Hirschegg, Jan. 20-24, 2025

Distribution of heavy element abundances generated by decay from a quasi-equilibrium state

Gerd Röpke, Rostock
David Blaschke, Wroclaw
Friedrich Konrad Röpke, Heidelberg

Outline

- Heavy element abundances: nuclear statistical equilibrium (NSE) or nuclear reaction networks (NRN)?

- Nuclei: nuclear clusters in a medium (hot and dense matter).
  In-medium effects: quasi-particle self-energy (relativistic mean-field) and Pauli blocking

- Non-equilibrium approach:
  freeze-out concept, Lagrange parameters

- Lagrange parameters for the solar system abundances

Solar system abundances

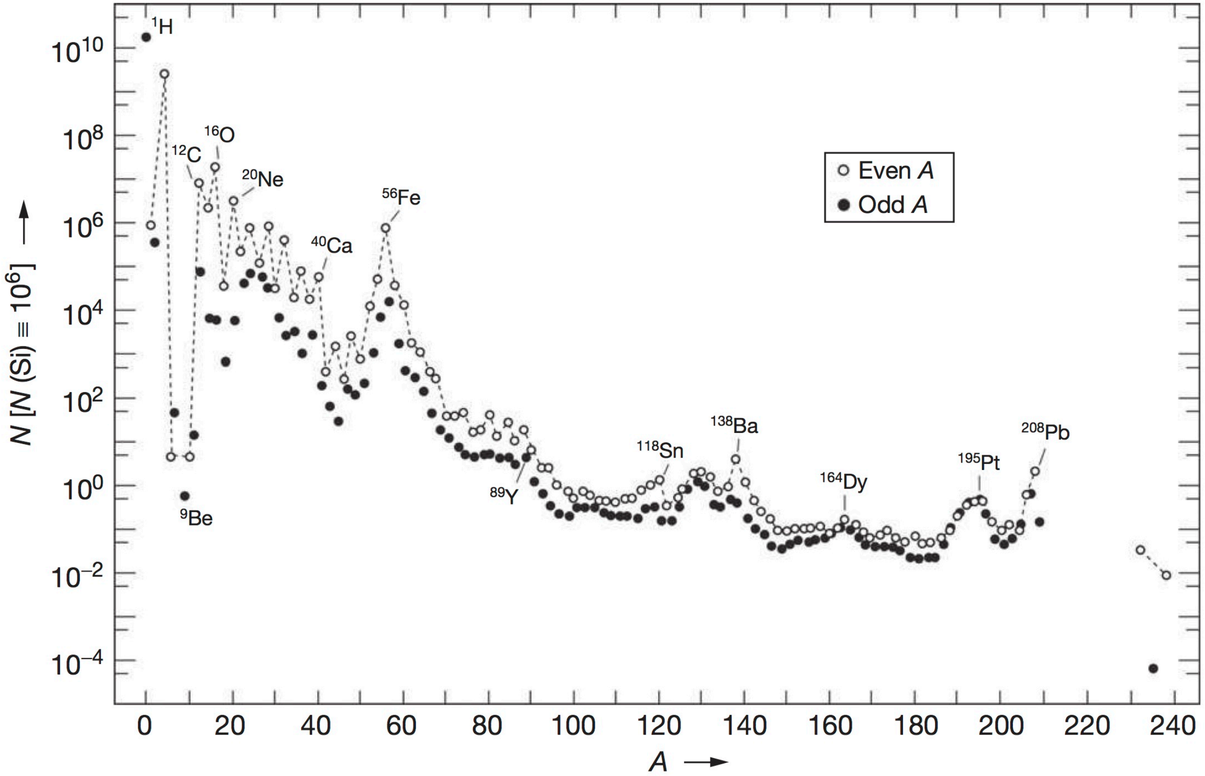
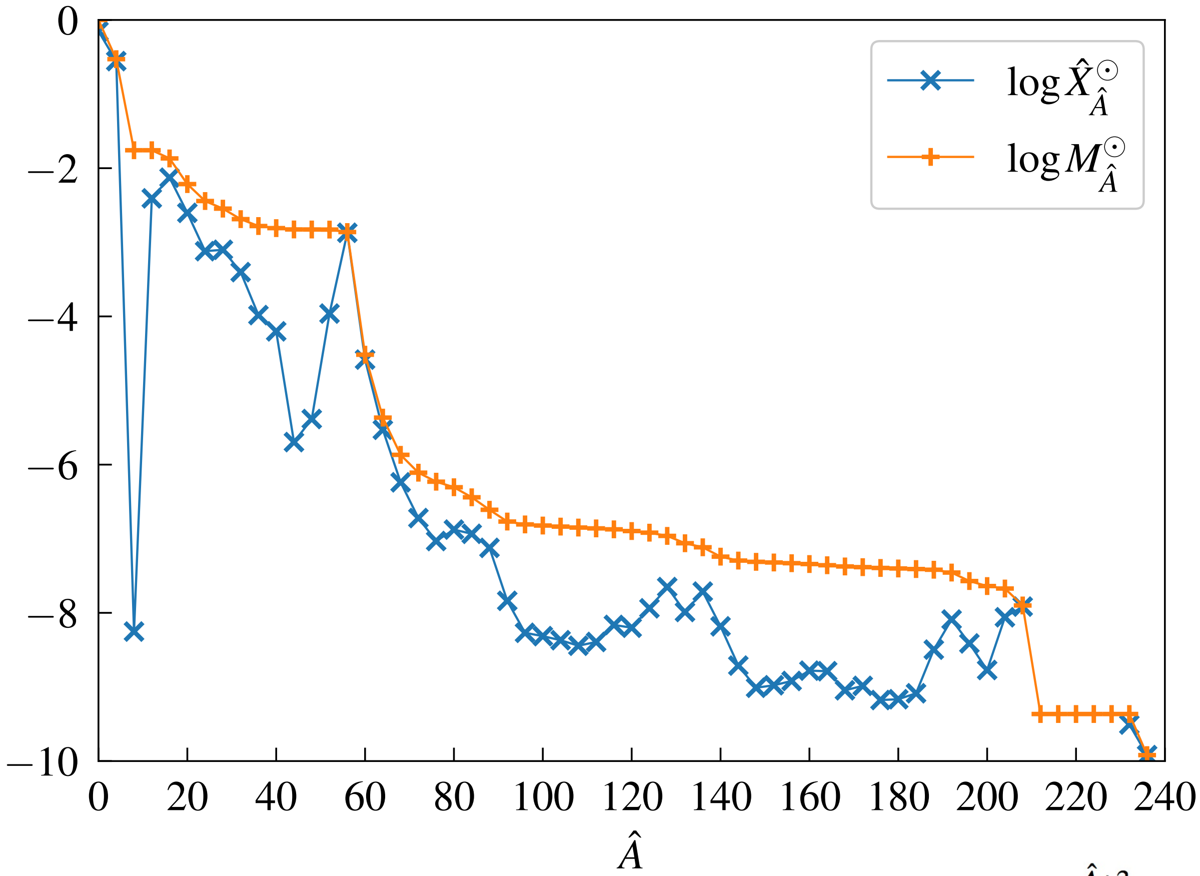


Figure 7 Solar system abundances by mass number. Atoms with even masses are more abundant than those with odd masses (Oddo–Harkins rule). There is no smooth dependence of abundances on mass numbers for even (e.g., ^{118}Sn and ^{138}Ba) or for odd masses (e.g., ^{89}Y).

H. Palme, K. Lodders, A. Jones, Solar System Abundances of the Elements (Elsevier 2014)

Origin of the heavy elements?

Solar system abundances



coarse-grained distribution: accumulated mass fraction $\hat{X}^{\odot}_{\hat{A}} = \frac{1}{n_{\mathrm{B}}} \sum_{A'=\hat{A}}^{\hat{A}+3} A' \sum_{Z,\nu} n_{A',Z,\nu}$

A-metallicity $M^{\odot}_{\hat{A}} = \sum_{\hat{A}' \geq \hat{A}} \hat{X}^{\odot}_{\hat{A}'}.$

The state of matter in simulations of core-collapse supernovae – Reflections and recent developments

Tobias Fischer[1,10], Niels-Uwe Bastian[1], David Blaschke[1,2,3], Mateusz Cierniak[1], Matthias Hempel[4], Thomas Klähn[5], Gabriel Martínez-Pinedo[6,7], William G. Newton[8], Gerd Röpke[3,9] and Stefan Typel[7,6]

Publications of the Astronomical Society of Australia (PASA), Vol. 34, e067, 19 pages (2017).

doi:10.1017/pasa.2017.63

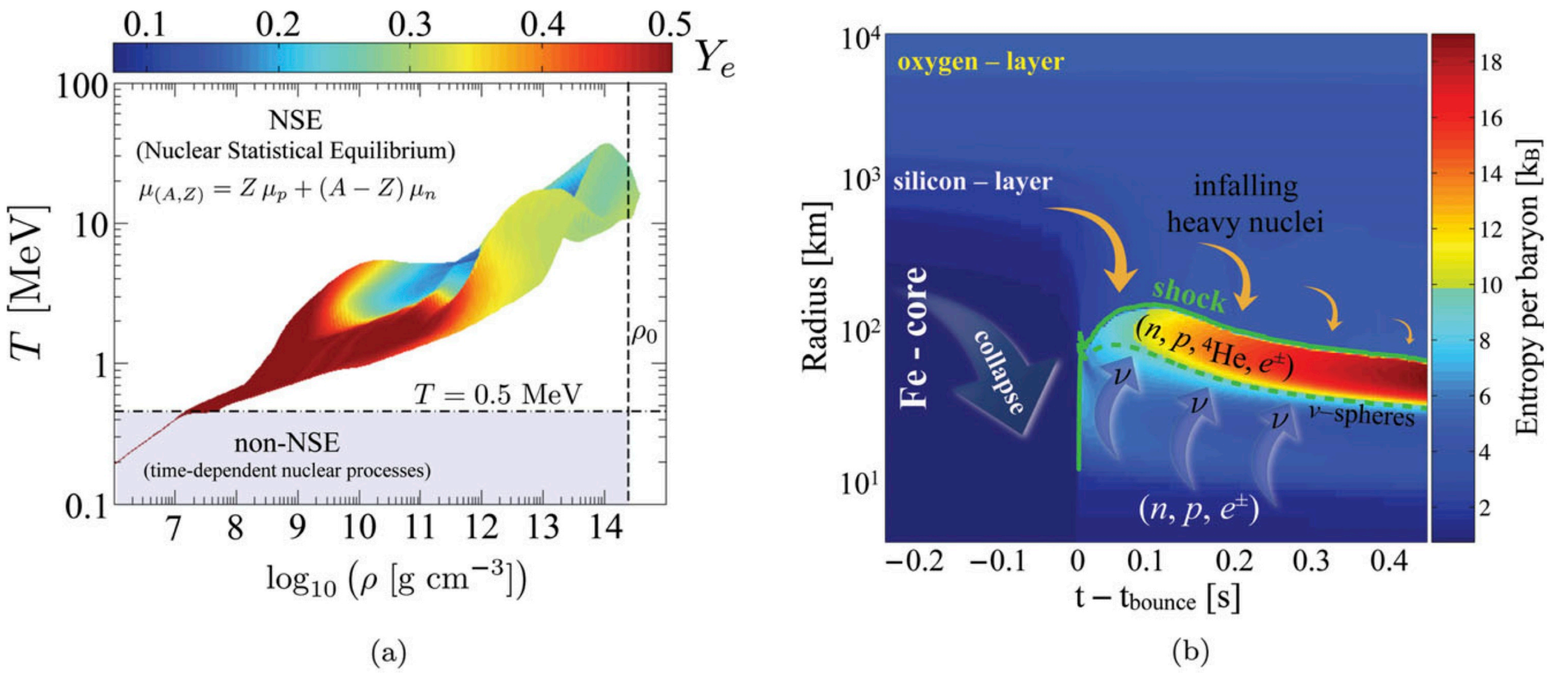


Figure 1. Supernova phase diagram (colour coding is due to the electron fraction Y_e) in graph (a) and space-time diagram of the supernova evolution (colour coding is due to the entropy per baryon) in graph (b), both obtained from the spherically symmetric core-collapse supernova simulation of the massive progenitor star of 18 $M_\odot$ published in Fischer (2016a). (a) Temperature-density domain of a supernova evolution. (b) Space-time diagram of the supernova evolution.

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\text{total}}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$|1\rangle = |\mathbf{p}_1, \sigma_1\rangle$$

which is related to the Green function and the self-energy as

$$A(1,\omega) = 2\,\mathrm{Im}\,G(1,\omega - i0) = 2\,\mathrm{Im}\frac{1}{\omega - E(1) - \Sigma(1,\omega - i0)}$$

$$E(1) = p_1^2/(2m)$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\text{quasi}}(1))}{1 - \frac{d}{dz}\mathrm{Re}\,\Sigma_e(1,z)|_{z=E_e^{\text{quasi}}-\mu_e}} - 2\mathrm{Im}\,\Sigma_e(1,\omega + i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega + \mu_e - E_e^{\text{quasi}}(1)}$$

$$E^{\text{quasi}}(1) = p_1^2/(2m) + \mathrm{Re}\Sigma(1,\omega)|_{\omega = E^{\text{quasi}}(1)}$$

We obtain the generalized Beth-Uhlenbeck formula (quasiparticles)
after calculating the self-energy in ladder approximation.
Bound states appear as solution of an in-medium Schrödinger equation.

