

E-by-E multiplicity fluctuations from HADES

MARVIN NABROTH
FOR THE HADES COLLABORATION

OUTLINE

■ MOTIVATION

■ ANALYSIS PROCEDURE

RESULTS

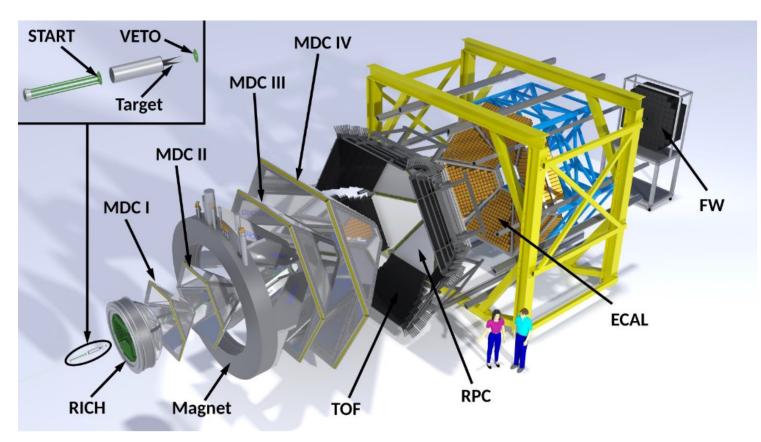
■ SUMMARY AND OUTLOOK



HADES

High-Acceptance-Di-Electron-Spectrometer

- ☐ Fixed target experiment at SIS-18
- ☐ Momentum reco. based on toroidal magnetic spectroscopy (MDCs and Magnet),
- ☐ Time-of-flight from START, RPC and TOF
- ☐ Energy-loss measurement from MDC and TOF
- ☐ ECAL and hadron-blind RICH detector
- ☐ Forward Wall for projectile spectator measurement
- ❖ Almost full azimuthal coverage
- Polar angle coverage between 18° and 85°



- ☐ New refined simulations with time-differential treatment of Delta-Electrons
- ☐ Up to about 10 % more yield for charged tracks around mid-rapidity

Reminder SIS18/Bevalac energies

Center-of-mass projectile/target velocities < 0.5 c

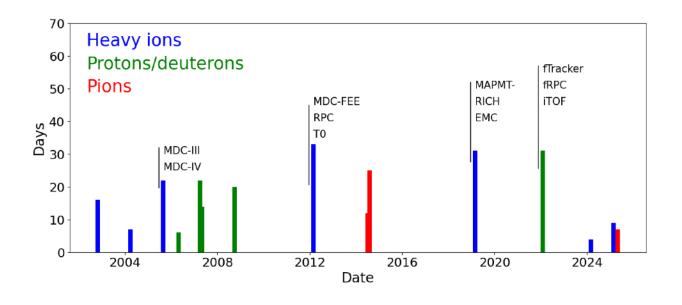
Reduced sensitivity on centrality because of moderate particle multiplicities

Charged-particle multiplicities largely due to stopped baryons (nucleons)

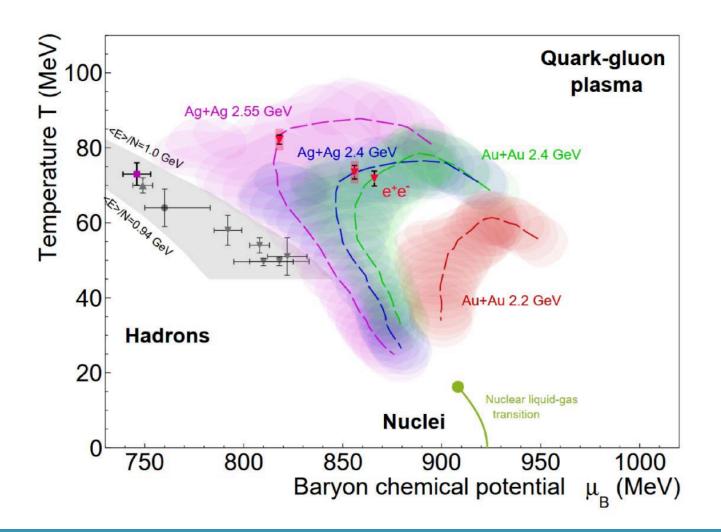
Moderate space-momentum correlation [2201.08486, 2404.00476]

HADES data sets:

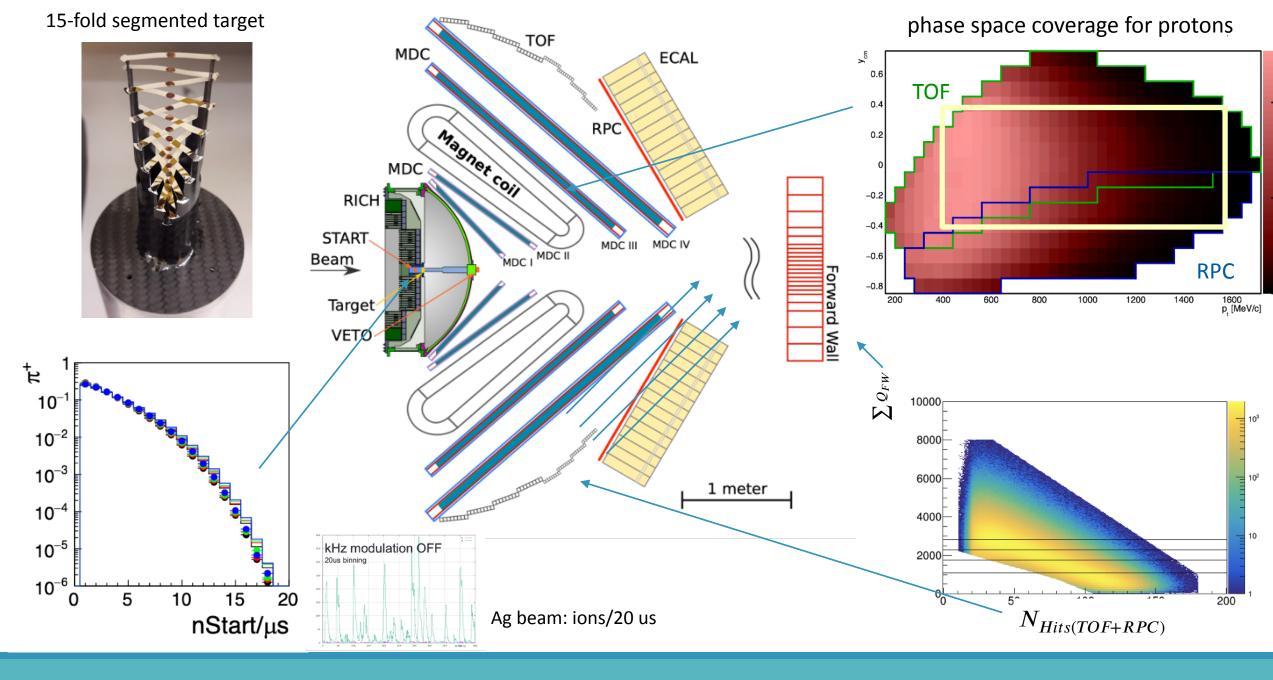
Au+Au	$E_{\rm kin} = 1.23 A {\rm GeV}$	(2012)
Ag+Ag	$E_{\rm kin} = 1.23 A {\rm GeV}$	(2019)
Ag+Ag	$E_{\rm kin} = 1.58 A {\rm GeV}$	(2019)
Au+Au	$E_{\rm kin} = 0.8 A {\rm GeV}$	(2019)



