

Exploring the QCD phase diagram with experiments in discrete space-time

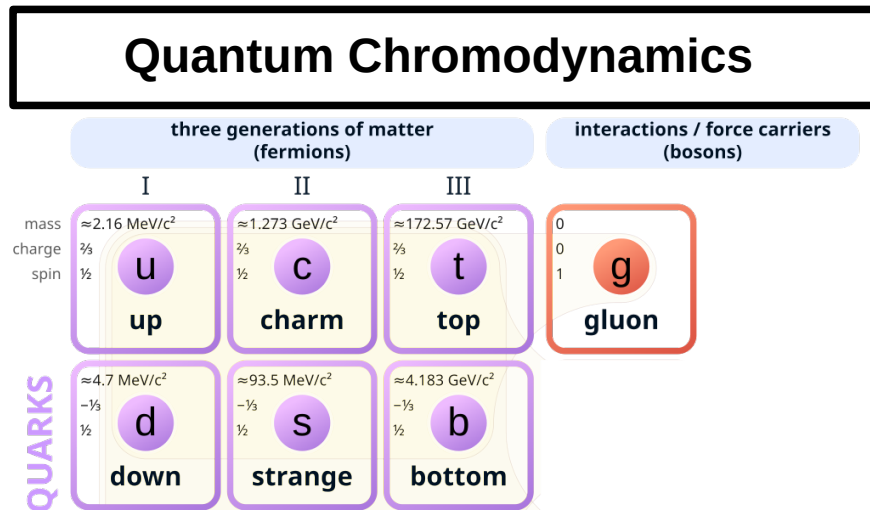


- the phase diagram on strongly interacting matter,
chiral symmetry restoration and the axial anomaly
Sabarnya Mitra
- screening masses and the axial anomaly
Tristan Ueding, Yu Zhang
- probing chiral symmetry restoration and deconfinement
in QCD with heavy quark cumulants
Sipaz Sharma



Deutsche
Forschungsgemeinschaft

When we entered University...



Goldstone-Modes

Y. Nambu (1960)
J. Goldstone (1961)
Y. Nambu, G. Jona-Lasinio (1961)

QCD

H. Fritzsch, M. Gell-Mann, H. Leutwyler, H. (1973)
D.J. Gross; F. Wilczek (1973)
H.D. Politzer (1973)
K.G. Wilson (1974)

Chiral symmetry restoration

D.J. Gross, R. D. Pisarski, L. J. Yaffe (1981)
J. Kogut et al. (1983)

Parity doubling and chiral symmetry restoration

T. Hatsuda, T. Kunihiro (1985)

J.B. Kogut, Three lectures on lattice gauge theory, in: *Many degrees of freedom in particle theory* (1976), 8th International Summer Institute on Theoretical Physics, University of Bielefeld, 1976 (edt. H. Satz) and *Rev. Mod. Phys.* 51 (1979)

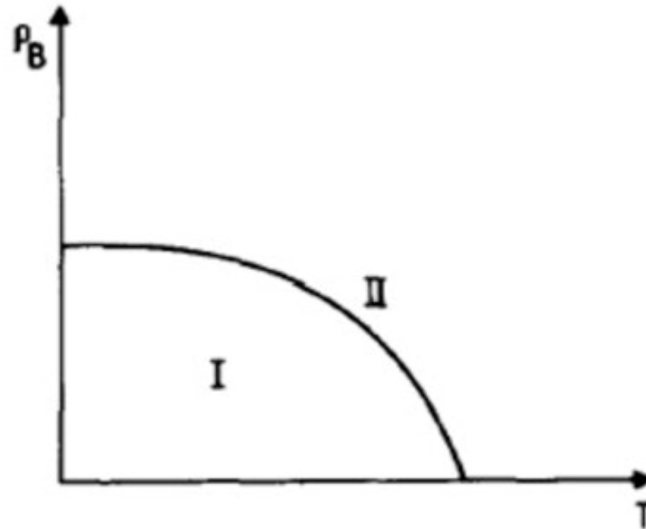
Strongly interacting matter in the '70s and early '80s

LGT~1980



Mike Creutz

Phase diagram of QCD



N. Cabibbo, G. Parisi,
Phys. Lett. 59B (1975) 67

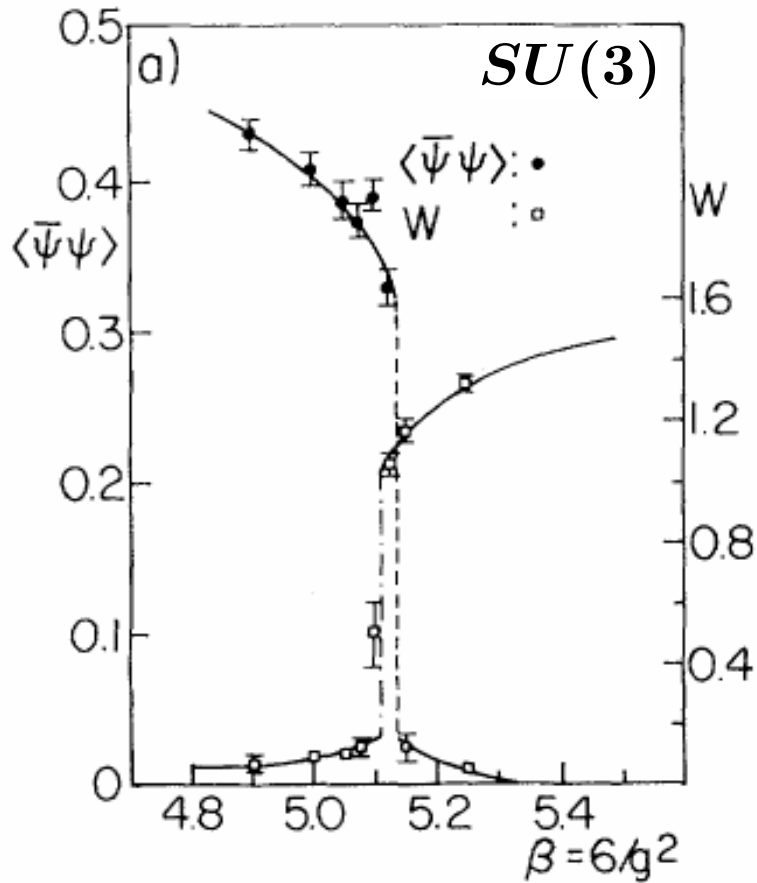
HRG~1964



Rolf Hagedorn:
Hadron resonance gas,
ultimate temperature?

- the physics/**thermodynamics of strong interaction matter** is described by the theory of strong interactions – **Quantum Chromo Dynamics (QCD)**
- understanding highly non-perturbative/collective effects like **phase transitions** requires the application of numerical techniques – **lattice QCD**

Deconfinement & Chiral Symmetry Restoration



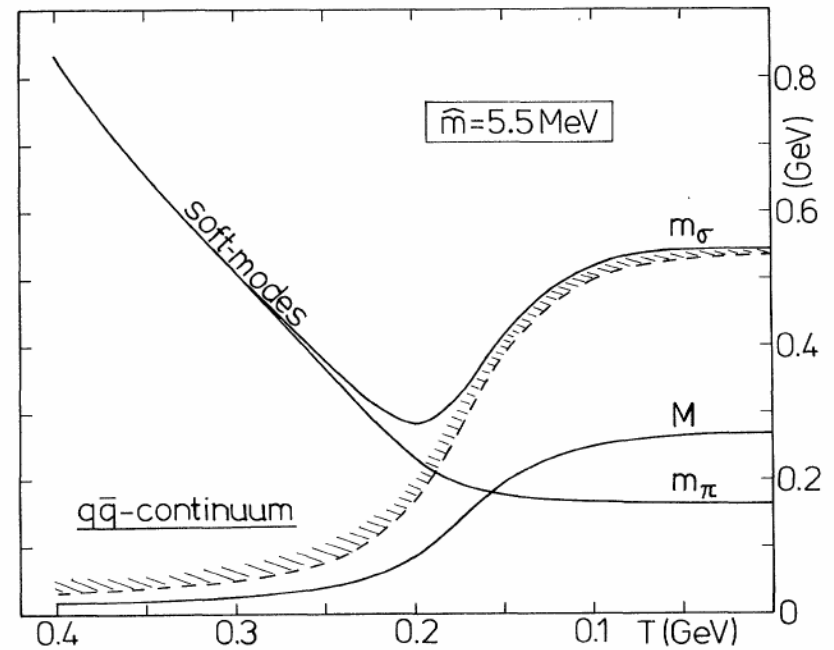
W: Polyakov loop expectation value

$\langle \bar{\psi}\psi \rangle$: chiral condensate

● deconfinement and chiral symmetry restoration are closely related in QCD

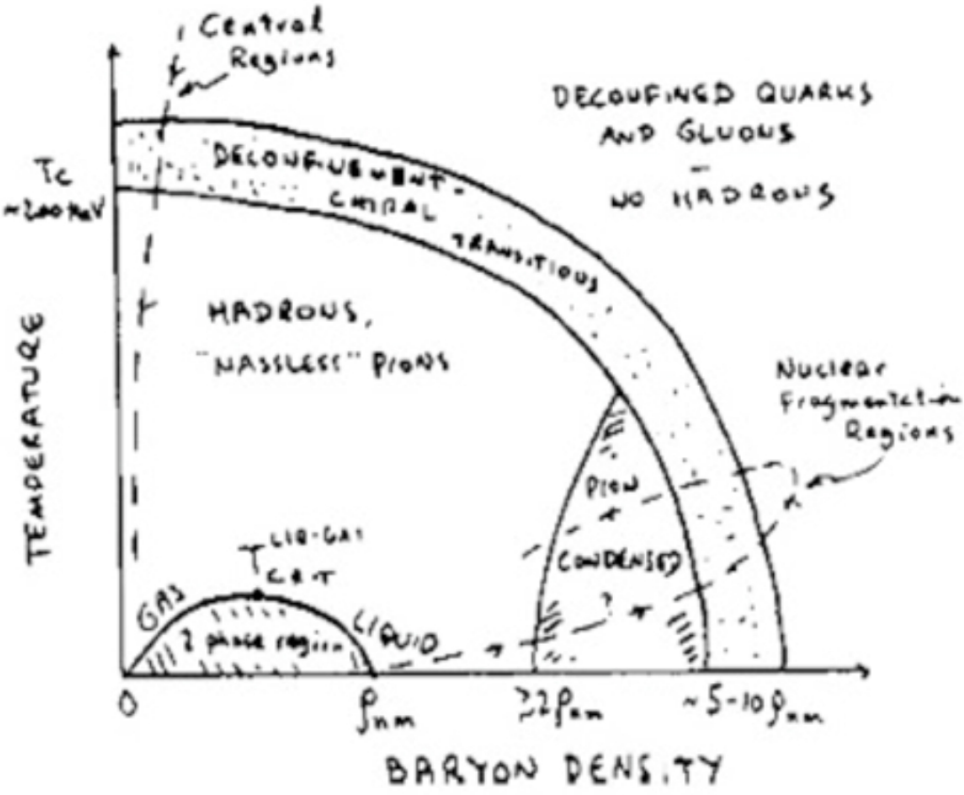
J. Kogut et al., PRL 50 (1983) 393

thermal screening masses
– parity partners degenerate close to T_c

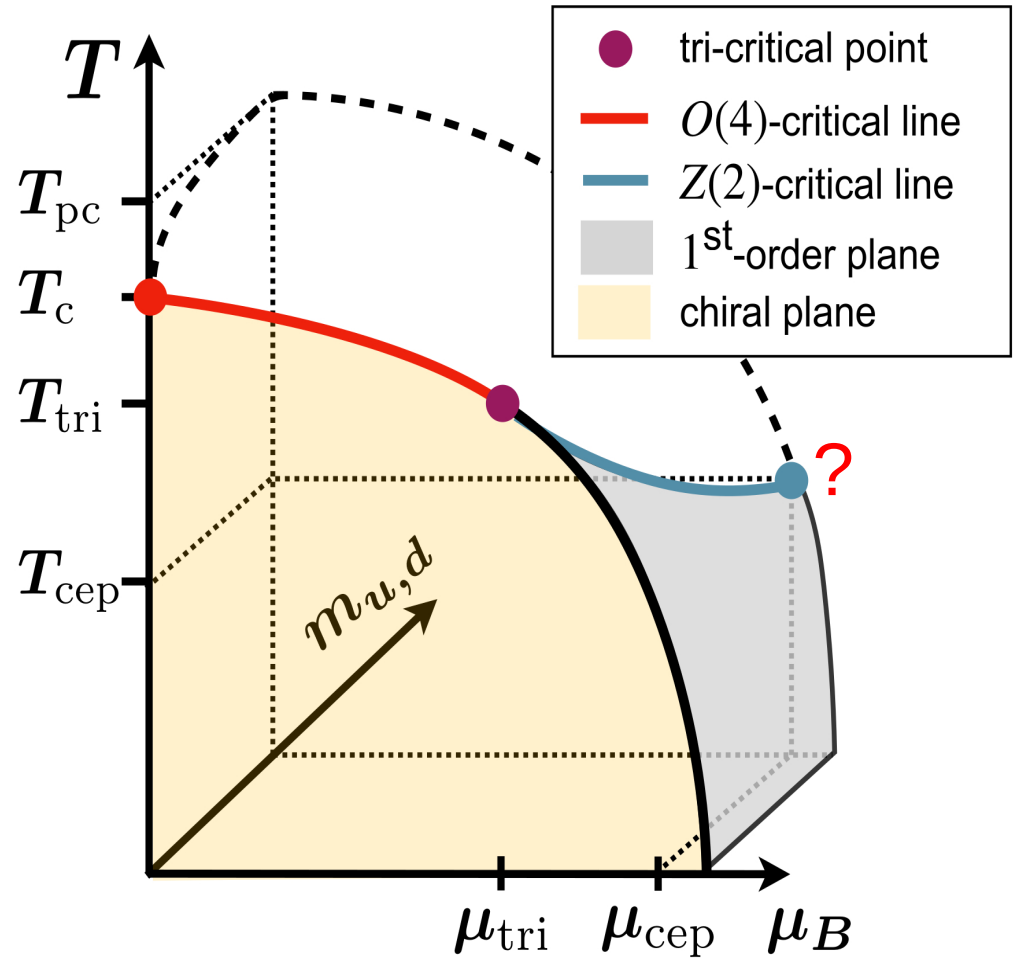


T. Hatsuda and T. Kunihiro, PRL 55 (1985) 158

The QCD phase diagram



Gordon Baym: Long Range Plan 1983



open question: What about the influence of the chiral anomaly?
 Is the chiral phase transition really in the $O(4)$ universality class?