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

self-energy

Pauli-blocking

phase space occupation:
Fermi distribution function

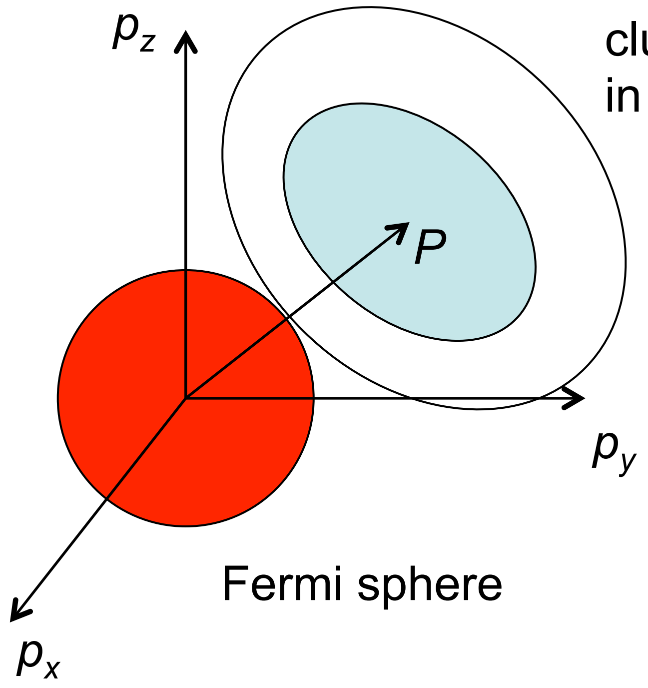
$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Thouless criterion

$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

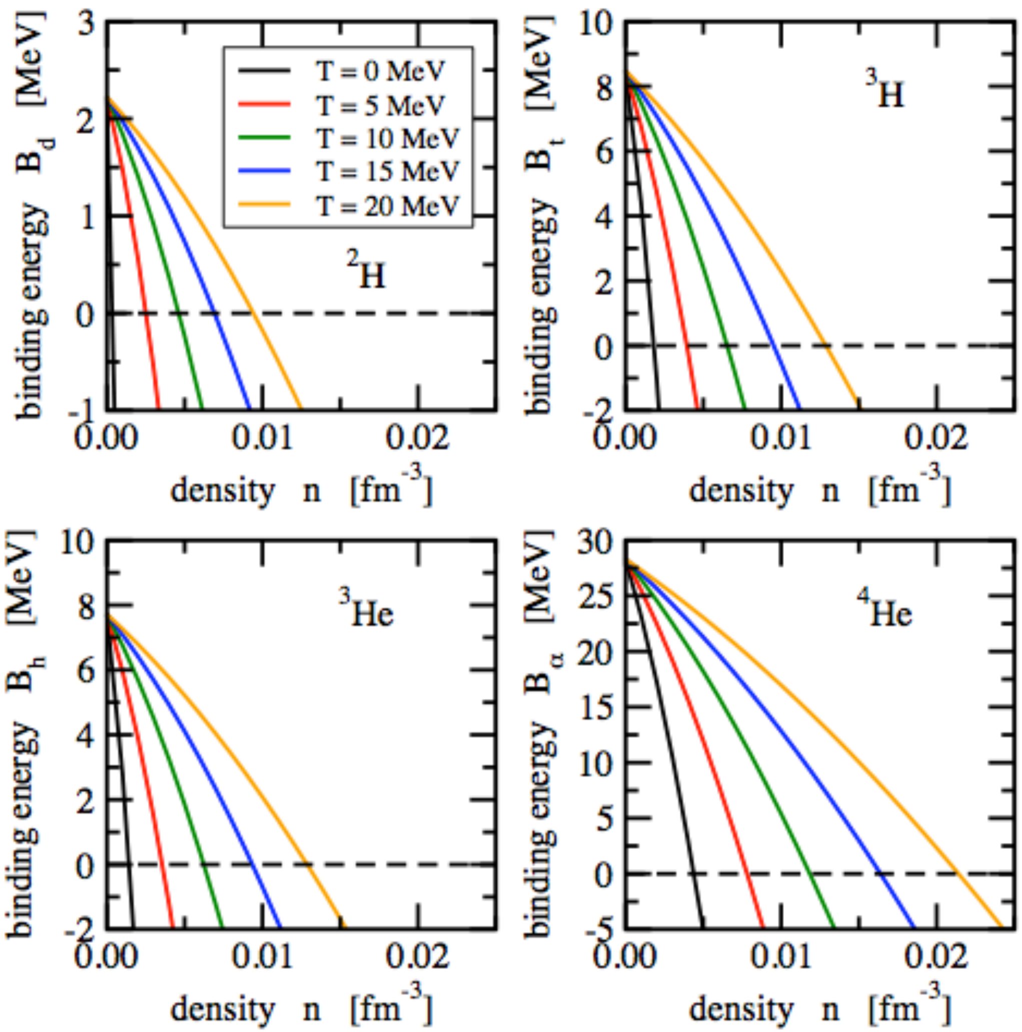
P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

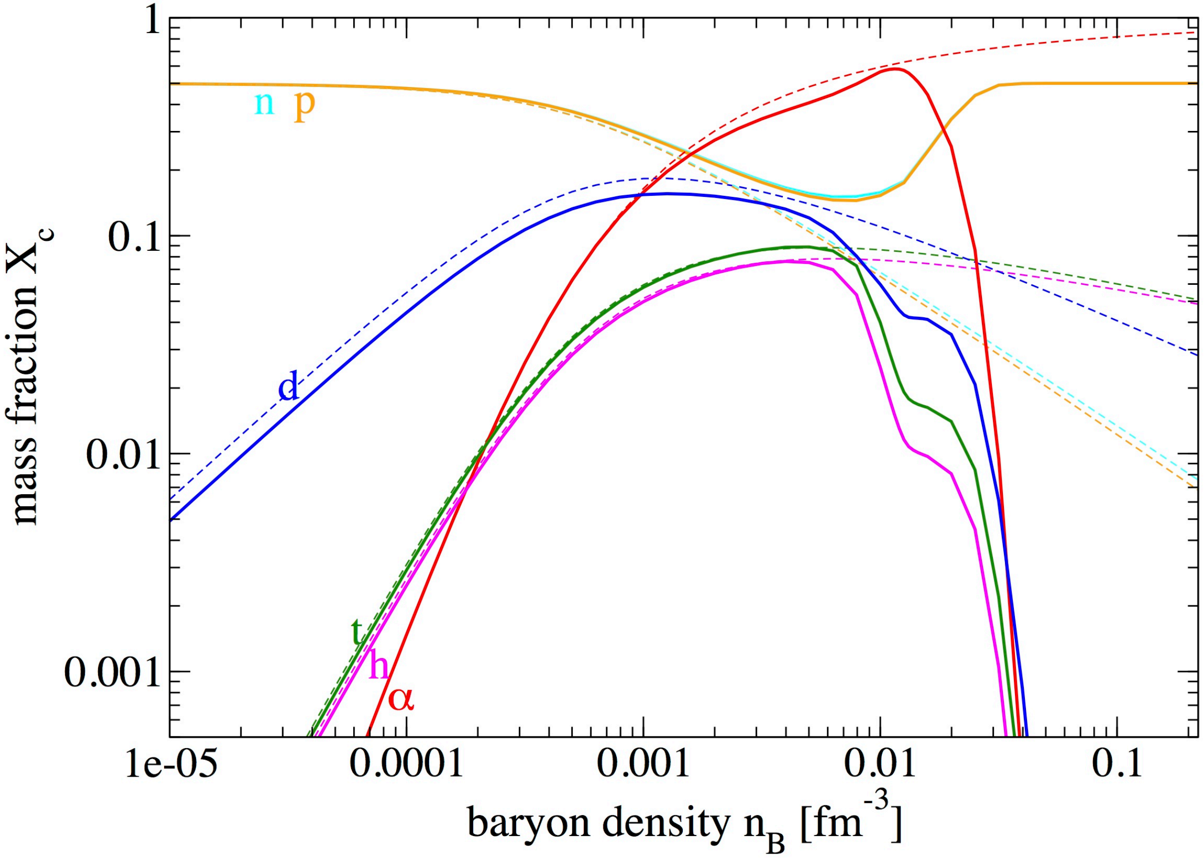
Shift of Binding Energies of Light Clusters



Symmetric matter

G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

G. R., PRC **92**, 054001 (2015)

EoS at low densities from HIC

PRL 108, 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

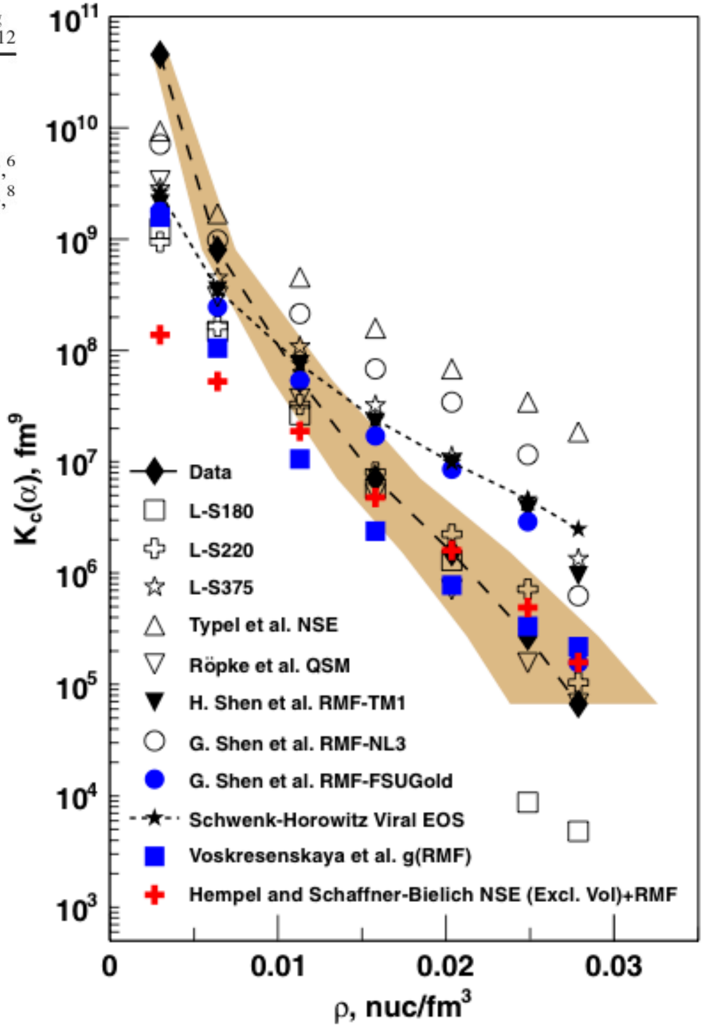
L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

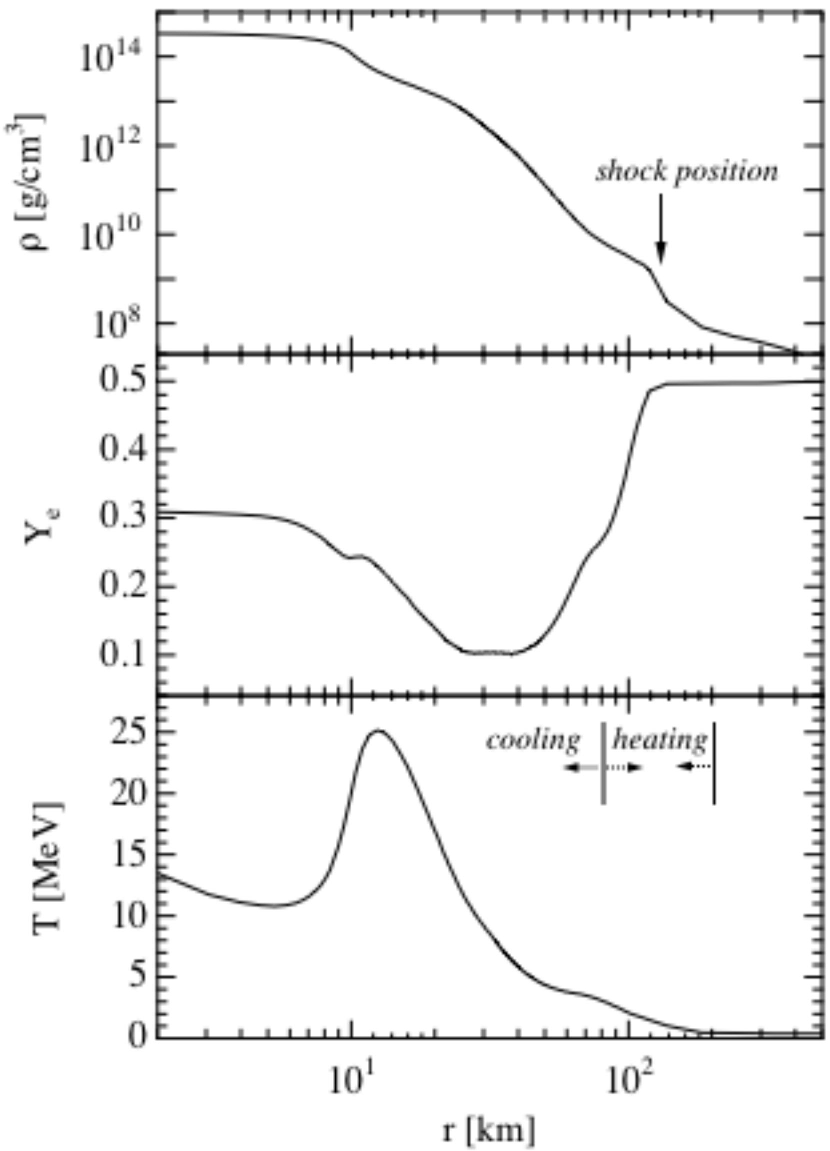
chemical constants

$$K_c(A, Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$

inhomogeneous,
non-equilibrium



Core-collapse supernovae



snapshot

density

proton fraction, and

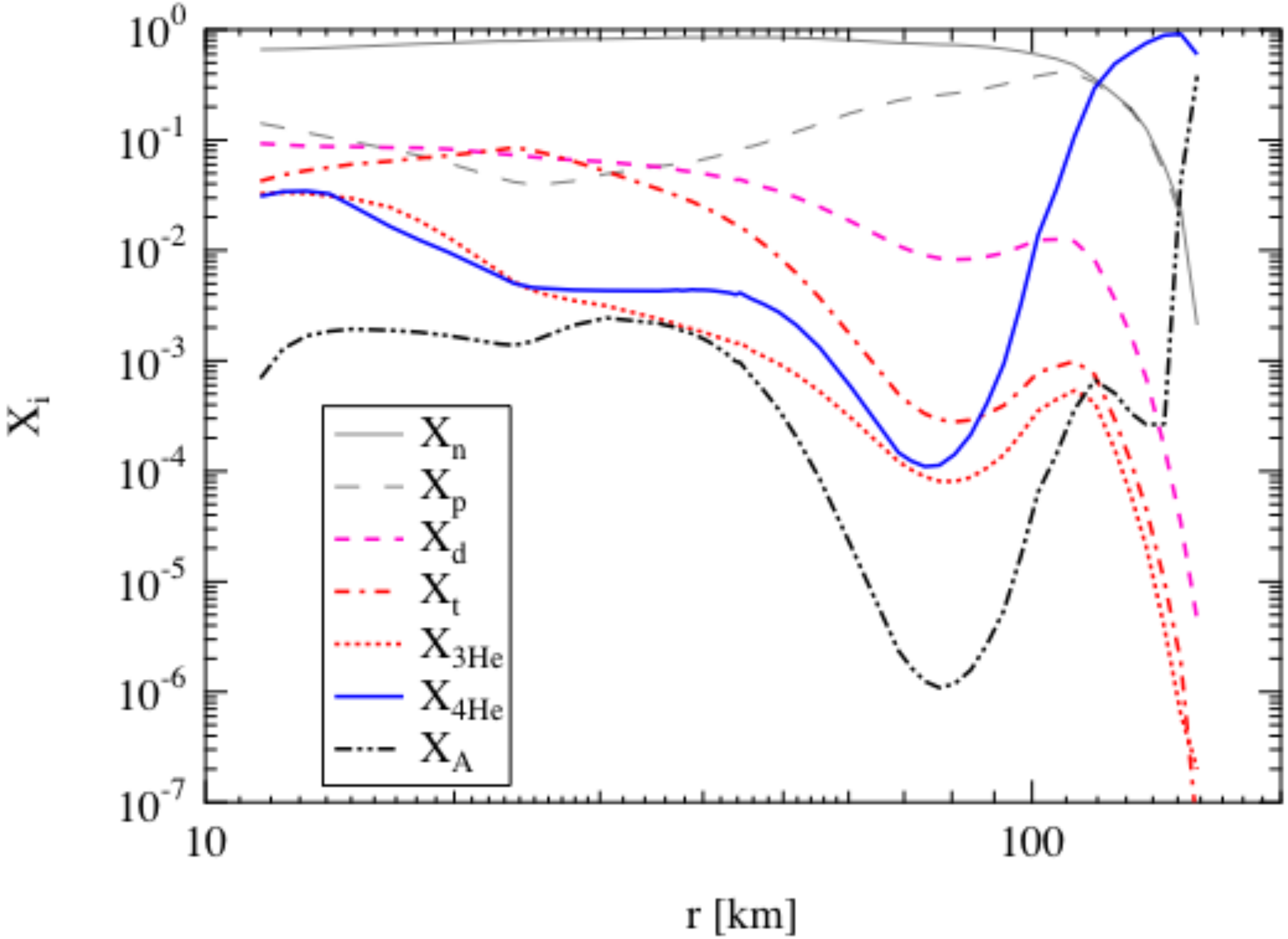
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi, G. R.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core



Mass fraction X
of light clusters
for a post-bounce
supernova core

K.Sumiyoshi,
G. R.,
PRC **77**,
055804 (2008)

Asymmetric nuclear light clusters in supernova matter

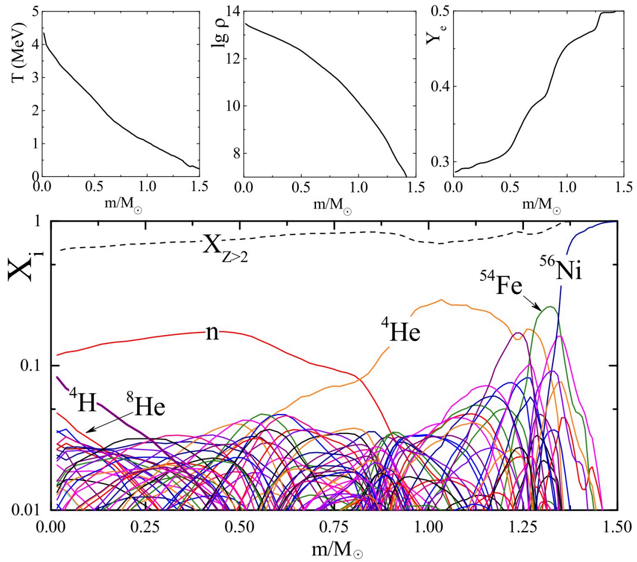


Figure 1. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g \cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions of of nuclei X_i as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of elements with $Z > 2$. EoS is pure NSE.

