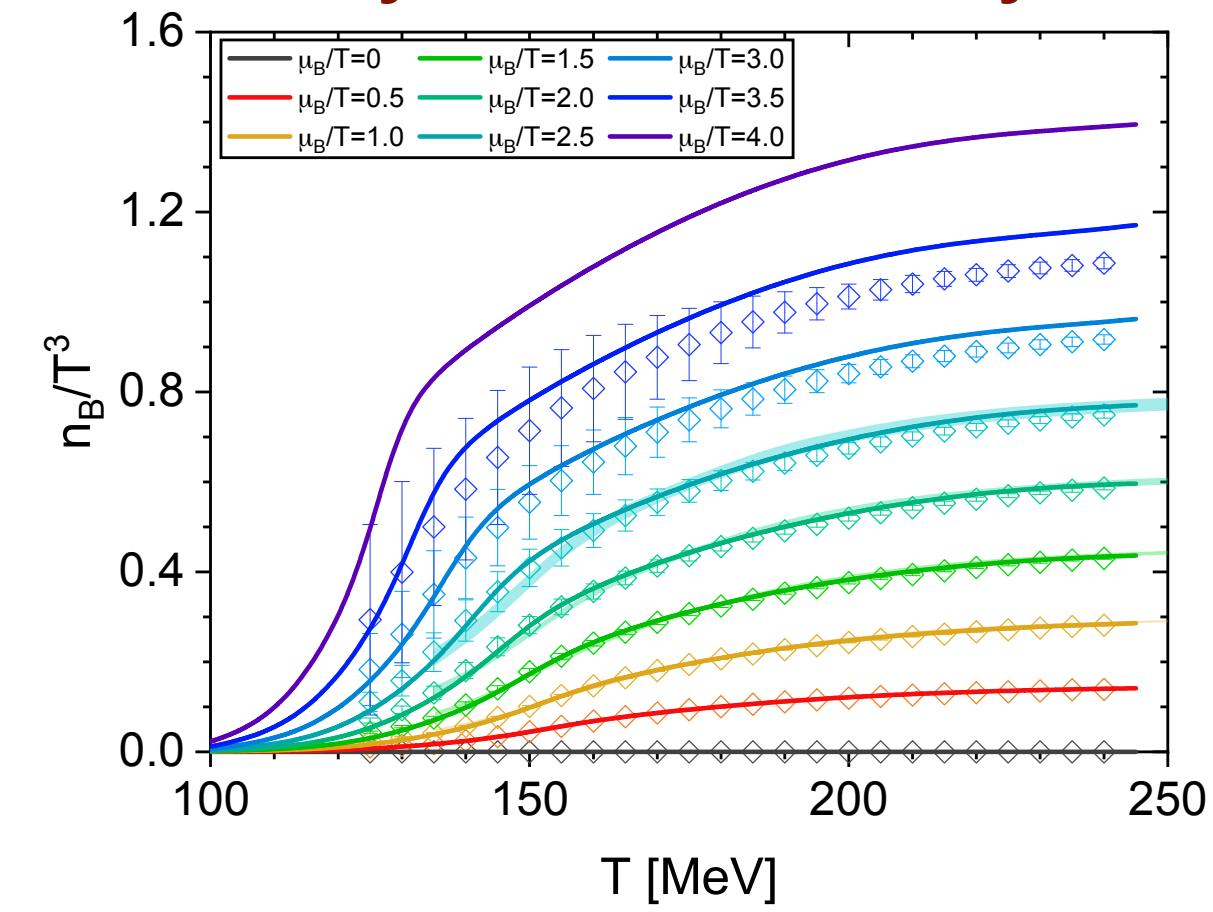


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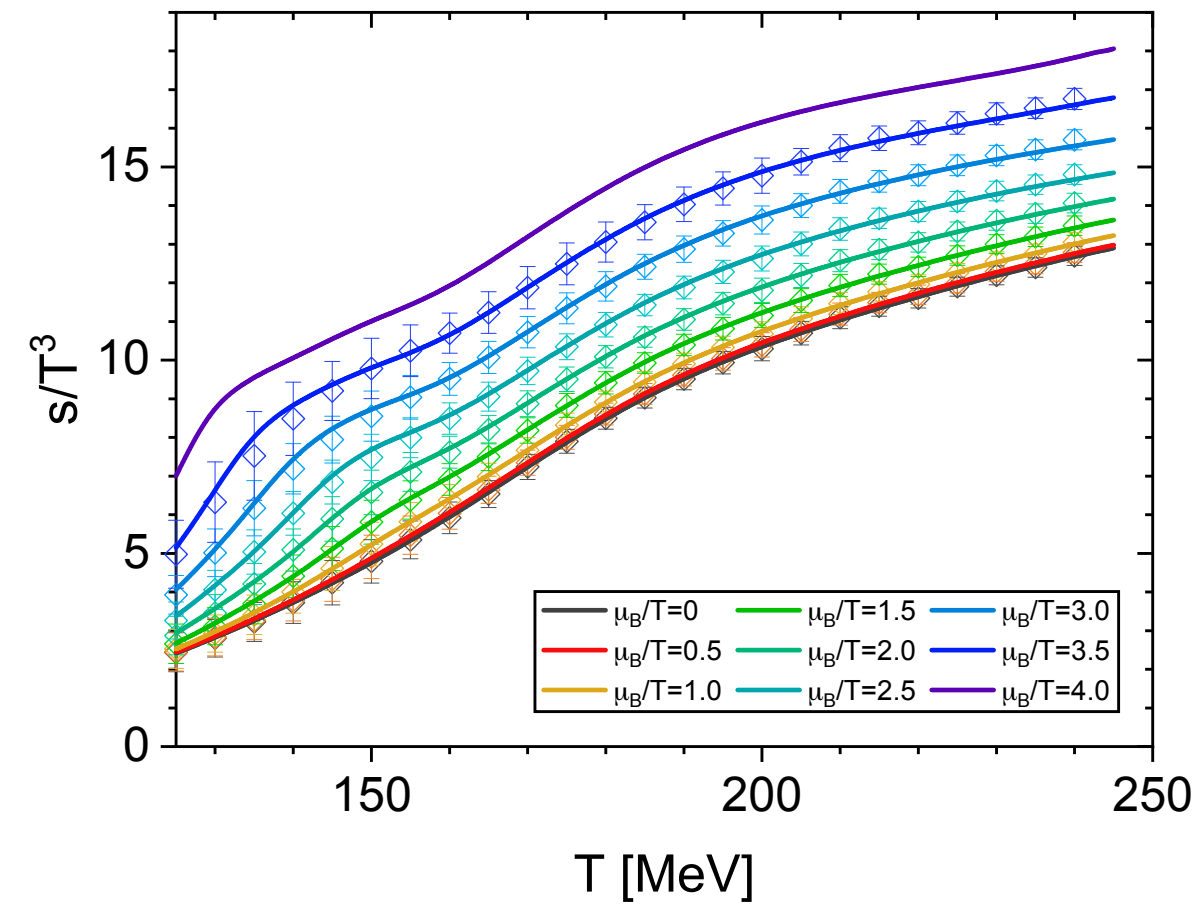
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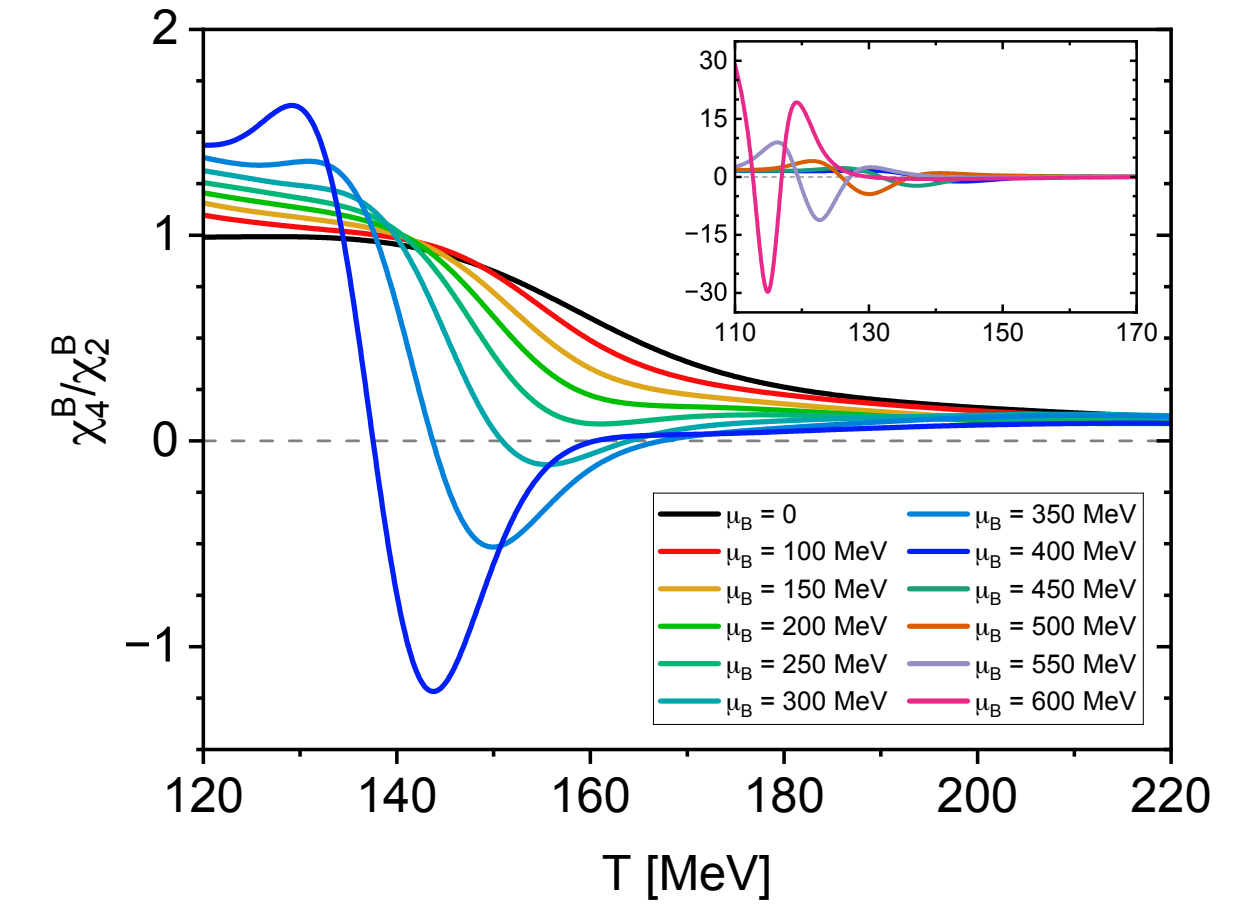


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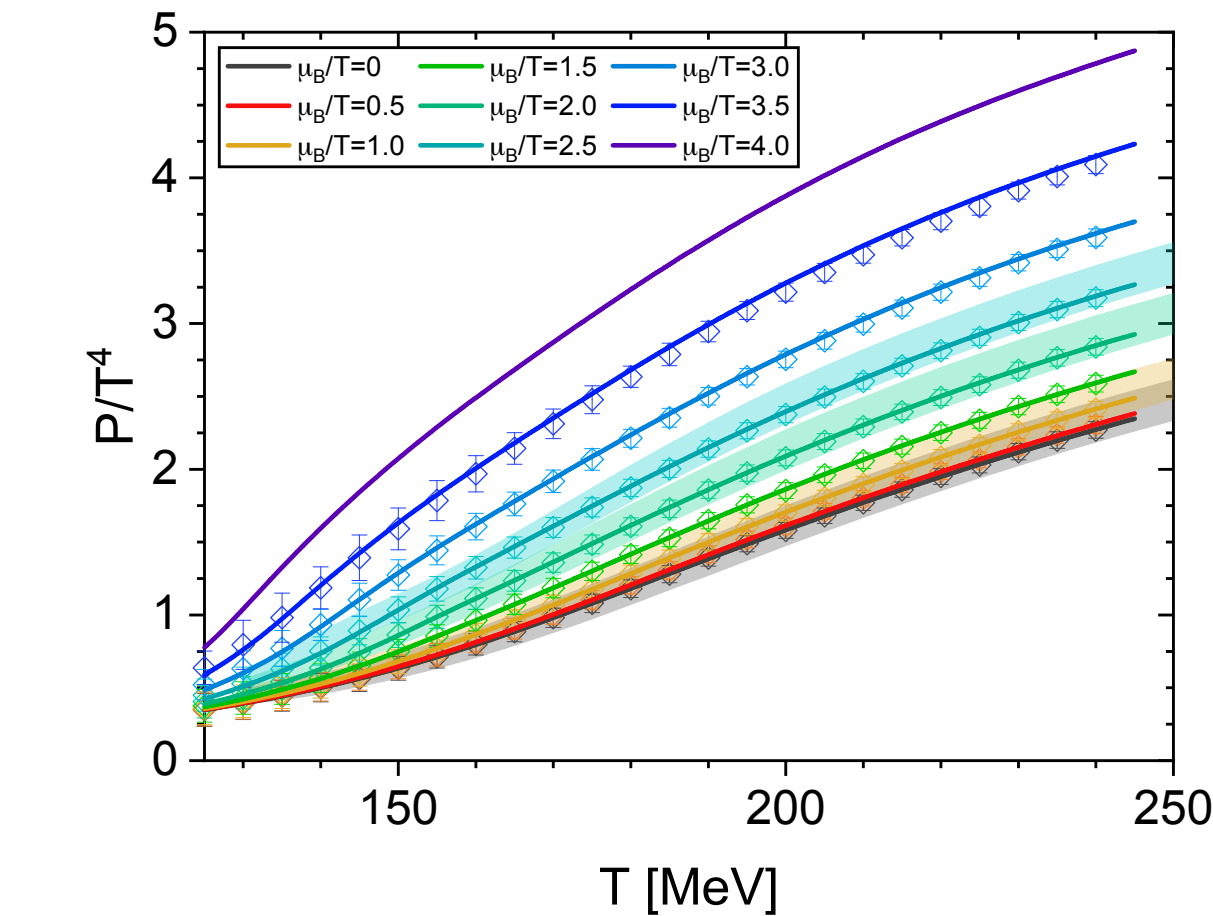


Fluctuations of net-baryon charge

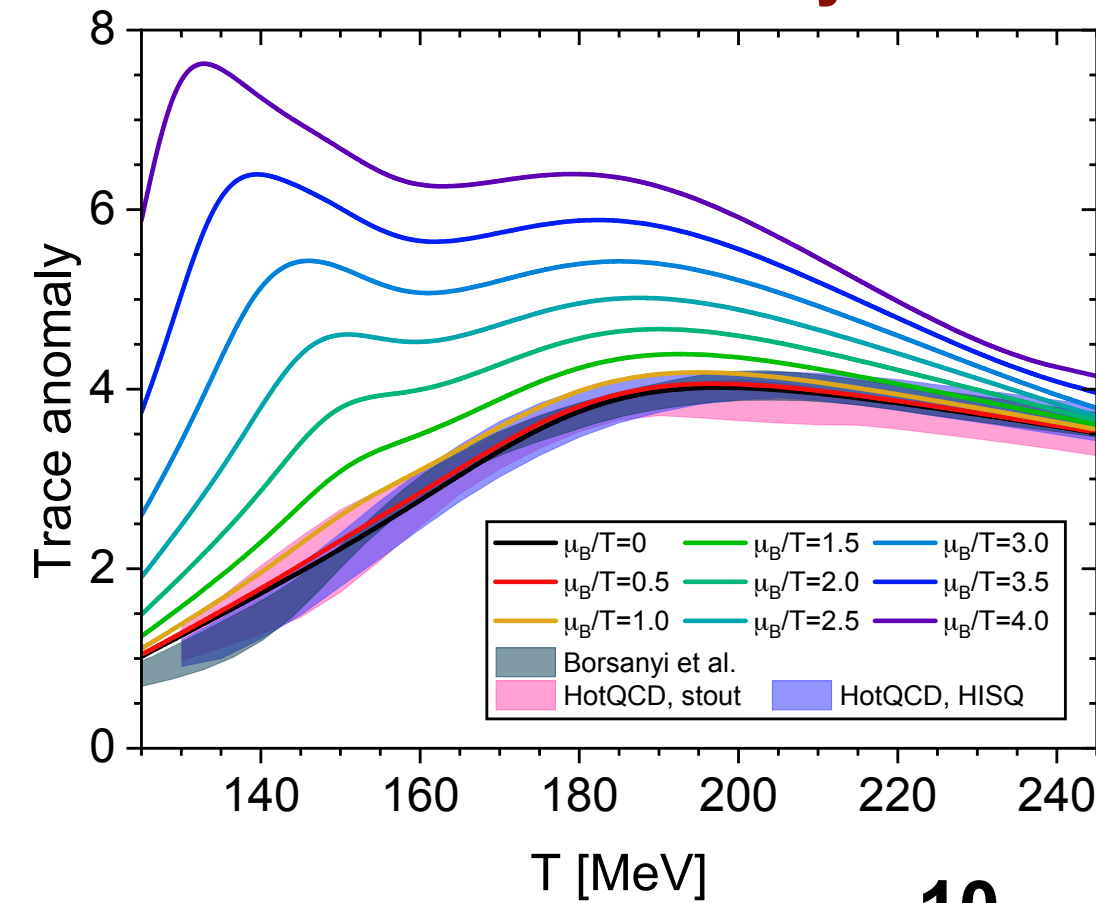
kurtosis



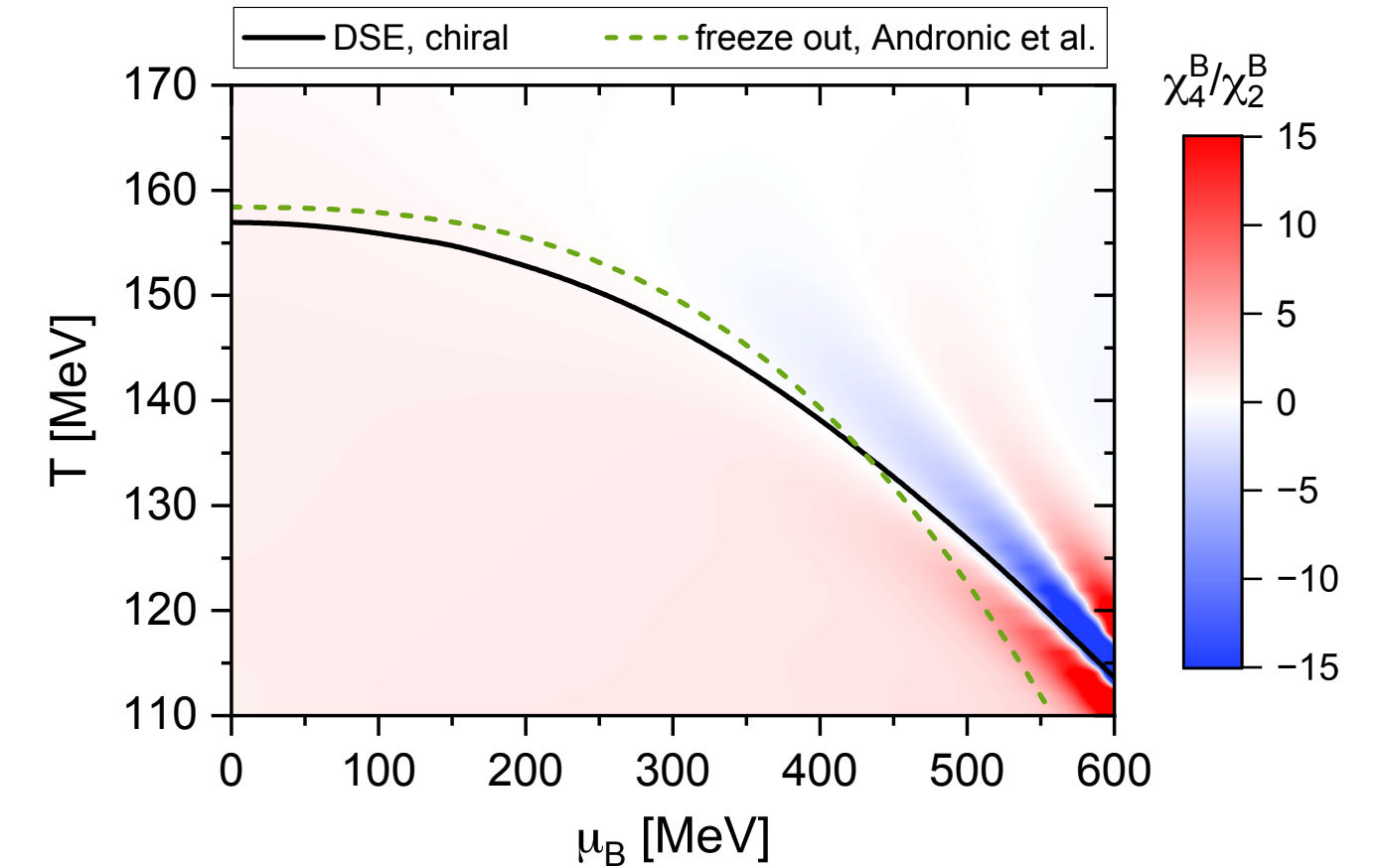
Pressure



trace anomaly



heat map kurtosis

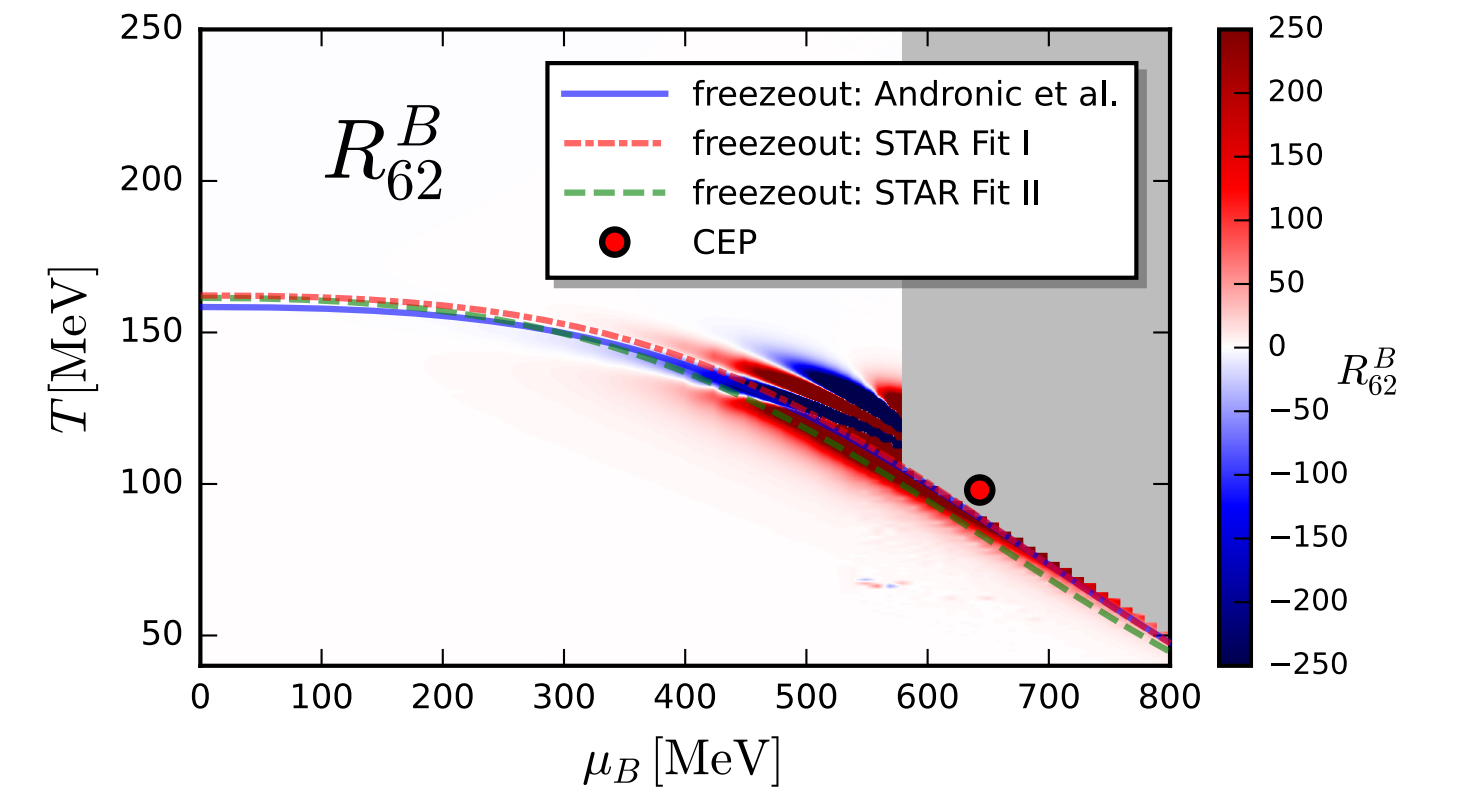
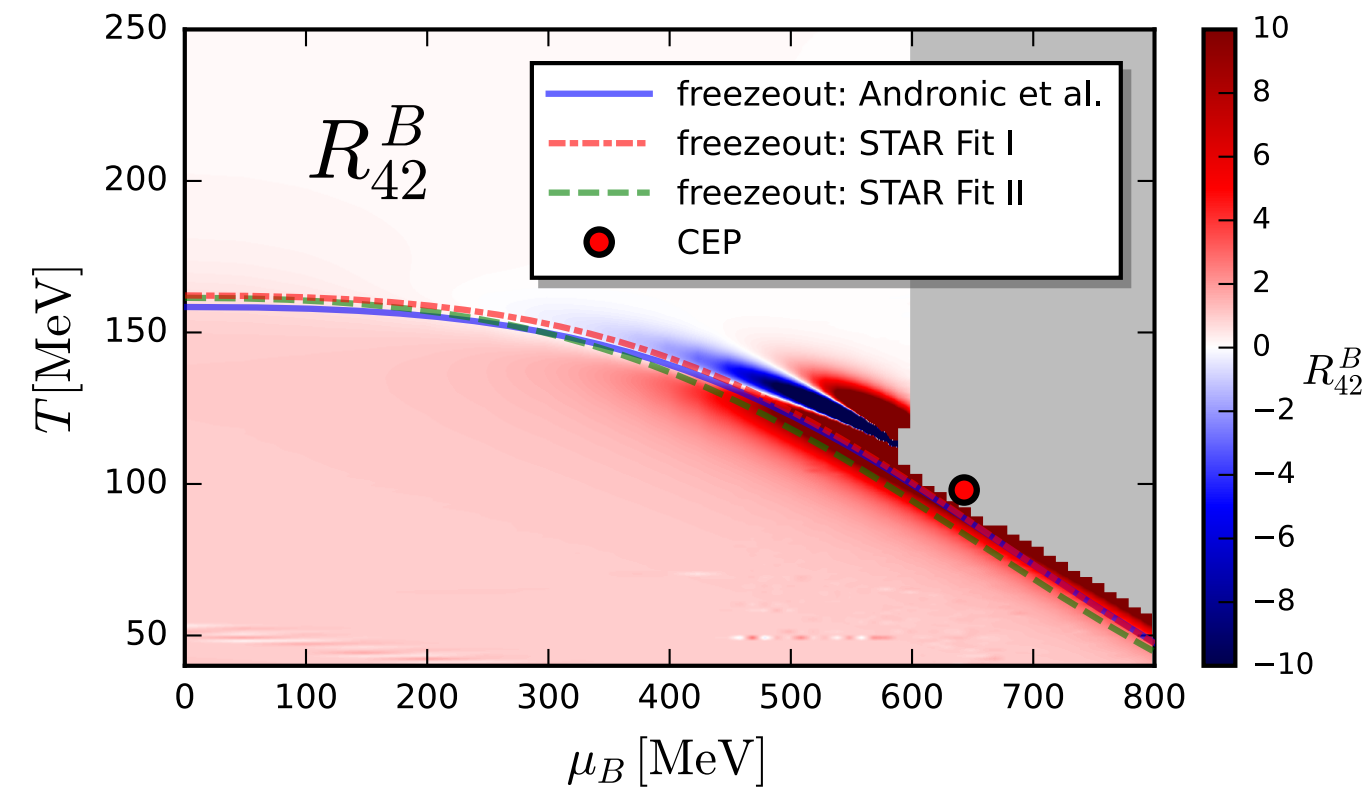


Ripples of the critical point

baryon number fluctuations in the phase structure

$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643) \text{ MeV}$$

QCD-assisted LEFT

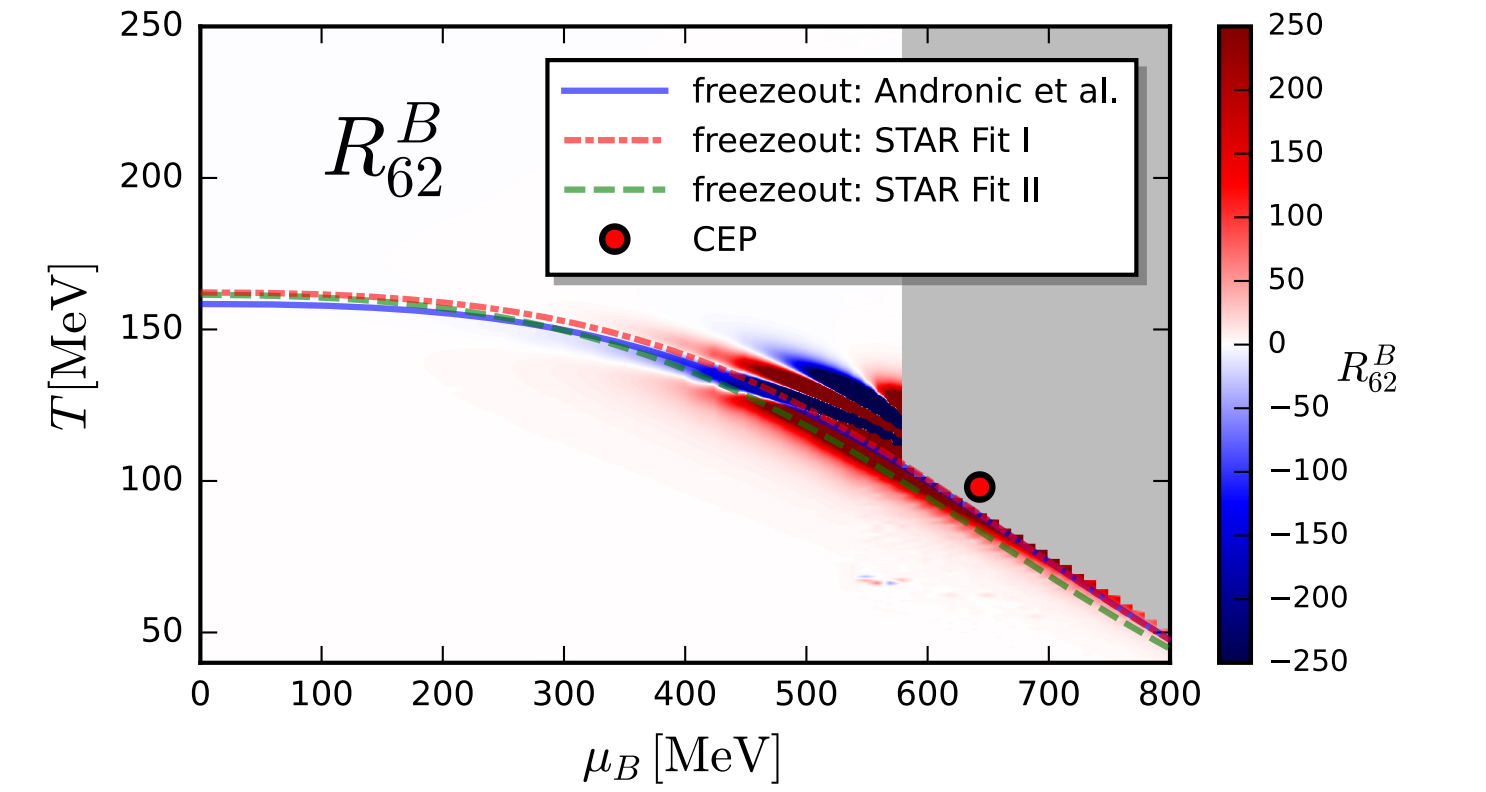
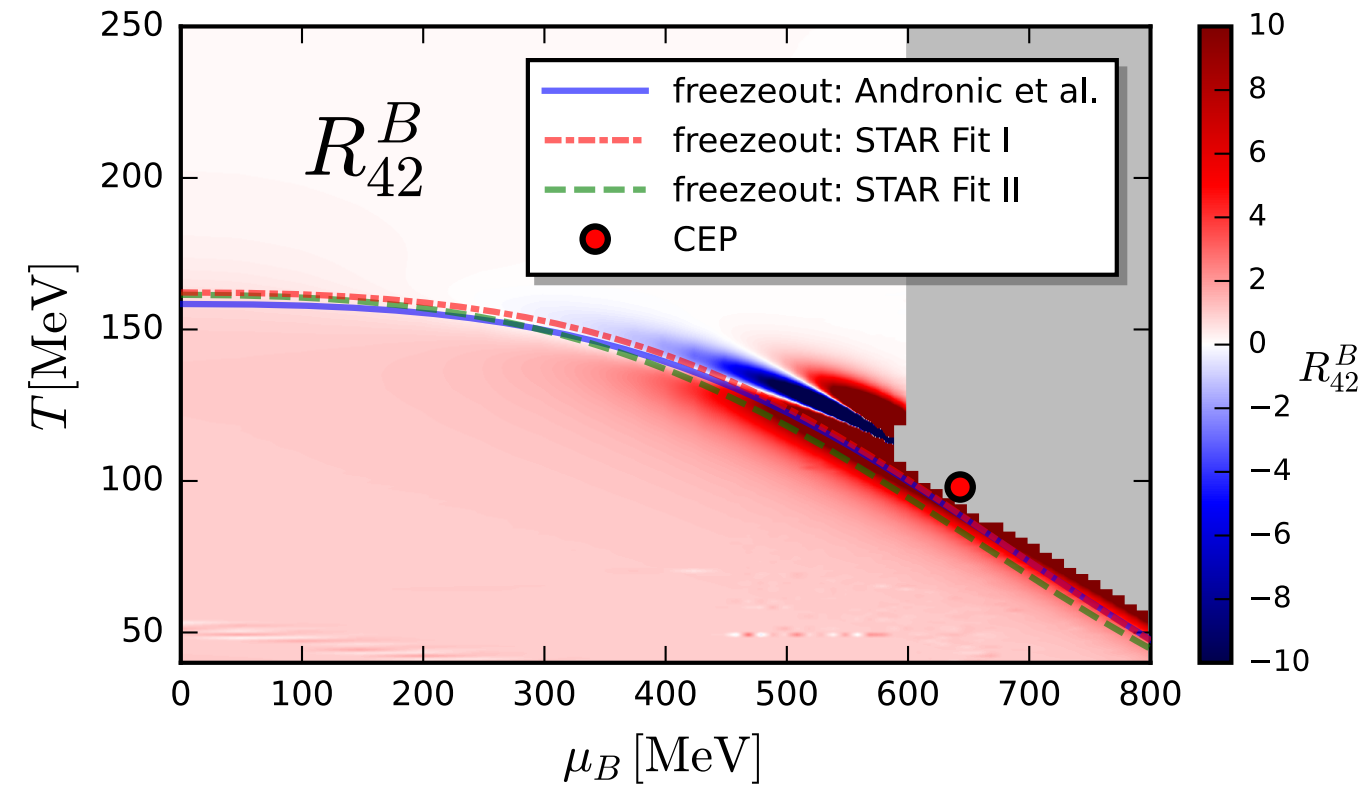


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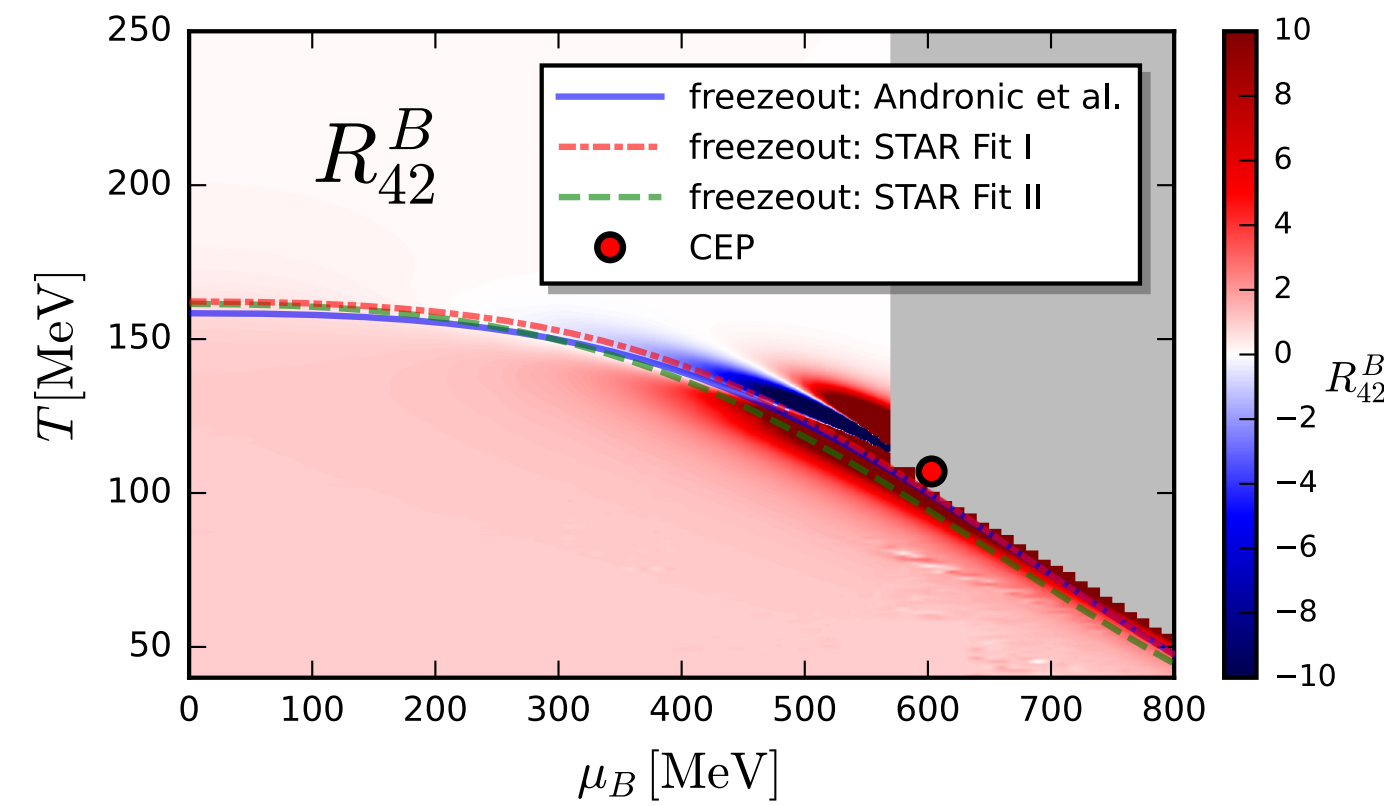
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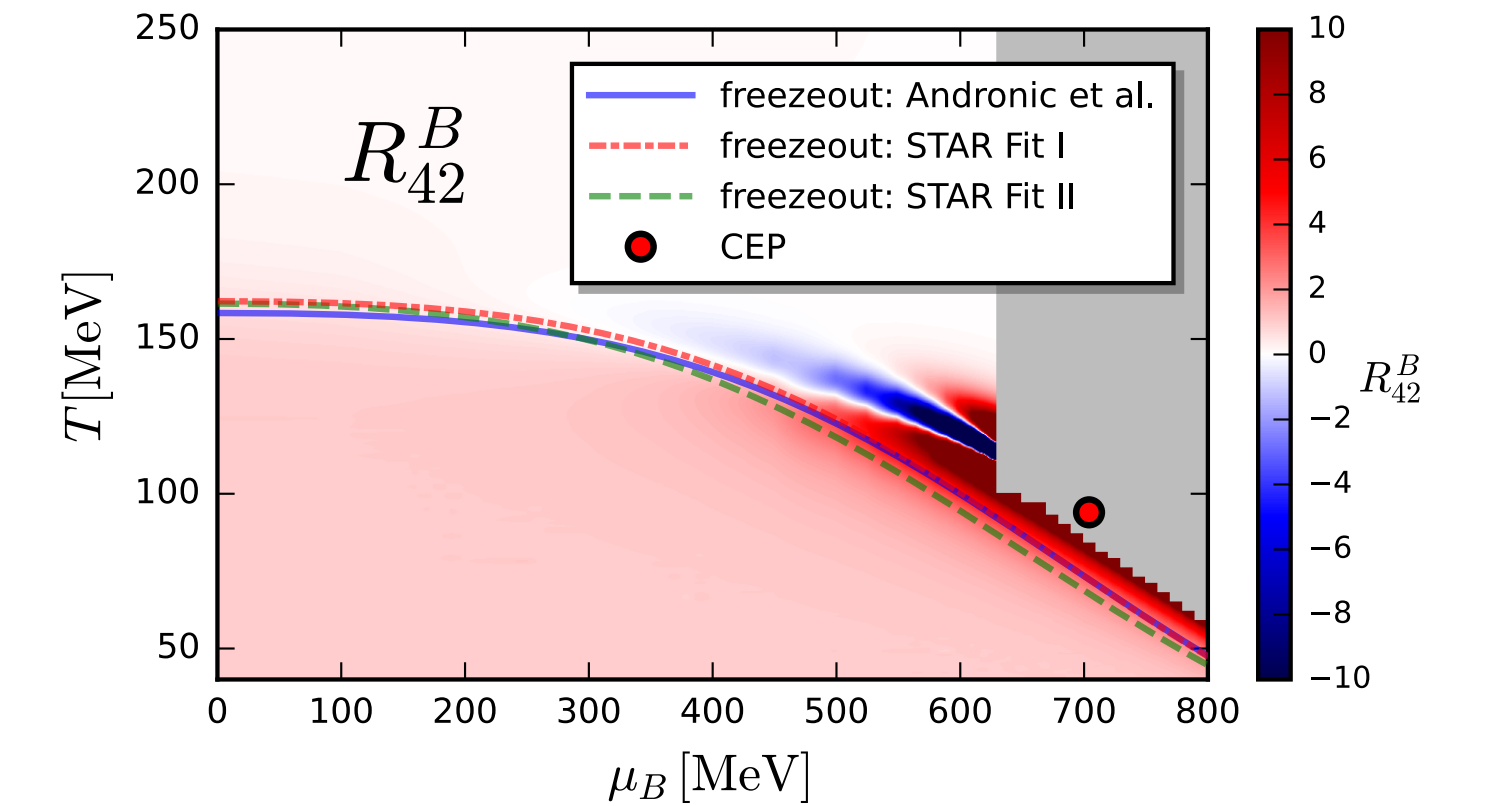
QCD-assisted LEFT



Variations of the CEP in the allowed estimate regime



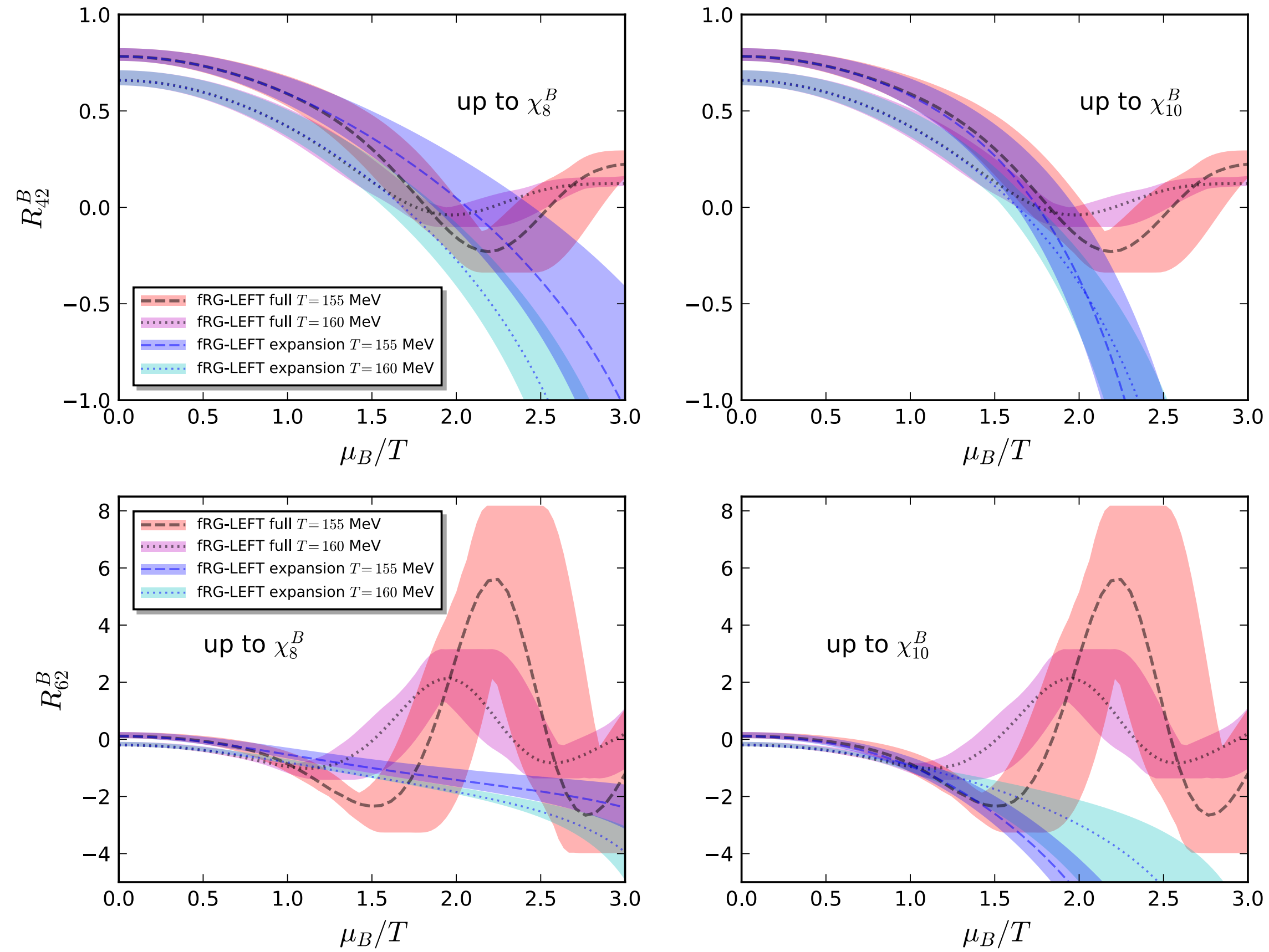
$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (108, 604) \text{ MeV}$$



$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (94, 704) \text{ MeV}$$

Fluctuations of conserved charges

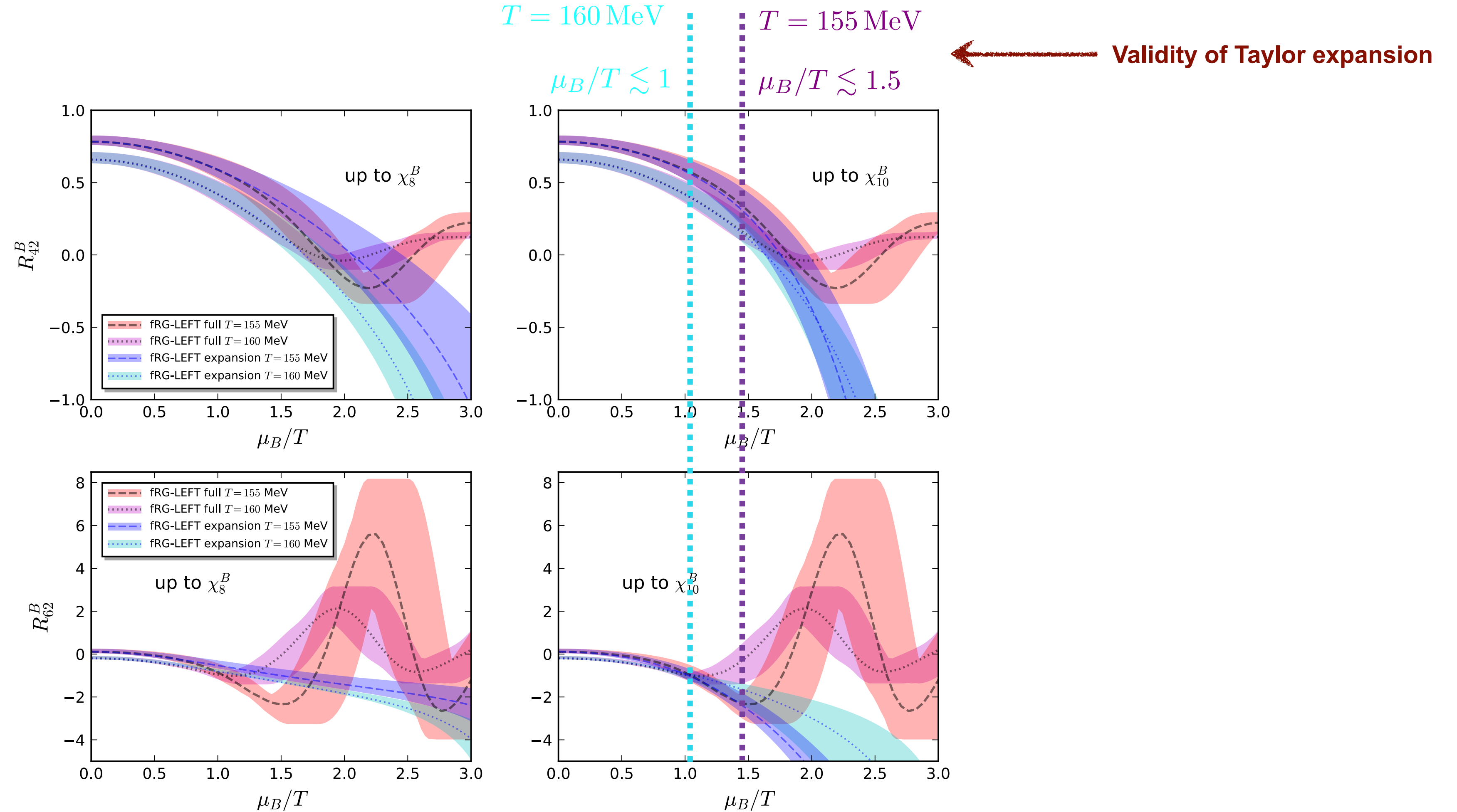
Validity regime of extrapolations



Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

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Canonical corrections via subensemble acceptance

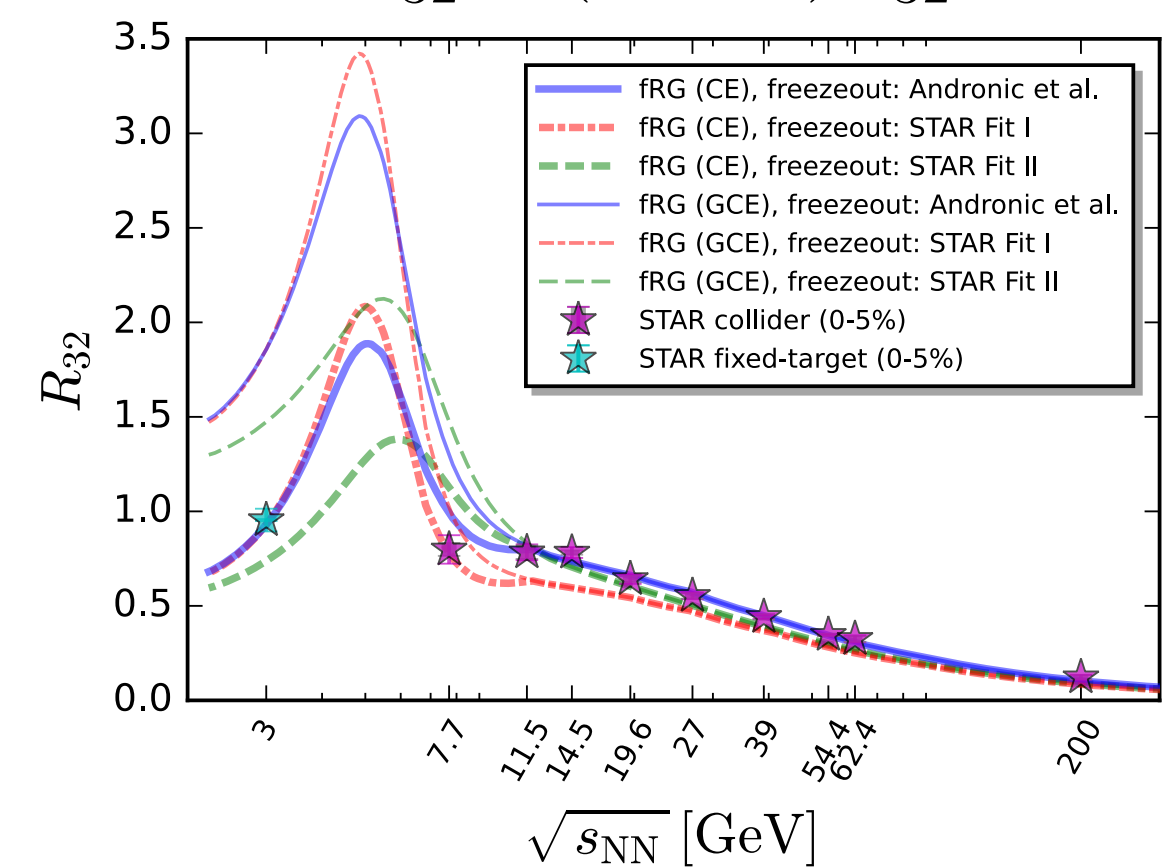
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha) R_{32}^B$$



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Canonical corrections via subensemble acceptance