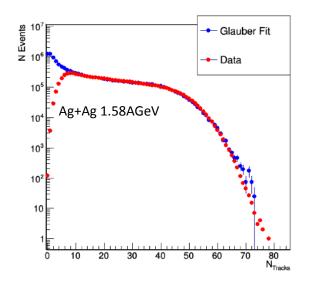
Trajectories from Coarse-grained UrQMD

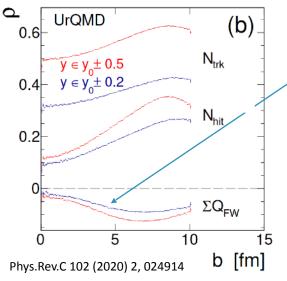


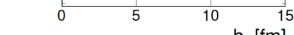
FO curve: J. Cleymans, K. Redlich, Nucl. Phys. A 661 (1999) 379 Au+Au 2.4 GeV data: HADES, Nature Phys. 15(2019) 1040 Eur.Phys.J.A 52 (2016) 5, 131 Phys.Rev.C 106 (2022) 1, 014904 Ag+Ag data: HADES preliminary figure: F.Seck, T.Galatyuk



Centrality Selection for fluctuation analysis



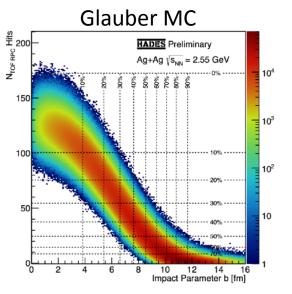


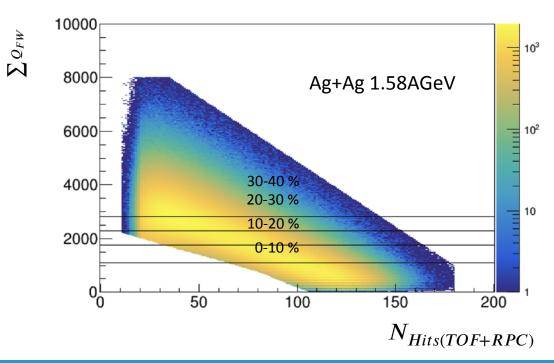


- ☐ Glauber x Neg. Binomial to **charged tracks** and **detector hits**
- ☐ Event selection corresponds to around 55 % most central events
- ☐ For fluctuation analysis, do further centrality binning based on FW signal
- ☐ Neg. Binomial for charged tracks at HADES is close to Poisson
 - reduce autocorrelation to protons

Smallest correlation with protons

Forward Wall Signal

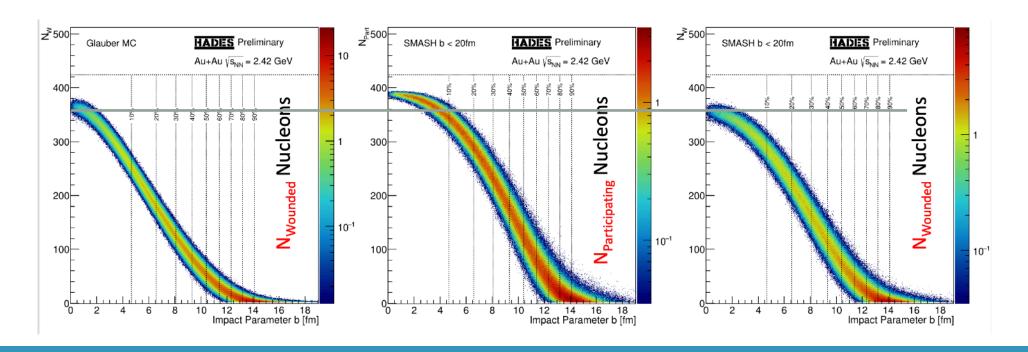




The challenge of reducing systematic uncertainties

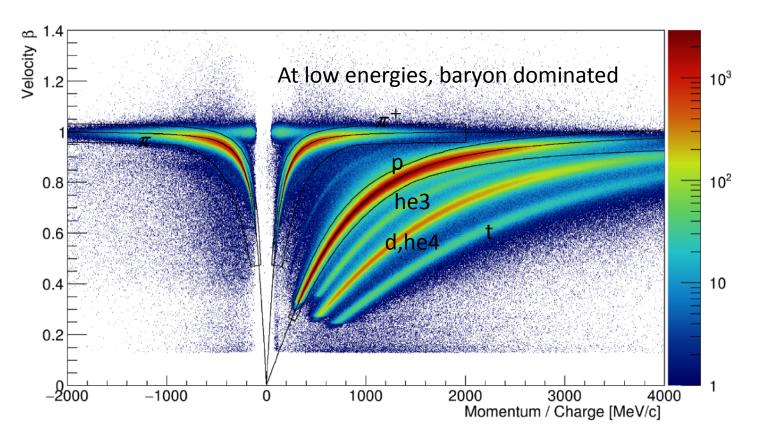
HADES has initiated a reprocessing of data – updated results were shown on QM2025:

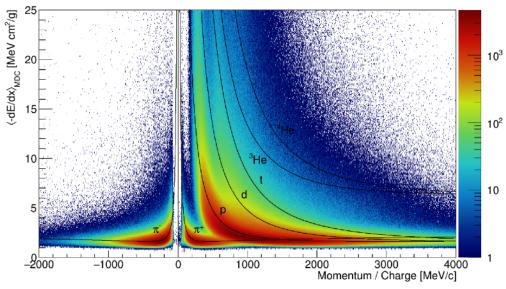
- Refined efficiency models using multi-hit information of T0 detector
- Further rejection of critical pile-up effects
- $^{\circ}$ HADES uses Glauber-MC which is comparable with Transport only if inelastic collisions are counted, i.e. $N_{\mathrm{wounded}} \equiv \mathrm{inel}$. only $N_{\mathrm{participant}} \equiv \mathrm{el}$. & inel .



PID

Particle identification at HADES



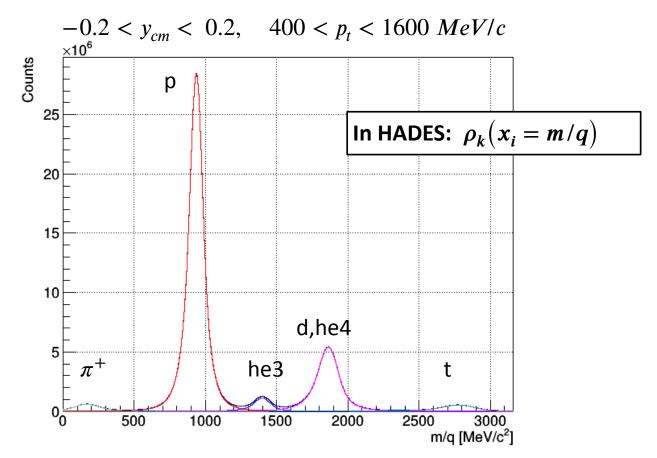


- → MDCs provide measurement of specific energy loss
- ☐ In deuteron analysis used as preselection

- \square PID via **time of flight** (β) and **momentum** measurement \rightarrow **m/** α
- ☐ Time-of-Flight measurement provides good separation

