The Nature of QCD Phase Transitions

From cumulants to the Metropolis Algorithm

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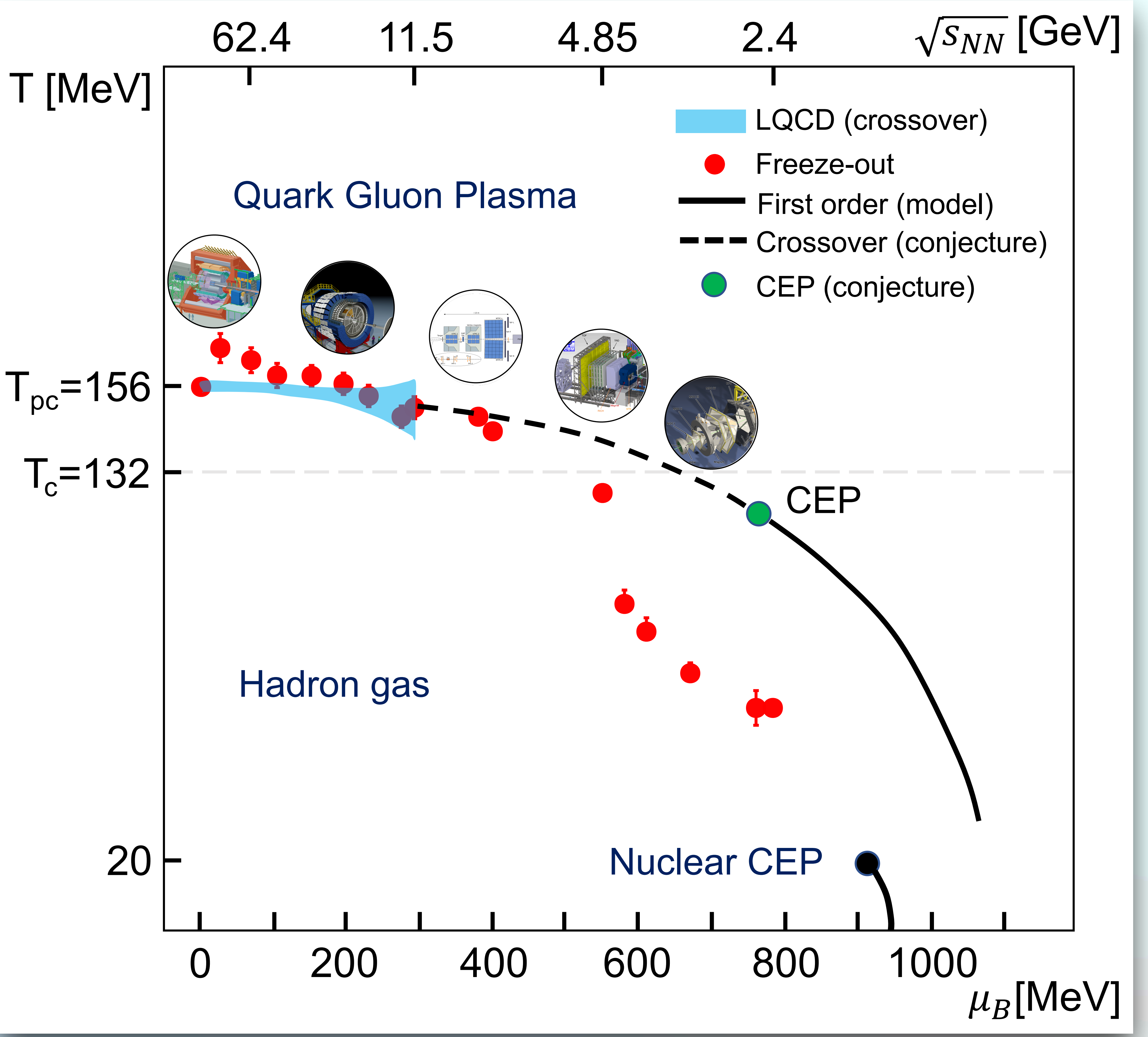
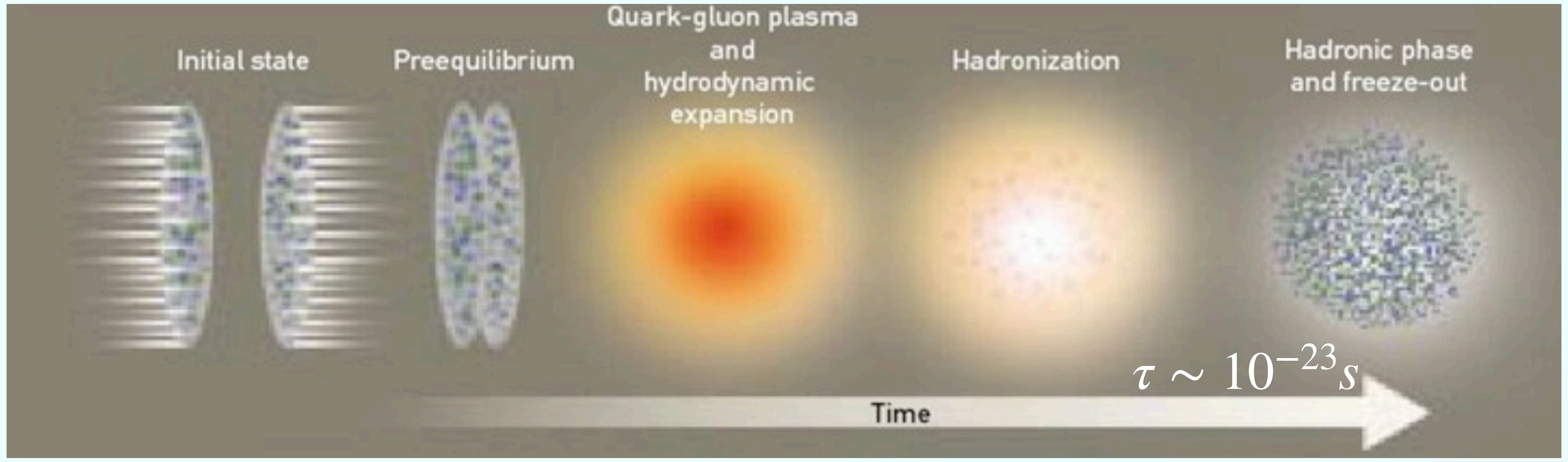
Phase diagram of QCD matter





$T_{fo}^{ALICE} = 156.5 \pm 1.5 \pm 3$ MeV(sys)

$\mu_B \approx 0, \; V \approx 5280 \text{ fm}^3$ (in one unit of rapidity) $\quad V_{Pb} \approx 1200 \text{ fm}^3$

$T_C^{LQCD} = 156.5 \pm 1.5$ MeV

for a thermal system of fixed volume V and temperature T

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} \equiv \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} = \frac{\partial^n P/T^4}{\partial\left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

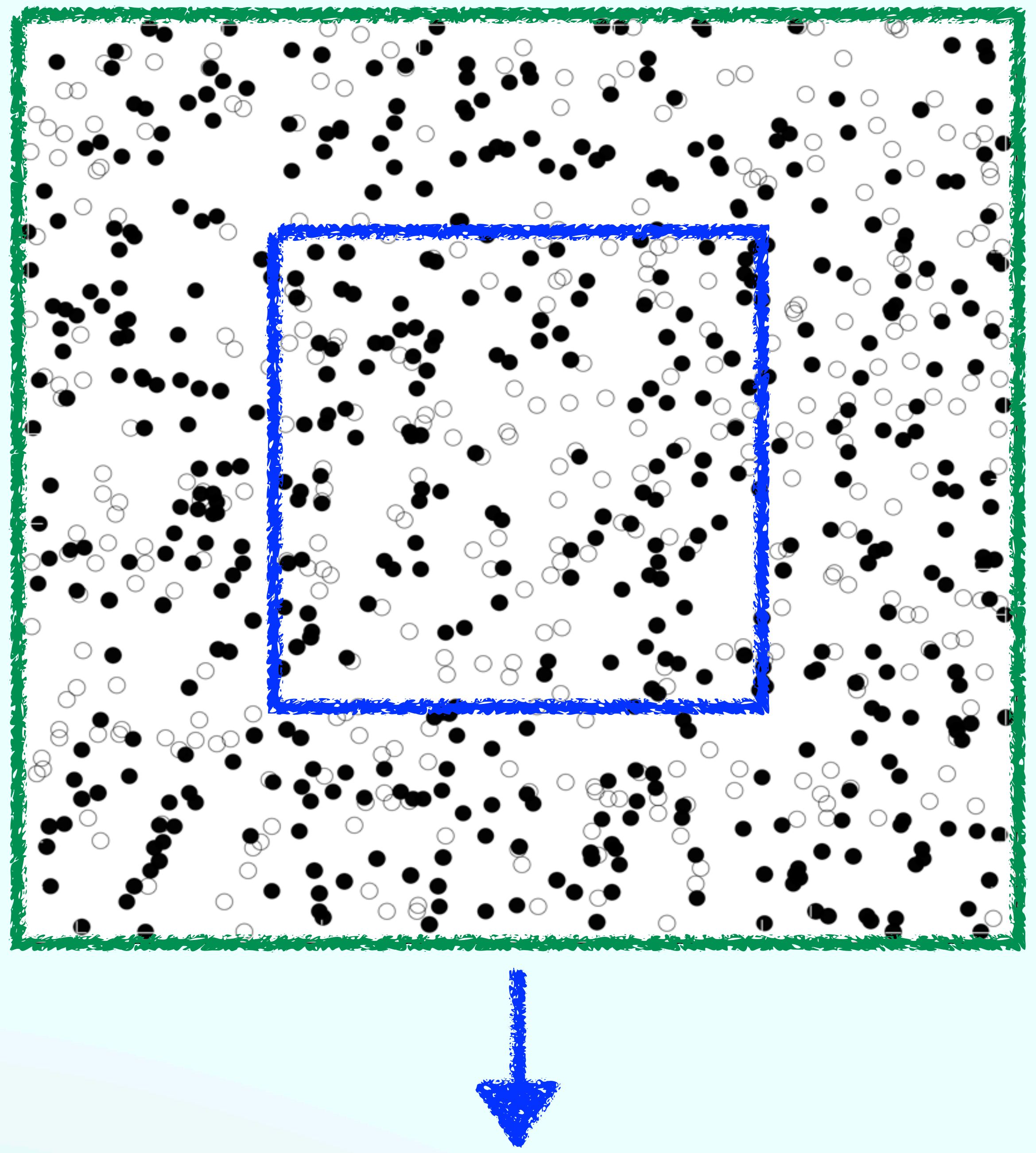
experiment **theory**

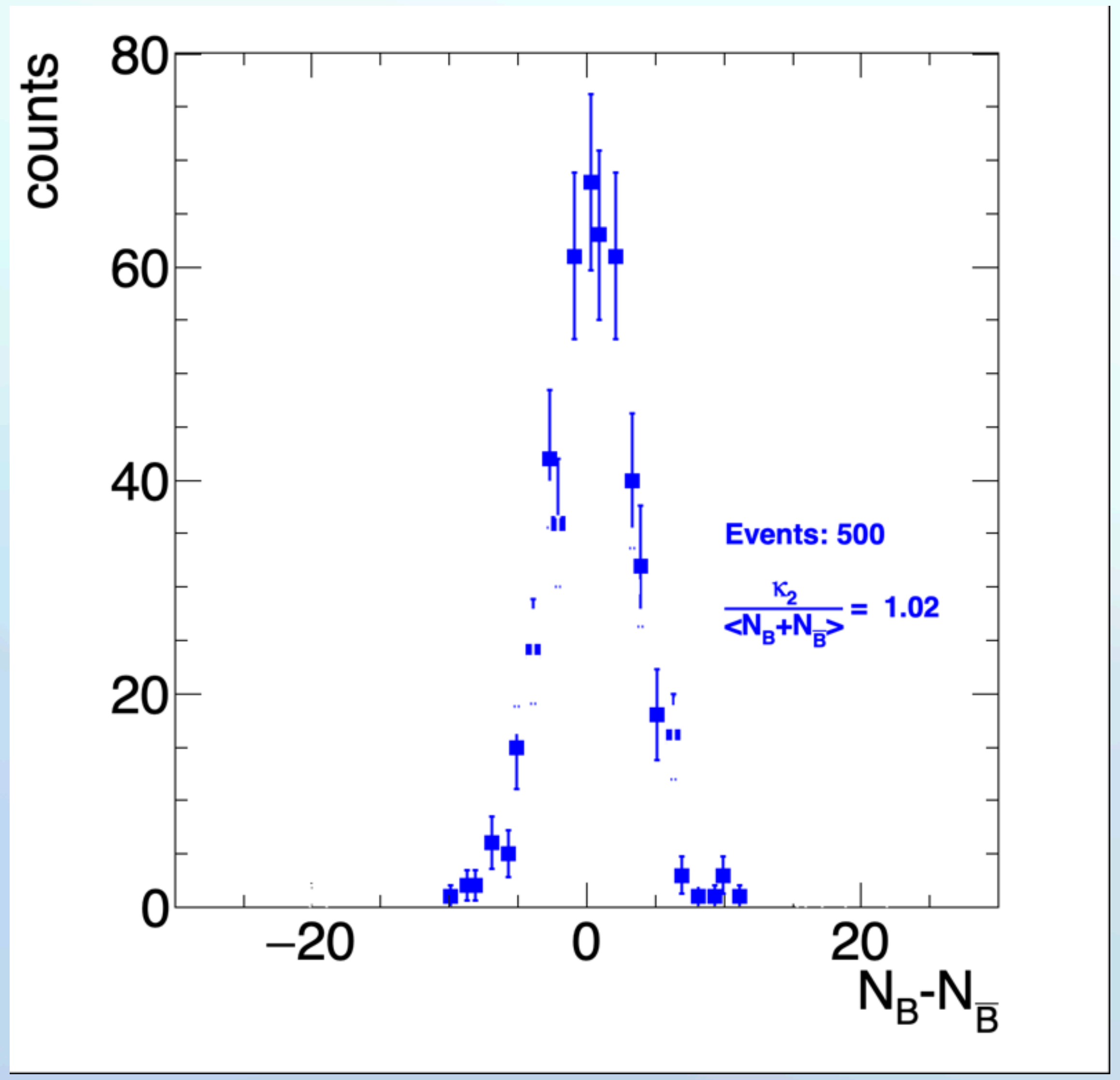
P. Braun-Munzinger, A.R., J. Stachel, 50 years of QCD, EPJ C 83 (2023) 1125

FO: A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)

lQCD: A. Bazavov et al., (HotQCD), PLB 795 (2019) 15-21

Probing EoS with event-by-event fluctuations





- $\Delta N = N_B - N_{\bar{B}}$ occurs with probability $p(\Delta N)$ (measured)
- r^{th} order central moment: $\mu_r = \sum_{\Delta N} (\Delta N - \langle \Delta N \rangle)^r p(\Delta N)$
- first 4 cumulants: $\kappa_1 = \langle \Delta N \rangle,\ \kappa_2 = \mu_2,\ \kappa_3 = \mu_3,\ \kappa_4 = \mu_4 - 3\mu_2^2$

$$\frac{\kappa_n(N_B - N_{\bar{B}})}{VT^3} \equiv \frac{1}{VT^3}\frac{\partial^n lnZ(V,T,\mu_B)}{\partial\left(\mu_B/T\right)^n} = \frac{\partial^n P/T^4}{\partial\left(\mu_B/T\right)^n} \equiv \hat{\chi}_n^B$$

experiment | **theory**

Experiment:
- *"volume"* fluctuates
- exact conservations
- measures **net-protons**

Theory:
- volume is fixed
- conservations on average
- predicts for **net-baryons**

Experimental challenges

- Volume fluctuations
- Conservation laws
- First ALICE results

Experiment vs. Theory

- Canonical Thermodynamics
- Comparison to STAR results
- Metropolis algorithm
- Comparison to ALICE results
- Outlook

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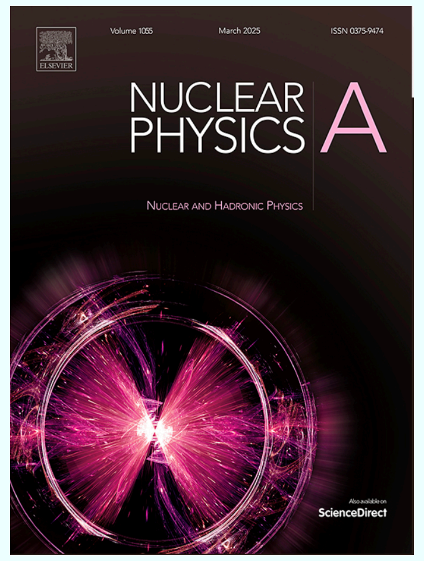
Bridging experiment with theory



Bridging the gap between event-by-event fluctuation measurements and theory predictions in relativistic nuclear collisions, P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 (2017) 114-130.

fixed volume/sources

$$\langle N \rangle \sim V, \quad \langle N \rangle \sim N_W$$

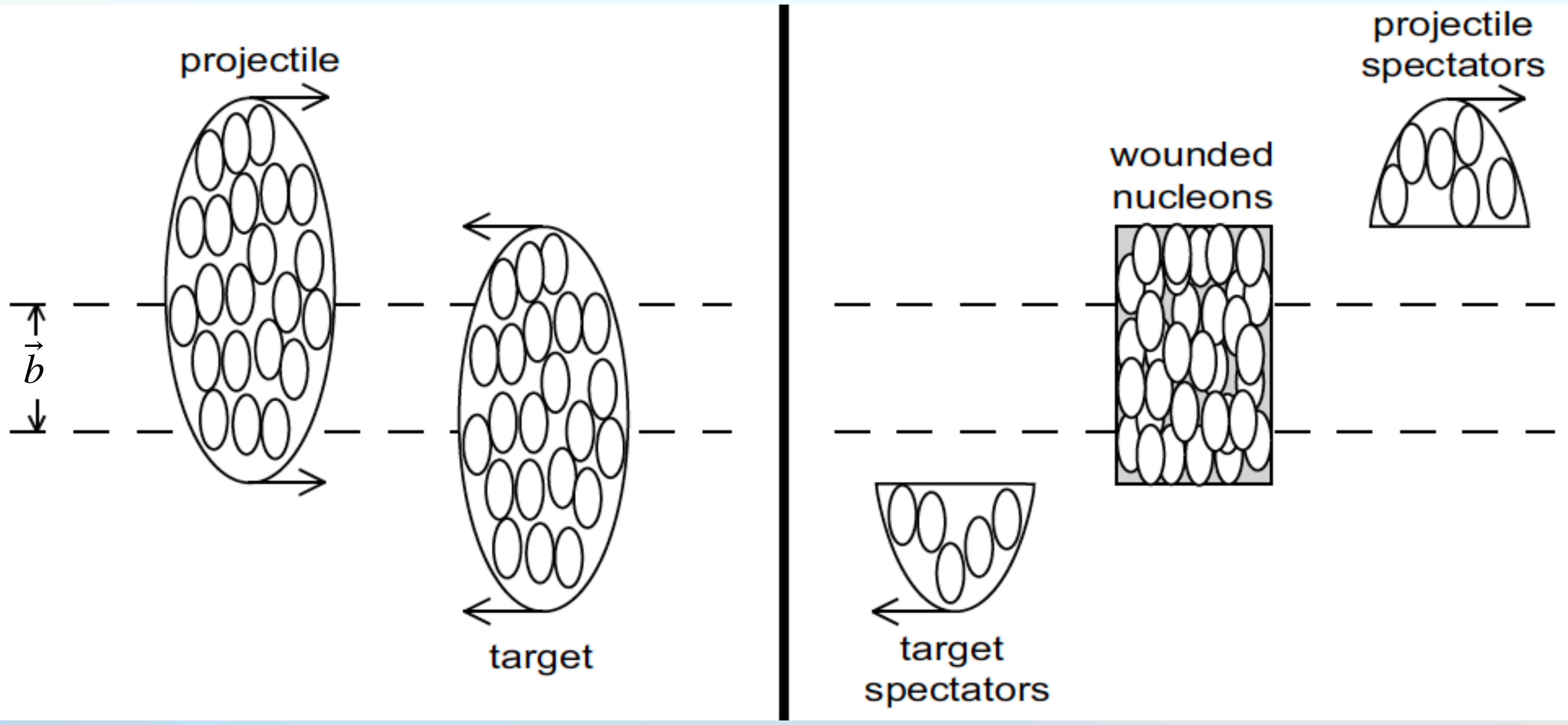
$$\kappa_2(N) \sim V, \; \kappa_2(N) \sim N_W$$

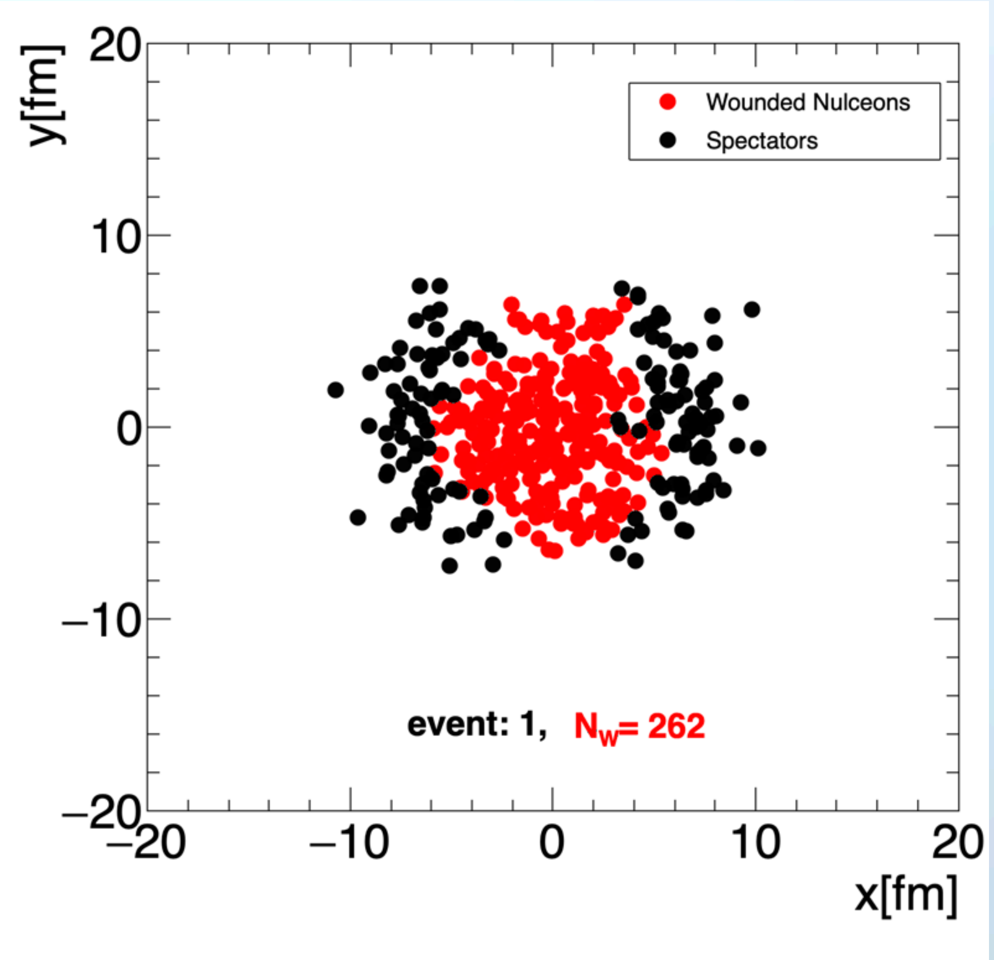
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

fluctuating sources

$$\langle N \rangle = \langle n \rangle \langle N_W \rangle$$

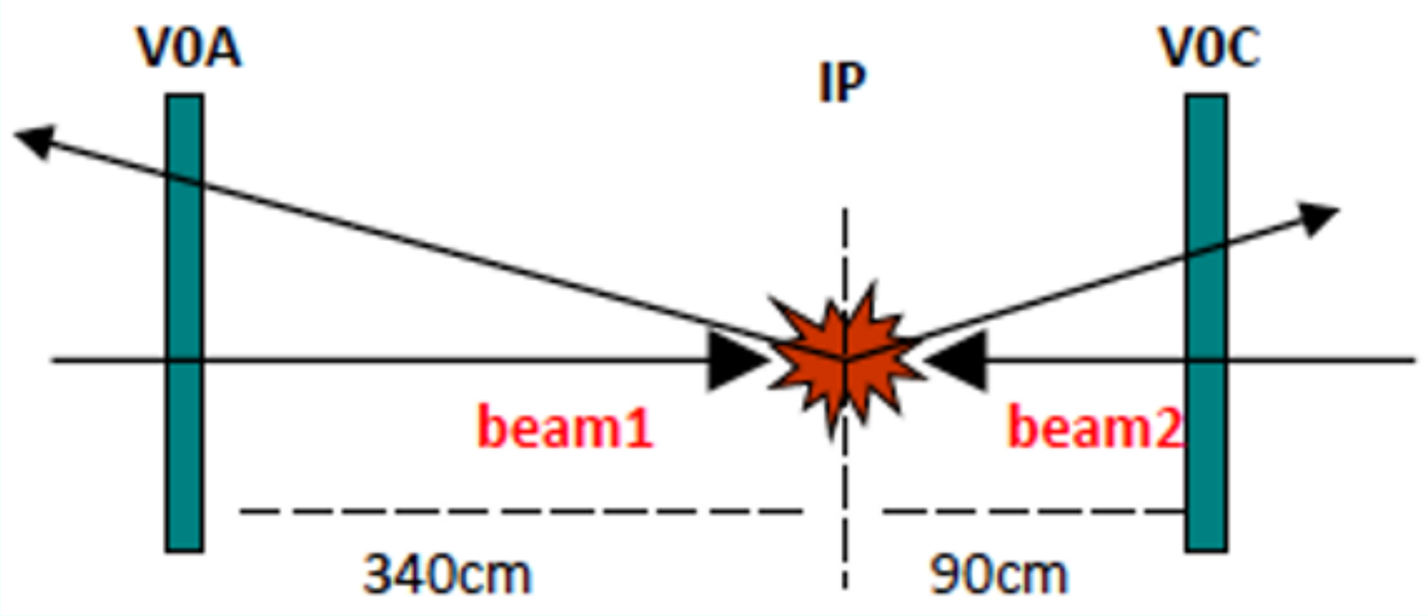
$$\kappa_2(N) = \kappa_2(n)\langle N_W \rangle + \langle N \rangle^2 \frac{\kappa_2(N_W)}{\langle N_W \rangle^2}$$