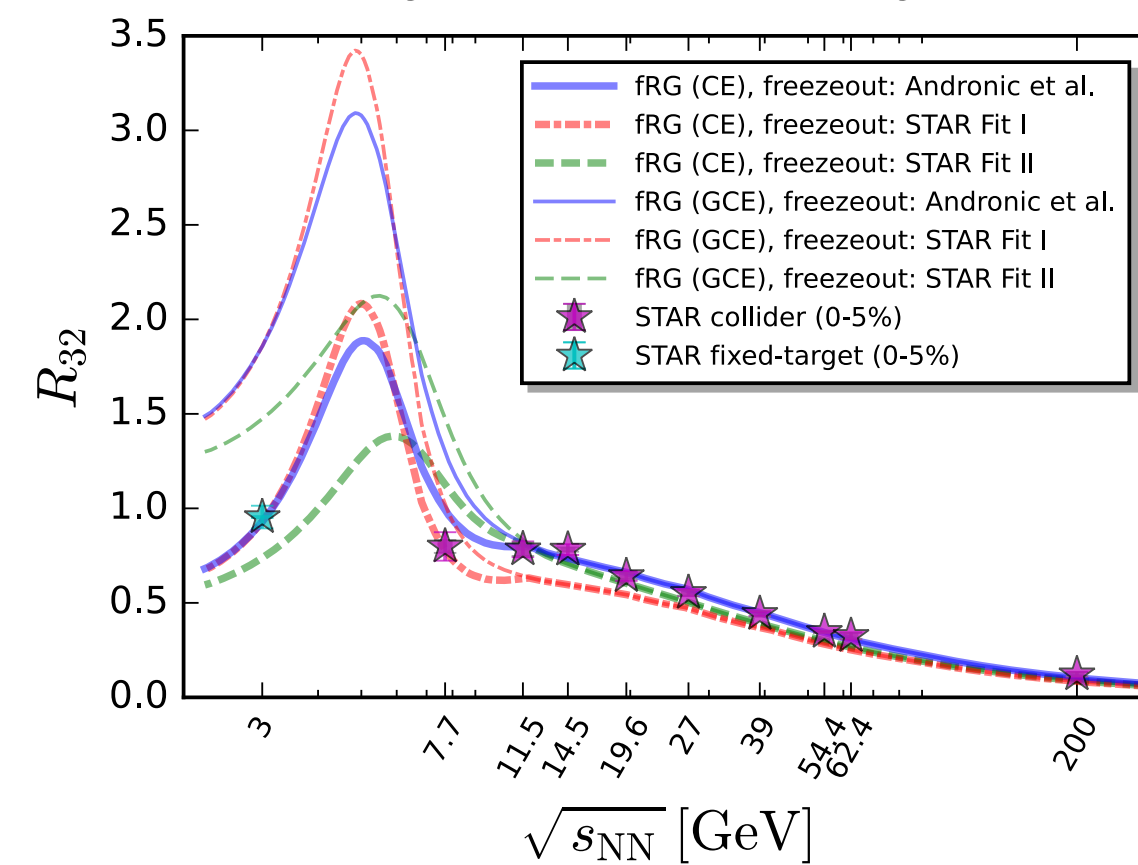
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qualitative adjustment

$$\alpha(\bar{s}) = a \left(1 - \sqrt{\bar{s}}\right) \theta(1 - \bar{s})$$

$$a = 0.33$$

$$\sqrt{\bar{s}} = \frac{\sqrt{s_{\text{NN}}}}{11.9 \text{ GeV}}$$

Ripples of the critical point

Canonical corrections via subensemble acceptance

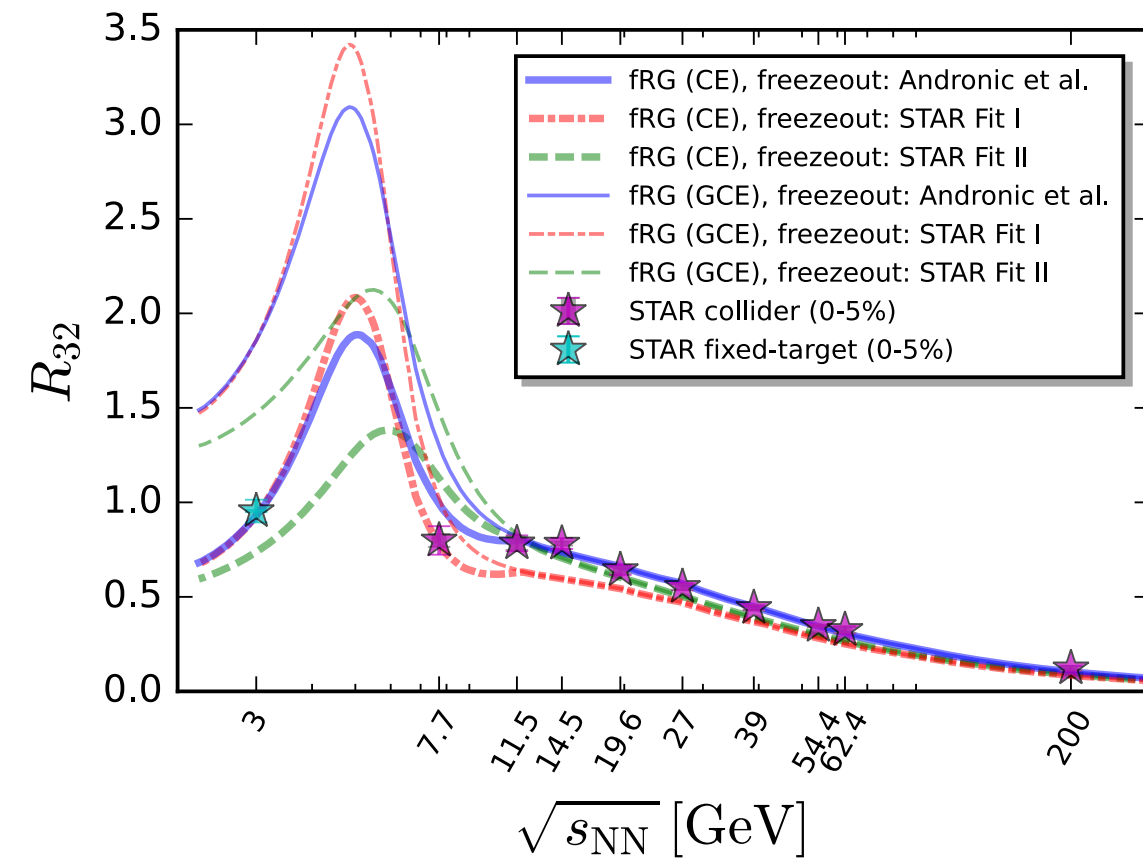
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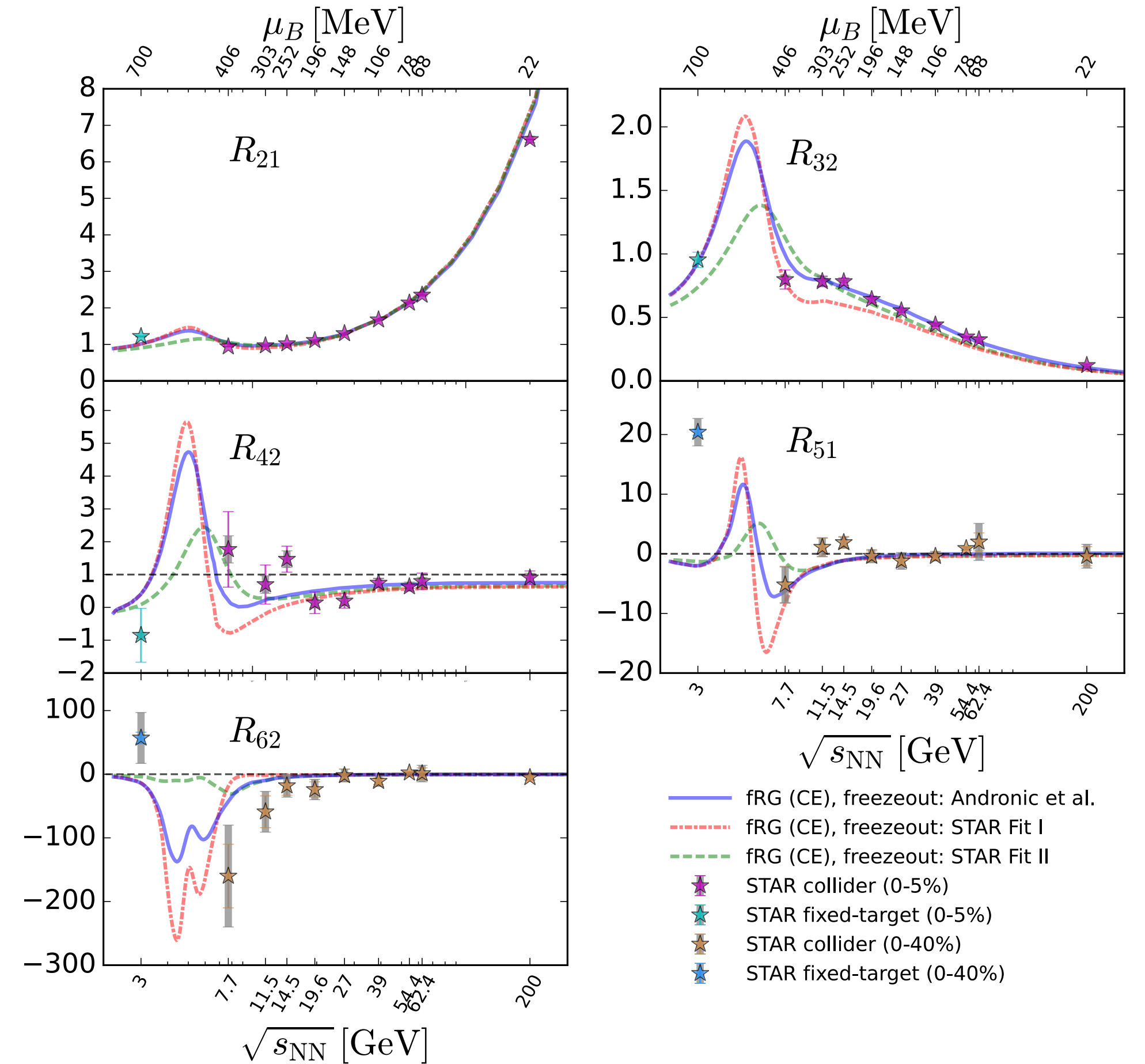
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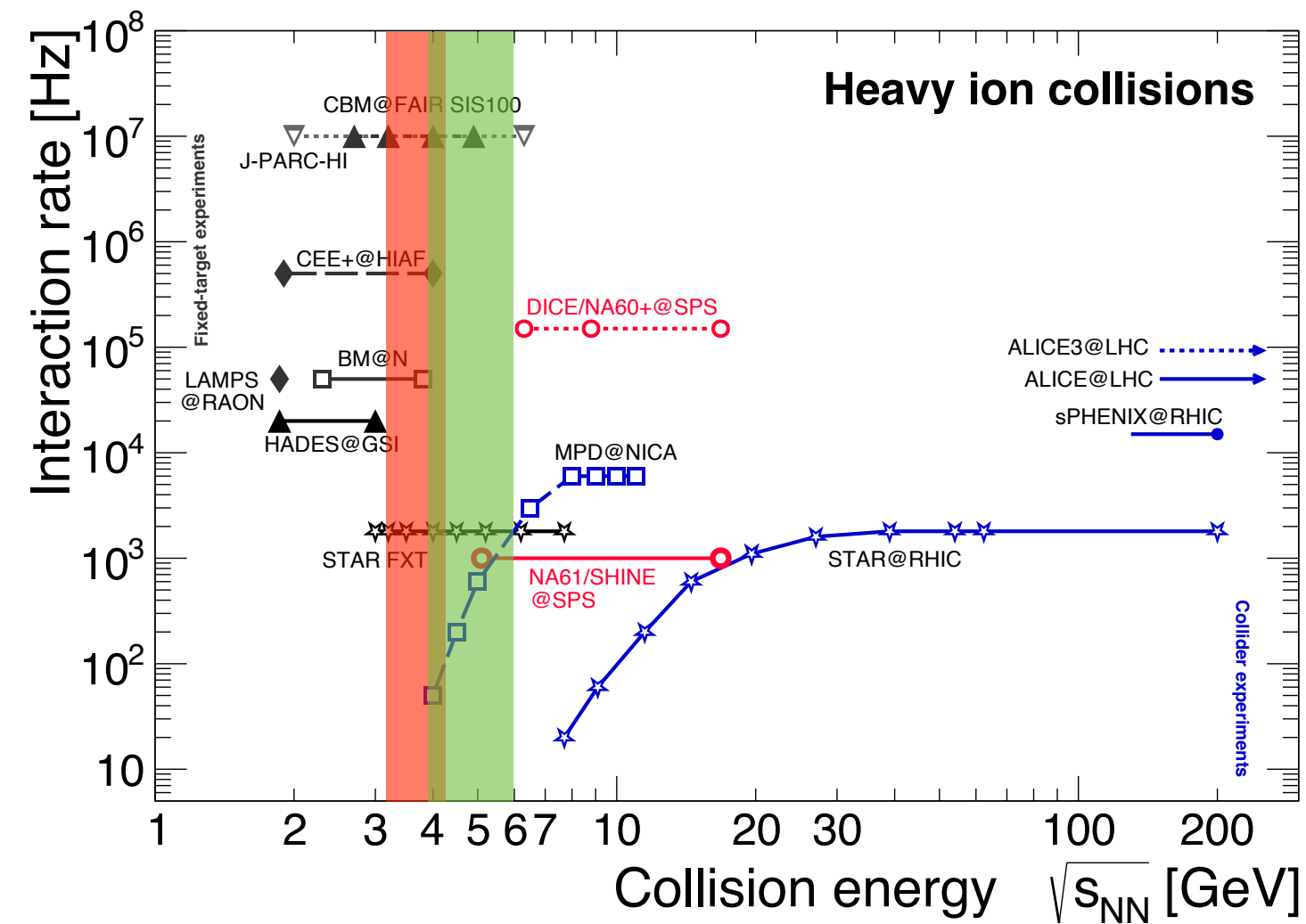
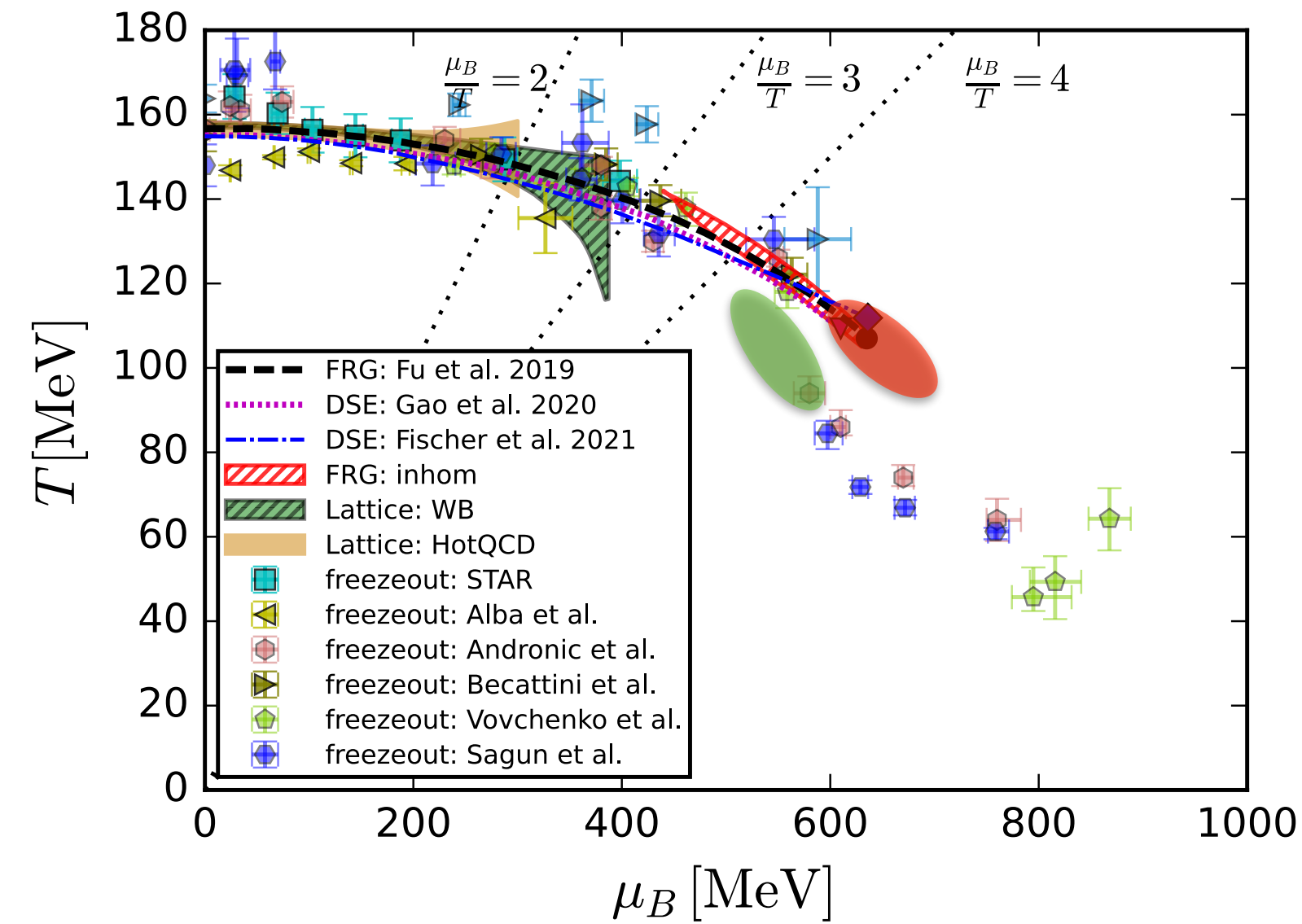
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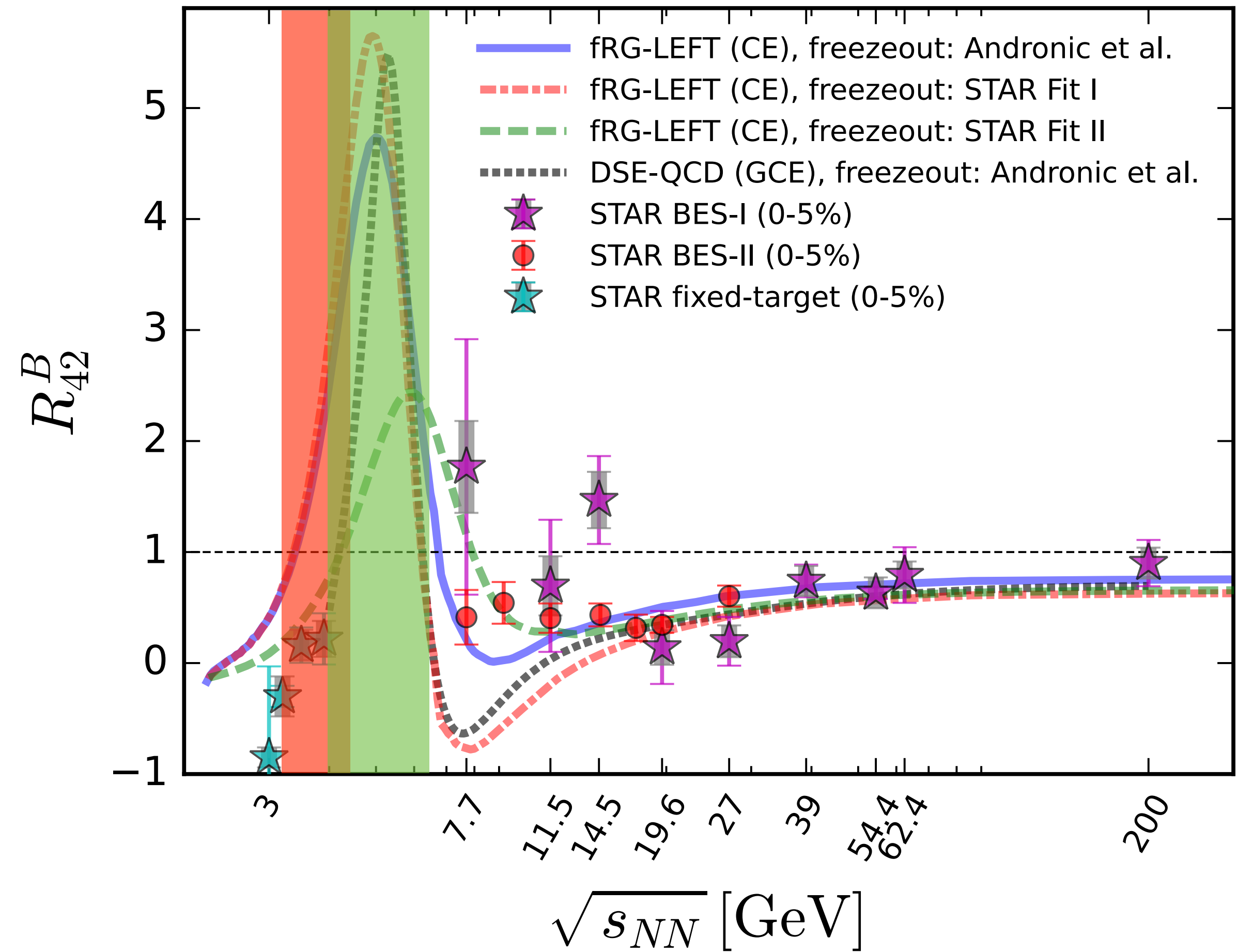
baryon & proton number fluctuations



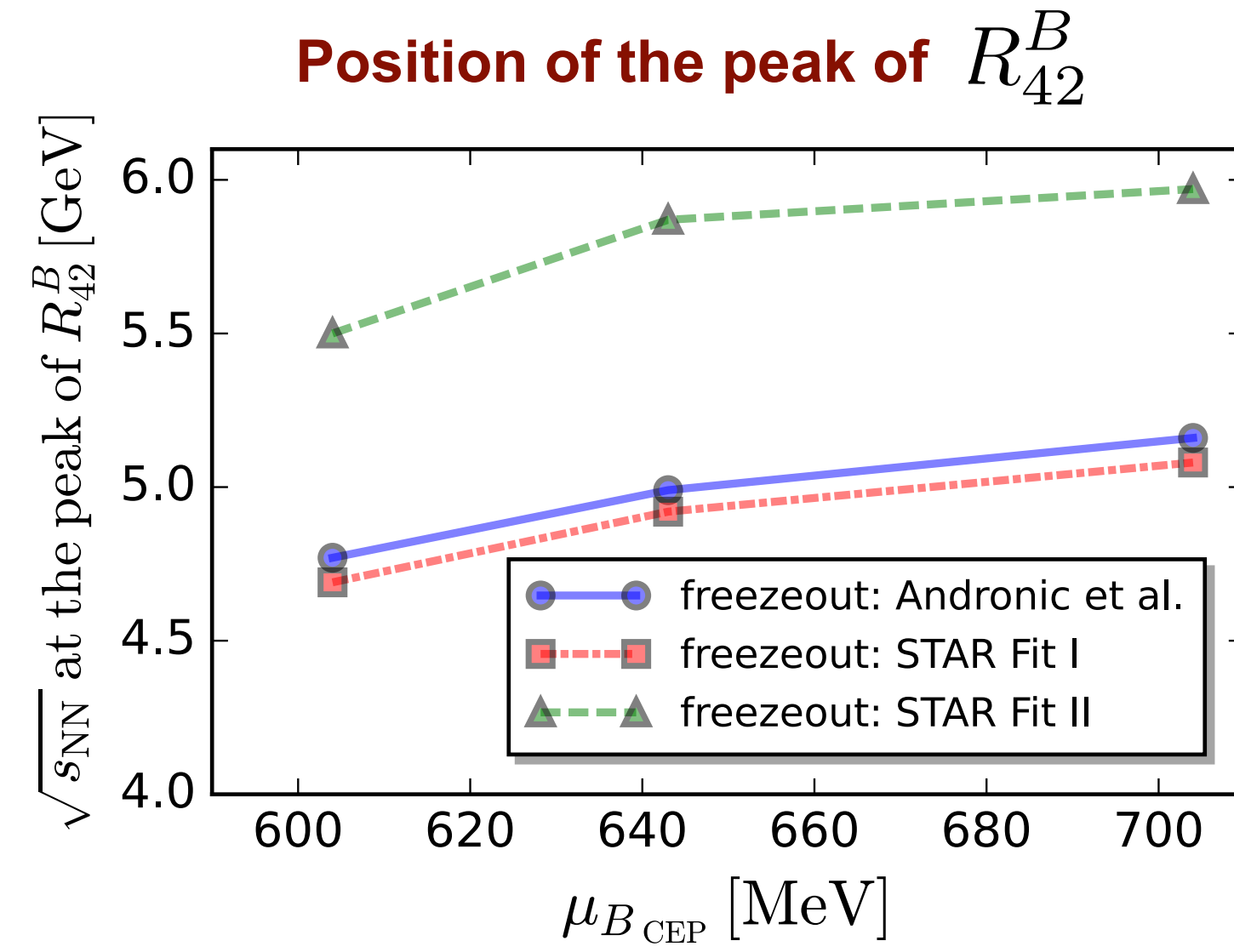
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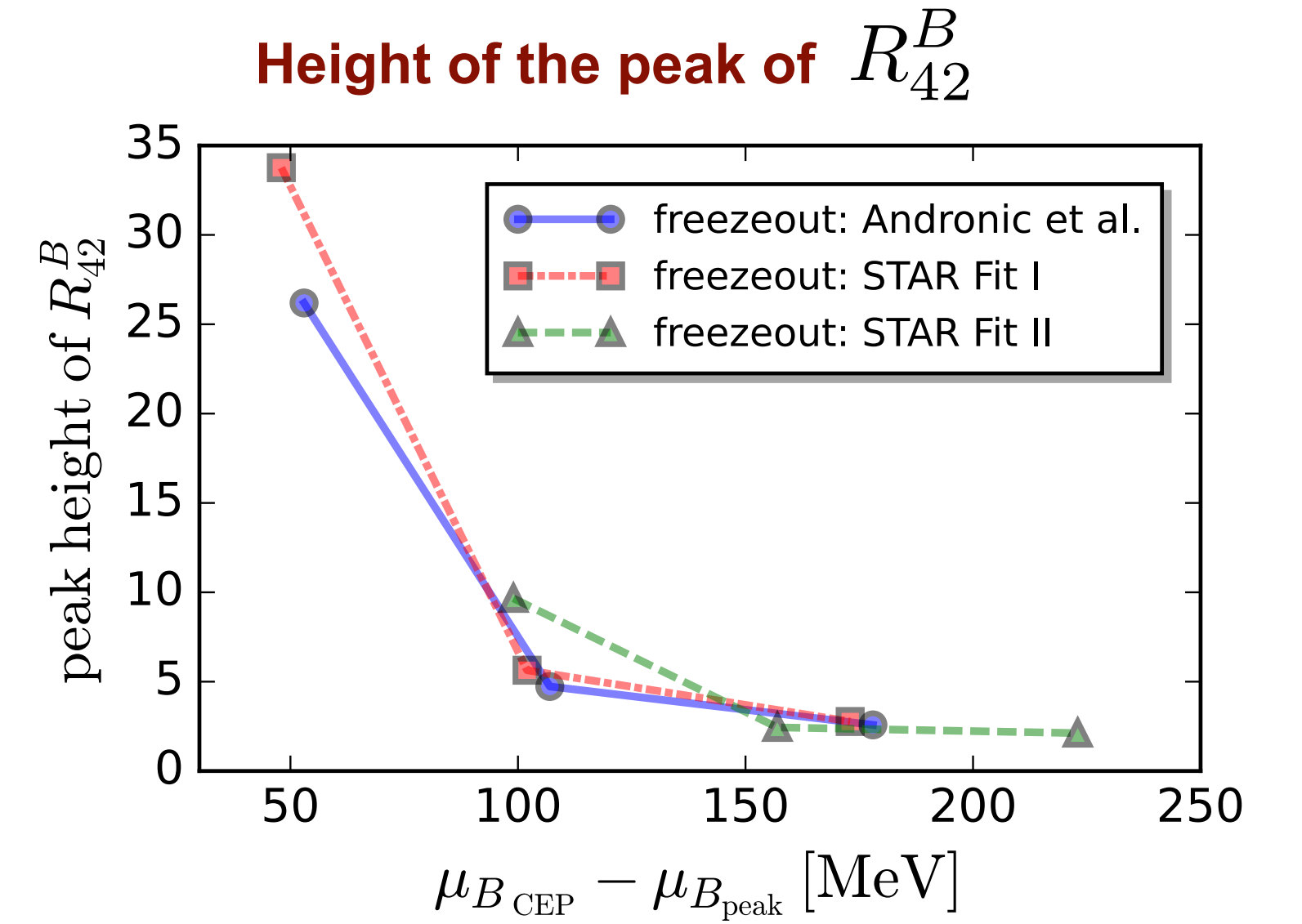
net-baryon fluctuations in QCD vs net-proton fluctuations at STAR



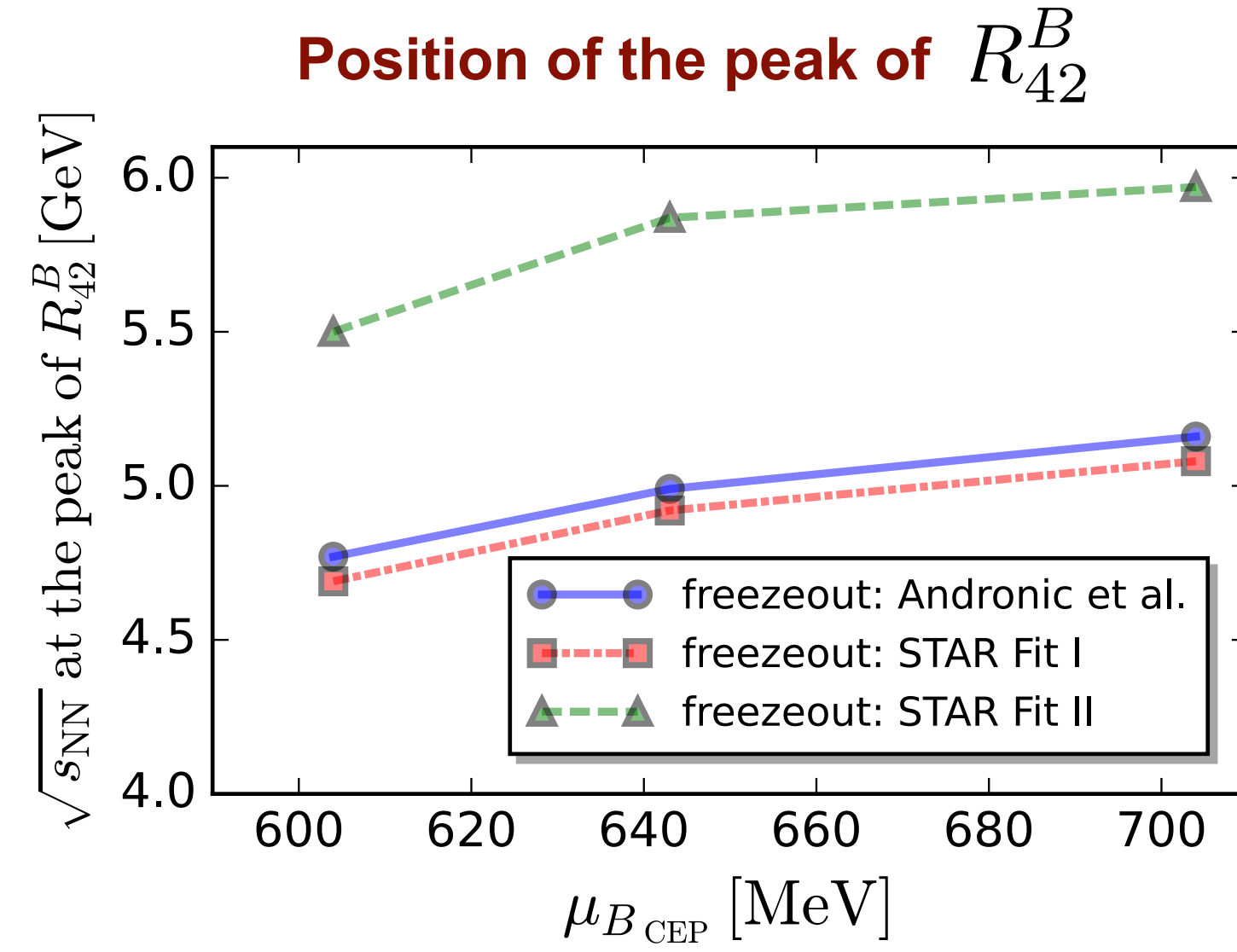
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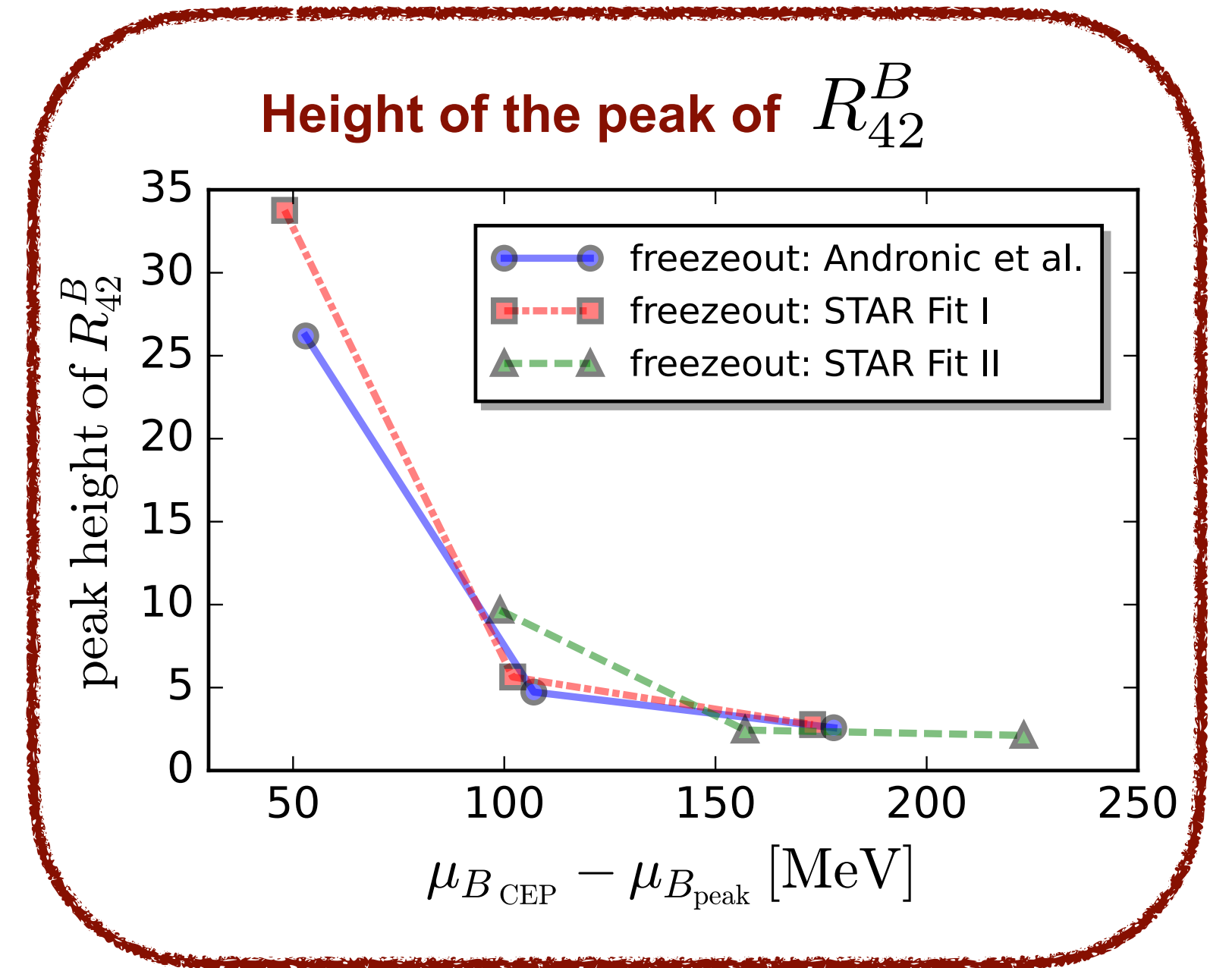
Reconstructing the CEP



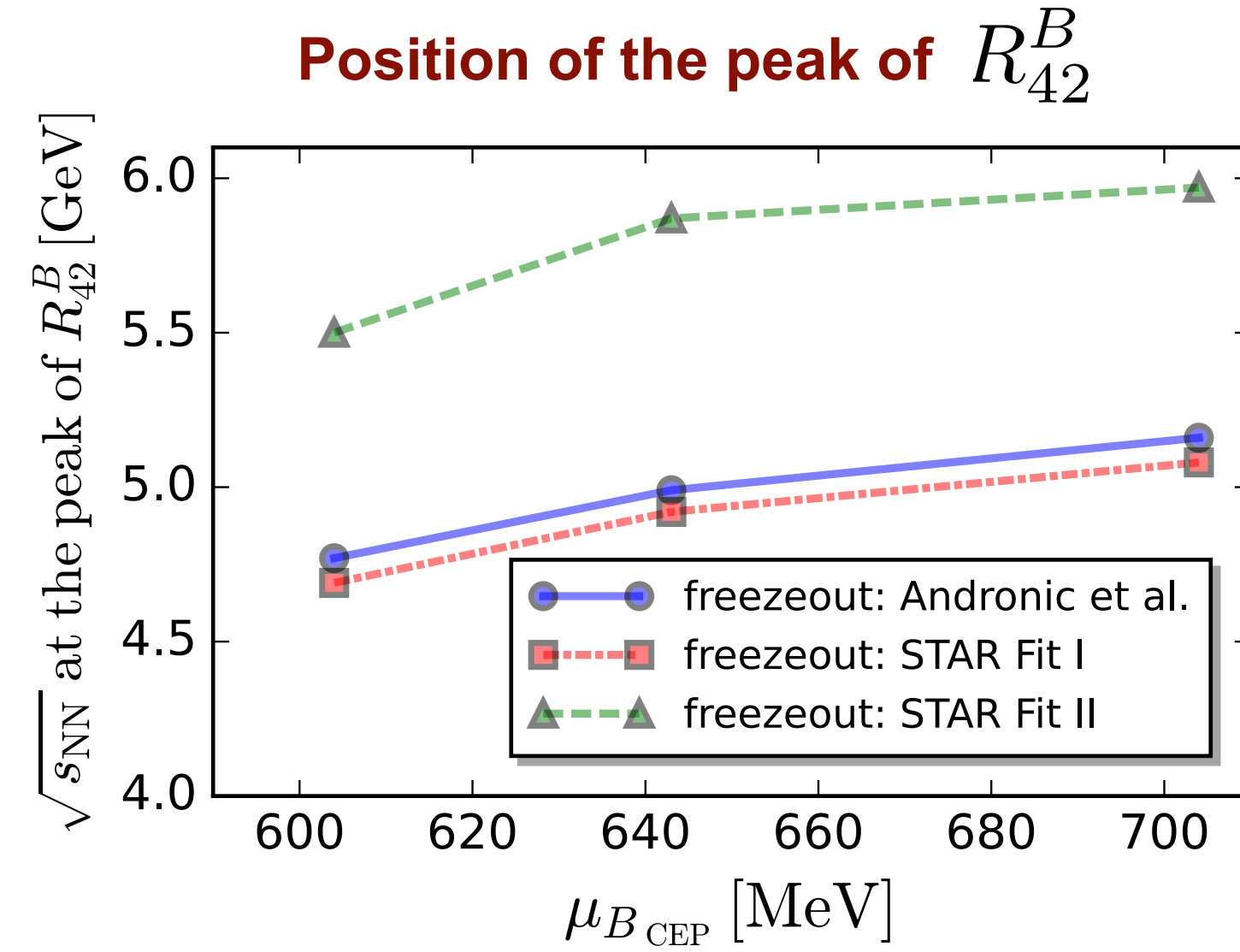
Ripples of the critical point



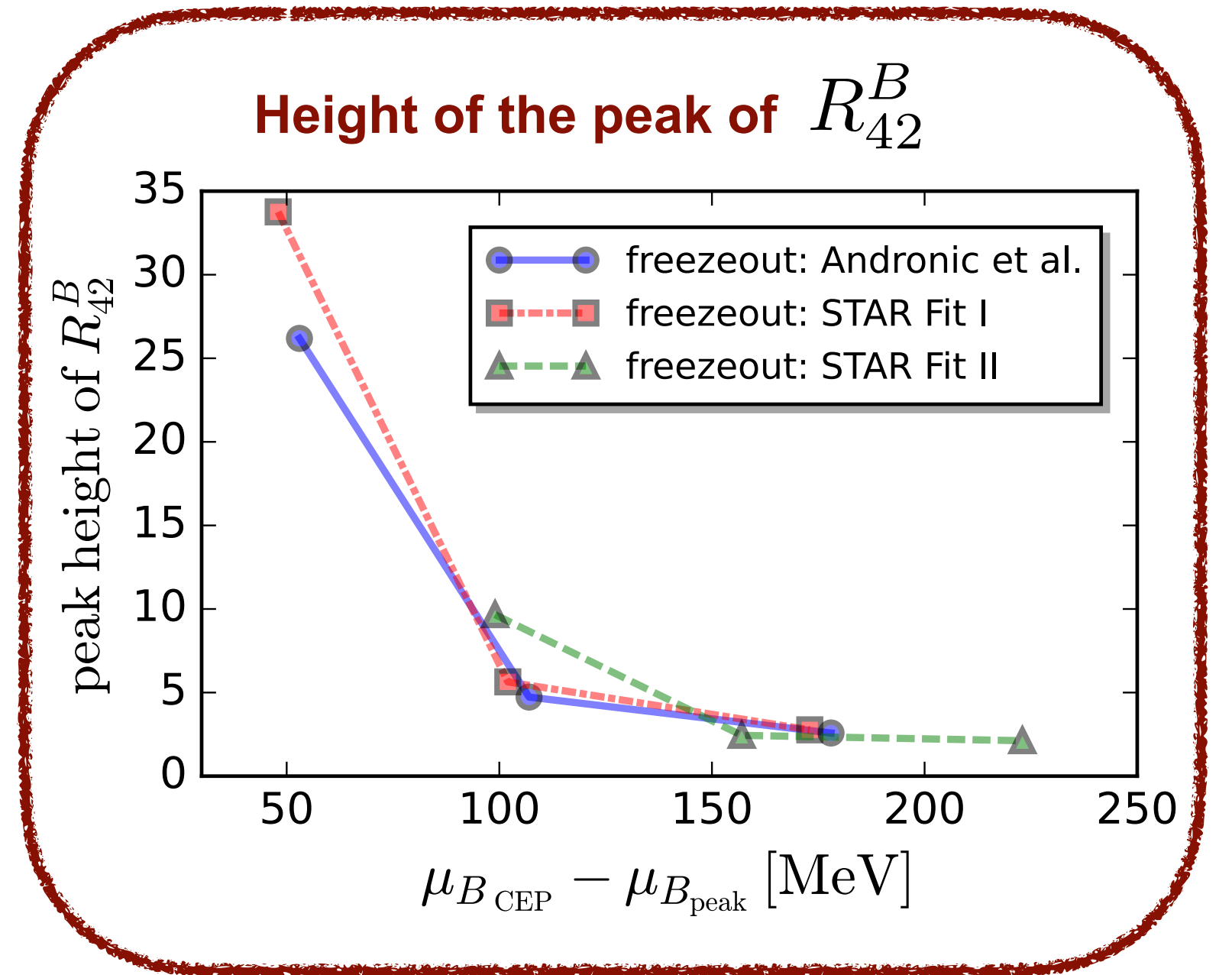
Reconstructing the CEP



Ripples of the critical point



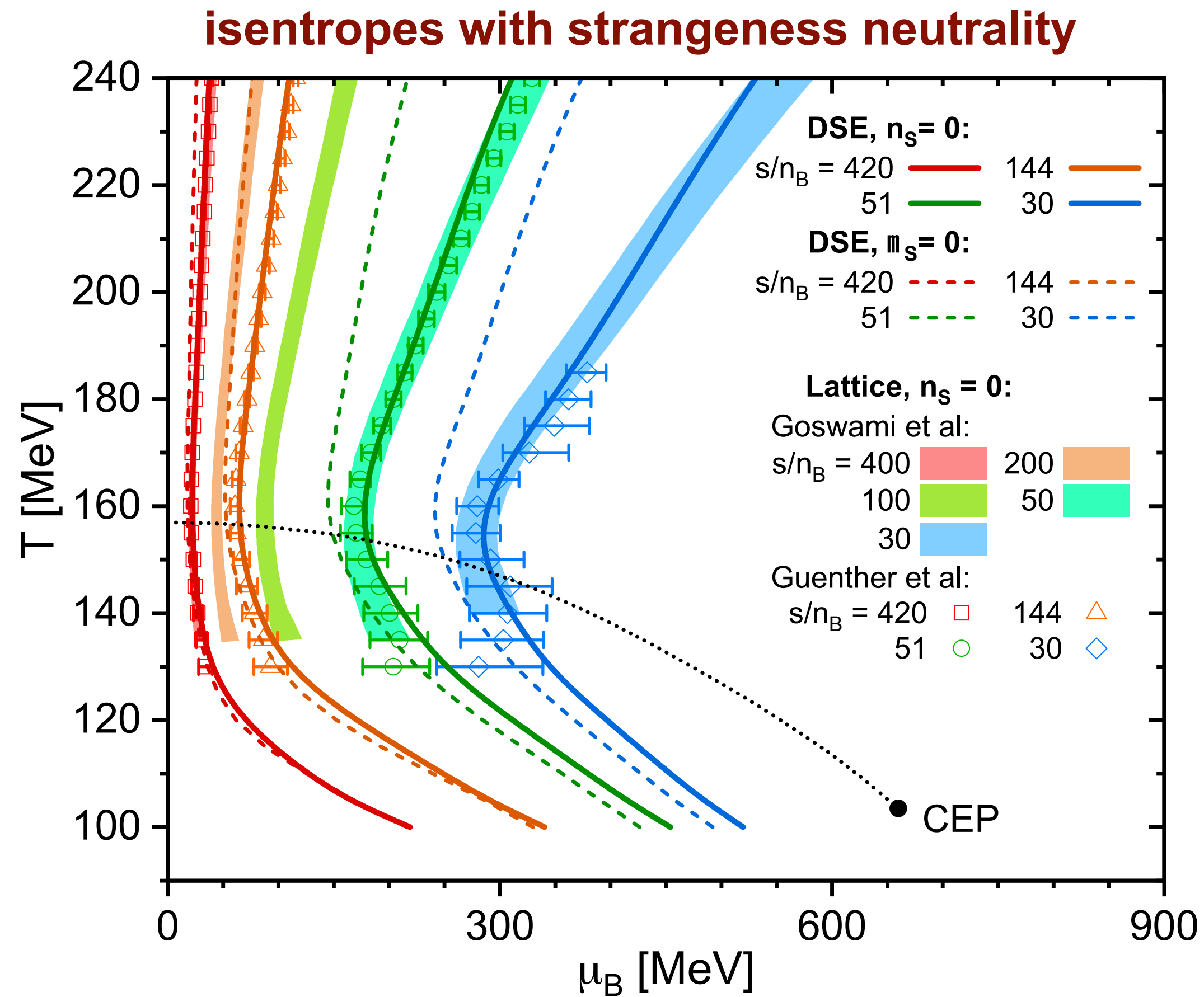
Reconstructing the CEP



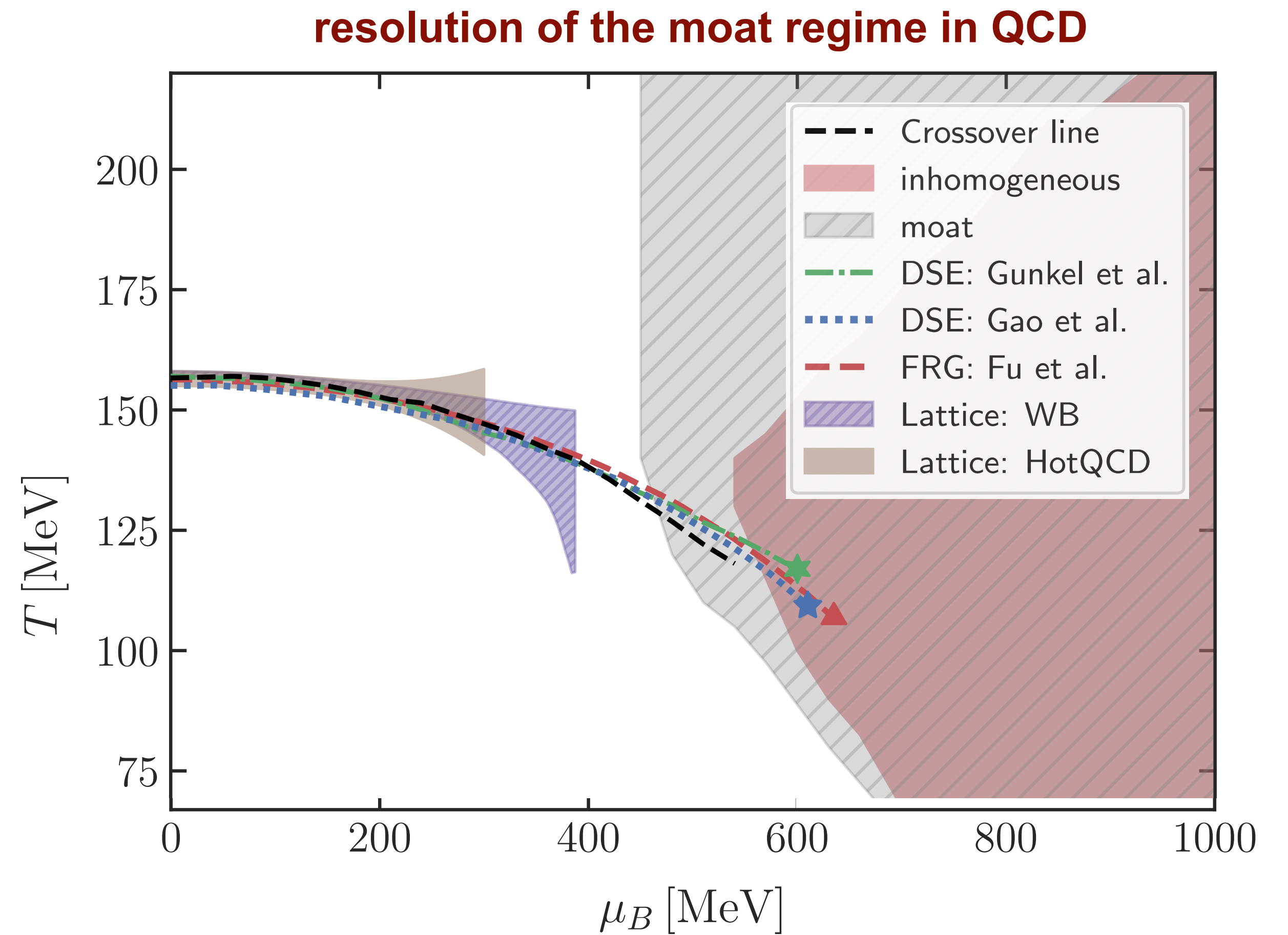
Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

Ripples of the critical point: Sneak preview



Lu, Gao et al, in preparation



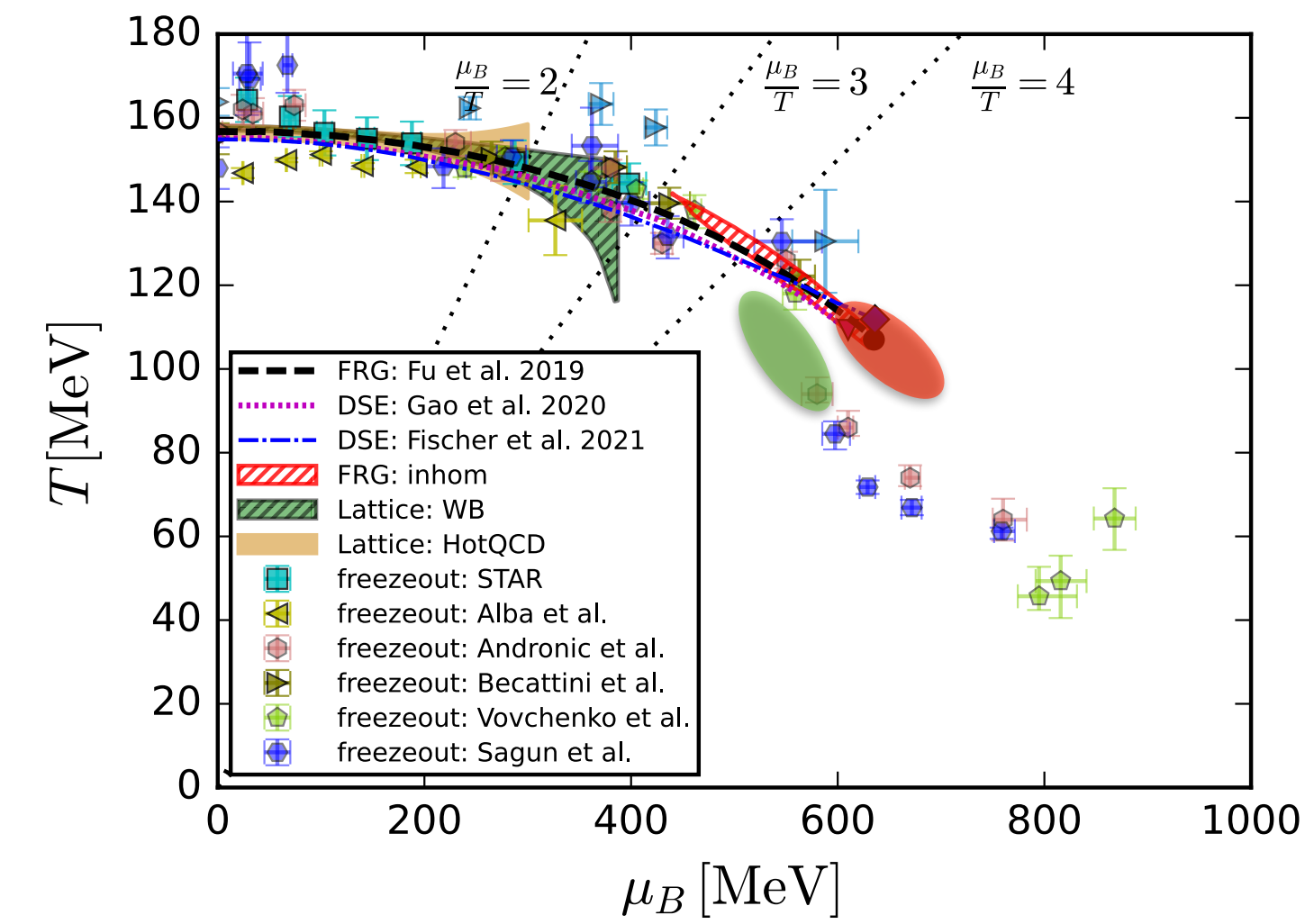
Sattler et al, in preparation

Summary

- Functional QCD provides direct 1st principle results for the phase structure at finite density

▪ **Predictions:** $\frac{\mu_B}{T} \lesssim 4$

▪ **Estimates:** $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



Summary

- Functional QCD provides direct 1st principle results for the phase structure at finite density