Correcting moments for mis-identification - Fuzzy Logic

Ag+Ag 1.58A GeV



☐ For deuterons, increase separation power by preselection based on dE/dx signal

- ☐ Traditional approach: Hard cuts
- ➤ Identity method/Fuzzy logic → Assign PID observable with degree of membership to different particles

$$\omega_j(x_i) = rac{
ho_j(x_i)}{\sum_j
ho_j(x_i)}$$



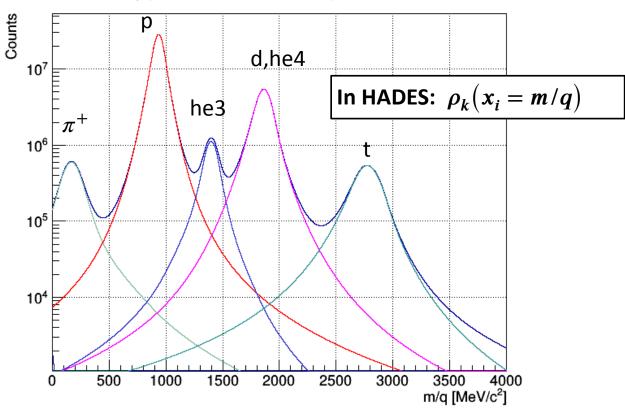
Event\Proxy quantities

$$W_k = \sum_{i=1}^{N_{tracks}} \omega_k(x_i)$$

Correcting moments for mis-identification - Fuzzy Logic

Ag+Ag 1.58A GeV

$$-0.2 < y_{cm} < 0.2$$
, $400 < p_t < 1600 MeV/c$



☐ For deuterons, increase separation power by preselection based on dE/dx signal

- ☐ Traditional approach: Hard cuts
- ➤ Identity method/Fuzzy logic → Assign PID observable with degree of membership to different particles

$$\omega_j(x_i) = \frac{\rho_j(x_i)}{\sum_j \rho_j(x_i)}$$

Event\Proxy quantities

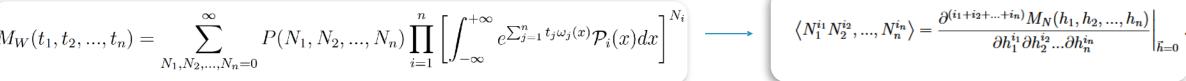
$$W_k = \sum_{i=1}^{N_{tracks}} \omega_k(x_i)$$

Correcting moments for mis-identification - Fuzzy Logic

A. Rustamov derived generalized relation between moments of W and N

Phys.Rev.C 110 (2024) 6

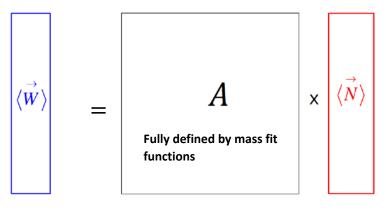
$$M_W(t_1, t_2, ..., t_n) = \sum_{N_1, N_2, ..., N_n = 0}^{\infty} P(N_1, N_2, ..., N_n) \prod_{i=1}^{n} \left[\int_{-\infty}^{+\infty} e^{\sum_{j=1}^{n} t_j \omega_j(x)} \mathcal{P}_i(x) dx \right]^{N_i}$$



☐ Inversion procedure to get from <W> to <N> implemented for the first time up to 4. order

For details, see poster session:

573. Fuzzy logic for reconstructing moments of multiplicity distributions Anar Rustamov, Joachim Stroth, Marvin Nabroth



vector of folded moments

response matrix

vector of true moments

- <W> are a linear combination of <N>
- For full efficiency corr. combine with **moment** exp. or unfolding (Cornish fisher expansion)

Idea of Moment Exp.: Establish from the detector response matrix a formal relation between corrected and uncorrected moments

T.Nonaka et. al., Nucl.Instrum.Meth.A 906 (2018) 10-17

Fuzzy logic

A.Rustamov, Phys.Rev.C 110 (2024) 6, 064910

- Relation between W and N is of linear nature
 - → Translation to regular matrix, inversion possible

$\begin{pmatrix} \langle N_a^2 \rangle \\ \langle N_b^2 \rangle \\ \langle N_a N_b \rangle \end{pmatrix} = \begin{pmatrix} \Omega_{a,a}^{a,a} & \Omega_{b,b}^{a,a} & 2\Omega_{a,b}^{a,a} \\ \Omega_{a,a}^{b,b} & \Omega_{b,b}^{b,b} & 2\Omega_{a,b}^{b,b} \\ \Omega_{a,a}^{a,b} & \Omega_{b,b}^{a,b} & \Omega_{P[a,b]}^{a,b} \end{pmatrix}^{-1} \begin{pmatrix} \langle W_a^2 \rangle - \sum_{i=a,b} \langle N_i \rangle \kappa_2(\omega_{a;i}) \\ \langle W_b^2 \rangle - \sum_{i=a,b} \langle N_i \rangle \kappa_2(\omega_{b;i}) \\ \langle W_a W_b \rangle - \sum_{i=a,b} \langle N_i \rangle \kappa_{11}(\omega_{ab;i}) \end{pmatrix}$

Calculating matrix elements by implementing permutation functions

$$\Omega_{P[i,j]}^{Q[(ab),c]} = \Omega_{i,j}^{(ab),c} + \Omega_{j,i}^{(ab),c} + \Omega_{i,j}^{(ac),b} + \Omega_{j,i}^{(ac),b} + \Omega_{j,i}^{(bc),a} + \Omega_{j,i}^{(bc),a} + \Omega_{j,i}^{(bc),a}$$

$$Q[(ab), c] \equiv [(ab), c] + [(ac), b] + [(bc), a]$$

$$Q[(ab), c, d] \equiv [(ab), c, d] + [(ac), b, d]$$

$$P[i, i, j] \equiv [i, i, j] + [i, j, i] + [j, i, i]$$

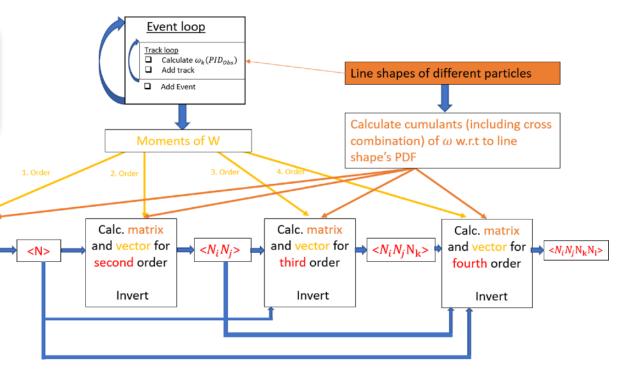
 $\Omega_{P[i,j]}^{a,b} = \Omega_{i,j}^{a,b} + \Omega_{i,i}^{a,b},$

$$\Omega_{i,j,k}^{a,b,c} = \kappa_1(\omega_{a;i})\kappa_1(\omega_{b;j})\kappa_1(\omega_{c;k}) ,$$

$$\Omega_{i,j,k,l}^{a,b,c,d} = \kappa_1(\omega_{a;i})\kappa_1(\omega_{b;j})\kappa_1(\omega_{c;k})\kappa_1$$