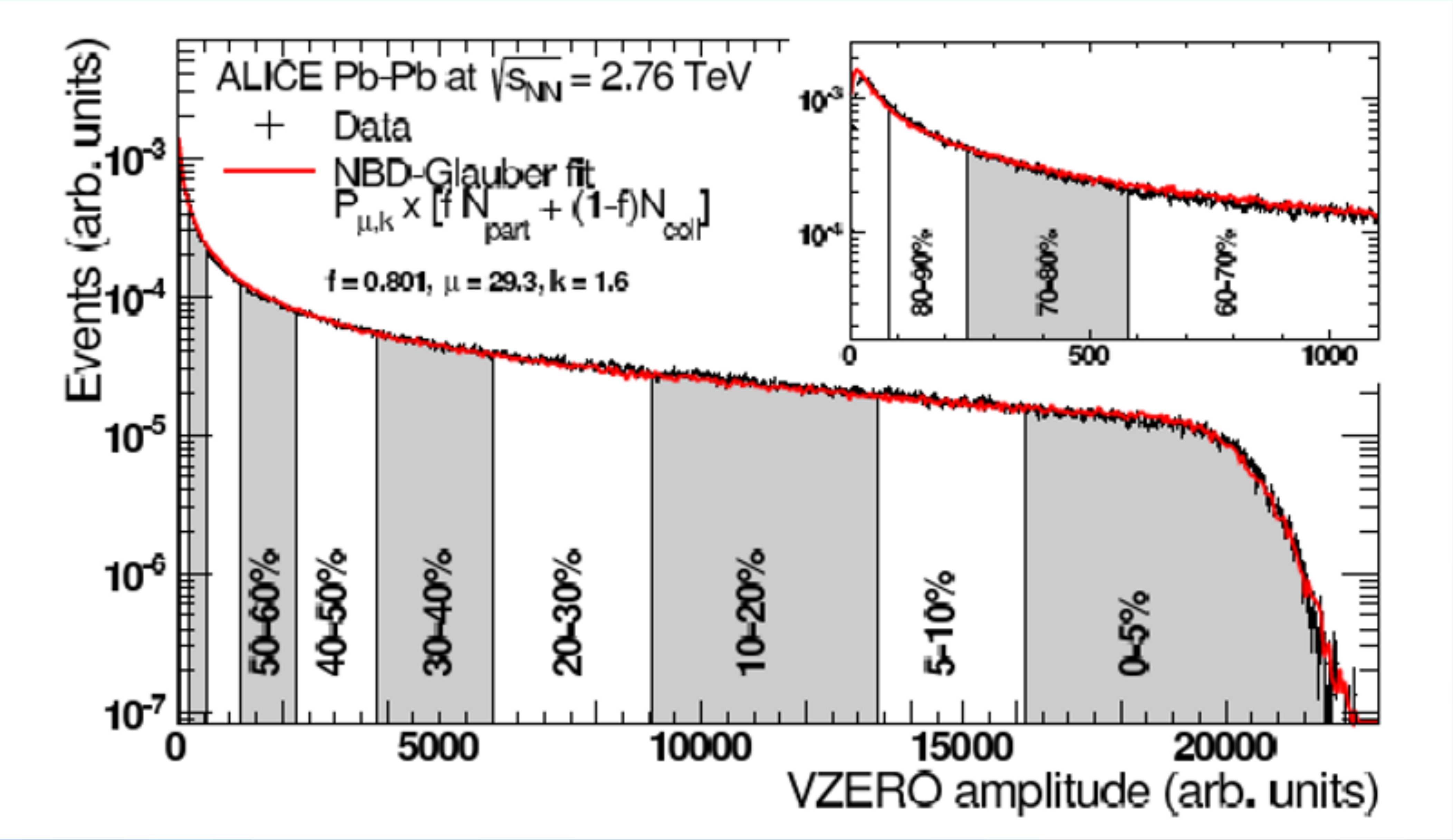


Wounded nucleons, N_W: Nucleons which collided at least once inelastically

Contributions from wounded nucleon fluctuations

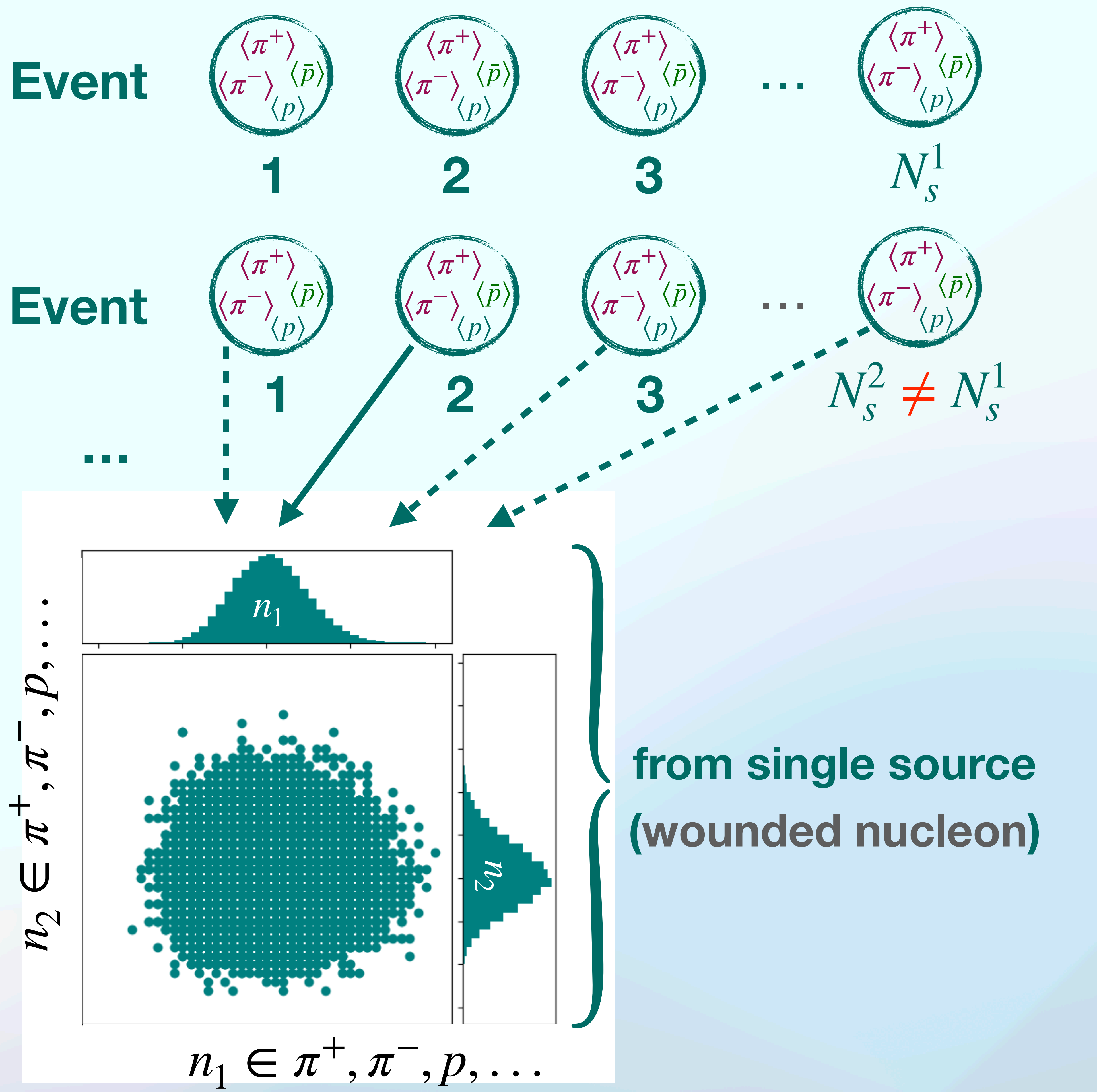


$2.8 < \eta < 5.1$ $\quad$ $-3.7 < \eta < -1.7$



ALICE Phys.Rev. C88 (2013) no.4, 044909

model of independent sources

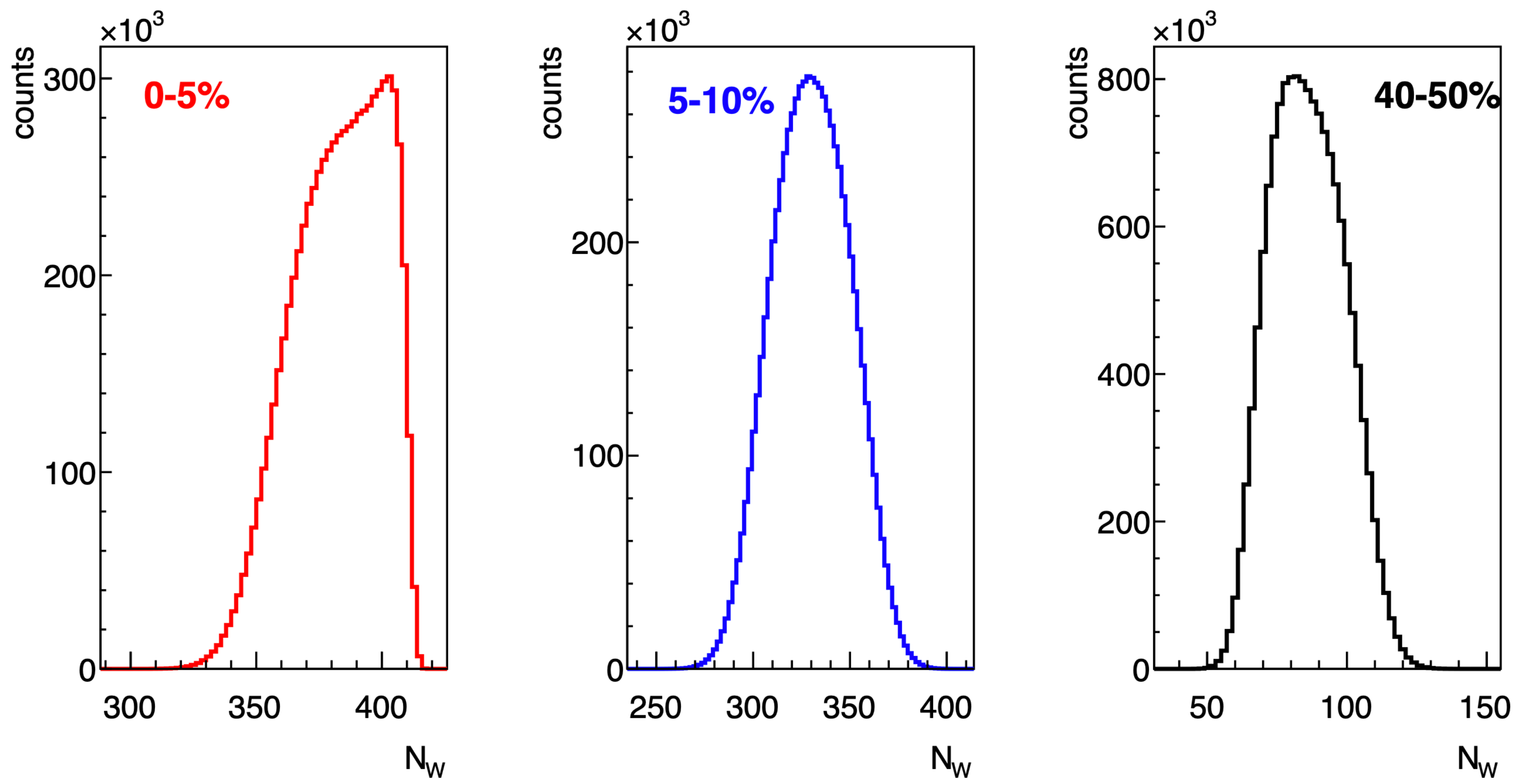


from single source (wounded nucleon)

A. R., R. Holzmann, J. Stroth, NPA 1034 (2023) 122641

V. Koch, R. Holzmann, A. R., J. Stroth, Nucl.Phys.A 1050 (2024) 122924

Volume fluctuations



$$\kappa_2(N-\bar{N}) = \kappa_2(n-\bar{n})\langle N_W\rangle + \langle N-\bar{N}\rangle^2 \frac{\kappa_2(N_W)}{\langle N_W\rangle^2}$$

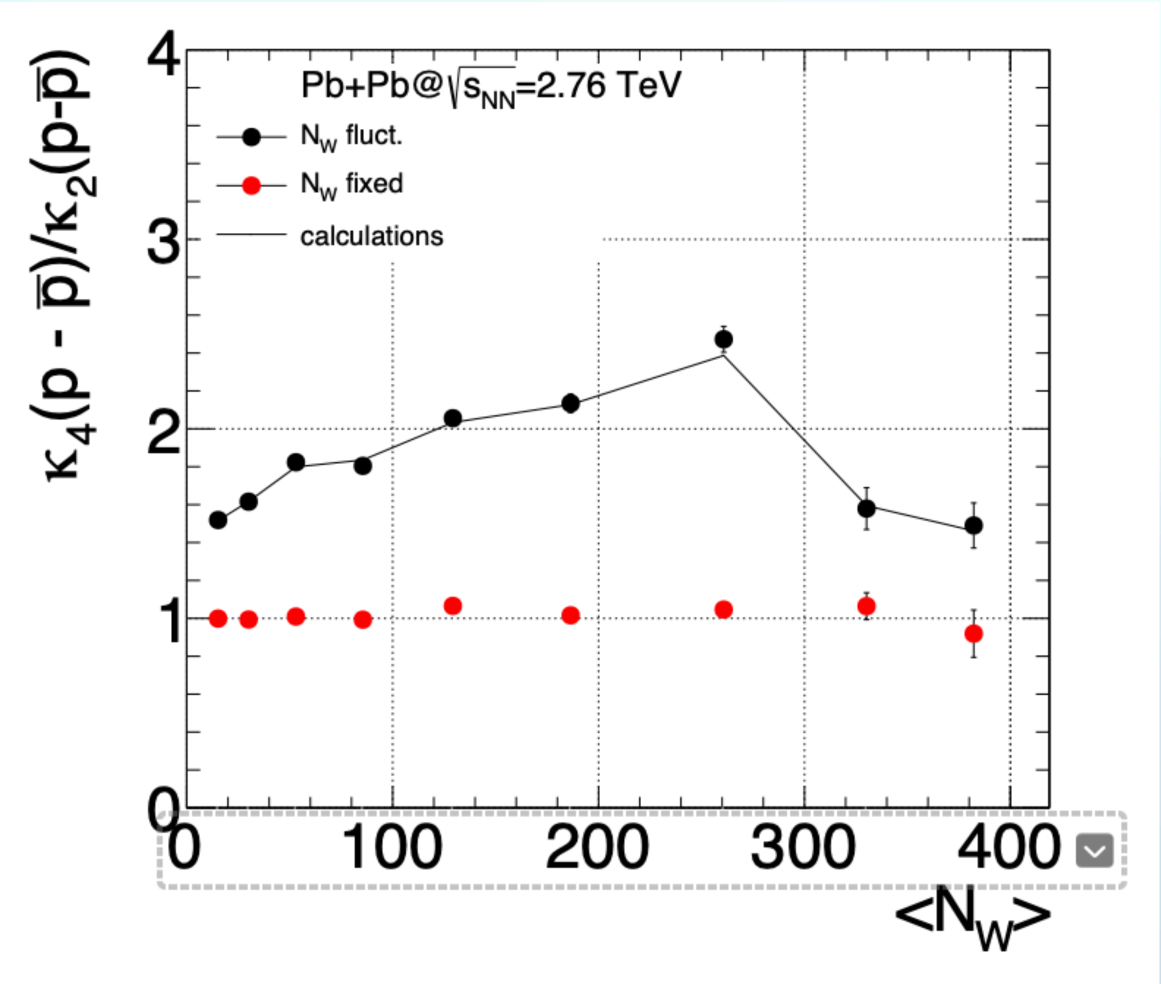
vanishes for ALICE

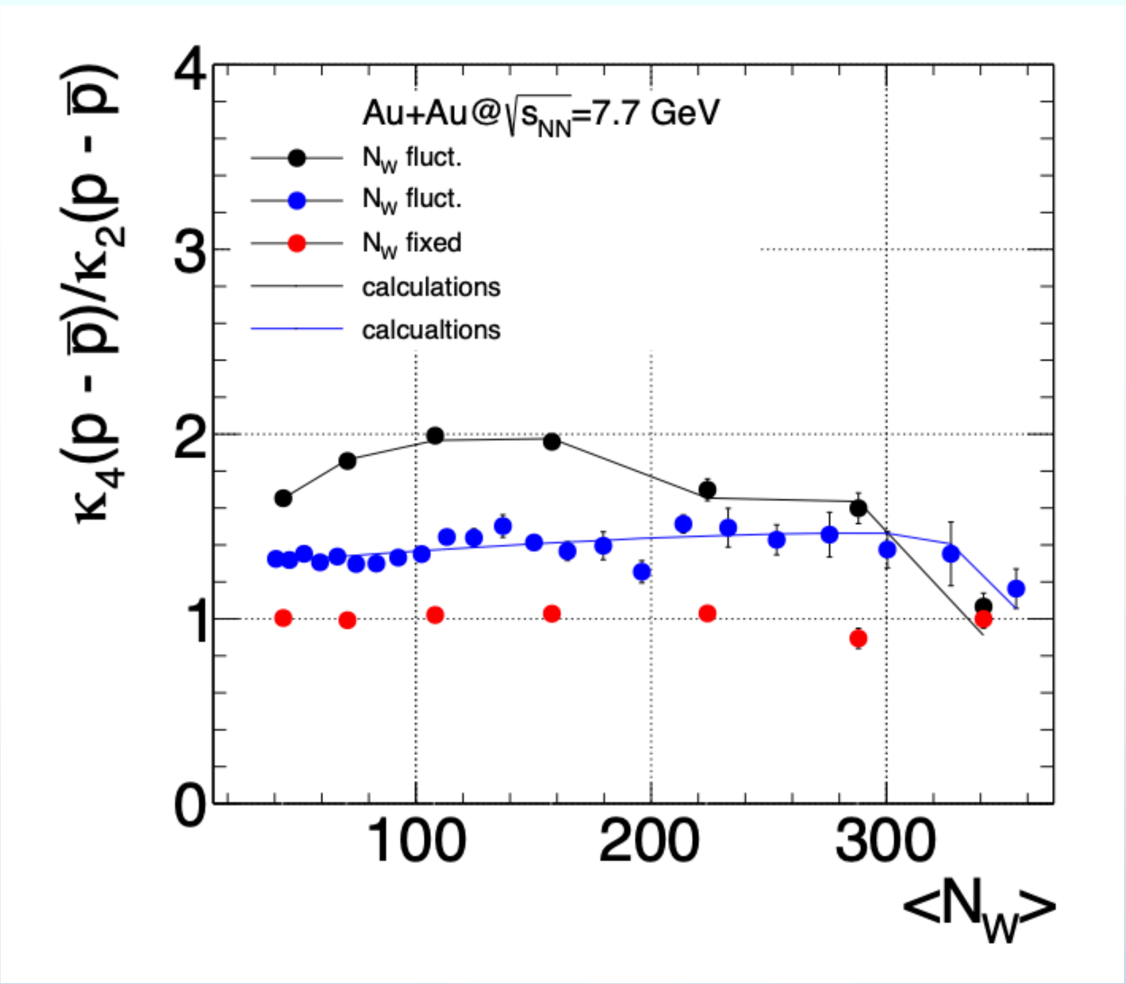
$$\kappa_4(\Delta N) = \langle N_W\rangle\, \kappa_4(\Delta n) + 4\,\langle \Delta n\rangle\, \kappa_3(\Delta n)\kappa_2(N_W) + 3\kappa_2^2(\Delta n)\kappa_2(N_W) + 6\,\langle \Delta n\rangle^2\, \kappa_2(\Delta n)\kappa_3(N_W) + \langle \Delta n\rangle^4 \kappa_4(N_W)$$

may be negative

Some predictions

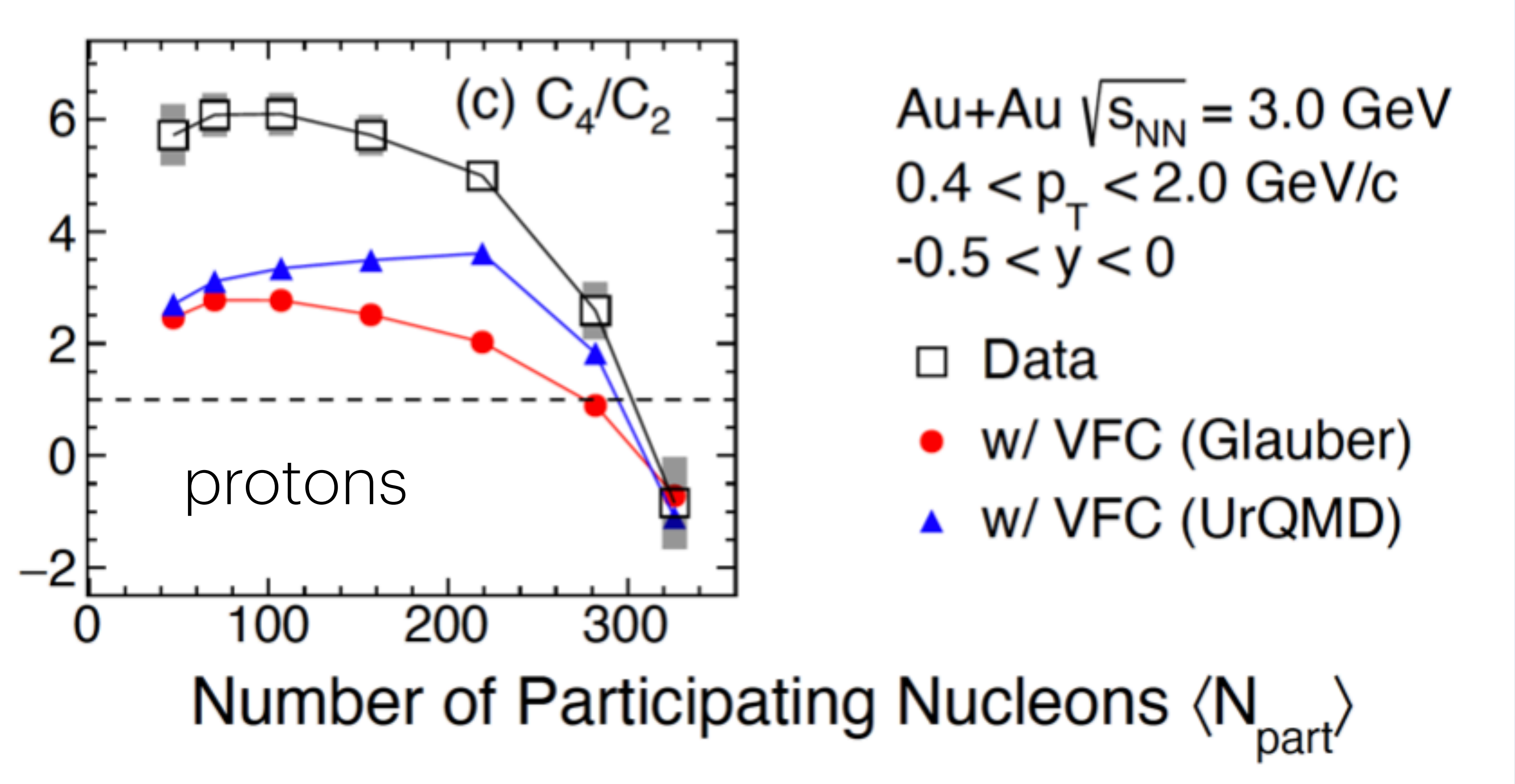
P. Braun-Munzinger, A. R., J. Stachel, NPA 960 (2017) 114-130





STAR experiment

PRL. 128 (2022) 20, 202303



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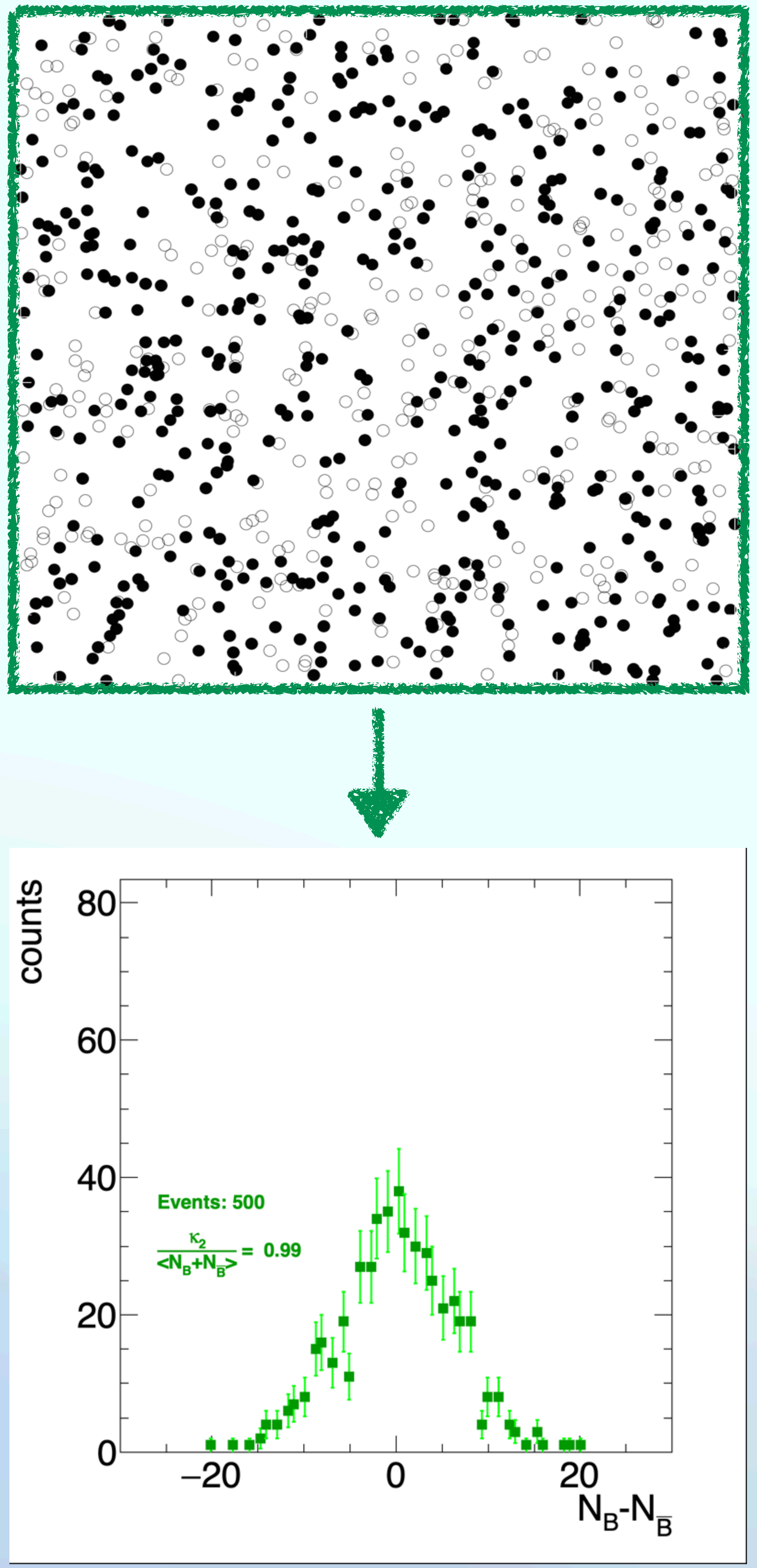
Ideal Gas in GCE



