Understanding Proton Interactions

interplay between attractive and repulsive correlations

Anar Rustamov

a.rustamov@gsi.de

a.rustamov@cern.ch

The interplay between attractive and repulsive correlations among protons plays a critical role in determining the conditions necessary for phase transitions

Non-critical baselines

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, NPA 1008 (2021) 122141 (**Global Conservations**)
P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113 (**Attractive local correlations**)
B. Friman, A. Rustamov, K. Redlich, **Repulsive interactions** (in preparation)

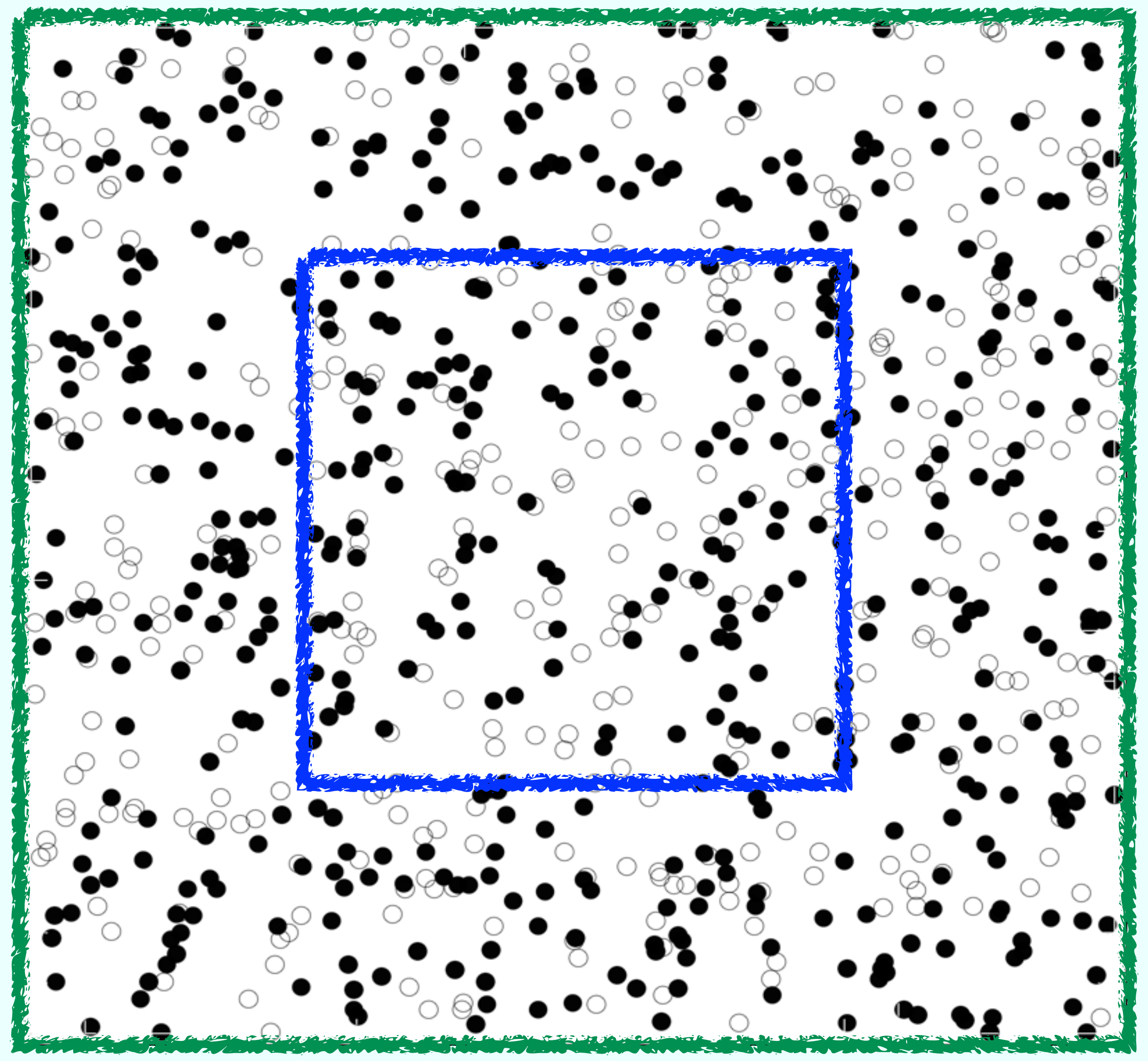
Volume Fluctuations

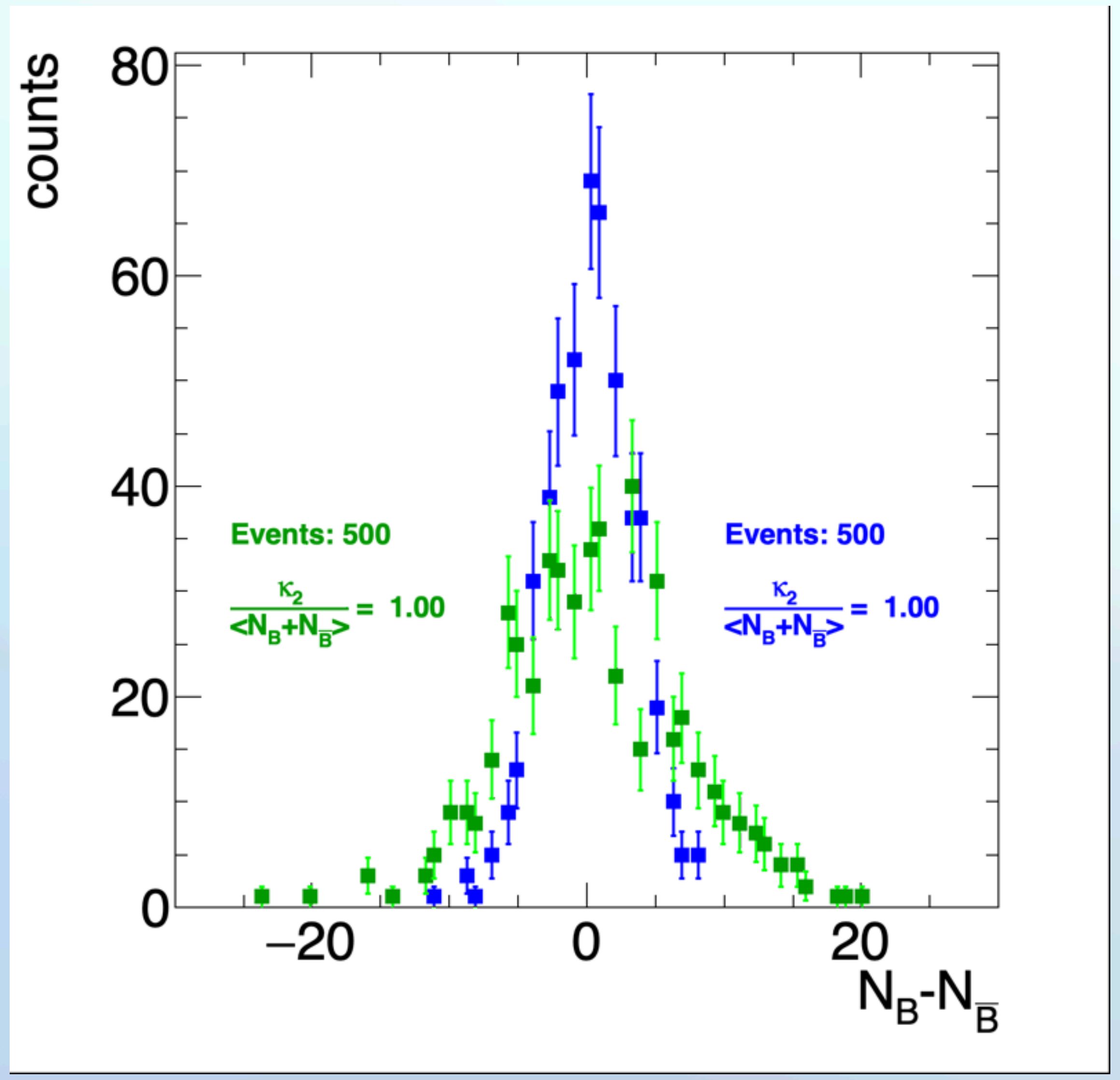
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114-130
A. Rustamov, R. Holzmann, J. Stroth , Nucl.Phys.A 1034 (2023) 122641 **(Event Mixing)**
A. Rustamov, R. Holzmann, V. Koch, A. Rustamov, J. Stroth, NPA 1050 (2024) 122924
V. Skokov, B. Friman, K. Redlich, Phys. Rev. C 88 (2013)

Analysis Methods

A. Rustamov, Phys.Rev.C 110 (2024) 6, 064910 **(Fuzzy Logic)**
A. Rustamov, M. I. Gorenstein, Phys.Rev.C 86 (2012) 044906
M. Gazdzicki et. al, Phys.Rev.C 83 (2011) 054907
M. Arslandok, A. Rustamov, Nucl.Instrum.Meth.A 946 (2019) 162622
…

Ideal Gas in GCE + conservation laws





$$\boxed{\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle}} = \alpha \boxed{\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle}} + 1 - \alpha$$

$\alpha = \langle N_B^{acc}\rangle / \langle N_B^{4\pi}\rangle$

Unity in GCE

If baryon number is conserved in full phase space

$$\boxed{\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle}} = 0$$

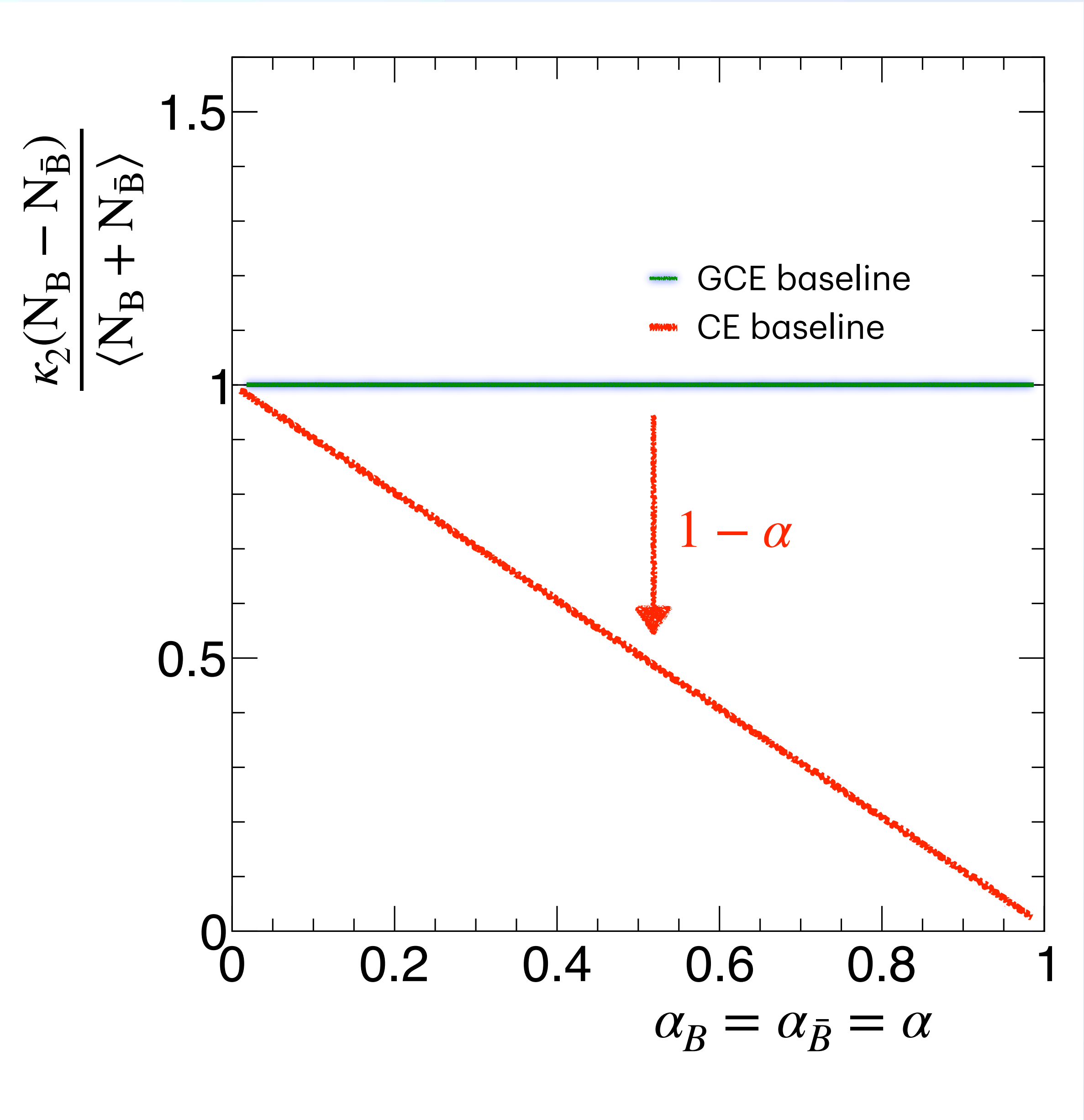
$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle} = 1 - \alpha$$

Grand Canonical
(conservation on average)

$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle} = 1$$

Exact conservation

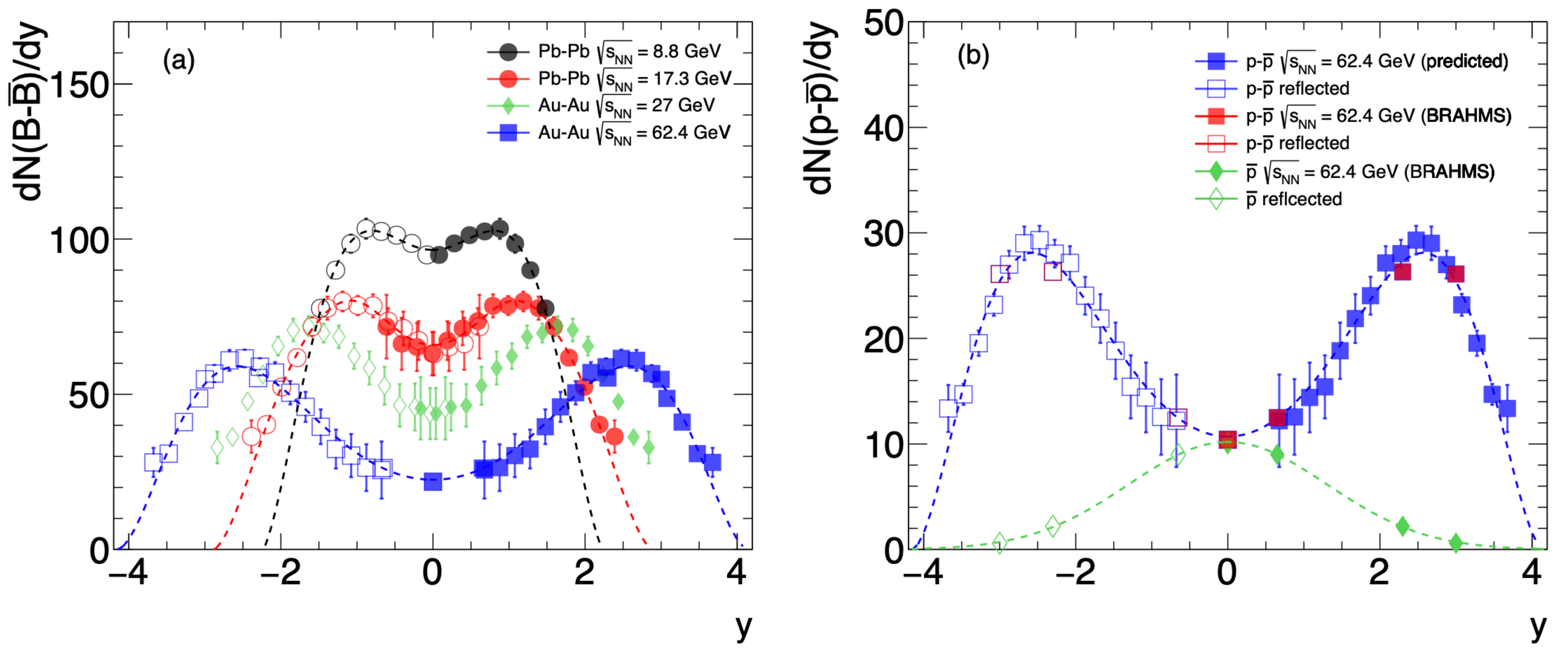
$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}}\rangle} = 1 - \alpha$$

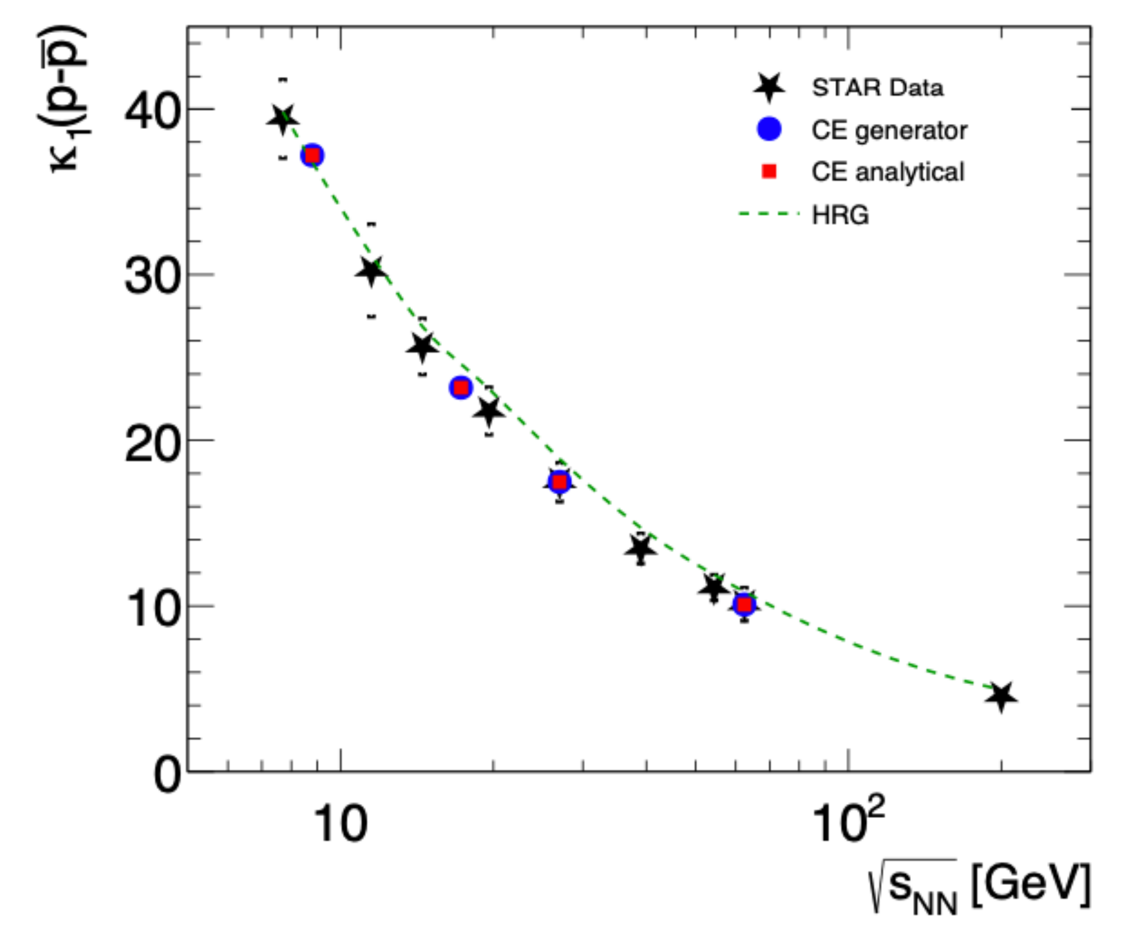


P. Braun-Munzinger, A.R., J. Stachel, NPA 960 (2017) 114-130
P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141
A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

Implementation of Canonical Ensemble

$$Z_B(V,T) = \sum_{N_B=0}^{\infty} \sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!} \frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2z\sqrt{\lambda_B \lambda_{\bar{B}}})$$





fixing B

fixing $\alpha_{B,\bar{B}}$

- **baryon rapidity distributions**
- **measured mean multiplicities**

fixing z

$z = \sqrt{z_B z_{\bar{B}}}$ **is calculated by solving**

$$\langle N_B \rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B, \lambda_{\bar{B}} = 1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

Symbolic derivation for all orders

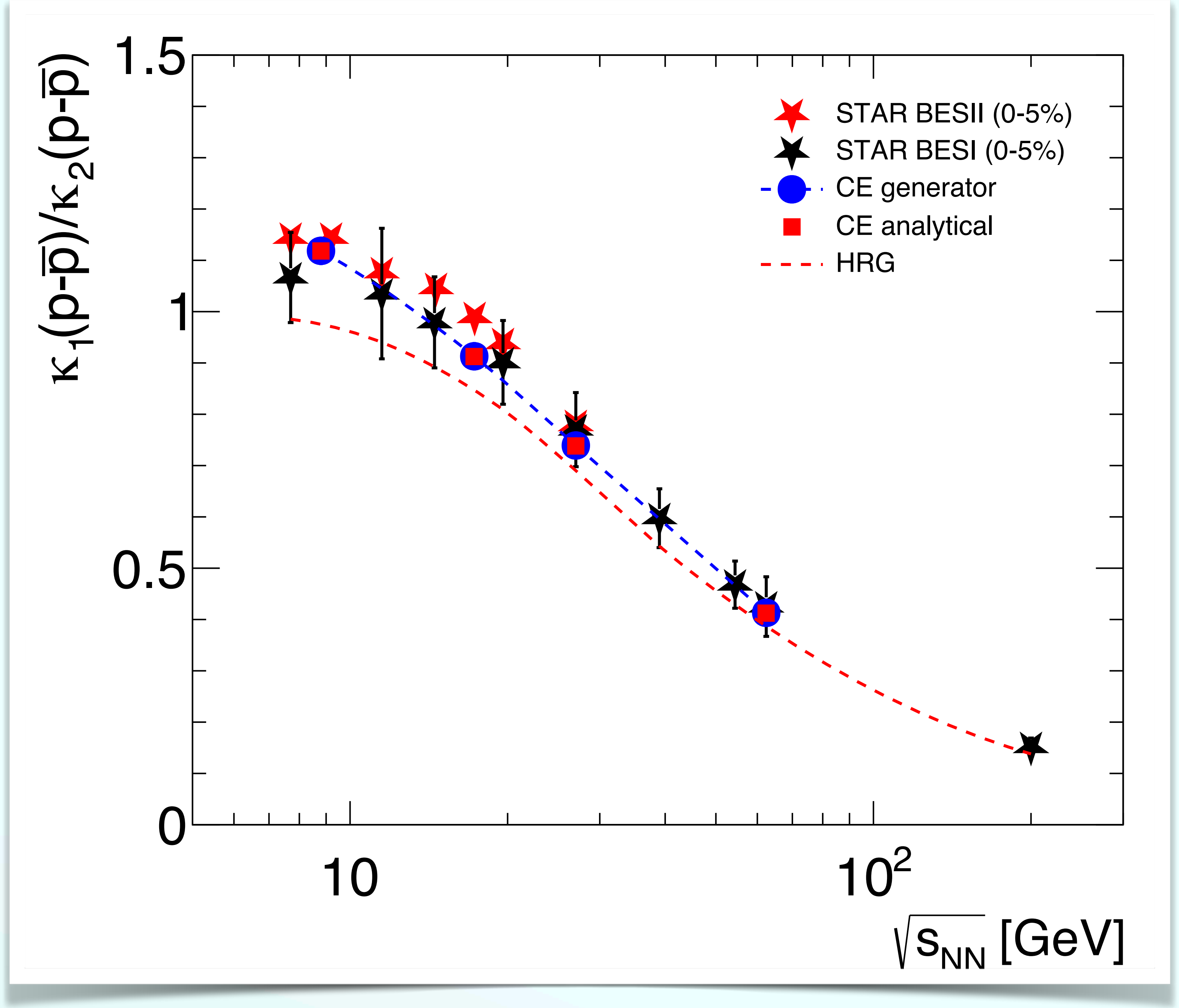
P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