Linear coefficients depended on lineshape properties only

Implemented inversion procedure up to 4th order



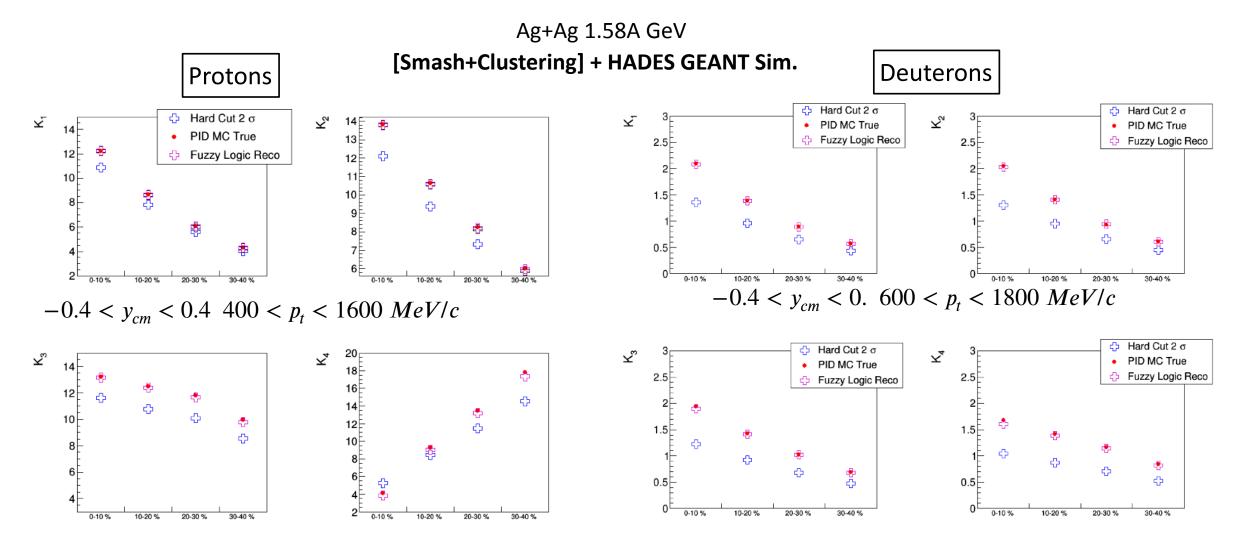
Calc. matrix

and vector for

first order

Invert

Fuzzy Logic - Performance study in simulation

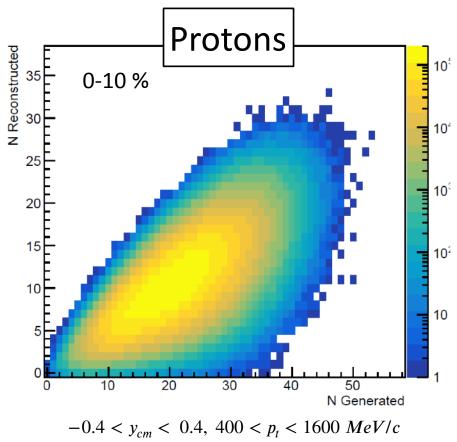


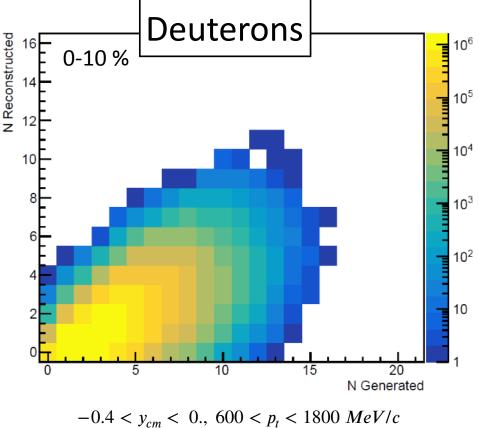
☐ Good agreement with MC truth, benefit from fuzzy logic compared to hard cuts especially visible for deuterons

Detection efficiency

Efficiency corrections - Response matrices

 $N_{generated}$ vs $N_{reconstructed}$ (requires good response simulation / digitisers)



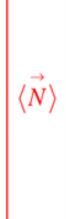


 $0.1 < y_{cm} < 0., 000 < p_1 < 1000 MeV$

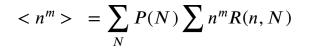
☐ 40-50 % average efficiency, non-binomial shape towards tails

Fuzzy-Logic + Efficiency correction

"PID" corrected moments from Fuzzy Logic/Identity Method



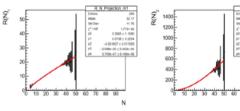
Moment Expansion



$$R(N)_m = \sum_N r_{mj} N^j$$

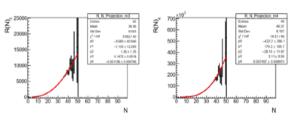
➤ Idea: Establish from the response matrix a formal relation between corrected and uncorrected moments

- True moments are encoded in column-wise moments $R_{m(N)}$
- Perform series expansion



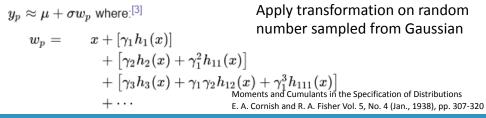
Requires for some bins 6 to 8 expansion terms \rightarrow Also required from fuzzy logic as input! (Poissoninan extrapolation possible for higher order)

T.Nonaka et. al.Nucl.Instrum.Meth.A 906 (2018) 10-17

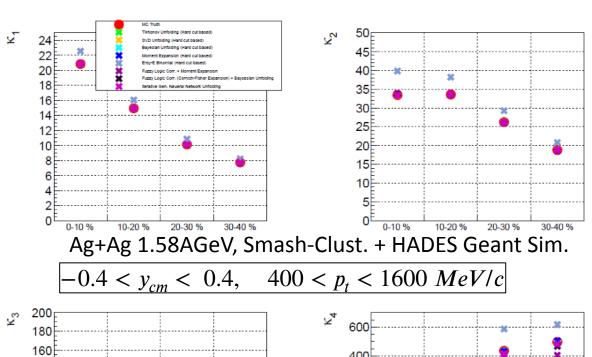


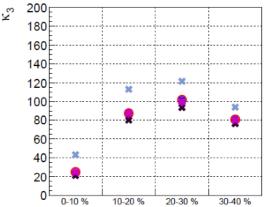
Cornish Fisher Expansion + Distribution Unfolding

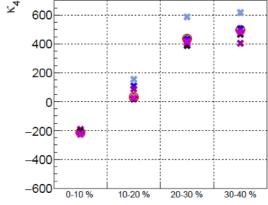
- Generate distribution having cumulants from fuzzy logic reconstruction
- ➤ Afterwards perform distribution unfolding → e.g. Bayesian Unfolding

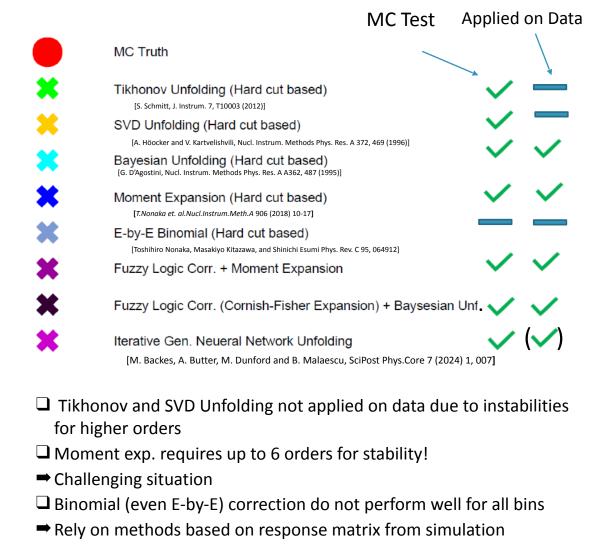


Efficiency corrections: Validation in simulation









Volume fluctuations

Volume Correction

- How to describe contribution of volume fluctuations in observed particle cumulants?
- Model with simplest assumption:
 - Sources of particle production are statistical independent
 - → Moment generating function factorizes [*][**]

$$M_N(t) = \left[M_N(t)\right]^{N_{src}}$$

$$\langle N^m(N_{src})\rangle = \frac{d^m}{dt^m}[M_N(t)]$$

$$k_m(N) := Oberserved particle cumulants$$

$$k_m(n) := Single source particle cumualnts$$

$$k_m(N_{src}) := Cumulants \ of \ sources \setminus participants$$

$$k_1(N) = \langle N_{Src} \rangle \langle n \rangle$$

$$k_2(N) = \langle N_{Src} \rangle k_2(n) + \langle n \rangle^2 k_2(N_{src})$$

$$k_3(N) = \langle N_{Src} \rangle k_3(n) + 3 \langle n \rangle k_2(n) k_2(N_{src}) + \langle n \rangle^3 k_3(N_{src})$$

$$k_4(N) = \langle N_{src} \rangle k_4(n) + 4 \langle n \rangle k_3(n) k_2(N_{src}) + 3k_2^2(n) k_2(N_{src}) + 6 \langle n \rangle^2 k_2(n) k_3(N_{src}) + \langle n \rangle^4 k_4(N_{src})$$