Symmetries of QCD

$$\mathcal{L}_E = \mathcal{L}_G + \mathcal{L}_F \quad , \quad \mathcal{L}_F = \sum_f \bar{\psi}_f (\gamma_\mu D_\mu + m_f) \psi_f$$

$$\psi_f \equiv (\psi_{f,L}, \psi_{f,R})$$

$$M_f = (\gamma_\mu D_\mu + m_f)$$

$$\mathcal{L}_F = \sum_f (\bar{\psi}_{f,L} \gamma_\mu D_\mu \psi_{f,L} + \bar{\psi}_{f,R} \gamma_\mu D_\mu \psi_{f,R} + m_f (\bar{\psi}_{f,L} \psi_{f,R} + \bar{\psi}_{f,R} \psi_{f,L}))$$

– diagonal in left and right handed fermions
 independent L/R rotations for each flavor

– mass term breaks
 chiral symmetry

chiral symmetry: $U(n_f) \times U(n_f)$



$$U(1)_V \times U(1)_A \times SU(n_f)_L \times SU(n_f)_R$$

$U(1)_A$

$$\psi_\theta(x) = e^{i\theta\gamma_5} \psi(x)$$

$$\bar{\psi}_\theta(x) = \bar{\psi}(x) e^{i\theta\gamma_5}$$

$SU(n_f)_L \times SU(n_f)_R$

$$\psi'_{L/R}(x) = U_{L/R} \psi_{L/R}(x)$$

$$\bar{\psi}'_{L/R}(x) = \bar{\psi}_{L/R}(x) U^\dagger_{L/R}$$

$$U_L, U_R \in SU(n_f)$$

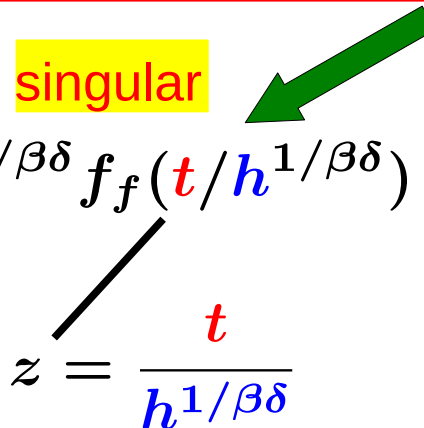
Universality class may be $U(2) \times U(2)$

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a **universal scaling function**

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f\left(\frac{t}{h^{1/\beta\delta}}\right) - f_r(V, T, \vec{\mu})$$

singular
regular



 $z = \frac{t}{h^{1/\beta\delta}}$

- symmetry breaking field \longleftrightarrow light quark masses m_ℓ : $H = \frac{m_\ell}{m_s}$, $h = \frac{1}{h_0} H$
- temperature-like field \longleftrightarrow does not break symmetry of the massless Hamiltonian

$$t = \frac{1}{t_0} \left(\frac{T - T_c}{T_c} + \kappa_2^\ell \mu_\ell^2 + \dots \right)$$

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a **universal scaling function**

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(\underbrace{t/h^{1/\beta\delta}}_z) - f_r(V, T, \vec{\mu})$$

singular ↙ regular

hyperscaling relations:
 $(2 - \alpha)/\beta\delta \equiv 1 + 1/\delta$

Pseudo-critical temperatures

response functions
2nd order cumulants

• magnetic

mixed

thermal

$$\frac{\partial^2 \ln Z}{\partial h^2}$$

$$\frac{\partial^2 \ln Z}{\partial h \partial t}$$

$$\frac{\partial^2 \ln Z}{\partial t^2}$$

O(4) critical exponents
 $\alpha = -0.21$
 $\beta = 0.38$
 $\delta = 4.82$

$$\chi_m \sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1} \uparrow \sim -0.79$$

$$\chi_t \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \uparrow \sim -0.34$$

$$c_V \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta} \uparrow \sim +0.11$$

divergence:

strong

moderate

none

Chiral order parameter and susceptibilities ($m_\ell \equiv m_u = m_d$)

chiral condensate: $\langle \bar{\psi}\psi \rangle_\ell = \frac{\partial P/T}{\partial m_\ell/T}$, $M_\ell = m_s \langle \bar{\psi}\psi \rangle_\ell / f_K^4$

chiral susceptibility: $\chi_\ell = m_s \frac{\partial M_\ell}{\partial m_\ell}$

chiral condensate needs additive and multiplicative renormalization

renormalized order parameter

$$M_{sub} = \frac{1}{f_K^4} [m_s \langle \bar{\psi}\psi \rangle_\ell - 2m_l \langle \bar{\psi}\psi \rangle_s] \quad \text{or} \quad M \equiv M_\ell - H\chi_\ell$$

renormalized susceptibilities $\chi_m^{M_{sub}} = m_s \frac{\partial M_{sub}}{\partial m_\ell}$

$$\chi_m^M = m_s \frac{\partial M}{\partial m_\ell}$$

magnetic

$$\chi_{t(T)}^M = -T_c \frac{\partial M}{\partial T}$$

mixed

$$\chi_{t(fg)}^M = -\frac{\partial^2 M_\ell}{\partial \hat{\mu}_f \partial \hat{\mu}_g}$$

The chiral phase transition in QCD

chiral (flavor) symmetry: $SU(2)_L \times SU(2)_R = O(4)$ spontaneously broken at $T=0$

Goldstone boson: $m_\pi \sim \sqrt{m_q}$

axial ($U(1)_A$) symmetry: $U(1)_A$

explicitly broken at $T=0$

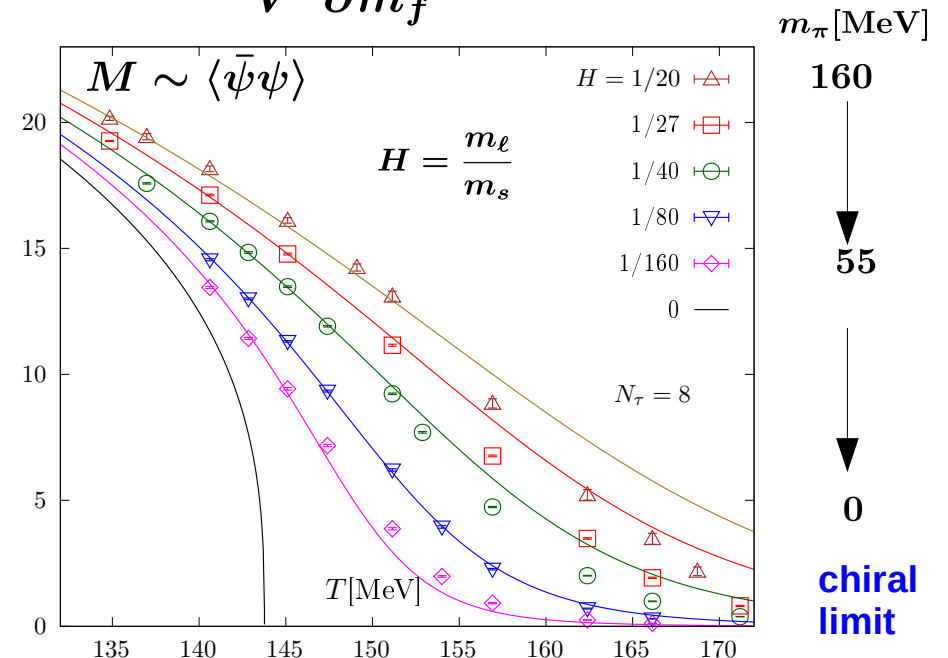
non-vanishing $\delta(a_0)$, η' masses

– chiral symmetry does get restored at high temperature: T_c

– is the $U(1)_A$ also "effectively restored" at $T_\chi \simeq T_c$?

order parameter for chiral symmetry breaking $\langle \bar{\psi}\psi \rangle_f = \frac{T}{V} \frac{\partial \ln Z}{\partial m_f} = \langle \text{Tr} M_f^{-1} \rangle$

$\langle \bar{\psi}\psi \rangle_f$ needs additive and multiplicative renormalization



staggered fermions do have a global $U(1) \times U(1)$ symmetry
(remnant of the chiral $SU(n_f) \times SU(n_f)$)

$U(1) \times U(1)$: independent phase transformations on
even and odd sites of the lattice
 $\sim O(2)$

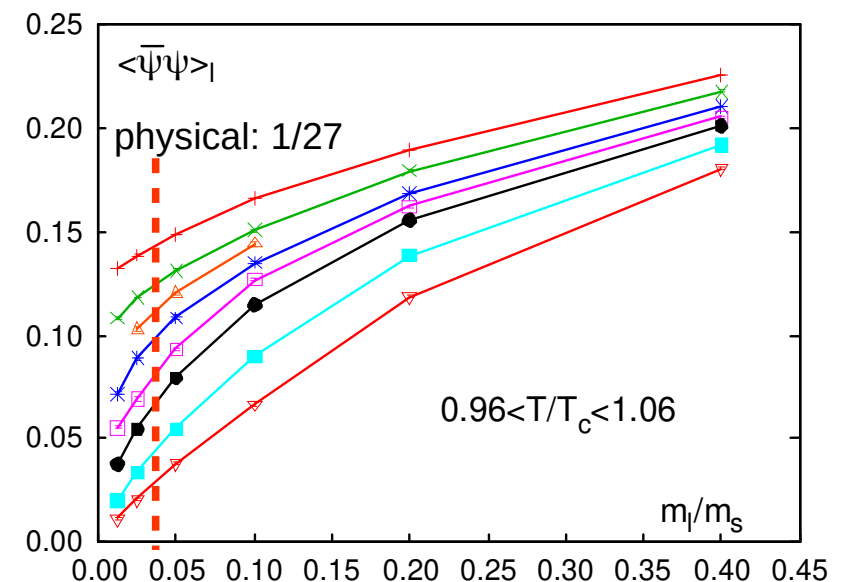
$$\psi'_e = e^{i\theta_1} \psi_e \quad , \quad \bar{\psi}'_e = e^{-i\theta_2} \bar{\psi}_e$$

$$\psi'_o = e^{i\theta_2} \psi_o \quad , \quad \bar{\psi}'_o = e^{-i\theta_1} \bar{\psi}_o$$

➔ its spontaneous breaking
generates one Goldstone pion

$$\langle \psi \psi \rangle \sim A(T) + B(T) \sqrt{m_\ell} + \mathcal{O}(m_\ell)$$

$$A(T) \begin{cases} > 0 & , T < T_c \\ = 0 & , T \leq T_c \end{cases}$$

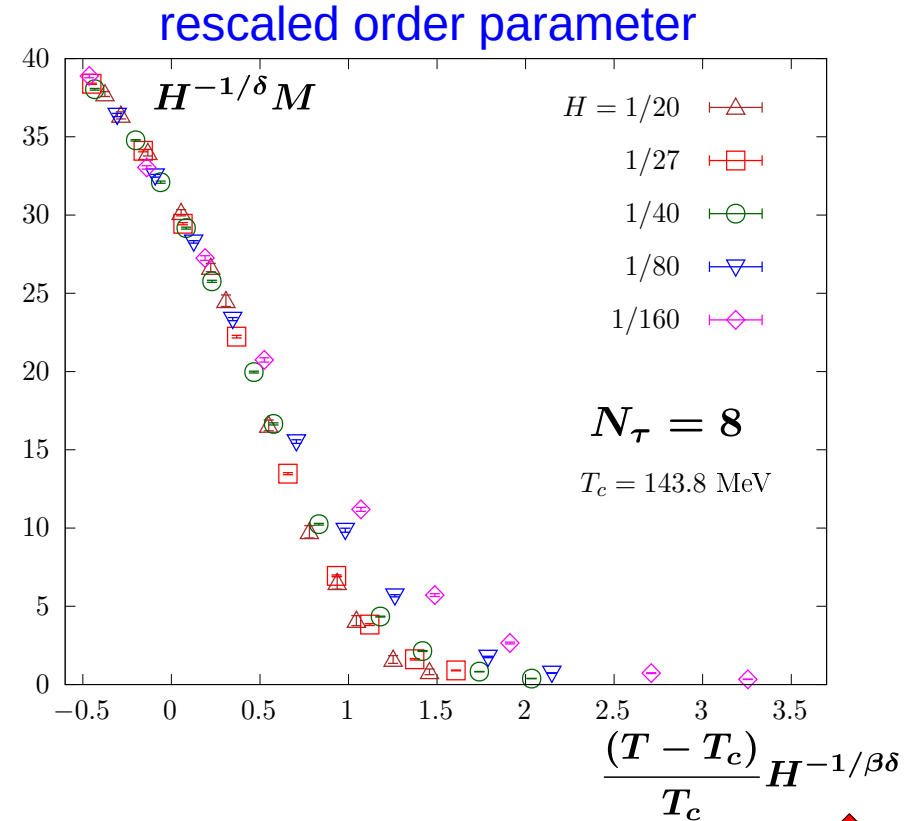
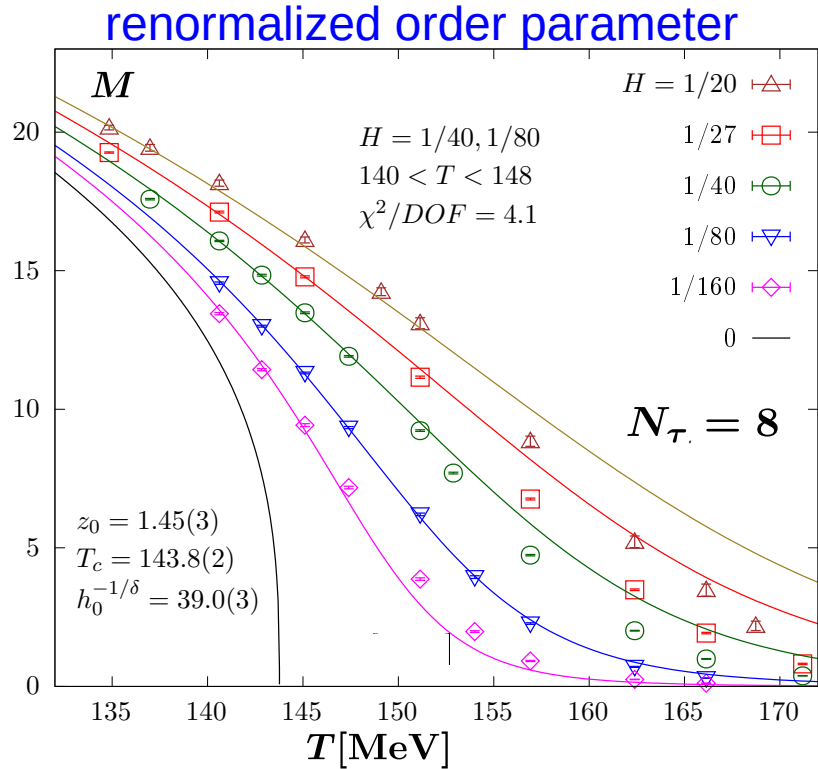


continuum limit with staggered fermions
recovers $O(4)$ flavor symmetry:

$$\lim_{m_\ell \rightarrow 0} \lim_{a \rightarrow 0} \lim_{V \rightarrow \infty}$$

Chiral symmetry restoration

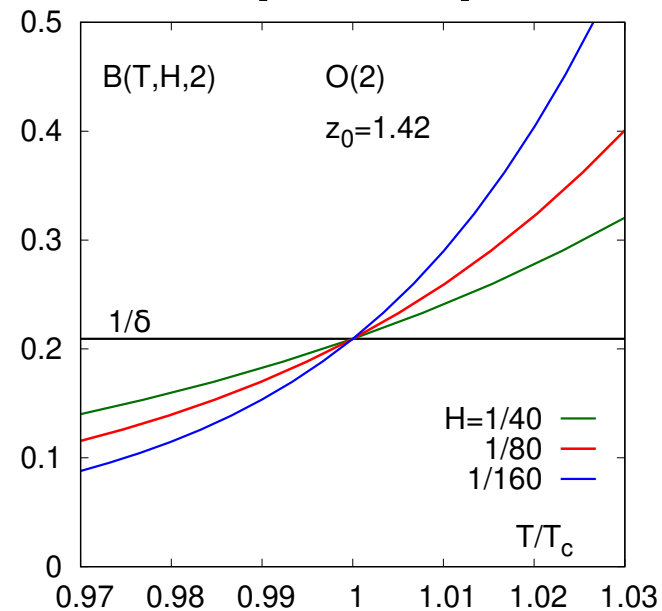
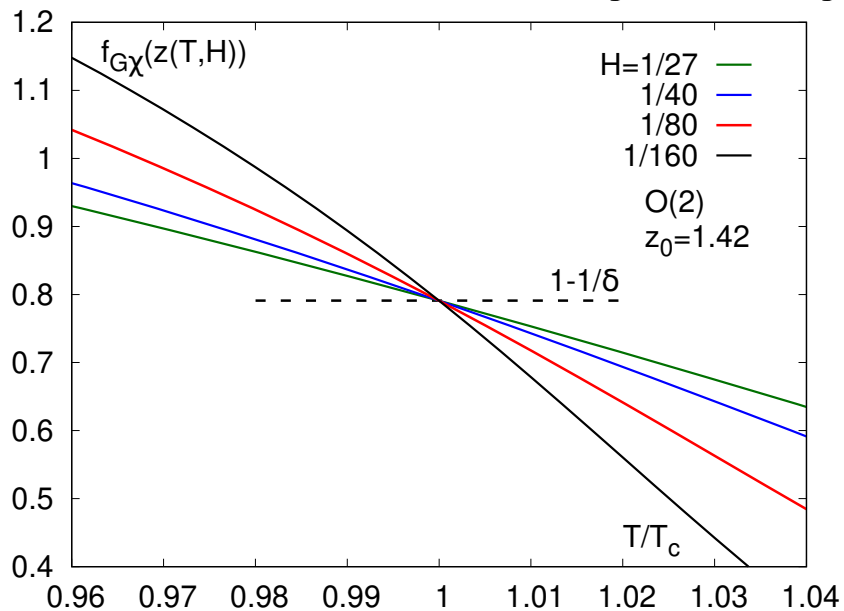
$m_{u,d} \rightarrow 0 \quad \longrightarrow \quad SU(2)_L \times SU(2)_R$ unbroken for $T \geq T_c$



$$M \equiv M_\ell - H\chi_\ell = h^{1/\delta} \underbrace{(f_G(z) - f_\chi(z))}_{f_{G\chi}(z)} + \tilde{f}_r(T, H)$$