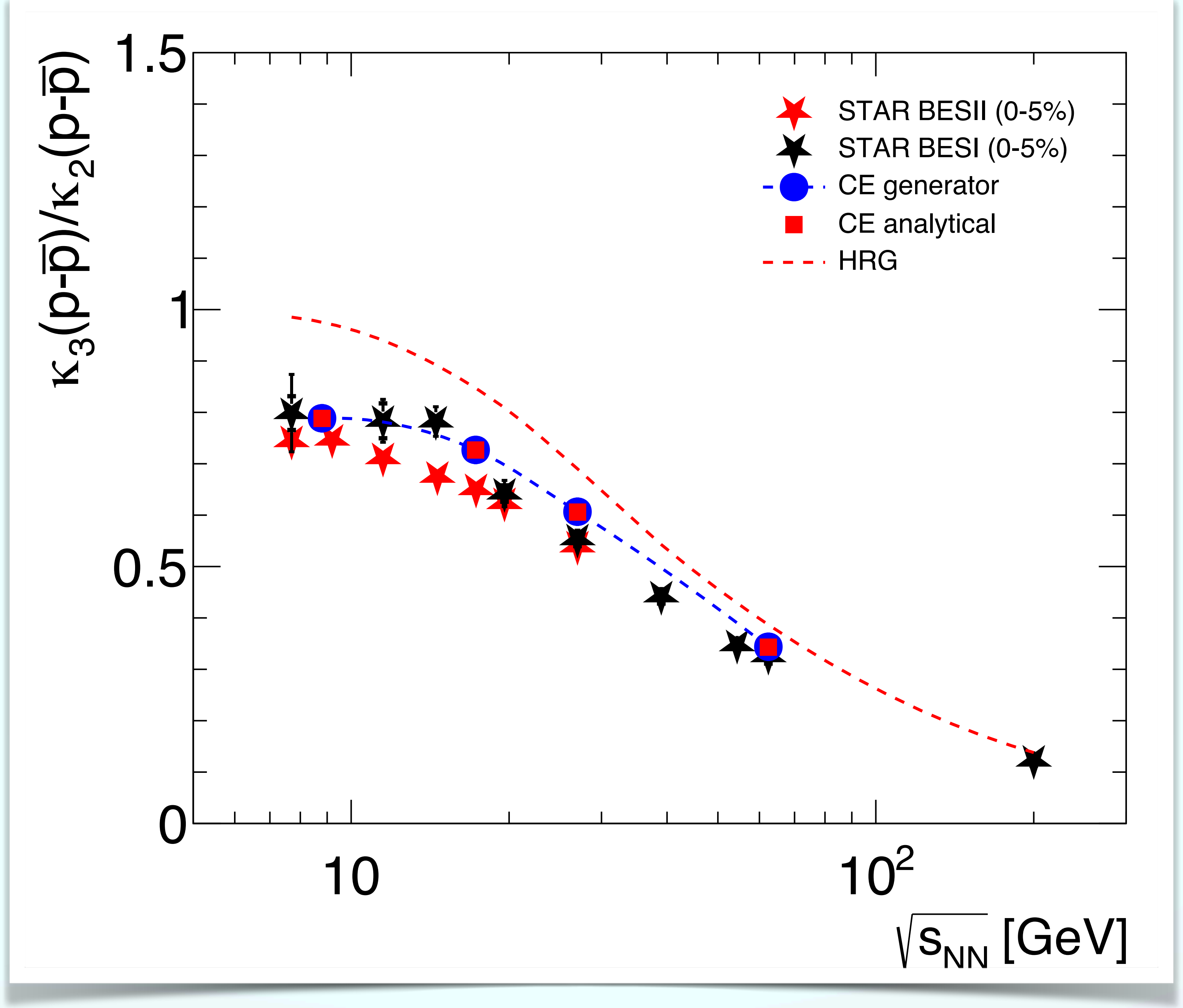
Canonical Baseline Calculator

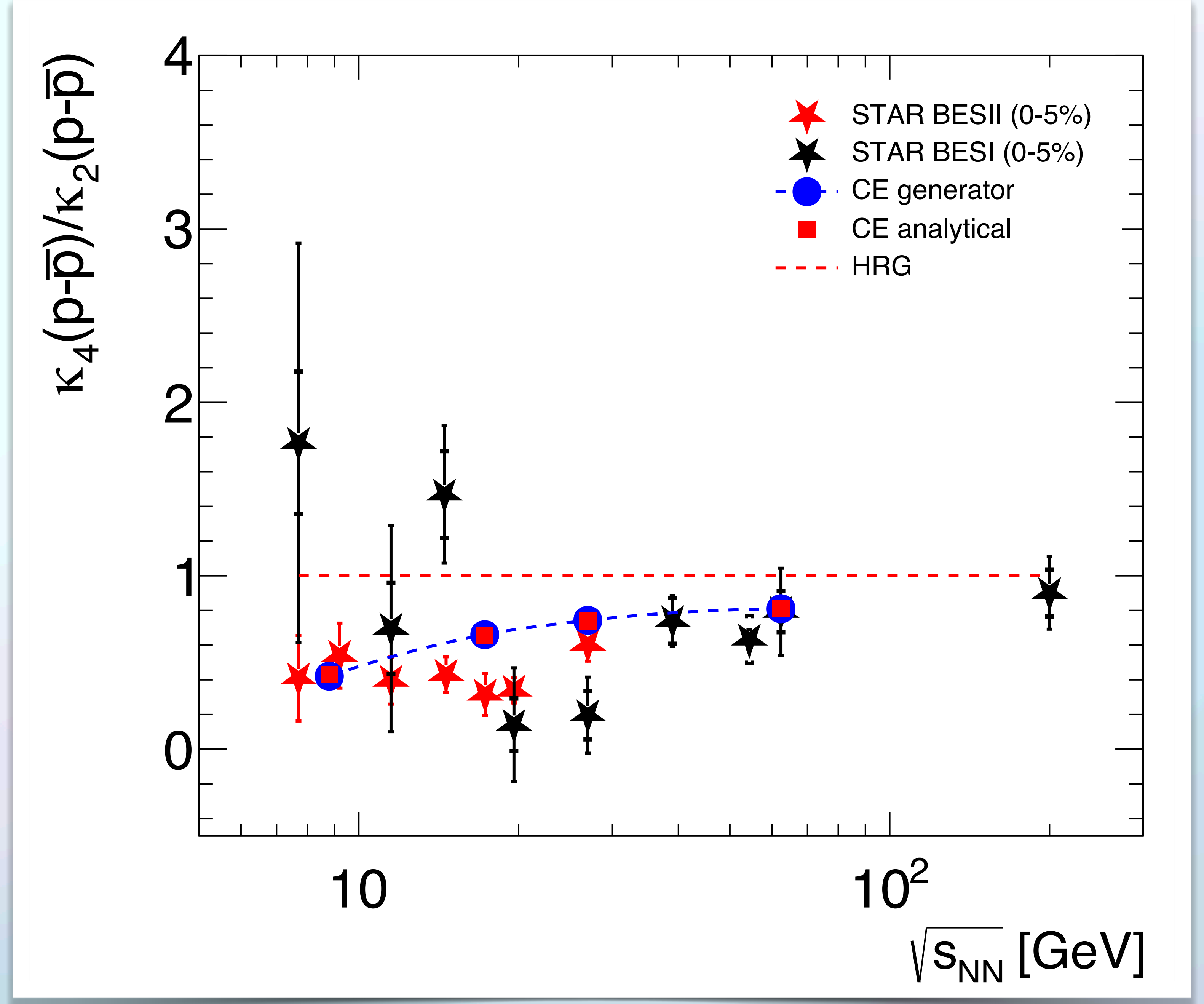
A. Rustamov, B. Friman

https://github.com/e-by-e/Cumulants-CE.git

Canonical baselines for cumulants vs. STAR data



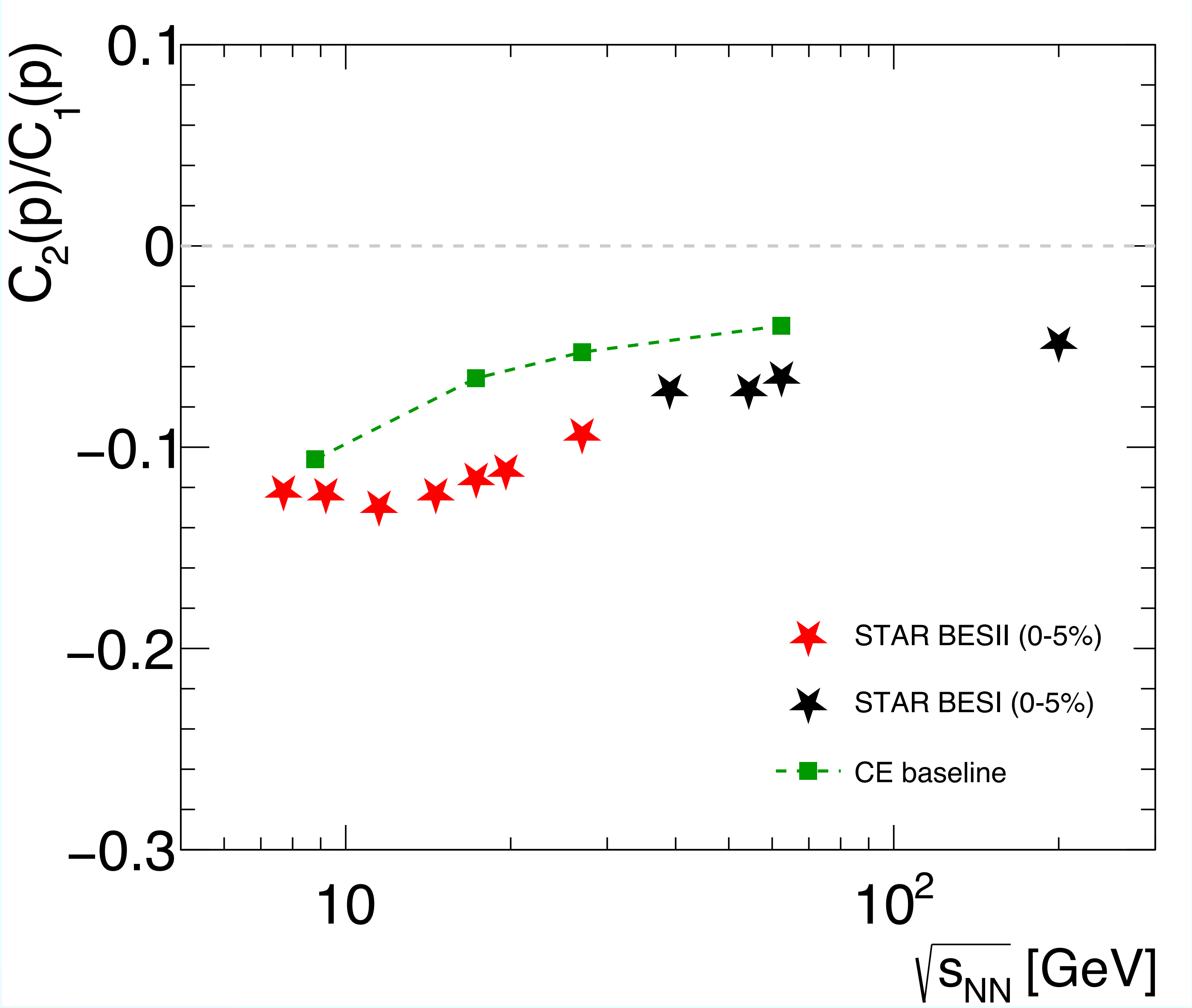


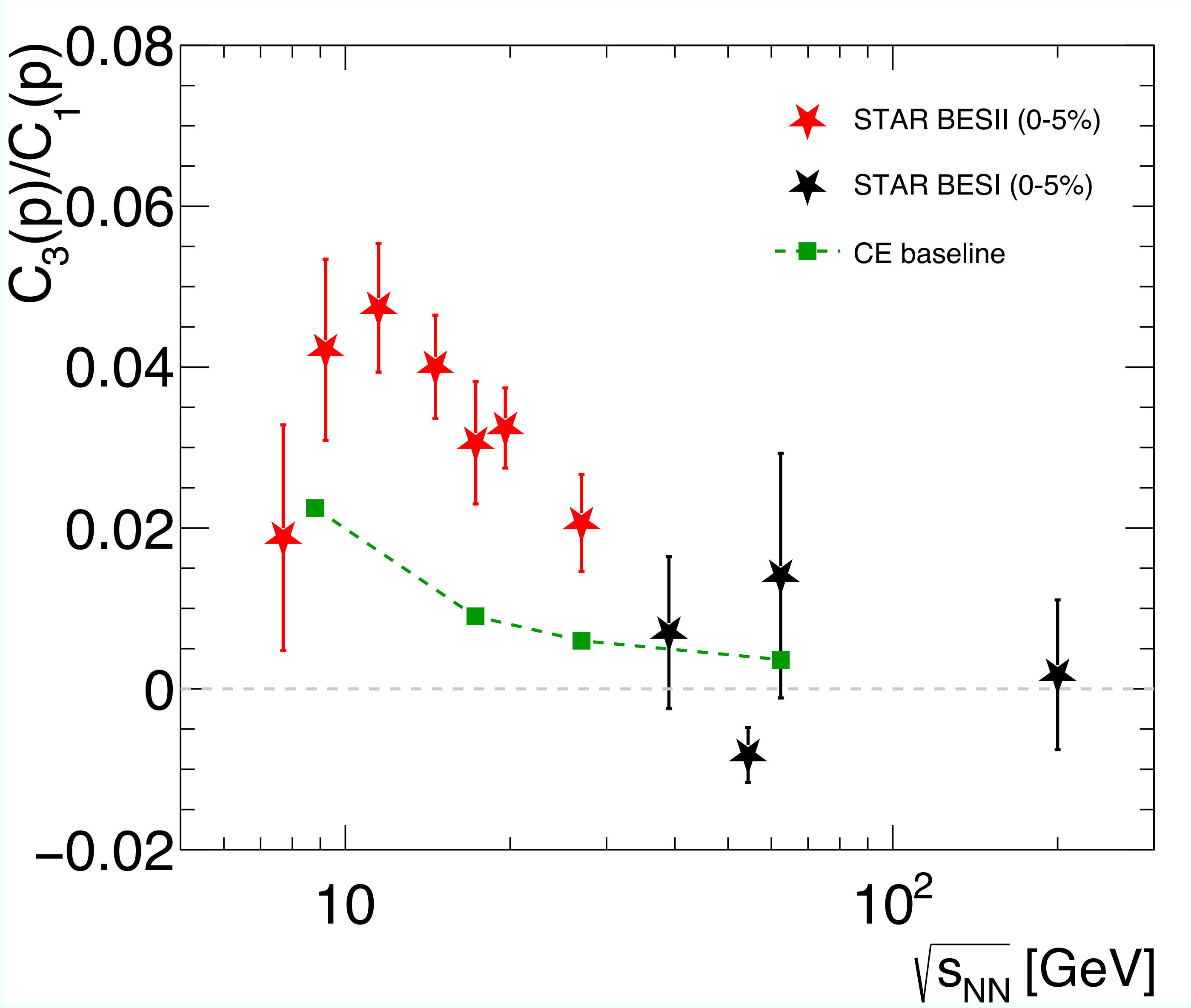


CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

The NEW STAR data are not consistent with the the CE baseline

Factorial cumulants





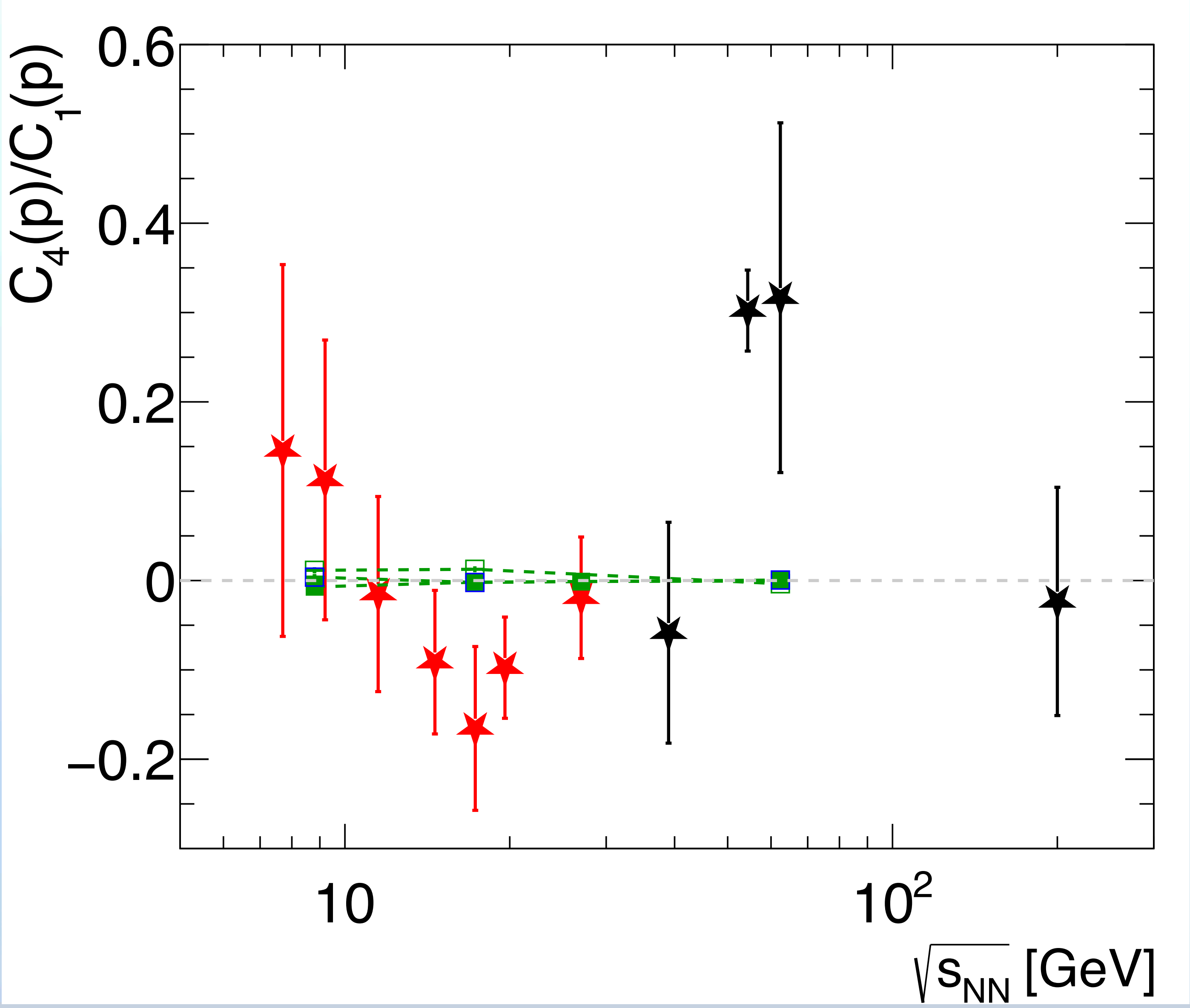
$$C_1(p) = \kappa_1(p) = \langle n_p \rangle$$

$$C_2(p) = -\kappa_1(p) + \kappa_2(p)$$

$$C_3(p) = 2\kappa_1(p) - 3\kappa_2(p) + \kappa_3(p)$$

$$C_4(p) = -6\kappa_1(p) + 11\kappa_2(p) - 6\kappa_3(p) + \kappa_4(p)$$

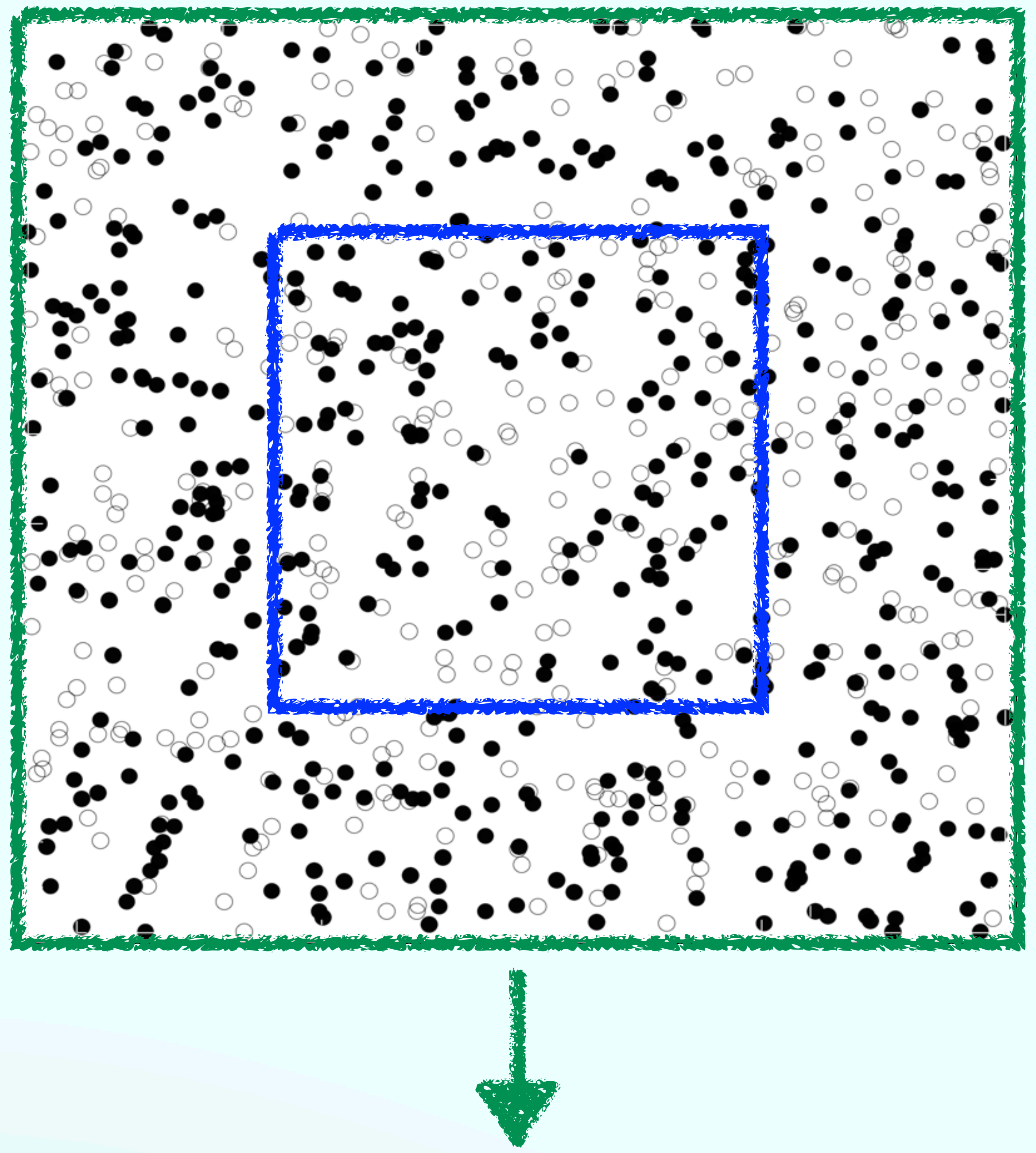
R. Holzmann, V. Koch, A. R., J. Stroth, 2403.03598 (2024)

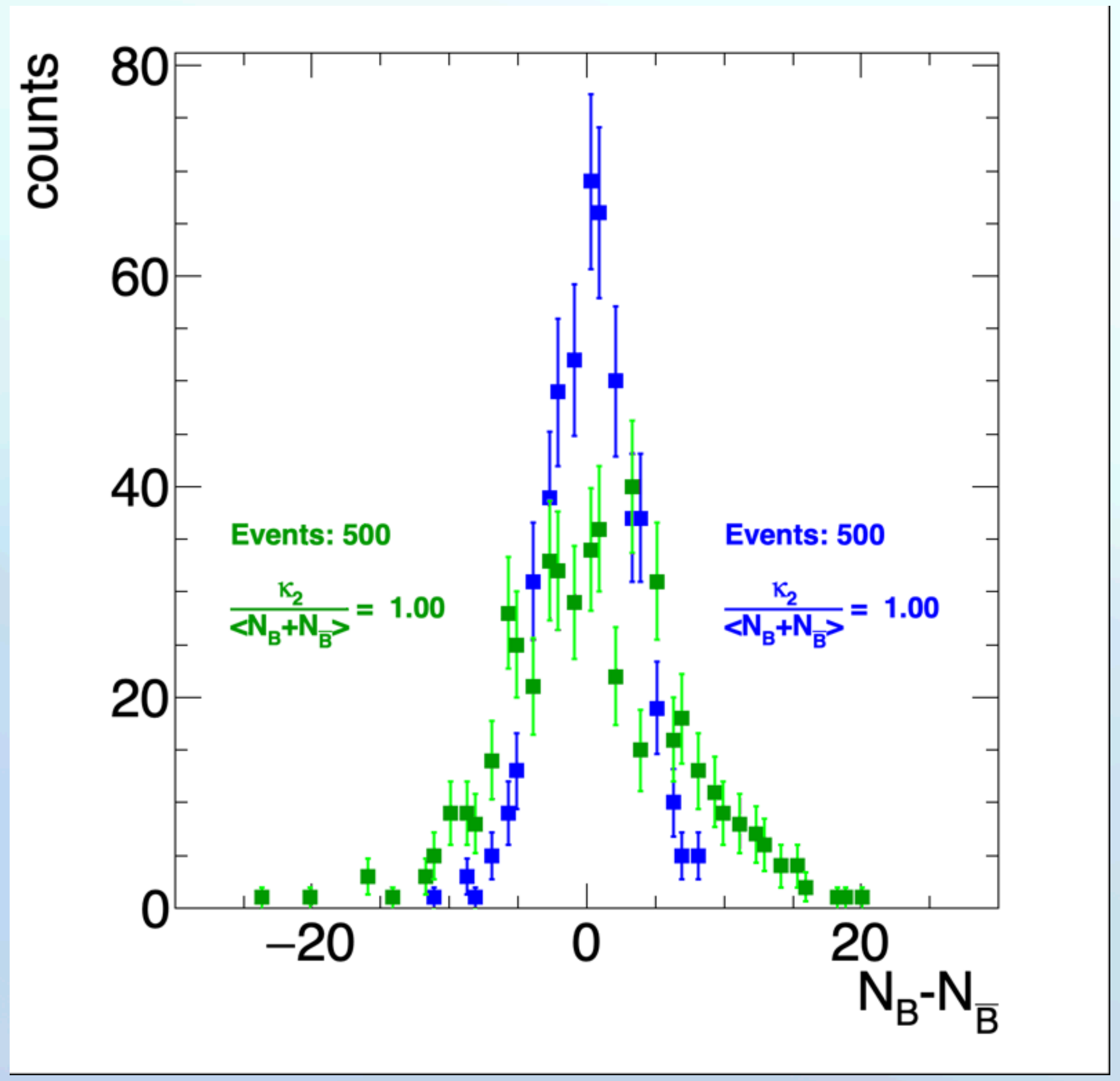


CE baseline: P. Braun-Munzinger, B. Friman, K. Redlich, AR., J. Stachel , NPA 1008 (2021) 122141

The NEW STAR data are not consistent with the the CE baseline

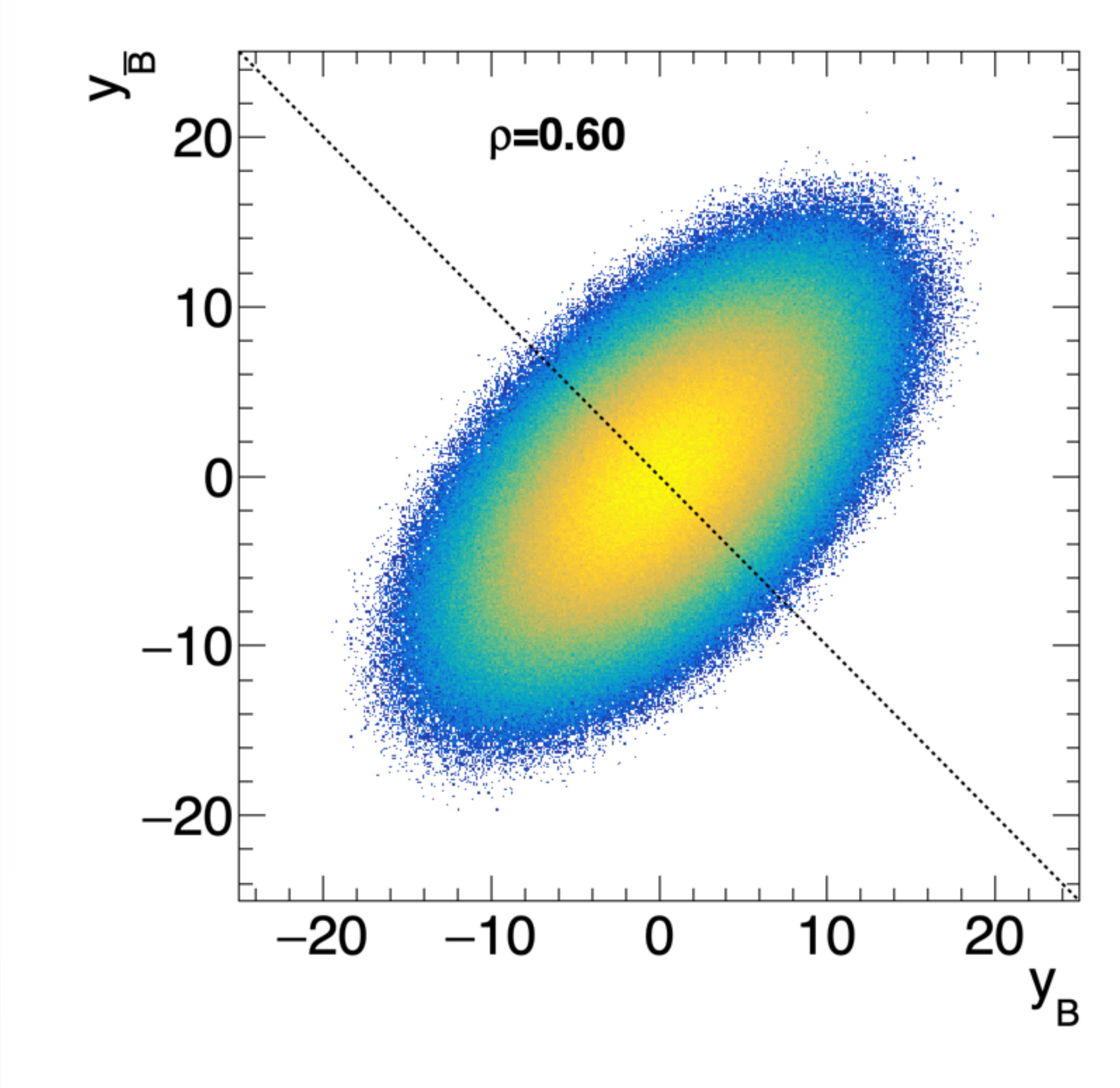
Ideal gas EoS plus local conservation laws





- exploiting **C**anonical **E**nsemble in the full phase space
  - no fluctuations in 4π (like in experiments)

+ **correlations in rapidity space (local conservations)**



P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

Correlated Random Number Generation

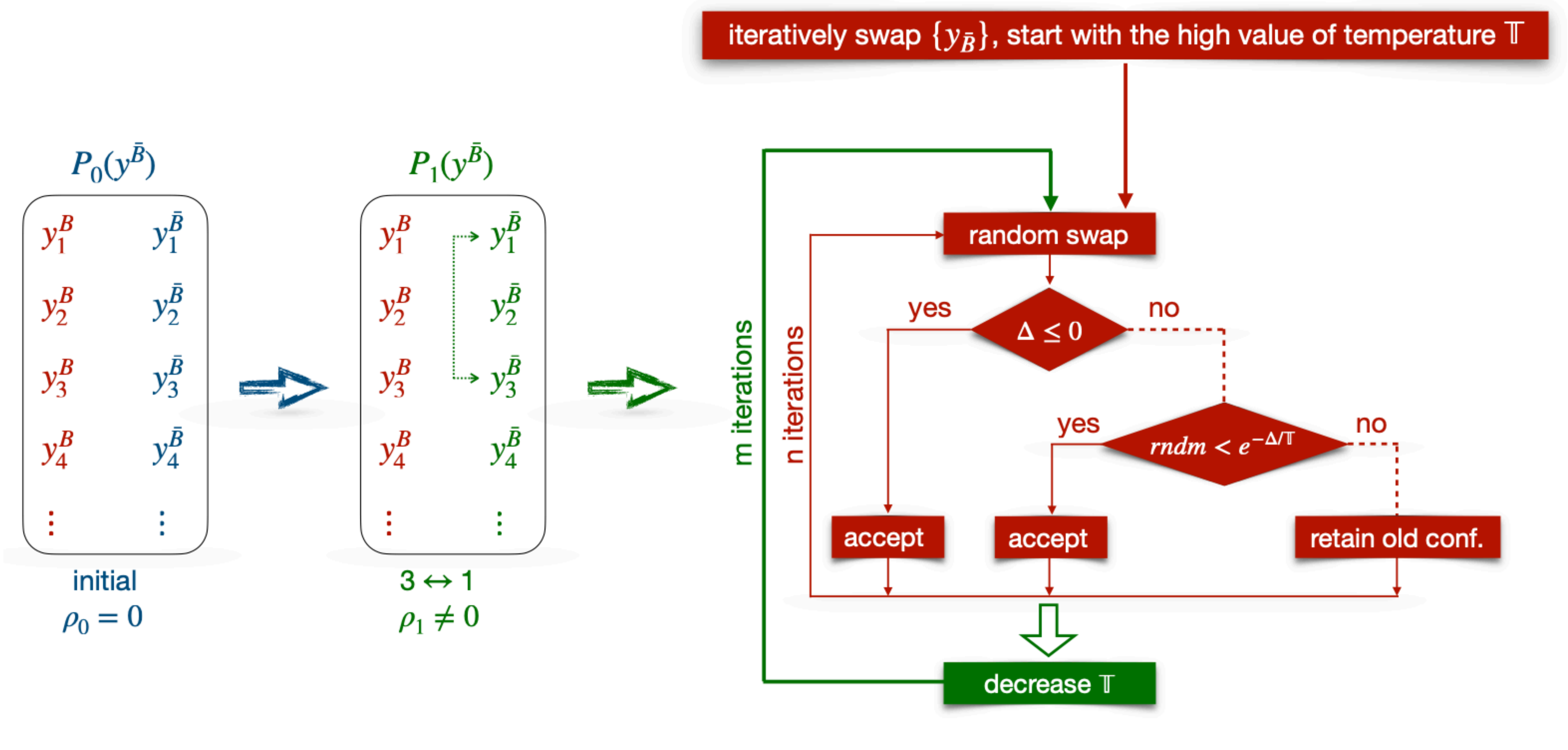


Figure 1. Implementation of the Metropolis algorithm.