- V. Skokov, B. Friman, K. Redlich, Volume Fluctuations and Higher Order Cumulants of the Net Baryon Number, Phys. Rev. C 88 (2013) 034911. arXiv:1205.4756, doi:10.1103/PhysRevC.88.034911
- Bridging the gap between event-by-event fluctuation measurements and theory predictions in relativistic nuclear collisions
 P. Braun-Munzinger (Darmstadt, EMMI and Heidelberg U.), A. Rustamov (Heidelberg U. and NNRC, Baku), J. Stachel (Heidelberg U.) Nucl.Phys.A 960 (2017), 114-130

Volume Correction – Data driven approach

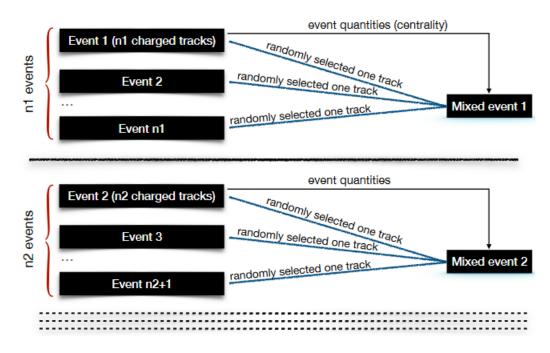


Figure 1: The strategy for event mixing used to remove correlations between particles while preserving participant fluctuations.

☐ Event mixing scheme preserves vol. fluctuations

A model-free procedure to correct for volume fluctuations in E-by-E analyses of particle multiplicities

Anar Rustamov, Joachim Stroth, Romain Holzmann

Nucl.Phys.A 1034 (2023), 122641

- ☐ Event mixing removes correlations between particles
- ➤ In case of **Poisson like behaviour** one expects for the emission per sources:

$$k_m(n) = \langle n \rangle , \quad cov(n_1, n_2) = 0$$

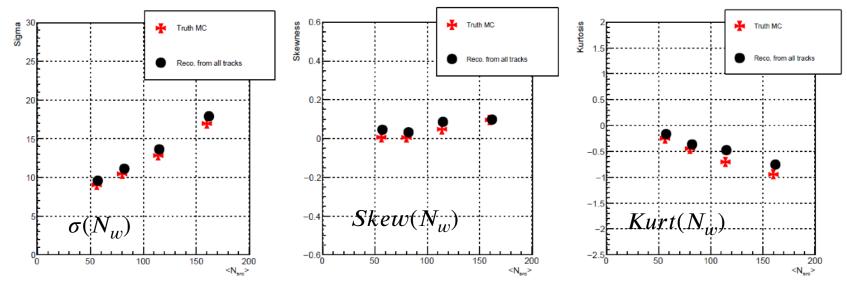
☐ Event mixing is equivalent to calculations based on charged tracks

$$\begin{split} \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} &= \frac{C_2[M] - \bar{C}_2[M]}{\langle M \rangle^2} \\ \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} &= -3 \frac{\bar{C}_2[M]}{\langle M \rangle^2} \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \frac{C_3[M] - \bar{C}_3[M]}{\langle M \rangle^3} \\ \frac{\kappa_4[N_w]}{\langle N_w \rangle^4} &= -6 \frac{\bar{C}_2[M]}{\langle M \rangle^2} \frac{\kappa_3[N_w]}{\langle N_w \rangle^3} - \frac{4\bar{C}_3[M]\langle M \rangle + 3\bar{C}_2[M]^2}{\langle M \rangle^4} \frac{\kappa_2[N_w]}{\langle N_w \rangle^2} + \frac{C_4[M] - \bar{C}_4[M]}{\langle M \rangle^4} \end{split}$$

Controlling volume fluctuations for studies of critical phenomena in nuclear collisions **Romain Holzmann, Volker Koch, Anar Rustamov, Joachim Stroth** e-Print: 2403.03598 [nucl-th]

Volume Correction — Data driven approach

☐ Reconstruction of volume cumulants using mixed particles \ charged tracks in Glauber toy model:



But, is nature at low energies really like Glauber?

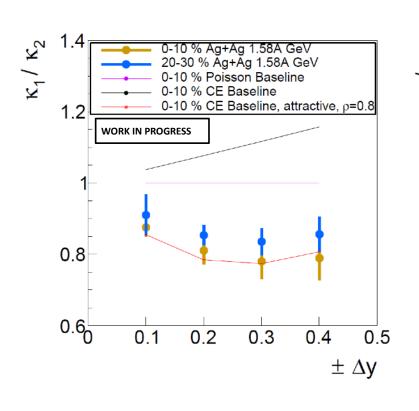
- ☐ Interaction of particles with spectators
- At low energies most of the protons are not produced, but rescattered → protons are significant part of the volume!
- ☐ In transport models, for Apart, assumption of independent emission violated for protons!

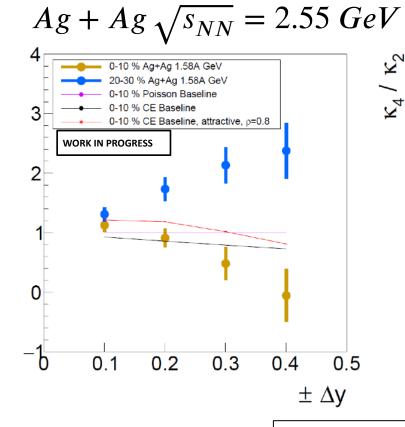
- ☐ In HADES: Negative Binomial toy model method exhibits good agreement with MC Truth volume → Negative Binomial close to Poisson
- ☐ For real data only use mixed-protons due to better understanding of eff. corrections for protons

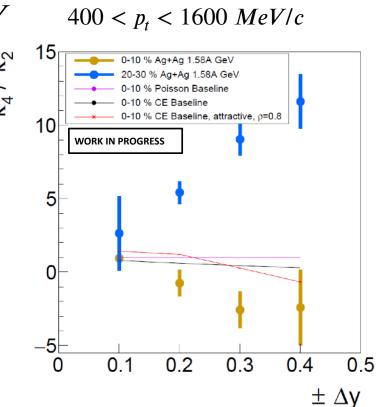
Event mixing still so far the best method available, as fully data driven, only independent source assumption!

Proton results for Ag+Ag

Eff. + Volume corrected – proton cumulant ratios



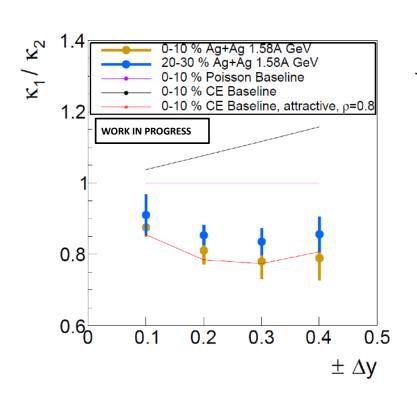


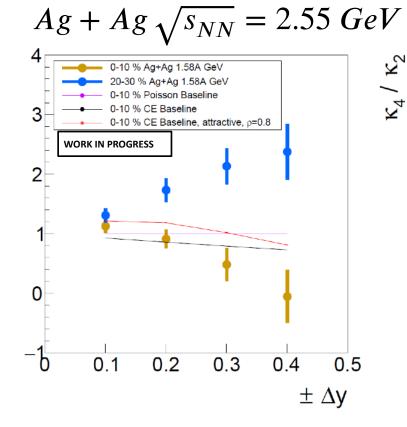


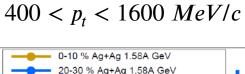
- ☐ In general convergence towards poisson limit (1) observed
- \blacksquare κ_3 now mainly positive as opposed to old HADES Au+Au data
- ☐ For higher order, different trend between central and semi-central events → Influence of spectators?
- \Box CE baseline considering acceptance window only can not describe data, different trend for κ_1/κ_2
- ☐ Trend of rapidity dependence can be described by Canonical baseline considering correlations with an attractive potential (P.