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Ideal Gas in Grand Canonical Ensemble

particles

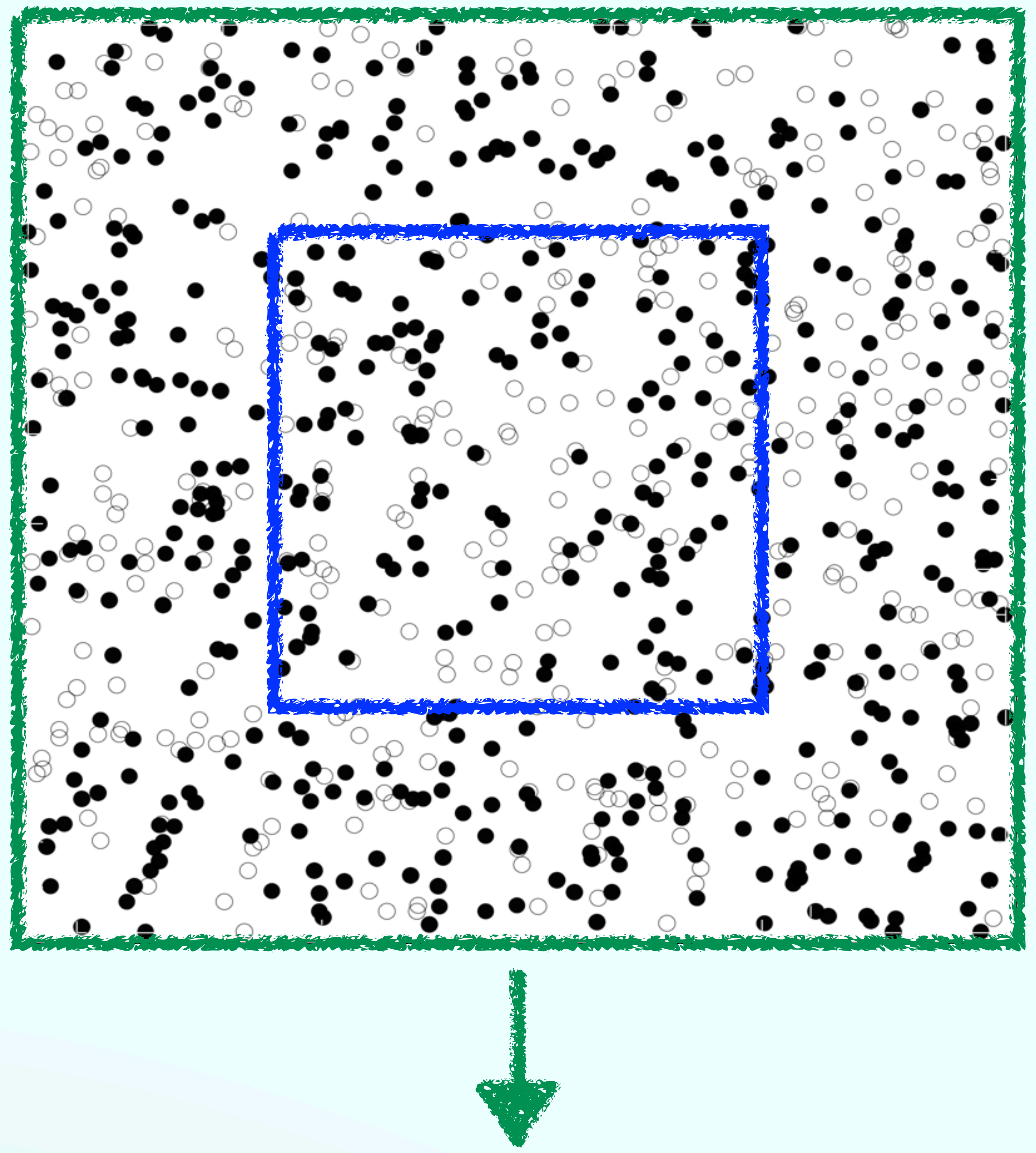
$\kappa_n(N) = \langle N \rangle$ (Poisson distribution)

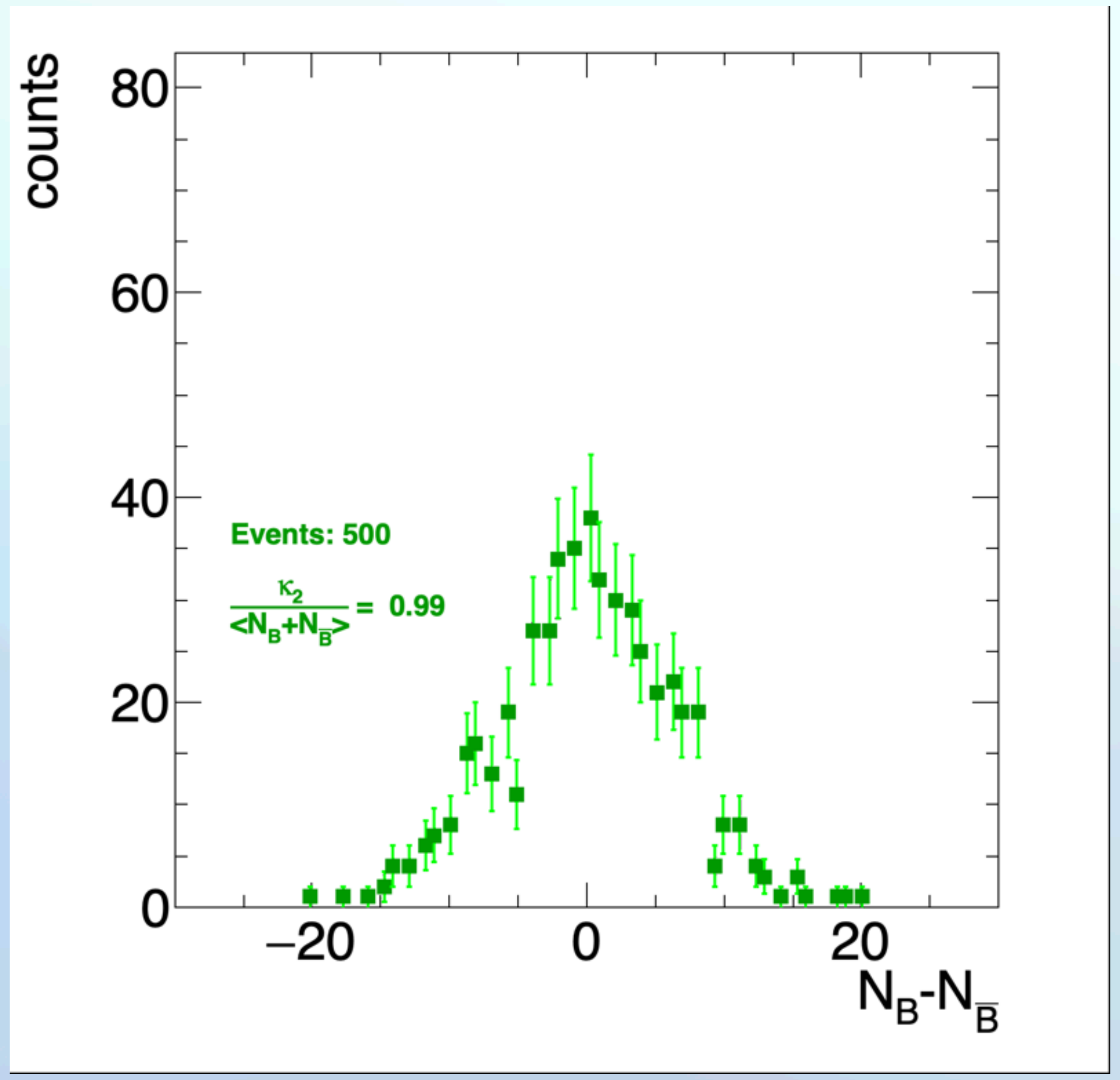
net-particles

$\kappa_n(N - \bar{N}) = \langle N \rangle + (-1)^n \langle \bar{N} \rangle$ (Skellam distribution)

for example: $\dfrac{\kappa_2(N - \bar{N})}{\langle N + \bar{N} \rangle} = \dfrac{\langle N + \bar{N} \rangle}{\langle N + \bar{N} \rangle} = 1$

Ideal Gas in GCE





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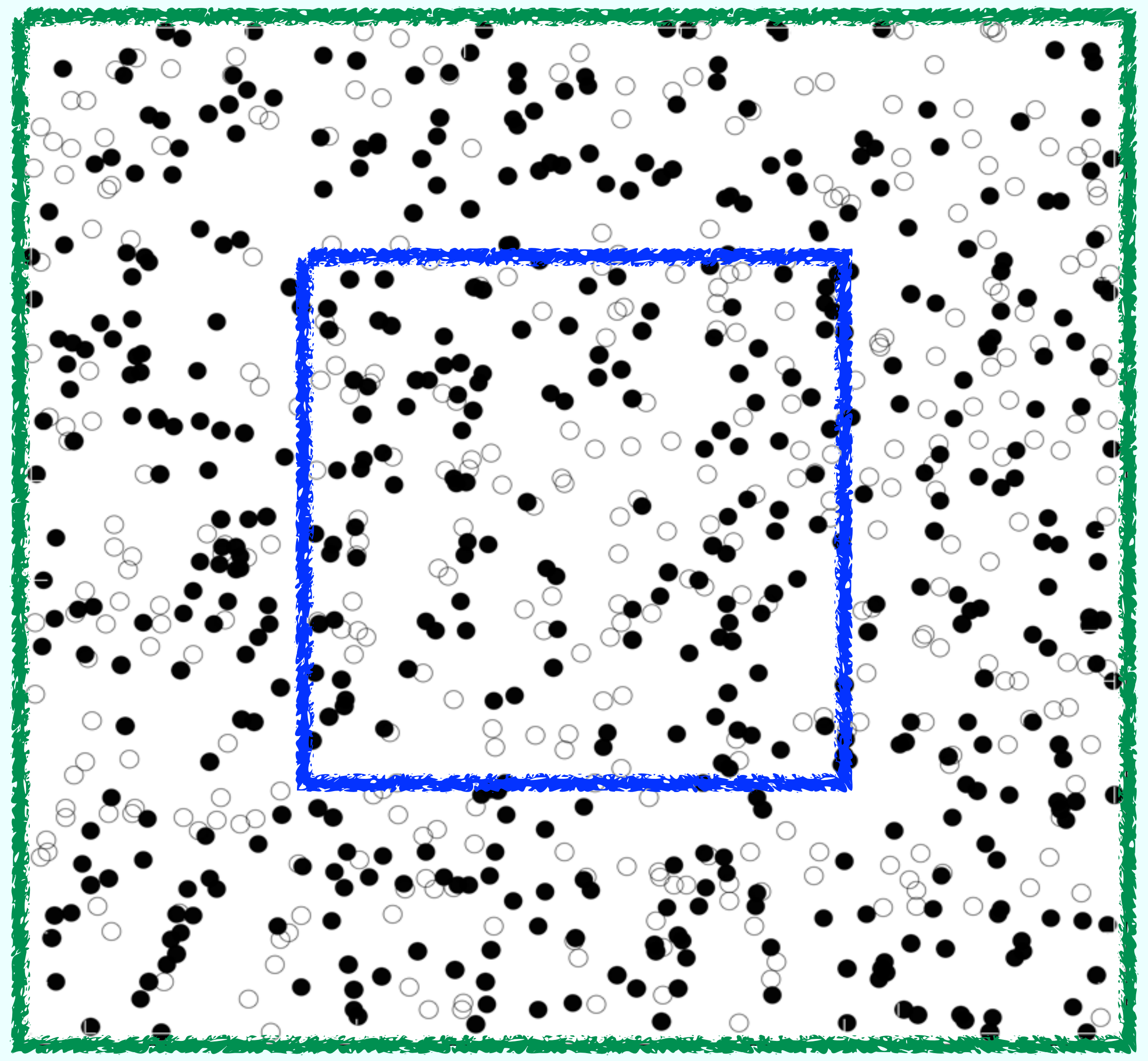
Ideal Gas in GCE + conservation laws

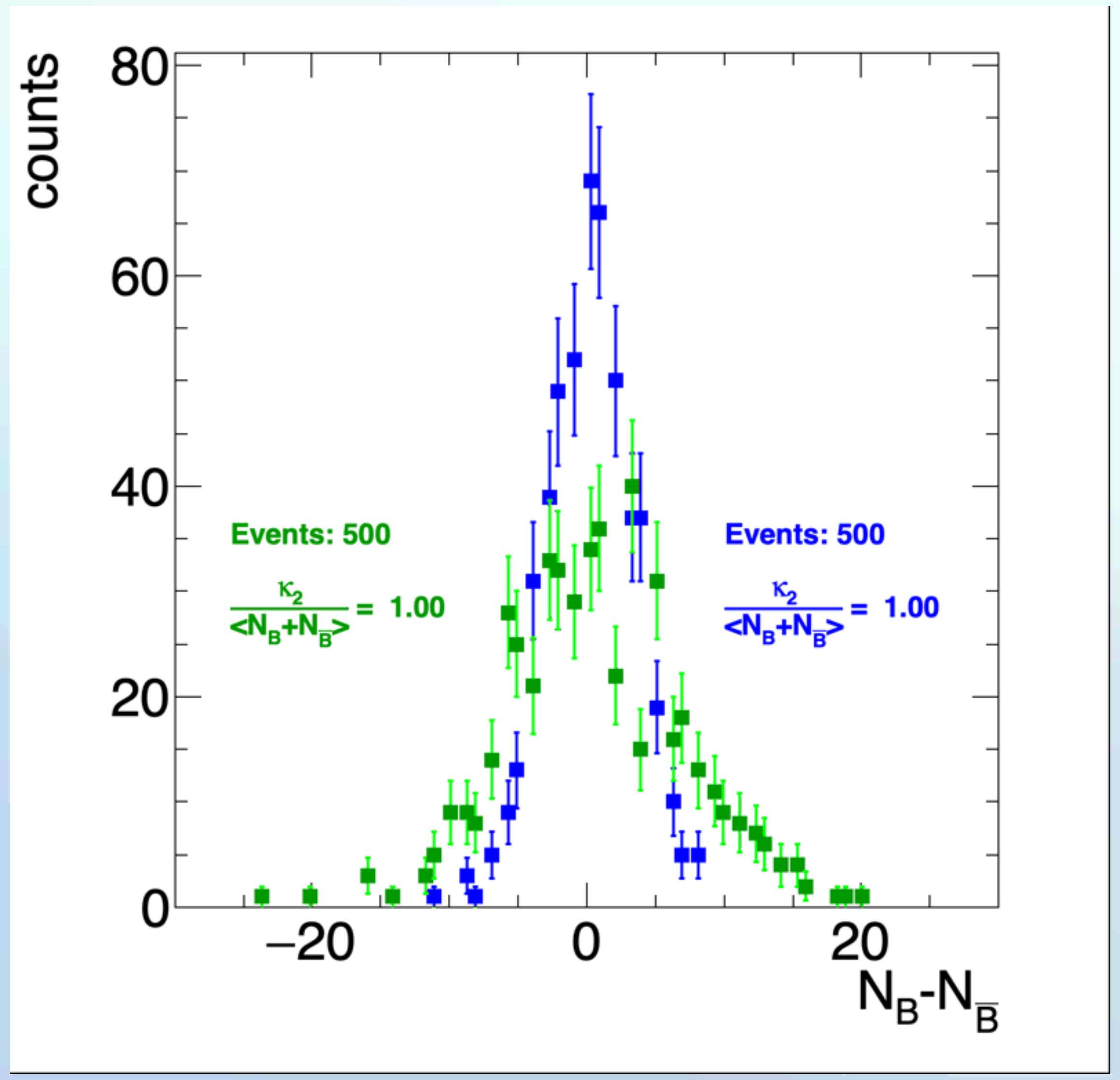


$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = \alpha \frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} + 1 - \alpha$$

$\alpha = \langle N_B \rangle / \langle N_B \rangle$

→ **Unity in GCE**

P. Braun-Munzinger, A.R., J. Stachel, NPA 960 (2017) 114-130

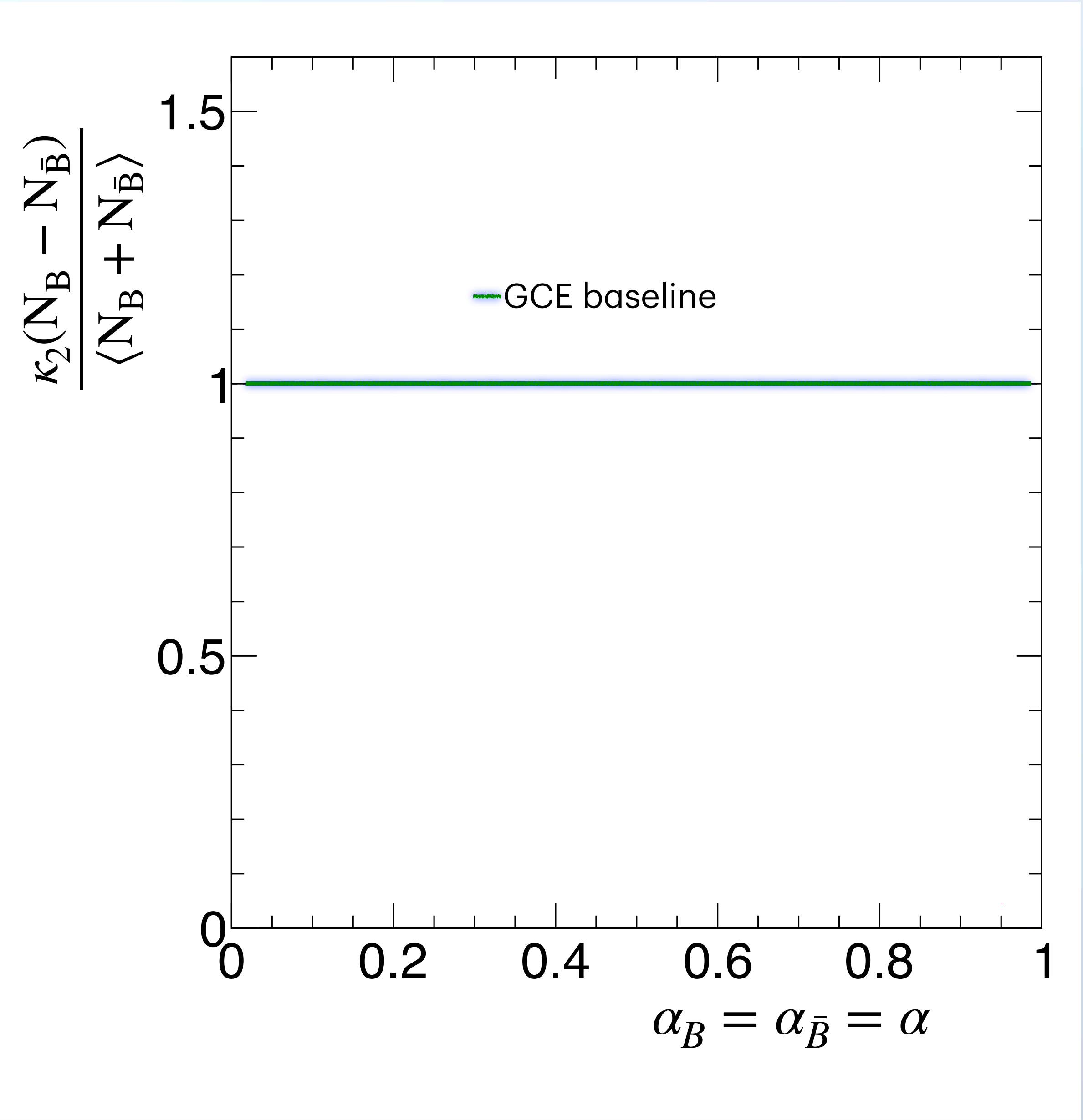


If baryon number is conserved in full phase space $\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = 0$

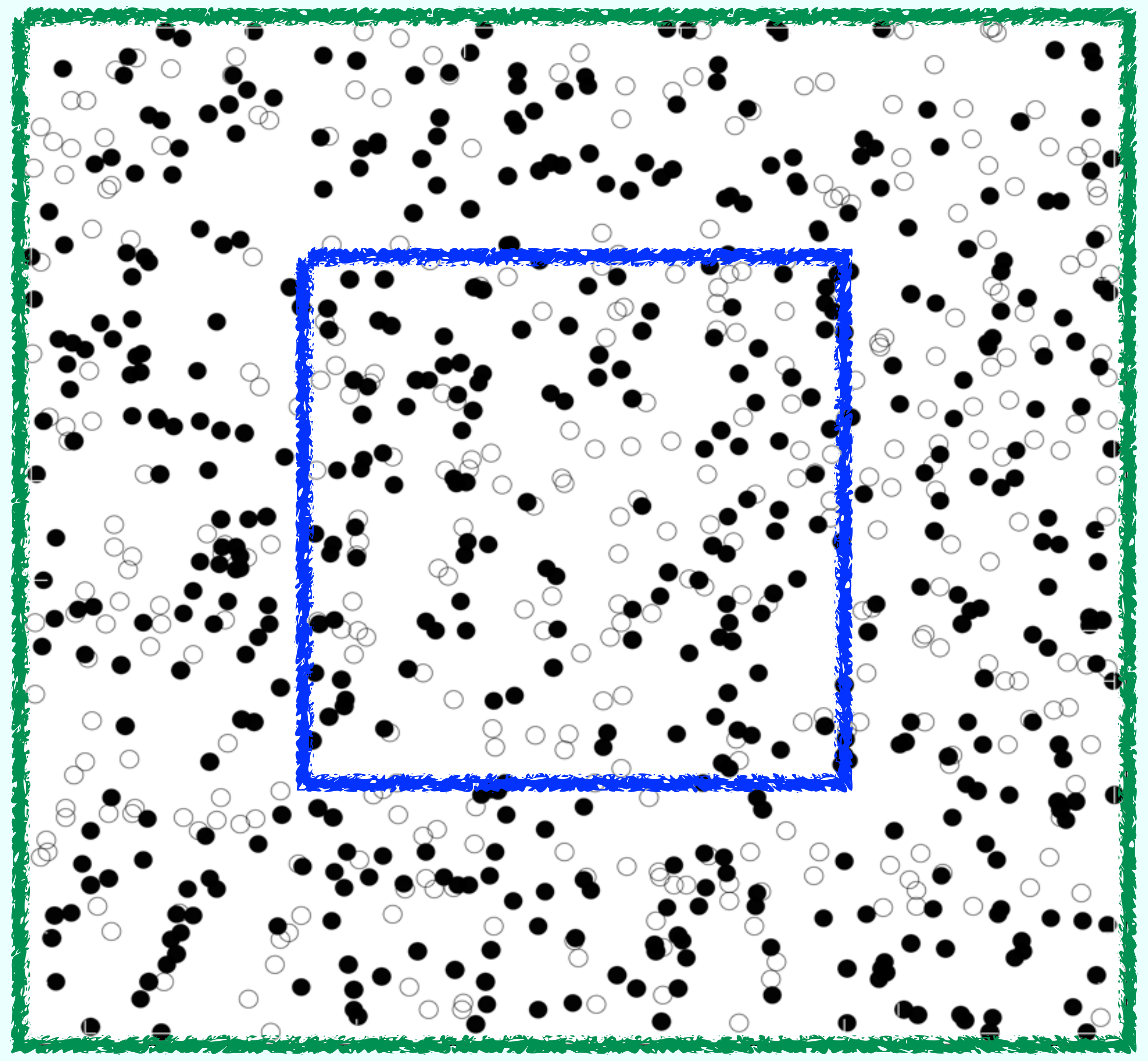
$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = 1 - \alpha$$

Grand Canonical
(conservation on average) $\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = 1$

Exact conservation $\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = 1 - \alpha$



Ideal Gas in GCE + conservation laws



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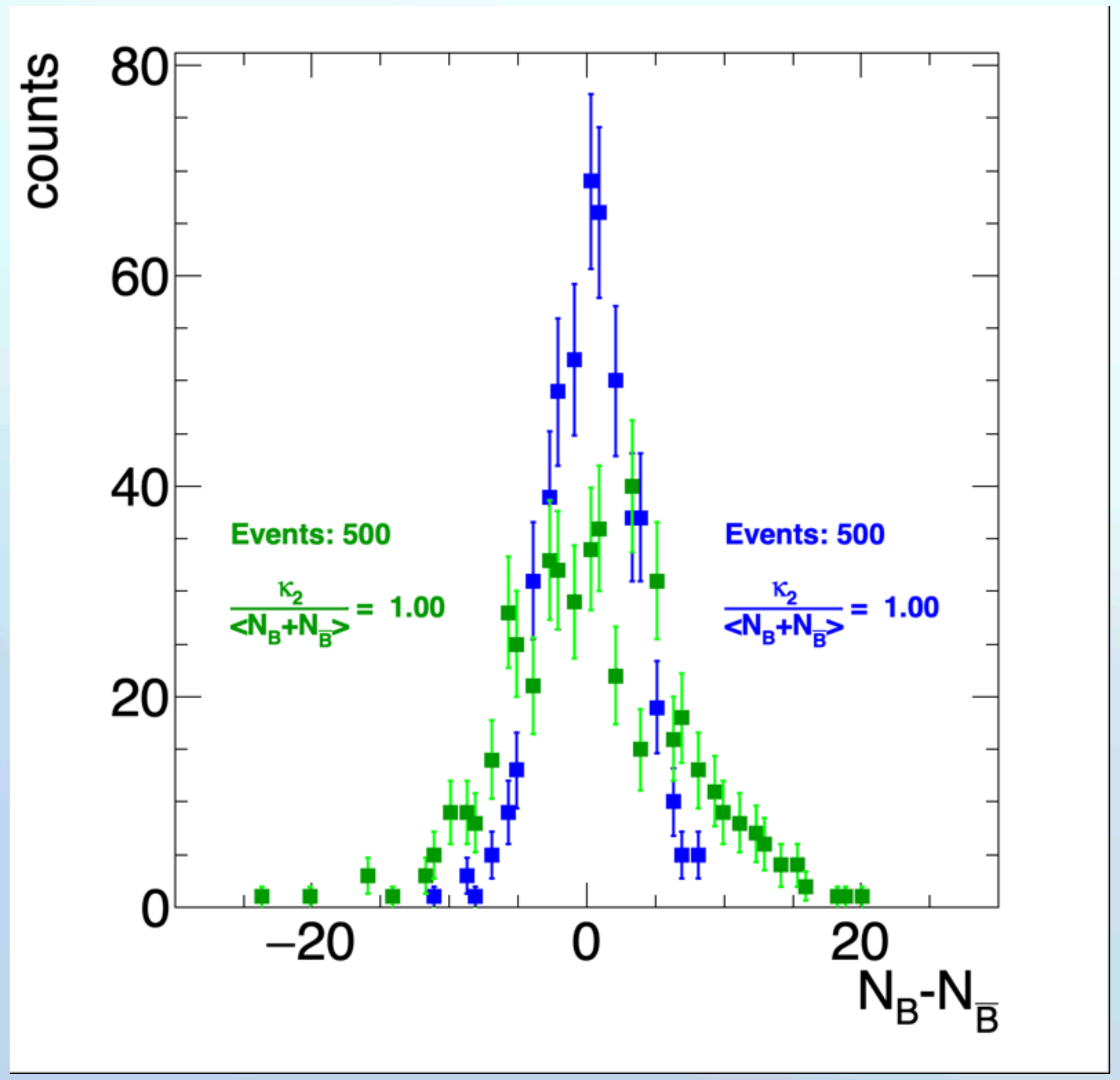