Minimizing the cost function

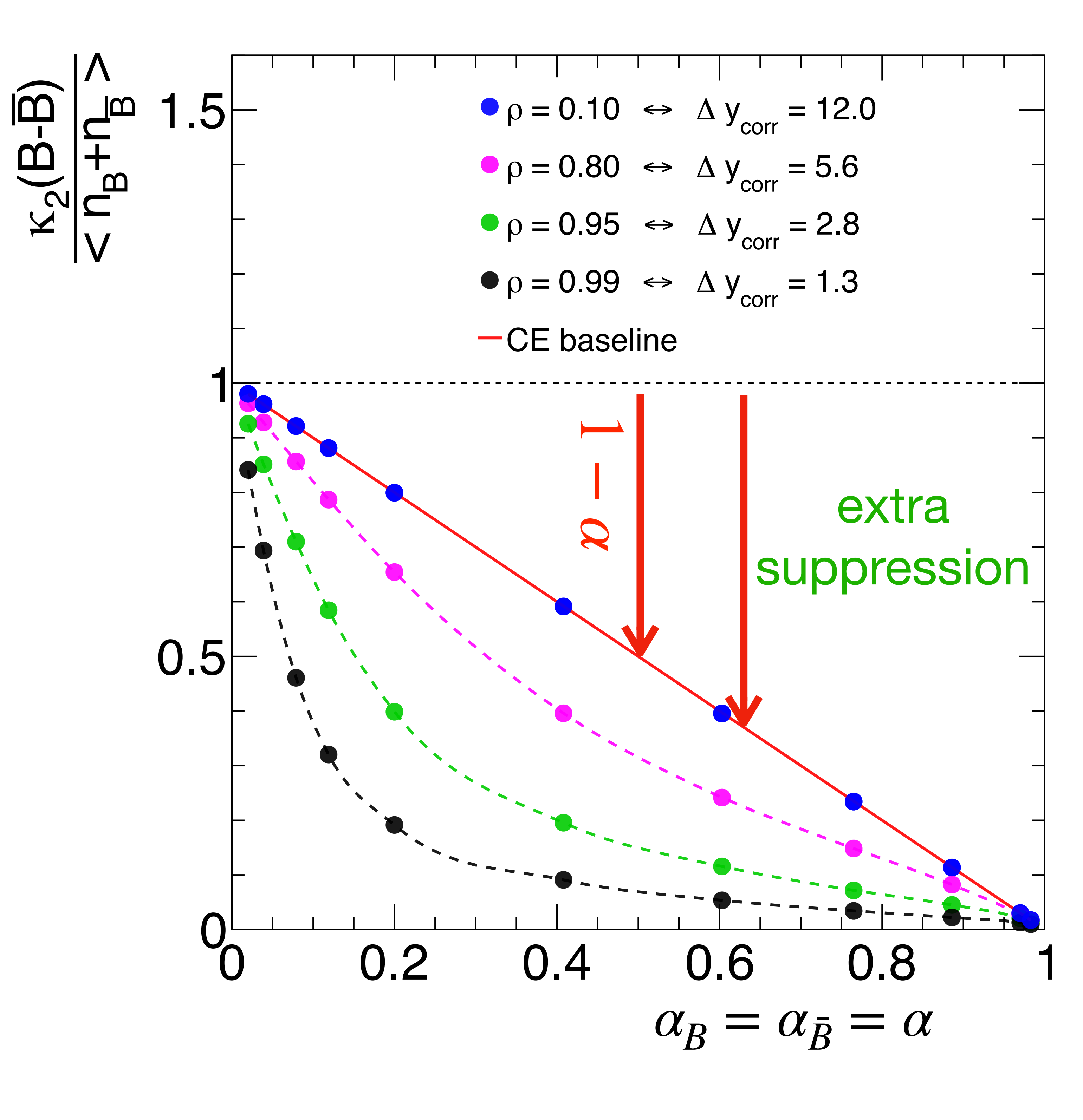
$$\Delta = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

$$\rho = \frac{Cov(y_B, y_{\bar{B}})}{\sigma_B \sigma_{\bar{B}}}$$

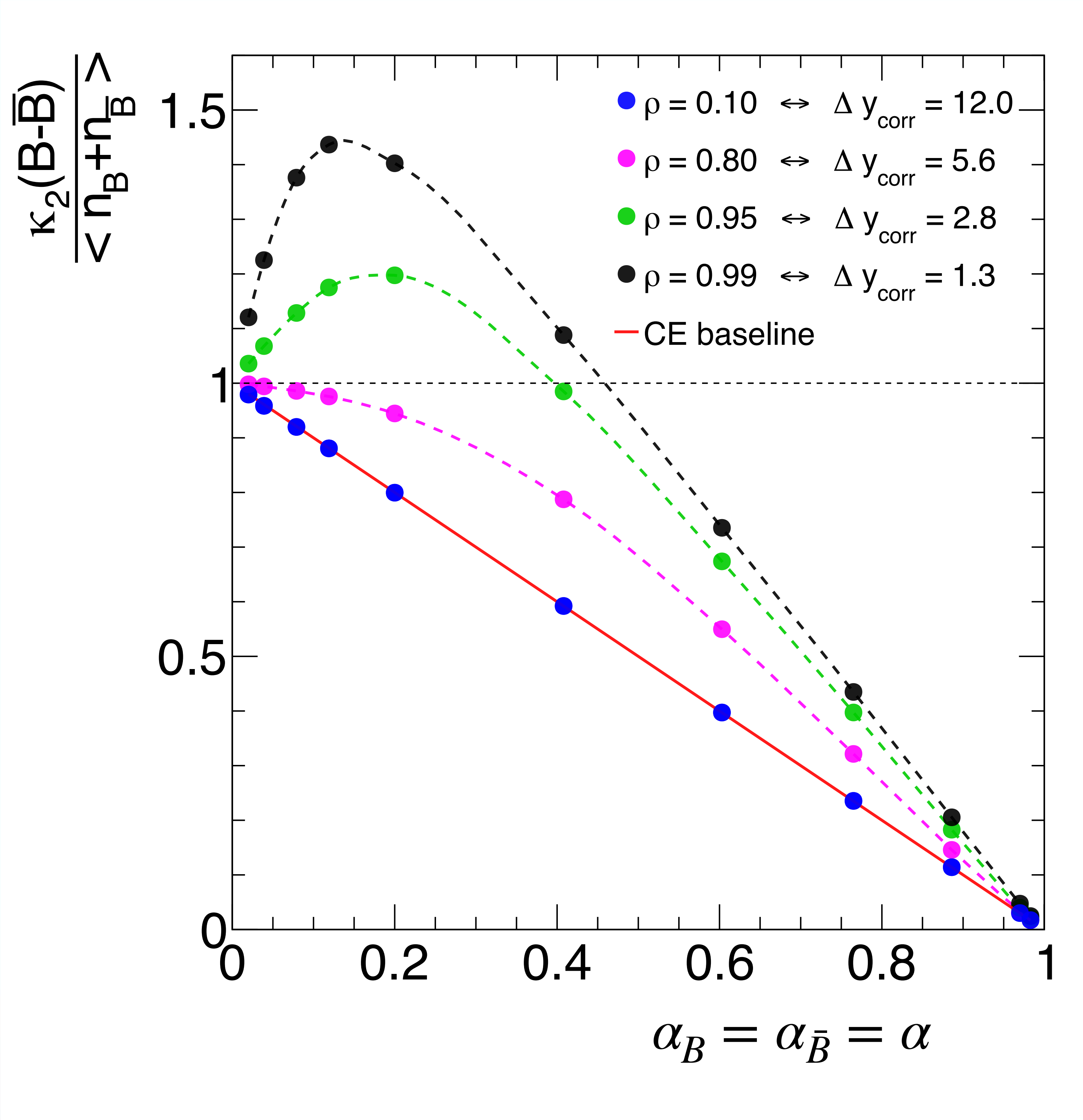
P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

Results on $B-\bar{B}$, $B-B$ and $\bar{B}-\bar{B}$ correlations, ALICE energy

Correlations between $B-\bar{B}$



Correlations between $B-B$ and $\bar{B}-\bar{B}$

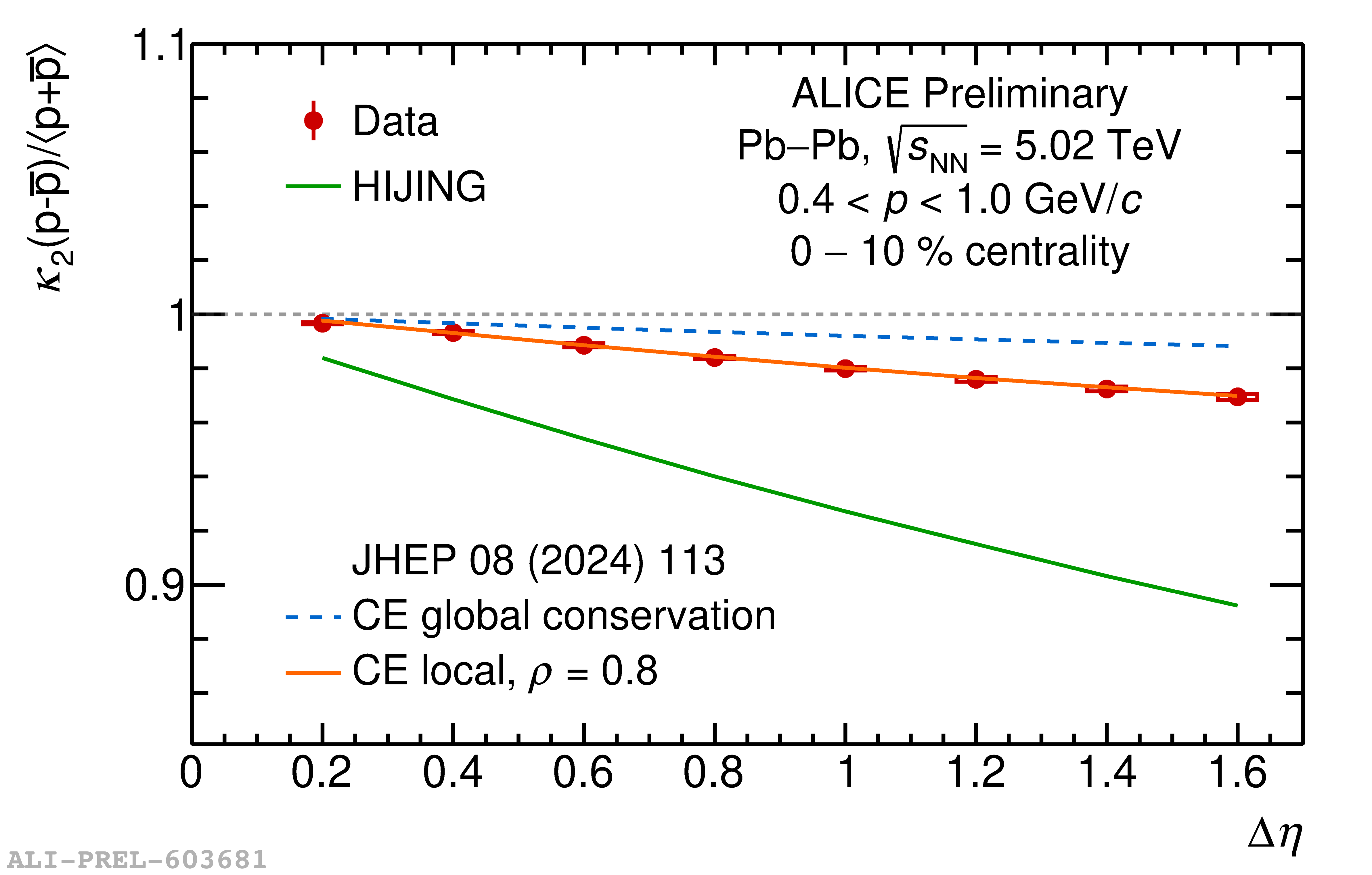


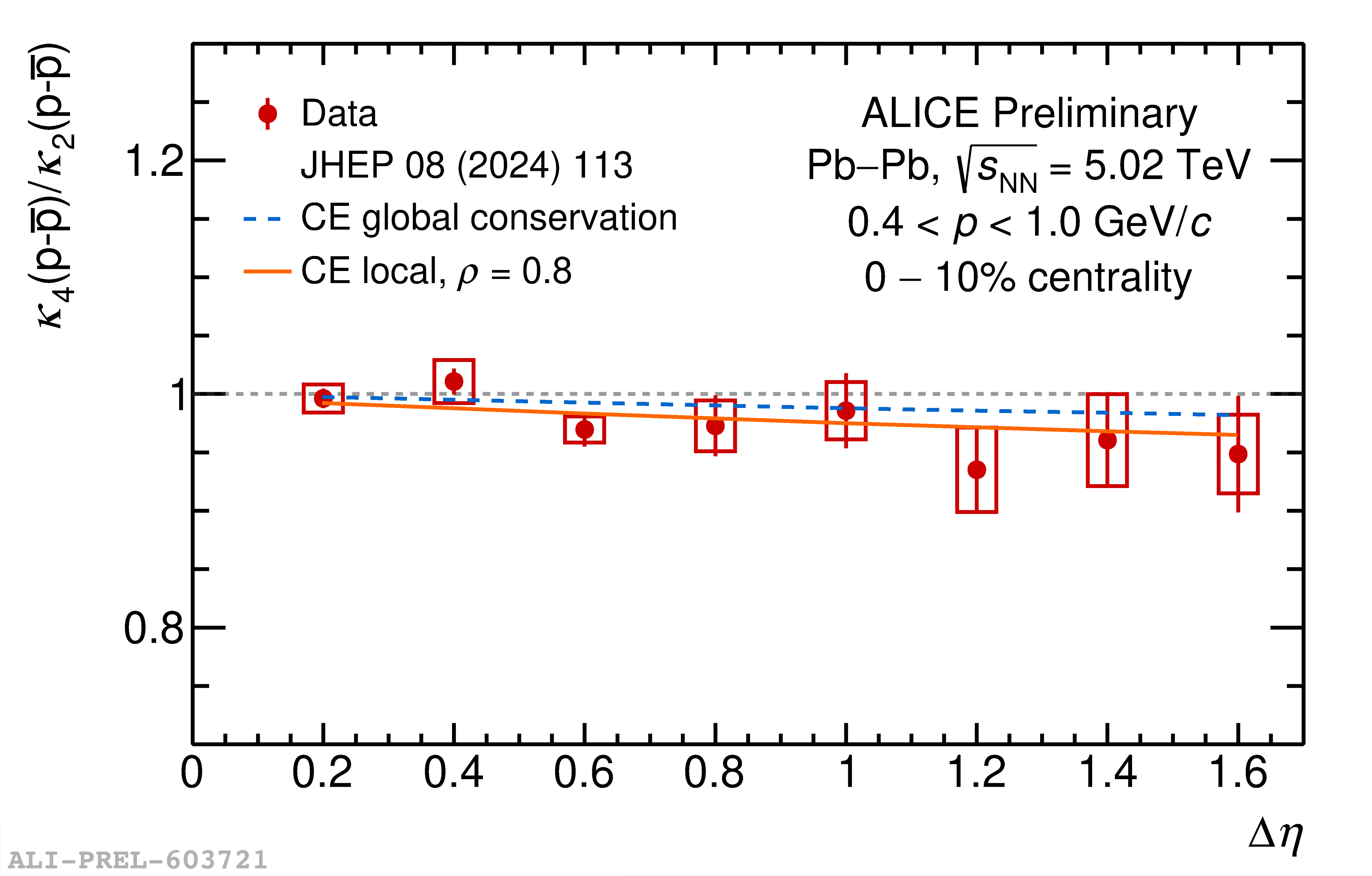
correlations between like-sign particles leads to cluster formation. Fluctuations increase.

P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

ALICE results

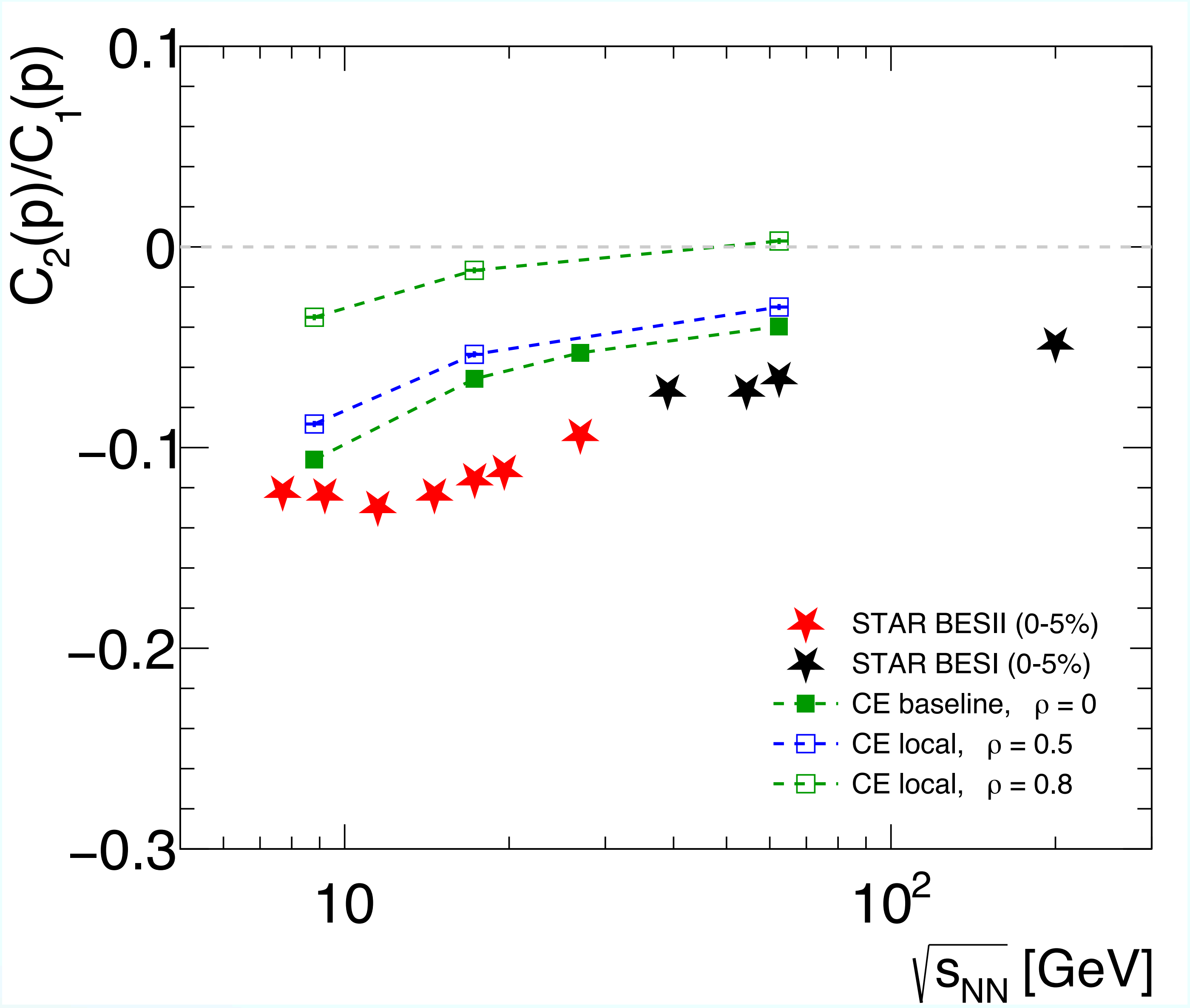
Ilya Fokin, Mesut Arslandok QM 25

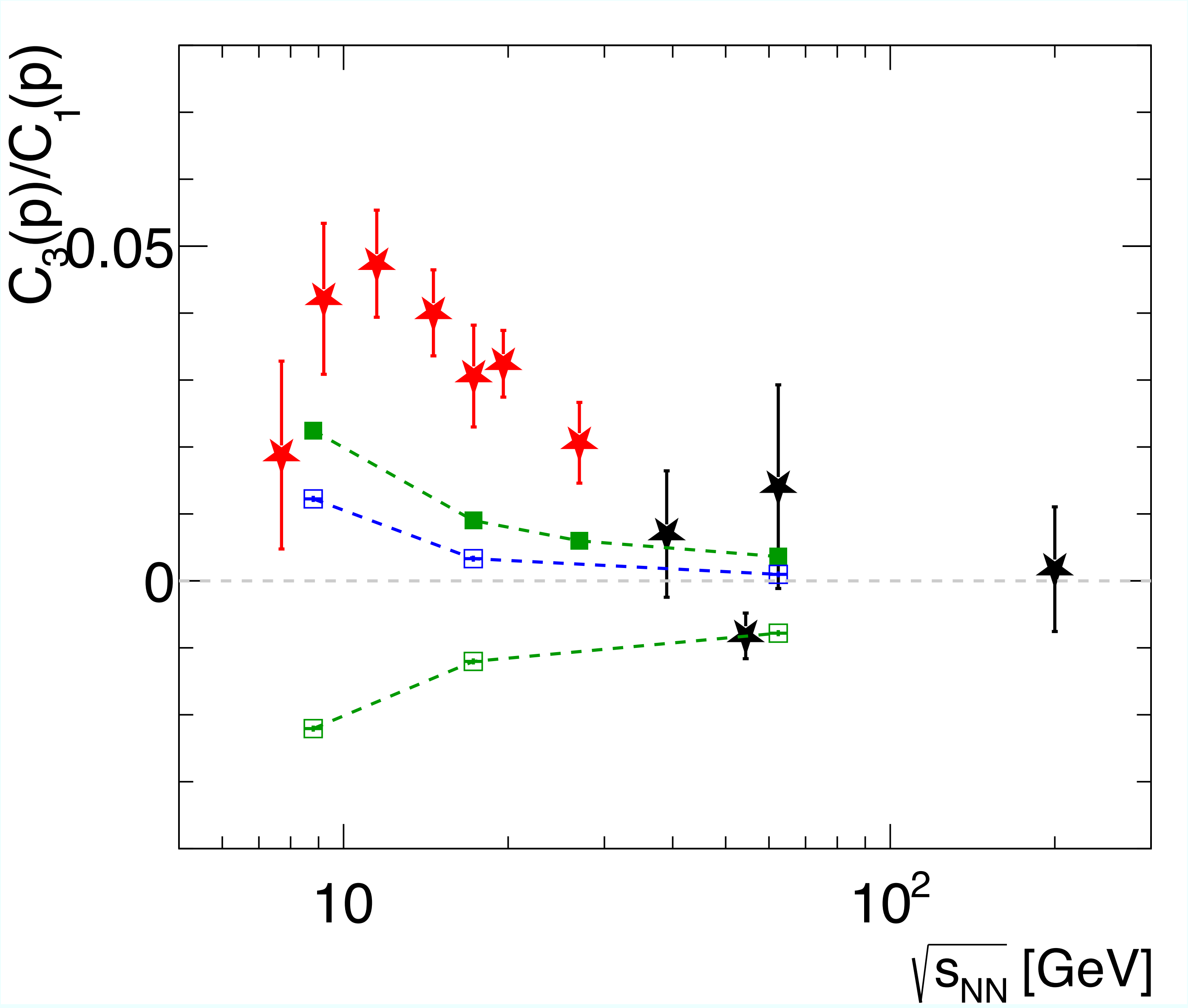




consistent with attractive local correlations between protons and antiprotons

Attractive correlations and STAR data





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$$C_2(p) = -\kappa_1(p) + \kappa_2(p)$$

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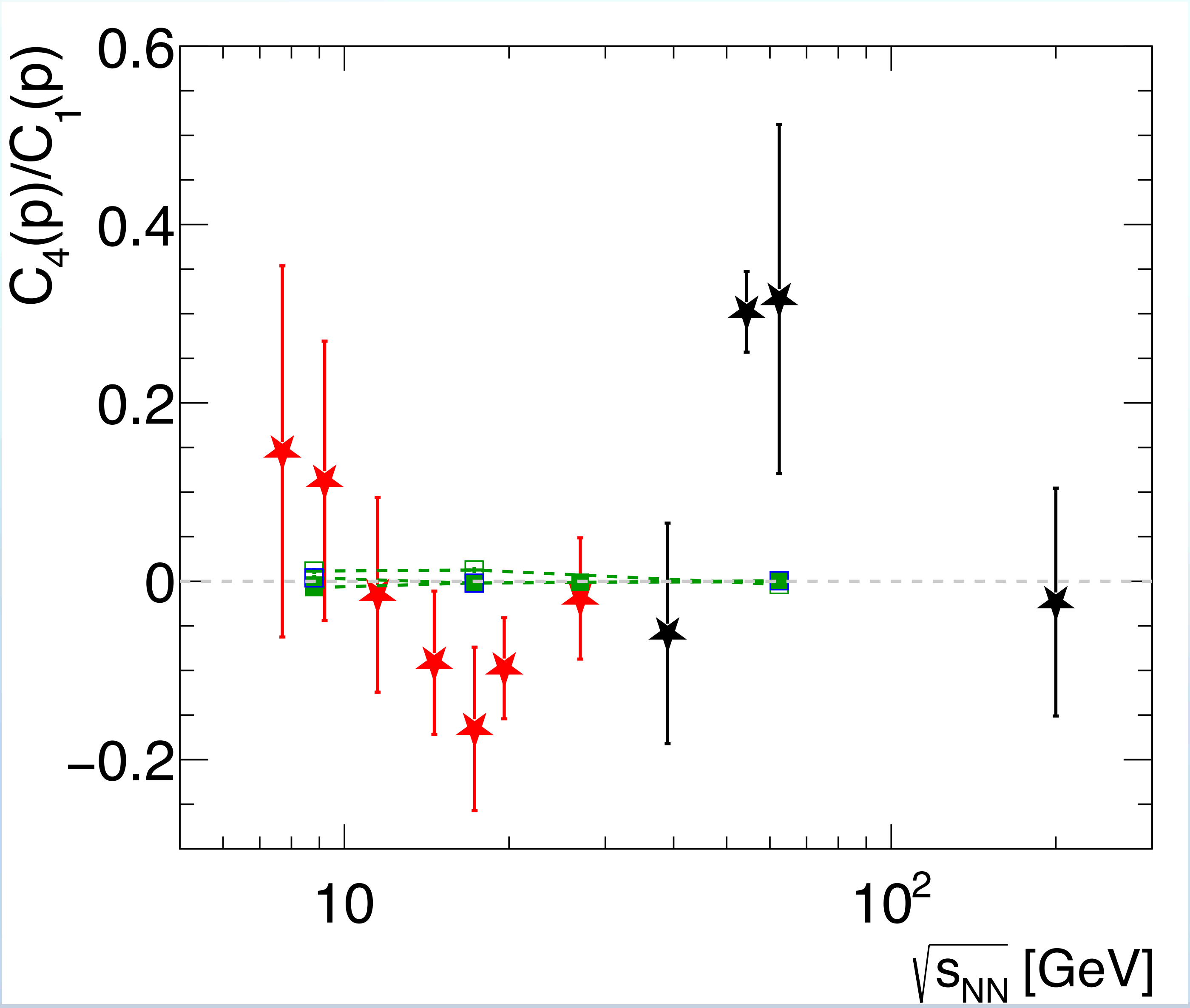
$$C_4(p) = -6\kappa_1(p) + 11\kappa_2(p) - 6\kappa_3(p) + \kappa_4(p)$$

R. Holzmann, V. Koch, A. R., J. Stroth, 2403.03598 (2024)



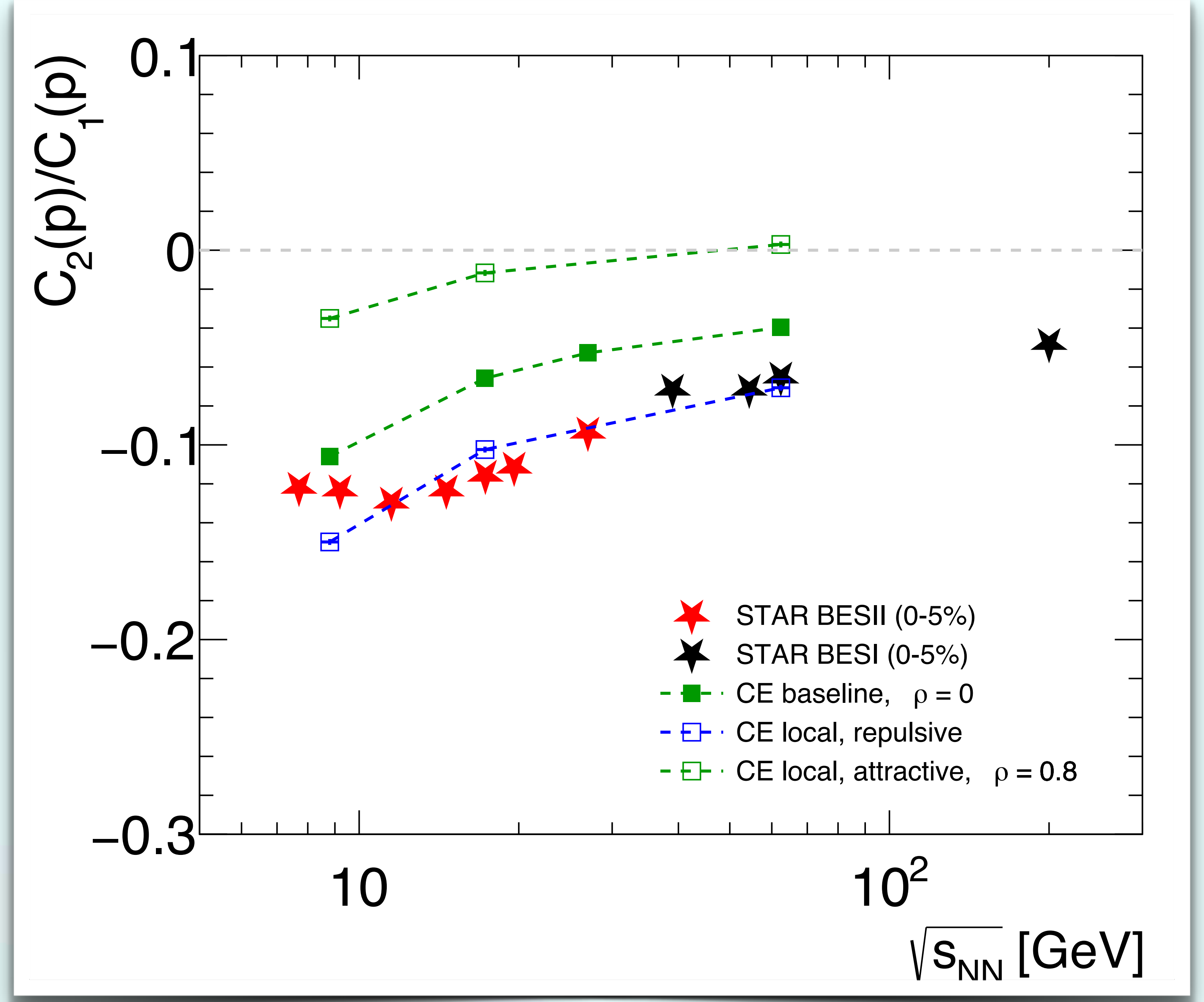
Canonical baselines, including attractive correlations, shows an opposite trend compared to the STAR data!