▪ Predictions: $\frac{\mu_B}{T} \lesssim 4$

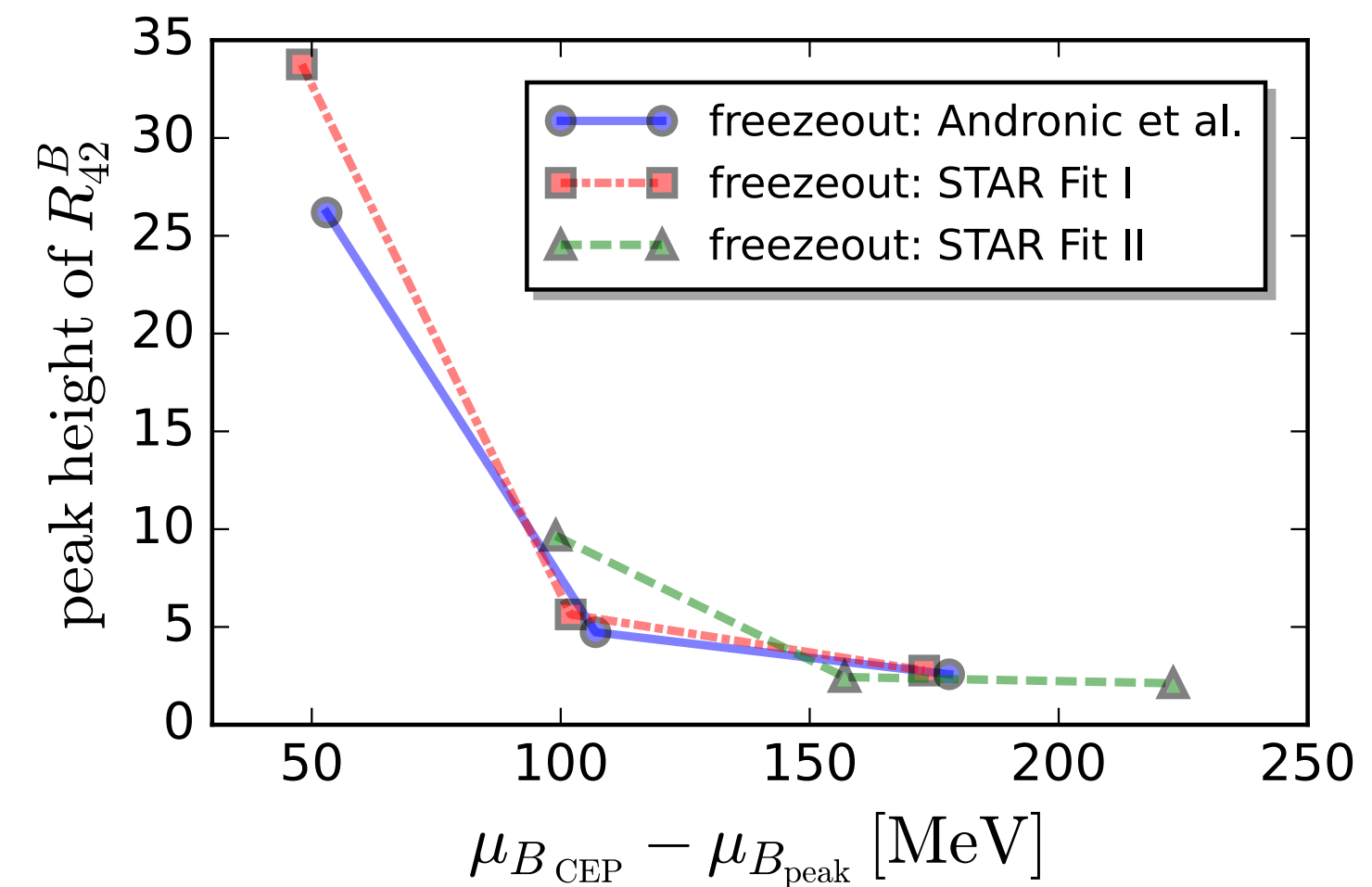
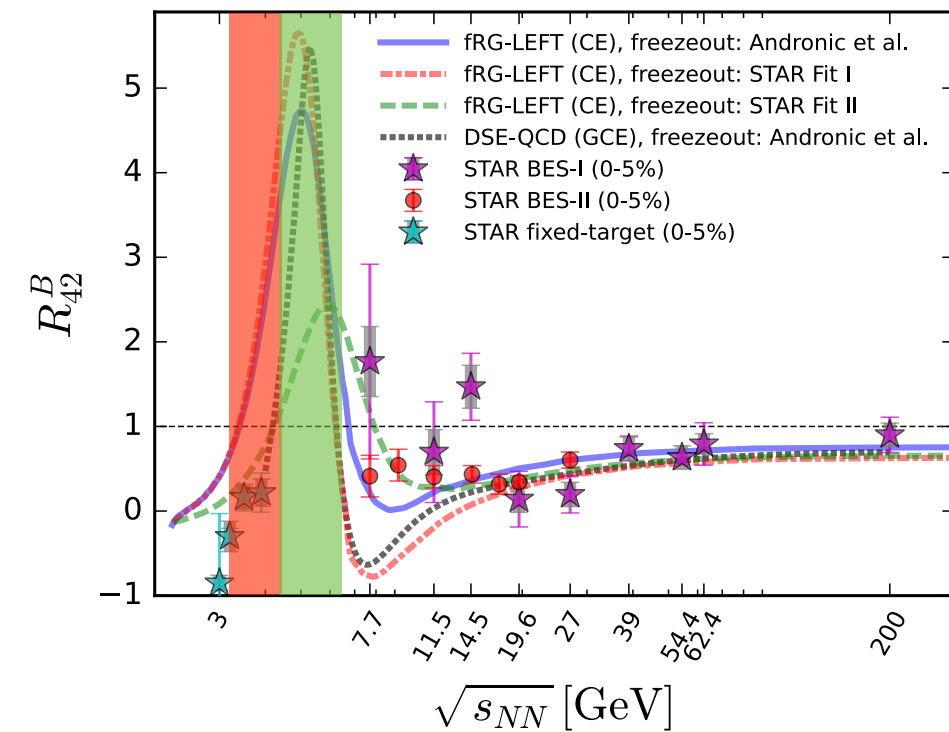
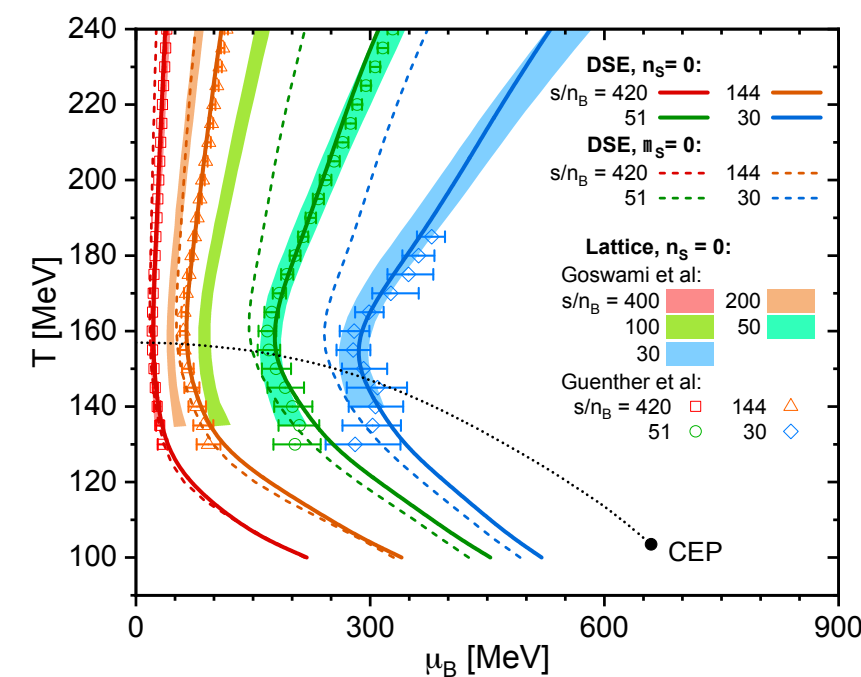
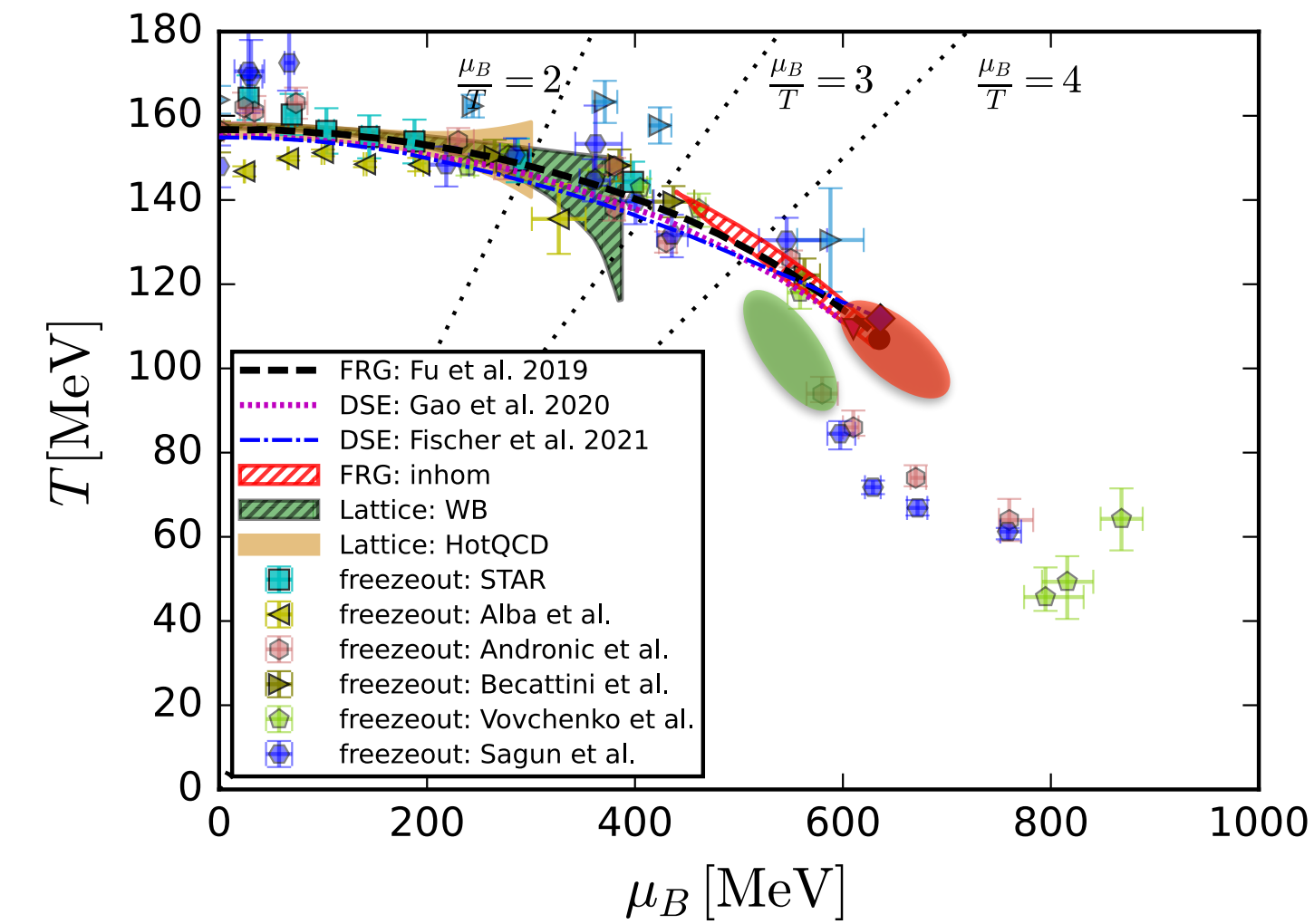
▪ Estimates: $4 \lesssim \frac{\mu_B}{T} \lesssim 8$

- Ripples of new phases aka of the critical end point

- Explanation for the convergence of CEP locations of extrapolation approaches

- Results & observables: EoS, fluctuations of conserved charges, tiny critical regime & onset of new phases; moat regime,

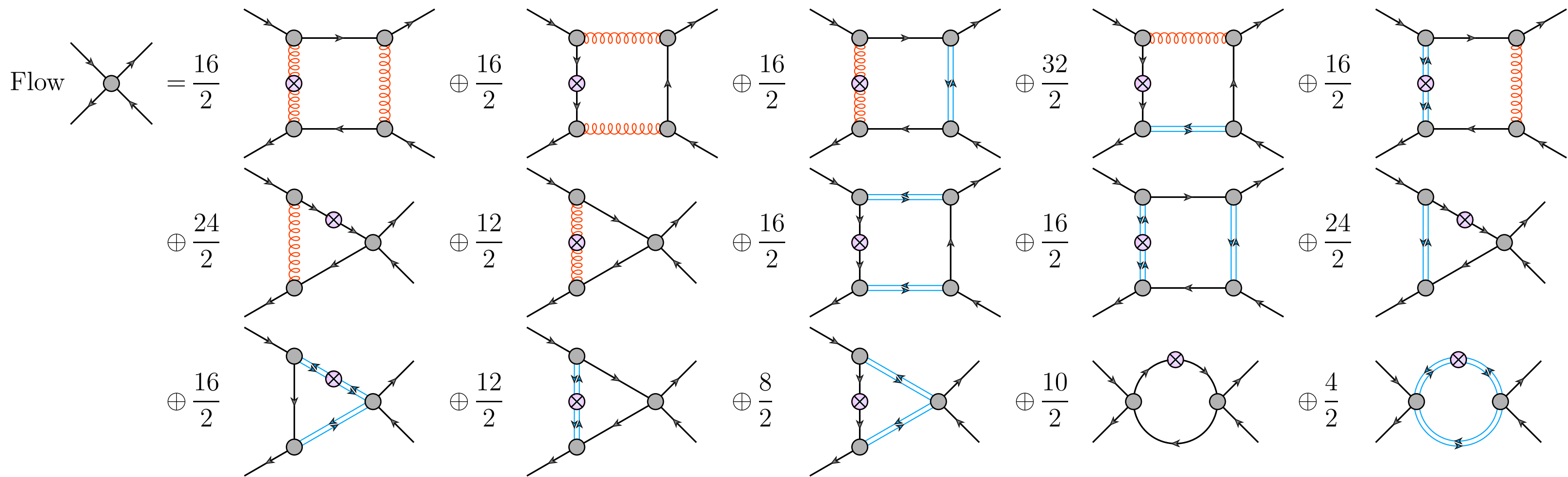
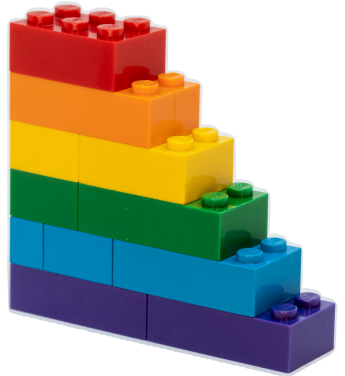
- Peak height of kurtosis carries the location of the CEP



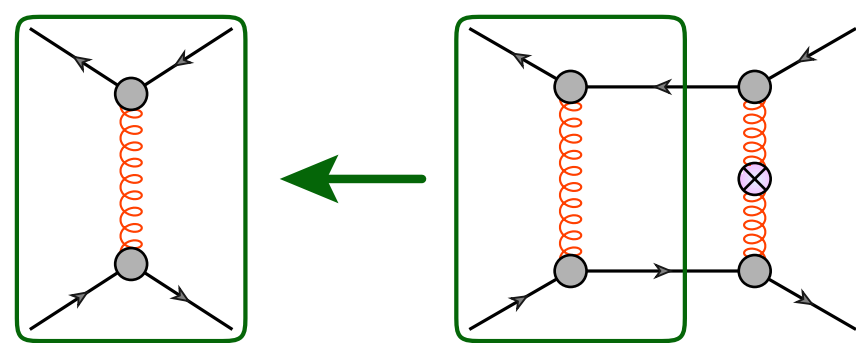
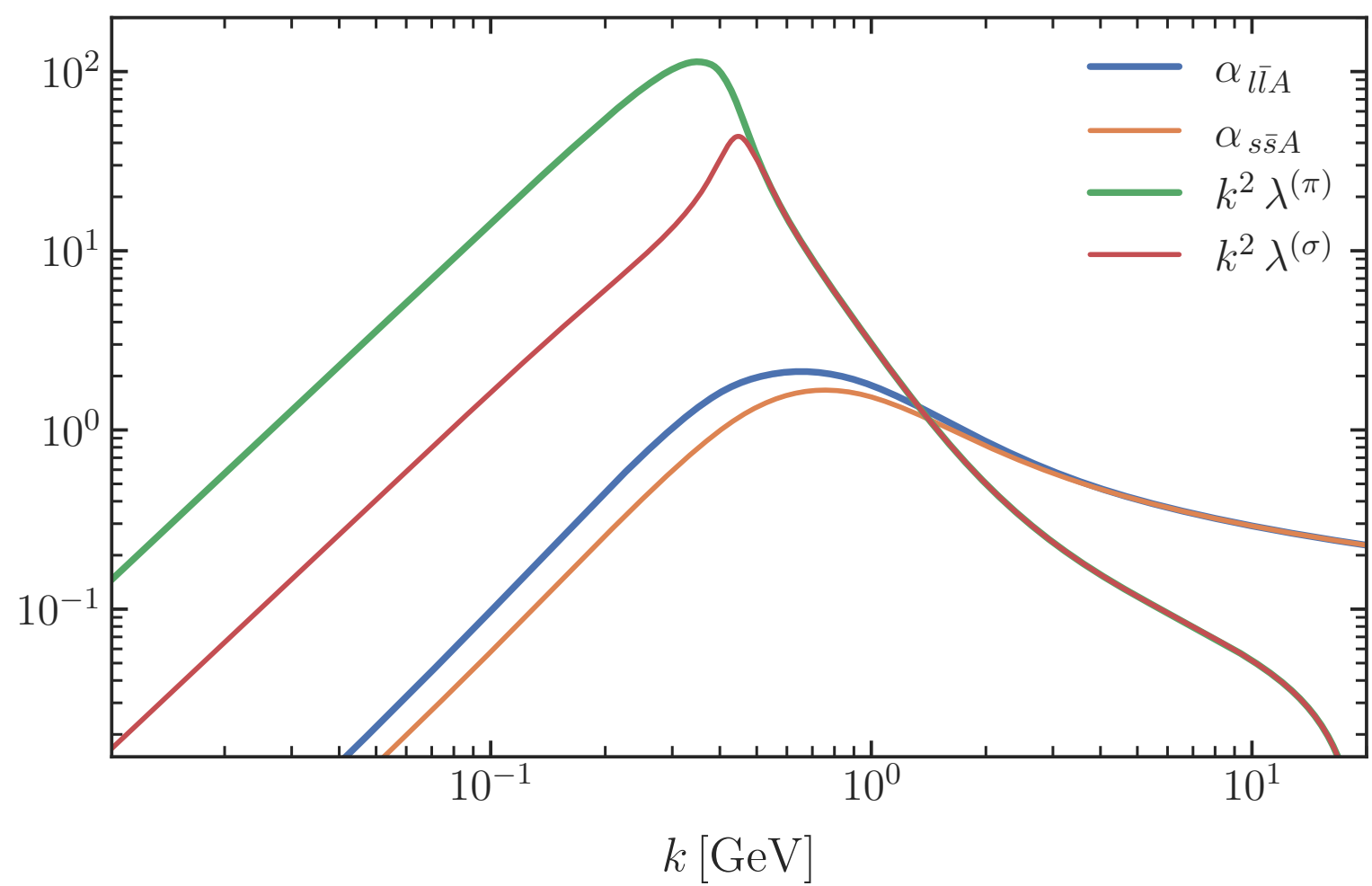
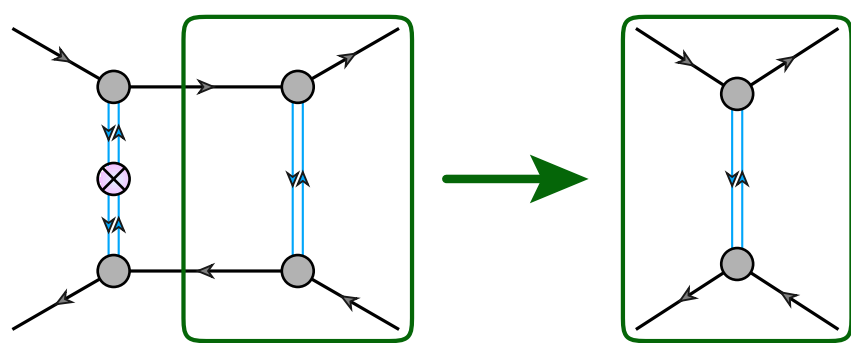
Supplementary material

Phase structure from functional QCD: how to

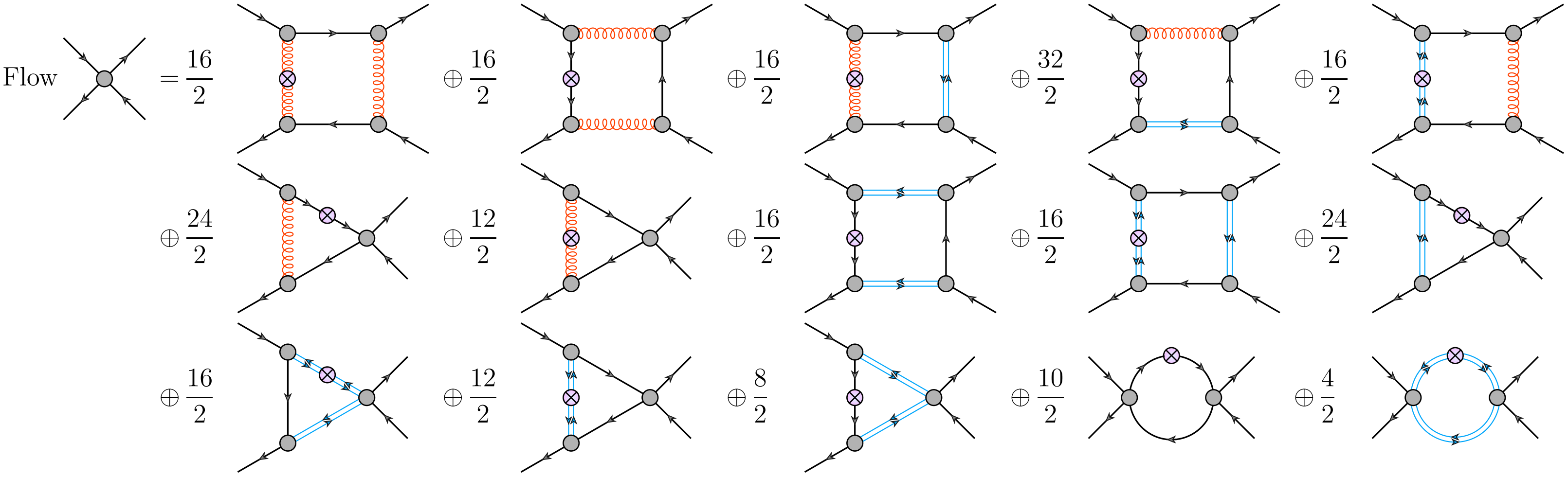
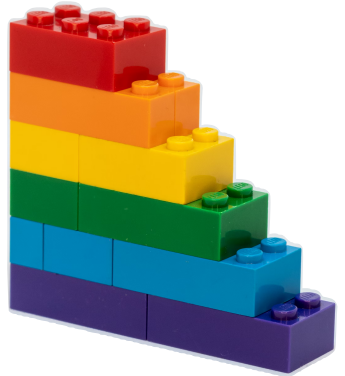
How to: systematic error estimates & the LEGO[®] principle



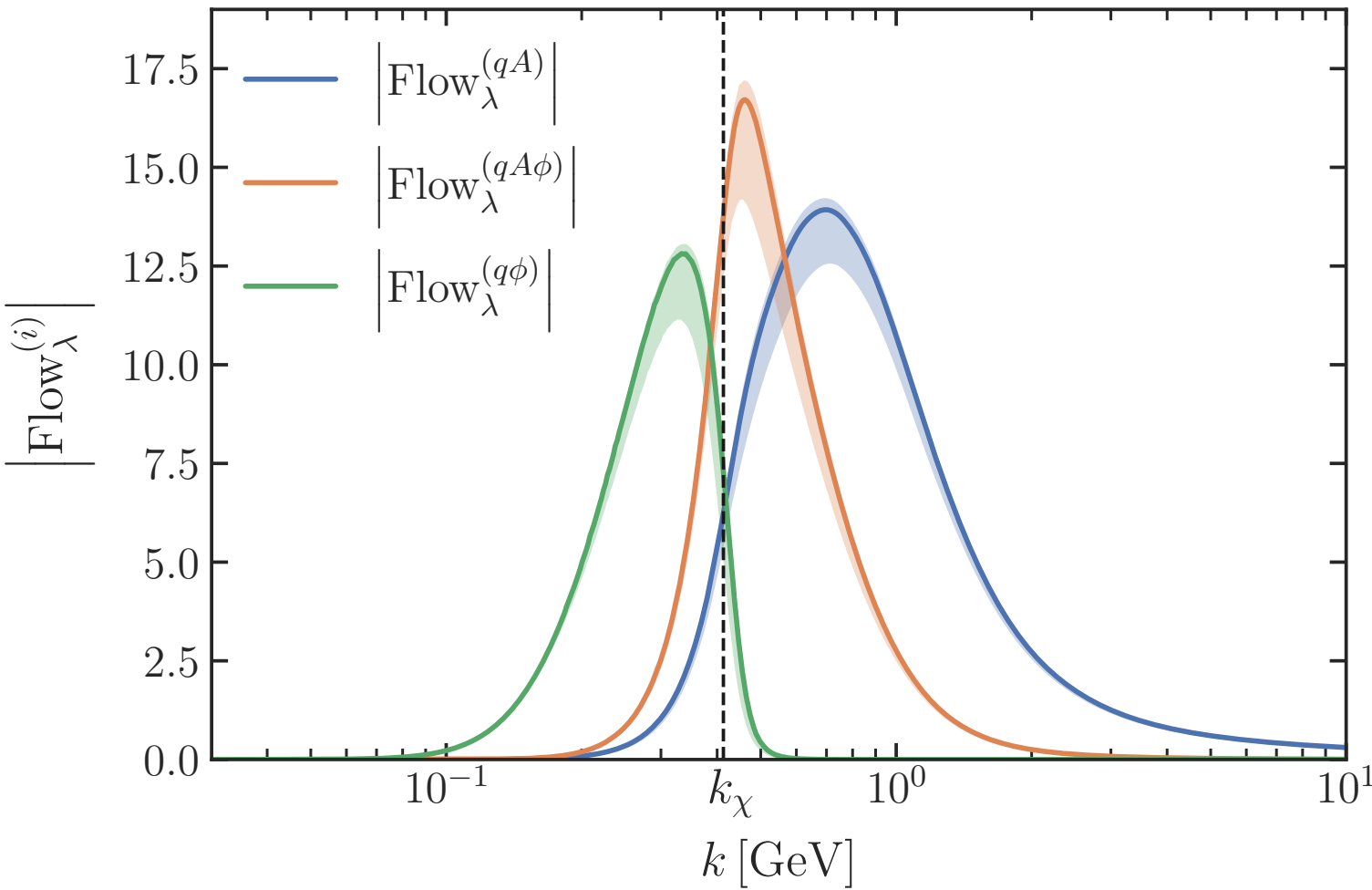
Example: 4-quark scattering vertex



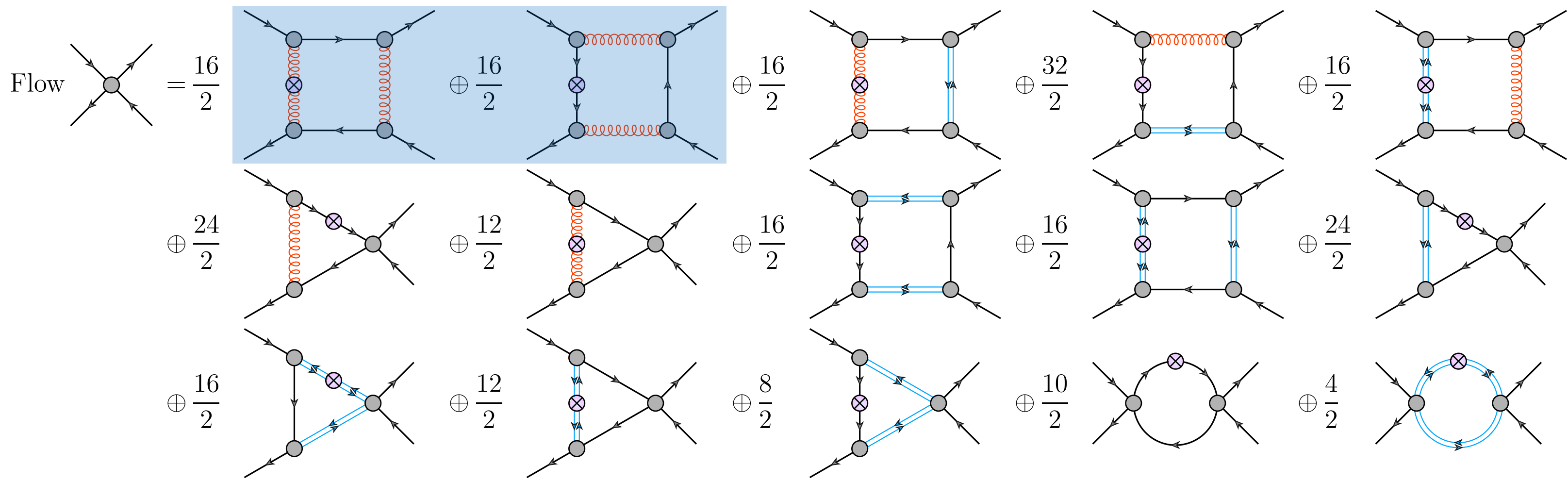
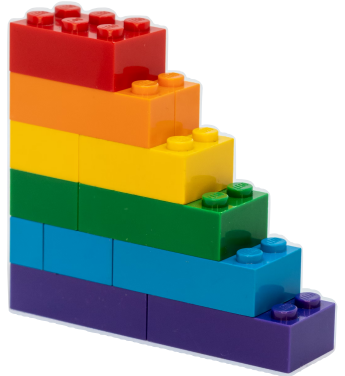
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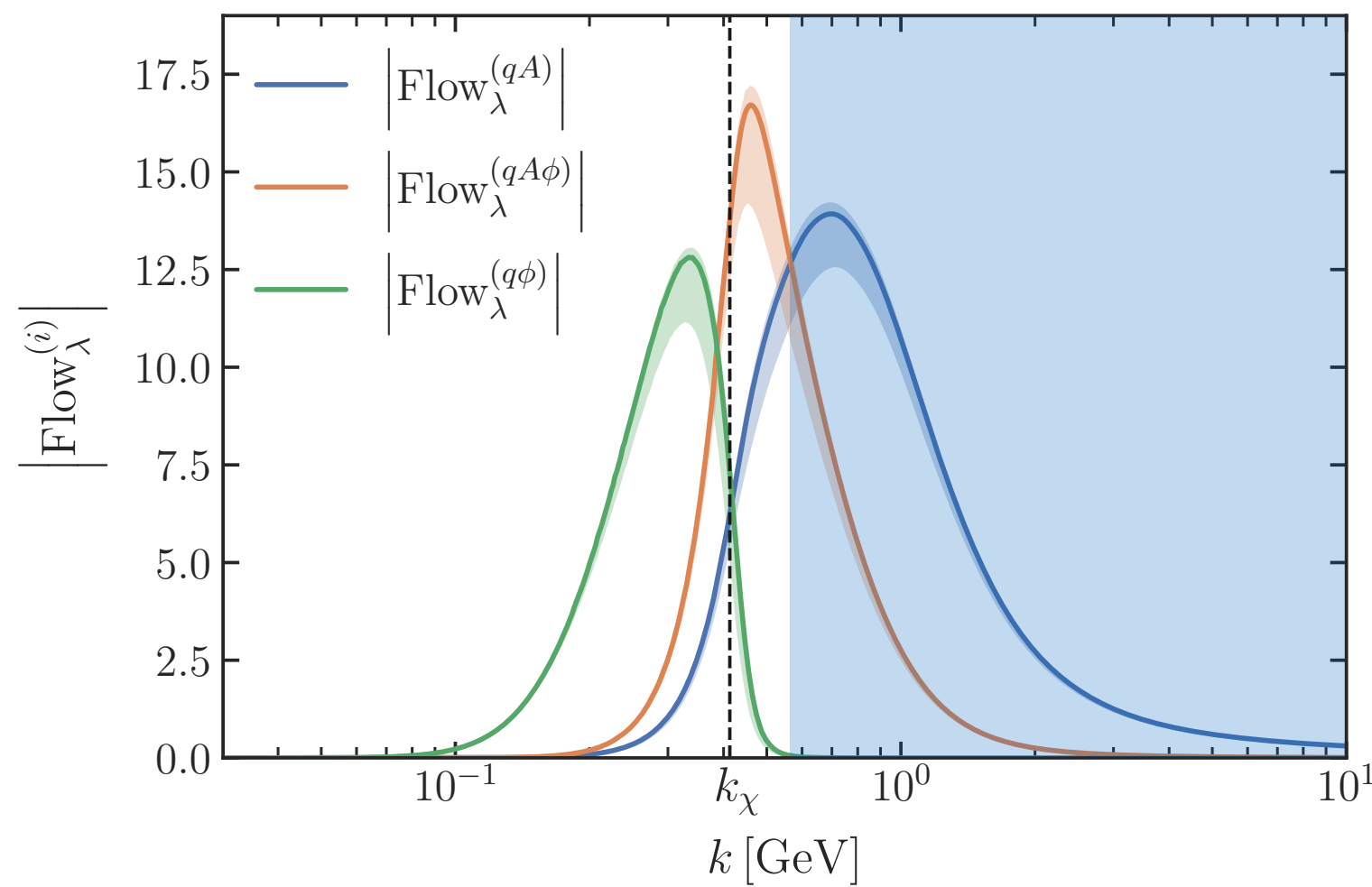
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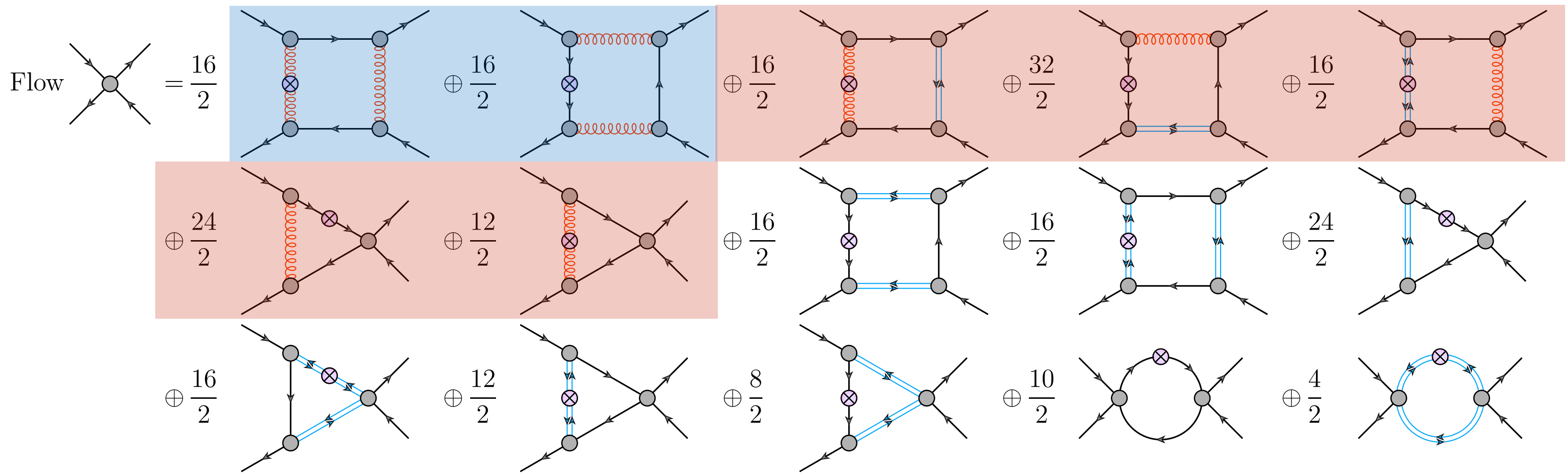
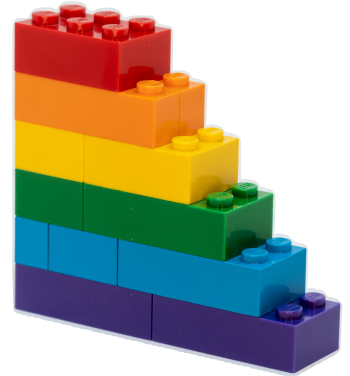
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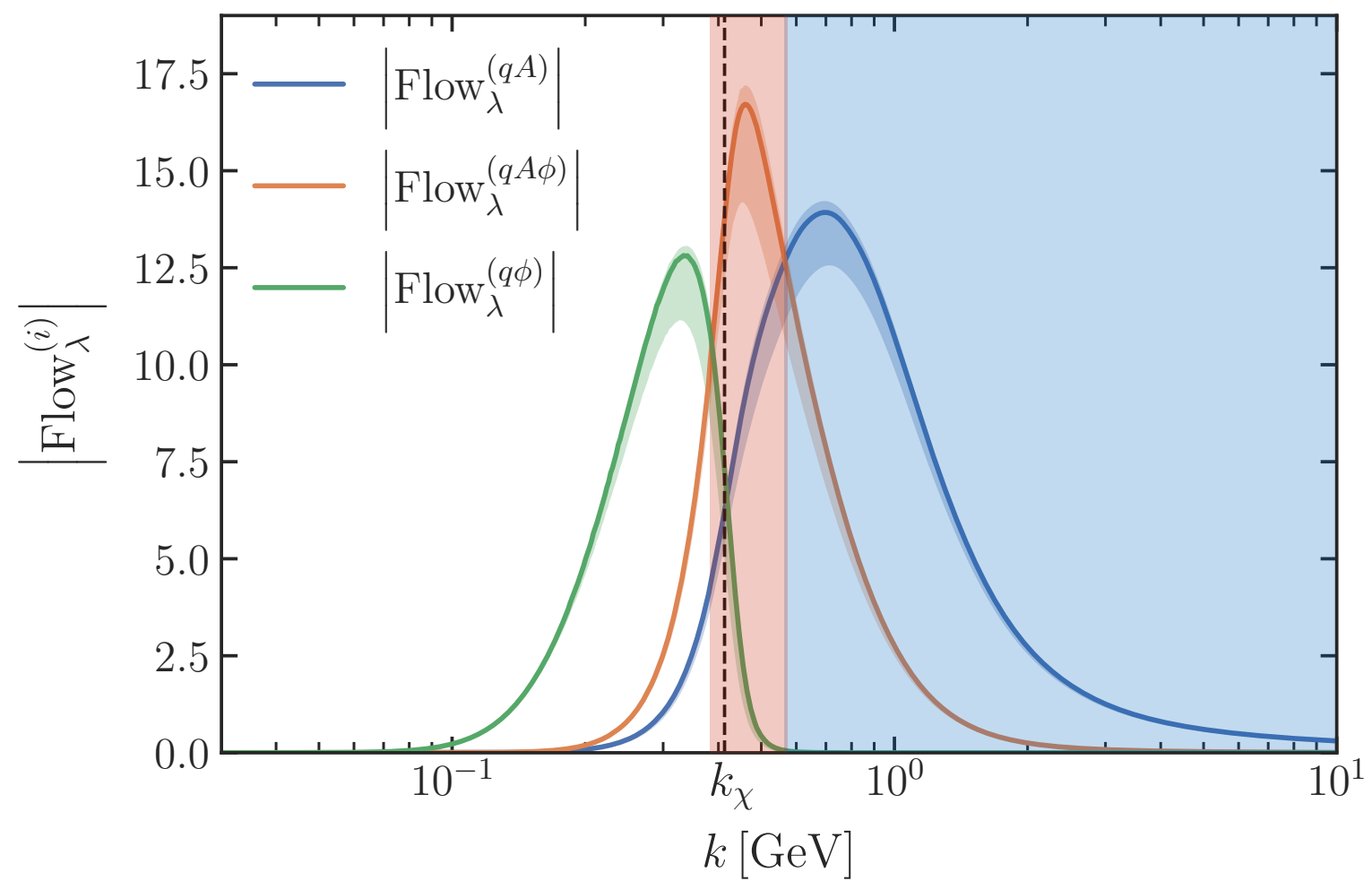
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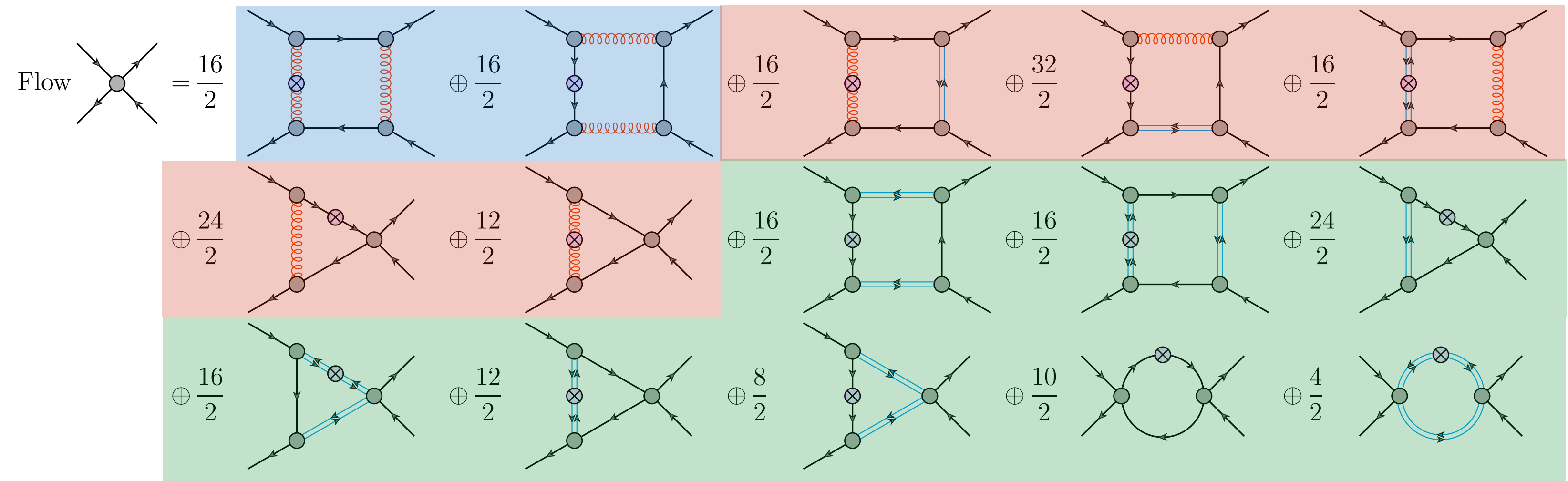
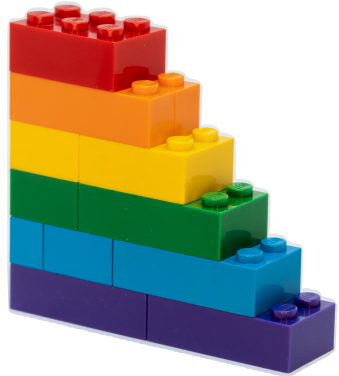
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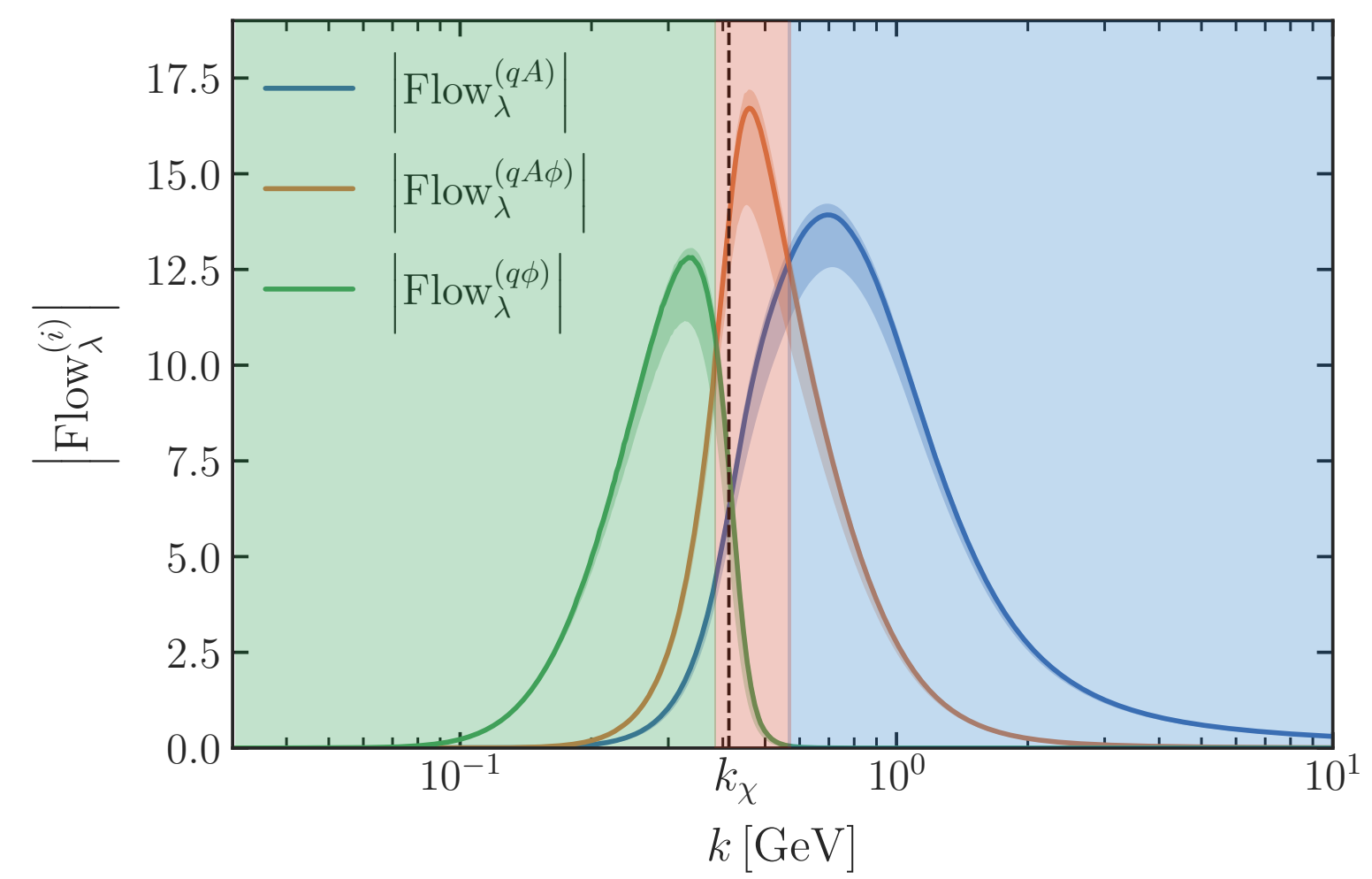
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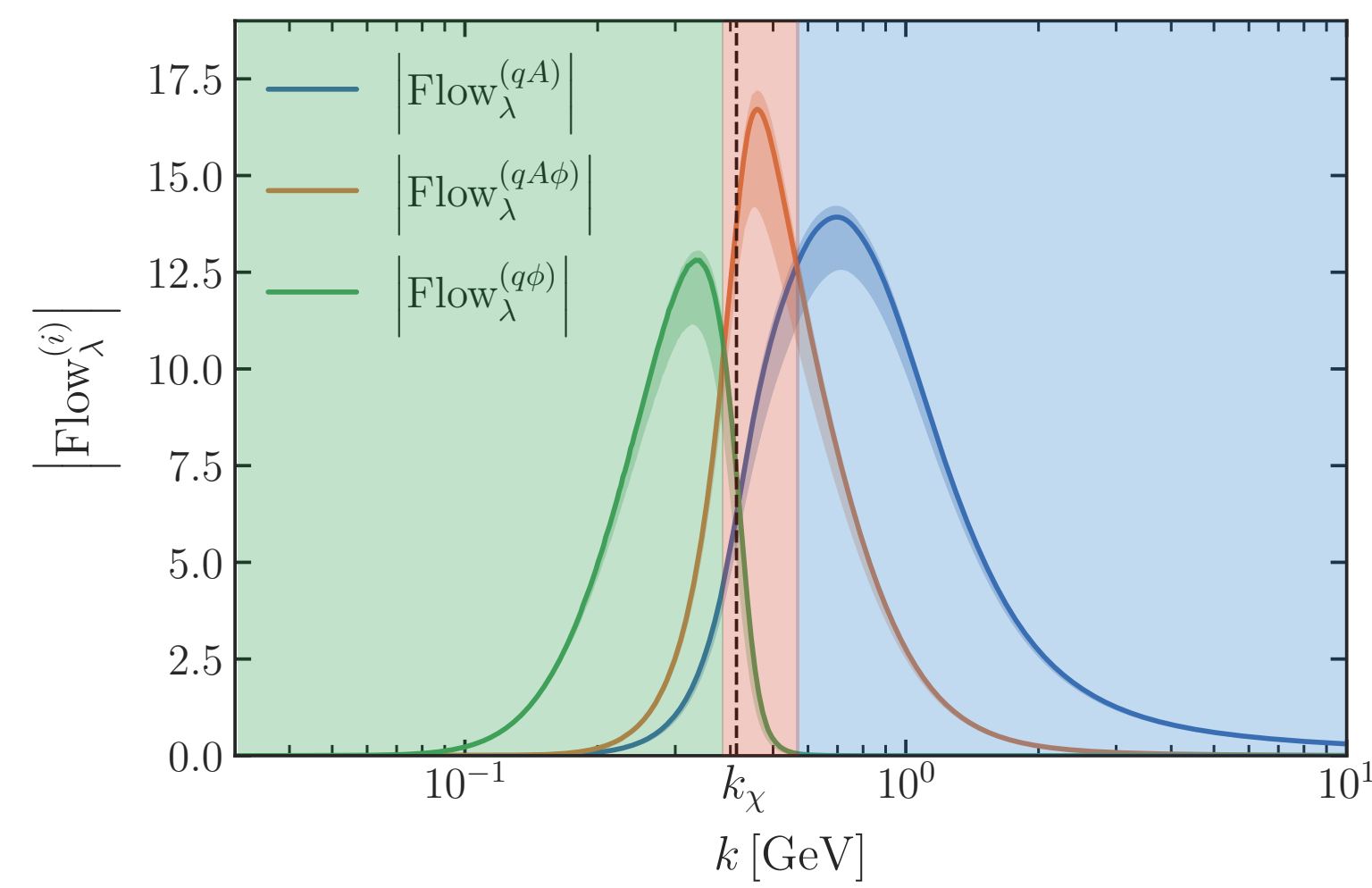
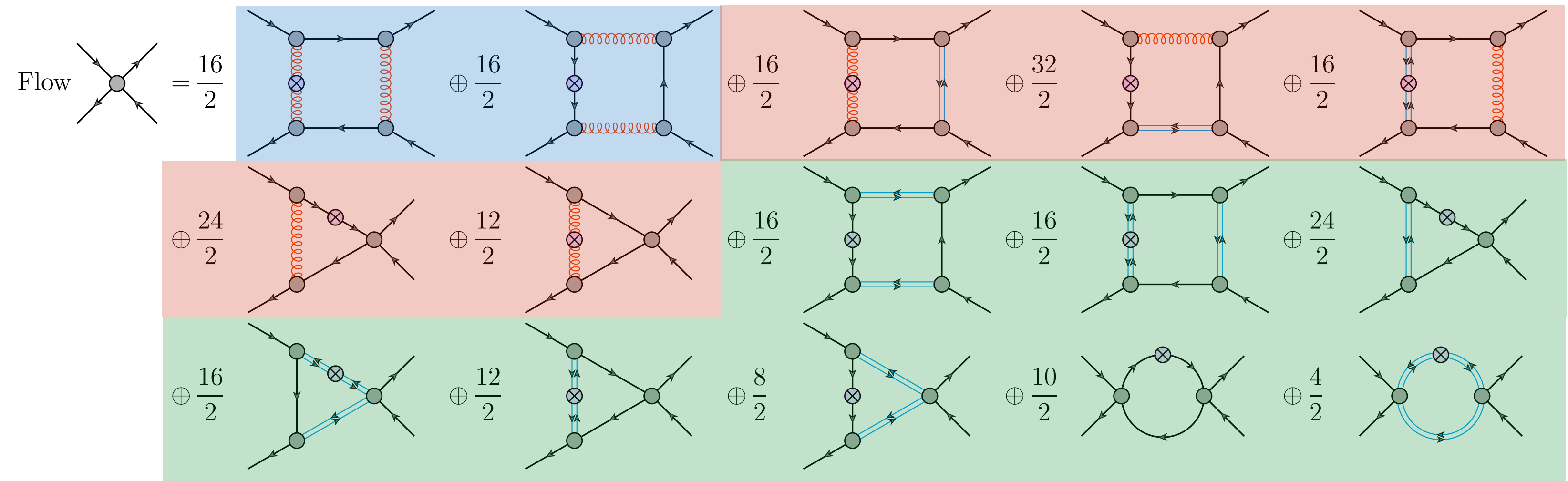
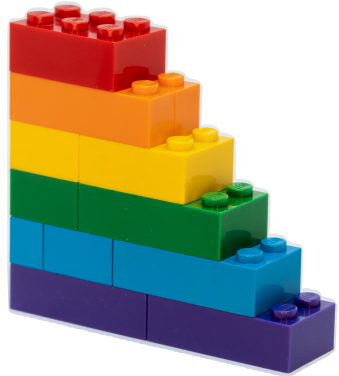
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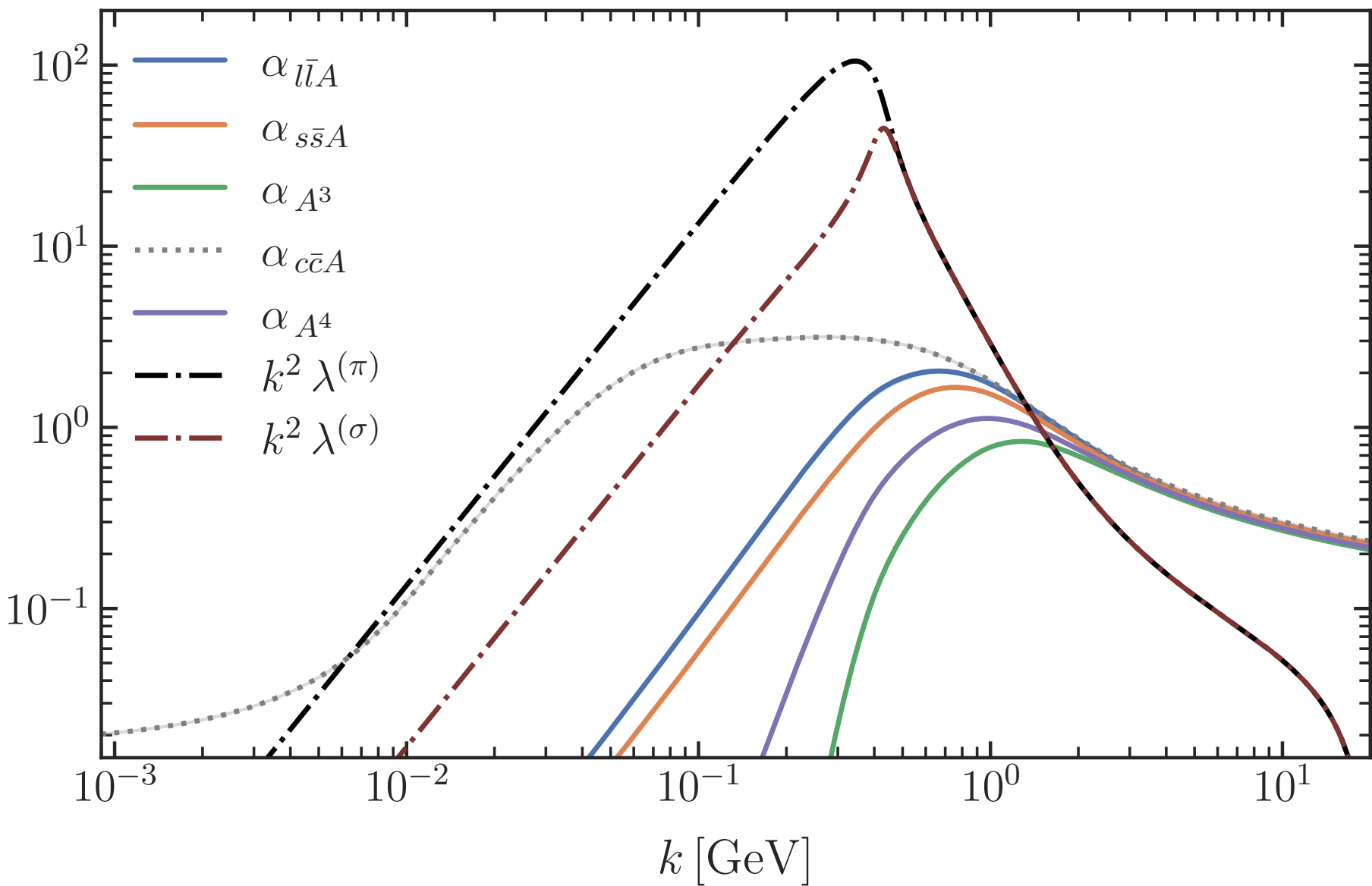
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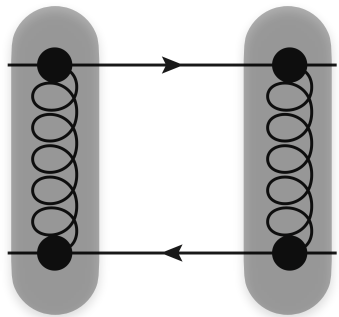
Access and combined use of
error estimates
from functional QCD & LEFTs

