

# Theoretical model of post acceleration and focalization of protons produced with TNSA

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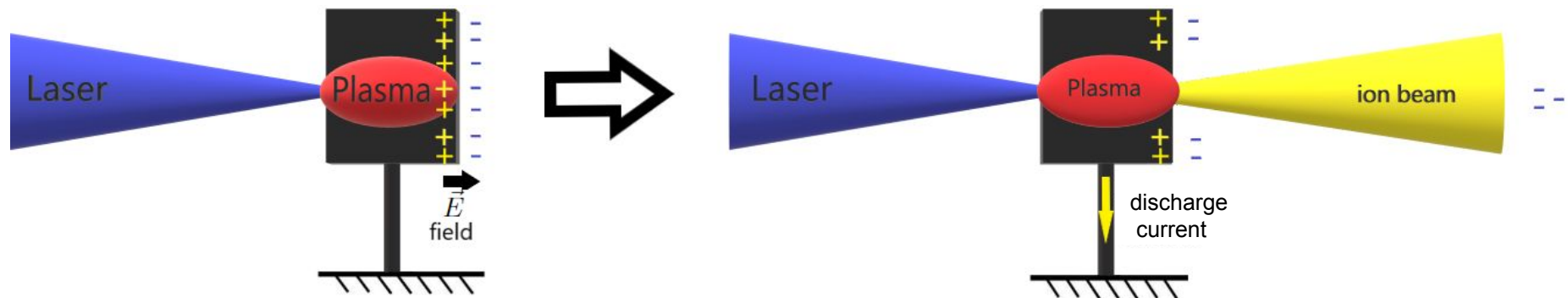


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# Introduction to Target Normal Sheath Acceleration (TNSA)



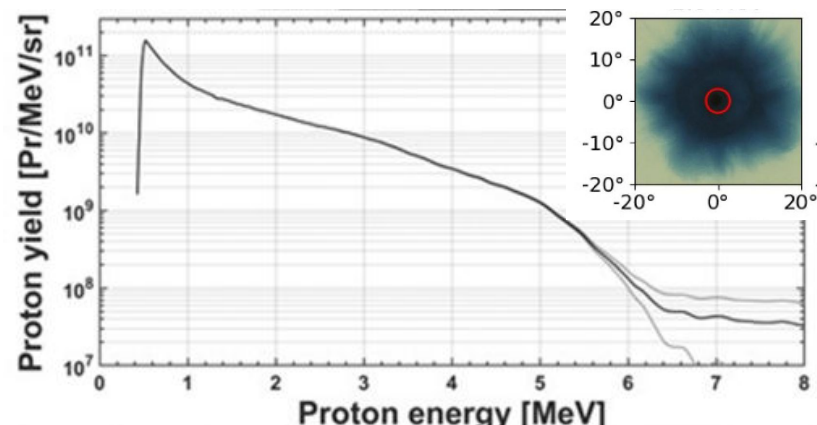
- 1) Laser ASE/prepulse => plasma
- 2) Main pulse => electron acceleration and creation of the sheath

- 1) Hot electrons escape
- 2) The charge separation electric field accelerates some protons (TV/m) [1,2]
- 3) The charging of the target induces a discharge current

[1] RA Snavely and al. PRL,85(14) :2945, 2000  
[2] S. Wilks and al. Phys.Plasmas, 8 :542-549, 2001.

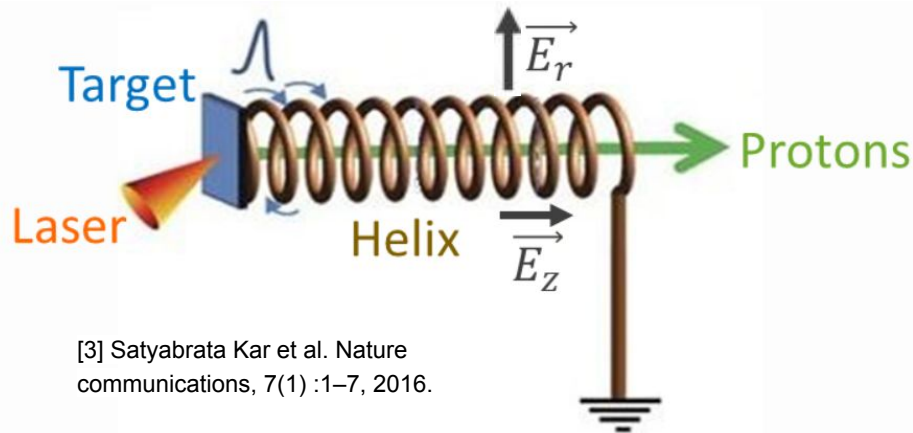
## Problems for applications

- Charge flux limited (dispersed due to opening angle)
- Maximum proton energy too low for certain applications (record ~100 MeV)
- Energy spectrum decreases exponentially



# Adding a helical coil

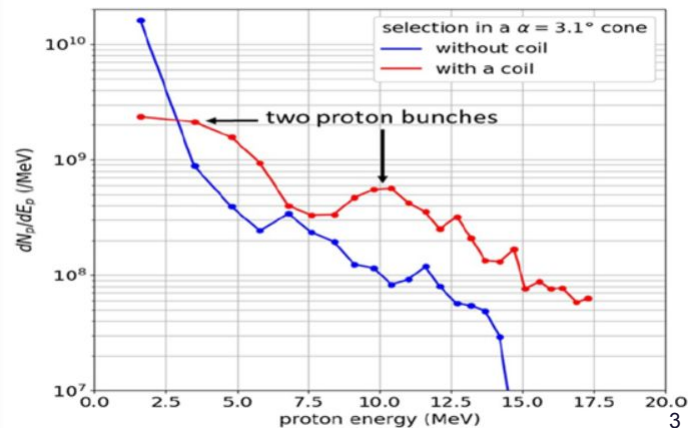
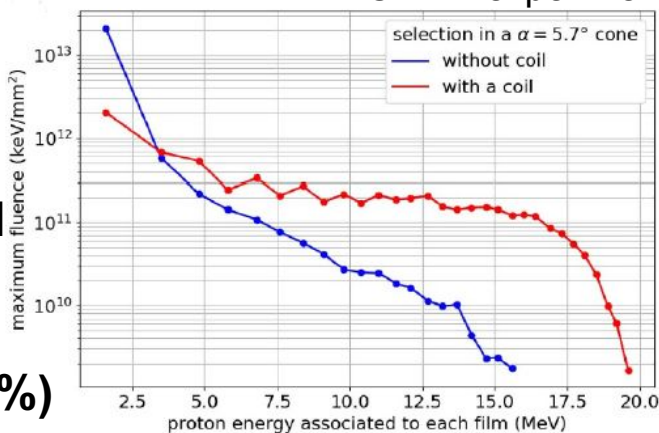
-Creation of E and B fields which post-accelerate, focus and bunch the protons (GV/m)



[3] Satyabrata Kar et al. Nature communications, 7(1) :1-7, 2016.

[4] M Bardon et al. Plasma Physics and Controlled Fusion, 62(12) :125019, 2020.