This indicates the need to include repulsive interactions

Introducing repulsive interactions

B. Friman, A. Rustamov, K. Redlich, Repulsive interactions (in preparation)

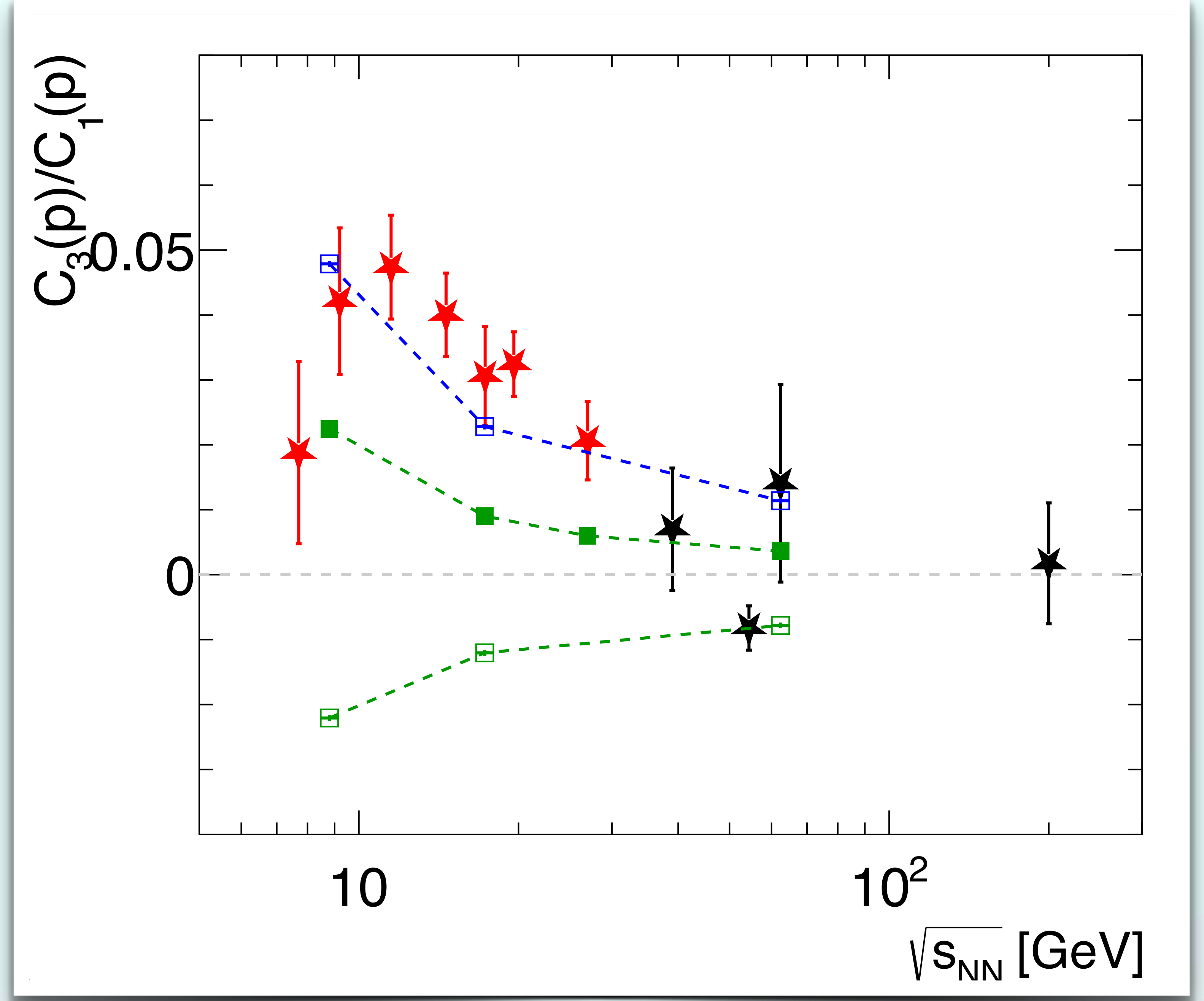
Repulsive correlations and factorial cumulants



P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, NPA 1008 (2021) 122141

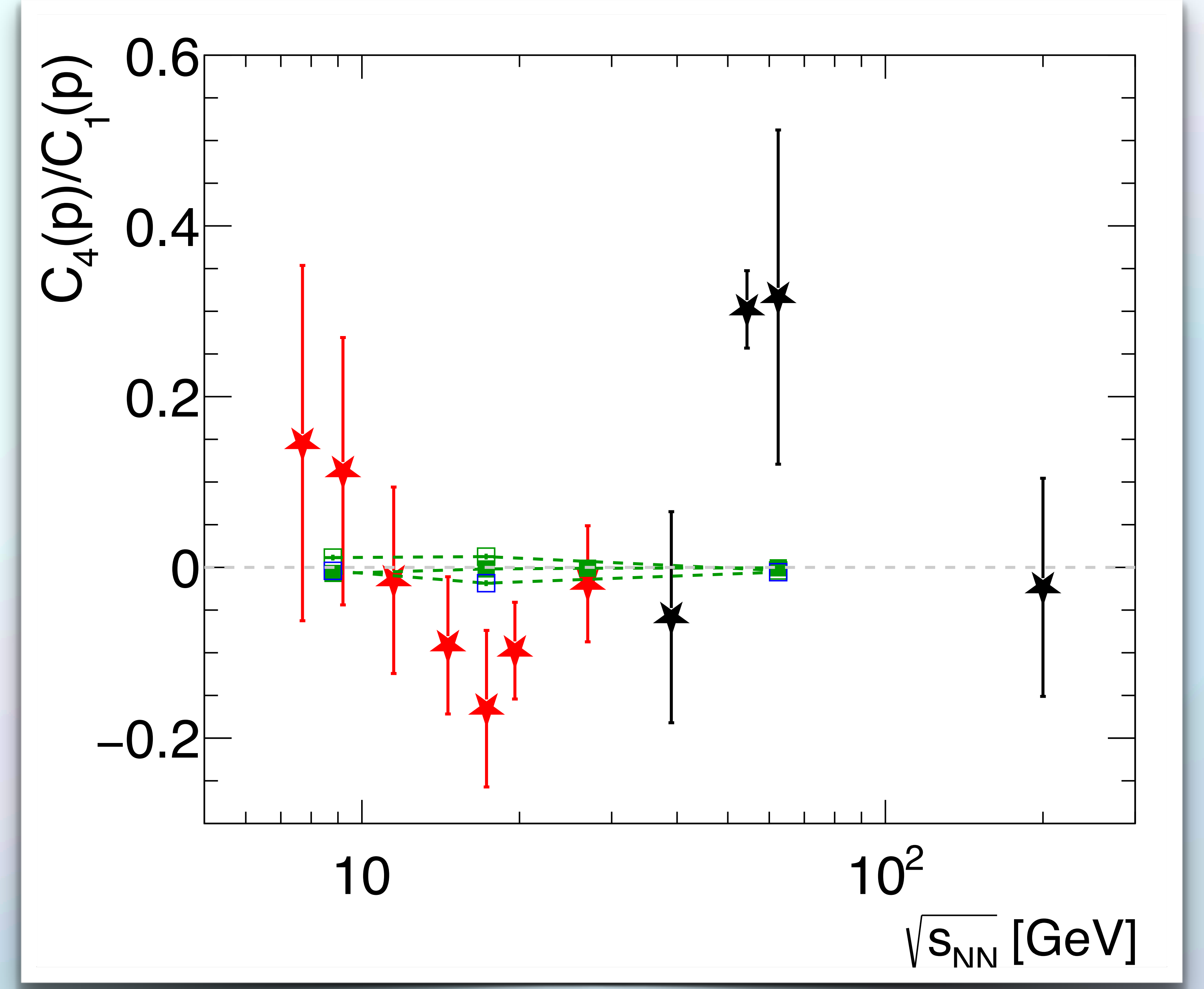
P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

B. Friman, A. Rustamov, K. Redlich, Repulsive interactions (in preparation)



STAR Data: A. Pandav, CPOD 2024

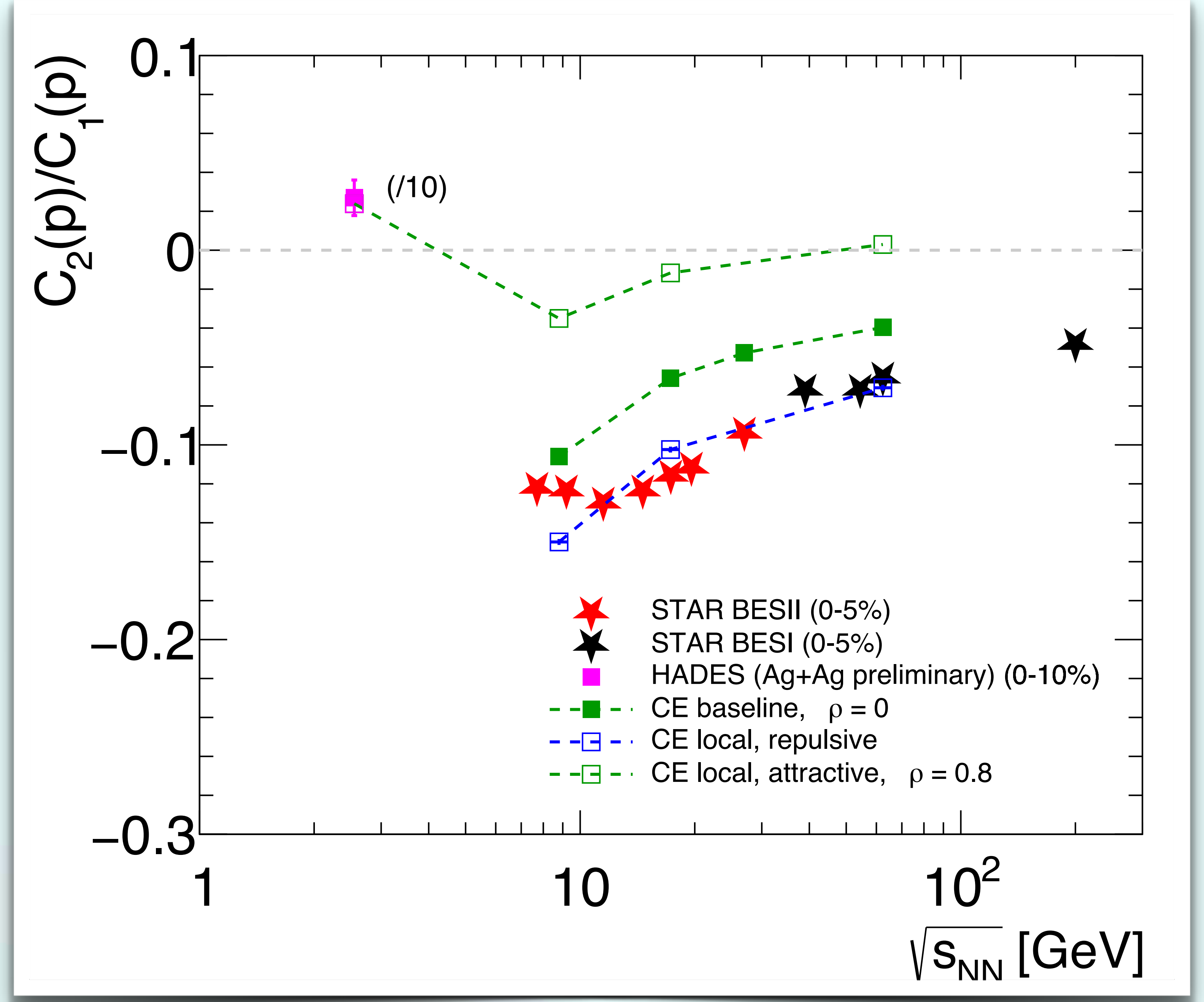
$-0.5 < y < 0.5, \quad 0.4 < p_t < 2$ GeV/c



Inclusion of repulsive correlations explains the STAR data!

Consistent with Volodymyr Vovchenko, QM 25

Adding results from HADES



- P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, NPA 1008 (2021) 122141
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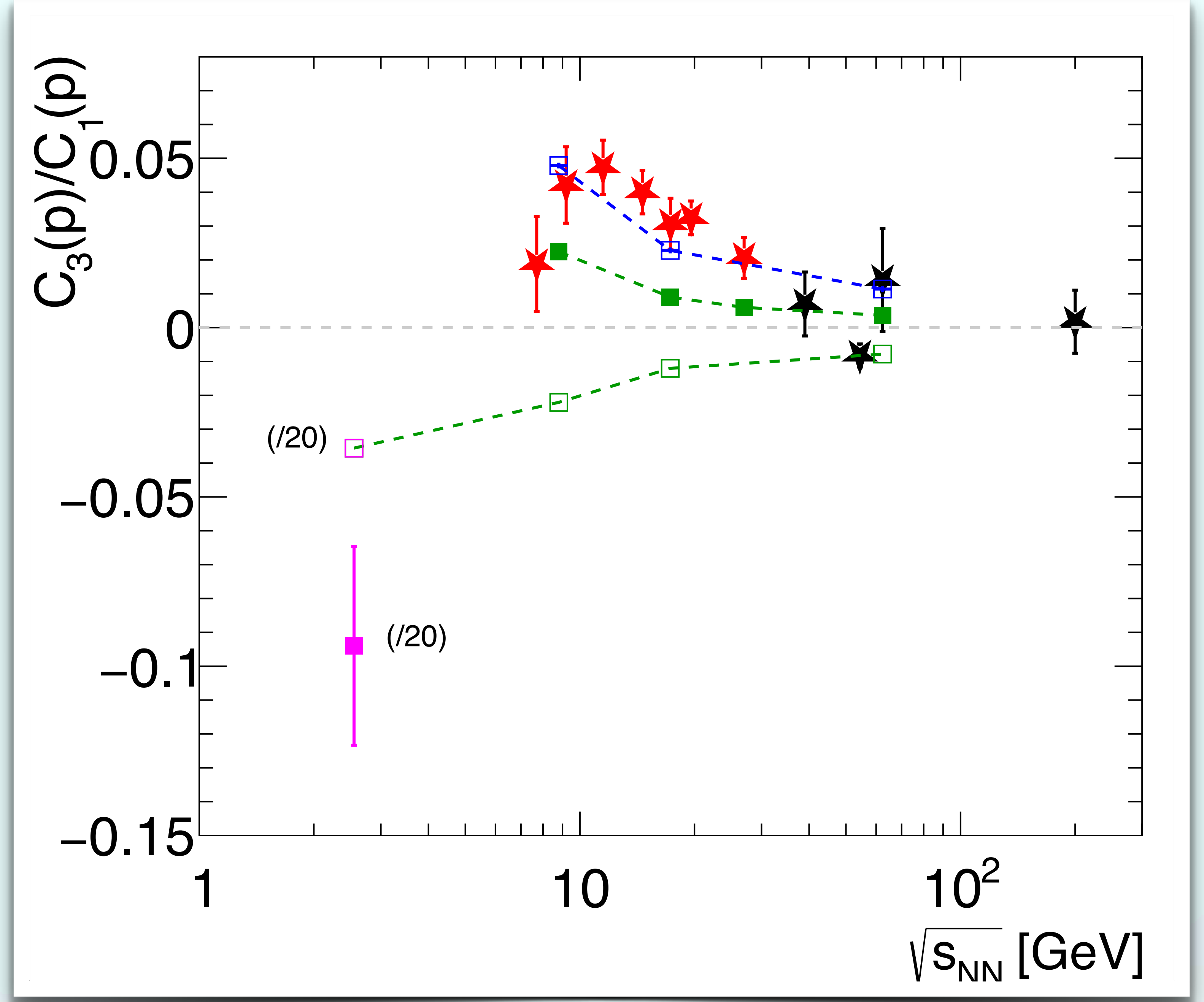
- B. Friman, A. Rustamov, K. Redlich (in preparation)

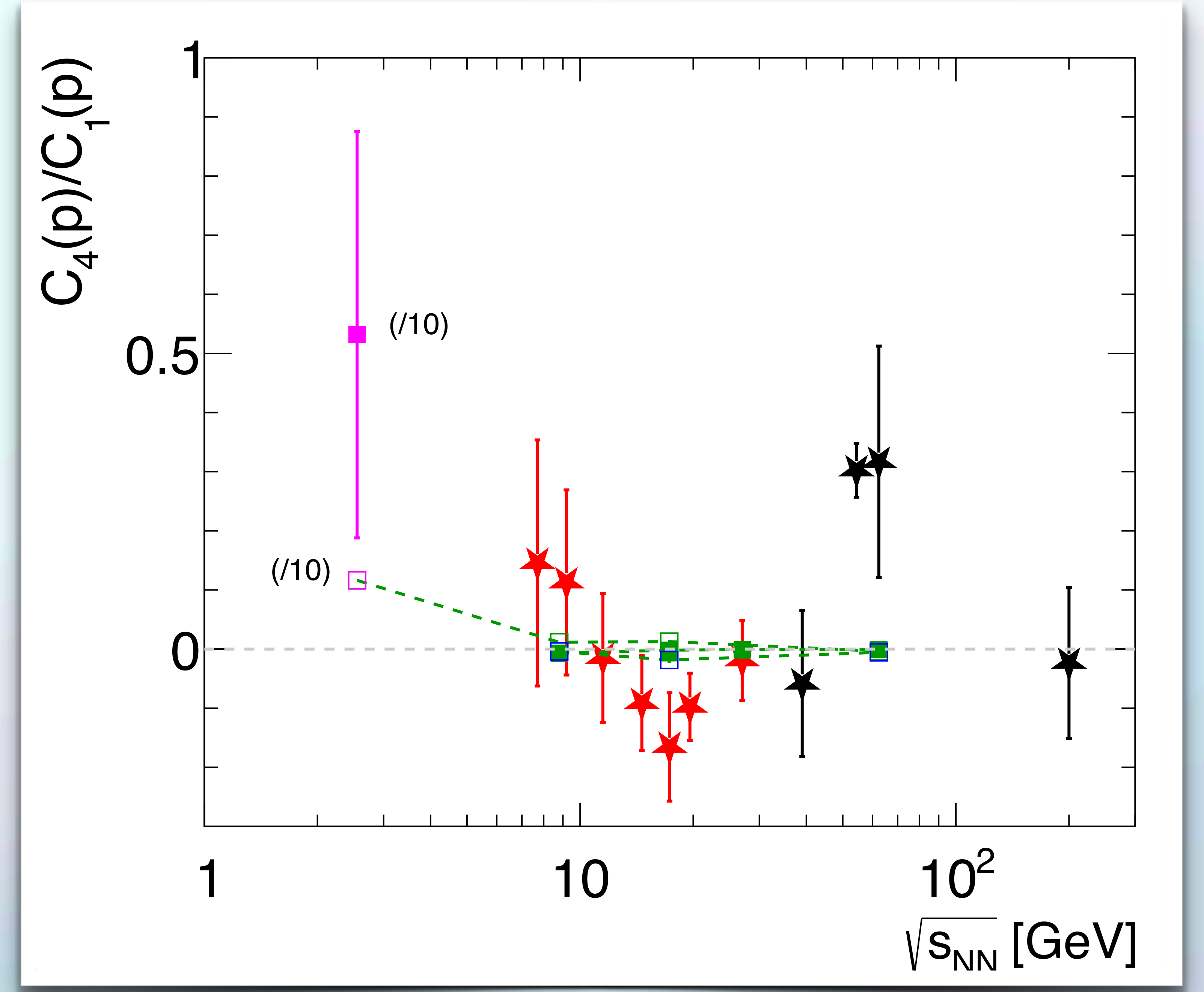


STAR Data: A. Pandav, CPOD 2024

$-0.5 < y < 0.5, \quad 0.4 < p_t < 2$ GeV/c

HADES:

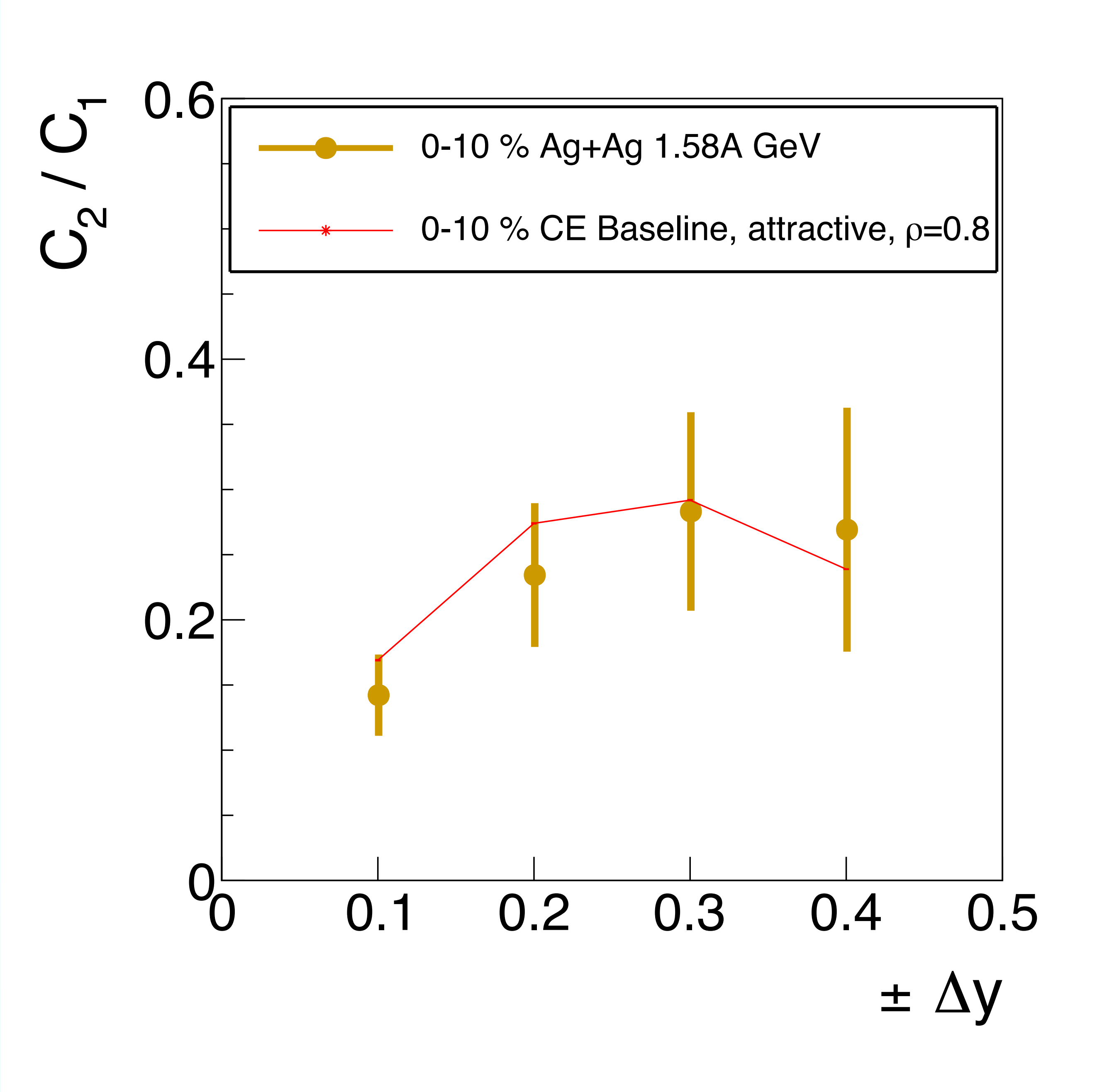
$-0.4 < y < 0.4, \quad 0.4 < p_t < 1.6$ GeV/c

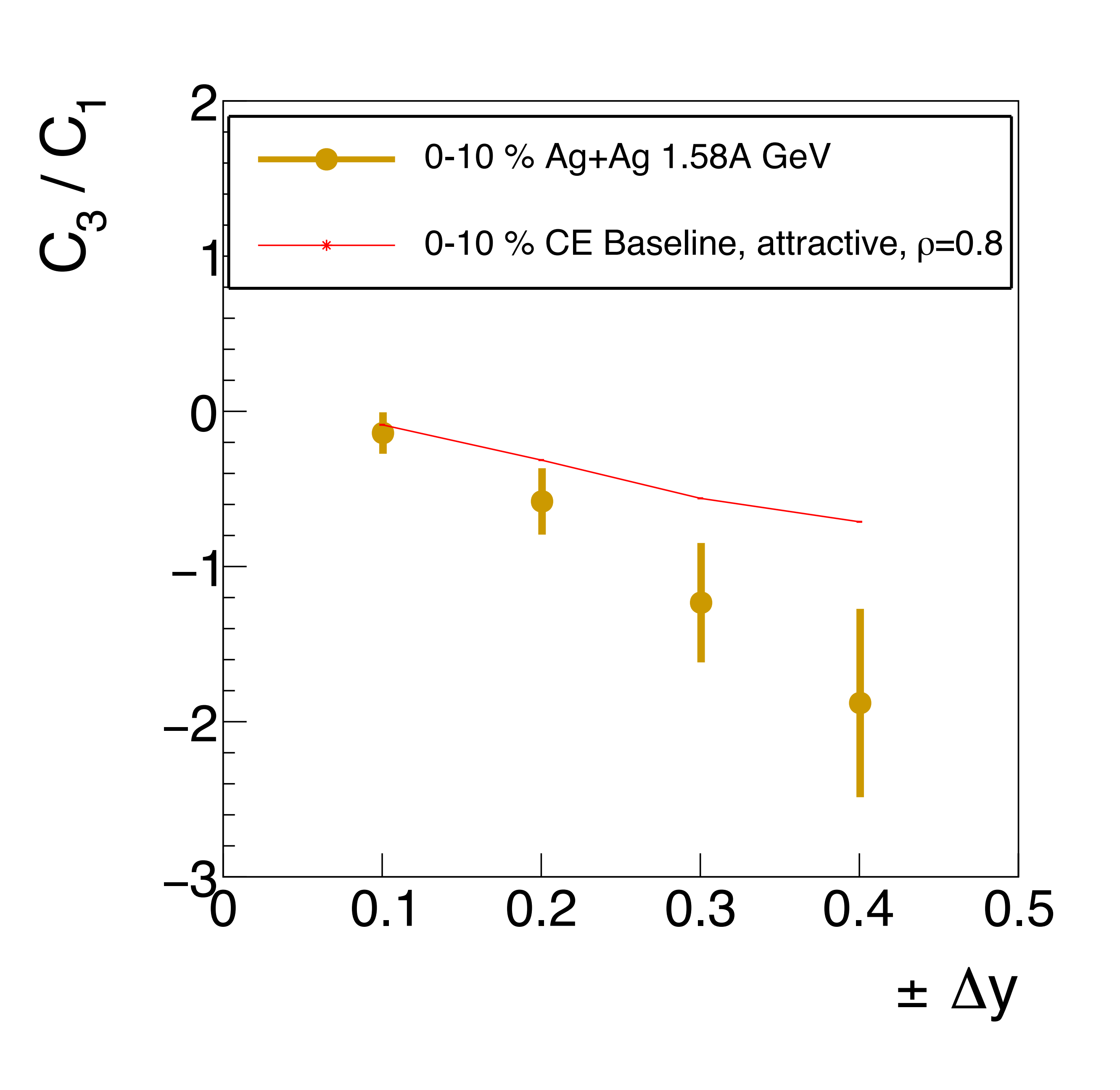


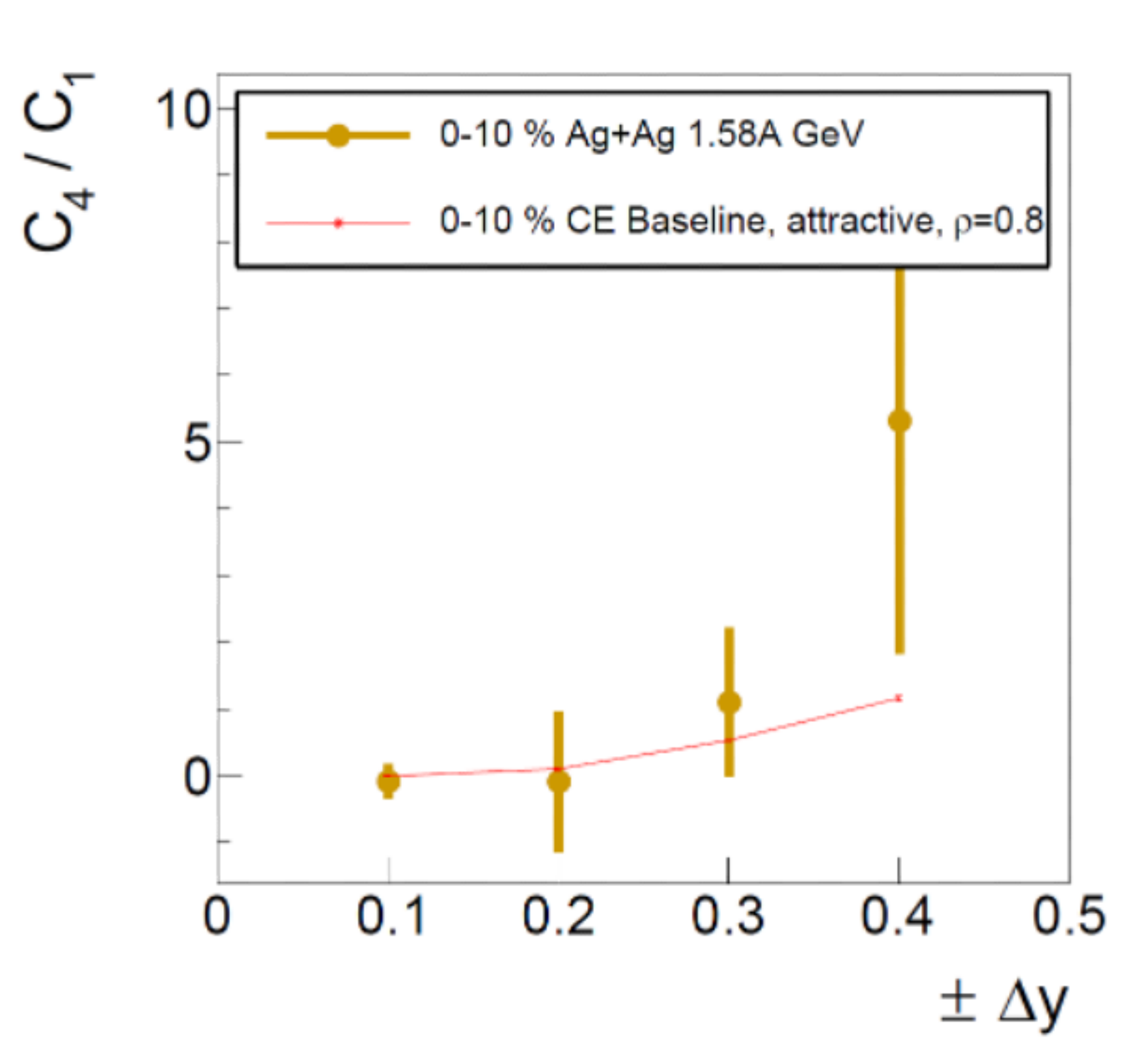
HADES data prefer attractive interactions!