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dashed loop} - \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

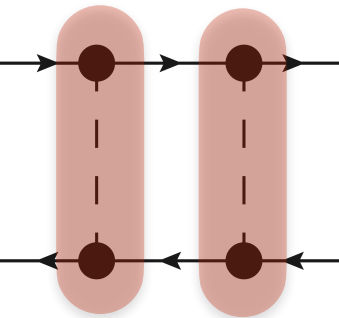
Sequential decoupling of gluon, quark, sigma, pion fluctuations



$$\frac{g_{lAl}^2}{g_{\bar{s}As}^2}$$



$$k^2 \lambda^{(\pi, \sigma)}$$



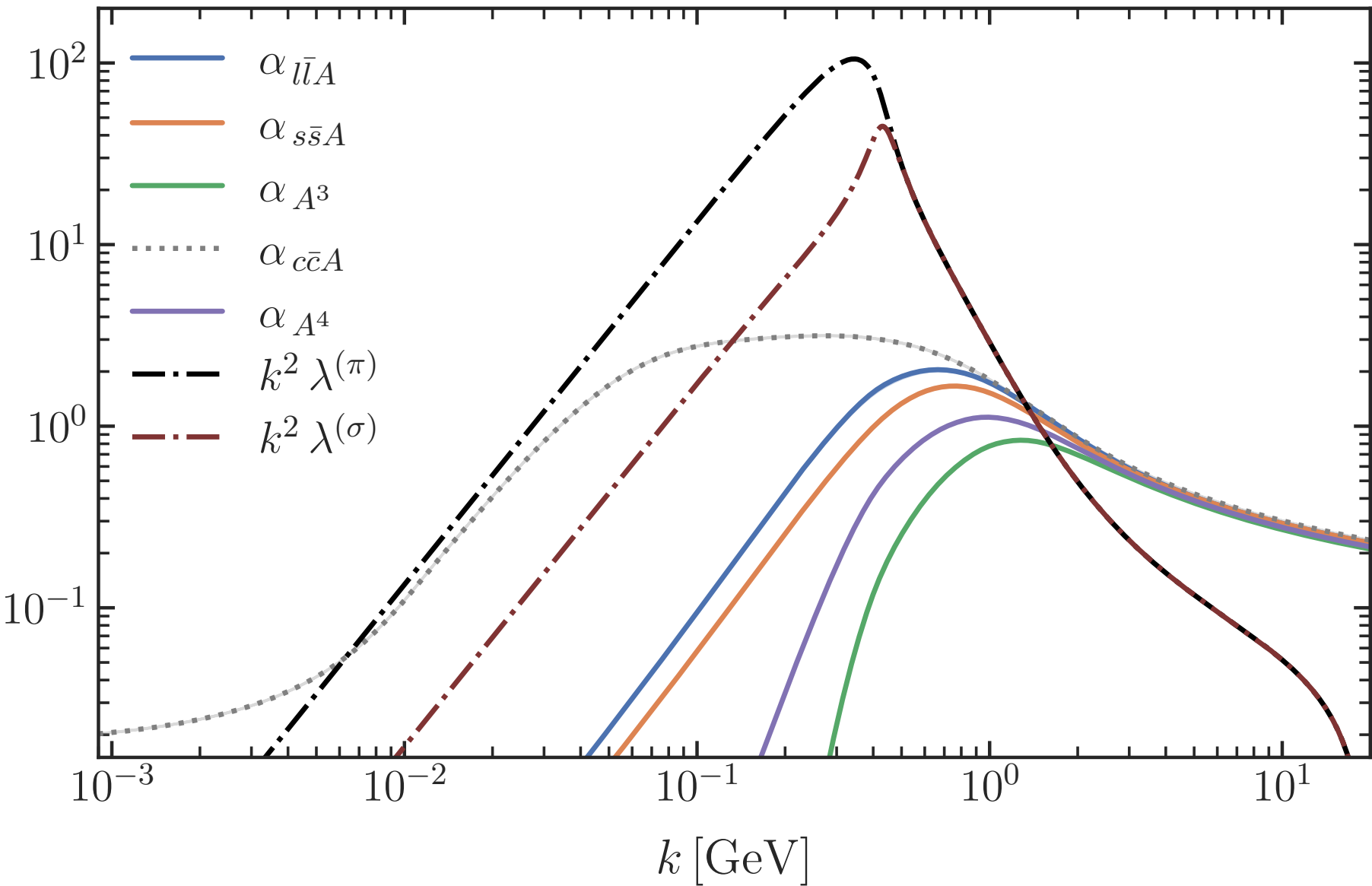
Vacuum: Ihssen, JMP, Sattler, Wink, arXiv:2408.08413

Phase structure: Fu, JMP, Rennecke, PRD 101, (2020) 054032

fQCD

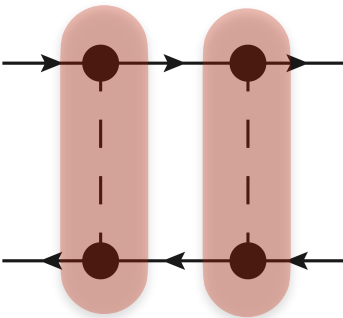
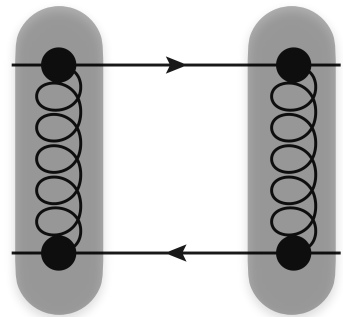
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PQM-model

$$\frac{1}{2} \left(\text{orange loop} - \text{dashed loop} + \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

PNJL-model

$$\frac{1}{2} \left(\text{orange loop} - \text{dashed loop} + \text{solid loop} \right)$$

QM-model

$$- \text{solid loop} + \frac{1}{2} \text{blue loop}$$

NJL-model

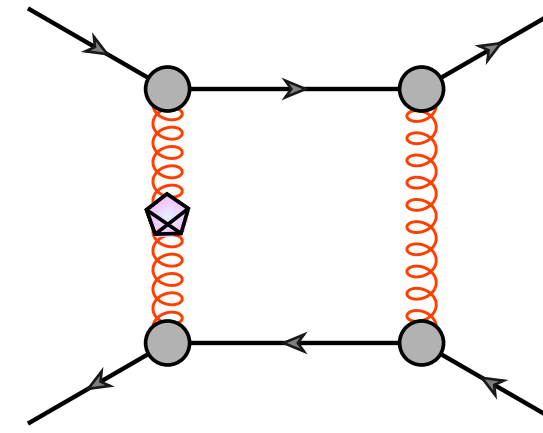
$$- \text{solid loop}$$

QCD-assisted low energy effective theories

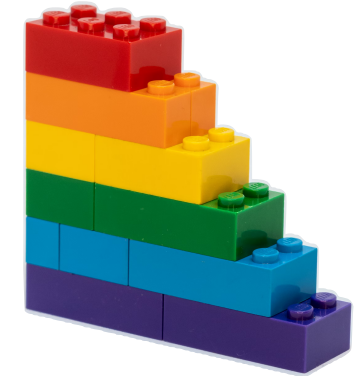
How to: systematic error estimates & the LEGO® principle



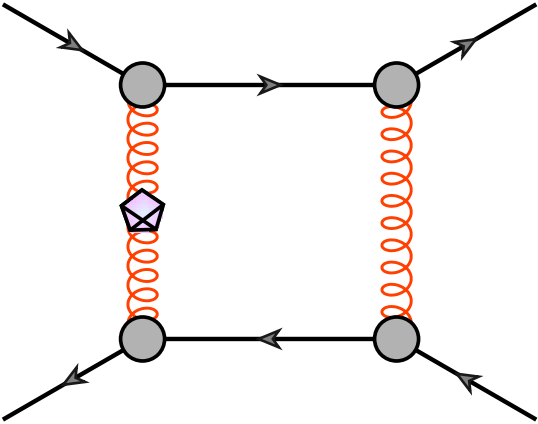
Example: Disect quark-gluon box



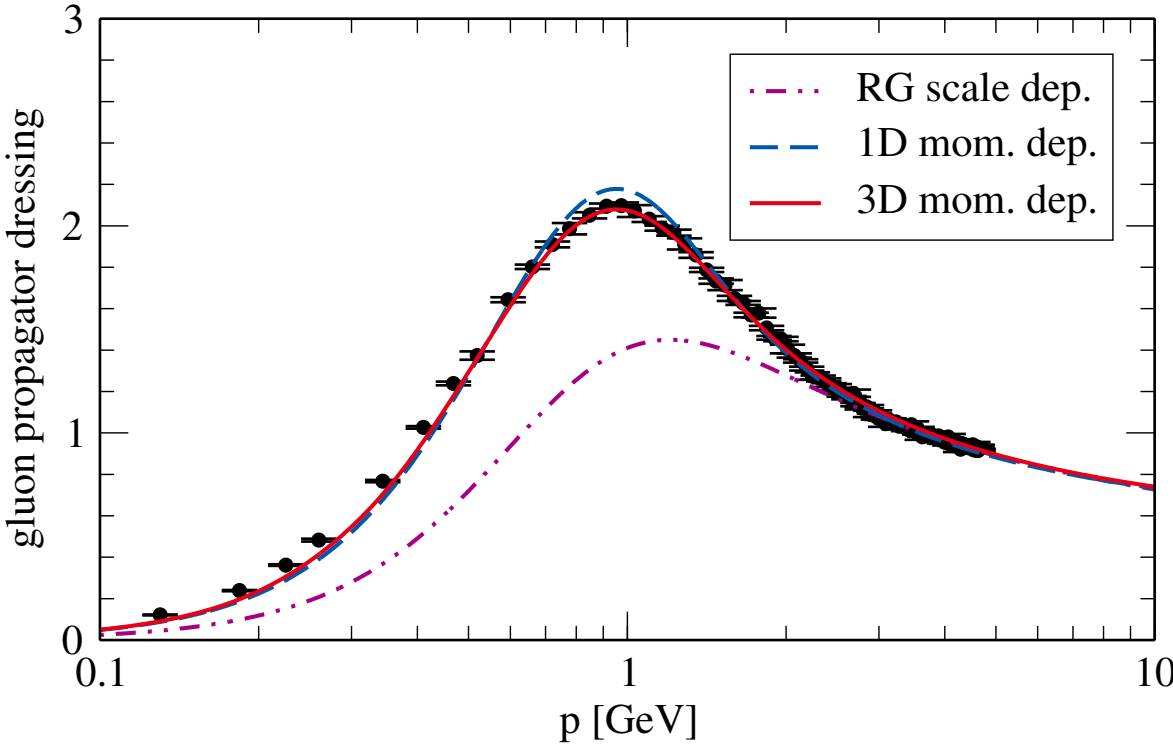
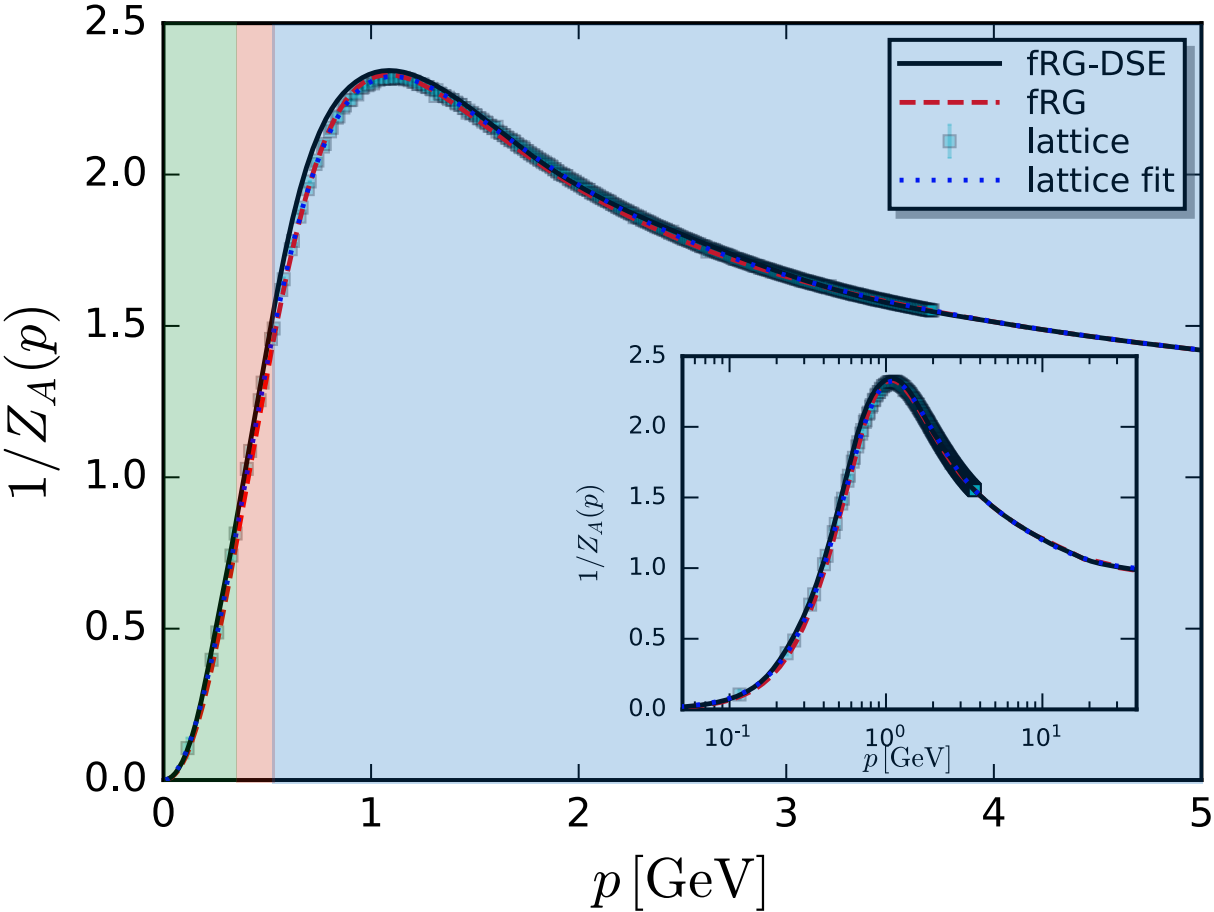
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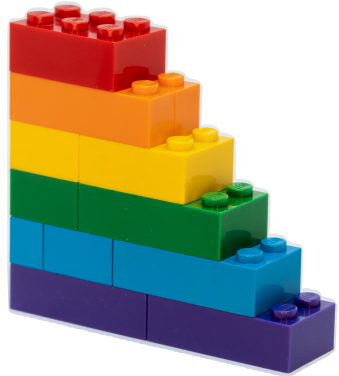
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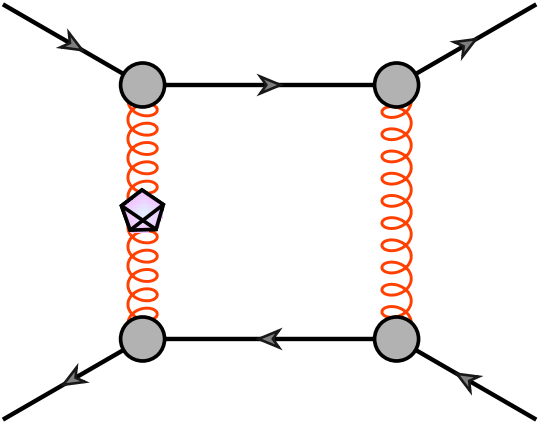
gluon two-point correlator



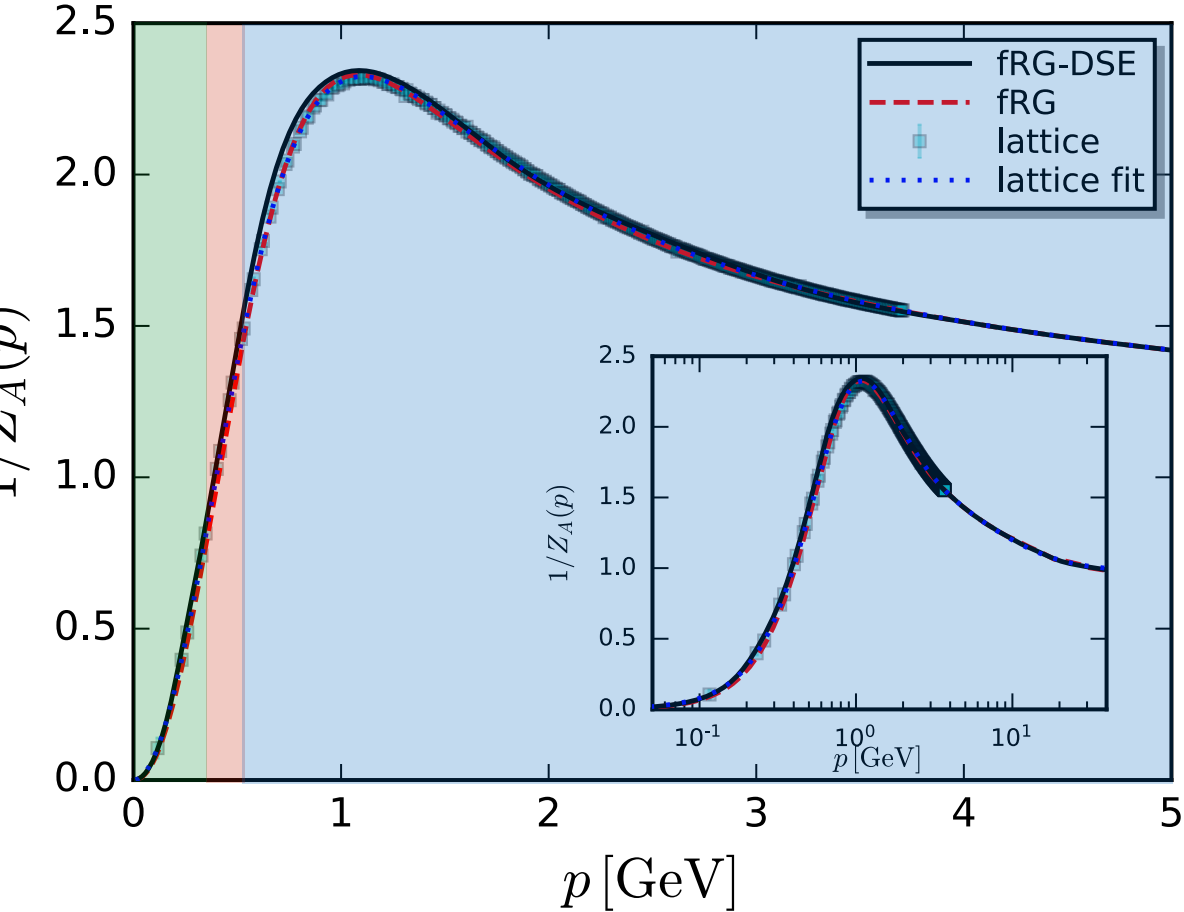
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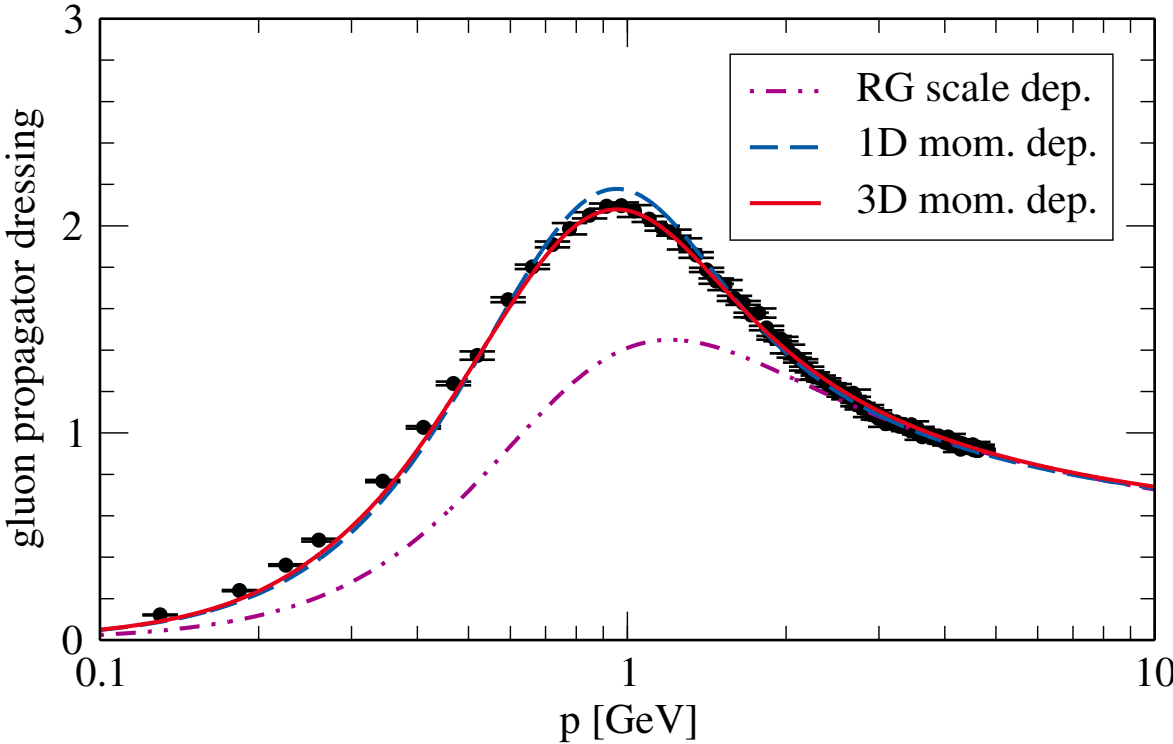
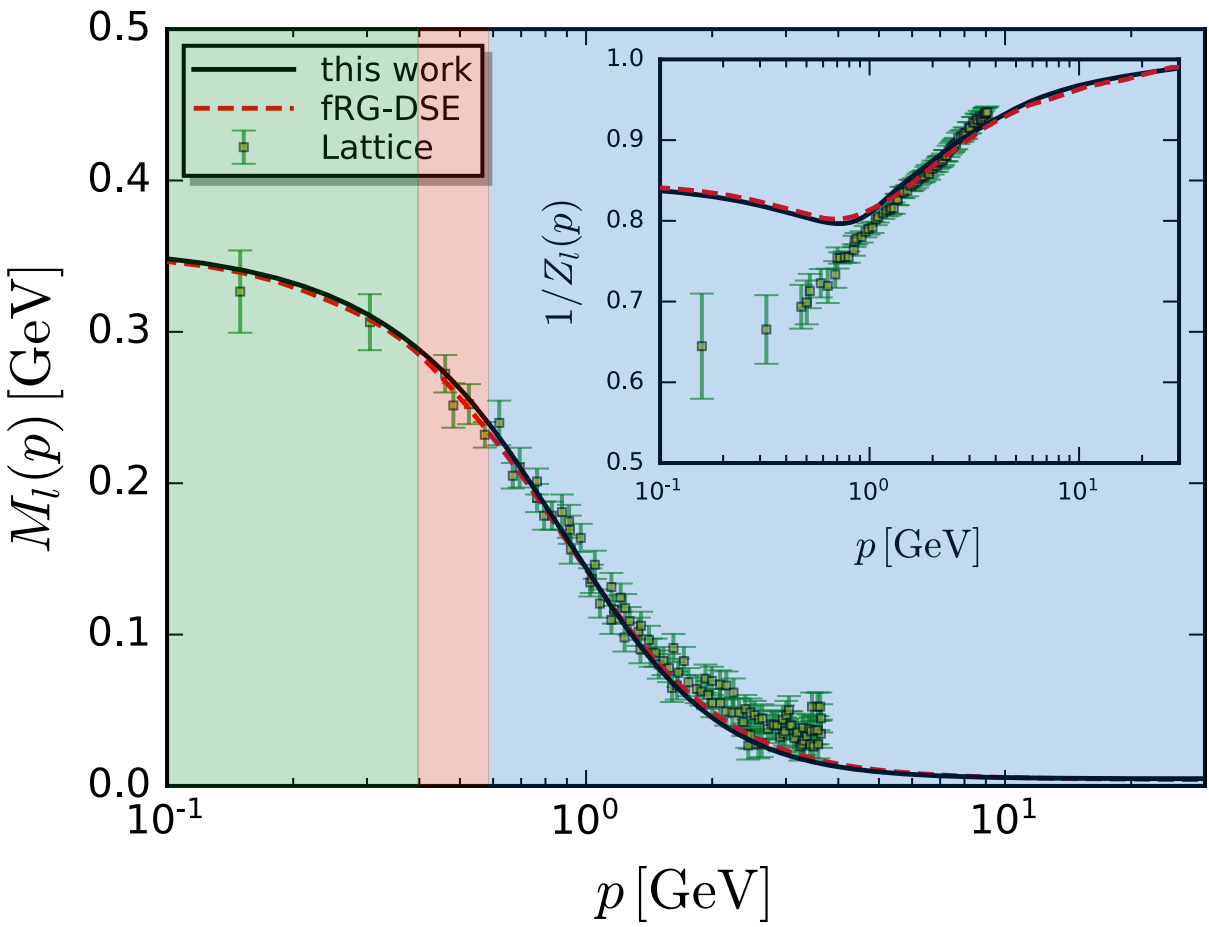
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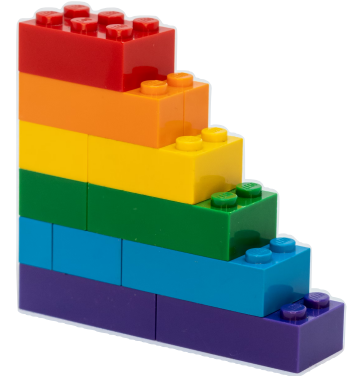
gluon two-point correlator



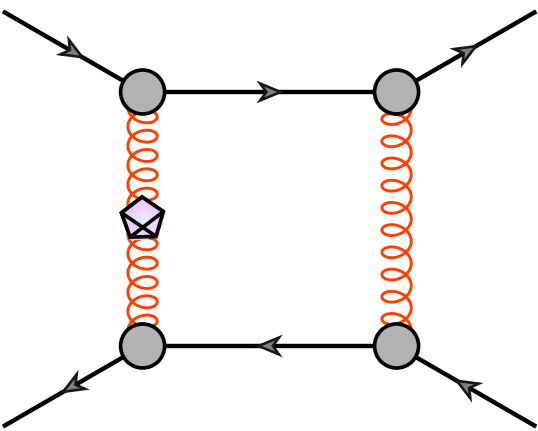
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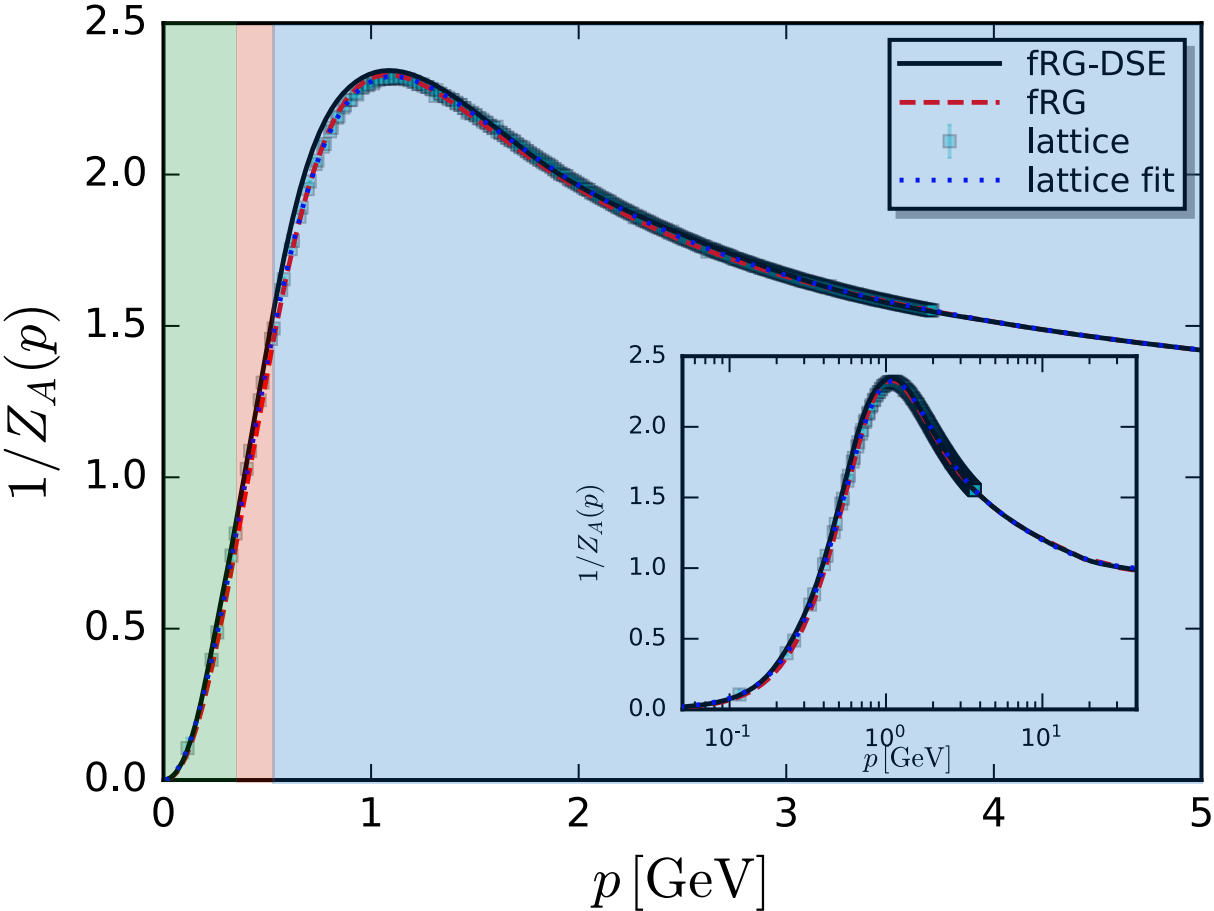
How to: systematic error estimates & the LEGO[®] principle



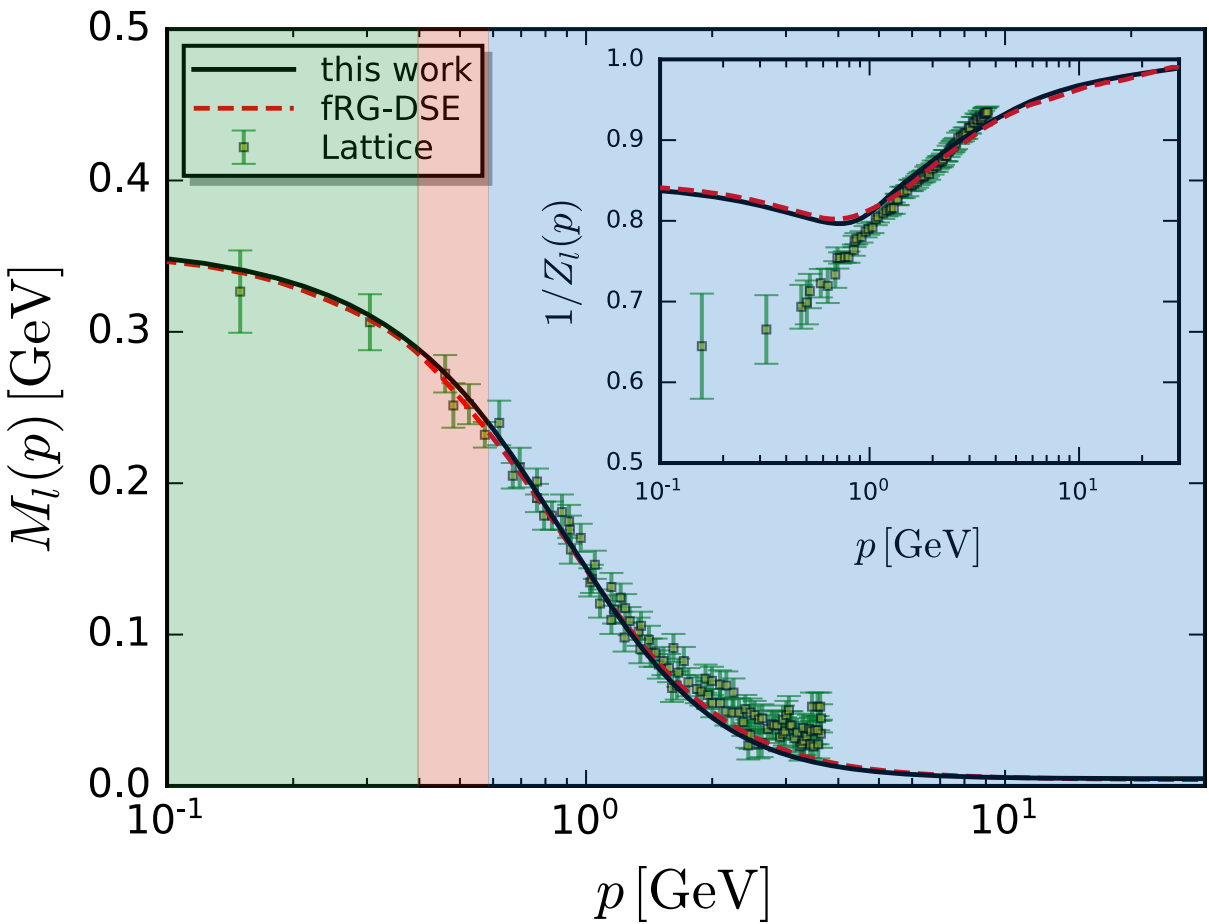
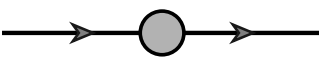
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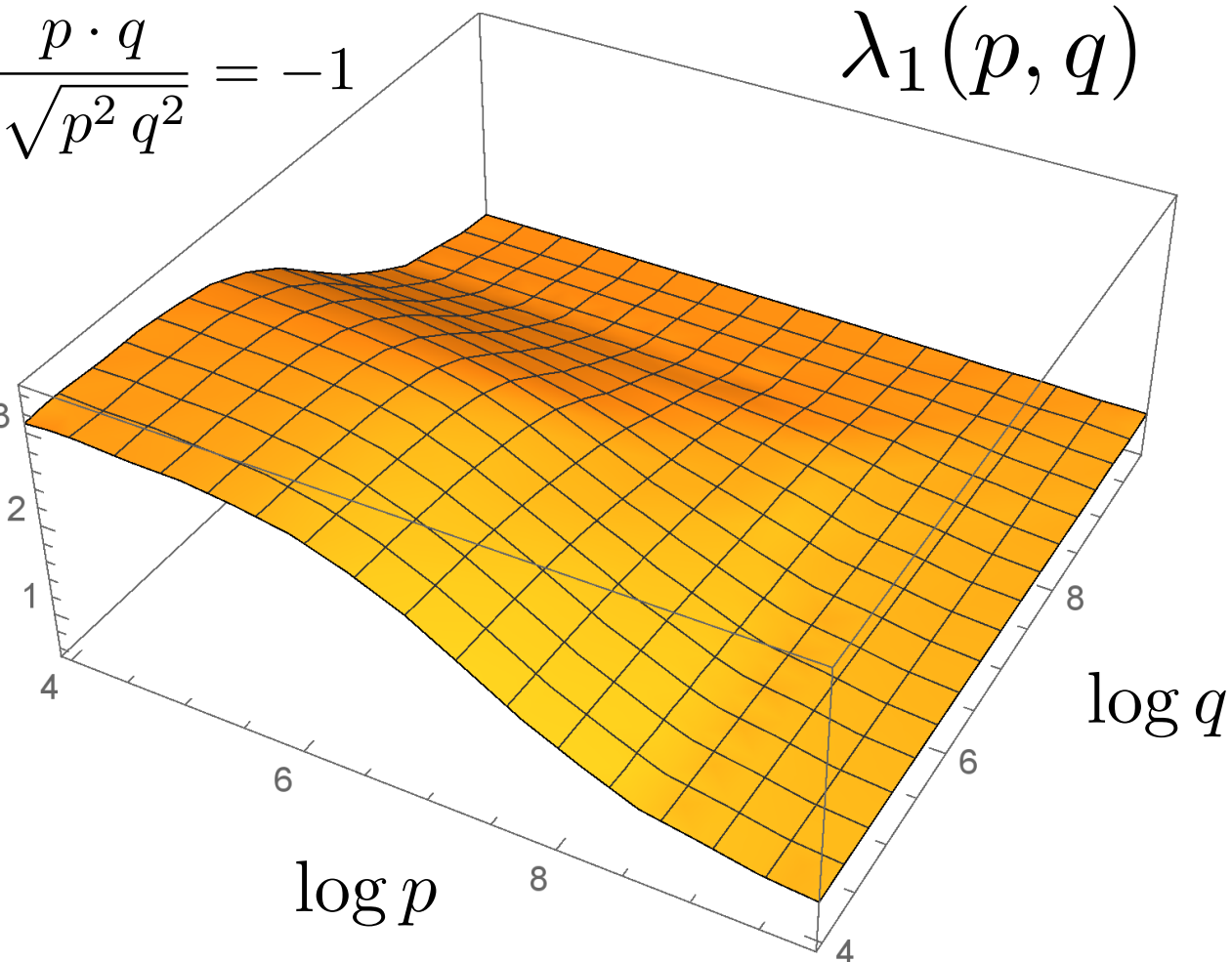
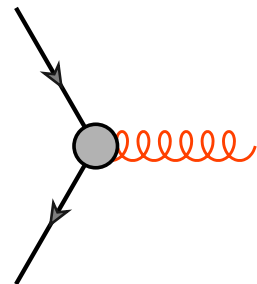
gluon two-point correlator



quark two-point correlator

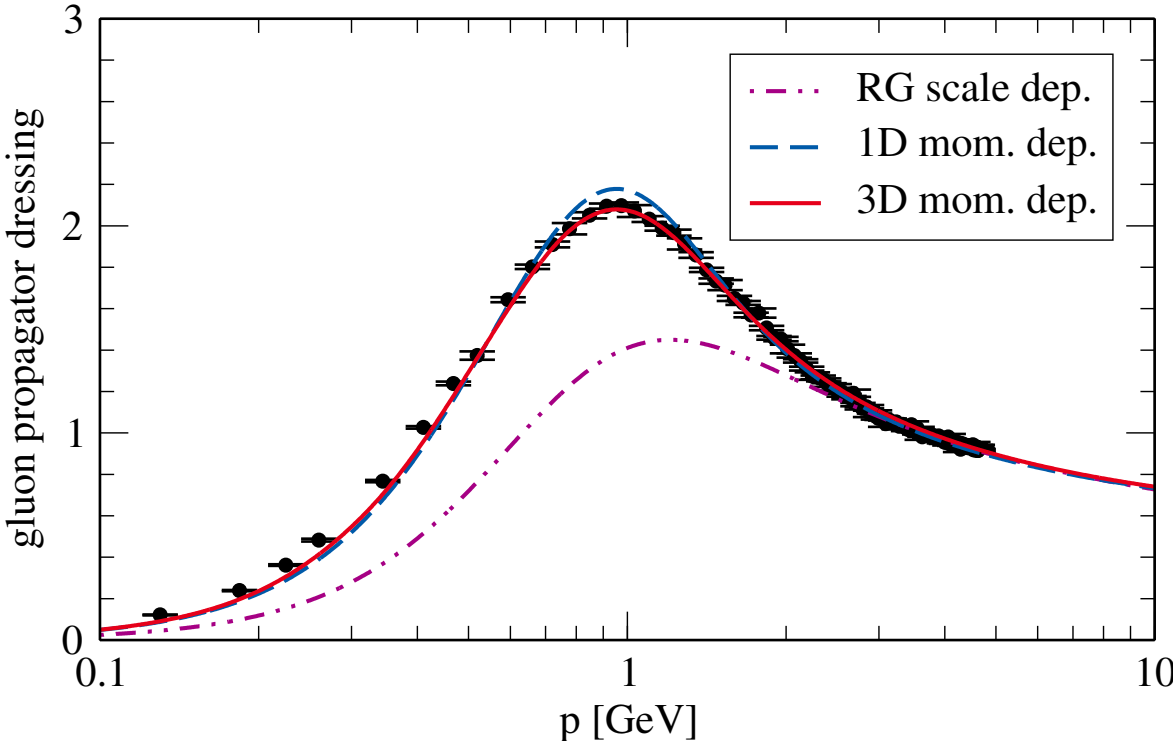


quark-gluon scattering



$$\lambda_1(p, q), \dots, \lambda_8(p, q)$$

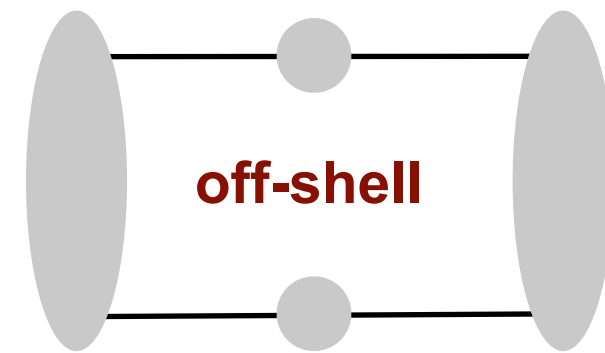
Relevant: $\lambda_1, \lambda_4, \lambda_7$



Three remarks on Functional Methods for QCD

- off-shell representation of thermodynamic observables

e.g. $\text{Tr} \langle q(x) \bar{q}(x) \rangle$

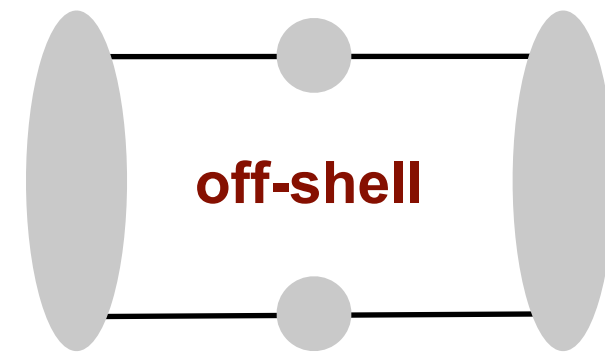


**pressure, trace anomaly,
fluctuations, volume flucs., ...**

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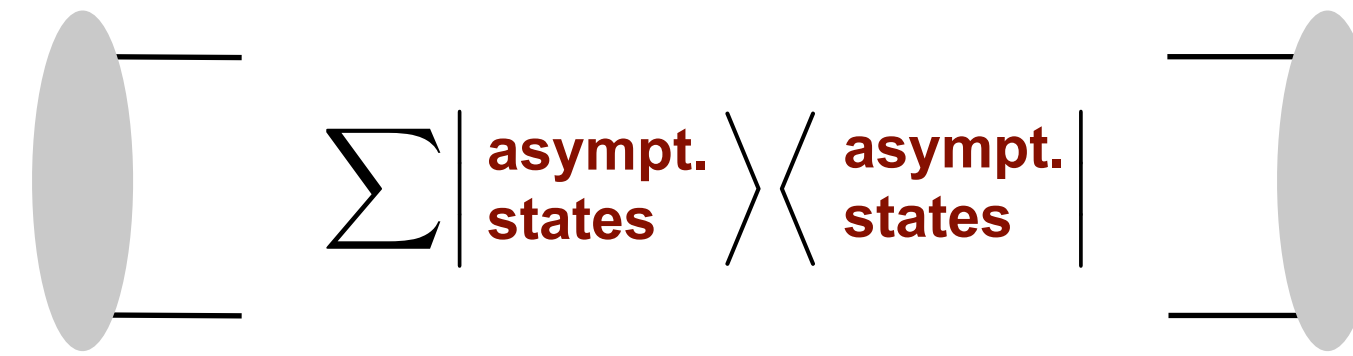
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on-shell

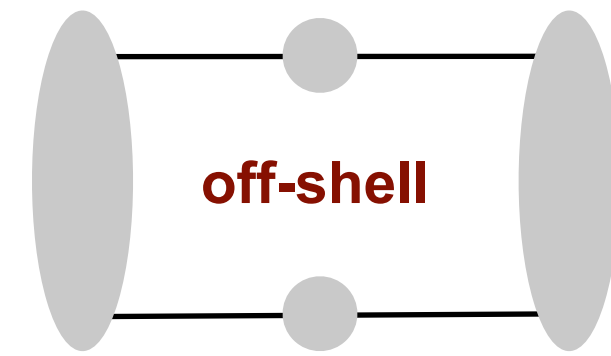


e.g. hadron resonances

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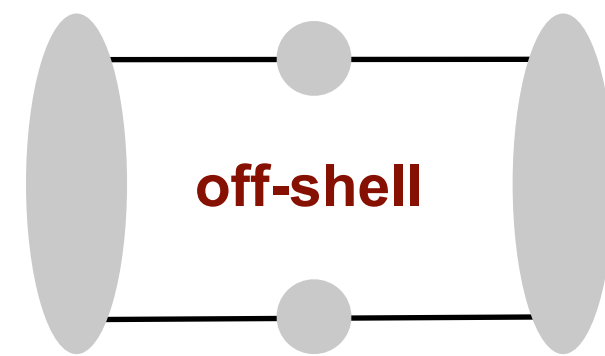
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on-shell



e.g. hadron resonances

- gauge fixing = parameterisation

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$

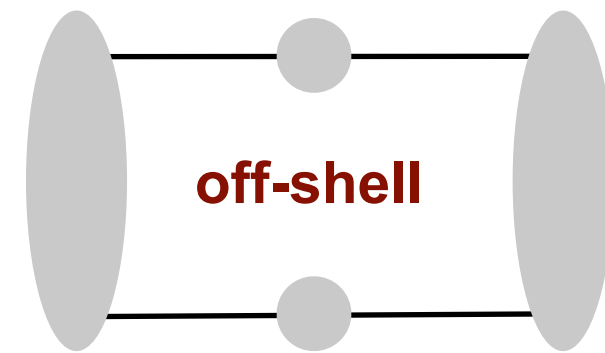
Consequences

I: simple correlations

Three remarks on Functional Methods for QCD

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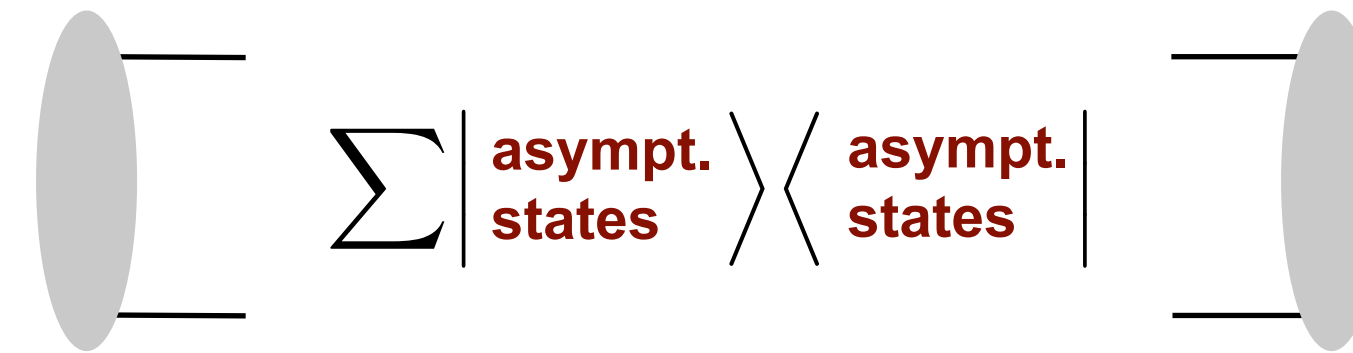
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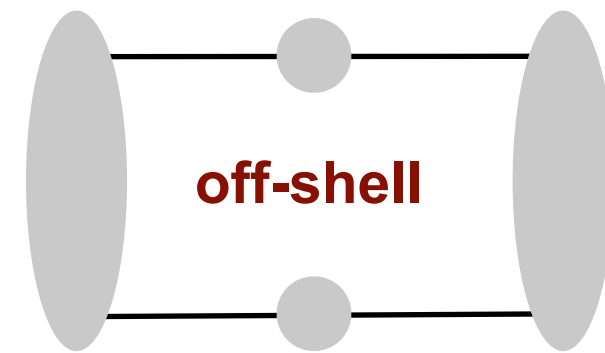
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II: Difficult access to some observables
'No free lunch theorem'

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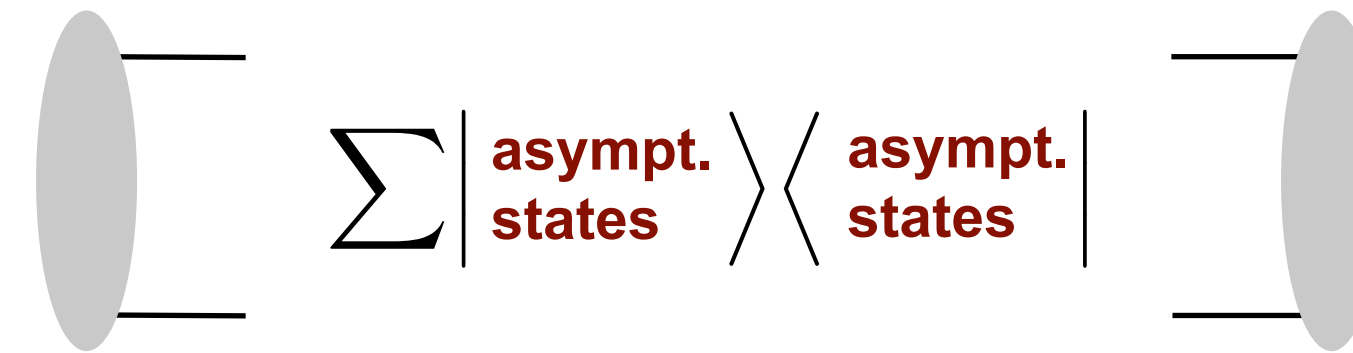
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Consequences

I: simple correlations

II: Difficult access to some observables
'No free lunch theorem'

- 'Your mean field is not my mean field'

$$\left. \frac{\delta S_{\text{cl}}[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}} = 0$$

$$\left. \frac{\delta \Gamma[\phi]}{\delta \phi} \right|_{\phi=\bar{\phi}_{\text{quant}}} = 0$$

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \left\langle q(x) \bar{q}(y) \right\rangle_{\mu_B} = \text{Loop} \left[\left\langle q(x) \bar{q}(y) \right\rangle_{\mu_B}, \left\langle q(x) A_\mu(y) \bar{q}(z) \right\rangle_{\mu_B}, \dots ; \mu_B \right]$$

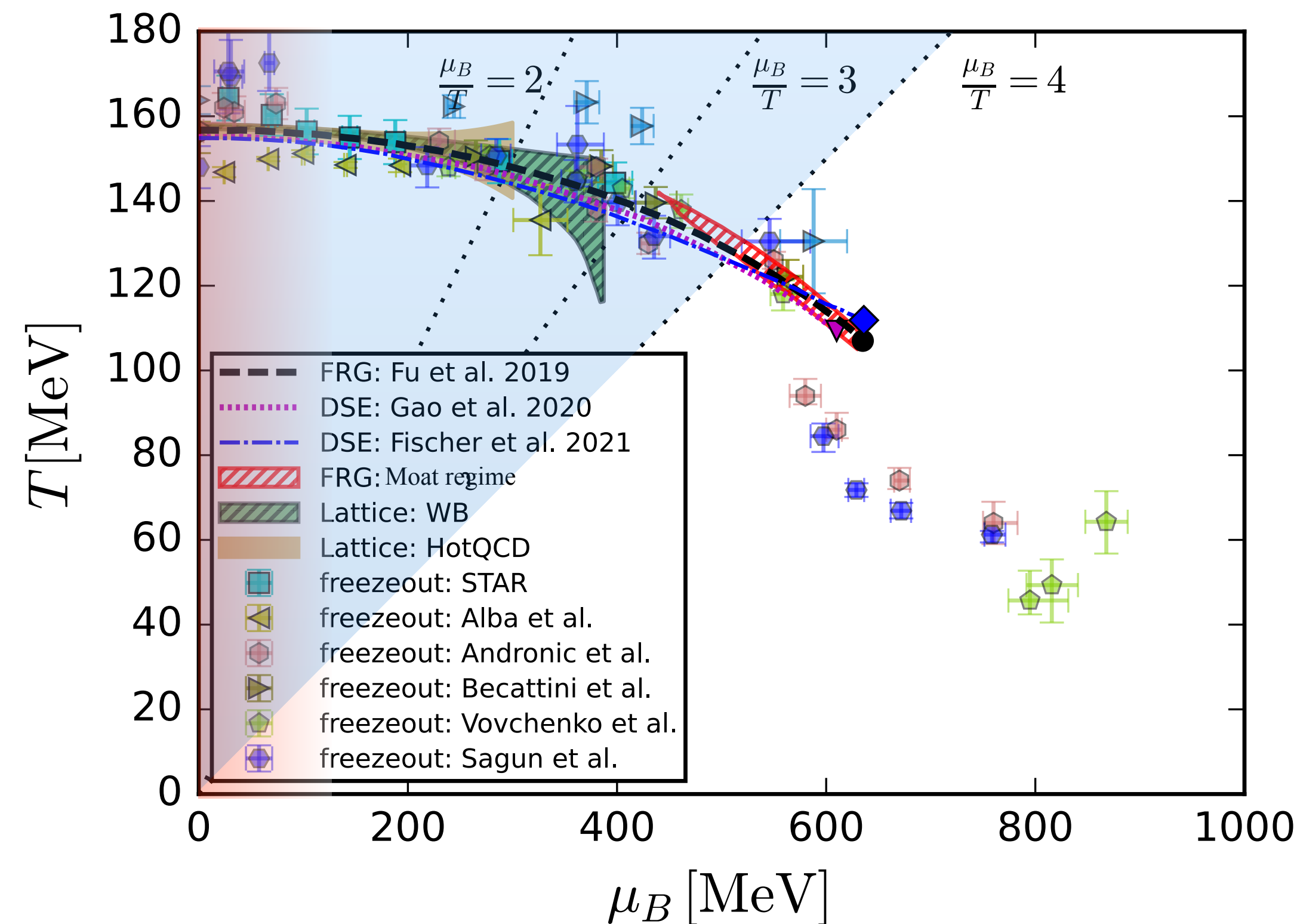
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- Consequences for functional QCD predictions at finite density



By now the best truncations to functional QCD pass lattice benchmark tests at vanishing and small chemical potential

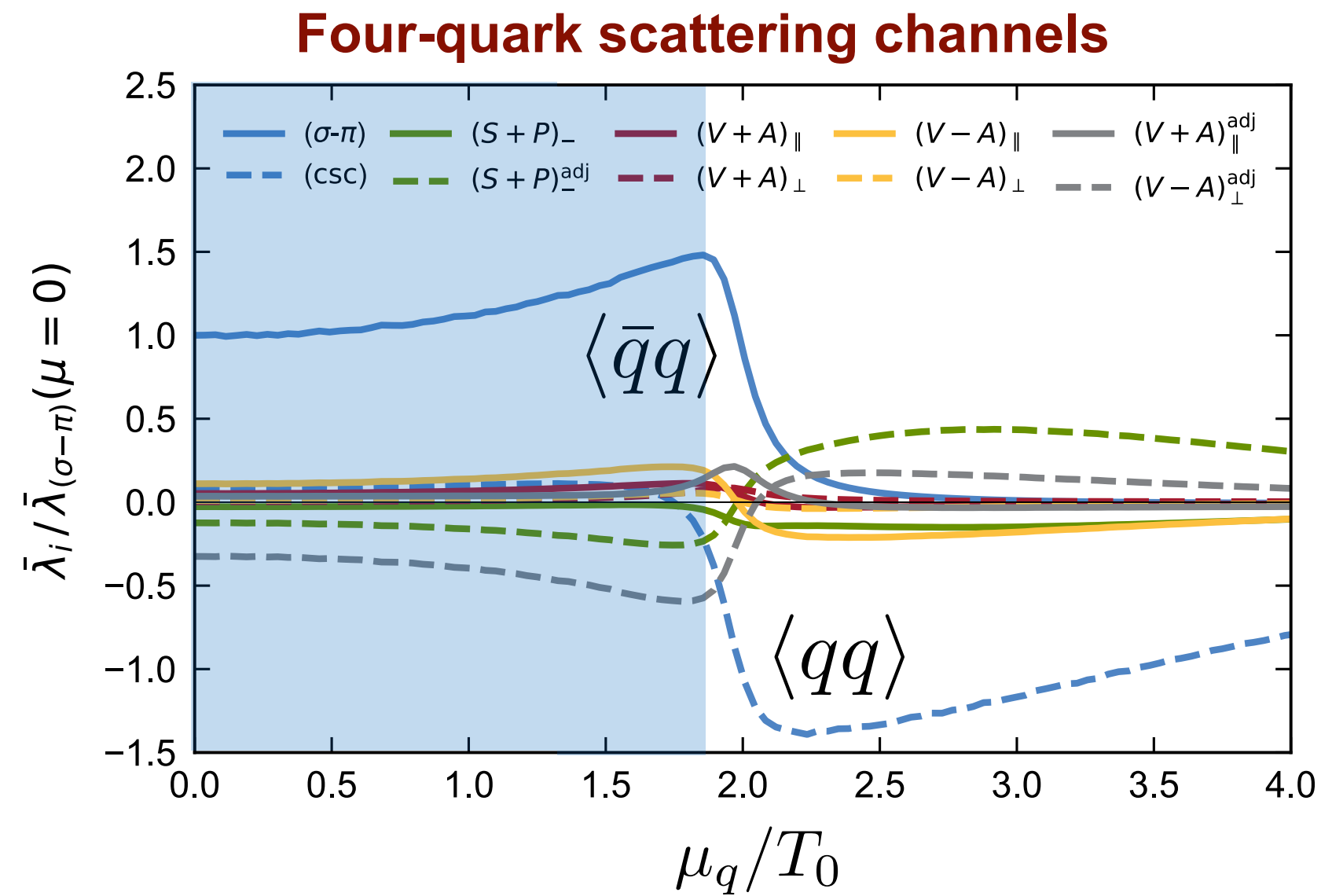
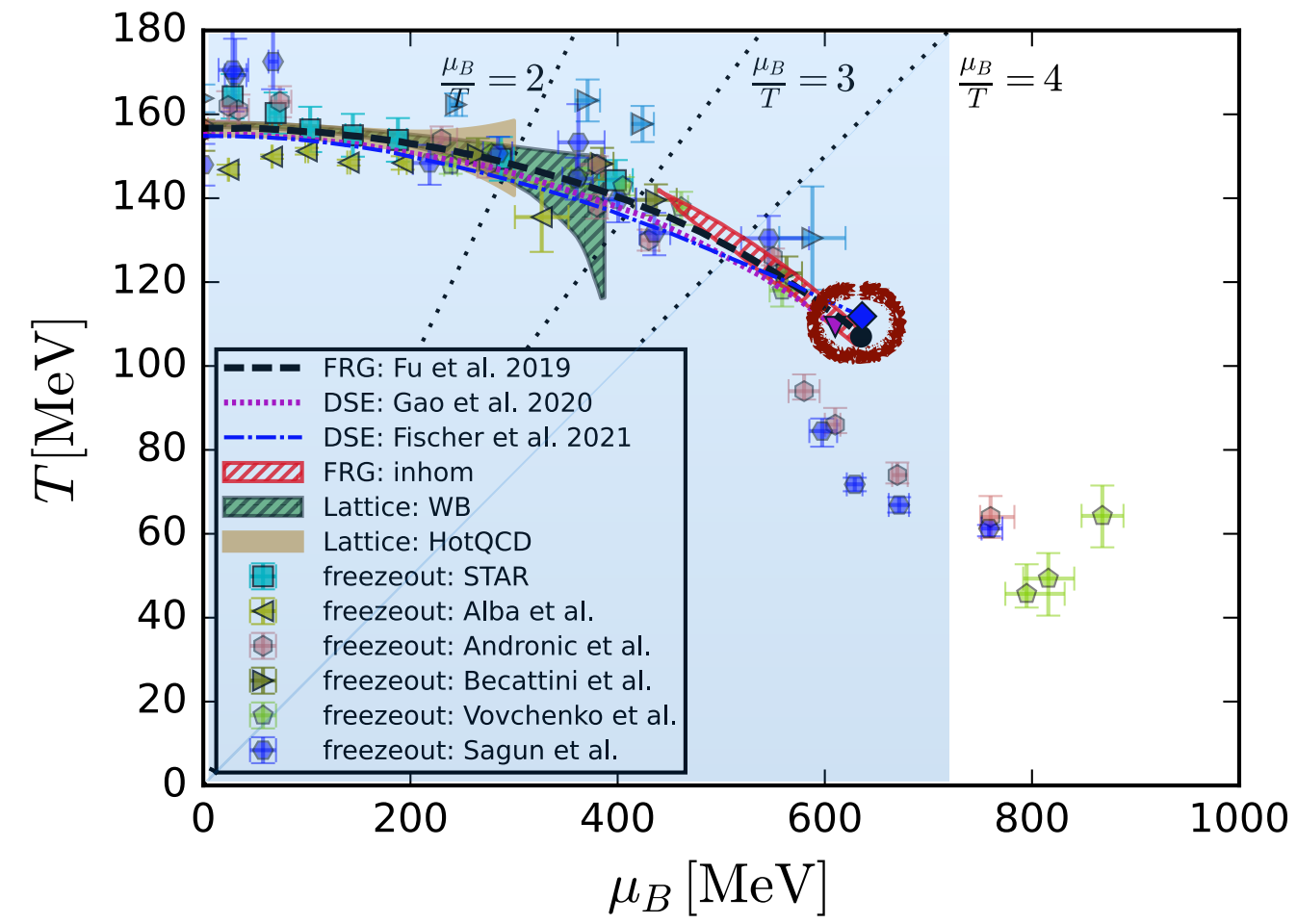
Regime of quantitative reliability of current best truncation

Unique: QCD-based analytic continuations that satisfy the lattice benchmarks at small chemical potential.

Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

Predictions & estimates

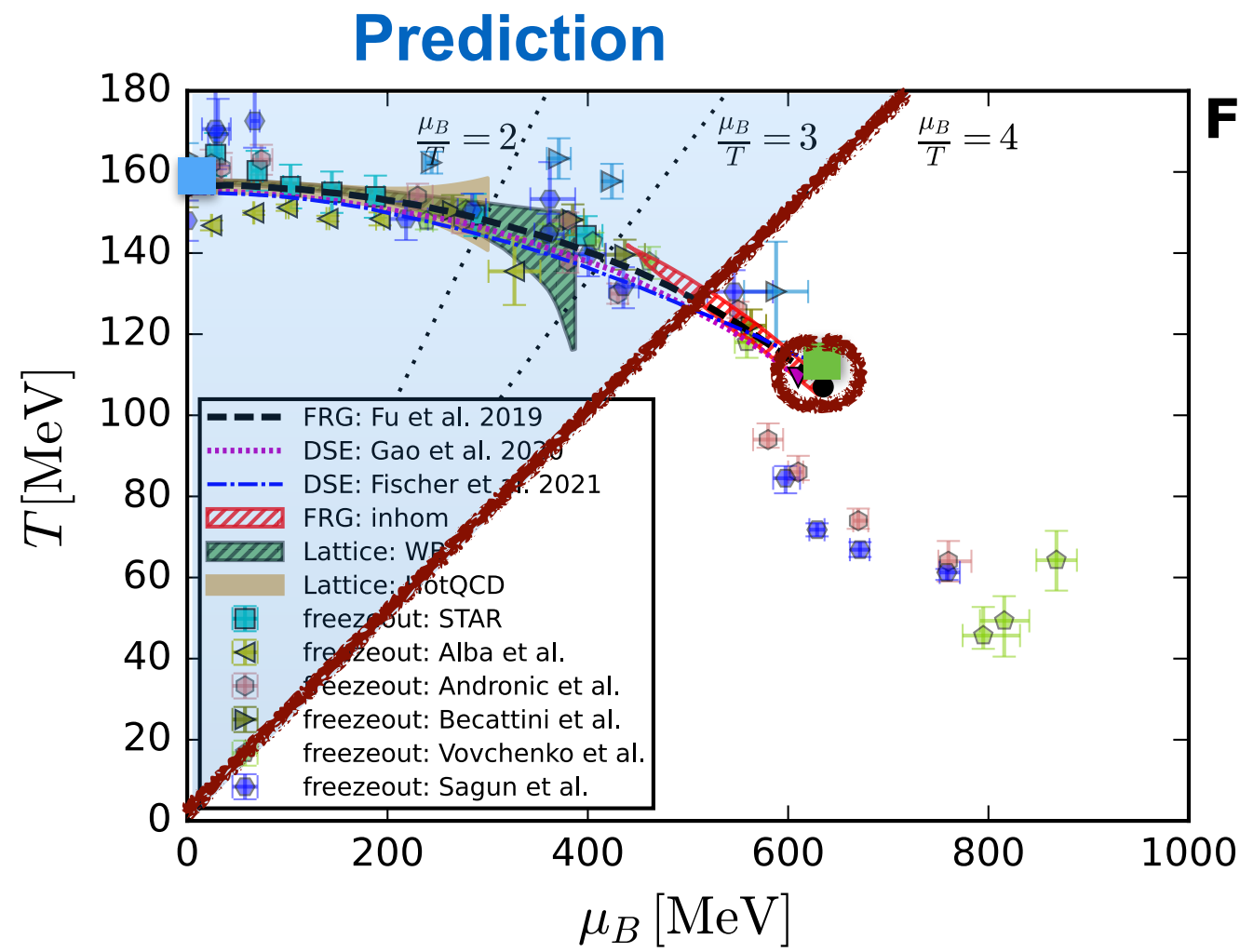


Dominance of scalar-pseudoscalar fluctuations
Pions & sigma mode

Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302



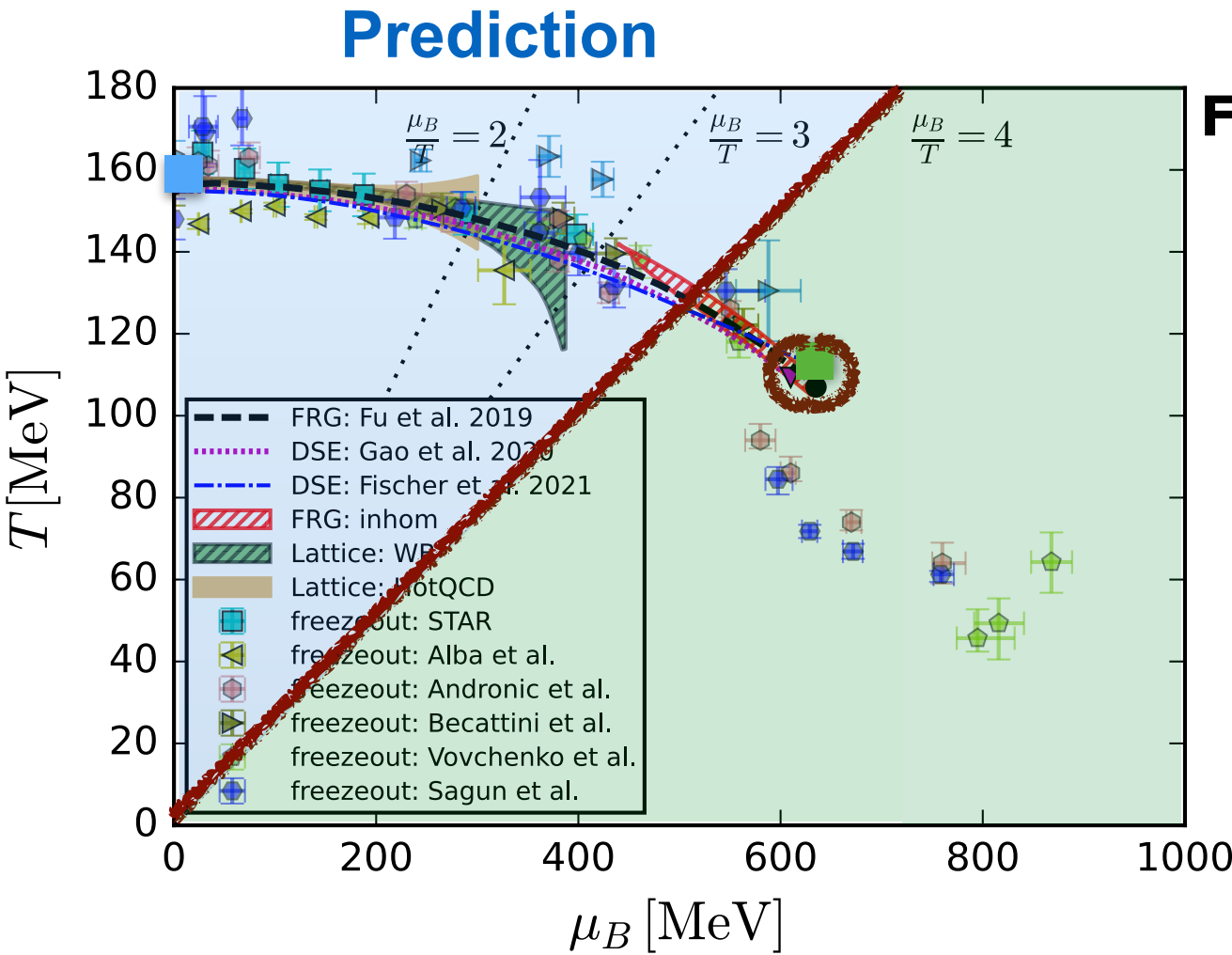
Fu, JMP, Rennecke, PRD 101 (2020) 054032

**Regime of quantitative reliability
of
current best truncation**

Predictions & estimates

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Pisarski, Rennecke, PRL 127 (2021) 152302



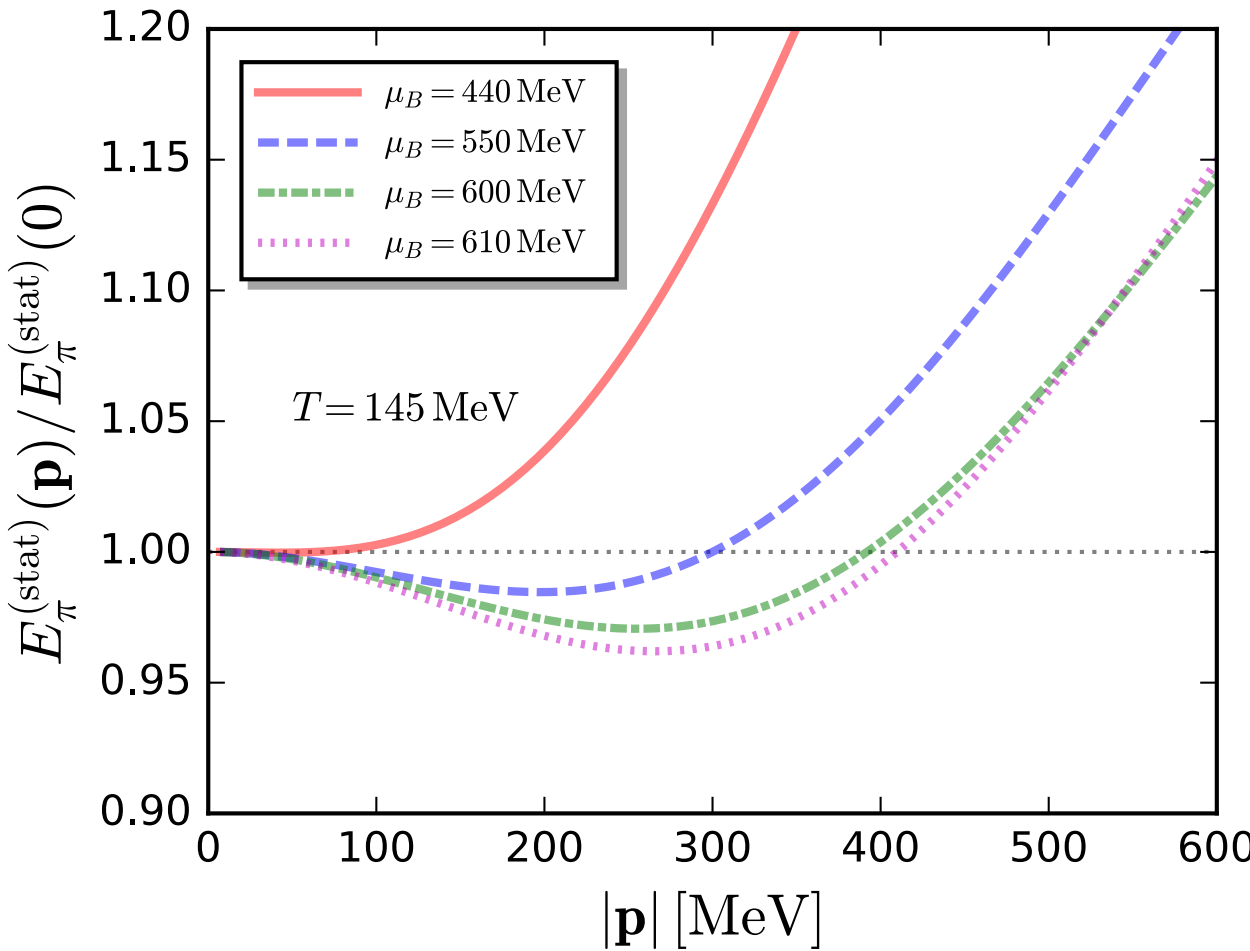
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Regime of quantitative reliability
of
current best truncation

Estimate

Moat regime is not captured quantitatively



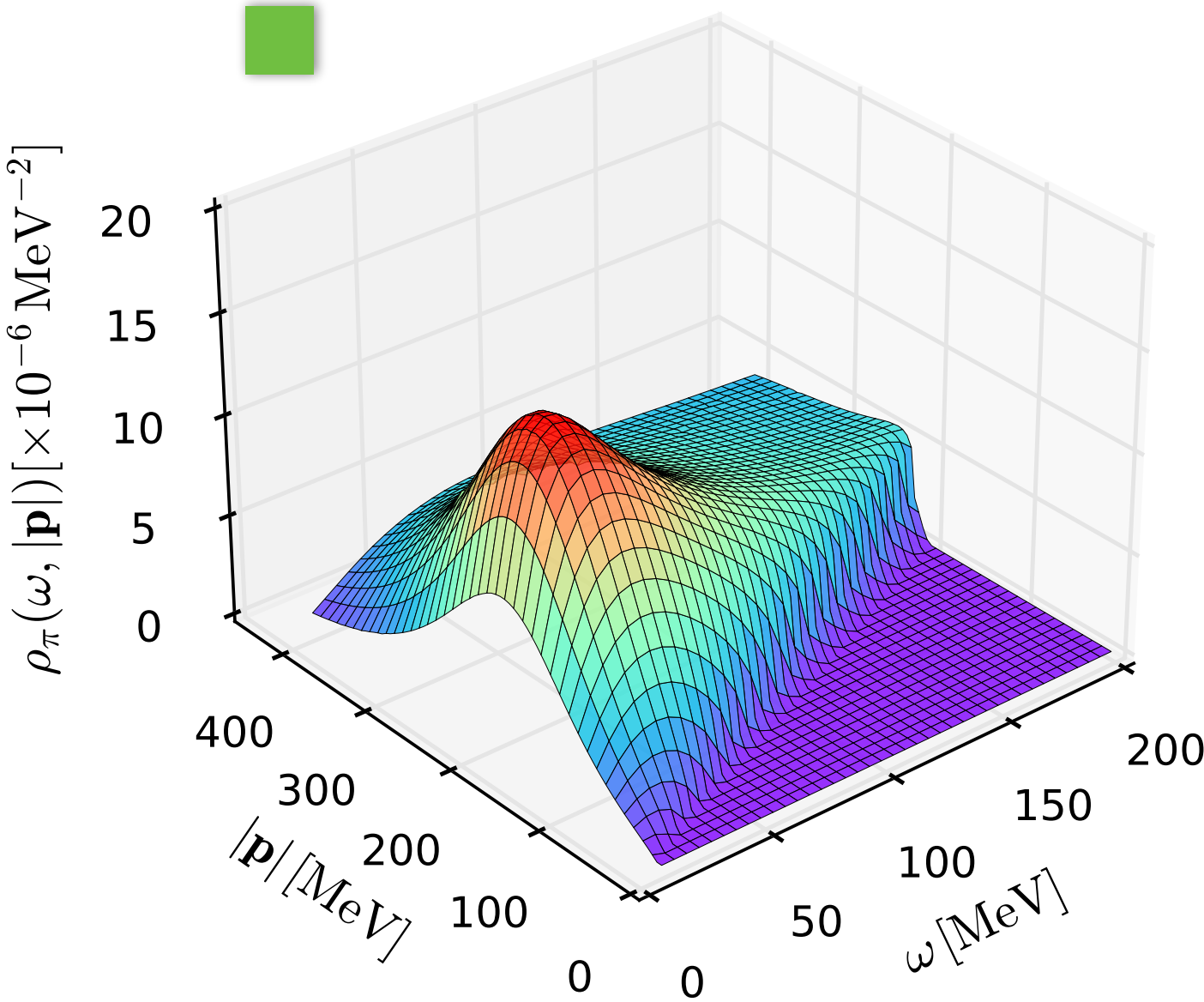
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

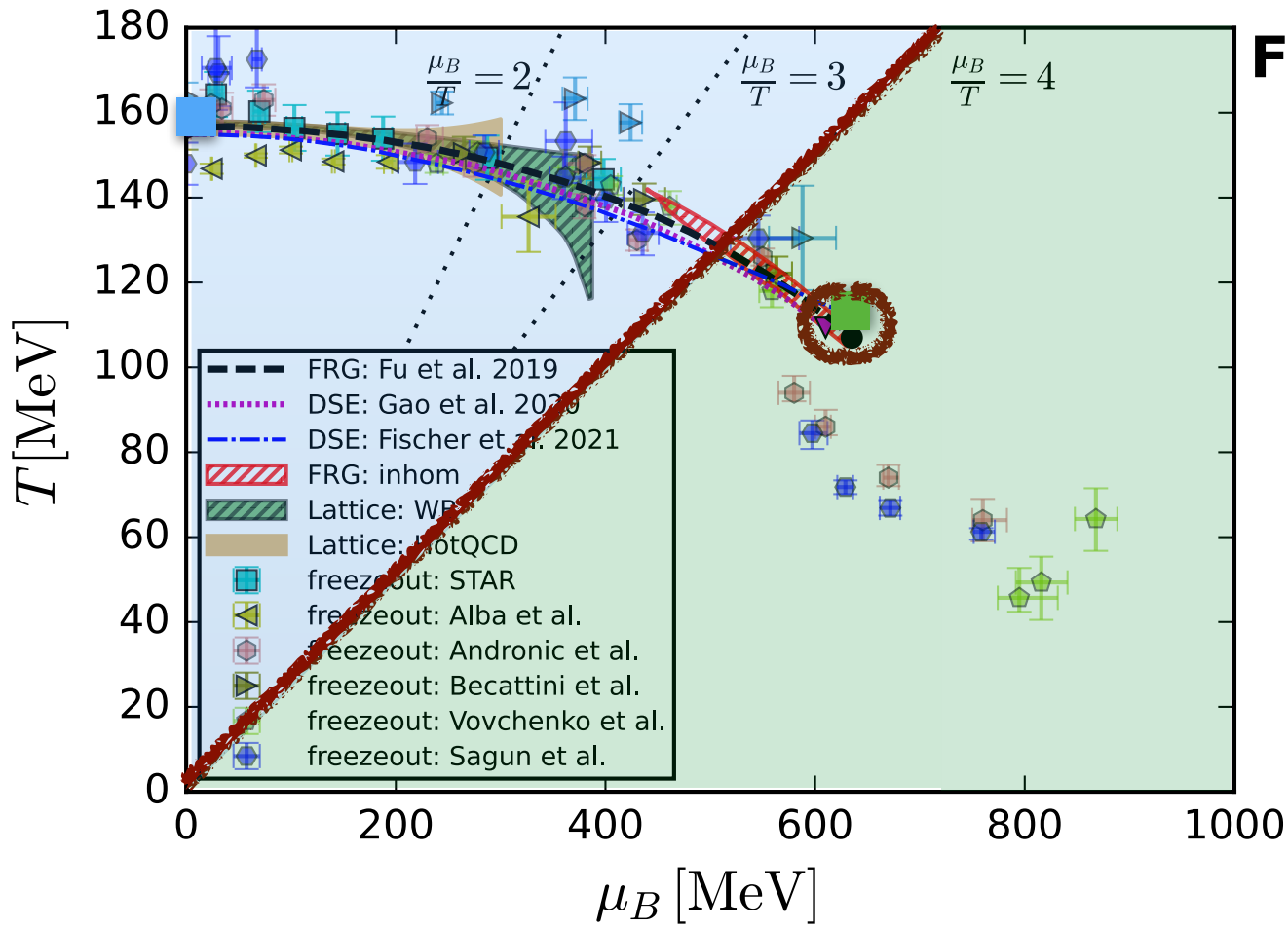
Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

T=114 MeV & μ_B =630 MeV



Prediction



Fu, JMP, Rennecke, PRD 101 (2020) 054032



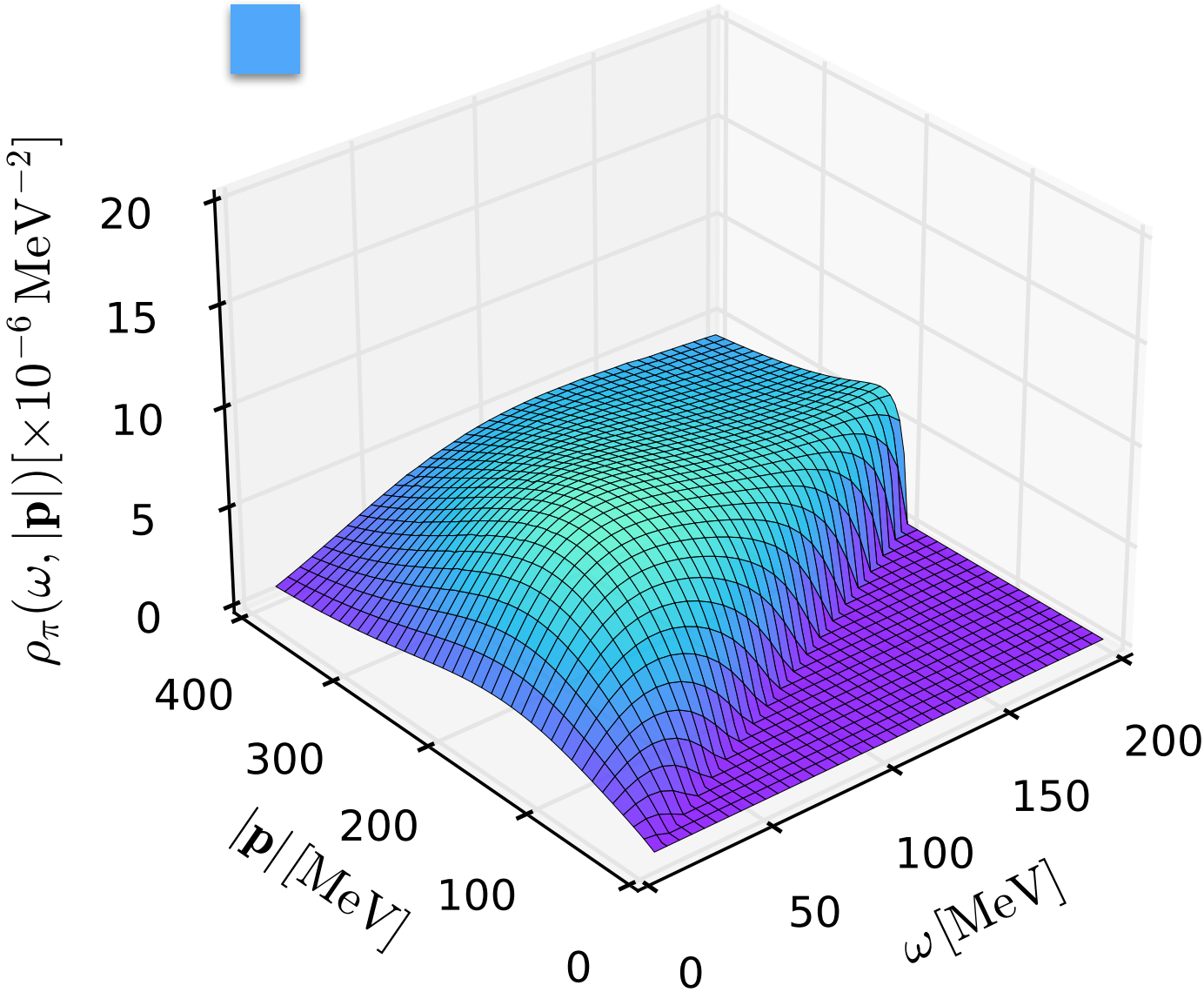
Regime of quantitative reliability
of
current best truncation

Estimate

Moat regime is not captured quantitatively

Pion spectral functions

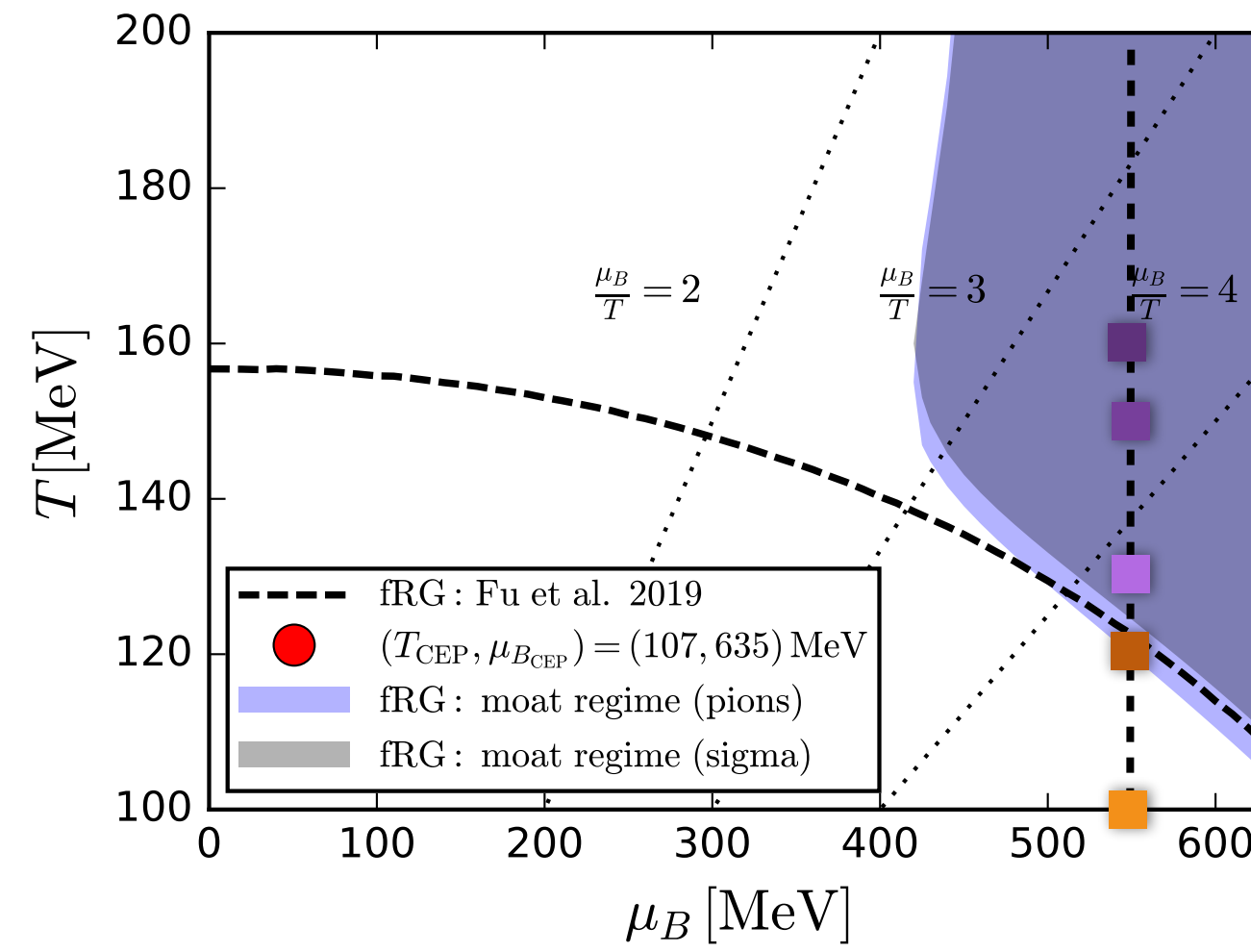
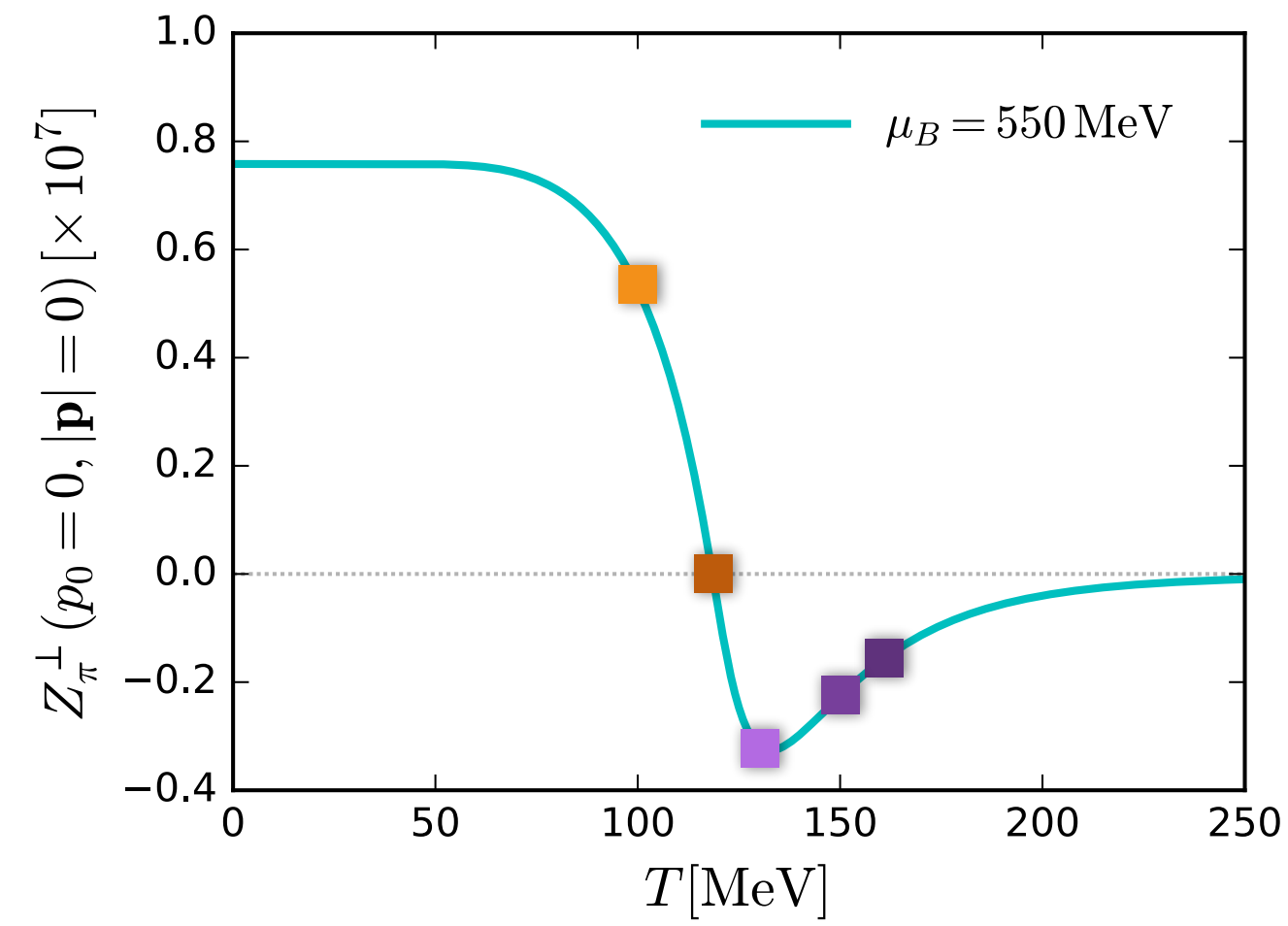
T=160 MeV & μ_B =0 MeV



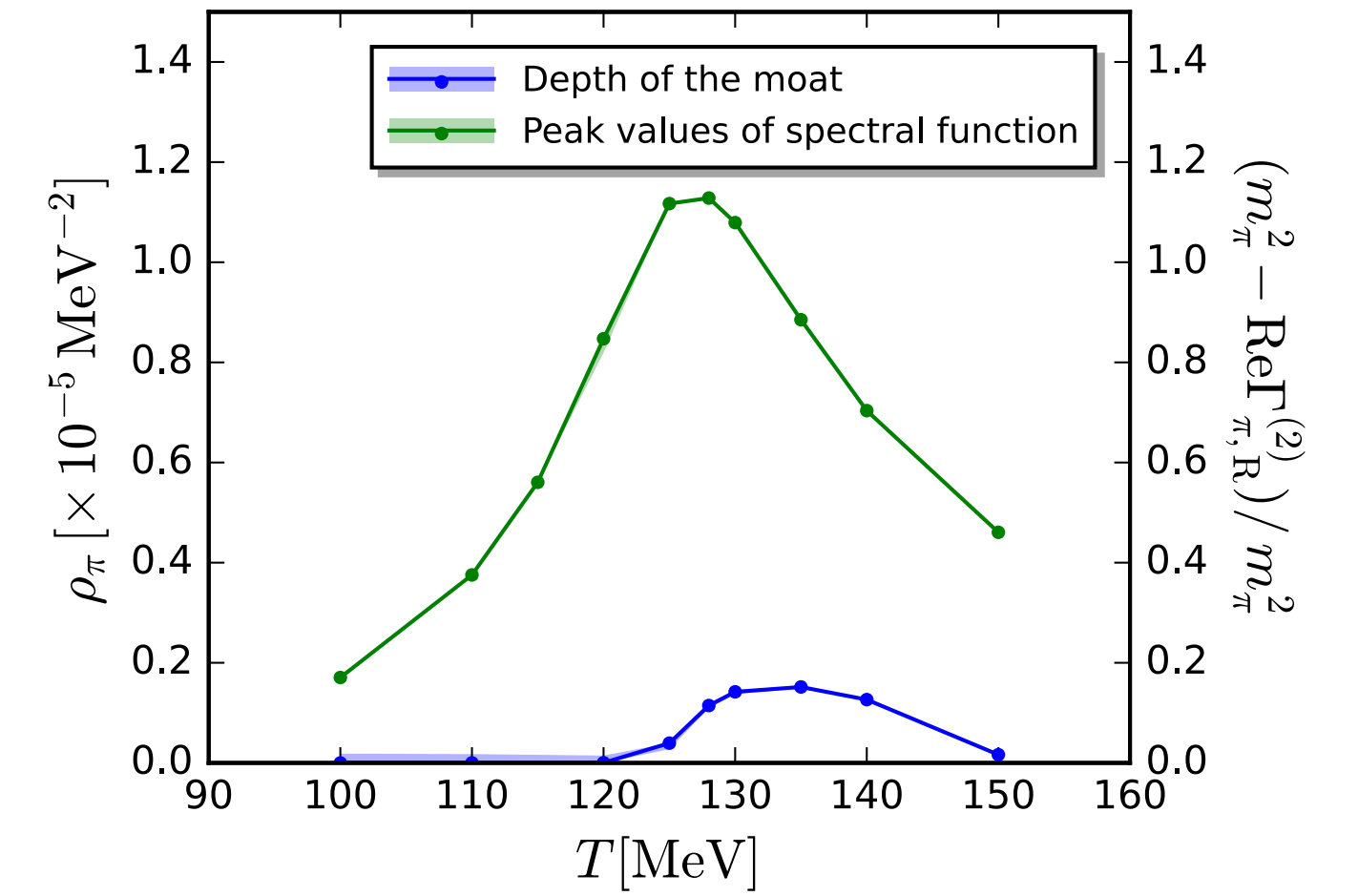
Fu, JMP, Pisarski, Rennecke, Wen, Shi Yin, 2412.15949

Predictions & estimates

Spatial wave function of the pion at $p=0$



Depth of the moat & spectral peak of the pion



Dissecting the moat & the moaton

$\mu_B = 650$ MeV

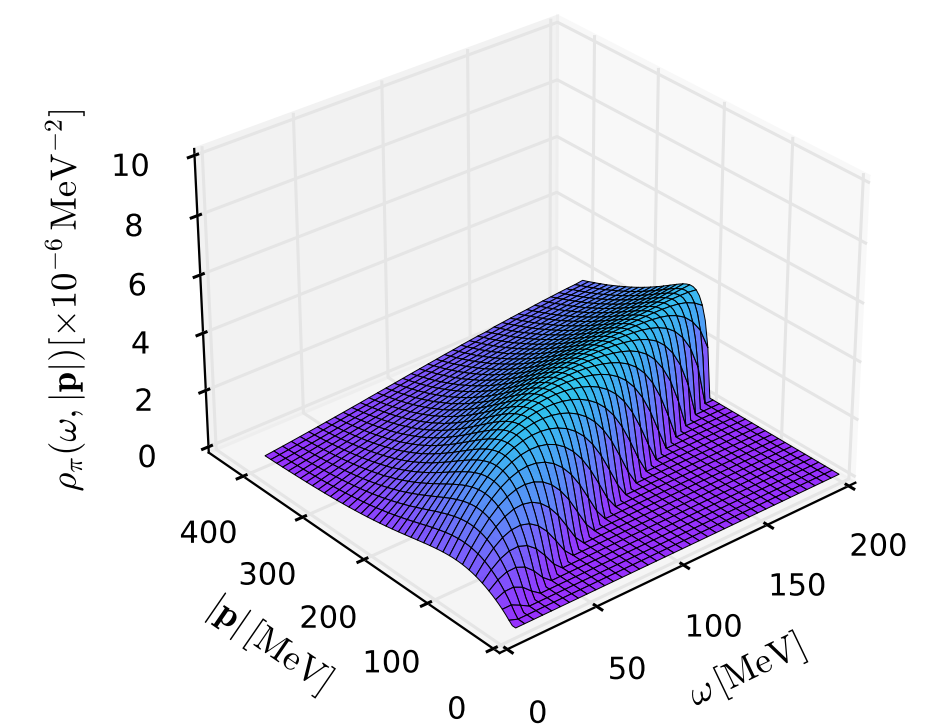
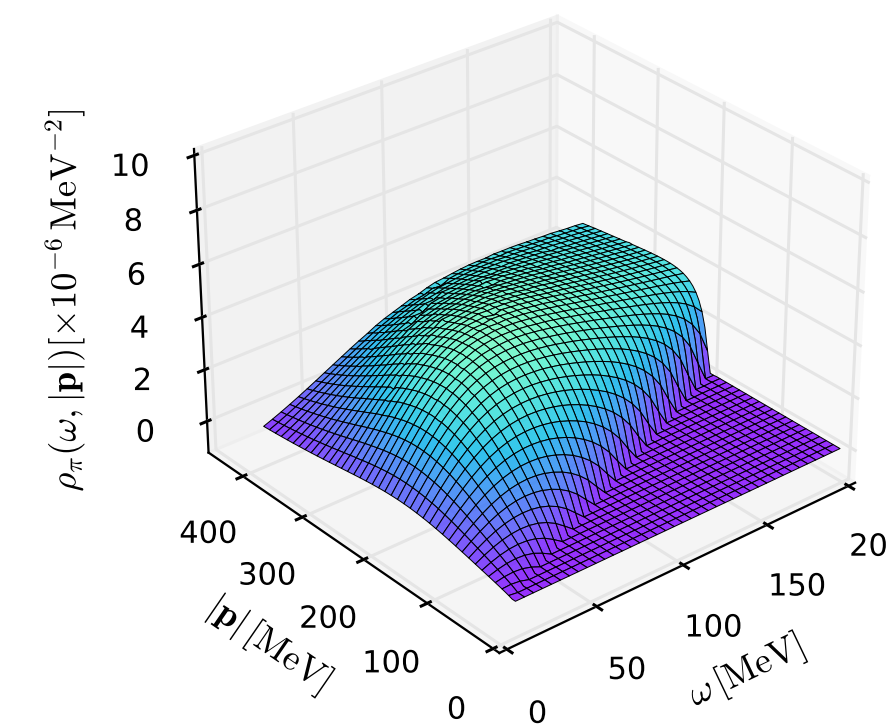
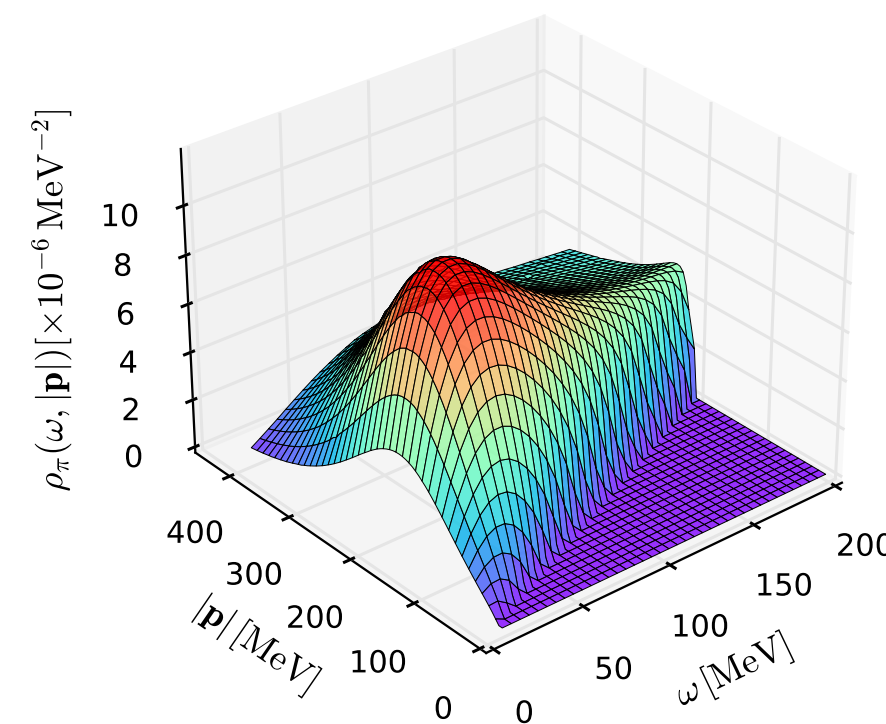
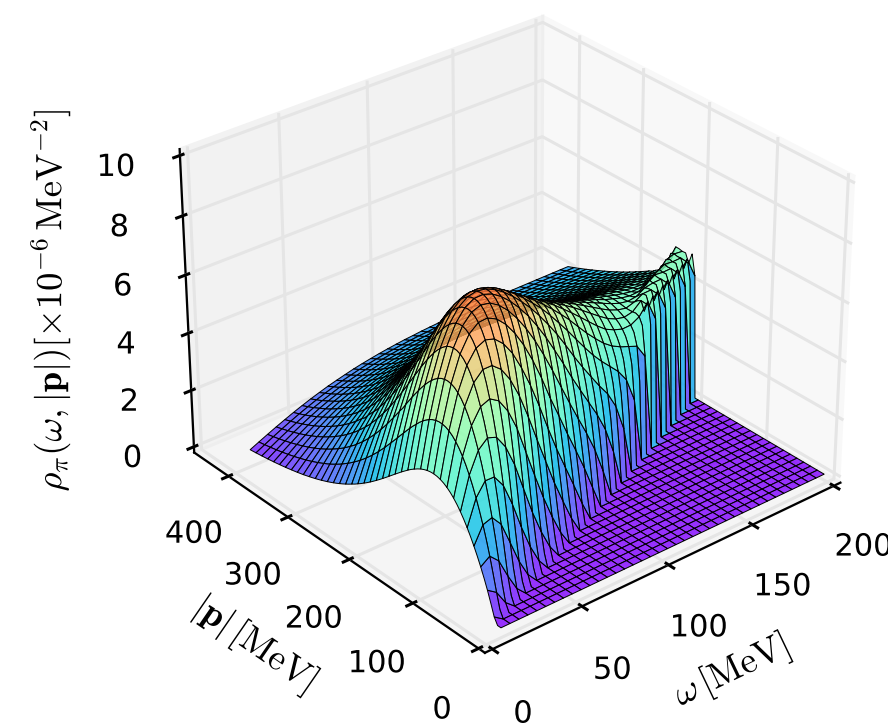
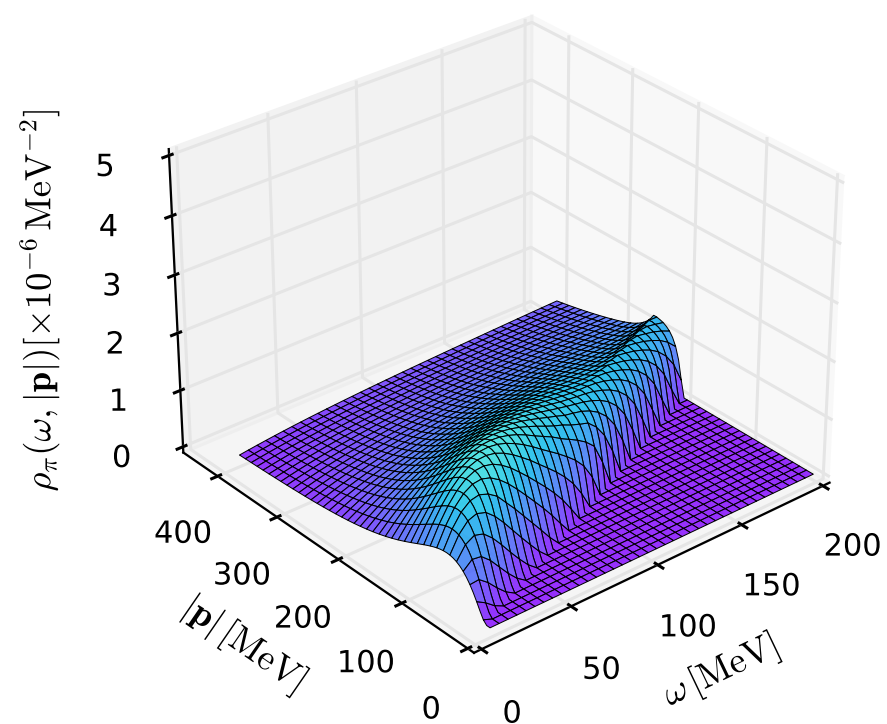
■ T=100 MeV