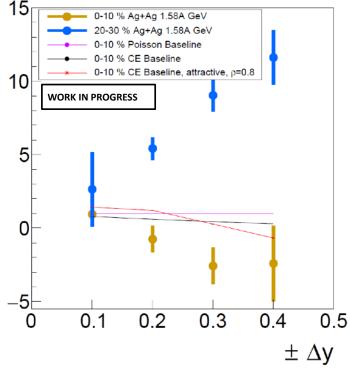
Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113

Eff. + Volume corrected – proton cumulant ratios



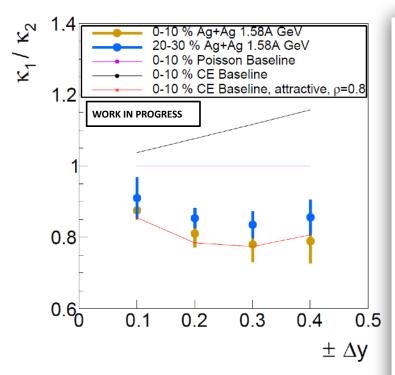






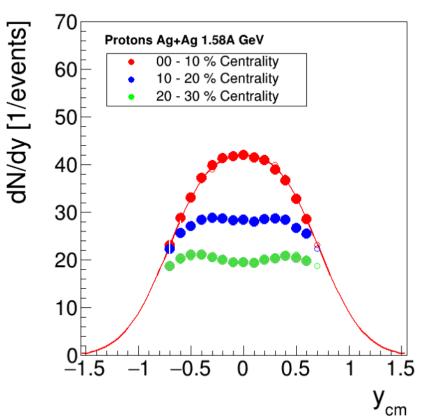
- ☐ In general convergence towards poisson limit (1) observed
- \mathbf{a} κ_3 now mainly positive as opposed to old HADES Au+Au data
- ☐ For higher order, different trend between central and semi-central events → Influence of spectators?
- ☐ Trend of rapidity dependence can be described by Canonical baseline considering correlations with an attractive potential (P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113)
- \square κ_1/κ_2 matched by CE model within uncertainties

Eff. + Volume corrected – proton cumulant ratios



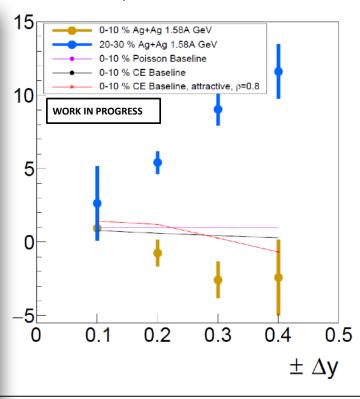
- ☐ In general convergence towards poi
- \mathbf{x}_3 now mainly positive as opposed
- ☐ For higher order, different trend be events → Influence of spectators

 $Ag + Ag\sqrt{s_{NN}} = 2.55 \; GeV$



Different pattern also apparent in rapidity spectra for semi-central events

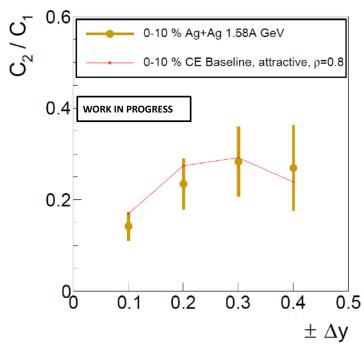
 $400 < p_t < 1600 \; MeV/c$



y dependence can be described by Canonical ering correlations with an attractive potential K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113) by CE model within uncertainties

Eff. + Volume corrected – proton factorial cumulant ratios

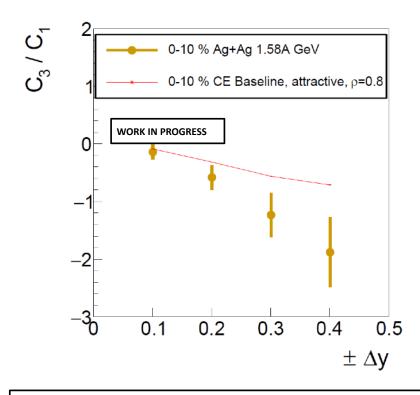
"Measure for multi-particle-correlations"

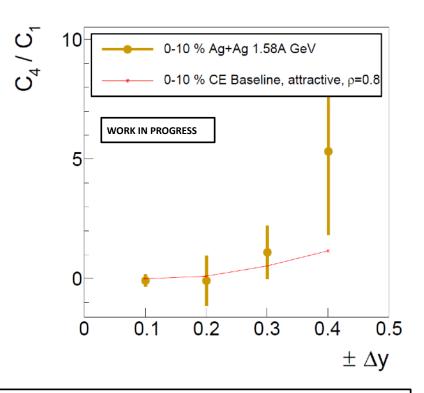


$$C_{2} = \kappa_{2} - \kappa_{1}$$

$$C_{3} = \kappa_{3} - 3\kappa_{2} + 2\kappa_{1}$$

$$C_{4} = \kappa_{4} - 6\kappa_{3} + 11\kappa_{2} - 6\kappa_{1}$$





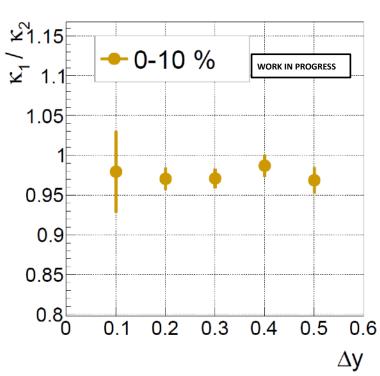
☐ Also trend of factorial cumulant ratios as a function of rapidity can be described by Canonical baseline considering correlations with an attractive potential

[P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113]

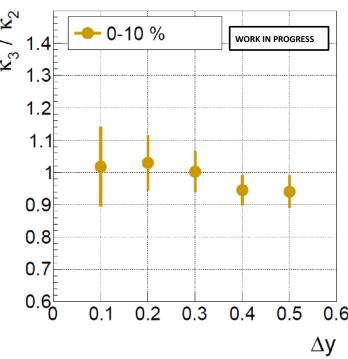
☐ Measurement matched by CE model within uncertainties

Deuteron results for Ag+Ag

Eff. + Volume corrected – deuteron cumulant ratios

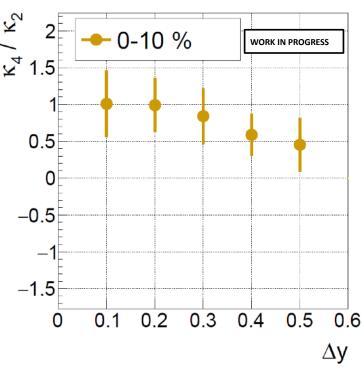


$$Ag + Ag\sqrt{s_{NN}} = 2.55 \; GeV$$



- Identity method allows to perform deuteron cumulant analysis
- ☐ Ratios are closer to poisson limit compared to protons
 - \rightarrow smaller phase space coverage ($-0.4 < y_{cm} < 0.1$)
- ☐ Need to establish proper CE baseline

$$600 < p_t < 1800 \frac{MeV}{c}$$



$$\Delta y = 0.1$$
: $-0.1 < y_{cm} < 0.0$

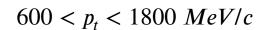
$$\Delta y = 0.4$$
: $-0.4 < y_{cm} < 0.0$
 $\Delta y = 0.5$: $-0.4 < y_{cm} < 0.1$