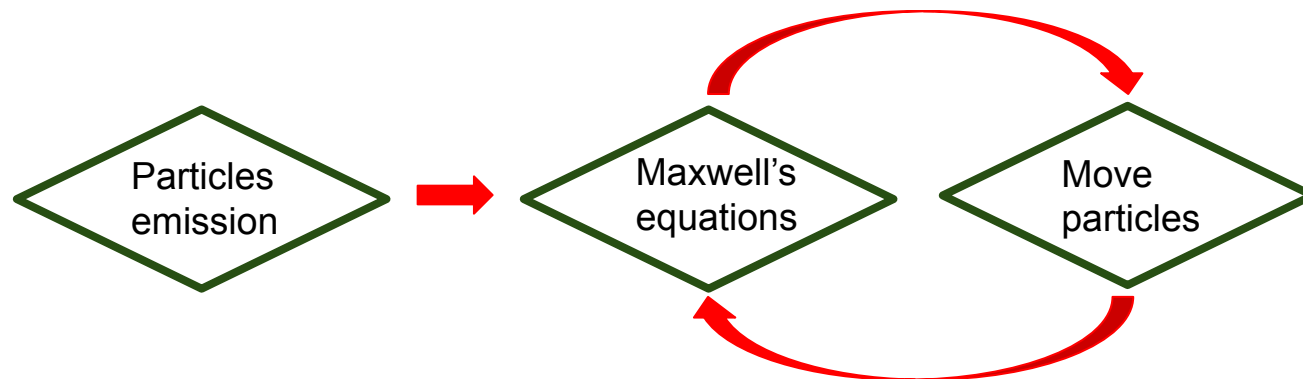
PACMAN experiment at LULI2000 Facility [4]



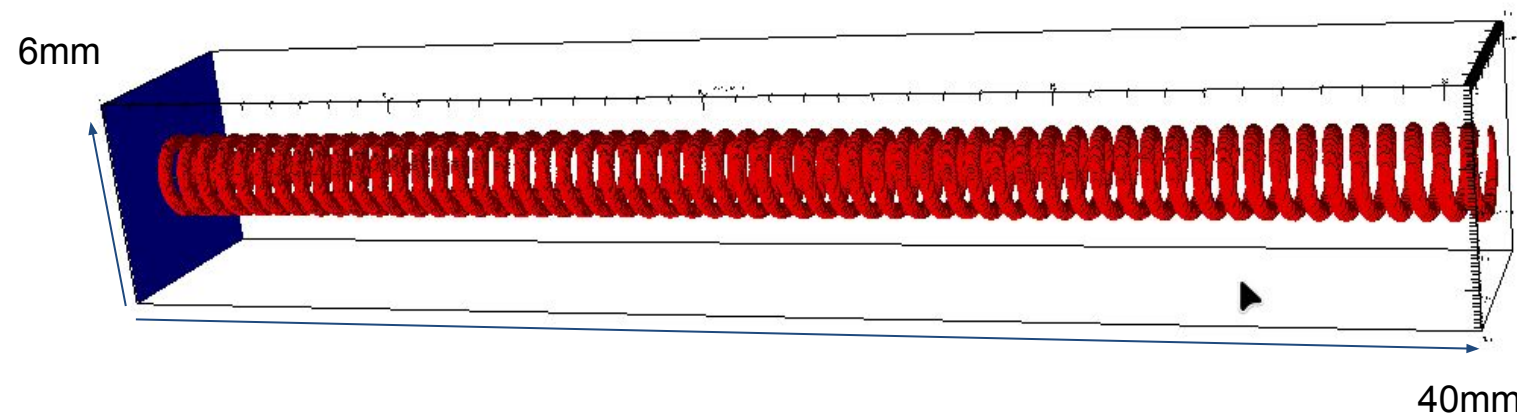
**-Good focusing**

**-Post-acceleration and bunching is limited**

**-Limited efficiency (15%)**



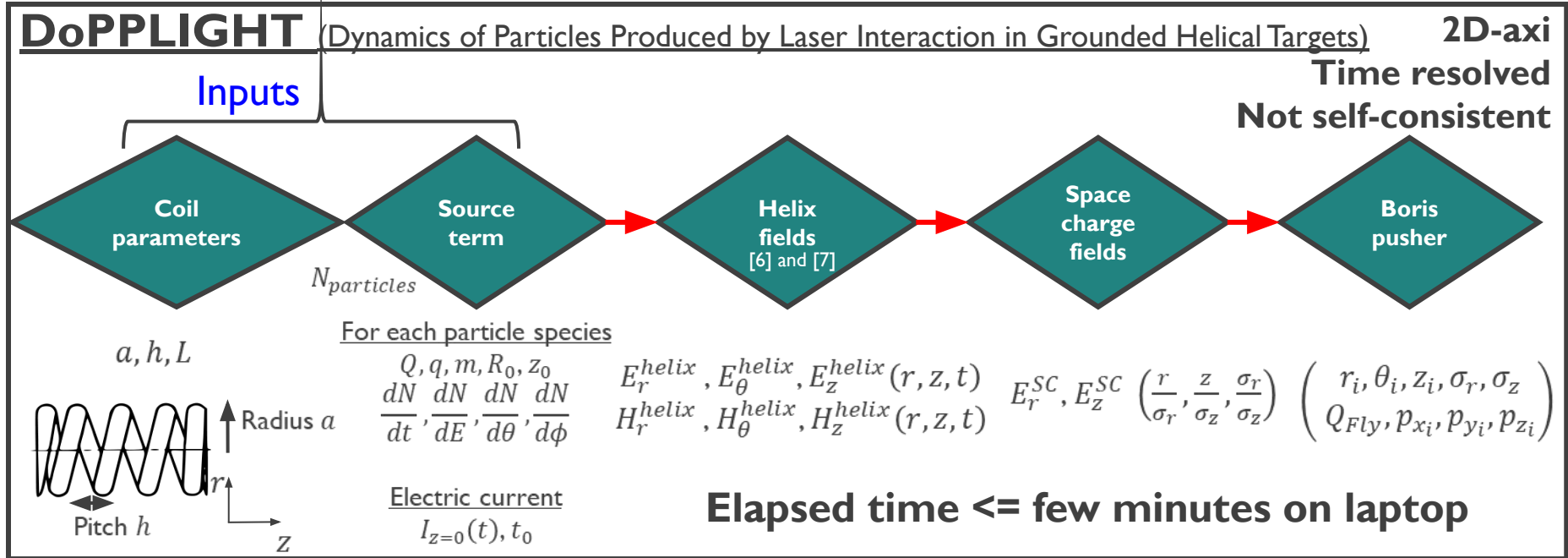
- Wide area
- Boundary conditions permit the propagation of fields in matter