Grand Canonical
(conservation on average)

$$\frac{\kappa_2(N_B - N_{\bar{B}})}{\langle N_B + N_{\bar{B}} \rangle} = 1$$

Exact conservation

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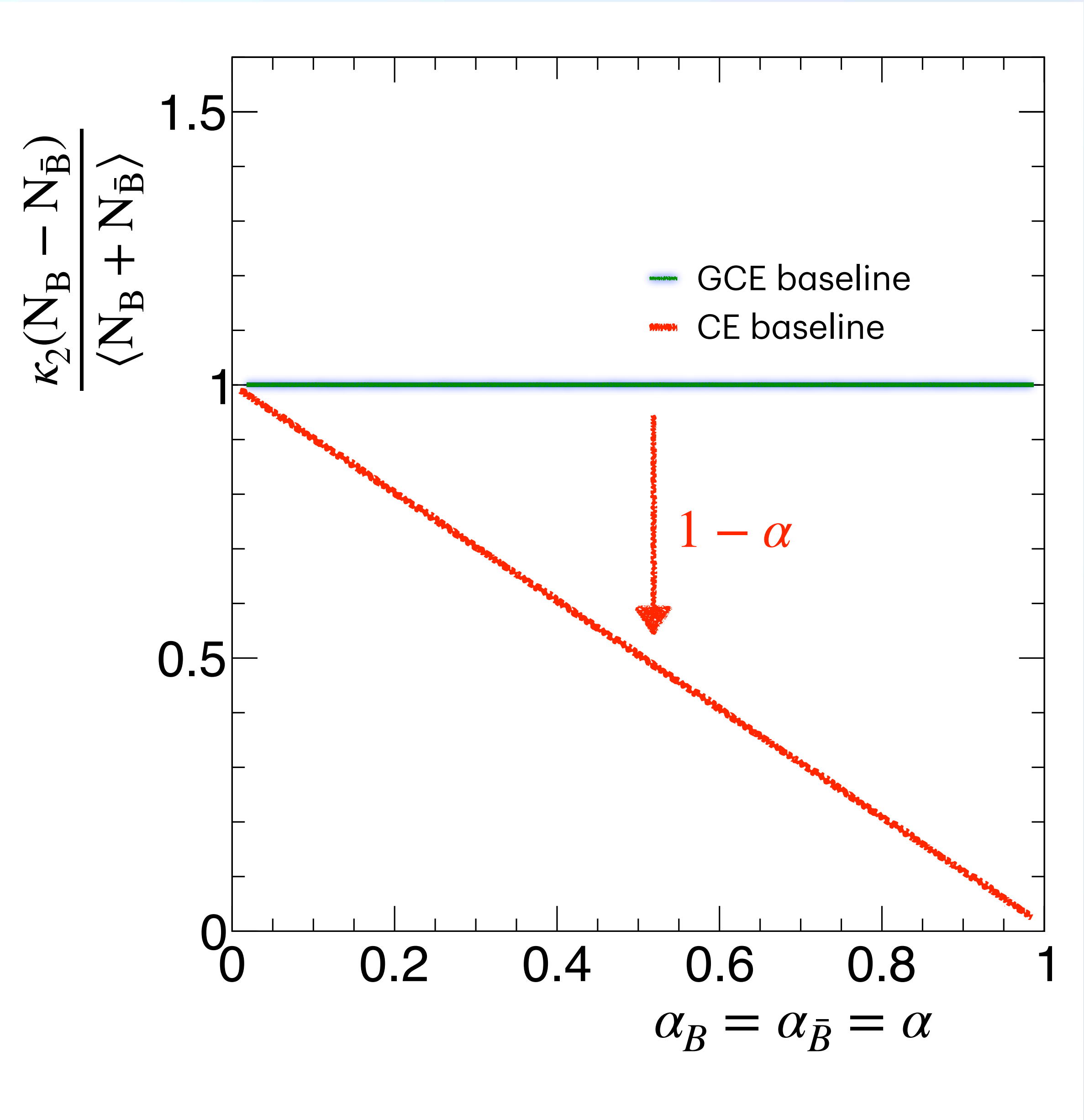
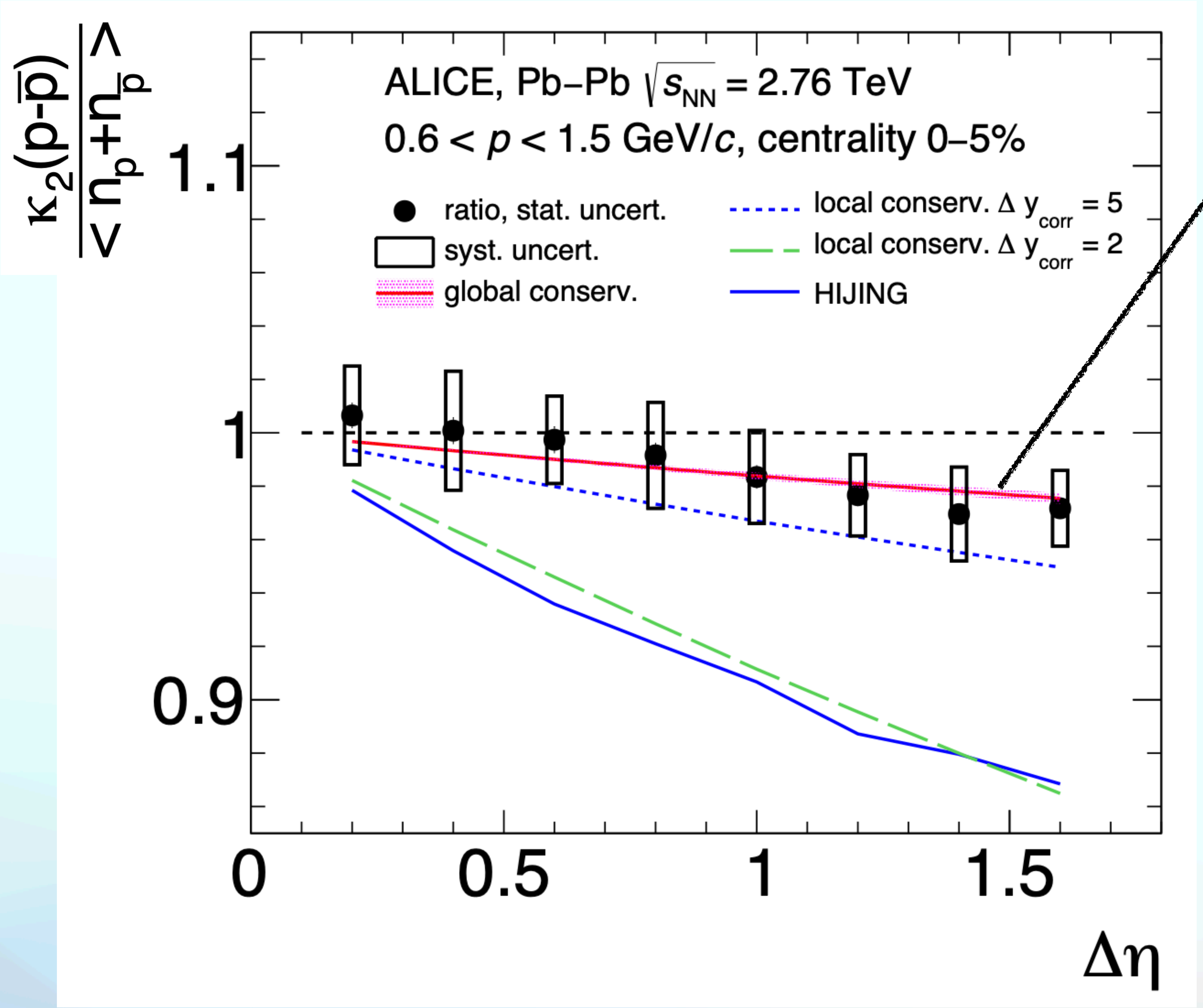
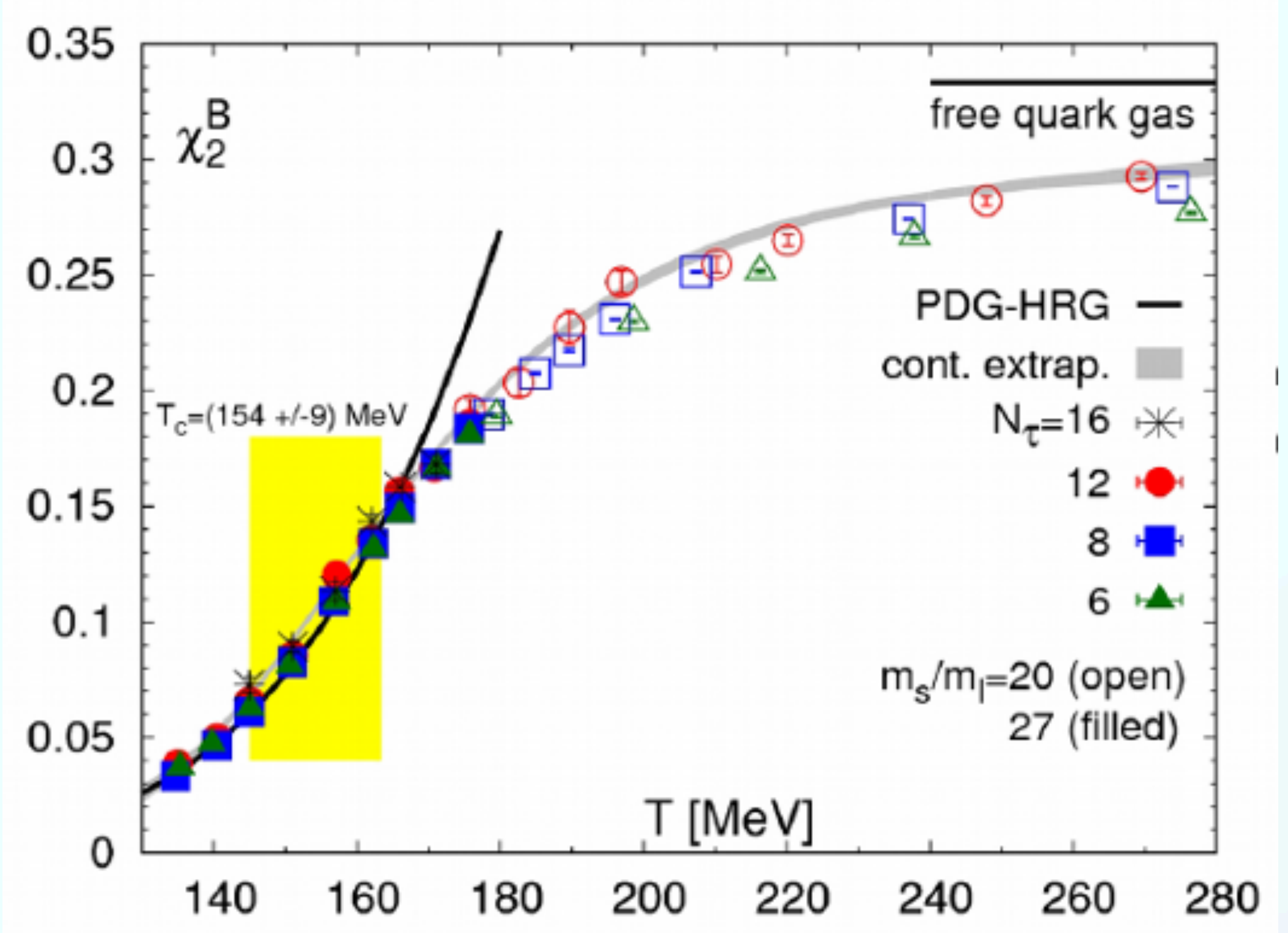


First Alice results (Identity Method)



$1-\alpha$

$$\alpha_{\Delta\eta} = \frac{\langle n_p \rangle^{acc}}{\langle N_B \rangle^{4\pi}}$$



A. Bazavov et al [HotQCD], PRD 101 (2020) 074502
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

first verification of LQCD results

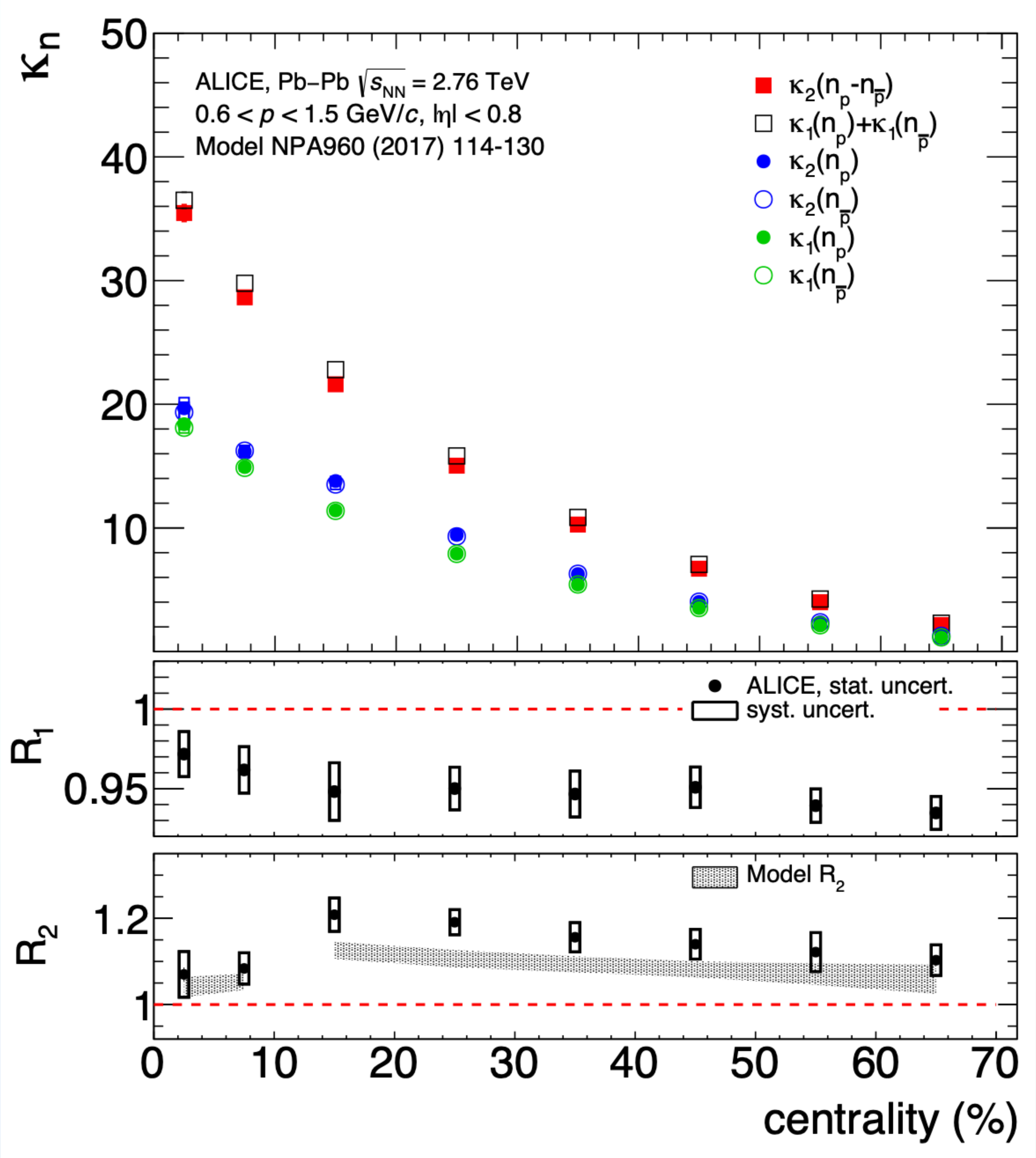
Consequences:

- Support for the validity of the HRG model
- Further support for freeze-out at the phase boundary

A. R., Nucl.Phys.A 967 (2017) 453-456 (QM 17)
ALICE: Phys. Lett. B 807 (2020) 135564,
Phys. Lett. B (2022) 137545

Identity Method A.R., M. I. Gorenstein, PRC 86, 044906 (2012)
M. Arslandok, A.R., NIM A946, 162622 (2019)
A. R., Phys.Rev.C 110 (2024) 6, 064910

First ALICE results (Identity Method)



$$\kappa_2(N-\bar{N}) = \kappa_2(n-\bar{n})\langle N_W\rangle + \langle N-\bar{N}\rangle^2 \frac{\kappa_2(N_W)}{\langle N_W\rangle^2}$$

P. Braun-Munzinger, A. R., J. Stachel, NPA 960 (2017) 114-130

A. R., Nucl.Phys.A 967 (2017) 453-456 (QM 17)
ALICE: Phys. Lett. B 807 (2020) 135564,
Phys. Lett. B (2022) 137545

Experimental verification:

$$R_1 = \kappa_2(p-\bar{p})/\langle n_p + n_{\bar{p}}\rangle$$

- not influenced by volume fluctuations

$$R_2 = \kappa_2(p)/\langle n_p\rangle$$

- affected by volume fluctuations

Experiment vs. Theory

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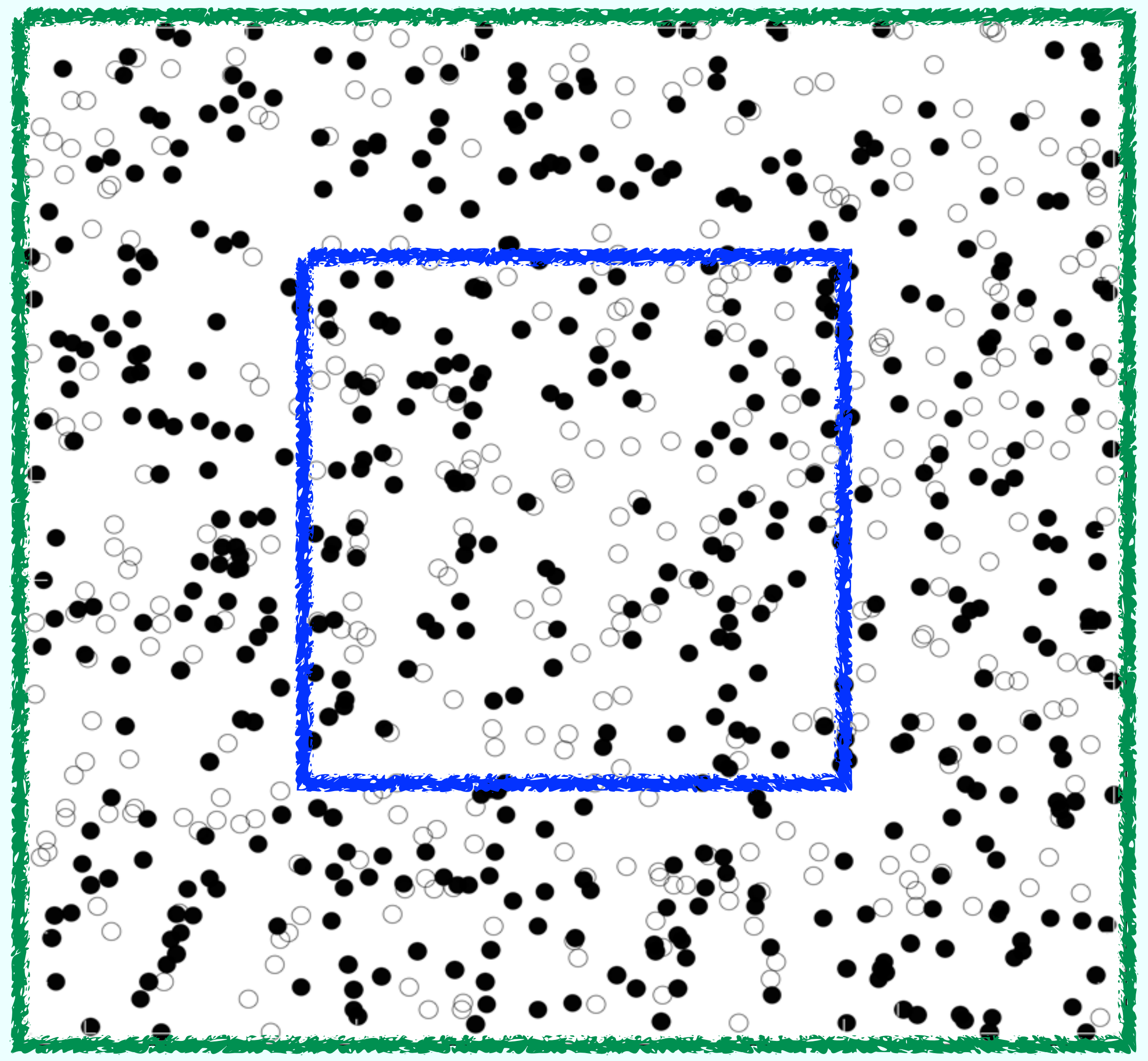
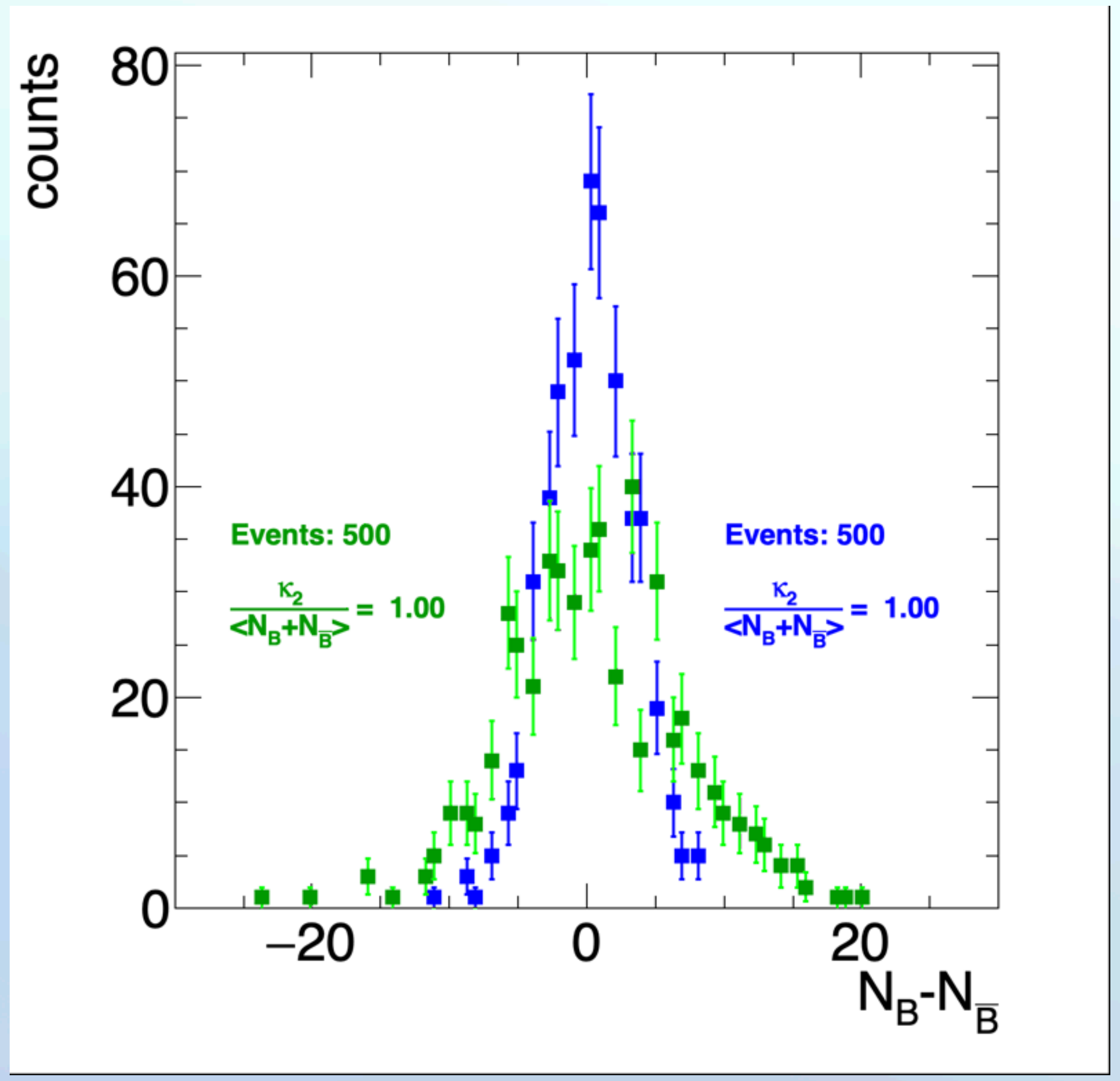
Experimental challenges

- Volume fluctuations
- Conservation laws
- First ALICE results

P. Braun-Munzinger, A.R., J. Stachel, NPA 982 (2019) 307-310 (QM 18)
P. Braun-Munzinger, A.R., J. Stachel, e-Print: 1907.03032 [nucl-th] (2019)
A. R., NPA 1005 (2021) 121858(QM 19)

Ideal gas EoS plus global baryon number conservation





- exploiting **C**anonical **E**nsemble in the full phase space
  - no fluctuations in 4π (like in experiments)

CE suppression

$$\frac{\kappa_2(B-\bar{B})}{\langle n_B + n_{\bar{B}}\rangle} = 1 - \frac{\alpha_B\langle n_B\rangle + \alpha_{\bar{B}}\langle n_{\bar{B}}\rangle}{\langle n_B + n_{\bar{B}}\rangle} + \left(z^2 - \langle N_B\rangle\langle N_{\bar{B}}\rangle\right)\frac{\left(\alpha_B - \alpha_{\bar{B}}\right)^2}{\langle n_B + n_{\bar{B}}\rangle}$$

$\langle N_B\rangle, \langle N_{\bar{B}}\rangle$ - in 4π

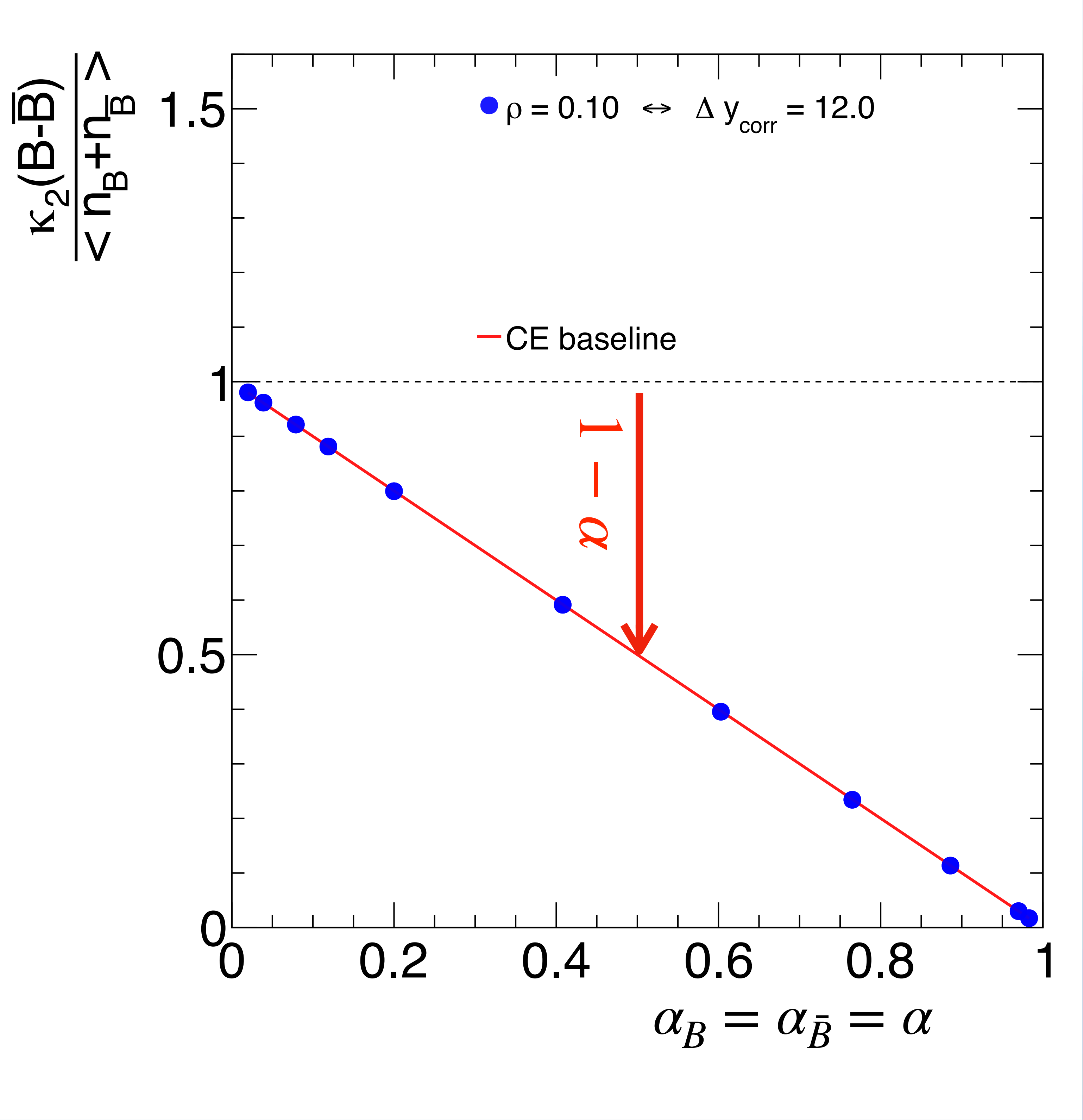
$\langle n_B\rangle, \langle n_{\bar{B}}\rangle$ - inside acceptance