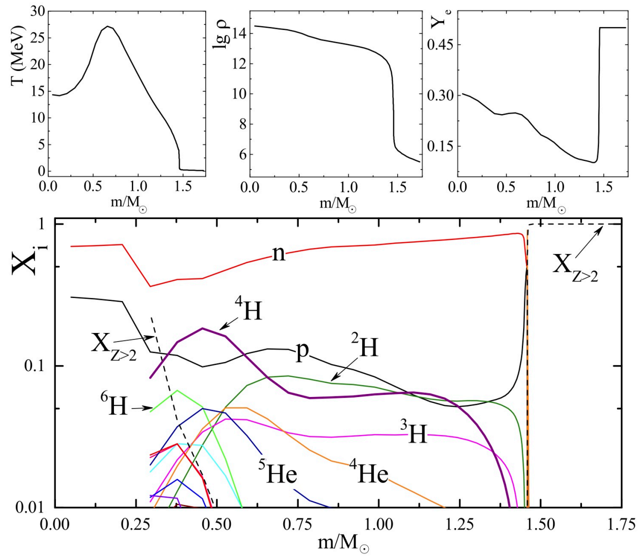
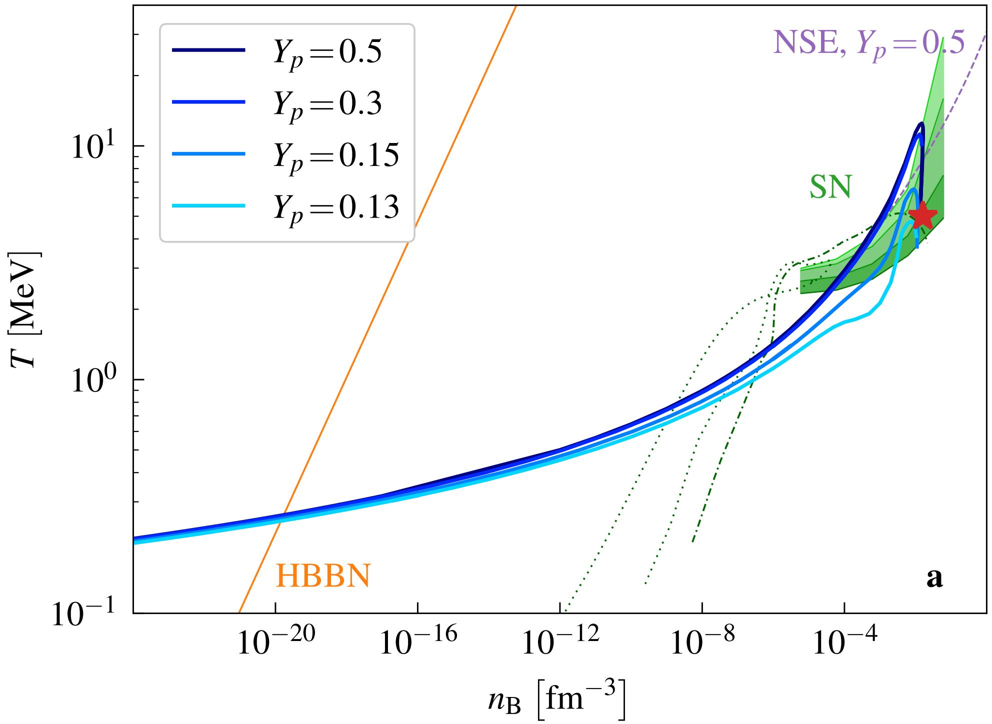


Figure 7. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $\mathrm{g \cdot cm^{-3}}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions X_i of of hydrogen and helium isotopes as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of all rest nuclei. Stellar profile corresponds to 200 ms after bounce approximately, calculations according to modified HS EoS.

A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov, *Monthly Notices of the Royal Astronomical Society* **483**, 5426 (2019)

T. Fischer, S. Typel, G. R., N. Bastian, G. Martinez-Pinedo, Phys. Rev. C **102**, 055807 (2020)

Density-temperature diagram

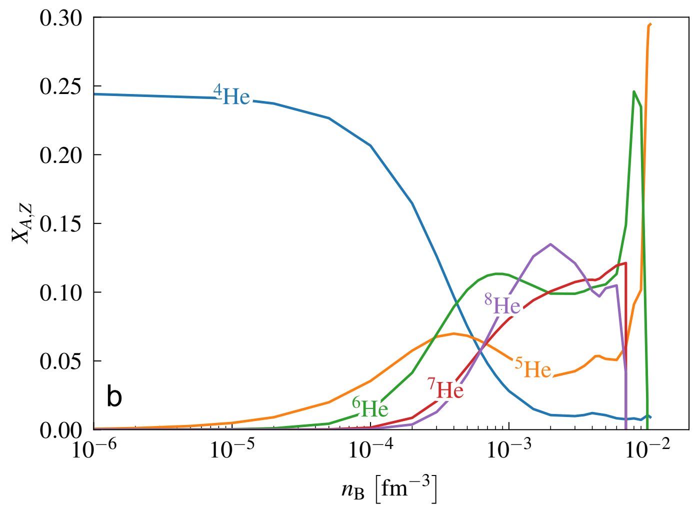


H-He mixture
"final" mass fraction of He: 0.245

curves show the evolution
of two tracer particles
placed in the supernova simulation
[T. Fischer et al., 2014/2017]

composition: He isotopes

$$X_{^4\mathrm{He}} = 0.245,\ Y_p = 0.13.$$



Nonequilibrium statistical operator (NSO)

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho(\hat{t}) = \lim_{\epsilon \to 0} \epsilon \int_{-\infty}^{\hat{t}} d\hat{t}' e^{-\epsilon(\hat{t}-\hat{t}')} e^{-\frac{i}{\hbar} H(\hat{t}-\hat{t}')} \rho_{\rm rel}(\hat{t}') e^{\frac{i}{\hbar} H(\hat{t}-\hat{t}')}$$

relevant distribution: maximum of information entropy (Gibbs)

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle_{\rm rel}^t = \langle B_n \rangle^t$

maximum of information entropy $S_{\rm rel}(t) = -k_{\rm B}\, \mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}$

generalized Gibbs distribution $\rho_{\rm rel}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$

$$\rho_{\rm rel}(\hat{t}) = Z_{\rm rel}^{-1}(\hat{t}) e^{-[H - \lambda_n(\hat{t}) N_n - \lambda_p(\hat{t}) N_p]/\lambda_T(\hat{t})}$$

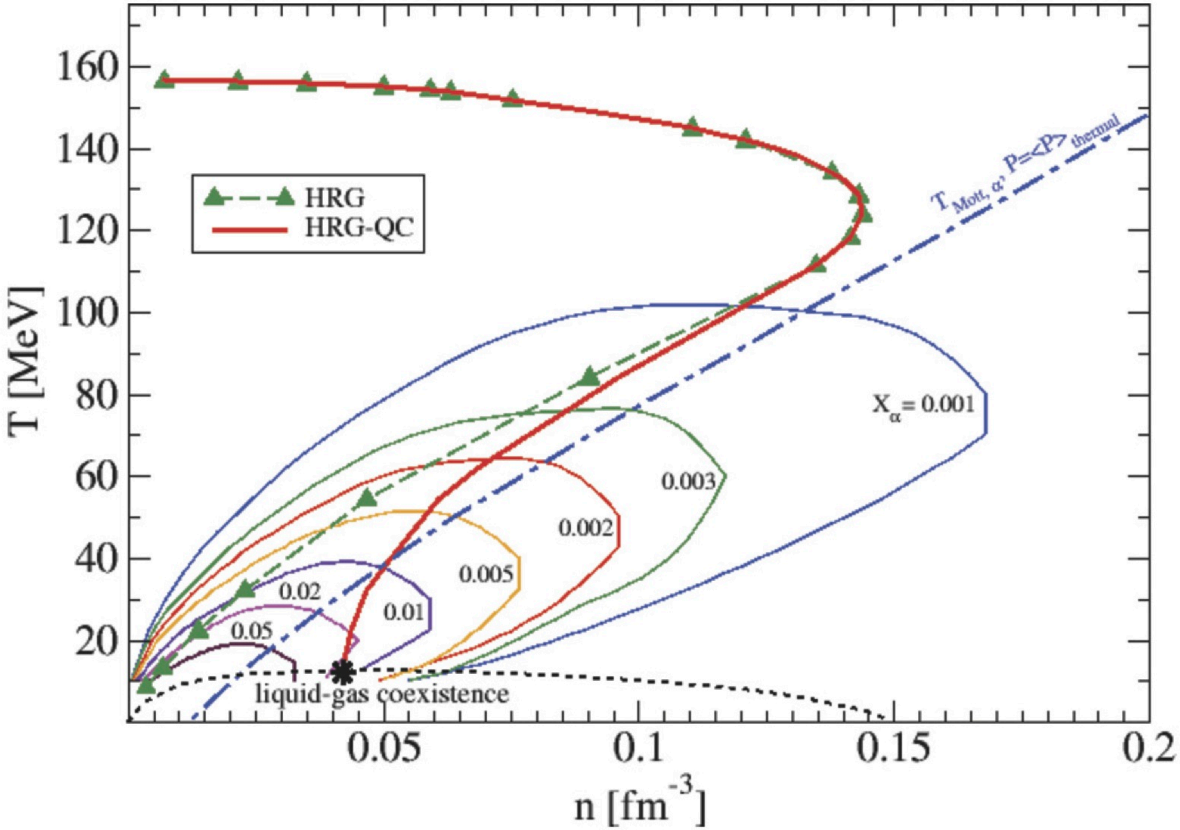
yields, partial densities (NSE)

$$Y_{A,Z}^{(0)} \propto n_{A,Z}^{(0)} = g_{A,Z} \left(\frac{2\pi\hbar^2}{A m \lambda_T^{(0)}} \right)^{-3/2} e^{(B_{A,Z} + (A-Z)\lambda_n^{(0)} + Z\lambda_p^{(0)})/\lambda_T^{(0)}}$$

Expanding nuclear matter: freeze-out and reaction processes (feed-down)

Freeze-out at heavy ion collisions

Mott line: bound state formation



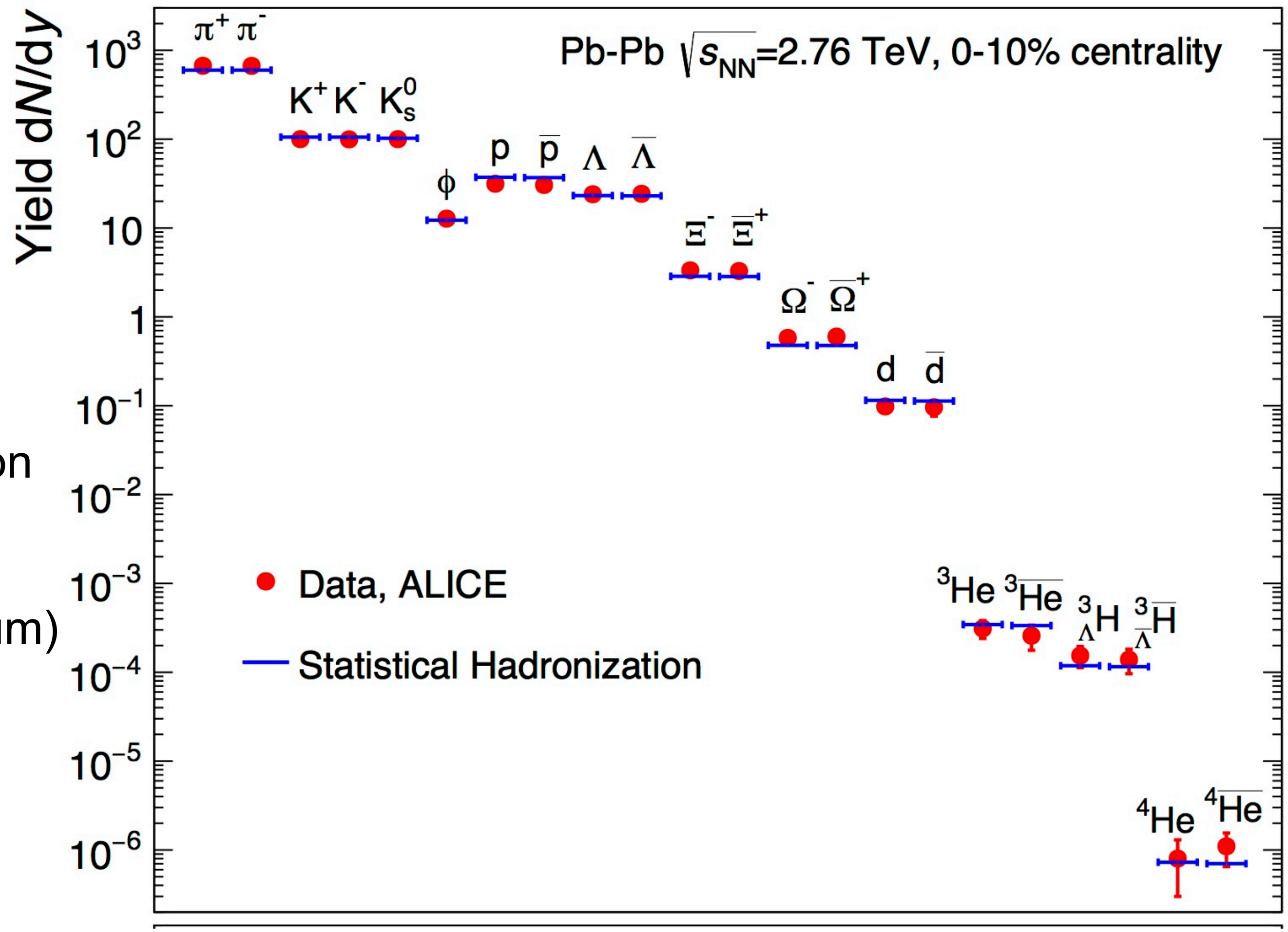
D. Blaschke, S Liebing, G.R., B. Doenigus, Phys. Lett. B **860** (2025) 139206

Cluster formation at LHC/CERN

ALICE@LHC

Excellent description of data by the Statistical model (chemical equilibrium)