**-Reference code**

**-High precision**

**-High cost in time calculation**

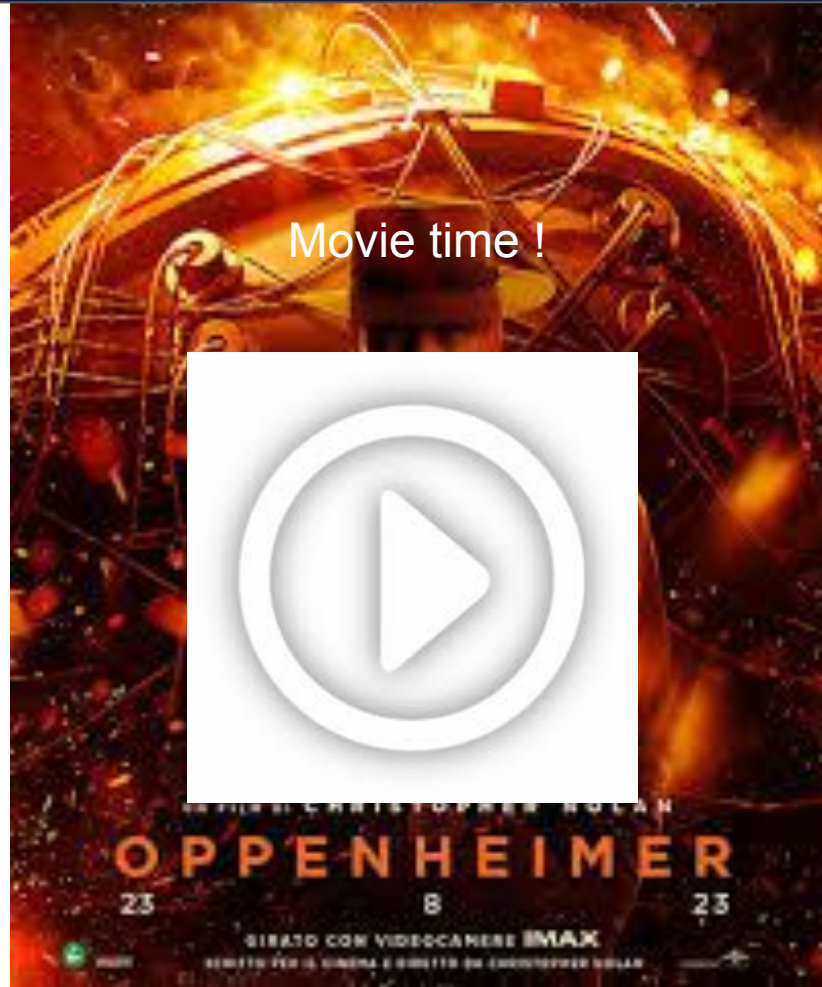


[6] John Robinson Pierce. The bell System technical journal, 29(2) :189–250, 1950.

[7] GS Kino and SF Paik. Journal of Applied Physics, 33(10) :3002–3008,1962.

**-Run in few minuts**

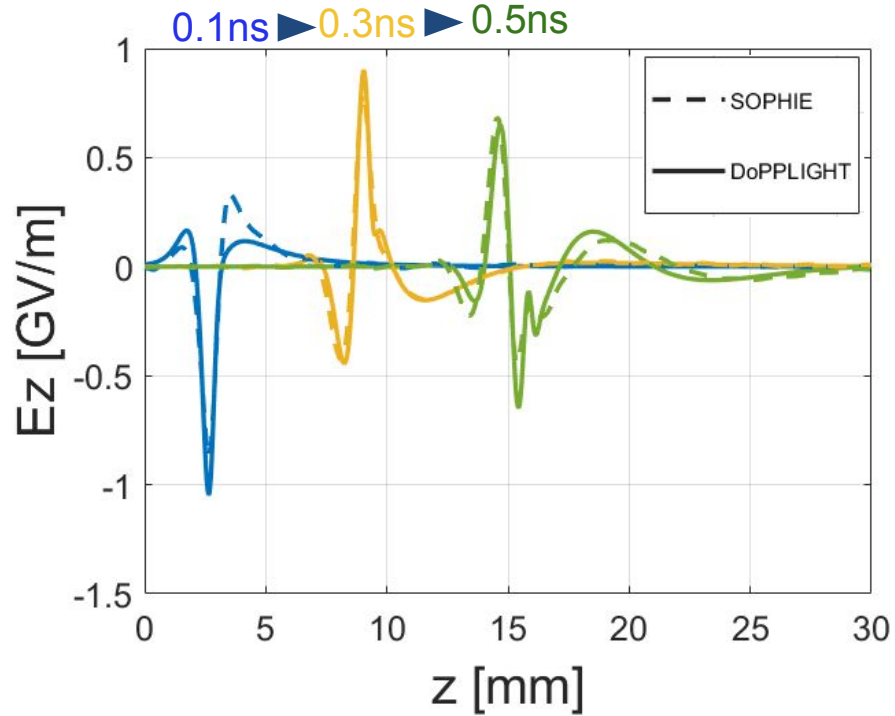
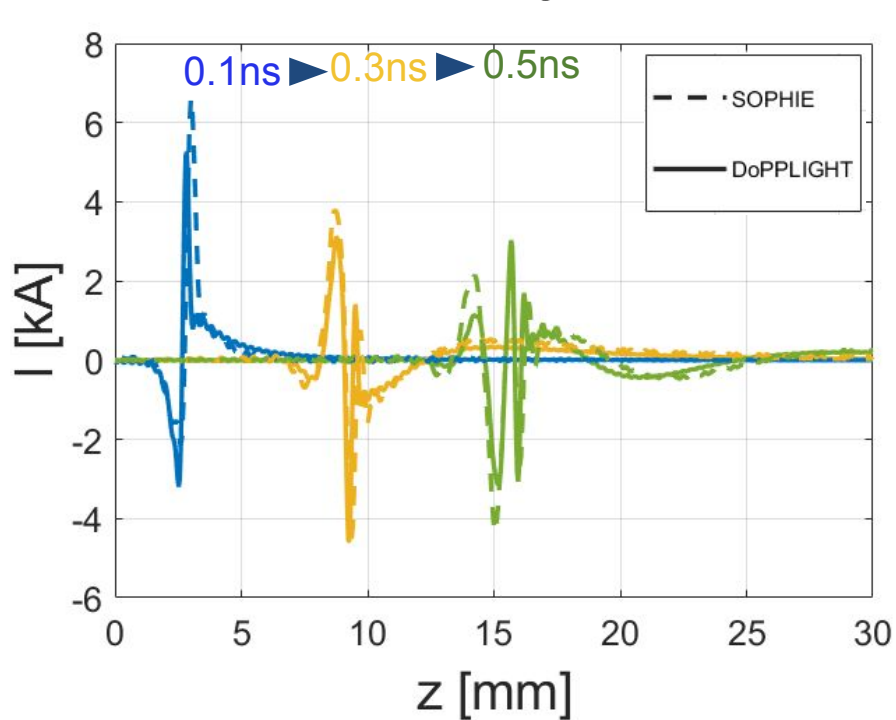
**-Less precise than SOPHIE**



Movie time !