■ T=120 MeV

■ T=130 MeV

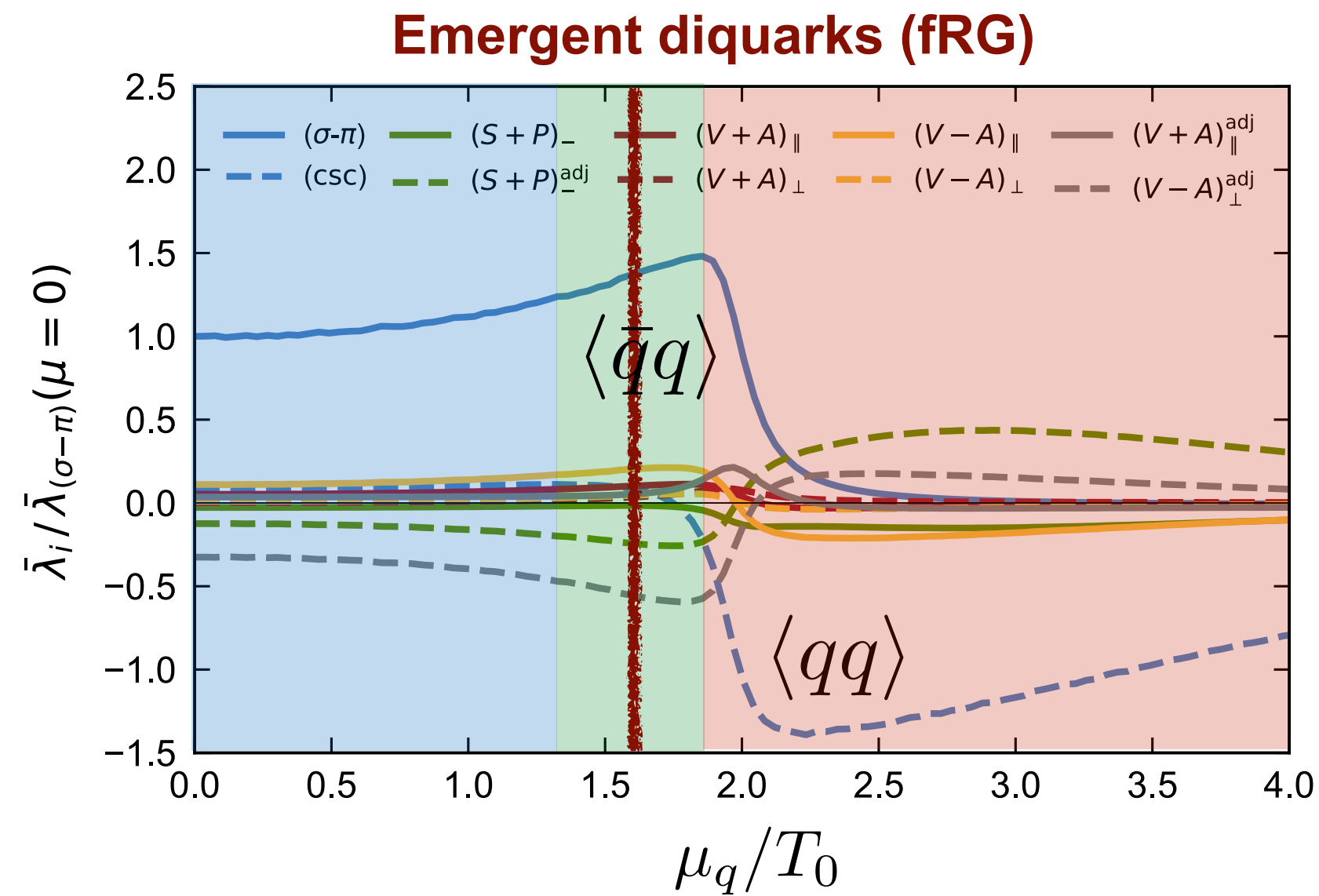
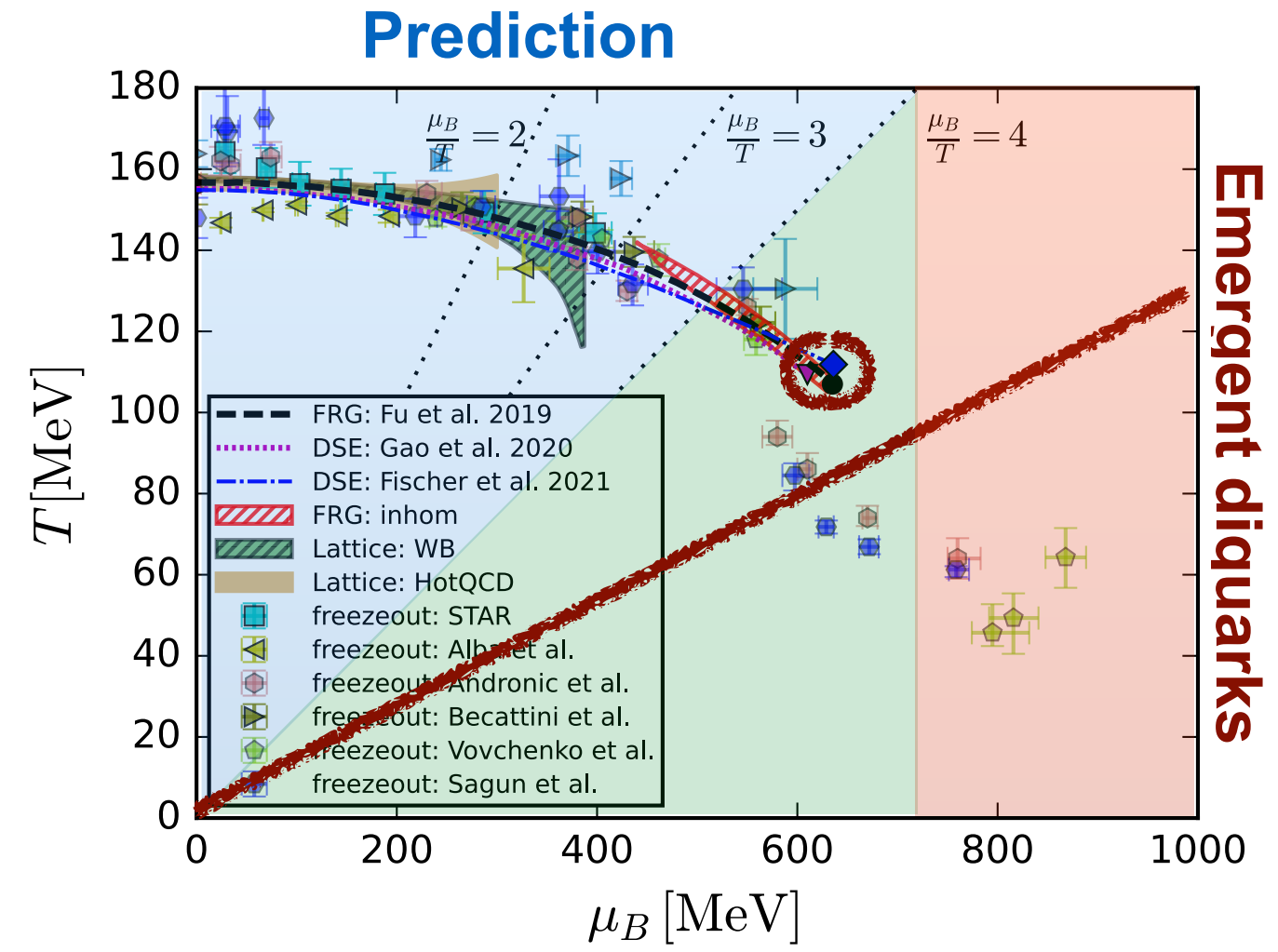
■ T=150 MeV

■ T=160 MeV



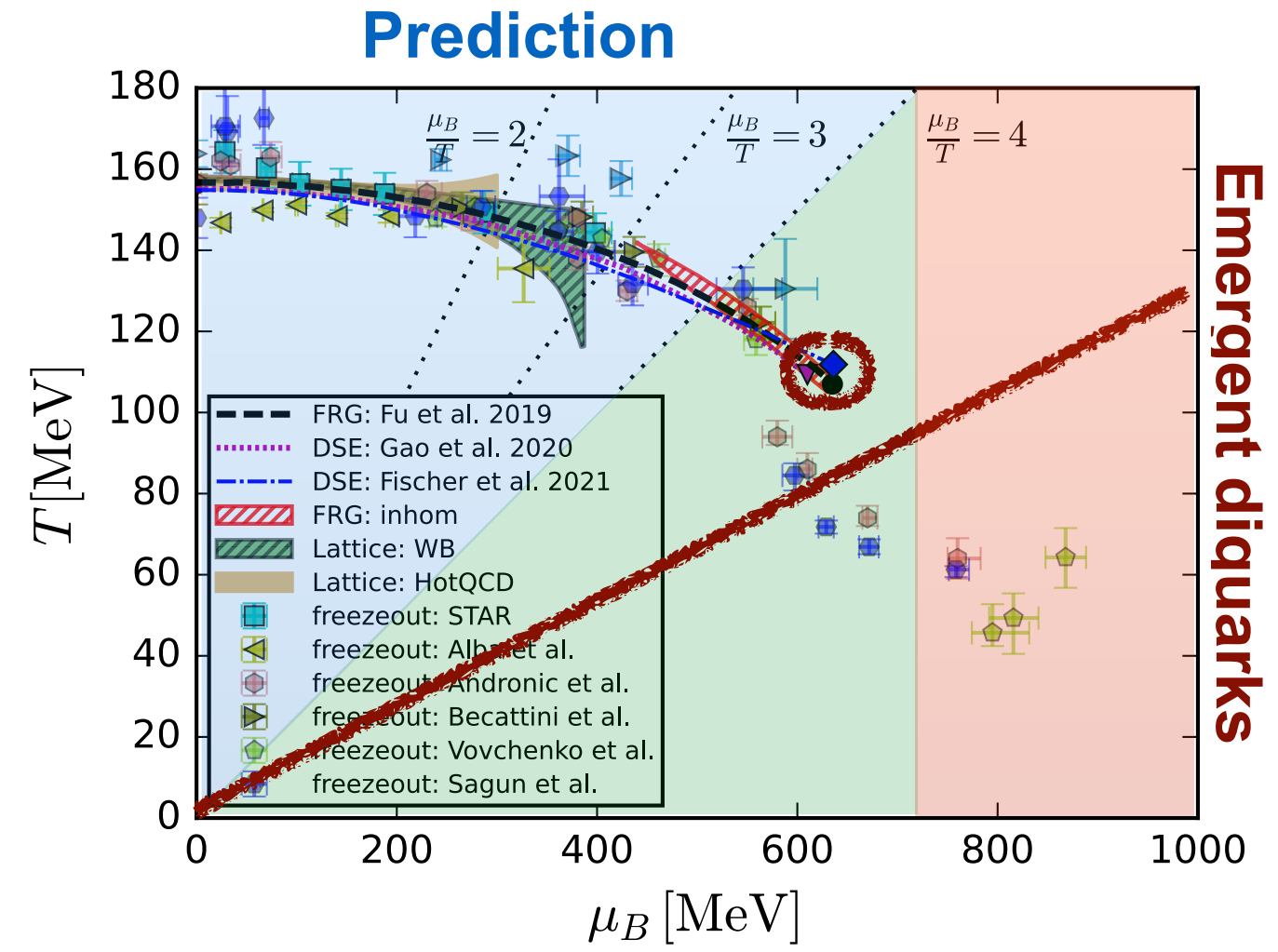
Predictions & estimates

Emergent diquarks

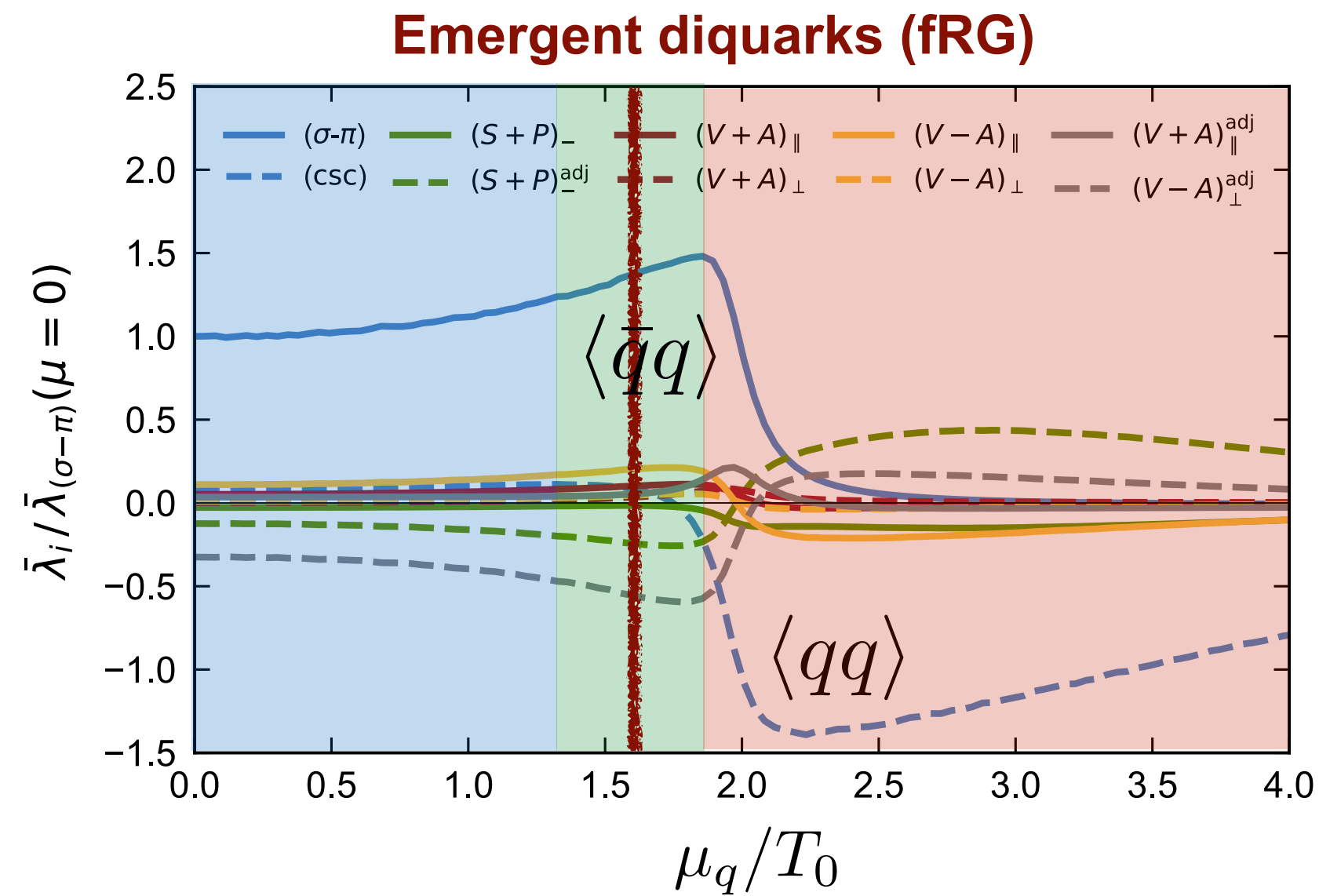


Predictions & estimates

Emergent diquarks



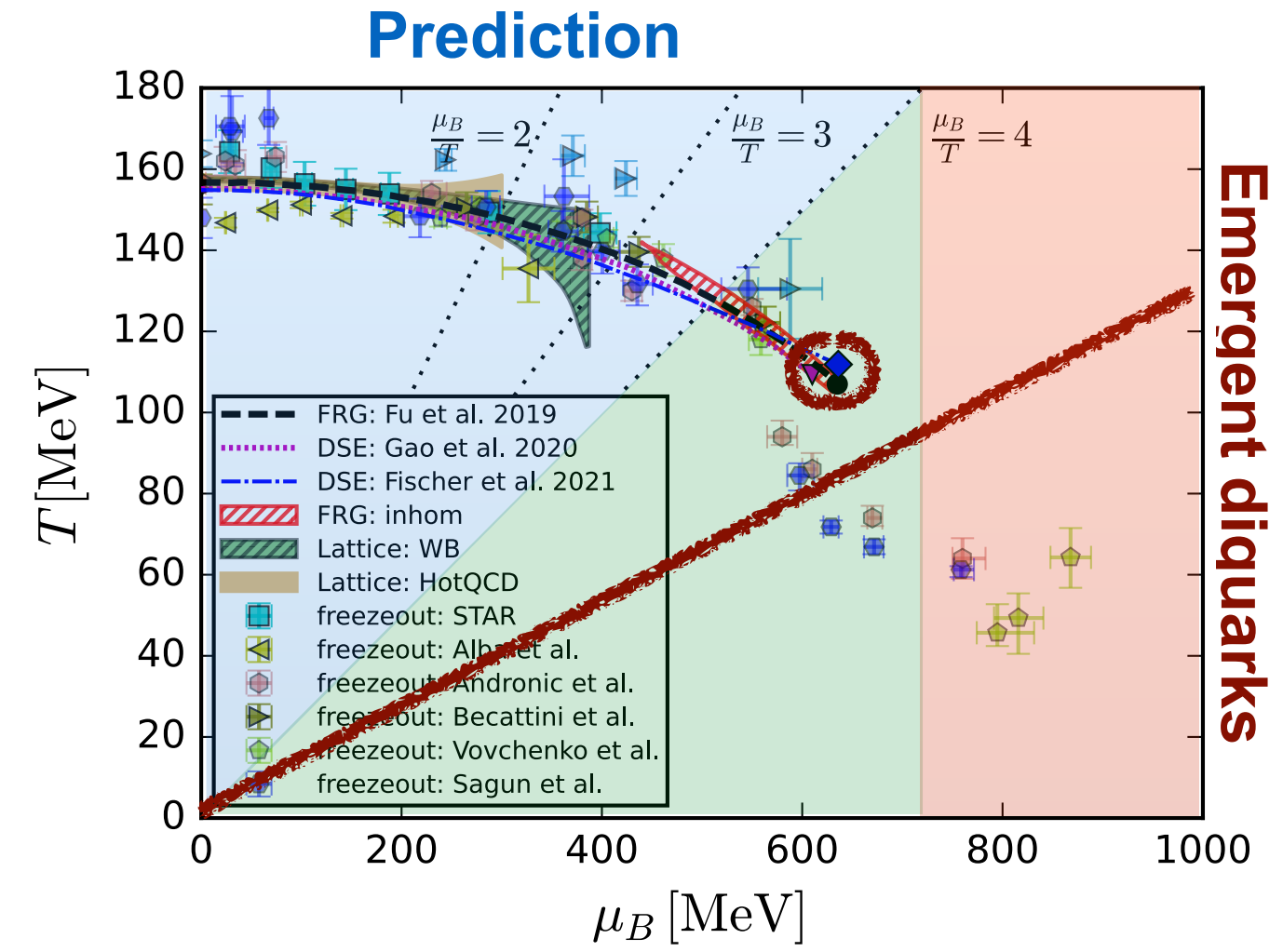
Estimate



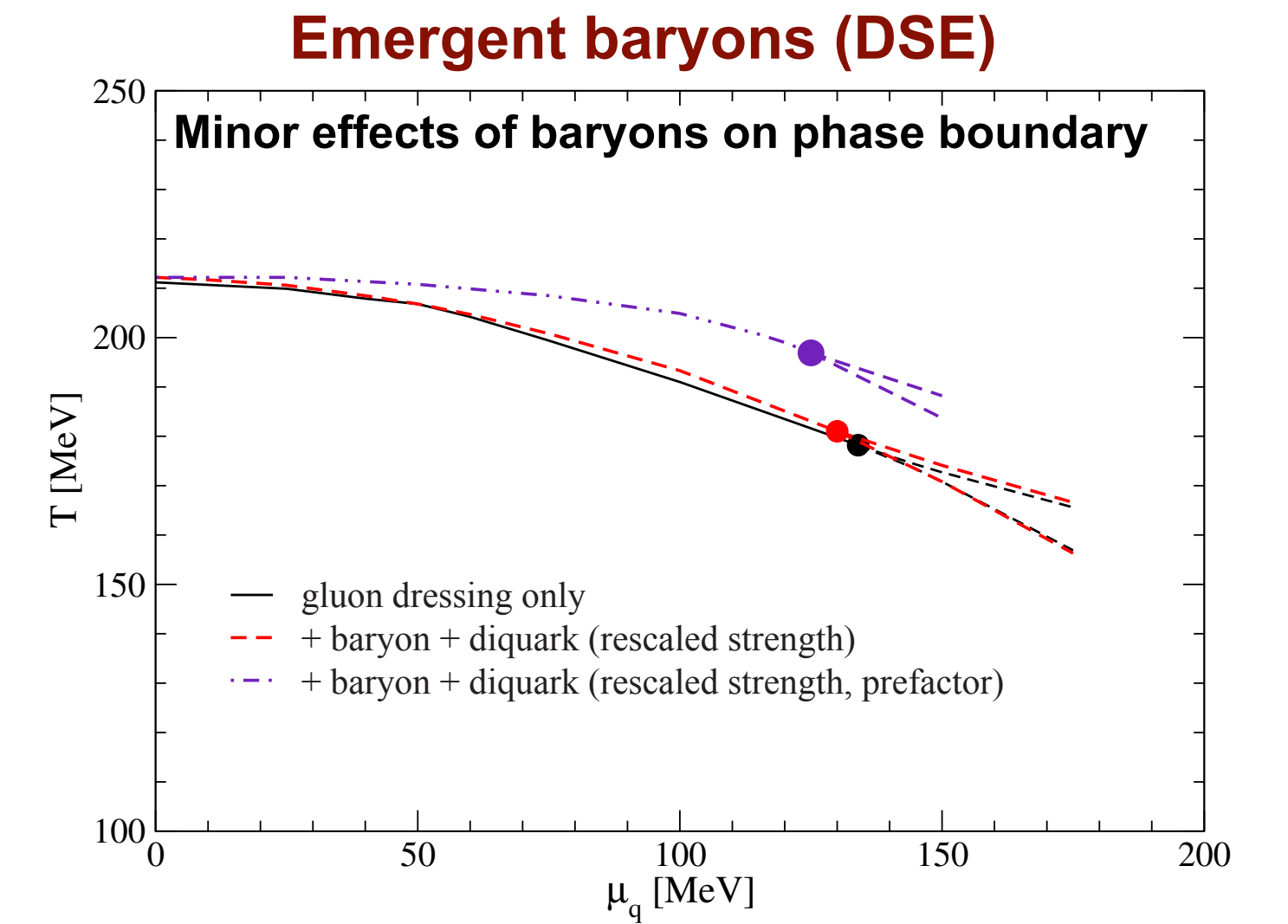
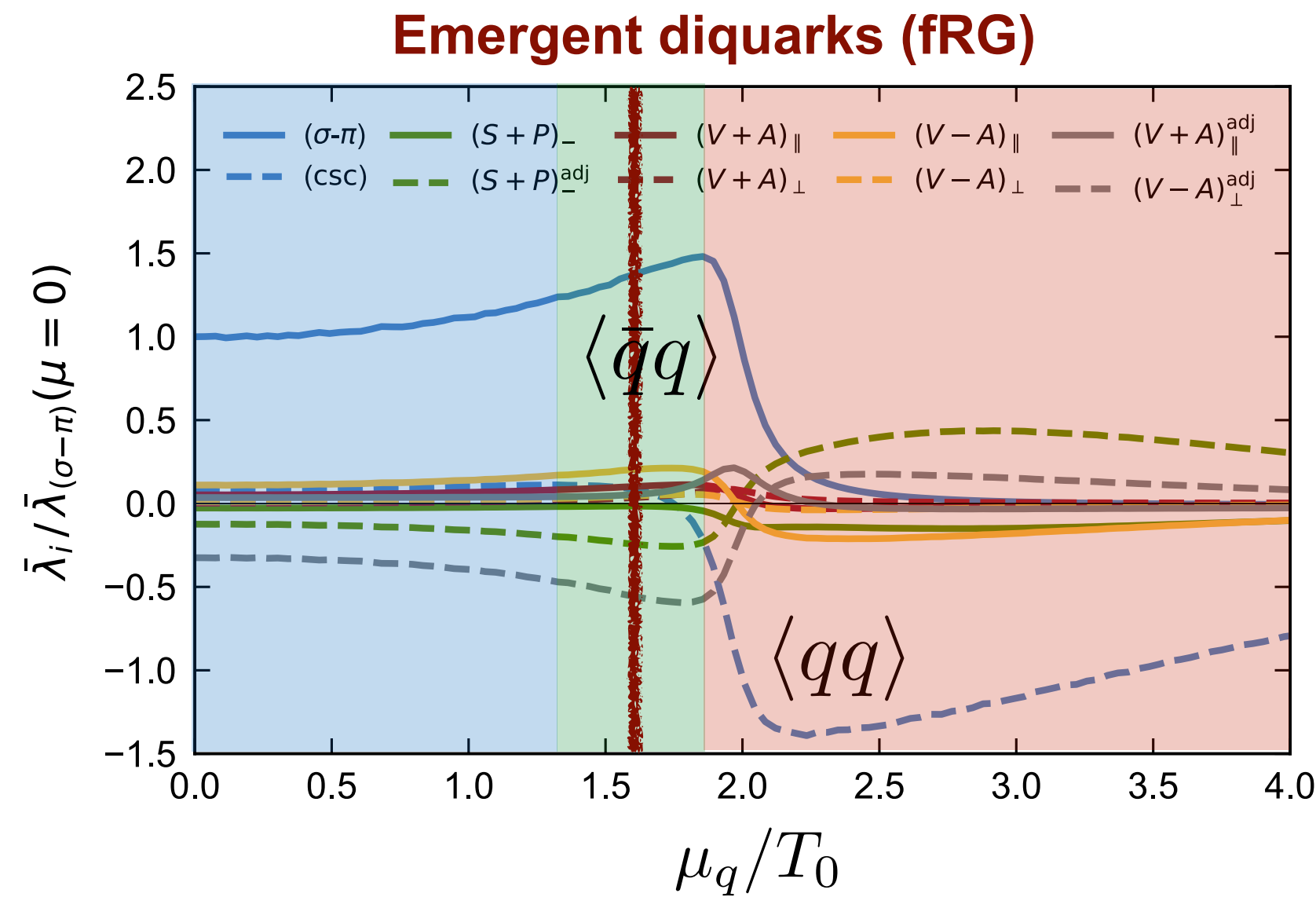
Emergent diquarks are not captured by extrapolations

Predictions & estimates

Emergent diquarks



Regime of quantitative reliability
of
current best truncation

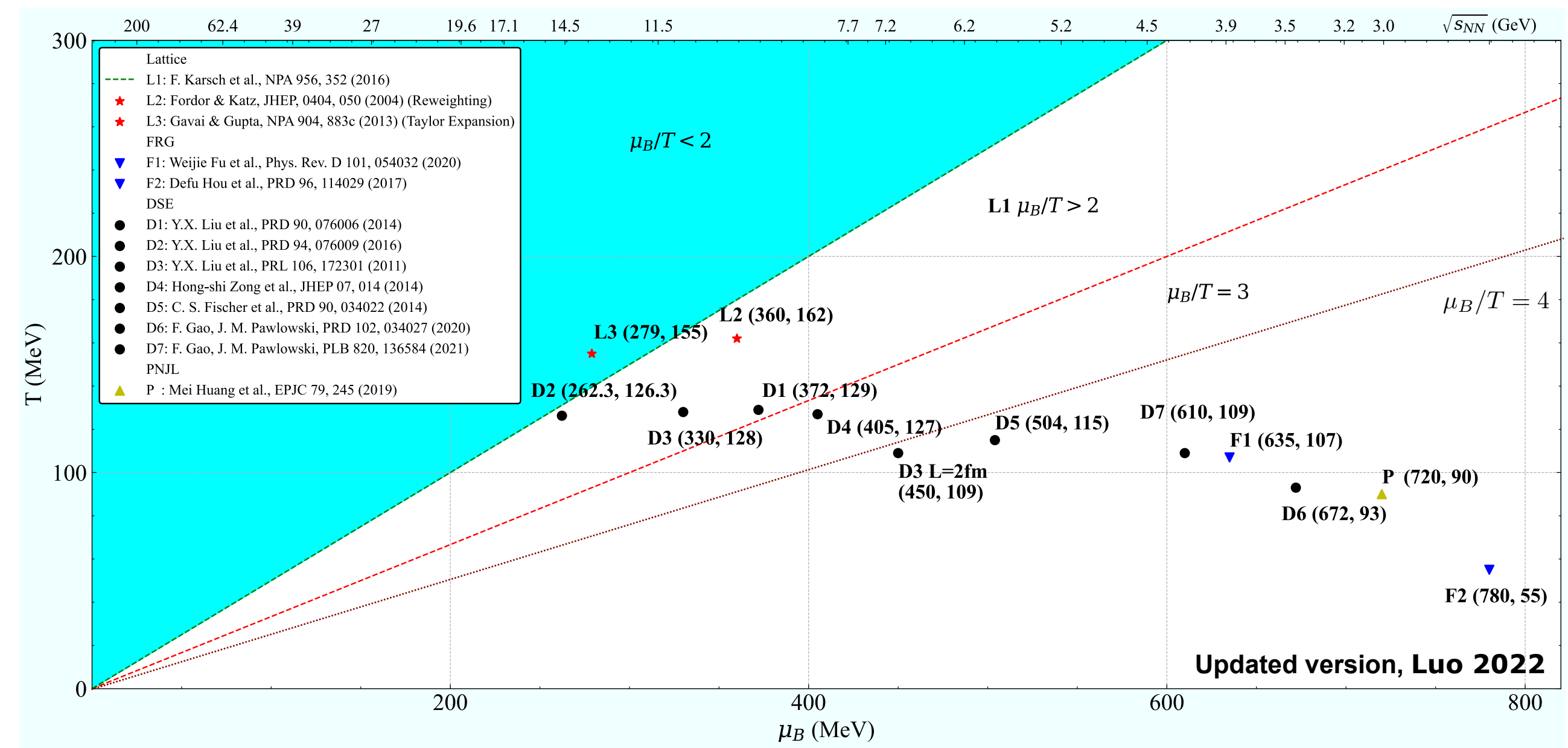


Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

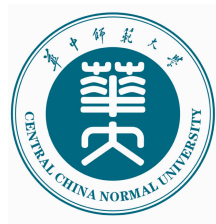
Disclaimer

Most functional computations
(LEFT or QCD) have not been
set-up for CEP-predictions!

Lack of predictive power
for CEP-predictions
is no quality measure!

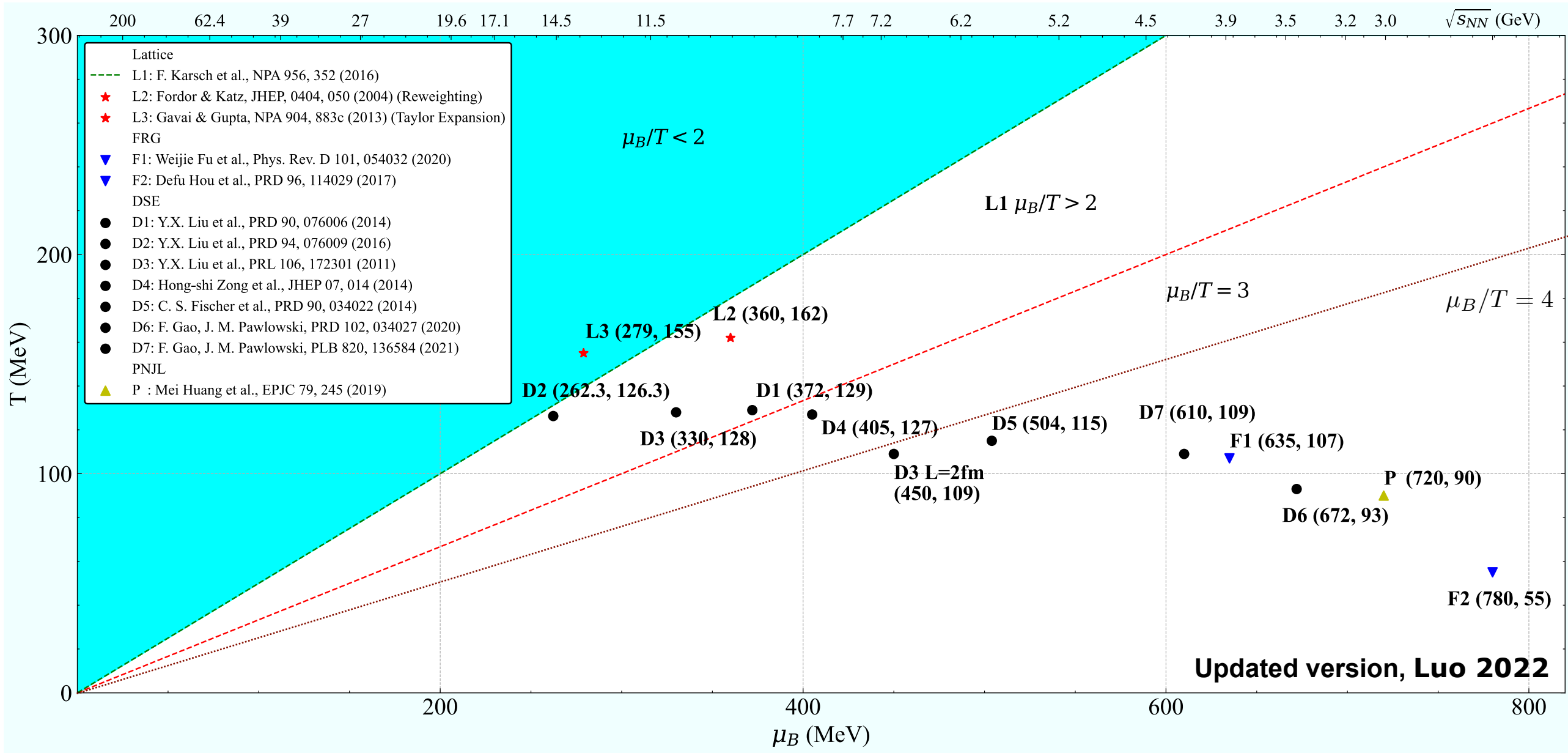
CEP is standing for
'regime with new physics'

Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Common folklore
since ~2004

Large uncertainties for the estimation of CP location.

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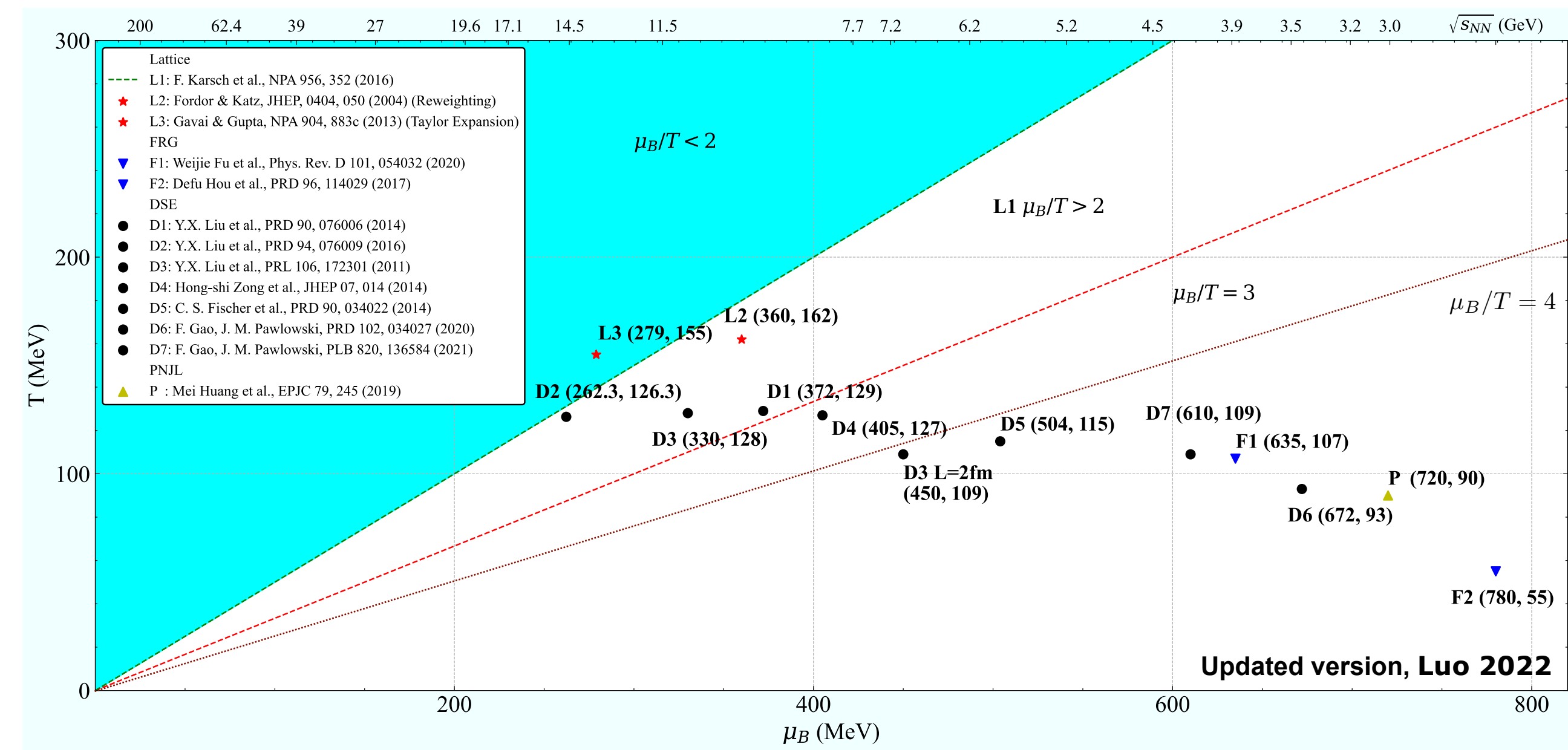
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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) ‘old’ CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

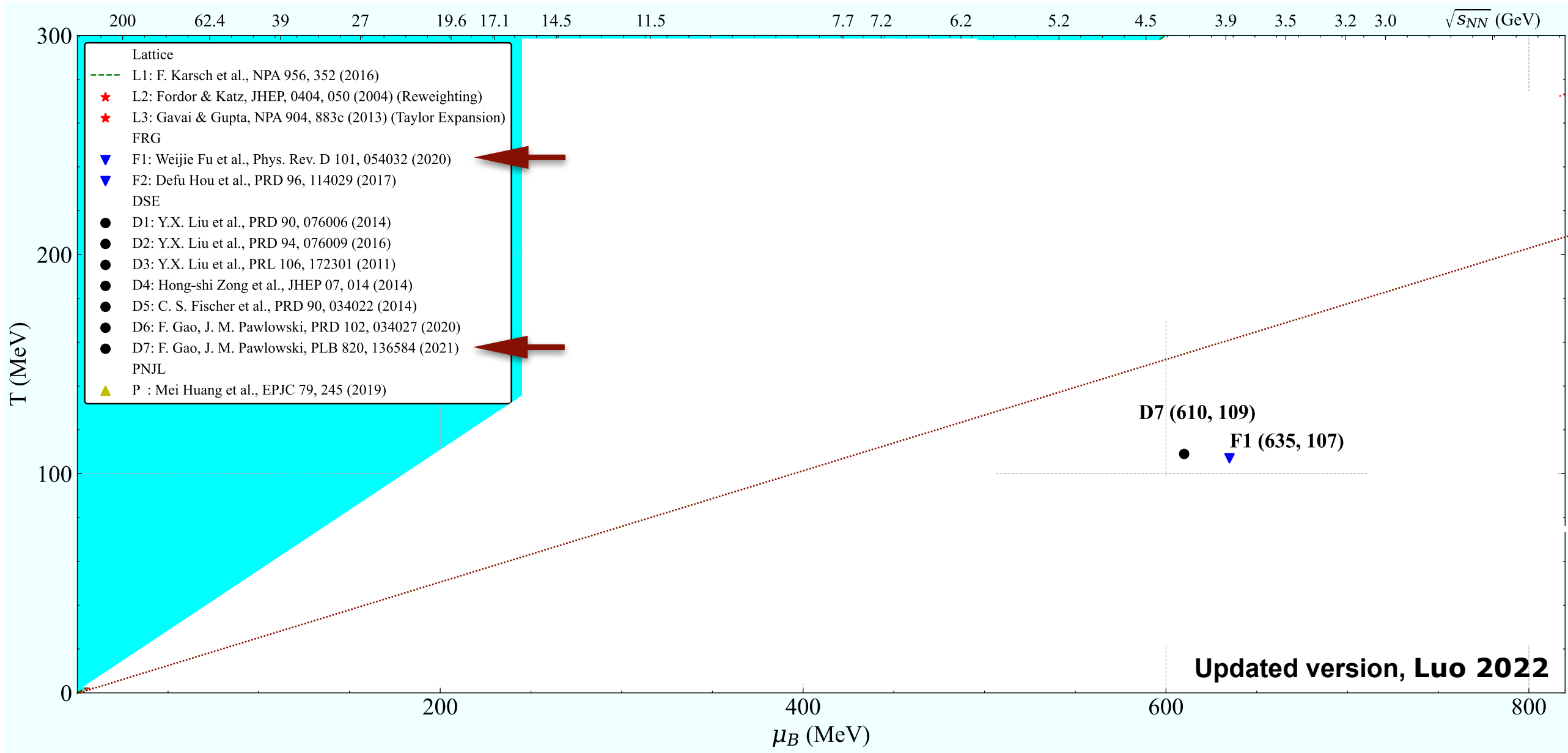
Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

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Functional QCD



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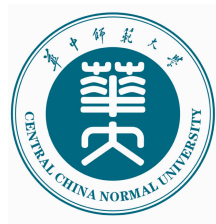
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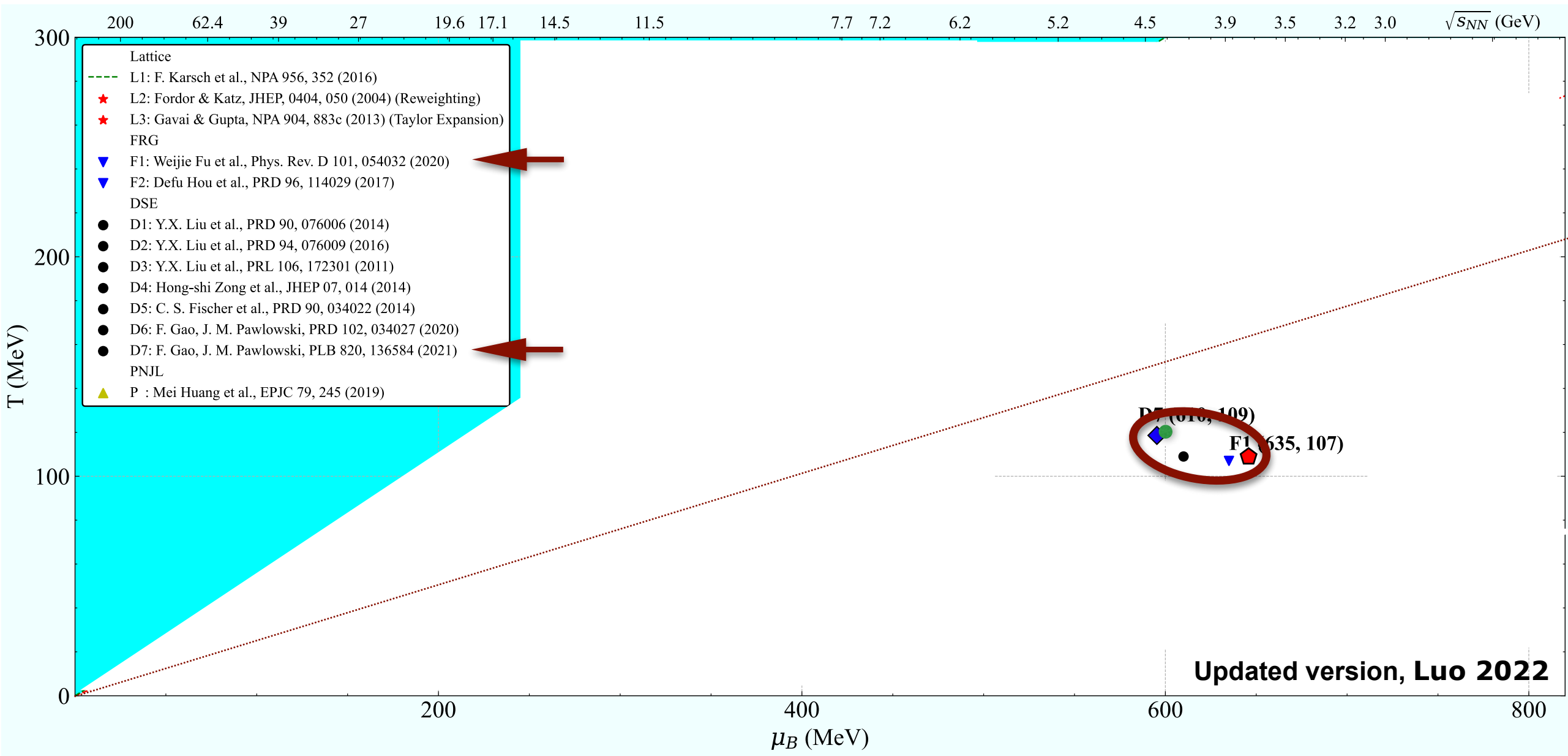


Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- ◆ Gao, Lu, JMP, Schneider, in prep (DSE)
- ◆ Fu, JMP, Rennecke, Wen, Yin, in prep (fRG)
- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



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Predictions, estimates & extrapolations and how to judge them

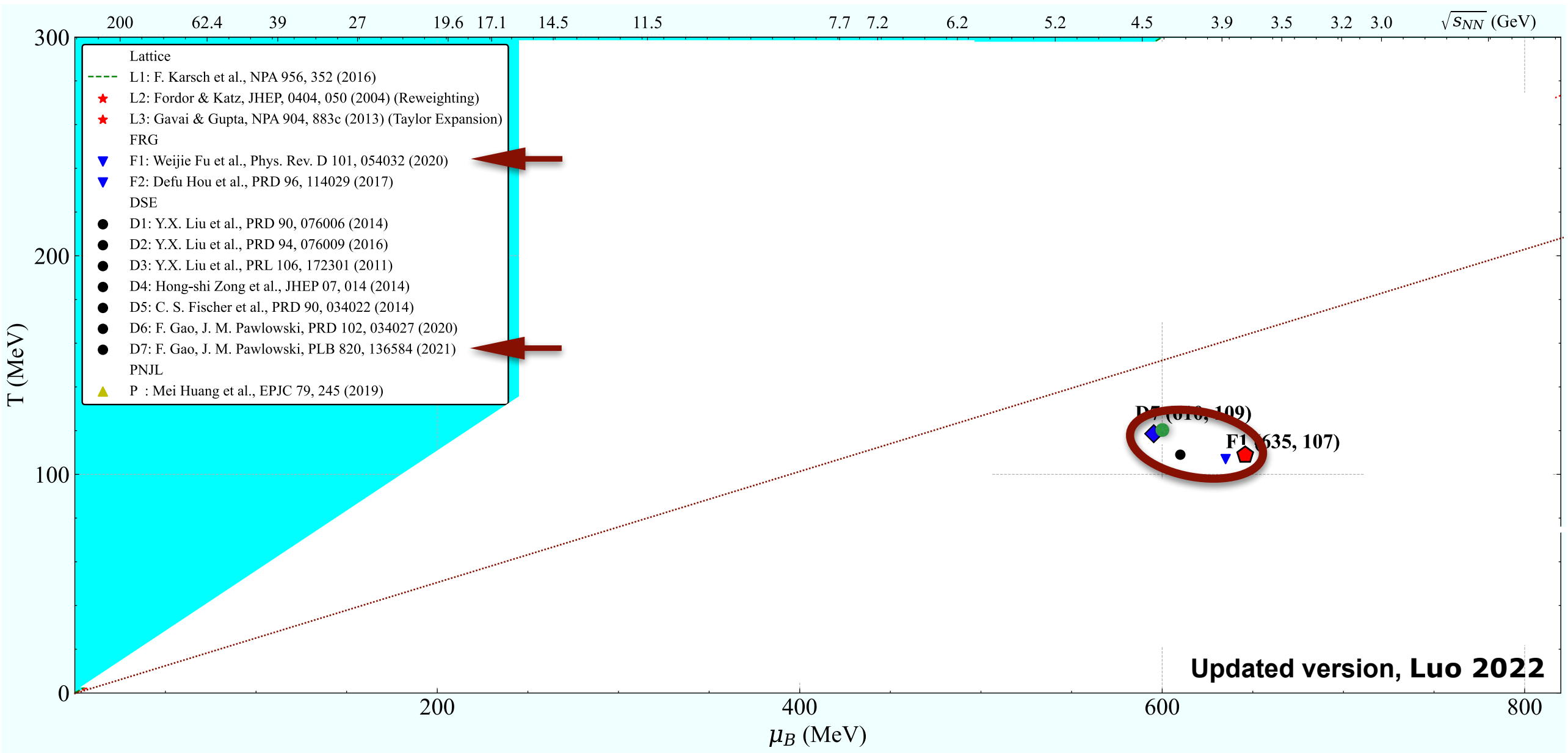


Location of CP : Theoretical Prediction

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Functional QCD

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- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)



Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

- (i) ‘old’ CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)
- (ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$
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Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2023)

Functional QCD

- ◆ Gao, Lu, JMP, Schneider, in prep (DSE)
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- Gunkel, Fischer, PRD 104 (2021) 054022 (DSE)

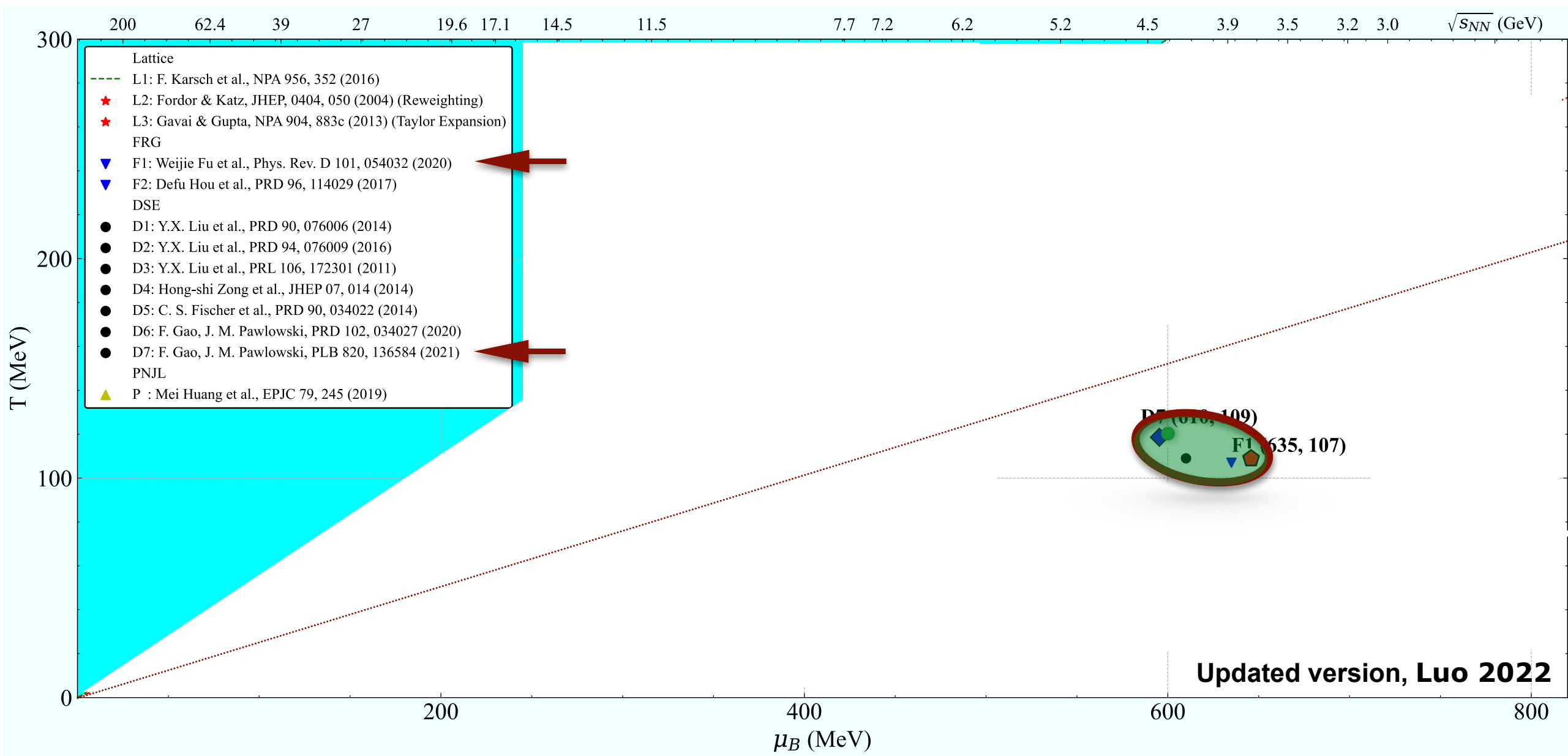
Extrapolations

Lattice extrapolations: Basar, PRC 110 (2024) 015203
Bielefeld-Parma, arXiv:2405.10196

⋮

Holographic models: Hippert, Grefa, Manning, Noronha,
Noronha-Hostler, Portillo Vazquez, Ratti,
Rougemont, Trujillo, arXiv: 2309.00579

⋮



Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

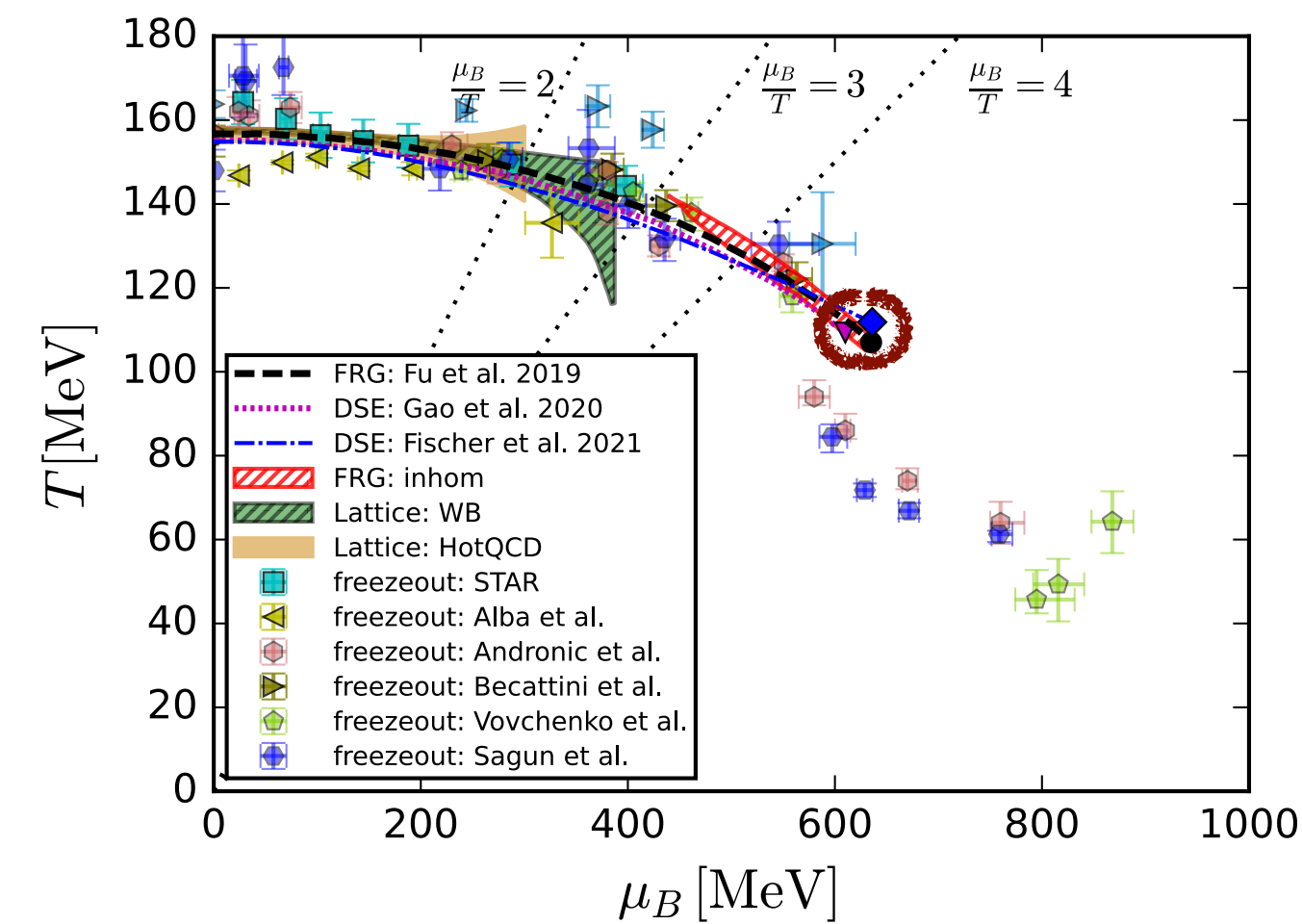
(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

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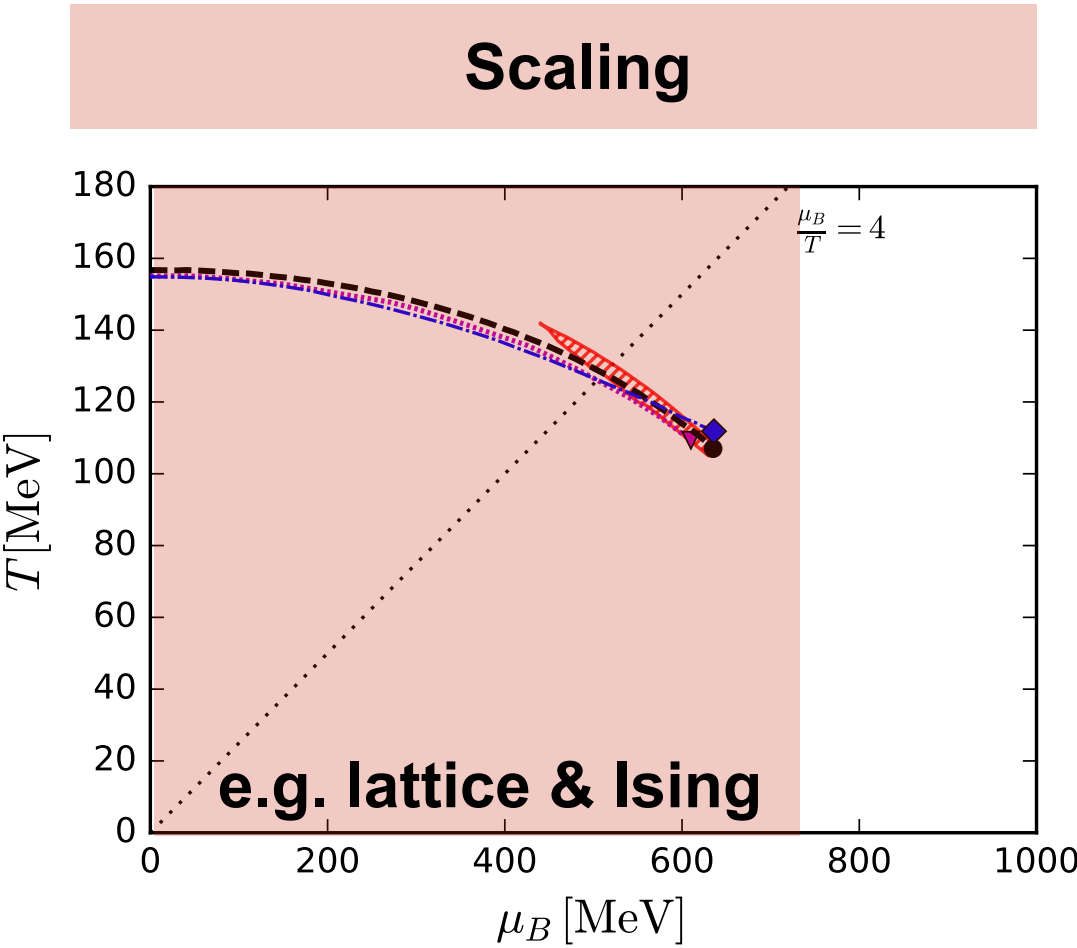
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Predictions, estimates & extrapolations and how to use them