T = 156 MeV

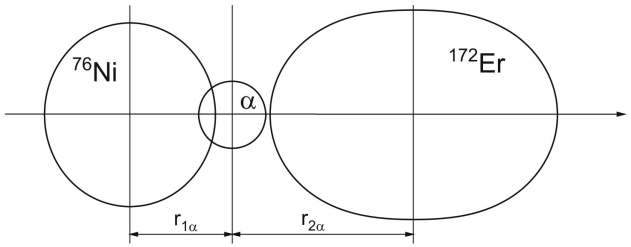


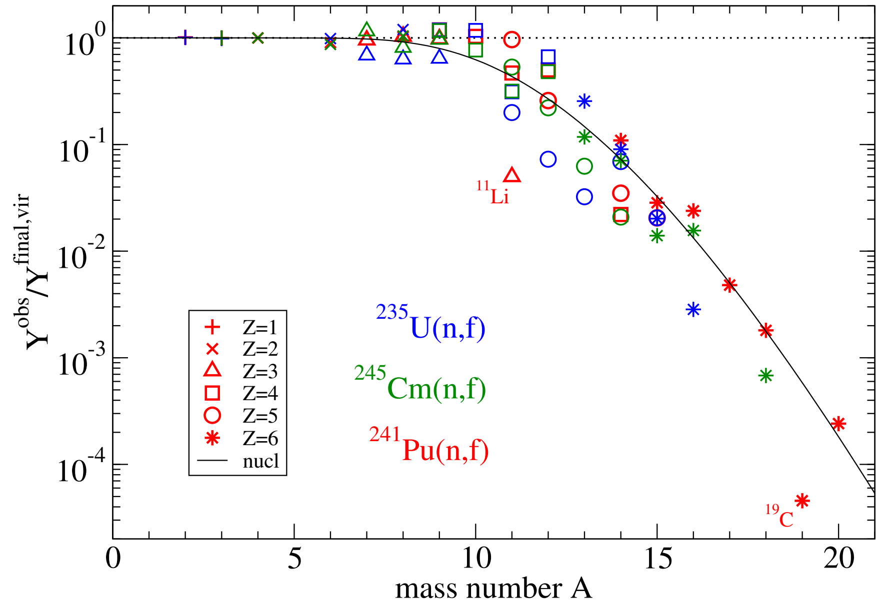
A. Andronic, P. Braun-Munziger, K. Redlich, J. Stachel, Nature **561**, 321 (2018)

Ternary fission: light cluster yields

Neck formation and scission: freeze-out, low-density neutron-rich matter

Nonequilibrium information entropy approach to ternary fission of actinides





$T = 1.24$ MeV

$\mu_n = -2.99$ MeV

$\mu_p = -16.35$ MeV

$n_B = 0.000067$ fm^{-3}

$Y_p = 0.035$

nucleation kinetics

$$Y^{\rm obs}_{A,Z}/Y^{\rm final,vir}_{A,Z} = \frac{1}{2}{\rm erfc}\left[b(\tau)(A^{1/3} - a(A_c,\tau))\right]$$

saddle-to-scission relaxation time about 7000 fm/c

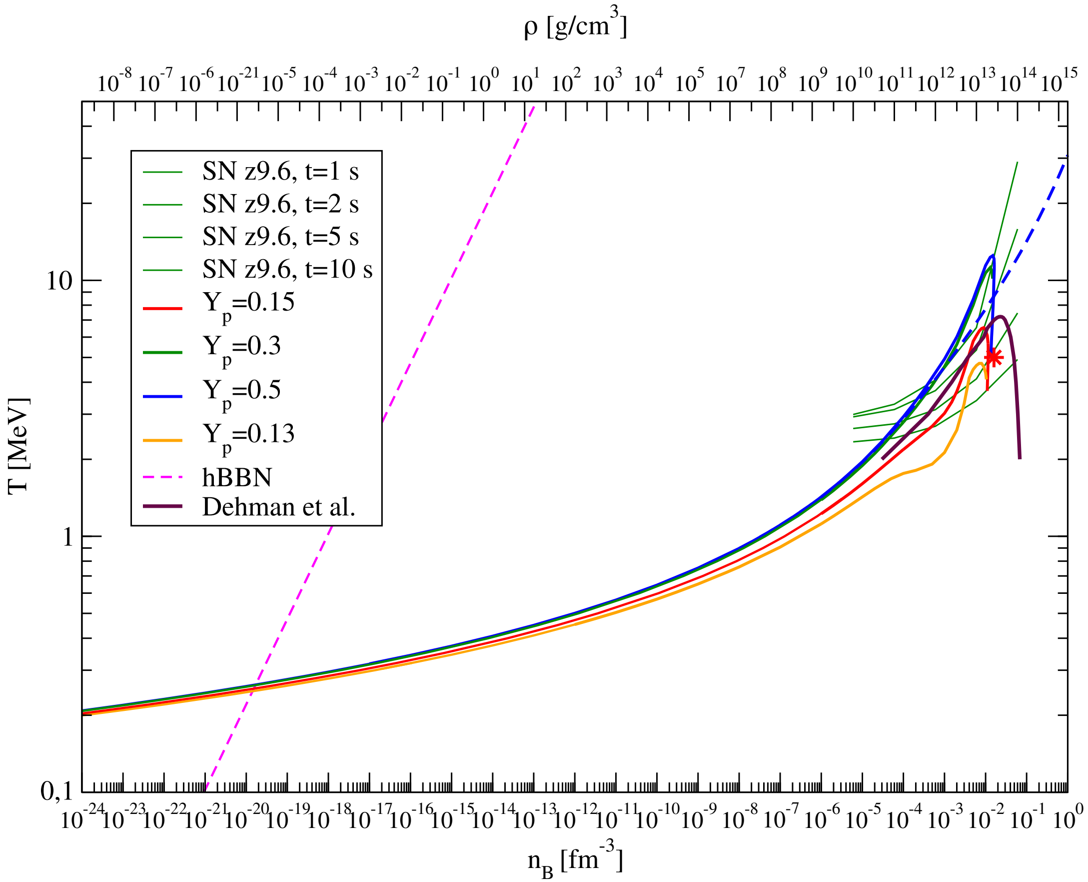
size effect?

G. R.,J. Natowitz, H. Pais, Phys. Rev. C **103**, L061601 (2021)

solar system Lagrange parameter

- Mass formula (Bethe-Weizsaecker mass formula, magic numbers: Duflo & Zuker, Koura & Chiba, …)
- Excited states (Rauscher,…)
- In-medium corrections (surface term, continuum correlations,…)
- Expanding hot and dense matter, freeze-out of M(A>76)
- Decay processes: Evaporation of nucleons, α decay, fission
- Lagrange parameter: T = 5.266 MeV, μ_n= 940.317 MeV, μ_p= 845.069 MeV (n_B = 0.013 fm^{-3} and Y_p = 0.13)

Phase transition?



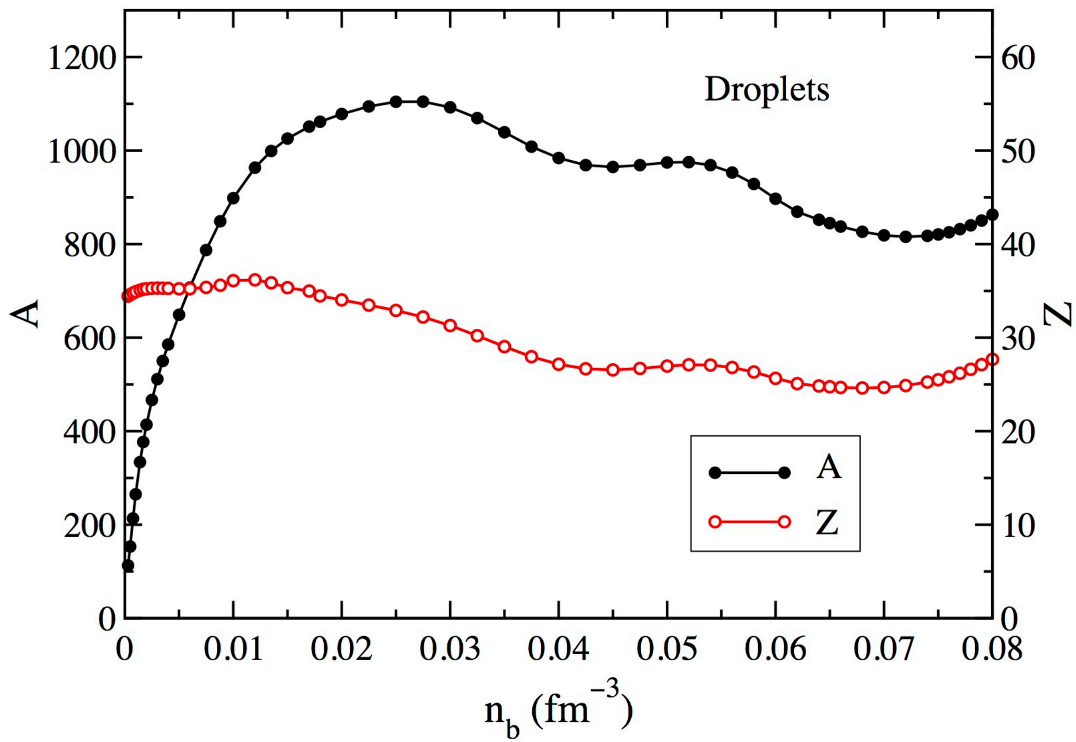
$X_{\alpha}=0.245$

Cluster region (hot inner crust of PNS)

C. Dehman et al., A&A **687**, A236 (2024)

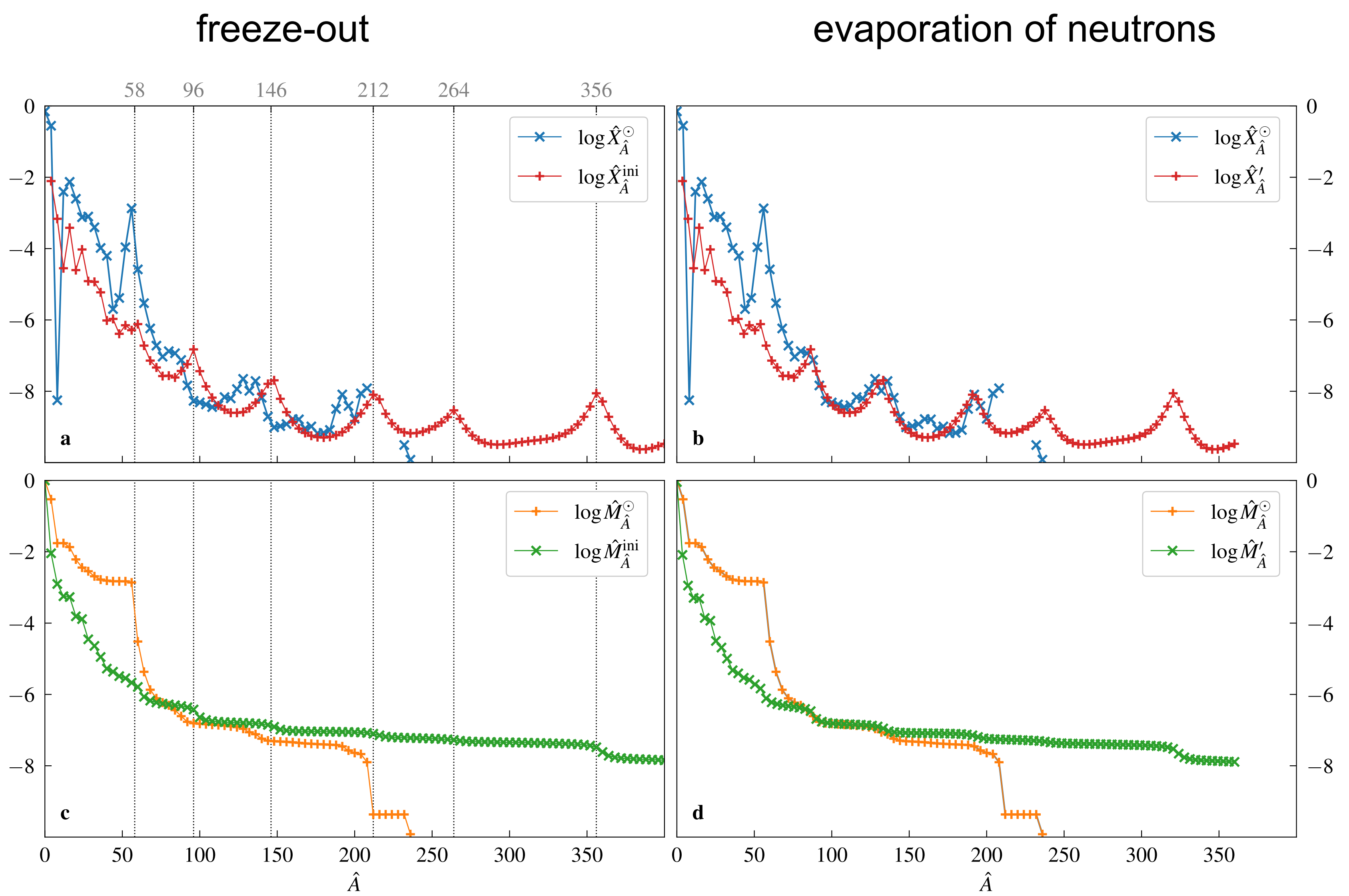
Neutron star inner crust

T = 0, 0.0003 fm^{-3} < n_b < 0.08 fm^{-3} : spherical nuclei (pasta phases)



B.K. Sharma et al., A&A **584**, A103 (2015)

Coarse-grained distributions

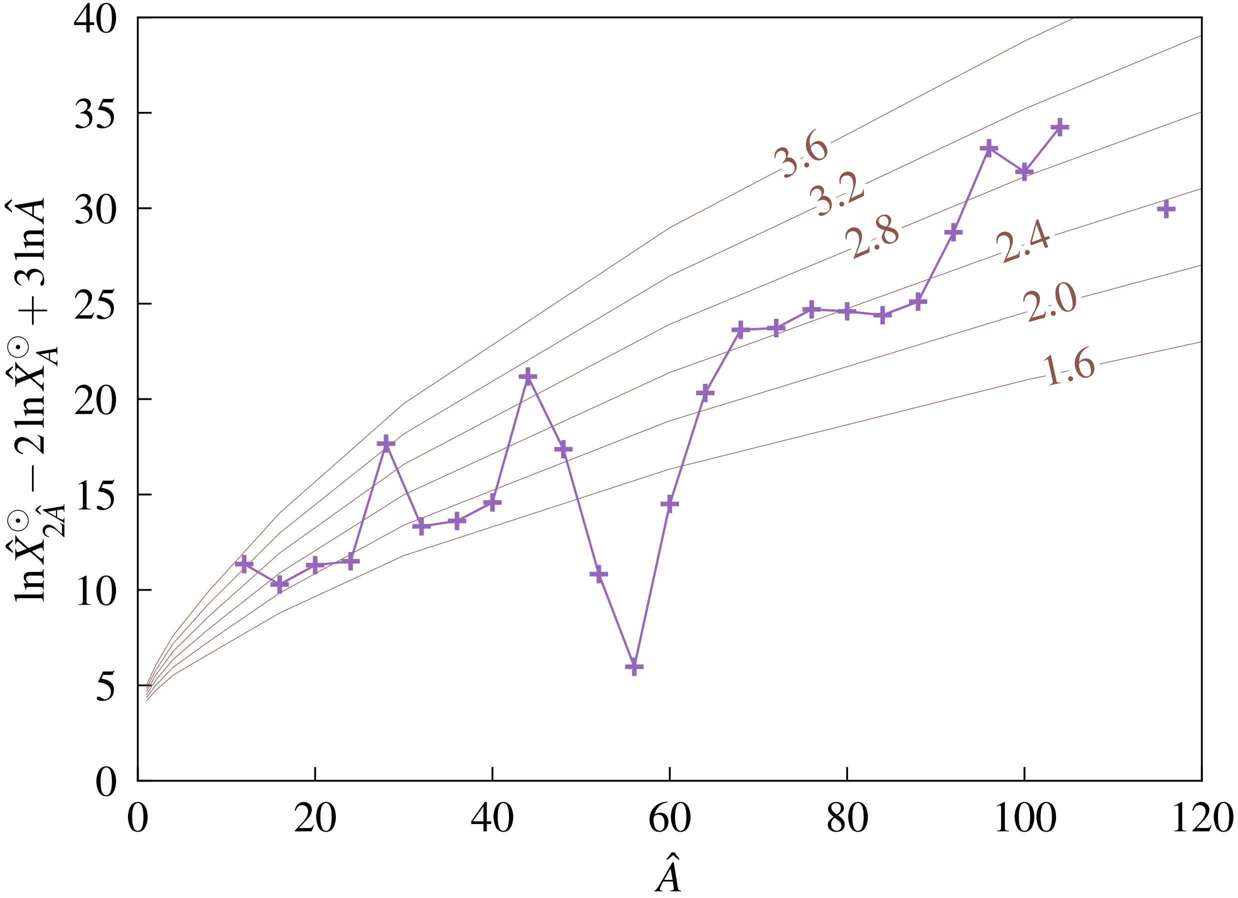


Conclusions

- Dense matter: improve the nuclear statistical equilibrium
  In-medium corrections, continuum correlations
- Improve the nuclear reaction networks, in-medium effects
- Nucleation theory for the heavy nuclei, thermodynamic stability?
- Lagrange parameters for the heavy-element freeze-out
- Astrophysical sites of heavy-element formation?