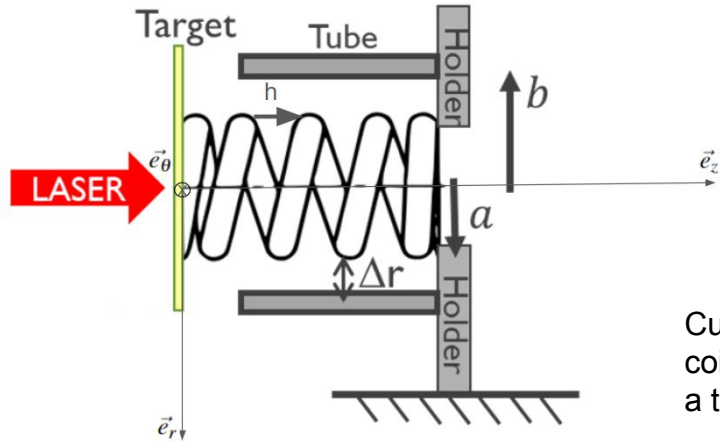


Current and longitudinal field at  $r=0.2$  mm for  $a=0.5$  mm  $h=0.35$  mm

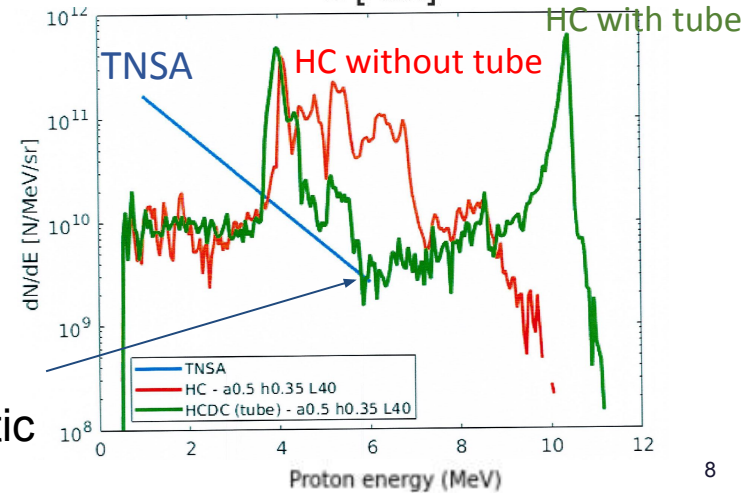
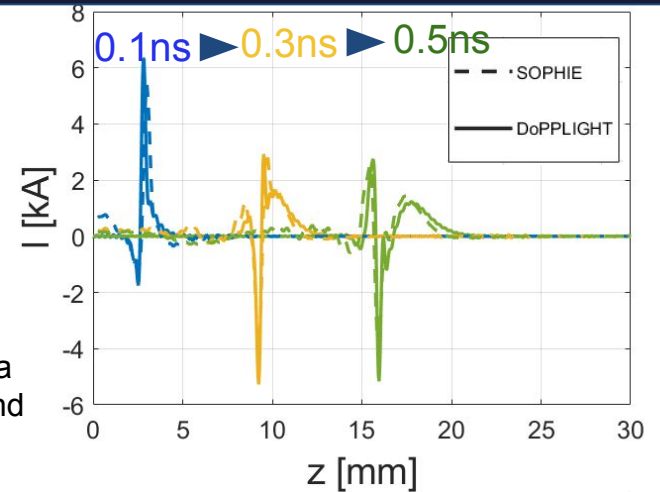


**acceleration and deceleration alternation of electric field**

# Adding a tube around the helical coil



Current and proton spectra for a coil of  $a=0.5$  mm  $h=0.35$  mm and a tube of radius  $b=0.7$ mm



## Helical coil with tube [8]:

-Strongly reduces the current dispersion. [9-10]

- Creation of strong bunches on proton spectra (around the geometric energy)

[8] A Hirsch-Passicos, CLC Lacoste et al. PRE accepted 2024

[9] JP Freund et al. IEEE transactions on plasma science, 20(5) :543–553, 1992.

[10] Han S Uhm et al. Journal of Applied Physics, 53(12):8483–8488, 1982

characteristic energy

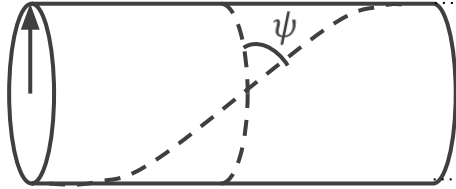


# Varying helical coil with tube - theoretical model

\*mobility at ELI beamlines  
with Vladimir Tikhonchuk

$$\psi = \arctan\left(\frac{h}{2\pi a}\right)$$

Radius  
 $a$



vacuum

vacuum

$$E_z^-(r=a) = E_z^+(r=a)$$

$$B_{\parallel}^+ = B_{\parallel}^-$$

$$E_{\parallel} = 0$$

$$B_{\perp}^+ - B_{\perp}^- = \frac{\mu_0 i(\omega, z)}{2\pi a \tan^2(\Psi)}$$

Boundary conditions

Continuity equation and propagation equation

Boundary conditions

Maxwell's equations

$i(z, t)$

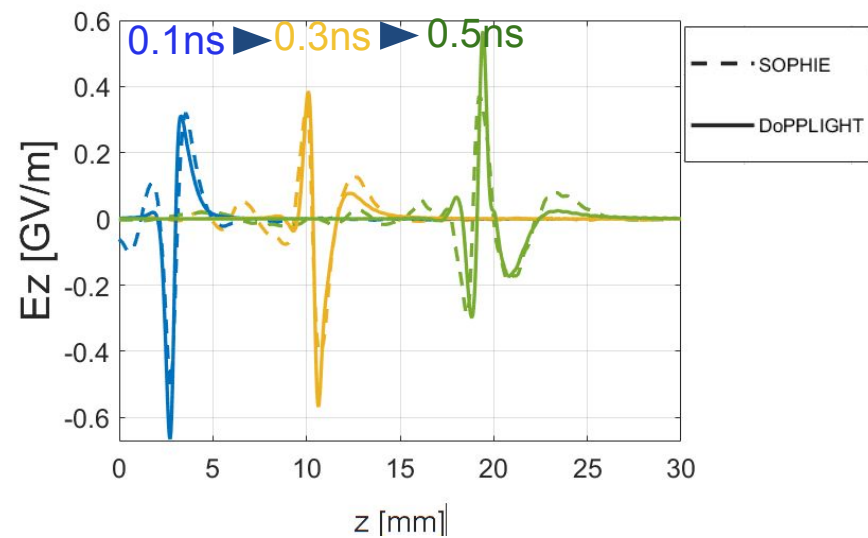
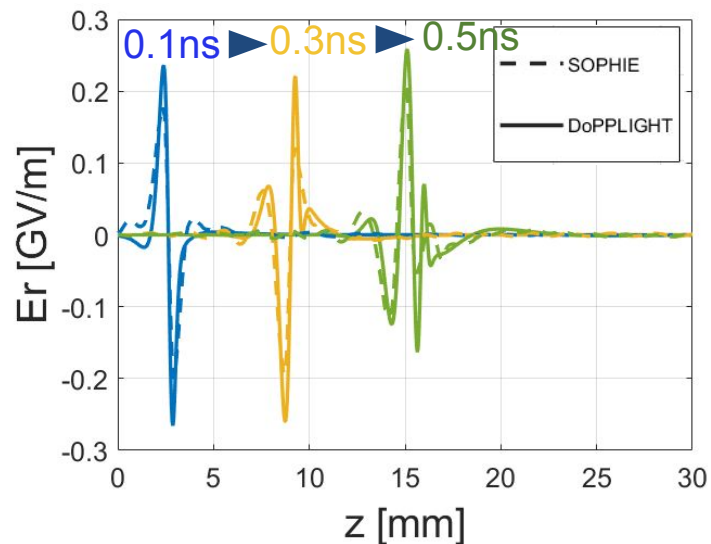
Constant of integration, dispersion relation  
and link between current and field