$\alpha_B = \langle n_B\rangle / \langle N_B\rangle$ - acceptance for B

$\alpha_{\bar{B}} = \langle n_{\bar{B}}\rangle / \langle N_{\bar{B}}\rangle$ - acceptance for $\bar{B}$

z - single baryon partition function

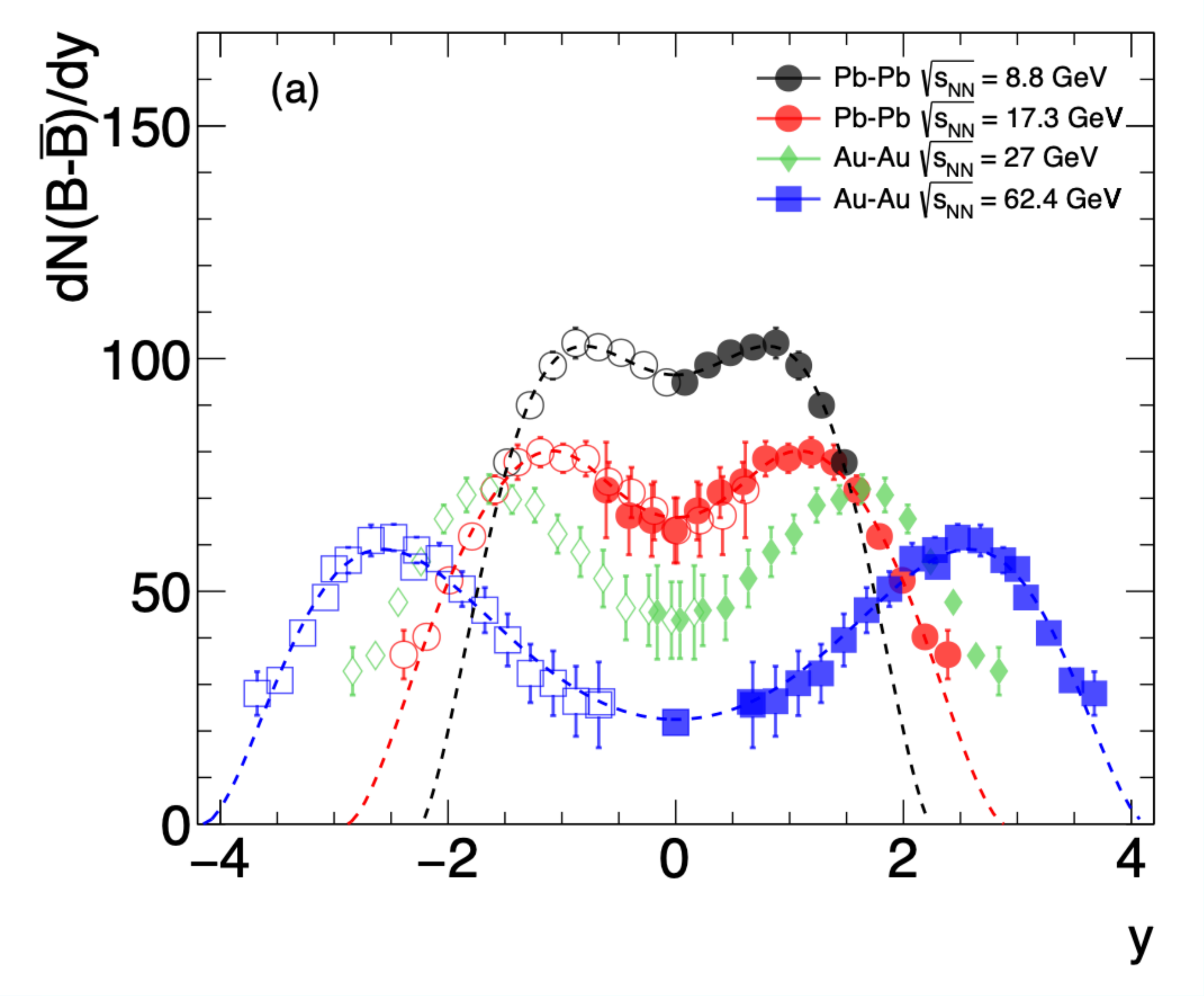
in general: $\alpha_B \neq \alpha_{\bar{B}}$

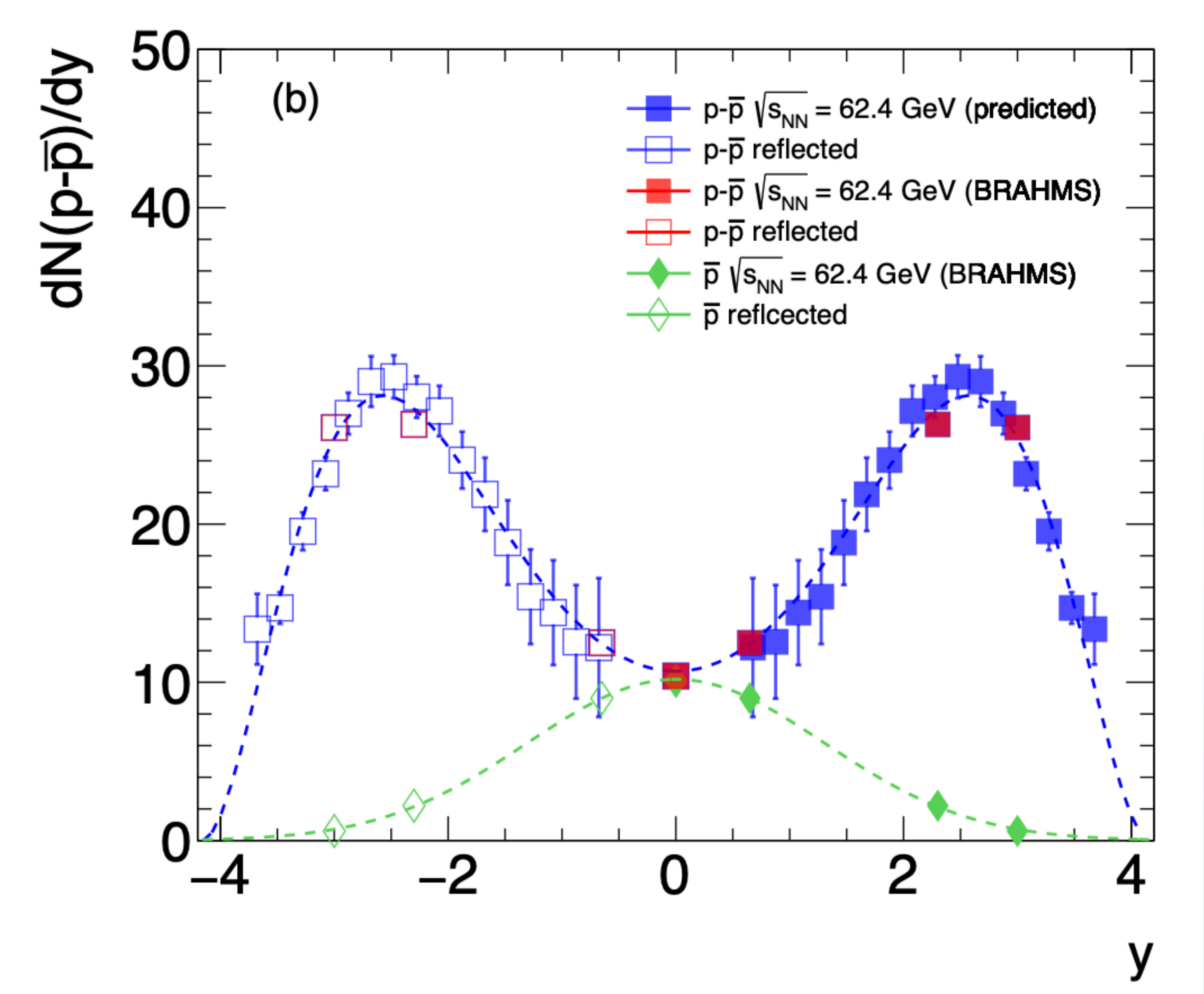


P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel , NPA 1008 (2021) 122141

A. Bzdak, V. Koch, V. Skokov, Phys.Rev.C 87 (2013) 1, 014901

Experimental acceptance





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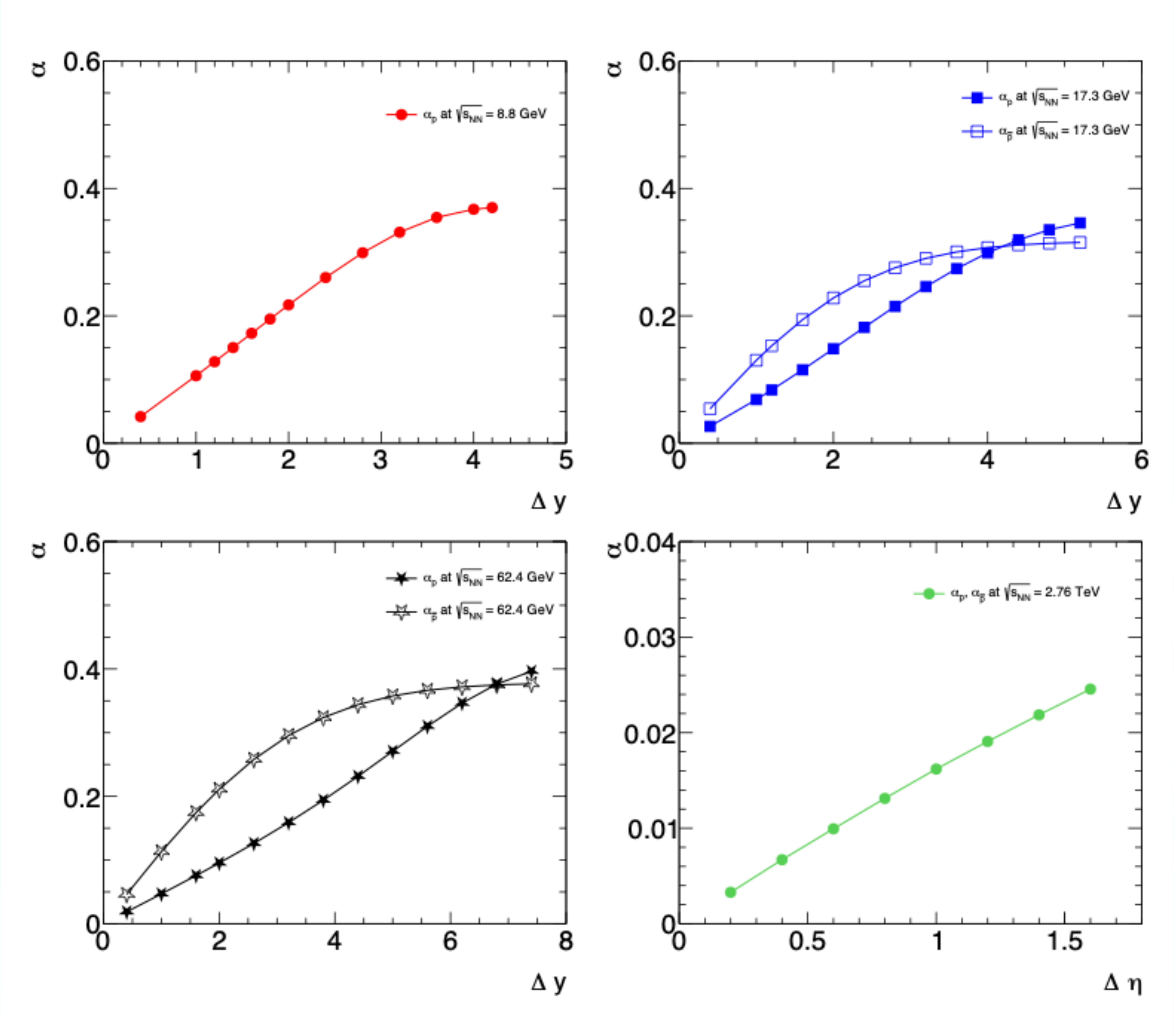


$$\alpha_p = \frac{\gamma_p \int_{y_{min}}^{y_{max}} \left[\frac{dn_p}{dy}\right] dy}{\langle N_B \rangle}$$

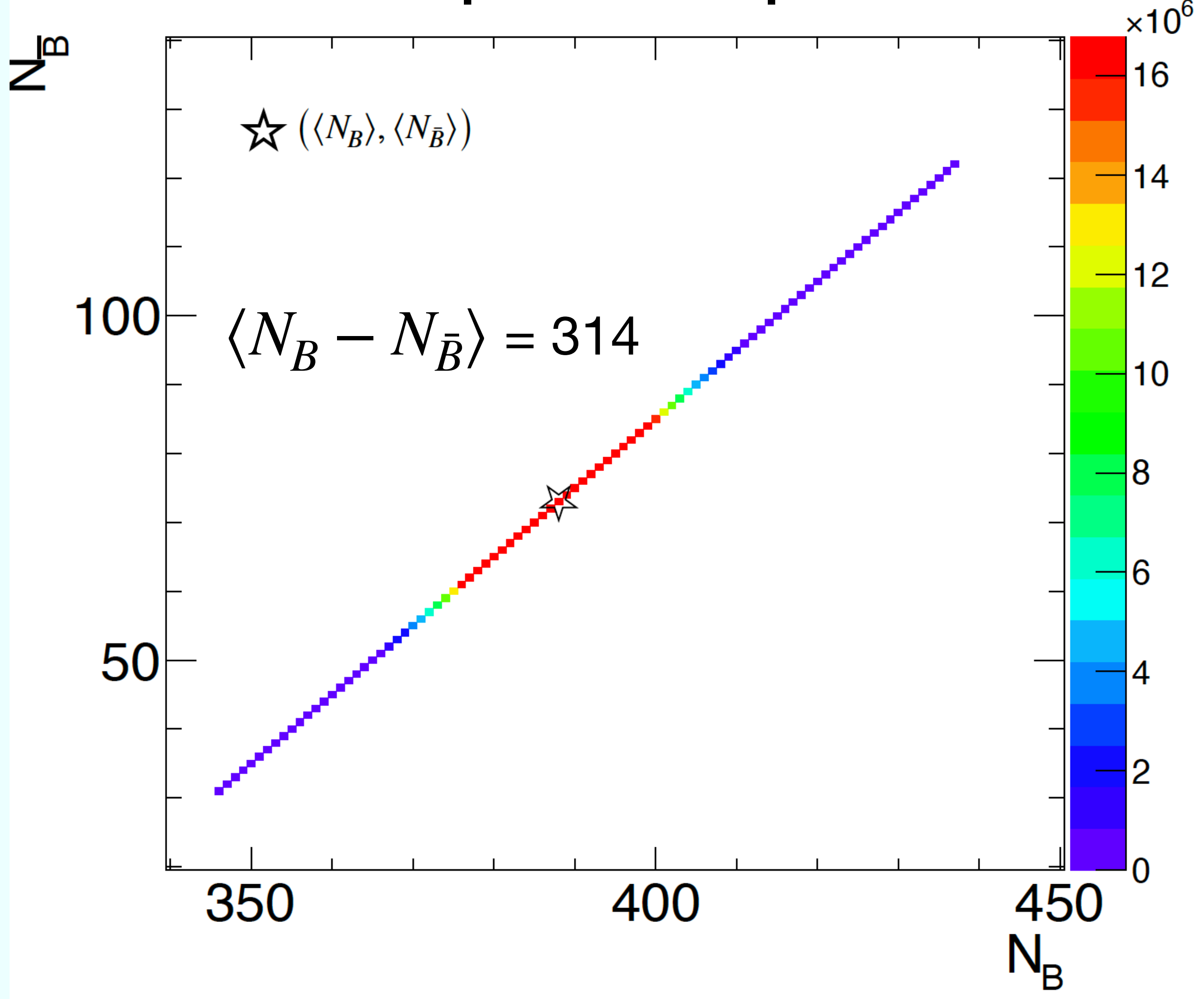
$$\alpha_{\bar{p}} = \frac{\gamma_{\bar{p}} \int_{y_{min}}^{y_{max}} \left[\frac{dn_{\bar{p}}}{dy}\right] dy}{\langle N_{\bar{B}} \rangle}$$