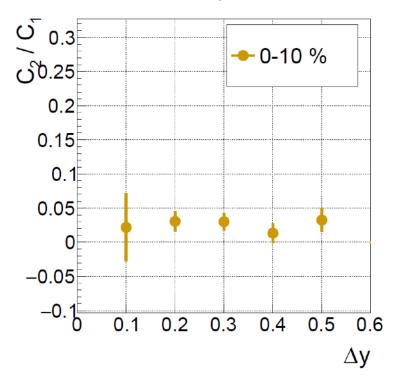
Eff. + Volume corrected – deuteron cumulant ratios

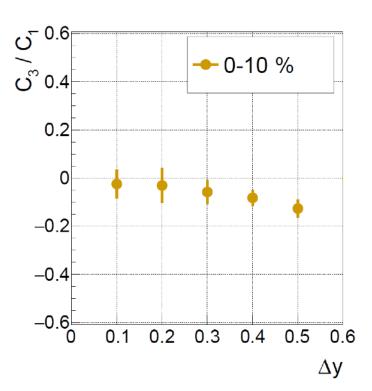
$$C_2 = \kappa_2 - \kappa_1,$$

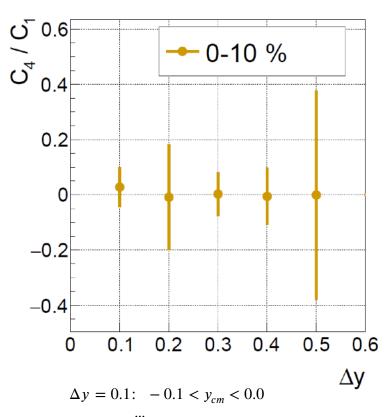
 $C_3 = \kappa_3 - 3\kappa_2 + 2\kappa_1,$
 $C_4 = \kappa_4 - 6\kappa_3 + 11\kappa_2 - 6\kappa_1$

$$Ag + Ag\sqrt{s_{NN}} = 2.55 \; GeV$$









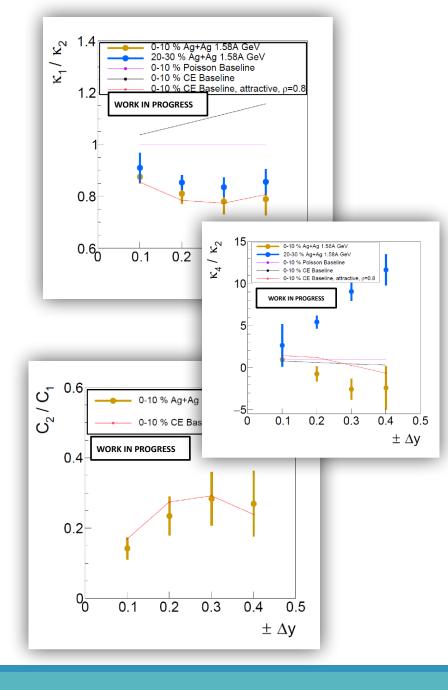
- Identity method allows to perform deuteron cumulant analysis
- Need to establish proper CE baseline

$$\Delta y = 0.4: -0.4 < y_{cm} < 0.0$$

$$\Delta y = 0.5: -0.4 < y_{cm} < 0.1$$

Summary and Outlook

- New refined efficiency corrections in HADES
- ☐ Fuzzy logic/Identity method for higher-orders to correct for particle mis-identification
- ☐ Full efficiency correction based on unfolding and moment expansion techniques
- Volume correction using mixed events
- \blacksquare Presented eff. and vol. corrected cumulants and factorial cumulants of protons and deuterons for $A\,g+A\,g\,\sqrt{s_{NN}}=2$. 55 GeV
- ☐ Trend of rapidity dependence of (factorial) cumulant ratios can be described by Canonical baseline considering correlations with an attractive potential
- Outlook
 - ◆ (Re)analysis of other HADES data ongoing (Au+Au 1.23A GeV, Ag+Ag 1.58A GeV, Au+Au 0.8A GeV)
 - p-d joint-cumulant analysis ongoing
 - ◆ Extend cumulants analysis to t and He3, establish baseline
 - ◆ Further investigate and refine volume correction method



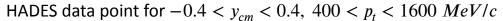


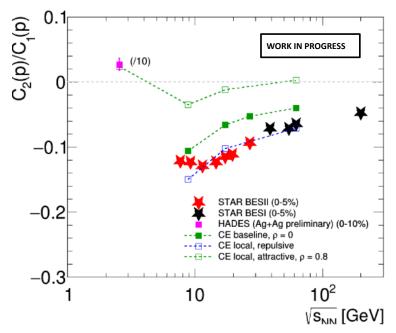


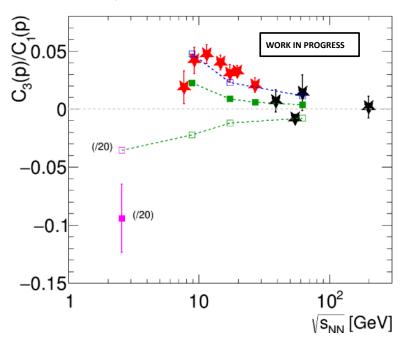
Thank you for your attention! THE HADES COLLABORATION

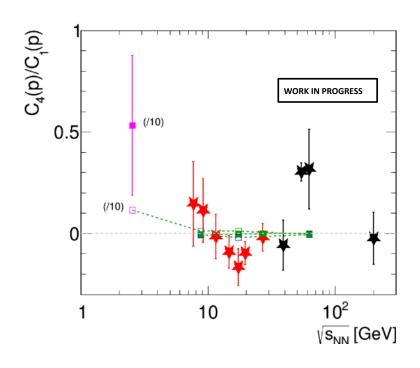
Back-Up

Proton Factorial Cumulants - Comparison with STAR



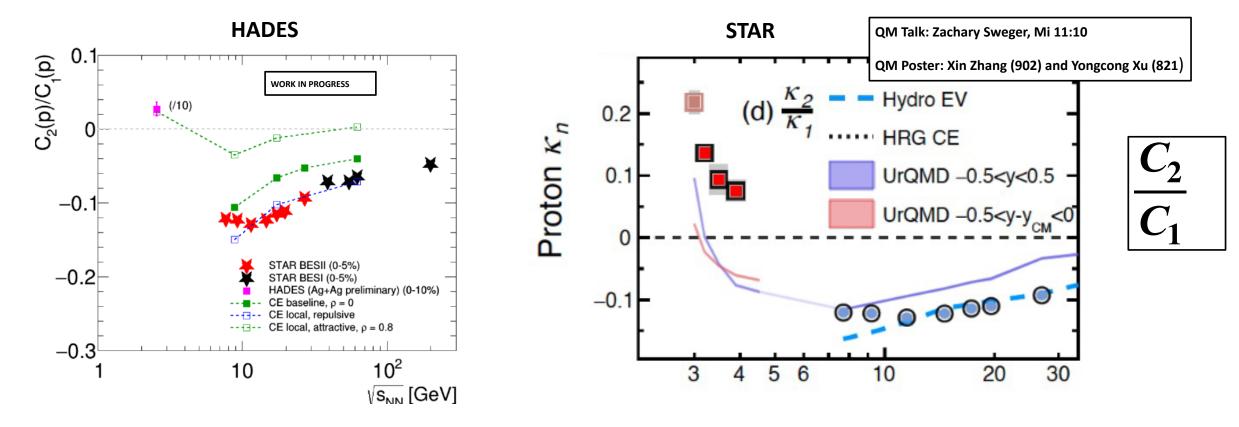






- ☐ In general, larger absolute factorial cumulant ratios at HADES energies compared to STAR points at higher energies ☐ For C_0/C_1 and C_0/C_2 . HADES points roughly continue trend
- \square For C_3/C_1 and C_4/C_1 , HADES points roughly continue trend observed at START towards lower energies
- Trend of **HADES** point described by correlated CE model with **attractive potential** [P. Braun-Munzinger, K. Redlich, A. Rustamov, J. Stachel, JHEP 08 (2024) 113]
- STAR points described by repulsive potential [B. Friman, A. Rustamov, K. Redlich (in progress)]
 - → Interplay between repulsive and attractive forces