$E(r, z, t)$  and  $B(r, z, t)$

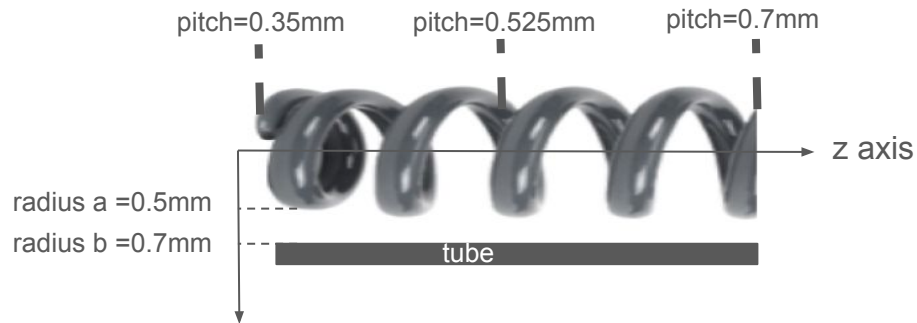
Theoretical model

$$E_z(r, z, t) = \frac{\mu_0 c}{2\pi^2 a} \int_0^{+\infty} \frac{j i_0(\omega) \omega \sqrt{k(\omega, z=0)} \kappa(\omega)}{\alpha \sqrt{k(\omega, z)}} \frac{I_1(\alpha) [K_1(\alpha b) I_1(\alpha) - I_1(\alpha b) K_1(\alpha)]}{I_0(\alpha) [I_1(\alpha) I_1(\alpha b) K_0(\alpha) + I_0(\alpha) I_1(\alpha b) K_1(\alpha)]} \cot^3(\Psi) \cos(\Psi) I_0(\alpha r) e^{j \int_0^z k(\omega, z_\eta) dz_\eta - \omega t} d\omega$$