THE INTERPLAY OF REPULSIVE AND ATTRACTIVE FORCES BETWEEN PROTONS EXPLAINS THE SYSTEMATIC TRENDS OBSERVED IN THE STAR BESII AND HADES DATA

Acceptance dependence of factorial cumulants





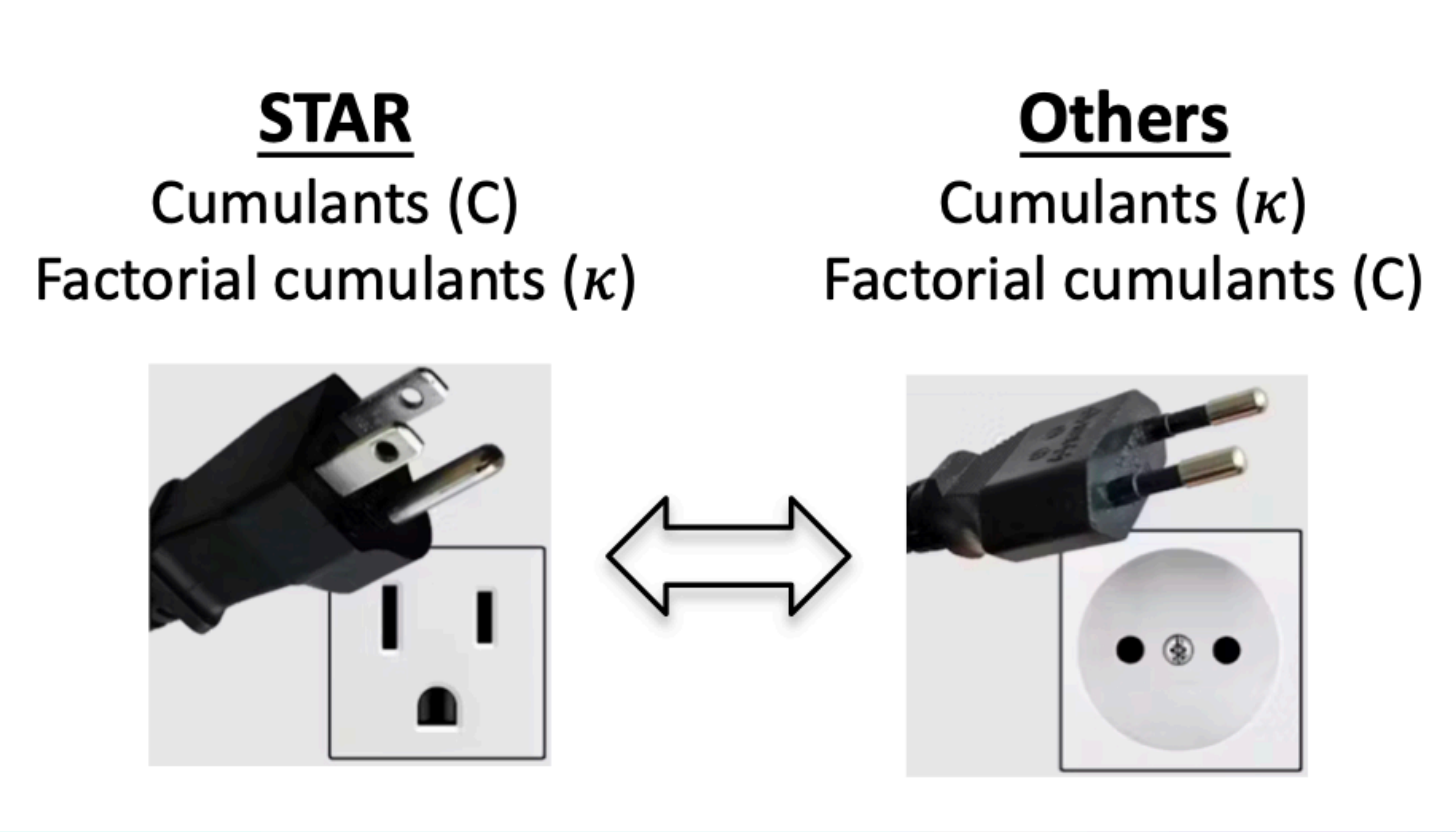


Marvin Nabroth, QM 2025

introduction of attractive correlations between protons provides an explanation for the HADES data

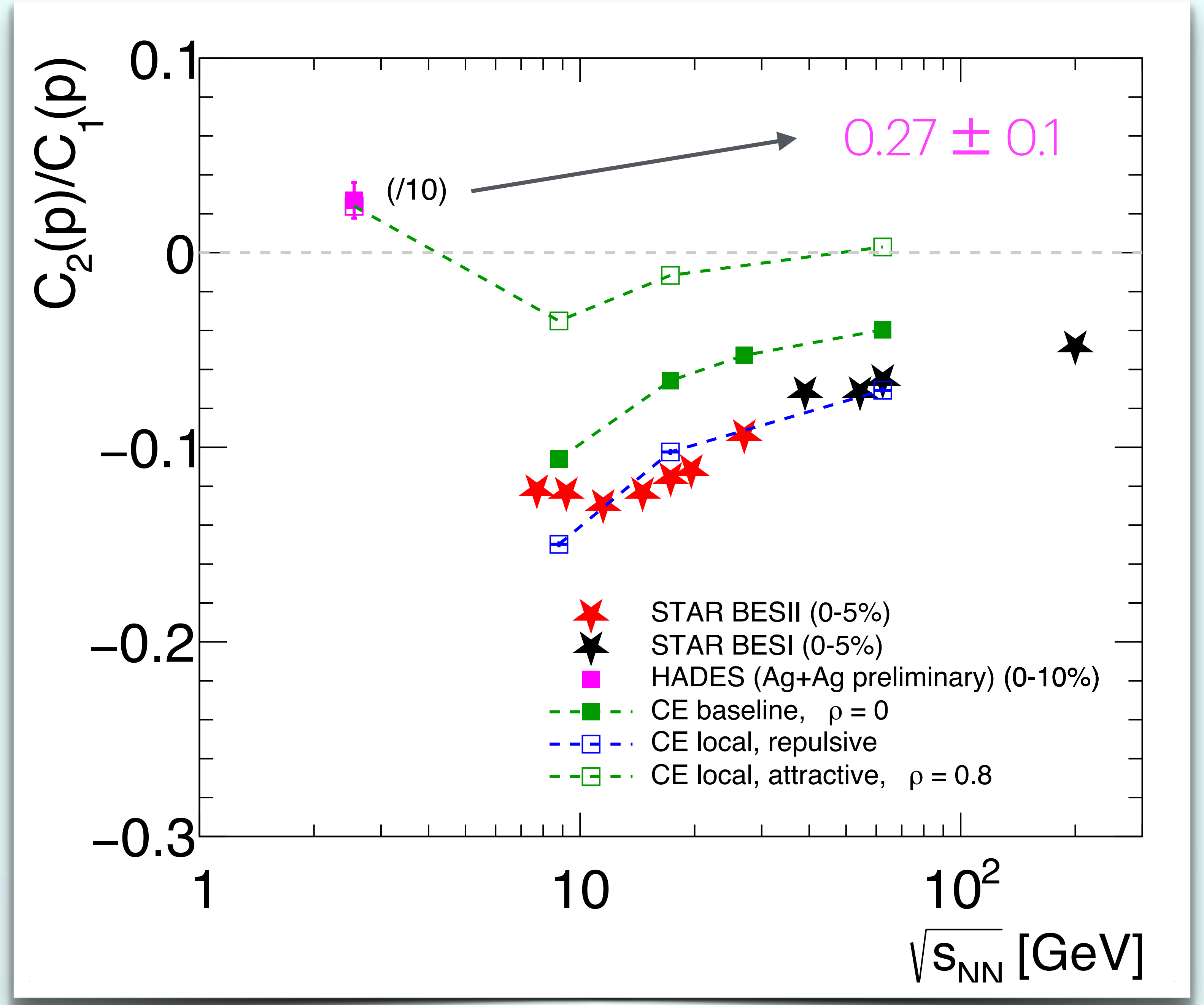
B. Friman, A. Rustamov, K. Redlich, Repulsive interactions (in preparation)

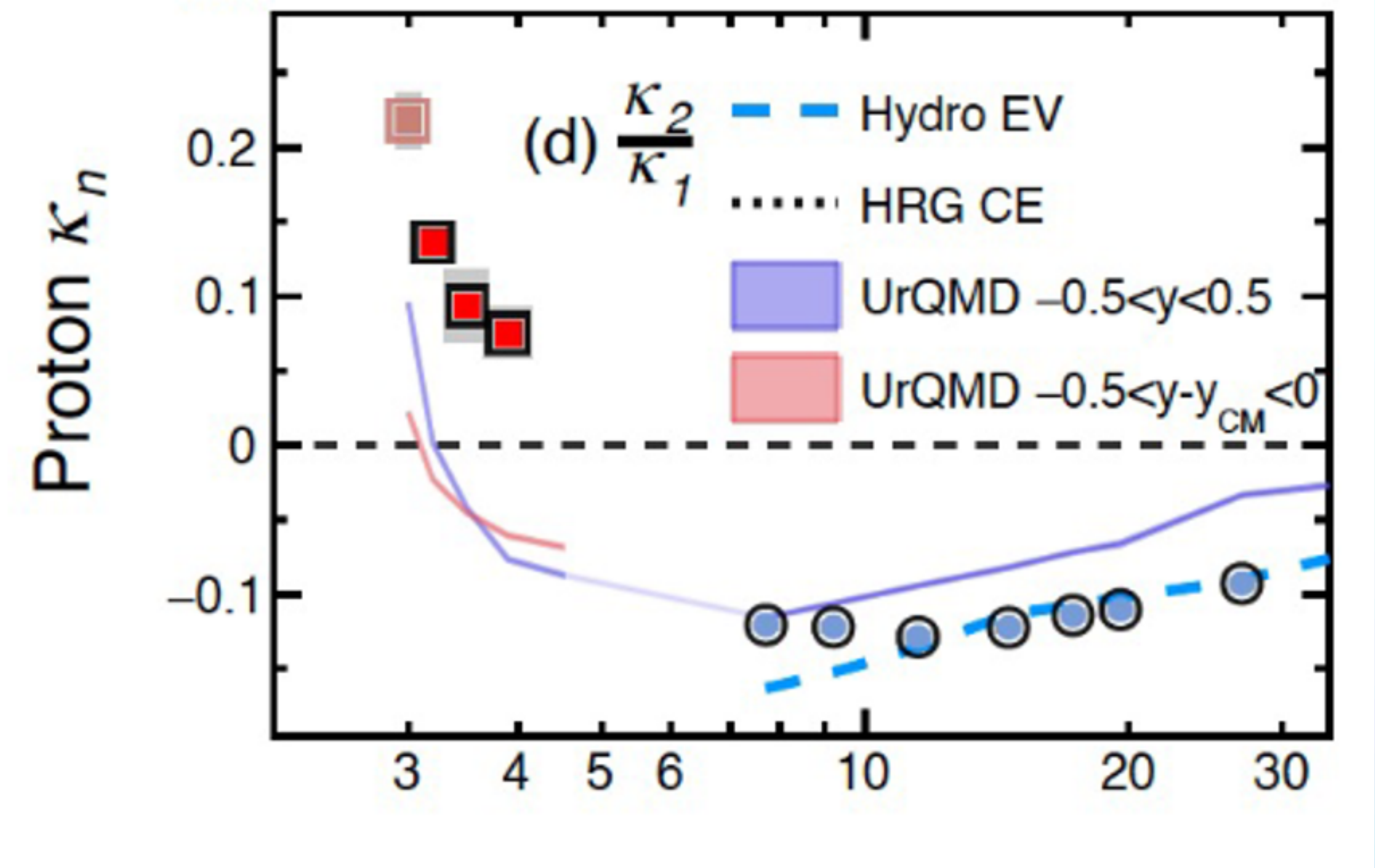
HADES vs. STAR FXT data



Mesut Arslandok, QM 2025

HADES vs. STAR FXT data, C_2/C_1





HADES: $-0.4 < y < 0.4, \quad 0.4 < p_t < 1.6$ GeV/c

STAR FXT Data: $-0.5 < y < 0, \quad 0.4 < p_t < 2$ GeV/c

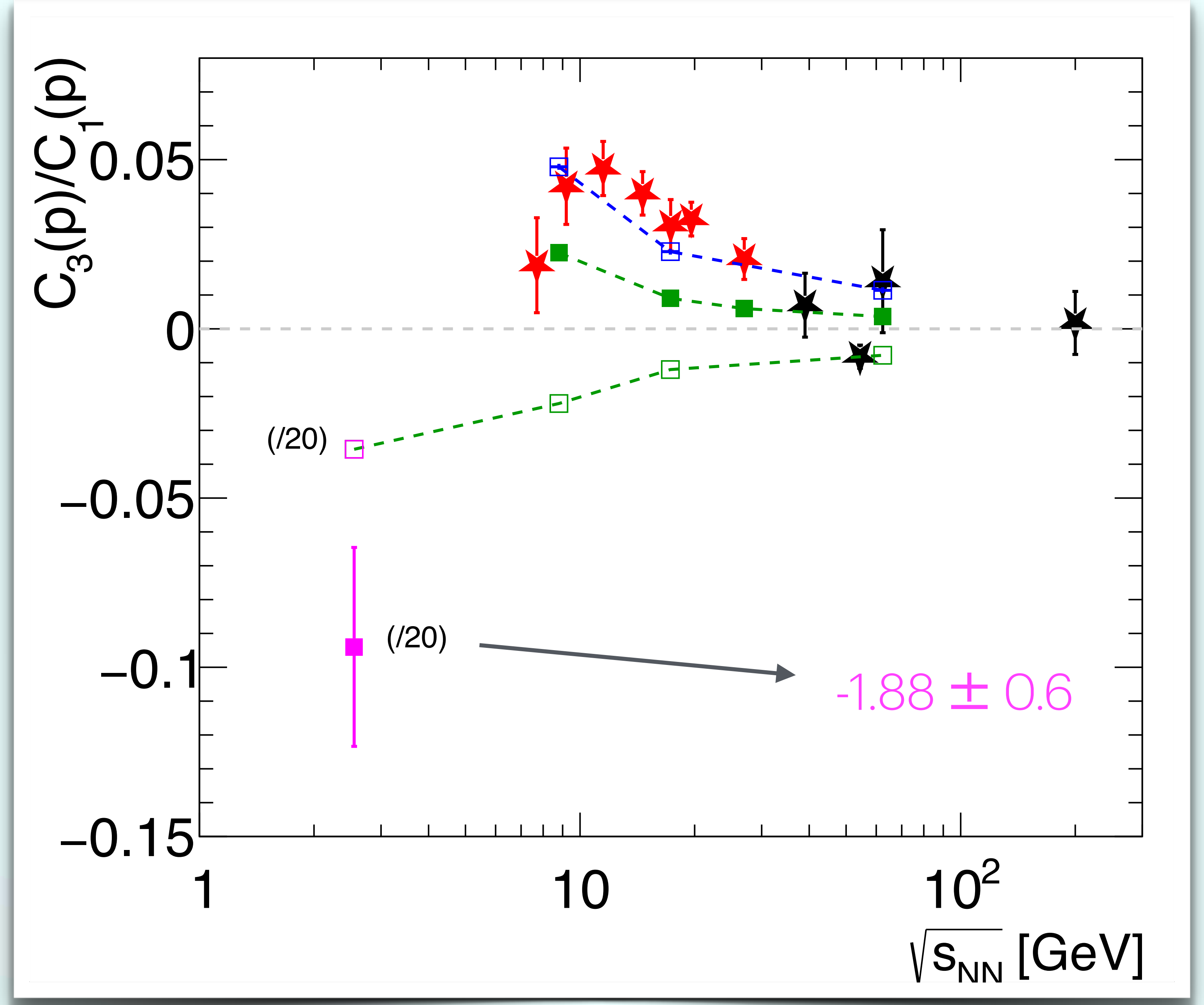
Both HADES and STAR FXT show sharp increase in C_2/C_1 and **sign change**

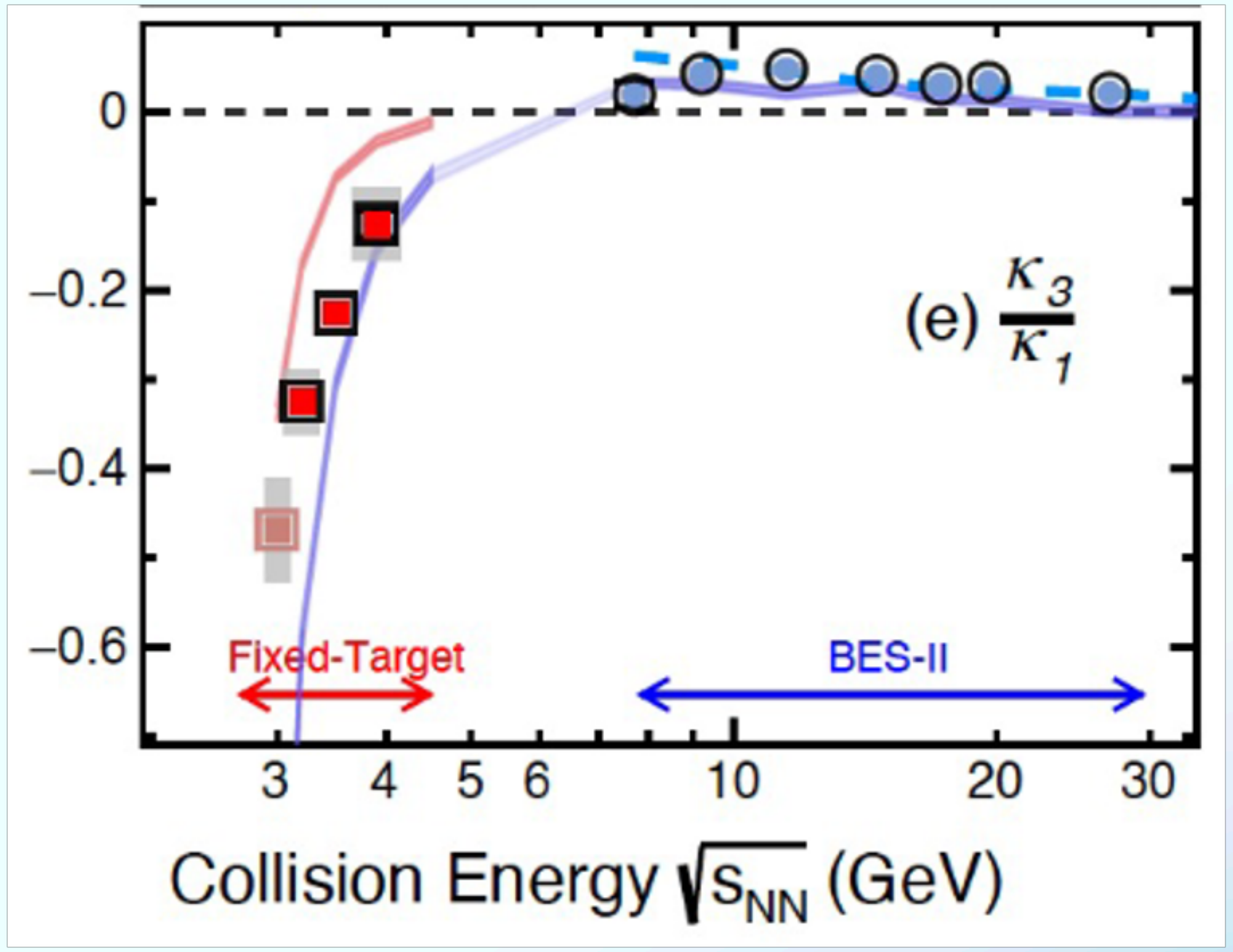
For a quantitative comparison

Differences in acceptance should be accounted for

Corresponding baselines need to be calculated

HADES vs. STAR FXT data, C_3/C_1





HADES: $-0.4 < y < 0.4, \quad 0.4 < p_t < 1.6$ GeV/c

STAR FXT Data: $-0.5 < y < 0, \quad 0.4 < p_t < 2$ GeV/c

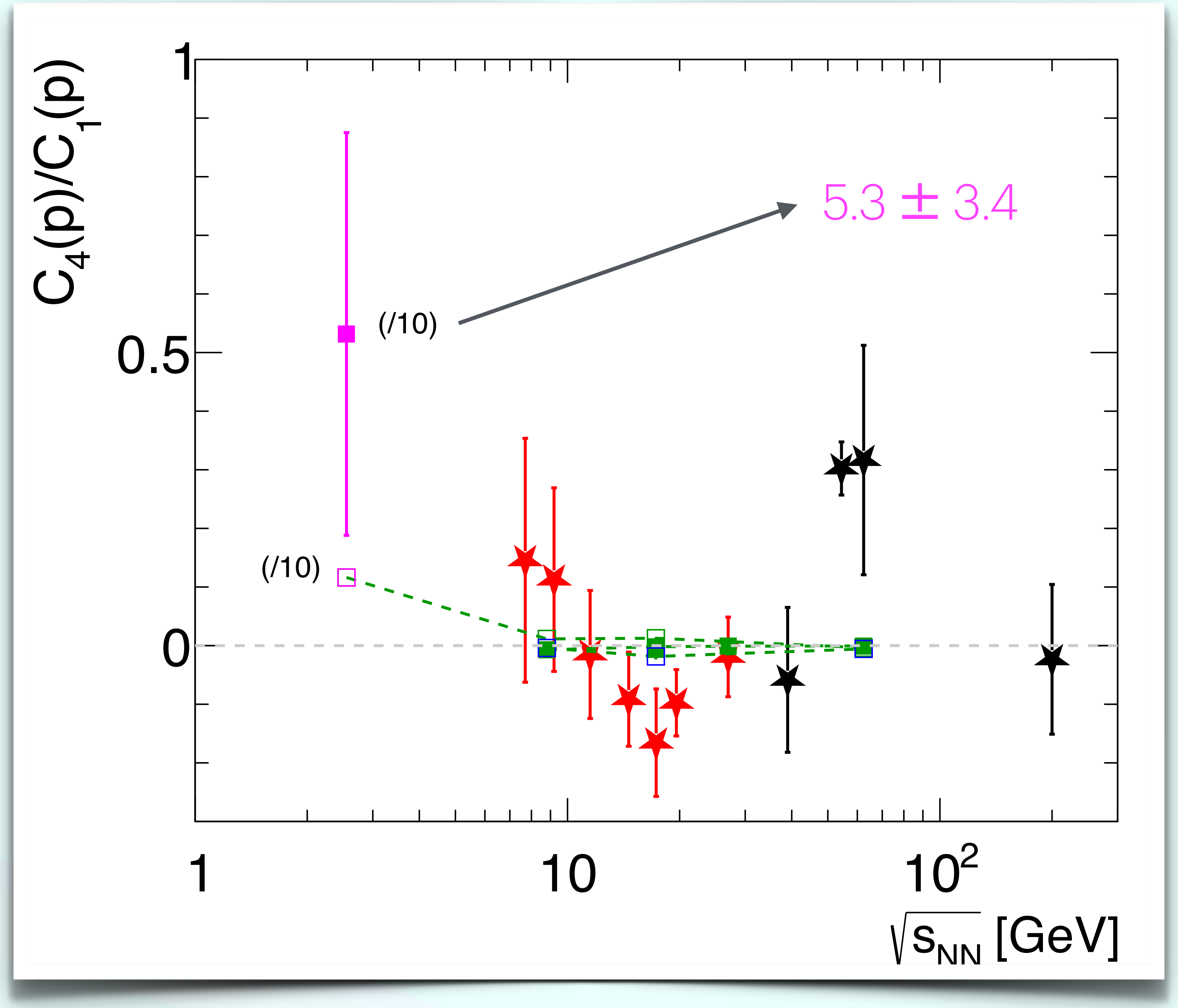
Both HADES and STAR FXT show sharp decrease in C_3/C_1 and **sign change**

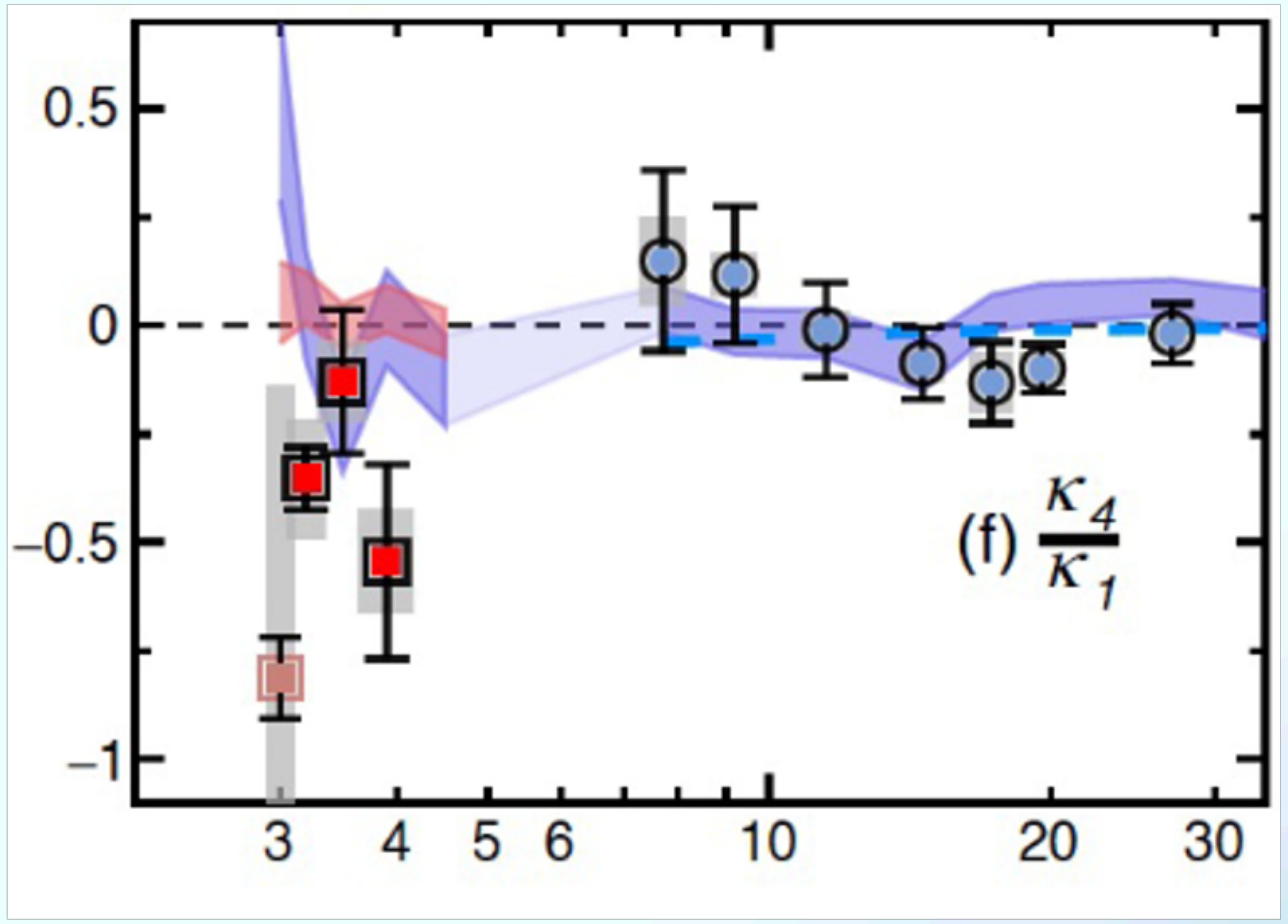
For a quantitative comparison

Differences in acceptance should be accounted for

Corresponding baselines need to be calculated

HADES vs. STAR FXT data, C_4/C_1





HADES: $-0.4 < y < 0.4, \quad 0.4 < p_t < 1.6$ GeV/c

STAR FXT Data: $-0.5 < y < 0, \quad 0.4 < p_t < 2$ GeV/c

Are differences in C_4/C_1 significant?