extraction of acceptances for **protons (solid symbols)**

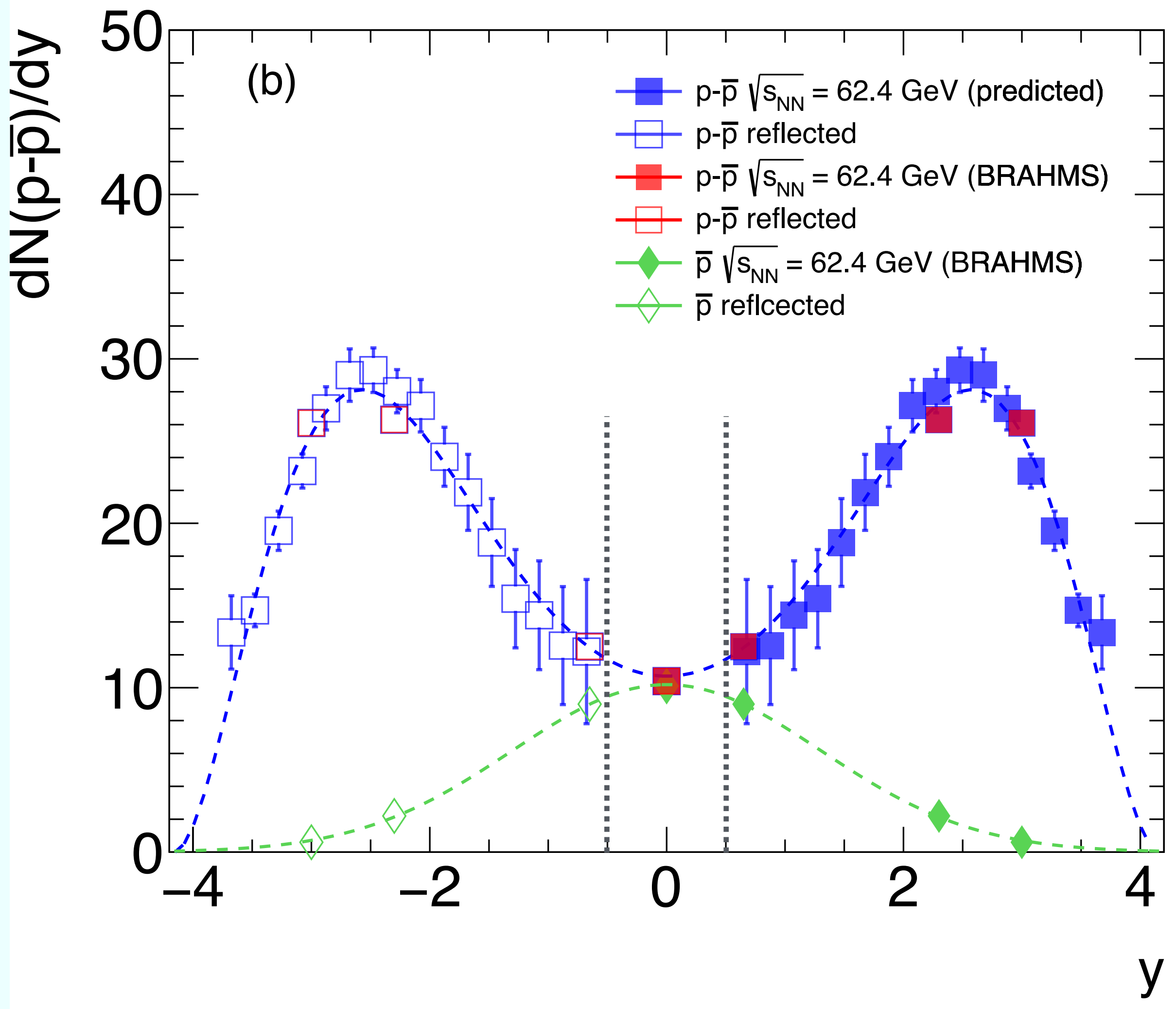
and **antiprotons (open symbols)**

The strategy

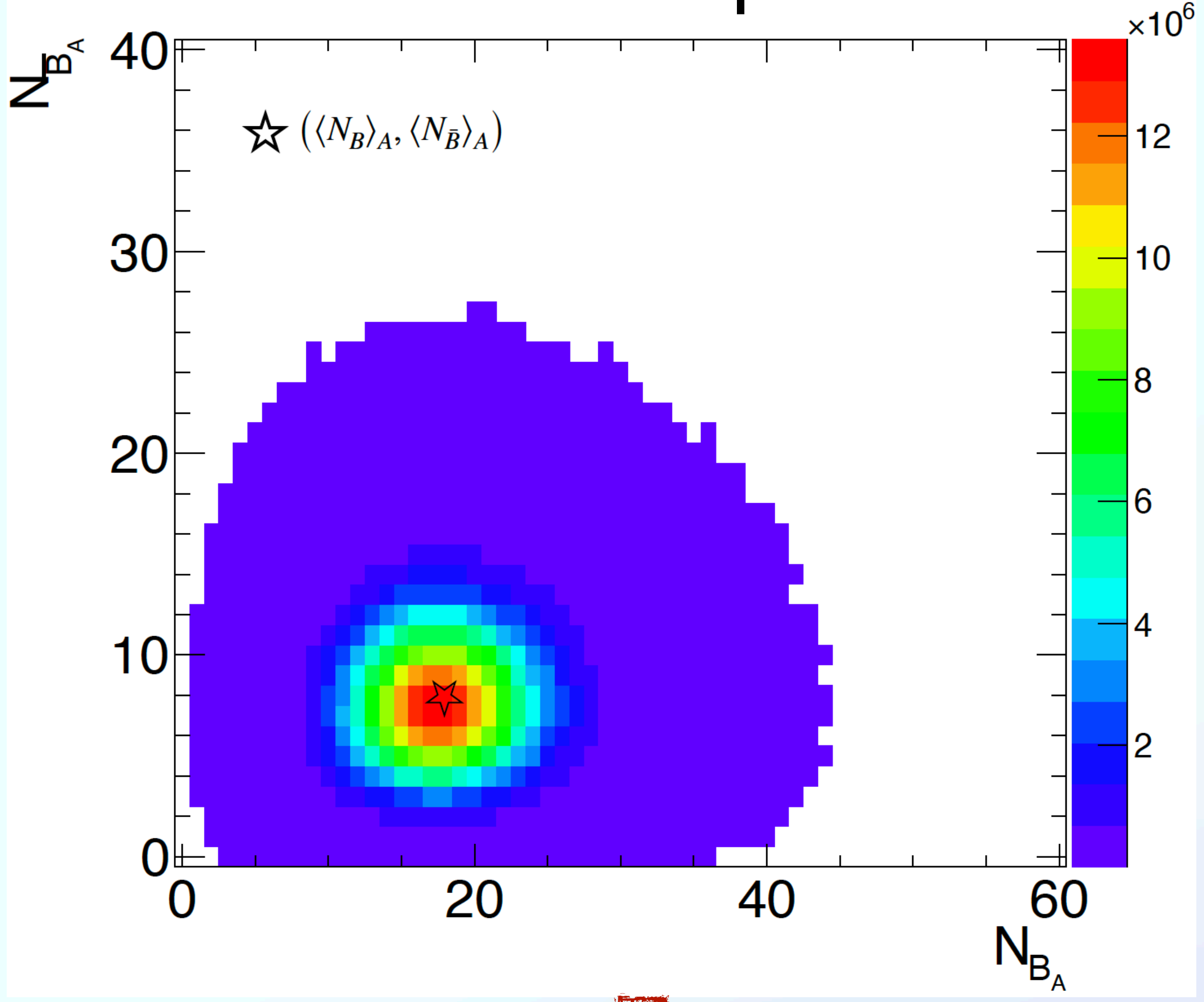
full phase space

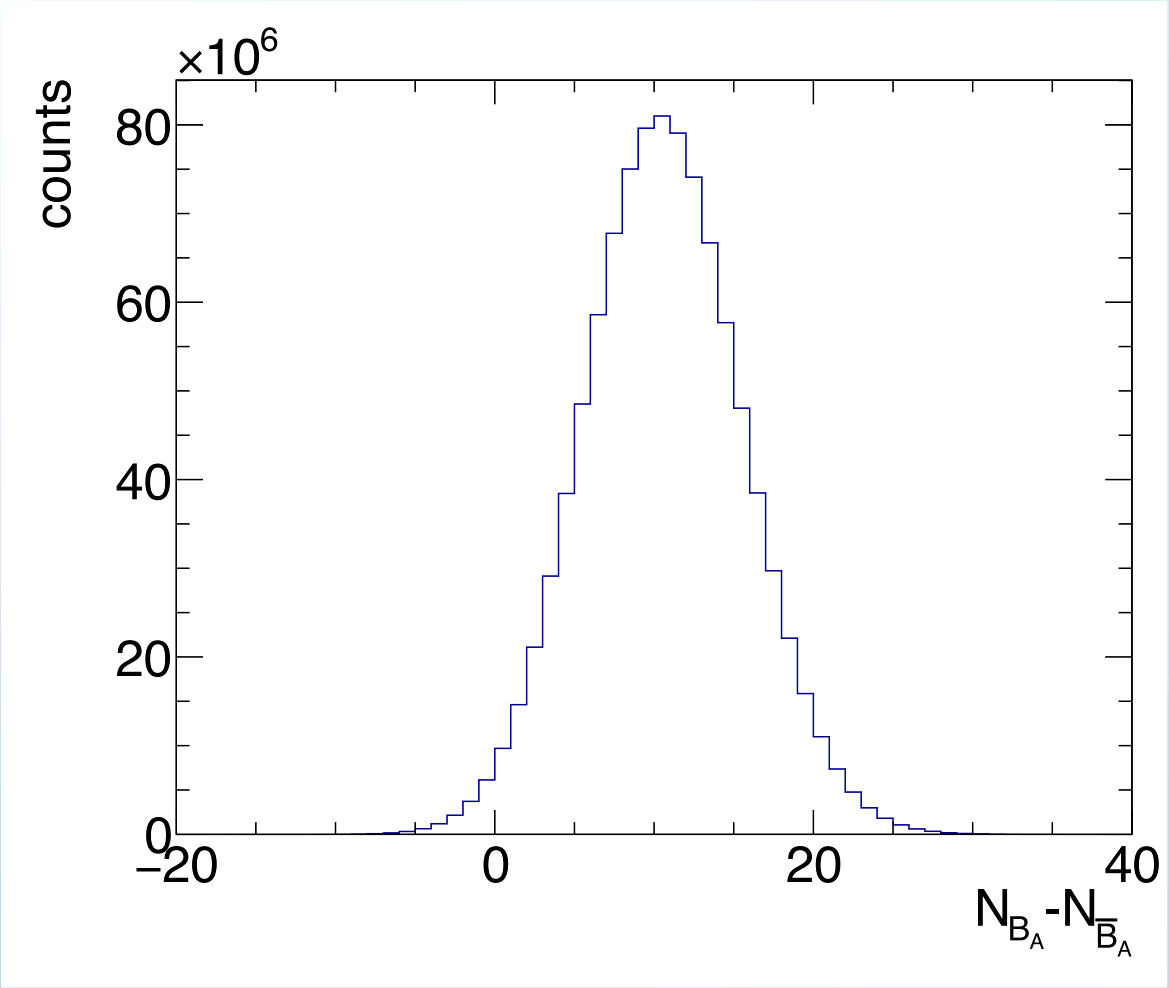


acceptance selection



within acceptance





Comparison to baseline

P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

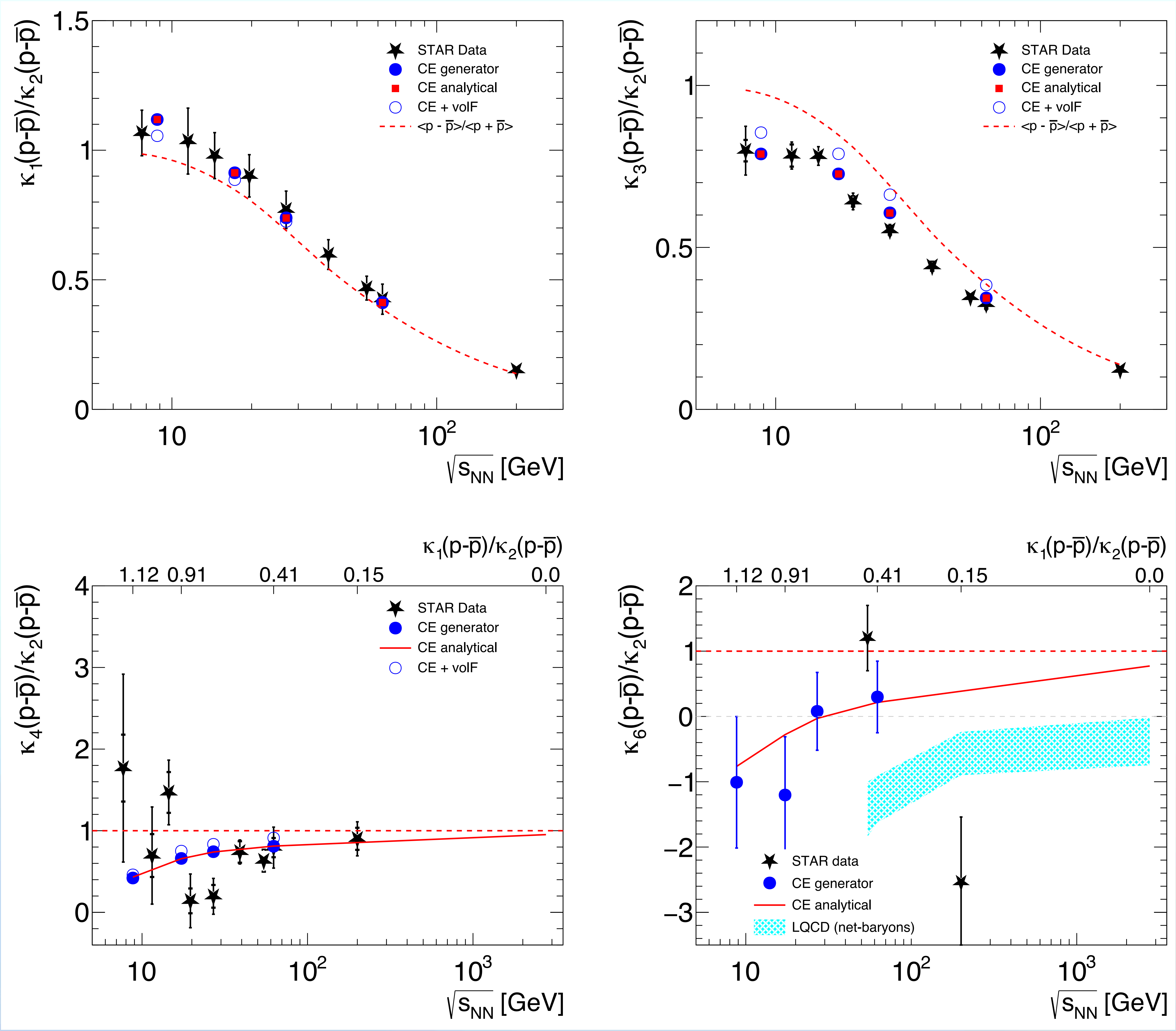
Baseline Calculator

A. Rustamov, B. Friman

https://github.com/e-by-e/Cumulants-CE.git

Results from STAR vs. canonical baseline

the first quantitative and most precise canonical baselines



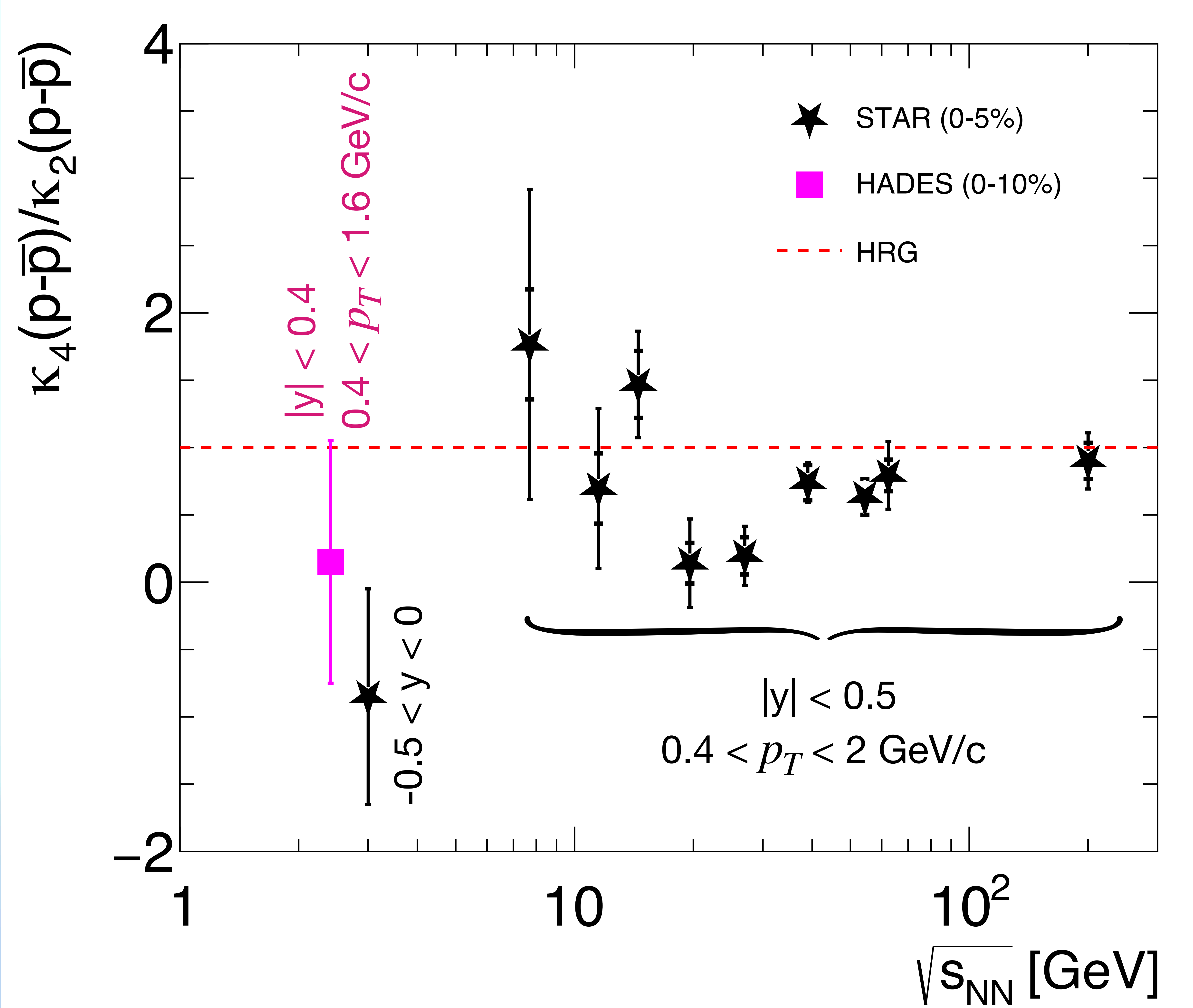
remarkable agreement between canonical baseline and STAR BESI data

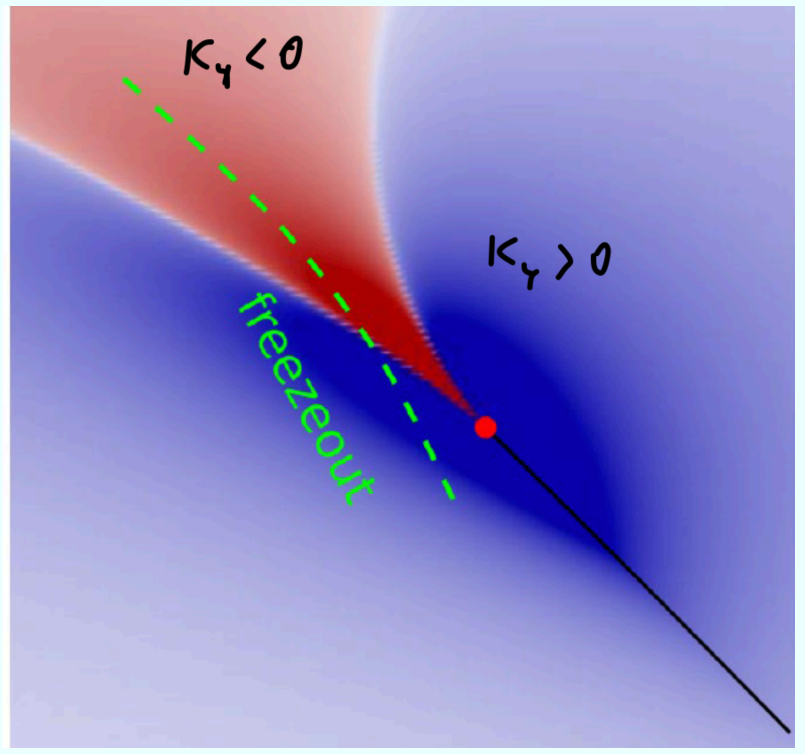
- significant reduction of canonical baseline for κ_6/κ_2 going from positive values at LHC to negative values at lower energies
- STAR DATA for κ_6/κ_2 is not consistent with the LQCD predictions

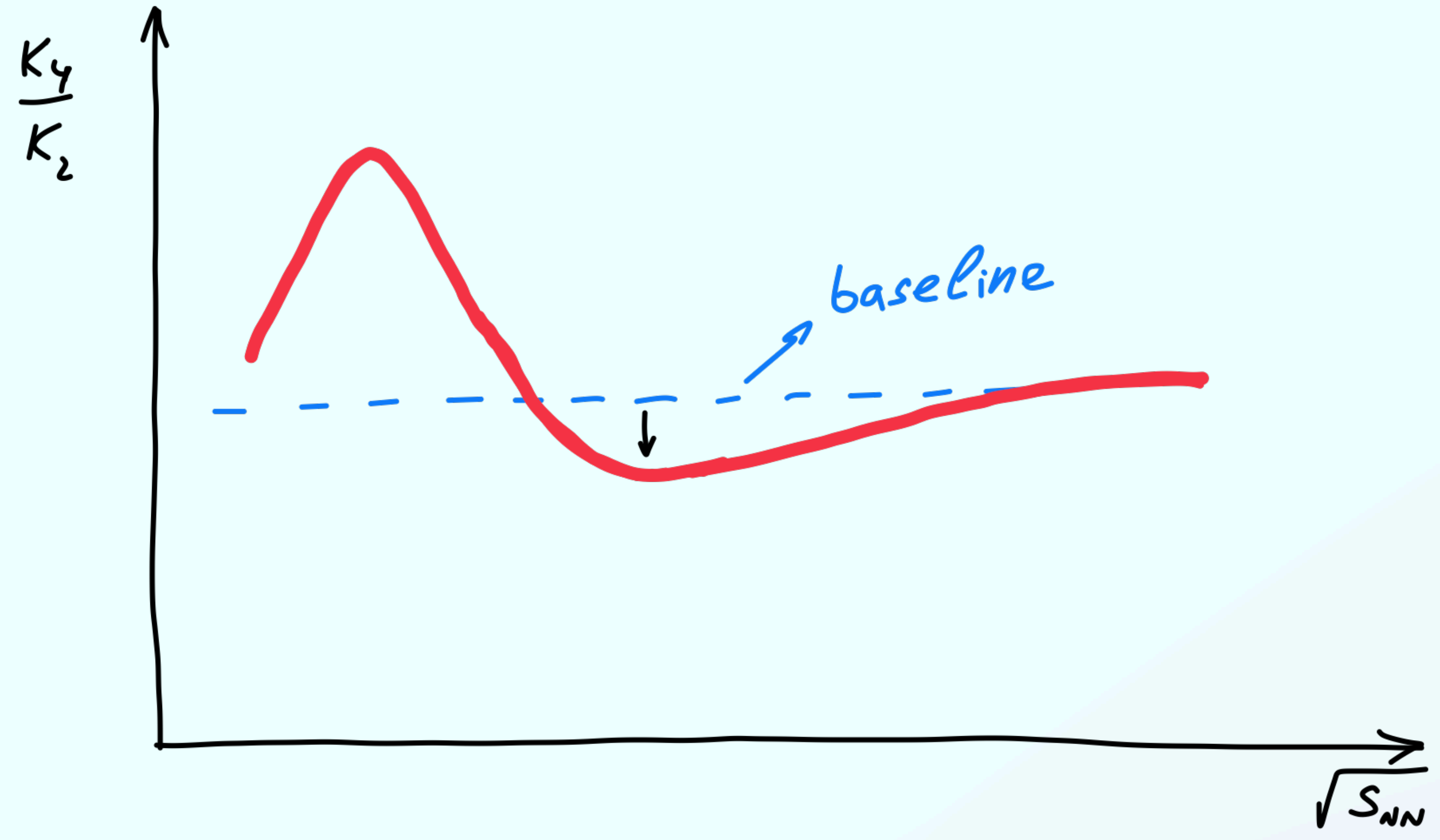
STAR: PRL 126 (2021) 9, 092301, PRL 130 (2023) 8, 082301
P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







a dip in the excitation function is generic

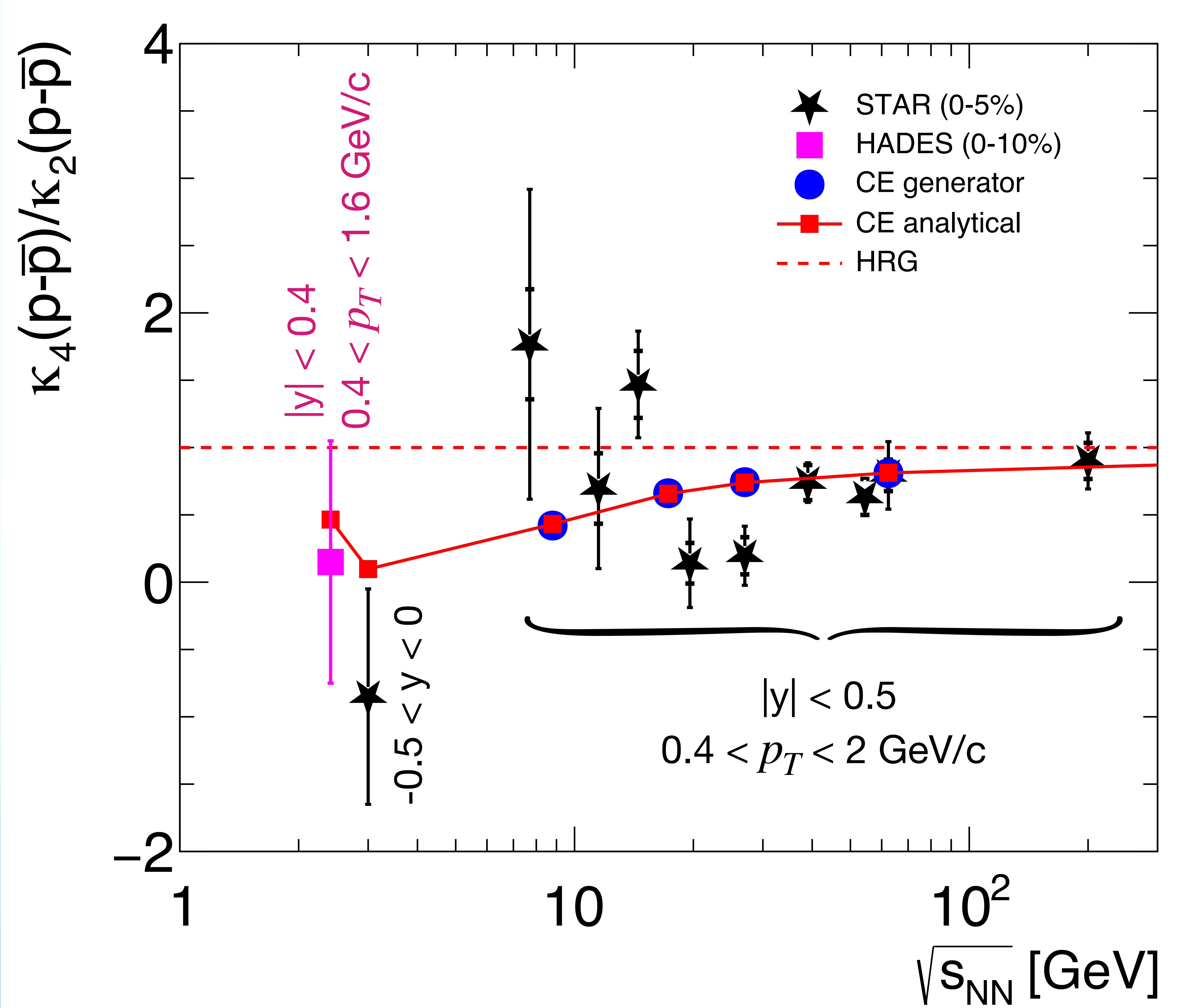
M. Stephanov, PRL102.032301(2009), PRL107.052301(2011)
M.Cheng et al, PRD79.074505(2009)

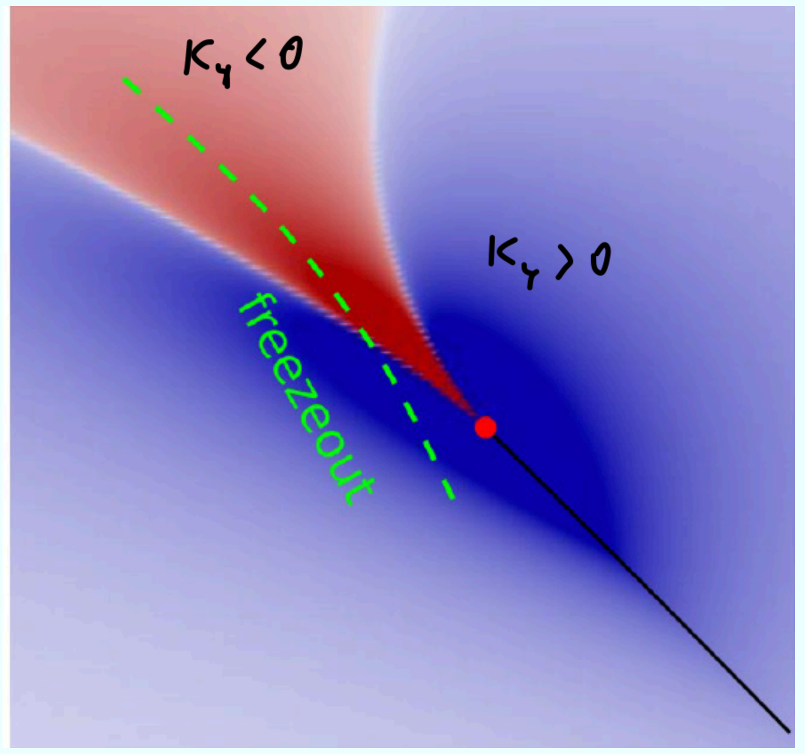
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301

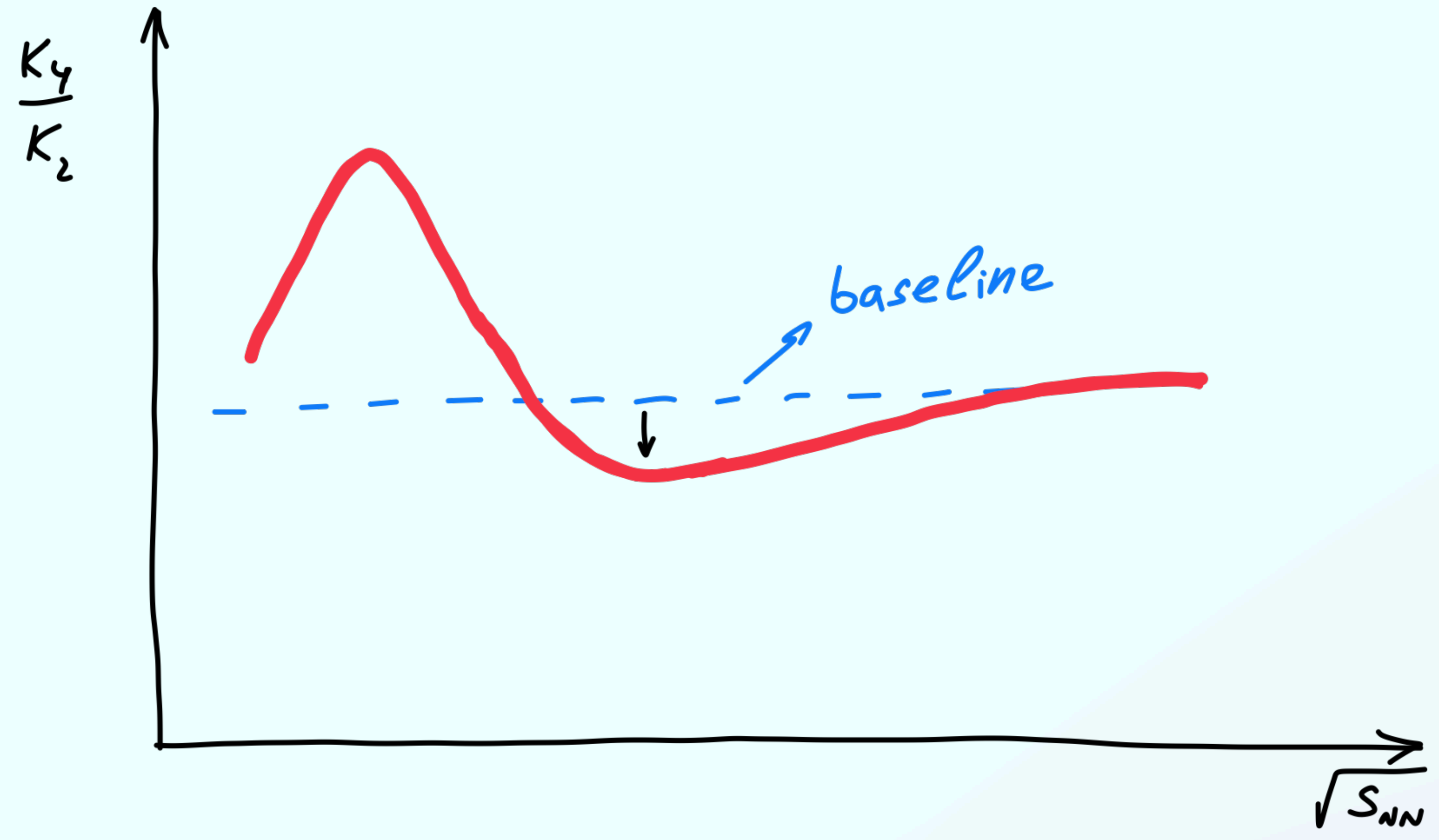
"non-monotonic behavior with a significance of 3.1σ relative to GCE expectation"

Energy excitation function of κ_4/κ_2 in central Au-Au collisions

HADES: Phys.Rev.C 102 (2020) 2, 024914
STAR: Phys.Rev.Lett. 126 (2021) 9, 092301







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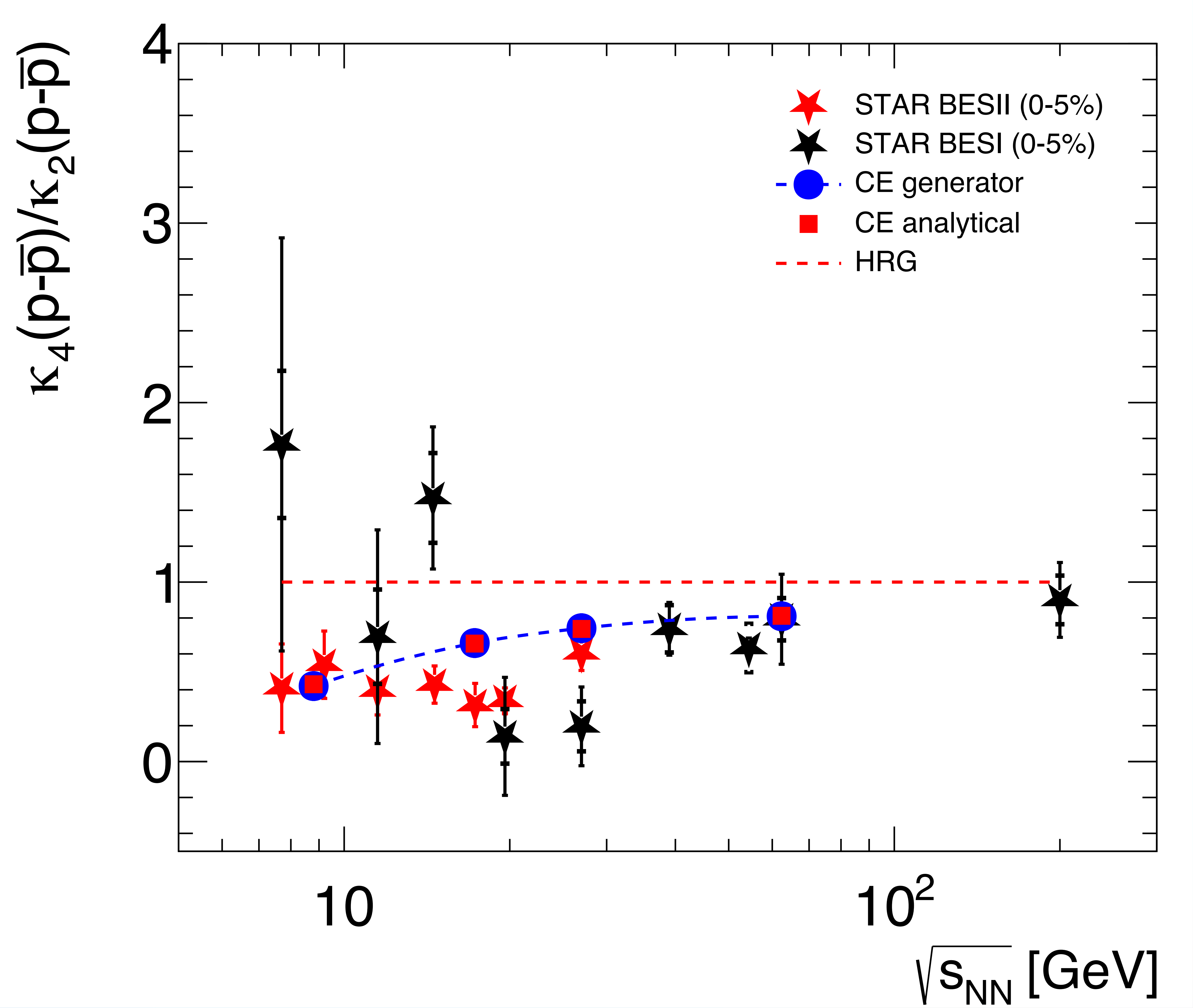
“non-monotonic behavior with a significance of 3.1σ relative to GCE expectation”

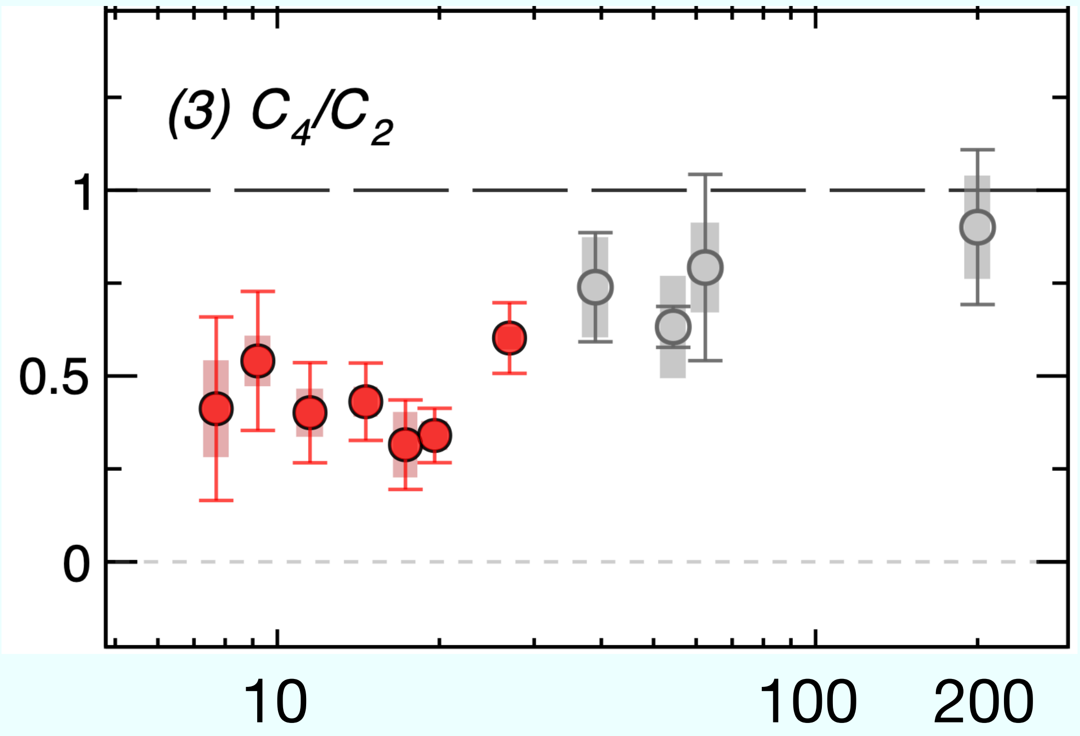
CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

no statistically significant difference between the data and the canonical baseline (KS test: 1.2σ, χ^2 test: 1.5σ)

STAR BES I vs. BES II DATA, κ_4/κ_2



NEW STAR data points are digitized from the pdf plot!