# Varying helical coil with tube - Theoretical results

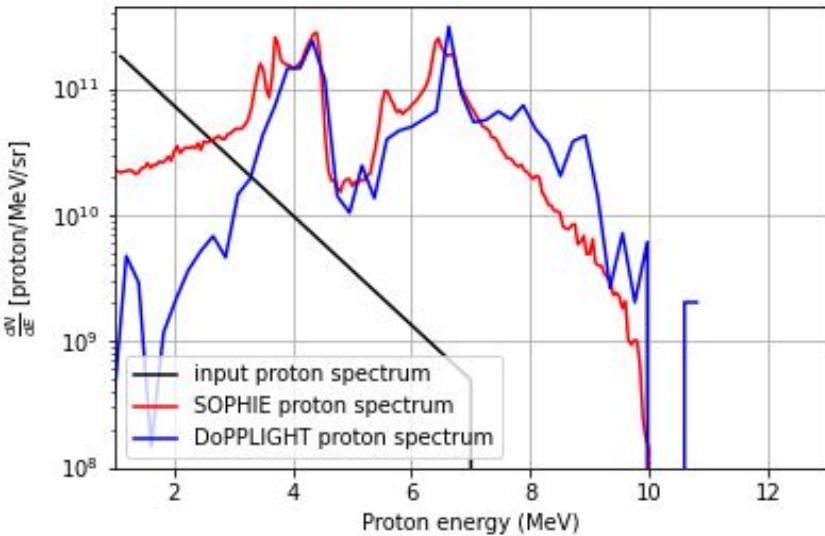


Length of helical coil  $L=40$ mm for ALLS facility

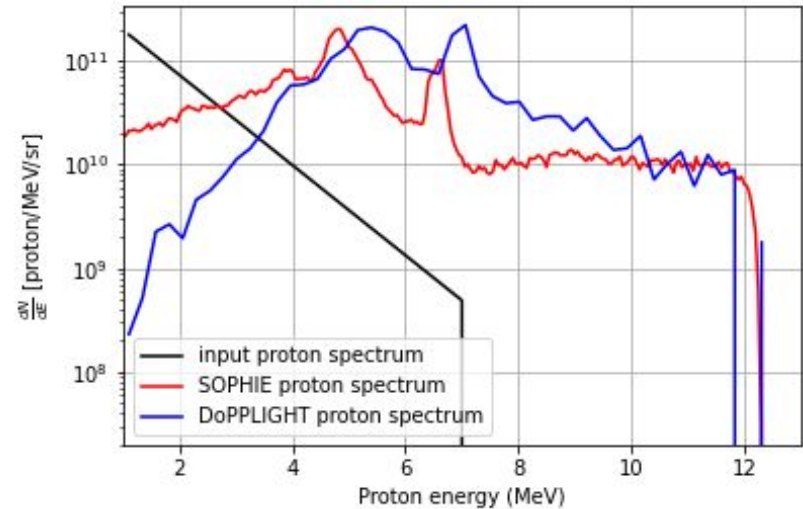


**good agreement !**

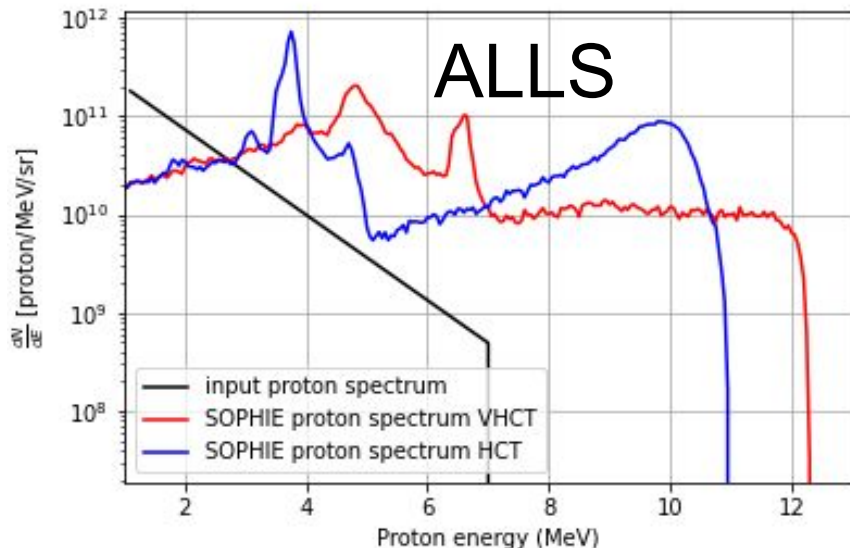
Varying helical coil



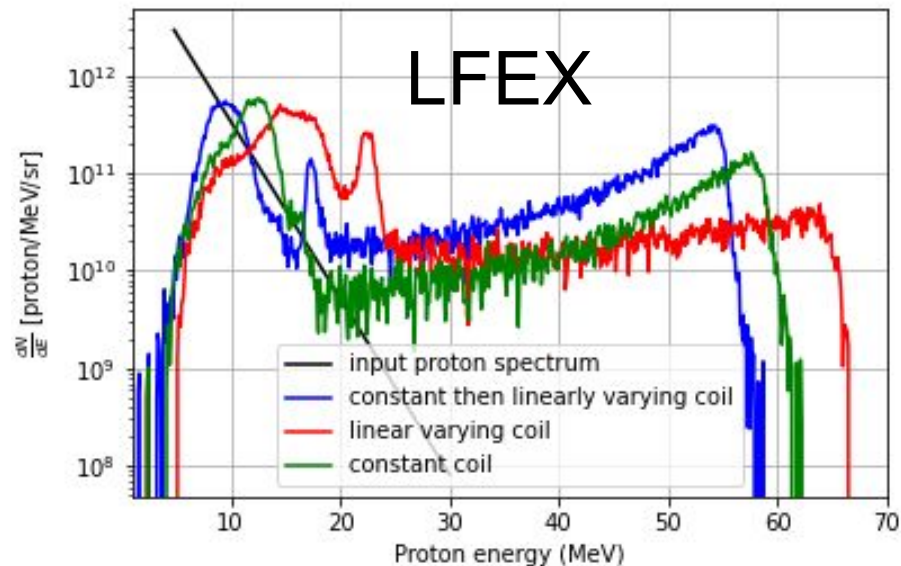
Varying helical coil with tube



**Good agreement !**

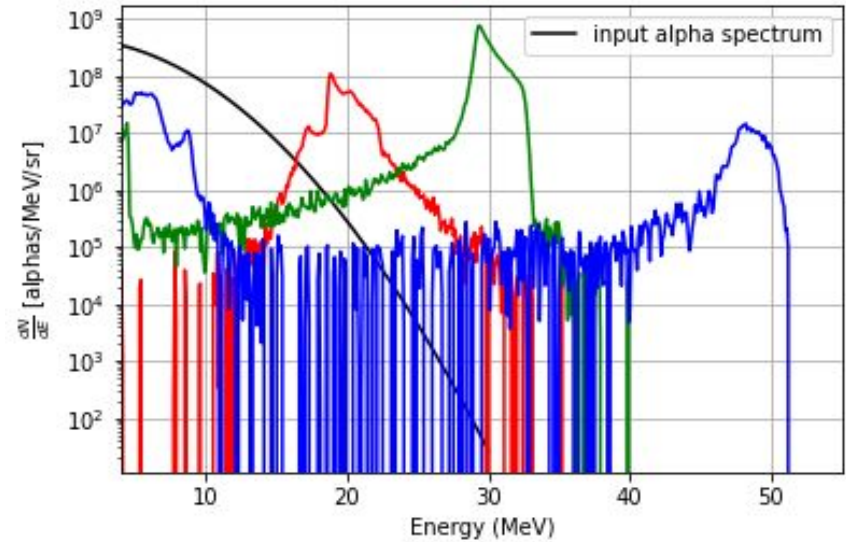
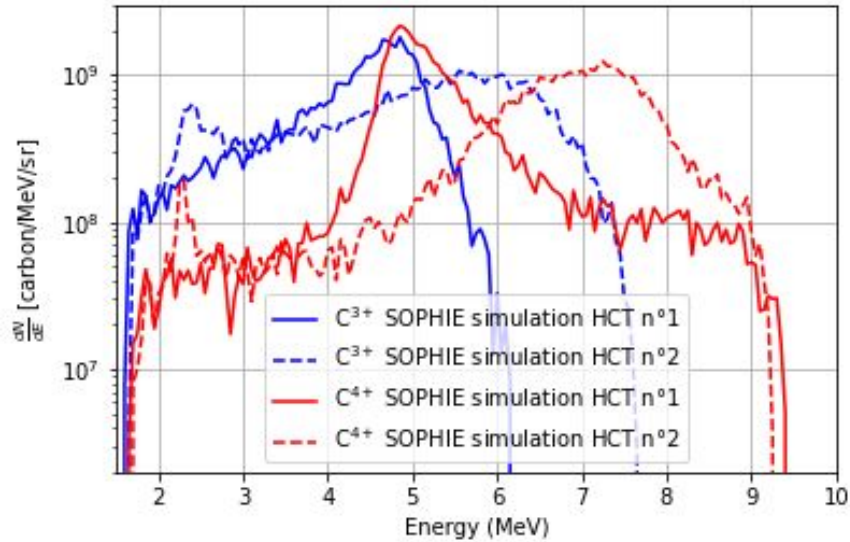


- **Doubles the cut-off energy 7 MeV to 12 MeV**
- **Increased number of protons per steradian  $E > 3.5$  MeV :**
  - \*TNSA =  $10^{10}$  protons/sr
  - \*Blue =  $3.7 \times 10^{11}$  protons/sr
  - \*Red =  $3 \times 10^{11}$  protons/sr
- **High repetition rate**
- **Proof of concept**

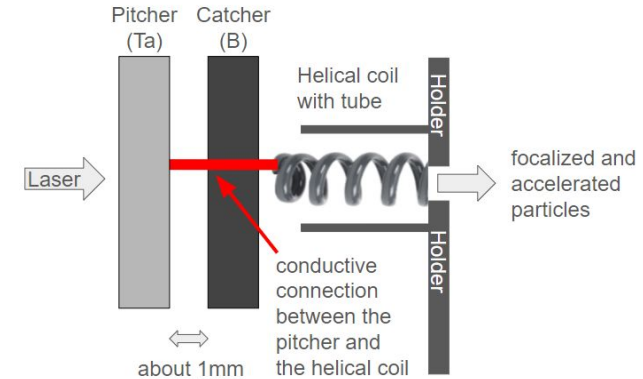


- **Doubles the cut-off energy 30 MeV to 60 MeV**
- **Increased number of protons per steradian  $E > 15$  MeV:**
  - \*TNSA =  $10^{11}$  protons/sr
  - \*Blue =  $2.7 \times 10^{12}$  protons/sr
  - \*Red =  $2.7 \times 10^{12}$  protons/sr
  - \*Green =  $1.3 \times 10^{12}$  protons/sr

## Some results



- Higher radius and lower pitch coil
- Bunch and post acceleration of heavy ions

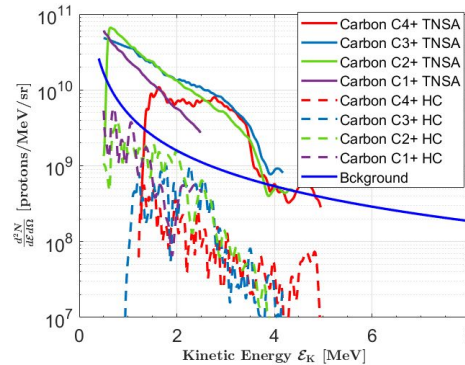


## Simulation results

- Flux charge increased for  $E > 3 \text{ MeV}$
- Double the cut-off energy
- Bunch in energy of protons
- Bunch and post-acceleration of heavy ions

## Computation

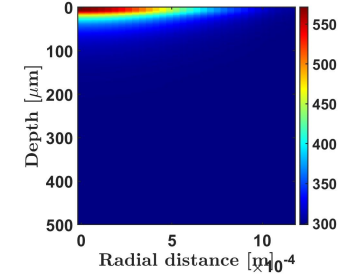
- Varying geometry developed and implement in DoPPLIGHT
- Experimental results



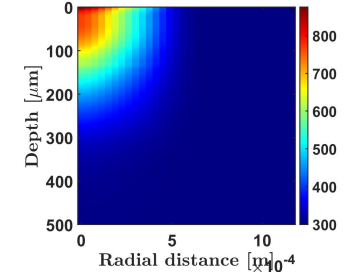
## Work in progress

- Optimization of varying helical coil
- Increase the energy of bunch
- Increase the energy cut-off
- **Experimental proof to validate our simulations**
- Application to heavy ions
- Application to isochoric heating