Thanks for attention

Liquid drop model: surface term



free value $a_s/T \approx 3.6$

Yields of H and He isotopes

isotope	$R^{\mathrm{vir}}_{A,Z}$	^{233}U(n_{th},f)	^{235}U(n_{th},f)	^{239}Pu(n_{th},f)	^{241}Pu(n_{th},f)	^{248}Cm(sf)	^{252}Cf(sf)
λ_T [MeV]	1.3	1.24177	1.21899	1.3097	1.1900	1.23234	1.25052
λ_n [MeV]	-	-3.52615	-3.2672	-3.46688	-3.02055	-2.92719	-3.1107
λ_p [MeV]	-	-15.8182	-16.458	-16.2212	-16.6619	-16.7798	-16.7538
^{1}n	-	560012	1.409e6	722940	1.8579e6	1.606e6	1.647e6
^{1}H	-	28.131	28.16	42.638	19.52	21.079	30.096
^{2}H$^{\mathrm{obs}}$	-	41	50	69	42	50	63
^{2}H	0.973	40.986	49.76	68.632	41.563	49.533	61.579
^{3}H$^{\mathrm{obs}}$	-	460	720	720	786	922	950
^{3}H	0.998	457.27	715.29	714.79	780.39	913.76	943.12
^{4}H	0.0876	2.7772	4.97	5.627	6.057	8.742	8.219
^{3}He	0.997	0.0124	0.0076	0.0235	0.00431	0.00645	0.00933
^{4}He$^{\mathrm{obs}}$	-	10000	10000	10000	10000	10000	10000
^{4}He	1	8858.46	8706.1	8615.7	8556.9	8313.98	8454.0
^{5}He	0.689	1130.75	1289.04	1374.7	1439.0	1680.75	1540.9
^{6}He$^{\mathrm{obs}}$	-	137	191	192	260	354	270
^{6}He	0.933	115.89	158.98	159.01	211.68	276.96	222.4
^{7}He	0.876	21.262	33.997	35.983	51.742	80.634	58.16
$Y^{\mathrm{obs}}_{^6\mathrm{He}}/Y^{\mathrm{final,vir}}_{^6\mathrm{He}}$	-	0.9989	0.9897	0.9846	0.9869	0.9899	0.9622
^{8}He$^{\mathrm{obs}}$	-	3.6	8.2	8.8	15	24	25
^{8}He	0.971	3.4725	6.764	6.4095	12.481	21.280	13.32
^{9}He	0.255	0.047077	0.105	0.111	0.219	0.455	0.258
$Y^{\mathrm{obs}}_{^8\mathrm{He}}/Y^{\mathrm{final,vir}}_{^8\mathrm{He}}$	-	1.0229	1.1936	1.3496	1.1811	1.1042	1.8409
^{8}Be	1.07	5.7727	2.594	5.147	2.188	2.819	2.544

Lagrange parameters

observed yieds,

primary yields,

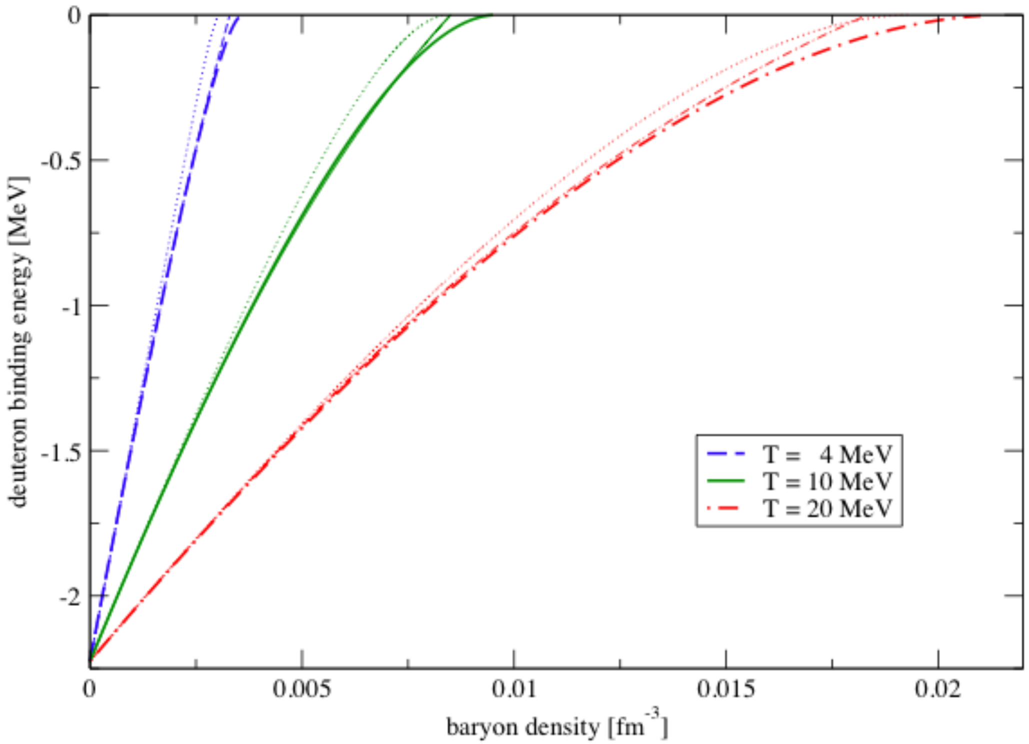
intrinsic partition function,
no density corrections:

6He is overestimated,
8He is underestimated:

Pauli blocking?

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures, zero center of mass momentum

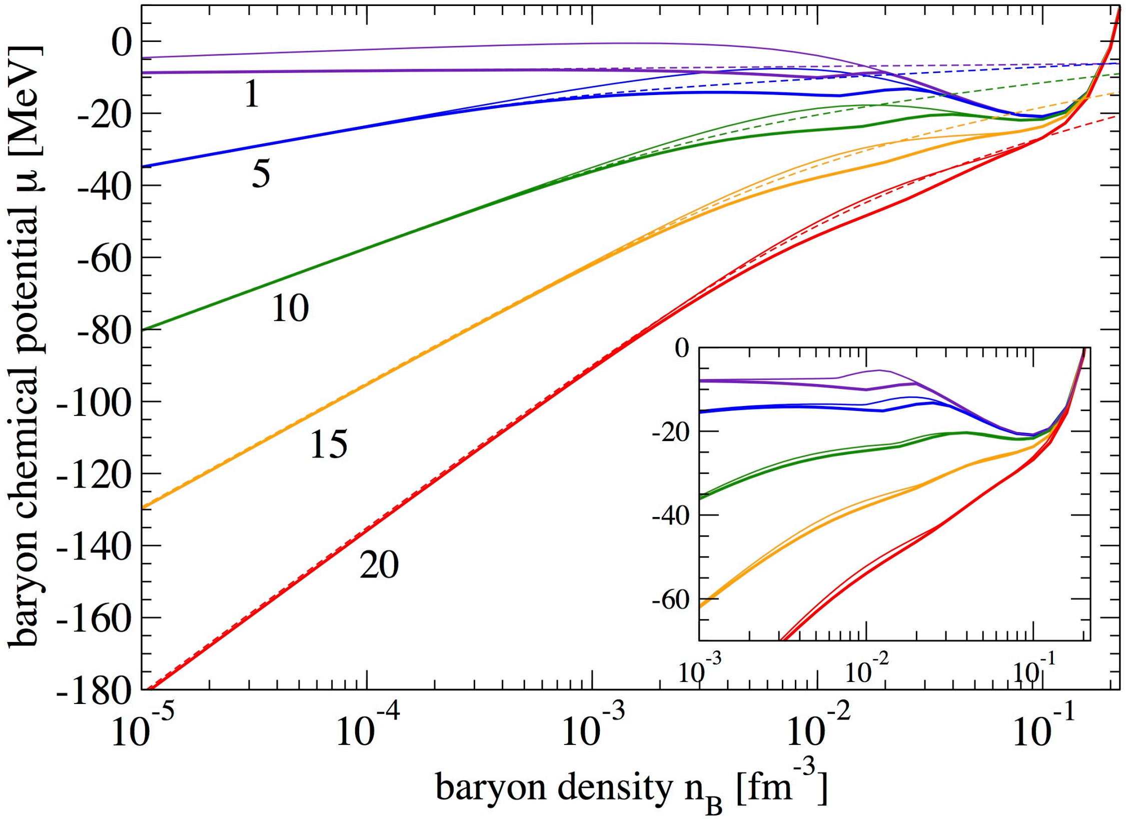


thin lines:

fit formula

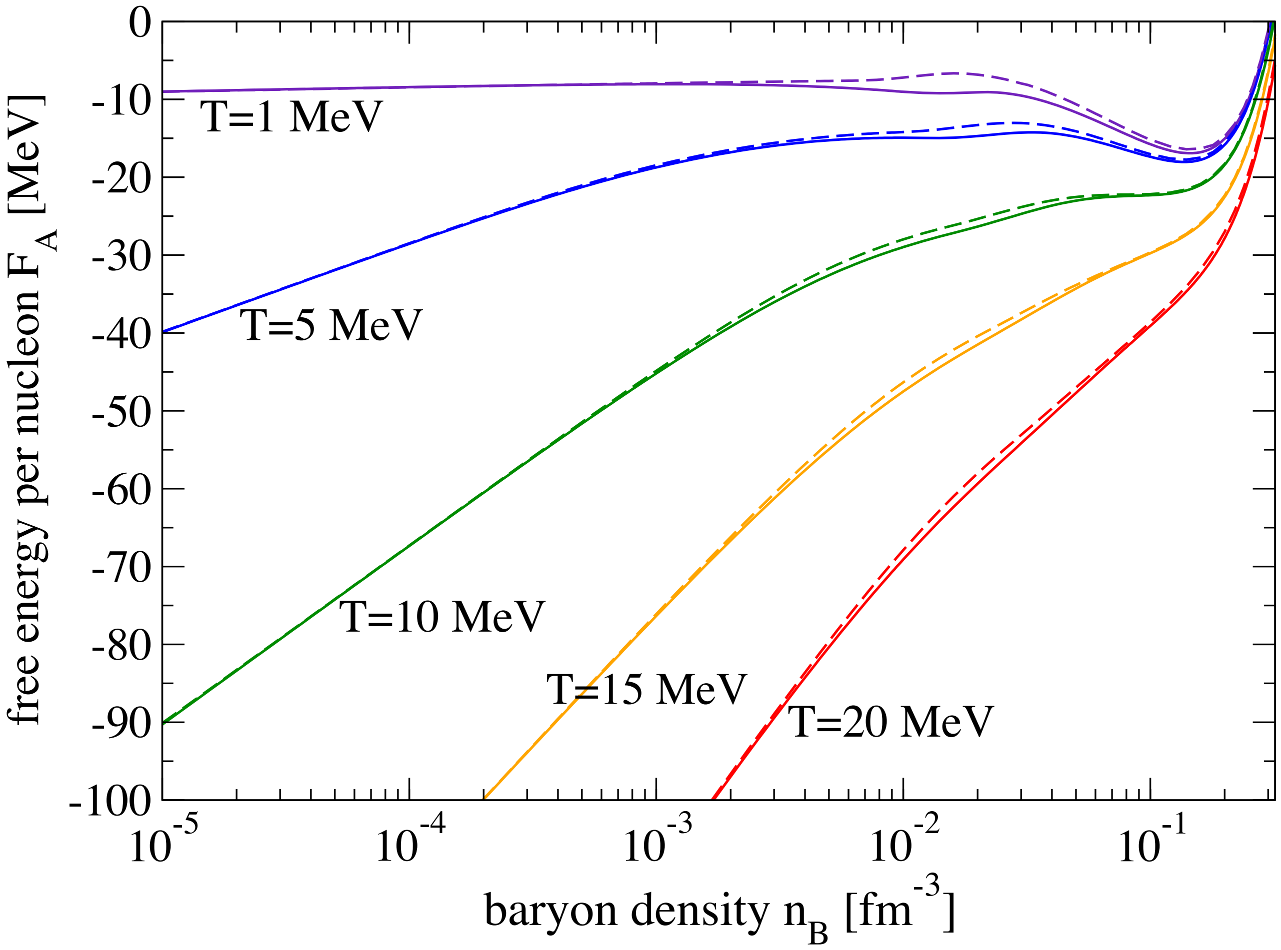
G.R., Nucl. Phys. A **867**, 66 (2011)

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

Symmetric matter: free energy per nucleon



Dashed lines: no continuum correlations

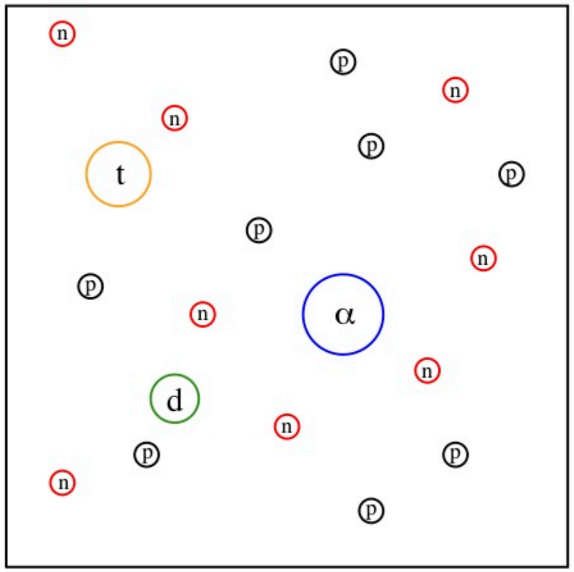
G. R., PRC **92**, 054001 (2015)