↑
rescaling makes
use of
O(2) universality

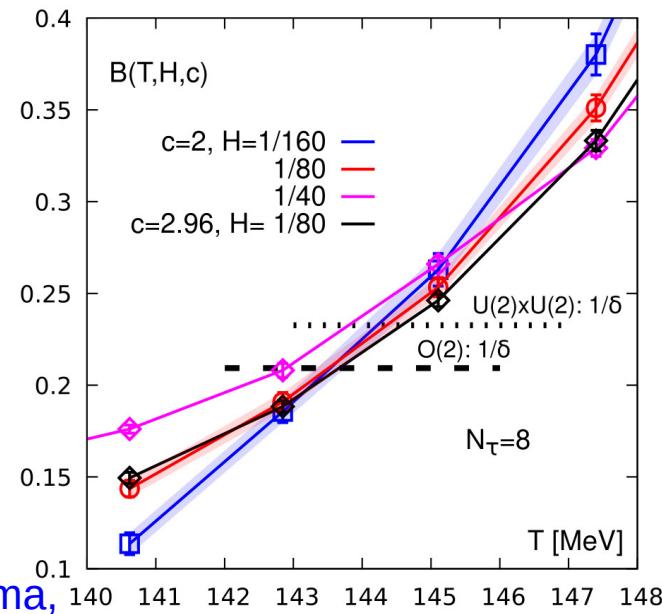
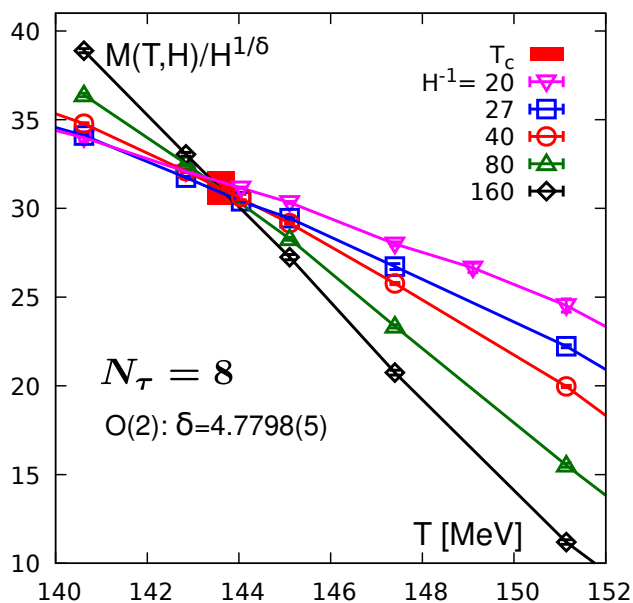
Goal: determine universality class for chiral symmetry restoration without prior input



$N_\tau = 8$

$$M(T,H)H^{-1/\delta} = h_0^{-1/\delta} f_{G\chi}(z)$$

$$B(T,H,c) = \ln(M(T,cH)/M(T,H))/\ln(c)$$



S. Mitra, FK, S. Sharma,
arXiv:2411.15988

The Chiral **PHASE TRANSITION** in (2+1)-flavor QCD

$$M \sim m_s \partial \ln Z / \partial m_l$$

$m_l \Rightarrow 0$

“magnetic”
susceptibility

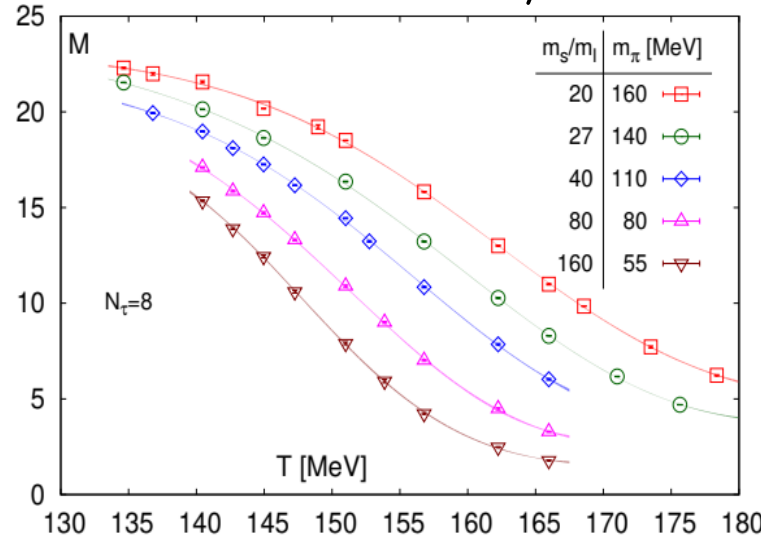
$$\chi_M \sim \frac{\partial^2 \ln Z}{\partial m_l^2} \sim (m_s/m_l)^{0.79}$$

$$(160/27)^{0.79} \sim 4$$

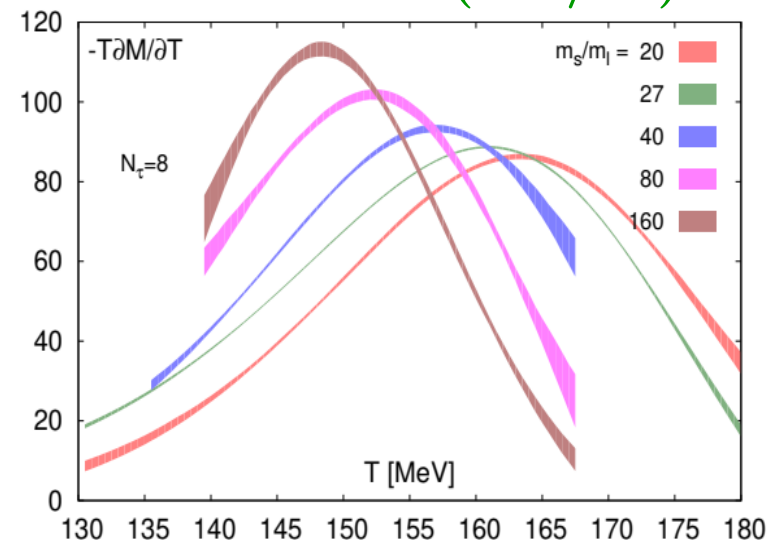
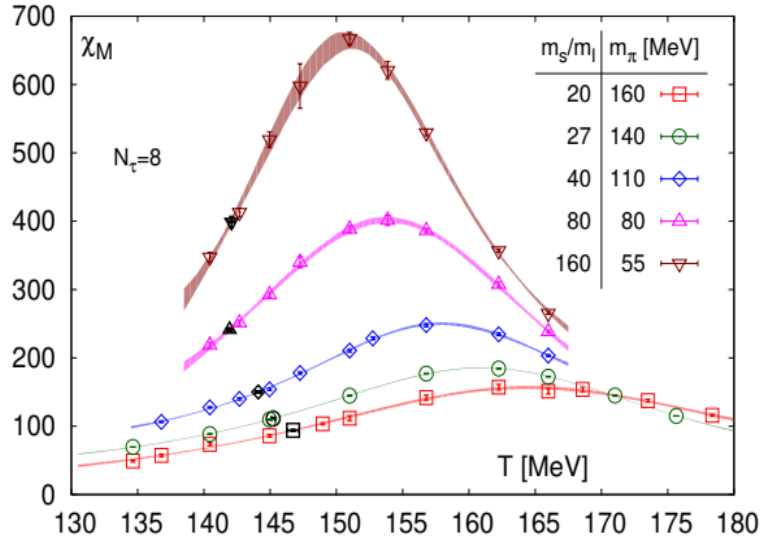
“mixed”
susceptibility

$$\chi_t \sim \frac{\partial^2 \ln Z}{\partial T \partial m_l} \sim (m_s/m_l)^{0.34}$$

$$(160/27)^{0.34} \sim 1.8$$



m_s fixed, physical
 $m_l \Rightarrow 0$

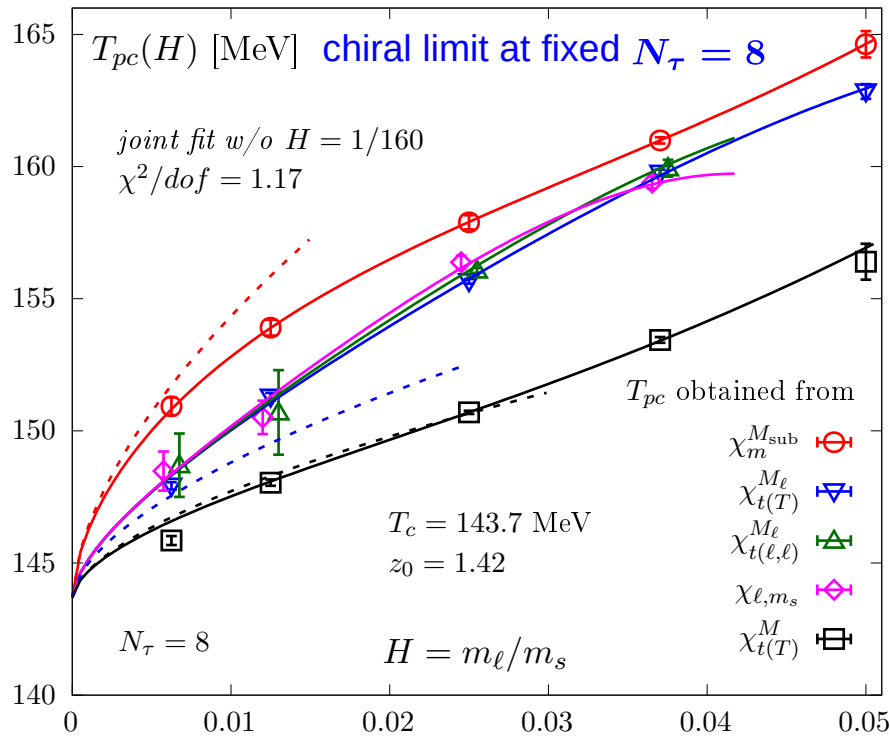


H.T. Ding et al. (HotQCD), PRL 123 (2019) 15, arXiv:1903.04801

Pseudo-critical and critical temperatures

$$T_{pc}(H) = T_c + z_x T_c H^{1/\beta\delta}, \quad x = \text{peak}, \delta, \dots$$

physical masses



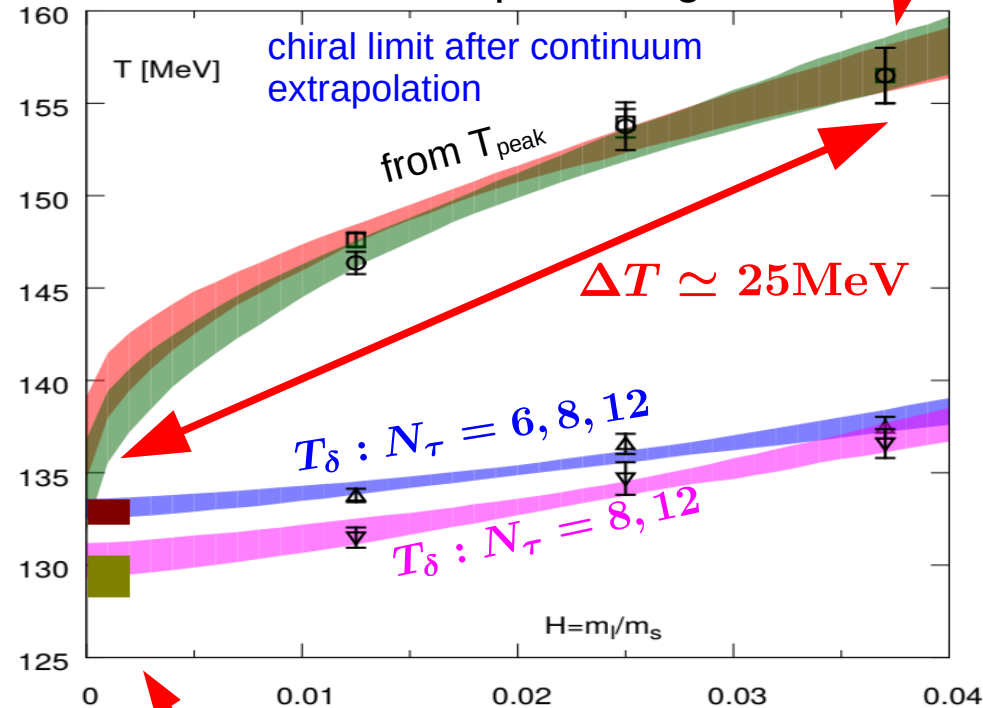
H.T.Ding, O. Kaczmarek, FK, P. Petreczky,
 Mugdha Sarkar, C. Schmidt, Sipaz Sharma,
 arXiv:2403.09390

physical masses

$$T_{pc}^{\text{phys}} = (156.5 \pm 1.5) \text{ MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

T_δ : T at $\sim 60\%$ of peak height



H.T Ding et al (HotQCD),
 arXiv:1903.04801
 Anirban Lahiri et al,
 arXiv:2010.15593

chiral limit extrapolations

$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

Critical temperatures at non-zero chemical potential – curvature of the critical line in the chiral limit –

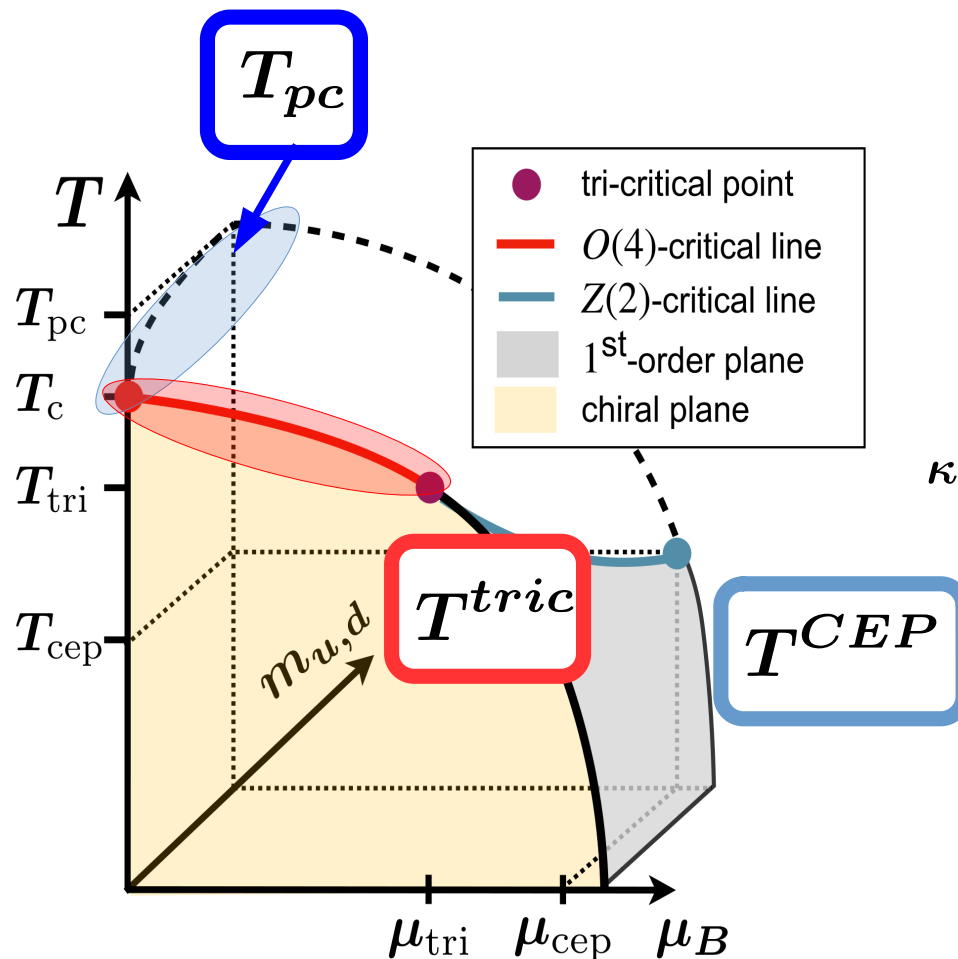
$$T_{pc,x}(H) = T_c^0 \left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} - \kappa_2^B (\mu_B/T)^2 + \dots \right), \quad H = \frac{m_\ell}{m_s}, \quad z_0 = \frac{h_0^{1/\beta\delta}}{t_0}$$

$$T_c(\mu_B, \dots) = T_c^0 \left(1 - \kappa_2^B \left(\frac{\mu_B}{T} \right)^2 - \kappa_{11}^{BS} \frac{\mu_B}{T} \frac{\mu_S}{T} - \kappa_2^S \left(\frac{\mu_S}{T} \right)^2 + \dots \right)$$

curvature coefficients

$$\kappa_2^f(H) = \frac{1}{2T_c} \left(\frac{\partial^2 M_\ell / \partial \hat{\mu}_f^2}{\partial M_\ell / \partial T} \right)_{(T_c, \vec{\mu}=0)} \quad f = B, S$$