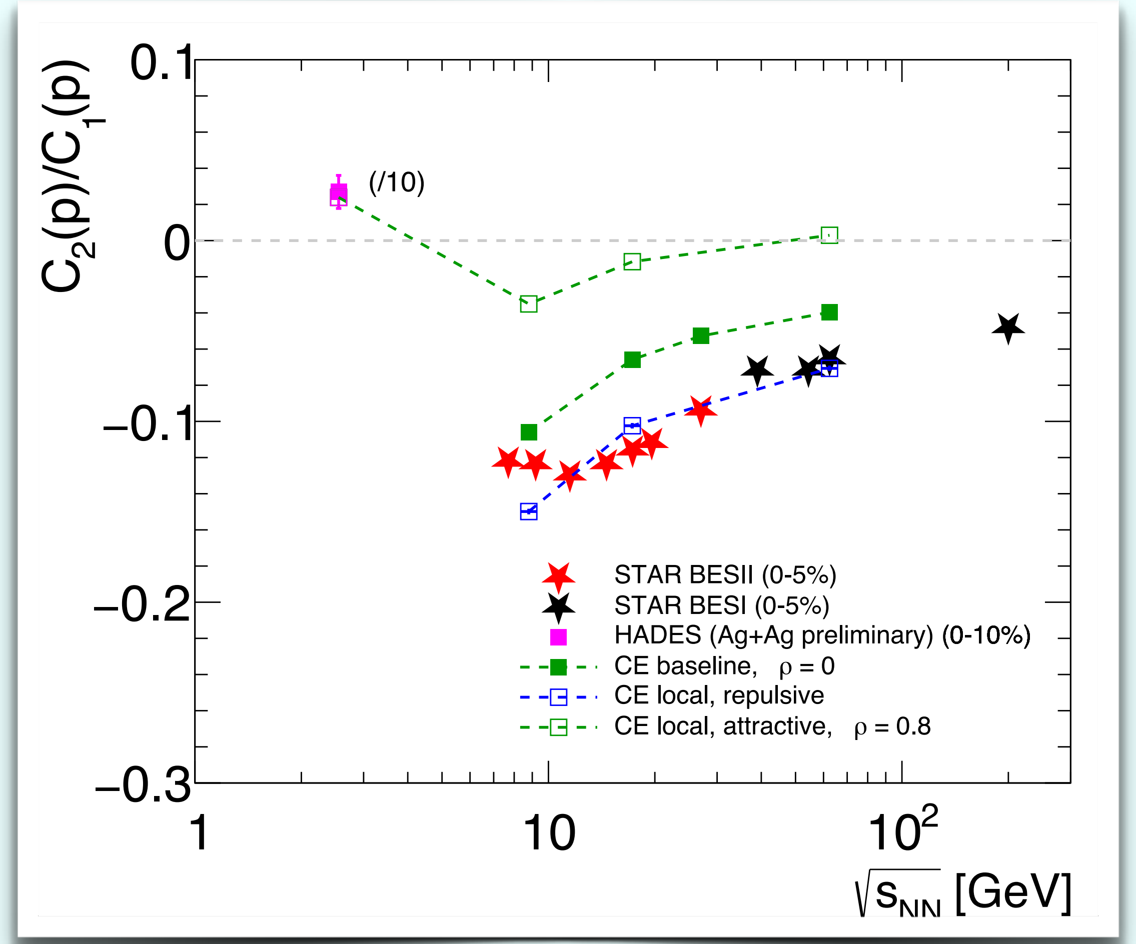
Zachary Sweger, Shinichi Esumi, QM 25
Marvin Nabroth, QM 25

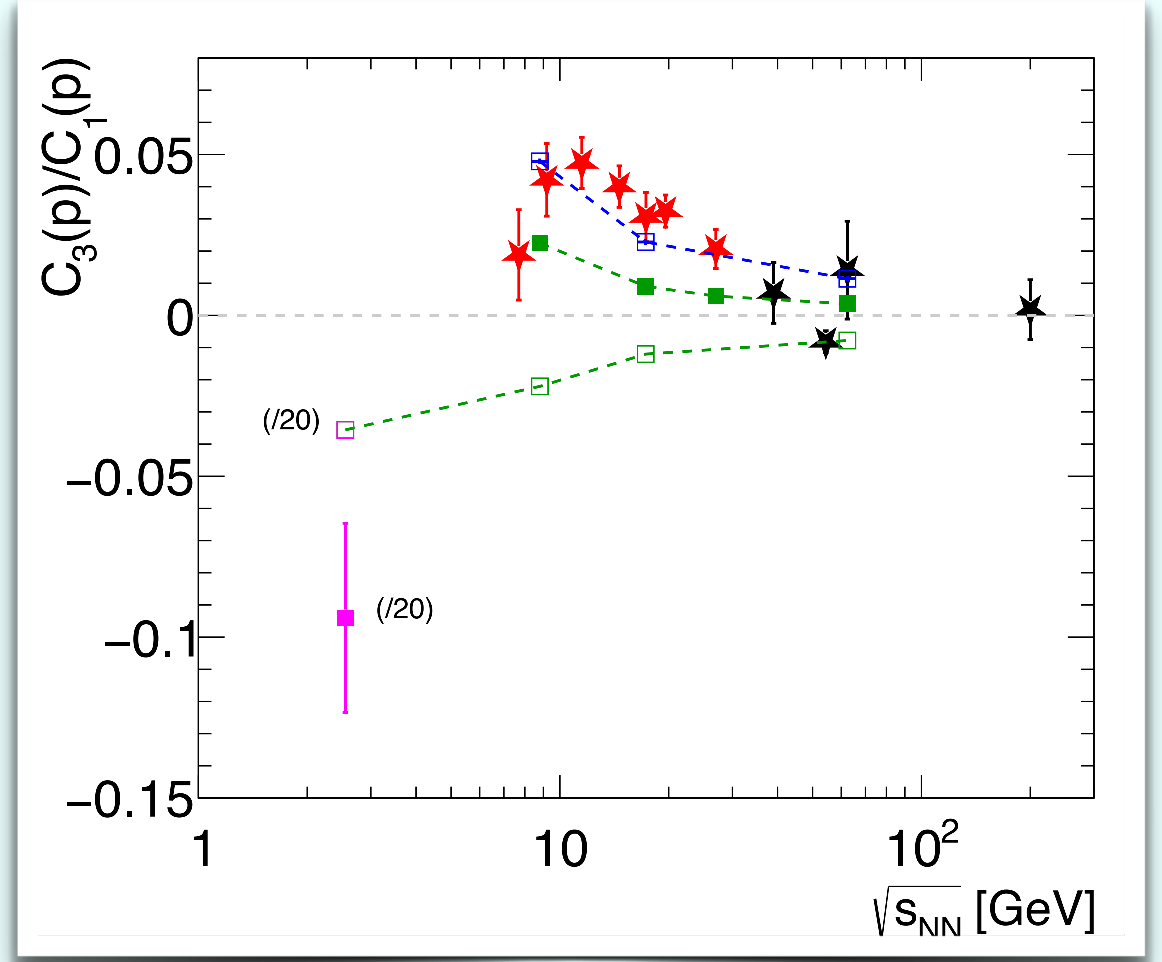
For a quantitative comparison

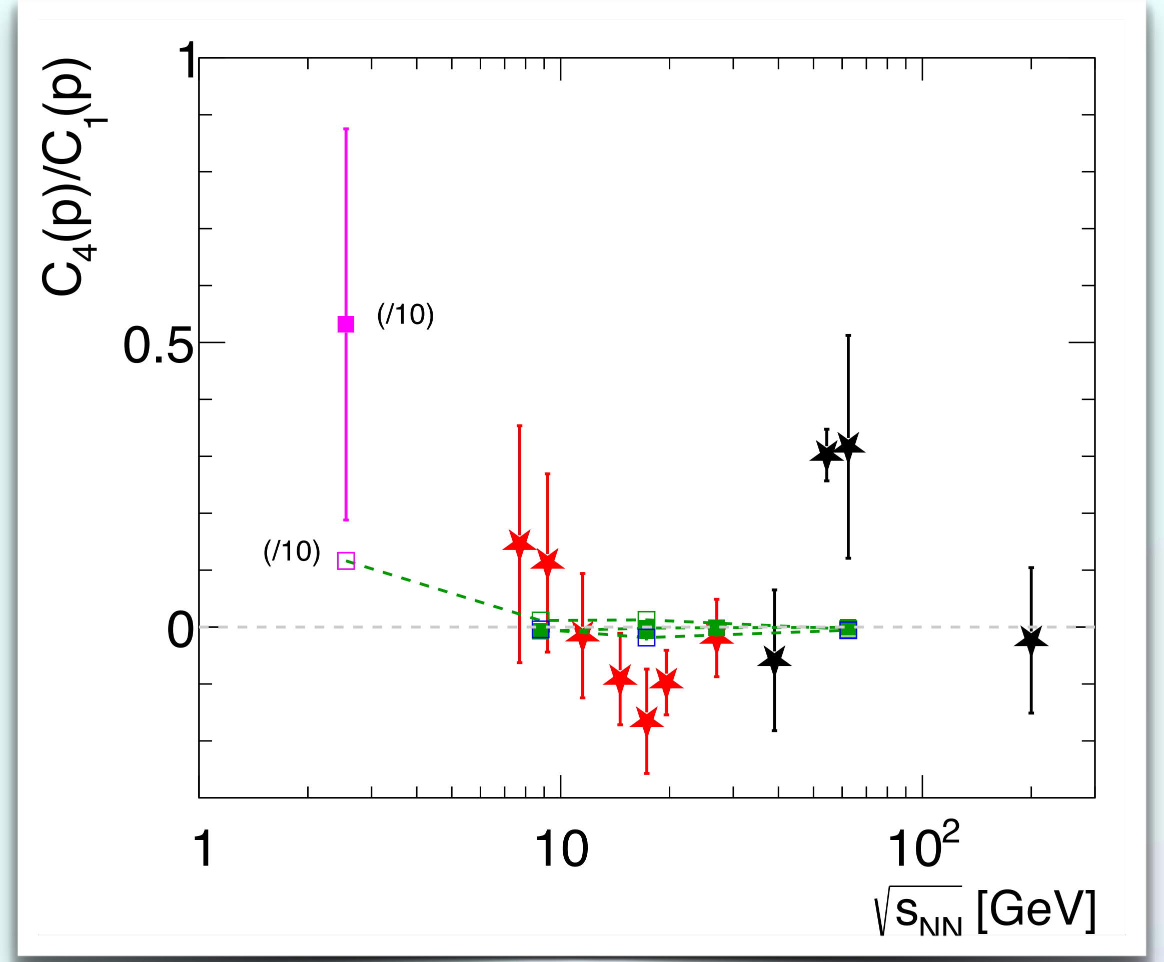
Differences in acceptance should be accounted for

Corresponding baselines need to be calculated

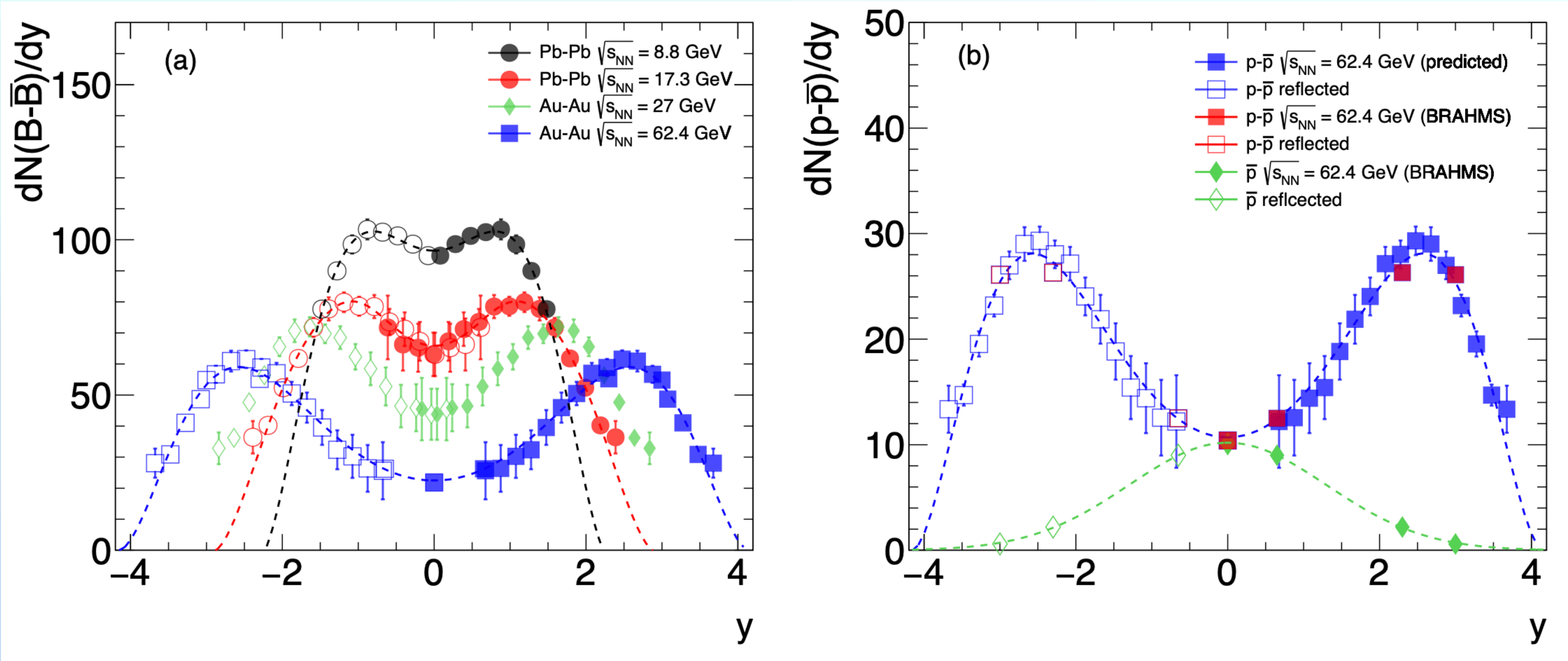
What do we see



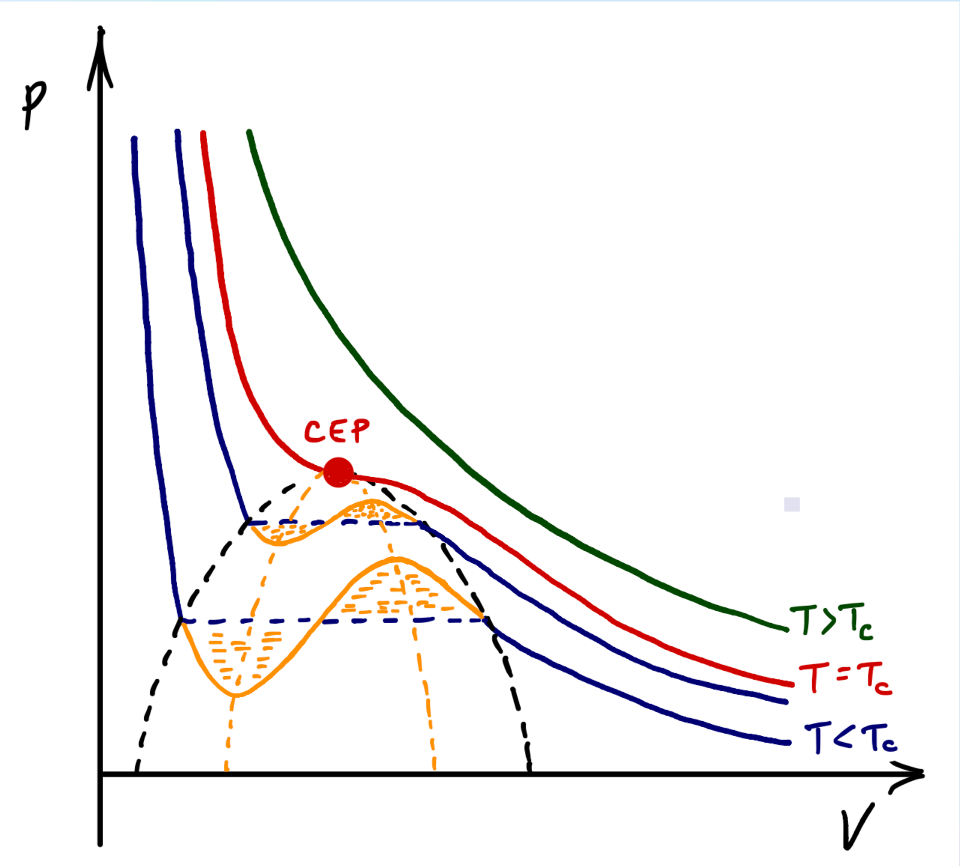




THE INTERPLAY OF REPULSIVE AND ATTRACTIVE FORCES BETWEEN PROTONS EXPLAINS THE SYSTEMATIC TRENDS OBSERVED IN THE STAR BESII AND HADES DATA



artifacts of baryon stopping
or
messengers of first-order phase transition ?



Volume Fluctuations

Volume fluctuations, wounded nucleon model

fixed volume/wounded nucleons

$\langle N\rangle \sim V, \ \ \langle N\rangle \sim N_W$

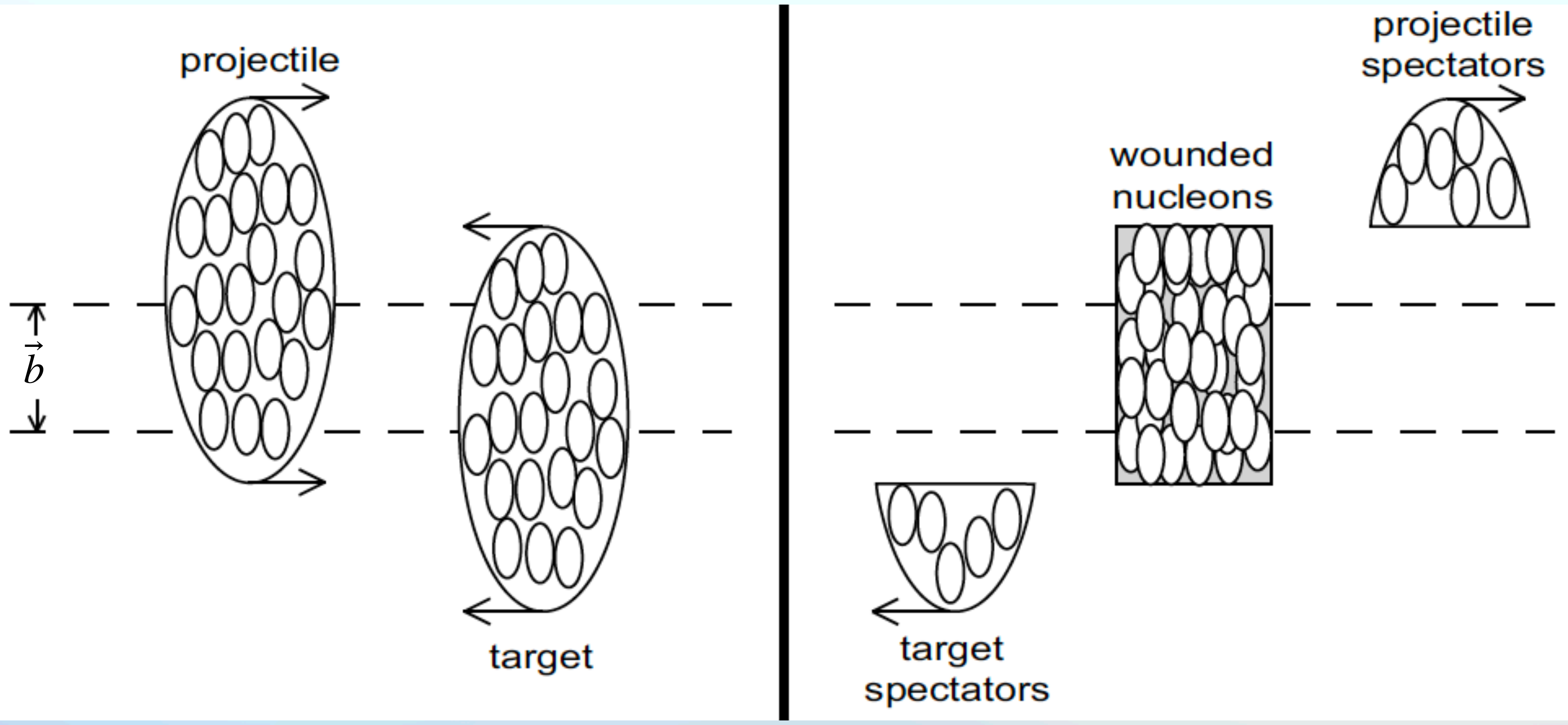
$\kappa_2(N) \sim V, \ \kappa_2(N) \sim N_W$

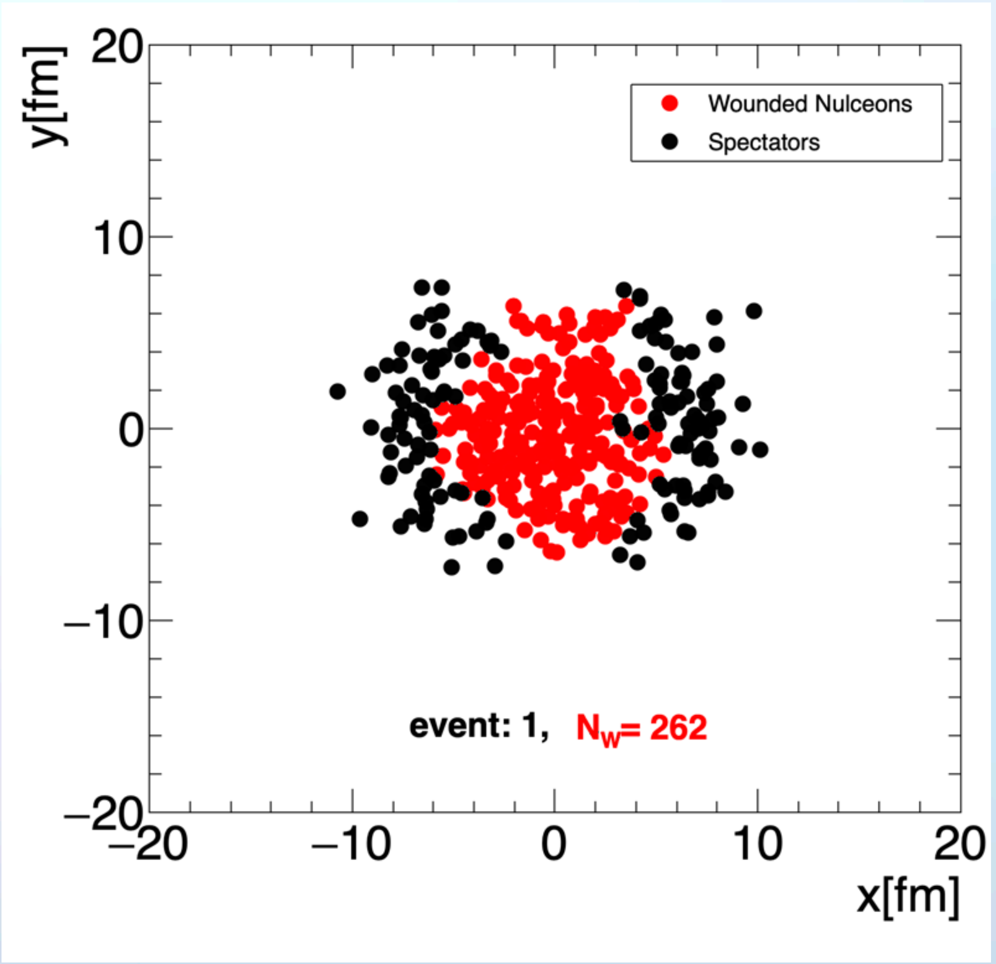
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114-130
V. Skokov, B. Friman, K. Redlich, Phys. Rev. C 88 (2013)

fluctuating sources

$$\langle N\rangle = \langle n\rangle\langle N_W\rangle$$

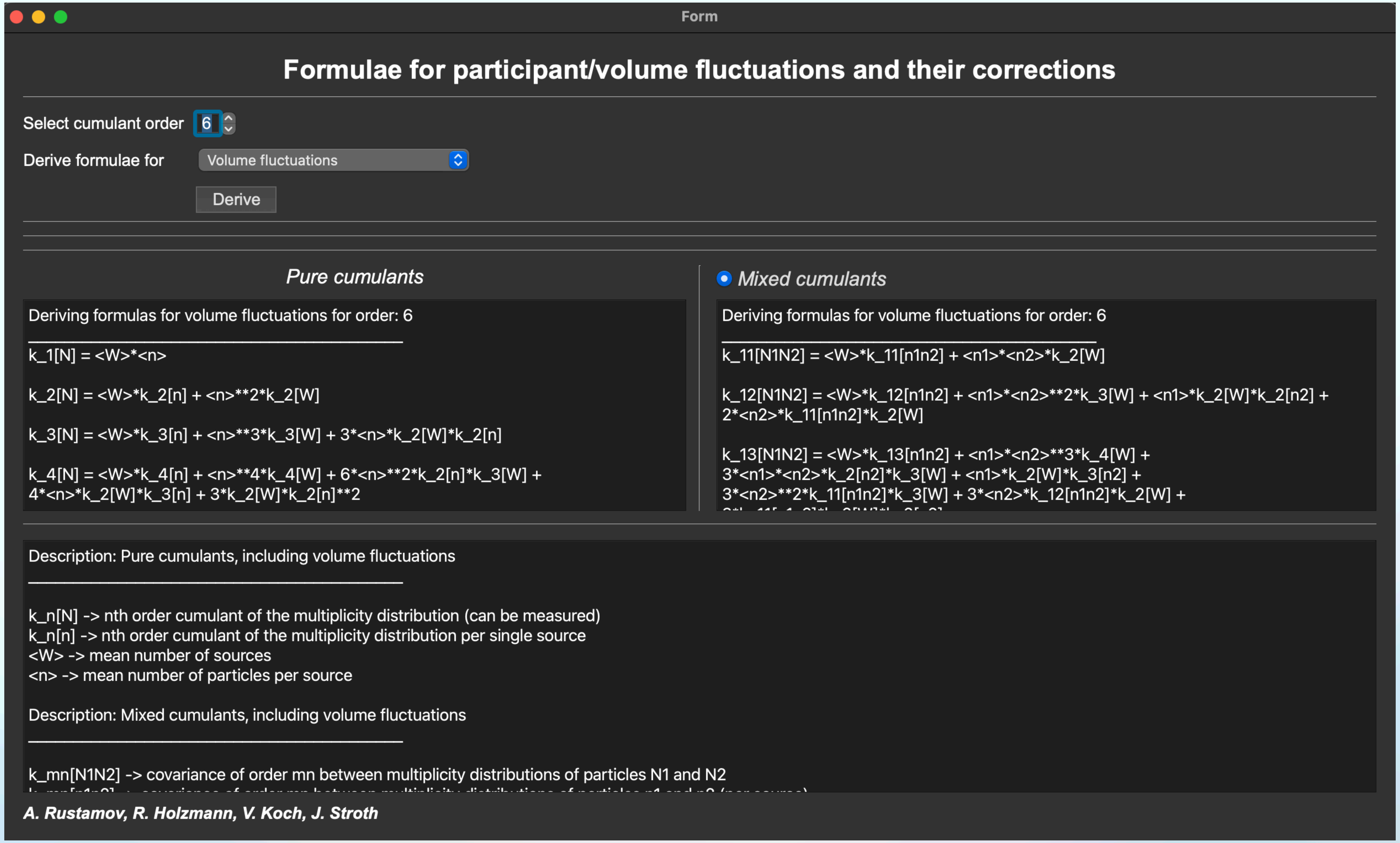
$$\kappa_2(N) = \kappa_2(n)\langle N_W\rangle + \langle N\rangle^2\frac{\kappa_2(N_W)}{\langle N_W\rangle^2}$$