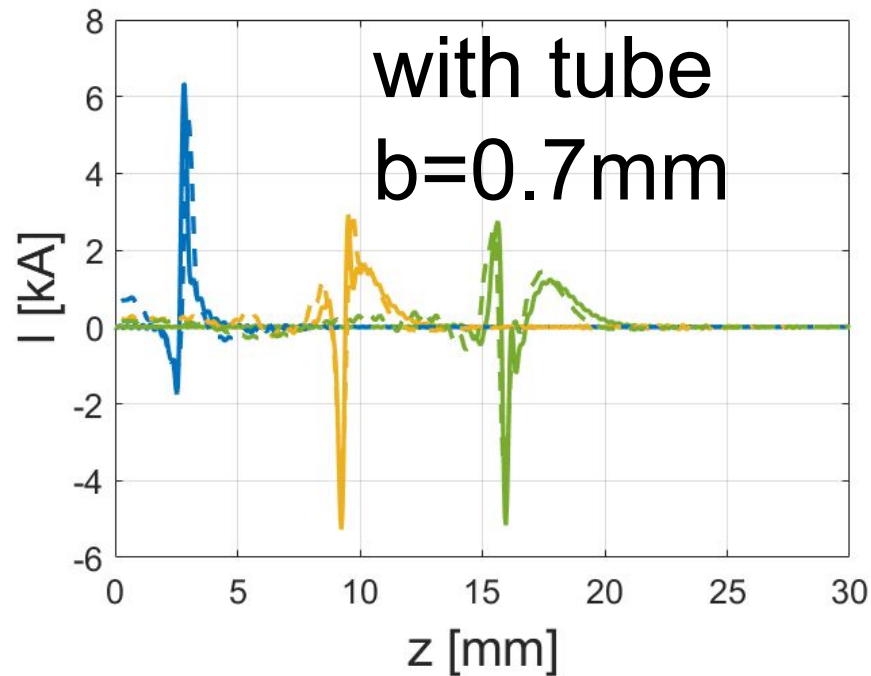
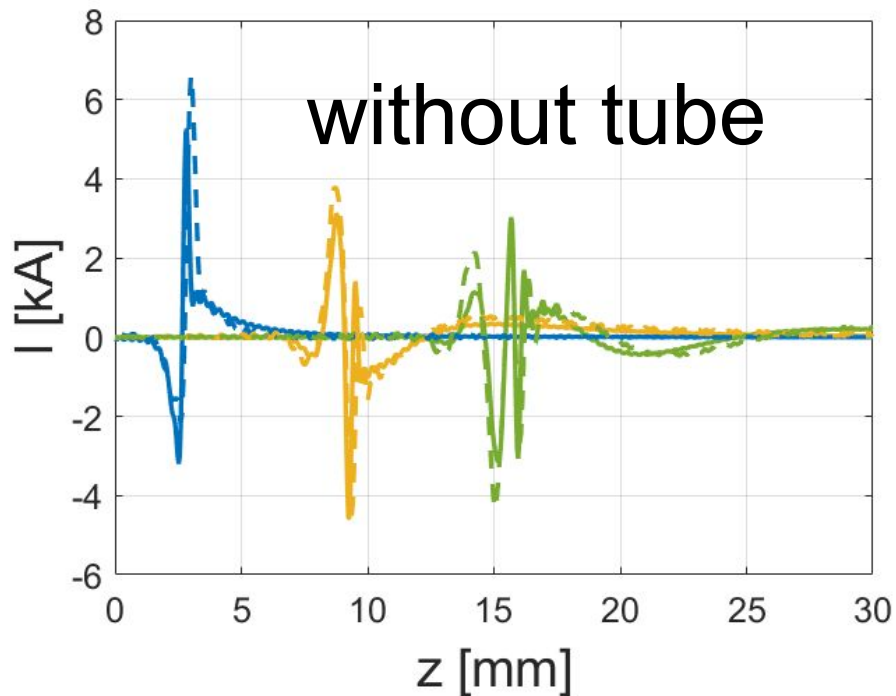
Projected temperature distribution in Al



Projected temperature distribution in Al

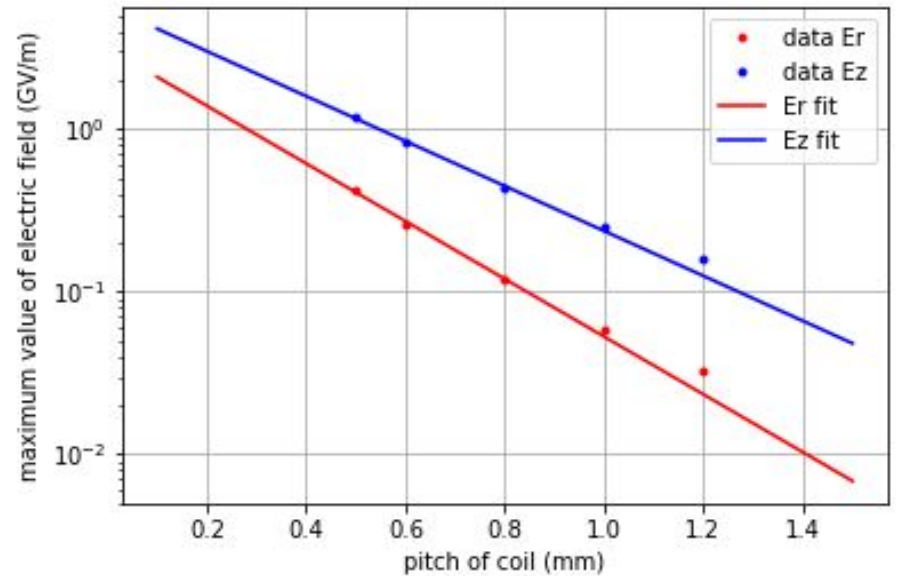
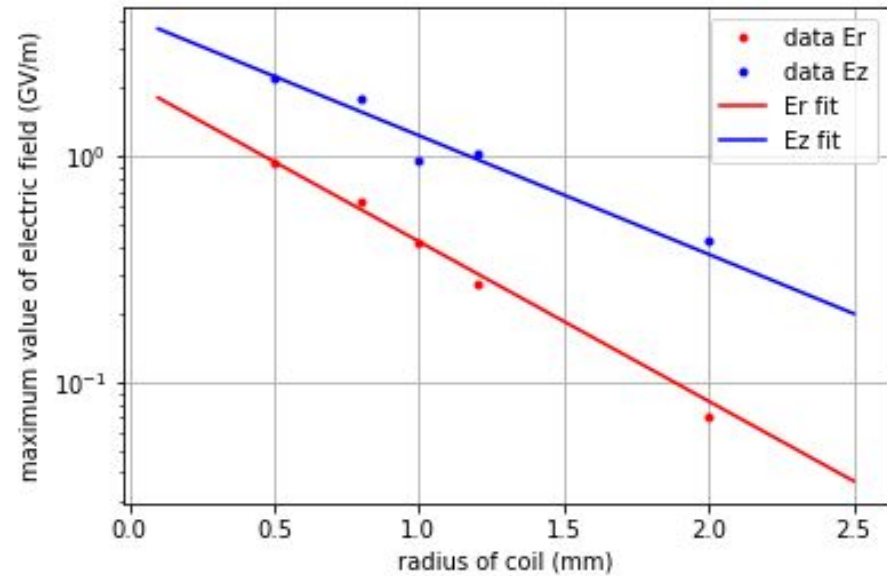


Current at  $r=0.2$  mm for  $a=0.5$  mm  $h=0.35$  mm



**acceleration and deceleration alternation of electric field**

# Propagation of the current along Helical coil axis





Dans le cadre d'un circuit à résistance nulle l'équation différentielle du courant est :

$$\frac{\partial^2 j(\omega, z)}{\partial z^2} + k^2(\omega, z)j(\omega, z) = 2j_0(\omega)\delta'(z)$$