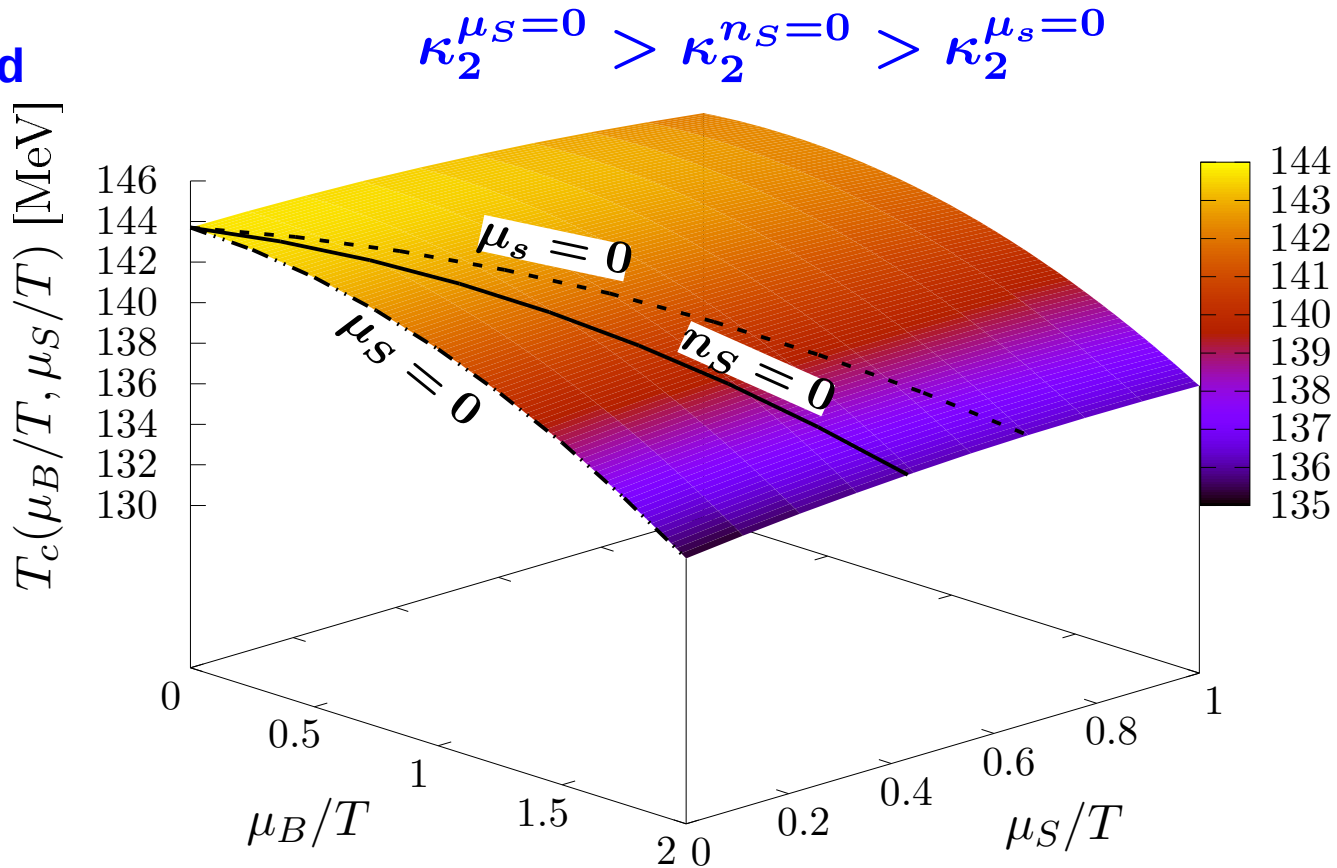
$$\kappa_{11}^{BS}(H) = \frac{1}{2T_c} \left(\frac{\partial^2 M_\ell / \partial \hat{\mu}_B \partial \hat{\mu}_S}{\partial M_\ell / \partial T} \right)_{(T_c, \vec{\mu}=0)}$$



H.T.Ding, O. Kaczmarek, FK, P. Petreczky,
Mugdha Sarkar, C. Schmidt, Sipaz Sharma,
arXiv:2403.09390

The critical surface in the $\mu_B - \mu_S$ plane

chiral limit
extrapolated

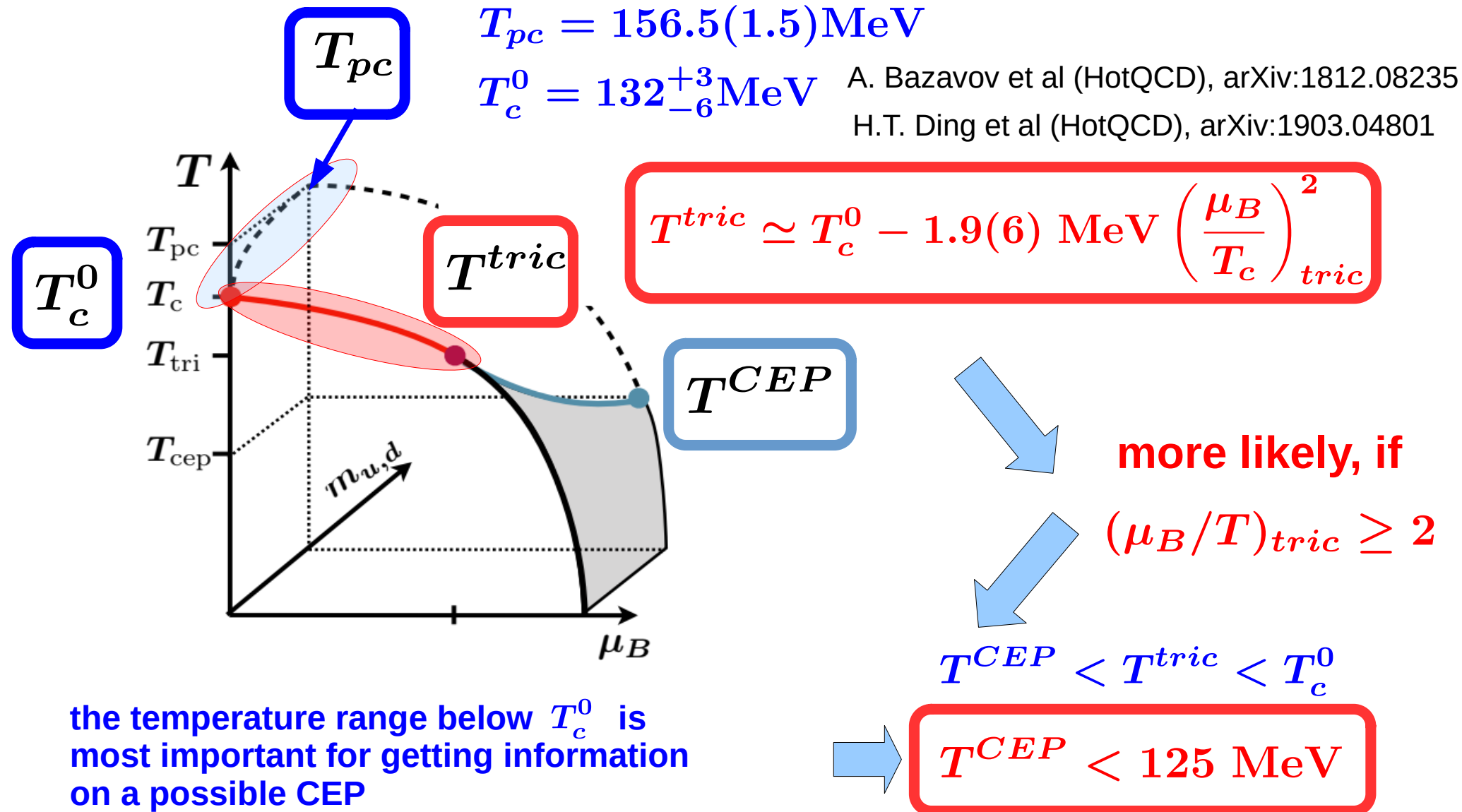


$$\mu_S = 0 : \quad \kappa_2^{\mu_S=0} = \kappa_2^B = 0.015(1)$$

$$n_S = 0 : \quad \kappa_2^{n_S=0} = \kappa_2^B \left(1 + s_1^2 \frac{\kappa_2^S}{\kappa_2^B} + 2s_1 \frac{\kappa_{11}^{BS}}{\kappa_2^B} \right) = 0.895(31) \kappa_2^B$$

$$\mu_s = 0 : \quad \kappa_2^{\mu_s=0} = \kappa_2^B \left(1 + \frac{1}{9} \frac{\kappa_2^S}{\kappa_2^B} + \frac{2}{3} \frac{\kappa_{11}^{BS}}{\kappa_2^B} \right) = 0.972(19) \kappa_2^{n_S=0}$$

determination of T_c^0 puts an upper limit on T^{CEP}



upper limit on T^{CEP} puts constraint on HIC searches for the CEP

– pseudo-critical temperatures at physical quark mass values

$$T_{pc}(\mu_B) = 156.5(1.5)\text{MeV} \left(1 - 0.012(4) \left(\frac{\mu_B}{T} \right)^2 - / + \dots \right)$$

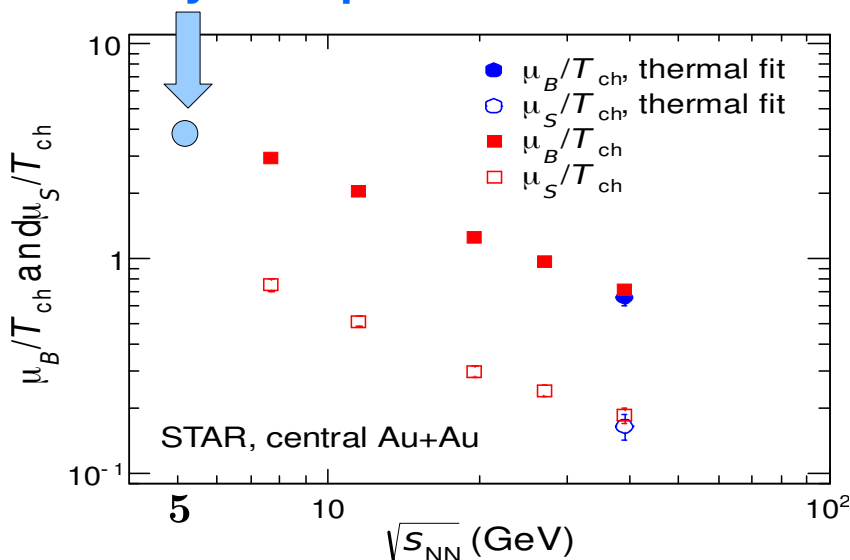
→ to reach $T < 125\text{MeV}$
need $\mu_B/T \simeq 4$

$T < 110\text{MeV}$
 $\mu_B/T \simeq 5$

$$\mu_B \geq 500\text{MeV}$$

$$\mu_B \geq 550\text{MeV}$$

my extrap.



Search for the Critical End Point requires beam energies

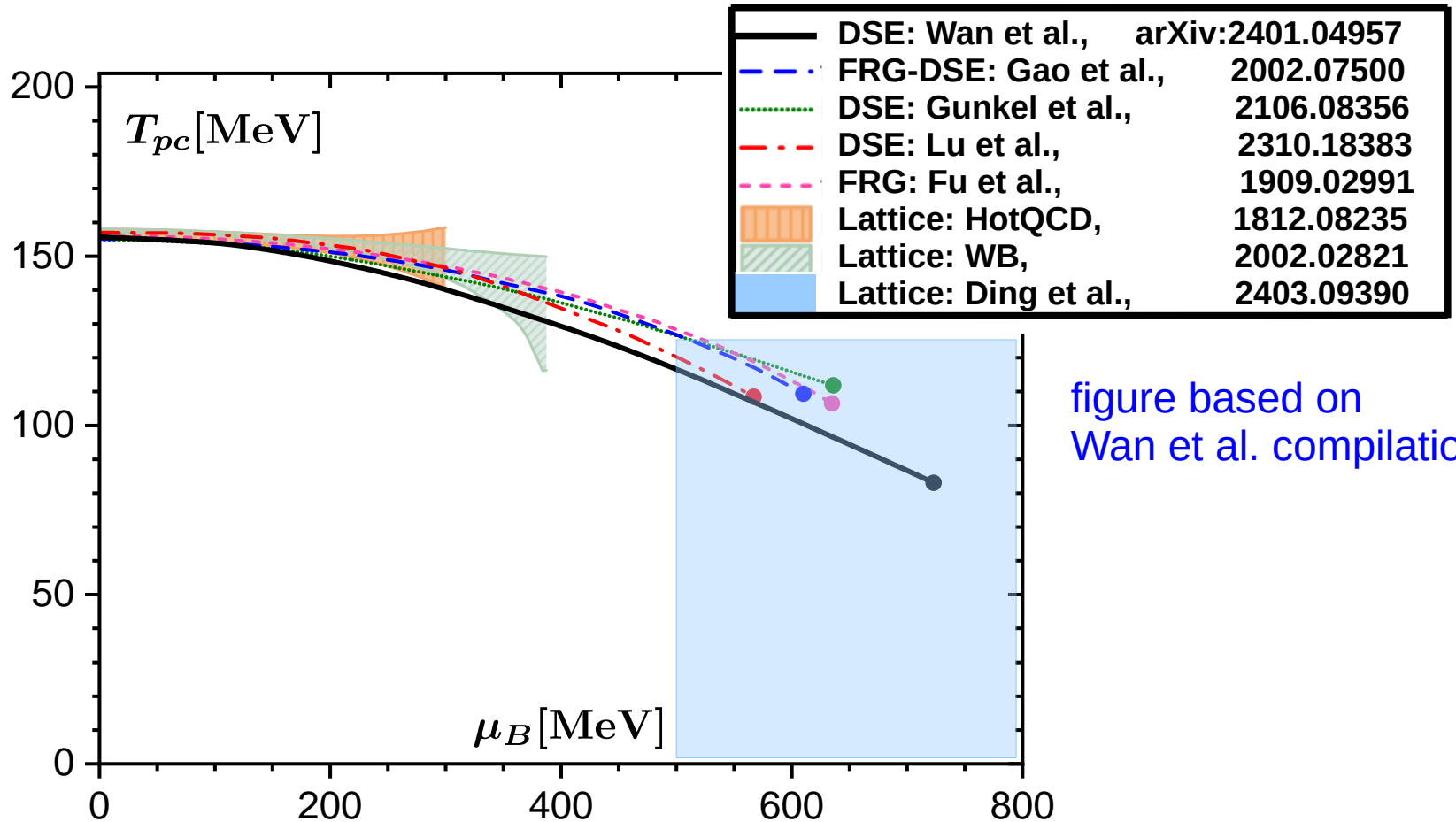
$$\sqrt{s_{NN}} \leq 5 \text{ GeV}$$

STAR freeze-out parameter, arXiv:1906.03732
also: HADES collaboration, arXiv:1512.07070

Constraint on allowed region for location of the CEP

Ding,..Mugdha Sarkar,... et al., arXiv:2403.09390

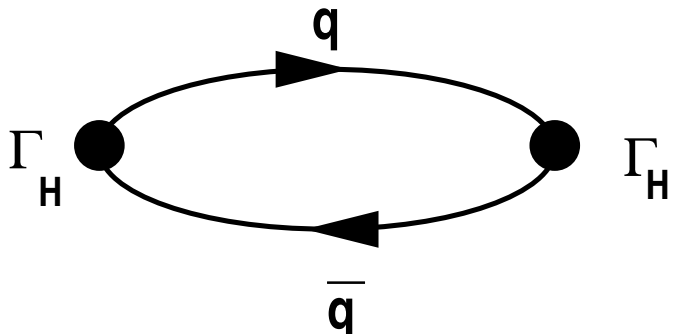
– based on determination of $T_c(\mu_B)$ and apparent convergence of Taylor series for $\mu_B/T \leq 2$