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law

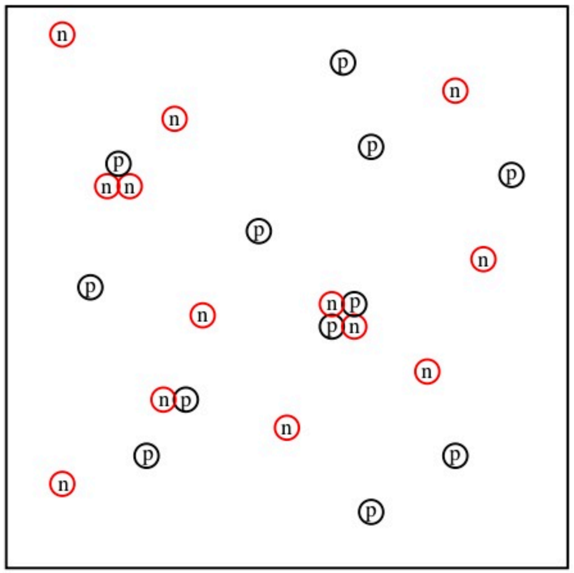


Interaction between the components
internal structure: Pauli principle

Physical picture:

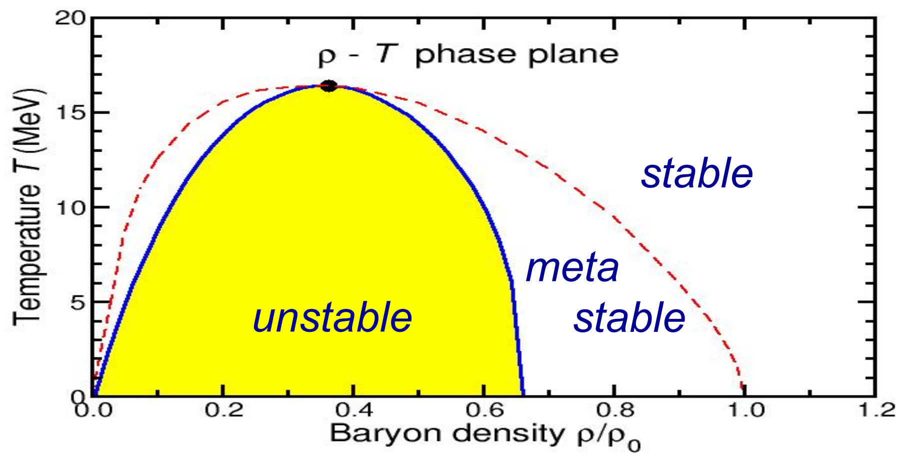
"elementary" constituents

and their interaction



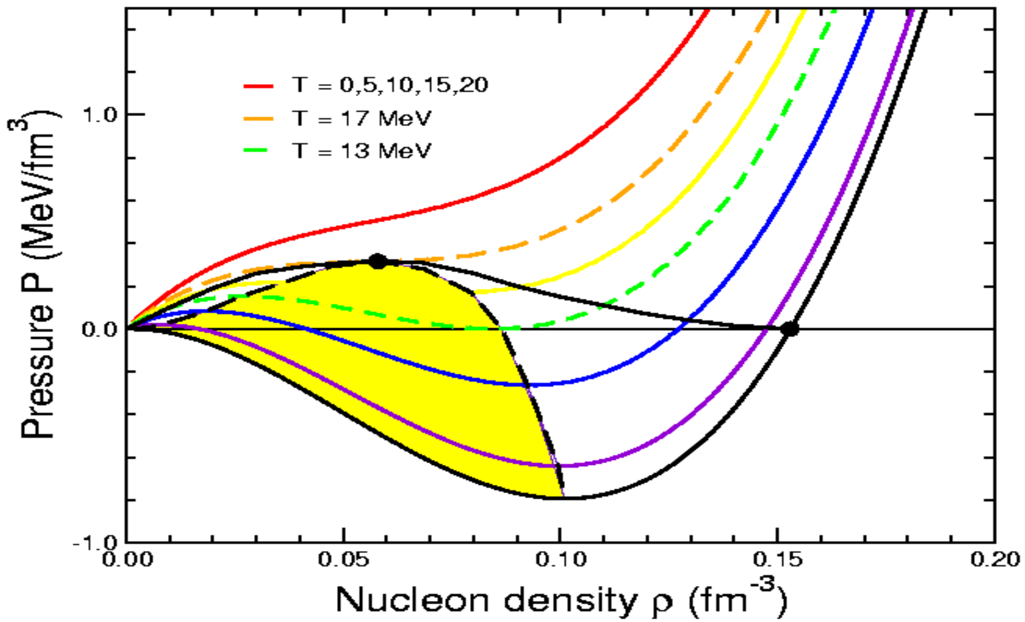
Quantum statistical (QS) approach,
quasiparticle concept, virial expansion

Nuclear matter



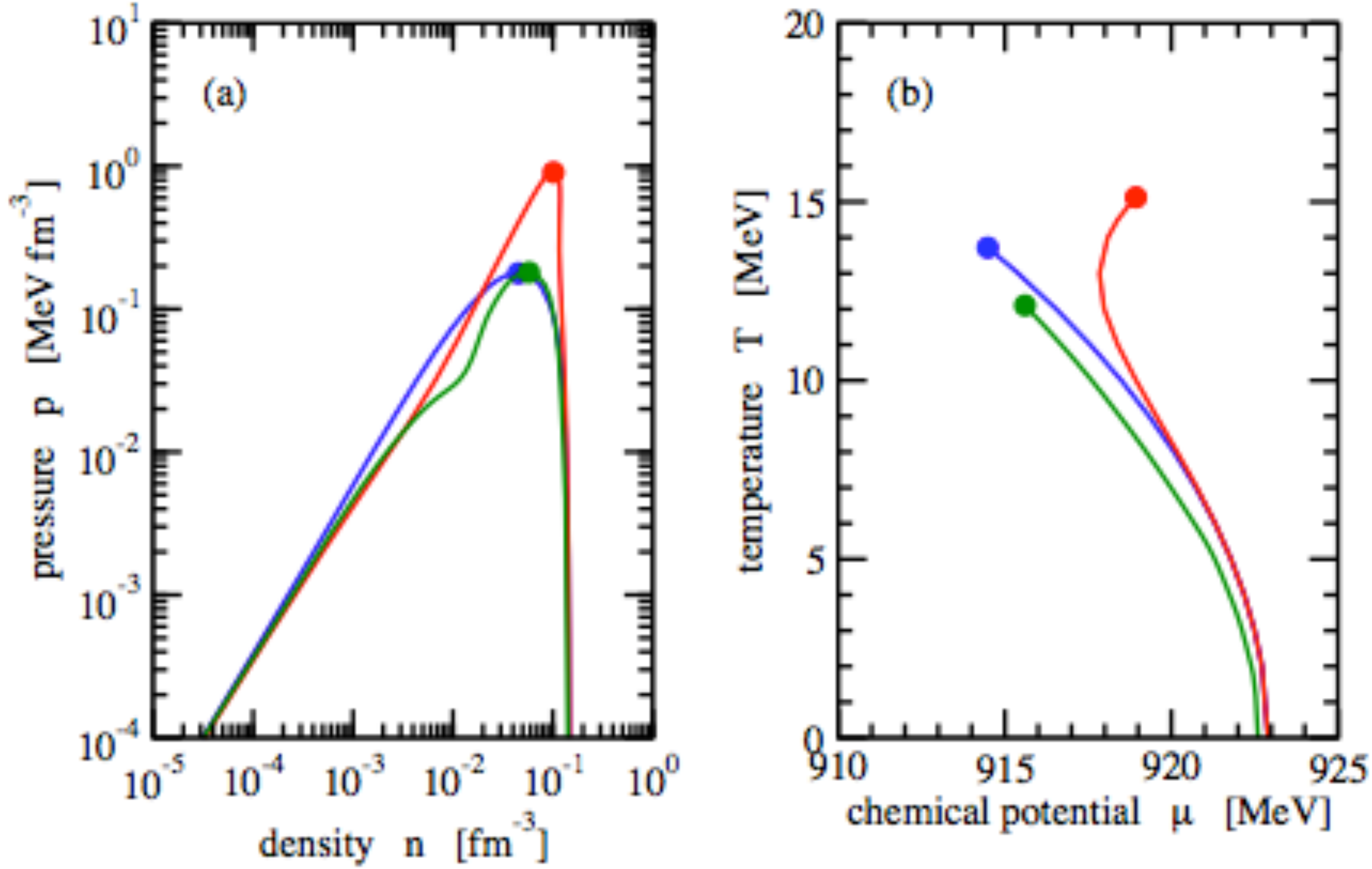
Phase diagram

$$\varepsilon(T;\rho) = \varepsilon_{\mathrm{FG}}(T;\rho) + w(\rho)$$



Equation of state:
$p_T(\rho)$

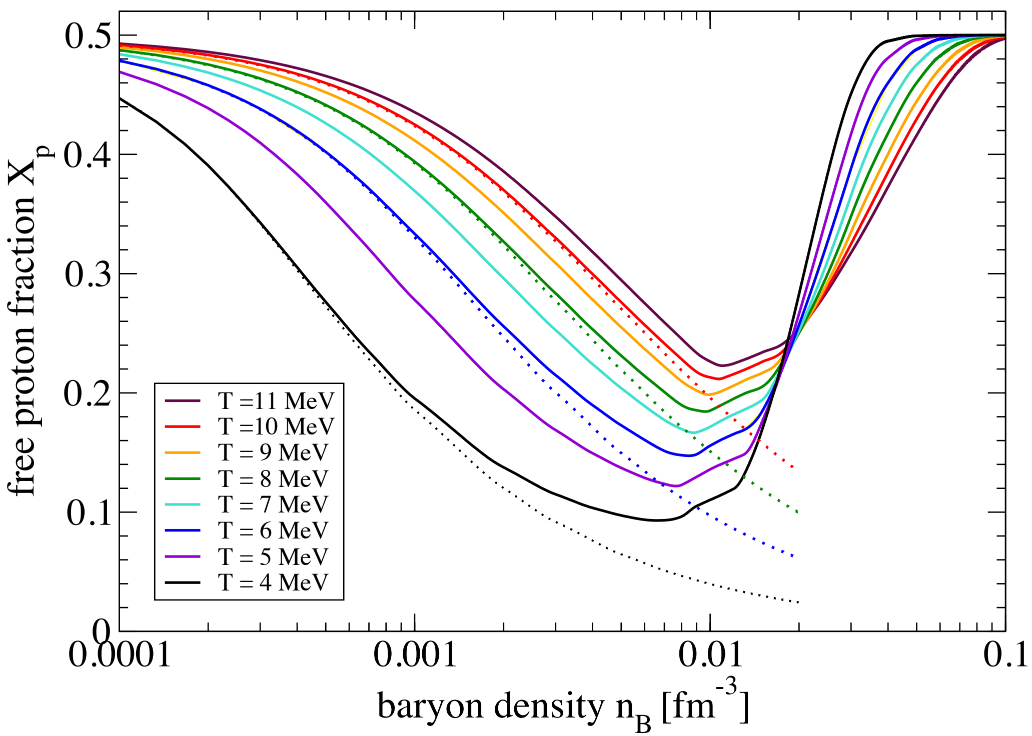
Liquid-vapor phase transition



blue: no light cluster, green: with light clusters, QS, red: cluster-RMF

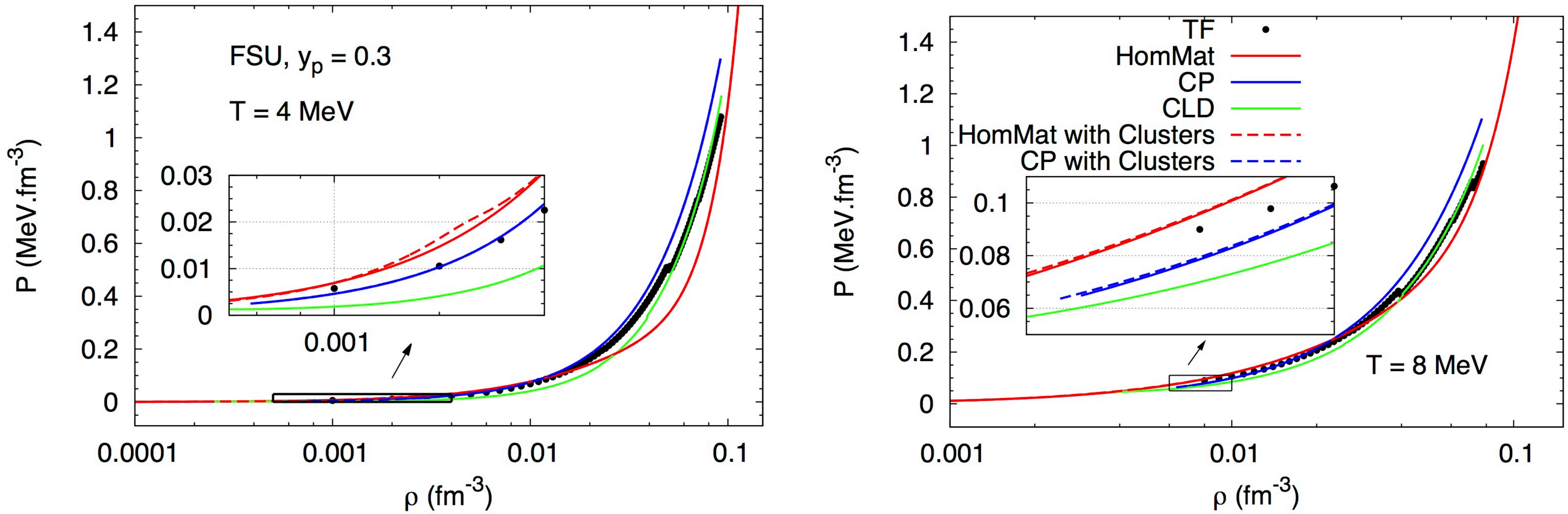
S. Typel et al., PRC **81**, 015803 (2010)

Pauli blocking in symmetric matter



Free proton fraction as function of density and temperature in symmetric matter.
QS calculations (solid lines) are compared with the NSE results (dotted lines).
Mott effect in the region $n_{saturation}/5$.