<u>Wounded nucleons</u>, N_W: Nucleons which collided at least once <u>inelastically</u>

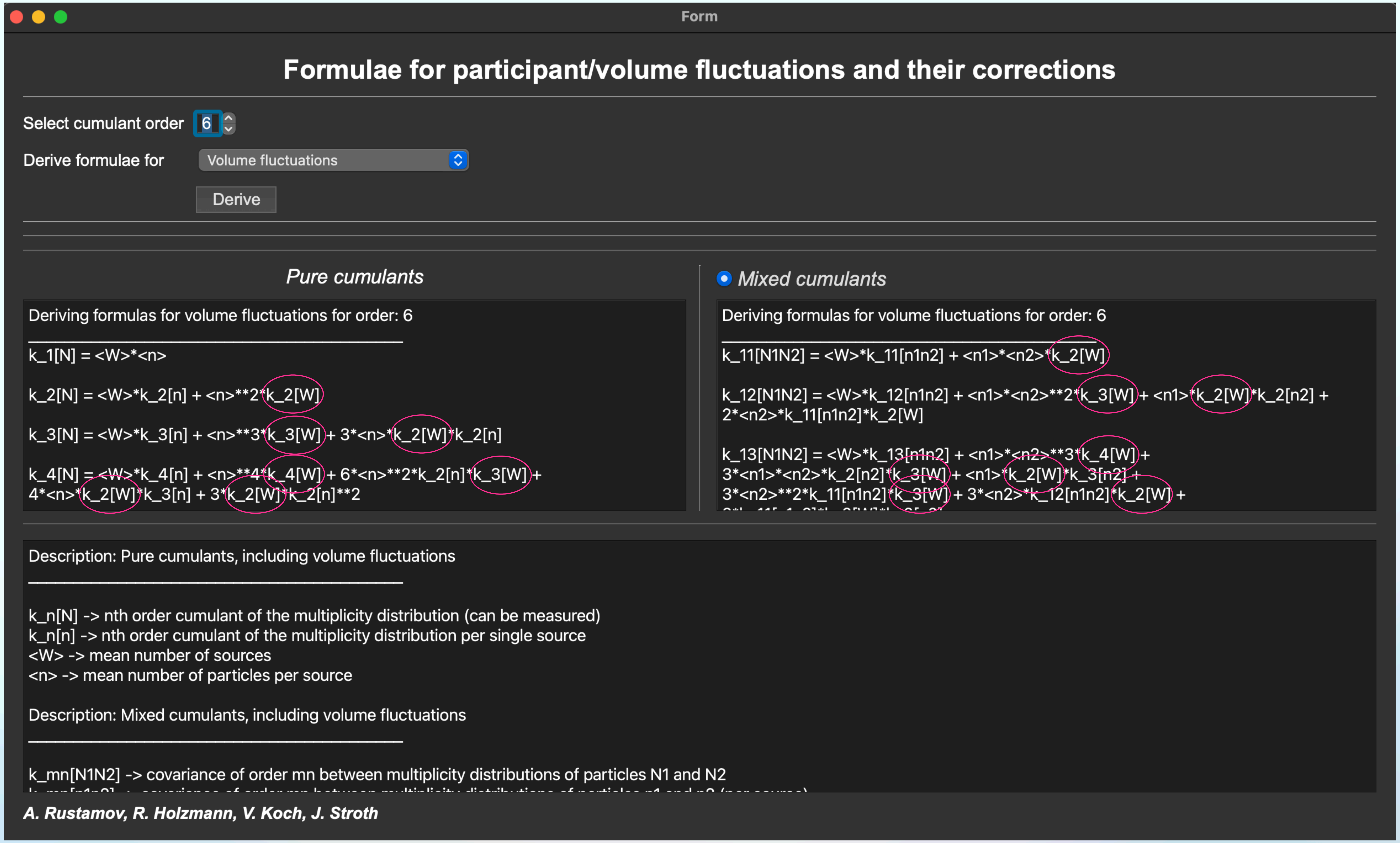
Higher moments are more involved



R. Holzmann, V. Koch, A. Rustamov, J. Stroth, NPA 1050 (2024) 122924

https://github.com/anarrustamov/VolumeFluctuations

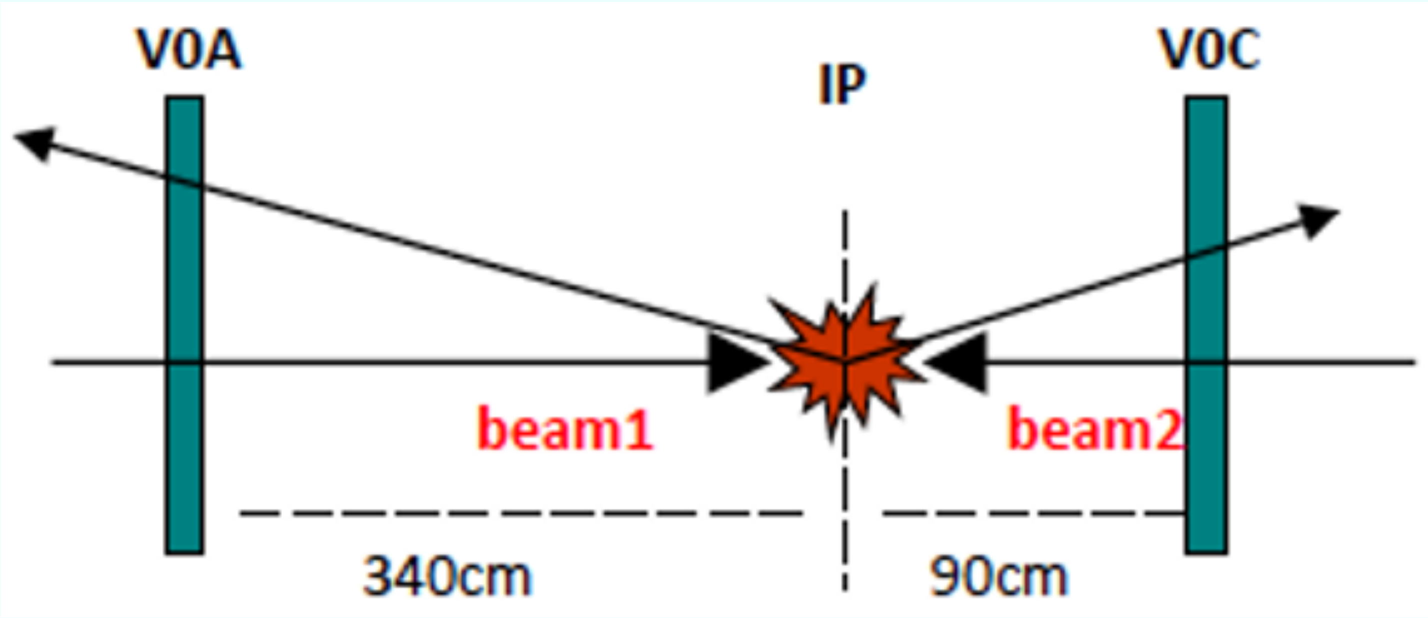
Higher moments are more involved



R. Holzmann, V. Koch, A. Rustamov, J. Stroth, NPA 1050 (2024) 122924

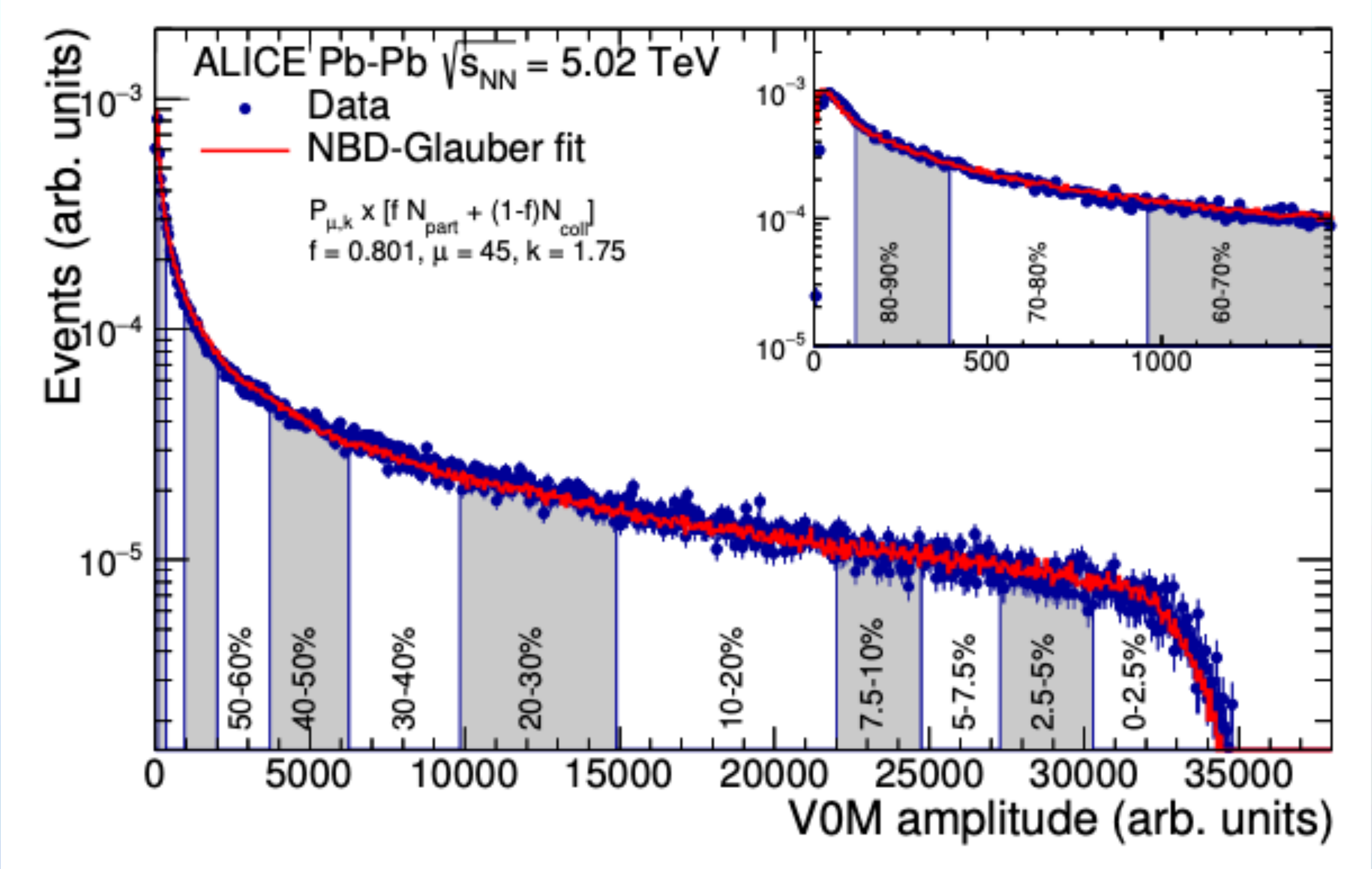
https://github.com/anarrustamov/VolumeFluctuations

Centrality determination

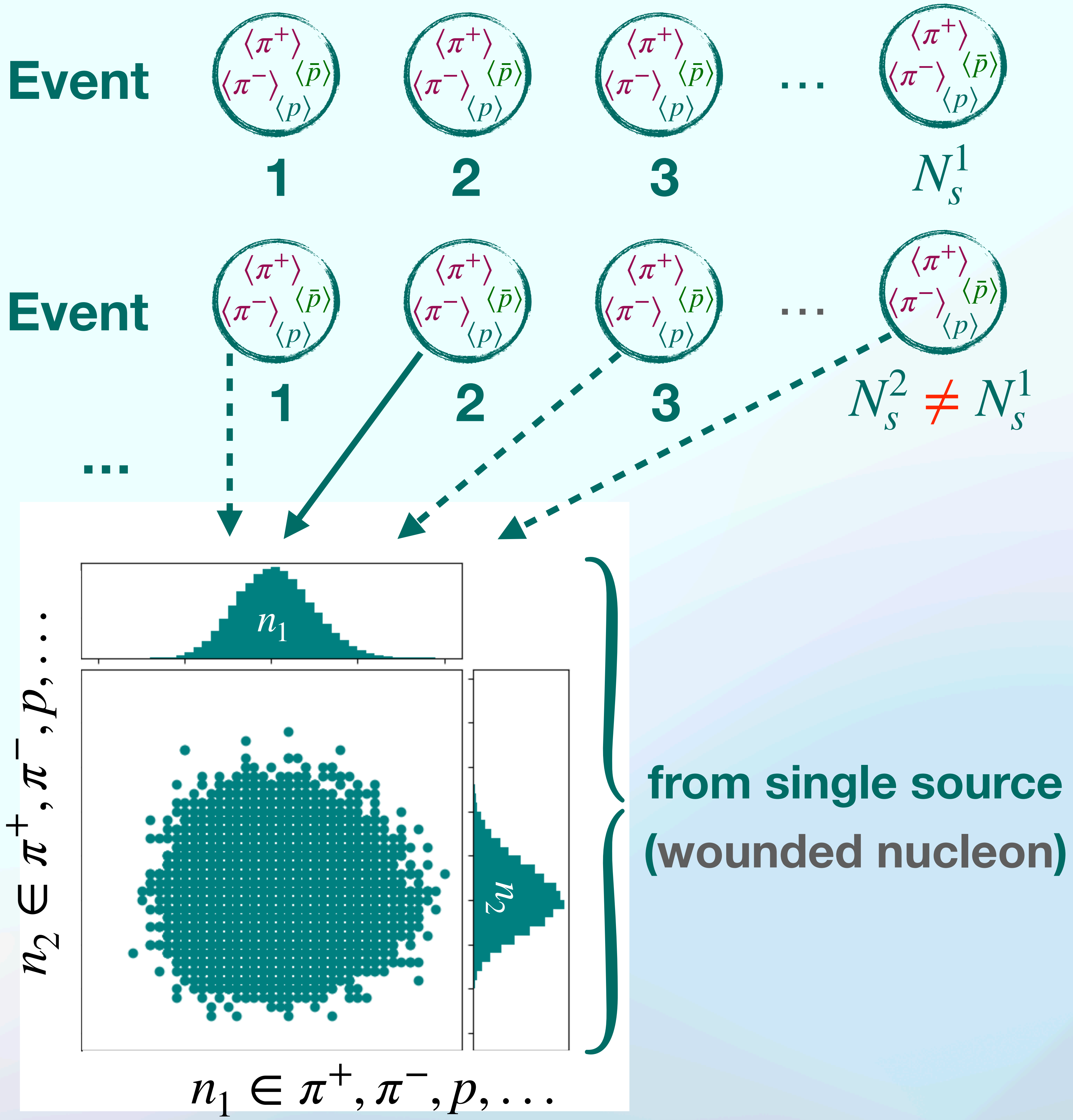


$2.8 < \eta < 5.1$ $\quad\quad$ $-3.7 < \eta < -1.7$

https://alice-notes.web.cern.ch/node/453



model of independent



A. Rustamov, R. Holzmann, J. Stroth, NPA 1034 (2023) 122641

V. Koch, R. Holzmann, A. Rustamov, J. Stroth, Nucl.Phys.A 1050 (2024) 122924

MC Glauber results

R = 6.62 fm, skin thickness a = 0.546, σ_{NN}^{Inel} = 70 mb

Exclusion distance d_{min} = 0.4 fm

f = 0.801, $\mu = 45$, $k = 1.75$

this analysis

Centrality	$\langle N_W \rangle$	RMS	$\langle N_{coll} \rangle$	RMS	$\langle N_s \rangle$	RMS
0-10%	358.89	31.50	1637.40	245.72	613.22	72.55
10-20%	263.82	27.38	1003.21	153.92	410.86	50.34
20-30%	188.99	21.73	603.72	103.37	271.43	35.75
30-40%	131.217	17.04	345.92	68.66	173.84	25.42
40-50%	86.94	13.09	184.45	43.49	106.24	17.83
50-60%	53.96	9.68	90.15	25.33	60.99	12.01
60-70%	30.59	6.80	40.01	13.33	32.33	7.71
70-80%	15.45	4.42	16.16	6.39	15.50	4.71
80-90%	6.80	2.62	5.97	3.09	6.42	2.78

ALICE note https://alice-notes.web.cern.ch/node/453

Centrality		RMS		RMS
0-10%	359	31.2	1636	246
10-20%	263	27.1	1001	154
20-30%	188	22.5	601	106
30-40%	131	19.1	344	74.7
40-50%	86.3	16.3	183	50.8
50-60%	53.6	13.6	89.8	32.4
60-70%	30.4	10.8	39.8	19.1
70-80%	15.6	7.83	16.2	10.5
80-90%	7.59	4.89	6.46	5.27

MC Glauber results

R = 6.62 fm, skin thickness a = 0.546, σ_{NN}^{Inel} = 70 mb

Exclusion distance d_{min} = 0.4 fm

f = 0.801, $\mu = 45$, $k = 1.75$

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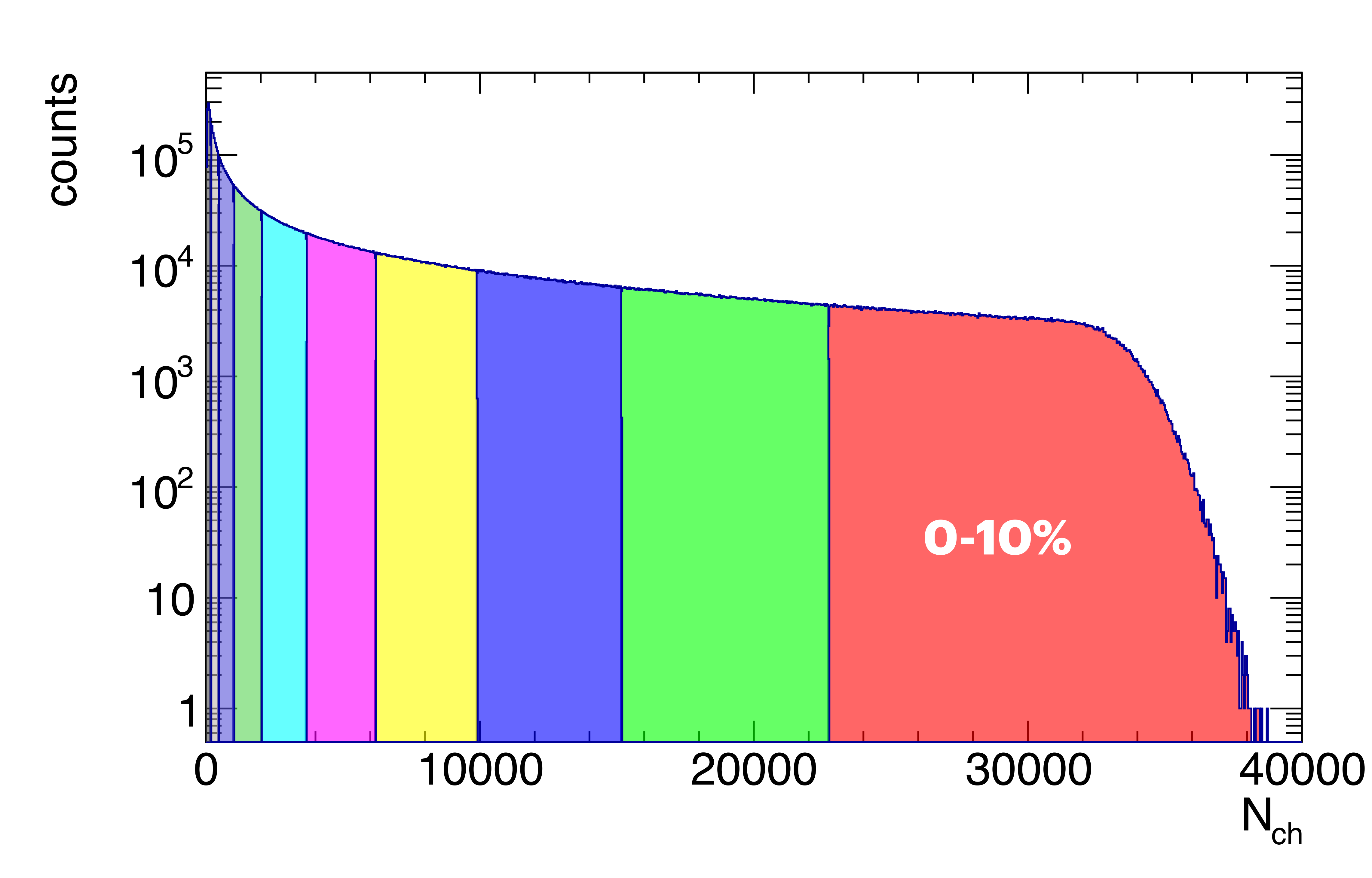
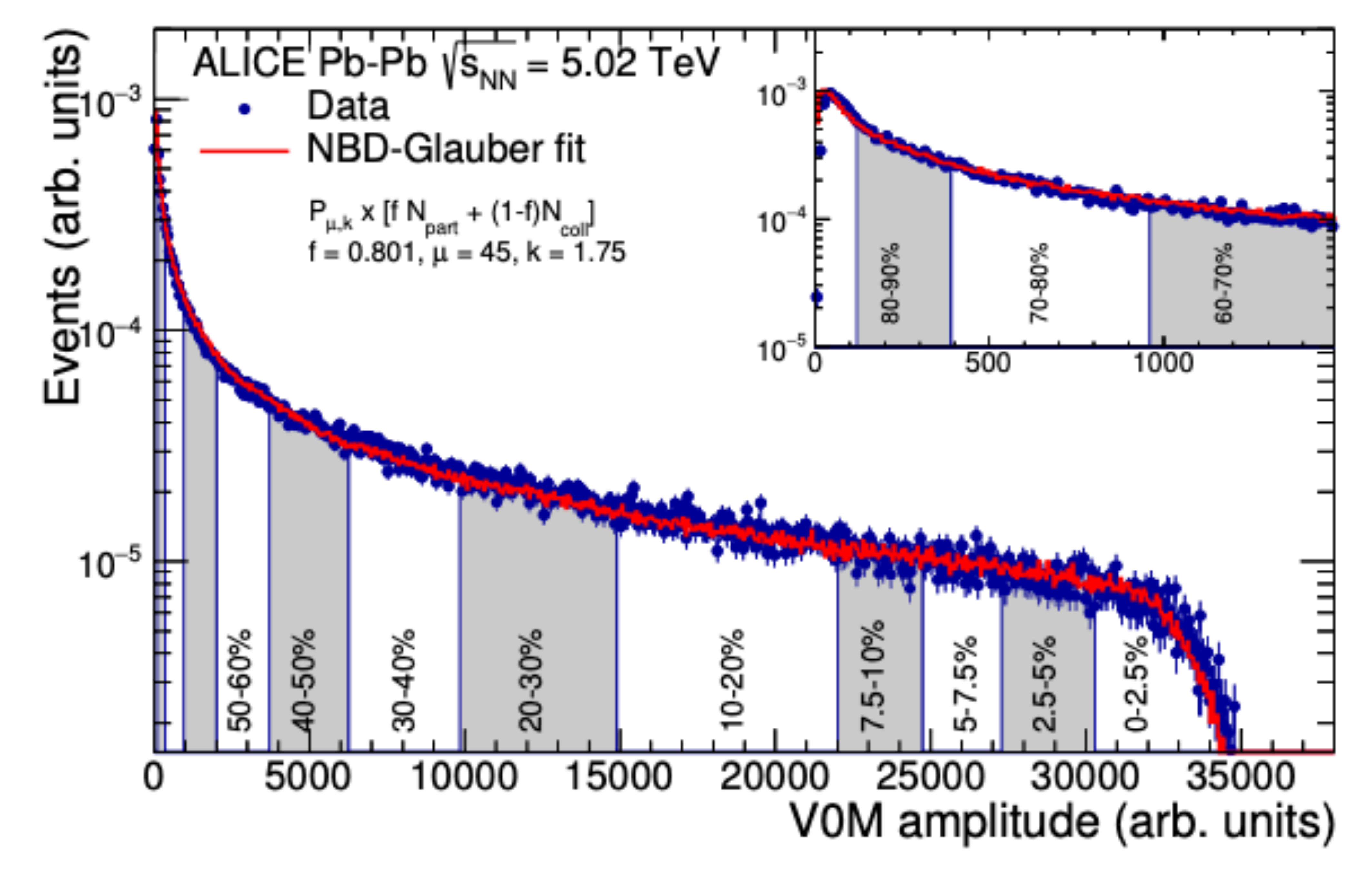
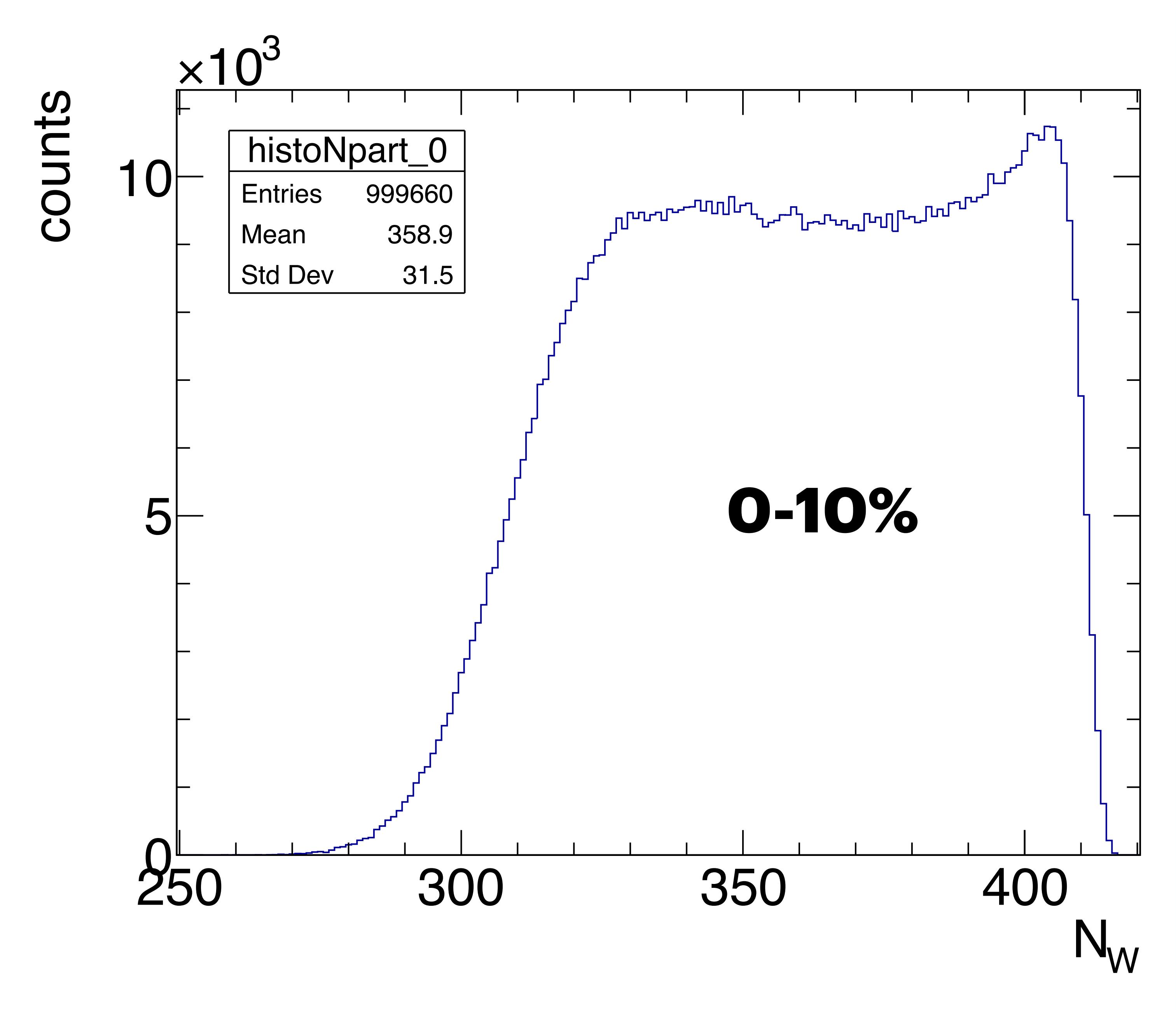
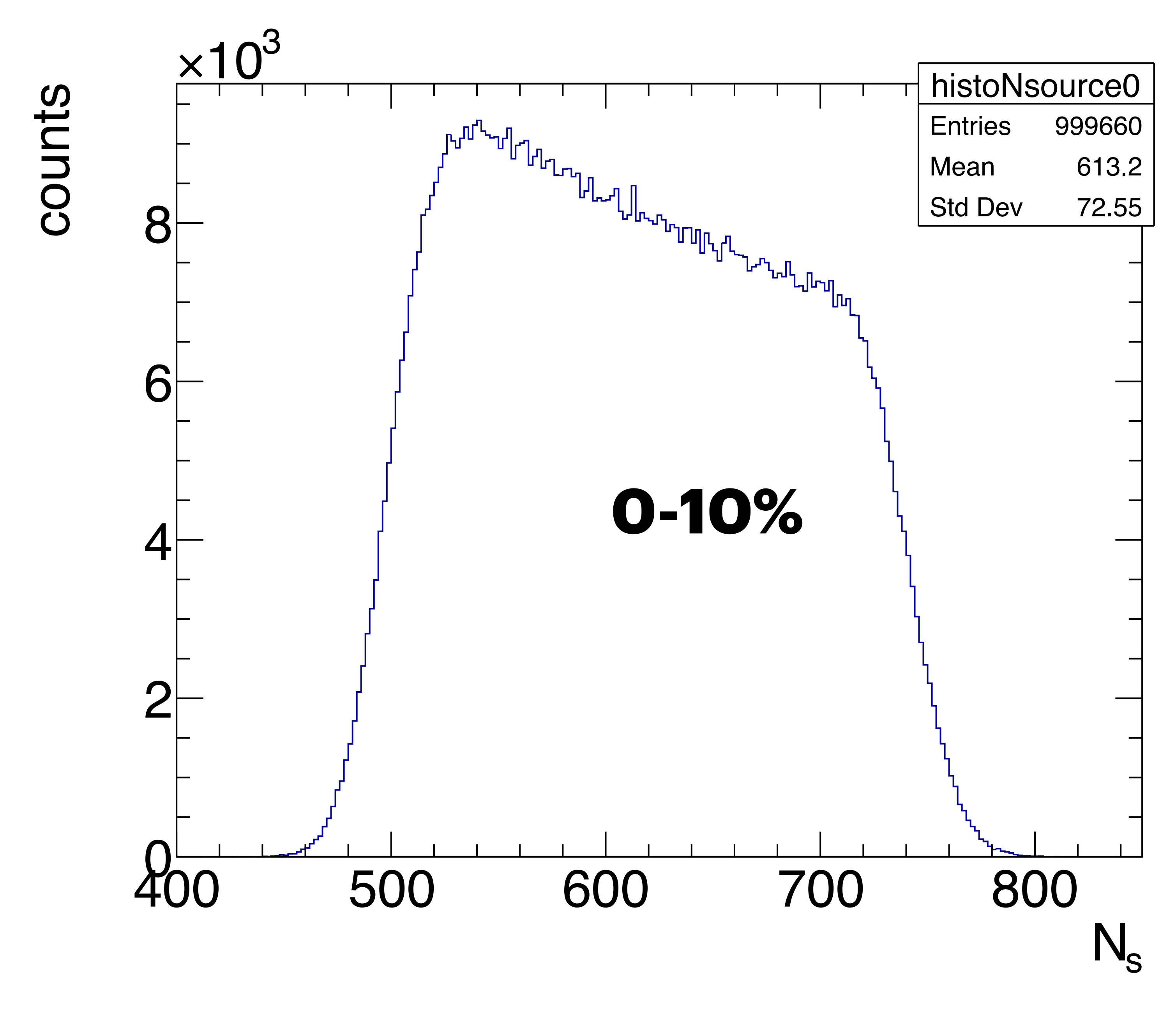