Screening masses, thermal susceptibilities and $U(1)_A$ breaking/restoration

FK, E. Laermann (in: QGP3), hep-lat/0305025

2-point function with hadronic currents that project onto various hadronic channels



$$\Gamma_H = \Gamma_D, \quad \Gamma_D = \begin{cases} 1 & \text{scalar} \\ \gamma_5 & \text{pseudo - scalar} \\ \gamma_\mu & \text{vector} \\ \gamma_\mu \gamma_5 & \text{pseudo - vector} \end{cases}$$

$$G_H^\beta(\tau, \vec{r}) = \langle J_H(\tau, \vec{r}) J_H^\dagger(0, \vec{0}) \rangle; \quad J_H(\tau, \vec{r}) = \bar{q}(\tau, \vec{r}) \Gamma_H q(\tau, \vec{r})$$

Spectral representation of thermal correlation functions

$$G_H^\beta(\tau, \vec{r}) = \int_0^\infty d\omega \int \frac{d^3 \vec{p}}{(2\pi)^3} \sigma_H(\omega, \vec{p}, T) e^{i\vec{p}\vec{r}} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

Spatial correlation functions:

$$\begin{aligned} G_H^S(z) &= \int_0^{1/T} d\tau \int dz_\perp \langle J_H(\tau, z_\perp) J_H^\dagger(0, \vec{0}) \rangle \\ &= \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma_H(p_0, \vec{0}_\perp, p_z)}{p_0} \\ &\sim e^{-m_H^{\text{scr}}(T)z} \end{aligned}$$

Thermal susceptibilities are integrated thermal correlation functions:

$$\chi_H = \int dz G_H^S(z)$$

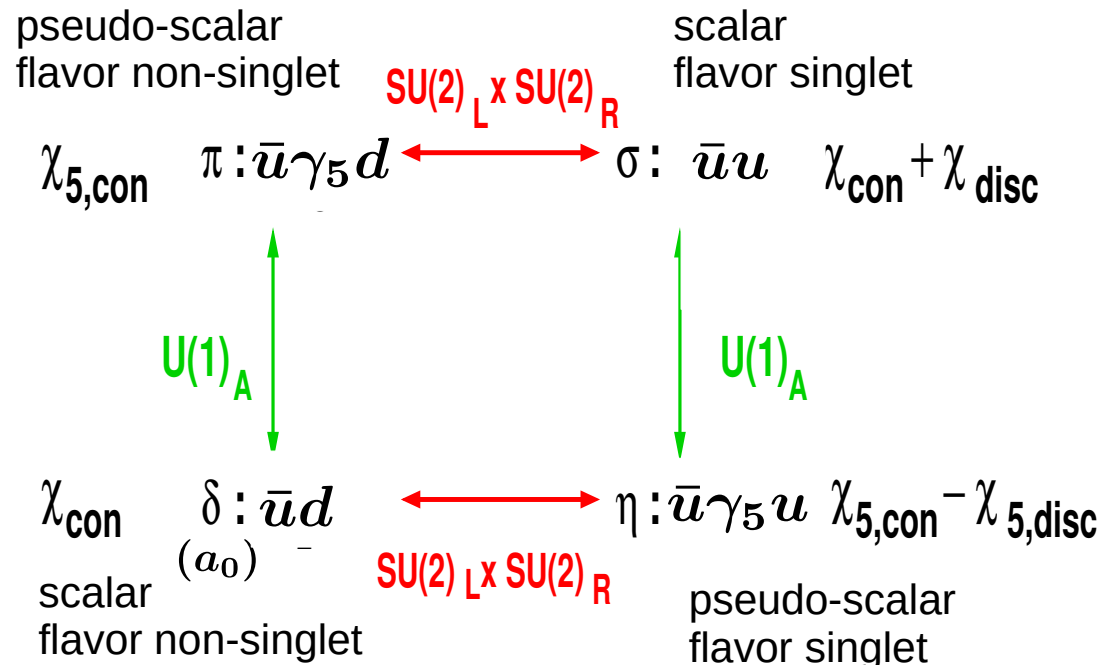
Scalar, flavor non-singlet 2-point function: $H \equiv \delta(a_0)$

$$\chi_\delta = \int dz G_\delta(z) = \int dz \bar{u} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \text{---} \end{array} u = \chi_{\text{con}}$$

Scalar, flavor singlet 2-point function: $H \equiv \sigma$

$$\chi_\sigma = \int dz G_\sigma(z) = \int dz \left(\bar{u} \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \\ \text{---} \text{---} \end{array} u + \left(\bar{u} \begin{array}{c} \text{---} \\ \text{---} \end{array} u + \begin{array}{c} \text{---} \\ \text{---} \end{array} \bar{u} \right) \right)$$

$$= \chi_{\text{con}} + \chi_{\text{disc}}$$

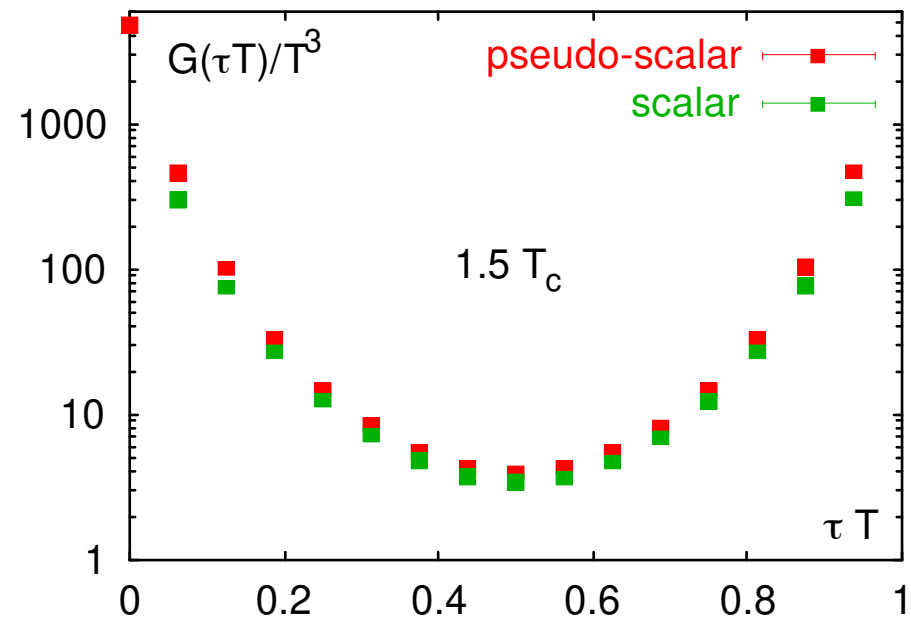
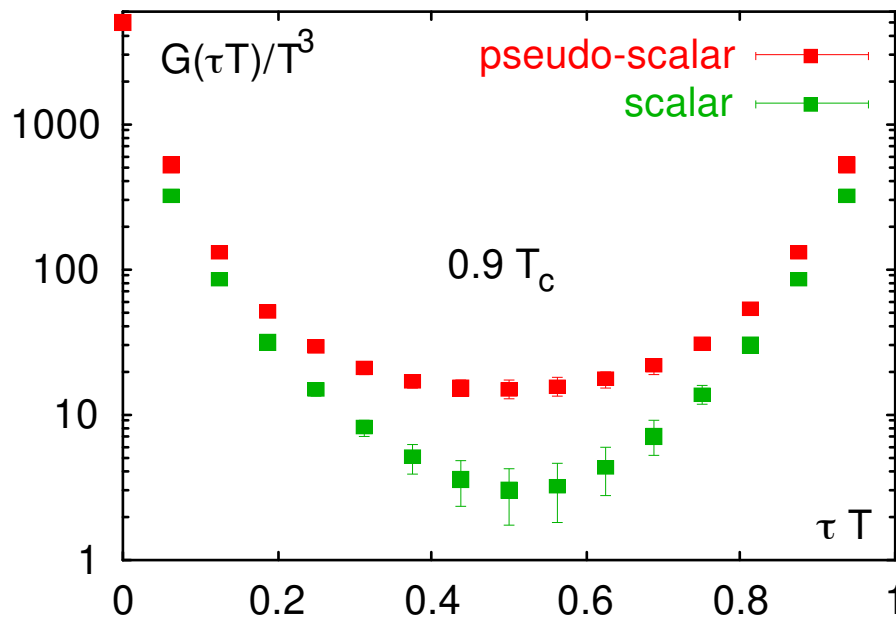
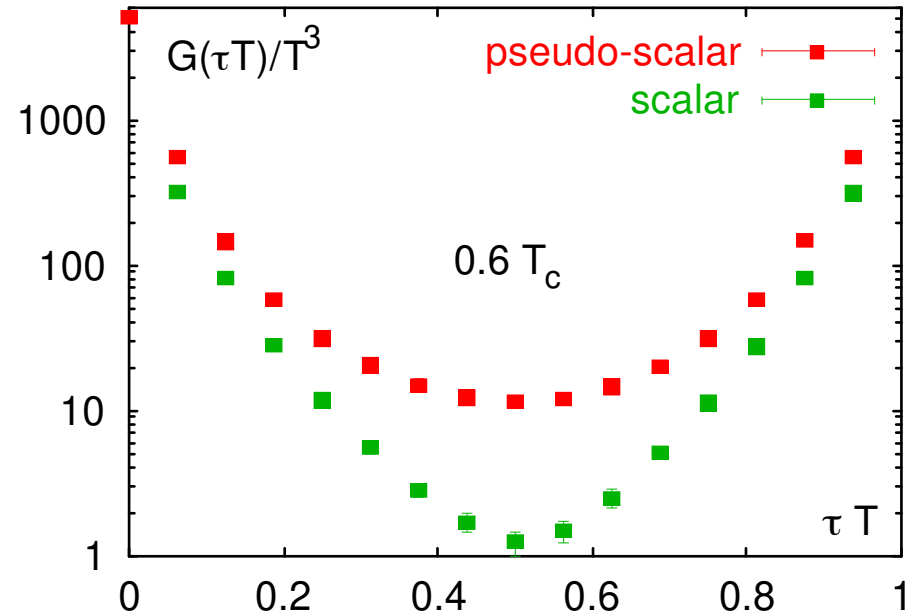


Effective $U(1)_A$ symmetry restoration above T_c

$$\pi : J_{PS} \sim \bar{u}\gamma_5 d \Leftrightarrow \delta(a_0) : J_S \sim \bar{u}d$$

temporal correlation functions:

effective $U(1)_A$ symmetry restoration at high T

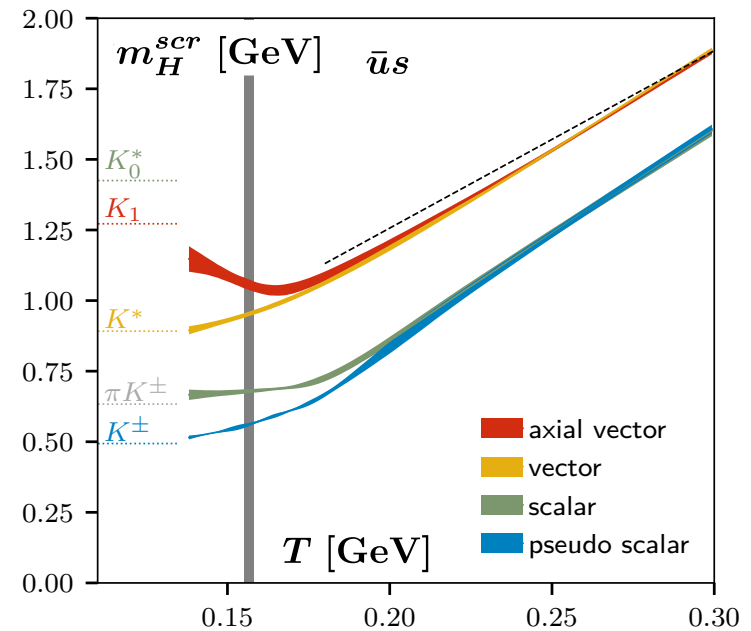
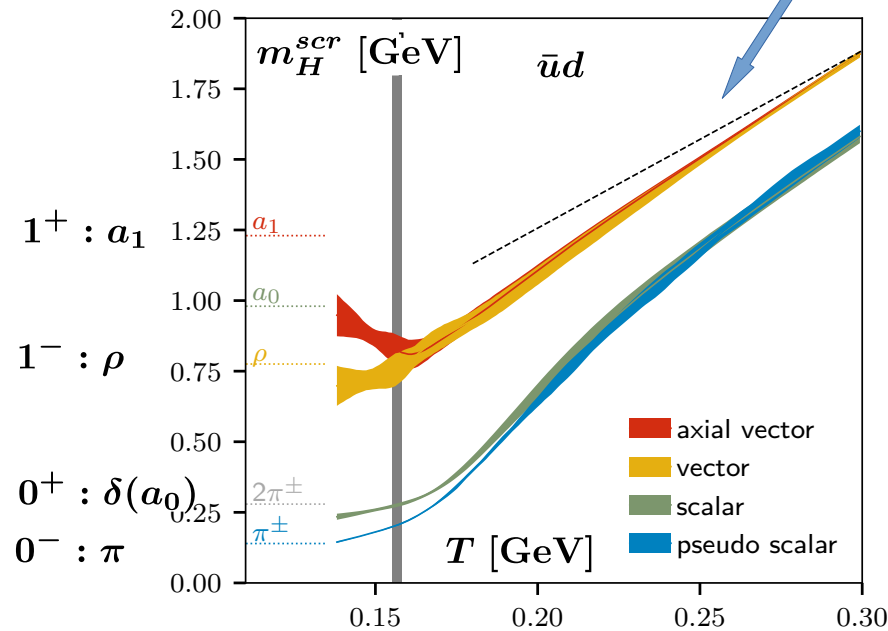


Effective $U(1)_A$ symmetry restoration above T_c

thermal screening masses

- parity partners degenerate close to T_{pc}

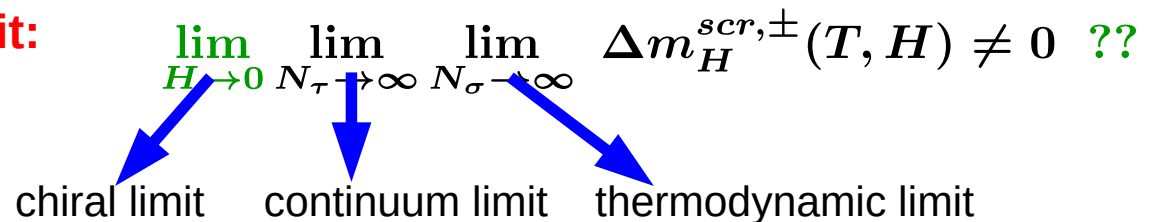
$$m_H^{scr}(T) \sim 2\sqrt{m_q^2 + (\pi T)^2}$$



A. Bazavov et al (HotQCD), arXiv:1908.09552

- (2+1)-flavor QCD calculation with physical light and strange quark masses

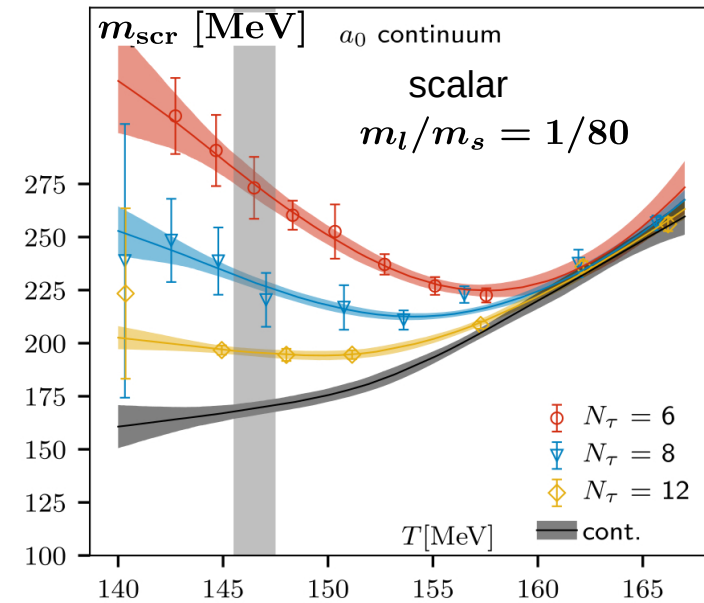
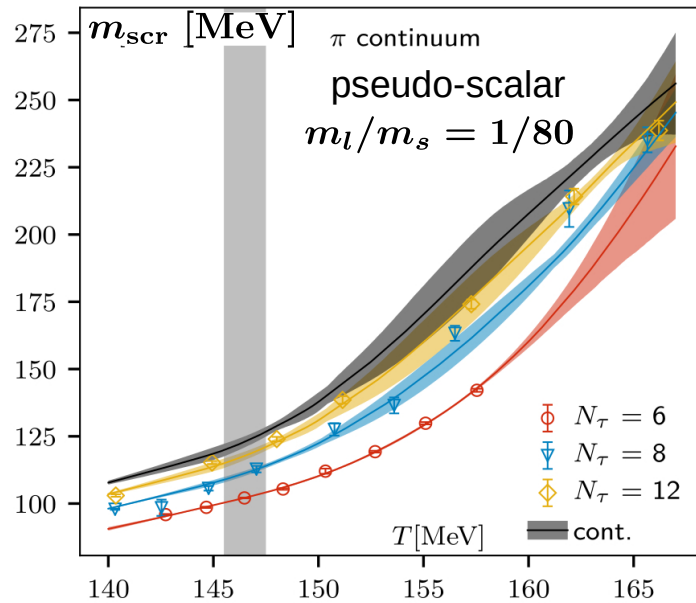
– need to take the chiral limit:



Effective $U(1)_A$ symmetry restoration above T_c : Screening Masses

(i) $N_\sigma/N_\tau \gg 1$ (ok)

(ii) continuum limit at fixed m_l/m_s



spatial correlation functions:

$$G_H^S(z) \sim e^{-m_H^{\text{scr}}(T)z}$$

→ degenerate screening masses

$$m_{a_0}(T) \rightarrow m_\pi(T)$$

effective $U(1)_A$ symmetry restoration
at $T \geq 1.15T_c??$

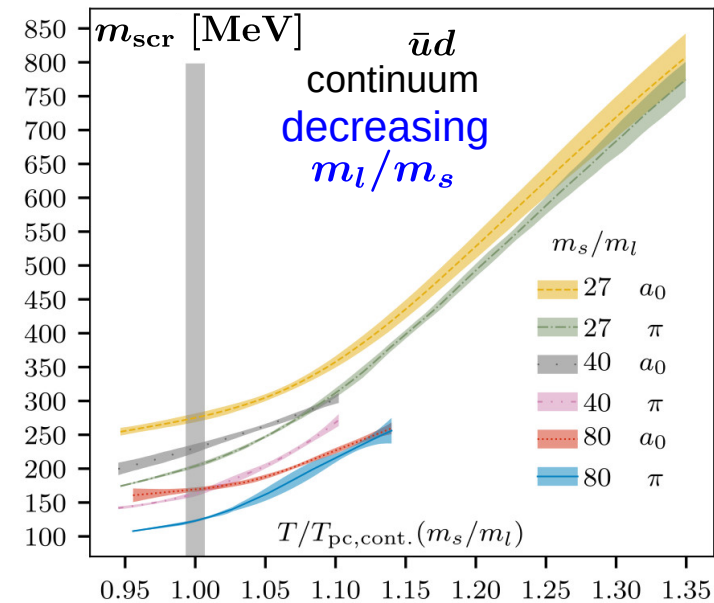
staggered fermions (HISQ)

S. Dentinger et al., arXiv:2102.09916

S. Dentinger, PhD thesis 2021,

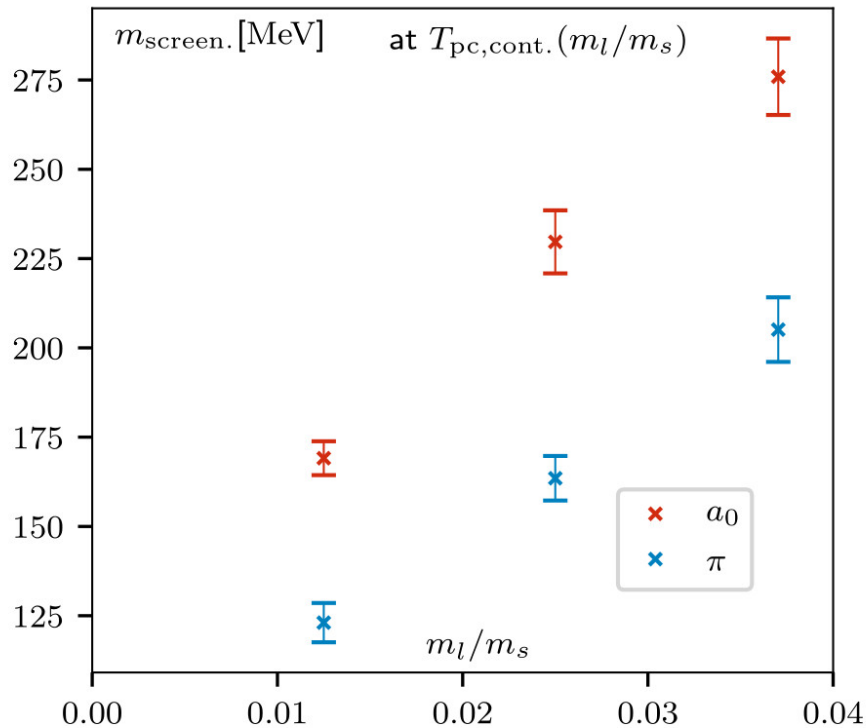
<https://pub.uni-bielefeld.de/record/2960222>

(iii) chiral limit in the continuum limit



Effective $U(1)_A$ symmetry restoration above T_c

– chiral limit after continuum limit at fixed $H=m_l/m_s$ –



thermodynamic limit and
continuum extrapolation for

$$H \equiv m_l/m_s \geq 1/80 \Leftrightarrow m_{\pi} \geq 80\text{MeV}$$

well controlled

m_{π}^{screen} will vanish for $m_l/m_s \rightarrow 0$
at T_c

$$\lim_{m_l/m_s \rightarrow 0} m_{a_0}^{\text{screen}} > 0 \quad ??$$

**need to get better control over
chiral limit extrapolation close to T_c**

Tristan Ueding, Yu Zhang,...

- use fermion discretization schemes with better chiral properties already at non-zero lattice spacing (Domain Wall Fermions, overlap fermions)
- explore eigenvalue spectrum of the Dirac operator (fermion matrix)

Open charm hadrons at finite temperature

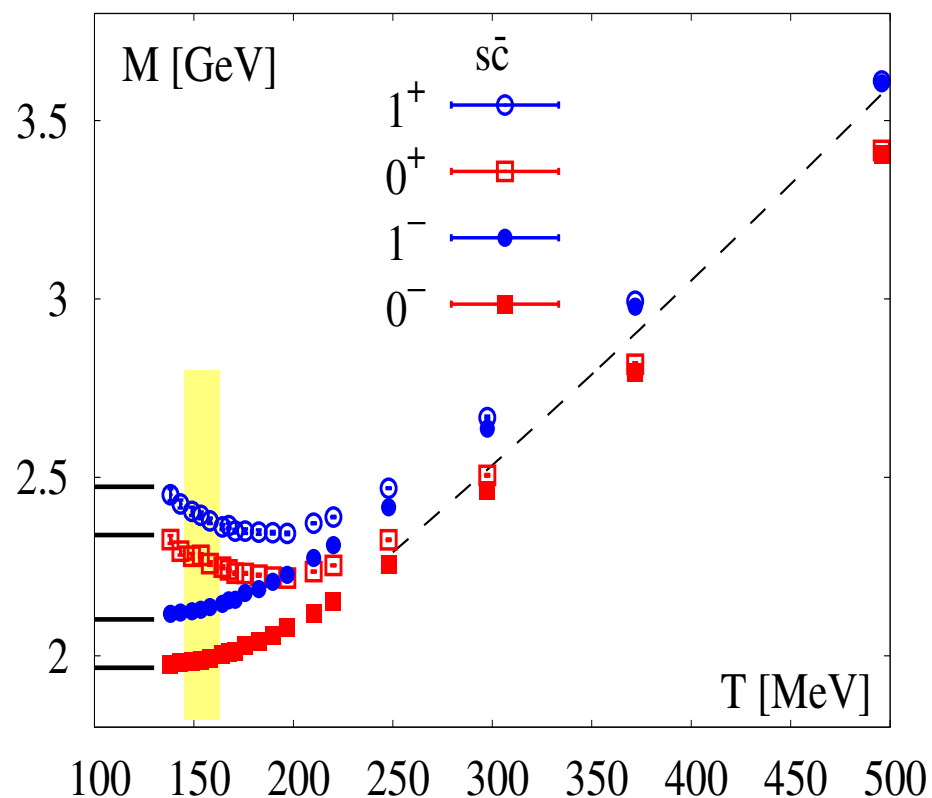
- a probe for deconfinement and chiral symmetry restoration -

A. Bazavov, Phys. Lett. B737 (2014) 210

C. Sasaki, Phys. Rev. D 90, 114007 (2014)

C. Sasaki, K. Redlich, Phys. Rev. D 91, 074021 (2015)

open charm screening masses



parity partners degenerate only at about $2 T_c$

S. Dentinger et al., arXiv:2102.09916

Probing the hadron spectrum using QCD thermodynamics

fluctuations of conserved (charm) charges

- construct QCD observables that would project onto specific quantum numbers, if QCD is approx. described a gas of non-interacting hadrons (HRG-model)

e.g.: HRG pressure:

$$\frac{P}{T^4} = \sum_{m \in \text{mesons}} \ln Z_m^b(T, V, \mu) + \sum_{m \in \text{baryons}} \ln Z_m^f(T, V, \mu)$$

chemical potentials: $\mu \equiv (\mu_B, \mu_Q, \mu_S, \mu_C)$

Boltzmann approximation: $\ln Z_m^{b/f}(T, V, \mu) = f_m^{b/f}(T) \cosh(B\mu_B + Q\mu_Q + \dots)/T)$

in a HRG charge fluctuations and partial pressures are related, e.g.

contribution of charged baryons to the total pressure,

$$\chi_{11}^{BC} = \sum_{\substack{m \in C- \\ \text{baryons}}} \left. \frac{\partial^2 \ln Z_m^f(T, V, \mu)}{\partial(\mu_B/T) \partial(\mu_C/T)} \right|_{\mu=0}$$

Probing the hadron spectrum using QCD thermodynamics

fluctuations of conserved (charm) charges

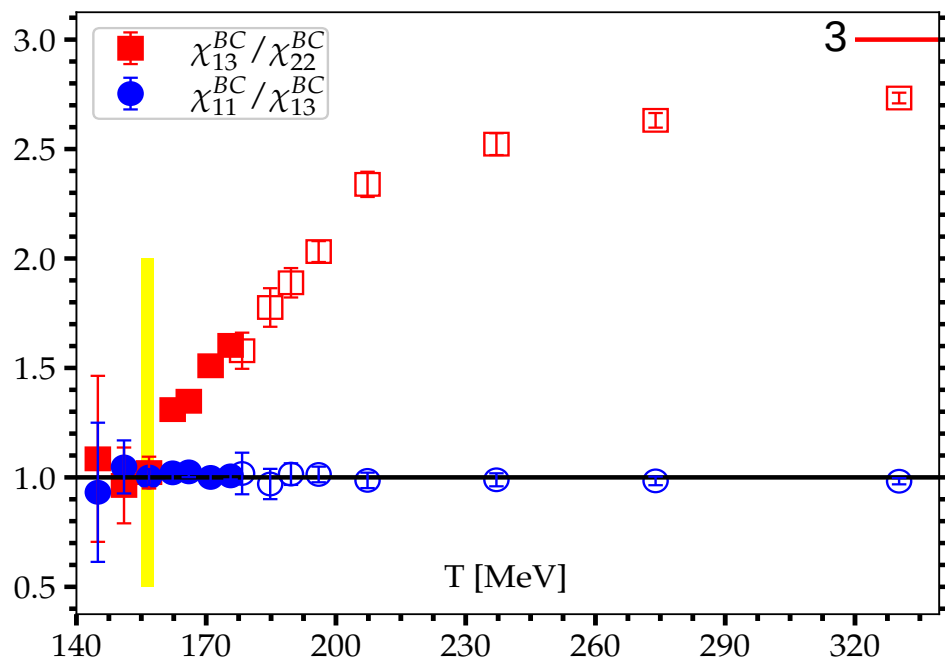
A. Bazavov,... Sayantan Sharma... et al,
Phys. Lett. B737 (2014) 210

– partial pressure resulting from charmed mesons or charmed baryons can be represented by various fluctuation observables (if the medium is well approximated by a HRG), e.g. iff $|B|=1$,

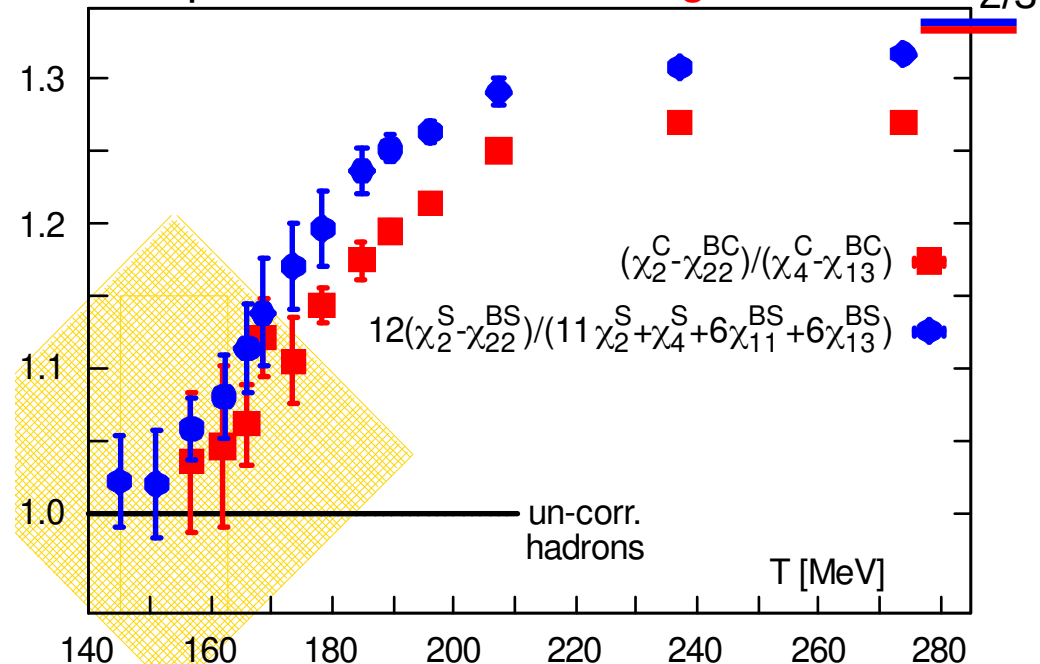
– proxies for charmed baryon pressure: $P^{C-baryons}/T^4 \simeq \chi_{11}^{BC} \simeq \chi_{13}^{BC}$