Light clusters and pasta phases in core-collapse supernova matter

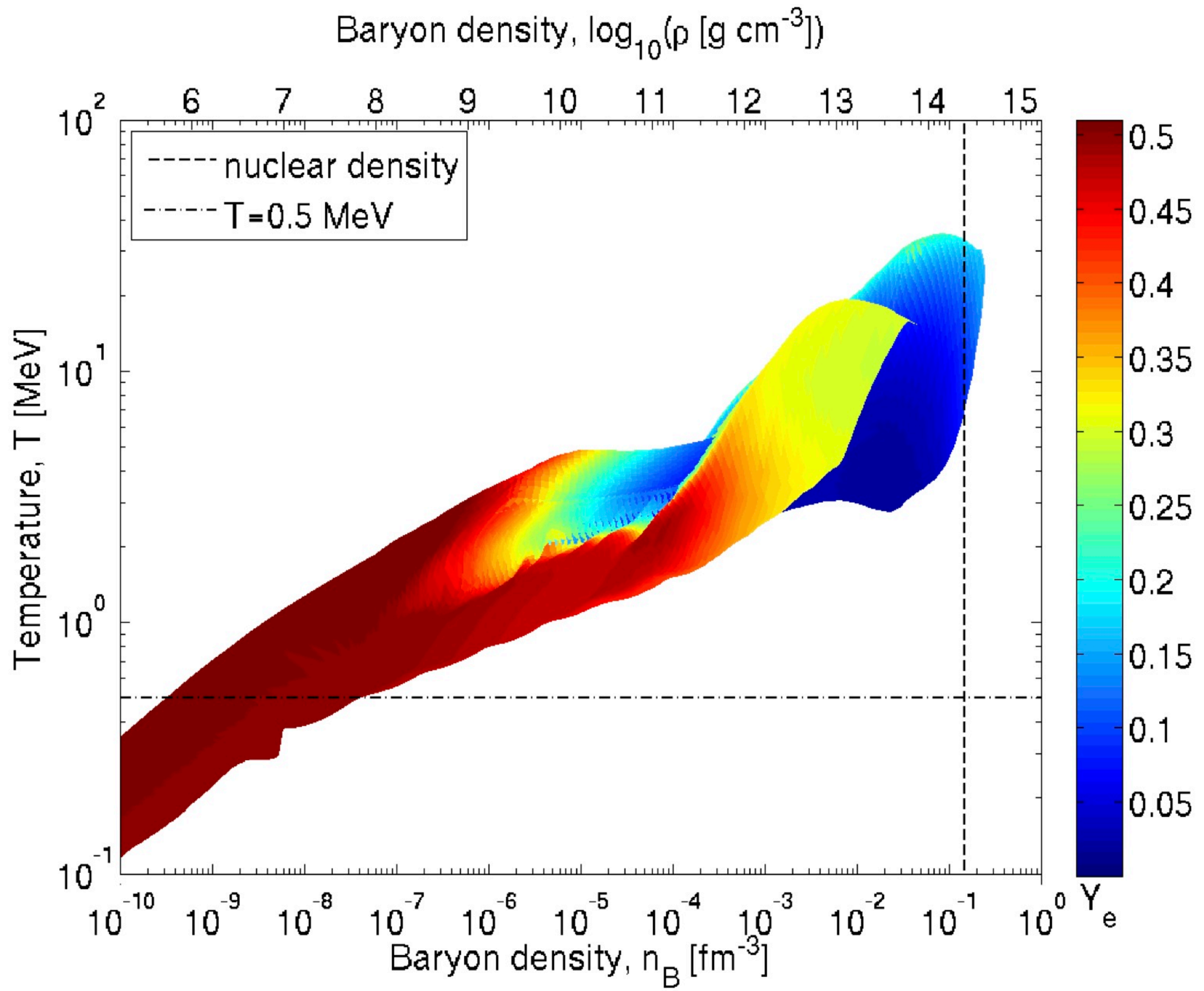


Pressure as function of density, Yp=0.3, T=4 MeV / 8 MeV.
With and without pasta, including or not clusters. TF - Thomas-Fermi,
CP – coexisting-phases method, CLD – compressible liquid drop

H. Pais, S. Chiacchiera, C. Providencia, PRC **91**, 055801 (2015)

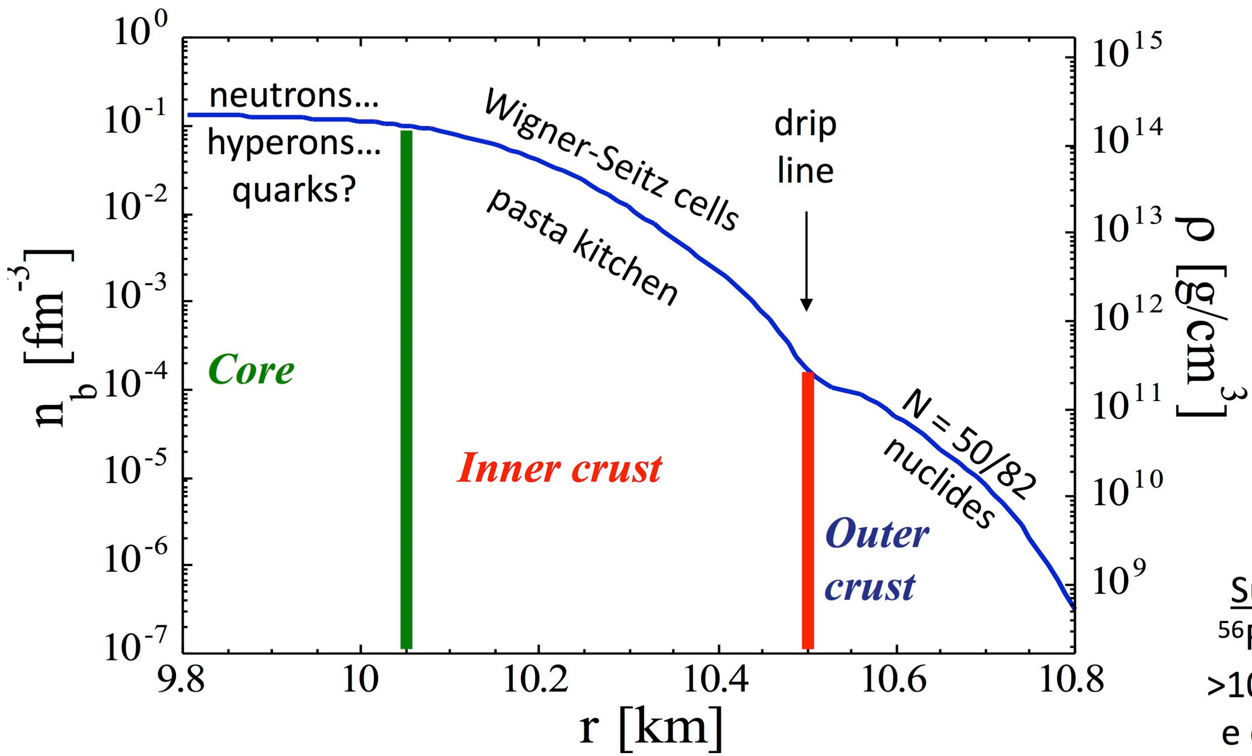
Nuclear matter phase diagram

Exploding supernova



T. Fischer et al., arXiv 1307.6190

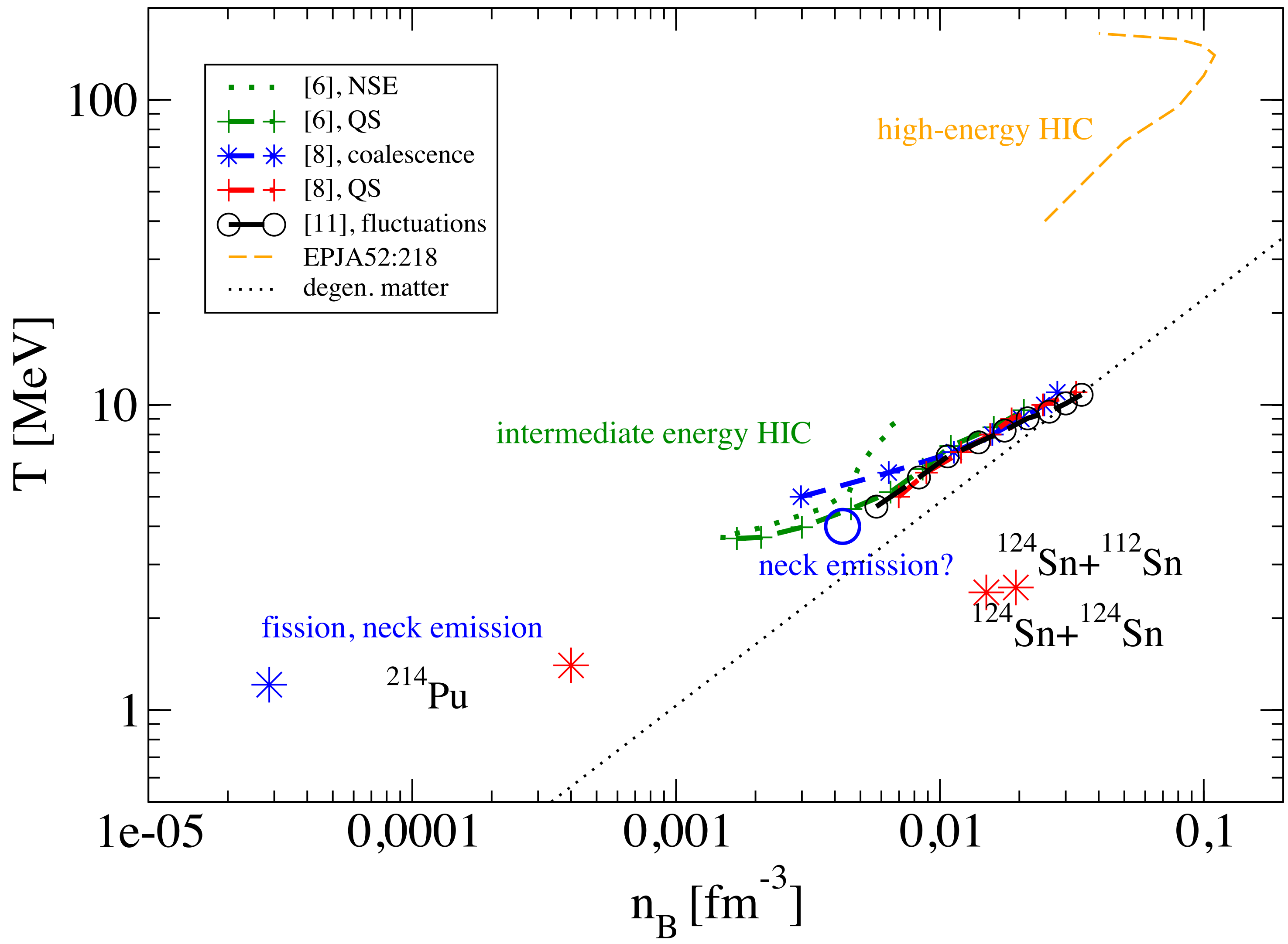
Density of neutron star crust



<u>Surface</u>:
^{56}Fe; ^{62}Ni;
$>10^7$ g/cm^3
e degen.:
p + e → n

J. M. Pearson *et al.*, PRC **83**, 065810 (2011)

Freeze-out temperatures and densities



Solar element abundances

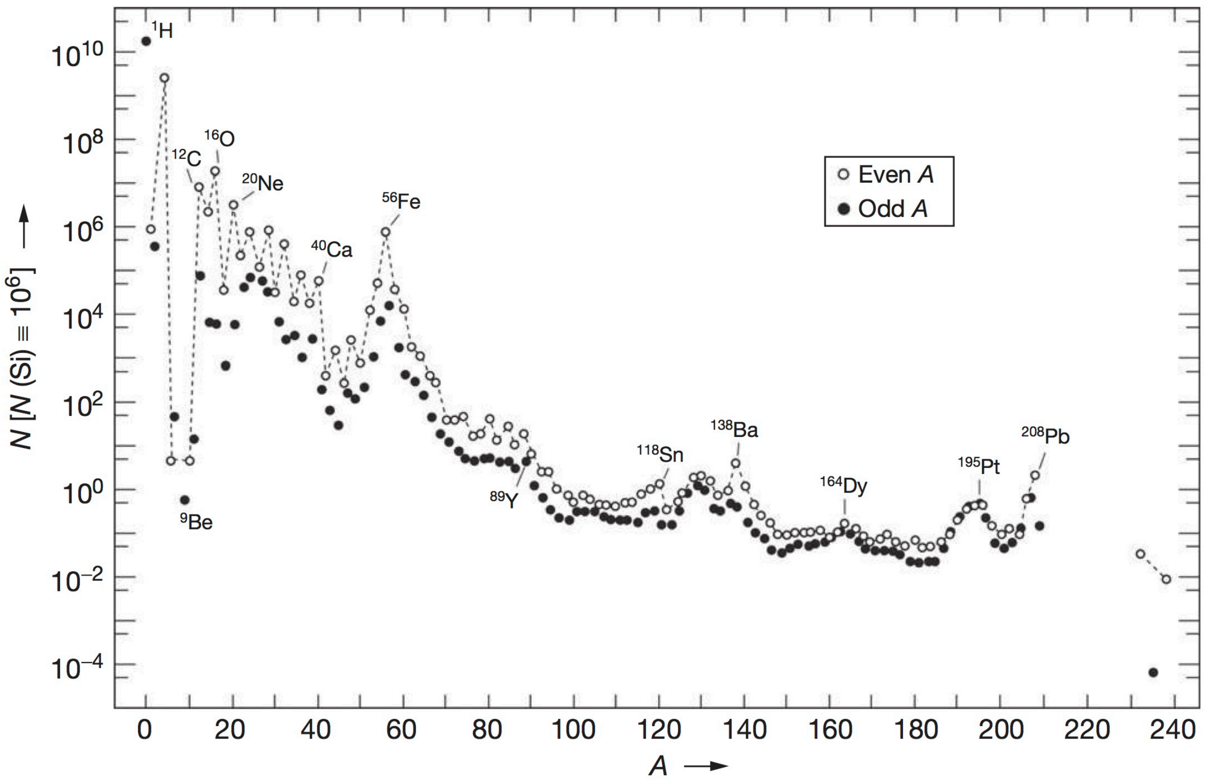
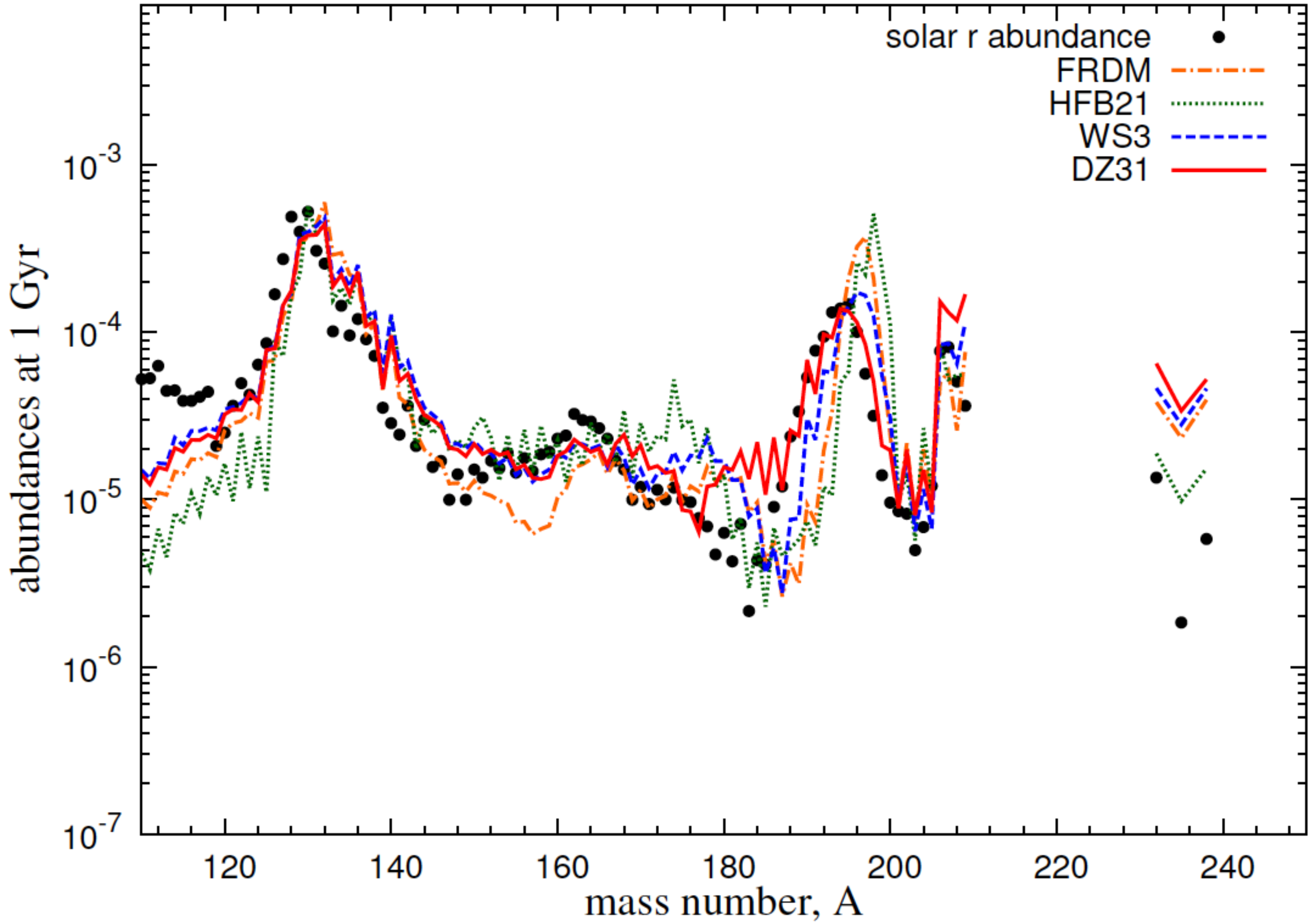


Figure 7 Solar system abundances by mass number. Atoms with even masses are more abundant than those with odd masses (Oddo–Harkins rule). There is no smooth dependence of abundances on mass numbers for even (e.g., ^{118}Sn and ^{138}Ba) or for odd masses (e.g., ^{89}Y).