Et considérant une solution de la forme :

$$j(\omega, z) = A(\omega, z)e^{i\phi(\omega, z)}$$

On obtient une solution similaire à l'approximation WKB:

$$j = j_0 \frac{\sqrt{k_0 \kappa(\omega)}}{\sqrt{k}} e^{i \int_0^z k(\omega, z, \eta) dz \eta}$$

Pour les champs, les équations différentielles sont:

$$-\Delta \vec{E} + \frac{\vec{\nabla} \rho}{\epsilon_0} = -\frac{\partial(\mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t})}{\partial t}$$

$$-\Delta \vec{B} = \mu_0 \vec{\nabla} \wedge \vec{J} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$$

Par la méthode de séparation de variables :

$$E_z(\omega, r, z) = f(r)g(z) \text{ et } B_z = h(r)l(z)$$

On résout donc pour  $E_z$  et  $B_z$ , puis grâce à Maxwell-Gauss et Maxwell-Faraday, il vient:

$$\begin{cases} B_r(\omega, r, z) = \frac{-1}{r} \frac{\partial l(z)}{\partial z} \int_0^r r h(r) dr \\ E_\theta(\omega, r, z) = \frac{j\omega}{r} l(z) \int_0^r r h(r) dr \\ B_z(\omega, r, z) = h(r)l(z) \end{cases}$$

$$\begin{cases} E_r(\omega, r, z) = \frac{-1}{r} \frac{\partial g(z)}{\partial z} \int_0^r r f(r) dr \\ B_\theta(\omega, r, z) = \frac{j}{r\omega} \frac{\partial^2 g(z)}{\partial z^2} \int_0^r r f(r) dr + \frac{j}{\omega} \frac{\partial f(r)}{\partial r} g(z) \\ E_z(\omega, r, z) = f(r)g(z) \end{cases}$$

On résout pour  $f(r)$ ,  $h(r)$ ,  $g(z)$  et  $l(z)$ , il vient alors :

$$\begin{cases} E_r(\omega, r, z) = \frac{\sqrt{k(\omega, z=0)}\sqrt{k(\omega, z)}}{\alpha} \Lambda(\omega, z) e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [-B_1 I_1(\alpha r) + B_2 K_1(\alpha r)] \\ B_\theta(\omega, r, z) = \frac{j\sqrt{k(\omega, z=0)}\omega}{\alpha\sqrt{k(\omega, z)}c} \aleph(\omega, z) e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [B_1 I_1(\alpha r) - B_2 K_1(\alpha r)] \\ E_z(\omega, r, z) = \frac{\sqrt{k(\omega, z=0)}}{\sqrt{k(\omega, z)}} e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [B_1 I_0(\alpha r) + B_2 K_0(\alpha r)] \end{cases}$$

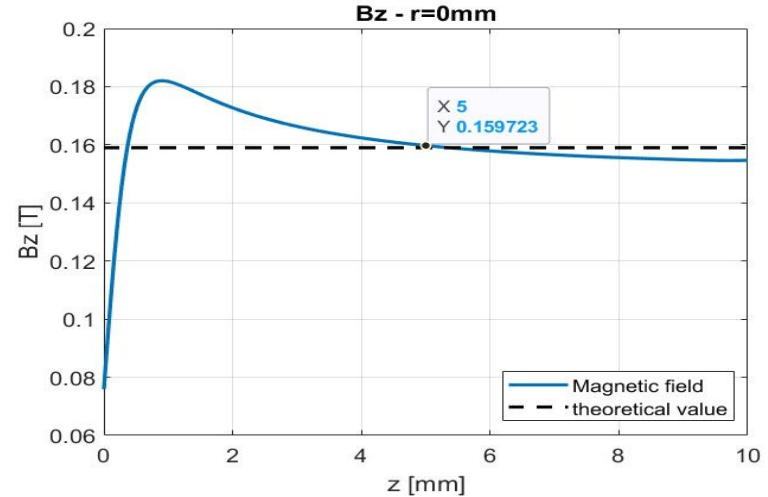
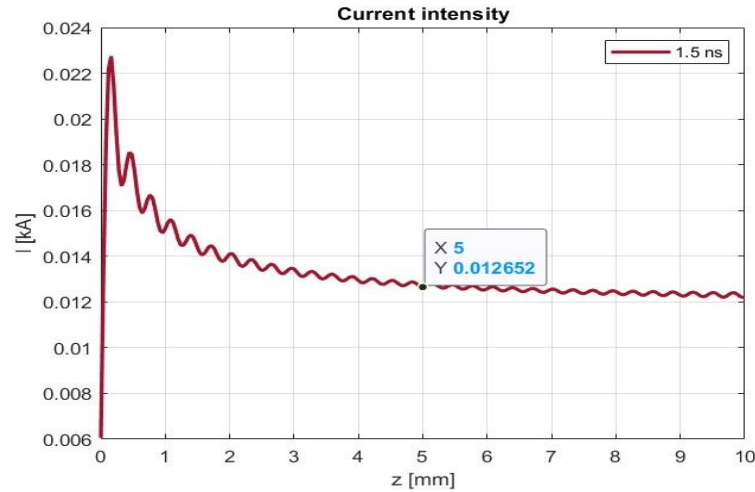
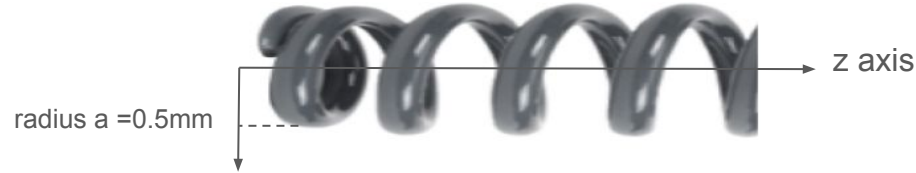
$$\text{avec } \aleph(\omega, z) = -\frac{k''(\omega, z)}{2k(\omega, z)\omega^2} + \frac{3(k'(\omega, z))^2}{4k^2(\omega, z)\omega^2} - 1$$

$$\text{et } \Lambda(\omega, z) = j - \frac{k'(\omega, z)}{2k^2(\omega, z)}$$

$$\begin{cases} B_r(\omega, r, z) = \frac{\sqrt{k(\omega, z=0)}\sqrt{k(\omega, z)}}{\alpha} \Lambda(\omega, z) e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [-B_3 I_1(\alpha r) + B_4 K_1(\alpha r)] \\ E_\theta(\omega, r, z) = j\frac{\omega\sqrt{k(\omega, z=0)}c}{\alpha\sqrt{k(\omega, z)}} e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [B_3 I_1(\alpha r) - B_4 K_1(\alpha r)] \\ B_z(\omega, r, z) = \frac{\sqrt{k(\omega, z=0)}}{\sqrt{k(\omega, z)}} e^{j \int_0^z k(\omega, z_\eta) dz_\eta} [B_3 I_0(\alpha r) + B_4 K_0(\alpha r)] \end{cases}$$

# Varying helical coil with tube - theoretical validation

pitch=0.1mm little pitch => approximation of a coil



Theoretical validation ; theoretical value of coil  $B_z(r, z, t) = \frac{\mu_0 N I_{\theta_0}}{L}$