– proxies for charmed meson pressure: $P^{C-mesons}/T^4 \simeq \chi_2^C - \chi_{22}^{BC} \simeq \chi_4^C - \chi_{13}^{BC}$

open charmed baryons



open charmed and strange mesons



A. Bazavov,... Sipaz Sharma... et al, Phys. Lett. B850 (2024) 138520

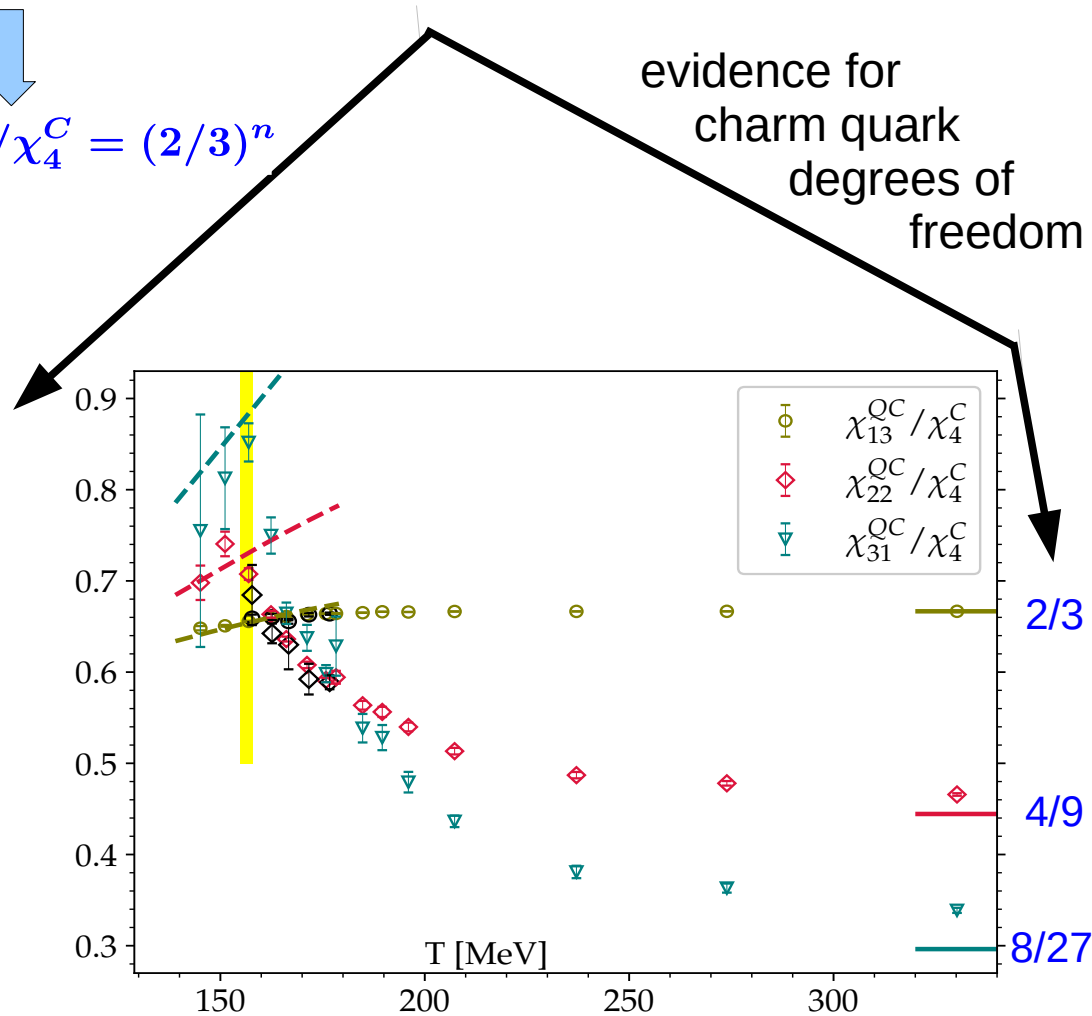
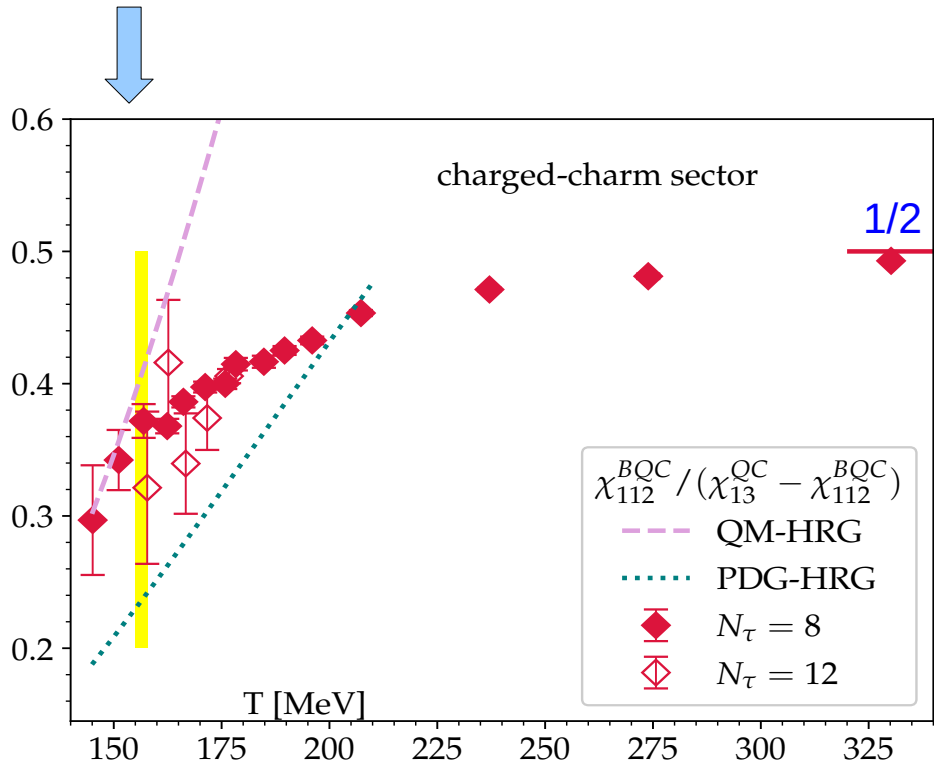
Evidence for charm quark degrees of freedom

$$P_q^C(T, \vec{\mu}) = \frac{3}{\pi^2} \left(\frac{m_q^C}{T} \right)^2 K_2(m_q^C/T) \cosh \left(\frac{2}{3} \hat{\mu}_Q + \frac{1}{3} \hat{\mu}_B + \hat{\mu}_C \right)$$

strong enhancement of charmed baryons over known states listed in PDG

$$\chi_{n(4-n)}^{QC} / \chi_4^C = (2/3)^n$$

evidence for charm quark degrees of freedom



F.K., Sipaz Sharma, P. Petreczky, in preparation

Quasi-particle model for open charm fluctuations

$m_H^C, m_q^C \gg T \sim T_{pc} \Rightarrow$ Boltzmann approximation is appropriate for excitations with charmed hadron as well as charmed quark quantum numbers below as well as above T_{pc}

partial charm pressure: $P^C = P_M^C + P_B^C + P_q^C$

proxies for partial charm pressures in different quantum number channels:

$$P^C = \chi_4^C \quad P_B^C \simeq \chi_2^C \simeq \chi_4^C \quad \text{for all } C$$

$$P_M^C = \chi_4^C + \underline{3\chi_{22}^{BC} - 4\chi_{13}^{BC}} \Leftrightarrow \text{eliminates } |B|=1 \text{ and } 1/3 \text{ contributions to } P^C$$

$$P_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2 \quad \Leftrightarrow \quad P_B^C \simeq 0 \quad \text{for } |B| = 1/3$$

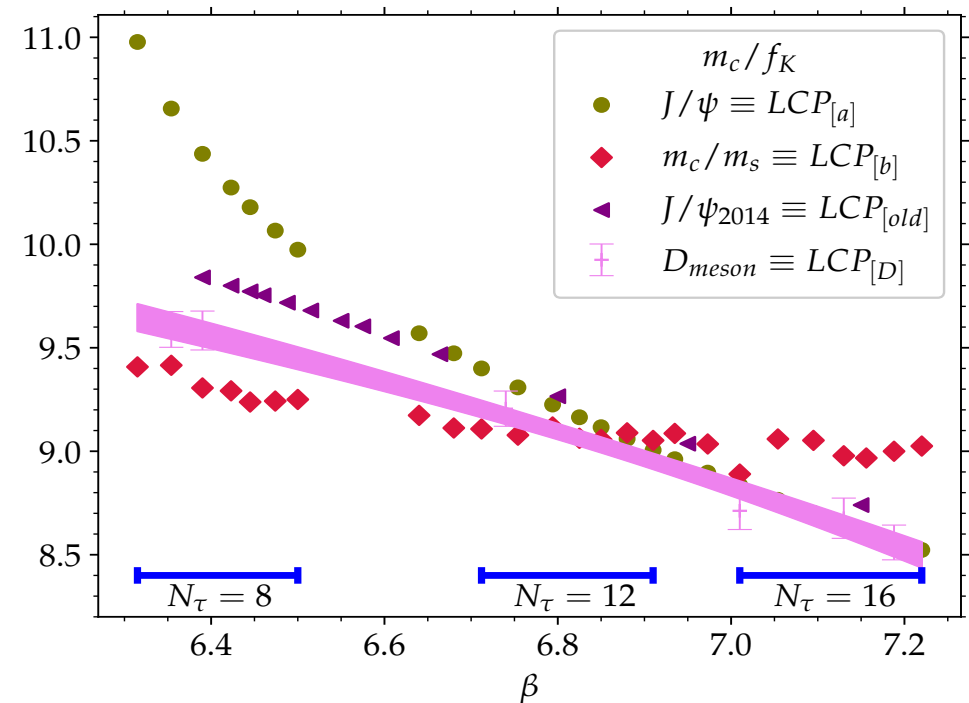
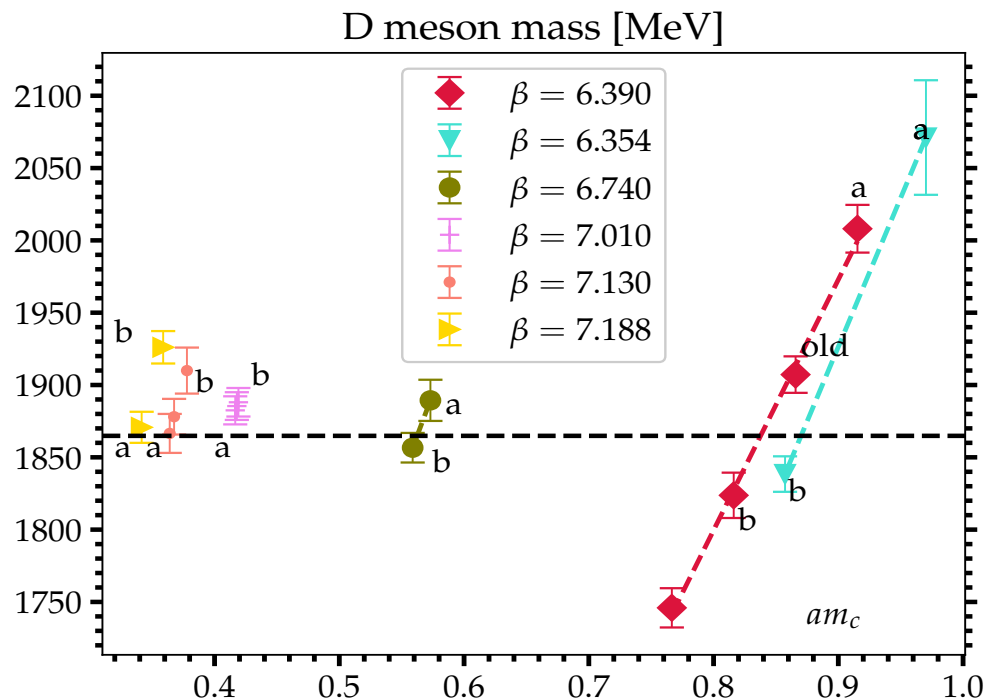
$$P_q^C = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2 \quad \Leftrightarrow \quad P_q^C \simeq 0 \quad \text{for } |B| = 1$$

A. Bazavov,... Sayantan Sharma... et al,
Phys. Lett. B737 (2014) 210

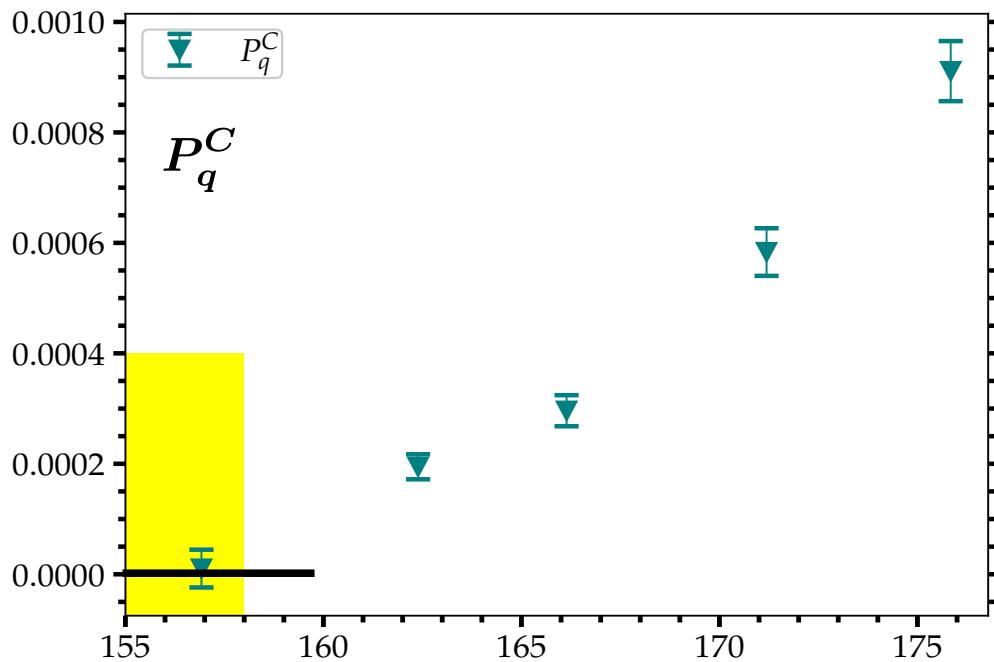
A. Bazavov...Sipaz Sharma... et al.,
Phys. Lett. B850 (2024) 138520

Open charm fluctuations as a probe for deconfinement

- going from cumulant ratios to absolute values of cumulants requires careful elimination of cut-off effects arising from tuning of the bare charm quark masses
- as charmed mesons dominate many cumulants we choose a line of constant physics (LCP) obtained by keeping the D-meson mass fixed (previously charmonium mass has been used)



F.K., Sipaz Sharma, P. Petreczky, in preparation

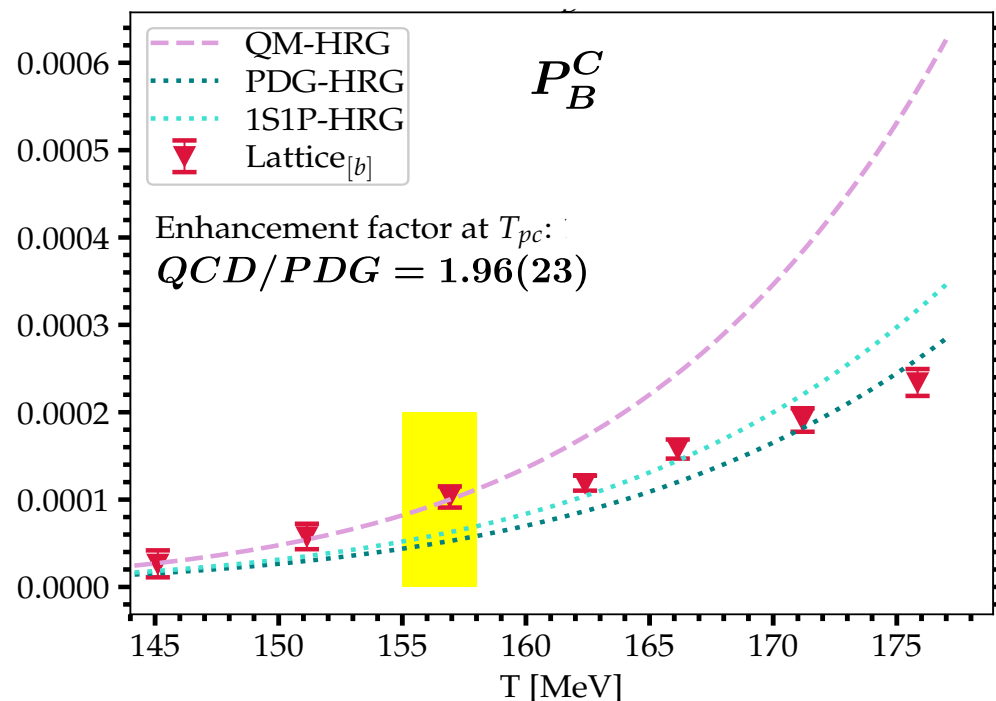
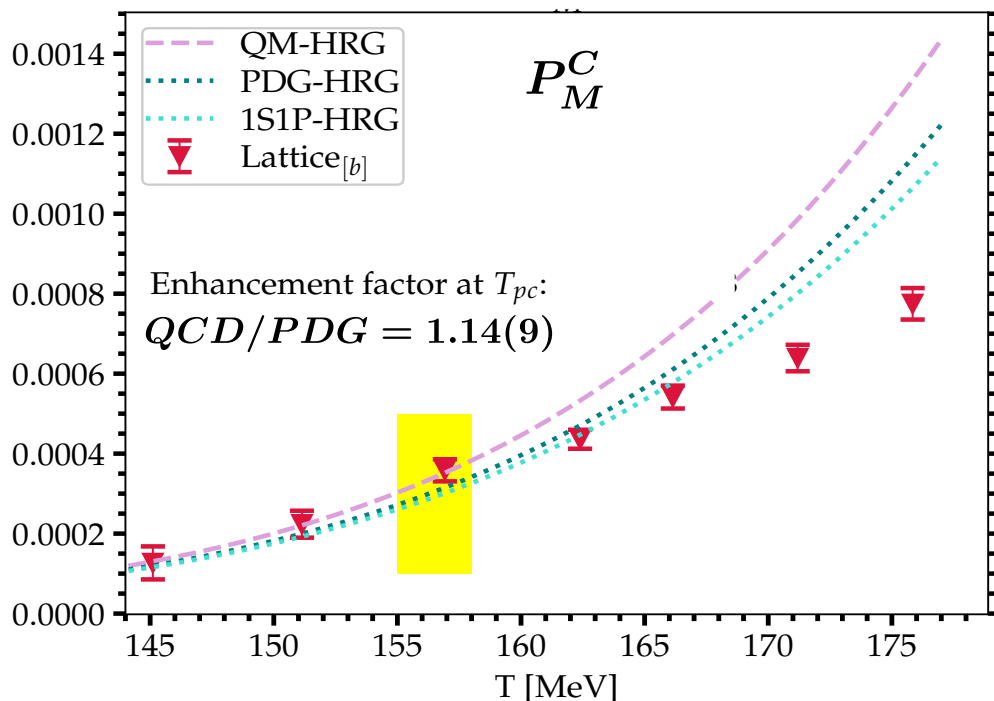