H. Palme, K. Lodders, A. Jones, Solar System Abundances of the Elements (Elsevier 2014)

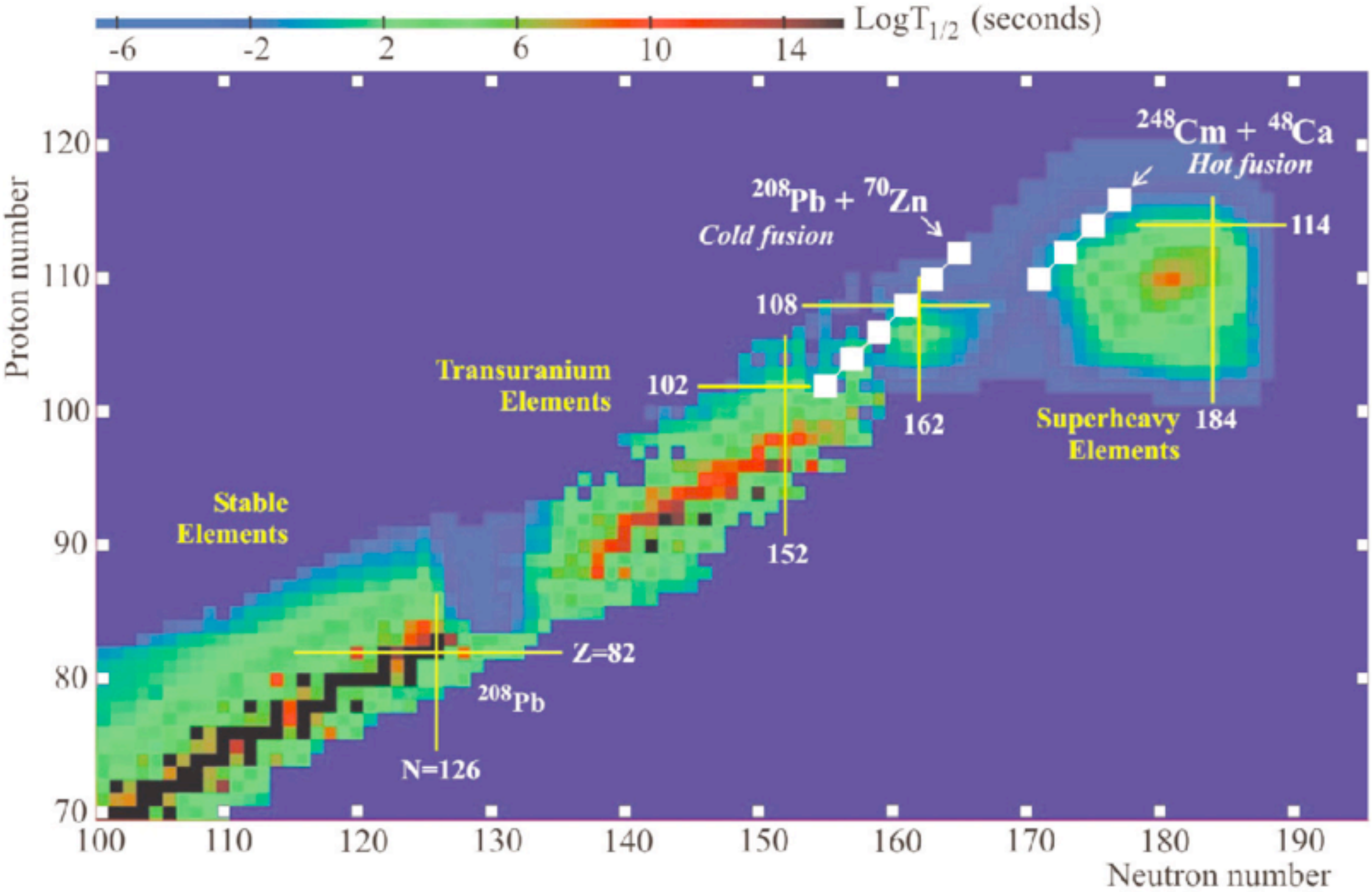
Elementsynthese aus der Verschmelzung von Neutronensternen: r-Prozess (Simulation)



Final mass-integrated r-process abundences obtained in a neutron star merger simulation using four different mass models compared to solar system r-process abundances [Mendoza-Temis et al. PRC 92 (2015) 055805].

J. Cowan, C. Sneden, J.E. Lawler A. Aprahamian, M. Wiescher, K. Langanke, G. Martinez-Pinedo, F.-K. Thielemann: "Making the heaviest elements in the Universe: A review of the rapid neutron capture process", arxiv:1901.01410

Island of Stability



Beth-Uhlenbeck formula

yields, partial densities

$$Y_{A,Z}^{(0)} \propto n_{A,Z}^{(0)} = g_{A,Z}\left(\frac{2\pi\hbar^2}{Am\lambda_T^{(0)}}\right)^{-3/2} e^{(B_{A,Z}+(A-Z)\lambda_n^{(0)}+Z\lambda_p^{(0)})/\lambda_T^{(0)}}$$

intrinsic partition function

$$R_{A,Z}^{\gamma}(\lambda_T) = 1 + \sum_i^{\mathrm{exc}} \frac{g_{A,Z,i}}{g_{A,Z}} e^{-E_{A,Z,i}/\lambda_T}$$

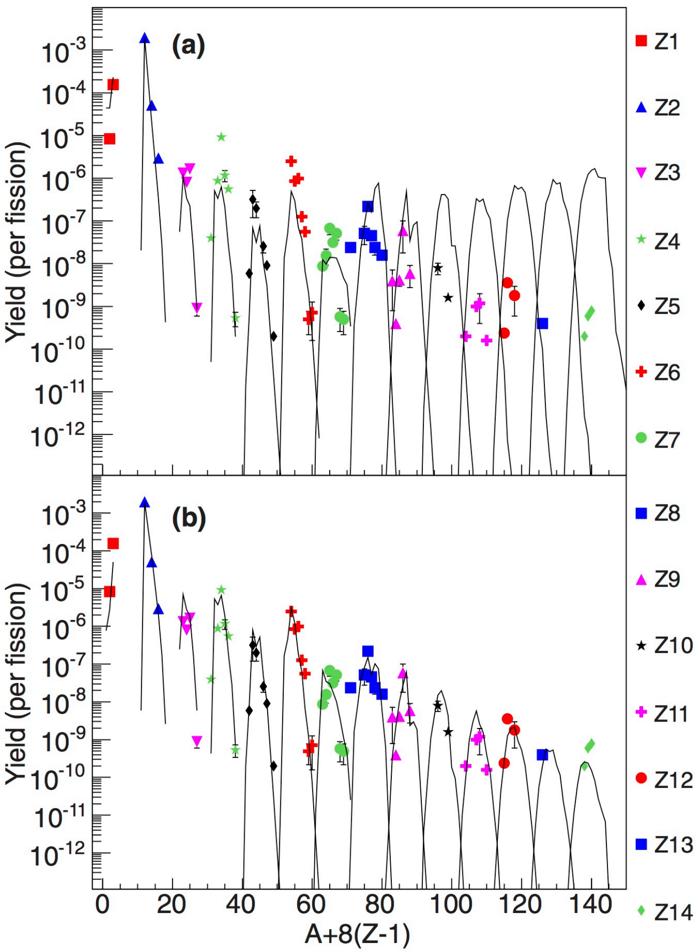
Inclusion of scattering states: Beth-Uhlenbeck formula: deuteron

$$R_d^{\mathrm{vir}}(\lambda_T) = 1 - e^{-E_d^{\mathrm{thresh}}/\lambda_T} + e^{-E_d^{\mathrm{thresh}}/\lambda_T}\frac{1}{\pi\lambda_T}\int_0^{\infty} dE e^{-E/\lambda_T}\delta_d(E)$$

effective binding energy: ^{4}H

$$R_{4,1}^{\mathrm{vir}}(\lambda_T) = e^{-E_{t,n}^{\mathrm{eff}}(\lambda_T,0)/\lambda_T + E_{4,1}^{\mathrm{thresh}}/\lambda_T}$$

Nucleation and cluster formation in low-density nucleonic matter



$$Y(A,\tau) = \frac{1}{2}\rho \exp\left[-\frac{G(A)}{T}\right] \times \operatorname{erfc}\left\{B(T,\sigma)\frac{[(A/A_c)^{1/3}-1]+\left(1-A_c^{-1/3}\right)\exp(-\tau)}{\sqrt{1-\exp(-2\tau)}}\right\}$$

$$\tau = \frac{3.967c\rho}{A_c^{2/3}\sqrt{T}}t$$

FIG. 2. (Color online) Yield per fission as a function of mass (A) and charge (Z) of products. Solid points represent $^{241}\mathrm{Pu}(n_{th}, f)$ experimental yields from Koester *et al.* [9]. Lines are theoretical predictions from NSE calculation [7]. NSE parameters are $T = 1.4$ MeV, $\rho = 4\times10^{-4}$ fm^{-3}, and $Y_p = 0.34$. (a) NSE calculation only. M^2 fit metric = 4.28. (b) NSE calculation with nucleation. Nucleation parameters are time = 6400 fm/c and $A_c = 5.4$. Fit metric = 1.18.

S. Wuenschel, H. Zheng, K. Hagel, B. Meyer, M. Barbui, E. J. Kim, G. Ropke, and J. B. Natowitz
PHYSICAL REVIEW C **90**, 011601(R) (2014)

Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

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(Received 13 September 2000; published 12 February 2001)

Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\text{gain}}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\text{loss}}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots .$$

loss rate

in-medium breakup transition operator

$$\mathcal{K}_d^{\text{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN\to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \qquad (3)$$

breakup cross section

$$\sigma_{\text{bu}}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3\rangle|^2 \times 2\pi\delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \qquad (4)$$

Quantum statistical approach

The total density as well as the DoS are given by the spectral function A,

$$n_e^{\rm total}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{f}_e(\omega) A_e(1,\omega) = \int_{-\infty}^{\infty} d\omega \hat{f}_e(\omega) D_e(\omega)$$

$$A(1,\omega) = 2\,{\rm Im}\,G(1,\omega - i0) = 2\,{\rm Im}[\omega - E(1) - \Sigma(1,\omega - i0)]^{-1}$$

A cluster decomposition for the self-energy is possible so that a quasiparticle (free) contribution can be separated,

$$A_e(1,\omega) \approx \frac{2\pi\,\delta(\omega - E_e^{\rm quasi}(1))}{1 - \frac{d}{dz}{\rm Re}\,\Sigma_e(1,z)|_{z=E_e^{\rm quasi}-\mu_e}} - 2{\rm Im}\,\Sigma_e(1,\omega+i0)\frac{d}{d\omega}\frac{\mathcal{P}}{\omega+\mu_e - E_e^{\rm quasi}(1)}$$

We obtain the generalized Beth-Uhlenbeck formula (quasiparticles)

$$n_e^{\rm total}(T,\mu_e,\mu_a) = \frac{1}{\Omega}\sum_1 f_e(E^{\rm quasi}(1))$$

$$+\frac{1}{\Lambda^3}\sum_{i,\gamma} Z_i e^{\beta\mu_i}\left[\sum_{\nu}^{\rm bound}(e^{-\beta E_{i,\gamma,\nu}} - 1) + \frac{\beta}{\pi}\int_0^\infty dE e^{-\beta E}\left\{\delta_{i,\gamma}(E) - \frac{1}{2}\sin[2\delta_{i,\gamma}(E)]\right\}\right]$$

In-medium Schrödinger equation for $E_{i,\gamma,\nu}(T,\mu)$, $\delta_{i,\gamma}(T,\mu)$, channel (spin…) γ

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Thouless criterion
for quantum condensate:

$E_{n,P=0}(T,\mu) = 4\mu$

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Physical properties

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

ν: internal quantum number
excited states, continuum correlations

- Medium effects: correct behavior near saturation
self-energy and Pauli blocking shifts of binding energies,
Coulomb corrections due to screening (Wigner-Seitz, Debye)

Relevant statistical operator

State of the system in the past $\mathrm{Tr}\{\rho(t) B_n\} = \langle B_n \rangle^t$

Construction of the relevant statistical operator at time t

$$S_{\mathrm{rel}}(t) = -k_{\mathrm{B}}\, \mathrm{Tr}\{\rho_{\mathrm{rel}}(t) \log \rho_{\mathrm{rel}}(t)\}$$ -> maximum

$$\delta[\mathrm{Tr}\{\rho_{\mathrm{rel}}(t) \log \rho_{\mathrm{rel}}(t)\}] = 0 \qquad \mathrm{Tr}\{\rho_{\mathrm{rel}}(t) B_n\} \equiv \langle B_n \rangle^t_{\mathrm{rel}} = \langle B_n \rangle^t$$

Generalized Gibbs distribution

$$\rho_{\mathrm{rel}}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$$

equations of state to eliminate $\lambda_n(t)$

$$\Phi(t) = \log \mathrm{Tr} \exp\left\{-\sum_n \lambda_n(t) B_n\right\}$$

$$\frac{\partial S_{\mathrm{rel}}(t)}{\partial t} = \sum_n \lambda_n(t) \langle \dot{B}_n \rangle^t$$

But: von Neumann equation?
Entropy?

Nonequilibrium statistical operator (NSO)

principle of weakening of initial correlations (Bogoliubov, Zubarev)

$$\rho_\epsilon(t) = \epsilon \int_{-\infty}^{t} e^{\epsilon(t_1 - t)} U(t, t_1) \rho_{\rm rel}(t_1) U^\dagger(t, t_1)\, dt_1$$

time evolution operator $U(t, t_0)$ relevant statistical operator $\rho_{\rm rel}(t)$

selection of the set of relevant observables $\{B_n\}$

self-consistency relations $\mathrm{Tr}\{\rho_{\rm rel}(t) B_n\} \equiv \langle B_n \rangle^t_{\rm rel} = \langle B_n \rangle^t$

maximum of information entropy $S_{\rm rel}(t) = -k_{\rm B}\, \mathrm{Tr}\{\rho_{\rm rel}(t) \log \rho_{\rm rel}(t)\}$

generalized Gibbs distribution $\rho_{\rm rel}(t) = \exp\left\{-\Phi(t) - \sum_n \lambda_n(t) B_n\right\}$

von Neumann equation

$$\frac{\partial}{\partial t} \varrho_\varepsilon(t) + \frac{i}{\hbar}[H, \varrho_\varepsilon(t)] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right)$$

$$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$$

Expanding nuclear matter: freeze-out and reaction processes (feed-down)