A. Pandav, CPOD 2024

Note: We prefer to plot C_1/C_2

Notation: $C_i \rightarrow \kappa_i$

The NEW data show significantly reduced uncertainties

CE Baseline: P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141

P. Braun-Munzinger, A. R., N. Xu, Annual Review of Nuclear and Particle Science (under preparation)

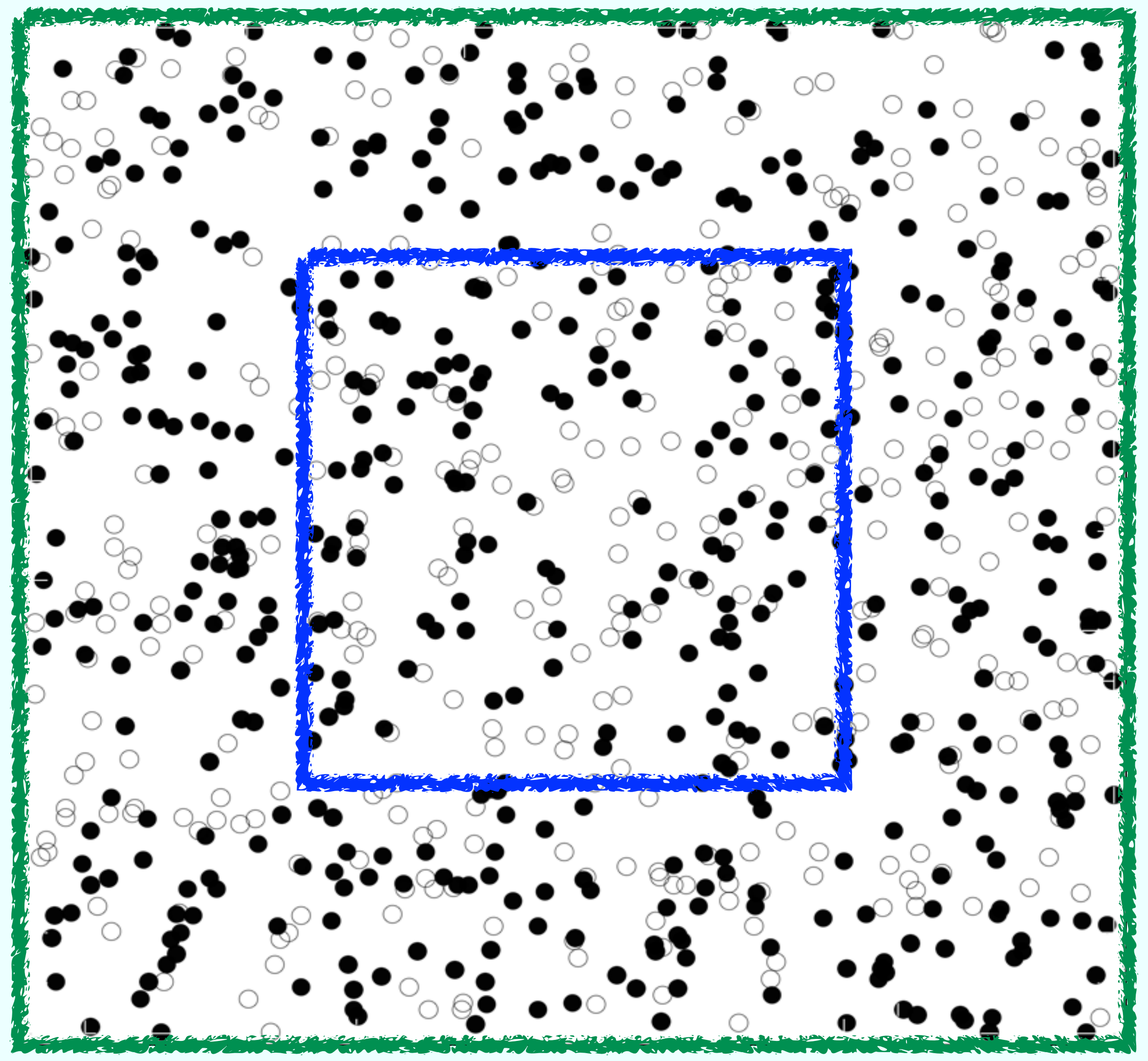
Experimental challenges

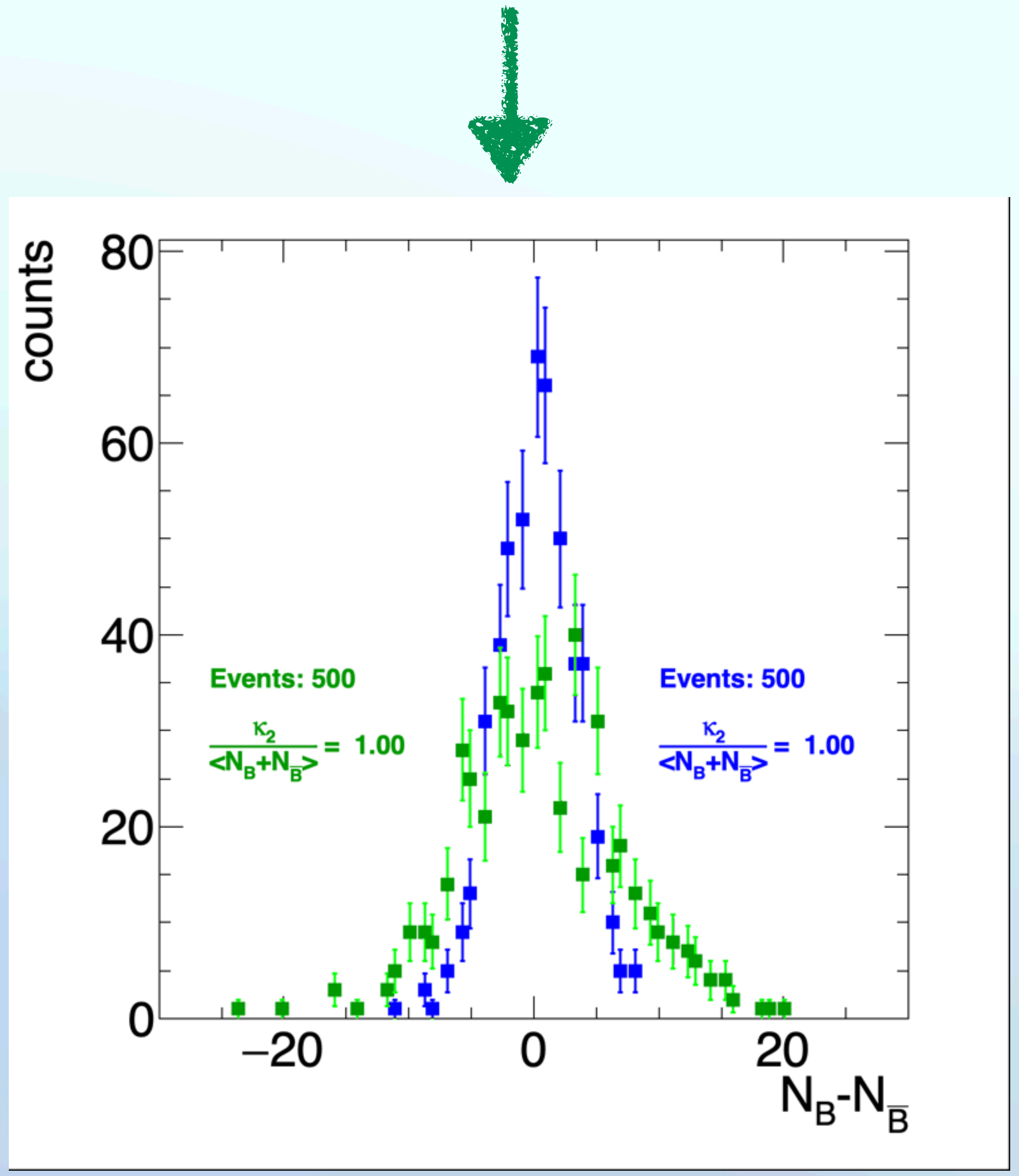
- Volume fluctuations
- Conservation laws
- First ALICE results

Experiment vs. Theory

- Canonical Thermodynamics
- Comparison to STAR results
- Metropolis algorithm
- Comparison to ALICE results
- Outlook

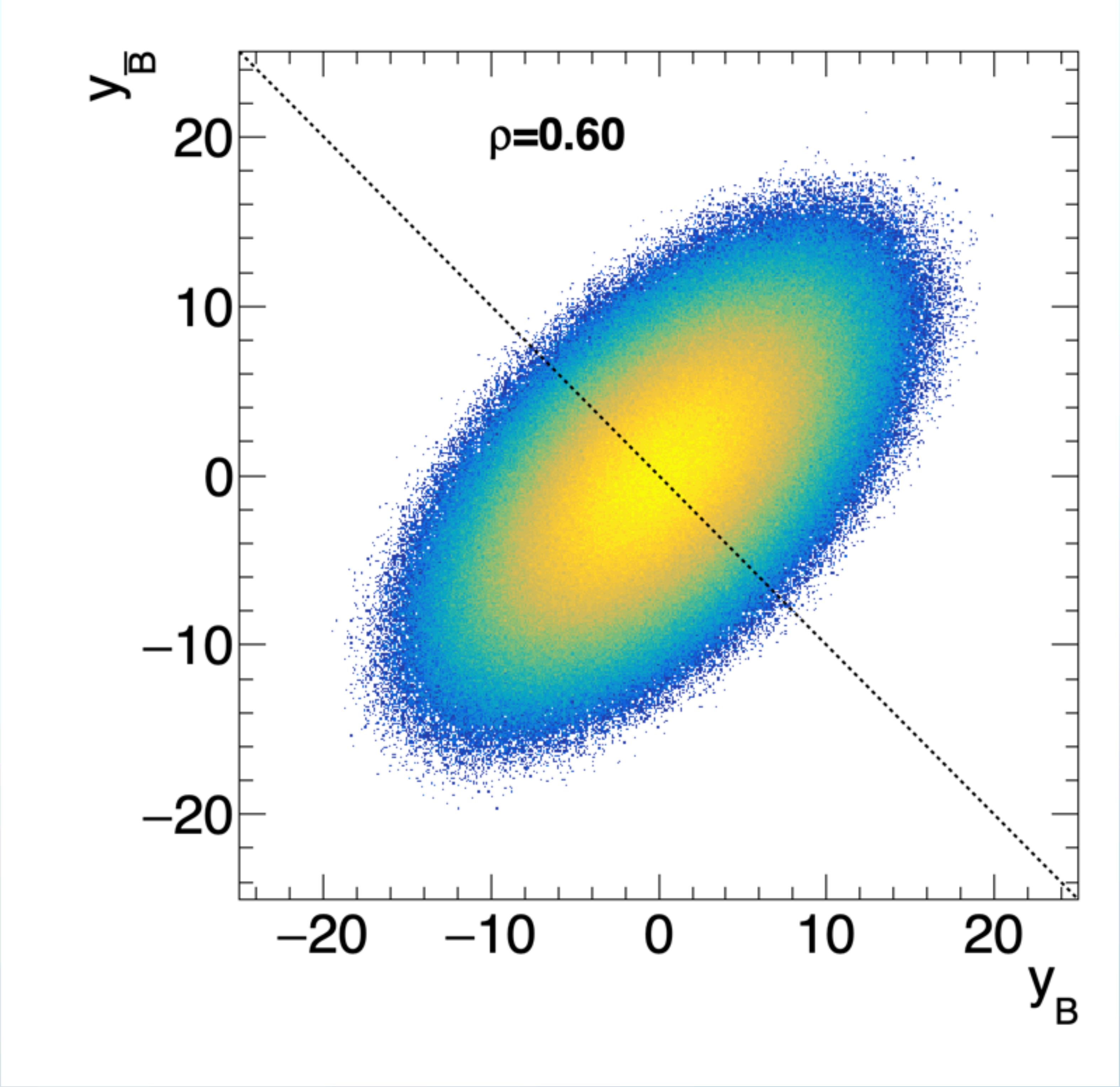
Ideal gas EoS plus local conservation laws



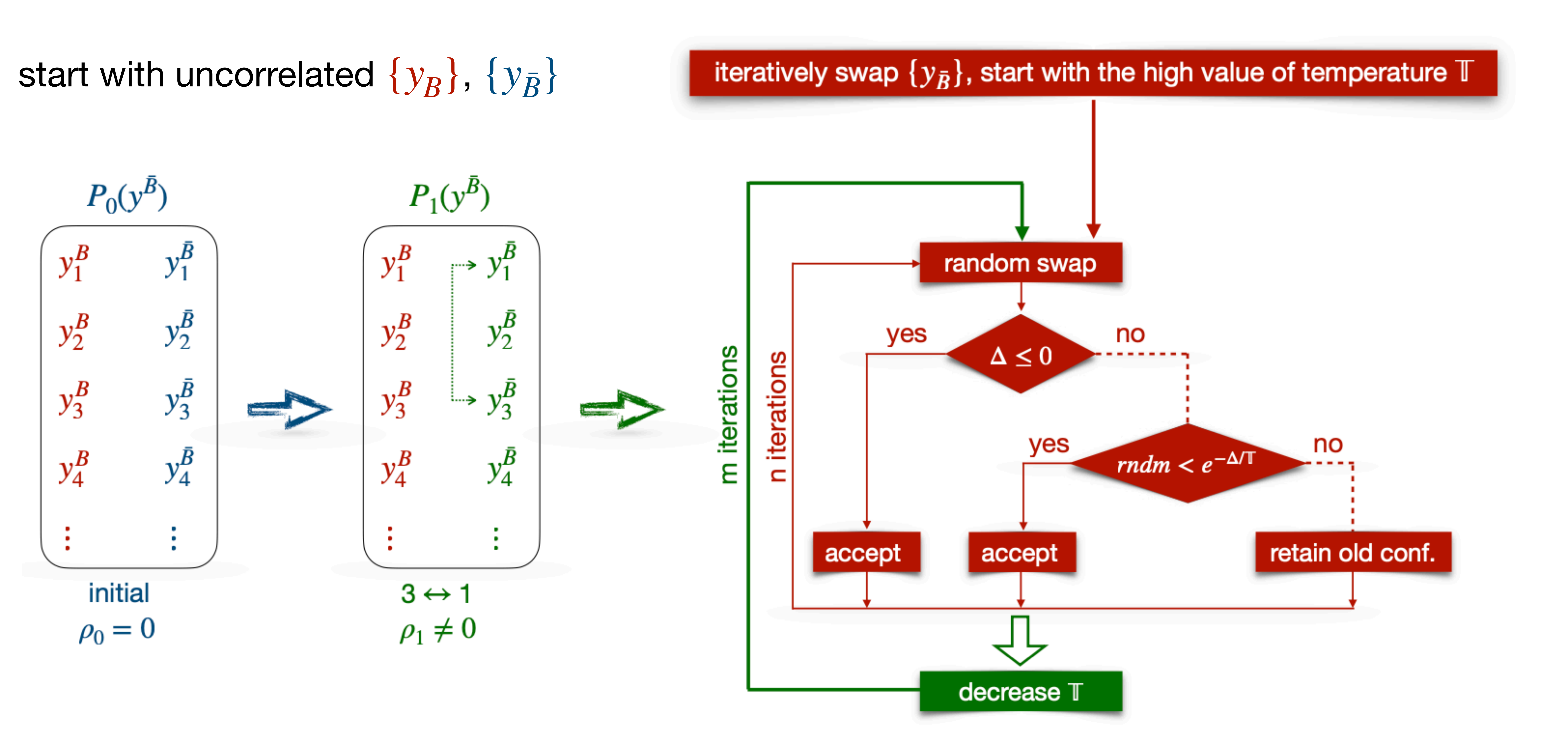


- exploiting **C**anonical **E**nsemble in the full phasespac
- no fluctuations in 4π (like in experiments)

➕ **correlations in rapidity space (local conservations)**



Metropolis algorithm (Simulated annealing)



$$\rho_n = \frac{cov[y_B, P_n(y_{\bar{B}})]}{\sigma_{y_B}\sigma_{y_{\bar{B}}}}$$

$$\Delta = |\rho_n - \rho| - |\rho_{n-1} - \rho|$$

ρ: desired corr. coefficient

works for arbitrary rapidity distributions

A. R., P. Braun-Munzinger, J. Stachel, QM 2022
P. Braun-Munzinger, K. Redlich, A. R., J. Stachel, JHEP 08 (2024) 113

Details of implementation

$$Z_B(V,T) = \sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B\, z_B)^{N_B}}{N_B\,!}\frac{(\lambda_{\bar{B}}\, z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}\,!}\delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B\, z_B}{\lambda_{\bar{B}}\, z_{\bar{B}}}\right)^{\frac{B}{2}} I_B(2\,z\sqrt{\lambda_B\,\lambda_{\bar{B}}})$$

B net baryon number, conserved in each event

I_B modified Bessel function of the first kind

z_B, $z_{\bar{B}}$ single particle partition functions for baryons, anti baryons

λ_B, $\lambda_{\bar{B}}$ auxiliary parameters for calculating cumulants of baryons, anti baryons

A. R., P. Braun-Munzinger, J. Stachel, QM 2022

P. Braun-Munzinger, K. Redlich, A. R., J. Stachel, JHEP 08 (2024) 113

baryon number conservation (CE partition function) + **correlations in y space** (Metropolis algorithm) →

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event 1

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event 2

... ...

$N_B, N_{\bar{B}}, y_B, y_{\bar{B}}$ event n

Input from experiments

- baryon rapidity distributions
- measured (canonical) $\langle N_B \rangle$, $\langle N_{\bar{B}} \rangle$

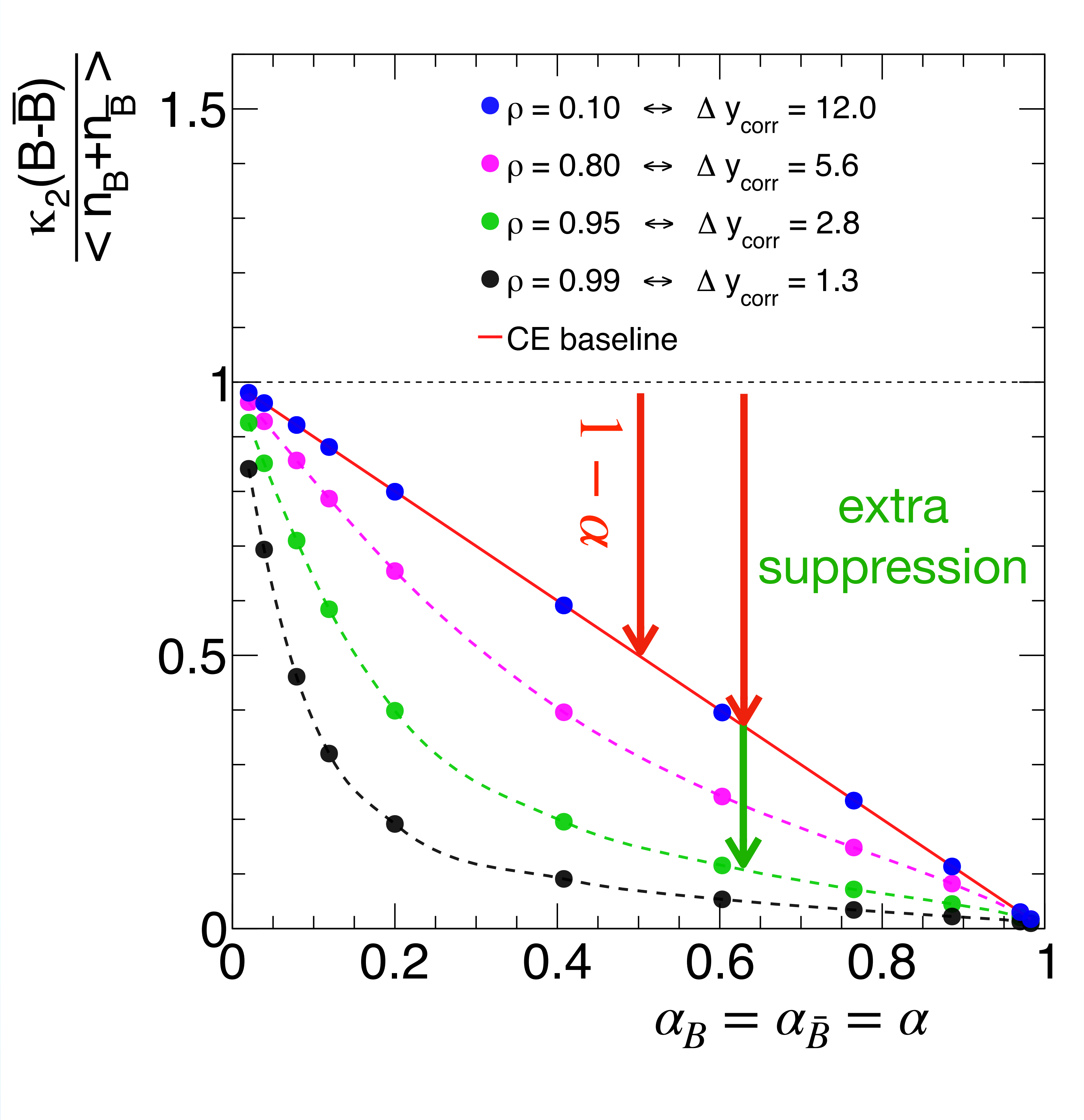
$z = \sqrt{z_B z_{\bar{B}}}$ **is calculated by solving**

$$\langle N_B \rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B,\,\lambda_{\bar{B}}\,=\,1} = z\frac{I_{B-1}(2\,z)}{I_B(2\,z)}$$

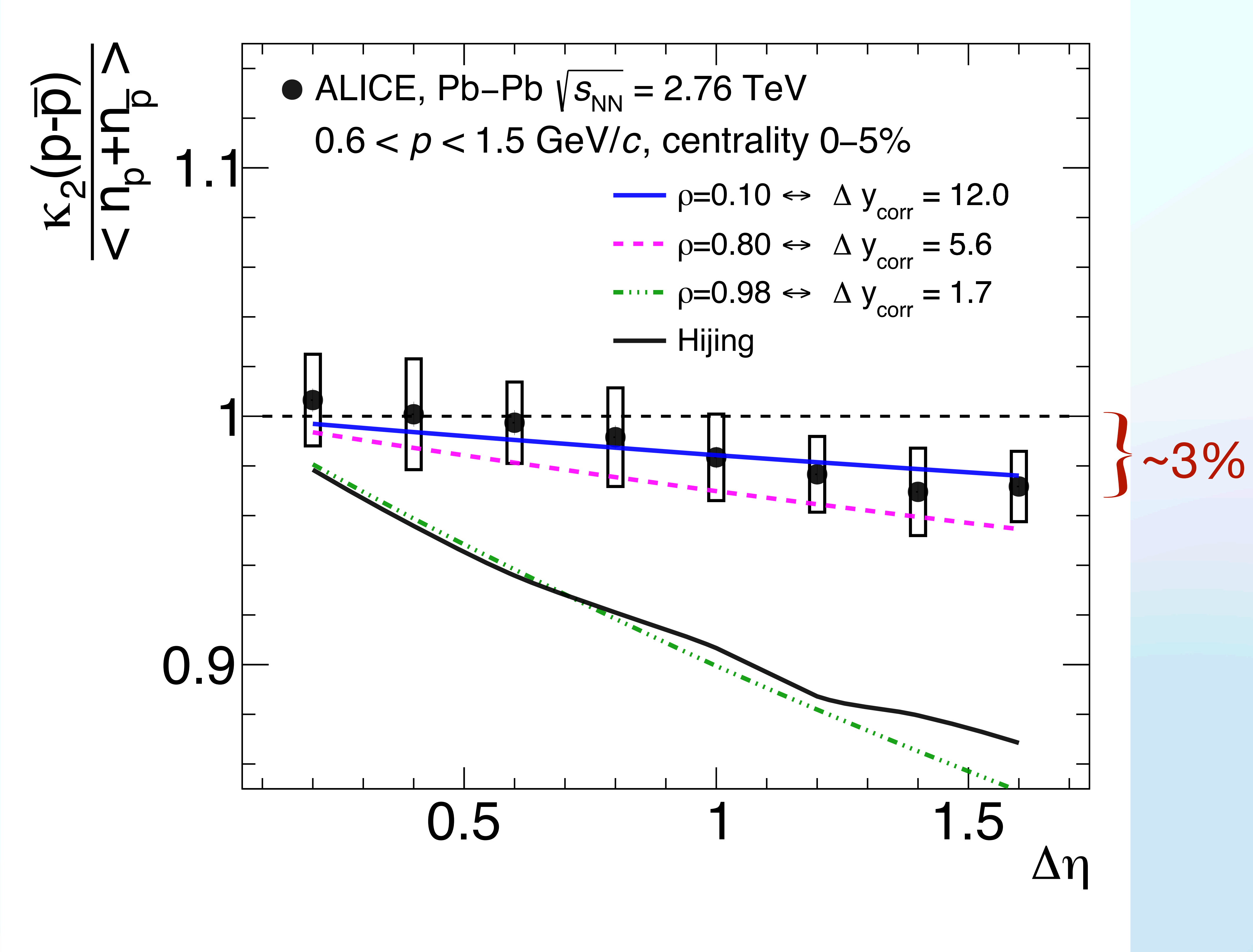
ALICE Results (Identity Method)

P. Braun-Munzinger, B. Friman, K. Redlich, A.R., J. Stachel, NPA 1008 (2021) 122141
P. Braun-Munzinger, K. Redlich, A.R., J. Stachel, JHEP 08 (2024) 113