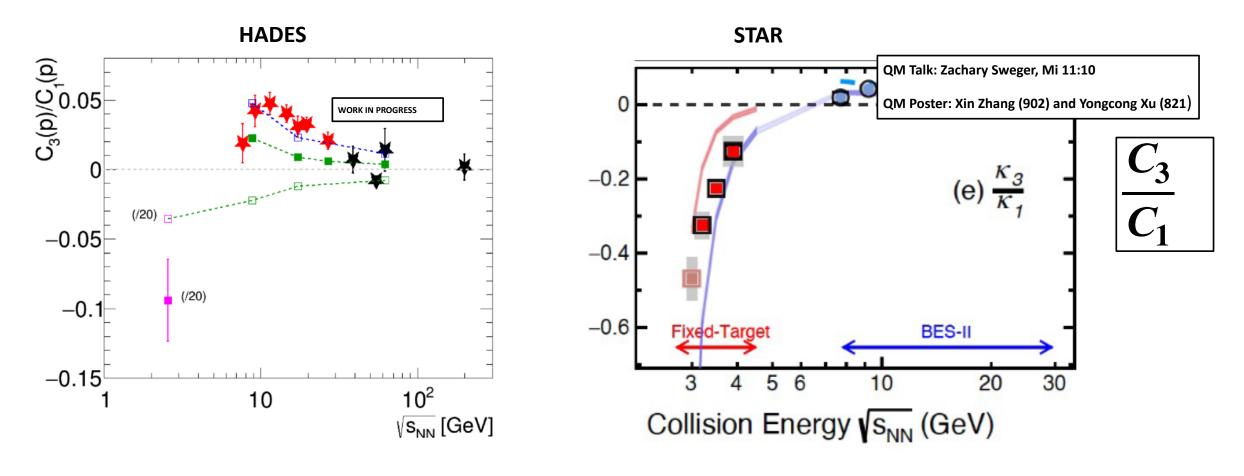
Proton Factorial Cumulants - Comparison with STAR at lower energies



- ☐ Also STAR measured positive C2/C1 for their lower energy points, values are similar to HADES
- → Towards lower energies transition to clustering

Proton Factorial Cumulants

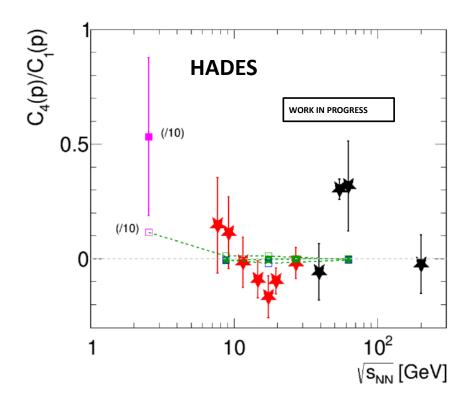
- Comparison with STAR at lower energies

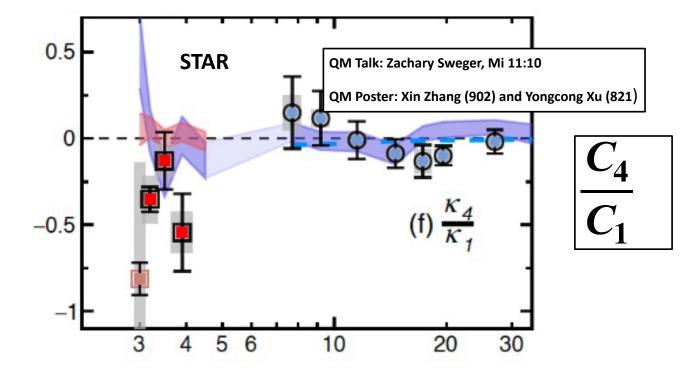


 \square Negative C3/C2 for lower energy points also observed by STAR \rightarrow In trend of HADES, but smaller absolute values in STAR

Proton Factorial Cumulants

- Comparison with STAR at lower energies





□Different sign for C4/C2

Potential reasons for deviations

- □Different collisions systems are compared, Ag+Ag (HADES) vs. Au+Au (STAR)
- □Not the same analysis ranges in pt and y
- $oldsymbol{\square}$ Remnants of volume fluctuations? Different methods: Event mixing vs. CBWC
- □..

Machine Learning based unfolding - Basics

$$t_{MC}(x) = Prior \ from \ MC \ Simulation$$

 $t_{Experiment}(x) = Prior \ from \ Experiment$

$$d(y) = \int p(y|x)t(x) dx$$
Bayse' Theorem
$$p(x|y) = \frac{p(y|x)t(x)y}{d(y)}(y,x) = p(y|x)$$

$$p(y|x) \text{ from HADES detector simulation}$$

- > Dependence on Prior!
- > Eliminate bias from MC prior by an iterative approach
 - ➤ 1. Start with prior from MC or any arbitrary prior
 - > 2. Apply detector response, compare with experimental measurement
 - > 3. Reweight distribution

Iterative Bayesian Unfolding

Disadvantages

- > Only works with binned data
- > Requires sufficient statistics
- > Unfolding of only one observable possible
- ➤ In recent years and month there has been development of brand new ML based unfolding techniques

The Landscape of Unfolding with Machine Learning

Nathan Huetsch, Javier Mariño Villadamigo, Alexander Shmakov, Sascha Diefenbacher, Vinicius Mikuni, Theo Heimel, Michael Fenton, Kevin Greif, Benjamin Nachman, Daniel Whiteson, Anja Butter, and Tilman Plehn,

arXiv:2404.18807v2 [hep-ph] 17 May 2024

Reweighting - Omnifold

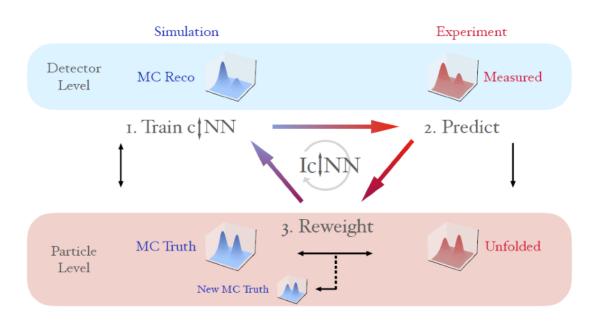
OmniFold: A Method to Simultaneously Unfold All Observables

Anders Andreassen,... Patrick T. Komiske, Eric M. Metodiev, Benjamin Nachman, and Jesse Thaler. arXiv:1911.09107v2 [hep-ph] 16.4pr 2020

Generative Unfolding

An unfolding method based on conditional Invertible Neural Networks (cINN) using iterative training Mathias Backesi, Anja Butter, Monica Dunford, and Bogdan Malaescu arXiv:2212.08674v3 [hep-ph] 10 Jan 2024

Iterative neural network unfolding



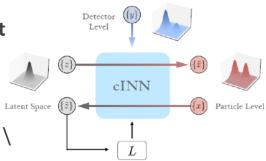
An unfolding method based on conditional Invertible Neural Networks (cINN) using iterative training Mathias Backes1, Anja Butter2,3, Monica Dunford1, and Bogdan Malaescu2 arXiv:2212.08674v3 [hep-ph] 10 Jan 2024

- Train generative neural network \rightarrow Learn $p(N_{Unfolded} | N_{reco})$
- ➤ After training predict for given measurement physics level distribution
- Train classifier to predict difference between predicted and initial MC truth
 - Reweight MC truth for next iteration

Generative neural network

➤ Generate from random noise (latent space) + condition (detector measurement) data samples for the truth\physics level

➤ Implemented as Invertible Network \
Normalizing flow network



Systematics