Predictions, estimates & extrapolations and how to use them



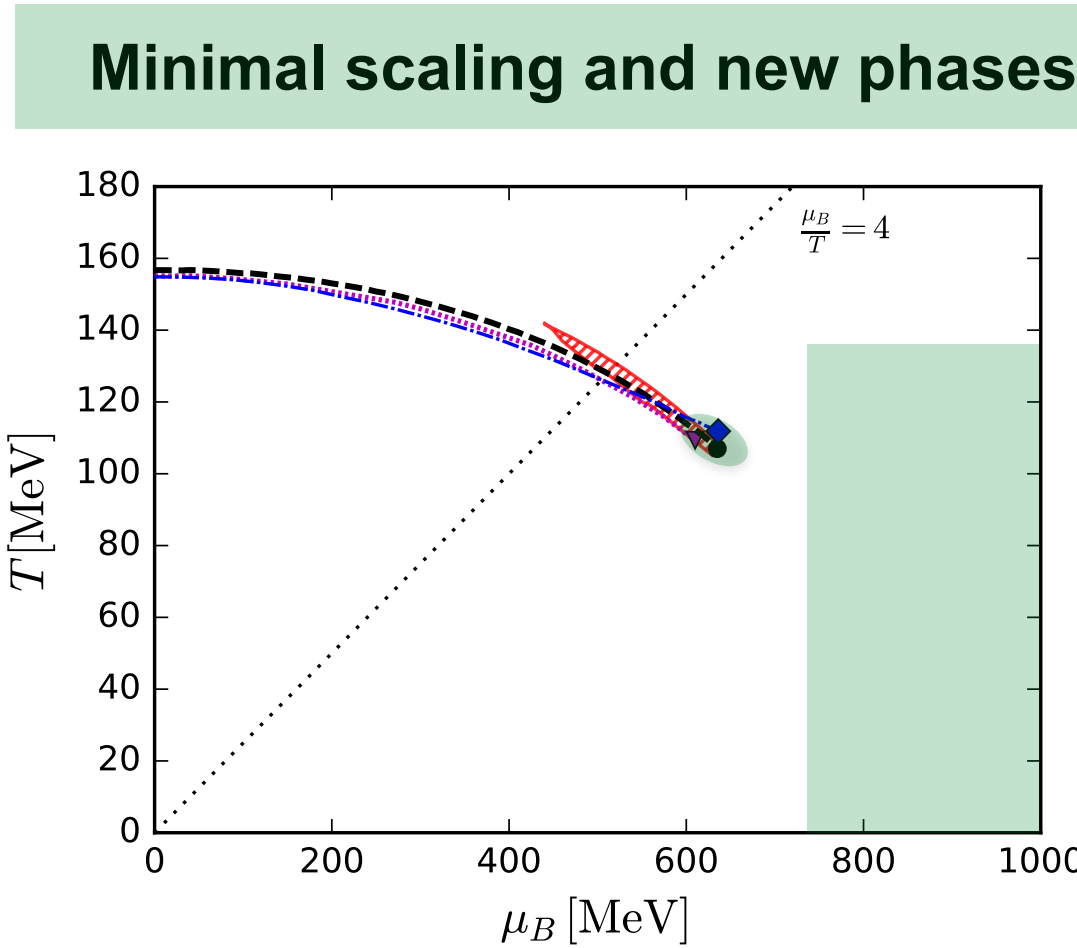
Scenario I



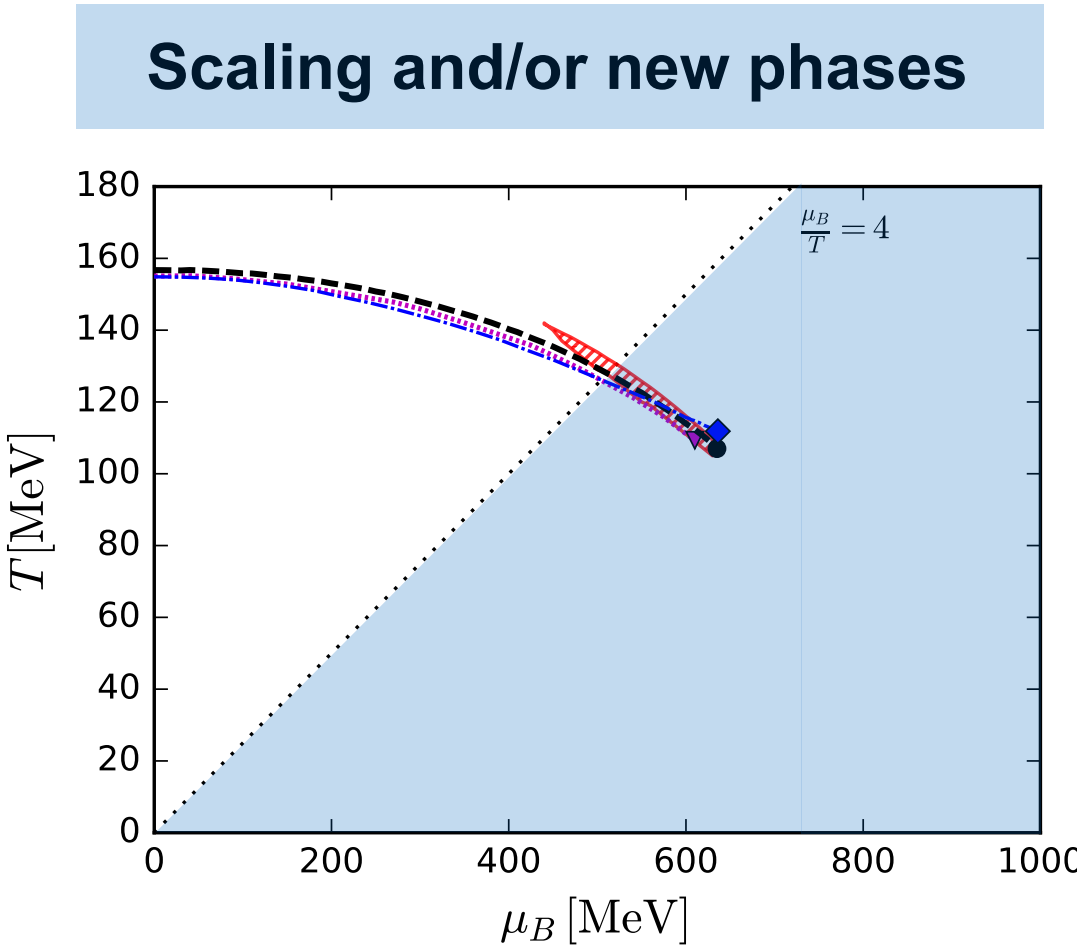
Extrapolations
for
Pheno



Scenario II



Scenario III



Predictions, estimates & extrapolations and how to use them

Out by the LEGO[®] principle

Fu, JMP, Rennecke, PRD 101 (2020) 054032

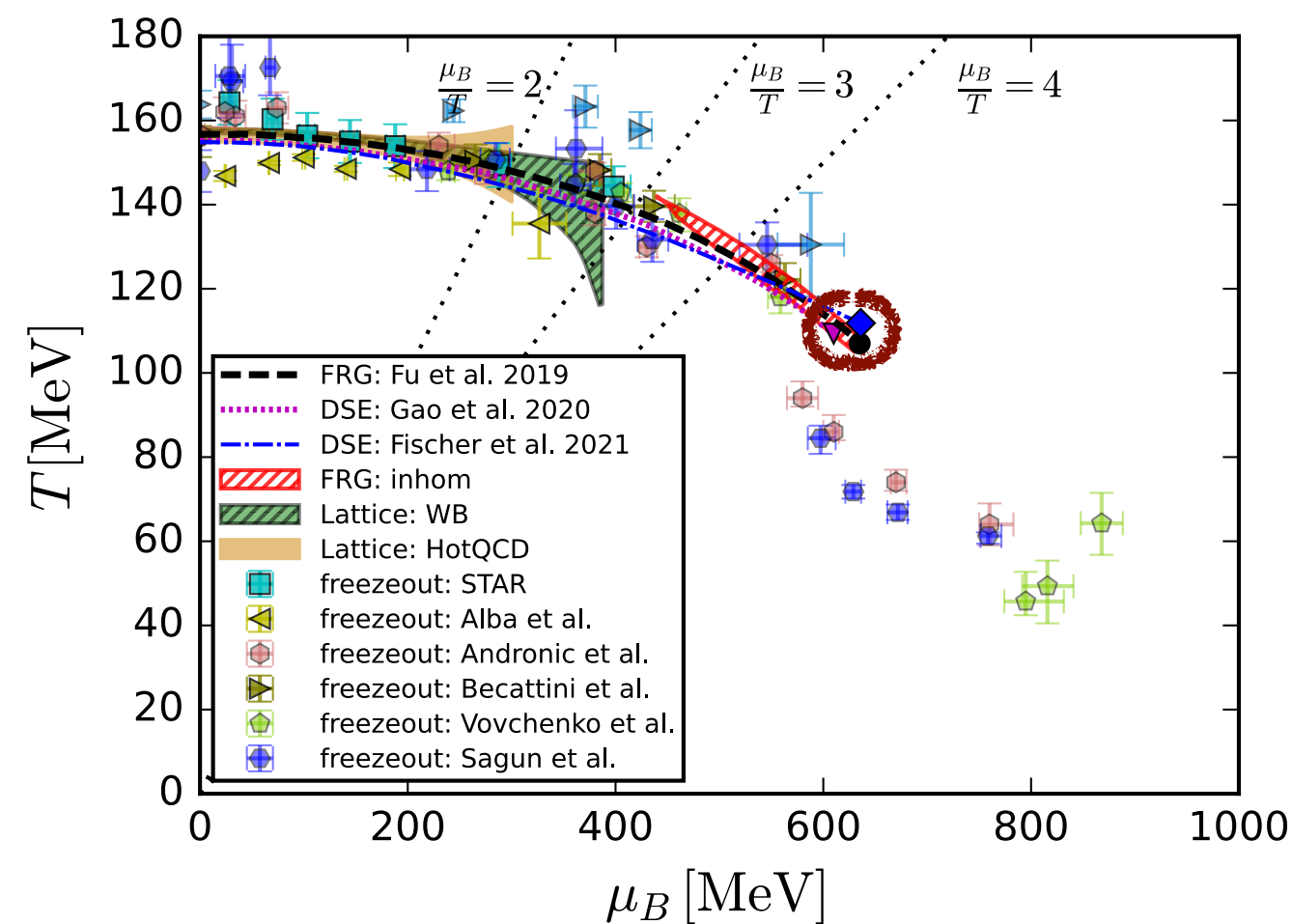
+

Size of scaling regime in LEFTs

Schaefer, Wambach, PRD 75 (2007) 085015

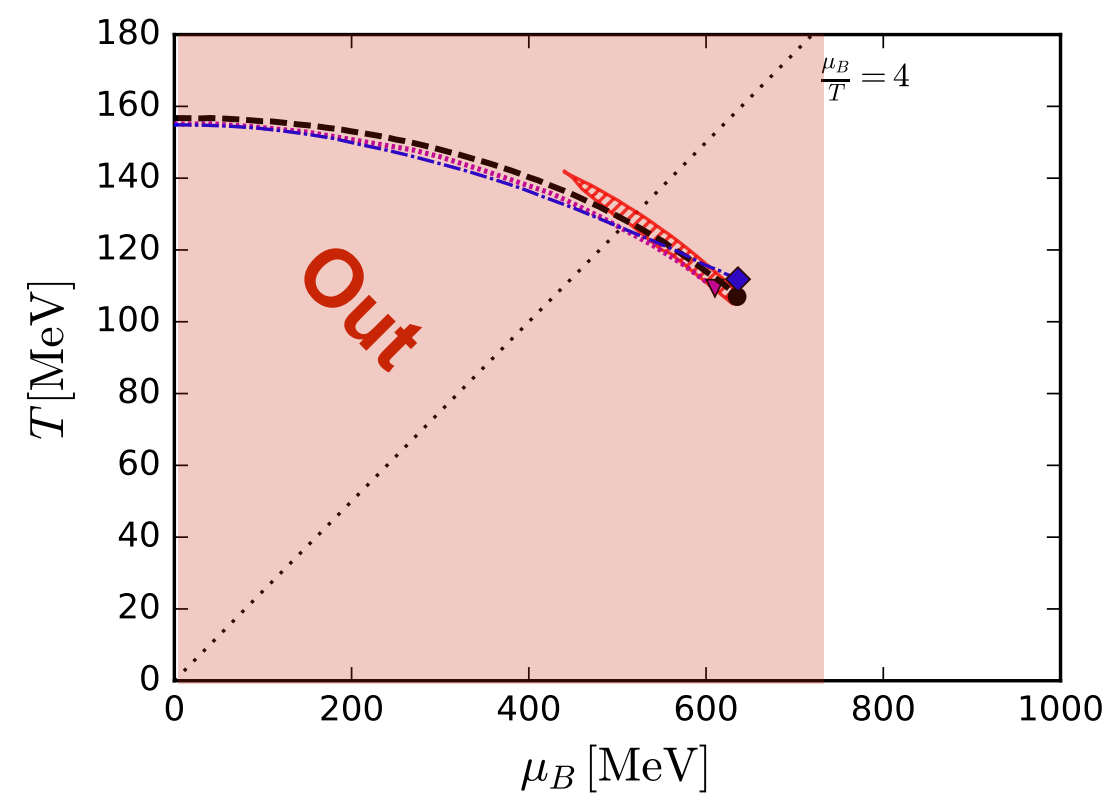
Braun, Klein, Piasecki, EPJC 71 (2011) 1576

⋮



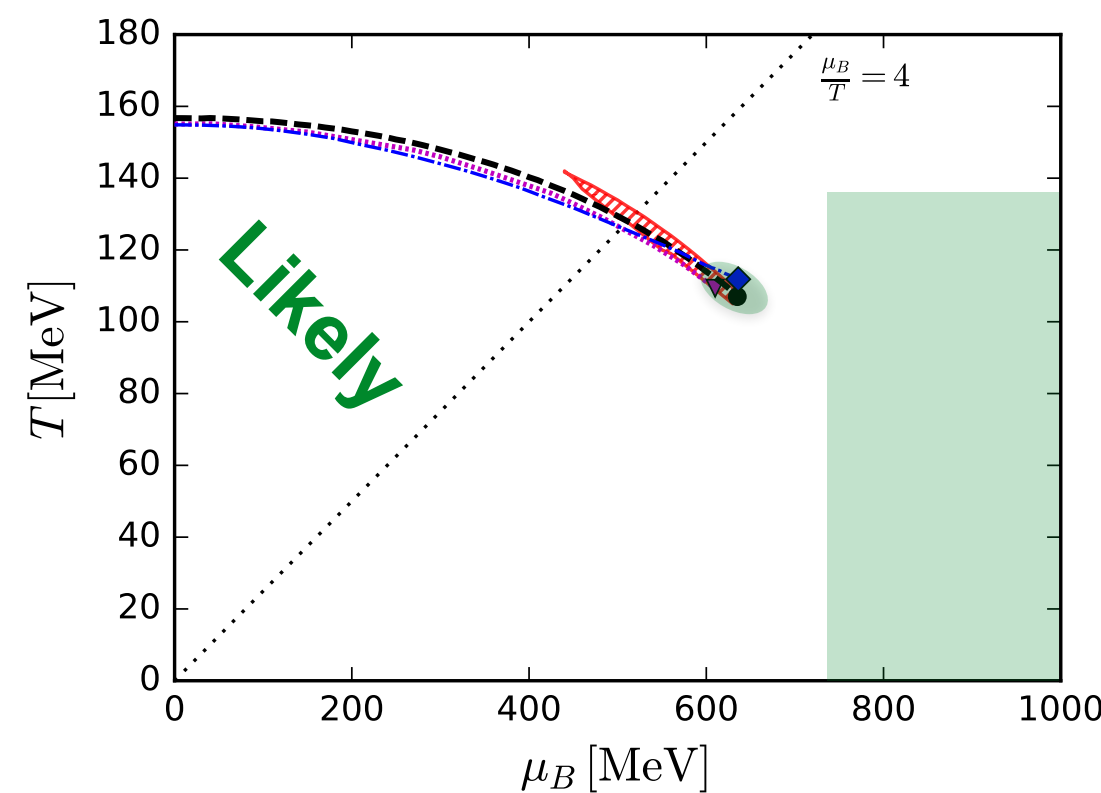
Scenario I

Scaling



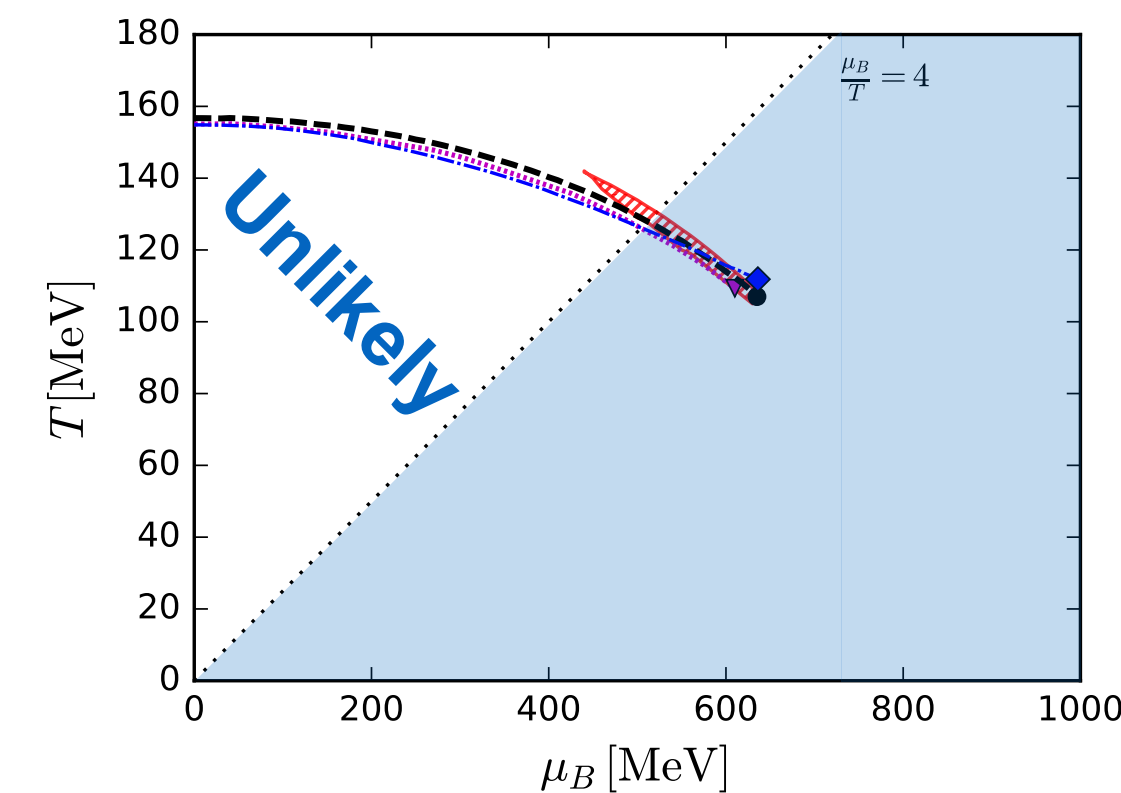
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

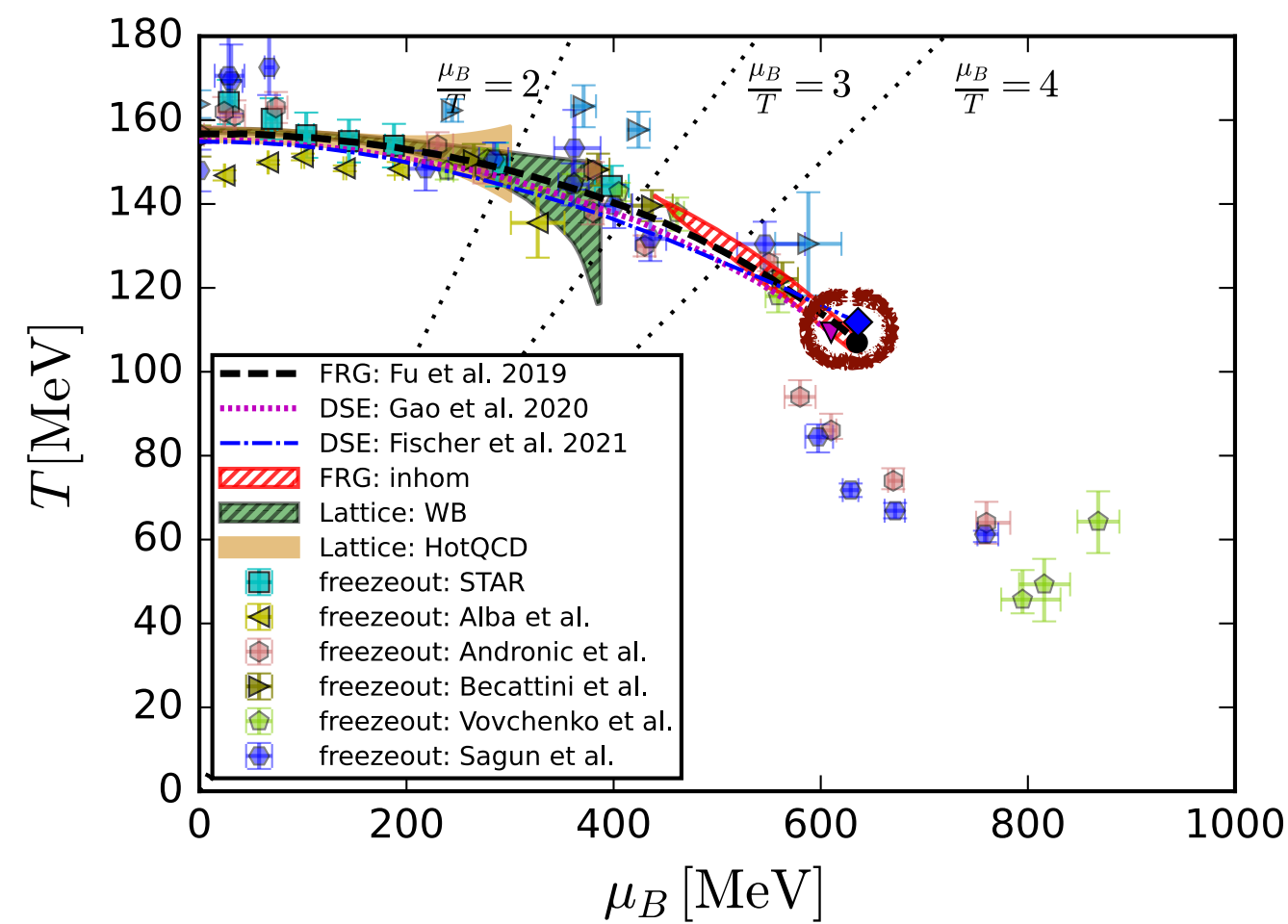
Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

**+ many results in dynamical
low energy effective theories**

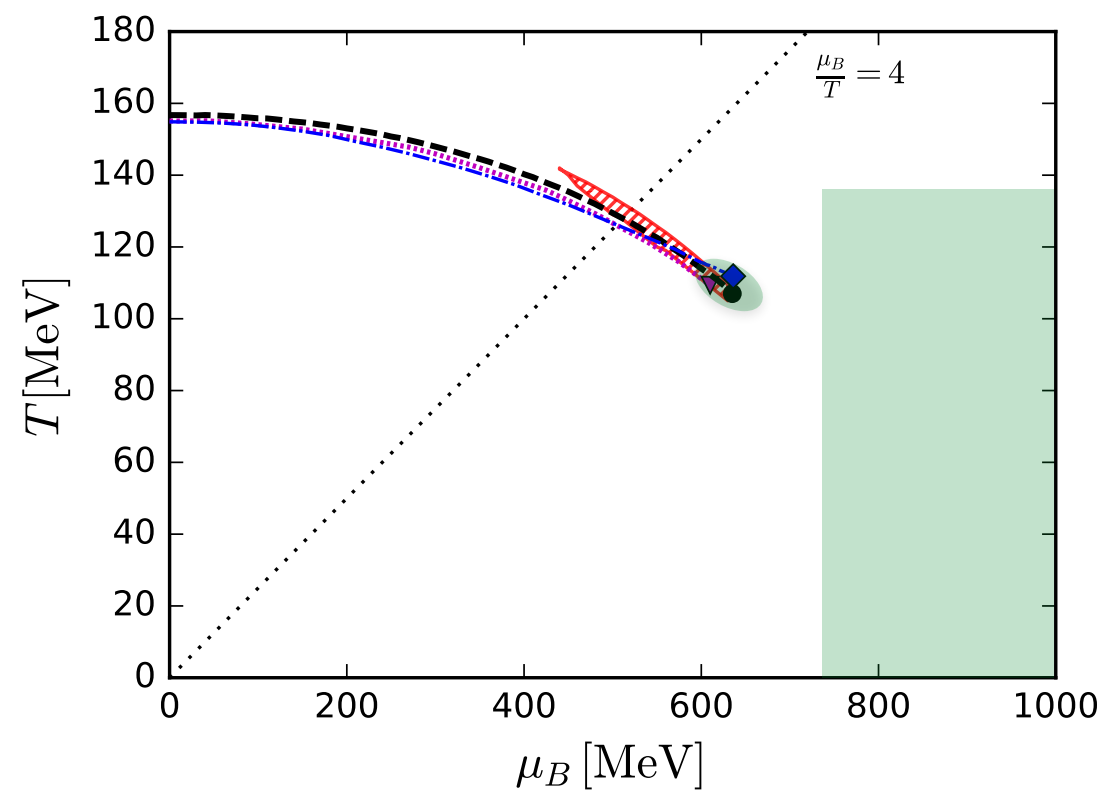
LEGO[®] principle

Predictions, estimates & extrapolations and how to use them



Scenario II

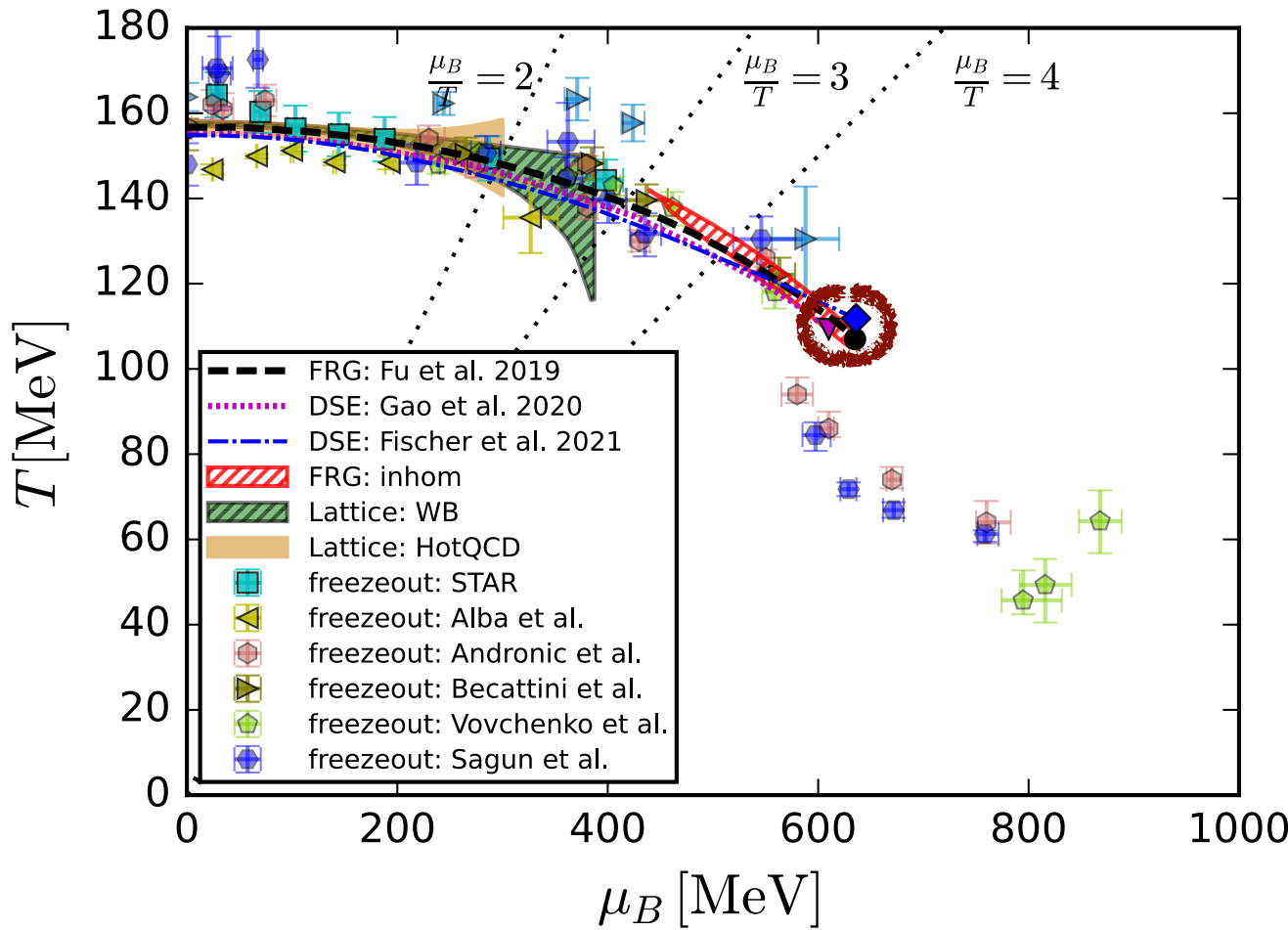
Minimal scaling and new phases



Extrapolations
for
Pheno

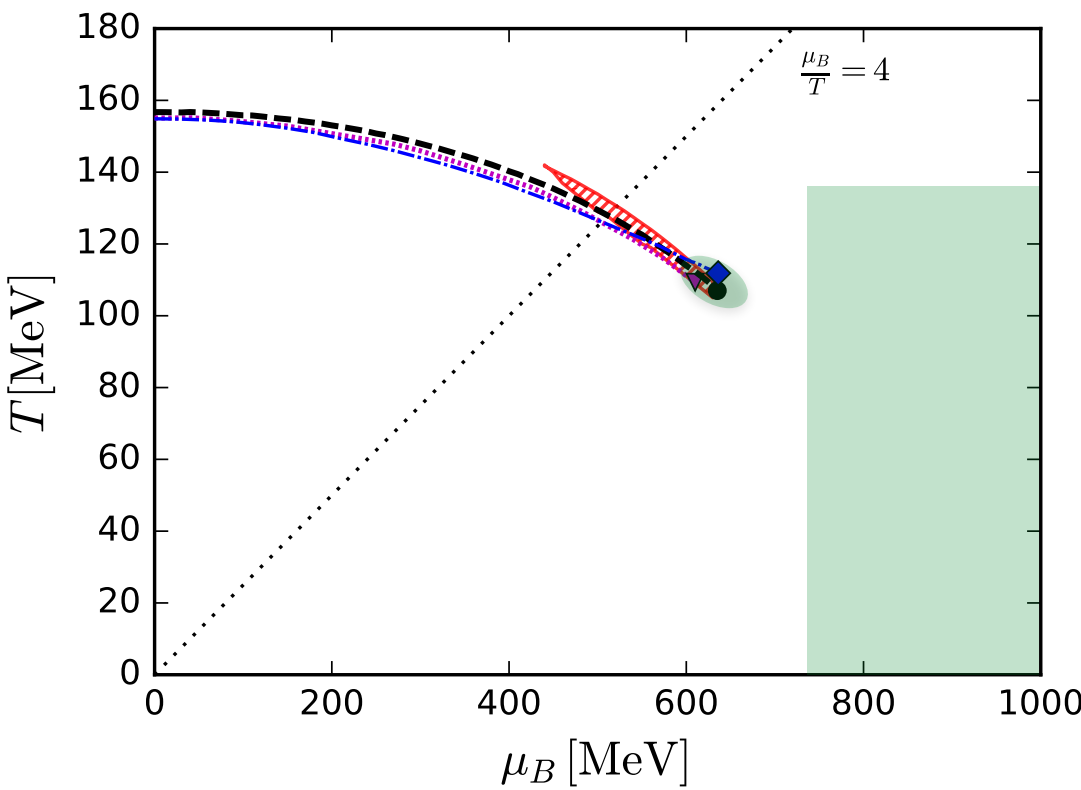


Predictions, estimates & extrapolations and how to use them



Scenario II

Minimal scaling and new phases

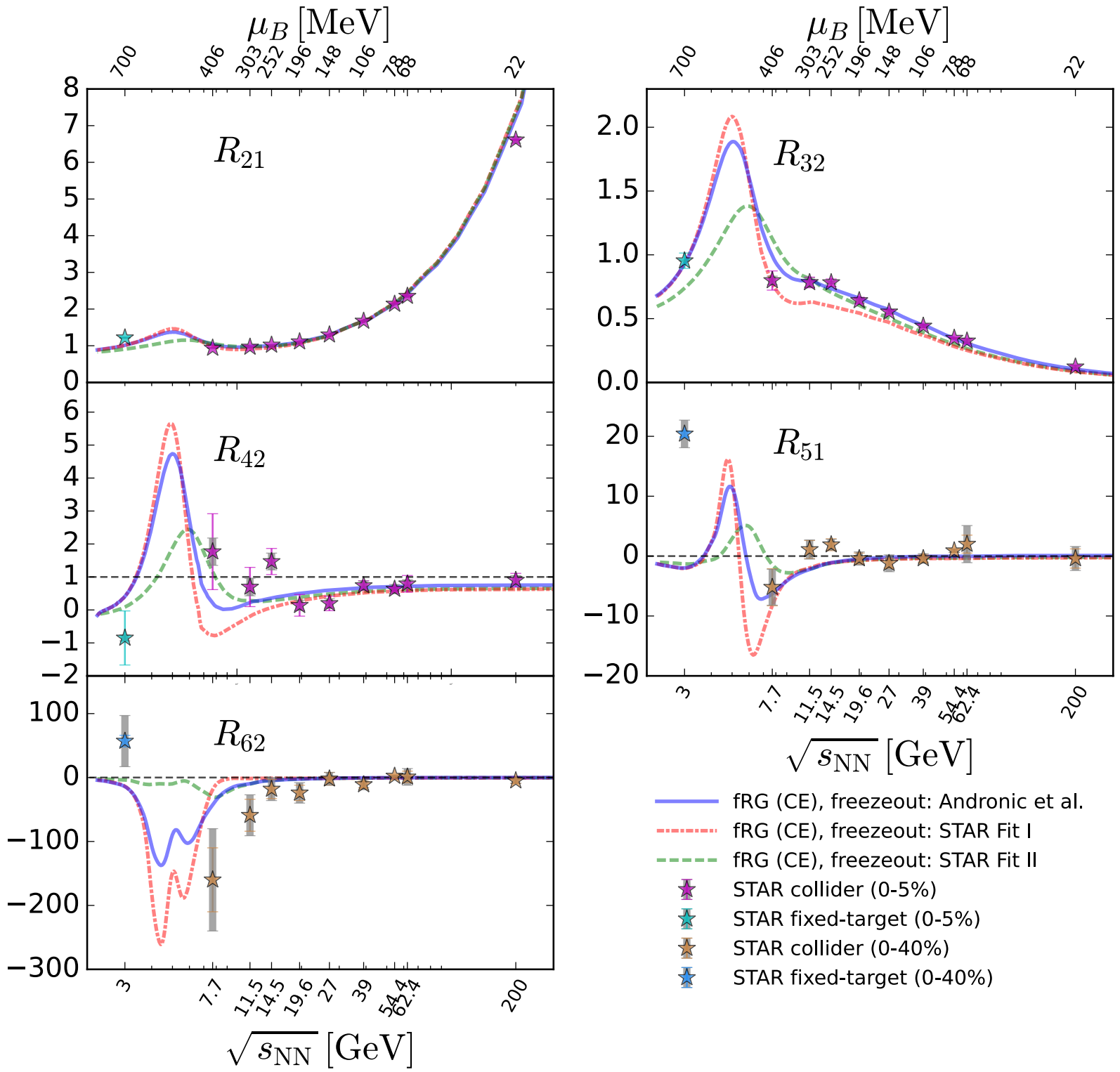


Extrapolations
for
Pheno



Ripples of the critical end point

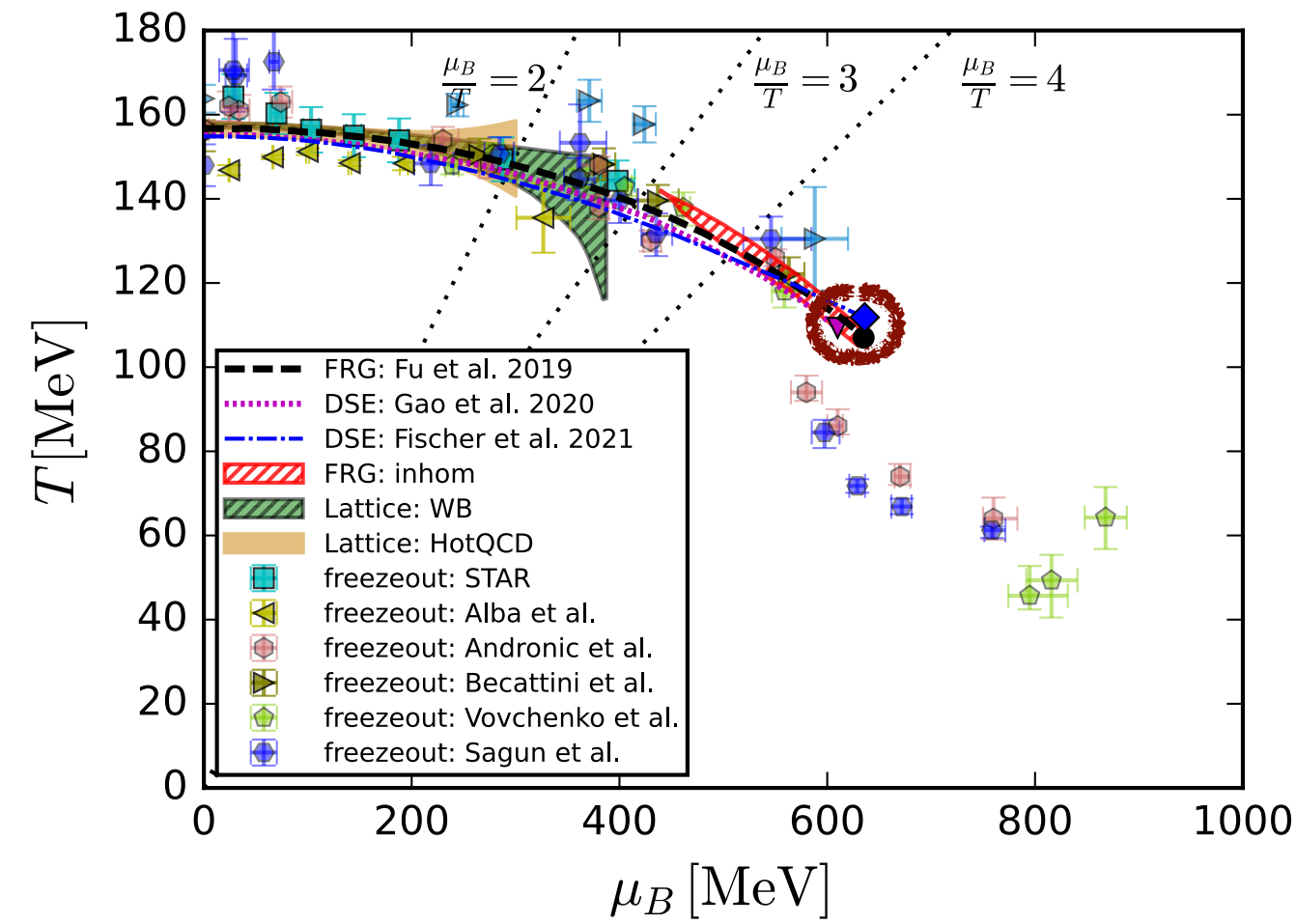
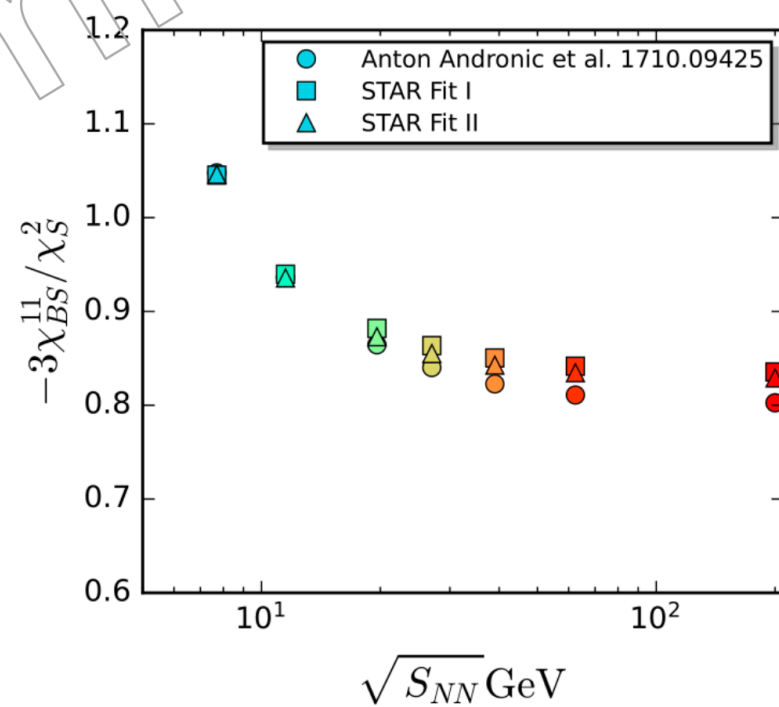
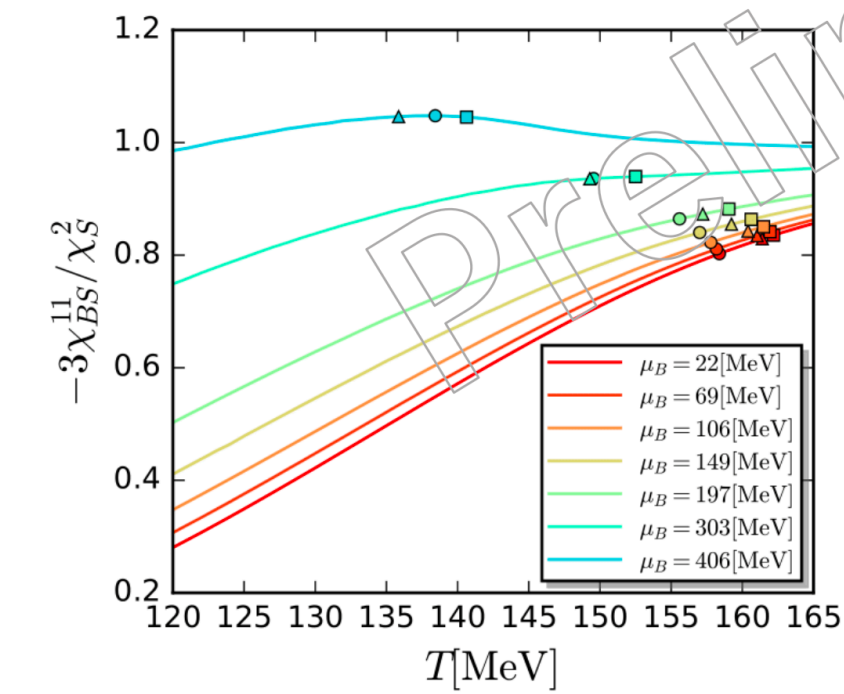
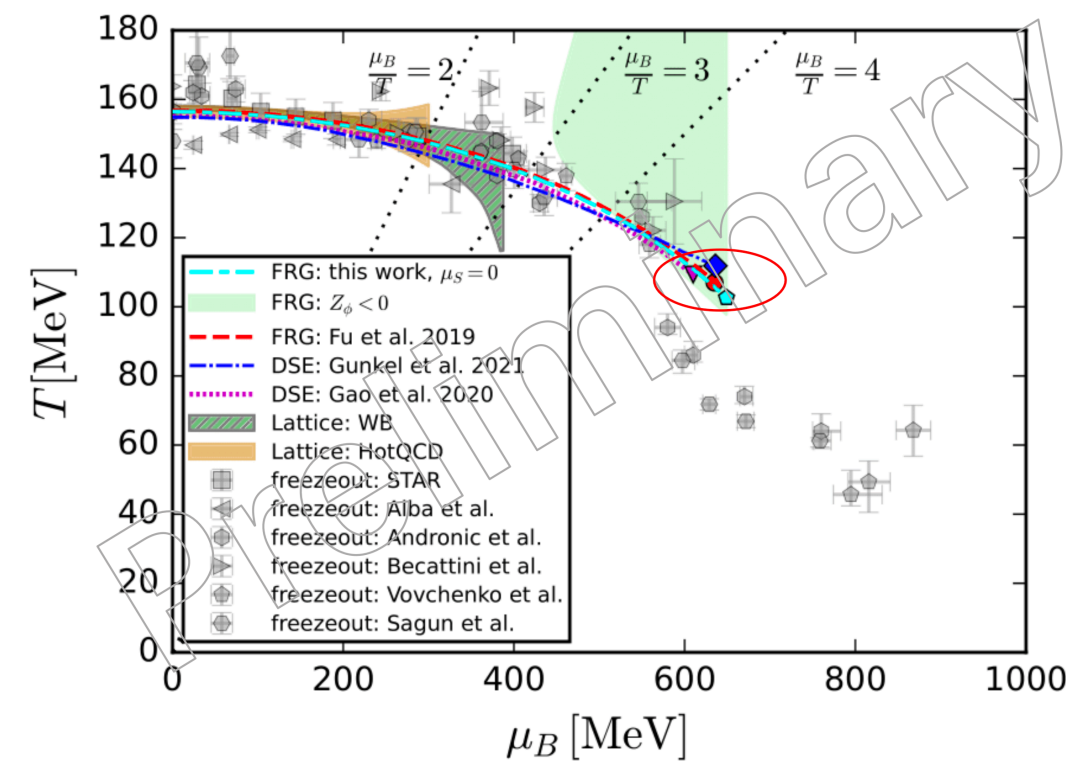
baryon & proton number fluctuations



Predictions, estimates & extrapolations and how to use them

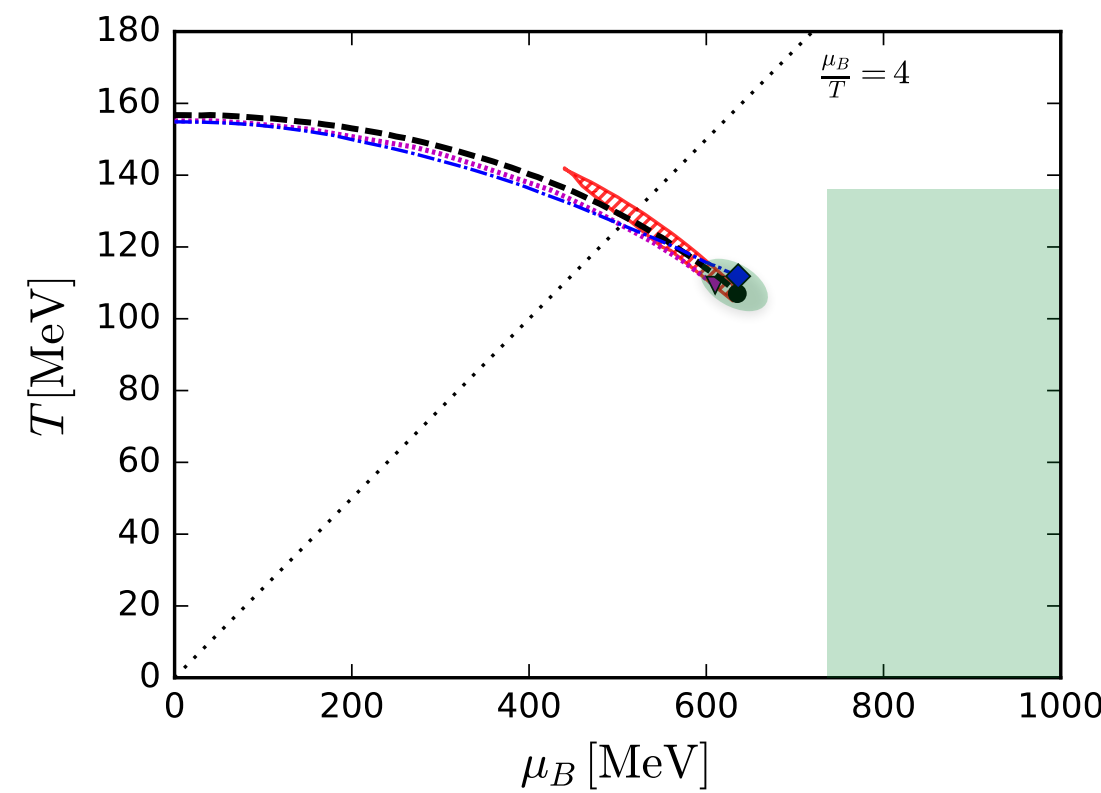
Strangeness neutrality

baryon & proton number fluctuations



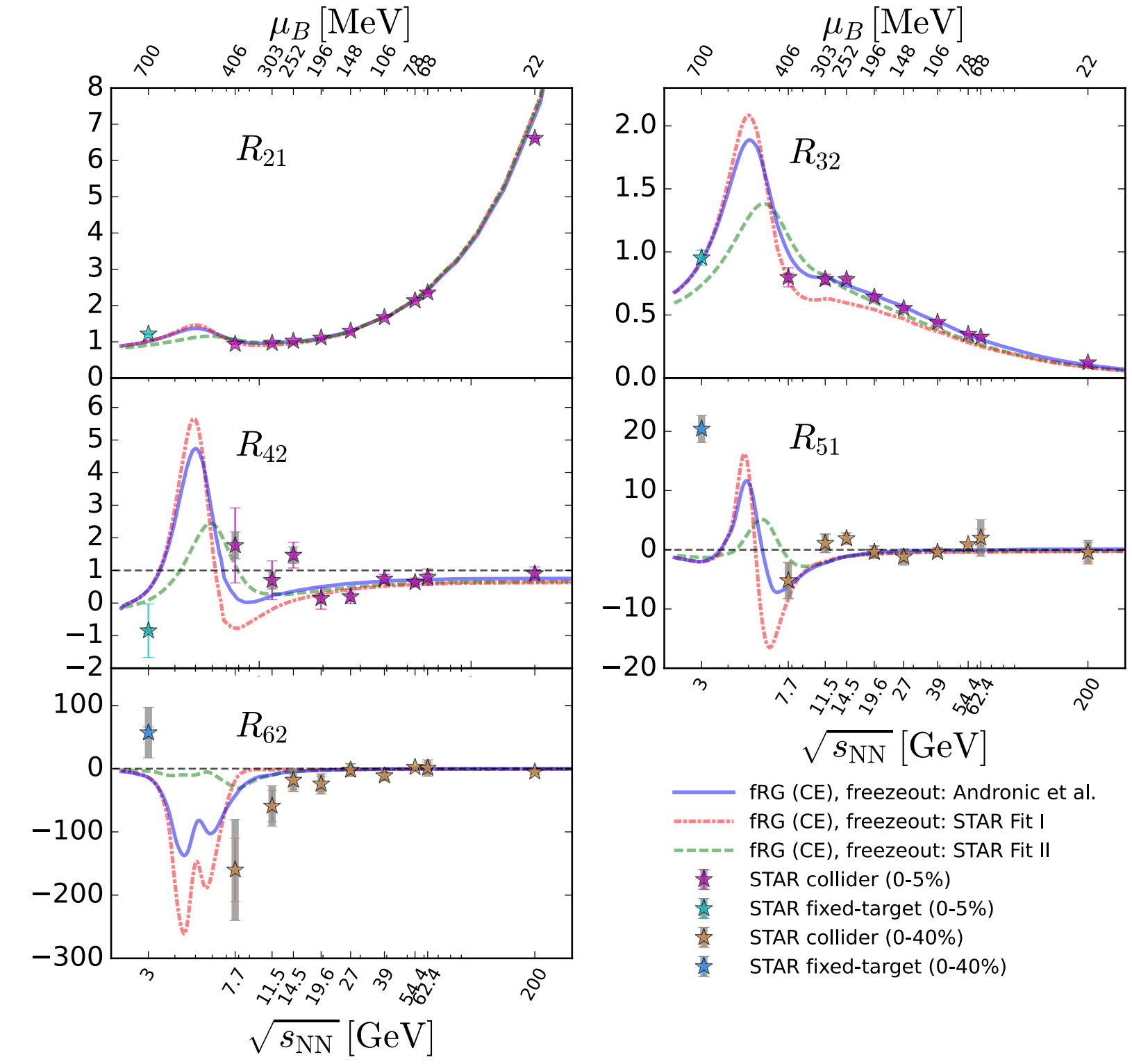
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations

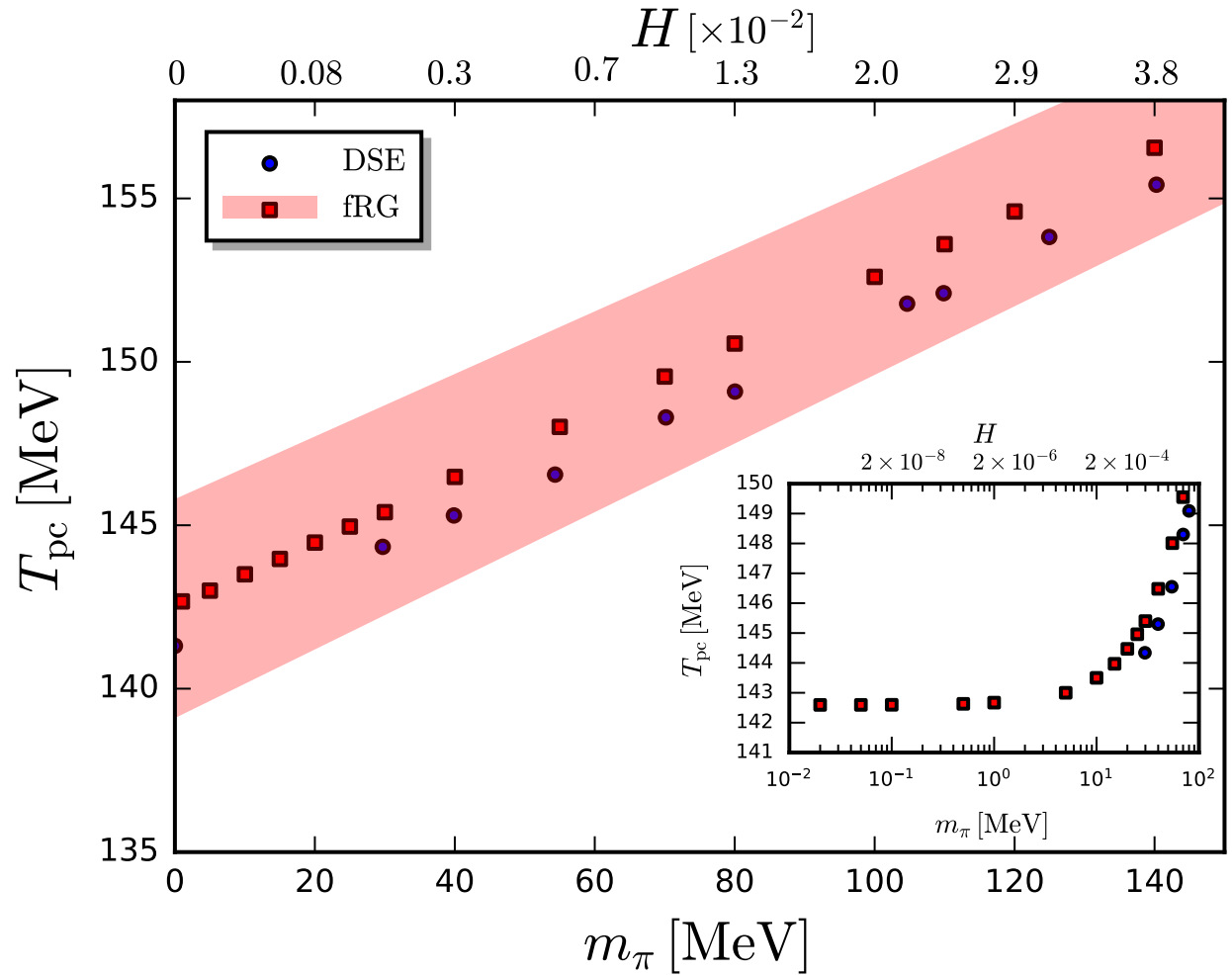
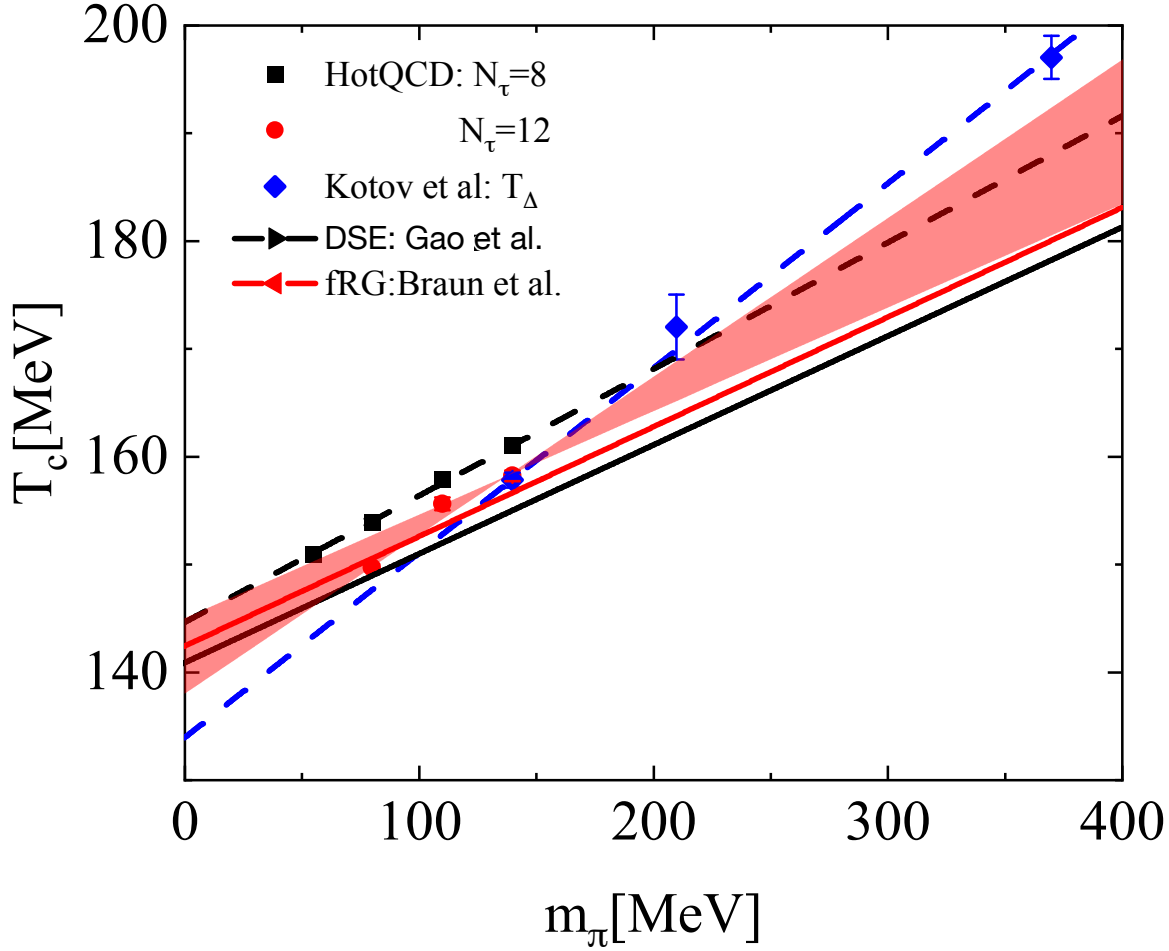
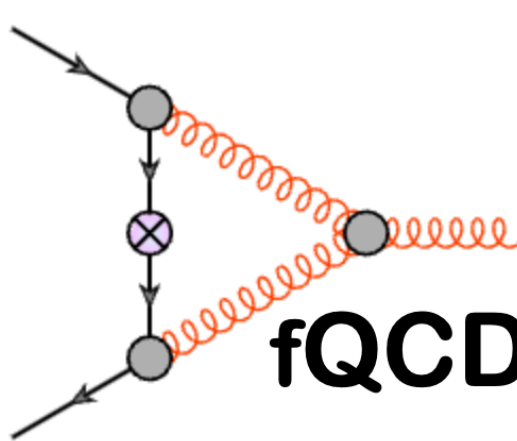


Chiral dynamics & soft modes

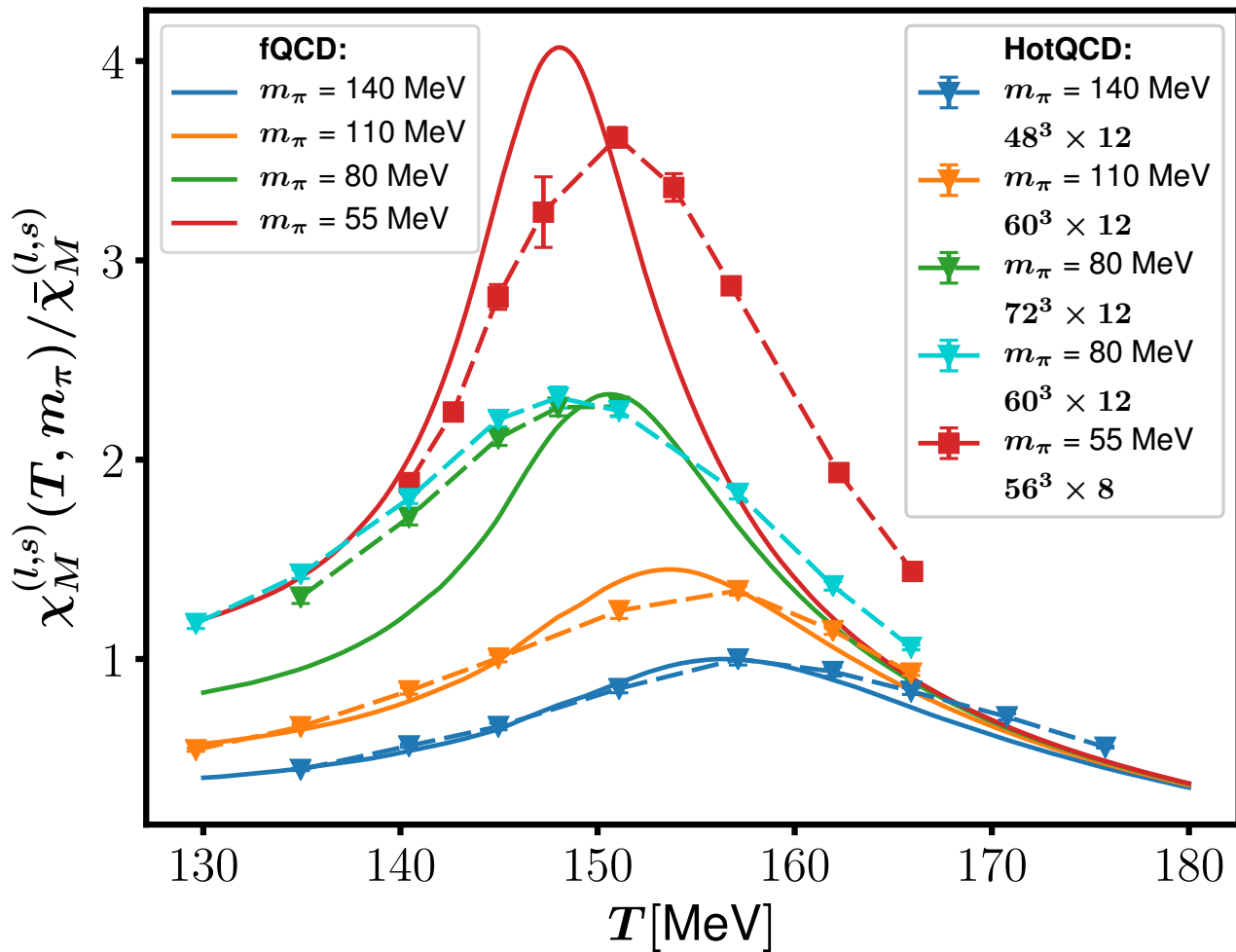
To be (critical) or not (to be)

Chiral transition temperature

$$H = \frac{m_l}{m_s}$$



Magnetic susceptibility

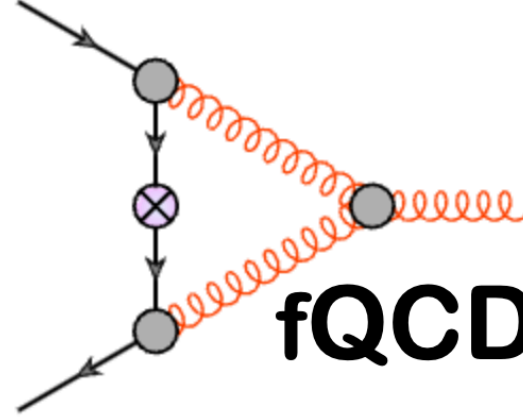


Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

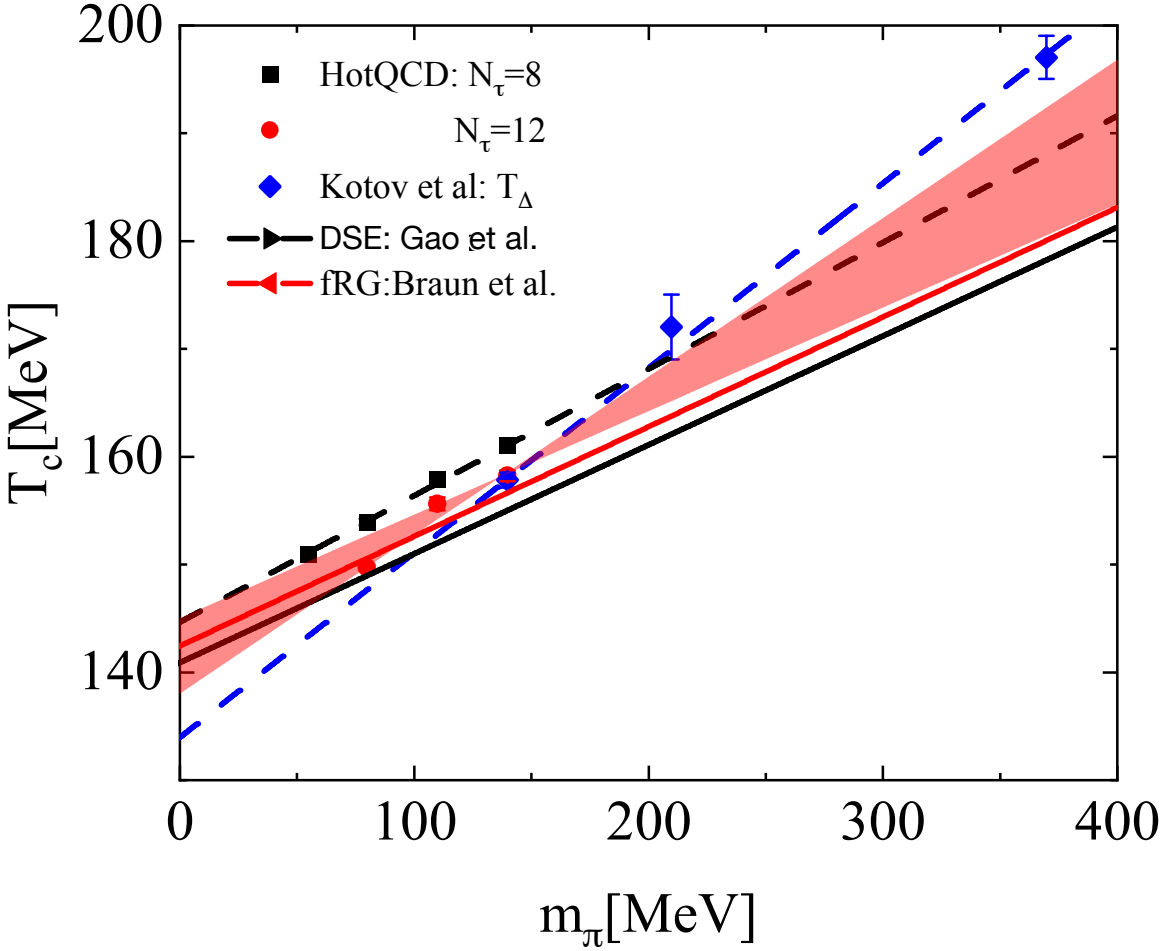
Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

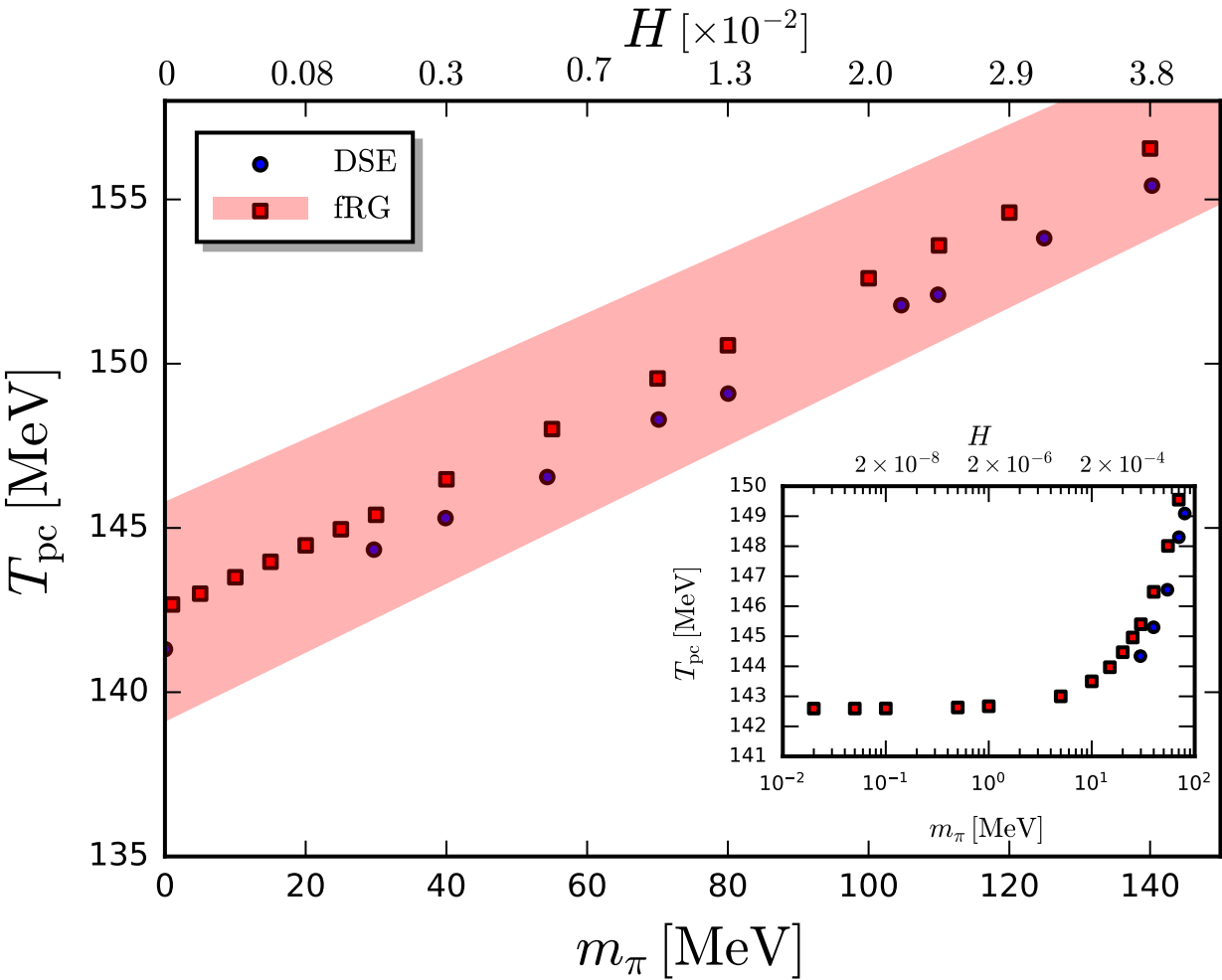
To be (critical) or not (to be)



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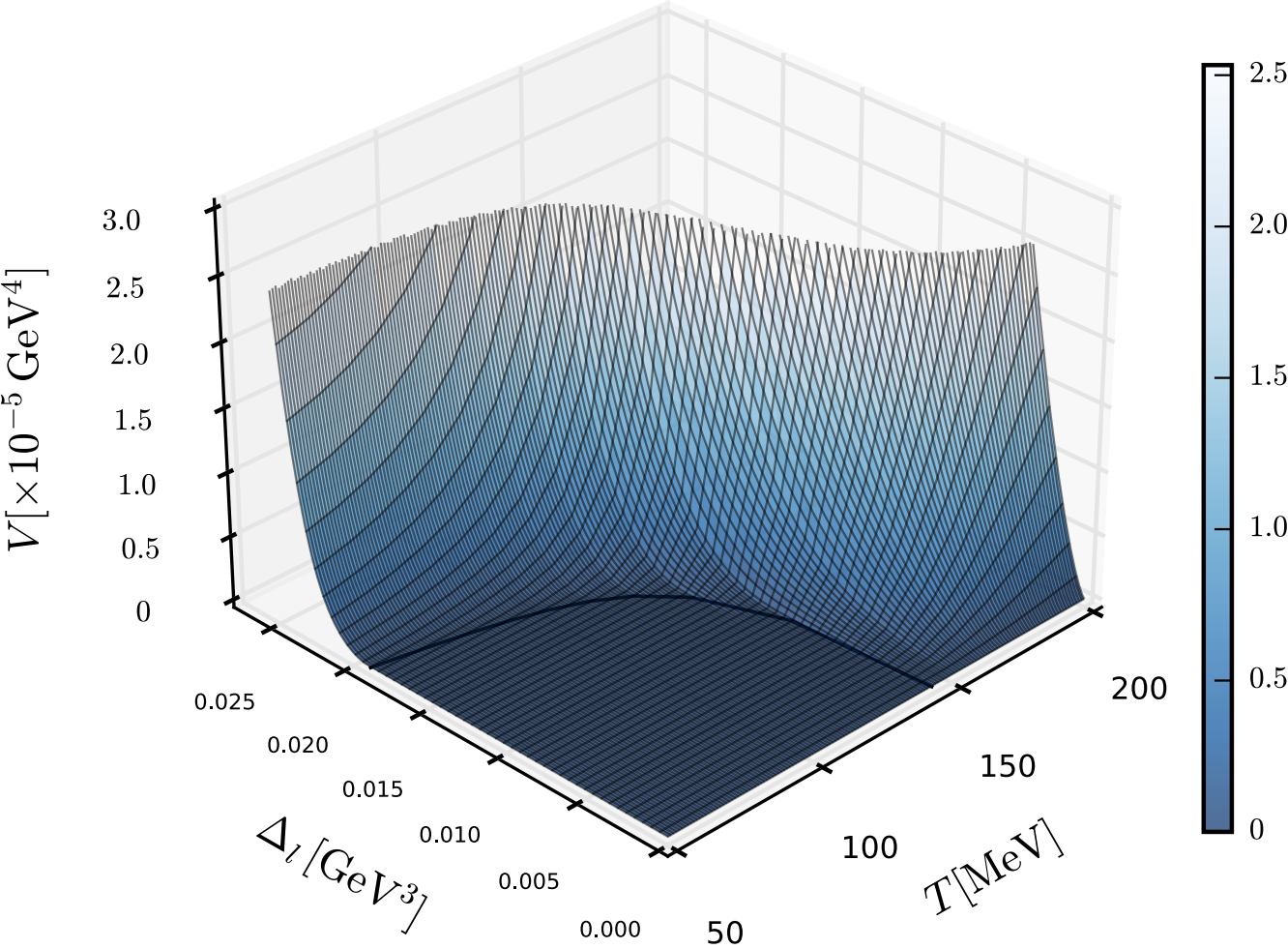
$$H = \frac{m_l}{m_s}$$



Order parameter potential & scaling

$$V_\chi \approx \Delta_l^n \longleftrightarrow \Delta_l(H) \propto H^{\frac{1}{n-1}}$$

(Critical) exponent: $\frac{1}{\delta} = \frac{1}{n-1}$

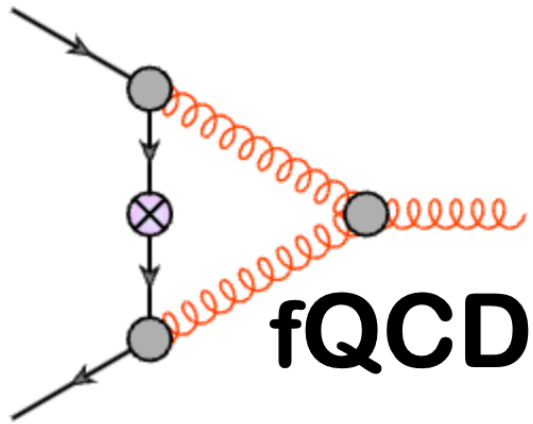


$$V_\chi^{(\text{fRG})} \approx V_\chi^{(\text{DSE})}$$

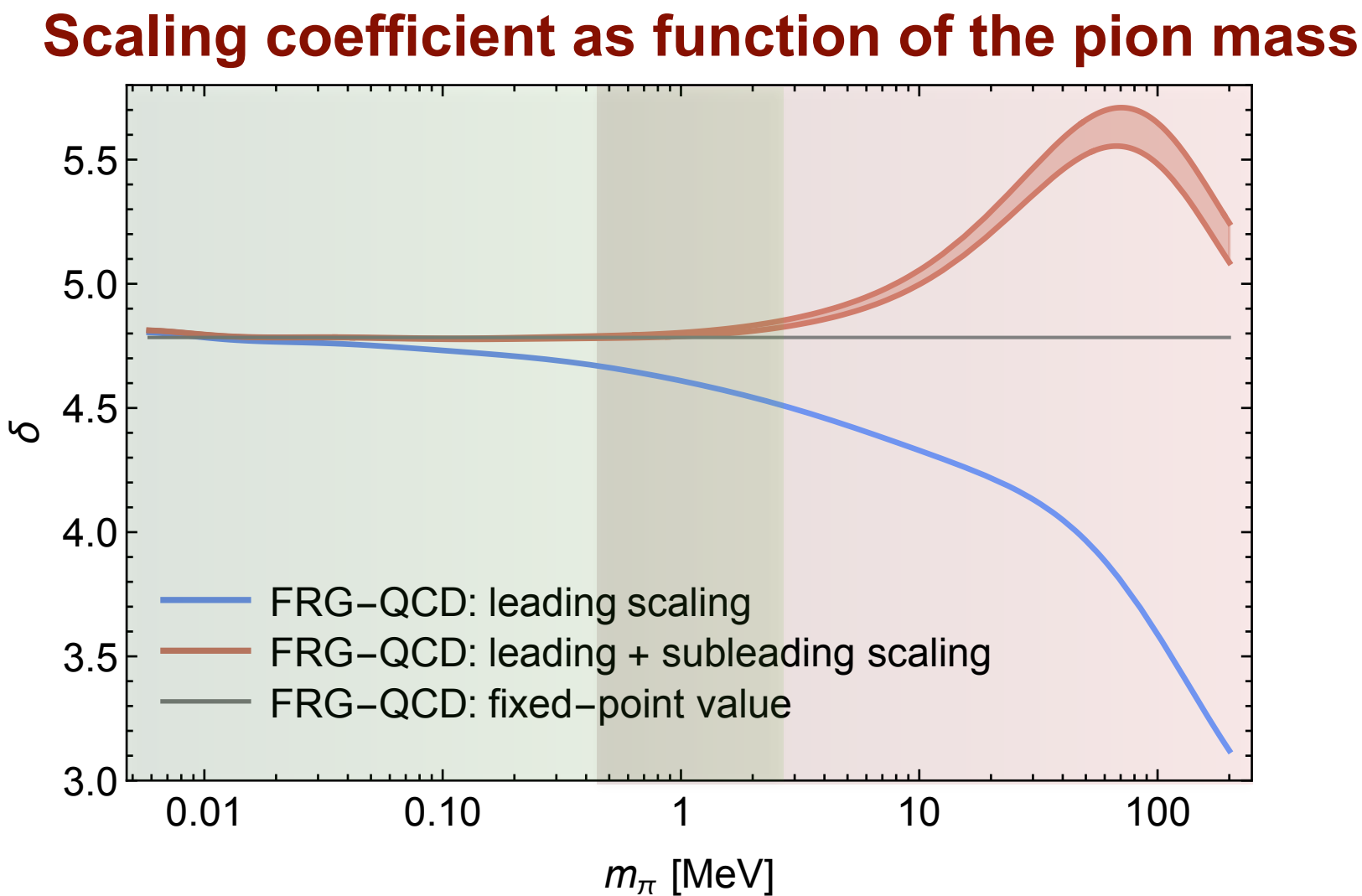
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010
Gao, JMP, PRD 105 (2022) 094020

Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, 2310.19853

Chiral dynamics & quasi-massless modes



Critical O(4) scaling

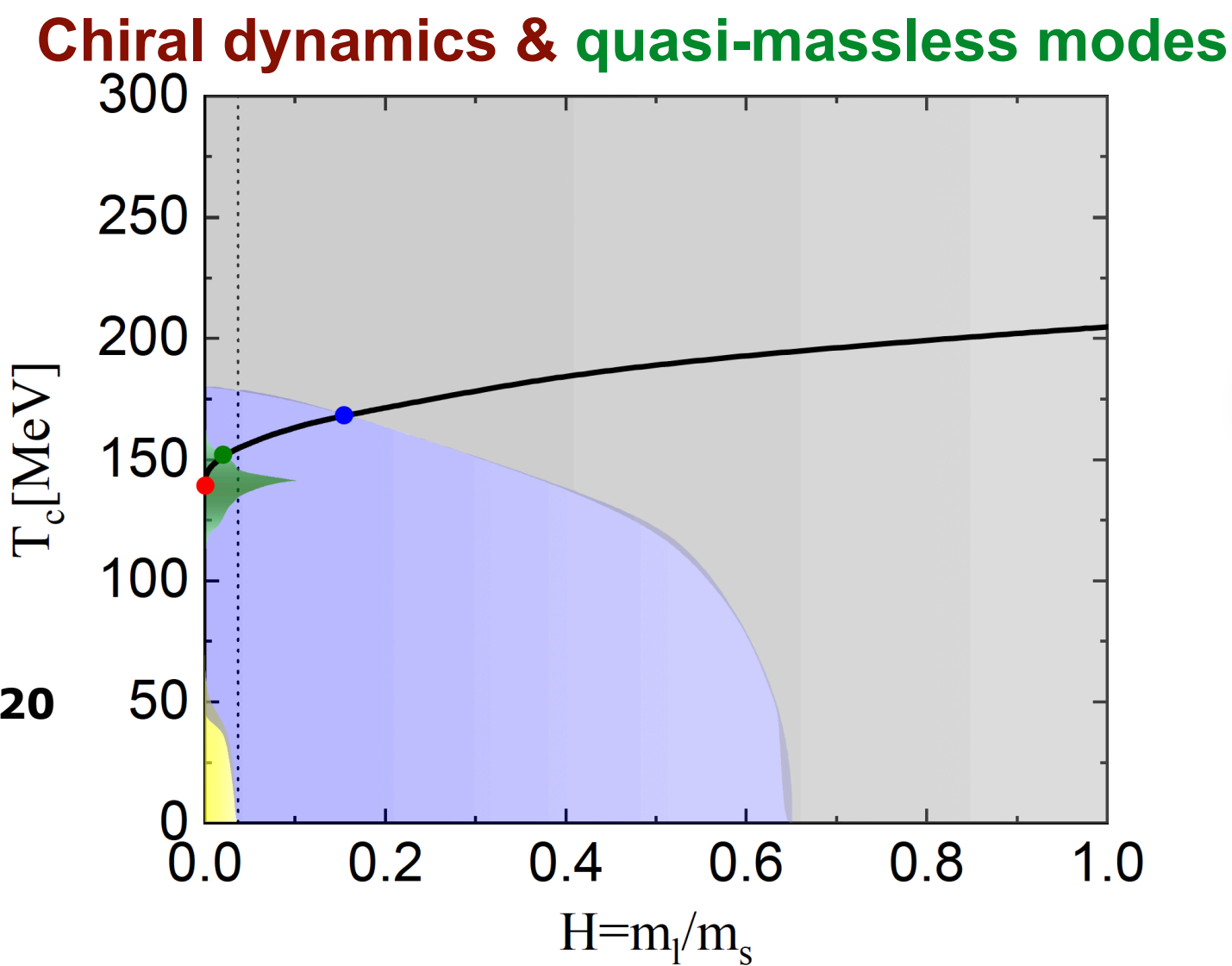


Trivial $\Delta_l^{1+\delta}$ scaling

Critical scaling



Gao, JMP, PRD 105 (2022) 094020



Far away from the critical regime for $m_\pi \gtrsim 1$ MeV

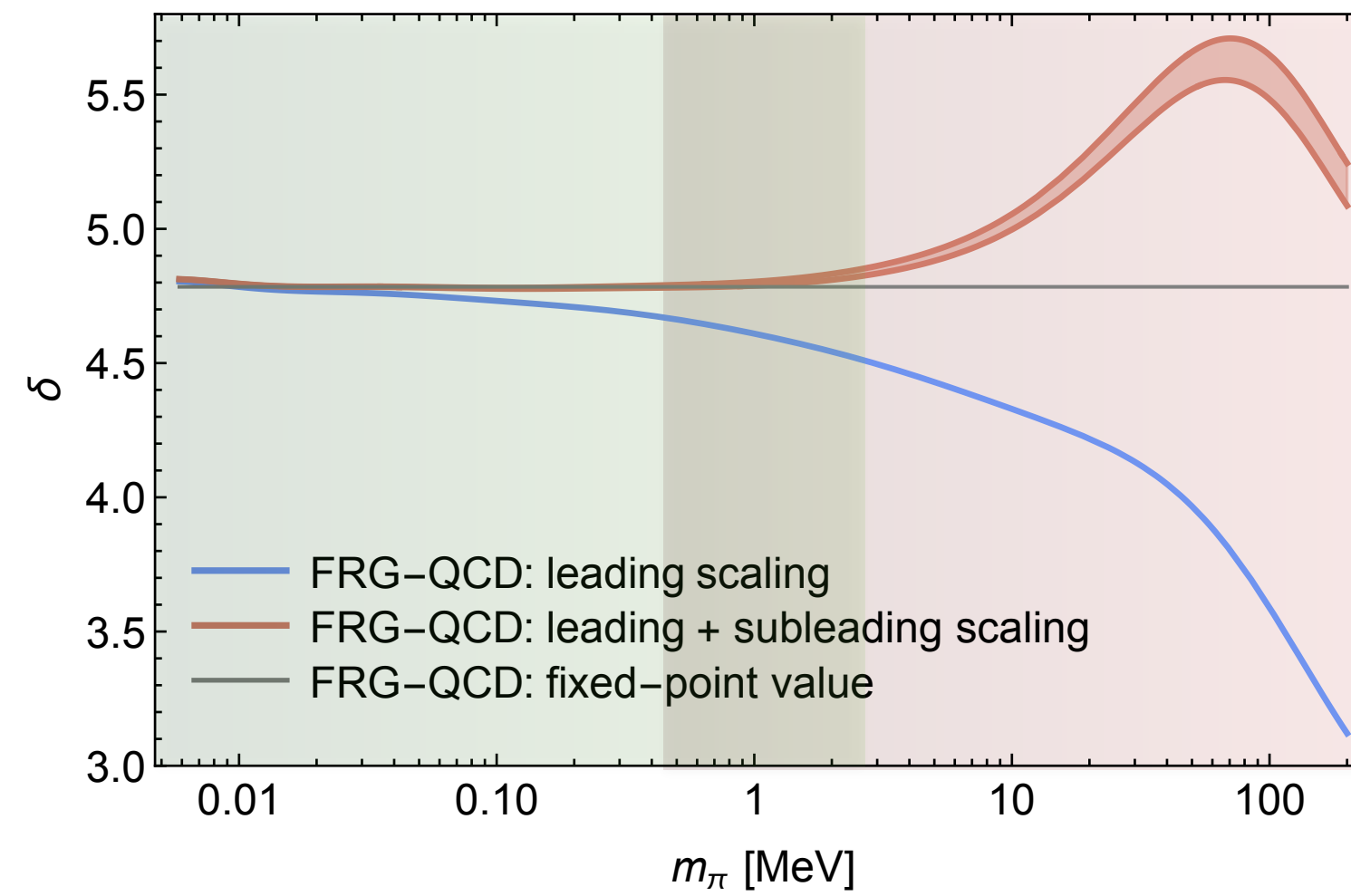
$$\Delta_l(T, H) \approx \Delta_{l,\chi}(0) \left(c_0 + c_{\frac{1}{5}} H^{\frac{1}{5}} + c_{\frac{1}{3}} H^{\frac{1}{3}} + c_1 H \right)$$

$V_\chi(\Delta_l) \propto$

Δ_l^6	Δ_l^4	Δ_l^2
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Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass

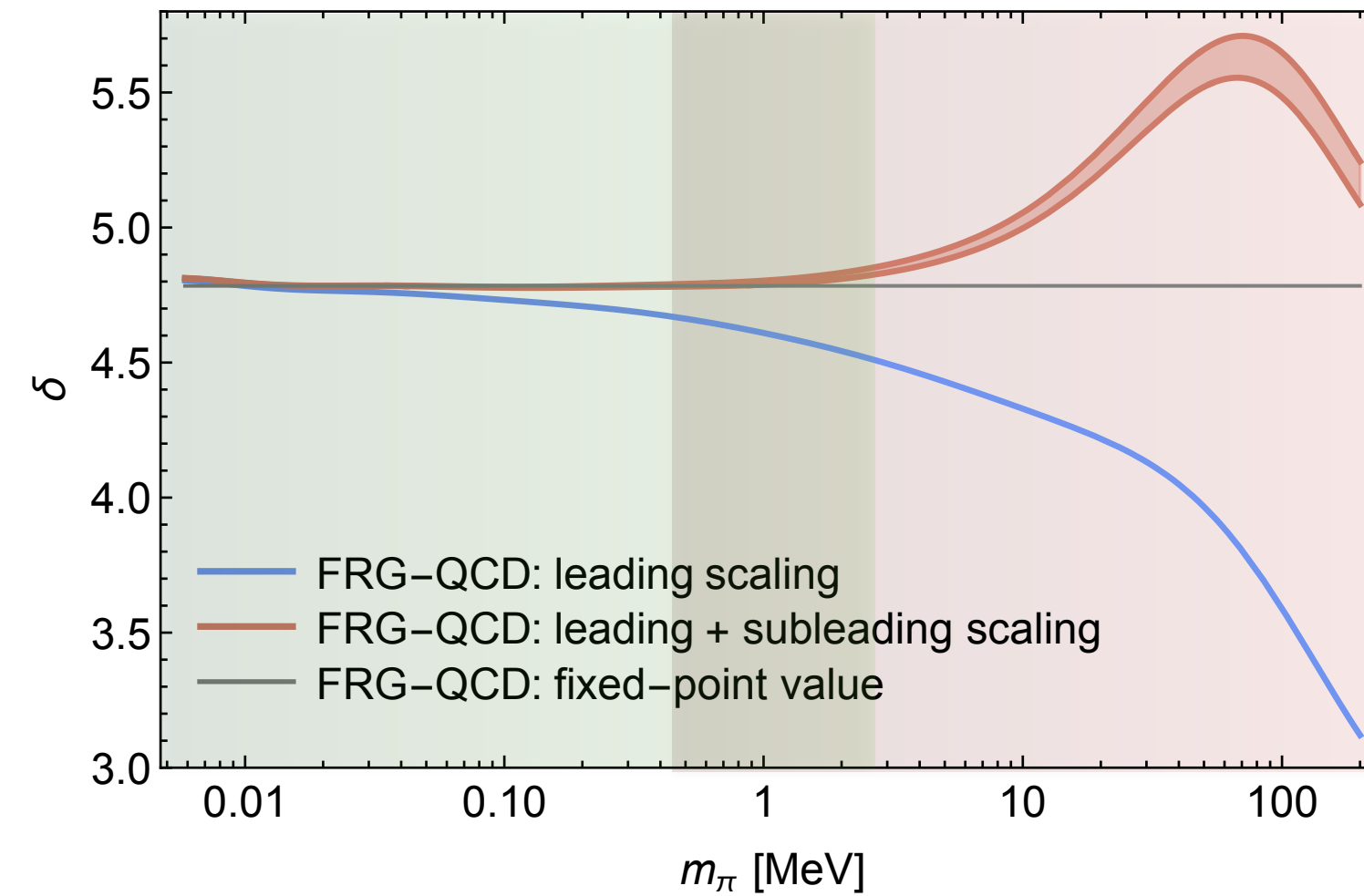


Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$

Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



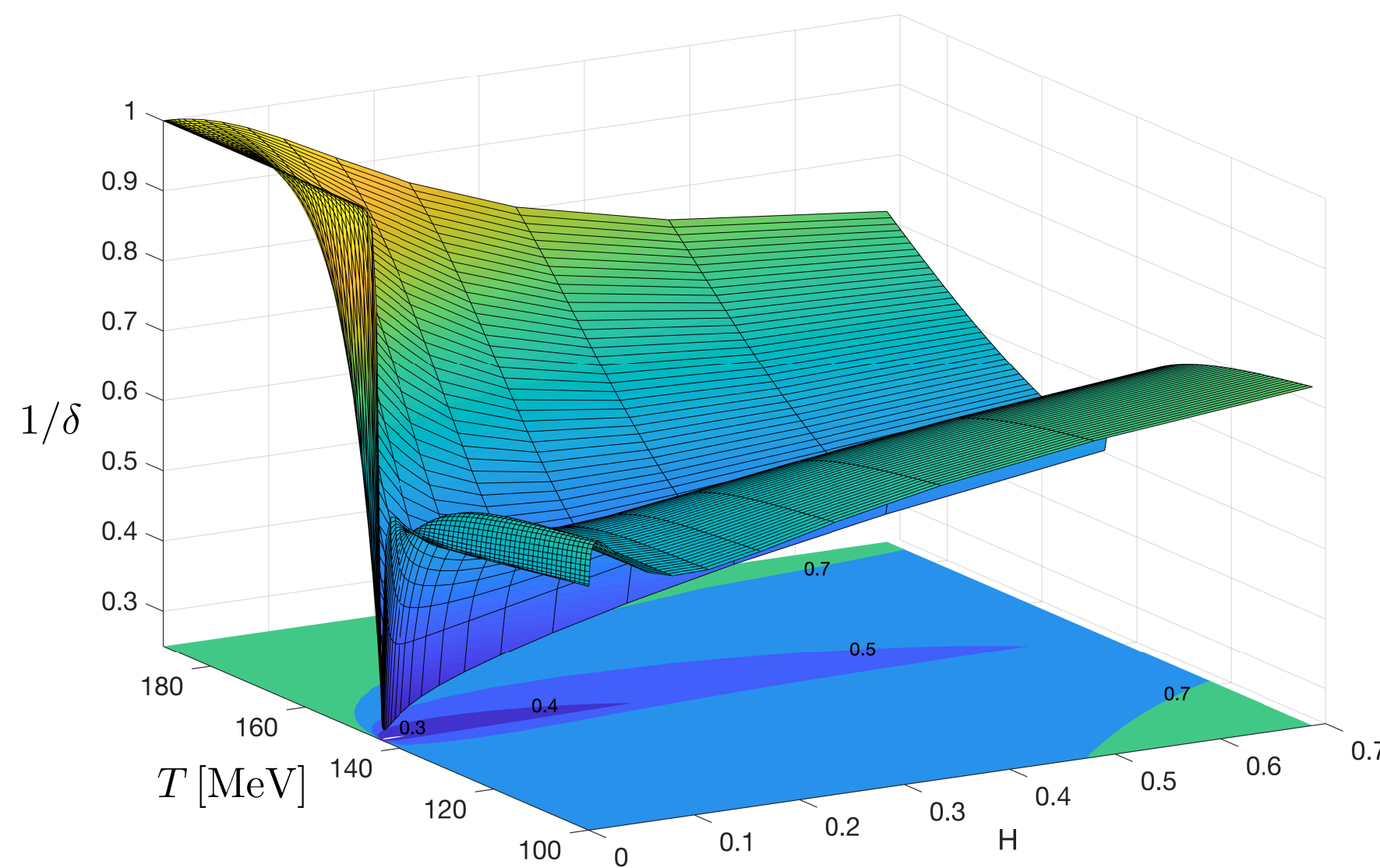
Trivial $\Delta_l^{1+\delta}$ scaling

$$\Delta_l(m_\pi) \propto m_\pi^{2/\delta} [1 + a_m m_\pi^{2\theta_H} + \dots]$$

Small chiral scaling regime

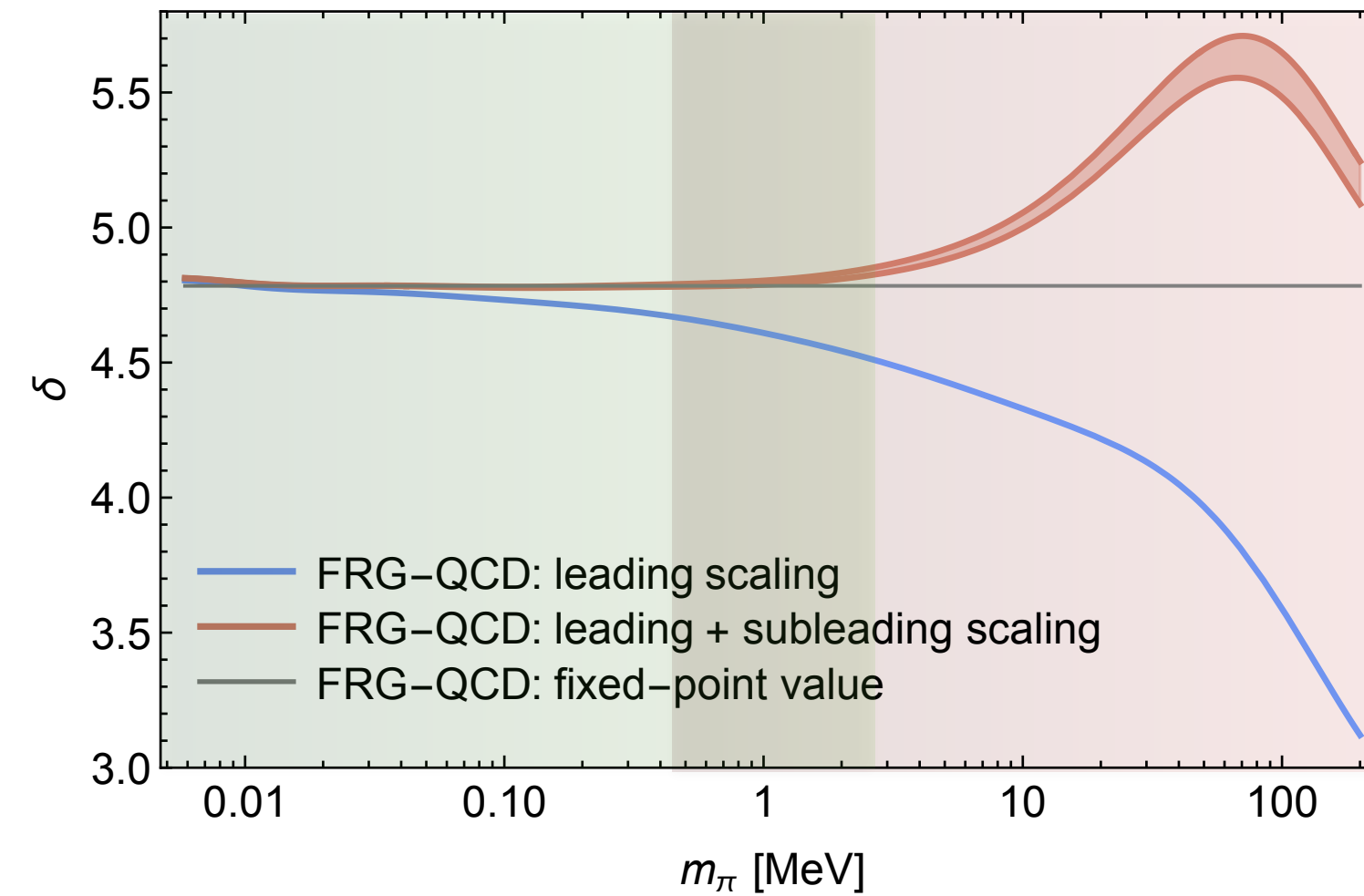


Small critical regime around pot. CEP



Chiral dynamics & quasi-massless modes

Scaling coefficient as function of the pion mass



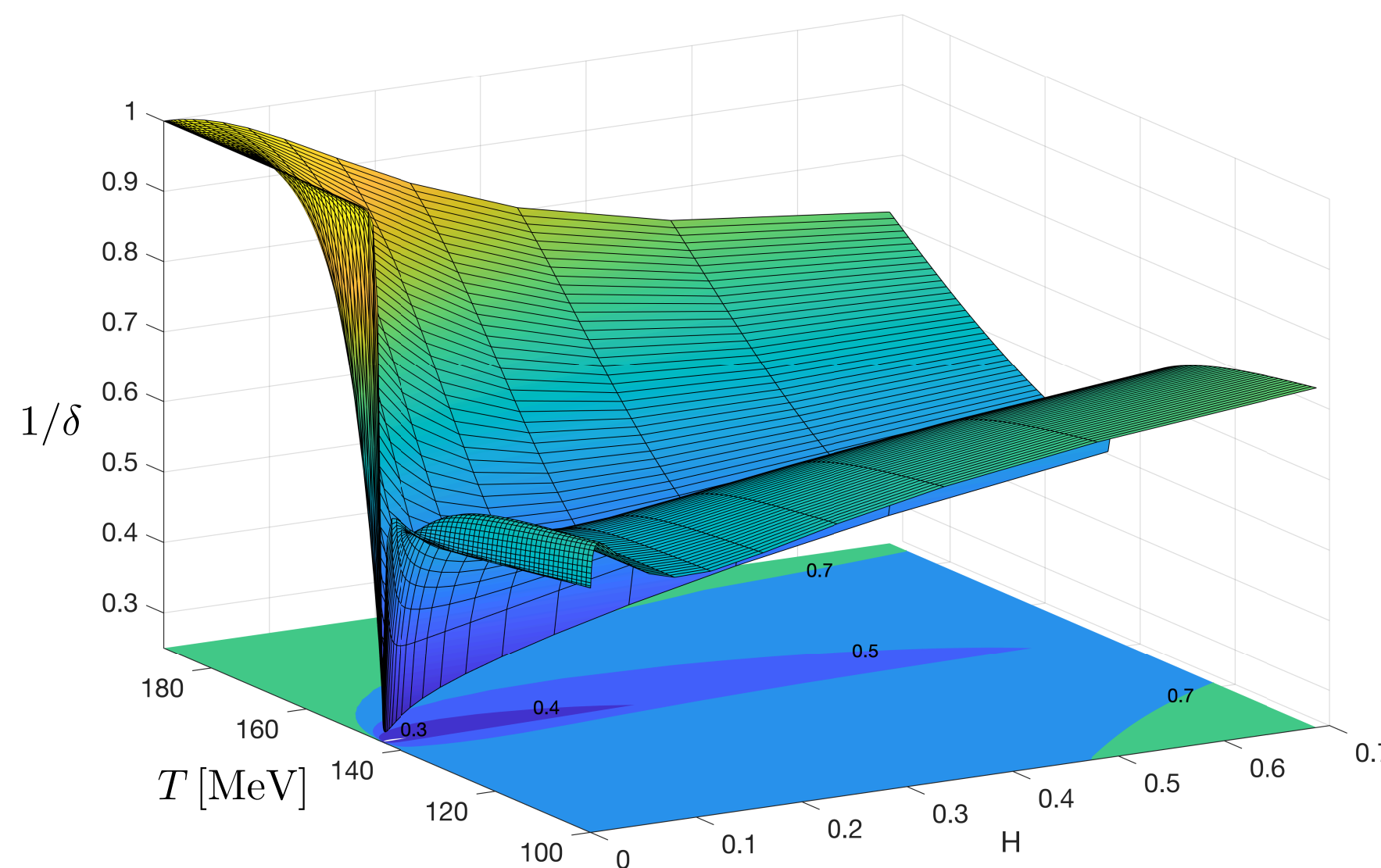
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Small chiral scaling regime



Small critical regime around pot. CEP

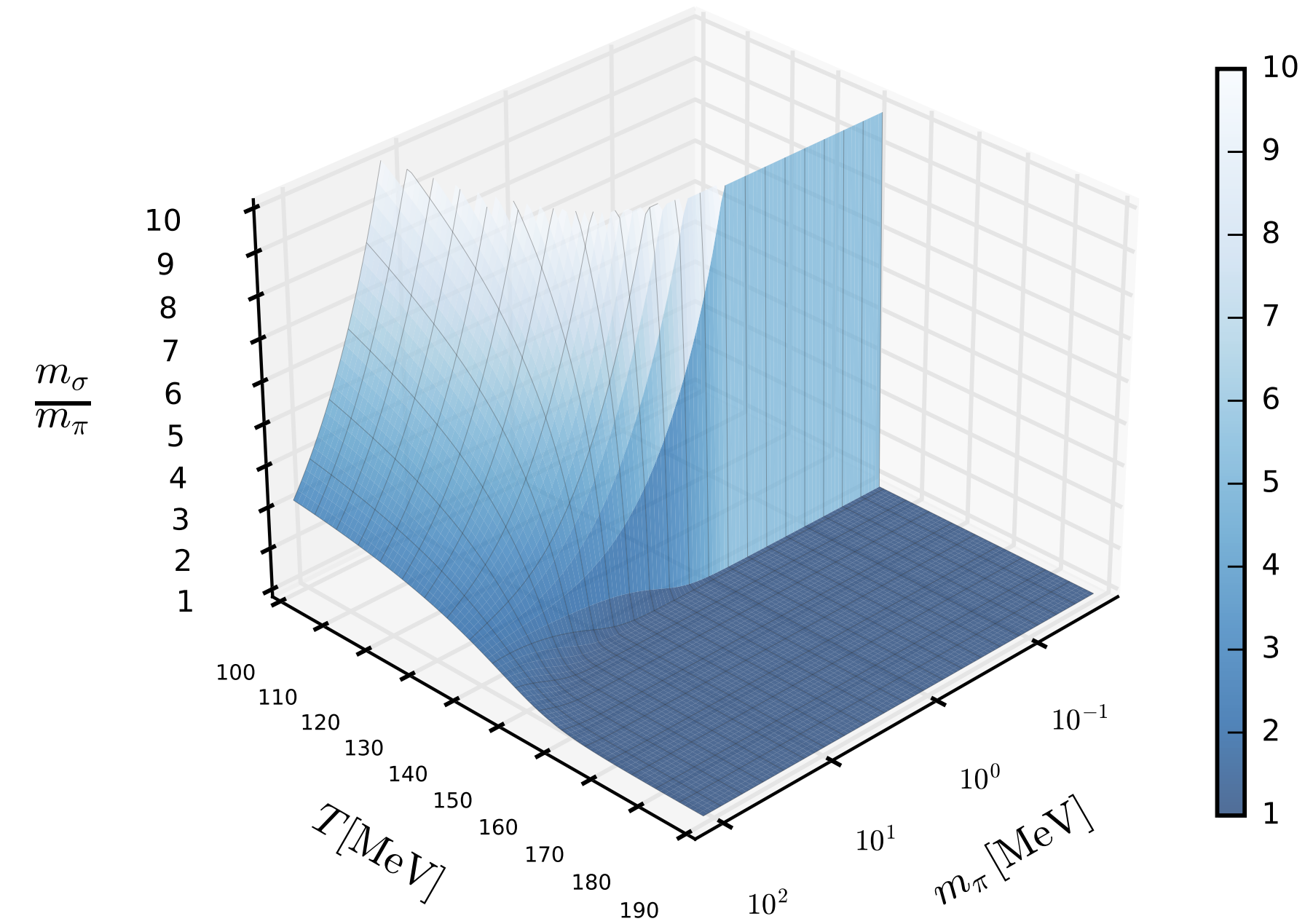
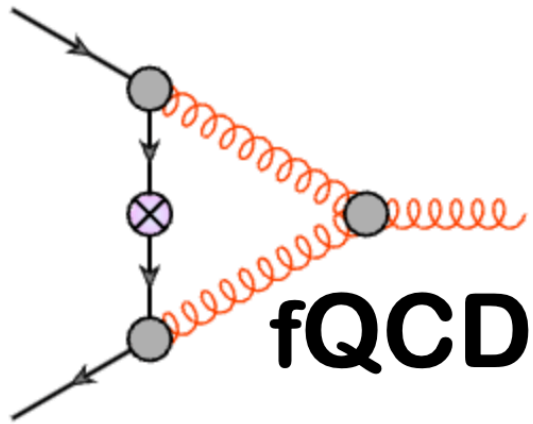


!!Great News!!

Location of CEP/New phase accessible via combination of precision measurements & computations

Full order parameter potential

Measure: correlation length



Use for chiral dynamics in heavy ion collisions