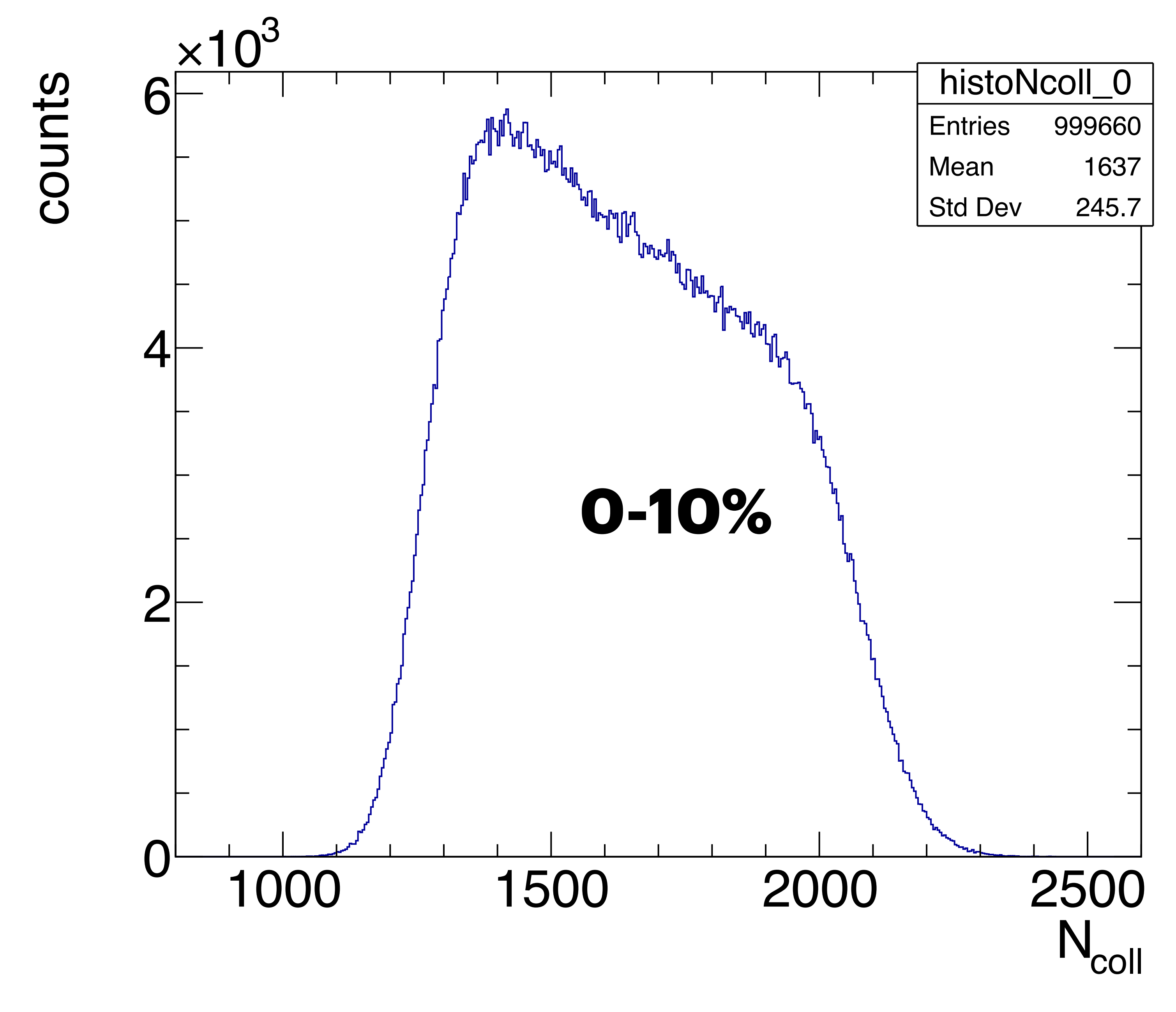
ALICE note https://alice-notes.web.cern.ch/node/453

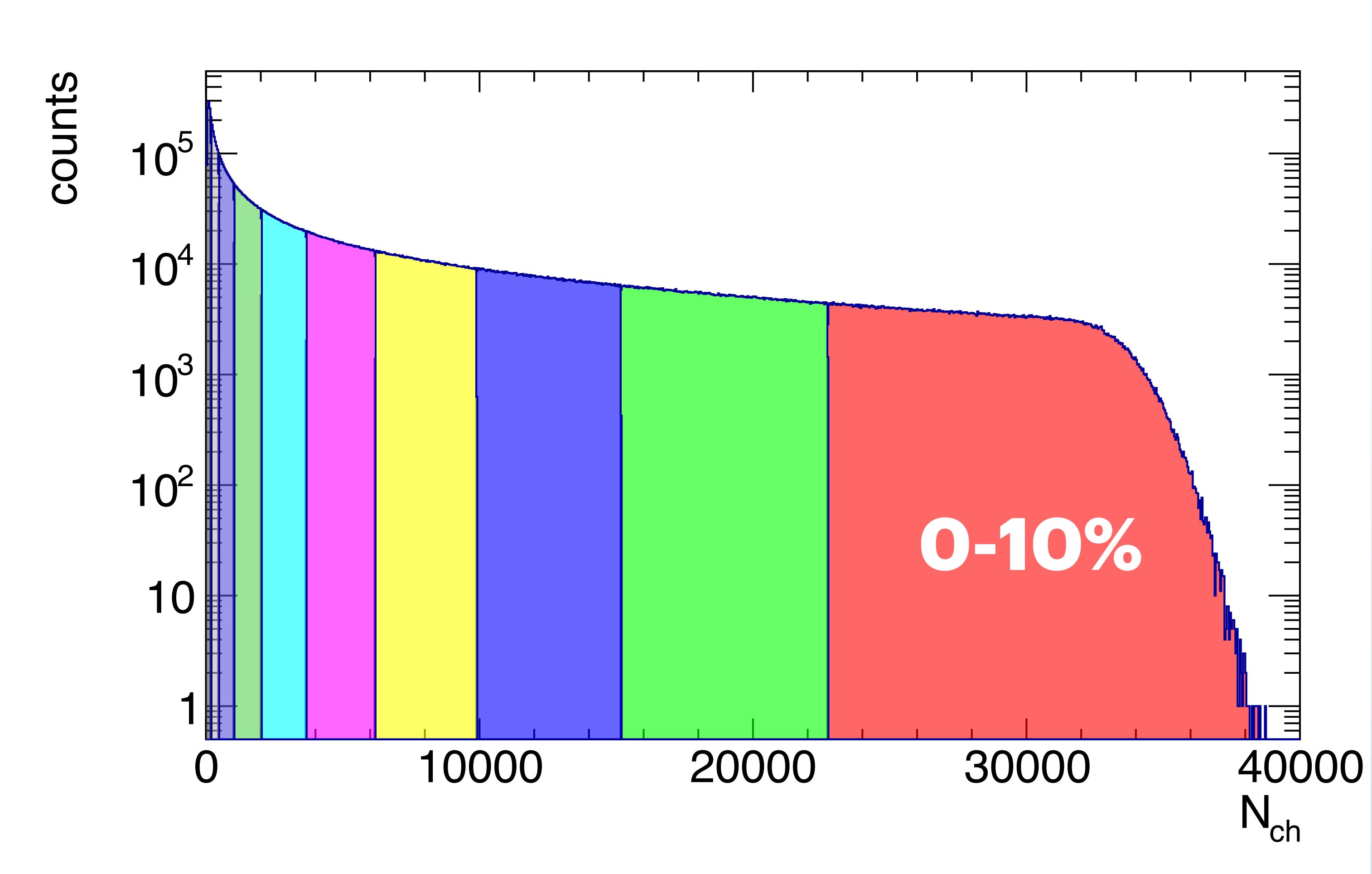


Volume fluctuations





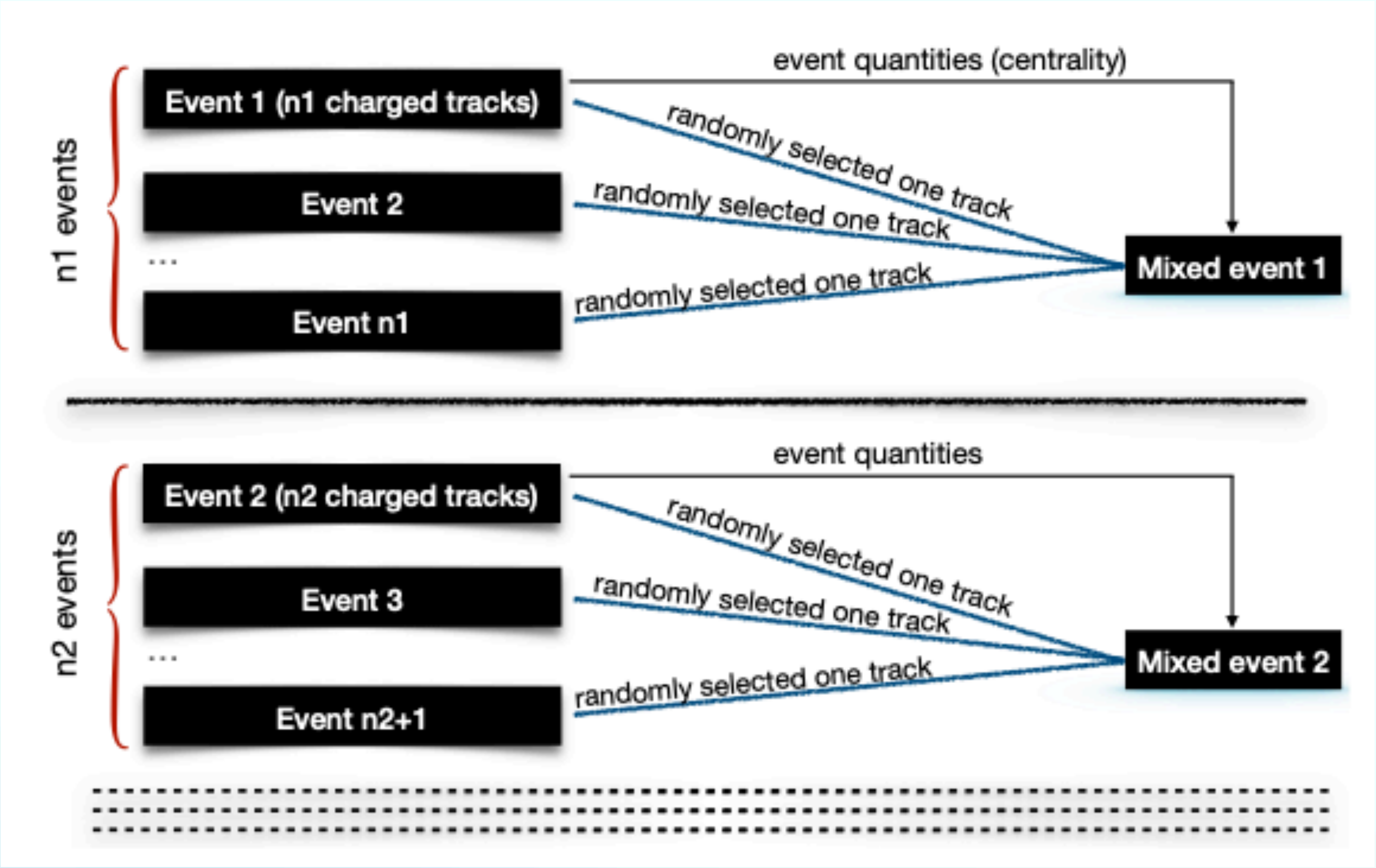




P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114-130

$$\kappa_2(N - \bar{N}) = \kappa_2(n - \bar{n})\langle N_W \rangle + \underbrace{\boxed{\langle N - \bar{N} \rangle^2}}_{\text{vanishes for ALICE}} \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$

Event mixing strategy



$$\frac{\kappa_2(N_W)}{\langle N_W\rangle^2} = \frac{cov(N_1, N_2)}{\langle N_1\rangle \langle N_2\rangle} \tag{13}$$

In a similar way, using Eq. 12, participant fluctuations can be extracted from the reconstructed second order cumulants for particle type N:

$$\frac{\kappa_2(N_W)}{\langle N_W\rangle^2} = \frac{\kappa_2(N)}{\langle N\rangle^2} - \frac{1}{\langle N\rangle} \tag{14}$$

The covariances and cumulants entering the right hand sides of Eqs. 13 and 14 are to be evaluated using mixed events. Note that, by exploiting Eqs. 8 and 9, higher-order cumulants of participant distributions can also be obtained.

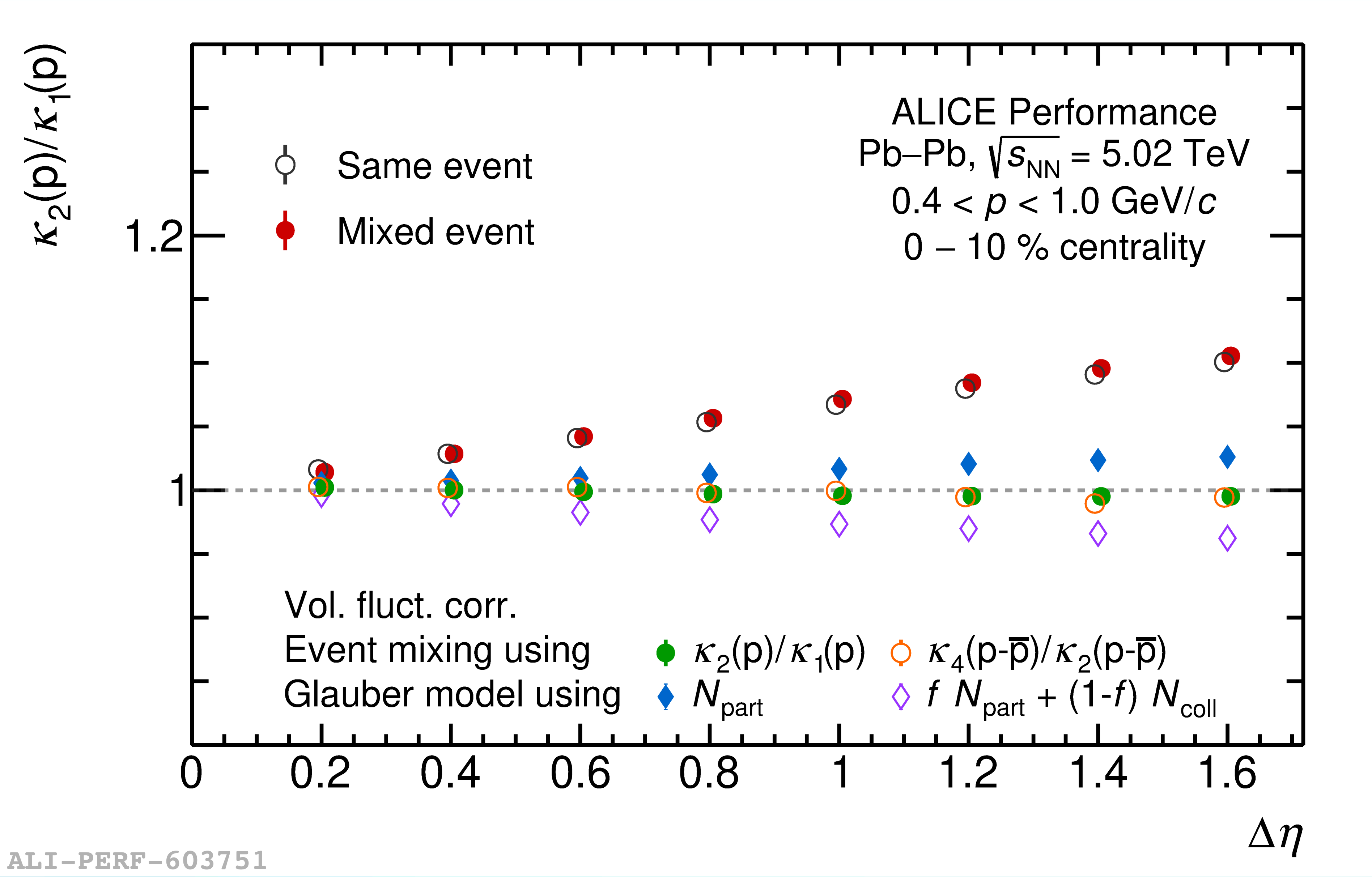
A. Rustamov, R. Holzmann, J. Stroth, NPA 1034 (2023) 122641

For example, using 4th order cumulants from ALICE

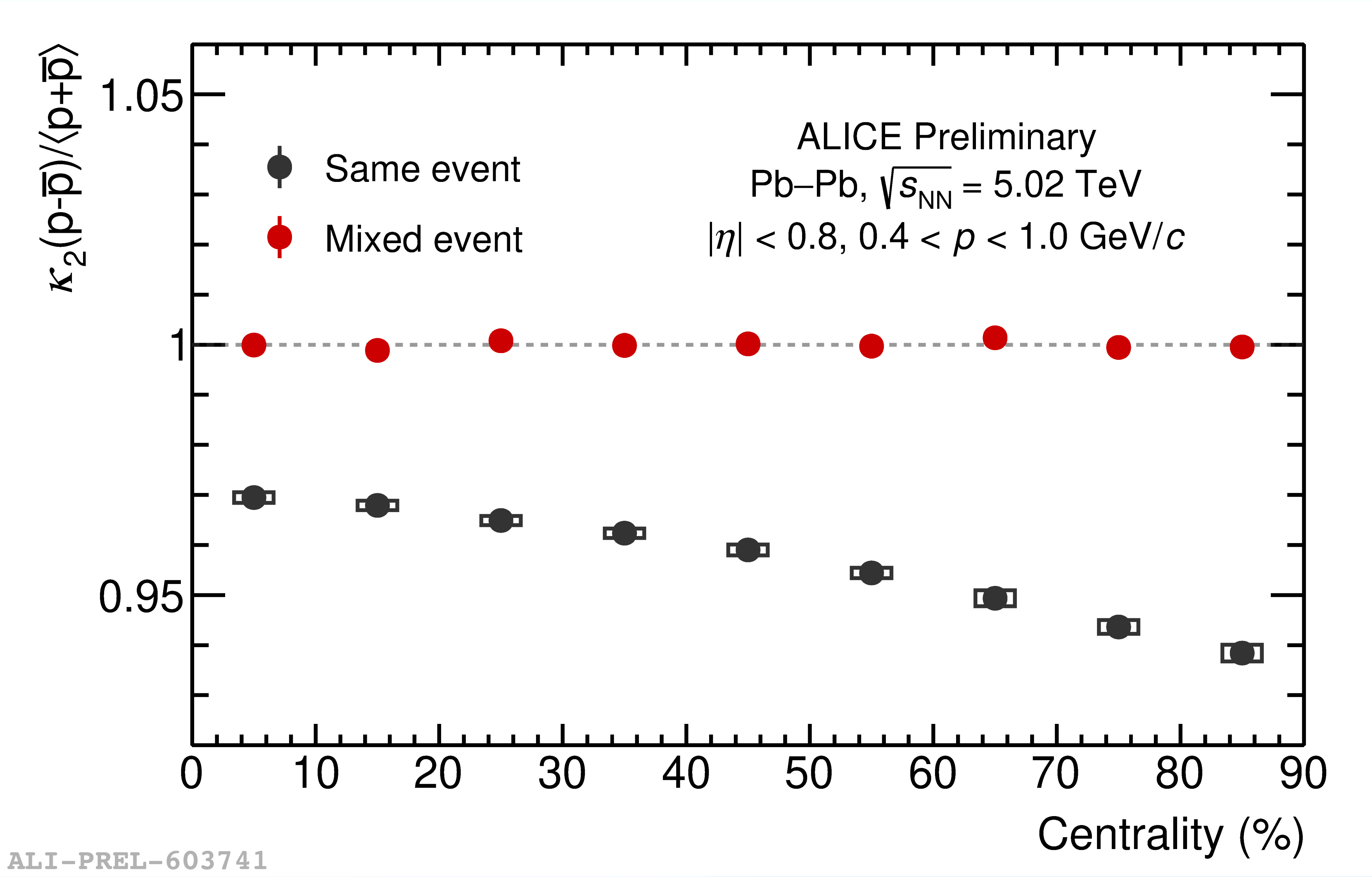
$$\frac{\kappa_2(N_s)}{\langle N_s\rangle^2} = \frac{k_4(\Delta N)/\kappa_2(\Delta N)}{3\langle N_p + N_{\bar{p}}\rangle}$$

Is event mixing working properly?

protons



net- protons



$$\kappa_2(N) = \kappa_2(n)\langle N_W\rangle + \langle N\rangle^2 \frac{\kappa_2(N_W)}{\langle N_W\rangle^2}$$

Recovering Poisson distribution

$$\kappa_2(N-\bar{N}) = \kappa_2(n-\bar{n})\langle N_W\rangle + \langle N-\bar{N}\rangle^2 \frac{\kappa_2(N_W)}{\langle N_W\rangle^2}$$

Recovering Skellam distribution

Fuzzy Logic

A. Rustamov, Phys.Rev.C 110 (2024) 6, 064910

Correcting moments for mis-identification - Fuzzy Logic

Marvin Nabroth, QM 2025

- A. Rustamov derived generalized relation between moments of W and N ***Phys.Rev.C* 110 (2024) 6**

$$M_W(t_1, t_2, ..., t_n) = \sum_{N_1, N_2, ..., N_n=0}^{\infty} P(N_1, N_2, ..., N_n) \prod_{i=1}^{n} \left[\int_{-\infty}^{+\infty} e^{\sum_{j=1}^{n} t_j \omega_j(x)} \mathcal{P}_i(x) dx \right]^{N_i} \longrightarrow \langle N_1^{i_1} N_2^{i_2}, ..., N_n^{i_n} \rangle = \frac{\partial^{(i_1+i_2+...+i_n)} M_N(h_1, h_2, ..., h_n)}{\partial h_1^{i_1} \partial h_2^{i_2} ... \partial h_n^{i_n}} \Big|_{\vec{h}=0}.$$

→ **<W> are a linear combination of <N>**

- Inversion procedure to get from <W> to <N> implemented for the first time up to 4. order

$$\langle \vec{W} \rangle = A \times \langle \vec{N} \rangle$$

Fully defined by mass fit functions

vector of folded moments — response matrix — vector of true moments

- For full efficiency corr. combine with **moment exp**. or **unfolding (Cornish fisher expansion)**

Idea of Moment Exp.: Establish from the detector response matrix a formal relation between corrected and uncorrected moments

T.Nonaka et. al.Nucl.Instrum.Meth.A 906 (2018) 10-17

For details, see poster session:

573. Fuzzy logic for reconstructing moments of multiplicity distributions

Anar Rustamov, Joachim Stroth, Marvin Nabroth

Marvin Nabroth, QM 2025

Fuzzy logic

A.Rustamov, Phys.Rev.C 110 (2024) 6, 064910

- **Relation between W and N is of linear nature**
  → Translation to regular matrix, inversion possible

$$\begin{pmatrix} \langle N_a^2\rangle \\ \langle N_b^2\rangle \\ \langle N_a N_b\rangle \end{pmatrix} = \begin{pmatrix} \Omega^{a,a}_{a,a} & \Omega^{a,a}_{b,b} & 2\Omega^{a,a}_{a,b} \\ \Omega^{b,b}_{a,a} & \Omega^{b,b}_{b,b} & 2\Omega^{b,b}_{a,b} \\ \Omega^{a,b}_{a,a} & \Omega^{a,b}_{b,b} & \Omega^{a,b}_{P[a,b]} \end{pmatrix}^{-1} \begin{pmatrix} \langle W_a^2\rangle - \sum_{i=a,b}\langle N_i\rangle \kappa_2(\omega_{a;i}) \\ \langle W_b^2\rangle - \sum_{i=a,b}\langle N_i\rangle \kappa_2(\omega_{b;i}) \\ \langle W_a W_b\rangle - \sum_{i=a,b}\langle N_i\rangle \kappa_{11}(\omega_{ab;i}) \end{pmatrix}$$

Calculating matrix elements by implementing permutation functions

$$\Omega^{a,b}_{P[i,j]} = \Omega^{a,b}_{i,j} + \Omega^{a,b}_{j,i},$$

$$\Omega^{Q[(ab),c]}_{P[i,j]} = \Omega^{(ab),c}_{i,j} + \Omega^{(ab),c}_{j,i} + \Omega^{(ac),b}_{i,j} + \Omega^{(ac),b}_{j,i} + \Omega^{(bc),a}_{i,j} + \Omega^{(bc),a}_{j,i}$$

$$Q[(ab),c] \equiv [(ab),c] + [(ac),b] + [(bc),a]$$

$$Q[(ab),c,d] \equiv [(ab),c,d] + [(ac),b,d]$$

$$P[i,i,j] \equiv [i,i,j] + [i,j,i] + [j,i,i]$$

$$\Omega^{a,b,c}_{i,j,k} = \kappa_1(\omega_{a;i})\kappa_1(\omega_{b;j})\kappa_1(\omega_{c;k}) \ ,$$

$$\Omega^{a,b,c,d}_{i,j,k,l} = \kappa_1(\omega_{a;i})\kappa_1(\omega_{b;j})\kappa_1(\omega_{c;k})\kappa_1$$

Linear coefficients depended on lineshape properties only

Implemented inversion procedure up to 4th order