$$P_M^C = \chi_4^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC}$$

$$P_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2$$

$$P_q^C = \chi_4^C - P_M^C - P_B^C$$

$$= 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2$$



charmed hadron and quark fluctuations above T_{pc}

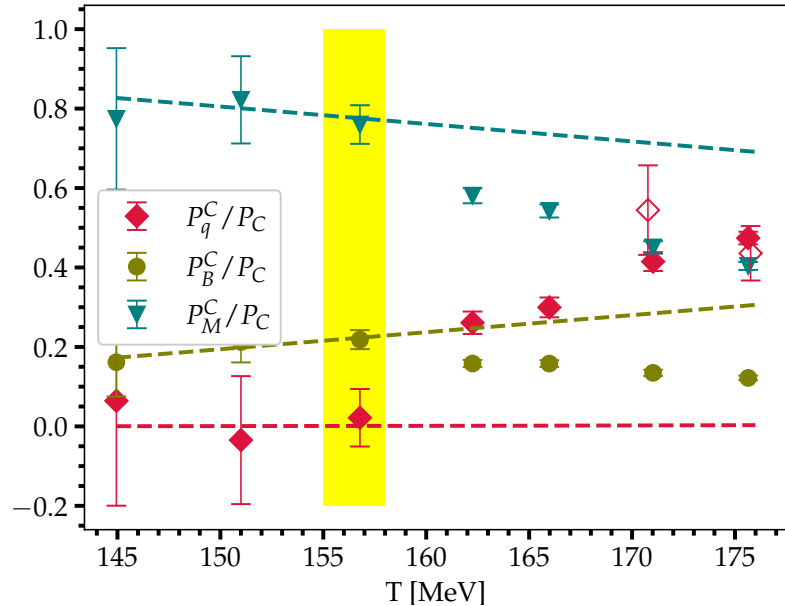
- partial pressure contributions to P^C
- excitations with quantum numbers of the charmed hadrons survive above T_{pc}
- at $T \simeq 1.1T_{pc}$ the partial charm pressure starts to be dominated by quasi-particle excitations with quantum numbers of charm quarks

$$P_C^{Q=2/3} = [54\chi_{13}^{QC} - 81\chi_{22}^{QC} + 27\chi_{31}^{QC}] / 8$$

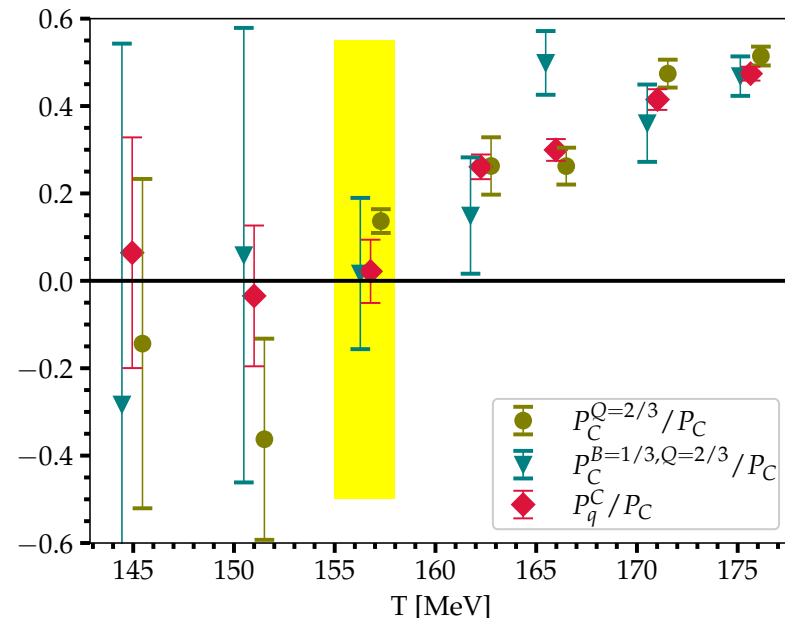
$$P_C^{B=1/3, Q=2/3} = 27[\chi_{112}^{BQC} - \chi_{211}^{BQC}] / 4$$

$$P_q^C = 9[\chi_{13}^{BC} - \chi_{22}^{BC}] / 2$$

- three independent observables project on excitation with quantum numbers of the charm quark

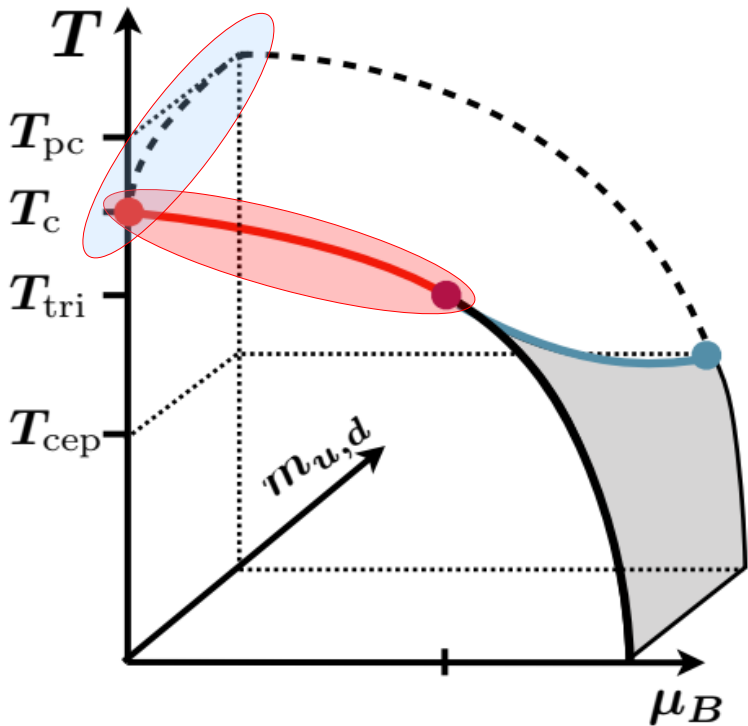


dashed lines:
QM-HRG



A. Bazavov...Sipaz Sharma... et al., arXiv:2312:12857

Conclusions



What we learned so far about the CEP in QCD from lattice QCD calculations:

I) the critical temperature is below $T_c = 132_{-6}^{+3}$ MeV

II) the corresponding critical chemical potential is likely to be above 500 MeV

→ Taylor expansions need to be resummed in order to reach higher μ_B/T

- no CEP for $\mu_B/T \leq 2.5$
- CEP not in the BES-II range (in collider mode)

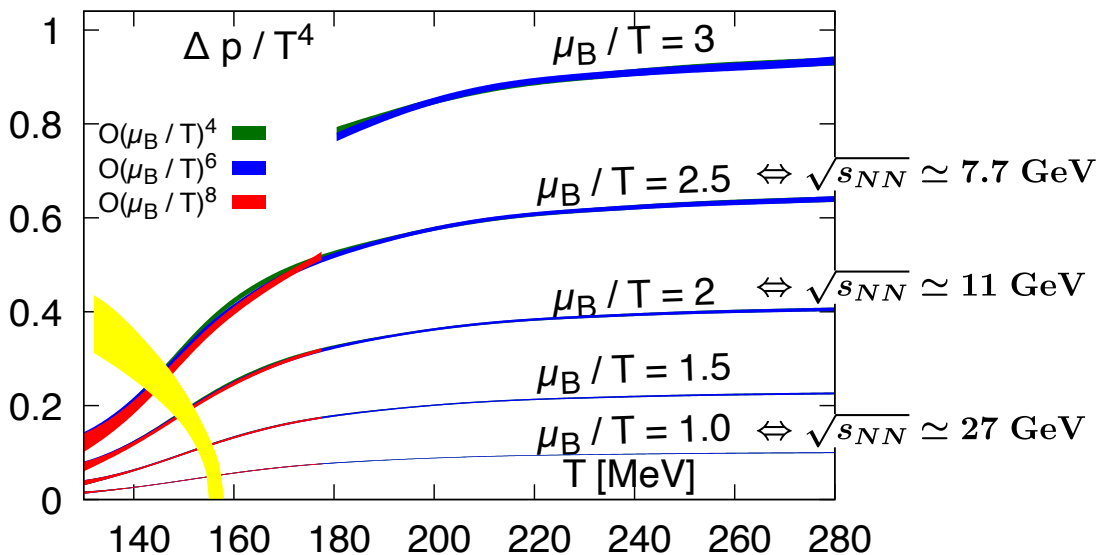
– EoS (pressure & number density) well controlled for

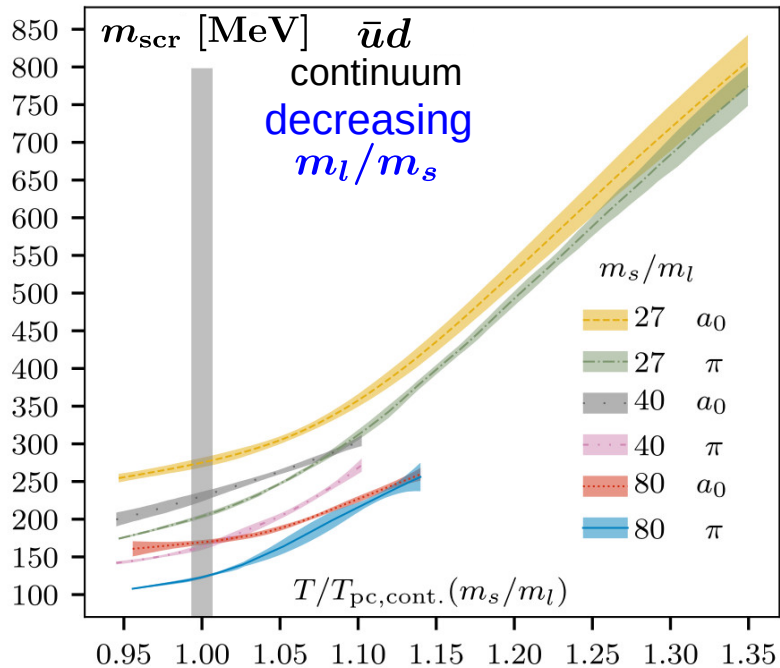
$$\mu_B/T \leq 2.0 \quad \forall T > 135 \text{ MeV}$$

(larger range for higher T)

– reliable μ_B - range is smaller for higher order cumulants, given only an 8th order Taylor series for the pressure

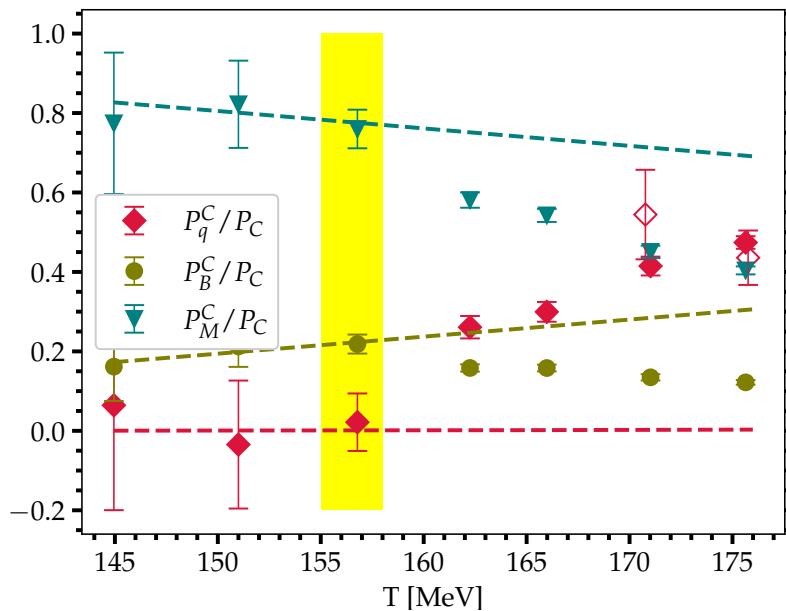
BES-II range



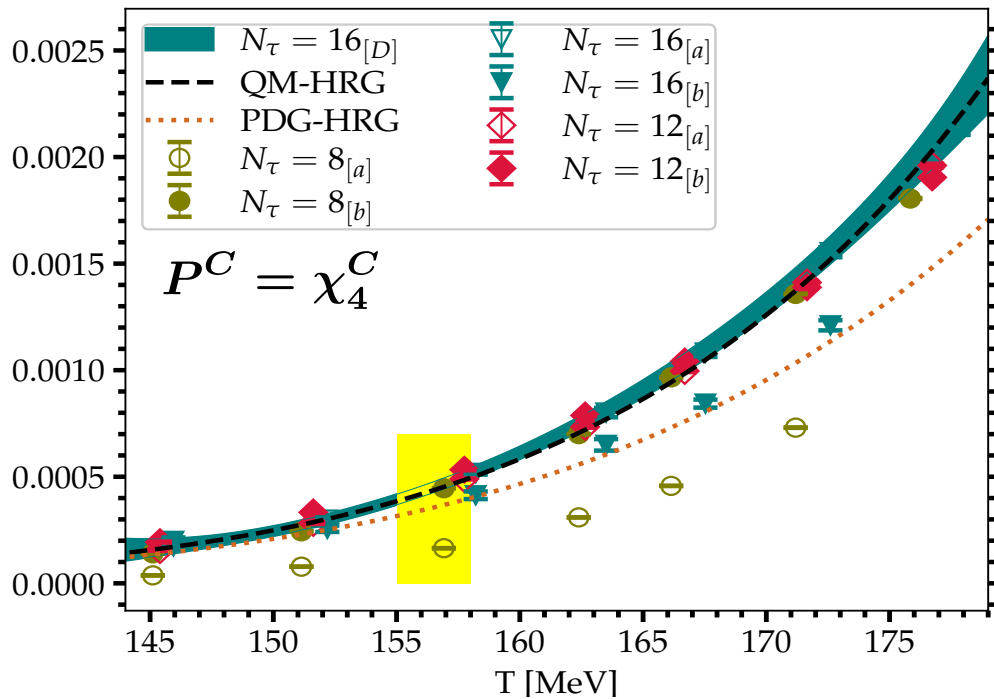


When approaching the chiral limit the gap between pseudo-scalar and scalar screening masses seems to stay non-zero at T_{pc}

$U(1)_A$ effectively restored at about $1.1T_{pc}$



at $1.1T_{pc}$ the partial charm pressure starts to be dominated by quasi-particle excitations with quantum numbers of the charm quark



$$P_M^C = \chi_4^C + 3\chi_{22}^{BC} - 4\chi_{13}^{BC}$$

$$P_B^C = (3\chi_{22}^{BC} - \chi_{13}^{BC})/2$$

$$P_q^C = \chi_4^C - P_M^C - P_B^C = 9(\chi_{13}^{BC} - \chi_{22}^{BC})/2$$