ALICE: Phys. Lett. B 807 (2020) 135564, Phys. Lett. B (2022) 137545

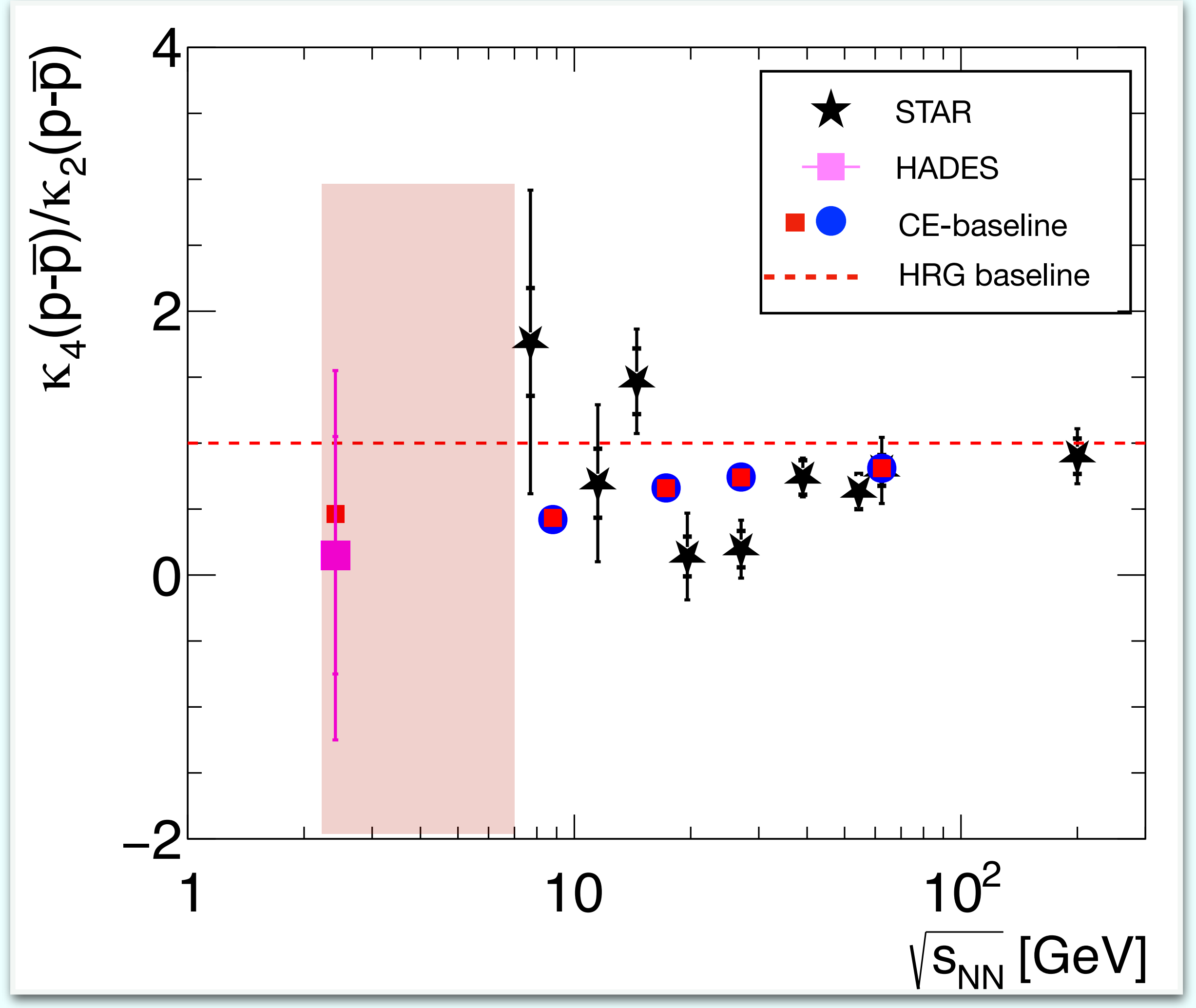


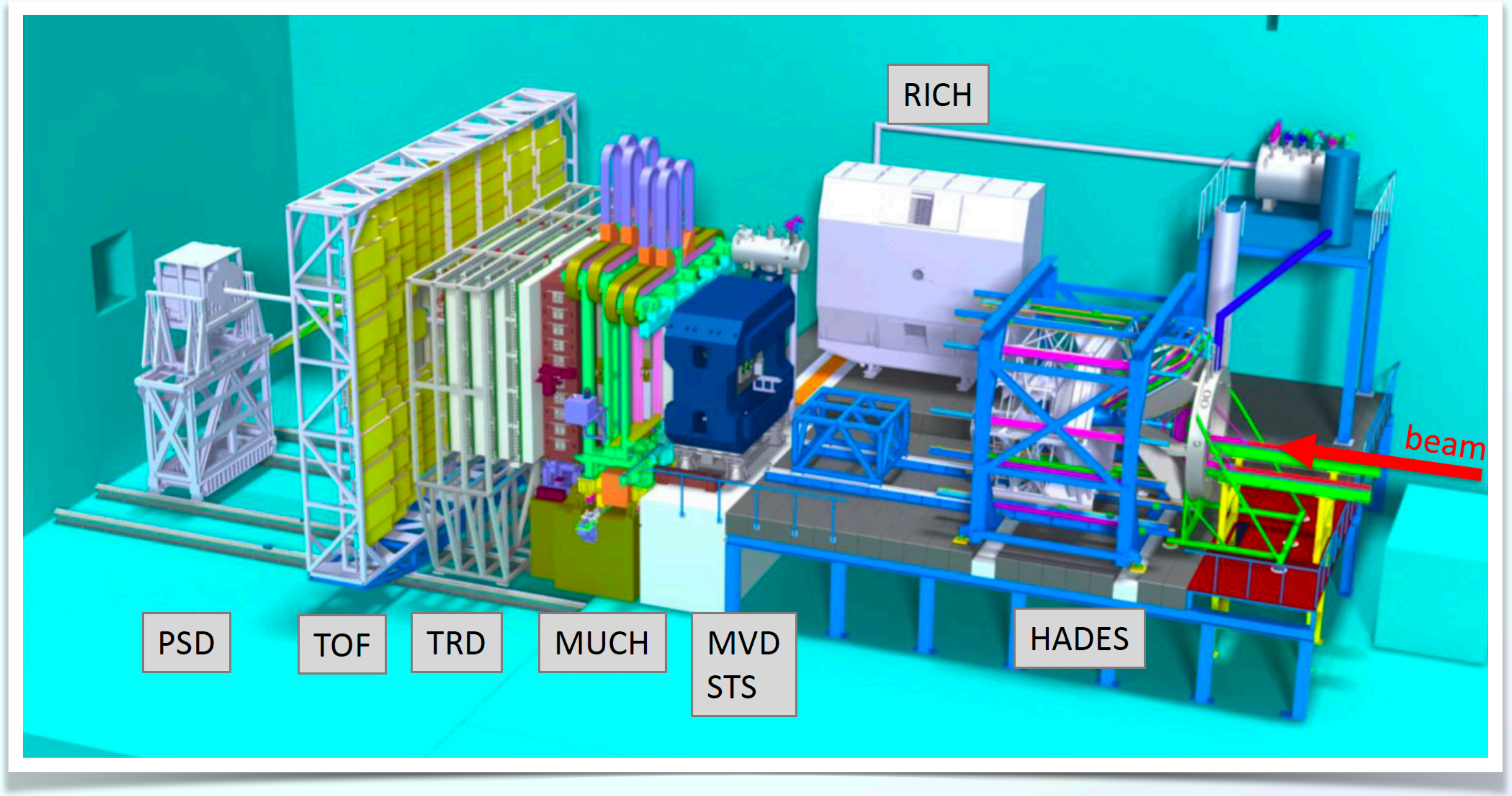
essential to constrain baryon production mechanism

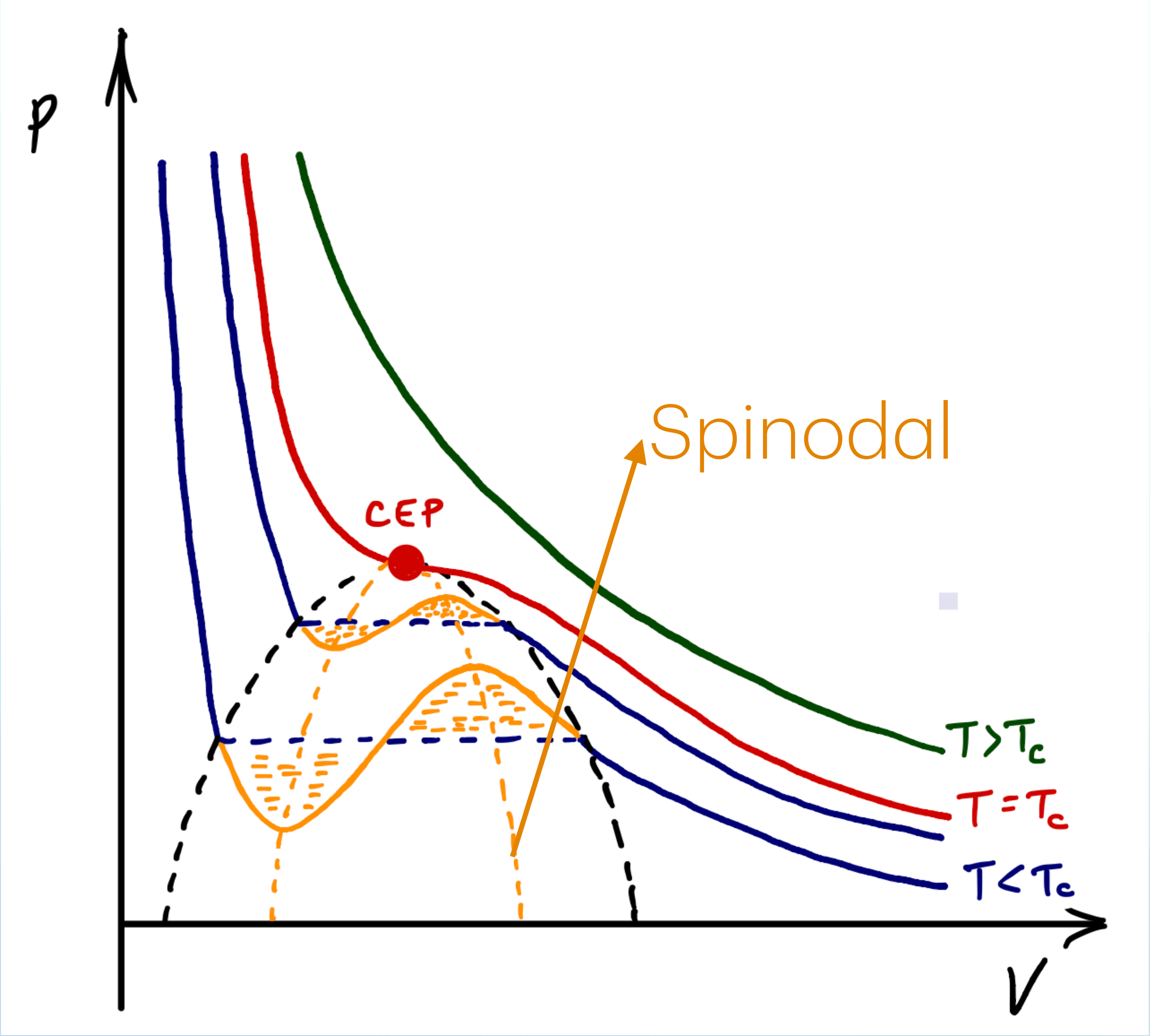


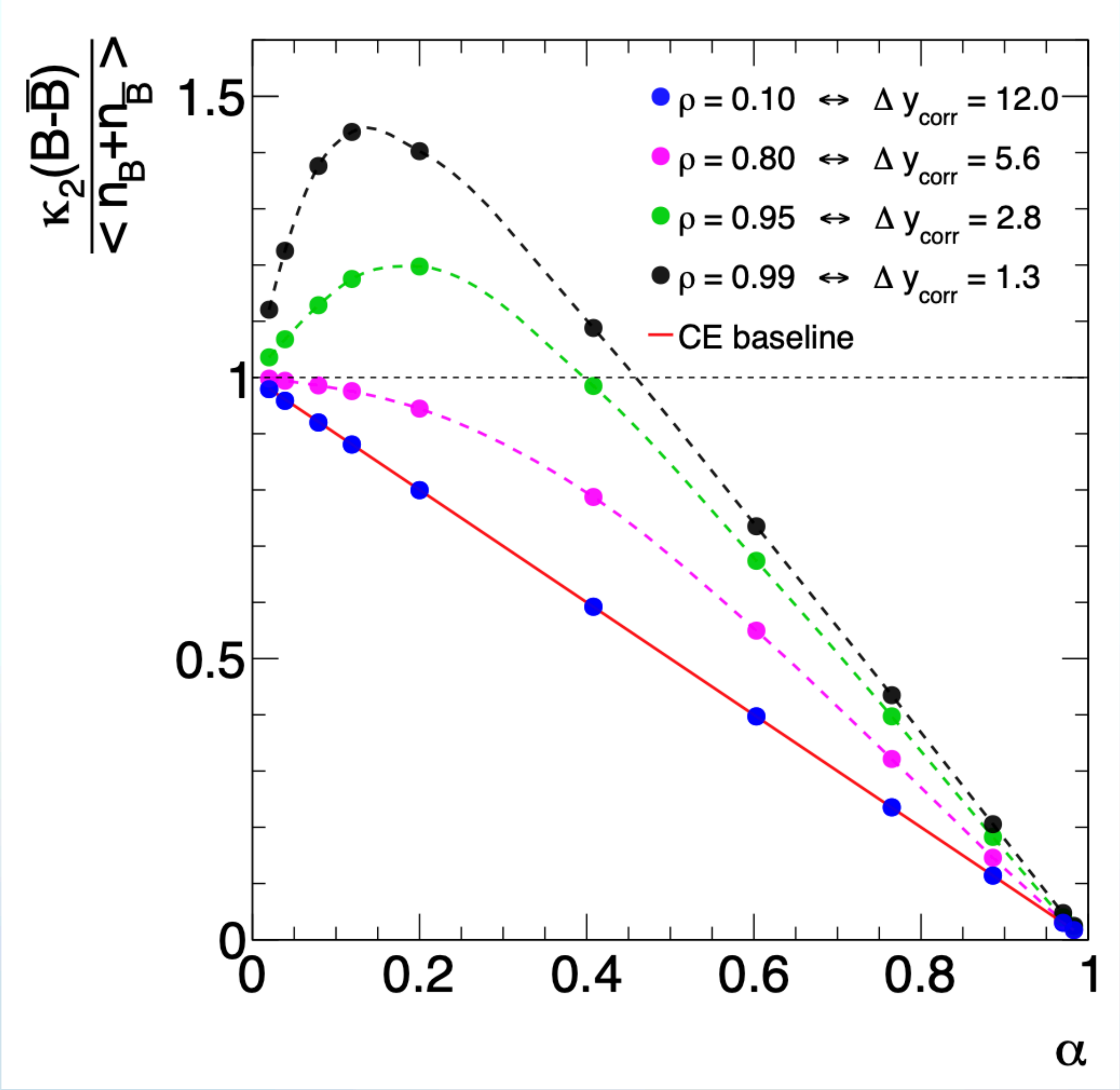
- Alice data: best description with $\rho = 0.1$ ($\Delta y_{corr} = 12$) ↔ **Global baryon number conservation**
- Agreement with LQCD predictions
- Calls into question baryon production mechanism in Hjing (Lund String Fragmentation)

Near future, CBM









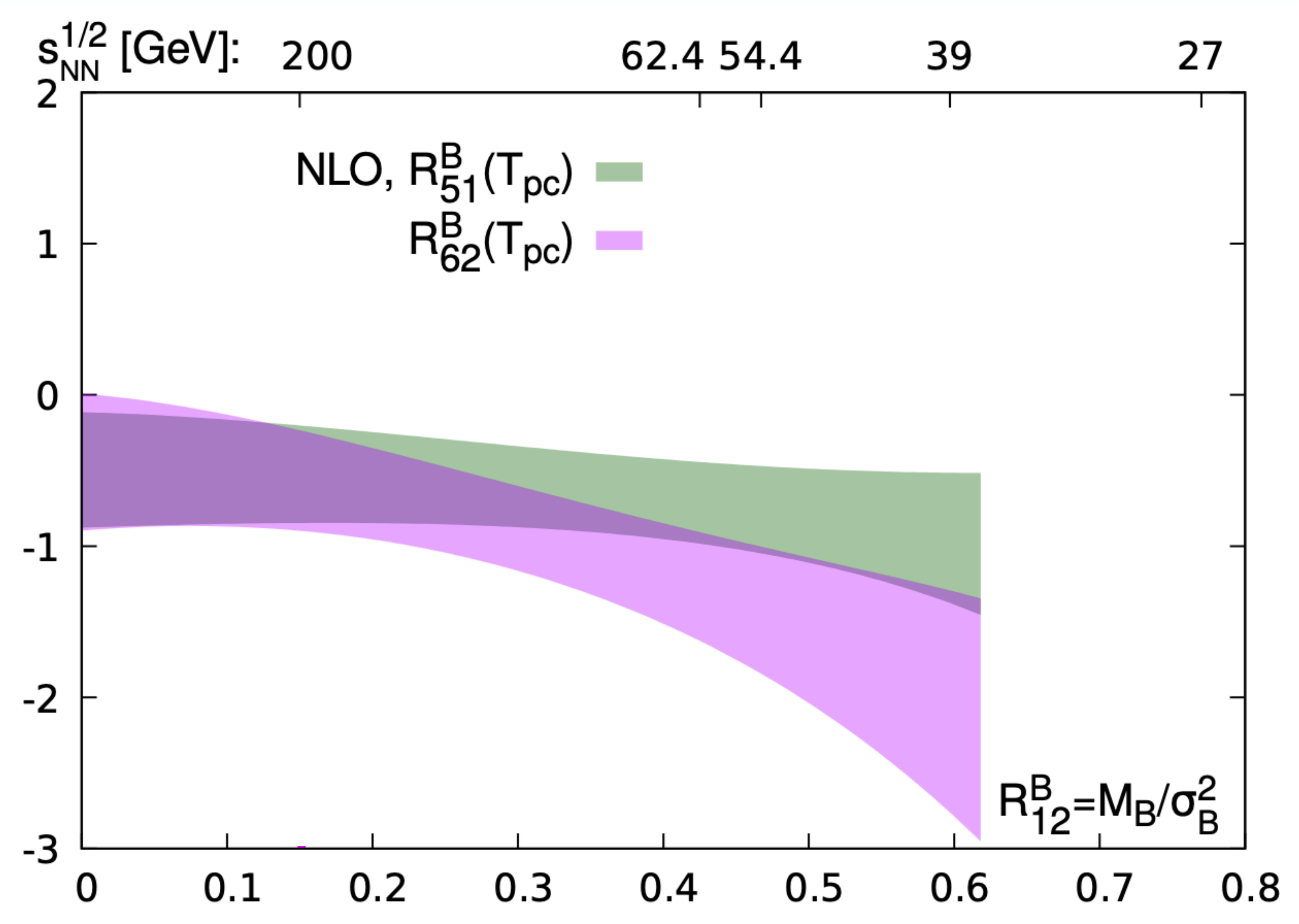
- Systematic measurements of fluctuations stemming from critical point
- Measuring fluctuations induced by spinodal decomposition
- Search for cluster formation

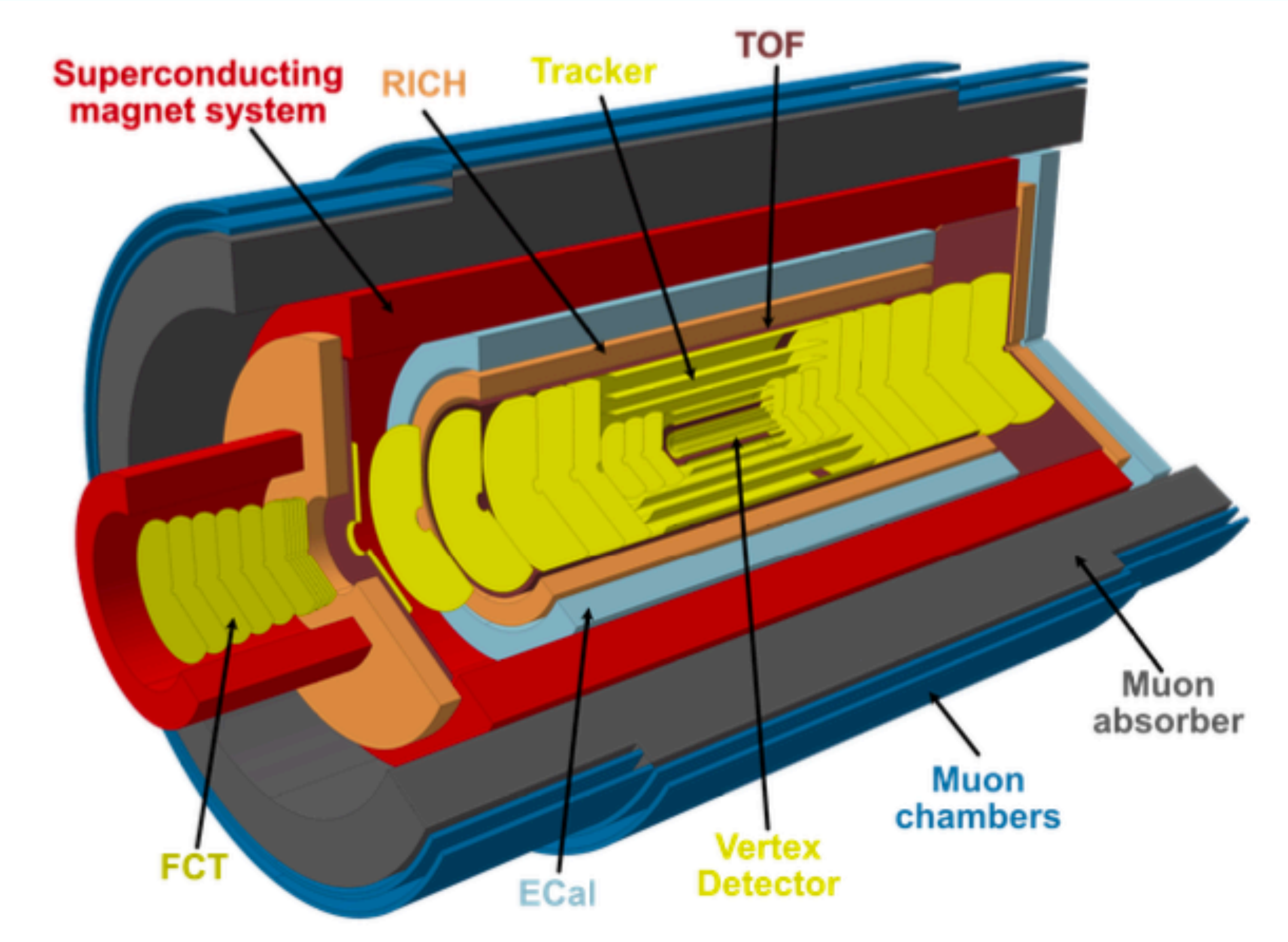
P. Braun-Munzinger, K. Redlich, A. R., J. Stachel, JHEP 08 (2024) 113

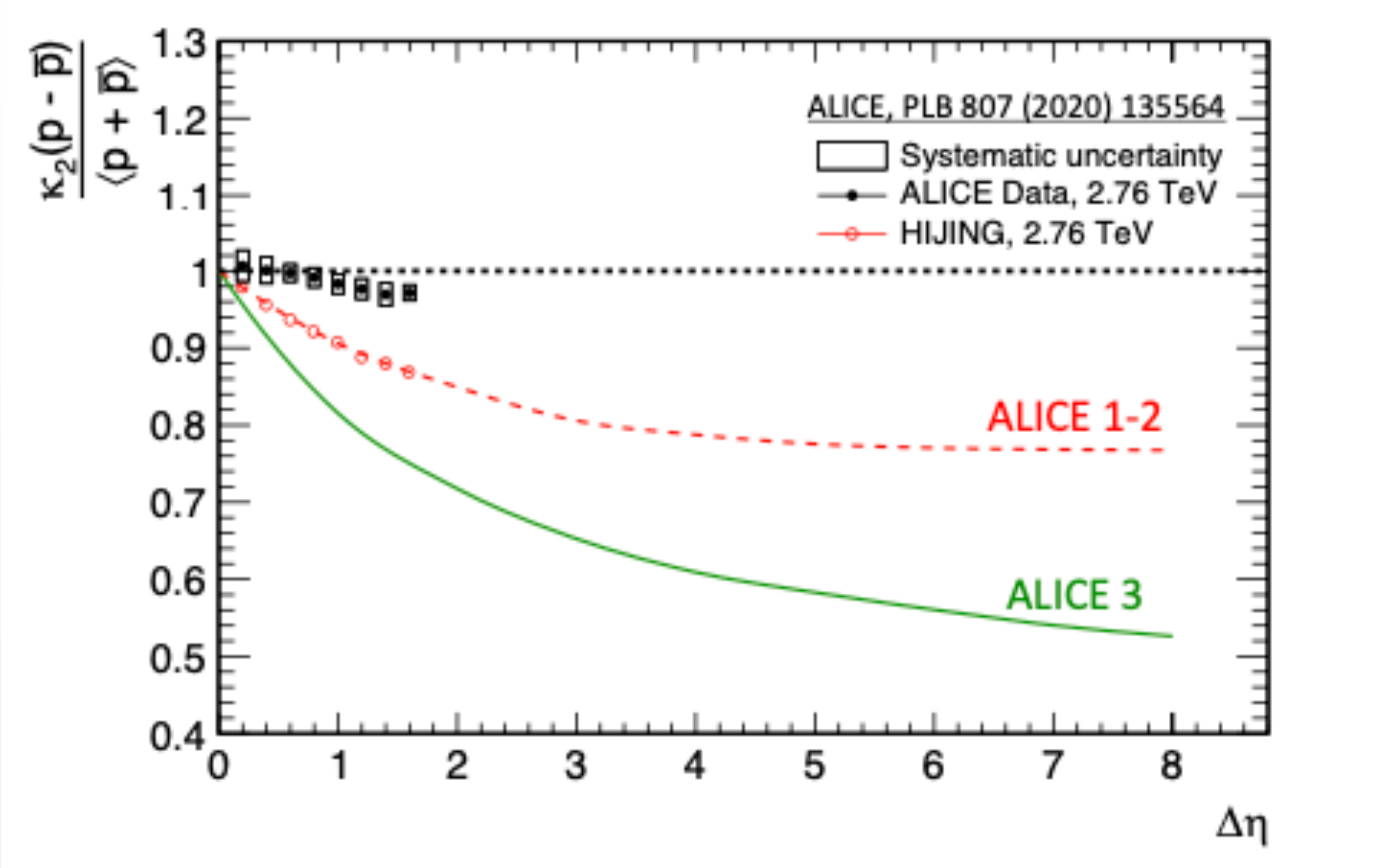
C. Sasaki, B. Friman, K. Redlich, Phys.Rev.D 77 (2008) 034024

Near future, ALICE3

e-Print: 2211.02491 [physics.ins-det]







Acceptance coverages

- ALICE 1-2: $0.6 < p < 1.5$ GeV/c, $|\eta| < 0.8$
- ALICE 3: $0.3 < p < 10$ GeV/c, $|\eta| < 4$

Opens new avenues, such as study of charm fluctuations

Happy Birthday, Dear Johanna!

May your journey, both in physics and in life, be filled with breakthrough moments, smooth trajectories, and just the right amount of **fluctuations** to keep things interesting.


THANKS FOR YOUR
ATTENTION